

Observing human gait and detecting falls

A model-based approach based on wearable sensors

Sachin A. Umans

Master of Science Thesis

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A model-based approach based on wearable sensors

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Sachin A. Umans

February 11, 2024

Faculty of Mechanical Engineering (ME) · Delft University of Technology



**BioMechanical
Engineering**



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DELFT UNIVERSITY OF TECHNOLOGY
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The undersigned hereby certify that they have read and recommend to the Faculty of Mechanical Engineering (ME) for acceptance a thesis entitled

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by

SACHIN A. UMANS

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Supervisor(s):

Tamás Keviczky

Katherine Poggensee

Heike Vallery

Reader(s):

Riccardo Ferrari

Abstract

Falling remains a large source of (traumatic) injuries and healthcare costs. Over the past years, different actuators have been developed in the field of wearable robotics to help prevent injuries from falling. To increase the wearability of these systems, the weight of power storage can be decreased with intermittent instead of continuous control. A fall detector is then needed for these systems to trigger the activation of the actuator to prevent the fall. A proof of concept for a model-based fall detector that is aimed at using only wearable sensor measurements is presented. The algorithm is based on a single inertial measurement unit placed on the lower back. The upper-body orientation and centre of mass velocity are estimated with two separate Kalman filters. The velocity is estimated with a gait model consisting of a spring-damper-legged point mass in three-dimensional space. The balance of the subject is evaluated with the velocity estimates and the extrapolated centre of mass method. The presented model is verified on a non-falling treadmill walking dataset of real humans and shows accurate estimates of the centre of mass velocity. Furthermore, a planar falling simulation is performed to show successful pre-impact fall detection. This resulted in no false alarms outside the initial estimation settling time, and a successful detection with a lead time of 680 ms. This lead time is long enough to provide a trigger for fall mitigation devices.

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Sachin A. Umans

Chapter 1

Introduction

Many people fall every year and a large fraction of these falls are caused by slips or trips. It is observed by multiple studies in different regions that most falls leading to injury happen during walking [1, 2]. More and older fallers are visiting the emergency department and more have been admitted since 1997 [3]. In line with this, the burden of disease from falling has increased sharply in the Netherlands since 1990 [4]. Fallers report a reduced quality of life up to nine months after sustaining an injury [5]. Even death due to falling is not uncommon: in the Netherlands it is found that 8% of hospital-admitted elderly fallers die because of it [6]. Apart from physical injuries, falling also incurs a mental load. There is a strong correlation between fear of falling and depression [7].

Apart from the physical and psychological burden of falling, the economic cost of the resulting healthcare is large. Of all healthcare costs over all ages in the Netherlands, 44% is attributed to falls [8]. The falls by the elderly alone amount to 21% of the total healthcare costs [5].

A solution to falling that yields a completely functional fall-prevention product will need to overcome many remaining problems. It would need to be wearable, lightweight, and low-power, but it would also need to be consumer-friendly and attractive to wear [9]. An issue in any wearable electronic device is balancing power storage versus product weight. A control scheme that does not continuously consume power, such as the output feedback controllers presented by Lemus et al. [10] for example, can be of great value to further reduce the weight of fall-prevention systems. Such an intermittent-actuation solution would require a trigger to switch the system from inactive to active when a wearer needs assistance during walking.

A trigger that enables such fall recovery or mitigation control schemes is conventionally created by a detection system designed to detect (the onset of) falls. The author is not aware of any existing publications on pre-impact, model-based fall detectors based on wearable sensors. Such a detector is expected to have three main advantages. Firstly, a decreased detection time, consequently an increased lead time, is expected of model-based approaches when compared to signal-based methods [11, 12]. Secondly, the differentiation between activities of daily life and (near-)falls is expected to be simplified with respect to signal-based methods. This is because it is possible to determine whether the measured system (the human subject) is adhering to the dynamics of walking through fault detection methods. Lastly, a small number of directly measurable or qualitative parameters are expected to fully define the

model, resulting in straightforward person-specific implementation, as opposed to the large amounts of data that are necessary for machine learning methods.

The introduction of such a model-based approach paves the way to gather information useful in different fields, such as: (i) the basic knowledge about posture this method yields at the moment of detection is significantly more in-depth than the traditional four way fall direction classification that current detection methods usually incorporate. (ii) The method introduces the possibility to analyse strategies applied to regain balance and incorporate this knowledge in controlled-environment balance training. (iii) It creates an objective way of reconstructing (near-)falls for the purposes of forensics, research, fall-risk assessment, or fall prevention training for example.

For this proof-of-concept detector, the assumed motion will be forward walking on a flat surface, with a constant velocity. The subject's preferred step length and cadence are such that walking could go on indefinitely. The group of interest is the healthy elderly. This excludes the groups affected by for example multiple sclerosis, Parkinson's disease or paraplegia. This choice to limit the target group gives a homogenous population to demonstrate the detector with for the first time, before having to consider confounding factors.

The investigated hypothesis is that wearable, model-based systems to detect falls are feasible. Moreover, these detectors are expected to perform better by resulting in longer lead times, and yielding more information than their signal-based or machine-learning-based counterparts.

Background

2-1 Gait Phase Cycle

Steady-state gait, with no falls, is a cyclical process and is often divided into two phases [13–18], the swing phase and the stance phase. A different division of the cycle takes both feet into account and divides the gait into single support, when one foot is on the ground, and double support, when both feet are in contact with the ground [17]. In the gait cycle, two instantaneous events at the start of phases are defined: Toe Off (TO) at the start of swing phase and Heel Strike (HS) at the start of stance phase [15, 19–21]. An overview is given in Figure 2-1.

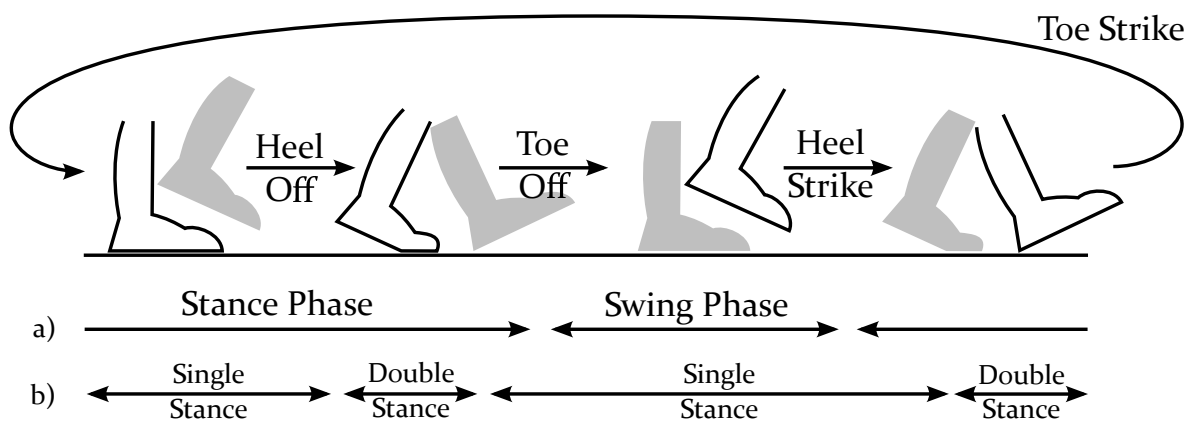


Figure 2-1: Visualisation of the gait cycle, including different segmentations of the cycle. (a) Monopedal stride segmentation of the outlined foot. (b) Bipedal stride segmentation of both the outlined and the shaded feet.

2-2 Defining Falls

Discrepancies in definitions for falling may lead to conflicting situations. To create uniformity, the World Health Organization has defined falling as

Fall: “an event which results in a person coming to rest inadvertently on the ground or floor or other lower level.” [22]

This definition is also broadly used in literature [23]. Detection schemes are split into two categories, pre- and post-impact detection. The impact itself is considered to be instantaneous at the moment a body part other than the feet contact the ground or floor.

Different types of falls are usually considered in experimentation. Most common are the body-relative directions, i.e. forward, backwards, or sideways [9, 24]. Relatively straight downwards is called a *collapse* or *crumple* [25]. Researchers have also included differentiation in preservative movements during the fall, such as twisting from the back to the side [26].

Most often, the causes of unintentional falls are divided into three types: slips, trips, and missteps [25]. *Slips* concern a perturbation of the stance leg, usually a sudden loss of support due to slipping of the foot. *Trips* do not have a conventional definition in the available literature. Broadly speaking, however, a trip is a perturbation of the swing leg, usually through unintended collisions of the swinging foot with an object or the floor. A *misstep* is the umbrella term used to capture all other types of falls [25].

2-3 Measures of Dynamic Stability

Stable gait can be divided into three separate control objectives: (i) stance, redirecting the Centre of Mass (CoM) through ground reaction forces; (ii) swing, cycling between stance legs; and (iii) balance, keeping the body kinematics within acceptable regions [27]. Assessing how well these three processes are operating, and if they successfully generate stable gait in humans, remains an unresolved topic. Many measures based on determining the stability of the limit cycle corresponding to gait exist. Examples of these measures are the maximum Lyapunov exponent [28], maximum Floquet multiplier [29], or fractional stability [30]. Other methods look to the variability of gait parameters [31–35] or to the long term correlation [36] to yield a measure of stability. Researchers have also applied tools such as spectral analysis to obtain more insight into the complex dynamics of gait [37]. All these methods yield global measures of stability, however, and do not produce measures that indicate the instantaneous assessment of stability. They are more suited to assess a subject’s risk of falling, but not necessarily if they are falling at any given moment.

The measures that do give information about instantaneous balance commonly originate from biomechanical principles. The review made by Bruijn et al. gives a practical and insightful overview of the existing measures of this type, including the method of the ‘extrapolated centre of mass’ [36]. To explain this method, an example model of a linear inverted pendulum is taken, as depicted in Figure 2-2. The linear inverted pendulum is a popular and accurate complexity reduction for modelling gait.

The extrapolated centre of mass method is based on extending static balancing principles to dynamic ones. It states that, if for static balance the CoM should be within the Base of Support (BoS), then for dynamic balance the projected CoM should stay within the BoS. With the example model

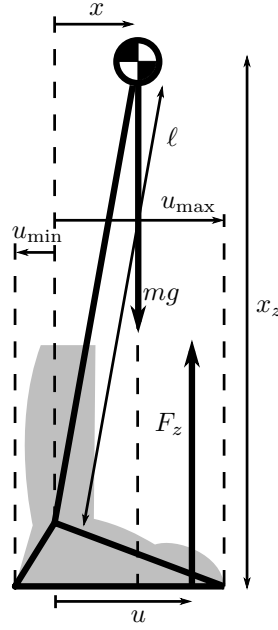


Figure 2-2: Inverted pendulum model of the body for the measure of dynamic stability

(Figure 2-2), Hof et al. present an implementation of the extrapolated centre of mass method [38]. In summary, the derivation is as follows; given the length ℓ of the pendulum, the linearised equations of motion can be constructed as

$$\ddot{x}(t) = -\frac{g}{\ell}(u(t) - x(t)), \quad (2-1)$$

where u and x are the horizontal positions of the ground reaction force and the tip of the pendulum, respectively, relative to the pivot point, as depicted in Figure 2-2. This is a second order differential equation that can be solved for arbitrary initial conditions. Considering the example where the initial velocity, $\dot{x}_0$, of the CoM is in the forward direction, then the physical limit of the ground reaction force is that it cannot move past the toes. In other words, the ground reaction force cannot move outside the BoS. As long as the CoM does not cross over the toe in the forward direction, the resultant moment will counteract the forward velocity of the CoM. However, if this crossing does happen, the resultant moment will accelerate the forward velocity, resulting in unstable growth, i.e. fall forward. As a constraint, this is expressed as

$$x(t) < u_{\max} \quad \forall t \in \mathbb{R}^+. \quad (2-2)$$

Knowing the analytical solution for the equations of motion of the CoM, assuming the ground reaction force is constant at its maximum and placed at the toes, the maximum velocity can be calculated such that the CoM trajectory stays behind the toes. This derivation starts with solving Equation 2-1 to obtain the CoM trajectory as

$$x(t) = u_{\max} + (x_0 - u_{\max}) \cosh(\omega_0 t) + \frac{\dot{x}_0}{\omega_0} \sinh(\omega_0 t), \quad (2-3)$$

where $\omega_0 = \sqrt{\frac{g}{\ell}}$ is the eigenfrequency of the non-inverted pendulum. Substituting into the constraint yields

$$(x_0 - u_{\max}) \cosh(\omega_0 t) + \frac{\dot{x}_0}{\omega_0} \sinh(\omega_0 t) < 0. \quad (2-4)$$

This can be transformed into

$$x_0 - u_{\max} < -\frac{\dot{x}_0}{\omega_0} \tanh(\omega_0 t). \quad (2-5)$$

Since the codomain of the hyperbolic tangent is $\langle -1, 1 \rangle$, this can be reduced to

$$x_0 + \frac{1}{\omega_0} \dot{x}_0 < u_{\max}. \quad (2-6)$$

Thus, any position and velocity measured that violates this inequality would theoretically result in a fall.

Two measures of stability can be extracted from this method, the margin of stability, b , and the time-to-contact, b_τ . The margin of stability is defined as the distance of the extrapolated centre of mass to the edge of the BoS, or

$$b = \left| u_{\max} - \left(x + \frac{1}{\omega_0} \dot{x} \right) \right|. \quad (2-7)$$

The time-to-contact is defined as the margin of stability divided over the current velocity of the CoM, or [36, 38]

$$b_\tau = \frac{b}{\dot{x}}. \quad (2-8)$$

2-4 Pre-Impact Fall Detection Methods

Many attempts at detecting falls before the individual makes impact with the ground have been quite successful, with recent publications presenting lead times (time between detection and impact) of up to nearly 900 ms. These pre-impact fall detectors are listed in Table 2-I, where the last three columns are performance measures.

Figure 2-3 shows a plot of the numerical values of Table 2-I for a more comprehensive comparison. To compare the performance through a scalar, the Euclidean norm of the sensitivity, specificity, and accuracy is calculated. Only sources that present at least two of these measures are included in the plot. For a fair, yet conservative comparison, the empty fields in Table 2-I are filled with the smaller value of the other two known fields (i.e. the accuracy of [52] is taken to be 95%). Model-based methods in wearable forms have not been found in the available literature, but the Kinect-based systems presented by Otanasap and Boonbrahm, and Li et al. perform relatively well [11, 12].

The results presented by Otanasap and Boonbrahm, and Li et al. support the hypothesis that model-based detectors perform relatively well [11, 12]. Moreover, the prevalence of wearable detector configurations in the literature suggests that wearable sensors yield enough information for a model-based approach.

Table 2-I: Pre-impact fall detectors gathered from the available literature. Se: Sensitivity, Sp: Specificity, Ac: Accuracy, Thr: Threshold-based method, Mod: Model-based method, SVM: Support Vector Machine, ML: Machine Learning method, MoCap: Motion Capture system.

Source	Sensors (amount, placement)	Method	Lead time (s)	Se (%)	Sp (%)	Ac (%)
[39]	IMU (1, front-waist)	Thr	0.07	-	-	-
[40]	IMU (2, back- and right-waist)	ML	0.15	-	100	97
[41]	IMU (1, waist)	Thr	0.26	93	98	-
[42]	IMU (1, sternum)	Thr	0.26	100	96	-
[26]	IMU (1, lower-back)	Thr	0.28	96	91	92
[43]	IMU (1, lower-back)	Thr	0.33	96	83	89
[44]	MoCap (39 markers, full-body)	ML	0.35	93	98	95
[45]	IMU (1, front-waist)	Thr	0.36	94	96	-
[46]	IMU (1, sternum)	Thr	0.37	99	95	97
[47]	IMU (1, shank)	Thr	0.37	-	-	-
[48]	IMU (2, front-waist, front-right-thigh)	ML	0.38	96	97	-
[43]	IMU (1, lower-back)	SVM	0.39	100	95	97
[43]	IMU (1, lower-back)	ML	0.40	99	99	99
[49]	IMU (1, front-waist)	Thr	0.40	100	78	87
[50]	IMU (waist, front)	Thr	0.40	100	100	-
[49]	IMU (1, front-waist)	Thr	0.43	100	84	90
[51]	IMU (1, waist or lower-back)	ML	0.50	99	-	98
[11]	Kinect	Mod	0.52	97	100	96
[52]	IMU (2, waist and thigh)	Thr	0.70	95	100	-
[12]	Kinect	Mod	0.87	100	81	92
[53]	IMU (1, front-waist)	SVM	-	97	94	95

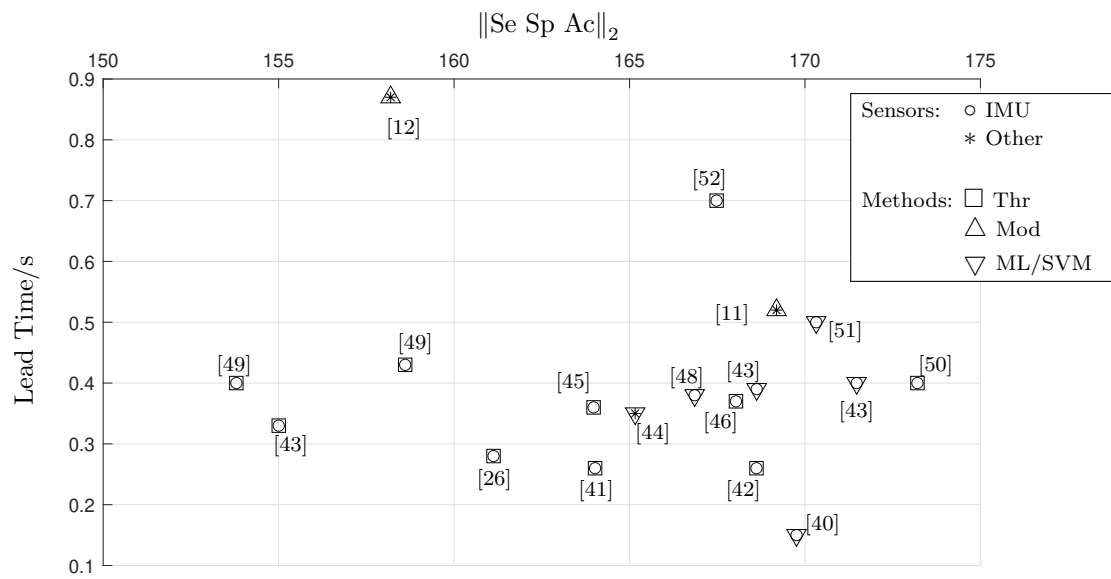


Figure 2-3: Comparison of existing pre-impact fall detectors. Se: Sensitivity, Sp: Specificity, Ac: Accuracy, Thr: Threshold-based method, Mod: Model-based method, ML/SVM: Machine Learning method or Support Vector Machine.

Chapter 3

Method

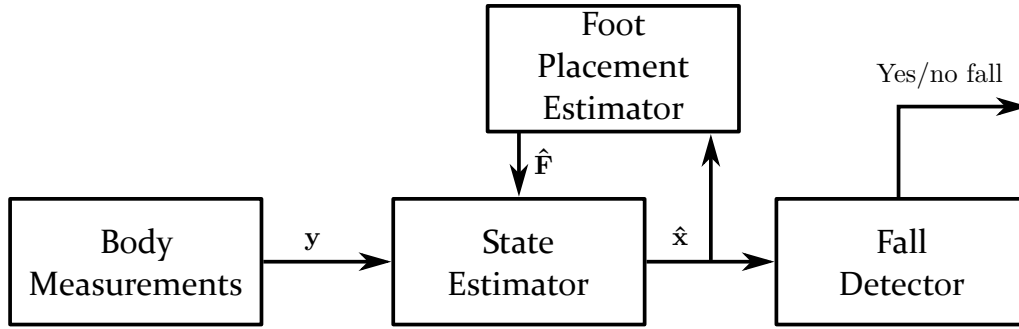


Figure 3-1: Block scheme of the detector

3-1 Overview

To answer the presented hypotheses, a detection scheme was used consisting of four main components, depicted in Figure 3-1. Firstly, the *body measurement* block consists of the walking human and sensors, and yields the sensor measurement stream, y . The *state estimator* reconstructs the state and orientation of the subject from the sensor measurements. These state estimates, consisting of the Centre of Mass (CoM) velocity estimate $\hat{x}$, are then used to determine if the person has initiated a fall by the *fall detector*. Lastly, it is assumed the time instances of heel strikes are known. Given these instances and the state estimate, the estimated foot position, $\hat{F}$, at heel strike is determined by the *foot placement estimator*. For clarity, the symbol x is used for the state in a filtering context and $\dot{O}_{B/\mathcal{N}}$ is used in a modelling context, but these symbols represent the same value, i.e.

$$x = \dot{O}_{B/\mathcal{N}}. \quad (3-1)$$

In order to define points in space and on the body, two coordinate systems are used. The first is the inertial world frame $\mathcal{N}$, where the placement of the origin is arbitrary. The second coordinate system

Table 3-I: Notation used throughout this thesis

Notation	Description
a	Scalar, $a \in \mathbb{R}$
$\mathbf{a}$	Vector, unspecified in what frame; $\mathbf{a} \in \mathbb{R}^3$
${}^{\mathcal{N}}\mathbf{a}$	Vector expressed in frame $\mathcal{N}$
$\mathcal{N}$	Frame, triplet of orthogonal unit vectors ($\mathcal{N}_x, \mathcal{N}_y, \mathcal{N}_z$); or coordinate system, quadruplet of three orthogonal unit vectors and an origin ($\mathcal{N}_x, \mathcal{N}_y, \mathcal{N}_z, \mathcal{O}_{\mathcal{N}}$)
$\mathcal{N}_x$	Unit vector x of frame/coordinate system $\mathcal{N}$
$\hat{\mathbf{a}}$	Matrix-vector notation of the cross product, skew antisymmetric; $\hat{\mathbf{a}} \in \mathbb{R}^{3 \times 3}$
$\underline{\mathbf{q}}$	Quaternion; $\underline{\mathbf{q}} \in \mathbb{R}^4$
${}^{\mathcal{N}}\underline{\mathbf{q}}^{\mathcal{B}}$	Rotation quaternion used to translate any ${}^{\mathcal{B}}\mathbf{a}$ to ${}^{\mathcal{N}}\mathbf{a}$
$\ \mathbf{a}\ $	The 2-norm of $\mathbf{a}$
$\hat{\mathbf{a}}$	Estimated value of $\mathbf{a}$
A	Matrix, $A \in \mathbb{R}^{n \times m}$
E_i	Identity matrix of size $i \times i$
a^-	The value of a at the previous timestep
a^+	The value of a at the next timestep
$\mathcal{O}_{\mathcal{B}/\mathcal{N}}$	The origin $\mathcal{B}$ relative to the world origin $\mathcal{N}$

is the body-fixed frame $\mathcal{B}$, with the origin relative to $\mathcal{O}_{\mathcal{N}}$, $\mathcal{O}_{\mathcal{B}/\mathcal{N}}$, placed at the CoM. The whole-body CoM is difficult to determine accurately [54], but is often approximated by markers on the pelvis (i.e. the average of the sacrum and the anterior superior iliac spines). The body-fixed coordinate system is defined such that the $\mathcal{B}_z$ axis is parallel to the spine and the x axis is in the anterior direction and perpendicular to $\mathcal{B}_z$ and the pelvis. This results in the $\mathcal{B}_y$ pointing to the left of the subject. Optical markers are used for training as well as finding the CoM. The symbol for the position of optical markers will be $\mathbf{r}$, subscripted with the corresponding abbreviation of the location defined in Table 3-II and depicted in Figure 3-2. In general, the notation conventions held onto are tabulated in Table 3-I.

The CoM and the origin of coordinate system $\mathcal{B}$, relative to the world origin, can then be estimated through

$${}^{\mathcal{N}}\mathcal{O}_{\mathcal{B}/\mathcal{N}} \approx \frac{1}{3}({}^{\mathcal{N}}\mathbf{r}_{\text{LASI}} + {}^{\mathcal{N}}\mathbf{r}_{\text{RASI}} + {}^{\mathcal{N}}\mathbf{r}_{\text{SACR}}). \quad (3-2)$$

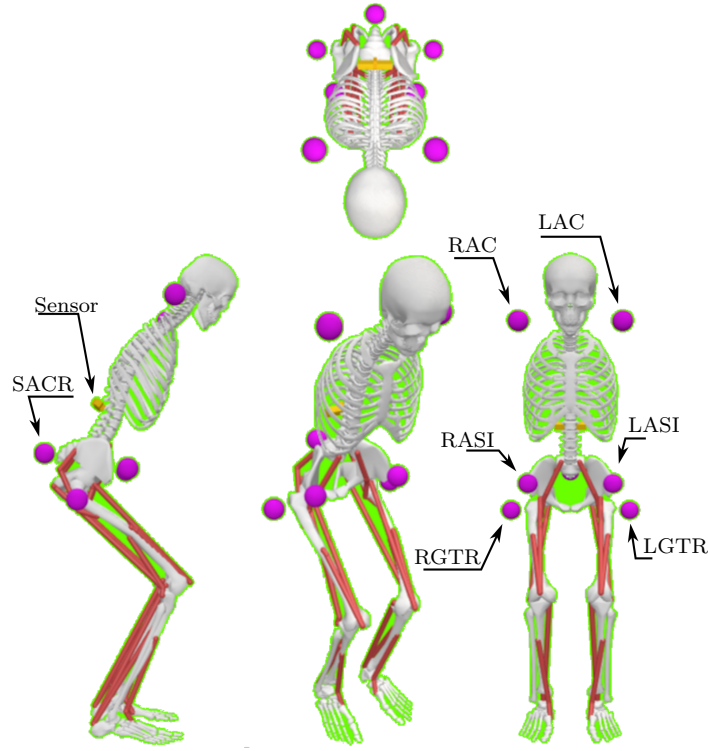


Figure 3-2: Positioning and names of optical markers indicated with purple spheres and positioning of sensor unit on the lower back indicated with an orange bar. Abbreviations are listed in Table 3-II. Adapted from model included with OpenSim [55].

3-2 Sensor Configuration

A single 6-Degrees Of Freedom (DOFs) Inertial Measurement Unit (IMU) is the only sensor necessary for the presented algorithm. The six DOFs are three axes of acceleration and three axes of angular velocity. An IMU is a fitting sensor for the target use-case, since it is low-cost, low-power, and small [24]. Additionally, these sensors are light-weight, user-friendly, and robust to environmental influences.

