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Maximum Likelihood Decoding for Channels with Uniform Noise and Offset

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Abstract

In storage and communication systems, noise is not the only disturbance during data transmission. Sometimes the error performance can also be seriously degraded by offset and/or gain mismatch. This paper derives a maximum likelihood decoding criterion using intersection distance for channels with uniform noise and offset distribution in the absence of gain mismatch. Under this framework, zero word error rate performance is achievable for various decoding criteria by different standard deviation constraints on noise and offset. Our results show that the intersection distance decoding criterion incorporates the advantageous perks from two pre-existing methods: the immunity to offset mismatch that highlights Pearson distance decoding as well as the higher noise resistance brought by Euclidean distance decoding.

1 Introduction

We consider transmitting a codeword $\mathbf{x} = (x_1, x_2, \ldots, x_n)$ from a codebook $\mathcal{S}$, where n, the length of $\mathbf{x}$, is a positive integer. It is assumed that the received vector

$$\mathbf{r} = a(\mathbf{x} + \mathbf{v}) + b\mathbf{1}$$

is hampered not only by noise $\mathbf{v} = (v_1, v_2, \ldots, v_n)$ but also by gain a and/or offset b, where $\mathbf{1}$ is the real all-one vector $(1, 1, \ldots, 1)$ of length n. The channels with gain and/or offset mismatch can commonly be found in storage and communication systems. Some examples are optical discs with fingerprints and scratches which may result in gain and offset variations of the retrieved signal [1], and charge leakage induced cell voltage shift in flash memory [2].

Traditional Euclidean distance decoding has been shown poor performances under such mismatches. Pearson distance decoding was proposed as an alternative to counter the effects of gain and/or offset mismatch [3]. Blackburn [4] investigated a maximum likelihood (ML) criterion for the channel with Gaussian noise and unknown gain and offset mismatch. In some applications, it is reasonable to assume a priori knowledge of the offset distribution. In [5], ML decision criteria are derived for Gaussian noise channels assuming the Gaussian or uniform distribution for the offset in the absence of gain mismatch. For Gaussian offset, the ML criterion turns out to be a weighted average of the Euclidean distance and the modified Pearson distance.

In this paper, we assume the absence of gain mismatch ($a = 1$), i.e.,

$$\mathbf{r} = \mathbf{x} + \mathbf{v} + b\mathbf{1},$$

as well as the uniform distribution of both the noise and the offset. For this channel model, we propose a maximum likelihood decoding criterion based on intersection distance (ISD), that possesses a property for countering both noise and offset mismatch.

Essentially, we combine the immunity to offset mismatch of Pearson distance decoding and the higher noise resistance of Euclidean distance decoding. We further show that, in this channel, zero word error rate (WER) performance is achievable by a small standard deviation of noise and/or offset.

The remainder of this paper is organized as follows. We start in Section 2 with a discussion of the prior art of decoding criteria. In Section 3, the ISD decoding criterion is presented. Zero WER performance is analyzed for Euclidean decoding, Pearson decoding, and intersection decoding in Section 4. Finally, we draw our conclusions in Section 5.

2 Preliminaries

For the noise vector $\mathbf{v} = (v_1, ..., v_n)$, assume the v_i are independently uniformly distributed with mean 0 and variance $\sigma^2 > 0$, i.e.,

$$\phi(\mathbf{v}) = \prod_{i=1}^{n} \phi(v_i) = \frac{1}{(2\sigma\sqrt{3})^n}, \; -\sigma\sqrt{3} < v_i < \sigma\sqrt{3}. \tag{1}$$

We assume that the offset b has a uniform probability density function ζ with mean μ and variance $\beta^2 > 0$. Since a receiver can subtract $\mu\mathbf{1}$ from $\mathbf{r}$ if the expected offset value is not equal to zero, we may assume $\mu = 0$ without loss of generality, which we will do throughout the rest of this paper. The probability density function of the offset is

$$\zeta(b) = \frac{1}{2\beta\sqrt{3}}, \; -\beta\sqrt{3} < b < \beta\sqrt{3}. \tag{2}$$

We use a codebook $\mathcal{S}$ which is a finite subset of $\mathbb{R}^n$. The receiver decodes the received vector $\mathbf{r}$ to a codeword which optimizes a certain criterion. Two well-known criteria are based on the (squared) Euclidean distance and the Pearson distance.

