

# INTEGRATING PUBLIC TRANSPORT NETWORKS IN THE AXIAL MODEL

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## **Abstract**

*This study presents a first step in the development of a model that integrates public transport networks with the space syntax axial model, towards a network model that can describe the multi-modal movement structure of a city and study its patterns and flows. It describes the method for building an integrated multi-modal network model of the London railway, underground and pedestrian movement networks, and the technique used to analyse the network centrality of the resulting graph. The initial results are presented, showing the configuration of London as a multi-modal city and testing the capacity of this model to predict movement levels of pedestrians and station usage. The lessons learned from this first model are discussed pointing to different possibilities of improving it.*

## 1. INTRODUCTION

This paper presents an initial exploration of the integration of public transport networks in the space syntax axial model used for centrality analysis of urban environments (Hillier 1997) and pedestrian and vehicular movement forecasting (Hillier and Iida, 2005). Previous studies have modelled individual public transport networks and analysed them using network centrality measures, namely the London underground (Chiaradia, 2005) and the South East of England railway (Schwander, 2009), but further work is necessary to integrate the various public and private transport modes into a single model.

- How can we model public transport networks?
- How can we integrate these with the pedestrian model?
- How can we measure the network centrality of this integrated model?

In order to address these questions it presents a model of the London railway and underground networks and explores different approaches to create an integrated multi-modal network model that includes the pedestrian network represented by a space syntax axial model. The topological configuration analysis results are compared with the axial model centrality analysis on its own to see the impact on pedestrian accessibility patterns of the public transport network. The results are then correlated against pedestrian counts and the usage levels of public transport network nodes. In the discussion, we reflect on these experiments pointing to several avenues of further research at the technical, modelling but also theoretical level.

## 2. LONDON'S MULTI-MODAL MOBILITY INFRASTRUCTURE

This study explores the case of London and its multi-modal mobility infrastructure. London is a city with some of the oldest public transport infrastructures in the world, with the first sections of railway and underground dating back to the mid 19<sup>th</sup> century. The different public transport infrastructures combined cater for the movement of its 7.8 million inhabitants and around 27 million visitors per year. Attesting the importance of these infrastructures, we can consider the case of the City, the business and financial centre of London, with over 320,000 daily commuters, 90% of which use public transport. The public transport infrastructure forms an established and comprehensive network comprising various modes, namely rail, underground, tram and bus. To these public transport networks, one must add the private transport infrastructure of roads and streets shared by cars, bicycles and pedestrians.

One starts examining the different networks using the space syntax approach, producing individual models representing the rail, underground and pedestrian mobility infrastructure, and examining their network configuration and data on network movement levels. The data for the models was obtained from a variety of sources and prepared in a GIS platform for analysis and visualisation.

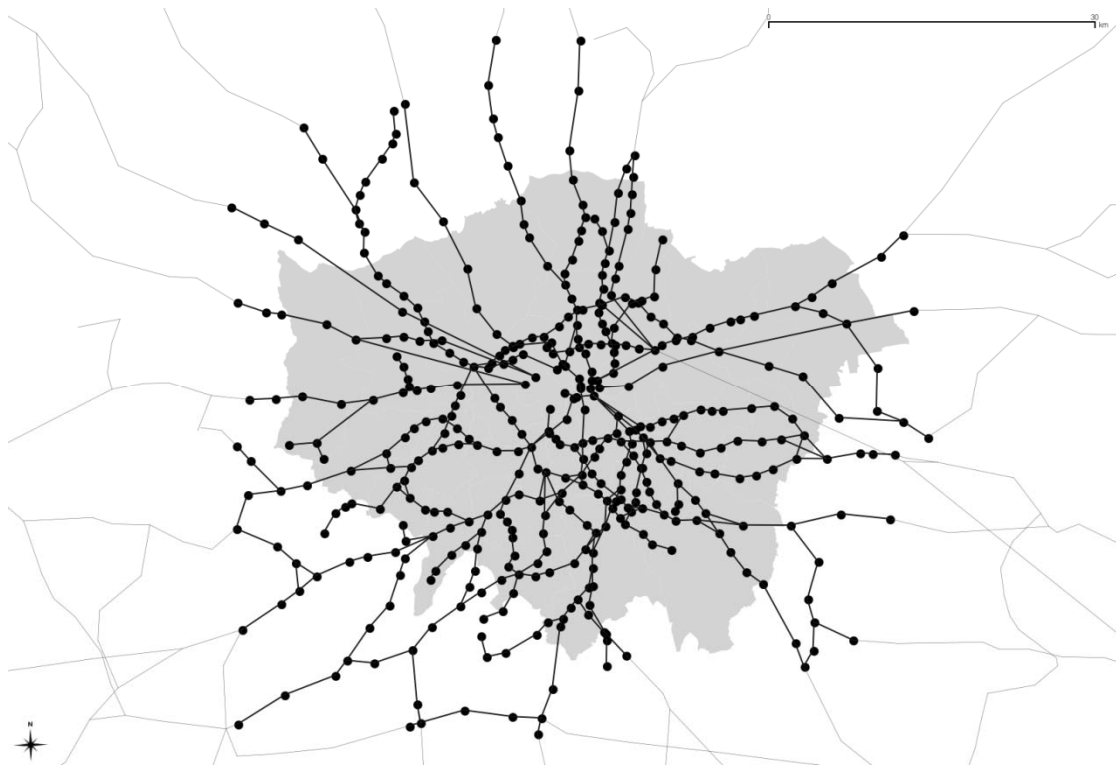
### 2.1 The rail network

The rail network model comprises the mainline railway stations that feature in the Transport for London's (TfL) "Oyster Rail services map" [<http://www.tfl.gov.uk/gettingaround/9444.aspx>, last accessed 15 August 2011] and is made of links between adjacent stations served by a rail service (figure 1).

The rail station information is available in the National Public Transport Access Node (NaPTAN) database from the Department for Transport (DfT) [<http://www.dft.gov.uk/naptan/>, last accessed 15 August 2011].

Detailed timetable information of train services and journeys was obtained from the National Public Transport Data Repository (NPTDR) database [<http://www.dft.gov.uk/data/release/10045>, last accessed 15 August 2011] from the DfT. Finally, the rail station usage data for 2009-10 was obtained from the Office of Rail Regulations (ORR) [<http://www.rail-reg.gov.uk/server/show/nav.1529>, last accessed 15 August 2011]. It is beyond the scope of this paper to describe how the different data sets have been combined to produce the rail network model, however it is important to note that despite the amount of information available certain steps were less automated than expected. The linking of stations with station usage data was done using the station name instead of using a unique identifier, because the different authorities use different identification systems. The extraction of rail station links from detailed timetable information proved complex and error prone, as the automatic aggregation process was not tolerant to errors or gaps in the original data, leading to missing links between stations.

**Figure 1** – Model of the London rail network with 432 stations and 473 links (below) and station usage levels, with red representing the highest level of use (next page).



