Delft University of Technology

Faculty of Architecture and the Built Environment
MSc Architecture, Urbanism and Building Science
Building Technology Track

Multi-Objective optimisation in grid-shell design

Structural, Fabrication and Sustainability trade-offs in grid-shells at early-stage design

MSc Graduation Thesis
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Abstract

The following work focuses on giving an appreciation of gridshell performances at an early stage of design. It has per goal to implement a tool that will orient the user toward a set of possible "best performing" designs. The aspects of interest in this trade-off exploration are the global warming potential, time of fabrication and construction, structure performance, and envelope repercussion.
Being able to funnel different design options at the early stage is a fundamental step in reducing the climate impact of our buildings and structures. The implementation of such a tool comes along with the effort to further integrate the different actors of the AEC industry to produce leaner and more efficient buildings.

To do so, the research divides the different design features of gridshells into parametrizable blocks of code. Each block is explored to generate a wide design space and produce the necessary data to track the desired performance traits. All together a working tool is assembled and its relevance is tested. For that, a graphical interface between the tool and the user is also developed to provide a quick and easy way to identify those trade-offs. The tool limits itself to the coarse structural part of the gridshell with a diamond grid and symmetrical shapes. It proposes to use wired line designs drawn at a schematic stage as a general base for the use of the tool.

The result is a working prototype that constitutes a quick and reliable way to assess the "best performing" designs to undergo the next stage of design. It proposes a new way to look at how to connect both the early stage of design and the late prefabrication construction stages. Normally directed by two different entities, this research provides a clear direction on where and how the industry could enhance its collaboration.
Finally, the tool provides an initial working prototype that highlights the limitations of the AEC and proposes the directions for future research and implementations to cover these needs.

Preface

This report is the results of my graduation project on multi-objective optimisation on gridshell design. This report is the last step towards obtaining the title of Master of Science in Building Technology at the faculty of Architecture and the Built Environment.

This research was done with the external advisory of Octatube. I want to thank Maria Meizoso and Daniel Van Kersbergen for their time and effort to support my work and help to steer the research in a more pragmatic direction.

I want to thank my two mentors, Mauro Overend, and Michela Turrin, for their guidance to filter and develop a methodology that could sustain all the different aspects and inputs of this complex topic.

I also want to thank Rayaan Ajouz from ABT for the support and feedback regarding the development of the connection design assessment.

I want to thank all the teachers from the Building Technology track who directly or indirectly participated in building a broader view of the threats, challenges, and opportunities of the AEC industry.

Finally, I want to especially thank my parents who provided the most idyllic conditions to pursue and complete my studies.

Thomas Lindemann
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1 Introduction

Shells have been used for their ability to cover long span with a renowned material efficiency. The term shell englobes thin structures which geometry can be doubly curve in a rather organic way. Being a doubly curved geometry adds a certain amount of complexity regarding the construction planification (Charest et al., n.d.-a).

The type of shell that will be researched in this thesis are the ones that can support tension and compression loads, also known as Gridshells (Adriaenssens et al., 2014). Gridshell are a sub-category of shell which discretize the thin layer of material into struts and joints. Based on a variety of possible discretization patterns, the complexity of the joints can vary drastically. Moreover, gridshell projects can easily reach a big scale or/and a high complexity due to amount of variety between its components and the needs to produce bespoke connections. This complexity both in detailed pieces and in amount have urged the necessity to deploy computational process. Those might be used to generate, quantify the different constituent of the structure, or finally inform about the impact of such design.

Assessing the efficiency of a design, digitally plan its fabrication, estimate the resources needed are action of capital interest nowadays which can radically lower the carbon emission of a particular design. According to Istructures, doing so at an early stage of the design can have drastic carbon reduction impact.

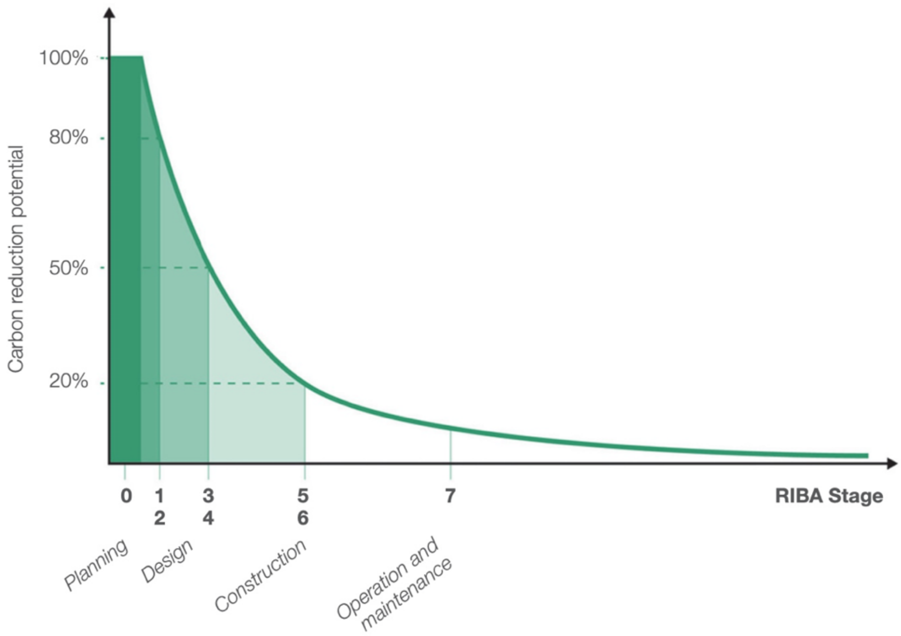


Figure 1-1 Carbon reduction potential over design stages (Istructures, 2020)

The work of this thesis aims build a multi-objective early-design tool that integrate fabrication, sustainable and structural aspects using computational design. The tool will be developed to quickly yet reliable estimate the performance of a specific design parametrizing a complex design problem, the gridshell structure.

1.1 Problem Statement

Despite their elegant design and material efficiency, gridshells can be attain high rate of bespoken elements. This can increase the complexity of the design, fabrication, assembly stages as well as making it harder to have a proper idea of the actual impact of these structures.

The built environment is in a tipping position where it should react quickly to face the challenges of climate change and reduce its impact on the environment. It has been shown that the carbon reduction potential is at its maximum at early stage of design and plummet as the stages pass. The problem here is that the Architectural, Engineering, Construction (AEC) industry is still not integrated at early stage of design. And many knowledge important for the designers at that stage is only provided at a later stage of design by

other experts. The knowledge could be purely fabrication practical such as a maximum size of a producible glass panels or more intricate such as the allowable deflection of the final structure.
Computational methods can help tackle this complexity by centralizing the production of information and reuse it for different purposes. Computational methods can be used to build integrated computational tool that allow to predict at an early stage the expected carbon value of different designs option as well as the linked fabrication complexity envisioned.
The fact that a lot of the elements of the gridshell can be bespoke and that assessing each element would be futile when deciding between different design options, the necessity to implement such methods becomes even more critical. With the use of computational tools, more sustainable structure could be created if the workflow between the designer and the contractor were more integrated.

1.2 Scope of Research

Being a building technology thesis, the function of the studied structure is kept undefined for this research as shells are commonly used to cover large span of different new or existing structure harvesting different function underneath them. The focus is rather on the carbon impact of the coarse steel structure and its related time to fabricate and assembly.
The research limit itself in creating a working generative tool that guides the designer towards an informed decision by tracking quantitative parameters throughout the generation of the different designs. The tool will be generated gridshell like geometries with a limited and stated amount of freedom in the design function covered. It will be including aspects from different field of expertise in the AEC industry into an integrated tool expert solution.
It will focus on the coarse structural elements of a gridshell. This regards the beams and their joint connections in the clothed part of the gridshell. Leaving the joints in the boundary edge out of scope. This is decided in a matter to keep the same valency and geometrical configuration between the assessed joints to limit the number of joint types of one only.
The objectives integrated in the tool will provide with quantitative metrics and only comment about how and which qualitative metrics could be important in this specific type of tool. The way the objectives are created is done as follow. The sustainable objectives will limit itself to track the embodied carbon within the structure and emitted carbon throughout the construction and provide a total global warming potential count (GWP). The fabrication objective will track the time spent to fabricate and assembly key parts of a gridshell and provide a value of total time of construction. The structural objective will track the specific performance indicators such as the structural mass ratio per covered area. In is to be noted that most of the objectives will have structural calculation as pre-requisite initial data and therefore these calculations will be in the scope of the research.

1.3 Research Question

Considering fabrication, sustainable, structural objectives in the early design of gridshell structures. These concerns translate into the following research question:

How can an early design optimisation tool integrate sustainable, structural and fabrication objectives when designing a grid-shell?

> The formulation implies that the research will focus in parametrizing a multi-variables design problem providing the what and the how to implement different design functions into an integrated tool.

The main research question is followed with sub-questions oriented towards understanding the output of the tool, assess its relevance and possible interaction between computer and Designer.

What impact has early geometric design on the constructability, assembly, and sustainability performance of a structure?

> This sub-question is intended as a quality control for the proposed tool. To distinguish the evolution type of the different metrics and ensure that the different results can be explained. It has per goal to estimate if the tool has the capacity to distinguish more or less performing designs and quantitatively temper between two similarly performing ones.

How to identify at an early phase the trade-offs between sustainability and fabrication?

This sub-question focuses on the relation that the tool and the designer can have. How to provide a display panel that inform the designer about a not met constraint or a specific design performance on a specific design. And how to store and compare the analytics of different design options that occur in the process.

How a strategy of pre-fabrication can be utilized in the process of finding this optimum?

The pre-fabrication strategy constitutes an import question when constructing a structure. Known for its drastic economy on time at the assembly process and its improvement in the quality of the different prefabricated parts, the interest is to highlight what and where are the benefit of this strategy, and how does it affect other performance traits. In general, this sub-question is intended to open the door into providing a way to test if the way the proposed tool is designed blocks the access a certain design space where a more optimized option could reside.

1.4 Objectives

The objective is, following the point made in the problem statement, to build a tool that joins the gap between fabrication considerations and early design geometry exploration. Provide a workflow that can put in relation early concept design of gridshells with their respecting fabrication time and global warming potential. The implemented tool will serve to give a rapid yet reliable insight about the different objectives assessed in the research (Sustainable, fabrication, structural) and provide a way to understand the possible trade-offs occurring between two or more design options. Ultimately, the objective is to provide an extra tool to help designers and engineers engage in concept verification to ensure a minimal impact of the environment.

1.5 Methodology

To answer the main research question, the following steps have been taken:
Create a fabrication and embodied carbon cost model: A model will be created to have a quick and reliable estimation of the parameters to validate an early design option. The research will follow the following steps.

- Research the design functions present in different gridshells designs
- Research about the design requirements in gridshells design which will help to properly inform the validity of a design options
- Gather information about the processes of fabrication via consultancies with external advisors

Create an optimisation ready model: Gather the above information into a parametric model. It will allow to automatically produce several iterations and compare them. This parametric model will allow to study the repercussion of the different parameters on the previously cited criterions.

To do so, grasshopper will be use as the main environment alongside other free and open-source plugins to generate and analyze the design iterations.

The sub-questions will be answered providing the following:

Sub-question 1 - Impact of early geometry variation on performance:
What impact has early geometric design on the constructability, assembly, and sustainability performance of a structure?
The impact that early geometries can have the fabrication, sustainability, and structural performance will be assessed via a set of three tests done of the tool itself. Each test will isolate specific design variables which will be tested on an array of different input value. On the back end of the generation model, the geometry as well as the score in the different aspects will be compared and plotted to quantify the impact. The result of the three different tests will altogether allow to reflect on the behavior of the tool and the consistency of the outputted performance score. The variation of the generated design space will allow to reflect on the relevance on the tool in a real-life case scenario.

Moreover, the results of each test will be analyzed following same methodology:

- A first graph plotting the indicators related to the embodied and emitted carbon of the design will allow to quantify the environmental impact of the different solutions.
- A second graph plotting the indicators related to the fabrication and assembly of each design will allow to quantify the impact on the time spent for the different fabrication steps for each iteration.
- A third set of graphs will clarify and explain the impact variation by plotting the total fabrication time and carbon count alongside other accessible metrics. Like so, certain steps or major change in the scores can be explained by one or a combination of highlighted factors. The behavior of the tool and the performance score tracking can then be understood and therefore validated.

Q2 Sub-question 2 – Identify the trade-offs between objectives at an early stage :
How to identify at an early phase the trade-offs between sustainability and fabrication?
Related to how accessible the tool outputted results can be assimilated by the designer to efficiently inform the decision process, the answer to this question will provide the creation of a graphical user interface (gui). The followed methodology will then search for heads-up display methods to provide the designer an efficient visualization during the design. A graphical extension will be implemented to save outputted metrics to then graph, as the design options grows, the scores of the different aspects of performance for each of the selected design iterations. The collection of graphs will allow to quickly visualize which design performs better in the different aspects.

Sub-question 3 – Reflection on a fixed process of the tool:
How a strategy of pre-fabrication can be utilized in the process of finding this optimum?
Intended as a fixed process in the proposed tool, answer this question will require change the way this part of script works.
A preliminary work will then have to be done to construct two different pre-fabrication strategies and constructive methods. With that, the main script will be cloned and used to produce two new variants. The tool metrics will be adapted to display accordingly the results of the different versions. Together, the variants will be tested with the same design variables and the scores and other metrics will be saved and used answer the question.
The set of graphs and comparison methodology done in the sub-question 1 will be repeated to analyze the different variants. The result will help defining if what is assumed as fixed process in the tool should be opened to different variables.
The question is intended to study if the pre-fabrication method could be intended as design variables that has a quantitative influence on the different objectives.
If positive, this confirms that a strategy of pre-fabrication can be used in the search to find an optimized relation between the stated objectives. Further research in its nature and implementation method would help fine tune trade-offs between fabrication time and assembly time with the related carbon count and structural performance.

1.6 Report Structure

The report will follow the following steps:
The chapters 2 and 3 are literature review which is used as anchor point. The first one focuses on the Gridshell on a physical level. Informing about the use of gridshell and the parameters and design functions to consider when designing this kind of structure. The second one focuses on the design of gridshells at a computational level. Looking for the different performative building simulation and the environment in which they can be used in order to build an integrated tool.
The chapter 0 will describe the method development, starting with an exploration of the different processes that could be uses and the design functions that can be covered. Following with the quantitative framework assessment definition. It will be then followed by the assumption and limitations used in the final proposed method as well as a stepped explanation of the different modules of the tool. This chapter will serve to partly answer the main research question.
The Chapter 5 will serve to estimate the impact and relevance of the proposed and built tool. It will answer the sub-questions and with it complete the main research question.
After that, chapter 6 will provide the last words, following recommendations for future research.

2 Gridshells in theory: Parametrizing a design space

2.1 Gridshell typology development

Shell structures can be defined by three-dimensional objects where one dimension is relatively thin compared to the other two. It can be simplified topologically to 2D surface (Nakahara, 2003). These types of structure are generally used to cover a span in most optimize pathway and approach the thrust line in order to minimize the surface thickness (Adriaenssens et al., 2014). These structures are commonly illustrated with the work of Gaudi in the Sagrada Familia, the concrete shell of Felix Candela or membrane Frei Otto in their research of hanging model (Figure 2-1).

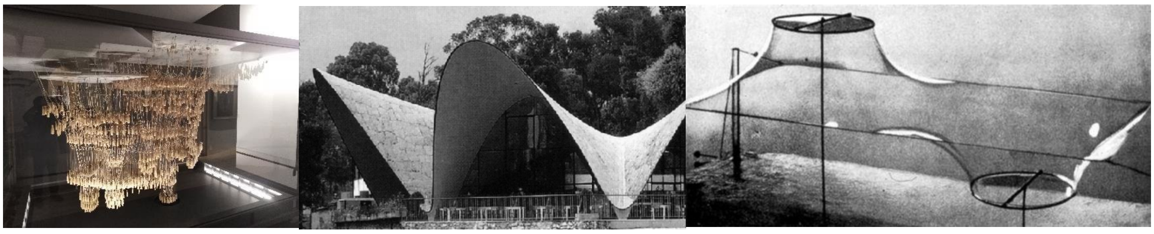


Figure 2-1 Hanging model of Gaudi, Concrete shell of Felix Candela, Soap surface of Frei Otto

In the family of shells, as stated in the introduction, this research paper focus on the shell with rigid elements, allowing for compression and tension loads. This eliminates the flexibles membrane and the masonry arches (Adriaenssens et al., 2014). It will also focus on the shell where the surface is discretized by nodes and struts called Gridshells. In this category on gridshell, there are two categories.

The elastic gridshell which is assembled flat on site with long elements called laths and uniformed nodes. Once the full grid mounted it is force fitted into the final optimum shape. A common example of those structure is the Mannheim multihalle in Germany (Figure 2-2). It is commonly a regular rectangular grid that is covered with a flexible cladding as the whole structure present no planar faces (Fernandes et al., 2020).

The rigid gridshell is a more faceted version of the gridshell. The division in faces happens following periodic patterns, which difference will be explained later on (Mesnil et al., 2017). The struts and nodes can be prefabricated and assembled in patches to ease the assembly process on site. This method required more interest in the last year as it can increase the complexity and variation in pattern, node and introduce more freedom in the façade design. It also introduces more precision and quality in the end result.

The complexity of these rigid gridshell have made its development almost unattainable until CAD techniques became more common. With the development of the later, it became the most common type of gridshell constructed for high end design (Charest et al., n.d.-b). The material and type of nodes are varied and represent a good field to run design exploration. A common example of such construction can be British museum, in UK or the Taiwan Botanical Garden domes in China (Figure 2-2).

Figure 2-2 Maanheim multihalle, British Museum, Botanical Garden domes

The rigid gridshell will be the structure category of interest throughout the research for several reasons:

- The shape reaction can be generalized in the same way than other shell and therefore can use the similar form finding methods and structural analysis
- The pattern of the gridshell itself can differ which offer more flexibility to the designer
- The complexity of the nodes makes it a good candidate for the envisioned research
- The material type can vary in use within the joints but also in struts
- Fabrication criteria and construction method can be assessed and discussed depending on design objectives

All these aspects will serve as structure for a more detail review specially focused on the rigid gridshells.

2.2 Shell surface

2.2.1 Surface generation

The gridshell can be shaped, like any shell, in a vast diversity, with one or more direction of curvature. They come in two categories, the regular shapes (differential geometry) and the free formed shapes displayed below:

One or the other can have their local or general curvature defined by the product of two crossing curves with the minimum and maximum curvature, called principal curvature.

If the product is positive, the surface is considered positive gaussian curvature and can be called clastic or synclastic. The surface is then doubly curved

If the product is negative, the surface is considered negative gaussian curvature and can be called anti-clastic. The surface is then doubly curved

If the product is equal to zero, the surface is considered zero gaussian curvature and can be called cylindrical or monoclastic or plane if both curvatures are zero. The surface is then simple curved or planar regarding the plane.

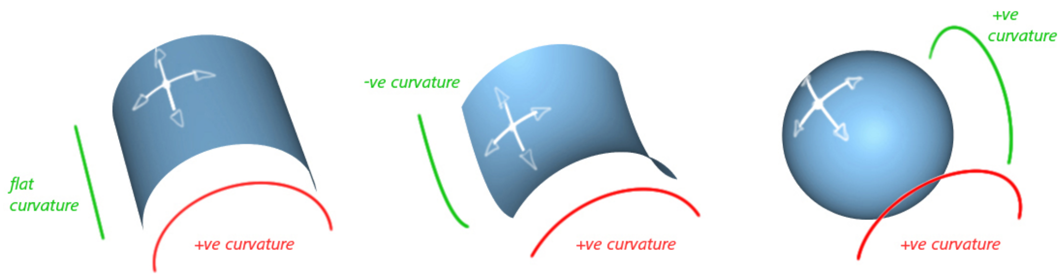


Figure 2-3 Gaussian curvature types in surfaces

Regarding a regular curvature, the generation can be done in two ways:

The surface of revolution: This type of generation is done by revolving a curve around an axis of rotation. The revolved curved is called meridian and its orthogonal curve called parallel. Examples of the surface are the dome (a), the cone (b) (with or without an oculus), the hyperboloid(c), and cylinder (d):

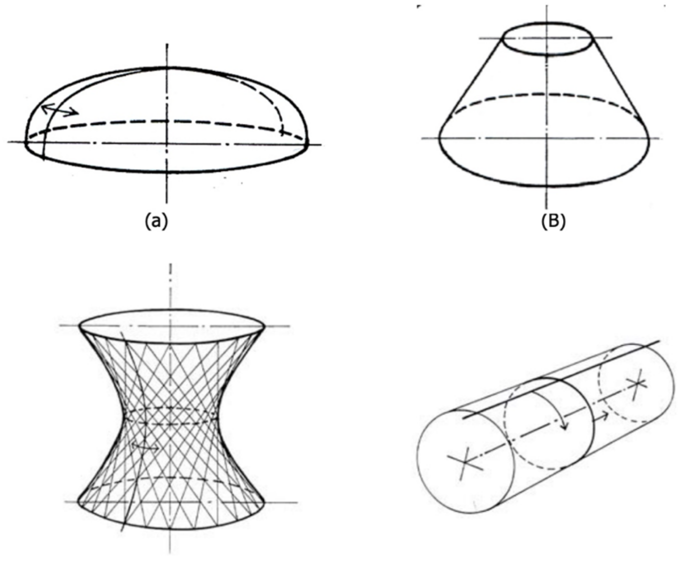


Figure 2-4 Surfaces of revolution (Toussaint, 2007)

The surface of translation: The generation is done by sliding one curve along another. The base line for the translation is called the generator its curvature defines the type of surface and recall to the same logic of the previous paragraph about gaussian curvature. If the generator is a straight line, the surface described is a plane or simple curved surface also called as cylindrical paraboloid (b). If it has a curvature, the doubly curved surface is called either Elliptical paraboloid (a) or Hyperbolic paraboloid (c), for respective positive and negative curvatures.

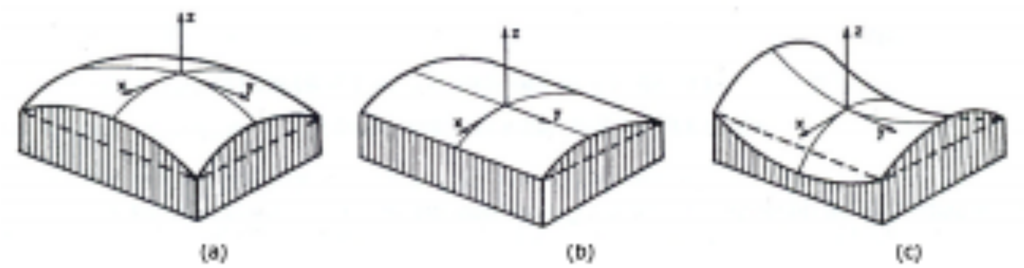


Figure 2-5 Surfaces of translation (Toussaint, 2007)

The freeform surfaces: This kind of surfaces can be seen as a series of positive and negative gaussian local surface stitched together. The parametrization of such surface has been the subject of exploration in many fields such as texture mapping or remeshing but it is quite complex and out of scope for this research (Adriaenssens et al., 2014). Most research limit themselves to produce tools to generate complex grid geometries on already made free-form surface (Oliveira, 2019)(Winslow et al., 2009).
Another way to achieve freeform surface is to relax a subdivided planar surface with diverse anchor points. The relaxing process, which will be detailed in chapter 3.2.2, allows to achieve surface such as the Solemar bath roof, in Bad Durrhaim.

2.2.2 Designers' role in the creation of the surface

The interest here is that most of the illustrated surfaces have already been used in gridshell constructions (see Table 3-1). The main question regarding the designer point of view is how he can interact with the design tool. Within the variety of the possible shapes, how the designer can interact together with an early design tool to fine tune, within an acceptable range, the different objectives that the project is subjected to. How the design tool can answer with quantitative evaluation each of the iterations that a designer qualitatively defines.

2.3 Mechanical definition

2.3.1 Plane stress in flat surface

In order to understand how curved surface work, it helps to undergo how flat surface mechanically work. We could also start with a curved line and go from arches to curved surface. Both approaches will bring some more understand to how shells work (Toussaint, 2007).

In a plane stress there are two normal stresses in the x and y direction noted σx and σy and their corresponding shear stress perpendicular to their normal, respectively, τxy and τyx. At equilibrium of moment between the normals means that $\tau xy = \tau yx$.

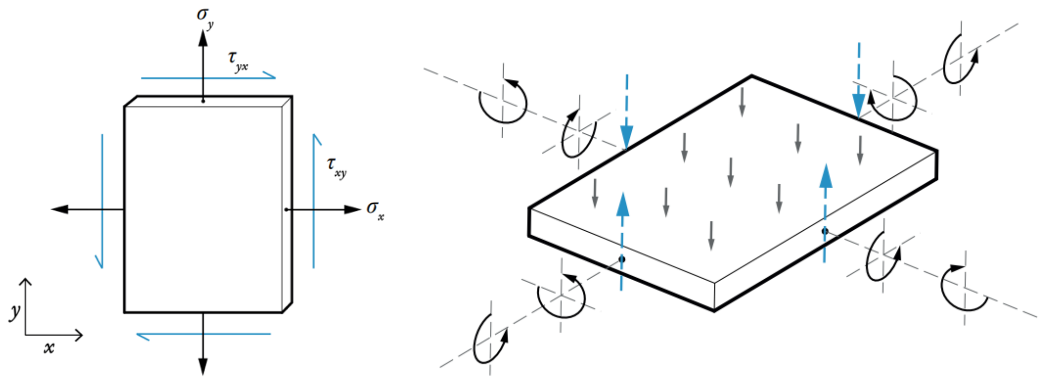


Figure 2-6 Plane stress and plate bending diagram (Adriaenssens et al., 2014)

In-plane stress correspond to axial stress in an arch, opposed to bending stress. This stress is central to the shell theory. As it is much easier the bend a plate than to stretch it, there are also shear stresses perpendicular to the plate.

2.3.2 Equilibrium in curved surfaces

In the shell, there is still three components with three related equations of equilibrium. Two are extracted from the plane stress, tangent to the shell. The third equation is normal to the shell, and its load represent the membrane stresses multiplied by the curvature of. The load is in kNm^{-2}, the membrane stress in kNm^{-1} and the curvature in m^{-1}. And unfortunately, even though we have three unknowns with three equations, the solution lies upon the shape and boundary conditions of the shell. It is a complicated area in mathematics, a simple way to solve it is by using coordinates find the horizontal equilibrium and then find the vertical equilibrium, more on that in chapter 3.2.2, following graphics static method.

Stiff structures, like shells, carry their design loads primarily by axial or membrane action, rather than by bending action. Their response usually involves very little deformation prior to buckling and total collapse. And the most efficient a shell is, the more sudden the buckling collapse.
Most of the load carried by shell are considered inextensional deformation. Eigenvalue buckling is generally used to estimate the critical buckling loads of stiff structures, but it is effectively impossible to estimate by hand, and still difficult to estimate with a computer.(Adriaenssens et al., 2014)

When transformed to gridshells, some local buckling effect can happen at the nodes, (Oliveira, 2019):
Shell-like buckling: Is commonly due to lack of bending rigidity in the nodes or connection points. But it could also happen in the beams-like elements if they are not properly sized.
Node rotation buckling: Produced by the dis-alignment of the loads within the node creating a in plane torsion moment. In that situation the rotation of the node reduces the length of its connected elements and the shell exit the state of inextensional deformation.

Numerical Analysis are then with physical testing the only practical solutions. The former use in most cases finite element methods. They can be applied to shell, and gridshells, by ability to be discretized in curvilinear coordinates models. It can use both an implicit method to find equilibrium such as the stiffness matrix, or an explicit method such as dynamic relaxation. The understanding of shell theory can only help in the choices of the shell shape and to interpret the computer models.

Funicular method can be used as a physical testing to achieve specific load cases for shell that by the material used cannot allow membrane or tensile stress. The method works as illustrated in Figure 2-1.Proceeding a horizontal mirror on a hanging chain model in pure tension produce a compression only arch. Similarly, the curved line can be extended to a curved surface and therefore produce a compression only shell.

Useful definition:
Inextensional deformation: An applied bending force that preserves unchanged the length of each line element and the measure of curvature at every point

Eigenvalue: Eigenvalue buckling is generally used to estimate the critical buckling loads of stiff structures (classical eigenvalue buckling).

2.4 Pattern

Often related to the shape generation in the design process, the pattern is, in gridshell construction, the filling method used to discretize the target shape. The pattern itself will have a strong mechanical and fabrication constraints and should therefore be included in the research. The type of pattern that can continuously fill a surface are considered periodic patterns and can be extracted from the tessellation pattern principal's m on Euclidian or hyperbolic geometries.

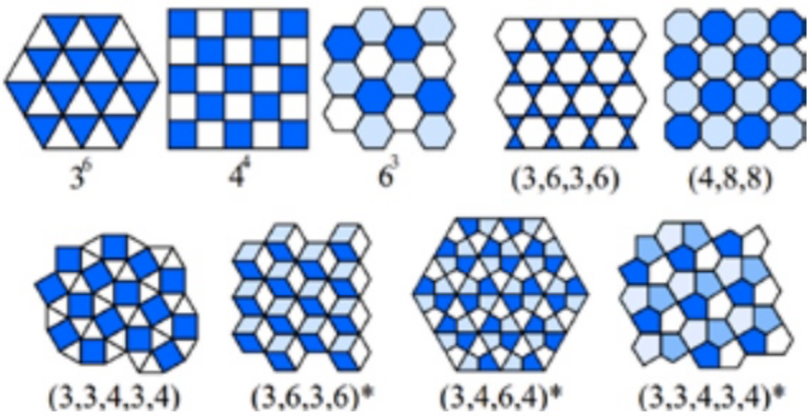


Figure 2-7 Periodic patterns with their vertex configurations (Mesnil et al., 2017)

For the structural part, as seen in the previous chapter, the pattern should allow the gridshell to have a bending rigidity, normal loads, as well as membrane action, axial loads. The choice of the pattern can have a significant influence on the weight of the final structure. It can also have an influence of the stress repartition within the gridshell. The same shape can have different pattern topologies using the same pattern base. Depending on the pattern used, singularities can make the topology different (Oval, 2019).

In the Kagome pattern, beside the faces varying between hexagons and triangles, the regular vertices have a valency of four. With a quad mesh the topology can vary without changing the pattern depending on the pole or partial pole addition, creating a singularity. The amount of valency of the singularity will depend on the shape in which the pattern is constructed. A valency of 5 or 7 in the Kagome pattern is singular (Ayres et al., 2018).
Singularities are vertices where the amount of valency is changed compared with the regular vertices of the used pattern. A singularity point will induce a different local flow and can reduce panels distortion and stress distribution on the overall shell (Oval, 2019).

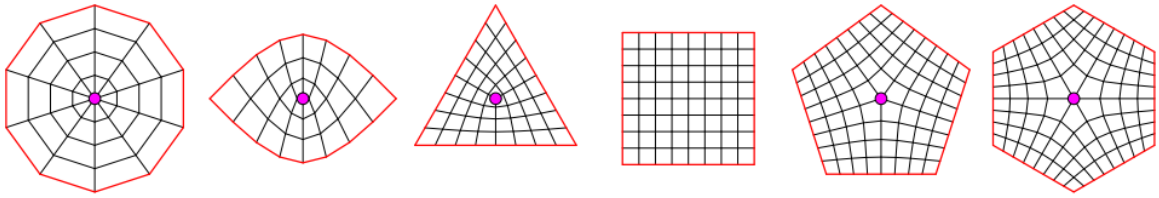


Figure 2-8 Singularities in quad meshes with a different valency count (Oval, 2019)

A pattern topology is unique if it cannot be deformed into an already existing topology. For structural design, the pole points are key. And a shape can be constructed with the same pattern using different pole points.

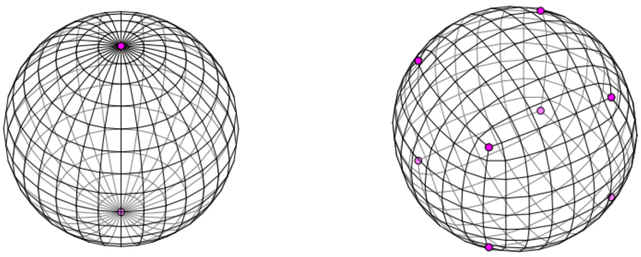


Figure 2-9 Same shape with different pattern topologies using a quad pattern (Oval, 2019)

For fabrication facets planarity and shape ratio accounts for the main requirements that can evolve in priority depending on the design choice. In the case of glass envelope, the planarity of the panels is key for the economy of the project (Schlaich & Schober, 1997). The choice of the pattern should also be according to the type of shape and therefore the direction and amount of offset that the pattern will suffer, and its consequence on the planarity of the final faces (Mesnil et al., 2017).

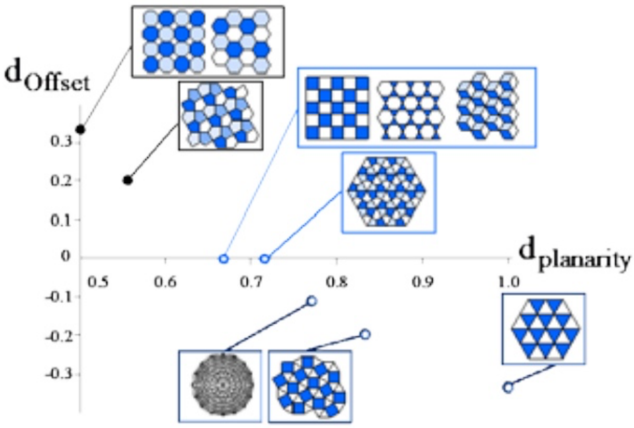


Figure 2-10 Classification of patterns regarding fabrication constraints (Mesnil et al., 2017)

The shape ratio can be seen in Figure 2-9, where a sphere with two poles has its meridians make the shape ratio goes higher as it gets closer to the poles, instead more subtle deformation when 8 poles points are introduced. Panel planarity can be taken into account using a parallel transformation in the form finding method (Mesnil et al., 2017). Different strategies of pole positioning can vary in the trade-off between mechanical compliance and panel planarity (see Figure 2-11).