The classical squared Euclidean distance between the received vector $\mathbf{r}$ and a codeword $\hat{\mathbf{x}} \in \mathcal{S}$ is defined as

$$\delta_E(\mathbf{r}, \hat{\mathbf{x}}) = \sum_{i=1}^{n} (r_i - \hat{x}_i)^2. \tag{3}$$

A Euclidean decoder chooses a codeword minimizing this distance. It is known to be optimal with regard to handling Gaussian noise.

The Pearson distance measure is used in situations which require resistance towards both offset and/or gain mismatch. For any vector $\mathbf{u} \in \mathbb{R}^n$, let $\bar{\mathbf{u}} = (1/n)\sum_{i=1}^{n} u_i$ denote the average symbol value, and let $\sigma_{\mathbf{u}} = (\sum_{i=1}^{n} (u_i - \bar{\mathbf{u}})^2)^{1/2}$ denote the unnormalized symbol standard deviation. The Pearson distance between the received vector $\mathbf{r}$ and codewords $\hat{\mathbf{x}} \in \mathcal{S}$ is defined as

$$\delta_P(\mathbf{r}, \hat{\mathbf{x}}) = 1 - \rho_{\mathbf{r},\hat{\mathbf{x}}} = 1 - \frac{\sum_{i=1}^{n} (r_i - \bar{\mathbf{r}})(\hat{x}_i - \bar{\hat{\mathbf{x}}})}{\sigma_{\mathbf{r}}\sigma_{\hat{\mathbf{x}}}}, \tag{4}$$

where $\rho_{\mathbf{r},\hat{\mathbf{x}}}$ is the well-known Pearson correlation coefficient. A Pearson decoder chooses a codeword minimizing this distance. As shown in [3], when there is no gain mismatch, i.e., $a = 1$, a modified Pearson distance criterion is obtained by removing the division by $\sigma_{\hat{\mathbf{x}}}$ and the irrelevant components $\bar{\mathbf{r}}$ and $\sigma_{\mathbf{r}}$ in the optimization process, i.e.,

$$\delta'_P(\mathbf{r}, \hat{\mathbf{x}}) = \sum_{i=1}^{n} (r_i - \hat{x}_i + \bar{\hat{\mathbf{x}}})^2. \tag{5}$$

3 Maximum Intersection Distance Decoding

In this section, we present a ML decoding criterion for channels with uniform noise and offset mismatch. For a codeword $\hat{\mathbf{x}} = (\hat{x}_1, ..., \hat{x}_n) \in \mathcal{S}$, we define its noise environment

$$U_{\hat{\mathbf{x}}} = \{\mathbf{y} = (y_1, ..., y_n) \in \mathbb{R}^n : \hat{x}_i - \sigma\sqrt{3} < y_i < \hat{x}_i + \sigma\sqrt{3}\}.$$

For a vector $\mathbf{r} \in \mathbb{R}^n$, we write

$$L_{\mathbf{r}} = \{\mathbf{r} - b\mathbf{1} : b \in (-\beta\sqrt{3}, \beta\sqrt{3})\}$$

for the line segment centered at $\mathbf{r}$ with length $2\beta\sqrt{3}$ and direction $\mathbf{1}$.

In order to achieve ML decoding, we need to choose the codeword of maximum probability given the received vector. Assuming all codewords are equally likely, this is equivalent to maximizing the probability density value of the received vector $\mathbf{r}$ given the candidate codeword $\hat{\mathbf{x}}$. From the channel model it easily follows that we should thus maximize

$$\int_{-\beta\sqrt{3}}^{\beta\sqrt{3}} \phi(\mathbf{r} - \hat{\mathbf{x}} - b\mathbf{1})\zeta(b)db. \tag{6}$$

Because of the uniform nature of both ϕ and ζ, this is tantamount to choosing a codeword $\hat{\mathbf{x}}$ for which the noise environment $U_{\hat{\mathbf{x}}}$ has the largest intersection with the line segment $L_{\mathbf{r}}$. Therefore, define the intersection distance $\mathrm{ISD}(\mathbf{r}, \hat{\mathbf{x}})$ between $\mathbf{r}$ and $\hat{\mathbf{x}}$ as the length of the intersection between the noise environment $U_{\hat{\mathbf{x}}}$ and the line segment $L_{\mathbf{r}}$. The most likely candidate codeword $\mathbf{x}_o$ for a received vector has the largest intersection distance, that is

$$\mathbf{x}_o = \arg\max_{\hat{\mathbf{x}} \in S} \mathrm{ISD}(\mathbf{r}, \hat{\mathbf{x}}). \tag{7}$$