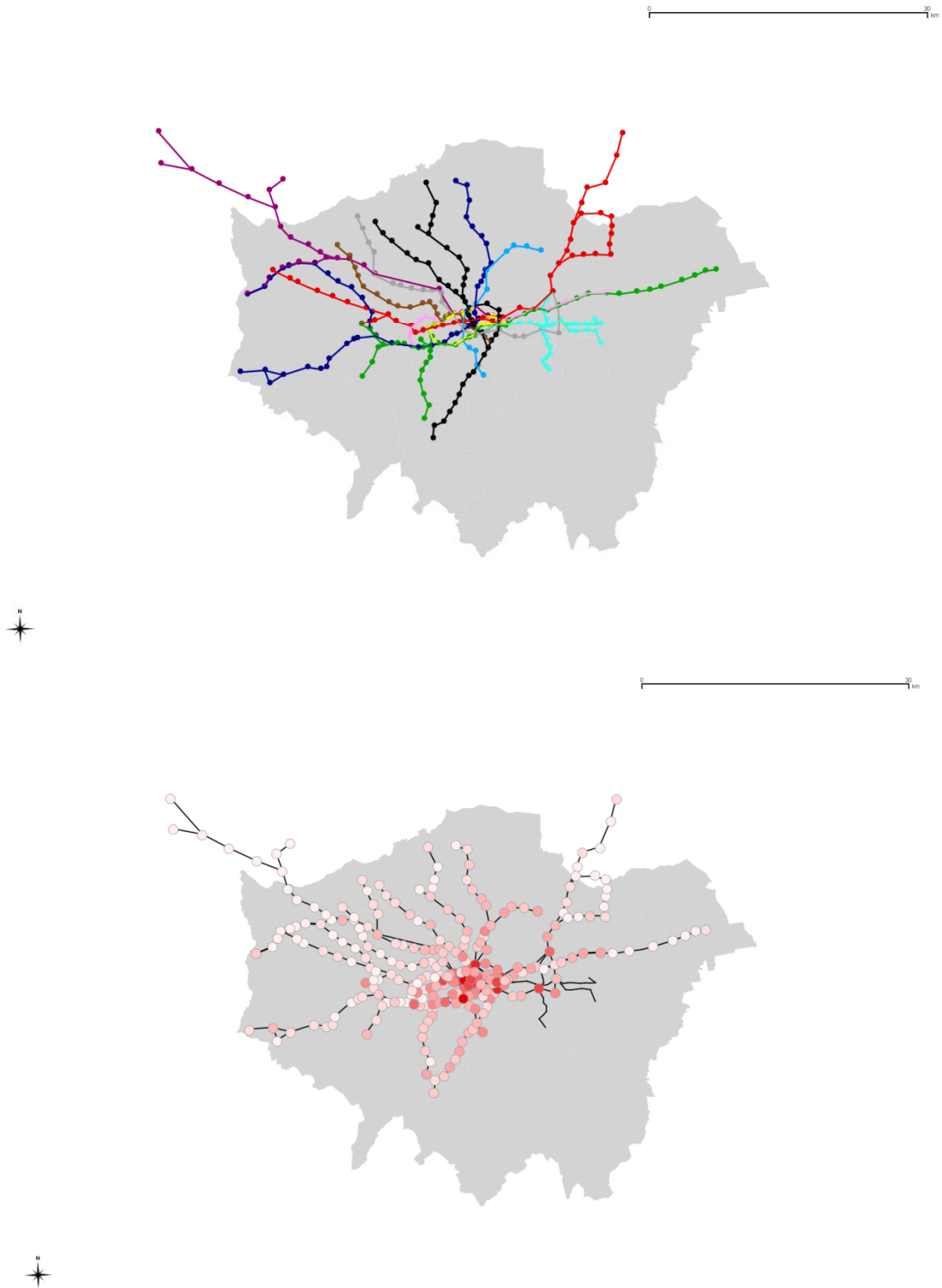


## 2.2 The London Underground

The London Underground model consists of all stations and lines, including the Docklands Light Railway (DLR) line, that feature in TfL's "Standard Tube Map" [<http://www.tfl.gov.uk/gettingaround/14091.aspx>, last accessed 15 August 2011] and is made of links between platforms on the same line and links between a station and the platforms of each line serving that station. The transfer between lines in any given station is thus achieved with two additional 'steps' of getting off a train and on to another line (figure 2).

The London Underground stations and platforms information was extracted from the NaPTAN database, and the links could be extracted from the NPTDR database. But once again this information proved difficult to handle and resulted in an incomplete network map. The "London Underground geographic maps/CSV" data from the Wikimedia Commons [[http://commons.wikimedia.org/w/index.php?title=London Underground geographic maps/CSV&oldid=17490779](http://commons.wikimedia.org/w/index.php?title=London_Underground_geographic_maps/CSV&oldid=17490779), last accessed 15 August 2011] provided a starting base of network line topology. This was edited to correct mistakes, update to the 2009-10 network status and match the station identifiers with the remaining data sets.

The "London Underground passenger counts" data for 2009-10 is available on TfL's "Developers' Area" [<http://www.tfl.gov.uk/businessandpartners/syndication/default.aspx>, last accessed 15 August 2011] and contains entry/exit totals at each station (except those on the DLR) for different time periods and days of the week.



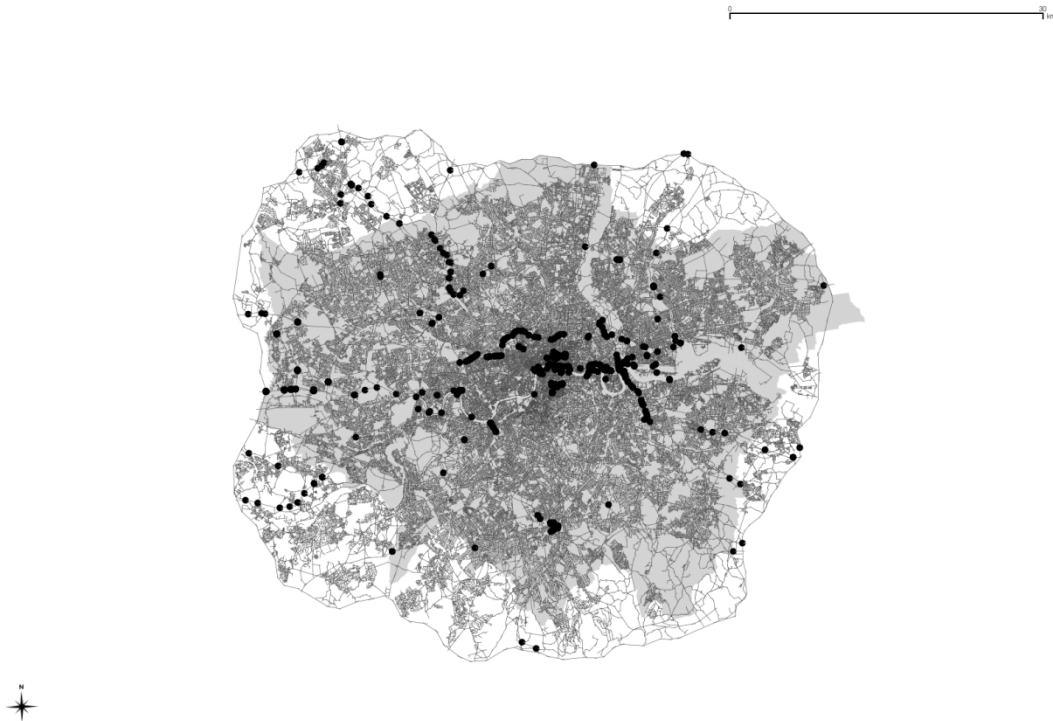
**Figure 2** – Model of the London underground with 310 stations, 412 platforms, 406 line links and 414 line interchange links (top) and station usage levels, with red representing the highest level of use (bottom).