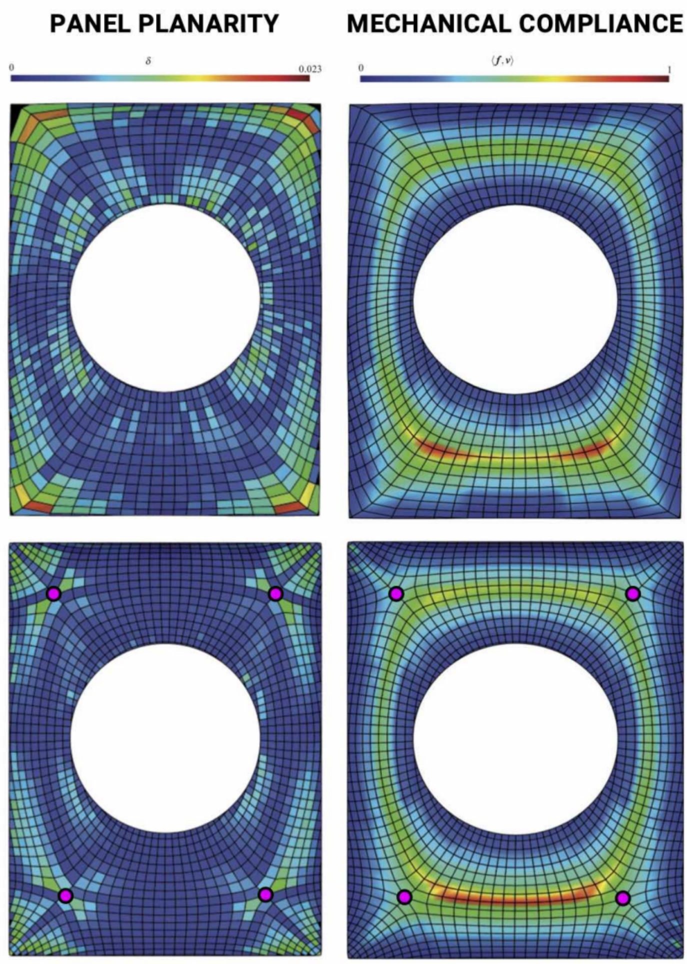


Figure 2-11 Comparison between two pattern topologies and their tradeoff between panels planarity and stress repartition (Oval, 2019)

2.5 Constituent elements in a grid shell

As the thesis goal is to integrate the fabrication process, their energy consumption and overall embodied carbon of a design iteration. It is important to explicitly count the different parts of the gridshell. Amount, material type and complexity should be able to assess in order to estimate the cost in material, time, energy consumption and carbon count.

2.5.1 Node classification in gridshells

When the shell is transformed into a gridshell and discretized into struts and nodes, the in-plane shear assumed in a continuous shell to activate the membrane action should be supported by the grid itself. The infinite loads paths are therefore limited to the ones that the grid describe. Depending on the type of pattern chosen, these new loads paths will create different forces and moments throughout the gridshell. The pattern type itself have inherent structural capacities and will behave differently when subjected to these forces. With a triangular pattern build, the grid will automatically be a rigid grid. Non rigid grids like quads or Kagome will need the transfer they in-plane stress through their nodes and will need rigid nodes to distribute the forces. (Van der Linden, 2015).
Another way to prevent such nodes is to provide other option to rigidify the grid by adding cross cables or secondary struts systems. This type of grid is calling a three-way grid, in a certain way a triangular grid.

In non-rigid grid the design of these joints become a crucial step. In order to structurally perform, the stiffness of the joint need to be properly designed.
Joints in gridshells can be classified based on their stiffness and on their moment capacity. The stiffness coefficient α which is the ratio of the node stiffness with the members stiffness and the moment capacity β the ratio of the maximum moment of the joint with the maximum plastic moment of the member connected to it. This gives us a combined classification as follow:

– Rigid: $\alpha \geq 5$ and $\beta \geq 0{,}5$
– Semi-rigid: $\alpha \geq 5$ and $0{,}5 < \beta < 0{,}01$ or $0{,}05 < \alpha < 5$ and $\beta \geq 0{,}01$
– Pinned: $\alpha \leq 0{,}05$ or $\beta \leq 0{,}01$

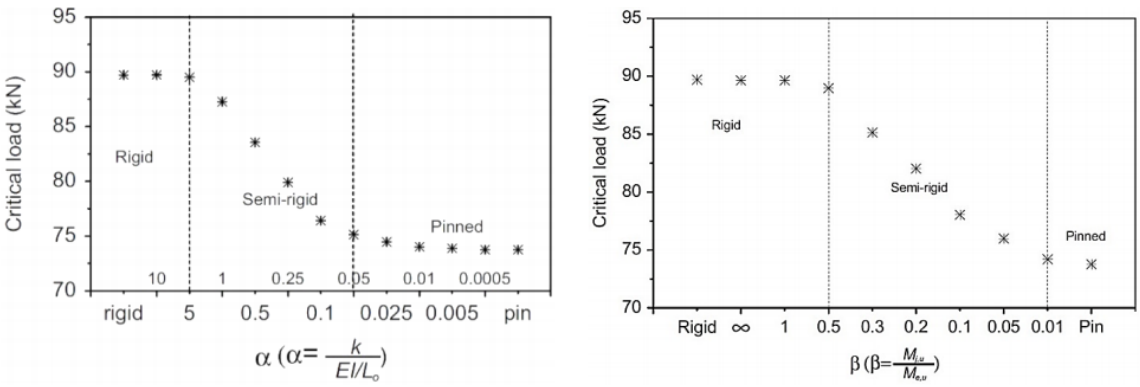


Figure 2-12 Critical load with different determination coefficients α and β (Van der Linden, 2015)

In the same paper, it is stated that a joint with α >> 5 and β>>0,5 behave the same way in a gridshell than a joint with α =5 and β=0,5. It is therefore more advantageous to produce the latter as the weight and manufacturing complexity will decrease. The searched of the adequate rigidity of the node can have impact of the structural performance and therefore the amount of material needed for a similar design (Isufi, n.d.).

Knowing their position, design requirements and number could also help the optimisation process to test only the node at the known key location instead of producing all the nodes in each design iterations. It could make the whole process more efficient without losing so much precision on the node assessment. The vertices reporting the same curvature, with the same load requirements and with similar angle distribution will produce, in number, the same type of node, see Figure 2-13. Digital fabrication, added with designer input, can in a later stage produce one by one the nodes of the elected best performing design.

Type	Typical zone	Axial forces	Moments	Number
1	Highly disturbed membrane field	Medium/High	High	200
2	Mediumly disturbed membrane field	Low/Medium	Medium	300
3	Pure (non-disturbed) membrane field	Low/Medium	Low	1800

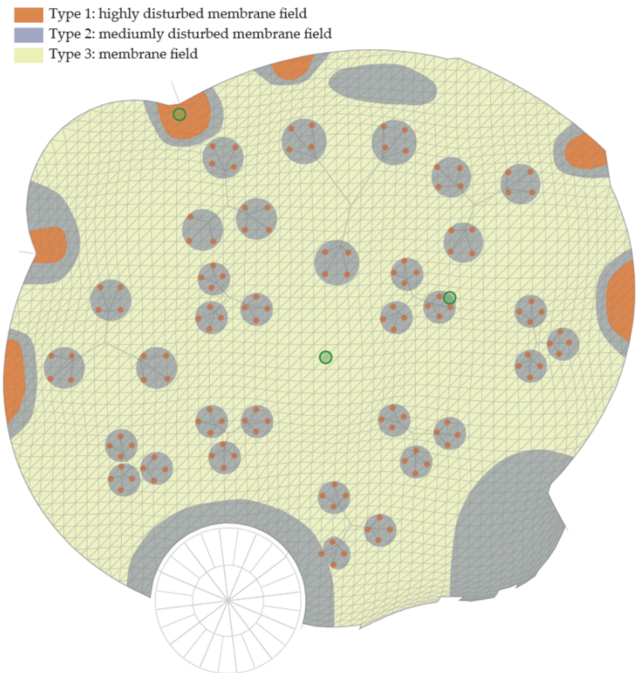


Figure 2-13 Overview of type of joints repartition in the Zlote Tarasy gridshell (Van der Linden, 2015)

Even though the last figure is made by visual inspection, some factors could be considered in order to automate the process. Therefore, only a dozen of nodes might be needed to be generated and extrapolated to the rest, in order to make right and quick assessment of the designs.

2.5.2 Node generation

In order to produce a parametric model that can provide a wide variety of design and provide an equal distribution between the factors that will play a role in the structural and embodied carbon optimisation, it is needed to be able to get as much information as possible regarding the final shape, function, and material of the intended node. The challenge could be to provide a node generator that allow a wide variety of shape curvature and pattern option such as singularities and valency adaptability, different material, and profile sizes. For that, a flexible way to approach node and beam connection, in term of material and detailing method, needs to be found. Also, a topological generator needs to be parametrized in order to follow the singularities.

A research on parametric modelling and digital fabrication for gridshell nodes propose this approach and produced a methodology where the nodes and beams are potentially reusable by computing the connector between them (Oliveira, 2019).

The research uses common circular profiles and clamped octanorm profile as node as main elements and produce costumed 3D printed connector based on the direction, and best incoming angle of the incoming into the node. It provides an effective response to triangular grids with vertices with valency of 6, but we could imagine it working with valency ranging from 2 to 8 as far as the octanorm node is concerned. In the Figure 2-14, the connector between the beam and the prefabricated octanorm profile is decomposed in three parts, (a)Proper connection to node, (b)Angle adjustment to allow proper connection, (c)proper connection to beam element, circular profiled steel profile in this case.

This strategy can be applied in this thesis as subcategory of the main elements of the gridshells. 1.Beams 2.Beam to node connector 3.Prefabricated node . The limit of such approach is to find a prefabricated node general enough that several solutions can be found with it, or to find a list of possible nodes and connector methodology in order to resolve the different design in comparable way. The risk here, if only one node type is chosen, is that the node design becomes the bottleneck in the whole optimisation. In terms of appearance and innovation of design this method as a finite method can be limited.

The tradeoff to think about is the ability to propose the designer different methods between the big families of joints type with an embodied carbon cost goal for it and let him develop it further.

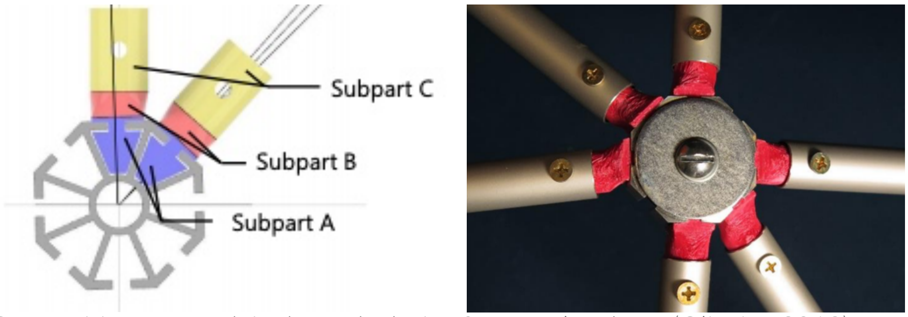


Figure 2-14 Composition approach in the node design for complex shape (Oliveira, 2019)

Open to more options, may be less specific but might be enough if it has as bases working sizes.
In order to allow any specific singularities to be tested throughout the parametric designs and research, the node should be topologically generated. A second paper on that topic propose to build a parametric model for topological generation (Rognes & Faugstad, n.d.).
The goal is to deliver a design parameter for the designer to be guided in the next steps, not to provide a fully resolved gridshell with imposed node type. This is what have been done in this digital workflow, where the several structural factors of the node have been integrated into a parametric model (Rognes & Faugstad, n.d.).

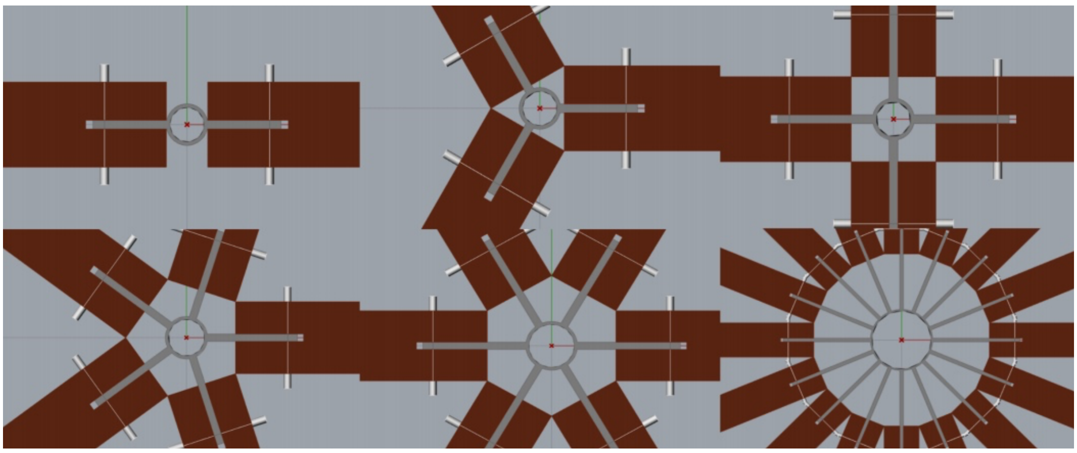


Figure 2-15 Topological generation following the number of valency in the node (Rognes & Faugstad, n.d.)

This method is a good approach for our research. It could still implement the method of composition of node + connector + connection to beam type, but in a more general way. Moreover, this parametric method proposes other parameters in order to align the beam element properly with the node, introduce 3D motion of the connector plate to adapt of specific cases.

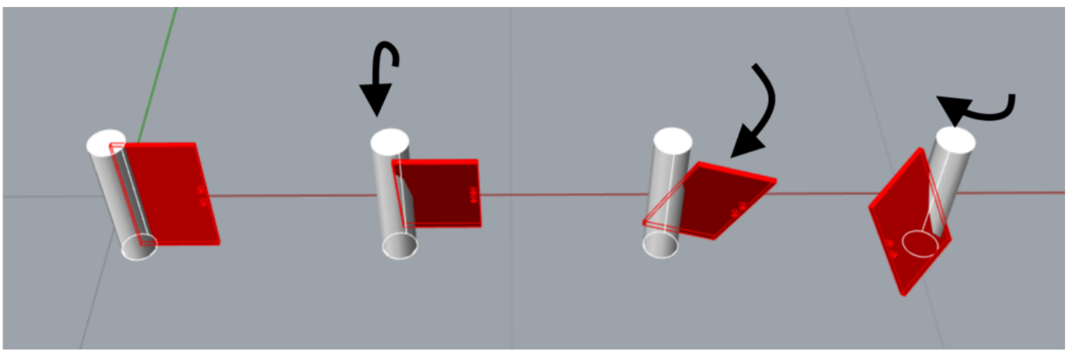


Figure 2-16Three rotational motions orienting the connector plate. From left to right; unrotated, roll (x axis), pitch (y axis) and yaw (z axis) (Rognes & Faugstad, n.d.)

A table would be needed to be stated and the possible outcome drawn out to ensure that the possible produced nodes reassemble the ones present in the real world and plausible design outcomes. It should include the different composition method and compatibility process. This account for the design feature and will be depicted in the method implementation, chapter 0.

2.5.3 Carbon Estimation

2.5.3.A Embodied Carbon

The ability to assess, the number of nodes, their requirements, the amount of material, complexity of manufacturing of a gridshell makes early-stage calculation of the embodied carbon an attainable goal.
The scope of the research will limit itself to the series A of the life cycle assessment. From cradle to gate steps only, accounting for Raw Material supply, transport, manufacturing, and assembly. These stages accounts for the carbon count only, later on, the scope for the fabrication estimation will be given.

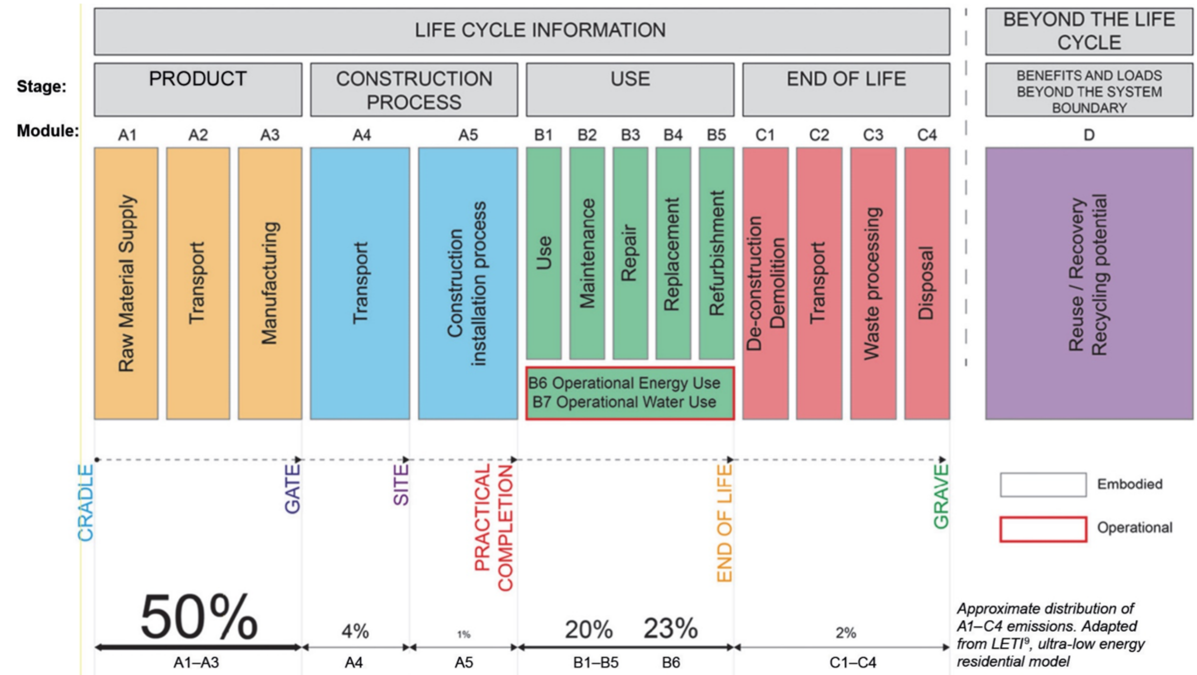


Figure 2-17 Life cycle Analysis: Stage of assessment (Istructures, 2020)

These few steps, also known as product stage, are the cause of 50% of the emission in a common project. The other big factor resides in the operational energy use and maintenance. It is therefore important to be able to minimize this part of the process as it have the most impact on the overall score Embodied energy score (Istructures, 2020).
The fundamental principle in embodied carbon calculation resides in the multiplication of the amount of material present in the project by the carbon factor of these material:

$$\text{material quantity (kg)} \times \text{carbon factor } (\text{kgCO}_2\text{e/kg}) = \text{embodied carbon } (\text{kgCO}_2\text{e})$$

Figure 2-18 Calculating embodied carbon (Istructures, 2020)

The goal of implementing such approach in the thesis is dual. First as a practical tool to orient decision toward less harmful solution. Secondly to create the need for designer to ask their different supplier to provide them with the EPDs of their different product.
An EPD is an environmental product declaration, which is bound to have passed a strict process that will ensure a Global warming potential (GWP) score measured in Kg CO_2 equivalent (KgCO_2e) per kg or m^3.

The certification of an EPD is a process that only third party approved entities can provide. It requires four main steps:

- Product category Rules (PCR): Rules and requirements of the product in order to compare it with similar functioning product afterwards
- Life cycle Inventory (LCI): Inventory of flows form and to nature
- Life cycle environmental Impact Analysis (LCIA): Assess the type and extent of the environmental impact. LCI and LCIA form together the Life cycle analysis (LCA).
- EPD registration: Compose, verify, and publish the EPD with an expiration date (5 years)

In the case of not having EPDs, the analysis can be done with empirical values of embodied carbon factors provided by the toolkit itself. It is to be noted that the carbon factor, for product stage, is dependent on the location, carbonation of the local grid and sourcing of the material, see Figure 2-19. In general, for a finished project, a common global warning potential per square meter for construction usually attain values between 150-400 $kgCO_2e/m^2$.

Material	Type	Specification/details	A1–A3 ECF ($kgCO_2e/kg$)	Data source
Steel	Reinforcement bars	UK: UK CARES Sector Average EPD	0.76	Ref. 18
		Worldwide: Worldsteel LCI study data, world average	1.99	Ref. 15
	PT strands	Assume the same as reinforcement bars		
	Structural sections	UK open sections: British Steel EPD	2.45	Ref. 19
		UK hollow sections: TATA EPD	2.50	Ref. 20
		Europe (excl. UK): Bauforumstahl[e] average EPD	1.13	Ref. 21
		Worldwide: Worldsteel LCI study data, world average	1.55	Ref. 15
	Plate	UK: Assume the same as 'structural sections — UK open sections'		
		Europe (excl. UK): Bauforumstahl average EPD	1.13	Ref. 21
		Worldwide: Worldsteel LCI study data	2.46	Ref. 15

Timber, excl. carbon sequestration[f,g]	Manufactured structural timber	CLT, 100% FSC/PEFC	0.437	Ref. 15
		Glulam, 100% FSC/PEFC	0.512	Ref. 15
	Studwork/framing/ flooring	Softwood, 100% FSC/PEFC	0.263	Ref. 15
	Formwork	Plywood, 100% FSC/PEFC	0.681	Ref. 15
Aluminium	Sheet	European consumption (31% recycled content)	6.58	Ref. 15
		Worldwide consumption (31% recycled content)	13.0	Ref. 15
	Extruded profiles	European consumption (31% recycled content)	6.83	Ref. 15
		Worldwide consumption (31% recycled content)	13.2	Ref. 15
Glass	General	Generic	1.44	Ref. 15
	Toughened	Generic	1.67	Ref. 15

Figure 2-19 Embodied carbon factors (ECFs) (Istructures, 2020a)

Different EPDs databases can be used depending on the country in which the design takes plane:
- Institut Bauen und Umwelt: https://ibu-epd.com/en/published-epds/
- Ins Environdec: https://www.environdec.com
- EPD Ireland: https://www.igbc.ie/epd-search/
- BRE Green Book Live: http://www.greenbooklive.com/
- Carbon Leadership Forum: http://www.carbonleadershipforum.org/resources/
- ECO Platform: https://www.eco-platform.org/list-of-all-eco-epd.html
- Transparency Catalog: https://www.transparencycatalog.com/
- Climate Earth: https://www.climateearth.com/greenmixselector/

2.5.3.B Carbon Emission

Creating our own EPDs require to reconstruct the logic of calculated the GWP. For that we need to know with what energy the different processes are done. As the research is done in Netherlands this will be the grid emission used in the tool. One could imagine what could happen if the country of pre-fabrication would change for example taking producing in France, reducing by six and a half the fabrication emission of the different parts.

Grid emission CO2eq per kWh in Netherlands: 328,4 g
Grid emission CO2eq per kWh in France: 51,1 g
(*European Environment Agency*, n.d.)

2.6 Manufacture processes

The research mainly focus on the welding volume as it is a good indicator of the complexity of the parts in a steel structure (Ajouz, n.d.). When performing a fillet weld the size is given by the leg. The speed of usage has been found to be around 100 cm3/h, (Ajouz, n.d.). The power consumption was found to be 12kWh.

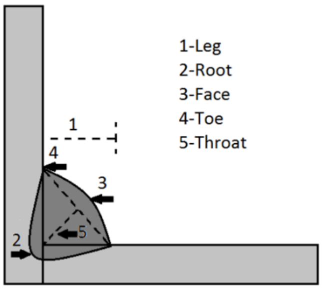


Figure 2-20 Diagram of the fillet welds parts name (Wikipedia, n.d.)

As the produced geometry will have various angle of arrival of the beam into the joints, the volume of the fillet weld will vary with the same leg size. It is important for both the fabrication time and structural matters to correctly estimate the weld volume. The size of the leg is calculated using the area of the face around the profile and applying the different forces that the weld is subjected too. The tool will then need to accurately estimate the area of the face around each welded connection as well as the angle of arrival.

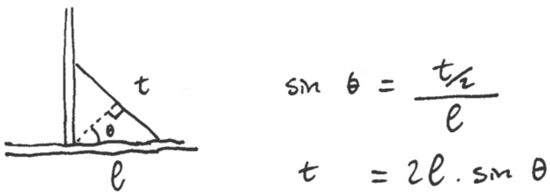


Figure 2-21 face length calculation in fillet welds

2.7 Design Criteria and Design requirements

Design Criteria constitutes the different parameters that a design includes in its search to distinguish the design options between them.
Design requirements are those specifications and design criteria contained in the Contract that specify the minimum acceptable technical standards and define the limits within which the design of the Project shall be developed further. The design criteria are joined into objectives which make part of the multi-objective optimisation.

In order to stipulate relevant objectives a literature review has been done on the different parts, function, and behavior of gridshell at different scale and their relative complexity to achieve together. Derived from the review done through these two chapters the following table of criteria has been drawn, see Table 2-1. The design requirement will be constituted with different coefficient of importance using the same criteria and will act as metric to quantify the performance of a specific design and consider it as "best preforming". The creation of the design requirement balance can be tweaked in importance depending on the designer choice.

General metrics	Spatial Aspects	Structural	Fabrication	Embodied Energy/Material effiency
Nbr vertices (valencies)	area	Max Serviceability limit state (SLS) deflection	Variation beam length / length disparity	Ratio structural mass to covered area
Nbr of faces	Density of grid / transparency	Max Ultimate limit state (ULS) stress utilisation	curvature of panels	Amount of material edges
Nbr edges	Usable area ratio	ULS first bucking load factor	Straigthness of panels (skewness)	Embodied carbon energy edges
Total edge length	height	Max displacement	Shape ratio of panels	Fabrication energy comsuption
area of panels	Volume	First buckling load factor	Variation Area of panels	Embodied carbon energy nodes
Nbr of nodes (classification)	Appearance of node	Strain energy	Angle tilt between segments and node	
Nbr vertices (valencies)		Stress repartition/distribution	Angle disparity of entries at nodes	
Nbr edges		membrane field disturbance	fabrication complexity	
Nbr of faces (two levels)		Rigidity of grid	time to manufacture	
		Moment at nodes	cost of manufacture	
		Load at support		
		Cross section height variations		
		weigth of node design		

Table 2-1 Possible criteria to integrate under each design objective

3 Gridshells in optimisation: Computational Design workflow

A computational design is set to be created in order to include, hierarchize, and monitor the design requirements and achieve the design criterion. This second part of the literature review has per goal to provide an intelligible framework describing the possible processes and practical steps that can be implemented in the proposed methodology. Provide a clear vision on the steps to take in order to produce a multi-objective optimisation on sustainable, structural and fabrication aspects of design in a gridshell.

3.1 Parametric model of a Design scenario

3.1.1 Parametric modelling: Object definition and Hierarchy of relations

Parametric modelling is often used in computational designs as a design process to subdivide a problem in different variable of a common equation, but in this case the set of values given to the variables will combine into generally a geometry proper to that set of variables inputted. It is commonly composed of two aspects: defining objects and their relationship and define parameters (range of numbers) to formulate these relationships. (Pan, n.d.-a).

The creation and relation of the objects created inside a parametric model are subject to an order and therefore a hierarchy of structure. This means that the model itself must be defined beforehand, knowing the constraints and the freedom of the parametric model (Turrin et al., 2012). The relation of hierarchy between elements can be illustrated by a directed acyclic graph, where a different set of values will produce another iteration of the parametric model (Christofides, 1975). Note that the creation of flowcharts prior to the parametric model retake the same idea but with a stepped approach of the generation done compared to the relation of values inputted.

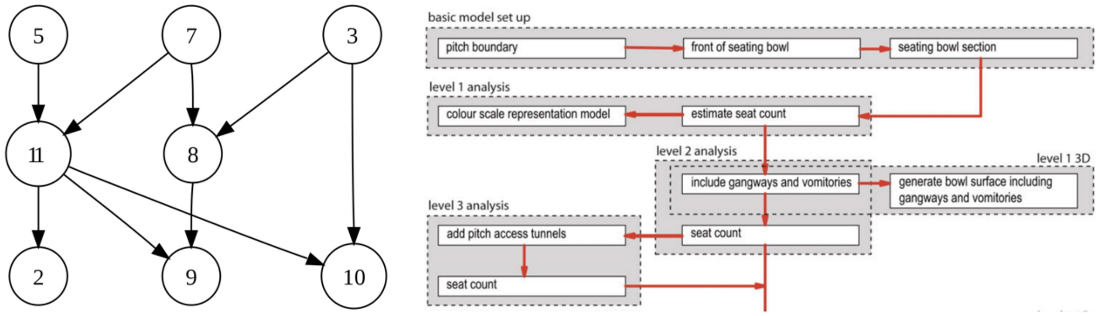


Figure 3-1 Directed acyclic graph, and flowchart reporting hierarchy in a parametric model (Pan, n.d.-a).

The array of possibilities that a parametric model can create by tweaking the different input values in called the design space. It represents the number of possible unique iteration from which the design can choose before selecting the final design iteration. The size of the design space will be as previously presented depending on the number of objects introduced in the model, their relationship and the range of the possible values that will quantify this relationship. As an overall design process, it is used to transcribe a design problem into smaller significant parameters that will produce quantifiable difference. The diversity of produced geometry should be controlled and in tune with the goal of the research or design. This quantifiable difference can be evaluated to help taking a decision on a specific type of geometry within the design space. The capacity to choose will depend on quantitative requirements produced through measuring the model itself or form the output value of building performance simulations (BPS).

The goal of the parametric model is to provide a good representation of the already constructed gridshells and their possible design variation in order to provide an extended and diverse design space. The creation as well as the assessment of that design space will require the use of BPS which can be integrated inside the parametric model as proper generation step. The use of BPS will be further exposed in section 3.2, and the quantification of the well preforming design will be explained in section 0.

3.1.2 Review on existing Gridshell construction

A study of the relevant and accessible elements on different constructed gridshell has been made on 11 different gridshell. This account for a non-representative demonstration of the state of constructed gridshell but inform on the different elements that are worth categorizing and at a later stage parametrize. Doing so helps understand nearly whole the shape types have been used. Beside the hyperboloid, their all share the function of coverage of an underlying area. It is interesting to note that the pattern commonly used in those examples are the quads, and the triangle but that a triangular pattern is often directed to triangular shapes when it comes to the structural paths or the cladding method (typically for glass). This can be explained as we discussed in section 2.5.1, as the necessity to rebuild a rigid grid in order to maintain a membrane effect in the shell. When it comes to the cladding, it can be paired with the lowering of the cost. Various design of node has been found with different rigidity system connected, such as secondary struts connection or secondary node to cable tensioning attach. These features are interesting and provide an interesting way to utilize quads and proposing rigidity methods afterwards. The construction is the less informed columns on this table and would increase the capacity of comparison between the different design as the assembly method is a serious matter regarding time, and therefore cost of a project.

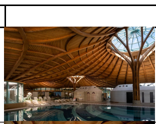
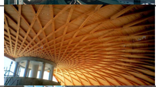
Project		Shape	Pattern	Structure	Joint type	Material	Construction
Solemar, Bad Dürrheim		Free form, with oculus	Quads and singularities. Tangancial pattern	Meridian ribs and Annular ribs. Supported during the span	cross lap joints	Skylight roof, glulam ribs	assembly on site
Piscine Saint-Quentin , Yvelines		Doubly curved, revolution, (synclastic)	Diamond grid, Quads, three poles	two way grid	cross lap joints	timber	assembly on site
Toskana Thermal Springs, Bad Sulza, Germany		Free form,	Quads	two way grid, punctually supported	coss lap joints	glued laminated timber	assembly on site
Toskana Thermal, Bad Orb		Free form	Quads	rectangular section, two way grid	cross lap joints	Glued laminated timber,HESS Timber	assembly on site
Capital C, Diamanterbeurs, Amsterdam		Free form, (nearly elliptical paraboloid)	Quads, two poles	Hollowed rectangular section, two way grid	welded & 3D lasered to snapfit	steel, glass	prefabricated patch, connected on site
AFAS Experience Center, Leusden, Nederland		Dome	triangular grid	circular section, three way grid, edge entirely supported	welded and bolted	steel, glass	assembly on site
Taiyuan Botanical Garden Domes, Taiyuan, China		Dome	square grid	three crossing layers, four way grid	notched + cable node	doubly curved glulam, steel tensioner	prefab panels and assembly on site
Kreod pavillion, London, UK		cylindrical	Kagome	two way grid	screwed	kebony Timber	Assembled in house
Chiddingstone castle orangery, Kent, UK		synclatic oval Elliptical paraboloid	quad structural, triangular grid for glass.	four crossing layers, four way grid, edge entirely supported	finger joint (lathes), steel clamping plate joint	Green chestnut, steel cables	assembly on site
British museum, London, UK		free form, (synclastic)	triangular grid	three way grid, edge entirely supported	welded	steel square profiles	partly prefab, partly welded on site
Crossrail place, Canary wahrf, London, UK		Simple curvature, cylinder shape	triangular grid	three way grid, (secondary ribs connection)	welded steel plates	timber, ethylene tetrafluoroethylene (ETFE)	assembly on site
Camp adventure tower, Denmark		Hyperboloid	Diamond grid, quads	three way grid, edge entirely supported	welded	weathered steel	assembly on site

Table 3-1 Constructed gridshell review of different design criteria

3.2 Building performance simulations

The building performance simulations can be implemented in order to achieve a check on a design criterion (structural analysis) or as part of the optimisation itself by providing a pre-defined set of change to either expand the design space (topology exploration) or achieve the theoretical optimum (form finding). Their outputs can be included as indicators to assess the different objectives of the optimization.

