ON THE USE OF THE FICTITIOUS WAVE STEEPNESS AND RELATED SURF-SIMILARITY PARAMETER IN METHODS THAT DESCRIBE THE HYDRAULIC AND STRUCTURAL RESPONSE TO WAVES

Daan Heineke[1] and Henk Jan Verhagen[2]

To assess the hydraulic performance of coastal structures – viz. wave run-up, overtopping and reflection – and to evaluate the stability of the armour layers, use is made of the dimensionless surf similarity parameter, as introduced by Battjes (1974). The front side slope of the structure and the wave steepness are combined in this parameter, also called the Iribarren number. The introduction of the wave steepness was based on the wish to include the effect of the wave period, T, in the surf similarity parameter and hence in the various methods that describe the hydraulic and structural response to waves. The wave steepness to be used in the various methods is the *fictitious* wave steepness: the ratio of the wave height at the toe of the structure (H) and the fictitious deep-water wavelength (L_o), or rather, the squared value of the local wave period, multiplied by $g/2\pi$. In deep water the fictitious wave steepness equals the **real** wave steepness (H_o/L_o), but this is not the case in shallow water, $H/L_o \neq H/L$. The characteristic wave period of a wave field travelling into shallow water is subject to change, due to bathymetry, initial wave breaking, etc. Using the real deep-water wavelength in the expression for the fictitious wave steepness may, therefore, lead to incorrect conclusions when evaluating the key response characteristics in (very) shallow water. To avoid ambiguities and mistakes, it is therefore suggested to refrain from using the wavelength in the expression of the fictitious wave steepness, but to rather only use the local wave period: $s_o = 2\pi H_s/(gT^2)$. A logical next step would be to use "s_f " as the notation for the fictitious wave steepness.

INTRODUCTION

Various methods have been developed in the last 25 years to assess the wave run-up and overtopping on coastal structures; and to evaluate the stability of rock and concrete armour layers on structures such as breakwaters, seawalls and revetments. Hydraulic performance and structural stability depend on the wave height and wave period as well as the structure front side slope. The wave parameters are in many instances described by the (fictitious) wave steepness parameter, $s_o = H/L_o$, where H is the wave height at the structure toe and L_o is the deep water wavelength, equal to $2\pi H/(gT^2)$, where T is the wave period. Combining the fictitious wave steepness or dimensionless wave period with the slope of the structure, given as $\tan\alpha$, results in a description of the way the waves break (Battjes, 1974). This parameter is called the surf-similarity parameter or the Iribarren number, given here as Equation 1:

---

[1] Ministry of Transport, Public Works and Water Management, Civil Engineering Division of Rijkswaterstaat, P.O. Box 20000, 3502 LA Utrecht, The Netherlands; e-mail: daan.heineke@rws.nl

[2] Delft University of Technology, Faculty of Civil Engineering and Geosciences, Section of Hydraulic Engineering, P.O. Box 5048, 2600 GA Delft, The Netherlands; e-mail: H.J.Verhagen@tudelft.nl

$$\xi = \tan\alpha \Big/ \sqrt{s_o} \qquad (1)$$

The introduction of the fictitious wave steepness was based on the wish to include the effect of the wave period in the surf similarity parameter and hence in the various methods that describe the hydraulic and structural response to waves. The wave period together with the wave height determine the energy in the wave train attacking a coastal structure. For relative deep-water conditions it was convenient to express the (fictitious) wave steepness in terms of wave height and wavelength, but this may easily lead to confusion in the case of conditions with shallow foreshores.

Considerations and motivation

Many authors conveniently express the fictitious wave steepness in shallow-water conditions also as: $s_o = H/L_o$, in which case L_o easily may be interpreted as the *real* deep-water wavelength (see Figure 1). Actually however, a fictitious local wavelength is meant, equal to $(g/2\pi)T^2$, with T being the local characteristic wave period. The reason to keep the wavelength in the expression of the fictitious wave steepness might be to show that this parameter is a (kind of) steepness, i.e. H/L. This may, however, easily result in mistakes in the case of shallow foreshores. Using the term "dimensionless wave period" would have solved this problem of misunderstanding and confusion. Even worse, there are also many authors who conveniently use the term "wave steepness", whereas actually the *fictitious* wave steepness is meant. Also this may easily lead to confusion and mistakes when shallow-water conditions are concerned. The real steepness of the waves is illustrated in Figure 1. The local fictitious wavelength, for the sake of clarity to be denoted as L_f, cannot be shown in this Figure 1, but its value is usually larger than that of the wavelength at the toe and at maximum equal to L_o.