There is no definitive consensus on the optimal placement of IMUs on the body [56]. However, multiple results indicate that proximity to the CoM is beneficial to the gathered information [9, 57, 58]. Following these findings, the IMU is placed on the lower back of the subject. Assuming the IMU is placed in the same sagittal plane as the CoM, the definition of the position relative to the CoM, $\mathbf{S}$, is given by

$${}^B\mathbf{S} = \begin{pmatrix} s_1 \\ 0 \\ s_3 \end{pmatrix}. \quad (3-3)$$

For subject-specific parameter determination these two parameters can be estimated from physical measurements. Given the approximation of the position of the CoM, and assuming the vertices

Table 3-II: Body location abbreviations and the full marker set necessary to perform the model training procedures.

Abbreviation	Location
LAC	Left acromion
RAC	Right acromion
LASI	Left anterior superior iliac spine
RASI	Right anterior superior iliac spine
LGTR	Left greater trochanter
RGTR	Right greater trochanter
SACR	Sacrum

$\{\mathbf{r}_{\text{RASI}}, \mathbf{r}_{\text{LASI}}, \mathbf{r}_{\text{SACR}}\}$ form an isosceles triangle, s_1 may be estimated as

$$s_1 = -\frac{2}{3}d_{\text{pelvis}}, \quad (3-4)$$

where d_{pelvis} is the body thickness in $\mathcal{B}_x$ direction at a height between the sacrum and the anterior superior iliac spines. The placement height on the back is highly dependent of the attachment method, i.e. belt, pelvis harness, shoulder harness. Under the same assumptions as for the estimation of s_1 , the height from the sacrum to the sensor can be measured and subtracting a third of the height difference between the sacrum and the anterior superior iliac spines will yield an estimate for s_3 .

3-3 State Estimator

3-3-1 Overview

The model-based approach relies on the estimation of the state of a dynamical model representing gait. As depicted in Figure 3-3, this starts with a (noisy) IMU measurement $\mathbf{y}$, that consists of the body-relative angular velocities, ${}^{\mathcal{B}}\boldsymbol{\omega}$, and acceleration including gravity, ${}^{\mathcal{B}}\mathbf{a}_g$. The angular velocity measurements are used to estimate the actual angular velocity ${}^{\mathcal{N}}\dot{\mathbf{q}}^{\mathcal{B}}$ in quaternion form, the upper-body orientation quaternion ${}^{\mathcal{N}}\mathbf{q}^{\mathcal{B}}$ that maps vectors in $\mathcal{B}$ to $\mathcal{N}$, and the angular acceleration ${}^{\mathcal{N}}\ddot{\mathbf{q}}^{\mathcal{B}}$. These estimates are stacked into

$$\hat{\mathbf{q}} = \left({}^{\mathcal{N}}\mathbf{q}^{\mathcal{B}} \quad {}^{\mathcal{N}}\dot{\mathbf{q}}^{\mathcal{B}} \quad {}^{\mathcal{N}}\ddot{\mathbf{q}}^{\mathcal{B}} \right)^{\text{T}} \quad (3-5)$$

and combined with the current foot position estimate, $\hat{\mathbf{F}}$, to obtain the gait model input

$$\hat{\mathbf{u}} = \left({}^{\mathcal{B}}\hat{\mathbf{F}} \quad \hat{\mathbf{q}} \right)^{\text{T}}. \quad (3-6)$$

This input and the previous state estimate, $\hat{\mathbf{x}}^-$, drive an Extended Kalman Filter (EKF) to obtain the velocity prediction $\hat{\mathbf{x}}'^-$, which is then updated with the IMU measurement to get the drifted velocity estimate $\hat{\mathbf{x}}'$. This estimate is dedrifted with a PI controller to obtain the final velocity estimate $\hat{\mathbf{x}}$. For an introduction to quaternions, see Vallery and Schwab [59, app. D] and Sabatini [60, app. A].

3-3-2 Orientation Estimation

The orientation estimation is performed by a Kalman Filter (KF) smoothing the angular velocity measurements. Taking inspiration from Joukov et al. [61], it is assumed that due to the cyclical

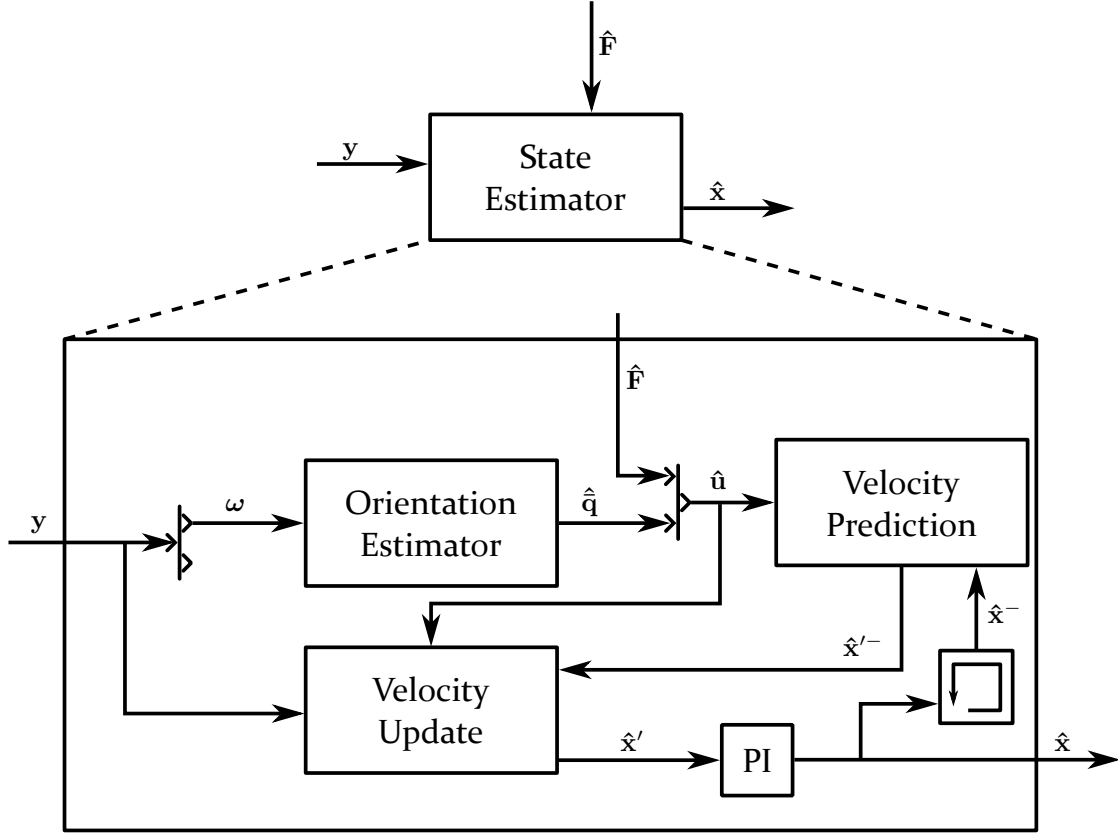


Figure 3-3: Block scheme of the state estimator subsystem

nature of walking, all elements of $\mathcal{N}\underline{\mathbf{q}}^B$, $\mathcal{N}\underline{\dot{\mathbf{q}}}^B$ and $\mathcal{N}\underline{\ddot{\mathbf{q}}}^B$ can be modelled as simple oscillators with an equal frequency, ϕ_{sway} . For example, the dynamical model for the first element of $\mathcal{N}\underline{\mathbf{q}}^B$, ${}^Nq_1^B$, is

$$\begin{cases} \dot{\mathbf{x}}_{q1} = \begin{pmatrix} 0 & 1 \\ -(2\pi\phi_{\text{sway}})^2 & 0 \end{pmatrix} \mathbf{x}_{q1} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u_{q,1} \\ {}^Nq_1^B = (1 \quad 0) \mathbf{x}_{q1}. \end{cases} \quad (3-7)$$

Appending four of these systems diagonally allows for filtering the whole quaternions with a single KF, such that the filtering procedure can be expressed as a function, $\text{KF}(\mathcal{N}\underline{\mathbf{q}}_{\text{meas}}^B)$, of the measured quaternion.

The estimation procedure at timestep k starts with calculating the measured quaternion derivative as

$$\mathcal{N}\underline{\dot{\mathbf{q}}}^B_{\text{meas}}[k+1] = \frac{1}{2}Q[k] \begin{pmatrix} 0 \\ {}^B\boldsymbol{\omega}[k] \end{pmatrix}, \quad (3-8)$$

where Q is the left quaternion multiplication matrix. For example, this matrix for a quaternion $\underline{\mathbf{q}}$ is expressed as

$$Q = \begin{pmatrix} q_0 & \mathbf{q}^T \\ \mathbf{q} & q_0 E_3 + \tilde{\mathbf{q}} \end{pmatrix}, \quad \underline{\mathbf{q}} = \begin{pmatrix} q_0 \\ \mathbf{q} \end{pmatrix}. \quad (3-9)$$

This measurement is then smoothed with the KF, such that

$$\hat{\underline{\mathbf{q}}}^{\mathcal{B}}[k+1] = \text{KF}(\hat{\underline{\mathbf{q}}}^{\mathcal{B}}_{\text{meas}}[k+1]). \quad (3-10)$$

Using the forward Euler integration method for a sampling period Δt , the measured orientation is obtained as

$$\hat{\underline{\mathbf{q}}}^{\mathcal{B}'}_{\text{meas}}[k+1] = \hat{\underline{\mathbf{q}}}^{\mathcal{B}}[k] + \Delta t \hat{\underline{\dot{\mathbf{q}}}}^{\mathcal{B}}[k] \quad (3-11)$$

and normalised to obtain a valid rotation quaternion,

$$\underline{\mathbf{q}}^{\mathcal{B}}_{\text{meas}}[k+1] = \frac{1}{\|\hat{\underline{\mathbf{q}}}^{\mathcal{B}'}_{\text{meas}}[k+1]\|} \hat{\underline{\mathbf{q}}}^{\mathcal{B}'}_{\text{meas}}[k+1]. \quad (3-12)$$

This is again smoothed as

$$\hat{\underline{\mathbf{q}}}^{\mathcal{B}'}[k+1] = \text{KF}(\underline{\mathbf{q}}^{\mathcal{B}}_{\text{meas}}[k+1] - \underline{\mathbf{q}}^{\mathcal{B}}_{\text{steady}}), \quad (3-13)$$

where $\underline{\mathbf{q}}^{\mathcal{B}}_{\text{steady}}$ is the steady state orientation. The quaternion $\hat{\underline{\mathbf{q}}}^{\mathcal{B}'}$ is then normalised as

$$\hat{\underline{\mathbf{q}}}^{\mathcal{B}}[k+1] = \frac{1}{\|\hat{\underline{\mathbf{q}}}^{\mathcal{B}'}[k+1]\|} \hat{\underline{\mathbf{q}}}^{\mathcal{B}'}[k+1] + \underline{\mathbf{q}}^{\mathcal{B}}_{\text{steady}}. \quad (3-14)$$

Lastly, the angular acceleration is calculated with forward differentiation as

$$\hat{\underline{\ddot{\mathbf{q}}}}^{\mathcal{B}}_{\text{meas}}[k+1] = \frac{1}{\Delta t} (\hat{\underline{\dot{\mathbf{q}}}}^{\mathcal{B}}[k+1] - \hat{\underline{\dot{\mathbf{q}}}}^{\mathcal{B}}[k]) \quad (3-15)$$

and smoothed as

$$\hat{\underline{\ddot{\mathbf{q}}}}^{\mathcal{B}}[k+1] = \text{KF}(\hat{\underline{\ddot{\mathbf{q}}}}^{\mathcal{B}}_{\text{meas}}[k+1]). \quad (3-16)$$

The exact formulation of $\text{KF}(\hat{\underline{\mathbf{q}}}^{\mathcal{B}}_{\text{meas}})$ is elaborated on in Appendix A.

3-3-3 Centre of Mass Velocity Estimation

Dynamical Model

The velocity estimates of the CoM are made with an EKF driven by a hybrid dynamical model, f , representing gait, and a measurement model, g , that links the state to sensor readings. The EKF algorithm is very similar to the regular KF algorithm, except that the system is linearised at every time step for the current state. The exact algorithm is given by Särkkä [62, p. 69, 70] and presented in Appendix A.

Furthermore, the model consists of four modes, p , with different dynamics: single and double stance for both left and right legs. The mode transitions are depicted in Figure 3-4a. The model switches from single stance to double stance at every heel strike, and after five sampling periods in double stance, continues to single stance again on the other leg. It is assumed that heel strike events occur at known time instances. The shifting of the centre of pressure from the back foot to the new stance foot, takes roughly these five sampling periods in double stance.

The continuous model, f' such that $\ddot{\mathcal{O}}_{\mathcal{B}/\mathcal{N}} = f'(\dot{\mathcal{O}}_{\mathcal{B}/\mathcal{N}}, \mathbf{u}, p)$, with gait phase p , is synthesised to be similar to the broadly-used Spring-Loaded Inverted Pendulum (SLIP)-model [63] and is similar to the model presented by Paiman et al.[64]. It consists of a point, $\mathcal{O}_{\mathcal{B}/\mathcal{N}}$, at the CoM in 3D-space, and two

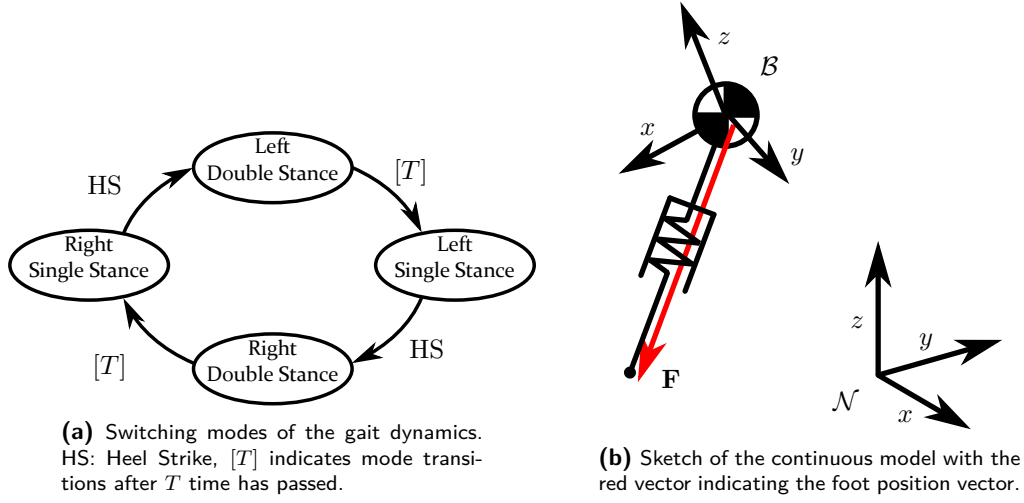


Figure 3-4: Switching modes and single mode dynamics of the dynamical model

spring-damper legs connected to it with ball joints. It is assumed there is only one foot exerting a Ground Reaction Force (GRF), $\mathbf{R}$, at any given moment and its origin relative to the CoM is the foot position denoted as $\mathbf{F}$. The model is sketched in Figure 3-4b. The state of this model is the velocity of the CoM, ${}^B\dot{\mathbf{O}}_{B/\mathcal{N}}$, and the input is

$$\mathbf{u} = ({}^B\mathbf{F} \quad \bar{\mathbf{q}})^T. \quad (3-17)$$

The GRF is the only force besides gravity acting on the CoM. The direction of the GRF is in the opposite direction of $\mathbf{F}$. Its magnitude is dependent upon the spring and damper constants and the untensioned length of the leg, K , b and ℓ_0 respectively. Denoting the leg length with ℓ_ℓ gives

$$\|\mathbf{R}\| = K(\ell_0 - \ell_\ell) + b\dot{\ell}_\ell. \quad (3-18)$$

The damper in the leg does not necessarily dissipate energy. If the training performance improves because the damper adds energy to the system, it is allowed to be a negative damper, i.e. b can be both negative and positive.

The derivative $\dot{\ell}_\ell$ is not easily determined, because for example the centre of pressure moves beneath the foot, or the foot can slip. However, if it is assumed that $\mathbf{F}$ is static in $\mathcal{N}$, or in other words, that the centre of pressure does not move, then

$$\dot{\mathbf{F}} = -\dot{\mathbf{O}}_{B/\mathcal{N}}. \quad (3-19)$$

Then the leg length derivative can be calculated by projecting this foot velocity onto the leg direction as

$$\dot{\ell}_\ell = \frac{1}{\|\mathbf{F}\|} (-\dot{\mathbf{O}}_{B/\mathcal{N}} \cdot \mathbf{F}). \quad (3-20)$$

The final force is then found through

$$\mathbf{R} = \|\mathbf{R}\| \frac{-\mathbf{F}}{\|\mathbf{F}\|}. \quad (3-21)$$

During double stance, the x component of $\mathbf{R}$ is set to zero to correct for the shifting of the centre of pressure from one foot to the other.

The gravitational force, $\mathbf{Z}$, is defined in the world frame as

$${}^{\mathcal{N}}\mathbf{Z} = m^{\mathcal{N}}\mathbf{g} = m \begin{pmatrix} 0 & 0 & -g \end{pmatrix}^T, \quad (3-22)$$

where g is the gravitational constant and m is the whole-body mass. With the inverse of the input ${}^{\mathcal{N}}\underline{\mathbf{q}}^{\mathcal{B}}$, this can be rotated to ${}^{\mathcal{B}}\mathbf{Z}$. The acceleration of the CoM can then be found by

$$m^{\mathcal{B}}\ddot{\mathcal{O}}_{\mathcal{B}/\mathcal{N}} = {}^{\mathcal{B}}\mathbf{R} + {}^{\mathcal{B}}\mathbf{Z} \quad (3-23)$$

$$\Rightarrow f'(\dot{\mathcal{O}}_{\mathcal{B}/\mathcal{N}}, \mathbf{u}, p) = {}^{\mathcal{B}}\ddot{\mathcal{O}}_{\mathcal{B}/\mathcal{N}} = \frac{1}{m} ({}^{\mathcal{B}}\mathbf{R} + {}^{\mathcal{B}}\mathbf{Z}). \quad (3-24)$$

This can then be discretised through forward Euler integration as

$${}^{\mathcal{N}}\dot{\mathcal{O}}_{\mathcal{B}/\mathcal{N}}^+ = {}^{\mathcal{N}}\dot{\mathcal{O}}_{\mathcal{B}/\mathcal{N}} + \Delta t^{\mathcal{N}}\ddot{\mathcal{O}}_{\mathcal{B}/\mathcal{N}} \quad (3-25)$$

$$\Rightarrow f(\dot{\mathcal{O}}_{\mathcal{B}/\mathcal{N}}, \mathbf{u}, p) = {}^{\mathcal{B}}\dot{\mathcal{O}}_{\mathcal{B}/\mathcal{N}}^+. \quad (3-26)$$

Only one discontinuous reset map is applied during state transitions. Due to how the model is defined, the GRF is too small during the last part of the swing phase. The CoM is therefore not accelerated upwards whilst measurement data indicates it should be. The $\mathcal{B}_z$ velocity is set to zero at every heel strike to compensate for this missing upward motion.

Measurement Model

To estimate the CoM velocity, the EKF requires a function that relates the velocity to the measurement. This function is $g(\dot{\mathcal{O}}_{\mathcal{B}/\mathcal{N}}, \mathbf{u}, p)$ and describes the relation between body movement and IMU measurements. Given the CoM acceleration $\ddot{\mathcal{O}}_{\mathcal{B}/\mathcal{N}} = f'(\dot{\mathcal{O}}_{\mathcal{B}/\mathcal{N}}, \mathbf{u}, p)$ and the orientation quaternion and its derivatives, the body-relative angular velocity and acceleration, $\boldsymbol{\omega}$ and $\dot{\boldsymbol{\omega}}$ respectively, can be found through [59, app. D]

$$\begin{pmatrix} 0 \\ {}^{\mathcal{B}}\boldsymbol{\omega} \end{pmatrix} = 2Q^{\mathcal{T}\mathcal{N}}\dot{\underline{\mathbf{q}}}^{\mathcal{B}}, \text{ and} \quad (3-27)$$

$$\begin{pmatrix} 0 \\ {}^{\mathcal{B}}\dot{\boldsymbol{\omega}} \end{pmatrix} = 2Q^{\mathcal{T}\mathcal{N}}\ddot{\underline{\mathbf{q}}}^{\mathcal{B}}. \quad (3-28)$$

Then, the acceleration of $\mathbf{S}$ is calculated as [59]

$${}^{\mathcal{B}}\ddot{\mathbf{S}} = {}^{\mathcal{B}}\ddot{\mathcal{O}}_{\mathcal{B}/\mathcal{N}} + {}^{\mathcal{B}}\dot{\boldsymbol{\omega}} \times {}^{\mathcal{B}}\mathbf{S} + {}^{\mathcal{B}}\boldsymbol{\omega} \times ({}^{\mathcal{B}}\boldsymbol{\omega} \times {}^{\mathcal{B}}\mathbf{S}) + {}^{\mathcal{B}}\mathbf{g}. \quad (3-29)$$

The complete IMU measurement function then becomes

$$g(\dot{\mathcal{O}}_{\mathcal{B}/\mathcal{N}}, \mathbf{u}, p) = \begin{pmatrix} {}^{\mathcal{B}}\boldsymbol{\omega} \\ {}^{\mathcal{B}}\ddot{\mathbf{S}} \end{pmatrix}. \quad (3-30)$$

Knowing the expressions of the dynamical and measurement models, the states can be estimated with an EKF. The precise filtering equations are described by Särkkä [62] and elaborated upon in Appendix A.

3-3-4 Dedrifting

Due to integration of measurement errors, the CoM velocities are prone to drift from their true nominal values. To counteract this drifting, it is assumed that the $\mathcal{B}_x$ velocity stays near to the nominal walking velocity, v_0 , and the $\mathcal{B}_y$ velocity is centred around zero. These assumptions allow the use of a diagonal PI-controller to compensate for the integration drift. The assumption that v_0 is known poses a practical limitation, since v_0 can not be obtained directly from accelerometry.

The two tuning parameters for this drift controller are κ_x and κ_y for x and y direction, respectively. The dedrifting is then accomplished by

$$\begin{cases} I_x^+ = I_x + (\hat{\mathbf{x}}'_x - v_0) \Delta t \\ \hat{\mathbf{x}}_x = \hat{\mathbf{x}}'_x - \kappa_x I_x \end{cases} \quad (3-31)$$

and

$$\begin{cases} I_y^+ = I_y + \hat{\mathbf{x}}'_y \Delta t \\ \hat{\mathbf{x}}_y = \hat{\mathbf{x}}'_y - \kappa_y I_y, \end{cases} \quad (3-32)$$

where I_x and I_y are the error integral terms for x and y velocities, respectively. Both I_x and I_y are bounded to the range $[-0.15, 0.15]$ to prevent integral windup during the initial transient.

3-4 Foot Placement Estimator

The Foot Placement Estimator (FPE) is similar to the Extrapolated Centre of Mass (XCoM) method in the sense that it relies on the CoM velocity [38]. Ignoring the vertical component, the new foot placement at heel strike, $\hat{\mathbf{F}}_{\text{new}}$, is determined in two steps where the first is

$$\hat{\mathbf{F}}'_{\text{new}} = \begin{pmatrix} 0.1 & 0 & 0 \\ 0 & \zeta_W & 0 \\ 0 & 0 & 0 \end{pmatrix} \dot{\mathcal{O}}_{\mathcal{B}/\mathcal{N}} + \begin{pmatrix} \zeta_L \\ 0 \\ 0 \end{pmatrix}. \quad (3-33)$$

Here, ζ_L and ζ_W are tuning parameters. The factor 0.1 in the diagonal matrix is a mean-removal approximation, where it is assumed the $\mathcal{B}_x$ velocity deviates roughly 10% around its nominal value. The final placement estimate is determined in the second step by finding the z component such that the leg at heel strike has length ℓ_{max} through

$$\begin{cases} \hat{\mathbf{F}}_{\text{new}} = \hat{\mathbf{F}}'_{\text{new}} + \begin{pmatrix} 0 & 0 & -\sqrt{\ell_{\text{max}}^2 - \|\hat{\mathbf{F}}'_{\text{new}}\|^2} \end{pmatrix}^T, & \|\hat{\mathbf{F}}'_{\text{new}}\| \leq \ell_{\text{max}} \\ \hat{\mathbf{F}}_{\text{new}} = \begin{pmatrix} 0.48 & 0 & -\sqrt{\ell_{\text{max}}^2 - 0.48^2} \end{pmatrix}^T, & \|\hat{\mathbf{F}}'_{\text{new}}\| > \ell_{\text{max}}. \end{cases} \quad (3-34)$$

The second option is added to prevent complex estimates, whilst still yielding a realistic output.

Outside of heel strike events, during the swing phase it is assumed the stance foot does not slip. The system $\mathcal{B}$ does move, so the foot moves with respect to $\mathcal{B}$ as [59]

$$\hat{\mathbf{F}}^+ = \hat{\mathbf{F}} + \Delta t \left(-\dot{\mathcal{O}}_{\mathcal{B}/\mathcal{N}}^+ + \boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \times \hat{\mathbf{F}} \right), \quad (3-35)$$

where

$$\begin{pmatrix} 0 \\ \boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \end{pmatrix} = 2Q^{\text{T}\mathcal{N}} \dot{\mathbf{q}}^{\mathcal{B}}. \quad (3-36)$$

The foot estimate is also prevented from crossing the zero line in the y direction to keep the model from accelerating in the wrong direction.

3-5 Fall Detector

The XCoM method as described in Section 2-3, is applied in 3-dimensional space for this model to detect falls given the CoM [38]. The eigenfrequency of the non-inverted pendulum of equal length, without spring or damper, is assumed to be

$$\omega_0 = \sqrt{\frac{g}{\ell_{\max}}}. \quad (3-37)$$

The XCoM, denoted by $\mathbf{x}_{\text{CoM}}$, is calculated as

$$\mathbf{x}_{\text{CoM}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \mathbf{x}_{\text{CoM}} \oslash \omega_0, \quad (3-38)$$

where $\oslash$ is the Hadamard (elementwise) division.