Note that a point $\mathbf{r} - t\mathbf{1}$ of $L_{\mathbf{r}}$ is in $U_{\hat{\mathbf{x}}}$ if and only if t satisfies

$$\begin{cases} r_i - \hat{x}_i - \sigma\sqrt{3} & < t < \quad r_i - \hat{x}_i + \sigma\sqrt{3}, \forall i = 1, ..., n, \\ -\beta\sqrt{3} & < t < \quad \beta\sqrt{3}. \end{cases} \tag{8}$$

Defining

$$\begin{aligned} t_0(\mathbf{r}, \hat{\mathbf{x}}) &= \min\left(\{r_i - \hat{x}_i + \sigma\sqrt{3} \,|\, i = 1, ..., n\} \cup \{\beta\sqrt{3}\}\right), \\ t_1(\mathbf{r}, \hat{\mathbf{x}}) &= \max\left(\{r_i - \hat{x}_i - \sigma\sqrt{3} \,|\, i = 1, ..., n\} \cup \{-\beta\sqrt{3}\}\right), \end{aligned} \tag{9}$$

we can express the intersection distance between $\mathbf{r}$ and $\hat{\mathbf{x}}$ as

$$\mathrm{ISD}(\mathbf{r}, \hat{\mathbf{x}}) = \sqrt{n(\max\{t_0(\mathbf{r}, \hat{\mathbf{x}}) - t_1(\mathbf{r}, \hat{\mathbf{x}}), 0\})^2}. \tag{10}$$

Note that maximizing $\mathrm{ISD}(\mathbf{r}, \hat{\mathbf{x}})$ is equivalent to maximizing the simplified measure

$$\mathrm{ISD}'(\mathbf{r}, \hat{\mathbf{x}}) = \max\{t_0(\mathbf{r}, \hat{\mathbf{x}}) - t_1(\mathbf{r}, \hat{\mathbf{x}}), 0\}. \tag{11}$$

In Figure 1, we give simulation results for WER of the code

$$\mathcal{S}_1 = \{(0, 0, 0), (1, 1, 0), (1, 0, 1), (0, 1, 1)\}$$

of length $n = 3$ and size 4, with different detectors and various values of the offset standard deviation β, while fixing the noise standard deviation $\sigma = 0.15$. Note that the Pearson decoder has stable performances because of its inherent resistance to offset mismatch, while performances of the Euclidean decoder get worse for increasing values of β. Further, note that in case neither the noise nor the offset is strongly dominating the other, the intersection distance based decoder is clearly outperforming both the Euclidean decoder and the Pearson decoder. For small values of β, the performance curves of the intersection decoder and the Euclidean decoder are gone, which suggests zero WER. This will be further investigated in the next section.