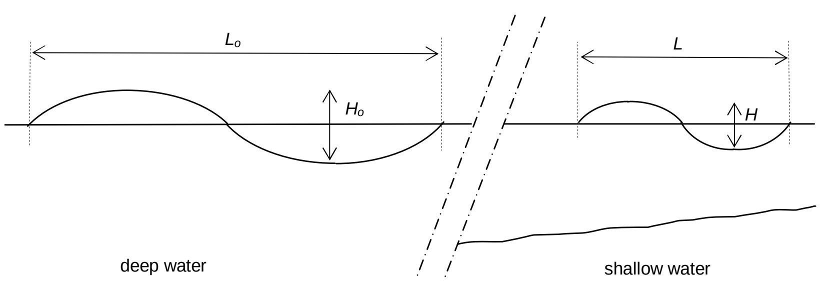

Figure 1. The real wave steepness in deep water and in shallow-water conditions. Note that the scale in this sketch is distorted by a factor of about 100; the wave steepness is usually in the range of 0.03 to 0.04, with a maximum of s = 0.14 for individual waves in deep water, derived from: $[H/L]_{max}$ = 0.14 tanh($2\pi h/L$), where h is the water depth (Miche, 1944).

In exceptional situations, also with shallow foreshores, the deep-water wave period is the same as its value at the toe of the structure: this is not only true for monochromatic waves, but also in some instances when a wave field (with a wave spectrum) is propagating towards the shore. An example is the situation occurring during tests in a 2-D wave flume: in many instances the change of the wave spectrum from the wave maker to the toe of the structure is limited to a general decrease of the top of the energy density. The wave period at the wave board is then called the 'deep' water wave period, and a related aspect is the fact that in such models no refraction, breaking over foreshore shoals and diffraction are occurring. The differences in the values of the characteristic wave period at the wave board and the toe are often that small, that the fictitious wave steepness is conveniently expressed as H_s/L_o (and this L_o value is even defined as the deep-water wavelength). In such cases the fictitious wave steepness, $s_o = 2\pi H/(gT^2)$, may also be expressed as: $s_o = H/L_o$. The near-shore wave conditions of the majority of the structures with shallow foreshores are, however, so much different from those offshore that it may be dangerous to indiscriminately use the general expression $s_o = H/L_o$ for the fictitious wave steepness in the surf-similarity parameter, as presented here as Equation 1.

Objectives

1. to illustrate the differences when using either the wave period at the toe of the structure with a shallow foreshore or the real deep-water wave period (L_o) when assessing the hydraulic performance of structures in shallow water and the stability of its armour layers.
2. to promote that the wave period at the toe of the structure is used in the expression of the fictitious wave steepness and the related surf-similarity parameter and that "s_f" is used as notation for the fictitious wave steepness.
3. to show that comparisons between methods specifically developed for deep-water and those for shallow foreshores may lead to incorrect conclusions, "*comparing apples and oranges*" may not be justified.

WAVE PARAMETERS AND NOTATION FOR WAVE STEEPNESS AND SURF SIMILARITY PARAMETER

Except for situations with monochromatic waves – one wave height, period and wavelength – the waves that travel from deep water towards the coast are part of a wave field, which can be described by a wave energy spectrum. Depending on which characteristic wave height at the toe of the structure and which wave period are to be used in which method, different expressions for both the fictitious wave steepness and the surf-similarity parameter are part of the different methods to evaluate the key response characteristics. A few examples are given, each with its specific definition of the surf-similarity parameter. An important aspect is that a distinction is (to be) made between deep-water conditions at the toe of the structure and shallow foreshores.