### 2.3 The pedestrian network

The pedestrian model is based on the space syntax axial map of London reaching as far as the M25 motorway that encircles the city. It is a map with more than 100 thousand lines representing the longest and fewest lines of unobstructed pedestrian movement forming a network of spaces. This network can be converted to a topological network by creating links between the spaces that intersect. In addition, there are 585 'unlinks' indicating spaces that are not connected despite intersecting in the map, such as the case of bridges, tunnels and viaducts.

The pedestrian movement data used in this study consists of the gate counts of the Standard Area Test Cases (SATC) compiled by Shinichi Iida and used in Hillier and Iida (2005) [<http://eprints.ucl.ac.uk/1232/>, last accessed 15 August 2011].

**Figure 3** – Model of the pedestrian network with 100,498 axial lines and 585 unlinks in black (below), and the 320 pedestrian gate counts in central London, with red representing the highest level of movement (next page).





### 3 AN INTEGRATED MULTI-MODAL NETWORK MODEL

The network models presented in the previous section can be combined in an integrated multi-modal network model in order to study the interactions of the different mobility networks, their effects on urban configuration and on movement levels (figure 4). An integrated multi-modal model is difficult to produce with the space syntax representation of the axial map and analyse using existing tools because it is assumed that all nodes in the graph are lines representing spaces and the links are all the intersections of such lines. In this section, we propose such an integrated model and describe a possible method for analysing it.

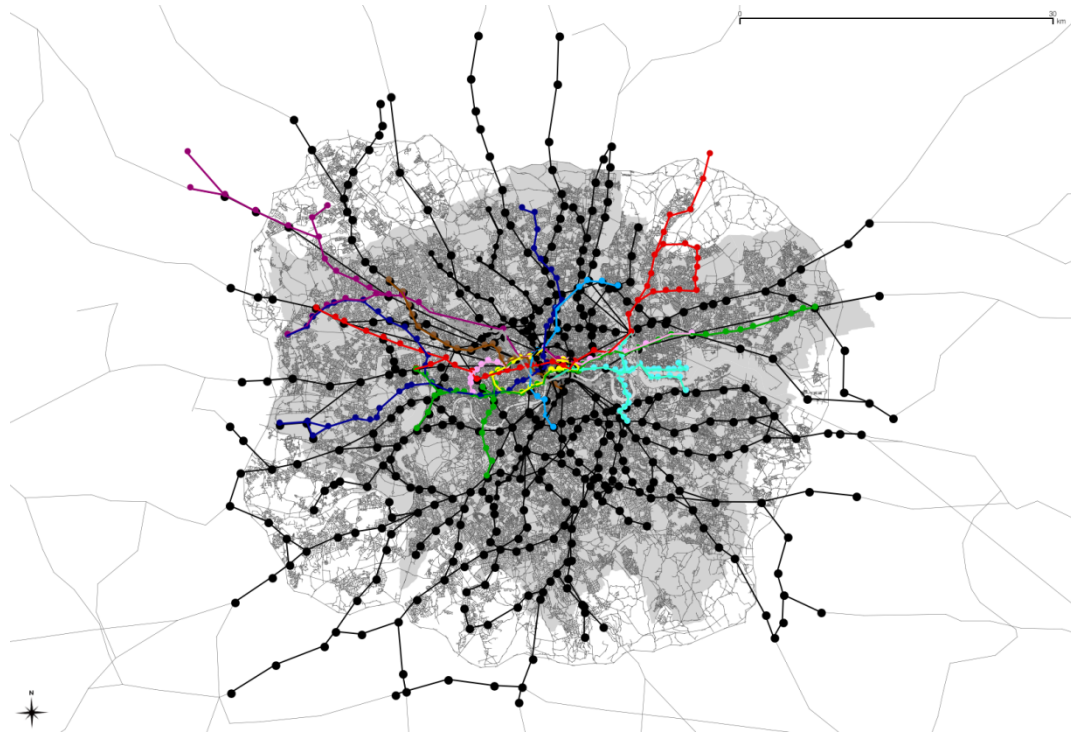


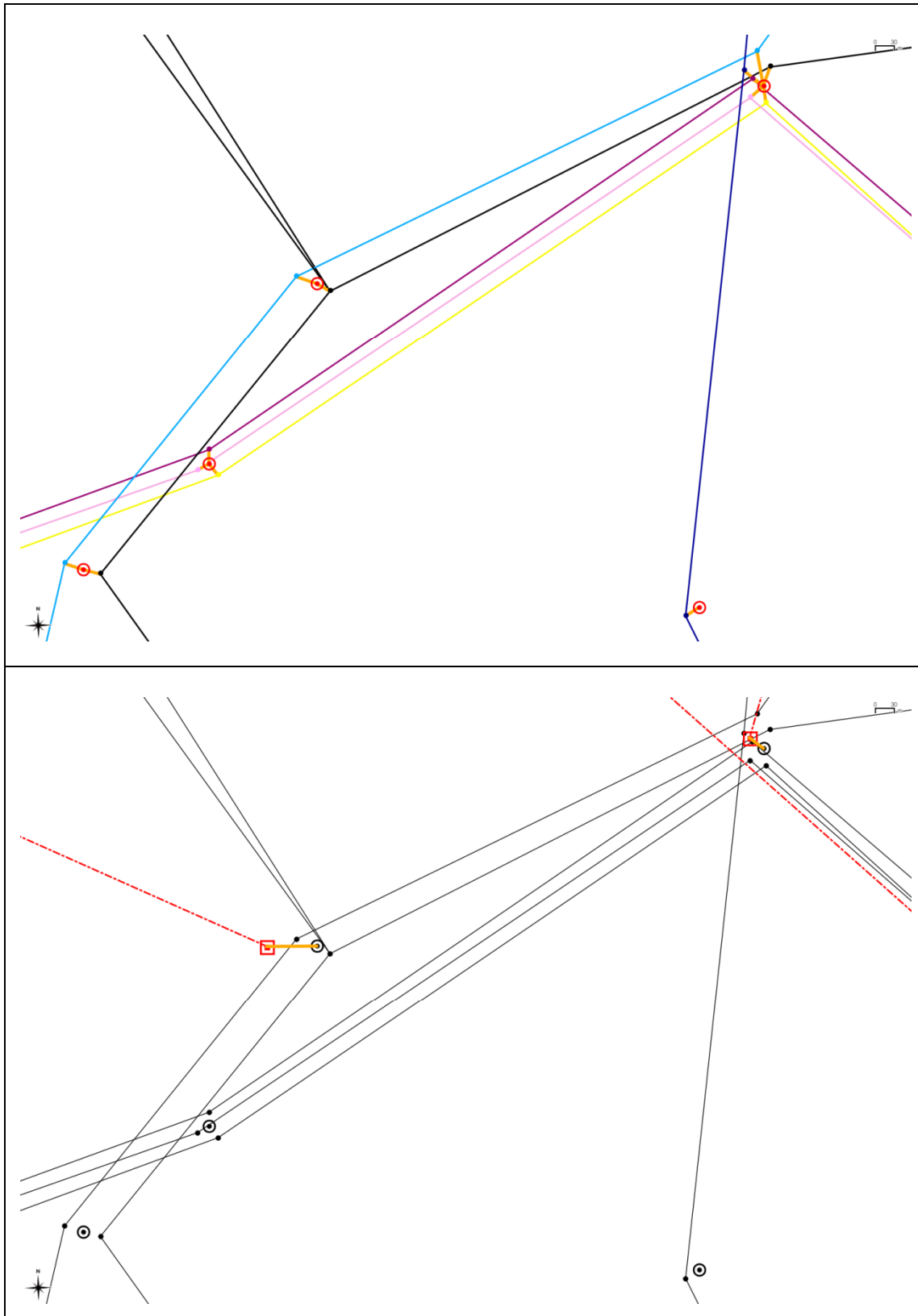
Figure 4 – The integrated multi-modal network model of London