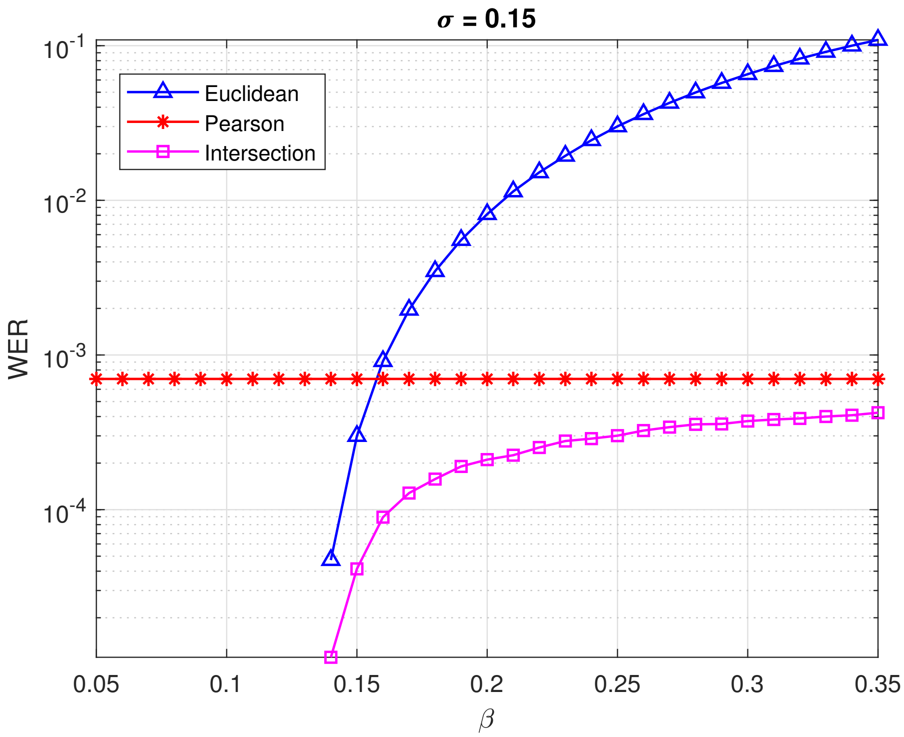

Figure 1: Simulated WER of Euclidean decoding, Pearson decoding, and intersection decoding for the code $\mathcal{S}_1$ without gain mismatch ($a = 1$), uniform offset mismatch with standard deviation β, and uniform noise with standard deviation $\sigma = 0.15$.

4 Zero WER Analysis

Since the noise and the offset are both assumed to be uniformly distributed, it is clear that a WER of zero will be achieved if σ and β are sufficiently small. In this section we present bounds on σ and β guaranteeing zero WER for Euclidean, Pearson, and intersection decoders.

4.1 Euclidean Decoder

When the sum of the noise and offset standard deviations is sufficiently small, the Euclidean decoder can achieve zero WER performance, as shown in the following result.

Theorem 1. *If*

$$\sigma + \beta \leq \min_{\mathbf{s},\mathbf{c}\in\mathcal{S},\mathbf{s}\neq\mathbf{c}} \left(\frac{\sum_{i=1}^{n} (s_i - c_i)^2}{2\sqrt{3}\sum_{i=1}^{n} |s_i - c_i|} \right), \tag{12}$$

then the Euclidean decoder achieves a WER equal to zero.

Proof. Assume that $\mathbf{x} \in \mathcal{S}$ is sent and $\mathbf{r} = \mathbf{x} + \mathbf{v} + b\mathbf{1}$ is received. Then, for all codewords $\hat{\mathbf{x}} \neq \mathbf{x}$, it holds that

$$\begin{aligned}
&\delta_E(\mathbf{r}, \hat{\mathbf{x}}) - \delta_E(\mathbf{r}, \mathbf{x}) \\
&= \sum_{i=1}^{n} (r_i - \hat{x}_i)^2 - \sum_{i=1}^{n} (r_i - x_i)^2 \\
&= \sum_{i=1}^{n} (r_i - x_i - \hat{x}_i + x_i)^2 - \sum_{i=1}^{n} (r_i - x_i)^2 \\
&= \sum_{i=1}^{n} (\hat{x}_i - x_i)^2 - 2\sum_{i=1}^{n} (\hat{x}_i - x_i)(r_i - x_i) \\
&= \sum_{i=1}^{n} (\hat{x}_i - x_i)^2 - 2\sum_{i=1}^{n} (\hat{x}_i - x_i)(v_i + b) \\
&\geq 2(\sigma + \beta)\sqrt{3} \sum_{i=1}^{n} |\hat{x}_i - x_i| - 2\sum_{i=1}^{n} |\hat{x}_i - x_i| \, |v_i + b| \\
&= 2\sum_{i=1}^{n} |\hat{x}_i - x_i| \, (\sigma\sqrt{3} + \beta\sqrt{3} - |v_i + b|) \\
&> 0,
\end{aligned}$$

where the first inequality follows from (12) and the last inequality from the fact that $|v_i + b| \leq |v_i| + |b| < \sigma\sqrt{3} + \beta\sqrt{3}$ for all i. Hence, if decoding is based on minimizing (3), the transmitted codeword is always chosen as the decoding result, leading to a WER equal to zero. □

4.2 Pearson Decoder

Since Pearson distance decoding features its immunity to offset mismatch, zero WER performance only requires a limited value of σ, as shown in the next theorem.

Theorem 2. *If*

$$\sigma < \min_{\mathbf{s},\mathbf{c} \in \mathcal{S}, \mathbf{s} \neq \mathbf{c}} \left(\frac{\sum_{i=1}^{n} (s_i - \bar{\mathbf{s}} - c_i + \bar{\mathbf{c}})^2}{\frac{n-1}{n} 4\sqrt{3} \sum_{i=1}^{n} |s_i - \bar{\mathbf{s}} - c_i + \bar{\mathbf{c}}|} \right), \tag{13}$$

then the Pearson decoder achieves a WER equal to zero.