Normal practice is to use the significant wave height, H_s (either from the record, equal to $H_{1/3}$, or H_{m0} from the wave energy spectrum, equal to $4\sqrt{m_0}$). Various researchers use the mean wave period, T_m. But the spectral peak wave period, T_p, is also used. More recently developed methods make use of the mean energy wave period, $T_{m-1,0}$, from the wave energy spectrum. $T_{m-1,0}$ is defined as the ratio of the wave energy spectral moments m_{-1} and m_0. Consequently, different parameters for both the (fictitious) wave steepness and the surf similarity parameter are to be used:

- s_{om} and ξ_m, when using H_s (from wave record) and mean wave period, T_m
- s_{op} and ξ_p, when using H_s (from wave record) and peak wave period, T_p
- $s_{m-1,0}$ and $\xi_{m-1,0}$, when using H_{m0} and the mean energy wave period $T_{m-1,0}$ from the wave spectrum
- $s_{s-1,0}$ and $\xi_{s-1,0}$, when using H_s (from wave record) and the mean energy wave period, $T_{m-1,0}$
- s_p, when indicating the real wave steepness at the toe of the structure: the ratio of H_s from wave record and the local wavelength, L_p, associated with the peak wave period, T_p.

Note 1: The subscripts of parameters used in this paper differ in some instances from those used in literature, but its use is not consistent across the range of references and books discussed here.

Note 2: Only in a limited number of methods use is made of the deep-water wave steepness, e.g. $s_{op} = H_{so}/L_{op}$, and related surf-similarity parameter ξ_{op}, while the structure is in shallow water. The same applies to the use of the local real wave steepness, e.g. H_s/L_p. The reader is therefore advised to be careful in applying a method that makes use of the fictitious wave steepness.

Note 3: The use of either $H_{1/3}$ (from the wave record) or H_{m0} (from the wave spectrum) hardly makes any difference when hydraulic or structural response characteristics are evaluated in deep-water conditions ($H_{1/3} = H_s \cong H_{m0}$). In shallow-water conditions, however, the values of $H_s = H_{1/3}$ and H_{m0} are no longer the same; the ratio of $H_{1/3}/H_{m0}$ may become as large as 1.2.

WAVE PERIODS

The fictitious wave steepness in deep water is equal to the real wave steepness, defined as H_o/L_o, where L_o is the wavelength, equal to $(g/2\pi)T^2$. For irregular waves typical characteristic values for the wave period are used, such as the mean value from the wave record or the peak period from the spectrum. Normal practise is to use the significant wave height, H_s, as characteristic value for the wave height.

The ratio of the different deep-water wave periods depends on the shape of the wave energy spectrum. Universal relationships between the mean wave period, T_m, and the spectral or mean energy wave period, $T_{m-1,0}$, or between the mean period, T_m, and the peak period, T_p, do not exist. The ranges of the ratios (or conversion factors) for these three wave periods in deep water are presented in Figure 2, based on work of Goda (2000) and the ratio of T_p and $T_{m-1,0}$ = 1.1 for single-peaked spectra.

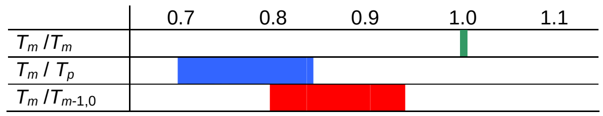

Figure 2. Ranges of ratios of three wave period measures, for single-peaked spectra in deep water

Effect on hydraulic performance and response

The relationship between the wave run-up (and overtopping) and the wave period (and hence the wave-similarity parameter), is more or less linear. This is shown by means of Equation 2, the method proposed by Owen (1980):

$$\frac{q}{T_m g H_s} = a \exp\left(-b\, R^* / \gamma\right) \quad \text{with } R^* = R_c \Big/ \left(T_m \sqrt{g H_s}\right) \tag{2}$$

where q is the specific overtopping discharge, R_c is the crest freeboard relative to still water level, a and b are empirically derived coefficients that depend on the profile and γ_f is correction factor for the influence of the slope roughness.

