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A choice-driven service network design and pricing including heterogeneous behaviors

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ABSTRACT

The design and pricing of services are two of the most important decisions faced by any intermodal transport operator. The key success factor lies in the ability to meet the needs of the shippers. Therefore, making full use of the available information about the demand helps to come up with good design and pricing decisions. With this in mind, we propose a Choice-Driven approach, incorporating advanced choice models directly into a Service Network Design and Pricing problem. We evaluate this approach considering both deterministic and stochastic choice models. To reduce the computational time for the stochastic instances, we propose a predetermination heuristic. The proposed models are compared to a benchmark, where shippers are solely cost-minimizers. Results show that the operator's profits can be significantly improved, even with deterministic models. The stochastic versions further increase the realized profits: in particular, considering shippers' heterogeneity allows a better estimation of the demand.

1. Introduction

In intermodal freight transport, planning at the tactical level is of key importance to make the best use of existing infrastructure and available assets and to ensure reliable transport plans. An appropriate way of managing this task is through Service Network Design (SND) problems, as they cover most of the tactical decisions (Crainic, 2000). They can support the decisions of intermodal operators about the itineraries to be served, the offered frequencies, and how demand should be assigned to these services.

Until recently, pricing was not explicitly covered in most SND models although it plays a crucial role in the success of the planning (Tawfik and Limbourg, 2018; Li et al., 2015). As pointed out by Macharis and Bontekoning (2004), intermodal transport pricing is a difficult task as costs must be accurately computed and some knowledge of the market situation has to be gained. Indeed, the costs faced by an intermodal operator are various (Li and Tayur, 2005): some of them, e.g. crew costs or contracts with infrastructure manager, are perfectly known by the operator but other variable costs are set by external companies, such as terminal operators for the handling costs or energy suppliers for the fuel costs. For the latter, not only do they depend on external actors, but also on the transport demand as they increase together with the carried load. Although transport operators have some control over the quantity of transported freight (via contract binding, for example), demand remains mostly stochastic in nature (Combes, 2013). As a result, variable costs can only be estimated from the expected transport demand.

Regarding the pricing decision itself, some knowledge about the targeted demand, such as the willingness to pay or the transport requirements, is also of key importance. Indeed, the cost of transportation is among the main drivers of shippers' mode choice. It would, however, be inadequate to consider that shippers are purely "cost-minimizers" as other factors (e.g., transport time, offered

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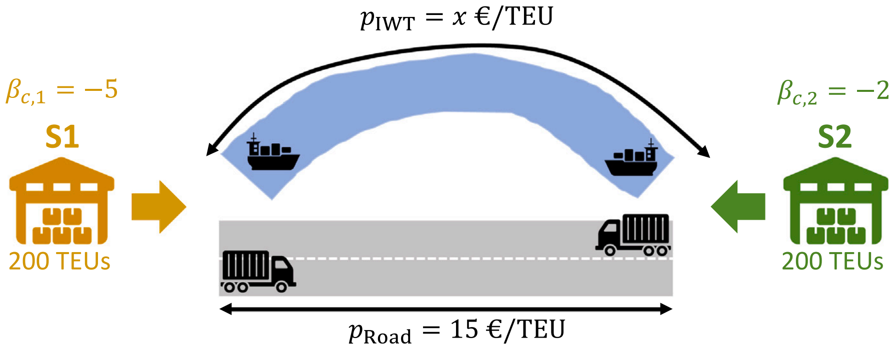

Fig. 1. Illustrative example with two shippers and two available transport modes.

quality, service frequency) play a role in the decision process, see for example Arencibia et al. (2015) or Ben-Akiva et al. (2013). On top of that, these factors and their importance can vary from shipper to shipper and the final decision of choosing a mode also depends on the available alternatives, hence making the planning and pricing process even more complex. On the other hand, there exists a great variety of mode choice models (see de Jong (2014) for a comprehensive review) that can be used to support the planning of intermodal operators. For example, Duan et al. (2019) include values of time and reliability, that are estimated from a stated preference survey, within the cost minimization of an SND model. This represents a step towards the integration of shippers' preferences within the planning process.

Our work aims at leveraging further the advantages of including a detailed mode choice model within this tactical decision-making setting. Therefore, we develop a Choice-Driven Service Network Design and Pricing (CD-SNDP) model, which includes an existing mode choice model to consider shippers' behavior directly in the decision-making of the transport operator. In the rest of this paper, we first emphasize the potential of our approach through an illustrative example. In Section 2, we review the existing literature on SNDP. Then, the proposed methodology is described in Section 3, where deterministic and stochastic formulations are covered. In Section 4, the proposed methodology is applied to a case study and several variations of the model are compared with each other. Finally, we conclude the paper in Section 5 and share some insights for future research.