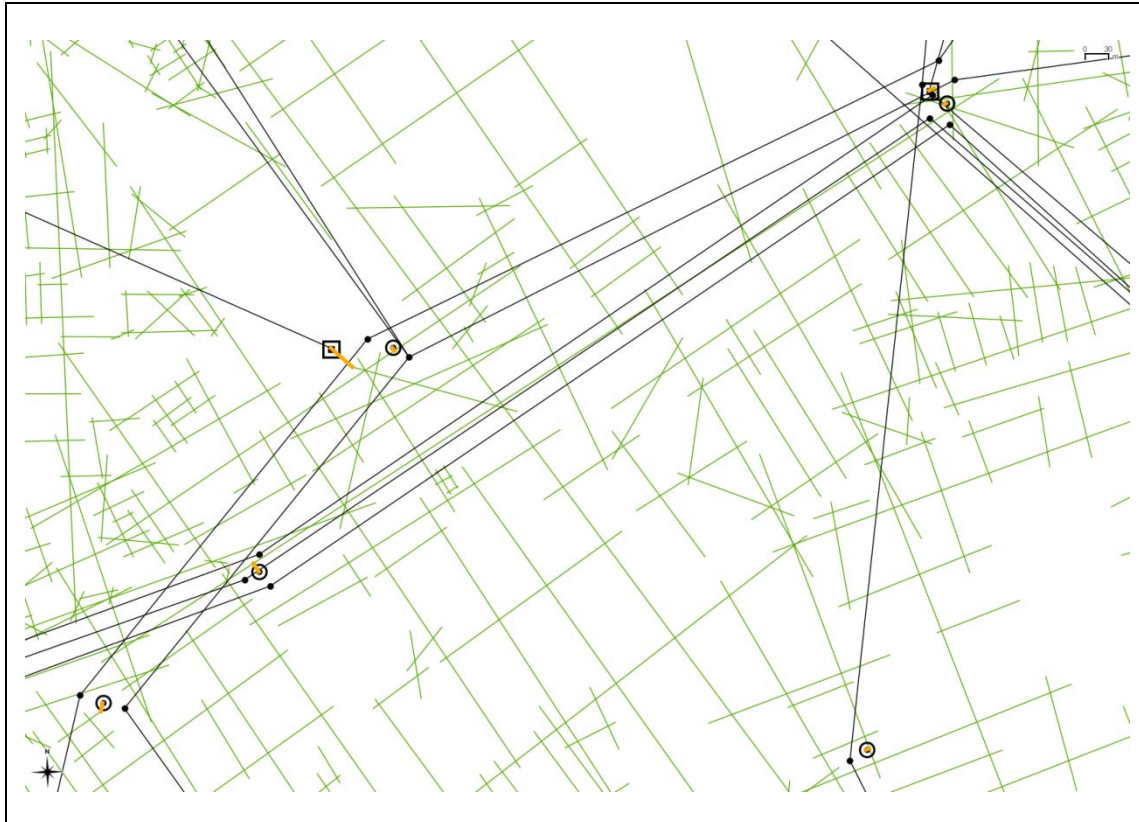
### 3.1 The construction of the model

The proposed model is a ‘hypergraph’ composed of different layers for each transport mode with additional layers for a type of link that we call ‘modal interface’ connecting the different modes. The ‘modal interfaces’ are similar to the concept of ‘superlink’ (Dalton 2007) or the link in the Depthmap software (Turner, 2007) used to connect different levels in buildings mapped side by side. It was built in a GIS platform because it facilitates the management of the various data sources and the complexity of the layered approach, while providing a continuous visual feedback on the model being constructed.

In the public transport network layers of the model, the nodes are represented as points and the links are the lines connecting them, because the configuration of the journey section between stops is not relevant to the topological configuration. The underground network is more complex because it is represented at the level of the line, having additional ‘intra-modal’ links between the line’s platforms and the underground station (figure 5 top). The rail stations and underground stations are then links using ‘modal-interfaces’ where the possibility of transfer is indicated in the public transport network maps and by the shared name of the station (figure 5 middle). Finally, all public transport nodes were linked to the pedestrian network by finding the nearest axial line to the public transport node (figure 5 bottom).

**Figure 5** – Maps showing in orange the links between the different layers of the integrated multi-modal network model. The ‘intra-modal interfaces’ of the underground network (top), the ‘modal interfaces’ between the rail and underground networks (bottom) and the ‘modal interfaces’ between public transport and the pedestrian network (next page).





All nodes in the different layers need to have an identifier. The NaPTAN 'ATCO code' was used for the public transport nodes and a synthetic code was created for the pedestrian model that contains a serial number and additional characters to identify the network. This way the links of the various layers and the links between layers can be combined to create the multi-modal network where every node can be uniquely identified. Additional attributes can be added to nodes and links of the various layers, and eventually used as weights in the network analysis. Once the multi-modal network has been modelled, it can be translated into a graph by extracting lists of links between nodes of the various layers and across layers, and combining these lists into a single list results in the hypergraph.

### 3.2 The configuration analysis of the model

A generic network analysis package called NetworkX (Hagberg et al., 2008) has been used to analyse the integrated multi-modal network model. This package offers a variety of graph/network management and manipulation tools and analysis algorithms. The main reason to use this package instead of conventional space syntax software, e.g. Depthmap, is that it takes a generic graph file as input, irrespective of the model that it represents or how it was produced.

The graph of the multi-modal network model was exported as a simple edges list containing the unique identifier of the two nodes of each edge and a column with a fractional weight value, by default 1.

The graph was measured in terms of various network centrality measures available: degree, closeness and betweenness (Brandes 2001). These measures are known to be equivalent to the space syntax measures of connectivity, integration and choice, respectively. This was verified using the pedestrian model and correlating closeness and betweenness centrality calculated in NetworkX with global integration and choice calculated in Depthmap. The correlation was indeed  $R^2=0.999$  for the axial analysis and for integration in 'fewest turns' ('topological') segment analysis, thus confirming that the network centrality measures can be used to obtain metrics consistent with previous space syntax findings. The only exception to this high correlation was betweenness and topological choice on the segment analysis. This can result from the way the 'fewest turns' distance concept was introduced in NetworkX and/or differences in the implementation of the algorithm.