As can be seen in Figure 2, a relative error of 15 to 20 percent in the wave overtopping discharge may occur if the mean wave period is used instead of the mean energy wave period. Similar effects occur when transferring a T_m value (e.g. in Owen's method) to a $T_{m-1,0}$ value to be used in the method developed by TAW (2002), given here as Equation 3 (for breaking waves, i.e. $\gamma_b \cdot \xi_{m-1,0} < \approx 2$):

$$q \Big/ \sqrt{g H_{m0}^3} = \frac{A}{\sqrt{\tan\alpha}} \gamma_b \xi_{m-1,0} \exp\left(-B \frac{R_c}{H_{m0}} \frac{1}{\xi_{m-1,0}\, \gamma_b \gamma_f \gamma_\beta}\right) \tag{3}$$

where A and B are coefficients, and γ_b and γ_β are factors for the influence of the existence of a berm and oblique wave attack respectively.

A similar influence applies to the stability of rock-armoured slopes of coastal structures. This is shown by means of Equation 4, the stability formula developed by van der Meer (1988), for plunging waves ($\xi_m < \xi_{cr}$):

$$\frac{H_s}{\Delta D_{n50}} = 6.2\, P^{0.18} \left(\frac{S_d}{\sqrt{N}}\right)^{0.2} \xi_m^{-0.5} \tag{4}$$

where Δ is the relative buoyant density of the armourstone, D_{n50} is the median nominal diameter of the stones, S_d is the damage level parameter, N the number of waves and P the notional permeability factor.

The relative error that may be made in assessing the required armourstone size is less than when assessing the wave overtopping, $D_{n50} \propto \sqrt{T}$ versus $q \propto T$, but still appreciable because the required mass is what counts: $M_{50} = (D_{n50})^3 \rho_r$, where ρ_r is the apparent mass density of the rock. For example, using a wave period measure that differs 15 percent from the measure that should be used, means that a relative error of 25 percent is made in the determination of the required mass of the armourstone, e.g. 3-6 tons grading based on a calculated M_{50} of 5.1 t versus 6-10 tons grading based on a 25 percent heavier M_{50} value, which may imply considerable cost consequences (higher quarrying, transport and handling costs).

Intermediate conclusions

- Each method has been developed with its own specific wave period measure. So, use the prescribed wave period parameter and be careful when applying another wave period measure;
- Each method (for assessing hydraulic performance and for evaluating stability) has been developed for certain conditions, i.e. a certain range of validity applies to each of them. Do not compare the various methods indiscriminately, in particular those developed for deep water with those developed for conditions with shallow foreshores.

From deep to shallow water

The change of the wave conditions and hence the wave energy spectrum when travelling into shallow water depends largely on the bathymetry; but also on the spectrum itself (single or double-peaked), initial wave breaking, on the occurrence of long-period waves near-shore (such as surf beats) and on the degree of peakedness and skewness of the waves in the surf zone, etc. The characteristic shallow-water wave period may become smaller (which is mostly the case), but due to e.g. surf beats and or refraction over shoals this is not always so.

The use of wave height and wave period parameters assessed at the toe of the structure is also normal practice when evaluating conditions with shallow foreshores. This approach is logical from physics point of view, but it has also a disadvantage: an advanced spectral wave propagation model (such as SWAN) is needed to calculate the local spectral wave parameters. An approximation using linear wave theory is only partly solving this problem: the shallow-water wave height may be approximated rather well, but this does not apply to the wave period. The deep-water value of a wave period measure (peak, mean or mean energy period) is not necessarily the same as its value in shallow water at the toe of the structure. The degree of deviation depends on the situation, which is shown here with two examples. Comparing results of overtopping and stability calculations with those of other methods that are based on deep-water wave period parameters, may lead to incorrect conclusions.

Example 1

A typical example of the decay of a wave energy spectrum for an estuary with offshore shoals is given in Figure 3. Use has been made of the spectral wave propagation model SWAN. Station 1 is offshore the Dutch coast and station 8 is near-shore – in the Haringvliet estuary (see Figure 3a). The deep-water peak period is 6-7 s (f_p = 0.15 Hz) and the near-shore value in station 8 is T_p = 4 s (see Figure 3b). A similar trend can be observed for the (spectral) mean energy period, $T_{m\text{-}1,0}$.