3.2.1 Topology exploration

The exploration of the topology is one of the first decision to take when designing a gridshell. Topology Exploration is a vast field, and the correct choice can have drastic effects on the performance of a particular design. Achieving a knowable relation between design goal and pattern topology, requires a few steps. In Robin Oval work, (Oval, 2019), a stepped increase in complexity help achieve a direct relation between input and measurable output. During its work each step reuses the knowledge built from the previous ones and introduce new concepts and methods in order to ensure that the measured results can be explained by a specific change in the initial topology during the exploration.

A few preliminary steps are needed to set up the initial conditions within which the whole process articulates. The boundary definition of the working area can be defined, as an example, with an outer boundary and one or more inner boundaries (oculus).

When the boundary is set, and the starting surface created, a process of skeletonization is done. A skeleton provides medial axis between two or more boundary edges. It defines the points that are equidistant from the boundaries, like a Voronoi operation on a set of points. This skeleton is composed of lines when equidistant of two edges or a node when equidistant of three or more edges.

It is generated by using the Delaunay mesh triangulation on the boundary edge subdivided into vertices. This computes the smallest possible triangles within the surface. The triangles that have two valid adjacencies are part of a segment of the skeleton. The triangles that have one valid adjacency account for the end face. The triangles that have three valid adjacencies account for nodes. The topological skeleton is then created joining the segment until the nodes and end points of the initial surface. The topological skeleton serves as bases for the surface decomposition, which can then be divided using a Catmull-Clark surface subdivision.

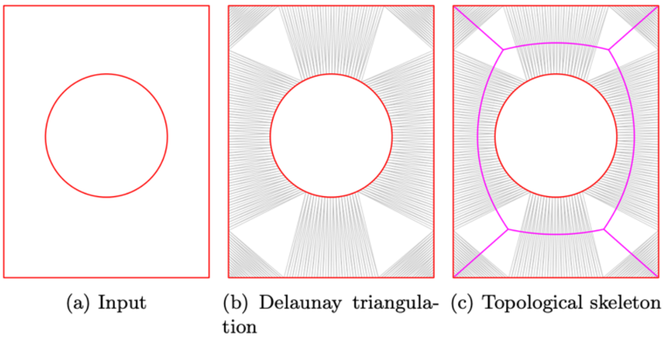


Figure 3-2 Creation of topological skeleton (Oval, 2019)

Geometry encoded:

The geometry encoded provide the possibility to add points and curves in the input as geometry accounted in the skeleton. This will have a series of repercussion on the outcome. These additions are called features. Point features allow orienting a pattern around a pole. Curve features allow orienting the pattern along with several directions. The placement, addition of these features is rather heuristic at this stage but can have a drastic change on the outcome performance.

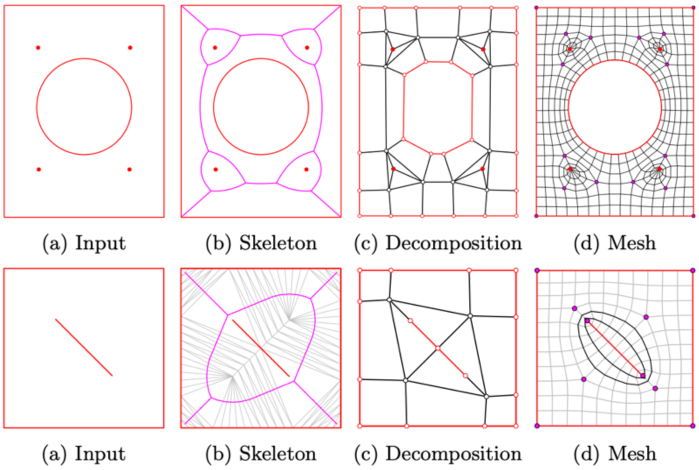


Figure 3-3 Point (top) and Curve (bottom) feature influence on pattern creation (Oval, 2019)

Graph encoded:
The generated primal skeleton can be defined by strips, sequence of second and third edge or second and fourth edge of the quad. These strips are inherent from the skeleton and graph can be drawn out which strip crosses with which other. With this is coupled a grammar that can add and delete the strips with selected boundary to boundary edge selection.

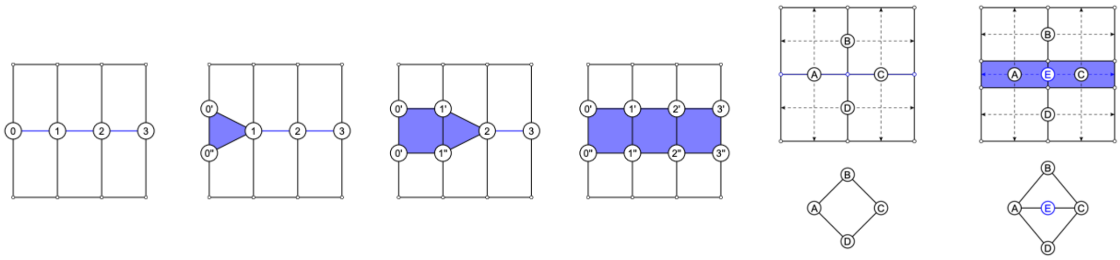
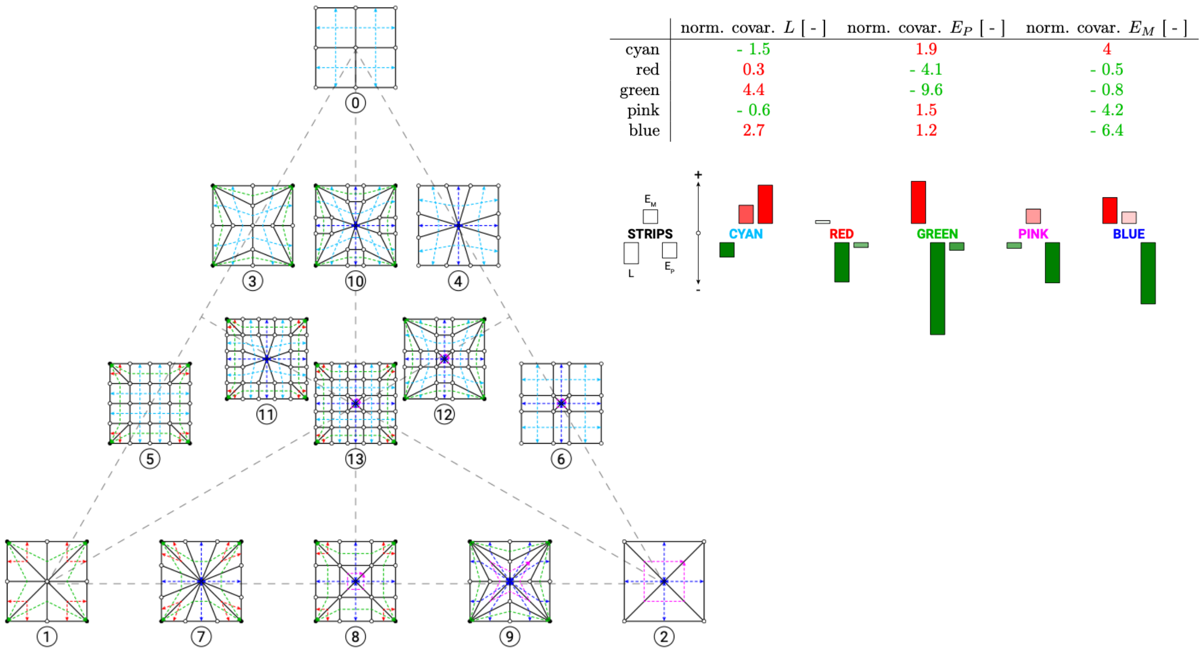


Figure 3-4 Adding a strip to a pattern and the formation of strip graph (Oval, 2019)

With this strip method, other coarse mesh can be freely created to achieve other pattern distribution. The leap here is that starting from the initial skeleton, the strip nature can be mapped using the isomorphism analysis of network X in python and linked to their influence in the pattern performance. The isomorphism of a strip is done by comparing topologically the strips between them.



	norm. covar. L [-]	norm. covar. E_P [-]	norm. covar. E_M [-]
cyan	- 1.5	1.9	4
red	0.3	- 4.1	- 0.5
green	4.4	- 9.6	- 0.8
pink	- 0.6	1.5	- 4.2
blue	2.7	1.2	- 6.4

Figure 3-5 Combination of strip structure organized by common colored strip and the influence of the different strips on the design requirements (Oval, 2019)

Here the presence of cyan strips tends to improve fabrication metric L but worsen the static metric Em and Ep. . With this covariance relation between strip and performance metric, the topology creation can be guided depending on the desired trade-off.

String encoded:
The string encoded metho creates even lower grammar encoding the strip rule. It provides a relation between phenotype (observable physical traits) and genotype (genetic information). It can formulate an interesting implement if the tool is to use this pattern creation method without having to generate the options of pattern by hand. The third encoding method will be studied in the next steps even though its implementation, as warned by its author, still require some manual processes.

Here the use of topology exploration using the proposed method is a heuristic generation, which means that is will have to be done prior to the optimisation, most necessarily requiring to come back to test other pattern configurations. It has been developed with quad mesh only, which means that if other pattern type needs to be tested, another method will need to be used. All the methods and process are gathered in library usable with the Compas Environment in Python called singular.

3.2.2 Form finding methods

Several tools can be used to proceed with the dynamic relaxation on a surface. Such method is used to achieve what has been described as the optimum path for the surface. It relies on the hanging model experiments with numerical techniques. The techniques reviewed here are the Thrust Network Analysis (TNA) for Compas environment and Kangaroo plugin for grasshopper.
The method uses projective geometry, duality theory and linear optimization (Li et al., 2016). The TNA decompose the equilibrium method is several steps. It first uses graphic static to approach the structural model with a Force Diagram and Form diagram. It uses form and force diagrams to calculate the equilibrium of pin-jointed structures graphically.

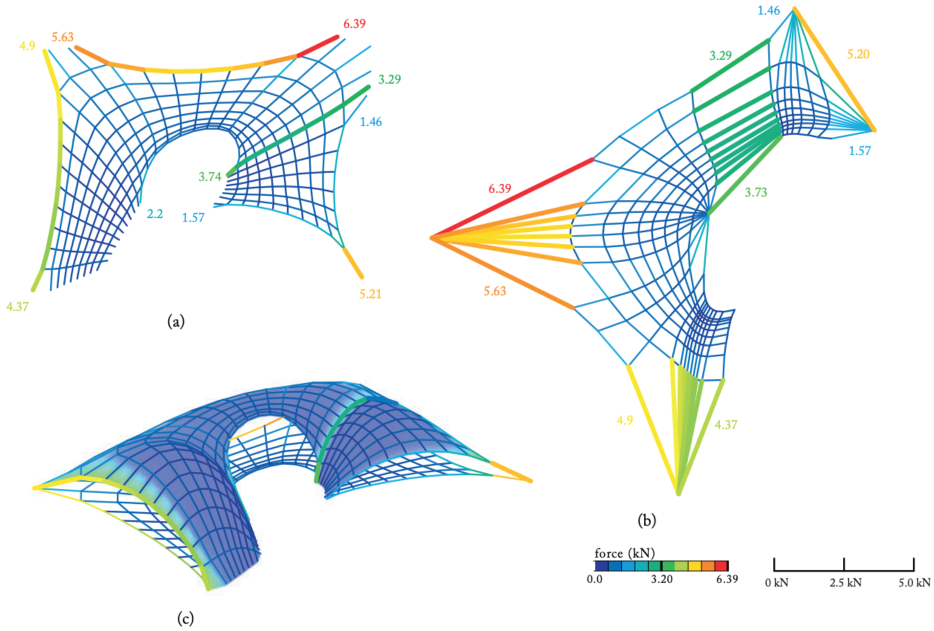


Figure 3-6 (a) Form diagram (b)corresponding force diagram (c) Compression only thrust network (Adriaenssens et al., 2014)

Both diagrams are reciprocal diagrams that are topological dual. The form diagram is the funicular model to be shaped, and the loads applied to each of its member can be directly measured on the corresponding element in the Force diagram. Controlling either form or force diagram allows an informed design exploration of the structural model. This method controls the shell stability as it is hung on the boundary condition supports, further test such as bucking, deflection must be studied in further test. As it is a two-step process, this method allows to first resolve the equilibrium with the horizontal loads and then find the vertical loads. This allows to study the reaction of the structural model based on a specific load type. The TNA process will result in any force, form diagram combination a specific convergence points, resulting in a network spatial wireframe. The TNA process can be visualized as follow,

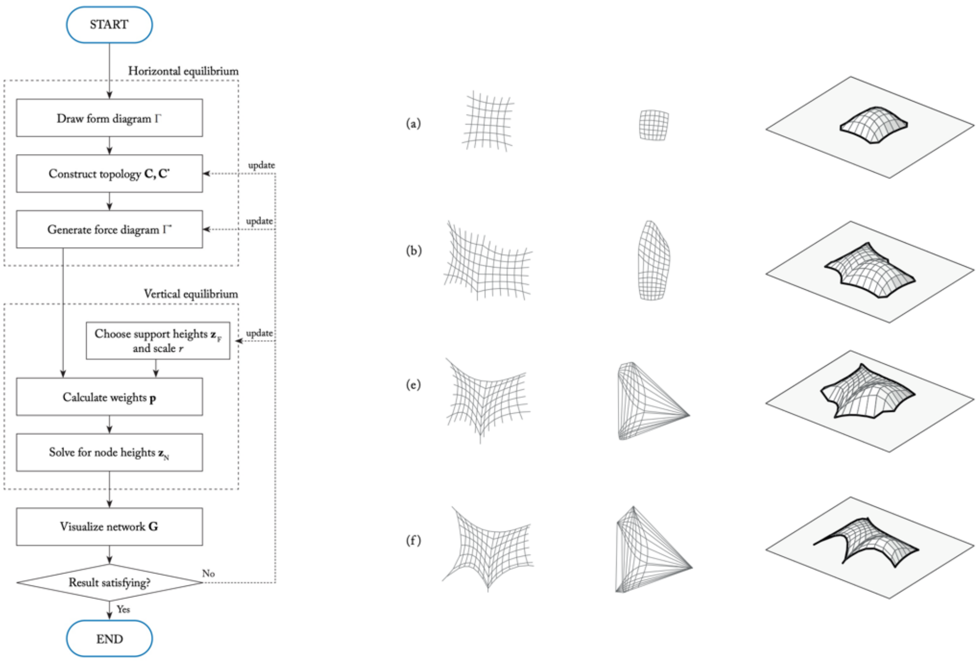


Figure 3-7 Overview of TNA process, and sequential step of the design exploration (Adriaenssens et al., 2014)

The TNA process, as shows, allows a direct exploration on the structural model. This allows the designer to have more interaction with the process as well as having informed feedback on its decision. The advantage is that both vertical and horizontal solving are two distinct processes where other process happening in between could be imagined, such as a loop between structural analysis and vertical relaxation.

To achieve compression only structure the cross section or thickness of the elements should be made that every point of the network grid is within the kern of the cross section, which means that it should be passing by the middle third. If it passes by the outside third, the shell will collapse inwards. If it passes by the inside third, the shell will burst open. This is applicable if the constructed structure is compression only, passing by the outside third will in other word produce tension loads, which can be assessed in a gridshell.
The question here regarding our set optimisation, is how much tension stress can the wireframe be designed with when designing a gridshell structure?

The main advantage of the overall TNA method is that it inputs a form diagram, a flat tessellation. Which is the object obtained from the topology exploration. It also outputs a network of straight lines and nodes which is ideal to continue with the next steps such as fabrication evaluations, p.150 (Adriaenssens et al., 2014). It can be used inside Rhino with the Rhinovault2 plugin.

The TNA process can be used to with the Form and force diagram as bases in order to generate a final shape, but it can also be reversed to fit a target shape. It is the goal of the Best-fit network Analysis paper in (Adriaenssens et al., 2014). The process lies in finding a specific load diagram that match a specific target shape. When inputted the shape and its related form diagram are subject to load cases and will optimize the force density at each node until converge is attained, see Figure 3-8.

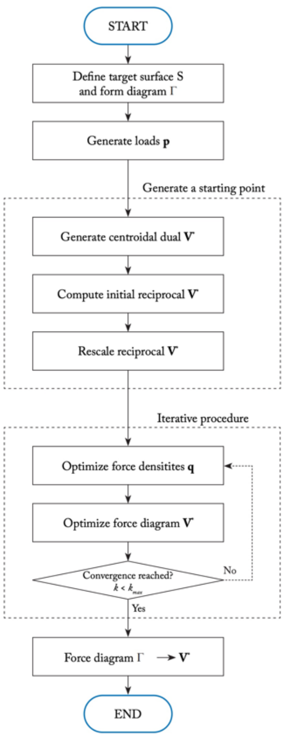


Figure 13.6 Flowchart of a complete implementation

Figure 3-8 Process overview to approach target shape (Adriaenssens et al., 2014)

The Kangaroo plugin is also a hanging chains models solving the funicular system on meshes. It applies the Hooke's law which link the elongation of the segments to the applied force in the segments. It can have some target points to achieve and some degrees of restriction in the form finding. This allows the process to properly integrate the design requirements and for the designer to easily set the boundary condition of certain areas of the shape. It also has as downside that if targeted, other shape potential optimum will be discarded in the form finding process.

3.2.3 FEM Analysis - Karamba

Karamba is a common structural analysis plugin for grasshopper for early structural design. It is used to analyze discretized linear models and mesh models using Final elements method analysis. Karamba is used in two steps, the assembly and analysis. First the geometrical model is transformed into a structural model where the lines become beams and vertices become nodes with specific degree of freedom. Theses degree of freedom need to be stated in all the vertices and will define the nature of node or support of the structural model. The freedom information is in number of six, three degrees for the translation freedom, in x, y and z axis, and three degrees for the rotation freedom, in x, y and z axis. Each type of freedom will have a different load scenario on the overall shell and node. It will make the node classification (2.5.1) range from a rigid to semi-rigid node depending on the moment loads acting on it.

In general, there are five structural factors to consider into mechanics analysis, there are five important factors: Structural geometry, structural topology, structural cross-section, structural material, and loads (Li et al., 2016).

The two first have been explained in 2.2 and 2.4. The other three are described directly in Chapter 4.

3.2.4 Sustainability tracking - Embodied Carbon estimation

As explained in section 2.5.3, the calculation of the embodied carbon is a calculation done on the current state of the model. The implementation of such calculation in term of parametrically created geometry will require the separation of each different elements in term of process of fabrication and material usage. This can be achieved by doing list manipulation in Python or grasshopper in order to keep track of the movement and modification in amount at each process steps for each element. The elements are then multiplied depending on their Embodied carbon factor for the material and embodied energy factor in term of fabrication processes.

Depending on the success of creation of the fabrication steps, time factor can be added in different node and beam fabrication methods and therefore other parameters of sustainability can be assessed such as time cost.

3.3 Optimisation practice

3.3.1 Optimisation processes

The research done around the different optimisation algorithm and the multi-objective optimisation stem the lecture and workshop given by Thomas Wortman teacher at University of Stuttgart participated on the 30th of November.

An optimisation process commonly relates in providing an equation to a solving algorithm that will test possible range of combinations for the equation and retrieve the best combination of parameters that will either maximize or minimize the result in the equation.

The important step in optimisation is to set up penalty functions in order to orient certain variables or certain part of the design problem to avoid a set of combinations. The penalty function also accounts to give the proper scale, or weight, to the different variables in order to make the optimisation "care" for the change in this variable. Several equations can be used to normalize, magnify small values (
Equation 1), or keep closest to a certain value the outcome (Equation 2).

$$f(x) = \sqrt{x}$$

Equation 1 – Squared root, Magnify small values

$$\sigma(x) = \frac{1}{1 + e^{-x}}$$

Equation 2 - Sigmoid Curve

Certain solver works better for specific scenario based on the amount of iteration needed for convergence. The convergence is the time that a solver takes to find a best result that it can before not being able to record any progress for a certain time. The advice is to use different solver for single objective optimisation (SOO). The solvers that have proved to be more efficient are:

RBFOpt for iterations =< 500

CMA-ES (Opossum) for iterations >= 100 *(n+1)

In the implementation of a multi-objective optimisation (MOO), the best trick is to avoid the MOO. Objectives are set as different when they compete against each other. For that the MOO is used to understand the tradeoff. If the objectives are mutually inclusive then two options are available to treat them as one objective, the weighted sum, and the weighted product, see Figure 3-9. For the weighted sum, all the objectives need to be normalized otherwise one variable would have more influence on the result that another one.

Weighted Sum
(Need to normalize objectives and to define weights)

$$\mathrm{w_1 + w_2 + \cdots + w_n = 1.0}$$

$$\mathrm{o_n = o_1 * w_1 + o_2 * w_2 + \cdots + o_n * w_n}$$

Weighted Product (No need to normalize objectives)

$$w_1 + w_2 + \cdots + w_n = 1.0$$

$$o_n = {o_1}^{w_1} * {o_2}^{w_2} * \cdots * {o_3}^{w_3}$$

Figure 3-9 Weighted sum and product in inclusive objectives implementation (Thomas Wortmann)

When implemented the MOO produce a pareto front curve where the best trade located the closest to the origin.
More work will need to be studied in the implementation and other option regarding the topic.

Also, the self-organizing map have been used in several of the reviewed literature as a support to visually represent the related designs, more about its implementation should also be studied as a next step. Follow two examples of it use.

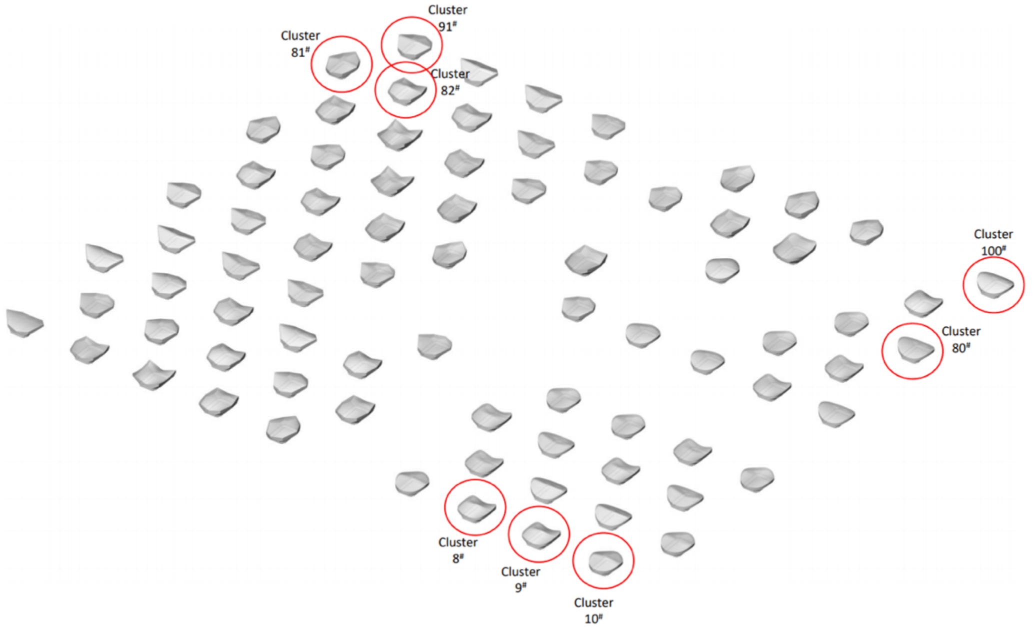


Figure 3-10 Clustering example on indoor arena generation (Pan, n.d.-a)

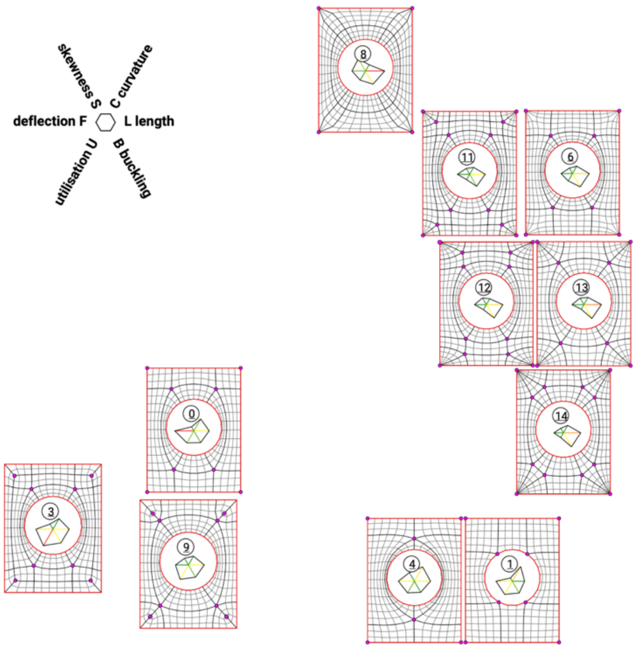


Figure 3-11 Clustering on different pattern generation (Oval, 2019)

3.3.2 Result Visualization methods

At the end of the whole process, some visualization board will need to be drawn to order the showcase the best performing design. Depending on the software used, if the method of exploration will finally be by hand or using a multi-objectives optimisation algorithm, several possibilities to display the results will be available.

The type of chart will have to be chosen according to the clarity of the data set and the purpose of use.

Polar chart has been used to quantify the different objectives and laid next to the clustering map, Figure 3-11. Dendrogram have been used to follow the design families and type defined along the optimisation process, it could be used to find a related cousin with specific performance traits (Pan, n.d.-b).

To only cite a few examples of libraries or plugins that could be used in the process:

Matplotlib in Python can be used to produce the related charts.

Conduit, Mandrill, or Human are also plugins that can be used for plot charts in Grasshopper.

All in all, a graphical interface needs to be developed or used in order to help the designer make an informed decision on a quantitative base.

4 Method development

This chapter explain the steps taken for the implementation of an integrated tool considering fabrication, structural and environmental objectives.

A quick summary on the iterations of the different options evaluated when setting up the tool will be done in the first place. Limitations and opportunities will be highlighted.
Secondly, a detail explanation of the processes implemented in the final version of the tool will be done.
It will follow the main steps of each process provided some detail in certain part which were important decision for the data structure, methods to attain a needed data or process to reduce the calculation time.

The development of the tool followed, in the main steps, a constructive method already implemented by Octatube for the design of their gridshell. One of the goals was to transcribe this manual process in an automated and centralized one where values could be tracked, and a design exploration could then be informed by direct feedback.

The General flowchart that guides the implementation of the tool is the following:

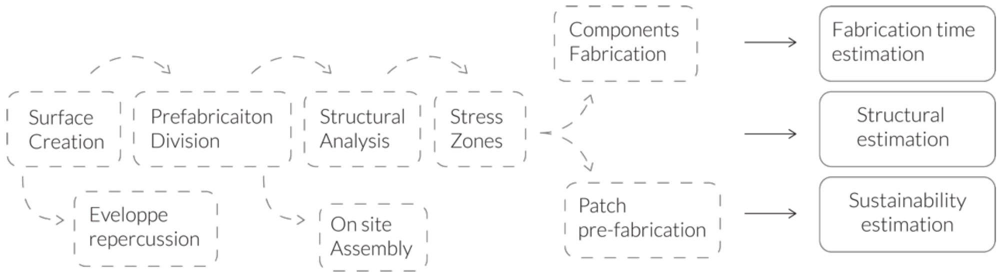


Figure 4-1 Flowchart of the tool divided in modular steps in order to integrate the different objectives

4.1 Generation and method exploration

4.1.1 Stepped Guideline of the exploration.

4.1.1.A Boundary condition & tessellation

As seen in the literature review, rigid gridshells are often used to cover extensive areas, non-convex, intricate and/or organic support line. It was a wish to support this design prospect and integrate a custom mesh for the Surface Creation. The work of Robin Oval on topology exploration has been the base to create a custom mesh creation process for the tool. Despite limiting the tool to work with quad meshes, this process could bring more flexibility in the use of the tool. As boundary curve or lines would need to be inputted and the mesh would be directly created.

Being quite complex to translate the process into grasshopper a first process of the cited work has been developed regarding the creation of a custom mesh based on the initial inputted edges. This represents only a portion of the work of Robin Oval and will serve to confirm if the process can be used in a grasshopper workflow. Topological Skeleton and coarse mesh generation. A specific method to start with any type of boundary condition and create a coarse mesh specific to the initial edge. This would introduce singularities which could be assessed as a specific type of node connection.

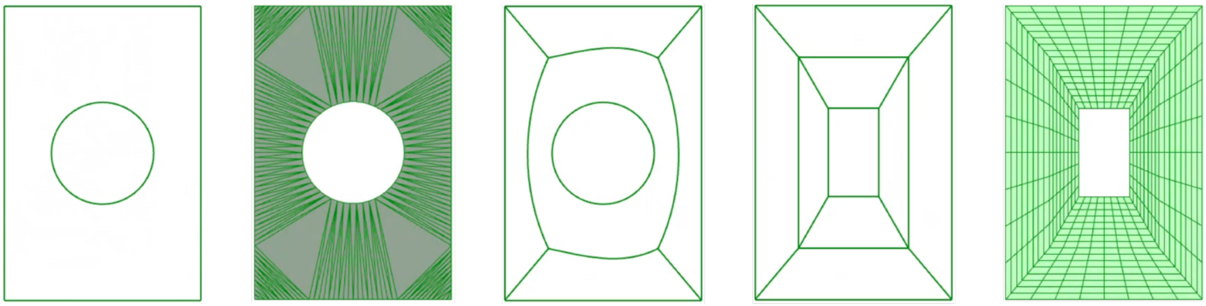


Figure 4-2 Topological skeleton and coarse mesh creation

It was commented by the author of the paper itself, that the method still needed some manual set-up at different stages. The produced script was not generalizable enough and the result on a specific example was already too distant from the original work. It seems that there is some operation that are not properly commented in the original report of culling some quads when divided the coarse mesh is performed.

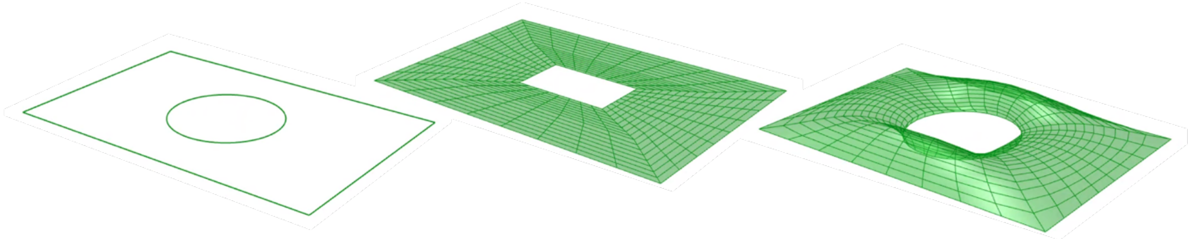


Figure 4-3 Dynamic relaxation of a topologically created mesh

This type of process would result a more form exploration type of tool whether the proposed tool is trying to focus in highlighting the first steps to connect early design decision and fabrication estimation. After defining that the generated design space of the tool will be limited to simpler geometries this process was left out to scope. In the proposed tool, the pattern generation is integrated in the Surface creation. Future implementation would greatly benefit of such topology exploration at the base mesh creation.

4.1.1.B Surface design: Base mesh creation & relaxation

This step is intended to produce a mesh that will be the base for the tool. Intended to be a replacement to the shape provided by the designer, it is expected from this part to produce an array of dynamically relaxed shape with different level of subdivisions. Each division will produce mesh edge which will directly be used for the struts of the gridshell. The main goal being to estimate how much impact would it have on the objectives when altering the subdivision and relaxation of a shape received by the designer.