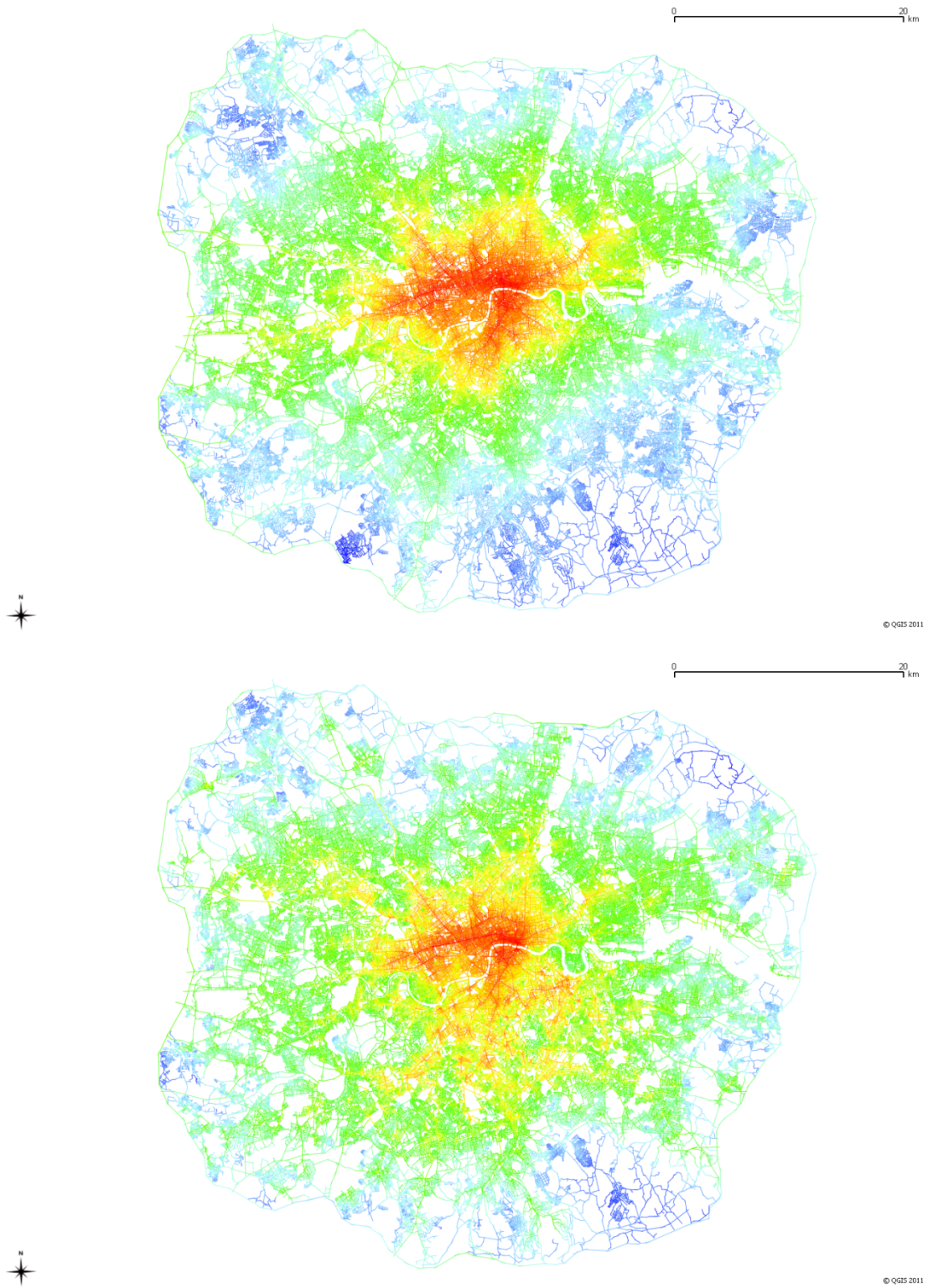
The weight of the edges in the NetworkX graph can control the type of distance used to measure shortest distance in the analysis. With an edge weight of 1 it calculates the equivalent of space syntax topological distance analysis. In the case of the segment map, to obtain 'fewest turns' distance analysis (Hillier and Iida, 2005), we set the edge weight to 0.001 if two segments are on the same axial line and 1 if they are on different axial lines. The edge weight value cannot be 0 (zero) due to a restriction in the algorithm implementation of NetworkX, and this small difference might explain some differences in the results, especially for very long lines with many segments and intersections.

An important restriction to the emulation of space syntax metrics in NetworkX is the absence of the concept of 'radius' that currently is not implemented as a cut-off distance parameter in the centrality algorithms. Therefore, the following analyses of the multi-modal network model always refer to global or radius N analysis.

#### **4. EXPLORING THE CONFIGURATION OF THE MULTI-MODAL NETWORK MODEL**

For the first time it was possible to study the configuration a complex multi-modal network as opposed to the land use accessibility measures based on metric distance and time of traditional transport models. In a first approach, we compare the effect of the multi-modal network on the space syntax model of London and evaluate to what extent this model can contribute to movement forecast models.