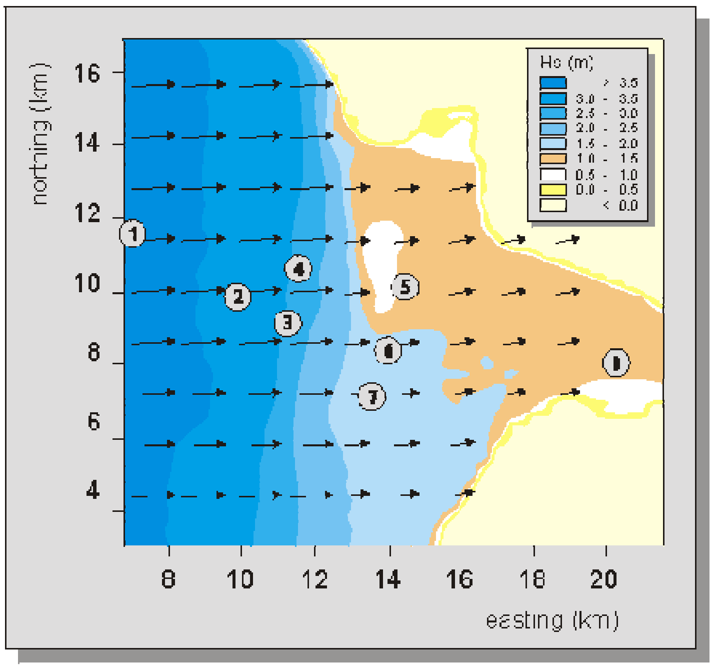

Figure 3a. Situation and location of wave gauges from offshore (Station 1) to inshore (Station 8) at the Haringvliet estuary, the Netherlands (courtesy WL|Delft Hydraulics)

From this prototype situation it is clear that the designer should make judicious use of the results of the model, when assessing the relevant values of the fictitious wave steepness, s_{op}, and the related surf-similarity parameter, ξ_p. The correct value of the fictitious wave steepness in station 8 (with H_s = 1.0 m) is: $s_{op} \cong 1.0/(1.56*4^2) \cong 0.04$. In the case of incorrectly using L_o ($\cong 1.56*6^2$), the result would be: $s_{op} \cong 0.02$; and the relative error in the ξ_p value would be 40%.

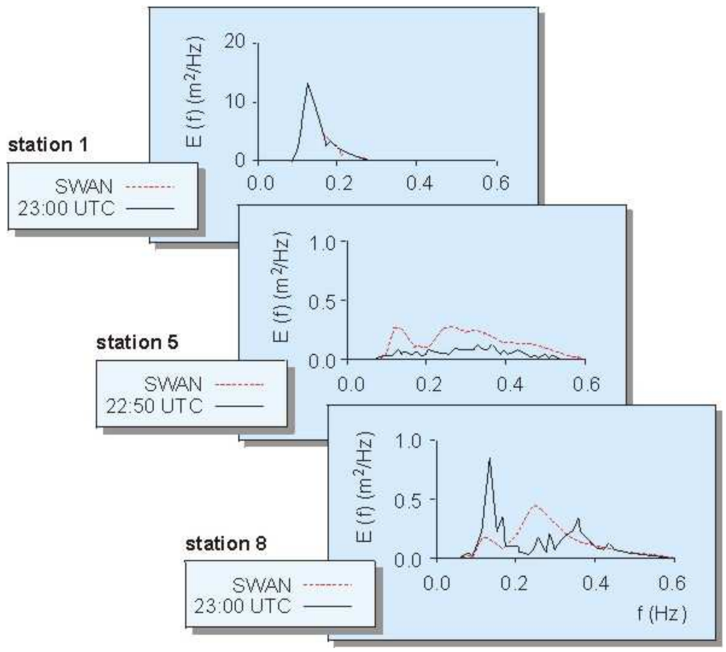

Figure 3b. Wave energy density spectra for station 1 – offshore (see Figure 3a), station 5 – near-shore at leeside of a shoal and station 8 – inshore (courtesy WL|Delft Hydraulics)

Example 2

This example refers to a coastline with a relatively steep foreshore. Figure 4a shows the cross sectional profile. For this profile also tests have been done in a physical model (scale 1:45). However, in this case the depth near the wave board is (on prototype scale) 27 m, so it is not really deep.