V1 – Quad Relaxation on rectangular shape.
Boundary conditions: Rectangle base, orthogonal pattern
Creation: Use Kangaroo solver to relax a mesh
Design variables: u v division on mesh, vertex load
Output: Relaxed quad mesh

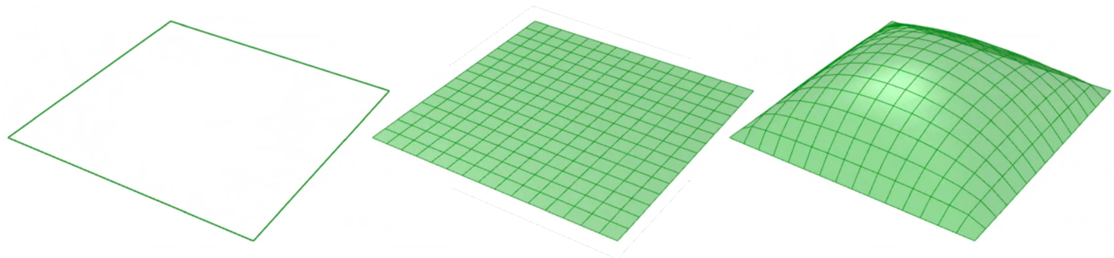


Figure 4-4 Orthogonal pattern relaxation

V2 – Quad relaxation on free form geometry.
Freeform baseline, orthogonal and triangle pattern, direct relaxation
Design variables: not great control on mesh division, vertex load, volume under the mesh

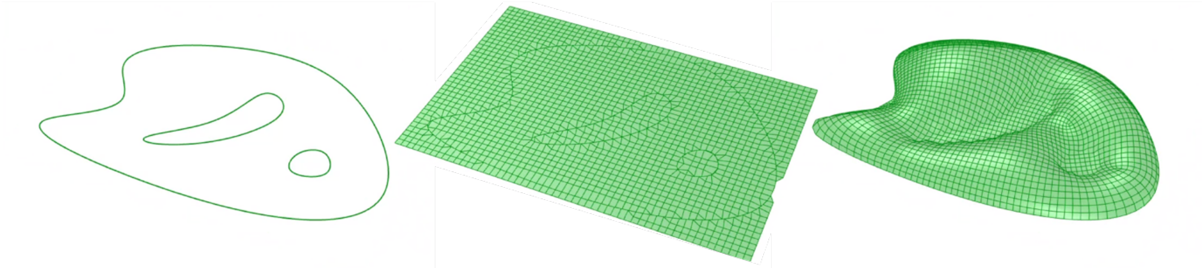


Figure 4-5 Quad mesh relaxation on freeform boundary curve

V3 – Mesh relaxation, Chebyshev diamond pattern net.
Rectangular base, diagrid mesh creation (tri mesh and quad mesh), relaxation of first mesh, and Chebyshev projection of the diamond grid on the first mesh. Had to take out one line in corners that were forming tetrahedron.
Design variables: Type of ending of the diamond grid on the rectangle, u v division, vertex load,

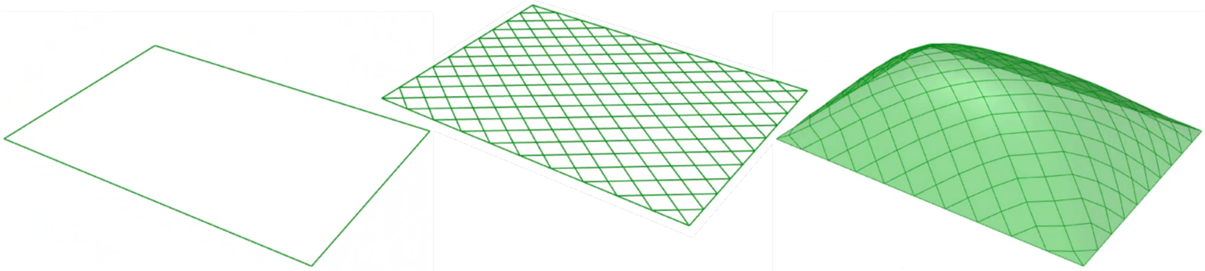


Figure 4-6 Diamond grid relaxation on rectangular shape with a Chebyshev net applied

Despite the ideal condition of the correctly lengthen lines, mesh type, and control possibility, this process revealed to be too unreliable and couldn't not be used automatically. The process starts failing with low and high u v division which reduce too much the design space. If used, the different possibilities would need to be made in advance and picked within the optimisation steps, where a relation between the iteration would have to be inputted into the solver for the algorithm to learn from each pick.

V4 – Orthogonal mesh relaxation, diagrid mesh creation.
Rectangle base, only one type of diamond pattern, relaxation + rebuild
Use of the plugin Ngon to create a regular triangular mesh on a relaxed surface, which is then reconstructed in quads and tri-mesh at the edge.
Design variables: u v division and vertex load.
In general, the range of the design variables is larger and can provide a greater diversity.

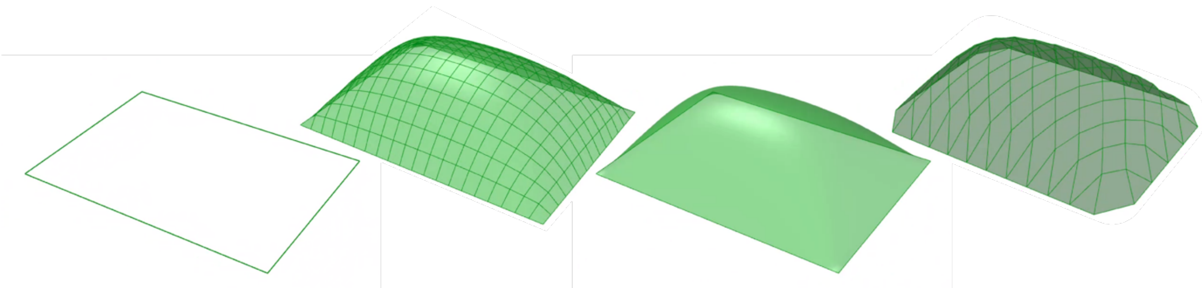


Figure 4-7 Diamond grid creation on a pre-relaxed surface

Here the lack of the Chebyshev net is noted, the lines on the edges of the gridshell are nearly twice the size of the other ones. This gives a large shape ratio disparity between the quad mesh inside the gridshell and the triangular mesh at the edge. The shift from orthogonal grid to diamond grid is done following the division system developed in the next step (C) and to get closer to the type of structure that Octatube fabricates. This approach would of course also work for orthogonal meshes as they are both quad-meshes.

4.1.1.C Prefabrication division

The division of a specific design into patch for refabrication is an essential step that will define the structural nature of the different nodes and beams in the design. As it come in a rather later stage of the design, this step is usually done by hand in professional practice. In order to be added to an optimisation process this research propose a way to automatize this step. The main goal in this section is to divide the structure in the biggest patch as possible that still fits in a specified truck size. The outcome of this part is to define which of the four members incoming a node will be translated as a bolted connection or welded connection. The bottleneck computationally is the way the patches are created. For that, several options have been tested searching for a pragmatic and efficient way to divide each iteration.

V1: Point search

Point cloud enclosure in boxes. Searching till a specific bounding box, size of transport trailer, cannot contain more points. Python script written to enclose the neighboring points after each successful enclosure. The way the code is written cuts the neighboring points list in half till it has a successful fit. It then iterates until all the points have been assigned to a patch. It will retrieve a list of lists contain the index of the points in each patch.

The search can be guided but is too specific for a specific type of pattern. It can also have some points remaining alone or forming together a non-conventional patch. The main downside of this process is that it is a brute-forcing algorithm and therefore can be computationally intensive.

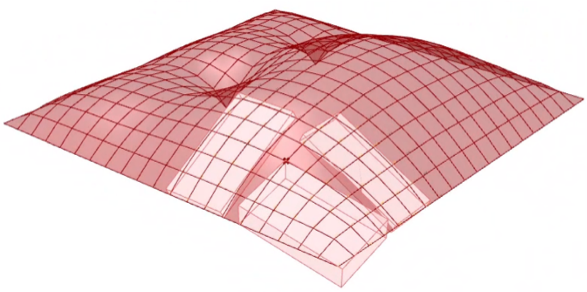


Figure 4-8 Point search algorithm for patch division

V2: Curve enclosure

Semi-automatic, quick way to divide but create unconventional patches. As it is not an iterable definition, there is also no action taken if the divided patch does not fit in a truck

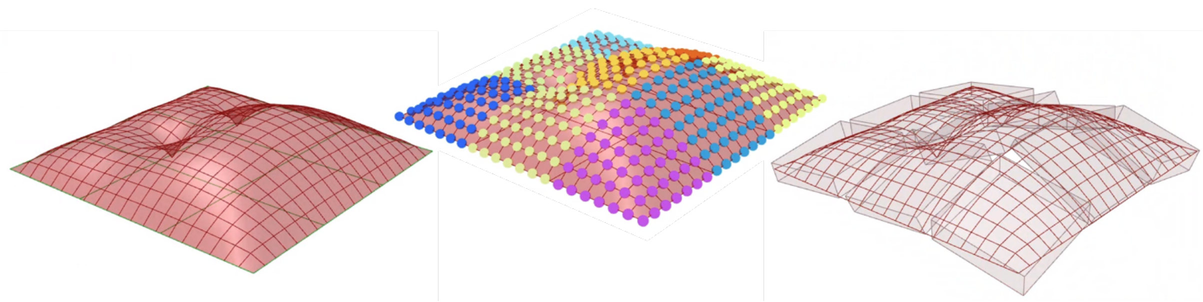


Figure 4-9 Curve enclosure script for patch division

V3: Strips Division
Divide the quad-mesh into strips. The remaining tri-mesh are connected to the closest strip. First, a minimum bounding box can estimate the width and length of each strip. In a second step, the strips will be divided in length and joined in width accordingly and form patches. This solution forms a consistent division of the mesh with customizable width division assemblies. This procedure is also more general, applicable to all quad-meshes, which means that it opens the door to test more intricate assemblies in the search of MOO. This process chosen as final method, will be explained in more detail in the next section, 4.3.1.

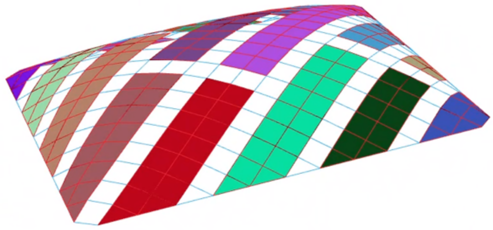


Figure 4-10 Strips division for prefabricated patches

4.1.1.D Structural Analysis

For the Structural Analysis, karamba is used throughout the whole script. Despite some iteration, none are worth mentioning per say. The detail of how the loads, materials and cross sections are assigned is explained in the next part under 4.3.1.

4.1.1.E Component Assessment

The whole thesis revolves on searching the different impacts of a specific design either structurally, constructability or in sustainability impact. For that, the estimation of the amount of material and amount of fabrication time is capital. Most of it, when focusing on the coarse structural elements is present in the node and the way the prefabrication and construction of the shell is undergo. The component assessment has per goal to estimate the amount of material needed for each node, their related fabrication processes, estimated in time and carbon emission. As the scope of the research states it, the connection design is reduced to one general type, image below. Based on the position of the node in the gridshell (inside a patch, at the edge or at a corner of the patch) the connection can have one or two faces that can receive bolted member.

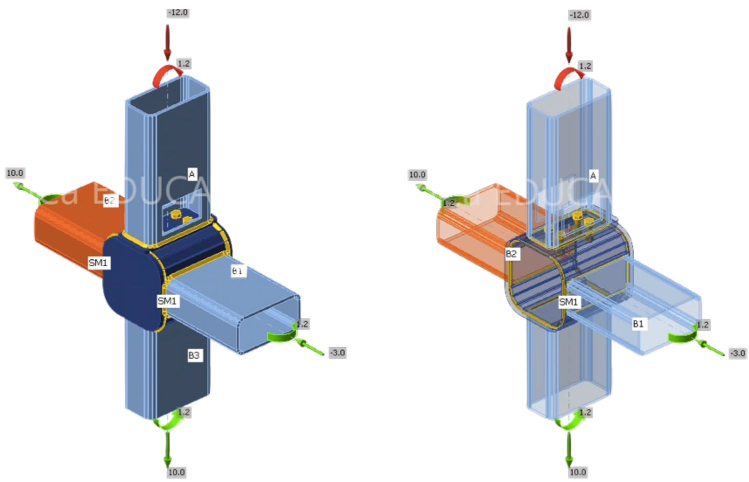


Figure 4-11 Selected connection design, version with one face as bolted connection

Knowing the type of connection that each beam needs with each node from the process (4.1.1.C) and the stresses and moments at the end of each beam thanks to Karamba, calculations can be done in order to estimate the amount of material in a structurally valid connection. In order to estimate these values two solution has been developed. Both solutions are kept in the general workflow as they provide different level of precision for their operating time.
The first method is exporting a model of a single connection created in grasshopper to IDEAStatica using KarambaIDEA API developed by Rayaan Ajouz, a plugin suite that allow to estimate the cost of connection. Composed of the stresses, orientations and cross section sizes generated in the main script, the model can be analyzed in IDEAStatica using a custom-made template. Calculation can be done in IDEAStatica or retrieved directly from the API. Such process will allow estimate the thickness of the endplates, the volume of the welds. The most important item provided by this software is the resulting stiffness of the connection, which is then used for the Karamba simulation in the hinged joints. This process being too slow (~30 s/connection) cannot be used during the process of optimisation but can be checked in a post analysis to validate a generated solution or as a preliminary check on the stiffness of a specific connection design.
The second method developed consist of estimate the size of the different elements of the connection by simpler formulas building an analytical model. Deconstructing the joint connection in simple stress state allows to estimate one by one the different elements constituting the connection, considering the incoming stresses. Two sets of calculations need to be done in order to estimate the welded connection and the bolted connection. The detailed steps and calculations are explained in the section 4.3.1.

4.1.1.F Fabrication of components, Prefabrication of patches & on-site assembly count.

From step A to B, information has been gathered regarding the amount of the different elements of the gridshell. Only to cite certain ones, they are the number of elements, their size, their linked steps of fabrication, their stress utilization. Altogether, this information can be compiled and provide analytics for the different stages of the fabrication and assembly of the gridshell. Coupled with data about fabrication processes and carbon factors, a life cycle analysis can be drawn. Similarly, the time spent for the prefabrication and the time assembly on site can be estimated. The data in question and the methods used are detailed in the chapter 4.3.1.

4.1.1.G Geometrical repercussion and estimation for the facade

In the literature, emphasis in given towards the general aspects of the resulting geometry of the structure. Specially on the panels of the gridshell. Even though the tool focuses on structural elements, indicators about the repercussion of the geometry on the facade panels can help orient the decision.

Two factors have been chosen to assess the repercussion on the façade:
The planarity of each panel, which will inform about the nature of the glass construction that can be foreseen in the with this geometry. The planarity deviation will inform whether cold bent glass or hot bent construction should be used, significantly increasing the estimated cost of the design.
The waste of glass produced when cutting panels into final shape. This amount can be calculated based on the geometry of the panels and be estimated in percentage. The process used to estimate such value will be explained in the part 4.3.1.

4.1.1.H Metrics to indicators

The Numerical Assessment is accounted as the main quantitative metric holder of the multi-objective optimisation (Pan, n.d.-a). It represents a separate part which reads the measurable relations between objects created in the generative part and records the outputs values of the simulations done throughout the process. All these metrics, which are the design criteria, have their objectives set up for the optimisation and possible constraint stated. The objective could be to keep to the minimum the tensile stress the structure. Or push to the maximum the usable area ratio of the space designed. The constraints can be set on the Any design code (Eurocode), and rule of thumbs value, as we are designing an early design tool. For the deflection has as objective to kept at minimum and as constraints to be less than the span/250.
The table following does not come to such detail as the objective of each criterion and their formula to assess them must be studied. It rather puts in relation the different steps and the moment where the criteria could be evaluated. It provides an informative landscape on what the criteria need, when they are going to be accessible in the script, and what type of objectives can we measure at which step of the tool.

# STEP	Parameters	Metrics - Objective performance/constraints				
		General	Spatial Aspects	Structural	Fabrication	Embodied Energy
0 Boundary Set-up Research parameters	Outside curve Supports information Nbr support Height, load allowed Oculus (inside curve) Nbr inside curve Height, load allowed Nbr of "best perfoming" design Parameters related to ranking		area			
1 Pattern Generation	Points feature addition Curve feature addition Skeleton creation no modification, Pruning, grafting Sequence of addition of strips	Nbr vertices (valencies) Nbr of faces Nbr edges	Density of grid / transparency			
2 Form finding	Height movement constraints at edges	Total edge length area of panels	Usable area ratio height Volume		Variation beam length / length disparity curvature of panels Straigthness of panels (skewness) Shape ratio of panels Variation Area of panels Angle tilt between segments and node	
3 Structural Analysis	Cross Section type Cross section dimensions Loads cases (list) type Material (list)			Max Serviceability limit state (SLS) deflection Max Ultimate limit state (ULS) stress utilisation ULS first bucking load factor Max displacement First buckling load factor Strain energy Stress repartition/distribution membrane field disturbance Rigidity of grid Moment at nodes Load at support Cross section height variations		Ratio structural mass to covered area Amount of material edges Embodied carbon energy edges
4 Node generator Change pattern in rigidity (semi loop)	Nbr of nodes to study yaw, roll, pitch of segment attached Type of node Type of connection Nbr of segment added type of rigidity feature	Nbr of nodes (classification) Nbr vertices (valencies) Nbr edges Nbr of faces (two levels)	Appearance of node	weigth of node design Tolerance of design	Angle disparity of entries at nodes fabrication complexity time to manufacture cost of manufacture	Fabrication energy comsuption Embodied carbon energy nodes
5 Structural verification	Final load cases			Rigidity of grid (check) same metrics…		

Table 4-1 Possible parameters and metrics to implement in the tool at each step of the design generation.

During the tool development, as few of the metrics above are estimated. These are then used to calculate indicators values that will be the base to inform the user about the score of their design. The relation between the design variable (parameters) and the indicator must be clear. The indicator must reflect the change of the parameter and vice versa. A relational map is drawn in Figure 4-14.

The impact of the design variables on the indicators are highlighted in the tests done in chapter 5.1.
The metrics produced along the 3d model are coupled with data gathered during the research to transform this simple metric in quantifiable values that can be comparable. The following diagram sums-up the output of each step from the script and how they are combined fabrication timing data, or embodied factors to build accurate indicator of each design iterations.

4.1.2 Reflection on the exploration

In the timeframe allowed for the research, the elaboration of a working tool can be limited by the time needed to implement each part of the code having with the adequate data type and structure for each input and output. Build a modular script so that every part can be enhanced or modified without affecting the whole script. Eventually, the proposed method is only a prototype. A first step when all excessive operations, initially build to allow some flexibility of change, can be revised to just match the previous output. The integrity of the data structure must be tracked and revised to suit the way each part of the script inputs and outputs their data adequately. Confirm that there is a consistent organization of data that can ensure a smooth exploration of geometry without losing information. This task can be quite time intensive and rigorously hard to implement. This can be a challenge when choosing this kind of thesis. For this reason, most of the features explored did not make their way in the final proposed method. The focus had to be set on a first working prototype covering the essentials in order to provide a proof of concept.

4.2 Quantitative assessment framework

In order to assess the performance of a specific design iteration and retrieve a set of values for each of the envisioned objectives. A set of metrics have to be put in relation with the geometry generation and retrieve a series of data called indicators. Together, these indicators will retrieve a quantitative evaluation of the design iteration and together constitutes the score of the different objectives. The placement of these indicators constitutes an important aspect of the optimisation set-up. Imagined as a black box script producing an intangible geometry, they are the only tool that will put light in some part of the process, to ensure it well functioning, and measure the different aspects of the produced geometry. The relation between indicators - Objective and indicators - design variable has to be assessed to ensure their correct positing and tracking.

The main objective when placing the indicators, is to ensure that all the design variables combinations are tracked with a unique set of resultant metrics and that a distinction can always be drawn between all the design iterations outputted by the script.

4.2.1 Design function selection

During the Literature review and the method exploration, several design aspects of gridshells have been explored jointly with-it computational implementation. Some implementation being more successful than others, the tool itself has been oriented by choosing the processes which input and output could be matched. During the method exploration, some explored features are crucial to generate geometries and quantify the different metrics needed for the objectives. Others, have been noted to provide more flexibility to the tool, expending the design space of the later without adding more detail to the assessment itself. These unique features can be categorized by design function and relate to the categories used to review the different constructed gridshell, done in Table 3-1.

As adding features adds complexity at analyzing the relevance of the tool and can be quite time intensive to implement. It has been decided to focus on the working processes found in the exploration and leave the other features to the side. Altogether, the different design functions with their different features can be seen in Figure 4-12. The features in grey are the ones mentioned in the review table in the literature review that have not been explored. Between the other ones, only the features in black have been selected. The ones in white, have been left aside due to a computational inconsistence, or because it coincides it only provide another way to provide a similar answer to the research question.

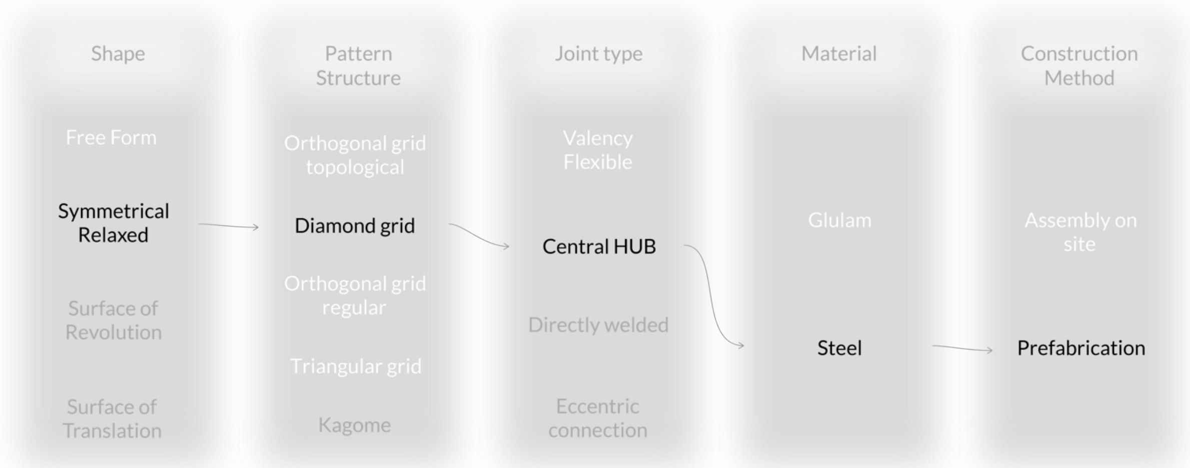


Figure 4-12 Design functions and features selected for the proposed method

4.2.2 Objectives and indicators

Alongside setting up the basic design functions that the tool will cover, the objectives envisioned are stated. As mentioned, the research focus on the fabrication, sustainability, and structural performance. Each of these three aspects have been divided in design requirements and criteria objectives specific to the metrics that will compose that performance trait.

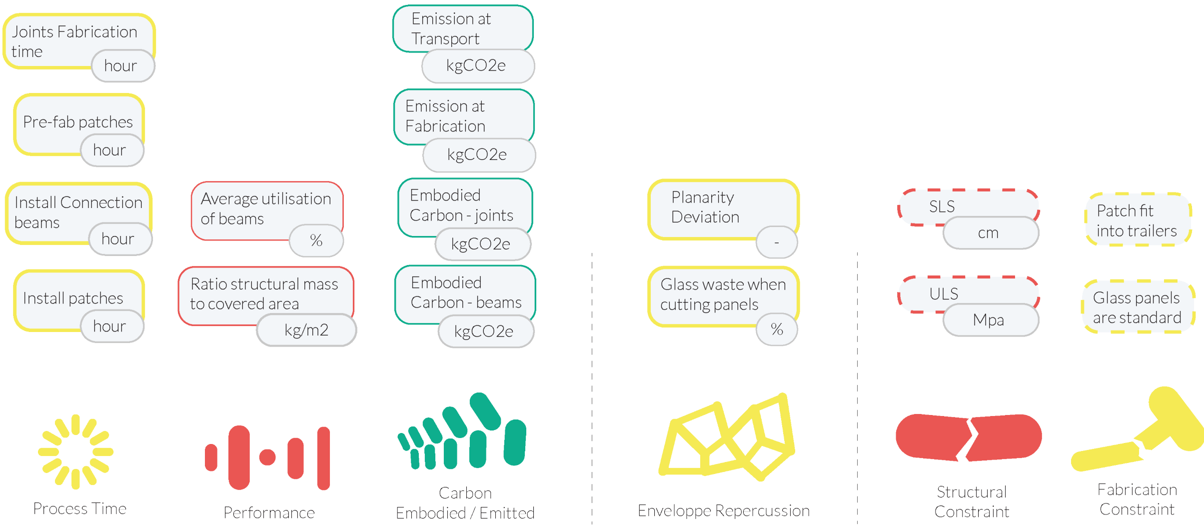


Figure 4-13 Objectives and constitutive indicator

Therefore, the fabrication aspect has been divided in three objectives. The first one calculated in hours that will track the time spent at the prefabrication and assembly of the structure. A second one regarding the envelope repercussion focusing on the resulting geometries of the façade panels. And a third one accounting the design requirement that the design need to check, assessed with a True or false event.
Similarly, the structural aspect is divided in two objectives. The first one, tracking the structural performance directly with to different technique. One is directly measured on the structure itself assessing the utilization of the beam. The other one is the structure compared to the context, where the weight of the structure I put in relation with the covered area. The latter is an efficient indicator when the design is changing in size, the former more useful when less boundary conditions are changing. Here, two indicators with two different scales are set to the same objectives and might be used in different case studies of trade-off search.
Finally, the sustainability aspect focuses on the global warning potential of the design. Measured in kgCo2 equivalent, the carbon count distinguishes what is emitted and what is embodied. This distinction will allow later on to mitigate the performance of different design by displaying a specific attribute of the total carbon count.

Once the objectives and their indicators are well defined, the required output of the different processes of the tool become clearer. It can also influence the depth or how much parameters should define the generation of a specific process. Defining the design functions and the related indicators orient how the script will be created in its entirety and inform the consequent flowchart of the tool. This aspect will be described in depth in part 4.3.

4.2.3 Relation between design variables, design requirements and design Objectives

When constructing a parametric model, the design variables are the parameters that will have, as their value change, a series of impact on the outcome of the model. As highlighted earlier, a well-defined indicator will be able to adequately track the variation of theses design variables and properly assess the performance of a specific design.

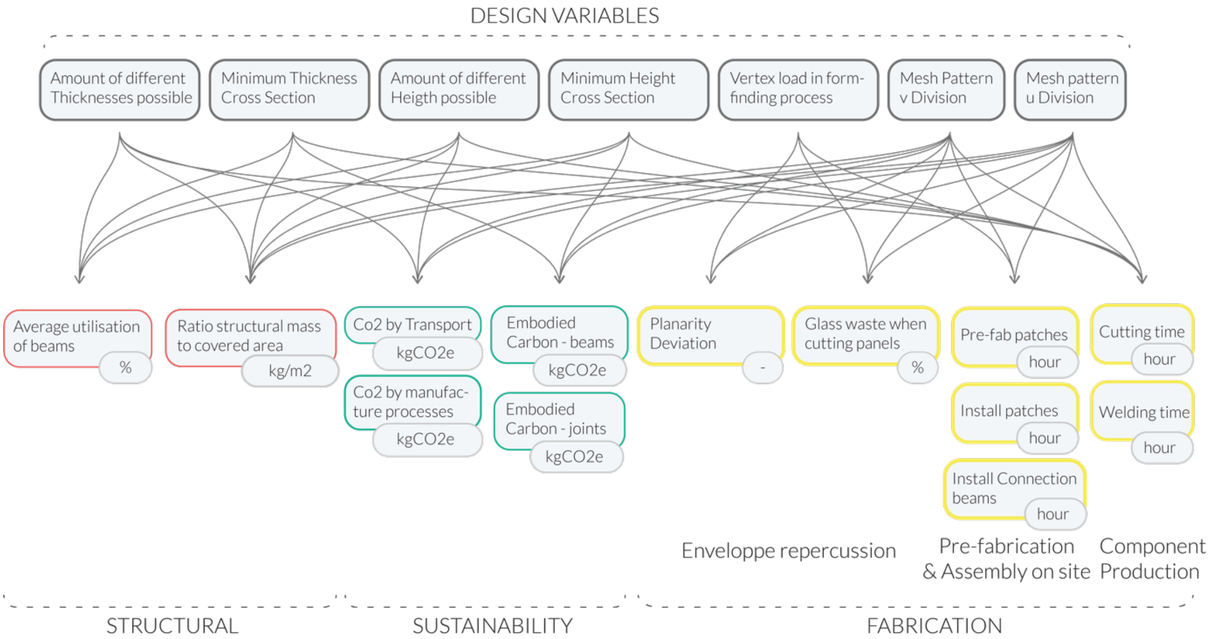


Figure 4-14 Link between Design Variables and indicators

To ensure this relation, beside running tests on the finished tool to see the weight of a specific design variable on an indicator, a relational map can help to conjecture the envisioned relations. Here the relation is drawn by a simple arrow disregarding the scale of the impact of the design variable. The weight of that arrow would help to inform to which extent the variation of a certain design variable will influence the metric recorded by the indicator. It is important when it comes to refine a performance trait in the design exploration.

4.2.4 Numerical assessment

The generated design alternatives are evaluated on a series of indicators. As mentioned in the scope of the research, the focus lies in sustainability, fabrication, and structural performance. Altogether, the indicators constitute a numerical assessment table that will allow to provide a quantitative score to a design.
The following table summarise under which category each of the indicators falls under, the method used to estimate them and to which optimum it should be brought to.

Aspect		Indicators	Analysing method	Criteria
FABRICATION	Fabrication of components	Cutting time related to connections fabrication	Measured in the 3D model, analytical calculation	objective: minimise
		Welding time related to connections fabrication	Measured in the 3D model, analytical calculation	objective: minimise
	Prefabricaiton	Size of prefabricated patches	Measured in the 3D model	Constraint: <dimension of trailer
		Prefabrication time of the patches	Measured in the 3D model, analytical	objective: minimise
	On site assembly	On site instalation time of the patch	Measured in the 3D model, analytical	objective: minimise
		On site installation time of the connecting beams	Measured in the 3D model, analytical	objective: minimise
	envoloppe repercussion	Glass waste when cutting the panels	Measured in the 3D model	objective: minimise
		Manufacturability of glass panels	Measured in the 3D model	Constraint: <3.1m in one dimension
		Planarity deviation	Measured in the 3D model	objective: minimise
SUSTAINABILITY	Embodied Carbon	Embodied Carbon in the beams	Karamba and Analytical calculation	objective: minimise
		Embodied Carbon in the joints	Analytical calculation	objective: minimise
	Emitted Carbon	Emitted Carbon at Fabrication	Measured in the 3D model, analytical Calculation	objective: minimise
		Emitted Carbon at Transportion	Measured in the 3D model, analytical Calculation	objective: minimise
STRUCTURAL		Serviceability load state	FEM based on Karamba 3D	constraint: < shortest span/250
		Ultimate load state	FEM based on Karamba 3D, Analytical calculation	constraint: < Yield stress of Material
		Average beam utilisation	FEM based on Karamba 3D,	objective: maximise
		Ratio of structural mass to covered area	FEM based on Karamba 3D, measured in the 3D model	objective: minimise

Table 4-2 Framework for a numeric assessment of generated design

4.3 Proposed parametric model

Having properly set up design functions that the script what to mimic and the metrics tracked, the flowchart of the tool is drawn to keep track where each part of the process is implemented. The following flowchart,

Figure 4-15, represents the script related to the design generation only. Another script in attached to the latter in order preview and track the performance of the different design iterations. This graphical part will be commented in chapter 0. A description in the implementation of the generation process will be explained step by step in this chapter. The different limitations, calculations and algorithms will also be stated. The description and explanation of the main passes will provide an answer on the necessary steps to take to properly assess the envisioned objectives research at an early design stage.

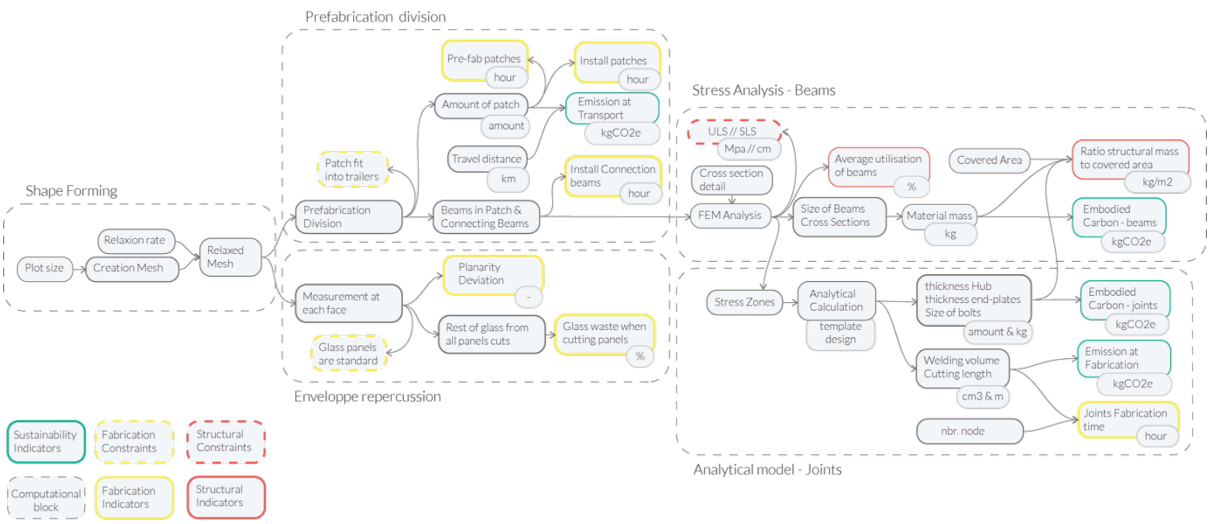


Figure 4-15 Proposed method process flowchart

4.3.1 Limitations

Assumptions were taken in the generation process of the tool. These limitations can have different impact in the precision and depth of the provided results. These limitations are stated altogether to remember that the proposed methodology composes only a first prototype on how to join and assess the stated objectives at an early design stage. Precautions and improvements can be taken to make it more accurate or cover more features, these are stated in chapter 6.3.

When counting the carbon LCA of the proposed , the total does not take into account the "waste" steel amount due to cutting the profiles with the proper length and angles. The amount of energy needed to put back this waste steel in the chain of production of future profiles should also be assessed at the LCA. Regarding the score and best performing geometries, it could have a negative impact of the carbon score of the geometries that have great angle deviation.
Similarly, the tool which focus on the series A of the life cycle assessment, account for only one of the traits on each of the sub-stages 1 to 5. This does not constitute a full life cycle analysis on that series. More depth can be attained by assessing the material wastage on site, (Istructures, 2020) or extrapolating the amount operating time of the different machinery used for the on-site works.

