

## EXPLICIT INVERSE DISTANCE WEIGHTING MESH MOTION FOR COUPLED PROBLEMS

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**Abstract.** An explicit mesh motion algorithm based on inverse distance weighting interpolation is presented. The explicit formulation leads to a fast mesh motion algorithm and an easy implementation. In addition, the proposed point-by-point method is robust and flexible in case of large deformations, hanging nodes, and parallelization. Mesh quality results and CPU time comparisons are presented for triangular and hexahedral unstructured meshes in an airfoil flutter fluid-structure interaction problem.

### 1 INTRODUCTION

In fluid-structure interaction simulations the dynamics of the structure and the flow are coupled by forces and displacements on their interface. Flow forces result through deformations of the structure in moving boundaries for the flow domain. There is, therefore, a need for accurate and efficient mesh motion algorithms to propagate these boundary displacements to the mesh in the interior of the flow domain. For structured meshes the efficient Transfinite Interpolation method is available. Methods for the deformation of unstructured meshes often use the connectivity of the flow mesh in algorithms based on spring analogy, body elasticity, or Laplacian and Biharmonic operators. These methods can be computationally intensive, since they result in solving a system of equations of the size of the number of flow points.

Point-by-point methods do not rely on connectivity information as they determine the displacement of each point in the flow mesh based on its relative position with respect to the domain boundary. Recently, a point-by-point mesh deformation method was developed based on radial basis function (RBF) interpolation [1]. The resulting flexible and robust mesh motion algorithm can deal with large deformations, hanging nodes, and is easily implemented in parallel. However, radial basis function mesh motion requires the

solution of a system of equations of the size of the number of boundary nodes. Solving this system can still be expensive as it accounts for a significant part of the computational time in large-scale three-dimensional simulations.

In this paper, we present an explicit mesh motion algorithm based on inverse distance weighting (IDW) interpolation [2], which does not lead to solving a system of equations. The proposed point-by-point method maintains the robustness and flexibility for dealing with large deformations, hanging nodes, and parallelization of the previous method. In addition, the explicit formulation results in a faster mesh motion algorithm and an easier implementation. The mesh deformation method is applied to unstructured triangular and hexahedral meshes in a NACA0012 airfoil fluid-structure interaction. Results show a similar accuracy at a reduction of computational costs up to a factor 10 compared to radial basis function mesh deformation.

## 2 INVERSE DISTANCE WEIGHTING INTERPOLATION

Inverse distance weighting interpolation [2] is an explicit method for multivariate interpolation of scattered data points. The interpolation surface  $w(\mathbf{x})$  through  $n$  data samples  $\mathbf{v} = \{v_1, \dots, v_n\}$  of the exact function  $u(\mathbf{x})$  with  $v_i \equiv u(\mathbf{x}_i)$  is given in inverse distance weighting by

$$w(\mathbf{x}) = \frac{\sum_{i=1}^n v_i \phi(r_i)}{\sum_{i=1}^n \phi(r_i)}, \quad (1)$$

with weighting function

$$\phi(r) = r^{-p}, \quad (2)$$

where  $r_i = \|\mathbf{x} - \mathbf{x}_i\| \geq 0$  is the Euclidian distance between  $\mathbf{x}$  and data point  $\mathbf{x}_i$ , and  $p$  is a power parameter.

## 3 TRIANGULAR GRID AROUND A NACA0012 AIRFOIL

First the mesh deformation of an unstructured triangular mesh around a NACA0012 airfoil in a square domain of size  $10c \times 10c$  is considered with  $c$  the airfoil chord, as shown in Figure 1. The mesh consists of 1524 cells with 112 nodes on the airfoil, 24 nodes on the outer boundary, and 694 internal nodes. The mesh is subject to a given displacement of the airfoil consisting of a translation in both directions of  $\Delta x = \Delta y = -2.5c$  and a rotation over  $\Delta\alpha = -60\text{deg}$  in one step from the equilibrium position of the initial mesh.

The mesh quality of the resulting mesh is shown in Figure 2. A combination of a size and skew measure between 0 and 1 is used as measure of the mesh quality. The proposed explicit mesh deformation method results in a mesh of good quality without degenerate cells even for this extreme displacement of the airfoil. Especially near the airfoil the grid is practically undistorted with a mesh quality close to 1, see Figure 2b. This is important for a robust deformation of the boundary layer cells and an accurate resolution of the aerodynamic forces on the airfoil. The overall quality of the mesh is in this case 0.483.

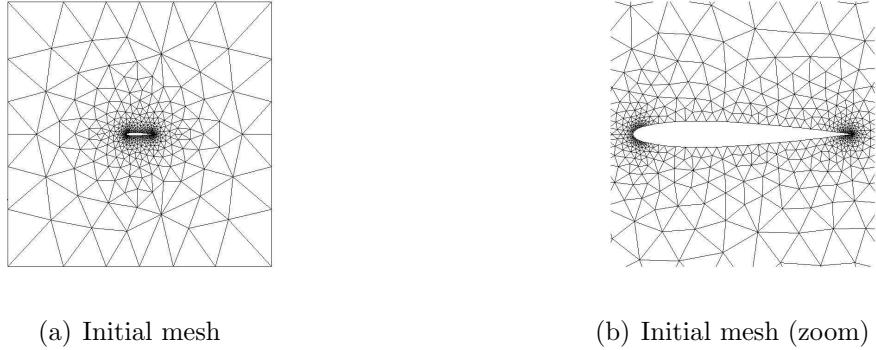


Figure 1: Unstructured triangular mesh around a NACA0012 airfoil with 1524 cells.