#### 4.1 The effect of the public transport network on the axial model

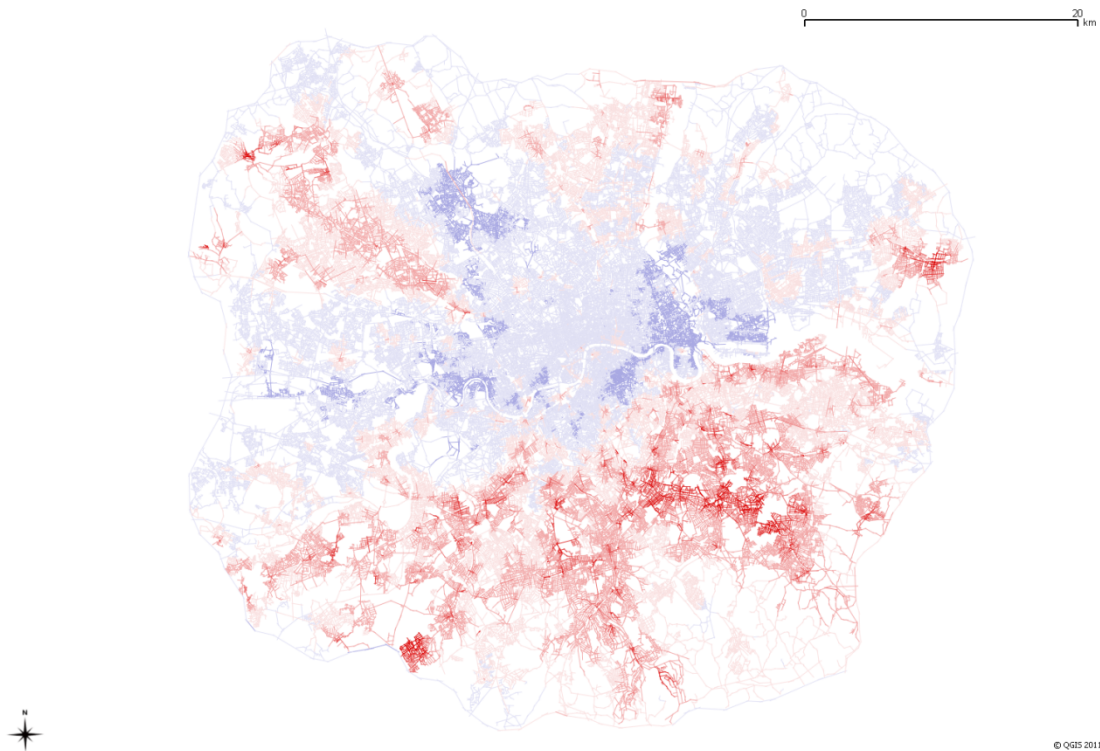


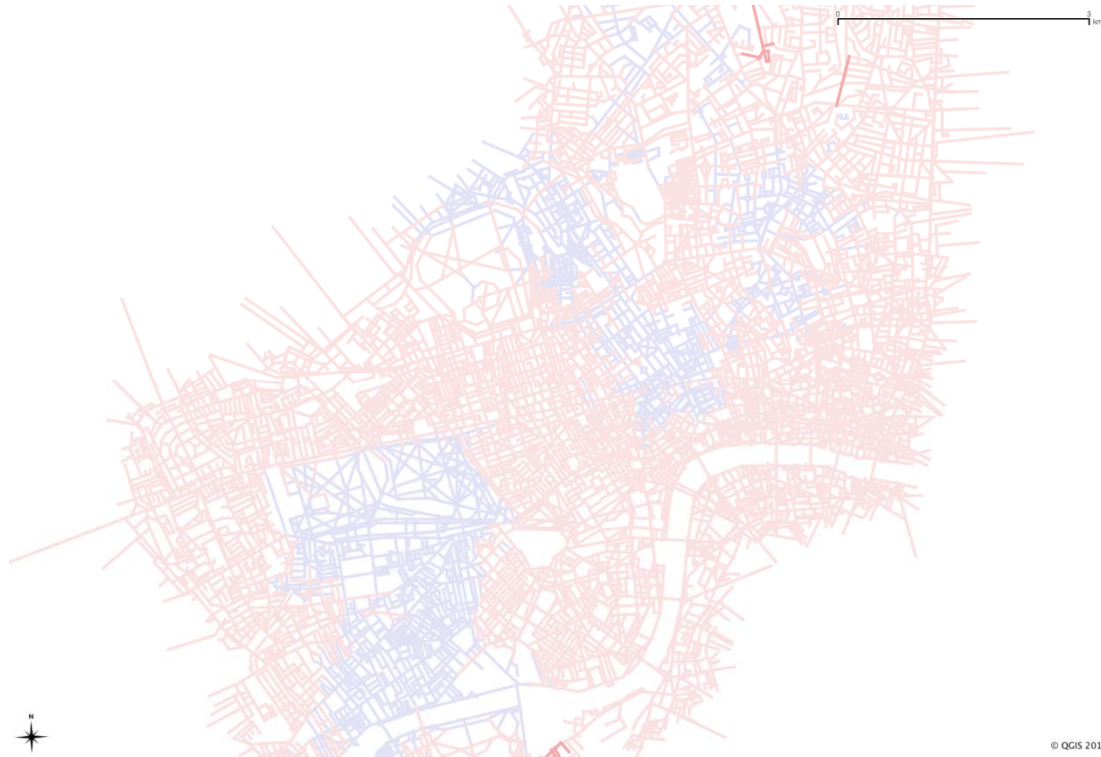
**Figure 6** – Maps of global integration of the pedestrian network, calculated for the individual network (top) and resulting from the multi-modal model (bottom).

Figure 6 presents the global integration measure (closeness centrality) mapped on the axial map (pedestrian network) of London. The top image maps integration measured on the pedestrian network alone and the bottom image maps integration measured on the multi-modal network. The impact of the structure of underground and rail network connection is quite clear. Firstly, we can observe a constellation of small, integrated centres emerging South of the river Thames. These are intermediate level centralities linked to central London but dependent on the public transport networks for that. To some extent, this confirms the perception of South London that seems to be 'far' to most Londoners, but once one learns about the public transport connections one realises that central London can be easily reached. We can then observe an 'erosion' of centrality in areas adjacent to central London, in particular East and South East London and the areas of Pimlico and Battersea in South West London, that are not so well served by both rail and underground networks. Finally, we can observe that the lower/mid integration values spread reaching further areas on the edges near the M25 orbital, with only very few local centres appearing completely segregated. This multi-modal integration map demonstrates the coverage of the London public transport network that integrates the whole city, but also its centralised structure that retains the strong core in central London.

To further analyse the impact of the rail and underground networks on the axial pedestrian network we calculate the difference in integration of every axial line of the two models and produce impact maps (figure 7). The integration values are scaled between 0 and 1 for comparison purposes and the differential represents a change in integration ranking with the introduction of the public transport networks.

**Figure 7** – Impact maps of the public transport network on the pedestrian model of London (below) and central London (next page). The colour scale is from blue (decrease in integration ranking) to red (increase in integration ranking), with colours close to white as the middle value for no change.





Overall, there are big improvements in integration ranking in South London and some fringe pockets in North London. The flip side of this is an even but only very slight drop in integration ranking in other areas, with bigger drops only in a few concentrated areas such as Edgware and Finchley in North London, Poplar, Bow, Clapton and Leyton in East London and Walworth and Peckham in South East London. But no areas are affected by drops of more than 25% in the ranking as opposed to areas that climb that much.

The bottom image of figure 7 shows the impact of the multi-modal network model on segment integration in the area of central London of the SATC pedestrian counts (figure 3 bottom). It becomes clear that the impact in central London is extremely limited with minor moves up and down the ranking of integration, and only few segments in the fringe of this model benefit from the connections offered by the public transport networks.

#### **4.2 The contribution of the public transport network on pedestrian flows estimation**

Based on the previous model of central London and the limited change in integration ranking with the introduction of the public transport networks it is not expected that this multi-modal network model improves on the correlations of the integration measures against local pedestrian flows. In fact, it only marginally affects positively the correlation between global fewest turns segment integration and pedestrian movement in the individual areas of the SATC data set (table 1). In the case of correlation against the counts of all areas combined, the multi-modal network model performs slightly worse.

**Table 1** – Correlation coefficient  $R^2$  of the pedestrian movement in different areas of the SATC dataset against global integration of the pedestrian and multi-modal network models.