Focusing on the coarse structure only, the tool lacks the data resulting from another building material with a high embodied carbon, the glass panels. Only counted in the envelope repercussion as a performance parameter the glass is used only for structural calculation as dead weight .

A fixed value of 24mm of thickness is used throughout the script. This account a triple glass unit of 8mm of interlayer. Being too general, it was decided to not add the embodied carbon of such elements. Nevertheless, two methods could be used to correctly estimate the thickness of the panels and therefore add the glass inside in the carbon count. The first one by approximating the size of the layers based on geometrical aspect of each panel such as the span or the area. It would provide a quick extrapolation customed to each iteration. A second method would be to proceed with a one-point load structural analysis based one or more load cases to estimate the necessary thickness. This last method composes a more computationally intensive calculation but would provide a more precise estimation.

The estimation of the joint stiffness composes the main assumption at the structural level. Based on the assumption that the proposed joint design is a semi-rigid design, the structural model builds in karamba use a rotational and translational stiffness based on a pre-study made in IDEAStatica, Figure 4-16. Therefore, an average stiffness value is used for all the connection of same nature. Using a properly estimated stiffness value of the different joints could have an impact of the structural repartition of the load and therefore the cross-section sizing and ultimately the carbon count, Figure 4-17.

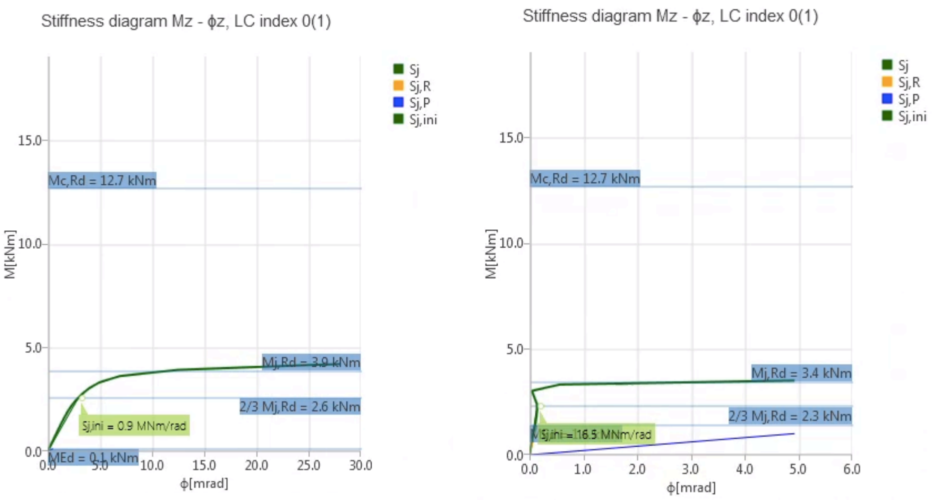


Figure 4-16 Rotational Stiffness Result (left to right) Bolted connection and Welded connection (IDEAStatica)

It constitutes an important step as the rotational stiffness has great effect of the deflection of the structure. This is mainly due to the way the constructive method chosen for the tool is designed. Assigning removable beam between the patches without welded connection to joint them, therefore breaking the structural continuity. As mentioned already, this is a consequence of having a simplified version of the fabrication division. In general, a feedback loop could be set in order to have the proper rotational stiffness inputted inside karamba and retrieve a more precise deflection of the structure. The way to tool is built, creates a lack of feedback in providing proper stiffness for all the different joints.

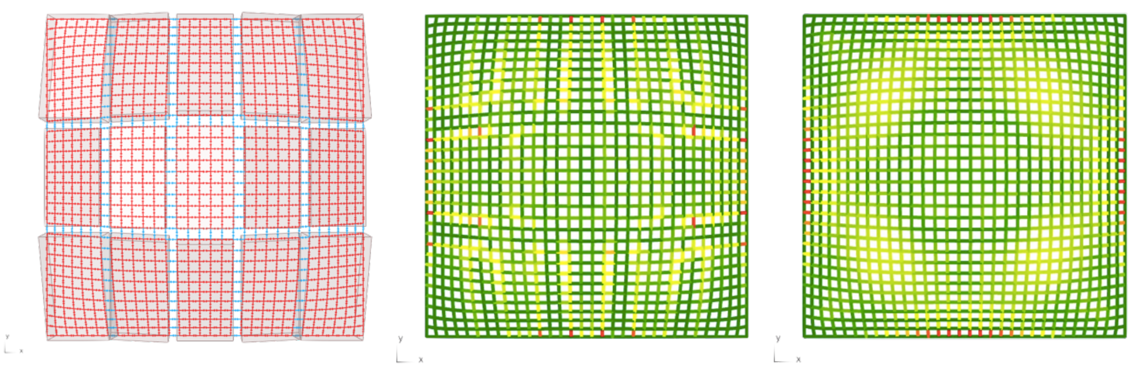


Figure 4-17 Beam utilization rate comparison varying the joint stiffness at the patch edge connection. Left to right: Reference patch division, pinned joints, fixed joints

As seen above, the fixed joints allow to distribute the moments through the shell unlike the pinned that concentrate the moment within each patch edges. Beside creating a scattered and heterogenous stress levels throughout the structure, the joint stiffness value has, within the constructive method stated, a drastic effect on the deflection of the structure. A scheme which would solve the problem of the feedback loop implementation and the miss-use of the joint stiffness is detailed in the reflection, chapter 6.3.

Finally, the effort set to estimate the fabrication time of the different processes of the construction of the structure was done using two different strategies. These strategies complement each other to retrieve a quantitative evaluation of the fabrication time but are done with different level of precision.
On one side, the scope of the research limited the parametric tool to assess each volume of the different weld volumes with high precision. On the other hand, extrapolated factors have been used to estimate blocks of process. For example, the time needed to prefabricate the different patch is estimated by counting 6h for each of the nodes present in the patch. This disparity in precision between the different process makes the uncertainty propagation ramp up and produce an outcome with potential high uncertainty. In the 6h value, estimated empirically by Octatube, the weld time is already integrated creating an intersection between assessment. This could be used to estimate the portion of the estimation done computationally on the volume weld on the total rule of thumb estimation.
Other parts of the prefabrication process such as setting up the scaffolding for prefabrication, selecting and positioning the different unique beam elements, warping the prefabricated patches, positioning the different cranes on site could be integrated in the tool, providing a score with less uncertainty.

4.3.2 Stepped Generation Process

4.3.2.A Surface Creation

The generation has been vastly modified as explained in the related chapter of Method Exploration, 4.1.1.B.
Input:
Boundary Conditions: width length of rectangular support edge
Design variables: u v division, vertex load

Output: Relaxed Mesh intended as each face being a panel, each node a connection and each edge a beam.
Mesh topology (node to node connectivity), set of all points.

Steps:
First relaxation of orthogonal mesh, used from the direct division of the
Rebuild of diamond grid on it

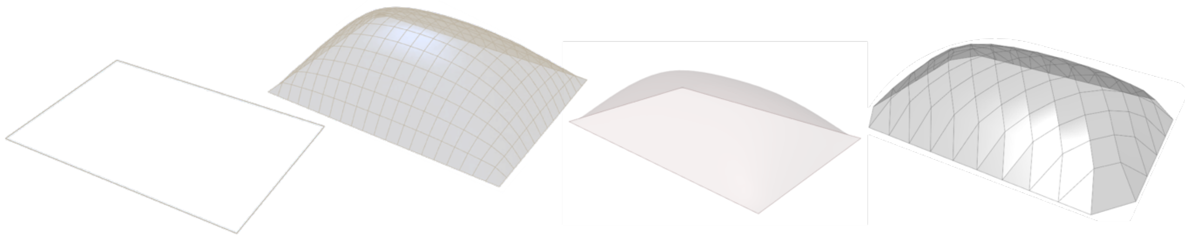


Figure 4-18 Surface creation, illustrated flowchart

Precision on the process:
The diamond grid, often used in design of gridshell produces a set of quad mesh and tri-mesh. It has to be converted back to tri-mesh only at the edges and quad mesh. As the stripping method only works with quad mesh.

4.3.2.B Prefabrication division

Input:
Data: Mesh, mesh topology
Boundary Condition: truck Type

Inputted data: Size of Trailers and Consumption of trucks

Output: Set of Patch each containing a set of nodes (index), list of connection type of each node, list of removable beams and fixed beams

Steps:
Rebuild strip out of trimesh and quads
Reorder the panels inside each strip
First minimum bounding box
Reads size of trailer and compares it to each strip dimensions (smaller, medium, and larger together)
estimate a division need to fit in length (conservative division, number is rounded up)
estimate a width merge possibility (conservative merge, number is rounded down)
Cross two information and produce an Assembly scene
Operate the assembly scene:
Join the strips together by width into patch
Divide the joined strips in length following this logic:
Divide the first strip of the patch
Divide the next strips of the patch by the closest panel.
Produce two (or more) smaller patches according to the assembly scene
Perform a second and last minimum bounding box to check is the division was successful:
The results become a Boolean constraint that can be applied as a penalty function on the design
Prepare needed the data for output:
Produce connection type list by reading the which of the nodes from the main topology (node//node) are also present in the list of indexes of nodes in each patch.

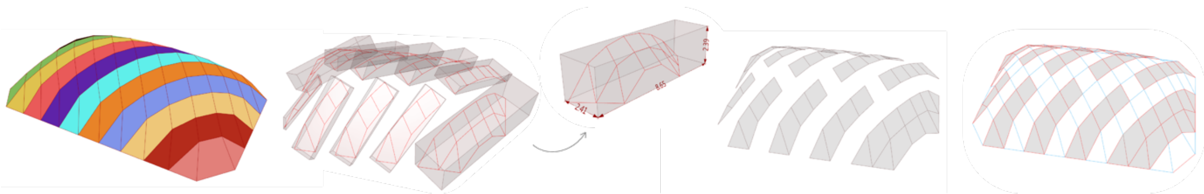


Figure 4-19 Patch division, illustrated flowchart

Precision on the process:
Assess if a strip fits in a truck's trailer:
Retrieve width, length, and height of the two volumes.
Xb, Yb, Zb of box and Xt, Yt, Zt of the Trailer
Sort them in ascending order, and compare the size one by one:
If $minB < minT \cap medB < medT \cap maxB < maxT$,then the evaluated elements fit in the trailer.

To find the minimum bounding box of each of the strip, a C# component crated by Rolf IRL is used, which reduce the rotation of the base plane at each iteration until no further change produce a tighter fit around the geometry.

4.3.2.C Structural Analysis

Input:
Data: removable lines, fixed lines, Topology
Design Variables: minimum height of profile, amount of height increasingly taller, minimum thickness of profile, amount of thickness increasingly thicker.
Inputted data: Installation load, thickness of glass units, stiffness of bolted connection, material
Output:
Moment and Forces at both extremities of each beam
Utilization, max tensile and shear stress of each beam
Height-width-thickness of each beam
Local coordinates system of each beam
Central hub vector of each connection

Steps:
Create two types of beams for each type of lines
Orient Beams on mesh
Prepare the attributes to build the Karamba model:
- Assign a stiffness on the joints connecting to removable beams
- Input anchor points
- Create Cross section list based on inputs
- Create loads base on design

Compute a cross section optimisation, provided by Karamba plugin
- Extract mass computed mass of beams
- Record maximum deflection of the structure

Recover all the necessary outputs:
- Moment and Forces at both extremities of each beam
- Utilization, max tensile and shear stress of each beam
- Height-width-thickness of each beam

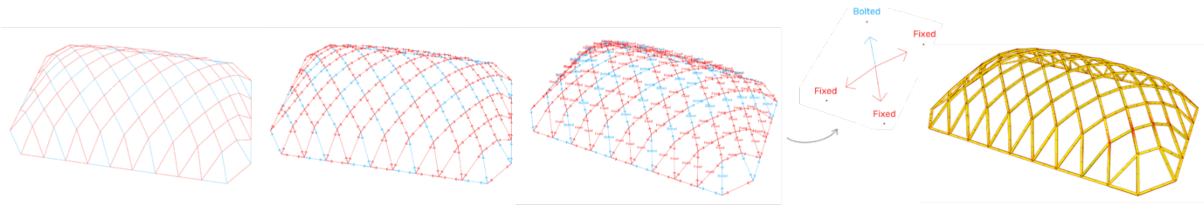


Figure 4-20 Structural analysis preparation, illustrated flowchart

Precision on the process:
Material used:
The material used throughout the whole tool is steel s235.

Load applied on structure:
Gravity applied globally, N/m:
G = 10 N/m

Panel deadweight [kN/m] applied on beams:

$$Panel\ deadweight\ = \frac{\rho glass * G * Aglass * tglass}{Beam\ total\ length}$$

Pglass: Density of glass, 2520 kg/m3
G: Gravitational constant 9.81 N/m
Aglass: Total area of glass panels, defined in model, m2
Tglass: estimate thickness of glass units, 24mm
Beam total length: Beam total length, m

Installation deadweight [kN/m] applied on beams:

$$Installation\ deadweight = \frac{Winst * Aroof}{Beam\ total\ length}$$

Winst: Deadweight per area for installations, 0,5 kN/m (Schoina, n.d.).
Aroof: Total area of gridshell, defined by model, m2

Sorting data properly:
An extensive data sorting must be done. At this level there is a list of beams, a list of nodes and several sub-list of four elements, for each face of the connection, that will contain several elements that will serve for the next calculation. These set of 4 is linked to the amount of beam incoming each node. The scope of this

research focused on valency of 4 but this could be easily extended to any valency, thanks to the main topology. The kind of data contained in these sub-lists will be various: Axial stress, moments, type of connection to the face of the node, heigh-width-thickness of the cross section of the incoming beam to only cite a few ones. The different elements will be further explained in.

4.3.2.D Stress zones and extrapolation

The goal of this part of the script is the extract from an analysis made in Karamba solely on beams the necessary information that will allow to perform more calculation on the connections.
Gridshells can be constituted from a few hundreds to a few thousands of different elements. In order to account for the scale of the analyzed elements, a system of grouping the similarly stress elements has been developed in order to reduce the calculation time.
This process is divided in three parts. First, giving a score to each node based on three selected criteria in order to see which node could be assessed together. These three scores put together produce an ID that characterize the node. Second, grouping the nodes with the same ID. At last, a maximum amount of node that can be analyzed at the same time is set, ~150, which is divided to pick an equal amount from each group for further calculations. This last step defines a selection list. Split into groups, it will serve to at the end calculate an average which will be multiplied to the inverse of that selection list in order to end up with results over all the connections. This method allows to increase the speed of future calculation made without losing the singularity of each connection

Input:
Data: Utilization, max tensile and shear stress of each beam
Inputted data: Amount of node to analyze at once

Output:
list index of joints to study in next process
Related data to each selected node, namely:
- About the nodes: (single value)
  - Hub vector
  - Coordinates of points
  - Index of neighbors
- About each face or incoming beams: (four values):
  - Vector of direction incoming beams
  - Local coordinates system of each incoming beams
  - Index of each beam connected to each node
  - Connection type of node

Steps:
ID Creation
- Step 1:
  Create Stress zones
  Deduct stress zones variety
  Add connection type
  Produce individual ID
  Define the difference that characterize groups of nodes of the same nature, namely of type of connection, amount of stress suffered by the connected beam, and the variety of that stress.
- Step 2:
  Group by ID
- Step 3:
  Select from each group a representative amount to perform future operations

With the index of the selected nodes prepare the data for next process
- Extract from step 4.3.2.B and 4.3.2.C the related information about the select nodes

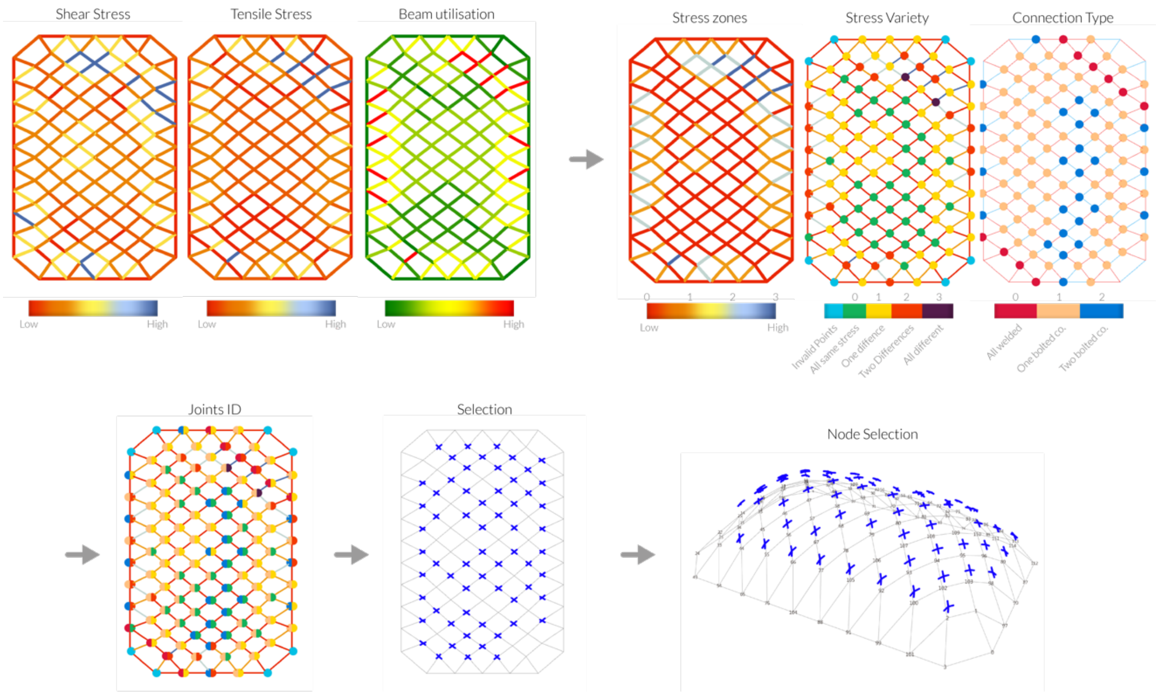


Figure 4-21 Stress zones and node selection, illustrated flowchart

4.3.2.E Component assessment

The goal of this part is to estimate the amount of material in the different type of joints and their related manufacture cost in time and energy consumption. The aimed information is the mass of material, the weld volume, cutting length of the beams, size of bolts. Some information can be extracted directly in the 3D model with geometrical operations. Others, such as the thickness of material and volume of welds, need some calculation.

Input:
Data:
About the nodes: (single value)
  Hub vector
  Coordinates of points
  Index of neighbors
About each side of node or incoming beams: (four values):
  Vector of direction incoming beams
  Local coordinates system of each incoming beams
  Index of each beam connected to each node
  Connection type of node
Inputted Data:
Ultimate Tensile strength of bolt class 10.9: 1000 MPa (Eurocode3, n.d.)
Yield strength S235 steel: 235 MPa (*GRANTA EduPack*, n.d.)
Weld Yield strength: 345 MPa (Budynas & Nisbett, 2011)

Output:
Volume of welds per face, per connection, per design
Thickness of ends plates, hub, and stiffening plates
Size of bolts

Steps:
Create best hub plane:
  Orient z axis of the plane to the inputted hub vector

Retrieve x axis from the local coordinates system of each opposite beams
Combine the vectors between them
And finally combine the remaining two vector to have the bisector angle
The bisector angle will be used to rotate the x axis of the hub plane in the best suited position

A square hub is created:
Based on each hub plane
The width is defined by the biggest width out of the widths of the incoming beams
The height is defined by the biggest height out of the heights of the incoming beams
2.5cm of clearance in added to each dimension
This provides as output the

Measure the cutting length for each beam by intersection of hub and beam

Measure the two angles of arrival of the beam to each face, cf :
Create first plan between vector H and vector F,
Measure the angle of arrival in the horizontal component, angle named β
Create a second plan between vector I and vector F
Measure angle of arrival in the vertical component, angle named α

Extract and order the loads applied to each face

Run analytical calculations based the type of connection:
The formulas are detailed in the next section

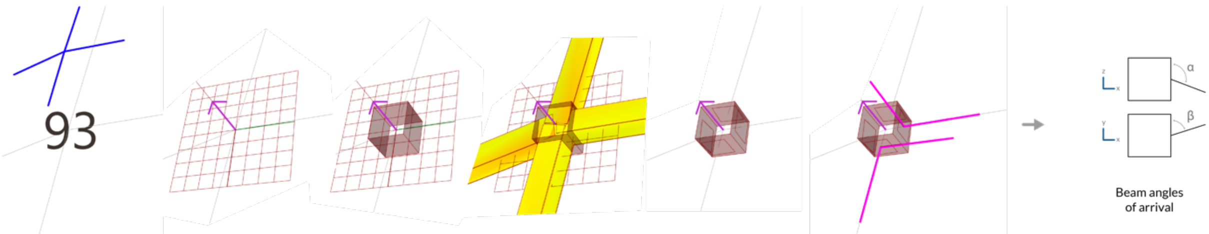


Figure 4-22 Custom measurement of angles of arrivals of each beam, illustrated flowchart

Precision on the process:

Connection design chosen for the research:

To ease implementation, the connection is a simplified version of central hub type joint. The type of design had to cope with a possible change in height in the profile from two or three directions. The goal of the picked design is not to take the most optimisation one term of material and manufacture complexity but rather have a design that can sustain the different change that the geometry can have and be able to quantify decompose into distinctive hand calculation.

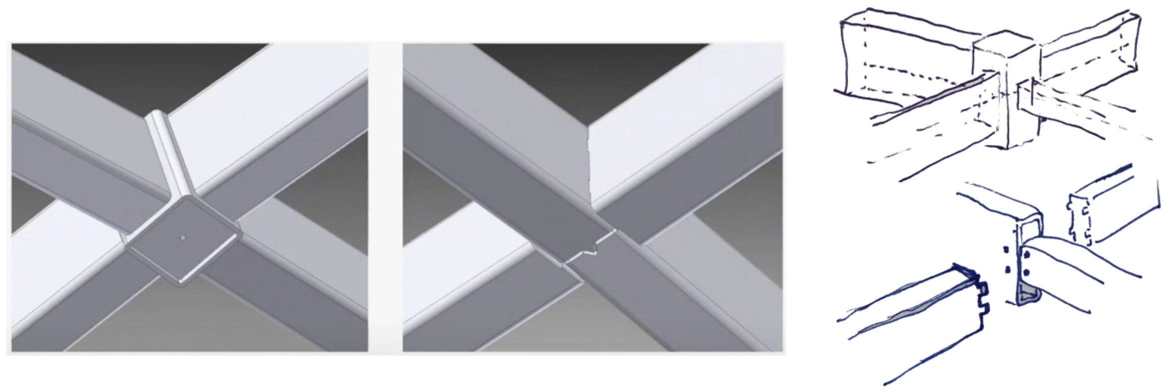


Figure 4-23 Connection comparison between a Hub design and Octatube's connection design in the cross-section variation

The design chose is then a square hub which connect to the different beam type as illustrated in the following figure. The node can vary from two bolted connection, to one or to all four being welded. The bolted connection is decided to have two*two rows of bolts in order to be able to sustain moments in both directions. A template built in IDEAStatica of a single bolted joint, is showed in Figure 4-11.

Bisector calculation:
The bisector calculation is a necessary step in order to have the best common alignment between the incoming members and the faces of square shaped hub. This feature has been implemented to optimize the angle of arrival of each beam.

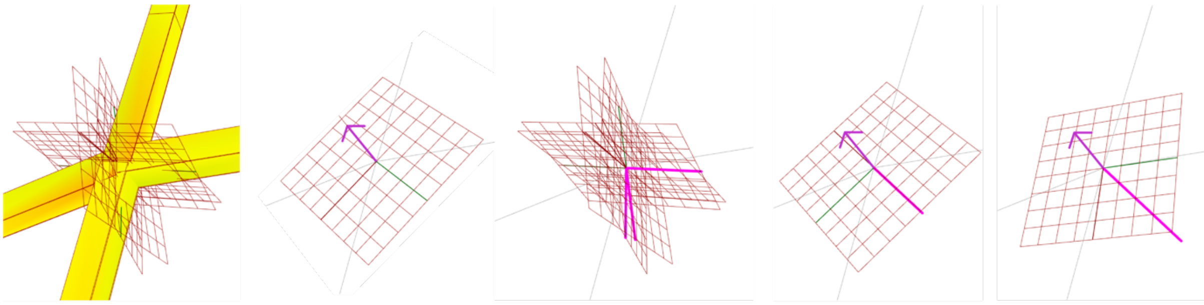


Figure 4-24 Creation of best custom hub plane, illustrated flowchart

Setting the width of the profiles:
The geometrical characteristics of the profile being set before performing the analytical calculation, the width must match some requirement.

Façade system & type of profile:
The minimum width of the profile can be defined by the façade system envisioned to be used. The usual demand of channeled system such as Raico's product in gridshell construction requires to have at least 60mm of flat surface for proper installation. The type of profiles used in tool are cold rolled extrusion. The radius of their fillet can be estimated by multiplying by 2 the thickness of the profile. Hot-rolled profiles can have a smaller radius but consume more energy at production than cold rolled ones. As the tool envisions to leave the thickness as a design variable with a range from 3mm to 10mm. The minimum width of the profile should be around 100mm to account for that change.

Eurocode application in spacing bolts:
The connection design chosen has, in the case of being a bolted connection, two rows of two bolts. In order to not have to build an iteration process between the needed size of bolts and the minimum width and height needed. It was decided to give the width of profiles a minimum width of 100cm. This allow regarding the Eurocode to be able to place two*two rows of bolt to up to M18, which results in a needed space of 92mm. This leave space to vary the thickness of the profiles.

	Normal round holes	Oversize round	Slotted holes							
Size	Minimum end distance along load direction e_1[mm] ($e_1 = 1.2d_0$)	Minimum edge distance perpendicular to load direction e_2[mm] ($e_2 = 1.2d_0$)	Minimum center-to-center spacing along load direction p_1[mm] ($p_1 = 2.2d_0$)	Minimum center-to-center spacing perpendicular to load direction p_2[mm] ($p_2 = 2.4d_0$)	Minimum end distance along load direction e_1[mm] ($e_1 = 1.2d_0$)	Minimum edge distance perpendicular to load direction e_2[mm] ($e_2 = 1.2d_0$)	Minimum center-to-center spacing along load direction p_1[mm] ($p_1 = 2.2d_0$)	Minimum center-to-center spacing perpendicular to load direction p_2[mm] ($p_2 = 2.4d_0$)	Minimum edge distance e_3 [mm] ($e_3 = 1.5d_0$)	Minimum edge distance e_4 [mm] ($e_4 = 1.5d_0$)
M12	16	16	29	32	18	18	33	36	20	20
M14	18	18	33	36	21	21	38	41	23	23
M16	22	22	40	44	24	24	44	48	27	27
M18	24	24	44	48	27	27	49	53	30	30
M20	27	27	49	53	29	29	53	58	33	33
M22	29	29	53	58	32	32	58	63	36	36
M24	32	32	58	63	36	36	66	72	39	39
M27	36	36	66	72	42	42	77	84	45	45
M30	40	40	73	80	46	46	84	92	50	50
M33	44	44	80	87	50	50	91	99	54	54
M36	47	47	86	94	53	53	97	106	59	59
M39	51	51	93	101	57	57	104	113	63	63

Table 4-3 Minimum end distance, edge distance, and spacing for bolt fasteners according to EN1993-1-8 Table 3.3 (rounded up to nearest mm) (Eurocode3, n.d.)

This pre-set of the fixed width could lead to a limitation of the possible outcome in the design. In order to have a proper assessed width, a recursive process would need to be place

Analytical Method:

An analytical calculation method is implemented in two distinct sequences of calculations defined by the type of connection between the face of the joint and the incoming beam. Each face of the connection is estimated separately following the following logic:

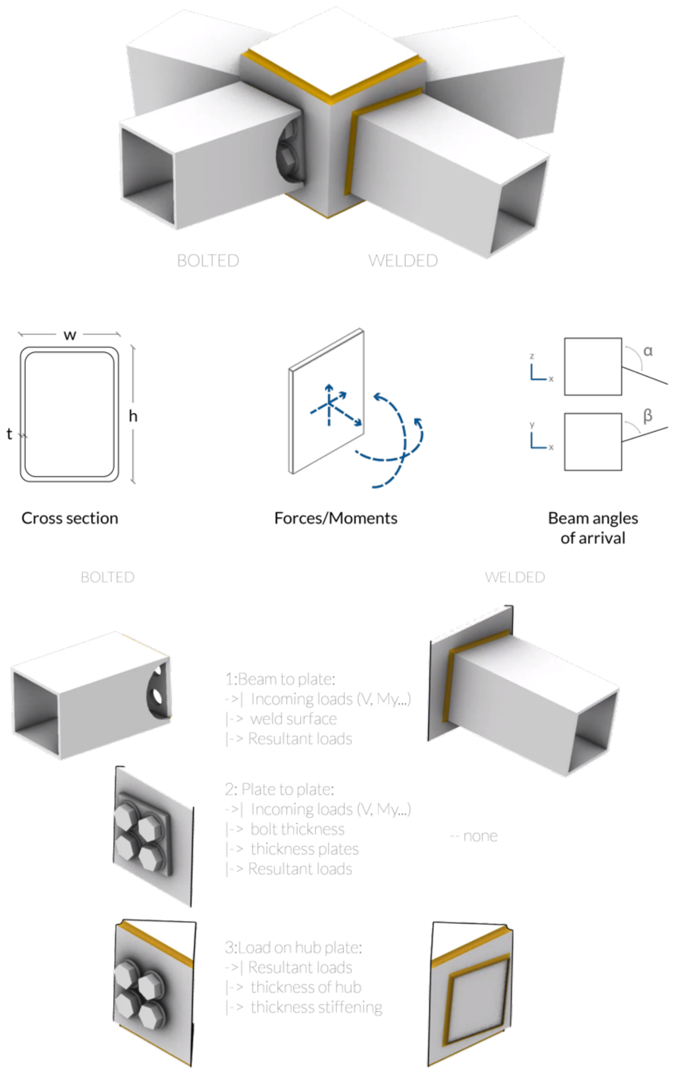


Figure 4-25 Two set of possible calculations done depending on the connection type

Before jumping into the formulas, a general logic has been applied to all the calculation. Only the more stressed area in each situation is interesting for the estimation of the different thicknesses and weld depth. Each calculation considers the two shear forces, the axial force and the two bending moments. As the calculation will estimate the maximum tensile stress of the different components, the maximum stress scenarios are the following. Note that the location of that stress is at this stage left aside.

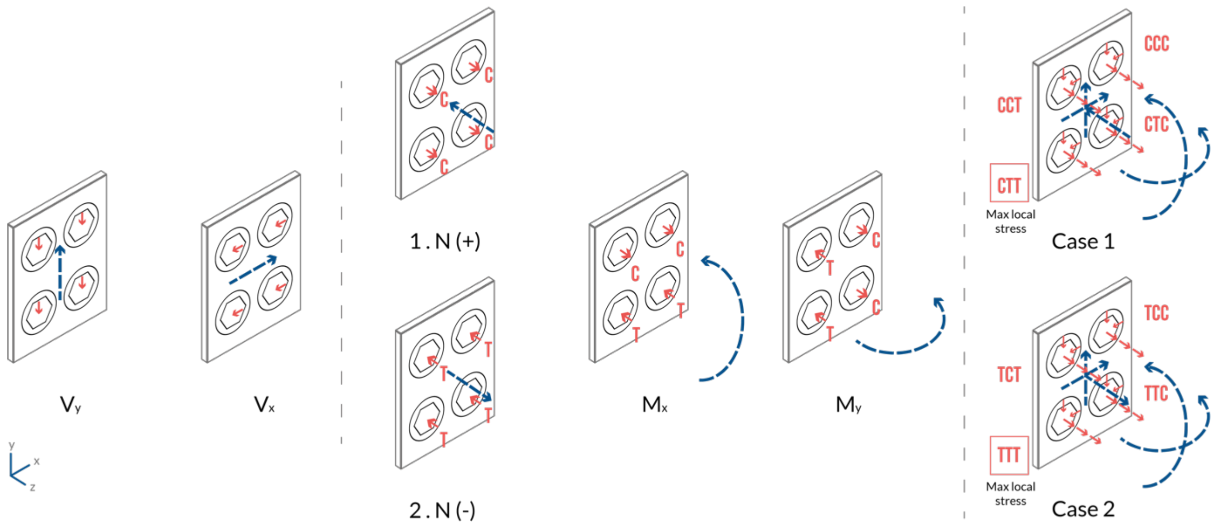


Figure 4-26 Tensile stress scenarios

As most of the calculation will be using all the fives forces/moments. It highlighted here that the sign of the Axial force will determine if it is added to the tensile stress build by the two moments, case 1, or subtracted, like in case 2. The shear for a matter of generalization, will always be added.

A) Welded connection

The size of the leg of the weld is estimated between the beam and the hub for a simple fillet join. This is achieved by calculating the face area of the weld needed of each of the side around the profile. As the angle of each beam varies in two directions, the same face area can result in different sizes of legs. It is important to notice that the minimum manufacturable weld leg size being 4mm, it will be replaced in every result that fall under that value. The following formulas to calculate the weld is the following.