The Base of Support (BoS), $\mathfrak{B}$, has no conventional definition for dynamic walking. By describing human gait as a static walking process instead, the BoS would include only the envelope of the feet contact areas. The BoS for a dynamic walking process turns the problem into a classification problem where it has to be determined what states (trajectories) belong to steady-state walking and unstable walking, respectively. In this detector proposal, these problems are avoided by simplifying the BoS based on observations from stable walking. The BoS is defined as a triangle with its vertices at

$$\left\{ \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \mathfrak{B}_x \\ \mathfrak{B}_y \\ 0 \end{pmatrix}, \begin{pmatrix} \mathfrak{B}_x \\ -\mathfrak{B}_y \\ 0 \end{pmatrix} \right\}. \quad (3-39)$$

The detector indicates falling by setting the value of a Boolean variable Λ , when $\mathbf{x}_{\text{CoM}}$ moves outside of $\mathfrak{B}$, or

$$\mathbf{x}_{\text{CoM}} \notin \mathfrak{B} \leftrightarrow \Lambda = \text{true}. \quad (3-40)$$

The reason for $\mathfrak{B}$ being defined in $\mathcal{B}$ is to decouple it from the orientation estimation. Even though $\mathfrak{B}$ will not coincide with the real-world floor, this is outweighed by the advantage of having it decoupled from the orientation estimate. This is further supported by the fact that the upper-body tilt is generally within small-angle magnitudes.

3-6 Parameter Training

The dynamical model described in Section 3-3-3 allows for subject-specific customisation to best reflect their gait. Aside from the dynamical model parameters, the other blocks have tunable parameters as well, which are listed in Table 3-III.

The system parameters are determined through experimental training data [65] that includes optical marker data retrieved through a motion capture system and the ground reaction force, including the centre of pressure position, obtained by a force plate sensor. The list of necessary markers is given in Table 3-II.

To train the dynamical model, the marker measurements are parsed to obtain the reference state and input trajectories first. This starts with obtaining the body-fixed frame $\mathcal{B}$, where $\mathcal{B}_z$ is defined as the

Table 3-III: Detection system parameters

Symbol	Description	Range	Unit
Dynamical Model			
m	Whole-body mass	$\mathbb{R}^+$	kg
K_L	Left leg spring constant	$[0, 10^4]$	N m^{-1}
b_L	Left leg damper constant	$[-5 \cdot 10^3, 5 \cdot 10^3]$	N s m^{-1}
ℓ_{0L}	Left leg untensioned length	$[0, 2]$	m
K_R	Right leg spring constant	$[0, 10^4]$	N m^{-1}
b_R	Right leg damper constant	$[-5 \cdot 10^3, 5 \cdot 10^3]$	N s m^{-1}
ℓ_{0R}	Right leg untensioned length	$[0, 2]$	m
FPE			
$\zeta_{W,L}$	Left leg step width correction	$\mathbb{R}$	s
$\zeta_{L,L}$	Left leg step length correction	$\mathbb{R}$	m
$\zeta_{W,R}$	Right leg step width correction	$\mathbb{R}$	s
$\zeta_{L,R}$	Right leg step length correction	$\mathbb{R}$	m
$\ell_{\max}$	Heel strike leg length	$\mathbb{R}^+$	m
Orientation Estimator			
ϕ_{sway}	Upper-body oscillation frequency	$[0.5, 2]$	Hz
Fall Detector			
$\mathfrak{B}_x$	Anterior boundary of base of support	$\mathbb{R}^+$	m
$\mathfrak{B}_y$	Lateral boundary of base of support	$\mathbb{R}^+$	m

direction from the CoM to the centre of the shoulders, such that

$${}^{\mathcal{N}}\mathcal{B}_z = \frac{1}{2}({}^{\mathcal{N}}\mathbf{r}_{\text{LAC}} + {}^{\mathcal{N}}\mathbf{r}_{\text{RAC}}) - {}^{\mathcal{N}}\mathcal{O}_{\mathcal{B}/\mathcal{N}}. \quad (3-41)$$

It is then assumed that the anterior direction, ${}^{\mathcal{N}}\mathcal{B}_x$, is perpendicular to the pelvis and ${}^{\mathcal{N}}\mathcal{B}_z$, and therefore

$${}^{\mathcal{N}}\mathcal{B}_x = ({}^{\mathcal{N}}\mathbf{r}_{\text{LASI}} - {}^{\mathcal{N}}\mathbf{r}_{\text{RASI}}) \times {}^{\mathcal{N}}\mathcal{B}_z. \quad (3-42)$$

The complete frame $\mathcal{B}$ is found with

$${}^{\mathcal{N}}\mathcal{B}_y = {}^{\mathcal{N}}\mathcal{B}_z \times {}^{\mathcal{N}}\mathcal{B}_x \quad (3-43)$$

and normalising the directions. Building the rotation matrix and converting it to a quaternion, gives the rotation quaternion ${}^{\mathcal{N}}\underline{\mathbf{q}}^{\mathcal{B}}$ that transforms vectors from frame $\mathcal{B}$ to $\mathcal{N}$.

The velocity of the CoM is numerically derived from the measured position via the symmetric derivative. Given the sampling time, Δt , the velocity is approximated as

$${}^{\mathcal{N}}\dot{\mathcal{O}}_{\mathcal{B}/\mathcal{N}}[k] = \frac{{}^{\mathcal{N}}\mathcal{O}_{\mathcal{B}/\mathcal{N}}[k+1] - {}^{\mathcal{N}}\mathcal{O}_{\mathcal{B}/\mathcal{N}}[k-1]}{2\Delta t}. \quad (3-44)$$

With the same derivative formula, ${}^{\mathcal{N}}\underline{\dot{\mathbf{q}}}^{\mathcal{B}}$ and ${}^{\mathcal{N}}\underline{\ddot{\mathbf{q}}}^{\mathcal{B}}$ are determined.

Secondly, the foot positions are obtained from the force plate measurements. Since the model has point feet and no-slip is assumed, the foot placement is determined to be the weighted average centre of pressure over a stride. The weights in this average are the GRF magnitudes. With these foot positions, the full state and input trajectories are known of the training data.

Orientation Estimation

The only parameter needed for the orientation estimation is the most prominent frequency in the upper body sway. This could be the stepping frequency or one of its harmonics. In this case, the parameter ϕ_{sway} is heuristically determined from the frequency composition of the measured orientations. The steady-state orientation is determined as the average orientation over the training period.

Dynamical Model

The model-training goal is to find a good initialisation point for a gradient-based nonlinear optimisation function. Knowing the stance leg length, ℓ_ℓ , and the GRF, $\mathbf{R}$, the spring, damper and untensioned length parameters can be found through a quadratic optimisation. The GRF magnitude parallel to the foot direction $\|\mathbf{R}_{//\mathbf{F}}\|$, is expressed as

$$\|\mathbf{R}_{//\mathbf{F}}\| = K(\ell_0 - \ell_\ell) + b\dot{\ell}_\ell. \quad (3-45)$$

Expanding and reordering yields the equation

$$K\ell_0 - K\ell_\ell + b\dot{\ell}_\ell - \|\mathbf{R}_{//\mathbf{F}}\| = 0. \quad (3-46)$$

Stacking this equation over time and taking the squared norm gives the optimisation problem

$$\min_{K,b,\ell_0} (K\ell_0\mathbf{1} - K\ell_\ell + b\dot{\ell}_\ell - \|\mathbf{R}_{//\mathbf{F}}\|)^T (K\ell_0\mathbf{1} - K\ell_\ell + b\dot{\ell}_\ell - \|\mathbf{R}_{//\mathbf{F}}\|). \quad (3-47)$$

Applying the substitution $\kappa = K\ell_0$ yields

$$\begin{aligned} & \min_{K,b,\ell_0} (\kappa\mathbf{1} - K\ell_\ell + b\dot{\ell}_\ell - \|\mathbf{R}_{//\mathbf{F}}\|)^T (\kappa\mathbf{1} - K\ell_\ell + b\dot{\ell}_\ell - \|\mathbf{R}_{//\mathbf{F}}\|) \\ & \iff \min_{K,b,\ell_0} \kappa^2\mathbf{1}^T\mathbf{1} - \kappa K\mathbf{1}^T\ell_\ell + \kappa b\mathbf{1}^T\dot{\ell}_\ell - \kappa\mathbf{1}^T\|\mathbf{R}_{//\mathbf{F}}\| - \kappa K\ell_\ell^T\mathbf{1} + K^2\ell_\ell^T\ell_\ell - Kb\ell_\ell^T\dot{\ell}_\ell \\ & \quad + K\ell_\ell^T\|\mathbf{R}_{//\mathbf{F}}\| + \kappa b\dot{\ell}_\ell^T\mathbf{1} - Kb\dot{\ell}_\ell^T\ell_\ell + b^2\dot{\ell}_\ell^T\dot{\ell}_\ell - b\dot{\ell}_\ell^T\|\mathbf{R}_{//\mathbf{F}}\| \\ & \quad - \kappa\|\mathbf{R}_{//\mathbf{F}}\|^T\mathbf{1} + K\|\mathbf{R}_{//\mathbf{F}}\|^T\ell_\ell - b\|\mathbf{R}_{//\mathbf{F}}\|^T\dot{\ell}_\ell + \|\mathbf{R}_{//\mathbf{F}}\|^T\|\mathbf{R}_{//\mathbf{F}}\|. \end{aligned} \quad (3-48)$$

Substituting $\varsigma = \begin{pmatrix} \kappa \\ K \\ b \end{pmatrix}$ allows for the matrix notation

$$\begin{aligned} & \min_{\varsigma} \frac{1}{2}\varsigma^T \begin{pmatrix} 2\mathbf{1}^T\mathbf{1} & -\mathbf{1}^T\ell_\ell - \ell_\ell^T\mathbf{1} & \mathbf{1}^T\dot{\ell}_\ell + \dot{\ell}_\ell^T\mathbf{1} \\ -\mathbf{1}^T\ell_\ell - \ell_\ell^T\mathbf{1} & 2\ell_\ell^T\ell_\ell & -\ell_\ell^T\dot{\ell}_\ell - \dot{\ell}_\ell^T\ell_\ell \\ \mathbf{1}^T\dot{\ell}_\ell + \dot{\ell}_\ell^T\mathbf{1} & -\ell_\ell^T\dot{\ell}_\ell - \dot{\ell}_\ell^T\ell_\ell & 2\dot{\ell}_\ell^T\dot{\ell}_\ell \end{pmatrix} \varsigma \\ & \quad + \begin{pmatrix} -\mathbf{1}^T\|\mathbf{R}_{//\mathbf{F}}\| - \|\mathbf{R}_{//\mathbf{F}}\|^T\mathbf{1} \\ \ell_\ell^T\|\mathbf{R}_{//\mathbf{F}}\| + \|\mathbf{R}_{//\mathbf{F}}\|^T\ell_\ell \\ \dot{\ell}_\ell^T\|\mathbf{R}_{//\mathbf{F}}\| - \|\mathbf{R}_{//\mathbf{F}}\|^T\dot{\ell}_\ell \end{pmatrix}^T \varsigma + (\|\mathbf{R}_{//\mathbf{F}}\|^T\|\mathbf{R}_{//\mathbf{F}}\|) \\ & \iff \min_{\varsigma} \frac{1}{2}\varsigma^T \begin{pmatrix} 2\mathbf{1}^T\mathbf{1} & -2\mathbf{1}^T\ell_\ell & 2\mathbf{1}^T\dot{\ell}_\ell \\ -2\mathbf{1}^T\ell_\ell & 2\ell_\ell^T\ell_\ell & -2\ell_\ell^T\dot{\ell}_\ell \\ 2\mathbf{1}^T\dot{\ell}_\ell & -2\ell_\ell^T\dot{\ell}_\ell & 2\dot{\ell}_\ell^T\dot{\ell}_\ell \end{pmatrix} \varsigma \\ & \quad + \begin{pmatrix} -2\mathbf{1}^T\|\mathbf{R}_{//\mathbf{F}}\| \\ 2\ell_\ell^T\|\mathbf{R}_{//\mathbf{F}}\| \\ -2\dot{\ell}_\ell^T\|\mathbf{R}_{//\mathbf{F}}\| \end{pmatrix}^T \varsigma + (\|\mathbf{R}_{//\mathbf{F}}\|^T\|\mathbf{R}_{//\mathbf{F}}\|). \end{aligned} \quad (3-49)$$

This is the standard quadratic programming problem, thus the optimal leg constants vector, $\boldsymbol{\varsigma}^*$, is found to be

$$\boldsymbol{\varsigma}^* = \begin{pmatrix} 2\mathbf{1}^T \mathbf{1} & -2\mathbf{1}^T \ell_\ell & 2\mathbf{1}^T \dot{\ell}_\ell \\ -2\mathbf{1}^T \ell_\ell & 2\ell_\ell^T \ell_\ell & -2\ell_\ell^T \dot{\ell}_\ell \\ 2\mathbf{1}^T \dot{\ell}_\ell & -2\ell_\ell^T \dot{\ell}_\ell & 2\dot{\ell}_\ell^T \dot{\ell}_\ell \end{pmatrix}^{-1} \begin{pmatrix} 2\mathbf{1}^T \|\mathbf{R}_{//\mathbf{F}}\| \\ -2\ell_\ell^T \|\mathbf{R}_{//\mathbf{F}}\| \\ 2\dot{\ell}_\ell^T \|\mathbf{R}_{//\mathbf{F}}\| \end{pmatrix}. \quad (3-50)$$

In the quadratic optimisation, κ and K are bound to be positive values, hence Equation 3-50 is only true if these bounds remain inactive.

For the final optimisation, $\boldsymbol{\varsigma}^*$ will be used as the initialisation point. The bounds for all parameters are given in Table 3-III. The objective function is defined as the weighted square error between the measured state trajectory and the modelled state trajectory, when the model state is corrected to the measured state at every heel strike. The weight is a user-defined parameter and indicates how important accurate modelling of the components of $\dot{\mathcal{O}}_{\mathcal{B}/\mathcal{N}}$ are relative to each other. The nonlinear, gradient-based optimisation function `fmincon` in MATLAB is used to retrieve the final parameters. In this last step, perfect input is not assumed. The measured feet positions are still used, but the velocity error is integrated and added to the foot displacement. This puts a slight preference for the early phase model fit, since these errors have a longer effect and incur a greater cost relative to the model fit at the end of the stance phase.

Foot Placement Estimator

The FPE parameters are calculated through comparing uncorrected and measured foot placements. The measured placements have already been derived from the force plate measurements. The heel strike leg length, $\ell_{\max}$, is defined as the maximum measured leg length, such that

$$\ell_{\max} = \max \|\mathbf{F}[k]\|. \quad (3-51)$$

The foot placements are stored in $\boldsymbol{\xi}_0$ and $\boldsymbol{\xi}_{\text{meas}}$, both in $\mathbb{R}^{N \times 3}$, for the uncorrected and measured heel strike positions. Then the step length correction is determined as

$$\zeta_L = \text{avg}(\boldsymbol{\xi}_{\text{meas},x} - \boldsymbol{\xi}_{0,x}). \quad (3-52)$$

The step width correction is determined similarly as

$$\zeta_W = \text{avg}(\boldsymbol{\xi}_{\text{meas},y} \oslash \boldsymbol{\xi}_{0,y}). \quad (3-53)$$

Fall Detection

The parameters for $\mathfrak{B}$ can be determined in two manners. The first option is to determine $\mathfrak{B}_x$ and $\mathfrak{B}_y$ heuristically, based on measured data of healthy gait. The second option is measuring a subject's healthy gait at different walking velocities and retrieving the maximum posterior excursions of $\mathbf{x}_{\text{COM}}$. Performing a linear regression on the retrieved data points yields a linear relation between the walking velocity and $\mathfrak{B}_x$.

3-7 Experiment Description

The functionality of the detection scheme is verified in two manners. Firstly, the state estimation is evaluated on the dataset published by van der Zee et al. [65]. This dataset consists of a full-body

marker set and a force plate sensor per foot, measured during treadmill walking. It includes 10 distinct subjects, who performed 33 walking trials each, with varying speed, step length, step frequency, and step width. To demonstrate the state estimation, the data of one example participant (21 years, 57.3 kg, 1.64 m) is used when walking at 1.6 ms^{-1} and all other trial parameters are self-selected (subject #2, trial #11). The data is 60 s long and measured at 120 Hz. This set is split up into training and verification data, and will be referred to as the Van der Zee [65] (VdZ) dataset. This subject is relatively short and lightweight, walking slightly fast considering the target group consisting of the elderly. The reason this subject is chosen is to make the test less advantageous to the algorithm, since a shorter person creates a shorter arm between the ground contact and sensor, resulting in smaller acceleration measurement magnitudes. The faster walking speed results in faster changes in velocity and orientation, making them more difficult to estimate.

Secondly, the simulation environment OpenSim is used to obtain measurements of a fall, since no real-life measurements are openly available [66, 67]. A secondary optimisation tool, SCONE [68], is used to generate realistic gait with the humanoid model, H0918M_osim4 [55].¹ This model is planar, meaning that there is no lateral movement possible. Two simulations are performed at 100 Hz to gather both healthy training data and measurements of a fall. A situational sketch of the walking and falling is given in Figure 3-5. The gait is generated by optimising the reflex-controller parameters presented by Geyer and Herr [70] for 40 generations, with an objective reflecting steady-state gait without falling. The same optimised muscle activations are used to drive the model in the tripping scenario.

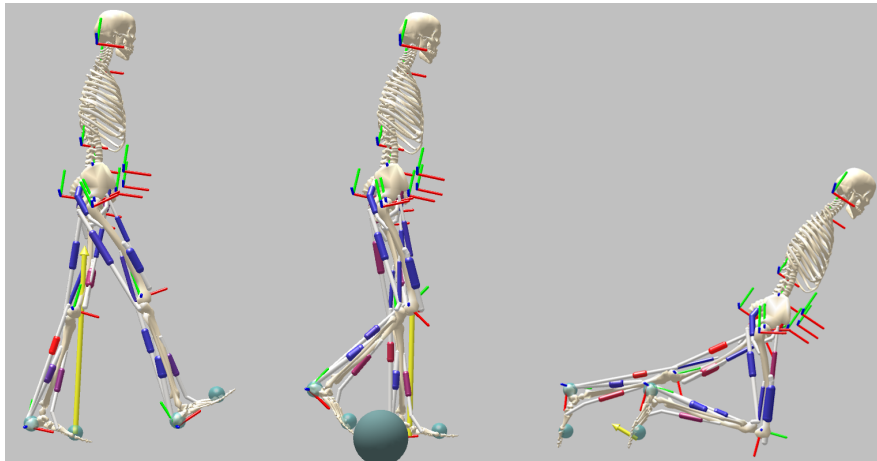


Figure 3-5: Situation sketch for gathered falling data. Retrieved from SCONE. From left to right is demonstrated: normal gait, contact instance, and impact instance.

¹In-file model description: “Planar model with 9 degrees-of-freedom and 18 muscles. Adapted for SCONE from Gait2392 by Thomas Geijtenbeek.” Gait2392 refers to [69].

Chapter 4

Results

4-1 Model Validation

4-1-1 Training Results

The verification of the models requires the training of the system parameters on steady-state gait. The training of each relevant part of the detector is elaborated upon to motivate and show what the process entails and what quantities and qualities may affect the training procedure. The length of the training dataset is 15 s for the Van der Zee [65] (VdZ) dataset. This training length is relatively short, but due to the limited amount of available data and the need to allow for a settling time during verification, this is the remaining available data.

Orientation Estimation

The choice of ϕ_{sway} is based on the frequency composition of the training data. The spectrum of the VdZ training data is graphed in Figure 4-1. The value at 1.2 Hz is annotated and because of the flatness of the middle two peaks, ϕ_{sway} is chosen to be 1.16 Hz.

This choice for ϕ_{sway} is the only parameter necessary to estimate the upper-body sway. These results are plotted and compared to the ground truth in Figure 4-2. The root mean square errors and variances accounted for are tabulated in Table 4-I. These performance measures are calculated as [71]

$$\text{RMSE}(\mathbf{x}, \hat{\mathbf{x}}) = \sqrt{\frac{\|\mathbf{x} - \hat{\mathbf{x}}\|^2}{N}}, \quad \mathbf{x}, \hat{\mathbf{x}} \in \mathbb{R}^N \quad (4-1)$$

and

$$\text{VAF}(\mathbf{x}, \hat{\mathbf{x}}) = \max \left(0, \min \left(1, 1 - \frac{\|\mathbf{x} - \hat{\mathbf{x}} - \text{avg}(\mathbf{x} - \hat{\mathbf{x}})\|^2}{\|\mathbf{x} - \text{avg}(\mathbf{x})\|^2} \right) \right). \quad (4-2)$$

The value for ϕ_{sway} obtained from the VdZ training is also used for the orientation estimation in the OpenSim environment. This is because the model used in this simulation focuses on the lower half of the body, but the upper body is of very low complexity.

Table 4-I: Performance measures of models. The results of the trained models are compared to the measurements through the Root Mean Square Error (RMSE) and the Variance Accounted For (VAF). Low RMSE and high VAF indicate good performance. Subscripts indicate the direction or element of the vector. Fields in the OpenSim columns are empty, because there was no training and the model is planar. The first 15 s of the OpenSim verification estimates are not considered.

		Van der Zee Training	Van der Zee Verification	OpenSim Verification	Unit
RMSE	$(\dot{\mathcal{O}}_{B/N})_x$	0.076	0.177	0.189	m s^{-1}
	$(\dot{\mathcal{O}}_{B/N})_y$	0.094	0.213		m s^{-1}
	$(\dot{\mathcal{O}}_{B/N})_z$	0.097	0.210	0.271	m s^{-1}
	$\mathcal{N}\bar{\mathbf{q}}_1^B$	0.001	0.001	0.000	-
	$\mathcal{N}\bar{\mathbf{q}}_2^B$	0.007	0.009	0.000	-
	$\mathcal{N}\bar{\mathbf{q}}_3^B$	0.014	0.013	0.007	-
	$\mathcal{N}\bar{\mathbf{q}}_4^B$	0.012	0.011	0.000	-
	$\mathcal{N}\dot{\bar{\mathbf{q}}}_1^B$	0.006	0.006	0.004	s^{-1}
	$\mathcal{N}\dot{\bar{\mathbf{q}}}_2^B$	0.069	0.069	0.002	s^{-1}
	$\mathcal{N}\dot{\bar{\mathbf{q}}}_3^B$	0.050	0.048	0.089	s^{-1}
	$\mathcal{N}\dot{\bar{\mathbf{q}}}_4^B$	0.117	0.115	0.002	s^{-1}
	$\mathcal{N}\ddot{\bar{\mathbf{q}}}_1^B$	0.256	0.262	0.175	s^{-2}
	$\mathcal{N}\ddot{\bar{\mathbf{q}}}_2^B$	1.517	1.437	0.035	s^{-2}
	$\mathcal{N}\ddot{\bar{\mathbf{q}}}_3^B$	1.985	1.931	4.024	s^{-2}
	$\mathcal{N}\ddot{\bar{\mathbf{q}}}_4^B$	5.356	4.995	0.037	s^{-2}
VAF	$(\dot{\mathcal{O}}_{B/N})_x$	0.996	0.984	0.975	
	$(\dot{\mathcal{O}}_{B/N})_y$	0.983	0.911		
	$(\dot{\mathcal{O}}_{B/N})_z$	0.975	0.905	0.848	
	$\mathcal{N}\bar{\mathbf{q}}_1^B$	1.000	1.000	1.000	
	$\mathcal{N}\bar{\mathbf{q}}_2^B$	0.999	0.999	1.000	
	$\mathcal{N}\bar{\mathbf{q}}_3^B$	0.998	0.997	1.000	
	$\mathcal{N}\bar{\mathbf{q}}_4^B$	0.999	0.998	1.000	
	$\mathcal{N}\dot{\bar{\mathbf{q}}}_1^B$	0.723	0.725	0.883	
	$\mathcal{N}\dot{\bar{\mathbf{q}}}_2^B$	0.485	0.508	0.875	
	$\mathcal{N}\dot{\bar{\mathbf{q}}}_3^B$	0.795	0.824	0.878	
	$\mathcal{N}\dot{\bar{\mathbf{q}}}_4^B$	0.810	0.823	0.874	
	$\mathcal{N}\ddot{\bar{\mathbf{q}}}_1^B$	0.000	0.000	0.216	
	$\mathcal{N}\ddot{\bar{\mathbf{q}}}_2^B$	0.128	0.164	0.147	
	$\mathcal{N}\ddot{\bar{\mathbf{q}}}_3^B$	0.000	0.019	0.189	
	$\mathcal{N}\ddot{\bar{\mathbf{q}}}_4^B$	0.111	0.147	0.173	

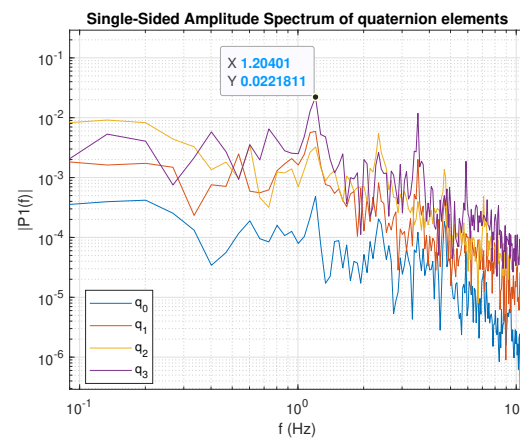


Figure 4-1: Frequency contents of the upper-body sway quaternion, analysed from the VdZ dataset.