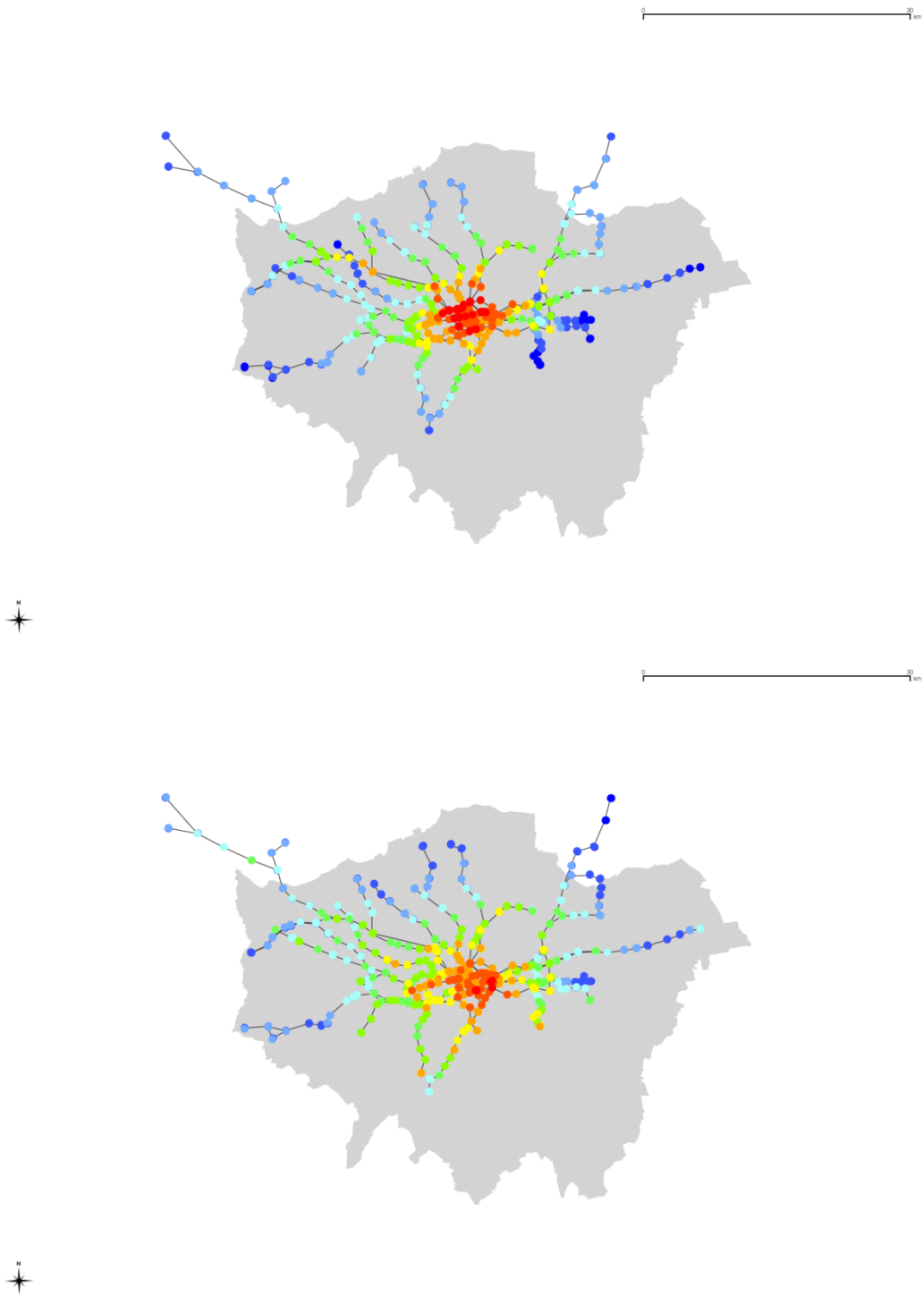
Area	Pedestrian integration $R^2$	Multi-modal integration $R^2$
Barnsbury	0.366	0.387
Brompton	0.477	0.477
Clerkenwell	0.552	0.555
South Kensington	0.233	0.239
All areas	0.157	0.152

To further test the use of the multi-modal network model against pedestrian flows one can calculate the fewest turns distance from every segment to the public transport nodes (underground and rail stations) and use this variable in a multivariate statistical model. The correlation coefficient with pedestrian flows of all areas combined in a model using integration and distance to the nearest station improves from  $R^2=0.157$  to  $R^2=0.350$ . This indicates that the presence of the public transport nodes has a local effect in the different areas that positively contributes to the description of the level of movement on the streets, as opposed to the more global configurational effect of an integrated multi-modal network.

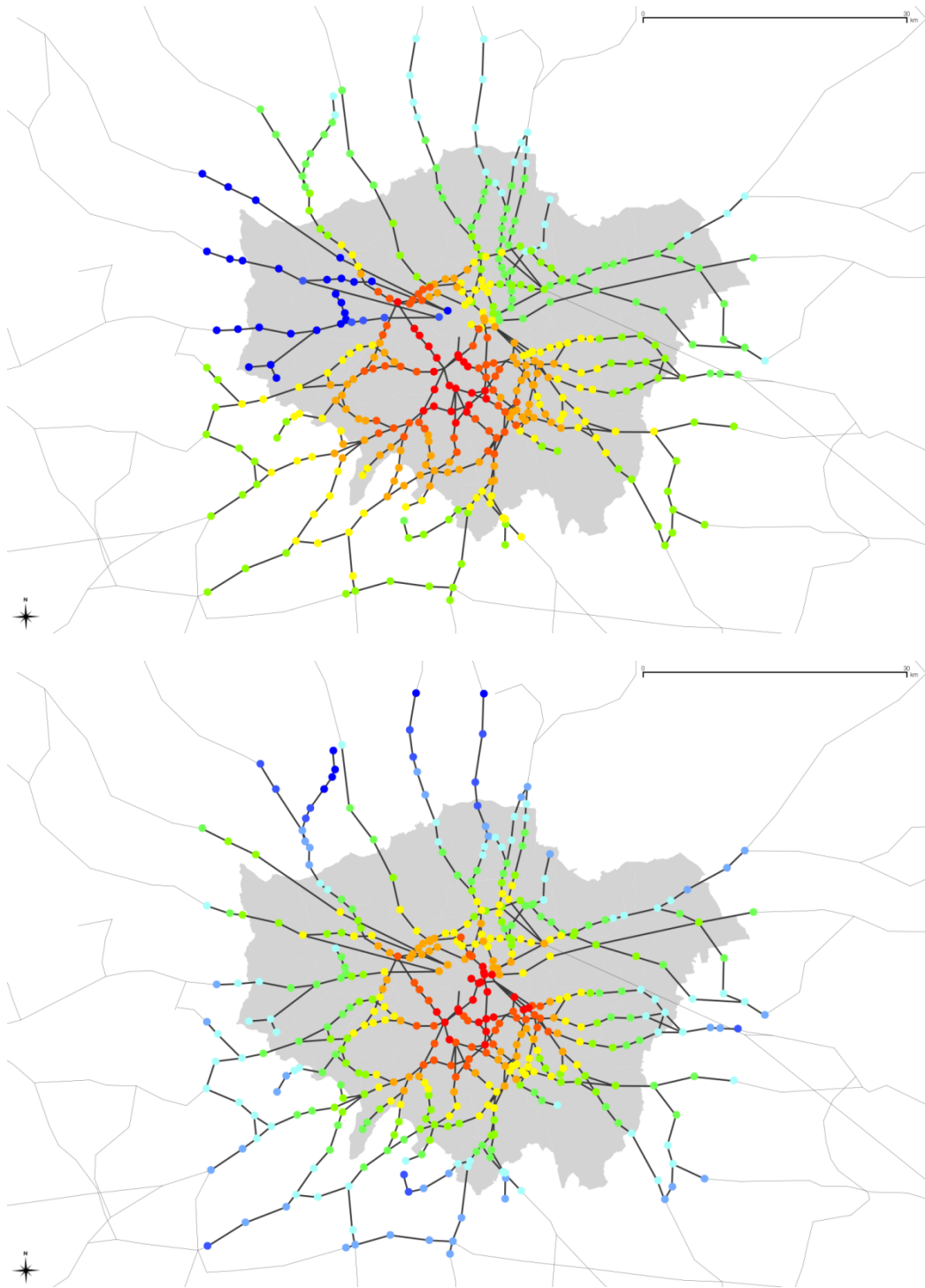
#### **4.3 The effect of the multi-modal network on public transport usage levels estimation**

To conclude the exploration of the multi-modal network model we test the effect it has on the configurational analysis of the public transport networks and the estimation of level of usage of the individual stations.