The calculate of stress in the weld is divide into two components:
On the first hand the shear stress, or stress applied on the whole area of the weld. This account for both the shear stress and the Axial stress.

$$\tau = \frac{V}{A} \tag{4.1}$$

τ: shear stress MPa
V: shear Force
A: Area

The area in question relates in this case to the face area which is estimated for fillet welds by the length of the weld multiplied by the length of the face, calculated as follow, cf Figure 2-21:

$$\sin\theta = \frac{\frac{t}{2}}{l} \tag{4.2}$$

$$t = 2l \, . \sin\theta \tag{4.3}$$

l: leg length of weld
t: face length of weld
θ: half of angle between

In 3D, each angle can be rewrite as follow:

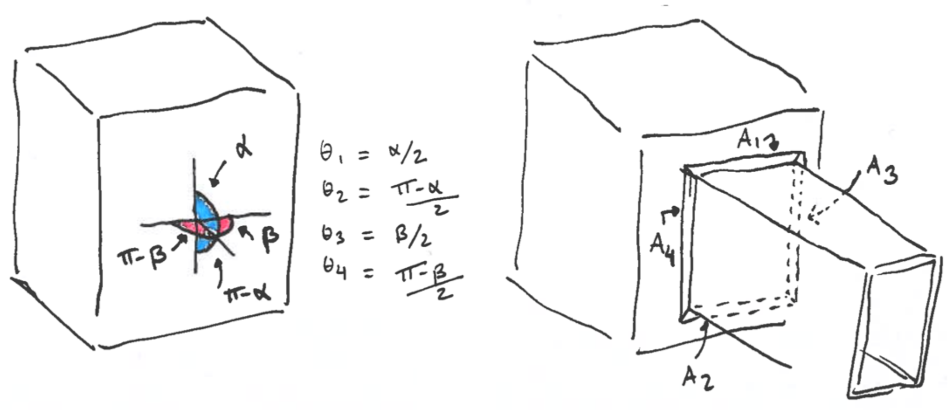


Figure 4-27 Angle to consider for each side of the fillet weld

The total area of face weld will be then:

$$A = A1 + A2 + A3 + A4 \tag{4.4}$$
$$with$$
$$A1 = 2.l.\sin\theta_1 * width$$
$$A2 = 2.l.\sin\theta_2 * width$$
$$A3 = 2.l.\sin\theta_3 * heigth$$
$$A4 = 2.l.\sin\theta_4 * heigth$$
$$rewrite\ as$$
$$A = 2l\big(w(\sin\theta_1 + \sin\theta_2) + h(\sin\theta_3 + \sin\theta_4)\big) \tag{4.5}$$

Replaced in the initial formula we have:

$$\tau = \frac{V}{2l\big(w(\sin\theta_1 + \sin\theta_2) + h(\sin\theta_3 + \sin\theta_4)\big)} \tag{4.6}$$

$$rewrite\ as$$
$$l = \frac{V}{\tau.2\big(w(\sin\theta_1 + \sin\theta_2) + h(\sin\theta_3 + \sin\theta_4)\big)} \tag{4.7}$$

Using eq. (4.6), the vertical shear and horizontal shear are combined as vector to give, (Budynas & Nisbett, 2011):

$$\tau = \sqrt{{\tau_{Vx}}^2 + {\tau_{Vy}}^2} \tag{4.8}$$

On the other hand, the stress created by the moment can calculated as such:

$$\sigma = \frac{M.c}{I} \tag{4.9}$$

M: Applied moment
c: distance to the neutral axis
I: second moment of Area

Same as the area in the previous formula, the second moment of area of the weld will be calculated with the average of the face length around the profile. The formula the second moment of area for rectangular hollow sections in used as it is the closest to this situation. (Gere & Goodno, 2012)

$$I_x = \frac{1}{3}.w.h^2.t_{av} \tag{4.10}$$

$$I_y = \frac{1}{3}.h.w^2.t_{av} \tag{4.11}$$

Ix, Iy: Second moment of area
w: width of profile
h: height of profile
tav: Average of face length

Using eq. (4.3), the average of the face length can be described as follow:

$$t_{av} = \frac{t_1 + t_2 + t_3 + t_4}{4}$$

$$using\ eq.(4.3), rewrites\ as$$

$$t_{av} = \frac{l(\sin\theta_1 + \sin\theta_2 + \sin\theta_3 + \sin\theta_4)}{2} \tag{4.12}$$

Using eq. (4.12), the second moment of area can be expressed with:

$$I_x = \frac{1}{3}.w.h^2.\frac{l(\sin\theta_1 + \sin\theta_2 + \sin\theta_3 + \sin\theta_4)}{2}$$

$$rewrite\ as,$$

$$I_x = w.h^2.\frac{l(\sin\theta_1 + \sin\theta_2 + \sin\theta_3 + \sin\theta_4)}{6} \tag{4.13}$$

Similarly, Iy is expressed with:

$$I_y = h.w^2.\frac{l(\sin\theta_1 + \sin\theta_2 + \sin\theta_3 + \sin\theta_4)}{6} \tag{4.14}$$

Replacing all the member is the initial eq. (4.9), we have for both moments:

$$\sigma_{Mx} = \frac{M_x.c_y}{I_x} = \frac{M_x.\frac{h}{2}}{w.h^2.\frac{l(\sin\theta_1 + \sin\theta_2 + \sin\theta_3 + \sin\theta_4)}{6}} \tag{4.15}$$

$$\sigma_{My} = \frac{M_y . c_x}{I_y} = \frac{M_y \cdot \frac{h}{2}}{h. w^2 . \frac{l(\sin\theta_1 + \sin\theta_2 + \sin\theta_3 + \sin\theta_4)}{6}} \qquad (4.16)$$

The is also the stress expressed by the Axial force, that is written:

$$\sigma_N = \frac{N}{A} = \frac{N}{2l\big(w(\sin\theta_1 + \sin\theta_2) + h(\sin\theta_3 + \sin\theta_4)\big)} \qquad (4.17)$$

As mentioned in Figure 4-26, the max stress scenario will depend on the sign of the Axial force. Therefore, the three stresses are combined as vector to give:

$$\sigma = \sqrt{\sigma_{Mx}{}^2 + \sigma_{My}{}^2 \pm \sigma_N{}^2} \qquad (4.18)$$

In order to calculate the total stress into the weld, both the shear stress and the tensile are combined again as vector to give the final expression:

$$\sigma_{total} = \sqrt{\tau^2 + \sigma^2} = \sqrt{\left(\sqrt{\tau_{Vx}{}^2 + \tau_{Vy}{}^2}\right)^2 + \left(\sqrt{\sigma_{Mx}{}^2 + \sigma_{My}{}^2 \pm \sigma_N{}^2}\right)^2} \qquad (4.19)$$

Decomposing all the members and solving with a maximum weld Yield strength of 345 MPa, (Budynas & Nisbett, 2011), the leg length is estimate as an average leg around the profile.

B) Bolt connection

For this step, calculation will serve to estimate the size of the bolt and the consequent cutting length to do in the laser cutter.

Two set of calculation have to be developed to estimate the tensile and shear stress in the bolt before adding the stress together like in eq.(4.19).

In the calculation of the bolt there is no influence of the angle of arrival of the beam, and the bolt are connecting the end plate and the hub at 90degrees.

The Shear stress can be defined with the following formula:

$$\tau = \frac{F}{NbrBolt\,.\,Ag} \qquad (4.20)$$

F: Shear Force applied [kN]
NbrBolt: Number of bolts that share that force
Ag: Nominal gross area of the bolt provided by the Eurocode (Eurocode3, n.d.)

The number of bolts is set to 4 for this particular design.
The class type of the bolt is chosen to be 10.9 giving a strength of 1000MPa, which with the safety values applied by the Eurocode based on the connection type of design is equivalent to 720MPa.
The Nominal gross area is defined by the following formula:

$$Ag = \frac{\pi}{4} . {t_{bolt}}^2 \quad (4.21)$$

t: the thickness of the bolt

Re-writing the formulas together give for each shear force the following:

$$\tau_{V_x} = \frac{V_x}{\pi . {t_{bolt}}^2}$$
$$\tau_{V_y} = \frac{V_y}{\pi . {t_{bolt}}^2} \quad (4.22)$$

Similarly, the tensile force applied using eq. (4.20) but with the Nominal tensile Area, As, instead of the Nominal gross area, Ag. The value of the nominal tensile area, As, is given as a table by the Eurocode for each bolt size. As the goal here is to find the most suited size of bolt, it adds complexity in the tool to have to retrieve a value from a table. Instead, a factor could be applied to approximate As based on Ag that is described with the value set to isolate, the thickness of the bolt.
A factor could be set to:

$$As \approx \frac{Ag}{1{,}32} \quad (4.23)$$

as seen in:

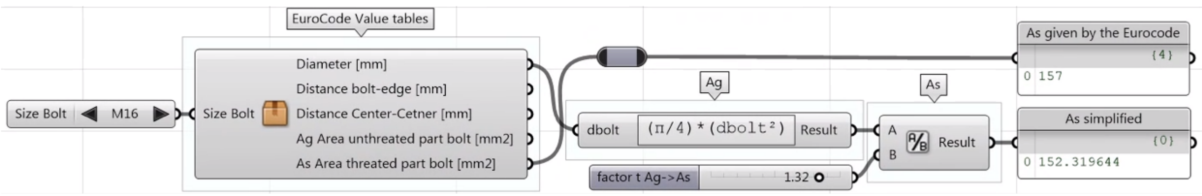


Figure 4-28 Conversion rate from Nominal gross area to Nominal tensile area in bolt calculation

Following the conversion rate, the Tensile stress in the bolt can be written:

$$\sigma_N = \frac{1{,}32 \, . N}{\pi . {t_{bolt}}^2} \quad (4.24)$$

The other two moments applied on the bolts can be done using the following formula to first calculate the tension force created by each moment. The Tension force is determined as follow:

$$F_M = \frac{M}{Nbrbolts_M . r} \quad (4.25)$$

Fm: Tension Force created by the moment
M: moment applied on the bolts
Number of bolts M: Are the amount of bolt receiving the moment, it would take out the bolt in the Neutral axis. In our case this number is also set to 4 as all the bolts of the design are not in the neutral axes.
R: Distance from the bolts to the neutral axis

The goal being the make the bolted connection as much moment resisting as possible, so the distance r must be maximized. The way the maximal distance to neutral axis can be defined is by looking at the minimal space required between a bolt and the edge of the plate, noted ed. Similar than with the Nominal tensile area, this value comes as a table and this type there is no reference value to multiply by a simple factor. The decision was to take the minimal edge distance for the bolt M18, which is the biggest applicable in the connection and use it for all the lower sizes. This provides a less moment resisting connection for the lower size of bolt but consist of the easier way of proceeding without adding too much complexity in this part. This minimal edge distance is set for M18 bolts to 24mm and will be then used to define the distance to the neutral axis as follow:

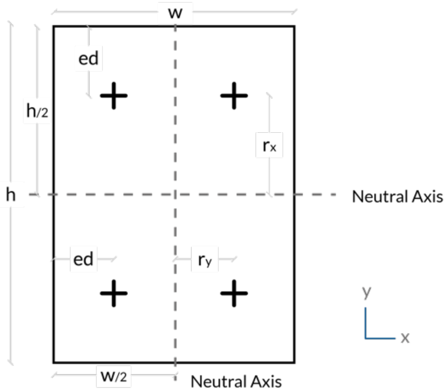


Figure 4-29 Diagram to estimate the Distance from the bolts to the neutral axis

Re-writing eq. (4.25) with the given information, we get the height and width function of the Moment and the Tension force:

$$F_{M_x} = \frac{M_x}{4\,.\left(\frac{h}{2} - ed\right)}$$

$$F_{M_y} = \frac{M_y}{4\,.\left(\frac{w}{2} - ed\right)} \qquad (4.26)$$

At last, the stress in the bolts resulting from the different moments can be written using eq.(4.24):

$$\sigma_{M_x} = \frac{1{,}32\,.\,F_{M_x}}{\pi.\,{t_{bolt}}^2}$$

$$\sigma_{M_y} = \frac{1{,}32\,.\,F_{M_y}}{\pi.\,{t_{bolt}}^2} \qquad (4.27)$$

Using the same logic described in the Weld calculation the different stresses can be added together, using the eq.(4.19). Knowing the maximum stress value of the bolt, the equation is solved to retrieve the thickness of the bolt. Having a step of 2mm between each bolt size, the resultant thickness is rounded up to the next bigger bolt thickness.

The same thickness is determining to calculate the perimeter of the hole needed for the bolt and added to the cutting length of the specific connection.

C) Load of profile on hub

Estimating the thickness of the hub is done when at the end of each process. This is done for all four side of the connection and the biggest value out of the four side continues as defined thickness of the hub in that particular joint. To estimate the thickness, a plate calculation is used with all edge fixed and a circle load at the center of the plate. The formula used is the following (Roark et al., 2002):

$$\sigma = \frac{3.W}{2.pi.t^2}.[(1+v).ln\frac{2b}{pi.r_0'} + \beta_1 \quad (4.28)$$

W: total load applied [N]
t: thickness of plate
v: poisson ratio of material,
r0': radius of circle , r0' = r0 if ro>= o.5*thickness (always the case in the connection)
B1: value issued from the table in, defined by the ratio of the height and width (Roark et al., 2002) see appendix Figure 10-1.
b: short edge, the width of the profile in the connection

Depending on whether the load applied on the plate comes from 4 bolts or a surface of weld. The total load applied, W, is estimated differently. The maximum force applied in the connection will be used as applied load to provide a pessimistic estimation of the plate thickness. The part of the connection where the most force is applied is also where there is the biggest stress, therefore, of each type of load, these maximums are denoted:

$$W_{bolt} = \frac{M_x}{4.\frac{h}{2}.ed} + \frac{M_y}{4.\frac{w}{2}.ed} \pm \frac{N}{4}$$

$$W_{weld} = \frac{M_x}{h} + \frac{M_y}{w} \pm N \quad (4.29)$$

In the case of the bolted connection, the plate calculation is done twice. Once with the plate dimension of the hub to estimate the thickness of the hub. And a second time with the dimensions of the cross section to estimate the thickness of the end plate.

4.3.2.F Verification of Connection calculation

The calculation about material thickness and volume size during the part 4.3.2.E are checked at one node at the edge of a patch by exporting it to IDEAStatica using the IDEAKaramba plugin. After applying the same thicknesses and weld volume calculated in the Analytical methods, a Stress Strain analysis as well as a stiffness analysis is performed. The results are positive as all the component estimated manually are validated in the Stress Strain analysis in IDEA.

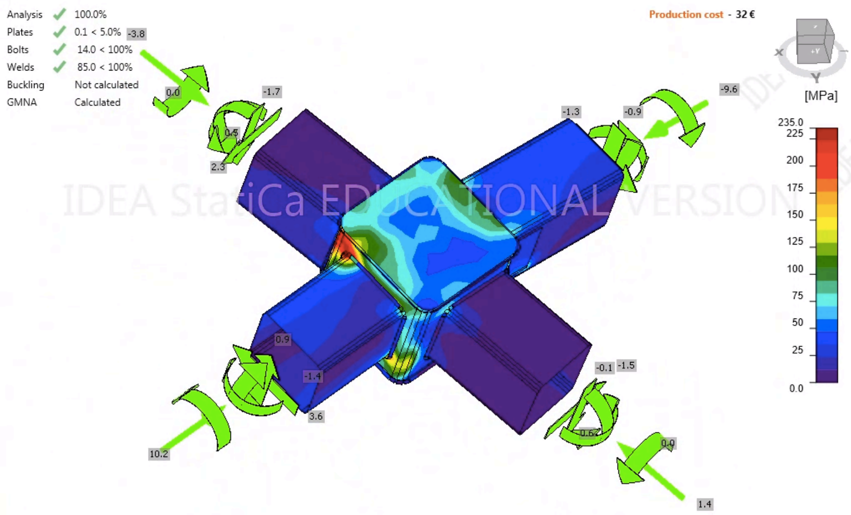


Figure 4-30 Stress Strain Analysis result in an example node

The analysis has been done on an iteration of design that has all the constraint set to valid. The cbFEM analysis show that the estimated component is done with acceptable considerations.
The same iteration will be used to inside the data structure in the next chapter.

4.3.2.G Envelope Repercussion

Input:
Data: Produced Mesh
Output:
Planarity deviation (indicator)
Glass waste at cutting the panels (indicator)

The Glass waste at cutting panels indicator is built to join several parameters that would need to be assesses individually. Using the same minimum bounding box script then at the prefabrication division step. Each panels constituting the gridshell were fitted and measure to see if at least of the x y dimension of the bounding would fit in a standards size of 3.1m of float glass. Facing the same problem than in the Component Assessment step, it would be unfeasible to search for the minimum bounding box for every panels. For that, the triangular panels were separated from the quadrangular shaped ones and just a portion of them were analyzed. Taking such steps is better than outputting the skewness and aspect ratio regarding the objective of reducing the waste purely.

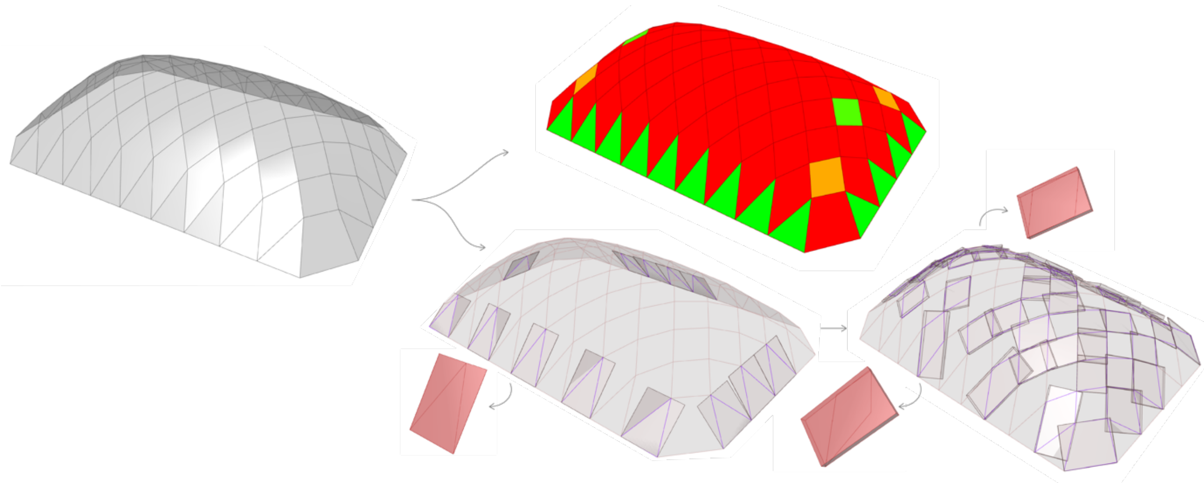


Figure 4-31 Envelope repercussion calculation, illustrated flowchart

4.3.2.H Data used throughout the tool

At the end of the different processes, output information is gathered to calculate the different indicators that will inform the objectives and score of the different design options. The fabrication time needed to prepare each component, prefabricate the patch, and assemble the parts on site. The sustainability count on the embodied carbon is done following Figure 2-18, once all the quantity of material is known. Finally, the Structural performance is extracted and summarized into indicators to testify the about the requirement and the performance of the design.

The data used in each of the calculation is accessible below. This list relates to all data input coming from outside the tool, not measured on the model but rather given by professional practice, material property and fabrication estimation counts. Certain value such as the emission of the grid could evolve producing different naturally less impactful structures.

DATA INPUTTED	VALUE	REFERENCE
Truck size [m, m, m]	var	Octatube advisory
Truck consumption [L/km]	var	(CTEu)
Max size Glass panels [m]	3.1	Octatube advisory
Deadweight installation [kN/m2]	var	(Schoina, n.d.)
Total Glass thickness of unit [mm]	var	Octatube advisory
Rotational thickness of Joint [MN/rad]	var	(Isufi, n.d.)
Yield Strength cold formed steel 235 [MPa]	235	(Eurocode3, n.d.)
Ultimate tensile strength bolt 10.9 [MPa]	1000	(Eurocode3, n.d.)
Yield Strength Weld [MPa]	345	(Budynas & Nisbett, 2011)
Density Steel 235 [kg/m3]	7850	(*GRANTA EduPack*, n.d.)
Fixed ed [m]	0,024	(Eurocode3, n.d.)
Washer Diameter [m]	0,034	(Eurocode3, n.d.)
Carbon factor steel 235		
Welding Speed [cm3/h]	100	(Ajouz, n.d.)
Energy Welding [kWh]	12	Lit.Rev
Laser Cutting speed [m/h]	12	Lit.Rev
Energy Laser [kWh]	23.8	Lit.Rev
Prefabricate patch:		
Hours/node in patch [unit/h]	6	Octatube advisory
Install patch on side [unit/h]	8	Octatube advisory
Install connection beam [unit/h]	1	Octatube advisory
Carbon Factor steel 235 [kgCO2e/kg]	2.975	(*GRANTA EduPack*, n.d.)
	2.29	EC3 av. on EPDs
Emission of burning Gasoline [kgCO2e/L]	2.3	(Pollution Probe)
Grid emission Netherlands [kgCO2e/kWh]	0.3284	European Environment Agency

4.3.3 Reflection on the implemented tool

The time for the tool to calculate what was aimed to, could be cut down to less than 3secondes. It is an important achievement as the tool is built to be used in optimisation software. It means that to produce a population size of 2000 iterations, the whole process would take around 1.5 hours. The finally selected tool and processes made the tool reliable within the range set for the design Variables. An image of the possible generated space can be seen in Figure 5-12.

On another note, the implemented tool provides only a proof of concept, and its generated design space could be considered quite narrow. There is only one type of support edge, rectangular. And only one type of pattern, diamond grid. Other features envisioned in the exploration part could greatly benefit the tool exploration into more irregular shapes.

5 Method relevance & use

5.1 Estimating impacts at early stage of design

Estimating the impact at early stage of gridshell design with the proposed tool implies two things. That the indicators chosen by the tool are sufficient to understand and evaluate the different performances given as score to a particular design. The second one, that the design exploration can be driven with informed decision. The two sub-chapters will detail the focus on the set of tests and visualization tools developed in that regard.

5.1.1 Design Variables impact on the Design Objectives

To study the variation of the scores attributed by the tool and how the objectives reflect the performance of the different design's, three test are done. The purpose of these tests is to check if the indicators allow to explain the values tracked by the objectives and if the design variables have a change in those objectives. Summarized below, the tests consist of isolating design variables in order to understand their impact over the design objectives. The Design variables isolated are the following:

- Increase the u v pattern division with the other design variables constant.
- Increase vertex load with the other design variables constant.
- Increase thickness amount created with the other design variables constant.

These tests do not cover over the design variables, Figure 5-10, as some of them would probably lead to same understanding. For example, when increasing of the amount of different thickness created a reaction on the amount of profiles height chosen as optimal by the tool is clear.

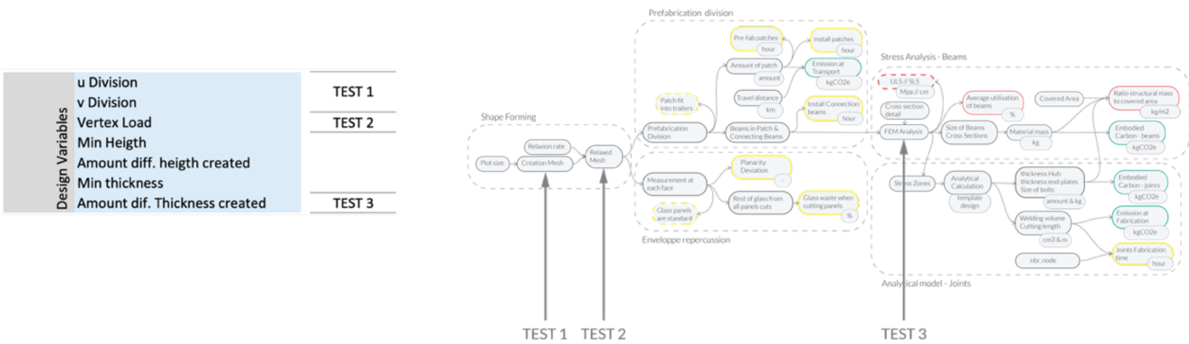


Figure 5-1 Design Variable focus for each test

The pool size will be of 10 designs for the first two experiments and three for the second one.
For each of them, a clustered stacked chart will report the score of the embodied and emitted carbon. The results will serve to assess the impact of a particular design choice regarding sustainability objectives.
Other graphs, with different focus, will highlight behavior occurred during the increase of the specified design variable. The punctual change in scoring will be explained by highlighting the parameter or group of parameters that made this subtle change in score. Doing so will also highlight the tendencies encrypted in the script.
The following statement as used as guide to analyze the outputted data:

- Understand the way each objective scales up. Be able to tell if the increase seems to be exponential or linear would be a start.
- Understand how the objectives scales and what could bound them into stepped increase and therefore possible clusters.
- Highlight is there a substantial incongruence in this evolution

<u>First Test: u v division increase</u>:
For the 10 samples produced for this test, the u v division of the produced mesh is increased with different combination. The design variables inputted for each generation can be seen in and their geometrical outputs with their related design requirements can be seen in
Figure 5-4.

Design Variables											
	u Division	10	12	12	16	20	20	22	24	26	28
	v Division	10	12	14	18	18	22	22	24	26	28
	Vertex Load	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7
	Min Heigth	10	10	10	10	10	10	10	10	10	10
	Amount diff. heigth created	15	15	15	15	15	15	15	15	15	15
	Min thickness	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3
	Amount dif. Thicnkess created	4	4	4	4	4	4	4	4	4	4

Figure 5-2 Design Variables set-up for test 1

It is to be noted that the design requirements were not fulfilled for all the generations. This can be checked directly on the on-screen display information in
Figure 5-4 or in the red areas displayed in the charts produced by the tool in Figure 5-5. At too low u v division the size of the panels is too big and therefore the design will have issues when arriving at that point of design. This is a case for the first three generations. Similarly for the 8th and 9th, it appears that the tool failed to split adequately the design into proposed sized patch. This issue has been explained in the tool implementation in Chapter 4.3.1, and reveal of a limitation taken at scripting. Regarding the Structural requirement, all are checked and therefore validates each generation for this test.
As the u v division is a combined values difficult track, the amount of node in each generation will be the base of the analysis. With the saved data accessible in the appendix, Figure 10-2, a first clustered stacked chart is plotted. This chart highlights how the embodied carbon is divided between the beams and the nodes at each step. The embodied carbon of the joints represents only a portion of the total, whereas it represents nearly a third in the last generations. This would make sense as the same physical space is divided more and more creating more intersection materialized by joints. The beams are then less and less represented as the number of joints increase. This evolution appears to be rather smooth beside the graph displayed each generation equally distant in the x axis even though the increase in node number is not constant. More about that in the next chart.
Similarly, the emitted carbon repartition between the transport and the fabrication processes is evolving but not with the same behavior. The emission linked by the fabrication itself seems to be increase constantly but the one by the transport appears to be increase in steps. Even the 7th and 8th generation appear to score worse than the two last. The reason to that is highlighted in Figure 5-5 c, as it is directly linked to the amount of patch defined at the prefabrication division.

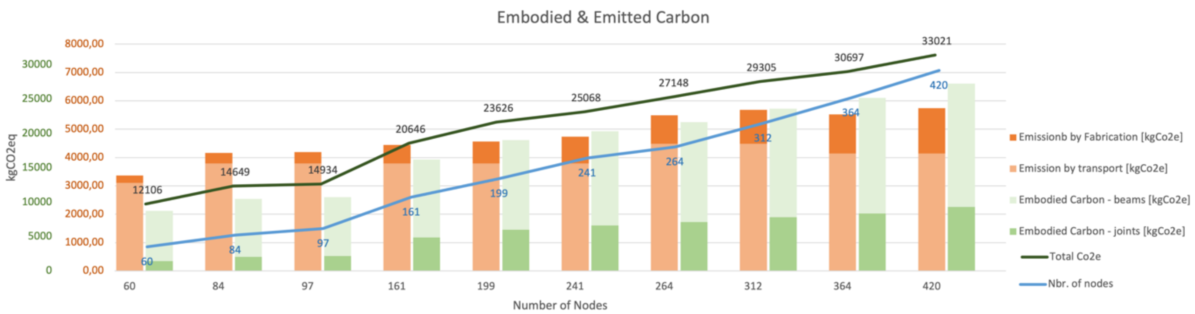


Figure 5-3 Embodied & Emitted Carbon evolution during u v change in pattern

The decrease implies that there is a limitation of the tool to find the optimum division, but it also has a geometrical relation. The tool divides the patch applying the same logic each time meaning that the two worse generations created a geometrical condition that were not favored by the way the division has been created. The part of script responsible to divide the strip by length encountered that these exact u v division coupled with the size of the plot created longer strips that had to be divided, therefore created extra patches, and consequently increasing transport and scoring a higher score for the emission.

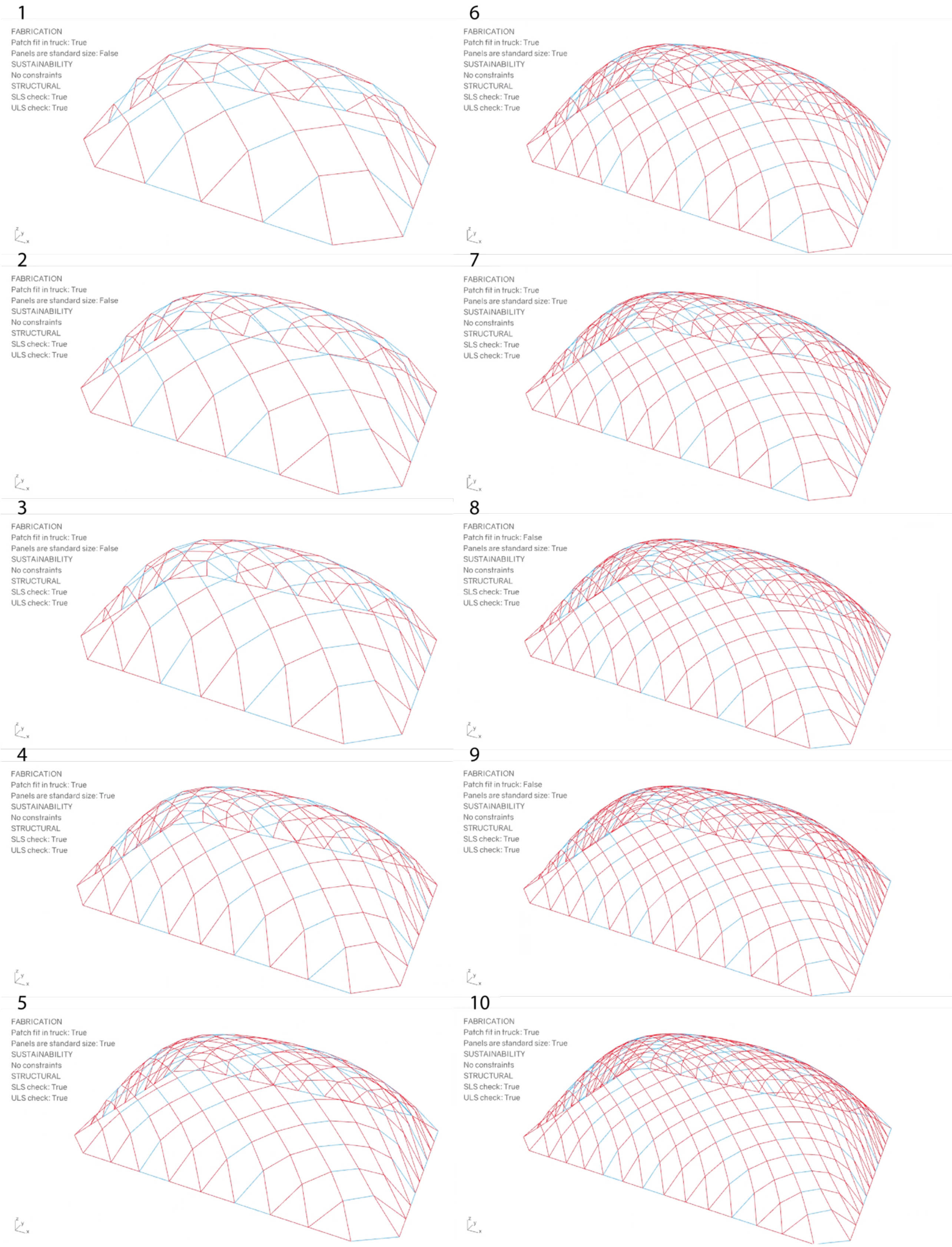


Figure 5-4 Design through increasing u v division

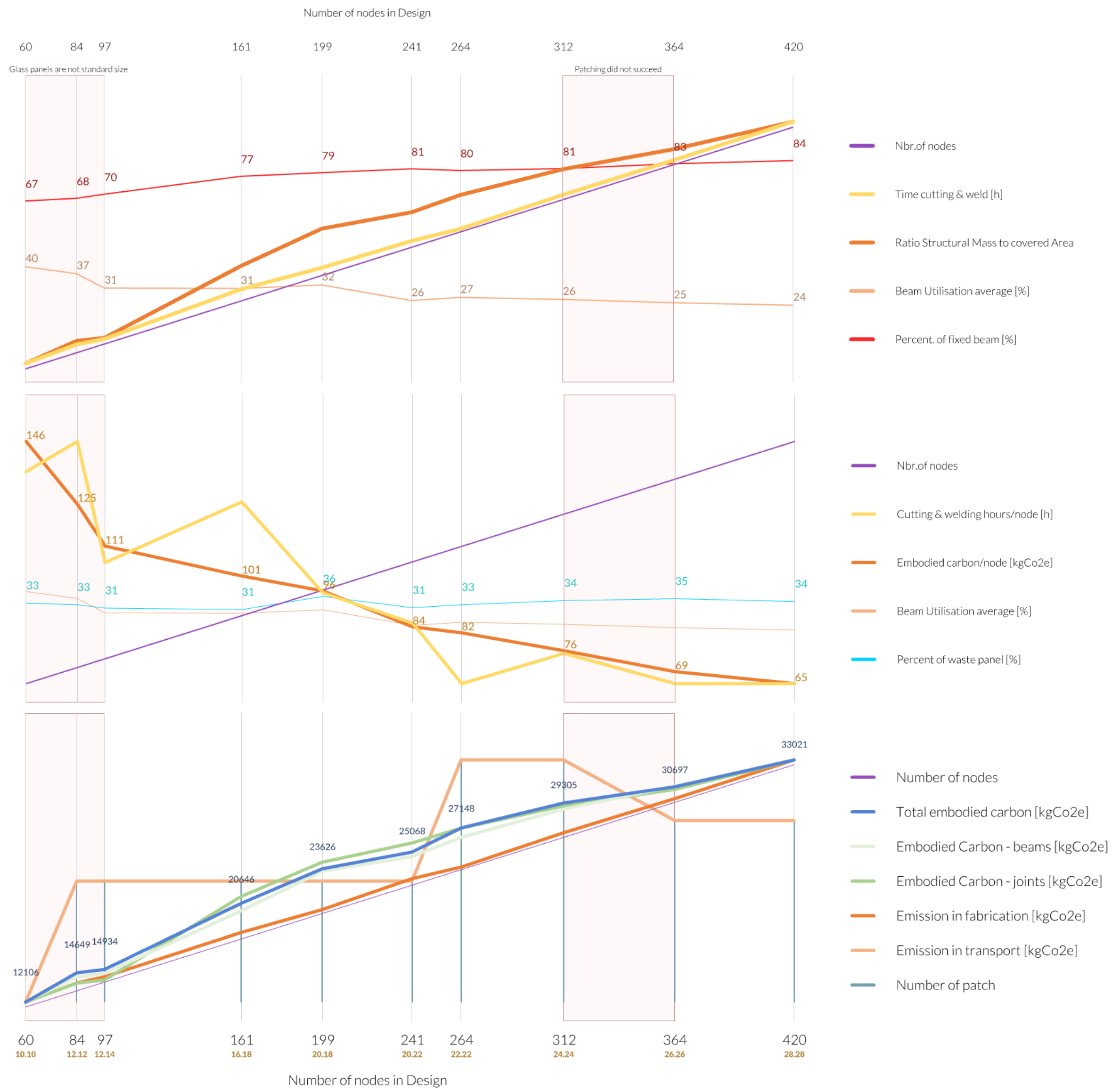


Figure 5-5 Normalized objectives data output from design exploration (top to bottom a, b, c)

In these graphs, produced as a quasi-automated output add-on by the tool itself, each series of data has been remapped to the same minimum to maximum values. This does not apply to the series in [%] which are plotted from 0 to 100. This kind of plot accentuate the changes within series of data and produce a more dynamic plot.
Also, the space between each generation, in the x axis, is set in order to show the increase in number of nodes as a linear function. The linear can be seen in purple as reference in each of the graphs.
This way in part c, the way each value increase can be better visualized.
The stepped increase of the emission linked to transport is clear and coincide with the number of patches. The other emission value appears to be nearly blended with the reference line attesting that the relation here is quasi proportional. In the case of the embodied carbon, in their own proportion the embodied carbon in the joints and the beams increase together. The space created between them and the refence suggest that there is nonlinear behavior between the increase in node amount and the embodied carbon. Plotting the Embodied Carbon of the joints divided by the number of nodes in each step, graph b), appear to suggest the same finding. For the same connection design, the embodied carbon per node decreases as the amount of node increasing. The same happens with the amount of time spent on fabricating those joints. It

suggests that with the increase of the number of nodes, less force is applied to each joint. It makes sense as the bending moment applied at the connection is reduced as the beam's length are reduced. Another item worth noting is the percentage of waste when cutting the glass panels that remains the same in the configuration. It can be concluded that the u v division coupled with the way the mesh is created, diamond pattern, does not help to significantly reduce glass waste when producing the panels.
More fixed beams are linked to a reduction in av. beam utilization. It means that even though more fixed connections are introduced it is not enough to have a more homogeneous distribution of forces in the shell.