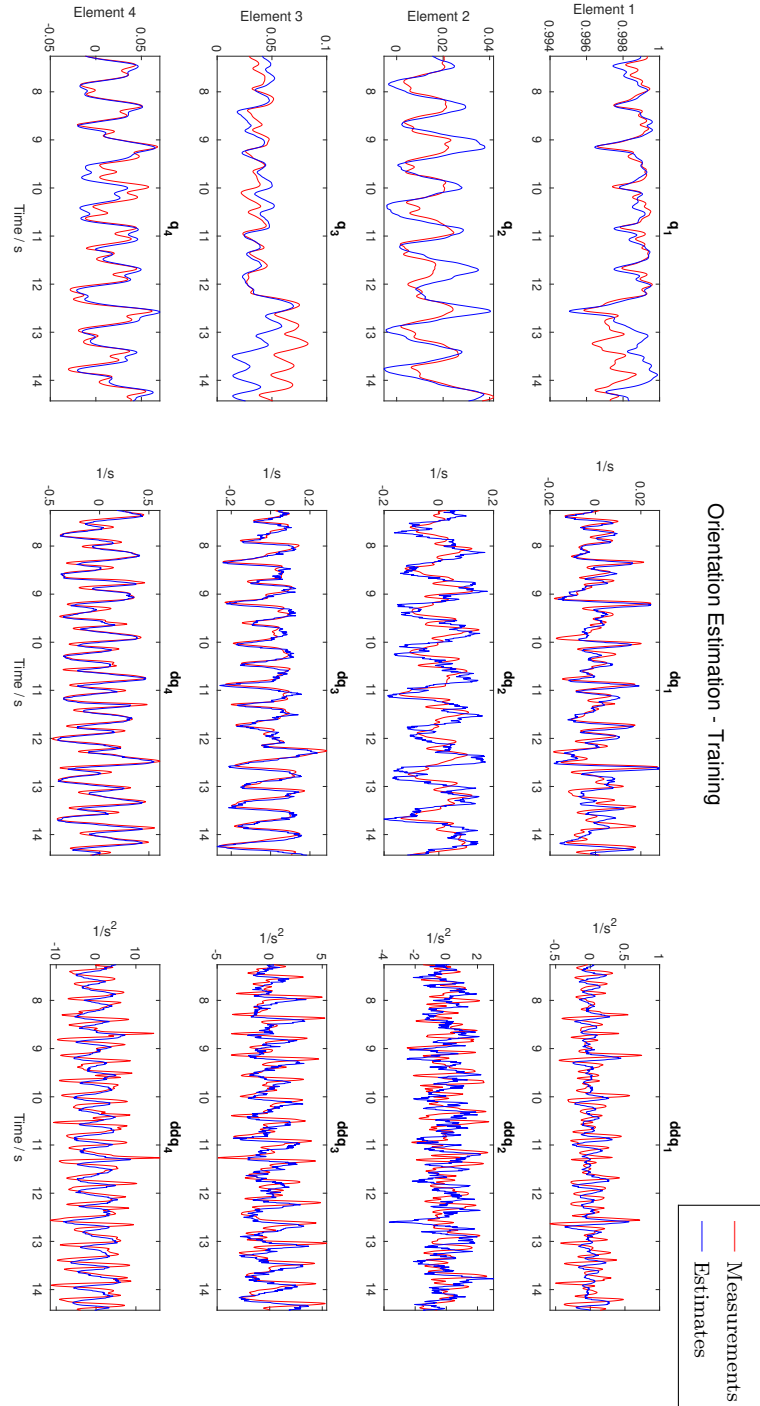


Figure 4-2: Training performance of the orientation estimator on the VdZ dataset. This is a representative window, the complete plot is provided in Appendix C, Figure C-1.

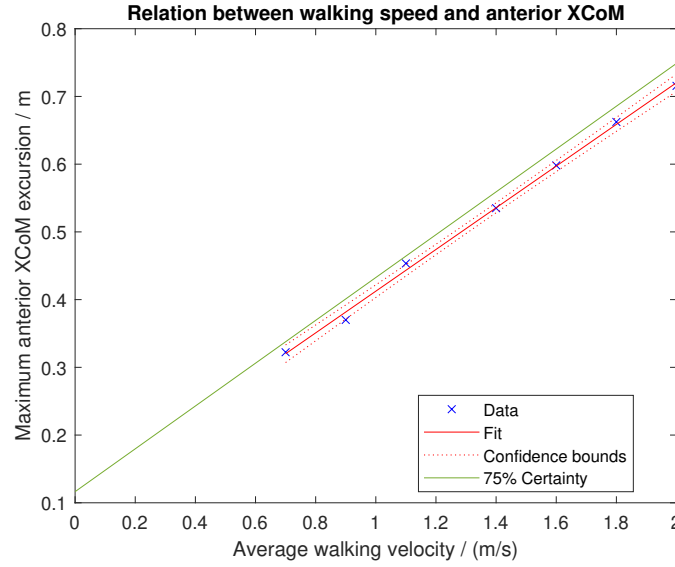


Figure 4-3: Linear fit between average walking velocity and anterior XCoM excursions in steady-state gait, analysed from the VdZ dataset.

Base of Support

The VdZ dataset contains multiple walking speeds with otherwise self-selected parameters and this enables performing the linear regression on $\mathfrak{B}_x$. The results of the linear regression are plotted in Figure 4-3. The obtained upper bound of the anterior excursion with a 75% confidence interval is

$$\mathfrak{B}_x = 0.116 + 0.316v_0 \quad (4-3)$$

for the subject in the VdZ data. Thus, the found value is

$$v_0 = 1.6 \text{ m s}^{-1} \rightarrow \mathfrak{B}_x = 0.622 \text{ m}. \quad (4-4)$$

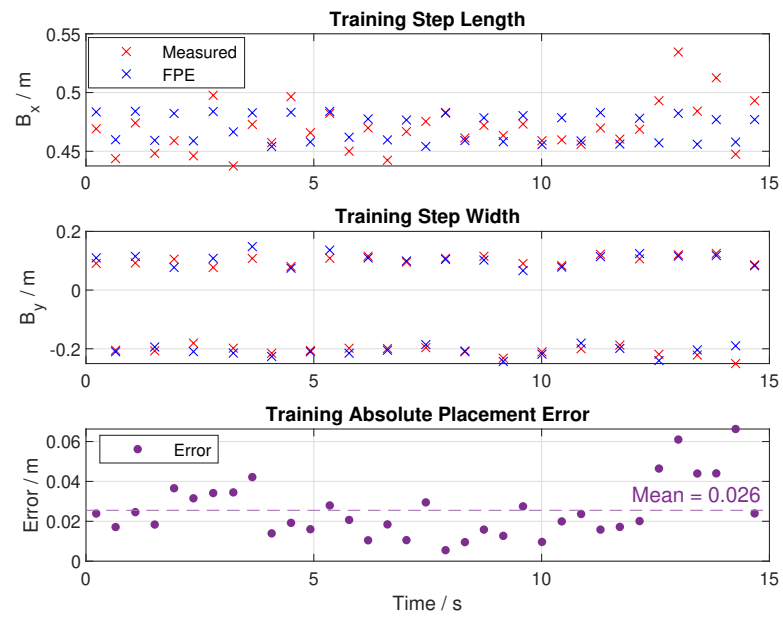
Foot Placement Estimator

The parameters for the Foot Placement Estimator (FPE) are found through comparing the untuned values with the measured values. Following the training procedure for both training datasets yields the estimates as plotted in Figure 4-4. The mean absolute placement errors after training are 0.026 m and 0.015 m for the VdZ and OpenSim sets, respectively.

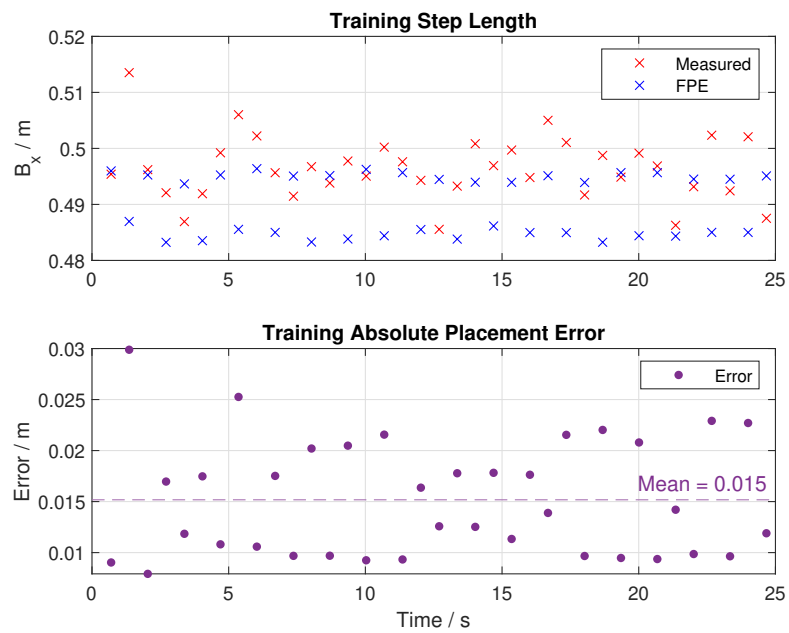
State Estimation

The model is trained through a combination of a quadratic programming problem on the Ground Reaction Force (GRF), followed by a nonlinear optimisation fitting of the model state trajectories to the measured trajectories. The errors in all states are weighed equally for this training. Executing this training procedure yields values for the spring-damper constants of both legs. The results of this

optimisation are plotted in Figure 4-5 for the VdZ data. This training method is not suitable for the planar nature of the OpenSim model, therefore the trained leg parameters obtained from the VdZ subject will also be used for the OpenSim dynamical model.



(a) Foot placement estimator training fit on VdZ dataset



(b) Foot placement estimator training fit on OpenSim dataset

Figure 4-4: Training performance of the foot placement estimator given heel strike instances

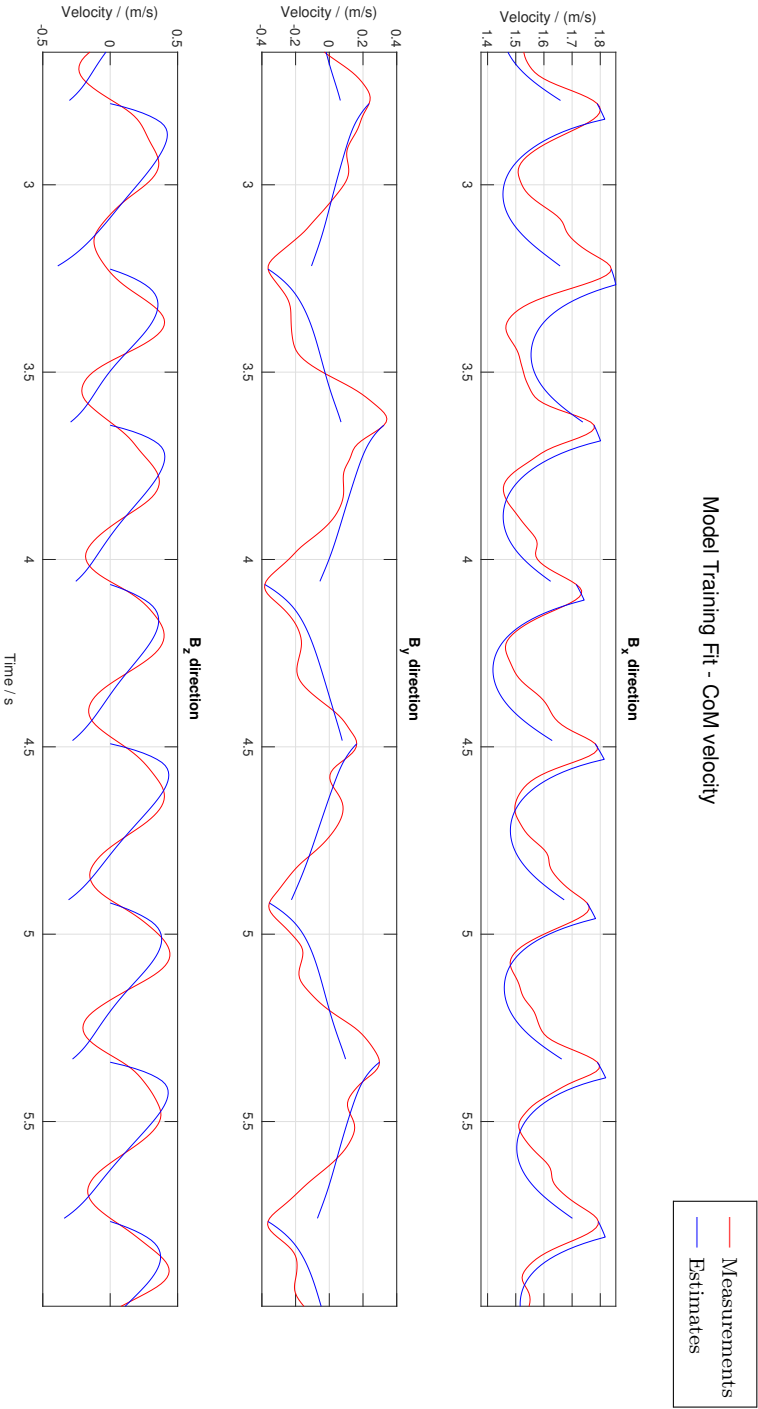


Figure 4-5: Trained model fit overlaid on measured state trajectories (CoM velocity) of the VdZ dataset. Top to bottom: x , y , and z velocities. This is a representative window, the complete plot is provided in Appendix C, Figure C-2.

Table 4-II: Obtained system parameters after training

Parameter	VdZ	OpenSim	Unit
m	57.3	74.5	kg
K_L	$4.51 \cdot 10^3$	$4.51 \cdot 10^3$	N m^{-1}
b_L	$-1.09 \cdot 10^3$	$-1.09 \cdot 10^3$	N s m^{-1}
ℓ_{0L}	1.078	1.078	m
K_R	$7.07 \cdot 10^3$	$7.07 \cdot 10^3$	N m^{-1}
b_R	$-1.57 \cdot 10^3$	$-1.57 \cdot 10^3$	N s m^{-1}
ℓ_{0R}	1.034	1.034	m
$\zeta_{W,L}$	0.458		s
$\zeta_{L,L}$	0.305	0.371	m
$\zeta_{W,R}$	0.591		s
$\zeta_{L,R}$	0.283	0.361	m
$\ell_{\max}$	1.038	1.092	m
ϕ_{sway}	1.16	1.16	Hz
$\mathfrak{B}_x$	0.622	0.550	m
$\mathfrak{B}_y$	0.178		m

All trained parameters are tabulated in Table 4-II. Completing the training procedure takes roughly 80s for the VdZ training set, which contains 15s of data. Table 4-I lists the training performance in terms of root mean square error and variance accounted for.

4-1-2 System Verification

Different data than the training set is employed to verify the state estimation algorithm using the entire system. After manual tuning, the values of the process noise covariance matrix, Q_{KF} , the measurement noise covariance matrix, R_{KF} , and the initial estimate covariance matrix, $P_{0,\text{KF}}$, are found as tabulated in Table 4-III. Using these parameters and inputting the Inertial Measurement Unit (IMU) measurements into the system, yields the orientation and Centre of Mass (CoM) velocity estimates of both verification sets. The IMU measurements for the VdZ set are emulated by twice deriving the marker positions to find the necessary acceleration of the point ${}^B\mathbf{S}$. The position of the sensor, ${}^B\mathbf{S}$, for the VdZ data is chosen to be

$${}^B\mathbf{S} = \begin{pmatrix} -0.07 \\ 0 \\ 0.042 \end{pmatrix}, \quad (4-5)$$

where 0.042 is 10% of the torso height, which is a suitable value for attaching the sensor to a waist belt. The IMU measurements of the OpenSim simulations are extracted directly from OpenSim. The position of the sensor is not known exactly, but is estimated from Figure 3-2 to be

$${}^B\mathbf{S} = \begin{pmatrix} -0.07 \\ 0 \\ 0.154 \end{pmatrix}, \quad (4-6)$$

where 0.154 is 30% of the torso height. For a more realistic experiment, Gaussian noise of zero mean and a variance of 10^{-3} and 10^{-2} is added to the accelerometer and gyroscope measurements, respectively for the VdZ dataset, and noise with a variance of 10^{-4} is added to all six measurement

Table 4-III: Obtained filter parameters after tuning

Parameter	VdZ	OpenSim
Q_{KF}	$10^{-3}E_3$	$10^{-3}E_3$
R_{KF}	$10^{-3}E_6$	$10^{-4}E_6$
$P_{0,KF}$	$10^{-2}E_3$	$10^{-2}E_3$
κ_x	$5 \cdot 10^{-2}$	$8 \cdot 10^{-2}$
κ_y	10^{-3}	

channels of the OpenSim data. The estimates given the verification measurements are plotted in Figure 4-6 through 4-9. Table 4-I gives the performance measures of the models.

Assessing the observability of the model analytically is difficult since the expressions for f and g are complex. However, the observability of the pair of Jacobians of f and g with respect to $\mathbf{x}$, (F_x, G_x) , can be assessed as the linearised system at every time step. This analysis of the VdZ verification data determines that at every time step the linearised system has one observable mode and two unobservable and unstable modes. This implies that the system is undetectable. However, at every time step the unstable unobservable modes are numerically found to have eigenvalues between 1 and $1 + 10^{-14}$. This means that the modes are either marginally stable or slightly unstable, depending on if the distance outside the unit circle is a numerical error.

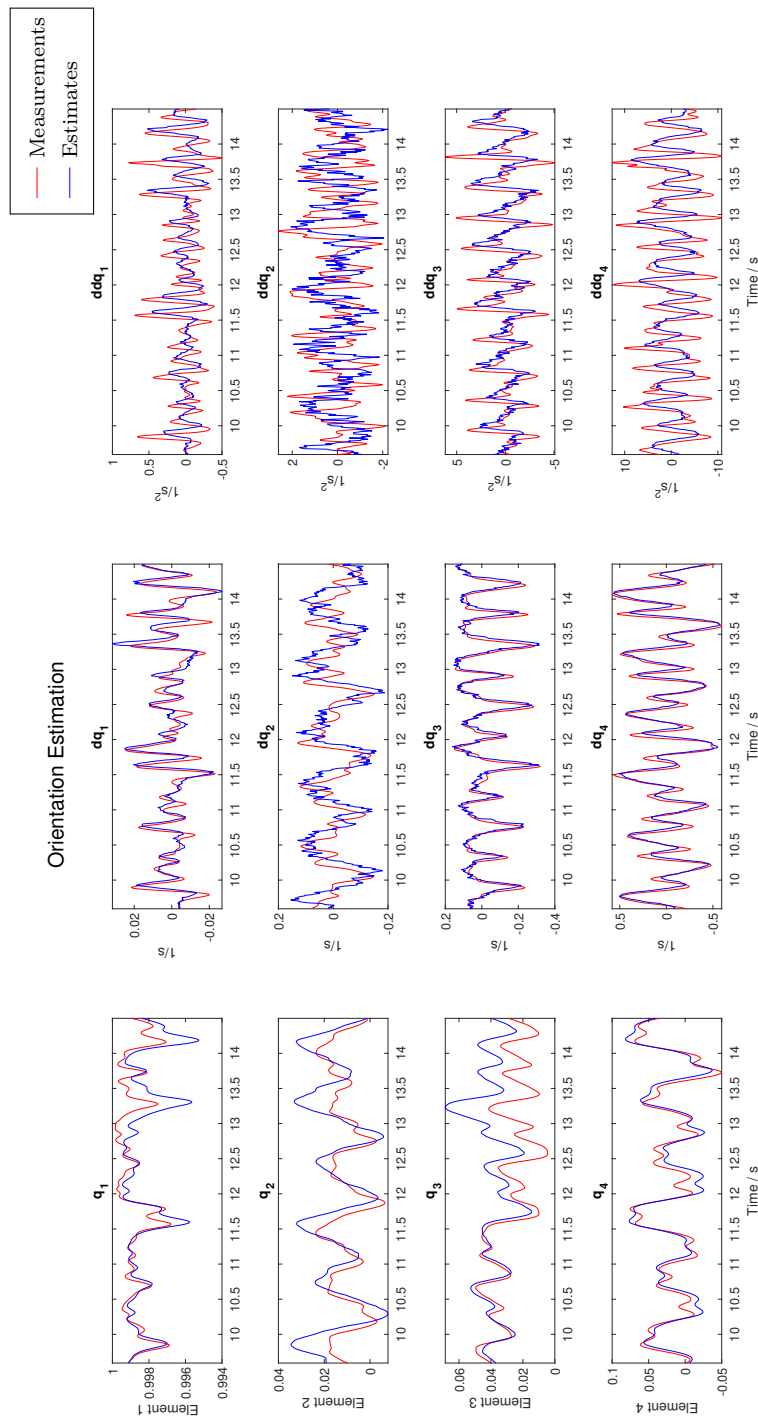


Figure 4-6: Orientation estimator performance on VdZ verification data. This is a representative window, the complete plot is provided in Appendix C, Figure C-3.

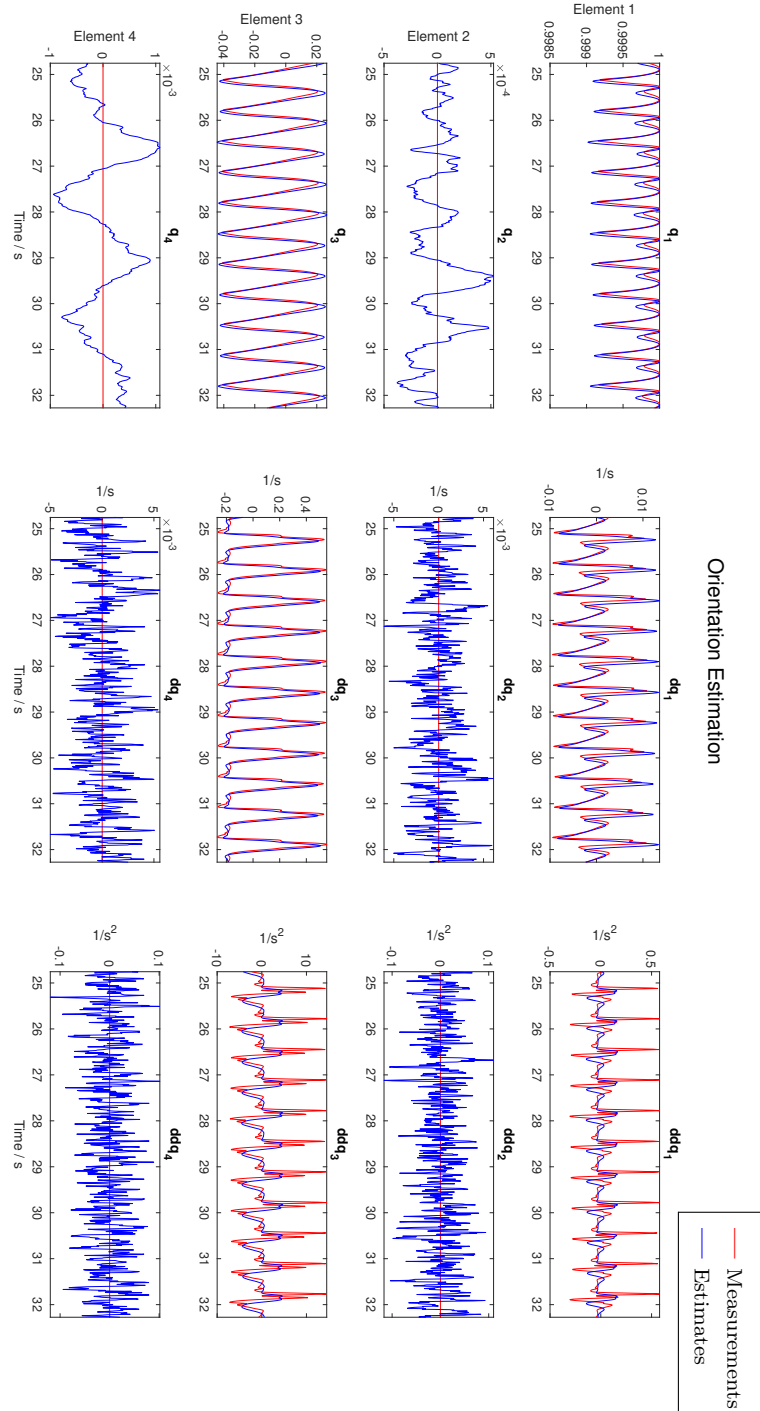


Figure 4-7: Orientation estimator performance on OpenSim verification data. Second and fourth elements are zero because of planar simulation model. This is a representative window, the complete plot is provided in Appendix C, Figure C-4.

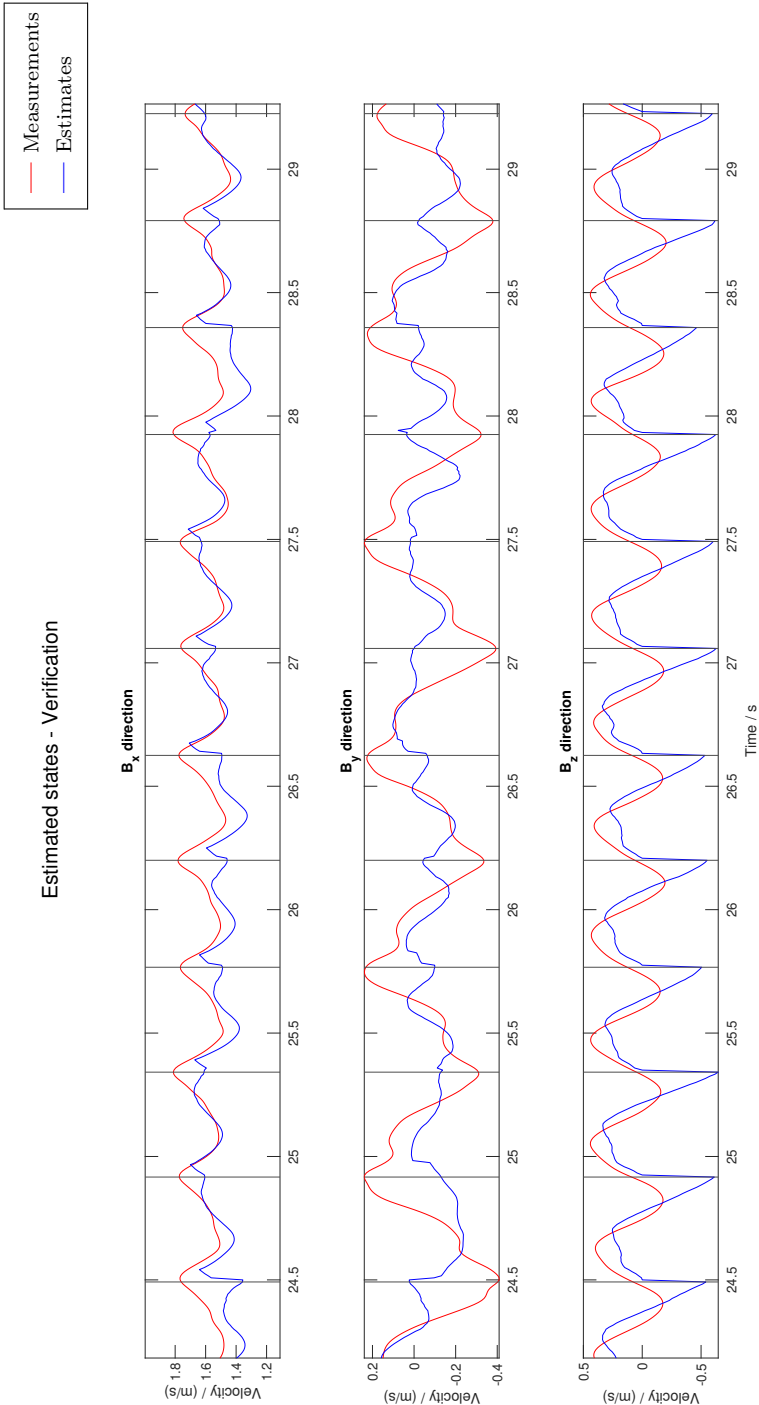


Figure 4-8: State estimation results overlaid on measured trajectories of the VdZ verification dataset. Vertical lines indicate heel strike events. This is a representative window, the complete plot is provided in Appendix C, Figure C-5.