1.1. Illustrative example

To highlight the benefits of using a mode choice model for the pricing decision, we consider the case in Fig. 1, where two shippers, S1 and S2, want to send 200 Twenty-foot Equivalent Units (TEUs) each. To do so, they have two alternatives: Road and Inland Waterway Transport (IWT). Each mode has the following utility function for each shipper i:

$$\begin{cases} V_i^{\text{IWT}} &= \beta_f f + \beta_{c,i} p_{\text{IWT}} = 1 \times 5 + \beta_{c,i} \times x, \\ V_i^{\text{Road}} &= \alpha_{\text{Road}} + \beta_{c,i} p_{\text{Road}} = 15 + \beta_{c,i} \times 15, \end{cases}$$

where α_{Road} is the Alternative Specific Constant (ASC) for Road, equal to 15, and the ASC for IWT is normalized to 0. p_{Road} is the cost of the Road alternative, set to 15 €/TEU, and $\beta_{c,i}$ represents the cost sensitivity of each shipper i: we assume that it is −5 for S1 and −2 for S2. β_f is the weight associated to the frequency of IWT services f, and assumed to be 1 for both shippers.

In this example, the decision-maker is the IWT operator that wants to set up a transport service running each working day (hence: $f = 5$) and to optimize their price x. The operator faces a fixed cost, c_{fix}, of 100 € per round trip and a variable cost, c_{var}, of 1 €/TEU. Assuming that the transport demand of shippers is split according to a logit model, the operator aims at setting a unique price to maximize their profits, expressed as:

$$\Pi(x) = \sum_i (200 \times \frac{e^{V_i^{\text{IWT}}}}{e^{V_i^{\text{IWT}}} + e^{V_i^{\text{Road}}}})(x - c_{\text{var}}) - f \times c_{\text{fix}} = \sum_i (200 \times \frac{e^{V_i^{\text{IWT}}}}{e^{V_i^{\text{IWT}}} + e^{V_i^{\text{Road}}}})(x - 1) - 500$$

The operator does not necessarily know the full utility specifications of the shippers. Therefore, it can opt for various demand models, here we consider three of them:

(A) Assume that shippers are homogeneous and purely cost-minimizers, the considered utilities may then be: $V_i^{\text{IWT}} = -1x$ and $V_i^{\text{Road}} = -1 \times 15\ \forall i$;
(B) Make more market study to come up with the same utility functions as above, but consider that shippers are homogeneous with a mean cost sensitivity, thus: $\beta_{c,i} = -3.5\ \forall i$;
(C) Consider also the heterogeneity regarding the cost sensitivity (ground truth model), thus: $\beta_{c,1} = -5$ and $\beta_{c,2} = -2$.

Finally, let us assume that the operator has a fixed vessel capacity of 20 TEUs. The resulting profits $\Pi(x)$ associated with price x are depicted in Fig. 2, together with the profits stemming from each individual shipper. Before the price reaches 10 €/TEU, the

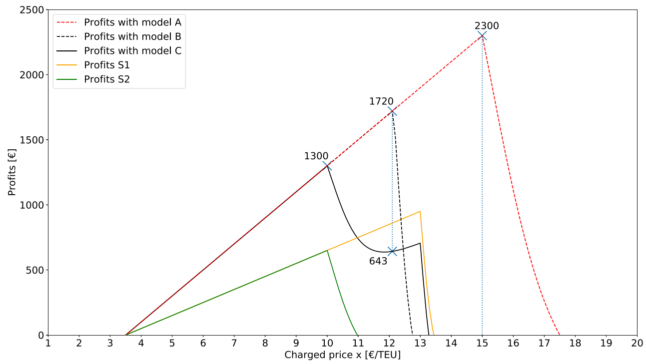

Fig. 2. Resulting profits for models A (pure cost minimization), B (homogeneous shippers), C (heterogeneous shippers), and individual profits for S1 and S2.

profits grow linearly. Indeed, IWT is much cheaper than Road so the demand assigned to IWT exceeds the capacity (only 100 TEUs per direction) and the profits only depend on the price.