Figures 8 and 9 present maps of the underground and rail networks analysed in terms of network integration in isolation and as part of the multi-modal network model. The underground network is a centralised system with the integration core in Zone 1 stations where lines cross and integration dropping along the various lines towards their terminals. However, when considered as part of a multi-modal network, certain terminal underground stations or interchanges clearly become more integrated as they link up with the rail network and the local pedestrian network. The rail network is a distributed system converging on central London in a series of terminal stations. The isolated rail network analysis fails to pick out the importance of the central terminals North of the river and the integration core is located on the South, where the network is more densely interconnected to compensate for the lack of an underground network. On the other hand, the rail network as part of a multi-modal network pulls the integration core to the rail stations in and around central London, including the terminals. It also integrates the rail branches going from Paddington and Euston that do not connect with other rail network branches within London but have local links to the underground and pedestrian networks.



**Figure 8** – Maps of integration of the London underground network as an isolated network (top) and integrated in the multi-modal network model (bottom).



**Figure 9** – Maps of integration of the rail network as an isolated network (top) and integrated in the multi-modal network model (bottom).

Despite this improvement in the description of the underground and rail network configuration and functioning, the multi-modal network model does not bring significant improvements in the estimation of

station usage (table 2). Only the rail network shows some improvement but remains far from being a strong explanatory variable with  $R^2=0.168$ .

**Table 2** - Correlation coefficient  $R^2$  of total station usage against global integration of the underground and rail networks considered in isolation or integrated in a multi-modal network.

Network	Isolated $R^2$	Integrated $R^2$
Underground	0.473	0.473
Rail	0.063	0.168

## 5. DISCUSSION AND FURTHER WORK

The discussion draws from the results of the experiment to address the questions set out in the paper's introduction.

### 5.1 How can we model public transport networks for configurational analysis?

The underground network representing the individual lines with 'intra-modal interfaces' linking the lines is starting to give a good representation of flows on the network, however the rail network is still far from achieving that. The next step should be to model the rail network in terms of services from stations, which includes fast services that only stop at important stations. This way one will more clearly represent the regional role of the rail network.

Because of the relational nature of the multi-modal network model, one can expect improvements in the performance of all modes by tweaking and improving the representation of another mode. For example, the analysis results of underground nodes should better describe the level of use of the underground stations by improving the representation of the rail network to which many of them are directly linked.

Furthermore, the inclusion of the bus network in the multi-modal network model should add a more local and detailed level of public transport linkages and should improve the description of the usage on all public transport modes.

### 5.2 How can we integrate these networks with the pedestrian model?

The analysis of local pedestrian movement levels was not improved using an integrated multi-modal network model instead of the isolated axial model.

One should consider improvements in the way the public transport nodes are linked to the pedestrian network. On the one hand, one should use a land use layer with a representation of the public transport stations to obtain their correct location and link to the surrounding streets, instead of a single link to the nearest axial line/segment. On the other hand, one should consider the location of underground station entrances because a single station can link to several different and distant streets simultaneously.

However, should one expect improvements in pedestrian movement predictive power from an integrated multi-modal network? The pedestrian network configuration already has embedded the presence of the public transport networks, and conversely the public transport nodes have been placed in locations of specific pedestrian network significance. This historical synergy cannot be untangled and an apparently simple model will already be a representation of a more complex and multi-modal reality.

### **5.3 How can we measure the network centrality of this integrated model?**

The limited impact of the multi-modal network on the integration measure of the pedestrian network needs to be discussed. The pedestrian network is formed of long axial lines, or segments derived from these long lines. At the same time, the links between nodes of the underground or rail networks in the denser and more integrated parts of central London tend to be located on and follow the most integrated lines. Performing fewest turns or least angle analysis on the pedestrian network eliminates the cost of moving along the longest lines, while the topological analysis of the public transport networks always gives the same cost for every 'step' on the network. Consequently, the public transport networks do not offer the expected 'shortcuts' and the integrated analysis of the multi-modal network might not distinguish the different scales and roles of the different networks in our use and experience of the city (Read 2009).

Further work is needed to understand the complexities of interaction between the levels of a multi-modal network model as a representation of a city, to enable its integrated configurational analysis. One can consider a differentiated analysis per mode instead of the same topological analysis across levels, and one would assign differentiated weights to the links of the different networks. Conceivably, one could use angular weights on pedestrian and cycle networks, topological weights on public transport networks and translate the cost of transfer between lines/services or between different modes (Guo and Wilson, 2011) into weights.

However, one would also need to investigate the possibility of using a unified unit for these weights and for measuring distance, such as time, that is conventionally used in other multi-modal transport models. One should also consider the introduction of analysis cut-off distances (radii) to limit the use and influence of specific networks, such as a walking distance limit for the pedestrian network. This would require the development of the algorithms in the network analysis tool currently used, or the adoption of a different tool that can take a custom made graph as input for analysis.

Finally, one should explore the use of local and simpler network metrics, such as the area of influence of transport nodes across levels, instead of an integrated configurational analysis. In this study, the proximity to stations was shown to contribute to the description of pedestrian movement levels in different areas. This can offer variables to combine with configurational analysis of individual networks to obtain an integrated multi-modal statistical model. In such a model, the public transport networks remain independent from each other, which can facilitate experimentation, calibration and ultimately description of their contribution to the overall multi-modal model.

## **6. CONCLUSION**

This paper presented a first attempt to integrate multi-modal public transport networks in the configurational analysis of the axial map. A method was developed to model and integrate the layers of

different public transport networks, with the introduction of the concept of 'modal interfaces' as a collection of links that connect the different layers at specific points to form the integrated multi-modal network. Furthermore, the technical challenge of configurational analysis of this network using space syntax equivalent metrics was addressed by creating a hypergraph and measuring its centrality using a standard network analysis package, which correlates with the integration and choice measures common in space syntax studies.

The results of this integrated analysis seem to improve the description of the individual networks, but with no contribution to the estimation of pedestrian movement flows and public transport station usage. Several technical improvements to the model are proposed but also theoretical issues that need to be addressed are raised.

In conclusion, the experiments with an integrated multi-modal network model raise some technical but also (more importantly) conceptual issues. Removing the boundaries between the experience of using and changing between different transport modes to make an integrated model does not seem to yield satisfactory results. It exposes the challenges of modelling complex systems and combining different levels and systems that operate at different scales.

## 7. ACKNOWLEDGEMENTS

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