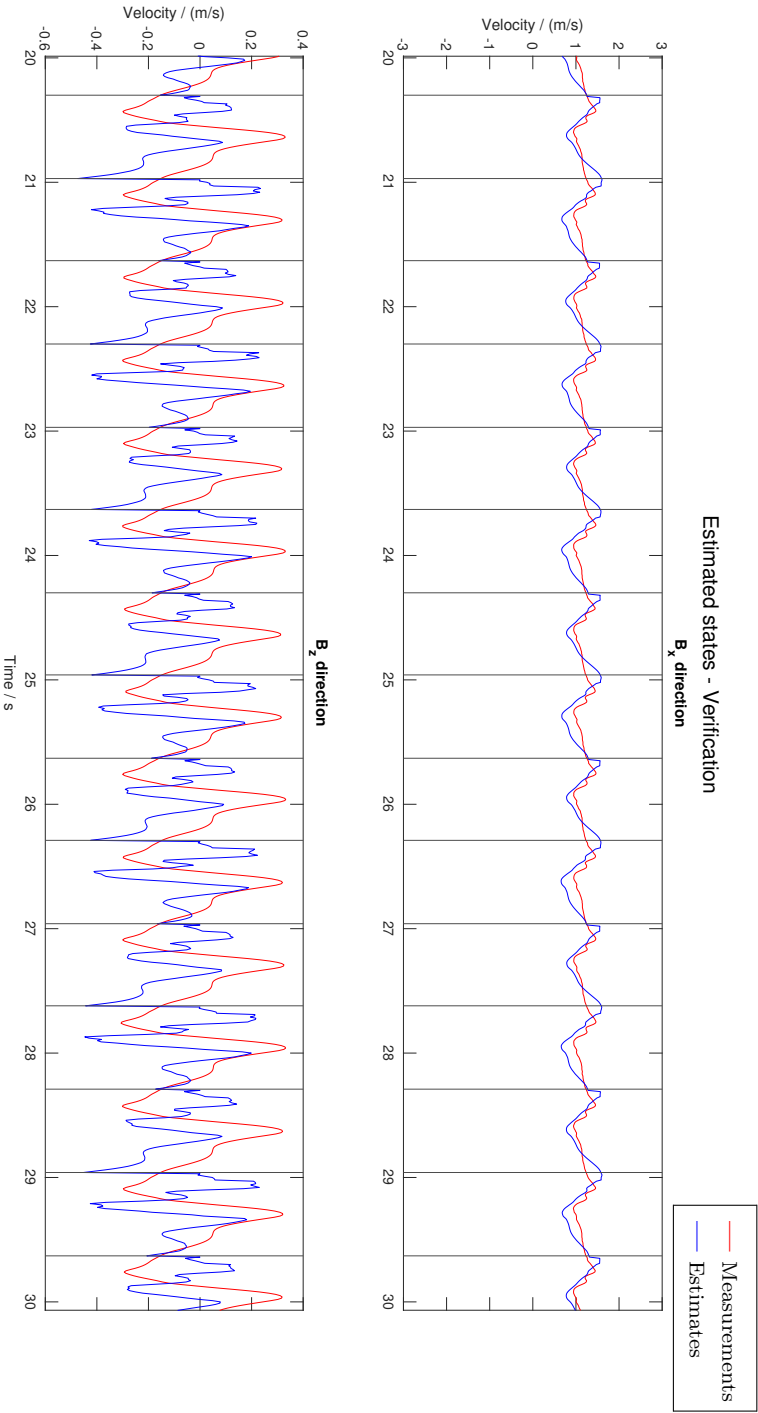


Figure 4-9: State estimation results overlaid on measured trajectories of the OpenSim verification dataset.. Vertical lines indicate heel strike events. This is a representative window, the complete plot is provided in Appendix C, Figure C-6.

The estimated output is also compared to the IMU measurements for both of these estimates. These comparisons are plotted in Figure 4-10.

The foot placement estimates at every heel strike of the VdZ measurements are plotted in Figure 4-11. Since the FPE is calculated from the estimated CoM velocity, its performance is largely dependent on the quality of the velocity estimate at the end of single stance.

The measured Extrapolated Centre of Mass (XCoM) over time is also plotted in Figure 4-12, in order to compare it to the Base of Support (BoS). The estimated XCoM is plotted in Figure 4-13 and it is shown that there are no erroneous detections, since the XCoM does not leave the BoS.

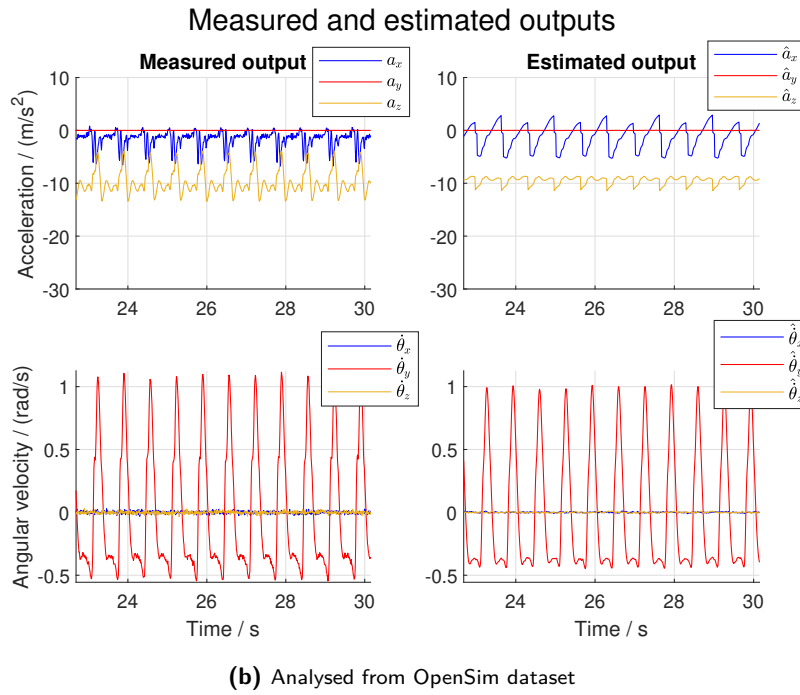
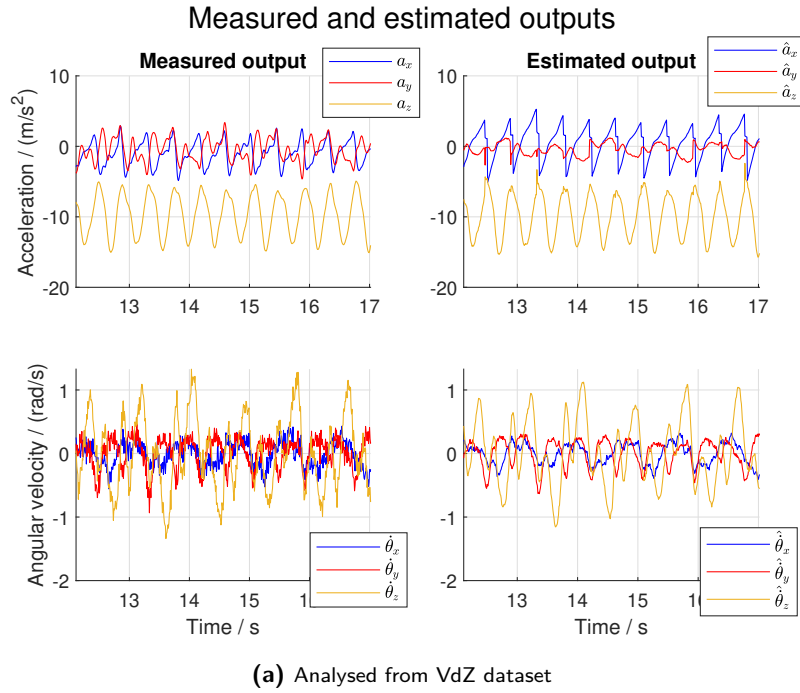


Figure 4-10: Comparison between actual and estimated output measurements. These are representative windows, the complete plots are given in Appendix C, Figure C-7.

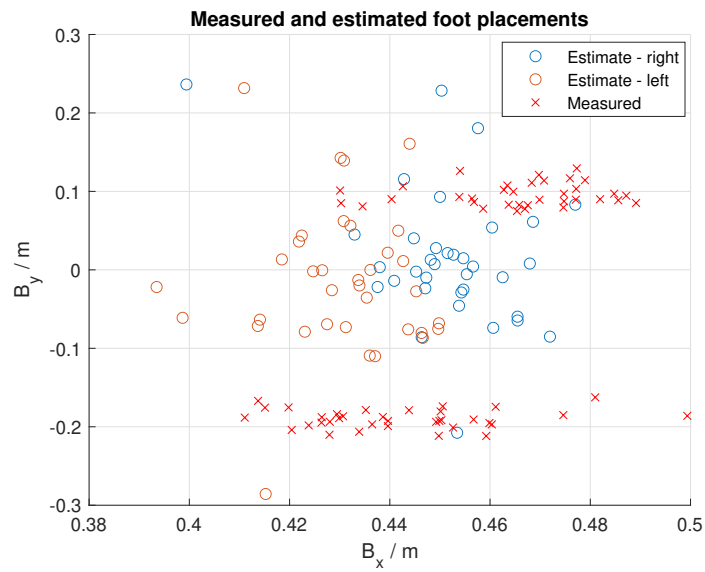


Figure 4-11: FPE performance on the VdZ verification data

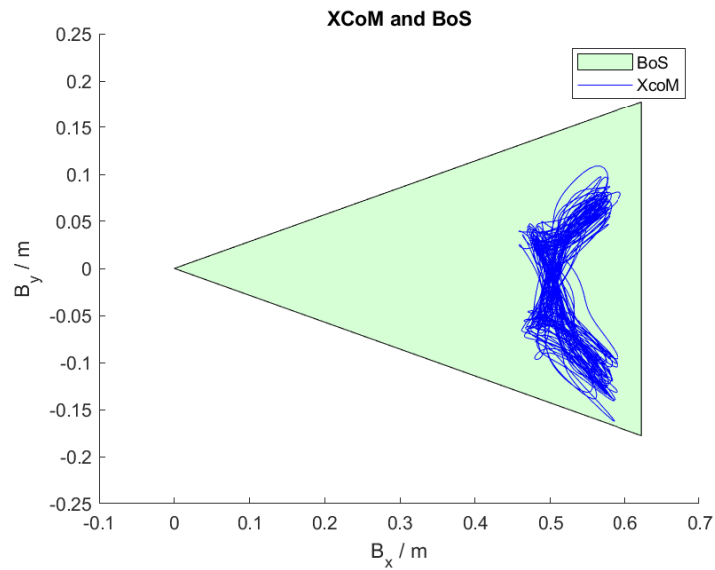


Figure 4-12: Measured extrapolated centre of mass trajectory during normal walking compared to the chosen base of support, analysed from the VdZ verification data.

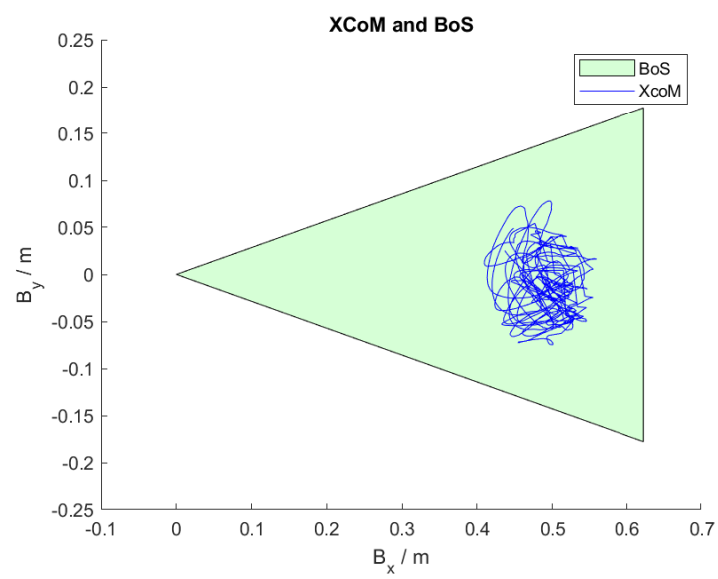


Figure 4-13: Estimated extrapolated centre of mass trajectory obtained from the VdZ verification dataset. The estimates during the settling time are excluded.

4-2 Detector Validation

IMU measurements from a falling scenario, as sketched in Figure 3-5, enable the verification of the entire detector system. The orientation and state estimations are plotted in Figure 4-14 and Figure 4-15. The initial contact with the tripping object is at time $t = 66.32$ s, and the first detection occurs at $t = 66.78$ s. The first contact of the knee with the floor is at $t = 67.46$ s. Therefore, in this case, the detection time is 460 ms and the lead time is 680 ms. The ratio of computation time to analysed time is

$$\frac{\text{Computation time}}{\text{Data time duration}} = \frac{66.4 \text{ s}}{67.4 \text{ s}} \approx 99\% \quad (4-7)$$

on a consumer-grade laptop with a 6-core, 2.60 GHz CPU.

To indicate the margin of detection, the anterior XCoM component is plotted over time in Figure 4-16. The detection threshold is shown as well in Figure 4-16, indicated by the black bar.

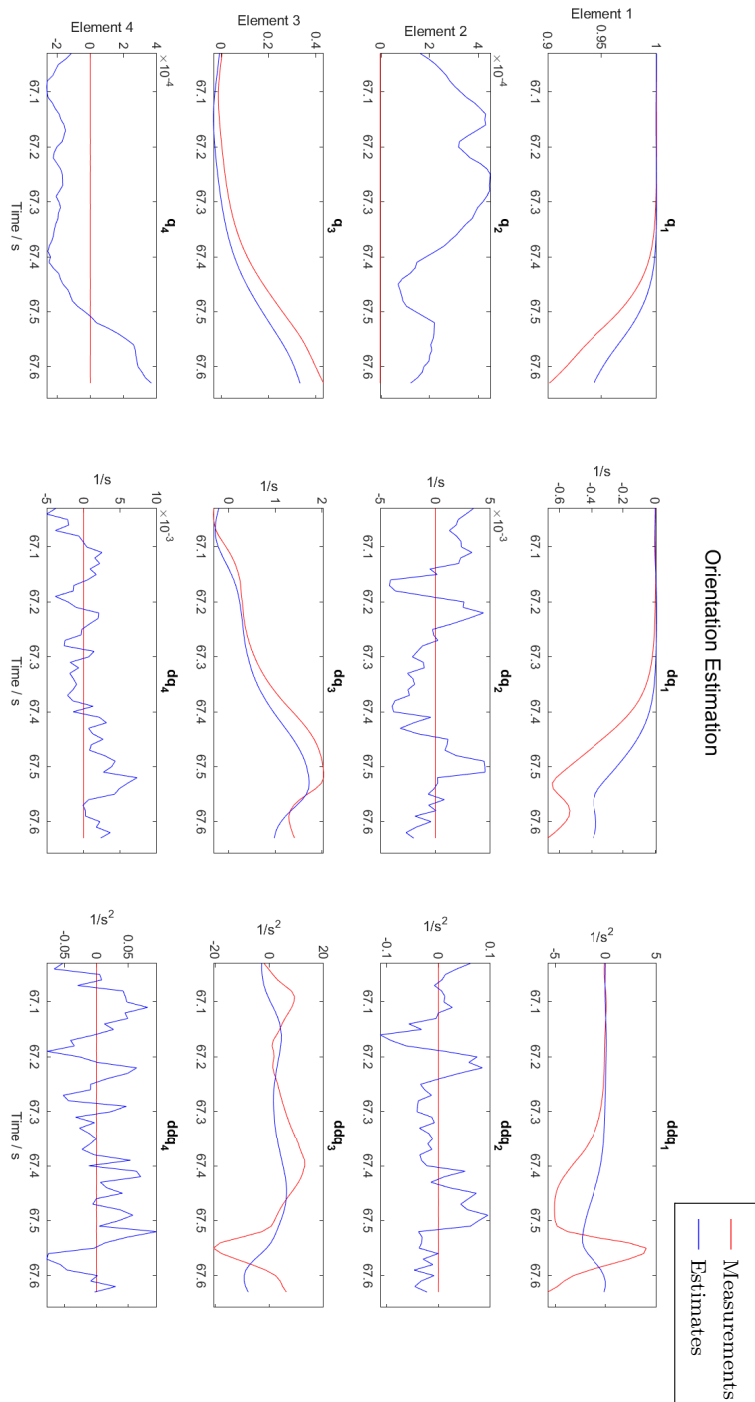


Figure 4-14: Orientation estimation around the time of falling in the OpenSim falling dataset

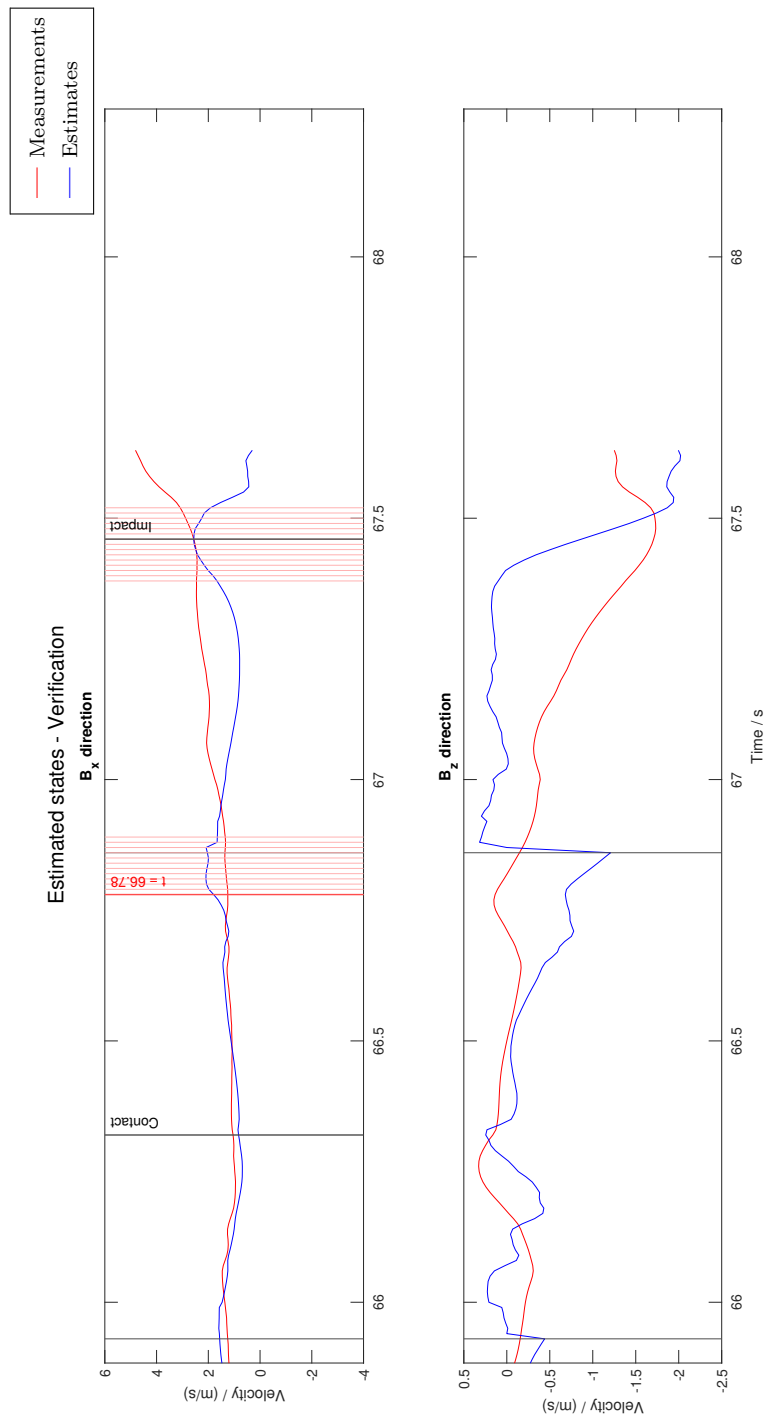


Figure 4-15: Detection of falls on the OpenSim falling dataset. Black vertical lines indicate heel strike events, or the contact or impact instances as annotated. Red vertical lines indicate instances where falling is detected.

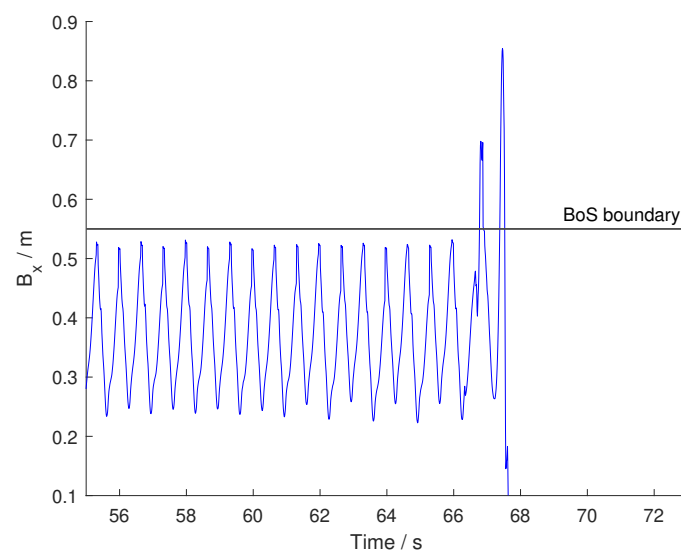


Figure 4-16: XCoM anterior excursion around the time of falling. This is the window around the falling event, the complete plot is provided in Appendix C, Figure C-8.

Chapter 5

Discussion

5-1 State Estimation Performances

As visible in Figure 3-3, all of the signals in the developed state estimator are physically measurable and the estimates are compared to the ground truths as presented in Chapter 4. Firstly, the model was fitted to a specific individual. The training of the parameterised model to this individual required little data (15 s), especially compared to machine learning-based methods. The presented method does, however, require training, whereas in signal-based methods, it is often not necessary to tune to individuals [46, 50]. One advantage of the detection method is that a small marker set is required. This entails that for real-life training procedures, the data collection time with the subject can be short, whereafter the data processing can be left to a non-healthcare professional.

The first estimate made in the detection algorithm is the orientation estimate. As can be seen in Figure 4-2, 4-6, and 4-7, the oscillation Kalman Filter (KF) described in Section 3-3-2 is capable of estimating the upper-body orientation sufficiently with a root mean square error in the order of 10^{-2} , 10^{-2} , and 10^0 for the zeroth, first, and second derivative respectively. The parameterisation of this filter results in low-complexity, intuitive tuning, based on measurable spectra. The filter is computationally more expensive than the often-used direct integration method employed for orientation estimation, but it does not suffer from integration drift. The orientation estimation shows good performance with low root mean square error and high variance accounted for, while the angular acceleration shows the opposite. This is likely due to the low-pass nature of derivators. The single-frequency assumption that holds for the orientation model may not hold for the angular acceleration model, or it may hold for a different frequency.

Using the orientation estimate, the Centre of Mass (CoM) velocity was estimated with the Extended Kalman Filter (EKF). From Figure 4-5 it is inferred that the model can capture the gait dynamics reasonably well with a root mean square error between 0.1 and 0.3 and a variance accounted for of more than 90%. It is also visible that most of the cyclical shape factors of the measured trajectory are also present in the estimated trajectory. Only the last moments of single stance, just before heel strike, are consistently estimated incorrectly in the $\mathcal{B}_z$ direction. The estimate keeps moving downwards, whilst the measurement bends up. This indicates that there is an unmodelled upward force acting on the CoM.

The training procedure for determining the spring-damper leg parameters was not fit for training with the planar data generated from the OpenSim scenario. However, the verification of the model using the Van der Zee [65] (VdZ) leg parameters for the OpenSim dynamical model, yielded comparable results to the training and verification results for the subject it was trained on, as tabulated in Table 4-I. This gives a strong indication of robustness to modelling errors for these parameters and also indicates that different variations of gait by the same subject may be captured by a single model.

The state estimation given the Inertial Measurement Unit (IMU) measurements, shown in Appendix C, Figure C-5 and C-6, have a relatively long settling time of around 20 s and 13 s respectively. After this settling time, the estimate stays near the ground truth, indicating that the filter is capable of accurately estimating the CoM velocity. The modelling error in the $\mathcal{B}_z$ direction at the end of single stance phase is also apparent here, meaning the filter is not able to compensate for this model-mismatch. The estimated outputs shown in Figure 4-10 also indicate that the filter is generating feasible estimates, as the estimated output magnitudes and cycle shape correspond closely to the sensor measurements. This also signifies that the filter is rejecting measurement noise. The fact that the estimates seem to settle around the true mean shows that the EKF and the PI dedrifting block have a balanced interaction. This is despite the fact that the model itself is determined to be undetectable by a small margin. The model shows to have two unstable, unobservable modes, highlighting the necessity for dedrifting two the state estimations.

The estimates around the falling scenario are plotted for the orientation estimation and CoM velocity estimation in Figure 4-14 and 4-16, respectively. These plots show that both estimation algorithms are flexible and fast enough to generate detectable changes in the estimates when not given walking measurements, but falling measurements.

The current orientation estimator is separate from the velocity estimator and it is not built on biomechanical principles. In the developing process of the presented method, the initial starting point was indeed to estimate the velocity and orientation at the same time, based on biomechanical models, such that a single filter would suffice. This necessitates a more complete model than the one presented for the velocity estimation, such that it includes angular dynamics equations. An approach to this more elaborate model was attempted, but abandoned during development due to the increased estimation difficulty. The model is described in Appendix B.

5-2 Foot Placement Estimator Performance

The plot in Figure 4-11 depicts the verification performance of the Foot Placement Estimator (FPE). It shows that the estimated step length is accurate compared to the real placements, but the step width is often too narrow. This could be caused by an interaction with the state estimate, where the lateral velocity magnitude is often underestimated at heel strike. This underestimation of the velocity results in a narrower foot placement estimation, which in turn results in more vertical modelled Ground Reaction Forces (GRFs). The model therefore suggests less lateral acceleration, resulting again in underestimation of the lateral velocity at the next heel strike.