Second test: Vertex load increase:
In this test, the amount of element will remain the same, but the relaxation of the mesh will be increase as the vertex load increase, generated a geometry that start rather flat and finish with a high curvature.

Similarly, then in the first test, the set of design variables used to produce each of the generation are displayed below and the geometrical output of the tool in the following figure. This time the increase rate between each generation could be stepped and have been stepped to 0.07. Looking at the generated geometries, it can be said that the range of options is rather large.

Design Variables	u Division	20	20	20	20	20	20	20	20	20	20
	v Division	20	20	20	20	20	20	20	20	20	20
	Vertex Load	0.1	0.17	0.24	0.31	0.38	0.45	0.52	0.59	0.66	0.73
	Min Heigth	10	10	10	10	10	10	10	10	10	10
	Amount diff. heigth created	15	15	15	15	15	15	15	15	15	15
	Min thickness	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3
	Amount dif. Thicnkess created	4	4	4	4	4	4	4	4	4	4

Figure 5-6 Design Variables set-up for test 2

Regarding the outputted data, the same chart is made as control test to evaluate the evolution of the embodied and emitted carbon for each design. Other graphs highlighting the impact of vertex load in the different objectives. As the number of elements does not change other behavior will be noticed.
Again, it is to be noted that design requirement is not fulfilled regarding the SLS check in the generation one to four. It should be taken into account in the analysis, nevertheless, this is not an issue per say as this behavior could be deducted as the gridshell does not direct very well nonplanar axis load as efficiently as shells and therefore build up more moments when the relaxation is low.

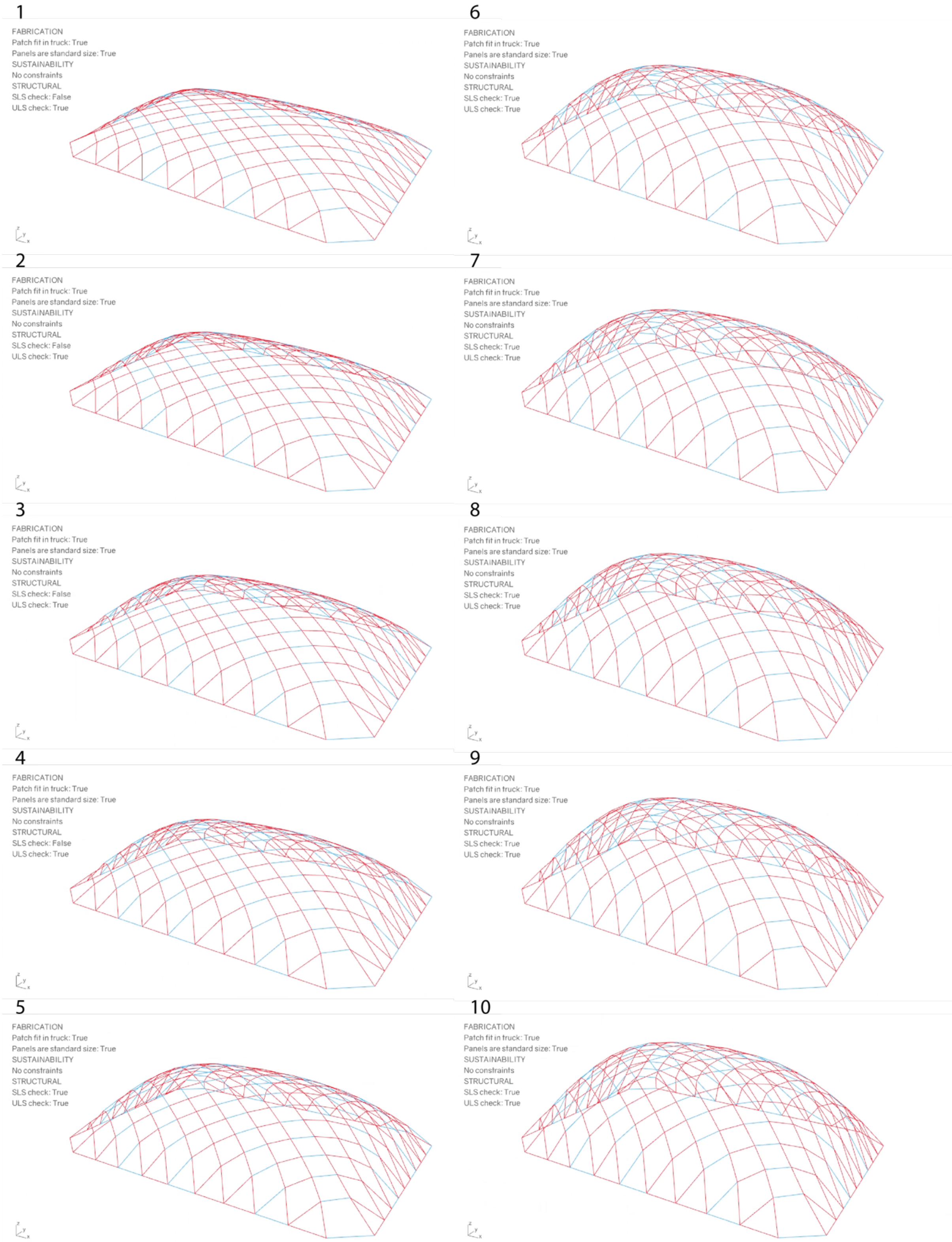


Figure 5-7 Design through increasing vertex load

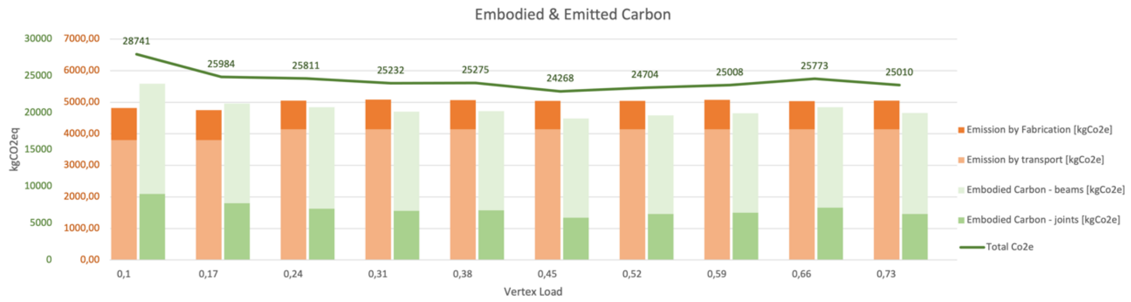


Figure 5-8 Embodied & Emitted Carbon evolution during vertex load increase at the relaxation process

Not changing the number of elements, other factors can be noticed with this test. With this test a notion of optimum start to arise. Looking only at the total Co2equivalent used for each design, the 6^{th} solution appears to be 15,6% more efficient than the first one and 5,8% more efficient than the ninth one, the worse performing within valid designs. The amount to emitted carbon linked by transport seem to stay within the same step beside the first two iterations and the one from fabrication varies slightly at that scale.

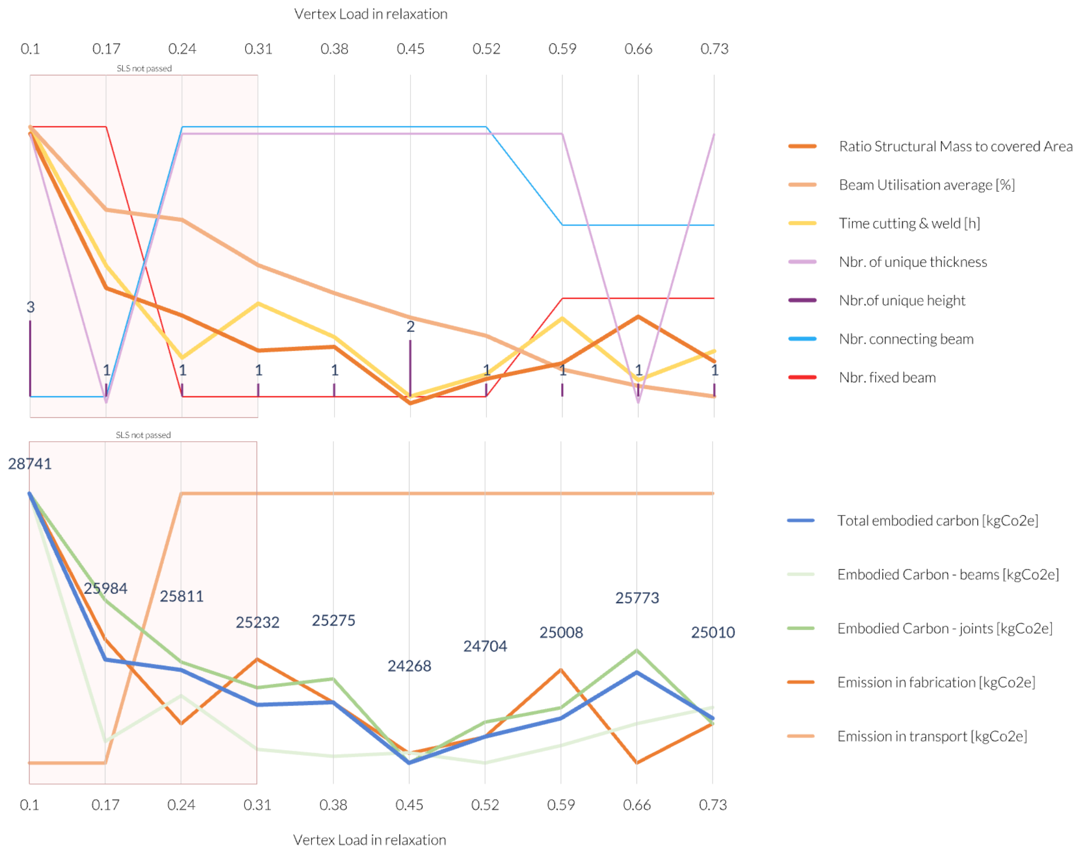


Figure 5-9 Normalized objectives data output from design exploration (top to bottom a, b)

Within this process of relaxing the mesh more and more. The amount of fixed beam and connecting beams varies. Within the valid designs, the amount of fixed beam increased, graph a, without changing the amount of patch, graph b. This means that the tool can produce design where the strips are more populated. It also means that the vertex load has a saying the prefabrication division process and therefore about the emitted carbon.

Coming back to the best performing design in term sustainability, design number 6. This can be explained with the increase of the amount of different height used by Karamba to build to structural model. Coupled with the information about the number of thicknesses used, it highlights that a strategy of one height of cross section with four different thickness has bigger impact than having two different heights and two different thicknesses. Regarding the numerical assessment, it is also noted that the cross section used should also be recording for each generation as is can provide valuable insight and great repercussion.

Similarly, in the 9th design the number of different thicknesses is reduced to three having great negative impact on the total embodied carbon as well as on the ratio of structural mass to covered area. The question here remains on why Karamba, which is the component used for the cross-section assignation, seems to be performing in a rather unreliable way. A hypothesis could be that its optimization process does not consider other objectives than a structural one.
On another note, the change of Relaxation does not seem to have an influence of the Waste of cut panels percentage either, Figure 10-3, reenforcing the statement made in the previous test.

<u>Test 3: Decrease of the amount of different thickness created</u>:
This test has influence on the behavior of Karamba when running the cross-section optimisation. What has been seen previously is that the couple height and thickness used can have a rather large impact. A hypothesis could be set on: Reducing the amount of different thickness possible to be selected by Karamba would make the amount of different height used increase and vice versa.
For this test only three samples have been selected with a range of 2 between each step. This have been chosen as there are rarely more thickness picked from Karamba.
As a reminder, 0 means that there are no more thicknesses created beside the initial one.

Similarly, then in the other tests, the set of design variables used to produce each of the generation are displayed below. In this test, all the geometrical parameters remain the same and only the possibility to change the cross section of the beams is allowed.

Design Variables	u Division	20	20	20
	v Division	20	20	20
	Vertex Load	0.45	0.45	0.45
	Min Heigth	10	10	10
	Amount diff. heigth created	15	15	15
	Min thickness	0.3	0.3	0.3
	Amount dif. Thicnkess created	4	2	0

Figure 5-10 Design Variables set-up for test 3

It is to be noted that all the three design options are considered as valid design, meaning that they all check the set design requirements.

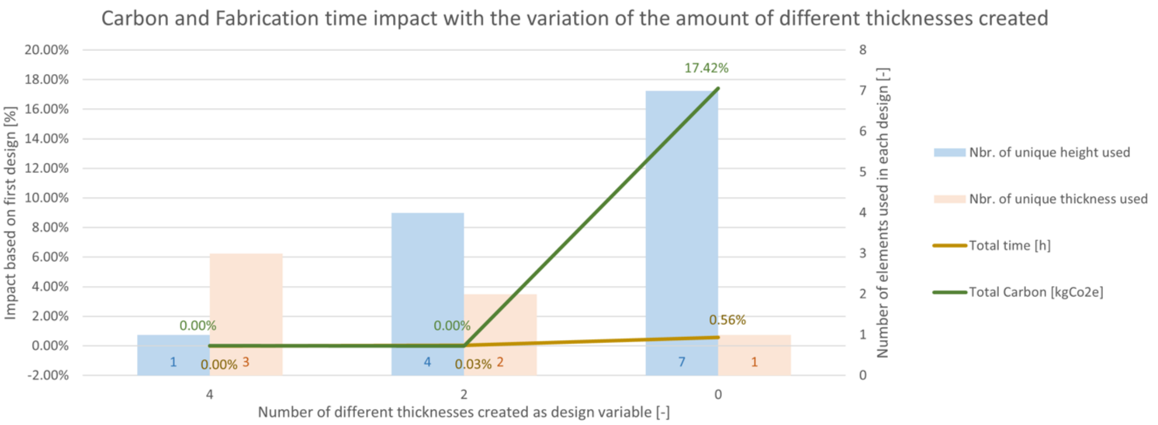


Figure 5-11 Carbon and Fabrication time impact with the variation of the number of different thicknesses created

For this test, another type of graph is used to compare the percentage of change based on the first design has been used. It plots on two axes the pair number of amount thickness and height used to materialize each beam and percentage of change on the total construction time, including fabrication and installation, and the total embodied Carbon, emitted and embodied kgCo2e.
This test provides an interesting result as it is a proof that the usage of different cross sections could end up with very similar Fabrication and Sustainable score. In fact, in this case and within the objectives stated by the scope of the thesis, the strategy of using only one height for three thickness give equal result than using 4 different height and 2 thicknesses. This information could be an idea for the next objective or parameter to consider in order to draw a definite line between the two designs.
As already noticed in the test 2, the lack of choice in the cross-section assignation can have a great impact.

In this case, depriving the number of different thicknesses to only one made the amount of height go to seven in order to account for the different stresses. This increase in height made the embodied carbon increase to a value 18% superior to the first two designs.
A last point should be brought to this test. As the minimum thickness in the tool is set to 3mm, the width set to 100mm and each step between of different height created is 20mm. Without knowing which height of the 16 possible options created at the beginning of the test where used, the profiles could be ranging from 100.100.3mm all the way to 380.100.3mm. Serious questions regarding the buckling of such profile could be of interest. Another design requirement, checking for the beam buckling, could be set in order to eliminate this design from the valid options.

Throughout the different test, several checks pointed to the fact that the tool outputs comprehensible data. The different behaviors can be explained by the indicators tracked and an informed decision on a specific design can, in most cases, be done without too much trouble. These tests provide an answer to the first sub question regarding the impact that different geometrical design can have on the stated objectives. It must be properly said that the geometry of the cross section must be included in this geometrical difference.
Within the limitation of the process used in the tool, it is clear to say that some design can score better than others in a certain objective, and that not all the design variables influence fully all the objectives. The graphs drawn when plotting the different results suggests that there is an optimum that lies somewhere in a balance point between the separated design variables. These local minimums, for each objective considered in the tool, could be part of a pareto front space if an optimisation process is run with a sufficiently large population size.

5.1.2 Interaction Tool/Designer in identifying trade-offs

This part will focus on the second sub question which is defining how the impact of the different design iterations can be assessed and used to make an informed decision over a potential trade-off in scoring.
In order to be able to take an informed decision, one must be able to retrieve the adequate information about his current and past design options. For that, two tools have been developed.
The first one is a on screen display that can help the designer to have live quantitative feedback over his design. It displays on one side the constraints that the design must fall into and the other the objectives that the design aims to minimize.

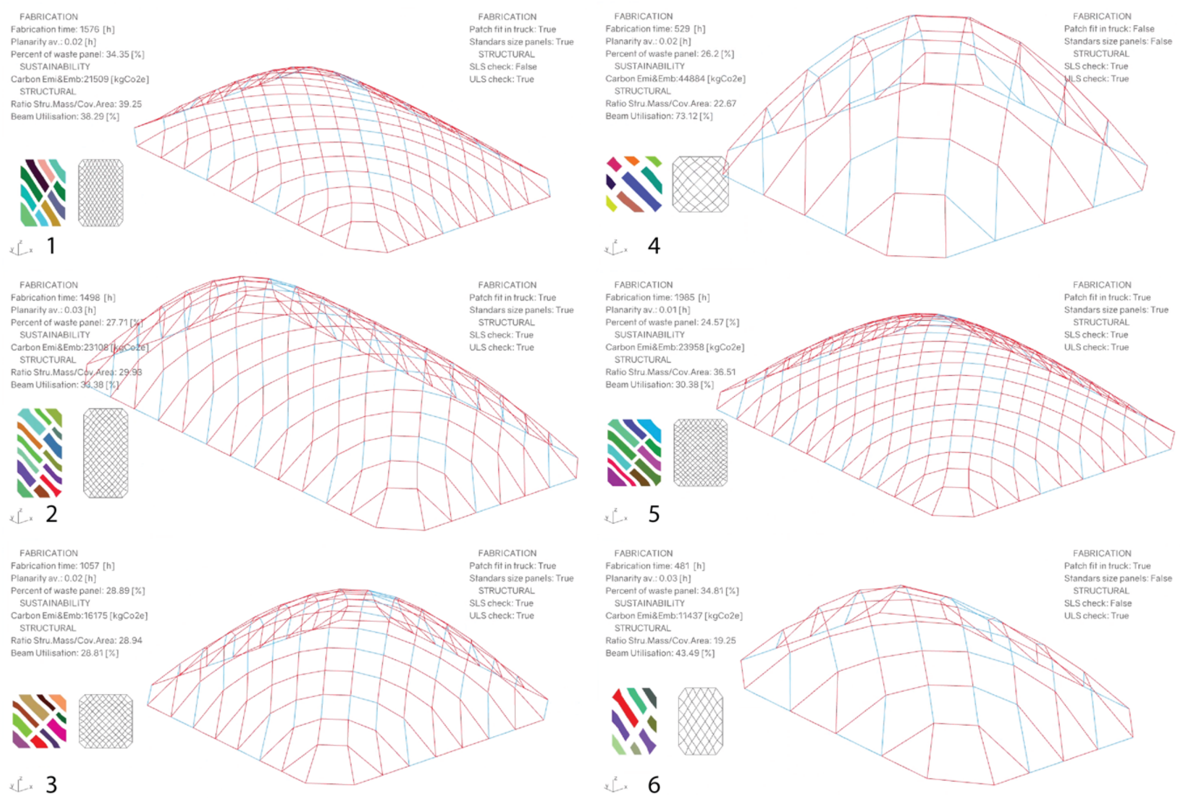


Figure 5-12 Design exploration guided by the on-screen value display

The second one is to be able to plot the history of design into dynamic graphs and be able to gradually the improvement and trade-off done within the design exploration.
To implement this feature, a graph script has been implemented with the script. The outcome of these scripts has been used already in the data analysis during the different test of this research. The range of plots can be seen in the different figures, Figure 5-5, Figure 5-9, Figure 5-19.
These graphs can highlight a particular objective within many and plotting them in this dynamic manner next to other ones able to deduct trends and refine the design until the goals or boundary of the project are met. The way this graphical part has been implemented is by using the lunchbox component to save the different indicators and other possible clue full information about the design into a spreadsheet, Figure 5-13. The script then imports back these values, and the indicators of interest can be tracked.
Percent of improvement at the different saved iterations can provide a quick overview of the different process with the first design saved as reference.

For now, the focus of the display is based on a manual exploration as the software envisioned to perform the optimisation is rather unhappy with the script without apparent or explained reasons. But the implemented tool would also work help the designer take an informed decision between best performing designs coming out a pareto front space by providing an understandable a graph and the score in the different objectives next to each design.

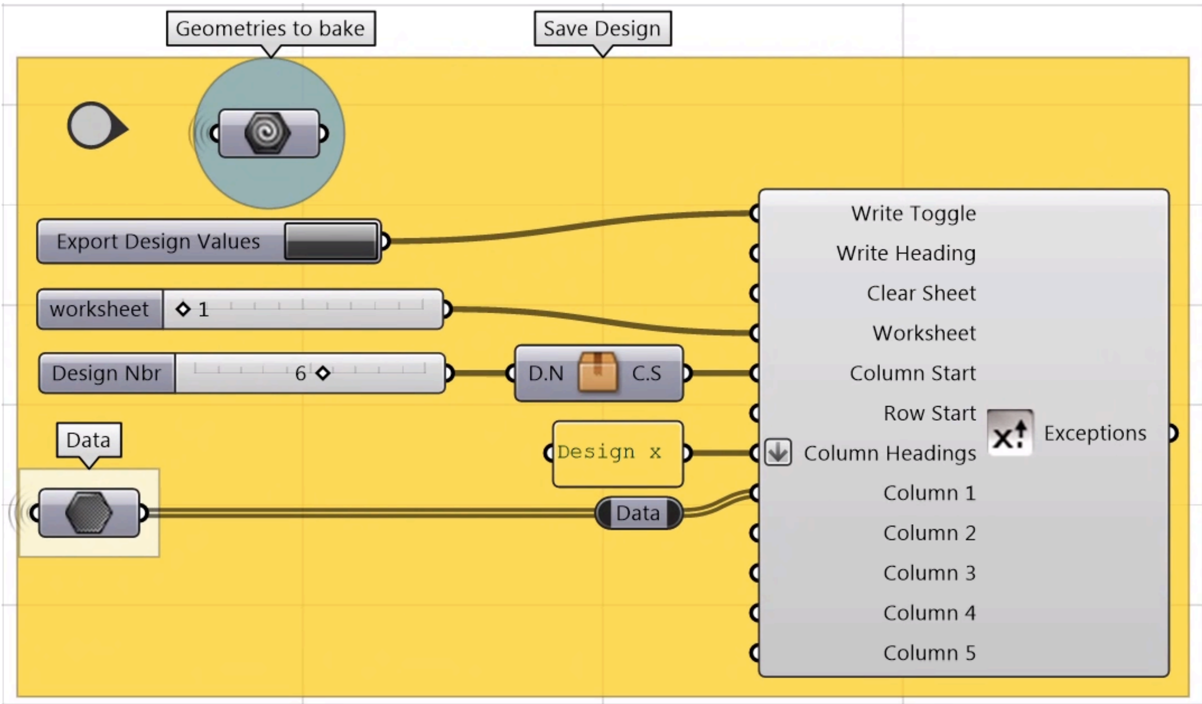


Figure 5-13 Saving specific design iteration to a spreadsheet

5.2 Construction methods comparaison

This study is done in order to evaluate the chosen constructive strategy of dividing the gridshell into welded patch and connect the patches between them with bolted beams. This part aim to answer the third sub question by comparing three constructive techniques.

For this study, three specimens are compared. Which reflects three different constructive strategies. The first one is fabricating the components in house and assembling all the beams and node directly on site. This method implies that all nodes are bolted connection on all four sides. The second one is the method proposed in the thesis. The third one follows the same logic than the proposed tool to the very difference that all the connecting beams will be welded on site instead of bolted. This strategy provides an answer to the undefined stiffness of the bolted joints in the proposed tool.

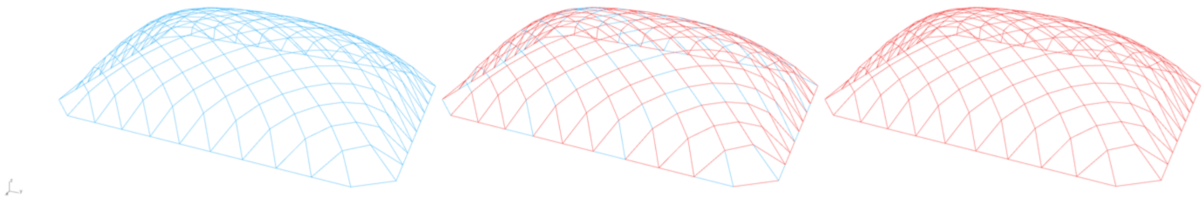


Figure 5-14 Constructive strategies, left to right: a) All bolted, b) proposed method, c) all welded

The set of design variable is kept the same for all the generation, and are the following:

Design Variables	u Division	20	20	20
	v Division	20	20	20
	Vertex Load	0.45	0.45	0.45
	Min Heigth	10	10	10
	Amount diff. heigth created	15	15	15
	Min thickness	0.3	0.3	0.3
	Amount dif. Thicnkess created	2	2	2

Figure 5-15 Design Variables set-up for test 4

Implementation of test:

Even though the tool has not been created to change constructive strategies, the change is, thanks to its modular construction, rather quick to set. The connection type at the end of each beam is defined at the early part of the script, going through the list of nodes inside each patch and checking for their initial topology organization in the mesh. If two nodes are neighbors in the topology and in the same patch, the connection between them is welded, noted 0. Similarly, if two nodes are neighbors in the mesh topology but not in the same patch, the connection between them will set to bolted, noted 1. This notation will define which face of the node undergoes the bolted set of calculation or the welded one.

For both extreme situations in this test, changing to all beams sharing bolted connection or welded connection implies overwriting that list to only zeros or only ones.

Inside Karamba the lines extracted from the relaxed mesh must be all set to the fixed beams or removable beams depending on the situation. Lastly, some precautions have be taken regarding the referenced inputted data for time estimation of the different prefabrication and assembly time.

Precaution in measuring the performance of each constructive method:

All bolted:

For the "all bolted", adaptations are:

All beams are counted 1 hours to install, like the proposed method.

There is no installation time per patch as there are no patches in the first place. Instead 6h is counted for each node installed, (scaffolding, alignment of the hub…), like the normal prefabrication time. The difference being that now the beams are counted again as 1h per beam, like in the proposed method. This accounts for one extra hour. This extra is accounted for the added complexity on site.

Prefabrication time is then equal to 0h. All the beams are defined to fit in 4 trucks.

The bolted process in the Analytical Calculation is added for all the sides of the joints.

As the connection design, being a semi-rigid connection, have not been designed to be used such construction method. The failing of the SLS check will not be considered.

Normal patch:

Normal count is done in the same way than the proposed method.

All fixed:

In "all fixed", adaptations are:

The prefabrication division will be done the same as the proposed method. Therefore, the same amount of truck will be needed. The welded process in the Analytical Calculation is done on all four sides of the joints.

Lastly, the removable beams will still be installed on site, but the installation time will be increased to 3h instead of 1h. This measure is taken to account to the time to set up the welding in position and secure the beam before proceeding with welding.

<u>Preliminary result</u>:
In a design produced with the proposed method, three types of joints are created. One solely composed of welded connection and two others with one or two faces which are bolted connections. At the end of each iteration, a direct comparison of the fabrication time and mass amount for each of these types of joints can be done.
This is a designed output of the tool that inform about the cost of each type in average. It is useful to determine which joint is more costly for the design and in which steps of fabrication of the components. For this test the comparative results between node type are the following.

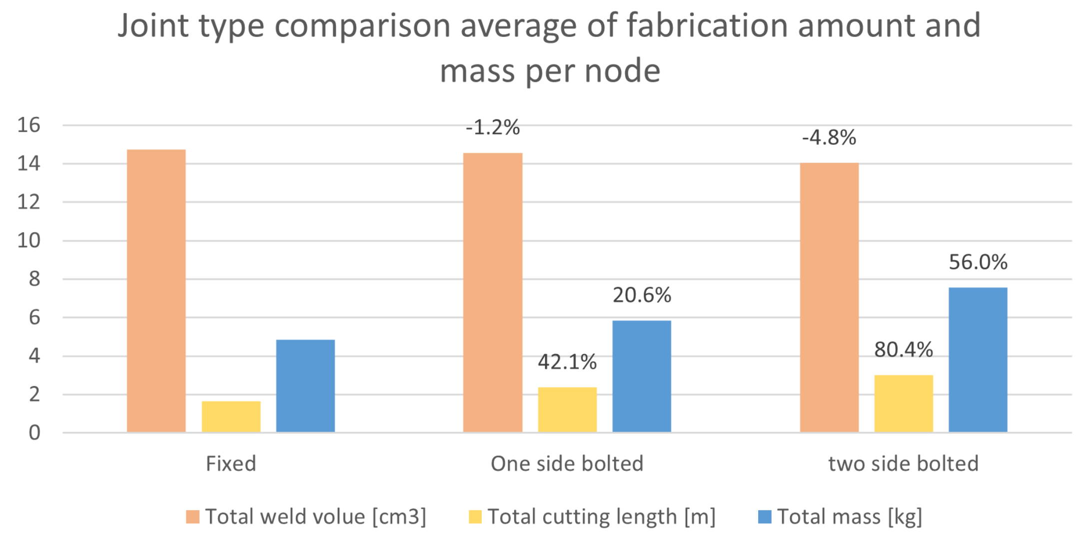


Figure 5-16 Node type comparison of average fabrication amout and mass per node

As in other chart in the analytics chapters, this chart is made taking the percentage of increase in of mass and fabrication time with the fully welded as reference value. In that design, and in the design, variables stated at the beginning the of test, it can be concluded that fully welded connection uses from 2 to 5% more weld but requires 40 to 80% less cutting in material. This is because the bolted connection as described in the analytical calculations can end up a thicker hub and have an extra end plate in their design. All together this create an increase in the mass of up to 56%, which should be seen in related embodied carbon.
With that in mind the result of the constructive strategies can be analyzed.