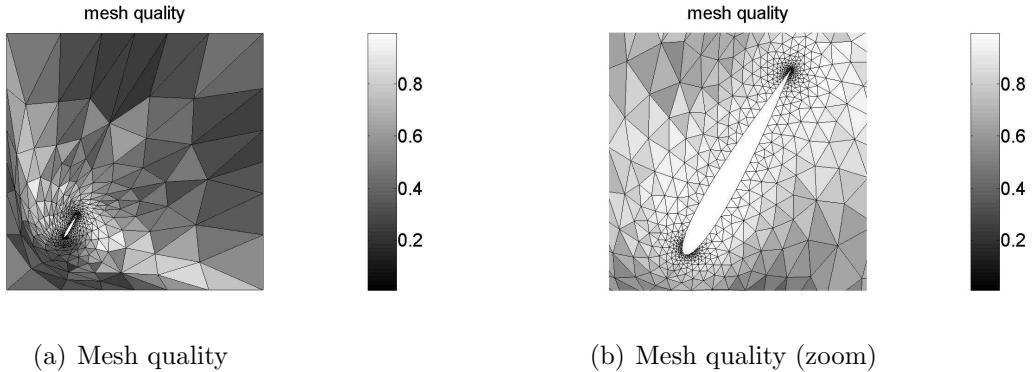


Figure 2: Mesh quality of the unstructured triangular mesh around a NACA0012 airfoil for a deflection of  $\Delta x = \Delta y = -2.5c$  and  $\Delta\alpha = -60\text{deg}$ .

### 3.1 TWO-DIMENSIONAL AIRFOIL FLUTTER

The second numerical example is the unsteady post-flutter simulation of an elastically mounted airfoil with an unstructured hexahedral mesh. The Euler equations of inviscid fluid mechanics are solved on a mesh with  $1.2 \cdot 10^4$  volumes in spatial domain  $D$  with dimensions  $30c \times 20c$ . An Arbitrary Lagrangian-Eulerian formulation is employed to couple the fluid mesh with the movement of the structure. The good quality of the deformed mesh is shown for a deflection of the airfoil by  $\Delta\alpha = 20\text{deg}$  in Figure 3.

The CPU time required for the mesh deformation routine in every time step for RBF and IDW mesh deformation is given in Table 1. Implicit RBF mesh motion results in solving a system of equations for obtaining the displacements of the internal mesh nodes. This results in an average computational time of per time step of 0.82s. The proposed explicit IDW method reduces the CPU time by more than 50% to 0.34s. The computational time further decreases to 0.05s when the displacements of the internal

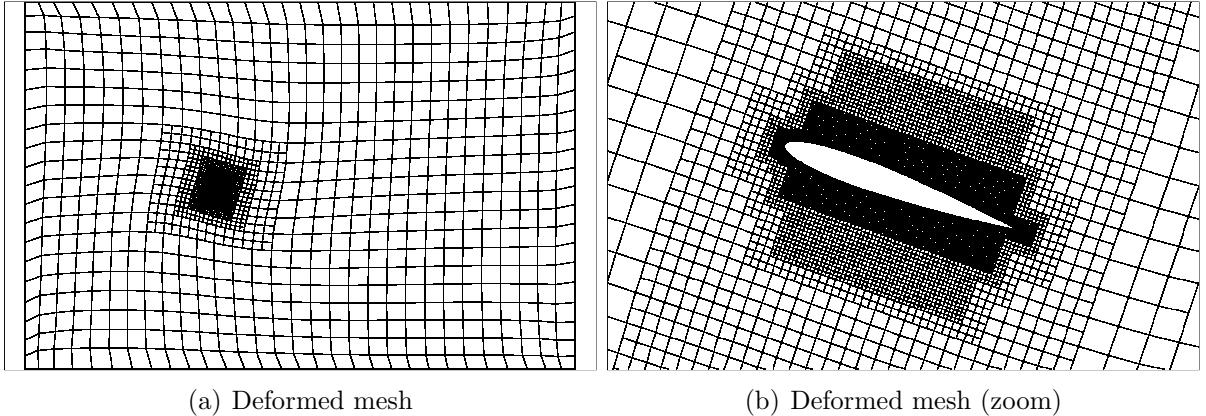


Figure 3: The deformed unstructured hexahedral mesh around a NACA0012 airfoil at an angle of attack of 20deg.

nodes are written explicitly as function of the 3 structural degrees of freedom instead of the displacements of the boundary nodes, see Table 1.

#### 4 CONCLUSIONS

The presented inverse distance weighting mesh motion algorithm results in a high mesh quality at a reduction of computational costs up to a factor 10 for unstructured triangular and hexahedral meshes in a NACA0012 fluid-structure interaction. The explicit point-by-point mesh deformation method leads to an easy parallel implementation and a robust treatment of large deformations and hanging nodes. The method is also easily applicable to structured and three-dimensional meshes.

#### REFERENCES

- [1] A. de Boer, M.S. van der Schoot, H. Bijl. Mesh deformation based on radial basis function interpolation. *Comput. Struct.* **85**, 784–795, (2007).
- [2] D. Shepard. A two-dimensional interpolation function for irregularly-spaced data. *Proceedings of the 1968 ACM National Conference*, 517–524, (1968).

Table 1: Average CPU time per time step for the RBF and IDW mesh motion routines for the two-dimensional airfoil flutter problem.

method	time [s]
RBF	0.82
IDW	0.34
eigenmode IDW	0.05