5-3 Detector Performance

The Extrapolated Centre of Mass (XCoM) based detector relies on an accurate and precise estimate of both the XCoM itself and the Base of Support (BoS). The plot in Figure 4-12 shows there is a

delimited region where the XCoM exists during normal walking. The definition of the BoS during walking is difficult to find for dynamic walking. The XCoM method was found to be hardly suitable for stability analysis during walking. However, the triangular choice of BoS used in this algorithm yields no false alarms during steady-state gait, excluding the estimation settling time, in both the verification data and falling data. The detector successfully detected the simulated fall with a lead time of 680 ms. Relating this result to the comparative algorithms plotted in Figure 2-3, it performs better than most other methods in this regard.

These two datasets are however not sufficient to perform quantitative analyses on either the state estimation or the fall detection. To the author's knowledge, there are no publicly available measurement sets containing all the necessary information to perform and test the presented method. Therefore, it would be required to obtain data through a new experiment, where people are intentionally brought to fall in a controlled setting and where their movements are measured.

5-4 Conclusions

In the considered literature, no model-based fall detection methods based on wearable sensors have been found. A proof-of-concept version of such a detector is presented, employing a single IMU placed on the lower back. The detection is determined through the extrapolated centre of mass method, where the CoM velocity is estimated through a combination of upper-body orientation estimation and foot placement estimation. The CoM velocity is estimated with an EKF driven by a gait model consisting of a three-dimensional point mass supported by spring-damper legs.

The presented method is not the first model-based fall detector, since Otanasap and Boonbrahm, and Li et al. have given examples of these methods already [11, 12]. However, it is the first method that has the possibility to function outside of laboratory settings, because it relies solely on wearable sensors. The exception is the training phase, which is necessary to perform in a laboratory setting.

The presented results prove the feasibility of a model-based pre-impact fall detector, using only a single wearable sensor. More specifically, it shows that a model-based approach is not too complex to implement, the estimation is accurate and fast enough to detect falls a relatively long time before impact, and that an individual's gait can be modelled with 12 parameters. Of these 12, only one (ϕ_{sway}) is determined heuristically based on measured data.

The falling simulation yielded a lead time of 680 ms. This shows that for at least this scenario the lead time would be sufficient to trigger a multitude of assistive machines. The detector could therefore be used in practical settings with existing actuators.

The computation time of the detector is found to be roughly equal to the sampling period. Combined with the facts that the implemented code is not optimised and the sampling time is relatively high compared to the dynamics, this shows a strong indication that the detector could run in real-time. These results suggest that the final goal of a functional wearable fall detector is feasible concerning the computation time. More broadly, it shows that model-based approaches for these applications do not always take too long to compute.

5-5 Future Work

A fundamental problem of the presented method lies in the theory of the XCoM. The definition of the BoS is important for the balance analysis of gait. Strictly, this BoS should be the envelope of

foot-ground contact areas when considering static walking. As can be seen in Figure 4-12, it is highly unlikely that the XCoM will reside in this stricter BoS definition and will therefore not work for balance evaluation. It is difficult to find a BoS to differentiate between stable and unstable dynamic walking. The alternative method of N -step capturability is suggested by Hof [38] in personal correspondence. This method relies on the principle of estimating if the body state resides in a subspace where it is possible to come to a halt in a finite number of steps [72]. It seems that this method is well suited to replace the detection method of XCoM in the presented system, because the state estimation procedure would not require many changes.

Another apparent issue lies in the interaction between the lateral velocity and the lateral foot placement estimation at heel strike. The model suggests a larger lateral acceleration during the stride when the foot placement at the start is placed further away laterally. Underestimation of the lateral velocity yields a too small estimate of step width, yielding an underestimation of the lateral velocity over the next stride. To accurately counteract this phenomenon, more information on the feet positions is necessary. An option is to add another IMU sensor to one foot or both feet. These measurements enable algorithm additions that could solve or improve multiple issues. For example, an input estimation algorithm could be added to the state estimation to correct initial foot placement estimation errors. Another problem that could be solved is that by doing zero velocity detection on the feet [15, 18, 73–75], the step length and cadence could be estimated. These two estimates could in turn yield an estimate of the walking speed and remove the assumption that this speed is known. Furthermore, a large assumption in the presented method is that the times of heel strike and the gait phase are known. This assumption can be made obsolete, by using the feet-IMU measurements and the resulting zero velocity detections to determine the gait phase and the transition events.

Moreover, the proposed dynamical gait model is not perfect, as can be seen from the training performance. In particular, the model yields a too small GRF at the end of single stance, causing the estimated CoM velocity in the $\mathcal{B}_z$ direction to wrongfully keep decreasing. Combined with the observation that the measured $\mathcal{B}_z$ velocity strongly resembles a sinusoid, the model for this channel might be replaced with the sinusoidal model that is also used for the orientation estimation (Equation 3-7). If the accelerations due to angular velocity and acceleration are small enough in the $\mathcal{B}_z$ direction, it might be sufficient to only use a direct integral of the z channel of the IMU accelerometry to estimate the $\mathcal{B}_z$ CoM velocity.

Lastly, for a practical implementation of the presented method, the robustness should be analysed. There are many influential parameters in the algorithm, where it is unknown how sensitive the results are to errors in these parameters. For example, the influences of sensor placement errors or model parameter errors could be investigated to obtain a measure of robustness of the detection results. Furthermore, in these results the measurement noise added is zero-mean Gaussian noise. This is not realistic, since the slack in the attachment interface between the sensor and the subject would introduce more low-frequency noise. The effects of this coloured noise should be investigated.

Appendix A

Filtering Equations

A-1 Kalman Filter

The filter used for the orientation estimation presented in Section 3-3-2 is implementable for linear systems with additive white noise [71, Sec. 5.3]. For a system

$$\begin{cases} \mathbf{x}[k] = A\mathbf{x}[k-1] + B\mathbf{u}[k] + \mathbf{q}[k] \\ \mathbf{y}[k] = C\mathbf{x}[k] + D\mathbf{u}[k] + \mathbf{r}[k], \end{cases} \quad (\text{A-1})$$

with state matrices A , B , C , D , state $\mathbf{x}[k]$, input $\mathbf{u}[k]$, output $\mathbf{y}[k]$, process noise $\mathbf{q}[k]$, and output noise $\mathbf{r}[k]$. The noise is zero-mean and normally distributed such that

$$\mathbb{E} \left[\begin{pmatrix} \mathbf{q}[k] \\ \mathbf{r}[k] \end{pmatrix} \begin{pmatrix} \mathbf{q}[j] \\ \mathbf{r}[j] \end{pmatrix}^T \right] = \begin{pmatrix} Q & 0 \\ 0 & R \end{pmatrix} \Delta(k-j) \geq 0 \quad (\text{A-2})$$

$$R > 0, \quad (\text{A-3})$$

where $\Delta(k)$ is the unit impulse. The covariance belonging to the estimate $\hat{\mathbf{x}}[k|k-1]$, $P[k|k-1]$, is

$$P[k|k-1] = \mathbb{E} \left[(\mathbf{x}[k] - \hat{\mathbf{x}}[k|k-1]) (\mathbf{x}[k] - \hat{\mathbf{x}}[k|k-1])^T \right] \geq 0. \quad (\text{A-4})$$

The measurement update equations are

$$\begin{cases} K = P[k|k-1]C^T (R + CP[k|k-1]C^T)^{-1} \\ P[k|k] = P[k|k-1] - KCP[k|k-1] \\ \hat{\mathbf{x}}[k|k] = \hat{\mathbf{x}}[k|k-1] + K(y - D\mathbf{u}[k] - C\hat{\mathbf{x}}[k|k-1]). \end{cases} \quad (\text{A-5})$$

The prediction equations are

$$\begin{cases} S = AP[k|k-1]C^T (R + CP[k|k-1]C^T)^{-1} \\ P[k+1|k] = AP[k|k-1]A^T + Q - S(AP[k|k-1]C^T)^T \\ \hat{\mathbf{x}}[k+1|k] = A\hat{\mathbf{x}}[k|k-1] + B\mathbf{u}[k] + S(y - D\mathbf{u}[k] - C\hat{\mathbf{x}}[k|k-1]). \end{cases} \quad (\text{A-6})$$

The proof for this filter is given by Verhaegen and Verdult [71, p. 137-139].

A-2 Extended Kalman Filter

The filter used to estimate the Centre of Mass (CoM) velocity as presented in Section 3-3-3 also works for nonlinear systems with additive white noise. For a nonlinear system

$$\begin{cases} \mathbf{x}[k] = f(\mathbf{x}[k-1], \mathbf{u}[k]) + \mathbf{q}[k] \\ \mathbf{y}[k] = g(\mathbf{x}[k], \mathbf{u}[k]) + \mathbf{r}[k], \end{cases} \quad (\text{A-7})$$

where Equations A-2, A-3, and A-4 hold, the prediction equations are

$$\begin{cases} \mathbf{m}^-[k] = f(\hat{\mathbf{x}}[k-1], \mathbf{u}[k]) \\ P^-[k] = F_x P[k-1] F_x^T + Q, \end{cases} \quad (\text{A-8})$$

where F_x is the Jacobian of f with respect to $\mathbf{x}$. The measurement update equations are then

$$\begin{cases} \mathbf{v} = \mathbf{y} - g(\mathbf{m}^-[k], \mathbf{u}[k]) \\ S = G_x P^-[k] G_x^T + R \\ K = P^-[k] G_x^T S^{-1} \\ \hat{\mathbf{x}}[k] = \mathbf{m}^-[k] + K \mathbf{v} \\ P[k] = P^-[k] - K S K^T, \end{cases} \quad (\text{A-9})$$

where G_x is the Jacobian of g with respect to $\mathbf{x}$. The proof for this filter is given by Särkkä [62, p. 70].

Appendix B

Dynamics: Complete Model

The model presented by Paiman et al. is adapted to three-dimensional space [64]. Furthermore, the upper body is modelled as a pair of body-fixed gyroscopes rigidly attached to a beam. The gyroscopes are necessary, because a rigid beam alone is not able to capture the oscillatory character of real-life upper-body sway. See Figure B-1 for a sketch of the model.

The Sagittal Virtual Pendulum Point (VPP-S), $\mathbf{P}_S$, is defined as a body fixed point above the Centre of Mass (CoM) in the sagittal plane. The virtual pendulum points are the body fixed points that define the direction of the Ground Reaction Force (GRF). When the virtual pendulum point is above the CoM, the upper body will behave like a pendulum [64, 76, 77]. In the lateral plane, Firouzi et al. presents a similar Lateral Virtual Pendulum Point (VPP-L), $\mathbf{P}_L$, below the CoM during single stance and above in double stance [78]. It is assumed the VPPs in different planes can be combined. For both theories, the proposed directions of the ground reaction force in two dimensions becomes a plane in three dimensions.

The VPPs are defined relative to the CoM as ${}^B\mathbf{P}_S = (0 \ 0 \ p_1)^T$ and ${}^B\mathbf{P}_L = (0 \ 0 \ p_2)^T$. On the same line lies the point ${}^B\mathbf{H}_C = -(0 \ 0 \ h)^T$, representing the center of the hip. The parameter W is defined as the physical distance between the greater trochanters, so ${}^B\mathbf{H}_L = {}^B\mathbf{H}_C + (0 \ \frac{W}{2} \ 0)^T$ and ${}^B\mathbf{H}_R = {}^B\mathbf{H}_C + (0 \ -\frac{W}{2} \ 0)^T$. The parameterised dynamics are symmetric, so points such as the left foot position, $\mathbf{F}_L$, or the right foot position, $\mathbf{F}_R$, will be denoted without subscripts, $\mathbf{F}$, when the side of the stance foot is of no consequence to the dynamics.

B-1 Direction of the Ground Reaction Force

The plane for the VPP-S goes through $\mathbf{F}$, $\mathbf{P}_S$, and a point on the floor in the direction of $\mathcal{B}_x$ with respect to $\mathbf{F}$.

Projecting $\mathcal{B}_x$ onto the floor gives this direction as

$$\mathbf{s}_x = \mathcal{B}_x - (\mathcal{B}_x \cdot \mathcal{N}_z)\mathcal{N}_z. \quad (\text{B-1})$$

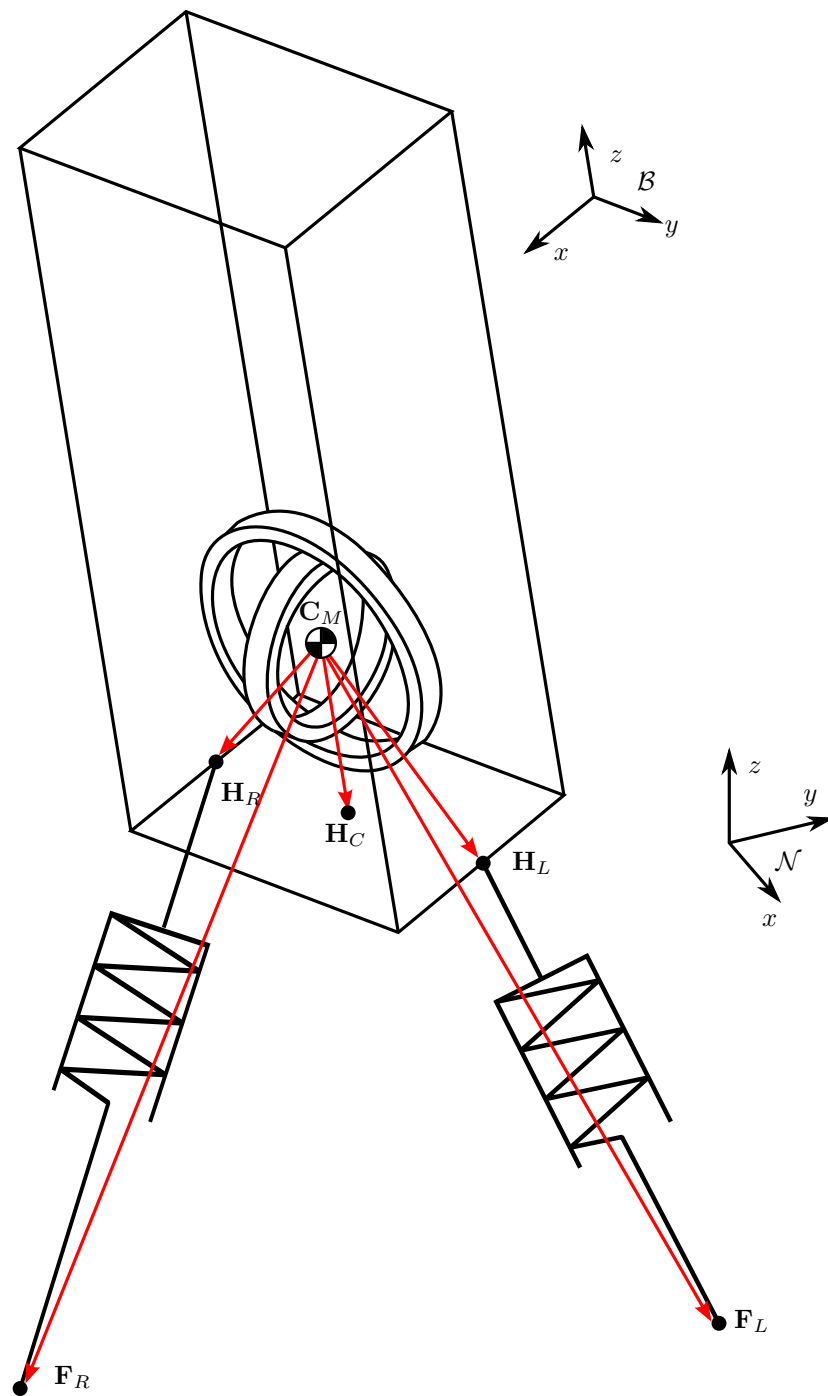


Figure B-1: Model sketch of the body

A normalvector, $\mathbf{n}_s$, of this plane is therefore

$$\mathbf{n}_s = (\mathbf{P}_S - \mathbf{F}) \times (\mathbf{F} + \mathbf{s}_x - \mathbf{F}). \quad (\text{B-2})$$

This simplifies to

$$\mathbf{n}_s = (\mathbf{P}_S - \mathbf{F}) \times \mathbf{s}_x. \quad (\text{B-3})$$

Suppose both single and double stance are considered and the VPP is interchangeable. Then the other plane belonging to the VPP-L goes through $\mathbf{F}$, $\mathbf{P}_L$, and $\mathbf{F} + \mathbf{s}_y$. Constructing a normalvector in a similar manner yields

$$\mathbf{n}_l = (\mathbf{P}_L - \mathbf{F}) \times \mathbf{s}_y, \quad (\text{B-4})$$

where

$$\mathbf{s}_y = \mathcal{B}_y - (\mathcal{B}_y \cdot \mathcal{N}_z) \mathcal{N}_z. \quad (\text{B-5})$$

The direction, $\mathbf{r}_G$, of this intersection, and therefore the direction of the ground reaction force $\mathbf{G}$, is orthogonal to both $\mathbf{n}_s$ and $\mathbf{n}_l$. Hence,

$$\mathbf{r}_G = \mathbf{n}_s \times \mathbf{n}_l. \quad (\text{B-6})$$

Normalising yields the unit direction

$$\hat{\mathbf{r}}_G = \frac{\mathbf{r}_G}{\|\mathbf{r}_G\|}. \quad (\text{B-7})$$

B-2 Magnitude of the Ground Reaction Force

The projection of the ground reaction force, $\mathbf{G} = \|\mathbf{G}\| \hat{\mathbf{r}}_G$, onto the ‘leg’, $(\mathbf{H} - \mathbf{F})$, should equal the spring-dampener force according to the theory of the VPP-S [76]. Suppose that the leg has a loadless length ℓ_0 and can only compress. The unit direction, $\hat{\mathbf{r}}_S$, from $\mathbf{F}$ to $\mathbf{H}$ is defined as

$$\hat{\mathbf{r}}_S = \frac{\mathbf{H} - \mathbf{F}}{\|\mathbf{H} - \mathbf{F}\|}. \quad (\text{B-8})$$

The magnitude of the spring-dampener force in a leg is then

$$k(\ell_0 - \|\mathbf{H} - \mathbf{F}\|) - b\left(\frac{\partial}{\partial t}(\mathbf{H} - \mathbf{F}) \cdot \hat{\mathbf{r}}_S\right), \quad (\text{B-9})$$

where k and b are the spring and dampener constants, respectively. The projection of the ground reaction force onto the leg can be expressed as

$$\text{proj}(\mathbf{G}, \hat{\mathbf{r}}_S) = \frac{\|\mathbf{G}\| \hat{\mathbf{r}}_G \cdot \hat{\mathbf{r}}_S}{\|\hat{\mathbf{r}}_S\|} \hat{\mathbf{r}}_S = \|\mathbf{G}\| (\hat{\mathbf{r}}_G \cdot \hat{\mathbf{r}}_S) \hat{\mathbf{r}}_S. \quad (\text{B-10})$$

Therefore, the magnitude is

$$\|\text{proj}(\mathbf{G}, \hat{\mathbf{r}}_S)\| = \|\mathbf{G}\| (\hat{\mathbf{r}}_G \cdot \hat{\mathbf{r}}_S), \quad (\text{B-11})$$

and this should equal the spring-dampener force magnitude

$$\|\mathbf{G}\| (\hat{\mathbf{r}}_G \cdot \hat{\mathbf{r}}_S) = k(\ell_0 - \|\mathbf{H} - \mathbf{F}\|) - b\left(\frac{\partial}{\partial t}(\mathbf{H} - \mathbf{F}) \cdot \hat{\mathbf{r}}_S\right). \quad (\text{B-12})$$

Solving for the ground reaction force magnitude yields

$$\|\mathbf{G}\| = \frac{1}{\hat{\mathbf{r}}_G \cdot \hat{\mathbf{r}}_S} \left[k(\ell_0 - \|\mathbf{H} - \mathbf{F}\|) - b\left(\frac{\partial}{\partial t}(\mathbf{H} - \mathbf{F}) \cdot \hat{\mathbf{r}}_S\right) \right]. \quad (\text{B-13})$$

The Time Derivative $\frac{d}{dt}(\mathbf{H} - \mathbf{F})$ can be calculated relative to either $\mathcal{B}$ or $\mathcal{N}$. Since the differentiation operator is distributive,

$$\frac{d}{dt}(\mathbf{H} - \mathbf{F}) = \frac{d\mathbf{H}}{dt} - \frac{d\mathbf{F}}{dt} = \dot{\mathbf{H}} - \dot{\mathbf{F}} \quad (\text{B-14})$$

and noting that $\mathbf{F}$ is assumed stationary in $\mathcal{N}$ and $\mathbf{H}$ relative to $\mathbf{C}_M$ stationary in $\mathcal{B}$, the choice seems arbitrary. However, since measurements are made in $\mathcal{N}$, the derivative will be calculated in this frame, relative to $\mathbf{C}_M$. The velocity of world-stationary $\mathbf{F}$ is then

$$\begin{aligned} {}^{\mathcal{N}}\dot{\mathbf{F}} &= \mathbf{0} \\ \Rightarrow \left({}^{\mathcal{N}}\dot{\mathbf{F}}\right)_{\mathbf{C}_M} &= -{}^{\mathcal{N}}\dot{\mathbf{C}}_M. \end{aligned} \quad (\text{B-15})$$

The velocity of $\mathbf{H}$ can be calculated with [59, Eq. 4.50], that states

$$\left({}^{\mathcal{N}}\dot{\mathbf{H}}\right)_{\mathbf{C}_M} = {}^{\mathcal{N}}\dot{\mathbf{C}}_M + {}^{\mathcal{N}}\left(\frac{d}{dt}(\mathbf{H} - \mathbf{C}_M)\right)_{\mathcal{B}} + {}^{\mathcal{N}}\omega_{\mathcal{B}/\mathcal{N}} \times ({}^{\mathcal{N}}\mathbf{H} - {}^{\mathcal{N}}\mathbf{C}_M), \quad (\text{B-16})$$

where $\omega_{\mathcal{B}/\mathcal{N}}$ is the angular velocity of $\mathcal{B}$ with respect to $\mathcal{N}$. Because $\mathbf{H}$ and $\mathbf{C}_M$ are body-fixed,

$${}^{\mathcal{N}}\left(\frac{d}{dt}(\mathbf{H} - \mathbf{C}_M)\right)_{\mathcal{B}} = \mathbf{0}. \quad (\text{B-17})$$

Therefore,

$$\left({}^{\mathcal{N}}\dot{\mathbf{H}}\right)_{\mathbf{C}_M} = {}^{\mathcal{N}}\dot{\mathbf{C}}_M + {}^{\mathcal{N}}\omega_{\mathcal{B}/\mathcal{N}} \times ({}^{\mathcal{N}}\mathbf{H} - {}^{\mathcal{N}}\mathbf{C}_M). \quad (\text{B-18})$$

The ultimate derivative is then expressed as

$$\begin{aligned} \left({}^{\mathcal{N}}\dot{\mathbf{H}} - {}^{\mathcal{N}}\dot{\mathbf{F}}\right)_{\mathbf{C}_M} &= {}^{\mathcal{N}}\dot{\mathbf{C}}_M + {}^{\mathcal{N}}\omega_{\mathcal{B}/\mathcal{N}} \times ({}^{\mathcal{N}}\mathbf{H} - {}^{\mathcal{N}}\mathbf{C}_M) + {}^{\mathcal{N}}\dot{\mathbf{C}}_M \\ &= 2{}^{\mathcal{N}}\dot{\mathbf{C}}_M + {}^{\mathcal{N}}\omega_{\mathcal{B}/\mathcal{N}} \times ({}^{\mathcal{N}}\mathbf{H} - {}^{\mathcal{N}}\mathbf{C}_M). \end{aligned} \quad (\text{B-19})$$

B-3 Body Dynamics

B-3-1 Cartesian Equations of Motion

The translational equations of motion are

$$m\ddot{\mathbf{C}}_M = \mathbf{G}_L + \mathbf{G}_R + \mathbf{Z}, \quad (\text{B-20})$$

where m is the body mass, $\mathbf{G}_L$ and $\mathbf{G}_R$ the ground reaction force of respectively the left and right feet, and $\mathbf{Z}$ is gravity. The moment around the CoM is then

$$\mathbf{M}_C = (\mathbf{F}_L - \mathbf{C}_M) \times \mathbf{G}_L + (\mathbf{F}_R - \mathbf{C}_M) \times \mathbf{G}_R. \quad (\text{B-21})$$

The inertia of the cylindrical gyroscopes are determined to be

$${}^{\mathcal{B}}J_{\text{gyr},x} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} \end{pmatrix} \alpha m r_x^2 \quad \text{and} \quad {}^{\mathcal{B}}J_{\text{gyr},y} = \begin{pmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{2} \end{pmatrix} (\alpha - 1) m r_y^2, \quad (\text{B-22})$$

where r_x and r_y are the radii of the gyroscopes and α is a distribution parameter. The inertia around the CoM is

$${}^{\mathcal{B}}J_C = {}^{\mathcal{B}}J_{\text{gyr},x} + {}^{\mathcal{B}}J_{\text{gyr},y}. \quad (\text{B-23})$$

Then the angular momentum, Ψ_C , is calculated as

$$\Psi_C = J_C \omega_{\mathcal{B}/\mathcal{N}} + J_{\text{gyr},x} \begin{pmatrix} \gamma_x \\ 0 \\ 0 \end{pmatrix} + J_{\text{gyr},y} \begin{pmatrix} 0 \\ \gamma_y \\ 0 \end{pmatrix}, \quad (\text{B-24})$$

where γ_x and γ_y are the angular velocities at which the gyroscopes spin, respectively. Taking the time derivative of this yields

$$\dot{\Psi}_C = \dot{J}_C \omega_{\mathcal{B}/\mathcal{N}} + J_C \dot{\omega}_{\mathcal{B}/\mathcal{N}} = J_C \dot{\omega}_{\mathcal{B}/\mathcal{N}}. \quad (\text{B-25})$$

Applying the transport theorem to Euler's second law gives

$$\mathbf{M}_C = (\dot{\Psi}_C)_{\mathcal{B}} + \omega_{\mathcal{B}/\mathcal{N}} \times \Psi_C. \quad (\text{B-26})$$

Substitution of the angular momentum and its derivative gives the equation

$$\mathbf{M}_C = J_C \dot{\omega}_{\mathcal{B}/\mathcal{N}} + \omega_{\mathcal{B}/\mathcal{N}} \times \left(J_C \omega_{\mathcal{B}/\mathcal{N}} + J_{\text{gyr},x} \begin{pmatrix} \gamma_x \\ 0 \\ 0 \end{pmatrix} + J_{\text{gyr},y} \begin{pmatrix} 0 \\ \gamma_y \\ 0 \end{pmatrix} \right). \quad (\text{B-27})$$