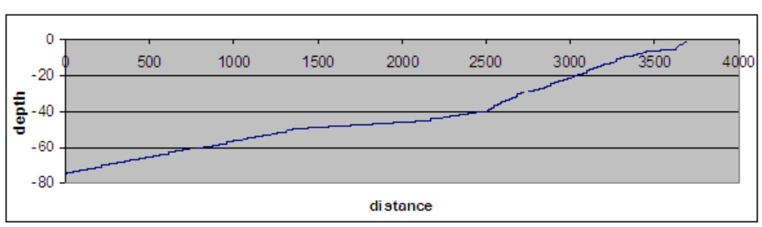

Figure 4a: Profile of Example 2

The real deep-water wave boundary condition is characterised by: H_s = 5.75 m and T_p = 8.8 s. Computations showed that the wave height at the –27 m depth contour should be 5.31 m, while at that point the period is taken equal to the deep-water wave period. A Jonswap spectrum is used. In the physical model the spectrum is measured directly in front of the structure (after removing the reflection from the data). This spectrum is given in Figure 4d.

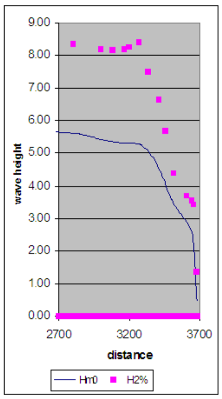

Figure 4b: Wave height H_{m0} and $H_{2\%}$ as function of the distance from the coastline

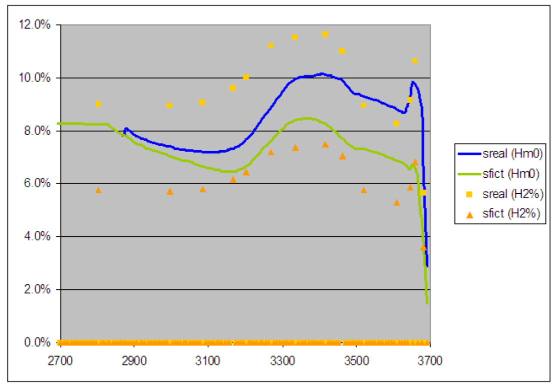

Figure 4c: Variation of the "steepnesses" as function of the distance from the coastline

In Figure 4b the variation of the wave height as function of the distance is given; as can be expected the wave height decreases. As a consequence also the fictitious steepness changes, and it is certainly not a constant value as can be seen in Figure 4c.

For this specific case the following ratios at the toe of the breakwater can be calculated from the measured wave spectrum:

- T_p/T_{m0} 1.08
- $T_{m-1,0}/T_{m0}$ 1.06
- $T_p/T_{m-1,0}$ 1.02

These values should certainly not be considered as "universal". They are not even constant for the whole coastal profile. So, one has to conclude that for a proper design of a coastal structure along a "non-standard" coastline, the local wave spectrum is needed. This can be determined either with a spectral wave model or with physical model tests. For a number of equations one needs to use the $H_{2\%}$ instead of the H_s. Also for the relation $H_{2\%}/H_s$ one cannot use a fixed value (see Figure 4b). But one should also realise that often in the same equation a fictitious steepness is used. This fictitious steepness has to be calculated with the local H_s and not the $H_{2\%}$.