The highest expected profits are reached with model A with a price of 15 €/TEU. Indeed, when only considering costs, the IWT operator can charge a higher amount as the cost of Road is relatively high. If the price exceeds 15 €/TEU, IWT becomes more expensive than Road and the IWT operator faces a rapid decrease in their demand, and thus their profit. Nevertheless, when shippers S1 and S2 will face a price of 15 €/TEU, they will both turn to Road as it has a higher utility. This will then end up in losses for the IWT operator.

The maximal expected profits with model B occur at 12 €/TEU. This is because the cost sensitivity of S2 is overestimated. With model B, it is as if the cost of Road was still too high for S2 despite the other advantages of this transport mode. But in reality, the profits stemming from S2 reach zero for a price of 12 €/TEU because Road advantages (included in α_{Road}) overcome the higher cost, thus making Road much more attractive. Therefore, it will result in profits reduced by half when the price of 12 €/TEU is charged to the actual heterogeneous shippers.

Applying model C returns the highest profits, as it considers the true cost sensitivity of each shipper. In reality, it will of course not be the case. But the purpose of this example is to showcase the potential consequences of using simplifying assumptions in the planning and pricing process of a transport operator. In this example, if they consider that their customers are purely cost-minimizers, then the optimal price under this assumption will eventually cause losses to the operator. When they consider a more detailed representation of shippers (as in model B), the optimal price is still overestimated but, at least, positive profits are achieved. So even if the exact parameters are not known, it is beneficial for the operator to incorporate more information about their customers.

Note that a revenue management strategy would be trivial to implement in this example with only two shippers and simple utility functions, then the optimal solution would be to set different prices for S1 and S2. However, segmentation may be difficult to identify when many more shippers are considered and less detailed information is available. In the remainder of this work, we will not consider revenue management, although we recognize that it can be an effective tool to optimize pricing decisions.

2. Literature review

In order to develop our CD-SNDP approach, we make use of "choice-driven (or choice-based) optimization". Although it has not been applied to SNDP models yet, choice-driven optimization is already used for other types of problems. Therefore, we first review the state of the literature on SNDP in intermodal transport, then investigate the existing choice-driven methods in related transportation fields, and finally present the main contributions of the present work.

2.1. Service network design and pricing problems in intermodal transport

The majority of existing studies on SND are formulated as a cost minimization of the transport operator and do not include the revenues of fulfilling the transport orders (Elbert et al., 2020; Wieberneit, 2008). Nevertheless, two models using cost minimization have addressed the pricing decision. Li et al. (2015) determine the price charged by an intermodal operator using a pre-defined profit margin, expressed as a given percentage of the operational costs. The price is the addition of the costs and the margin and cannot exceed a given market price. Dandotiya et al. (2011) include a target for the minimal profit (per transported unit) to be achieved by an intermodal operator: this translates into a constraint assuring that the applied rate is greater or equal to that target added to the operating costs. The authors also include a cost sensitivity factor representing the willingness to pay for intermodal transport rather than road and enforce that the rate difference between road and intermodal transport has to be greater or equal to this factor.

For the works applying a profit maximization, some of them do not include the pricing decision but rather assume fixed tariffs that are included as parameters in the model. Andersen and Christiansen (2009) apply an SND model to explore new rail services along a Polish freight corridor. The demand is represented as contracts generating a given revenue when served. The operator then decides to serve or not the contracts in order to maximize their profit. It also decides on the services' frequency and the vehicles and demand assignment to these services under vehicle balancing and capacity constraints. Braekers et al. (2013) are interested in designing a barge transport service along a Belgian canal considering empty container repositioning. Their SND model decides at which inland ports to stop and in which sequence as well as the fulfillment of transport demand from different clients. Bilegan et al. (2022) also apply an SND model to barge transport with detailed fleet management and revenue management considerations. Different customer segments are considered as well as two different service levels (standard or express) with a given fare. The operator then decides which services to operate, what percentage of the demand to serve, and how to assign the vessels and demand to the services to maximize their profits. The model has been developed further to include the possibility of bundling services and penalties for early and late distribution (Taherkhani et al., 2022). Teypaz et al. (2010) treat similar models and propose decomposition algorithms for computational efficiency. Zetina et al. (2019) capture demand elasticity using a gravity model, where the demand is considered inversely proportional to the transport costs faced by the transport operator. The decisions are whether or not an arc (or a path) is used and in which sequence to visit the demand nodes. Finally, Scherr et al. (2022) use SND to conceive a new platooning service of autonomous vehicles. They come up with a two-stage stochastic model considering scenarios to represent the demand variation. The first stage designs the services performed by "manually operated vehicles" and assigns rates to the different customers over all scenarios, whereas the second sets the flow of autonomous vehicles for each particular scenario.