Reordering of this equation yields

$$J_C \dot{\omega}_{\mathcal{B}/\mathcal{N}} = \mathbf{M}_C - \omega_{\mathcal{B}/\mathcal{N}} \times \left(J_C \omega_{\mathcal{B}/\mathcal{N}} + J_{\text{gyr},x} \begin{pmatrix} \gamma_x \\ 0 \\ 0 \end{pmatrix} + J_{\text{gyr},y} \begin{pmatrix} 0 \\ \gamma_y \\ 0 \end{pmatrix} \right), \quad (\text{B-28})$$

and since the cross product is distributive over addition

$$J_C \dot{\omega}_{\mathcal{B}/\mathcal{N}} = \mathbf{M}_C - \omega_{\mathcal{B}/\mathcal{N}} \times (J_C \omega_{\mathcal{B}/\mathcal{N}}) - \omega_{\mathcal{B}/\mathcal{N}} \times \left(J_{\text{gyr},x} \begin{pmatrix} \gamma_x \\ 0 \\ 0 \end{pmatrix} + J_{\text{gyr},y} \begin{pmatrix} 0 \\ \gamma_y \\ 0 \end{pmatrix} \right). \quad (\text{B-29})$$

Expanding the rightmost term gives

$$J_C \dot{\omega}_{\mathcal{B}/\mathcal{N}} = \mathbf{M}_C - \omega_{\mathcal{B}/\mathcal{N}} \times (J_C \omega_{\mathcal{B}/\mathcal{N}}) - \omega_{\mathcal{B}/\mathcal{N}} \times \begin{pmatrix} \gamma_x \alpha m r_x^2 \\ \gamma_y (\alpha - 1) m r_y^2 \\ 0 \end{pmatrix} \quad (\text{B-30})$$

and substituting

$$\begin{pmatrix} \gamma_x \alpha m r_x^2 \\ \gamma_y (\alpha - 1) m r_y^2 \\ 0 \end{pmatrix} = {}^{\mathcal{B}}\mathbf{\Gamma} \quad (\text{B-31})$$

yields

$$J_C \dot{\omega}_{\mathcal{B}/\mathcal{N}} = \mathbf{M}_C - \omega_{\mathcal{B}/\mathcal{N}} \times (J_C \omega_{\mathcal{B}/\mathcal{N}}) - \omega_{\mathcal{B}/\mathcal{N}} \times \mathbf{\Gamma}. \quad (\text{B-32})$$

B-3-2 Expressing in Quaternions

It is desired to express these equations of motion in quaternion space instead of Cartesian space. A derivation similar to [59, App. D] is presented. The norm constraint, C_q , on the unit quaternion is expressed as

$$C_q = {}^{\mathcal{N}}\underline{\mathbf{q}}^{\mathcal{B}^T} {}^{\mathcal{N}}\underline{\mathbf{q}}^{\mathcal{B}} - 1 = 0. \quad (\text{B-33})$$

The virtual constraint rate is therefore

$$\delta \dot{C}_q = 2 {}^{\mathcal{N}}\underline{\mathbf{q}}^{\mathcal{B}^T} \delta {}^{\mathcal{N}}\underline{\dot{\mathbf{q}}}^{\mathcal{B}}. \quad (\text{B-34})$$

By introducing the Lagrangian multiplier λ and expressing all terms in the body fixed frame, the resulting virtual power equation is

$$\left({}^{\mathcal{B}}\underline{\mathbf{M}}_C - {}^{\mathcal{B}}J_C {}^{\mathcal{B}}\dot{\omega}_{\mathcal{B}/\mathcal{N}} - {}^{\mathcal{B}}\omega_{\mathcal{B}/\mathcal{N}} \times {}^{\mathcal{B}}J_C {}^{\mathcal{B}}\omega_{\mathcal{B}/\mathcal{N}} + {}^{\mathcal{B}}\omega_{\mathcal{B}/\mathcal{N}} \times {}^{\mathcal{B}}\underline{\Gamma} \right)^T \delta {}^{\mathcal{B}}\omega_{\mathcal{B}/\mathcal{N}} = \lambda \delta \dot{C}_q = 2\lambda {}^{\mathcal{N}}\underline{\mathbf{q}}^{\mathcal{B}^T} \delta {}^{\mathcal{N}}\underline{\dot{\mathbf{q}}}^{\mathcal{B}}. \quad (\text{B-35})$$

Stacking a row of zeros on top of the left hand side of the equation and substituting

$${}^{\mathcal{B}}J_C = \begin{pmatrix} 0 & 0 \\ 0 & {}^{\mathcal{B}}J_C \end{pmatrix} \quad (\text{B-36})$$

yields

$$\left(\begin{pmatrix} 0 \\ {}^{\mathcal{B}}\underline{\mathbf{M}}_C \end{pmatrix} - {}^{\mathcal{B}}J_C \begin{pmatrix} 0 \\ {}^{\mathcal{B}}\dot{\omega}_{\mathcal{B}/\mathcal{N}} \end{pmatrix} - \begin{pmatrix} 0 \\ {}^{\mathcal{B}}\omega_{\mathcal{B}/\mathcal{N}} \end{pmatrix} \circ {}^{\mathcal{B}}J_C \begin{pmatrix} 0 \\ {}^{\mathcal{B}}\omega_{\mathcal{B}/\mathcal{N}} \end{pmatrix} - \begin{pmatrix} 0 \\ {}^{\mathcal{B}}\underline{\Gamma} \end{pmatrix} \circ \begin{pmatrix} 0 \\ {}^{\mathcal{B}}\omega_{\mathcal{B}/\mathcal{N}} \end{pmatrix} \right)^T \delta \begin{pmatrix} 0 \\ {}^{\mathcal{B}}\omega_{\mathcal{B}/\mathcal{N}} \end{pmatrix} = 2\lambda {}^{\mathcal{N}}\underline{\mathbf{q}}^{\mathcal{B}^T} \delta {}^{\mathcal{N}}\underline{\dot{\mathbf{q}}}^{\mathcal{B}}, \quad (\text{B-37})$$

where $\circ$ is the quaternion multiplication operator as defined by Vallery and Schwab [59]. Substitution of

$$\begin{aligned} \begin{pmatrix} 0 \\ {}^{\mathcal{B}}\dot{\omega}_{\mathcal{B}/\mathcal{N}} \end{pmatrix} &= 2Q^T {}^{\mathcal{N}}\underline{\dot{\mathbf{q}}}^{\mathcal{B}} + 2 \begin{pmatrix} \| {}^{\mathcal{N}}\underline{\dot{\mathbf{q}}}^{\mathcal{B}} \|^2 \\ \mathbf{0} \end{pmatrix}, \\ \begin{pmatrix} 0 \\ {}^{\mathcal{B}}\omega_{\mathcal{B}/\mathcal{N}} \end{pmatrix} &= 2Q^T {}^{\mathcal{N}}\underline{\dot{\mathbf{q}}}^{\mathcal{B}}, \\ \begin{pmatrix} 0 \\ {}^{\mathcal{B}}\underline{\mathbf{M}}_C \end{pmatrix} &= {}^{\mathcal{B}}\underline{\mathbf{M}}_C \quad \text{and} \\ {}^{\mathcal{B}}\underline{\Gamma} &= \begin{pmatrix} 0 & -{}^{\mathcal{B}}\underline{\Gamma}^T \\ {}^{\mathcal{B}}\underline{\Gamma} & {}^{\mathcal{B}}\tilde{\underline{\Gamma}} \end{pmatrix}, \end{aligned}$$

where

$${}^{\mathcal{N}}\underline{\mathbf{q}}^{\mathcal{B}} = \begin{pmatrix} q_0 \\ \underline{\mathbf{q}} \end{pmatrix} \Rightarrow Q = \begin{pmatrix} q_0 & -\underline{\mathbf{q}}^T \\ \underline{\mathbf{q}} & q_0 E_3 + \tilde{\underline{\mathbf{q}}} \end{pmatrix} \Rightarrow {}^{\mathcal{N}}\underline{\mathbf{q}}^{\mathcal{B}} \circ \underline{\mathbf{p}} = Q \underline{\mathbf{p}} \quad \forall \underline{\mathbf{p}} \in \mathbb{R}^4$$

into the equations of motion returns

$$\left({}^{\mathcal{B}}\underline{\mathbf{M}}_C - {}^{\mathcal{B}}J_C \left(2Q^T {}^{\mathcal{N}}\underline{\dot{\mathbf{q}}}^{\mathcal{B}} + 2 \begin{pmatrix} \| {}^{\mathcal{N}}\underline{\dot{\mathbf{q}}}^{\mathcal{B}} \|^2 \\ \mathbf{0} \end{pmatrix} \right) - (2Q^T {}^{\mathcal{N}}\underline{\dot{\mathbf{q}}}^{\mathcal{B}}) \circ {}^{\mathcal{B}}J_C (2Q^T {}^{\mathcal{N}}\underline{\dot{\mathbf{q}}}^{\mathcal{B}}) - 2 {}^{\mathcal{B}}\underline{\Gamma} Q^T {}^{\mathcal{N}}\underline{\dot{\mathbf{q}}}^{\mathcal{B}} \right)^T 2Q^T \delta {}^{\mathcal{N}}\underline{\dot{\mathbf{q}}}^{\mathcal{B}} = 2\lambda {}^{\mathcal{N}}\underline{\mathbf{q}}^{\mathcal{B}^T} \delta {}^{\mathcal{N}}\underline{\dot{\mathbf{q}}}^{\mathcal{B}}. \quad (\text{B-38})$$

Using that

$${}^B J_C \begin{pmatrix} \|\dot{\mathbf{q}}^B\|^2 \\ 0 \end{pmatrix} = \mathbf{0}, \quad (\text{B-39})$$

this simplifies to

$$\left({}^B \underline{\mathbf{M}}_C - 2{}^B J_C Q^{\text{T}\mathcal{N}} \ddot{\mathbf{q}}^B - 4(Q^{\text{T}\mathcal{N}} \dot{\mathbf{q}}^B) \circ {}^B J_C (Q^{\text{T}\mathcal{N}} \dot{\mathbf{q}}^B) - 2{}^B \Gamma Q^{\text{T}\mathcal{N}} \dot{\mathbf{q}}^B \right)^{\text{T}} 2Q^{\text{T}} \delta^{\mathcal{N}} \dot{\mathbf{q}}^B = 2\lambda^{\mathcal{N}} \underline{\mathbf{q}}^{\text{B}\text{T}} \delta^{\mathcal{N}} \dot{\mathbf{q}}^B. \quad (\text{B-40})$$

Moving all terms to the left hand side gives

$$\left(\left({}^B \underline{\mathbf{M}}_C - 2{}^B J_C Q^{\text{T}\mathcal{N}} \ddot{\mathbf{q}}^B - 4(Q^{\text{T}\mathcal{N}} \dot{\mathbf{q}}^B) \circ {}^B J_C (Q^{\text{T}\mathcal{N}} \dot{\mathbf{q}}^B) - 2{}^B \Gamma Q^{\text{T}\mathcal{N}} \dot{\mathbf{q}}^B \right)^{\text{T}} 2Q^{\text{T}} - 2\lambda^{\mathcal{N}} \underline{\mathbf{q}}^{\text{B}\text{T}} \right) \delta^{\mathcal{N}} \dot{\mathbf{q}}^B = 0 \quad (\text{B-41})$$

and therefore

$$\left({}^B \underline{\mathbf{M}}_C - 2{}^B J_C Q^{\text{T}\mathcal{N}} \ddot{\mathbf{q}}^B - 4(Q^{\text{T}\mathcal{N}} \dot{\mathbf{q}}^B) \circ {}^B J_C (Q^{\text{T}\mathcal{N}} \dot{\mathbf{q}}^B) - 2{}^B \Gamma Q^{\text{T}\mathcal{N}} \dot{\mathbf{q}}^B \right)^{\text{T}} 2Q^{\text{T}} - 2\lambda^{\mathcal{N}} \underline{\mathbf{q}}^{\text{B}\text{T}} = 0. \quad (\text{B-42})$$

The transpose of this equation is

$$2Q \left({}^B \underline{\mathbf{M}}_C - 2{}^B J_C Q^{\text{T}\mathcal{N}} \ddot{\mathbf{q}}^B - 4(Q^{\text{T}\mathcal{N}} \dot{\mathbf{q}}^B) \circ {}^B J_C (Q^{\text{T}\mathcal{N}} \dot{\mathbf{q}}^B) - 2{}^B \Gamma Q^{\text{T}\mathcal{N}} \dot{\mathbf{q}}^B \right) - 2\lambda^{\mathcal{N}} \underline{\mathbf{q}}^{\text{B}} = 0. \quad (\text{B-43})$$

Expansion of the brackets and rearranging gives

$$4Q^B J_C Q^{\text{T}\mathcal{N}} \ddot{\mathbf{q}}^B + 2\lambda^{\mathcal{N}} \underline{\mathbf{q}}^{\text{B}} = 2Q^B \underline{\mathbf{M}}_C - 8Q(Q^{\text{T}\mathcal{N}} \dot{\mathbf{q}}^B) \circ {}^B J_C (Q^{\text{T}\mathcal{N}} \dot{\mathbf{q}}^B) - 4Q^B \Gamma Q^{\text{T}\mathcal{N}} \dot{\mathbf{q}}^B. \quad (\text{B-44})$$

Then using the fact that

$$QQ^{\text{T}} = E_4, \quad (\text{B-45})$$

this equation simplifies to

$$4Q^B J_C Q^{\text{T}\mathcal{N}} \ddot{\mathbf{q}}^B + 2\lambda^{\mathcal{N}} \underline{\mathbf{q}}^{\text{B}} = 2Q^B \underline{\mathbf{M}}_C - 8\dot{Q}^B J_C Q^{\text{T}\mathcal{N}} \dot{\mathbf{q}}^B - 4Q^B \Gamma Q^{\text{T}\mathcal{N}} \dot{\mathbf{q}}^B. \quad (\text{B-46})$$

Application of the identity

$$Q^{\text{T}} \dot{q} = -\dot{Q}^{\text{T}} q, \quad (\text{B-47})$$

yields

$$4Q^B J_C Q^{\text{T}\mathcal{N}} \ddot{\mathbf{q}}^B + 2\lambda^{\mathcal{N}} \underline{\mathbf{q}}^{\text{B}} = 2Q^B \underline{\mathbf{M}}_C + 8\dot{Q}^B J_C \dot{Q}^{\text{T}\mathcal{N}} \dot{\mathbf{q}}^B - 4Q^B \Gamma Q^{\text{T}\mathcal{N}} \dot{\mathbf{q}}^B. \quad (\text{B-48})$$

Matricising the left hand side gives

$$\begin{pmatrix} 4Q^B J_C Q^{\text{T}} & 2{}^{\mathcal{N}} \underline{\mathbf{q}}^{\text{B}} \end{pmatrix} \begin{pmatrix} {}^{\mathcal{N}} \ddot{\mathbf{q}}^{\text{B}} \\ \lambda \end{pmatrix} = 2Q^B \underline{\mathbf{M}}_C + 8\dot{Q}^B J_C \dot{Q}^{\text{T}\mathcal{N}} \dot{\mathbf{q}}^B - 4Q^B \Gamma Q^{\text{T}\mathcal{N}} \dot{\mathbf{q}}^B. \quad (\text{B-49})$$

Addition of the constraint on acceleration obtained from

$$\begin{pmatrix} 0 \\ {}^B \dot{\omega}_{\text{B}/\mathcal{N}} \end{pmatrix} = 2Q^{\text{T}\mathcal{N}} \ddot{\mathbf{q}}^B + 2 \begin{pmatrix} \|\dot{\mathbf{q}}^B\|^2 \\ 0 \end{pmatrix} \Rightarrow 0 = {}^{\mathcal{N}} \underline{\mathbf{q}}^{\text{B}\text{T}} \ddot{\mathbf{q}}^B + \|\dot{\mathbf{q}}^B\|^2 \quad (\text{B-50})$$

yields the equations of motion

$$\begin{pmatrix} 4Q^B J_C Q^{\text{T}} & 2{}^{\mathcal{N}} \underline{\mathbf{q}}^{\text{B}} \\ {}^{\mathcal{N}} \underline{\mathbf{q}}^{\text{B}\text{T}} & 0 \end{pmatrix} \begin{pmatrix} {}^{\mathcal{N}} \ddot{\mathbf{q}}^{\text{B}} \\ \lambda \end{pmatrix} = \begin{pmatrix} 2Q^B \underline{\mathbf{M}}_C + 8\dot{Q}^B J_C \dot{Q}^{\text{T}\mathcal{N}} \dot{\mathbf{q}}^B - 4Q^B \Gamma Q^{\text{T}\mathcal{N}} \dot{\mathbf{q}}^B \\ -\|\dot{\mathbf{q}}^B\|^2 \end{pmatrix}. \quad (\text{B-51})$$

The Lagrangian multiplier is identified by left-multiplying Equation B-49 with ${}^{\mathcal{N}}\underline{\mathbf{q}}^{\mathcal{B}}$ and using that

$${}^{\mathcal{N}}\underline{\mathbf{q}}^{\mathcal{B}\text{T}}Q = \begin{pmatrix} 1 & 0 & 0 & 0 \end{pmatrix},$$

then

$$2\lambda = 8{}^{\mathcal{N}}\underline{\mathbf{q}}^{\mathcal{B}\text{T}}\dot{Q}^{\mathcal{B}}J_C\dot{Q}^{\text{T}\mathcal{N}}\underline{\mathbf{q}}^{\mathcal{B}} - 4\begin{pmatrix} 0 & -{}^{\mathcal{B}}\mathbf{\Gamma}^{\text{T}} \end{pmatrix}Q^{\text{T}\mathcal{N}}\dot{\underline{\mathbf{q}}}^{\mathcal{B}}. \quad (\text{B-52})$$

Implementing the identity in Equation B-47 again and extracting common factors yields

$$\lambda = \left(4{}^{\mathcal{N}}\underline{\mathbf{q}}^{\mathcal{B}\text{T}}\dot{Q}^{\mathcal{B}}J_C - 2\begin{pmatrix} 0 & {}^{\mathcal{B}}\mathbf{\Gamma}^{\text{T}} \end{pmatrix} \right) \dot{Q}^{\text{T}\mathcal{N}}\underline{\mathbf{q}}^{\mathcal{B}}. \quad (\text{B-53})$$

Moving the Lagrangian term to the right hand side of the equations of motion gives

$$\begin{pmatrix} 4Q^{\mathcal{B}}J_CQ^{\text{T}} \\ {}^{\mathcal{N}}\underline{\mathbf{q}}^{\mathcal{B}\text{T}} \end{pmatrix} {}^{\mathcal{N}}\ddot{\underline{\mathbf{q}}}^{\mathcal{B}} = \begin{pmatrix} 2Q^{\mathcal{B}}\underline{\mathbf{M}}_C + 8\dot{Q}^{\mathcal{B}}J_C\dot{Q}^{\text{T}\mathcal{N}}\underline{\mathbf{q}}^{\mathcal{B}} - 4Q^{\mathcal{B}}\mathbf{\Gamma}Q^{\text{T}\mathcal{N}}\dot{\underline{\mathbf{q}}}^{\mathcal{B}} - 2\lambda{}^{\mathcal{N}}\underline{\mathbf{q}}^{\mathcal{B}} \\ -\|{}^{\mathcal{N}}\dot{\underline{\mathbf{q}}}^{\mathcal{B}}\|^2 \end{pmatrix} \quad (\text{B-54})$$

and defining

$$E = \begin{pmatrix} 4Q^{\mathcal{B}}J_CQ^{\text{T}} \\ {}^{\mathcal{N}}\underline{\mathbf{q}}^{\mathcal{B}\text{T}} \end{pmatrix} \quad \text{and} \quad D = \begin{pmatrix} 2Q^{\mathcal{B}}\underline{\mathbf{M}}_C + 8\dot{Q}^{\mathcal{B}}J_C\dot{Q}^{\text{T}\mathcal{N}}\underline{\mathbf{q}}^{\mathcal{B}} - 4Q^{\mathcal{B}}\mathbf{\Gamma}Q^{\text{T}\mathcal{N}}\dot{\underline{\mathbf{q}}}^{\mathcal{B}} - 2\lambda{}^{\mathcal{N}}\underline{\mathbf{q}}^{\mathcal{B}} \\ -\|{}^{\mathcal{N}}\dot{\underline{\mathbf{q}}}^{\mathcal{B}}\|^2 \end{pmatrix},$$

such that

$$E{}^{\mathcal{N}}\ddot{\underline{\mathbf{q}}}^{\mathcal{B}} = D, \quad (\text{B-55})$$

where $E \in \mathbb{R}^{5 \times 4}$ and $D \in \mathbb{R}^5$. The structure of ${}^{\mathcal{B}}J_C$ shows that $Q^{\mathcal{B}}J_CQ^{\text{T}}$ is rank deficient, due to the zero row and column. Because the calculation of the analytical pseudo-inverse of E is computationally heavier than a normal analytical inverse, E^* is defined as the last four rows of E . The right-hand side D^* is defined likewise, such that

$$\begin{cases} E^* = E_{2:5,:} \\ D^* = D_{2:5}. \end{cases} \quad (\text{B-56})$$

This square matrix E^* can be analytically inverted, by which the explicit equations of motion are found as

$${}^{\mathcal{N}}\ddot{\underline{\mathbf{q}}}^{\mathcal{B}} = E^{-*}D^*. \quad (\text{B-57})$$

B-3-3 State-Space Formulation

Taking the velocity and orientation of the body and its first derivatives as the state, $\mathbf{x}$ such that

$$\mathbf{x} = \begin{pmatrix} \dot{\mathbf{C}}_M & {}^{\mathcal{N}}\underline{\mathbf{q}}^{\mathcal{B}} & {}^{\mathcal{N}}\dot{\underline{\mathbf{q}}}^{\mathcal{B}} \end{pmatrix}^{\text{T}},$$

and the input, $\mathbf{u}$, is the feet position

$$\mathbf{u} = \begin{pmatrix} \mathbf{F}_L & \mathbf{F}_R \end{pmatrix}^{\text{T}}.$$

The dynamics can then be expressed as the function f defined as

$$\dot{\mathbf{x}} = f(\mathbf{x}, \mathbf{u}) = \begin{pmatrix} \frac{1}{m}(\mathbf{G}_L + \mathbf{G}_R + \mathbf{Z}) \\ {}^{\mathcal{N}}\dot{\underline{\mathbf{q}}}^{\mathcal{B}} \\ E^{-*}D^* \end{pmatrix} \quad (\text{B-58})$$

Appendix C

Full-time-length Plots

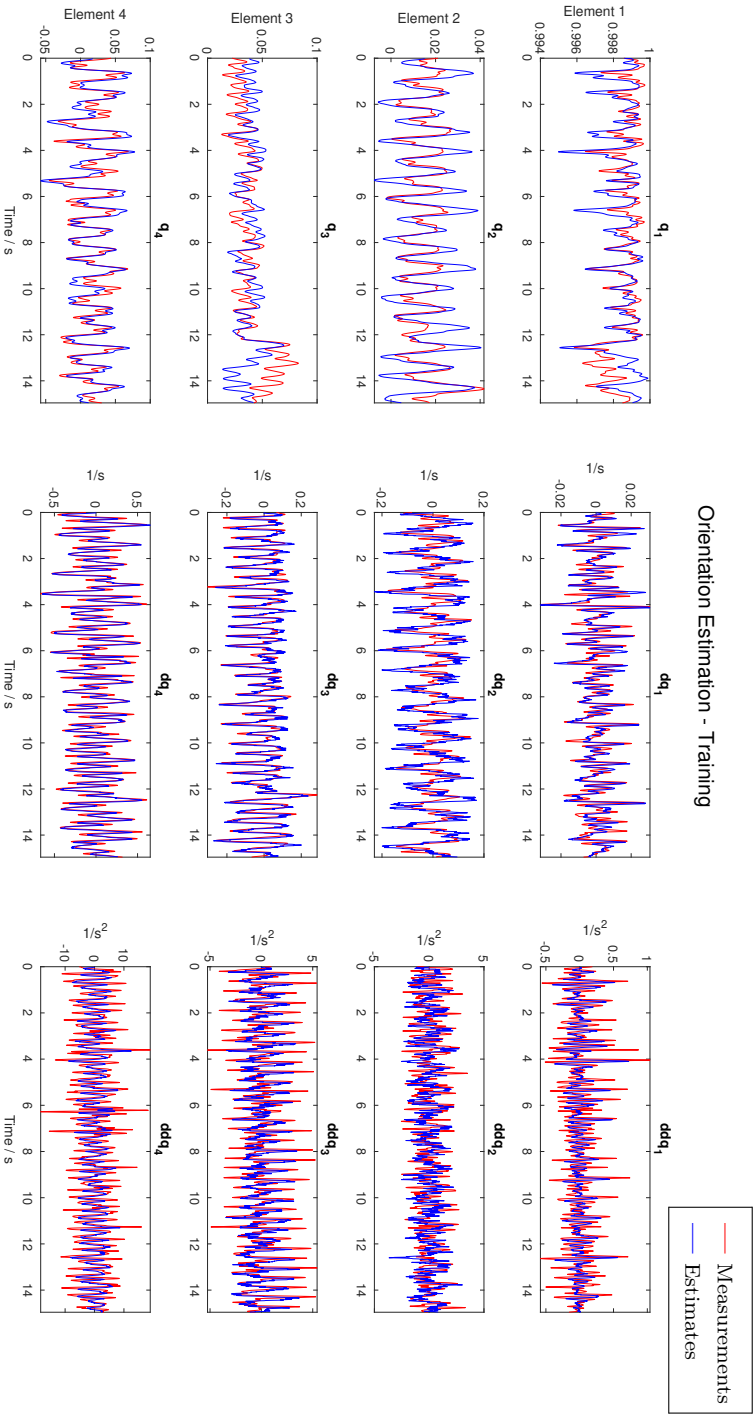


Figure C-1: Training performance of the orientation estimator on the VdZ dataset.