Proof. Assume that $\mathbf{x} \in \mathcal{S}$ is sent and $\mathbf{r} = \mathbf{x} + \mathbf{v} + b\mathbf{1}$ is received. Then, for all codewords $\hat{\mathbf{x}} \neq \mathbf{x}$, it holds that

$$\begin{aligned}
&\delta'_P(\mathbf{r}, \hat{\mathbf{x}}) - \delta'_P(\mathbf{r}, \mathbf{x}) \\
&= \sum_{i=1}^{n} (r_i - \hat{x}_i + \bar{\hat{\mathbf{x}}})^2 - \sum_{i=1}^{n} (r_i - x_i + \bar{\mathbf{x}})^2 \\
&= \sum_{i=1}^{n} (r_i - \hat{x}_i + \bar{\hat{\mathbf{x}}} - \bar{\mathbf{r}})^2 - \sum_{i=1}^{n} (r_i - x_i + \bar{\mathbf{x}} - \bar{\mathbf{r}})^2 \\
&= \sum_{i=1}^{n} (r_i - x_i + \bar{\mathbf{x}} - \bar{\mathbf{r}} + x_i - \bar{\mathbf{x}} - \hat{x}_i + \bar{\hat{\mathbf{x}}})^2 - \sum_{i=1}^{n} (r_i - x_i + \bar{\mathbf{x}} - \bar{\mathbf{r}})^2 \\
&= \sum_{i=1}^{n} (x_i - \bar{\mathbf{x}} - \hat{x}_i + \bar{\hat{\mathbf{x}}})^2 + 2\sum_{i=1}^{n} (x_i - \bar{\mathbf{x}} - \hat{x}_i + \bar{\hat{\mathbf{x}}})(r_i - x_i + \bar{\mathbf{x}} - \bar{\mathbf{r}}) \\
&= \sum_{i=1}^{n} (x_i - \bar{\mathbf{x}} - \hat{x}_i + \bar{\hat{\mathbf{x}}})^2 + 2\sum_{i=1}^{n} (x_i - \bar{\mathbf{x}} - \hat{x}_i + \bar{\hat{\mathbf{x}}})(x_i + v_i + b - x_i + \bar{\mathbf{x}} - \bar{\mathbf{x}} - \bar{\mathbf{v}} - b) \\
&= \sum_{i=1}^{n} (x_i - \bar{\mathbf{x}} - \hat{x}_i + \bar{\hat{\mathbf{x}}})^2 + 2\sum_{i=1}^{n} (x_i - \bar{\mathbf{x}} - \hat{x}_i + \bar{\hat{\mathbf{x}}})(v_i - \bar{\mathbf{v}}) \\
&> \tfrac{n-1}{n} 4\sigma\sqrt{3} \sum_{i=1}^{n} \left| x_i - \bar{\mathbf{x}} - \hat{x}_i + \bar{\hat{\mathbf{x}}} \right| - 2\sum_{i=1}^{n} \left| x_i - \bar{\mathbf{x}} - \hat{x}_i + \bar{\hat{\mathbf{x}}} \right| |v_i - \bar{\mathbf{v}}| \\
&= 2\sum_{i=1}^{n} \left| x_i - \bar{\mathbf{x}} - \hat{x}_i + \bar{\hat{\mathbf{x}}} \right| \left(\tfrac{n-1}{n} 2\sigma\sqrt{3} - |v_i - \bar{\mathbf{v}}| \right) \\
&\geq 0.
\end{aligned}$$

where the first inequality follows from (13) and the last inequality from the fact that $|v_i - \bar{\mathbf{v}}| < \frac{n-1}{n} 2\sigma\sqrt{3}$ for all i. Hence, if decoding is based on minimizing (5), the transmitted codeword is always chosen as the decoding result, leading to a WER equal to zero. □

4.3 Intersection Decoder

In this subsection, we show that zero WER is achieved for the intersection decoder in the case that σ or $\sigma + \beta$ is sufficiently small.