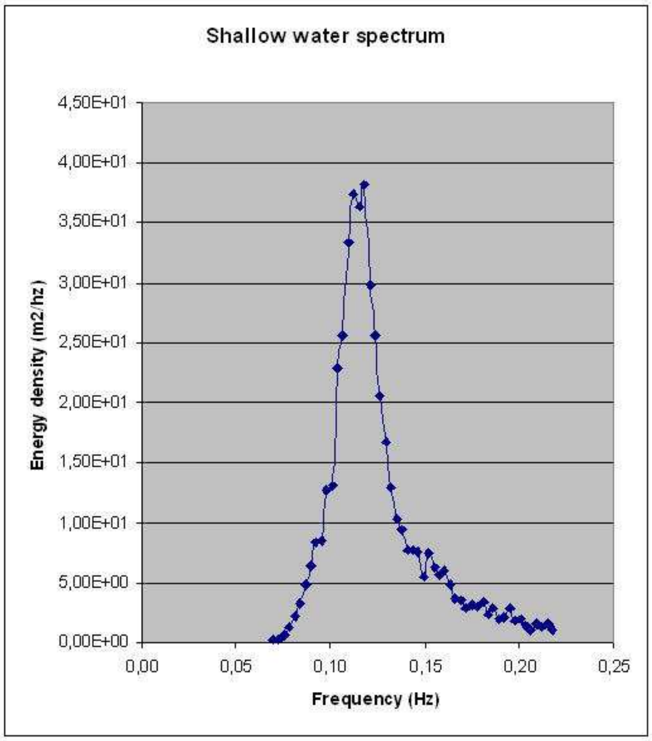

Figure 4d: Wave energy spectrum near the breakwater

CONCLUSIONS AND RECOMMENDATIONS

The use of the wavelength, L_o, in the expression for the fictitious wave steepness may introduce confusion and may lead to incorrect conclusions, in particular for situations with shallow foreshores. It would be good practise to only make use of a *fictitious* wavelength, $L_f = (g/2\pi)T^2$, where T is the characteristic wave period just in front of the structure. Similarly, the use of the expression "wave steepness" should be avoided, in particular for shallow-water conditions, as this gives the impression that the real wave steepness, H_s/L, at the toe is meant instead of $2\pi H_s/(gT^2)$.

Recommendations for researchers:

- Make those methods user-friendly that contain both wave parameters to be deduced from a wave record (time series) and parameters to be determined from the wave energy spectrum;
- Use the expression "fictitious wave steepness" when this is meant and define this as $2\pi H_s/(gT^2)$ instead of H_s/L_o, in order to prevent confusion;
- Do not compare (results of) methods applicable to deep-water conditions with those developed for shallow-water conditions; it may proof to be dangerous to compare apples and oranges.

Recommendations for users and researchers:

- Use the local wave period when defining the fictitious wave steepness;
- Use 's_f' as the notation for the fictitious wave steepness instead of 's_o', in order to avoid ambiguities and possible mistakes.

REFERENCES

Battjes, J.A., 1974. *Computation of set-up, longshore currents, run-up and overtopping due to wind generated waves*, Comm on Hydraulics, Report 74-2, Dept of Civil Eng, Delft University of Technology, Delft.

Goda, Y., 2000. *Random seas and design of maritime structures*, Advanced Series on Ocean Eng, Vol 15, World Scientific Publishing, Singapore.

Owen, M.W., 1980. Design of seawalls allowing for wave overtopping, HR Wallingford, Report EX 1809

TAW, 2002. *Technical Report Wave Run-up and Overtopping at Dikes*, Technical Advisory Committee on Flood Defence, Delft

Van der Meer, J.W., 1988. *Rock slopes and gravel beaches under wave attack*, PhD thesis, Delft University of Technology, Delft. Also Delft Hydraulics publication No 396

KEYWORDS – CSt07
Wave steepness
Wave similarity parameter
Fictitious wave steepness
Structural stability
Wave overtopping methods
Iribarren number
Relative wave period
Fictitious wavelength

Abstract acceptance number 48
ON THE USE OF THE FICTITIOUS WAVE STEEPNESS AND RELATED SURF SIMILARITY PARAMETER IN METHODS THAT DESCRIBE THE HYDRAULIC AND STRUCTURAL RESPONSE TO WAVES

1st Author: Daan Heineke
2nd Author: Henk Jan Verhagen …..