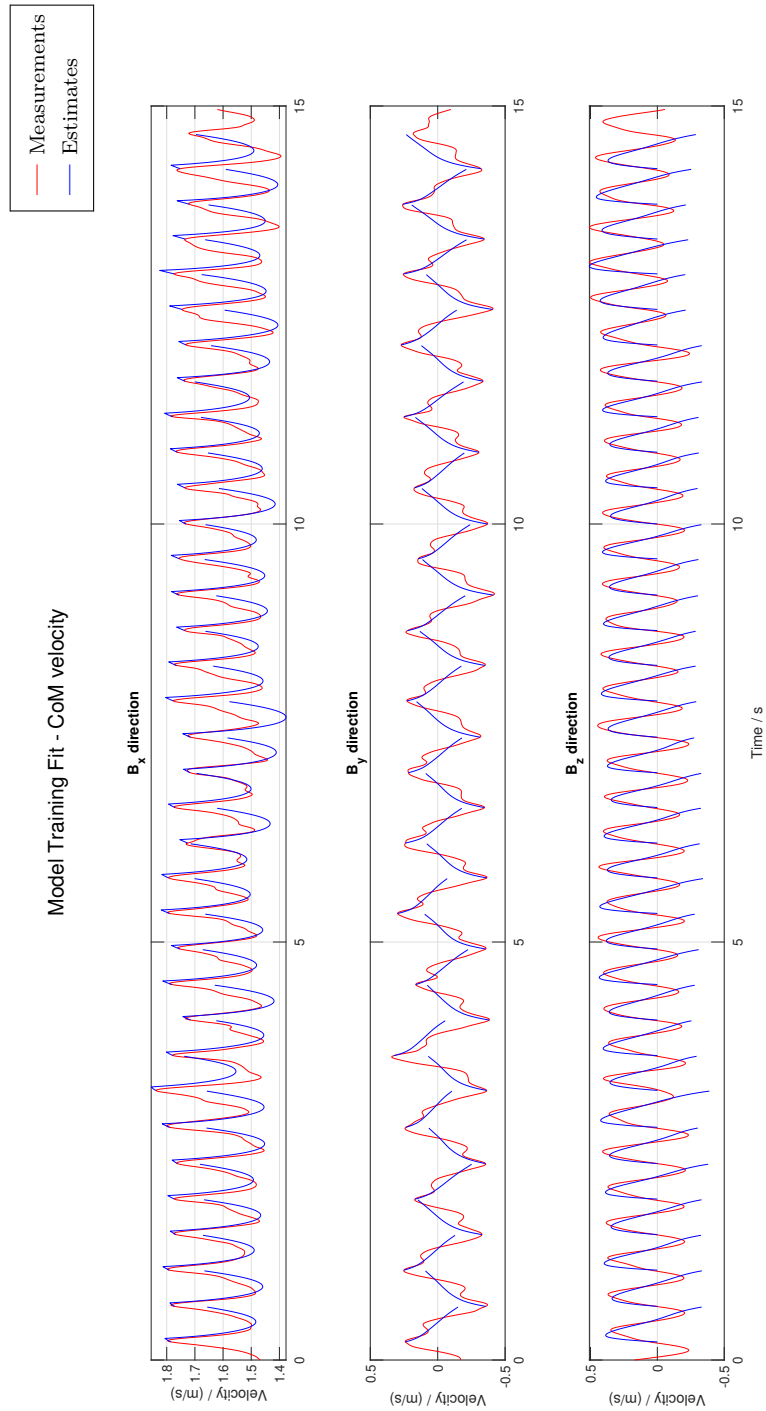


Figure C-2: Trained model fit overlaid on measured state trajectories (CoM velocity) of the VdZ dataset. Top to bottom: x , y , and z velocities.

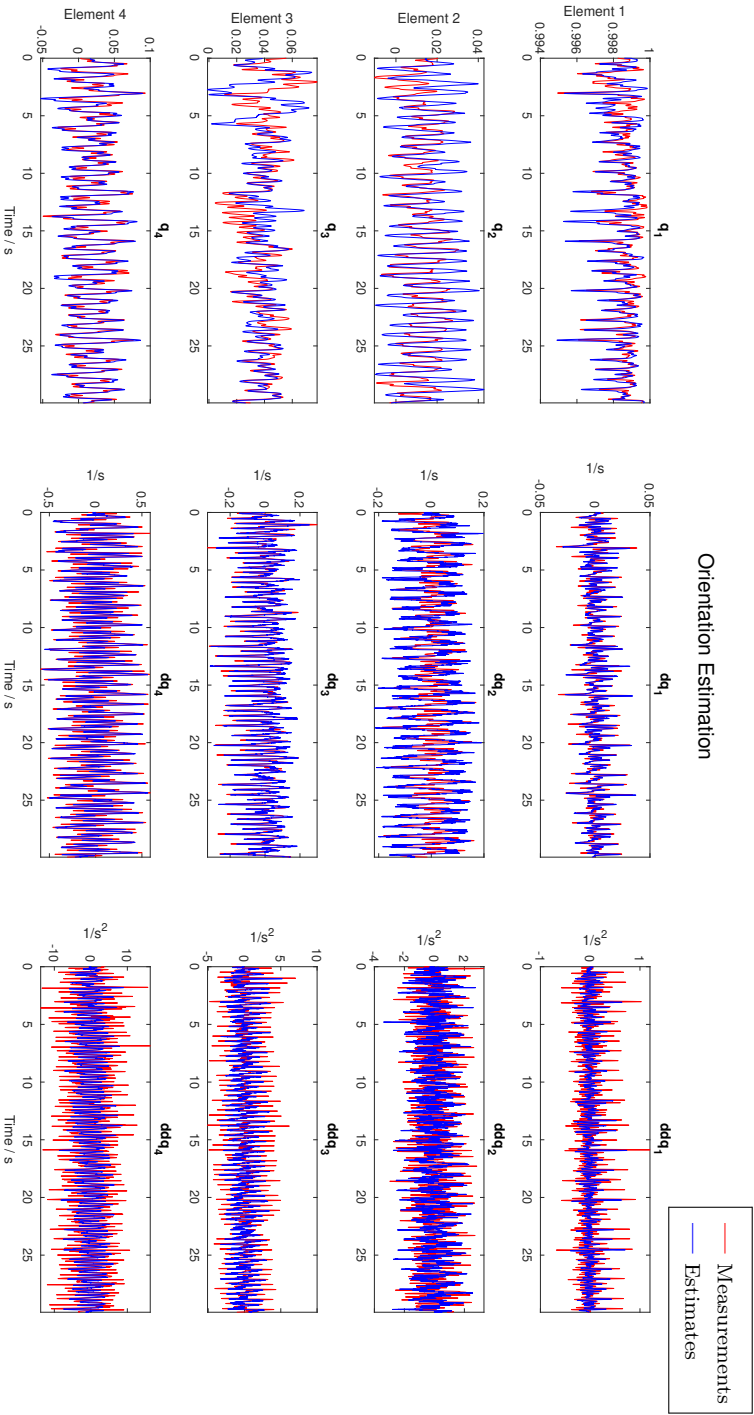


Figure C-3: Orientation estimator performance on VdZ verification data.

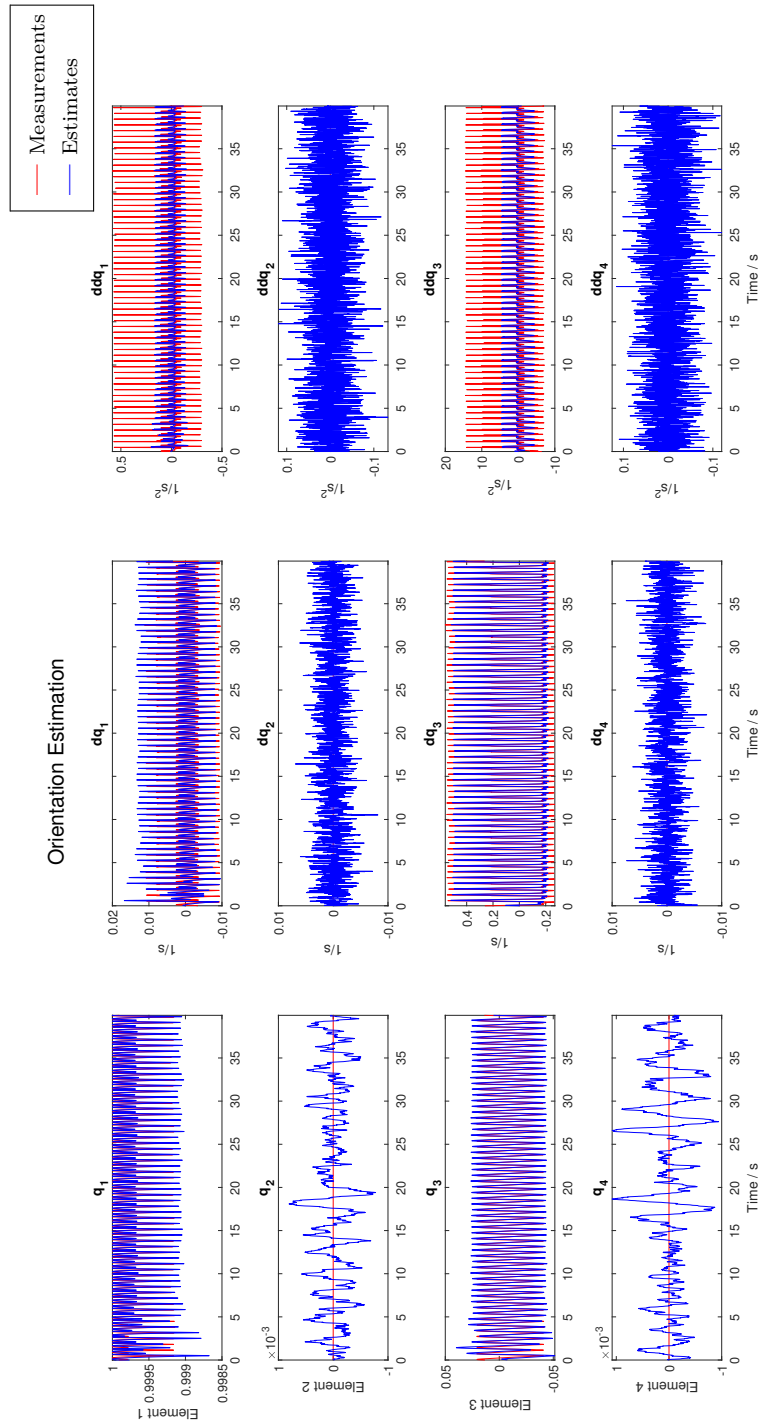


Figure C-4: Orientation estimator performance on OpenSim verification data. Second and fourth elements are zero because of planar simulation model.

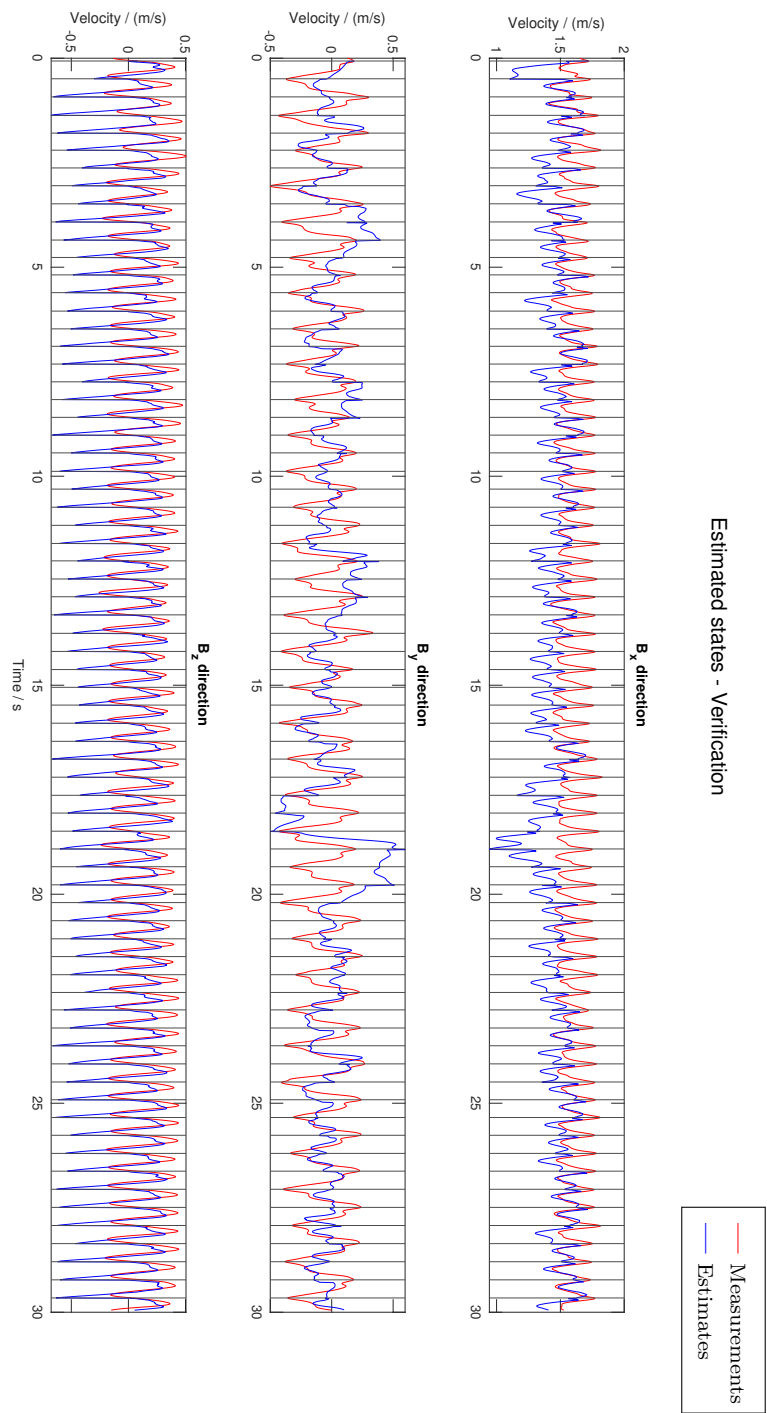


Figure C-5: State estimation results overlaid on measured trajectories of the VdZ verification dataset. Black lines indicate heel strike events.

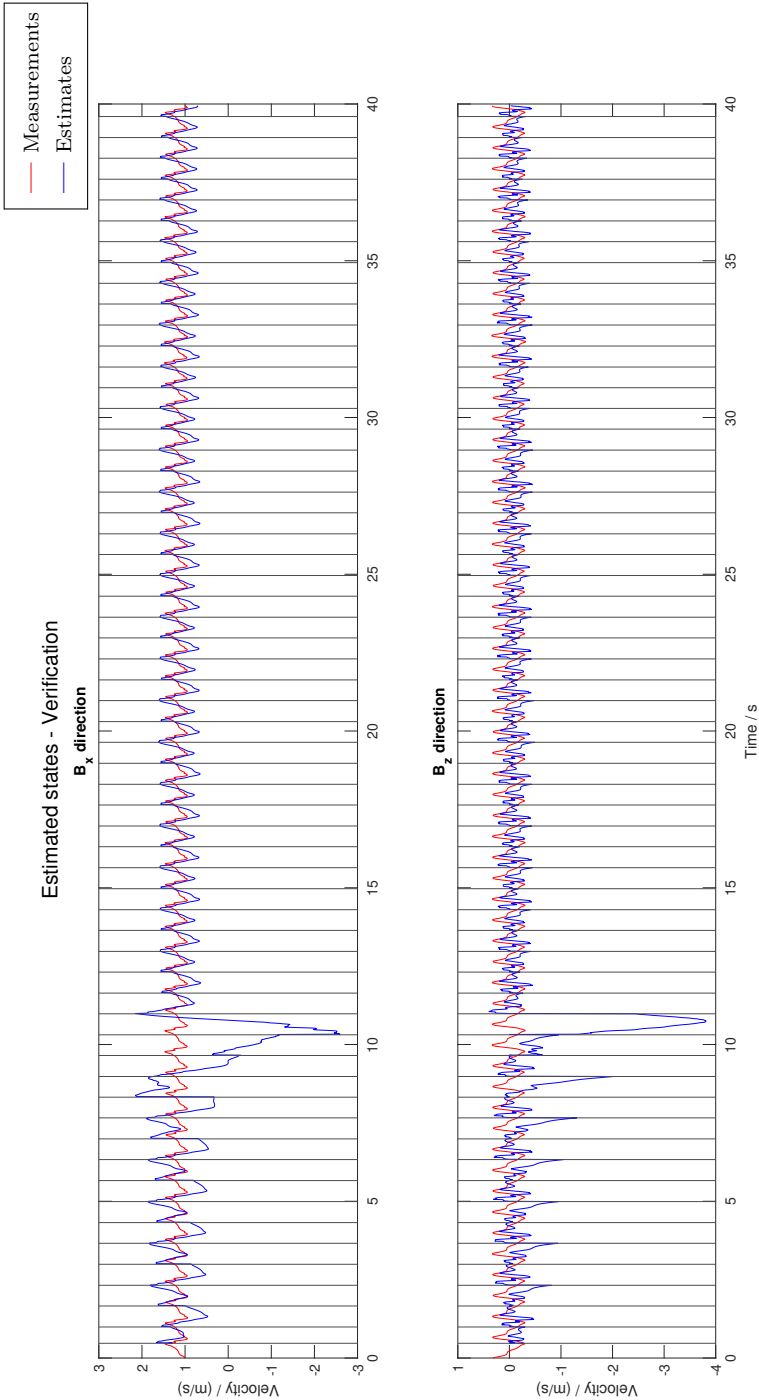


Figure C-6: State estimation results overlaid on measured trajectories of the OpenSim verification dataset. Black lines indicate heel strike events.

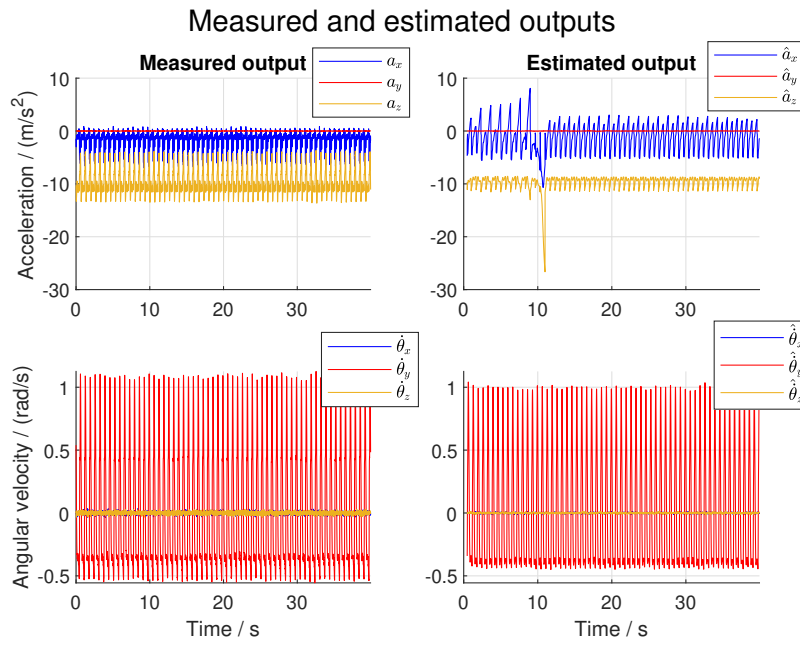
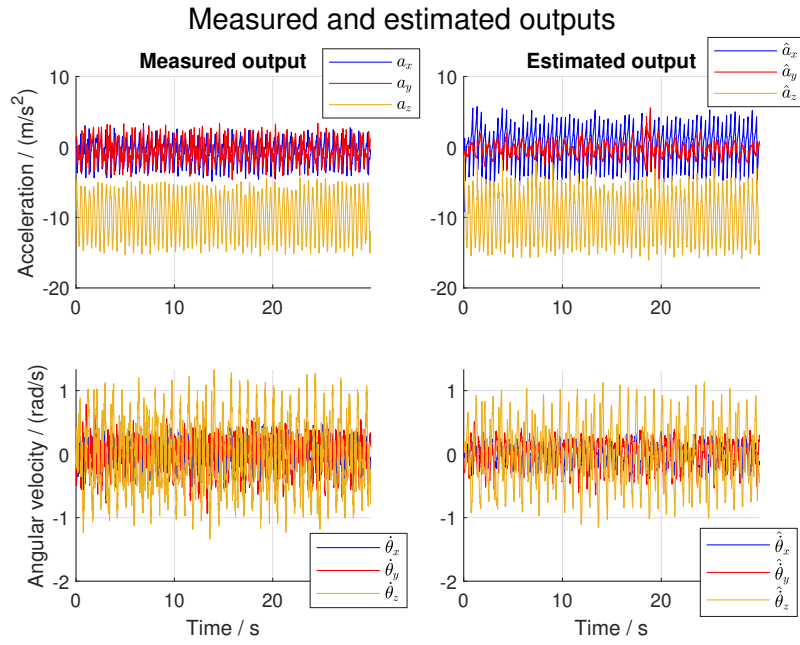


Figure C-7: Comparison between actual and estimated output measurements.

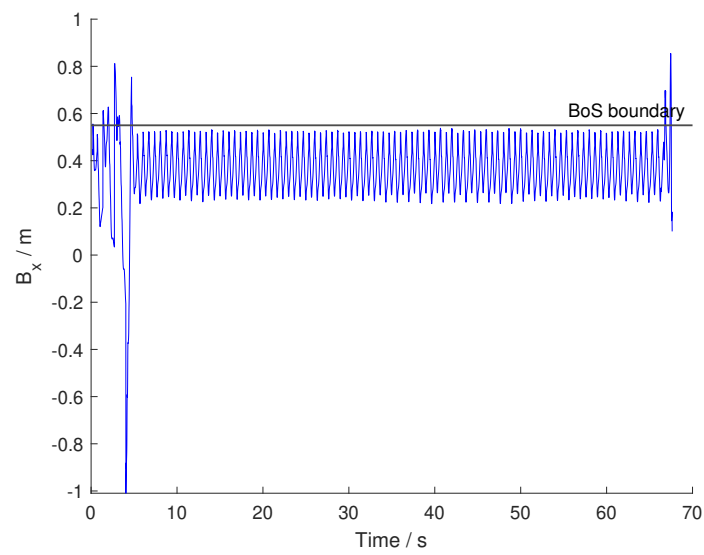


Figure C-8: XCoM anterior excursion around the time of falling compared to the BoS threshold.

Appendix D

Software User's Guide

This appendix gives a user's guide accompanying the code archived at [79] that contains the software used to obtain the results of this paper. It will exclude the OpenSim dataset and only show the three-dimensional case, since this is the default use-case. Known bugs and minor improvements to be done are tracked in [80].

D-1 Training

To train the model, open `System>Model>TrainModelAndFPE.m`. The first 62 lines have to be adjusted to fit the data structure of the gathered optical marker measurements. In these lines the optimisation weight for the model states are also defined, as well as the ground reaction force threshold used to detect heel strikes. The rest of this script should function without adjustments. Note that the script will run significantly faster if the `Parallel Computation Toolbox` in MATLAB is installed. The user will be prompted with the option to tune the initial leg parameters manually if desired, but this is often unnecessary. After the training procedure, the gait phase transitions are analysed and plotted to check there are no missing reset maps. On line 348, the value of ϕ_{sway} is entered as a function input. The trained parameters are saved on line 354.

The value of ϕ_{sway} can be determined by running `System>Model>OrientationEstimationTuner.m`. The start of this script needs to be adjusted to the data structure again. After running once, the frequency spectrum of the sway will be plotted and the value of ϕ_{sway} can be chosen. Then, on line 124 the determined ϕ_{sway} can be entered to check if the estimation works.

The Base of Support (BoS) values can either be determined heuristically, or if measurements at multiple walking speeds are available, `System>SystemBlocks>XCoM>TrainXCoM.m` can be used to analyse the measurements. It is likely that the first 50 lines will need to be adjusted to the specific datastructure. The analysis itself, however, is performed on lines 55 through 95.

Before running any script containing an observer, the explicit equations of motion have to be updated by running `System>Model>SymbolicModelFunctions>Create_EoM.m`.

D-2 Running the Detector

The script `System>ExecutableScenarios>FallDetection>DetectFall.m` is currently configured to simulate a sensor failure due to the lack of falling data. If the user inputs falling data in lines 52 to 84, the lines 20, 21, 220-229, and 343-345 should be removed. The filter parameters are defined in lines 30 to 50 and the sensor position is set on line 105. These should be the only adjustments necessary to run this script and the result will be plots of the estimated state, including detection time.

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Glossary

List of Acronyms

TO	Toe Off
HS	Heel Strike
SLIP	Spring-Loaded Inverted Pendulum
IMU	Inertial Measurement Unit
CoM	Centre of Mass
DOF	Degree of Freedom
VPP-L	Lateral Virtual Pendulum Point
VPP-S	Sagittal Virtual Pendulum Point
GRF	Ground Reaction Force
FPE	Foot Placement Estimator
KF	Kalman Filter
EKF	Extended Kalman Filter
XCoM	Extrapolated Centre of Mass
BoS	Base of Support
VdZ	Van der Zee [65]

List of Symbols

ω	Upper-body angular velocity
ς	Optimisation variable in quadratic optimisation problem for leg parameters for the model
Δt	Sampling period
$\ell_{\max}$	Estimated leg length at heel strike
κ	Integral term in PI-controller
Λ	Detection Boolean
ℓ_0	Untensioned spring length
$\bar{\mathbf{q}}$	Stacking of the 0th to 2nd time-derivative of ${}^{\mathcal{N}}\underline{\mathbf{q}}^{\mathcal{B}}$
$\mathbf{F}$	Foot position relative to CoM
$\mathbf{g}$	Gravity vector
$\mathbf{R}$	Ground reaction force
$\mathbf{r}$	Optical marker position
$\mathbf{S}$	Sensor position relative to the CoM
$\mathbf{u}$	Input signal to the EKF, consists of $\bar{\mathbf{q}}$ and $\mathbf{F}$
$\mathbf{x}$	Dedrified state of model, CoM velocity
$\mathbf{x}'$	Drifted state of model, CoM velocity
$\mathbf{x}_{\text{CoM}}$	Extrapolated centre of mass position
$\mathbf{y}$	IMU measurements, 6-DoF, three channels acceleration and three channels angular velocity
$\mathbf{Z}$	Gravity force
$\mathcal{B}$	Body fixed-coordinate system with its origin at the CoM

$\mathcal{N}$	Inertial world-coordinate system
$\mathfrak{B}$	Base of support
ω_0	The eigenfrequency of the non-inverted pendulum in the XCoM method
ϕ_{sway}	Prevalent upper-body sway frequency
ζ_L	Step length correction parameter
ζ_W	Step width correction parameter
b	Damper constant
f	Discrete-time dynamical model function
f'	Continuous-time dynamical model function
g	Measurement model function
I	Error integral in PI-controller
K	Spring constant
m	Body mass
p	Gait phase
v_0	Nominal walking velocity
${}^{\mathcal{N}}\underline{\mathbf{q}}^{\mathfrak{B}}$	Rotation quaternion from $\mathfrak{B}$ to $\mathcal{N}$
$\mathcal{O}_{\mathfrak{B}/\mathcal{N}}$	Centre of Mass, origin of $\mathfrak{B}$