Theorem 3. *If*

$$\sigma \leq \min_{\mathbf{s},\mathbf{c} \in \mathcal{S}, \mathbf{s} \neq \mathbf{c}} \left(\frac{\max_{1 \leq i,j \leq n} \{(s_i - c_i) - (s_j - c_j)\}}{4\sqrt{3}} \right) \tag{14}$$

or

$$\sigma + \beta \leq \min_{\mathbf{s},\mathbf{c} \in \mathcal{S}, \mathbf{s} \neq \mathbf{c}} \left(\frac{\max_{i=1,\ldots,n} (|s_i - c_i|)}{2\sqrt{3}} \right) \tag{15}$$

then the intersection decoder achieves a WER equal to zero.

Proof. Assume that $\mathbf{x} \in \mathcal{S}$ is sent and $\mathbf{r} = \mathbf{x} + \mathbf{v} + b\mathbf{1}$ is received. We will show that if (14) or (15) holds, then $\text{ISD}'(\mathbf{r}, \hat{\mathbf{x}}) = 0$ for all codewords $\hat{\mathbf{x}} \neq \mathbf{x}$. First of all, note that

$$\begin{aligned}
&t_0(\mathbf{r}, \hat{\mathbf{x}}) - t_1(\mathbf{r}, \hat{\mathbf{x}}) \\
&= \min\left(\{ r_i - \hat{x}_i + \sigma\sqrt{3} \,|\, i = 1, \ldots, n \} \cup \{\beta\sqrt{3}\} \right) \\
&\qquad - \max\left(\{ r_i - \hat{x}_i - \sigma\sqrt{3} \,|\, i = 1, \ldots, n \} \cup \{-\beta\sqrt{3}\} \right) \\
&= \min\left(\{ r_i - \hat{x}_i + \sigma\sqrt{3} \,|\, i = 1, \ldots, n \} \cup \{\beta\sqrt{3}\} \right) \\
&\qquad + \min\left(\{ -(r_i - \hat{x}_i) + \sigma\sqrt{3} \,|\, i = 1, \ldots, n \} \cup \{\beta\sqrt{3}\} \right) \\
&= \min(\{2\beta\sqrt{3}\} \cup \{ \min_{i=1,\ldots,n} \{ -|r_i - \hat{x}_i| \} + \sigma\sqrt{3} + \beta\sqrt{3} \} \\
&\qquad \cup \{ \min_{1 \leq i,j \leq n} \{ (r_i - \hat{x}_i) - (r_j - \hat{x}_j) \} + 2\sigma\sqrt{3} \}).
\end{aligned} \tag{16}$$

Next, we will show that if (14) or (15) holds, this expression is negative whenever $\hat{\mathbf{x}} \neq \mathbf{x}$.

If (14) holds, then

$$\begin{aligned}
&\min_{1\leq i,j\leq n}\{(r_i-\hat{x}_i)-(r_j-\hat{x}_j)\}+2\sigma\sqrt{3}\\
&=\min_{1\leq i,j\leq n}\{(r_i-\hat{x}_i)-(r_j-\hat{x}_j)\}-2\sigma\sqrt{3}+4\sigma\sqrt{3}\\
&<\min_{1\leq i,j\leq n}\{(r_i-\hat{x}_i)-(r_j-\hat{x}_j)-(v_i-v_j)\}+4\sigma\sqrt{3}\\
&=\min_{1\leq i,j\leq n}\{[(r_i-\hat{x}_i)-(r_j-\hat{x}_j)]-[(r_i-x_i-b)-(r_j-x_j-b)]\}+4\sigma\sqrt{3}\\
&=\min_{1\leq i,j\leq n}\{(x_i-\hat{x}_i)-(x_j-\hat{x}_j)\}+4\sigma\sqrt{3}\\
&=-\max_{1\leq i,j\leq n}\{(\hat{x}_i-x_i)-(\hat{x}_j-x_j)\}+4\sigma\sqrt{3}\\
&\leq 0.
\end{aligned} \tag{17}$$

where the first inequality follows from the fact that $v_i - v_j \leq |v_i| + |v_j| < 2\sigma\sqrt{3}$ and the second inequality from (14).