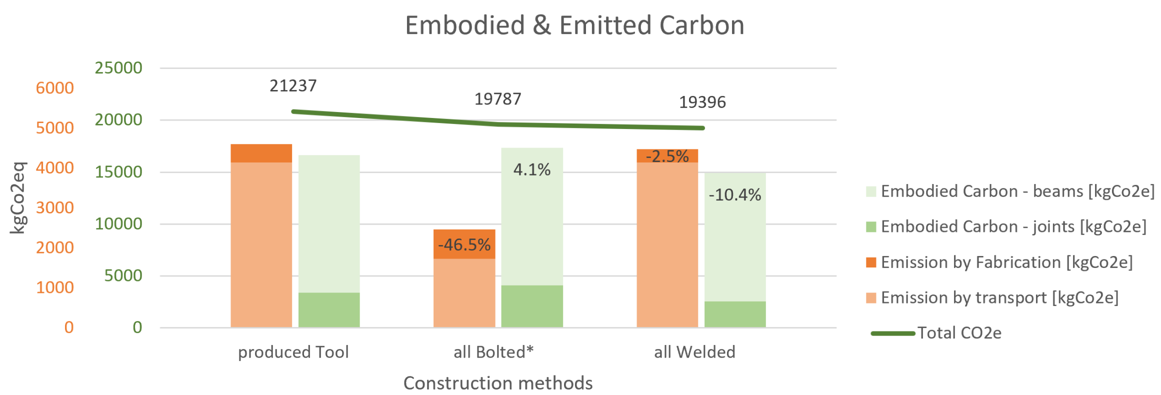


Figure 5-17 Embodied & Emitted Carbon evolution with different construction methods

The previous comment can indeed be seen in this test with an increase of 17% in the embodied carbon linked to the joint. Coupled with the beam embodied carbon the increase is reduced to 4%. With four sides bolted, the expectation for the embodied carbon count of the joint was at least 56% as estimated in the previous chart. This lack in scale can be explained with the fact that the design generated was in no possible way a valid design. The SLS check set the design to invalid with a maximum deflection of nearly 6cm. Against 3.5cm for the proposed tool and less than one centimeter for the "all welded" version.
Even though the embodied carbon of the of the second constructive could not be assessed properly, the biggest change noted here relate without surprise to the amount of emission due by the transport. Having only 4 trucks instead of 10 make to total emission drop 46%. The emission due to the fabrication of the joints follows more the expectation with a 40% increase.
Regarding the third constructive system which aim to weld to beam on site, it can be noted that the strategy indeed pays off regarding the sustainability objective as the embodied drop 10.4% and emission drop 2.5%, mainly due because there is still welded to do on site but not the end plate to cut anymore. Within the embodied carbon count, 6% decrease is measured on the beams and 25% decrease in the joints.

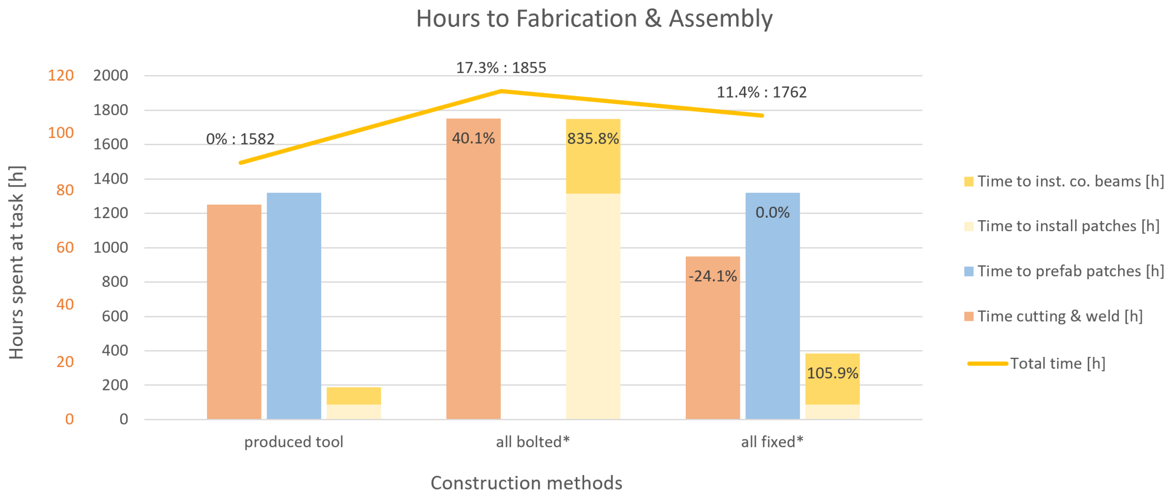


Figure 5-18 Fabrication & assembly time evolution with different construction methods

Regarding the sustainability objectives the third constructive method is measured to be 6.8% more efficient than the proposed tool. But the impact on the fabrication and on-site assembly can also be measured. With this second chart plotted in the same logic, the three steps of the fabrication are separated in, fabrication basic components (cutting the beam and manufacture the connection), prefabrication of patches, and works done on site (installation of patches and connecting beams).
The proposed method scores from 11 to 17% better than the other constructive system in term of total time put into the different steps of the construction. In the fabrication of components, the proposed method arrives second due to the manufacture of some bolted connection behind the "all welded" version 24% quicker in that regard. Where the proposed method scores the best is at reducing the tasks done on site. In fact, with the inputted data for the calculation, it is nearly 15 times quicker than the "all bolted" version and more than 2 times quicker than "all fixed" one. Accounting that the hours spent on site are more costly than the ones spent in house the proposed method accounts for a good strategy to allocate construction time in the less costly steps.

With these normalized graphs, Figure 5-19 exported from the graphic process proposed in the tool, the same conclusions can be drawn. In c, the transport decrease of the second constructive method is the only value that set apart this design from the other ones, scoring worse in all the other indicators. Similarly in the fabrication time, the "all fixed" strategy scores better than all the other in the indicator but the failure in installation time of the connection beam on site makes this strategy be in the end more time consuming.
Plotting the two total with the other tracked indicators, more conclusions can be drawn from that final test. With the same design variables, the amount of height used by Karamba to populate the cross section changed from 4 in proposed method to 1 in the "all fixed" one. Same goes for the number of thicknesses that passed from 2 to 1. It means that for this design and with all connection welded, only two types of thickness and height were used. This confirms a possible direction for improvement that would search

between creating a more rigid joint for the bolted connection and welding some connecting beams on site. This balance would help to find an optimized constructive system that optimize both objectives.

This test provides an answer to the third sub question regarding the use of a pre-fabrication strategy in the search of an optimum between sustainability and fabrication. This last test showed that the strategy chosen for the tool already constitutes a valid option to find this optimum, but that an improvement described above is possible.

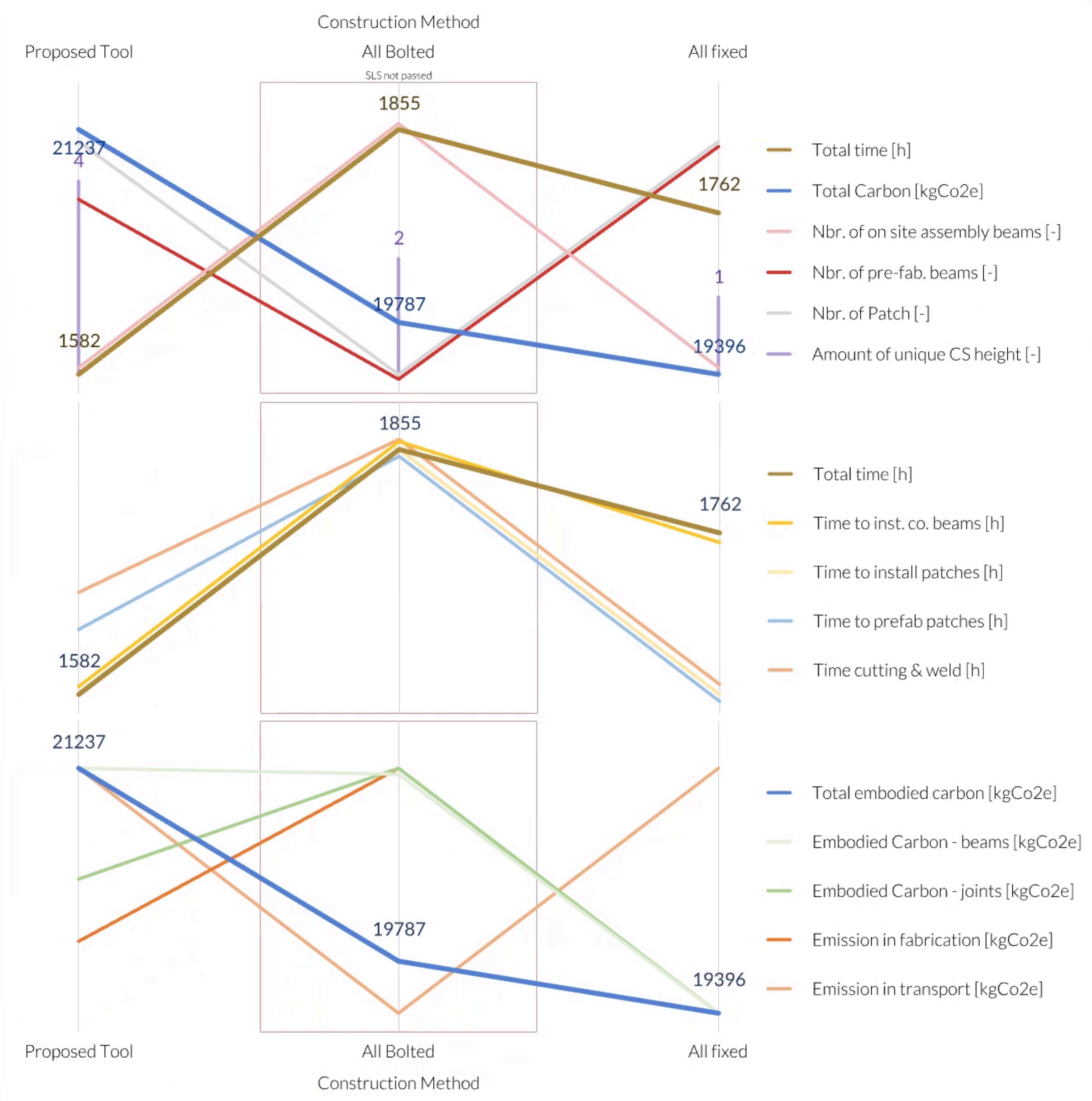


Figure 5-19 Normalized objectives data output from design exploration (top to bottom a, b, c)

6 Conclusion, Reflection & Recommendations

6.1 Conclusion

The translation of a multi-objective problem into an integrated tool was successful.
The basic features could be developed to follow a type of gridshell like the ones produced in the professional environment and therefore the tool can be a good proof of concept regarding the behavior of the different parameters considered in the scope of the research. Having an implemented tool that can be relied on in order to proceed with informed decision in different case scenarios provide an answer to the main research question.
Nevertheless, the proposed tool can only be seen as a prototype and the reflection and result from the tests provide a critical but positive guideline for the next future of research.
Within this guideline come the development of an improvement version or an alternative of the following processes.

- Prefabrication division can sometime disfavor a certain design leading for it to be out of the competition. It is one of the processes noted as the bottleneck of the tool, a diagram of possible improvement is exposed in Figure 6-1 in the future research sub-chapter.
- The use of the Karamba cross section optimizer seems to be not fully adequate for a multi-objective analysis and can sometime penalize a certain design by choosing to certain amount of possible choice even though it the design variables of the script provided the condition to do so. As noted in the results of test 3, the cross-section assignment appeared to have valuable impact on the different objectives.
- Along the different tests, the time estimated in the tool only account for the welding volume to be done and cutting length. This value appeared to be around 10% of the tendered time estimated with Octatube value of 6h per node when planning a prefabrication. This means that this way more elements that the ones considered. It testifies that other fabrication steps could be integrated to get a closer approximation be a competitive method to tender a prefabrication. This could be done by measuring real time taken to do different tasks. This tool is a first step in providing a more precise tendering of time of fabrication in an integrated tool
- The stiffness of the joints inputted into Karamba could not be defined because of the way the prefabrication division has been chosen. A note on that in the future research.

The tool is a reliable way to estimate tradeoffs within the scopes taken for the research, but it can only be used in a portion of the design cases seen in the literature review. In order to have a wider use, it should be able account for the following change in input:

- The possibility to not be able to vary the pattern. The chosen diamond grid pattern resulted that some indicator could evolve as good because of the natural limitation of the pattern. It would benefit greatly to be able to track different type of pattern with different valency count.
- As seen the shell review in the literature review, gridshell is often used to cover area with intricate supporting edge. It seems important to be able to support more freedom in the boundary condition.
- Often used in renovation to cover patios, adding possibility the assess the support reaction force with specific goals would also help to cover more design cases.

The behavior of the different objective can be understood and tracked, which means that the chosen indicators were correctly selected. There was only one situation where no indicators could be brought to provide an informed decision between too remaining designs. This fact implies a success as many designs could already be set aside. This limitation attest of the clarity of analytics and graphs outputted by the tool that grant the capacity to distinguish the most refined designs and even spot the limitations. This impossibility the bring another indicator also means that there is still room for improvement.

Based on the test done on different scenarios, it revealed that some features not implemented but envisioned reveals to be had could be potentially useful to nuance scores between the different objectives during the design exploration. This feature is, as already mentioned, the pattern variation. The diamond grid had the same effect on the planarity of the panels and amount of waste at cutting panels.

6.2 Reflection

6.2.1 Graduation Process:

Topic:
Computational processes, simulations and digital fabrication are starting to be common activities regarding the computational design process of any complex structure in the field of construction. The search of efficiency in material, energy and design iterations are gaining more interest as we approach the climate goals of 2050. The implementation of computer guided optimisation to choose the "best performing" is being widely used in multi-objective designs. Inform early design through decisional factors is the key to ensure efficiency. Accountability and material value-chain becomes the new vector of improvement and despite its late appearance in the design propose, common design workflows are providing possibilities to implement it in order to have earlier estimation of their design impact. The emerges of generalized EPDs databases coupled with mature computational tools and simulations makes the creation of integrated workflow reachable in term of embodied carbon estimation, fabrication complexity and structural analysis. Gridshells structures and their design have varied along the years carried by those improvements and represent a good study case for an integrated design.

Method:
The method followed to achieve the aimed result was a long process of iteration at each of the different step of the tool. A sort of research by design where each design is piece of code that will have to coincide properly with the next step. It was driven by parametrizing a certain situation and finding a technical way to implement into a feasible quick code. The complexity of transcribing a reality scenario into an automated process can lead of getting lost in added feature and lose track of the bigger picture. Similarly, the computational implementation of such a code or a computational process can become quite overwhelming as there many ways to parametrize a problem and some might be more time consuming to implement than others. Eventually after learning about the need of the different process and refining the strategy to reach the different gaps computationally a promising result could be achieved. The used method brought a lot of confidence and knowledge on the way and resulted in a working prototype.

About the produced output:
Within the boundary stated along the report, the produced tool provides a way to understand the different objectives that a project of sustainable design would have. It has its own limitation and provides only a proof of concept linking a process which concerns many disciplines and expertise

6.2.2 Societal Impact

Applicability of result:
The result has shown great capacity to assess a specific gridshell design and retrieve the necessary information in order to empower the designer to make an informed decision. With implementation proposed in the futures research and the precautions stated in the conclusion, this tool could become a powerful early design companion for any gridshell manufacturer.

Achieved innovation:
There was no innovation achieved per say, the research quest was simply to implement a process done manually and involving many disciplines into an automated, trackable, and centralized one. The research is escribed in the effort to make the AEC industry work together and provide a more sustainable built environment for the next generation.

Impact:
The impact of this research is, as described in the introduction, massive for the built environment and the way we design gridshell structure. As the research concern an early-stage design tool, its use could drastically reduce the emission of the built environment. It provides an easily implementable workflow for construction firm as well as architecture studios as it is made using a quite popular tool in both industries.

6.3 Future research

The futures research has been divided in three which relates to the different research direction of the research. The first regards the design functions covered by the tool, and how they could be enlarged or improved. The second one relates to the indicators that assess the performance scores of the design. The third propose future directions at the implementation level regarding features implementation and coding.

6.3.1 Design Functions

Highlighted in test 4, the constructive strategy can have a great impact of the studied objectives. It can also provide a base to create nuanced design that are able to balance between embodied Carbon and time of construction. Future research is given on that note.
On prefabrication division 4.3.2.B, the length division is done directly orthogonally to the strips. Even though the structural analysis is done taking this information into account. This implies a certain stiffness of the node, which is not fully processed the proposed tool with a feedback loop after the component assessment in the analytical calculation method. Another assembly scene could be produced, and take this issue out the problem list, by creating a sort of reciprocal frame of patch. This could be done by stripping the mesh in the other direction and produce a reciprocal frame that will be welded on site.

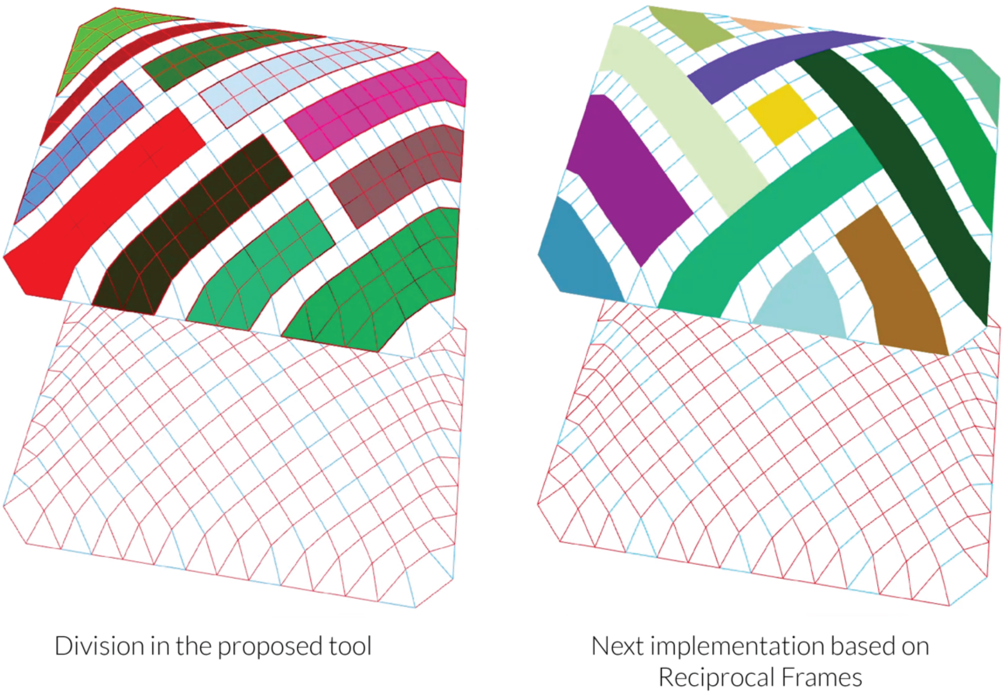


Figure 6-1 New method for an automated fabrication division

This has been done in the de Diamantbeurs gridshell by Octatube and have been proven in test 4, 5.2, to a better structural model, will have less invalid design options due to the SLS check caused by the bolted connection. It also provides a way, by introducing the distinction between beams connected welded or bolted on site, to balance between embodied carbon and time of construction. It would introduce another type of node connection, "welded on site". It would have its own fabrication steps and time count and could be integrated in the search of finding a tradeoff between embodied carbon and fabrication time/assembly time. It has already been shown that the connectivity of the face of a joint could be changed rather easily and therefore the current data structure would already allow such step-up in the process.

6.3.1 Numerical assessment

Following the previous comment, the addition of such new connection type "welded on site" would probably require the addition of other indicators to be able to distinguish between the welds done in house or on site. The introduction of qualitative indicators could make the tool give more preference by score to welds done in house against welds made on site based on a quality standard. As well as the time and end cost, quality is also a primal parameter when answering a client need. The introduction of such indicators would be a first good approach to cover this third aspect of early-stage tendering and designing.

The life cycle analysis could be extended incorporating some aspects of the end of Life (EoL) in sustainability objectives. The concept of reusability and circularity would add another lecture to the joint connection type. In that regard a welded joints which use less material and perform structurally better would receive a penalty due to the number of steps needed to take in order to disassemble, reuse into a circular design process. This aspect would have greater impact if the joints could vary designs and produce a gradient of pinned to rigid connections. Produced by changing some of their constitutive characteristic following a components-based construction. Such addition could greatly benefit the environmental score of the different design and provide a fertile ground to find a balanced relation between the amount of fabrication complexity added for circularity and the actual sustainable benefits.

6.3.2 Code structure and feature implementation

As a general concerns, when building such computational tool grasshopper can be a good prototyping board as its natural flexibility allow to test individually different parts of the script before merging blocks together but lack processing speed compared to other language such as Python or even more C#. Grasshopper provided a flexible environment to compile modules when the output and inputs of the different items are still in search and now that the input and output of the different modules is clearer, the calculation speed for each iteration would greatly benefit from passing the entire grasshopper script to C# coding language. C# is a .Net framework compatible language that allow to build clean and modular code for general purposes. When adding more and more features, the subsequent composition file will be more understandable and cleaner on an IDE environment than on a grasshopper Canvas. Using concept of class to build modular code would allow to implement new features without affecting other parts of the code. And produce different test set-up, as the test number 3 set-up by hand before using the results of the tool.
Moreover, coding the different tool processes in C# would allow to export them as individual components and build a grasshopper plugin that would allow to prepare custom tradeoff search with the performance traits of interest at the early stage of design.

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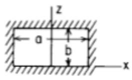
8 Figures

9 Tables

10 Appendix

8. Rectangular plate, all edges fixed



8a. Uniform over entire plate

(At center of long edge) $\sigma_{max} = \frac{-\beta_1 q b^2}{t^2}$

(At center) $\sigma = \frac{\beta_2 q b^2}{t^2}$ and $y_{max} = \frac{\alpha q b^4}{E t^3}$

a/b	1.0	1.2	1.4	1.6	1.8	2.0	∞
β_1	0.3078	0.3834	0.4356	0.4680	0.4872	0.4974	0.5000
β_2	0.1386	0.1794	0.2094	0.2286	0.2406	0.2472	0.2500
α	0.0138	0.0188	0.0226	0.0251	0.0267	0.0277	0.0284

(Refs. 7 and 25 and Ref. 21 for $\nu = 0.3$)

TABLE 11.4 Formulas for flat plates with straight boundaries and constant thickness (*Continued*)

8b. Uniform over small concentric circle of radius r_o (note definition of r'_o)

(At center) $\sigma_b = \frac{3W}{2\pi t^2}\left[(1+\nu)\ln\frac{2b}{\pi r'_o} + \beta_1\right]$ and $y_{max} = \frac{\alpha W b^2}{E t^3}$

(At center of long edge) $\sigma_b = \frac{-\beta_2 W}{t^2}$

a/b	1.0	1.2	1.4	1.6	1.8	2.0	∞
β_1	−0.238	−0.078	0.011	0.053	0.068	0.067	0.067
β_2	0.7542	0.8940	0.9624	0.9906	1.0000	1.004	1.008
α	0.0611	0.0706	0.0754	0.0777	0.0786	0.0788	0.0791

(Ref. 26 and Ref. 21 for $\nu = 0.3$)

Figure 10-1 Beta1 factor for plate calculation, page 508 (Roark et al., 2002)

		D1	D2	D3	D4	D5	D6	D7	D8	D9	D10
Boundary Condition	x	15	15	15	15	15	15	15	15	15	15
	y	10	10	10	10	10	10	10	10	10	10
	Distance to site	300	300	300	300	300	300	300	300	300	300
	Truck type	0	0	0	0	0	0	0	0	0	0
Design Variables	u Division	10	12	12	16	20	20	22	24	26	28
	v Division	10	12	14	18	18	22	22	24	26	28
	Vertex Load	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7
	Min Heigth	10	10	10	10	10	10	10	10	10	10
	Amount diff. heigth created	15	15	15	15	15	15	15	15	15	15
	Min thickness	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3
	Amount dif. Thicnkess created	4	4	4	4	4	4	4	4	4	4
Constraints Results	FABRICATION										
	Patch fit in truck	1	1	1	1	1	1	1	0	0	1
	Panels are standard size	0	0	0	1	1	1	1	1	1	1
	STRUCTURAL										
	SLS check	1	1	1	1	1	1	1	1	1	1
	ULS check	1	1	1	1	1	1	1	1	1	1
	All constraints	0	0	0	1	1	1	1	0	0	1
Objectives Results	FABRICATION										
	Time cutting & weld [h]	40.58	57.68	62.58	107.22	126.63	150.67	161.61	192.16	223.35	257.91
	Time to prefab patches [h]	360	504	582	966	1194	1446	1584	1872	2184	2520
	Time to install patches [h]	64	80	80	80	80	80	96	96	88	88
	Time to inst. co. beams [h]	38	52	57	72	83	93	106	120	126	135
	Planarity av.	0.03	0.03	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
	Percent of waste panel [%]	33.26	32.56	31.19	30.58	36.02	31.31	32.58	34.28	35.06	33.93
	SUSTAINABILITY										
	Emission by transport [kgCo2e]	3105.00	3795.00	3795.00	3795.00	3795.00	3795.00	4485.00	4485.00	4140.00	4140.00
	Emissionb by Fabrication [kgCo2e]	260.09	367.03	400.87	646.25	773.04	944.01	1008.19	1197.97	1386.48	1600.97
	Embodied Carbon - joints [kgCo2e]	1455.99	2077.14	2180.56	4893.06	6004.21	6627.69	7112.25	7834.11	8368.95	9326.20
	Embodied Carbon - beams [kgCo2e]	7285.06	8409.43	8557.91	11311.31	13054.07	13701.67	14542.68	15787.85	16801.90	17953.45
	STRUCTURAL										
	Ratio Structural Mass to covered Area	20.49	24.34	24.85	37.13	43.52	46.31	49.28	53.66	57.09	61.79
	Beam Utilisation average [%]	40.01	37.11	31.18	31.01	32.46	26.01	27.32	26.47	25.10	24.07
	Amount of Elements										
	Nodes	60	84	97	161	199	241	264	312	364	420
	Fixed beam	78	112	133	246	311	385	418	500	598	701
	Removable beam	38	52	57	72	83	93	106	120	126	135
	Nbr of Patch	7	9	9	9	9	9	11	11	10	10
	Amount of unique Height	1	1	1	1	1	1	1	1	1	1
	Amount of unique thickness	3	2	1	3	2	2	2	2	2	2

Figure 10-2 Raw data for test 1: Increasing u v division

		D1	D2	D3	D4	D5	D6	D7	D8	D9	D10
Boundary Condition	x	15	15	15	15	15	15	15	15	15	15
	y	10	10	10	10	10	10	10	10	10	10
	Distance to site	300	300	300	300	300	300	300	300	300	300
	Truck type	0	0	0	0	0	0	0	0	0	0
Design Variables	u Division	20	20	20	20	20	20	20	20	20	20
	v Division	20	20	20	20	20	20	20	20	20	20
	Vertex Load	0.1	0.17	0.24	0.31	0.38	0.45	0.52	0.59	0.66	0.73
	Min Heigth	10	10	10	10	10	10	10	10	10	10
	Amount diff. heigth created	15	15	15	15	15	15	15	15	15	15
	Min thickness	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3
	Amount dif. Thicnkess created	4	4	4	4	4	4	4	4	4	4
Constraints Results	FABRICATION										
	Patch fit in truck	1	1	1	1	1	1	1	1	1	1
	Panels are standard size	1	1	1	1	1	1	1	1	1	1
	STRUCTURAL										
	SLS check	0	0	0	0	1	1	1	1	1	1
	ULS check	1	1	1	1	1	1	1	1	1	1
	All constraints	0	0	0	0	1	1	1	1	1	1
Objectives Results	FABRICATION										
	Time cutting & weld [h]	183.12	163.38	150.30	158.01	153.25	144.79	147.95	155.90	147.13	151.25
	Time to prefab patches [h]	1320	1320	1320	1320	1320	1320	1320	1320	1320	1320
	Time to install patches [h]	80	80	88	88	88	88	88	88	88	88
	Time to inst. co. beams [h]	88	88	99	99	99	99	99	95	95	95
	Planarity av.	0.01	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
	Percent of waste panel [%]	34.77	34.54	34.63	34.37	34.65	34.73	34.47	34.74	34.83	34.73
	SUSTAINABILITY										
	Emission by transport [kgCo2e]	3795.00	3795.00	4140.00	4140.00	4140.00	4140.00	4140.00	4140.00	4140.00	4140.00
	Emissionb by Fabrication [kgCo2e]	1025.71	954.18	912.98	944.59	923.42	898.49	906.55	939.43	893.78	913.00
	Embodied Carbon - joints [kgCo2e]	8989.72	7714.02	6977.52	6671.28	6776.01	5772.20	6259.60	6431.22	7117.93	6242.96
	Embodied Carbon - beams [kgCo2e]	14930.51	13521.12	13780.57	13475.74	13435.98	13456.86	13398.10	13497.17	13621.60	13713.80
	STRUCTURAL										
	Ratio Structural Mass to covered Area	54.55	48.43	47.34	45.95	46.10	43.86	44.83	45.45	47.30	45.52
	Beam Utilisation average [%]	43.31	39.54	39.08	37.02	35.75	34.64	33.81	32.28	31.53	31.05
	Amount of Elements										
	Nodes	220	220	220	220	220	220	220	220	220	220
	Fixed beam	348	348	337	337	337	337	337	341	341	341
	Removable beam	88	88	99	99	99	99	99	95	95	95
	Nbr of Patch	9	9	10	10	10	10	10	10	10	10
	Amount of unique Height	3	1	1	1	1	2	1	1	1	1
	Amount of unique thickness	4	3	4	4	4	4	4	4	3	4

Figure 10-3 Raw data output for test 2: Vertex load increase

		D1	D2	D3
Boundary Condition	x	10	10	10
	y	15	15	15
	Distance to site	300	300	300
	Truck type	0	0	0
Design Variables	u Division	20	20	20
	v Division	20	20	20
	Vertex Load	0.45	0.45	0.45
	Min Heigth	10	10	10
	Amount diff. heigth created	15	15	15
	Min thickness	0.3	0.3	0.3
	Amount dif. Thicnkess created	4	2	0
Constraints Results	FABRICATION			
	Patch fit in truck	1	1	1
	Panels are standard size	1	1	1
	STRUCTURAL			
	SLS check	1	1	1
	ULS check	1	1	1
	All constraints	1	1	1
Objectives Results	FABRICATION			
	Time cutting & weld [h]	74.60	75.06	83.48
	Time to prefab patches [h]	1320	1320	1320
	Time to install patches [h]	88	88	88
	Time to inst. co. beams [h]	99	99	99
	Planarity av.	0.02	0.02	0.02
	Percent of waste panel [%]	34.72	34.72	34.72
	SUSTAINABILITY			
	Emission by transport [kgCo2e]	4140.00	4140.00	4140.00
	Emissionb by Fabrication [kgCo2e]	454.39	456.00	491.03
	Embodied Carbon - joints [kgCo2e]	3130.77	3385.41	3655.15
	Embodied Carbon - beams [kgCo2e]	13513.21	13256.01	16651.36
	STRUCTURAL			
	Ratio Structural Mass to covered Area	37.96	37.95	46.31
	Beam Utilisation average [%]	34.70	34.77	32.31
	Amount of Elements			
	Nodes	220	220	220
	Fixed beam	337	337	337
	Removable beam	99	99	99
	Nbr of Patch	10	10	10
	Amount of unique Height	1	4	7
	Amount of unique thickness	3	2	1

Figure 10-4 Raw data output for test 3: Amount of different thickness created

		D1 Tool produced	D2 all Bolted	D3 all Welded
Boundary Condition	x	10	10	10
	y	15	15	15
	Distance to site	300	300	300
	Truck type	0	0	0
Design Variables	u Division	20	20	20
	v Division	20	20	20
	Vertex Load	0.45	0.45	0.45
	Min Heigth	10	10	10
	Amount diff. heigth created	15	15	15
	Min thickness	0.3	0.3	0.3
	Amount dif. Thicnkess created	2	2	2
Constraints Results	FABRICATION			
	Patch fit in truck	1	1	1
	Panels are standard size	1	1	1
	STRUCTURAL			
	SLS check	1	0	1
	ULS check	1	1	1
	All constraints	1	0	1
Objectives Results	FABRICATION			
	Time cutting & weld [h]	75.06	105.16	56.97
	Time to prefab patches [h]	1320	0	1320
	Time to install patches [h]	88	1314	88
	Time to inst. co. beams [h]	99	436	297
	Planarity av.	0.02	0.02	0.02
	Percent of waste panel [%]	34.72	34.72	34.72
	SUSTAINABILITY			
	Emission by transport [kgCo2e]	4140.00	1725.00	4140.00
	Emissionb by Fabrication [kgCo2e]	456.00	735.73	339.99
	Embodied Carbon - joints [kgCo2e]	3385.41	4091.47	2531.36
	Embodied Carbon - beams [kgCo2e]	13256.01	13234.40	12384.40
	STRUCTURAL			
	Ratio Structural Mass to covered Area	37.95	39.51	34.02
	Beam Utilisation average [%]	34.77	37.97	20.40
	Amount of Elements			
	Nodes	220	220	220
	Fixed beam	337	0	436
	Removable beam	99	436	0
	Nbr of Patch	10	4	10
	Amount of unique Height	4	2	1
	Amount of unique thickness	2	2	1

Figure 10-5 Raw data output for test 4: different constructive methods