If (15) holds, then

$$\begin{aligned}
&\min_{i=1,\ldots,n}\{-|r_i-\hat{x}_i|\}+\sigma\sqrt{3}+\beta\sqrt{3}\\
&=\min_{i=1,\ldots,n}\{-|r_i-\hat{x}_i|\}-\sigma\sqrt{3}-\beta\sqrt{3}+2\sqrt{3}(\sigma+\beta)\\
&<\min_{i=1,\ldots,n}\{-|r_i-\hat{x}_i|-|v_i+b|\}+2\sqrt{3}(\sigma+\beta)\\
&=\min_{i=1,\ldots,n}\{-|r_i-\hat{x}_i|-|r_i-x_i|\}+2\sqrt{3}(\sigma+\beta)\\
&\leq\min_{i=1,\ldots,n}\{-|x_i-\hat{x}_i|\}+2\sqrt{3}(\sigma+\beta)\\
&=-\max_{i=1,\ldots,n}\{|x_i-\hat{x}_i|\}+2\sqrt{3}(\sigma+\beta)\\
&\leq 0
\end{aligned} \tag{18}$$

where the first inequality follows from the fact that $|v_i + b| \leq |v_i| + |b| < \sigma\sqrt{3} + \beta\sqrt{3}$ and the last inequality from (15).

Combining (11), (16), (17), and (18), we find that indeed $\mathrm{ISD}'(\mathbf{r}, \hat{\mathbf{x}}) = 0$ for all codewords $\hat{\mathbf{x}} \neq \mathbf{x}$. By definition, $\mathrm{ISD}'(\mathbf{r}, \mathbf{x}) > 0$. This implies that if decoding is based on maximizing (11), the transmitted codeword is always chosen as the decoding result, leading to a WER equal to zero. □

Simulated WER with various values of σ and β for the example code $\mathcal{S}_1$ from the previous section are shown in Fig.2. On one hand, we find for $\beta = 0.2$ and $\beta = 0.15$ that zero WER is achieved when $\sigma < 1/(4\sqrt{3}) = 0.144$, which agrees with the bound from (14). On the other hand, we find for $\beta = 0.1$ and $\beta = 0.05$ that zero WER is achieved when $\sigma + \beta < 1/(2\sqrt{3}) = 0.288$, which agrees with the bound from (15).

5 Conclusion

In this paper, maximum likelihood decoding for channels with uniform noise and offset mismatch has been presented. This method, which is based on the intersection distance, combines the immunity to offset mismatch of Pearson distance decoding and the higher noise resistance of Euclidean distance decoding. It has been shown that

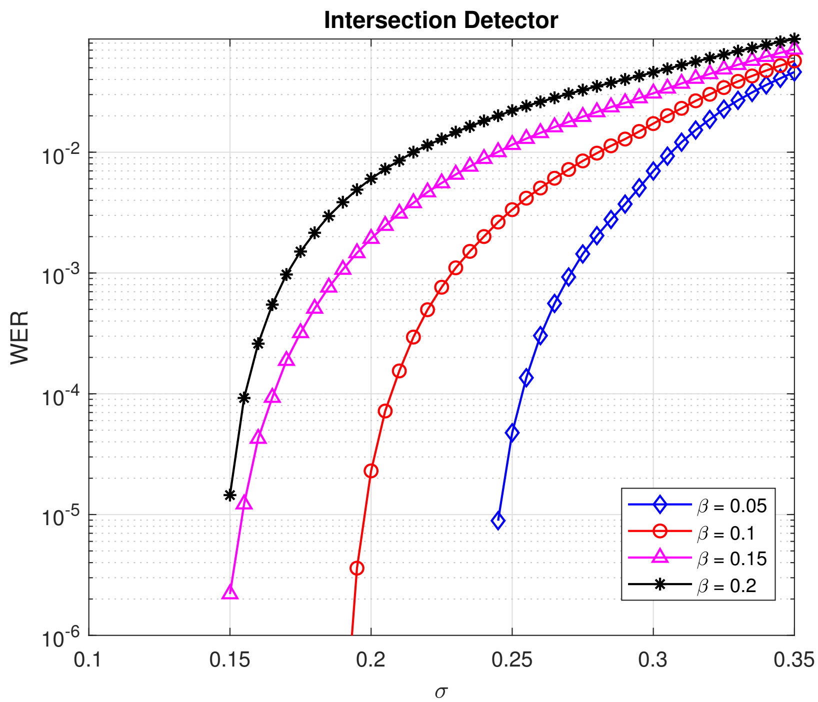

Figure 2: Simulated WER of intersection decoding for the code $\mathcal{S}_1$ in the case no gain mismatch ($a = 1$), uniform offset mismatch with standard deviation β, and uniform noise with standard deviation σ.

for sufficiently small standard deviations of the noise and/or offset zero WER can be achieved.

For future work, we are interested in investigating maximum likelihood decoding for noise channels with known distribution of both offset and gain mismatch.

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