Other works include demand functions in the profit maximization to capture the influence of prices on transport volumes. Li and Tayur (2005) design a railroad network using a concave inverse demand function. In this case, the demand for each service and each itinerary are the decision variables and the corresponding prices are computed using the inverse demand function. Mozafari and Karimi (2011) represent two competitive road carriers within a non-cooperative game model. Each carrier has to set their price to maximize their own profit and the demand is represented as a linear function of the carrier's price and the competitor's price. Shah and Brueckner (2012) also investigate competition between carriers: each of them fixes their price, frequency, and capacity. The demand of shippers for a given carrier is represented as a function of price and frequency. The inconvenience of demand functions is that they become hard to obtain when the number of shippers or alternatives increase (Li and Tayur, 2005), thus requiring a numerical estimation or some simplifying assumptions.

An increasingly common way to model SNDP problems is using Stackelberg game or bilevel programming. This formulation was first proposed in intermodal freight transport by Tsai et al. (1994). The intermodal operator is the leader and sets the price of their services to maximize their profit. Truck carriers are followers that will adjust their prices based on the leader's decision and the exogenous demand is split between the carriers using a logit model, where the considered attributes are the prices, travel times, and reliability. A general formulation for the Joint Design and Pricing (JDP) on a network has been proposed by Brotcorne et al. (2008). The network operator decides on the network design and prices to maximize their profits. The network and rates of the competitors are assumed known and exogenous. The followers are the network users who seek to minimize their costs by selecting the services of the operator or those of the competitors. The authors propose an iterative procedure to solve the JDP. Crevier et al. (2012) propose a similar formulation, with the addition of capacity constraints and revenue management considerations. Ypsilantis and Zuidwijk (2013) extend the JDP formulation to include time constraints, as well as capacity constraints. Their model is used to design and price the hinterland barge services of an extended gate operator. In their work, Tawfik and Limbourg (2019) include some level-of-service attributes in the JDP formulation. In particular, the lower level costs are more detailed as they not only consider transport costs but also the cost of capital: each cost component is weighted by a coefficient estimated using a random utility model. An iterative heuristic is later proposed to solve large instances of the JDP (Tawfik et al., 2022). A similar formulation is adopted by Zhang and Li (2019) to design and price rail container transport. The lower level objective is to minimize the generalized costs, made of price, transport time, convenience, and security. Only the price is endogenous to the model. The same authors also propose a time-varying model (Zhang et al., 2019; Li and Zhang, 2020). A single-level formulation is used and the demand follows a logit model with price as a single attribute. The model proposed by Wang et al. (2023) extends the JDP of Tawfik and Limbourg (2019) with the introduction of additional cost components. The transport operator faces some waiting costs and penalties for an under-utilization of their capacity, while the lower level costs also embed heterogeneous shipper classes through different values of time and reliability.

Finally, there also exist a few different versions of Stackelberg game. A monopoly setting is proposed by Qiu et al. (2021) where a hinterland carrier sets services and prices in multiple planning horizons. The followers are represented by a set of captive consignees that minimize their transport and storage costs. Lee et al. (2014b) consider three different actors as leaders and all shippers as followers. The upper level itself is represented as a three-level program where ocean carriers are leaders of terminal operators which, in turn, are leaders of land carriers (Lee et al., 2014a). At the lower level, shippers set their production, consumption, and transportation demand using "spatial price equilibrium".

The relevance of bilevel models is questioned by Martin et al. (2021), especially because of the simplifying assumptions regarding demand modeling (pure cost minimizers and homogeneous preferences). They propose an SNDP model applied to an express shipping service by airplanes and trucks. In their profit maximization problem, the transport operator has to set prices for some given service times that can be selected by their customers. The service time chosen by each customer is the one providing a welfare greater or equal to all the other options.

The novelty of our CD-SNDP is that it includes a stochastic demand model considering heterogeneity, within a bilevel optimization setting. The proposed formulation is inspired by the work of Tawfik and Limbourg (2019), where the cost minimization of shippers is replaced by the maximization of their utility. In our work, besides the costs, the utility functions also consider the transport time, the accessibility of a mode, and the frequency of intermodal services. This last element implies that now, both the price and frequency decisions of the transport operator influence the shippers. This CD-SNDP formulation then allows for a more detailed and realistic representation of the shippers' characteristics and behavior towards the prices and services designed by the operator. To include stochasticity and heterogeneity in our model, we make use of choice-based optimization: hereafter are presented some applications of this method to other transportation problems.

Table 1
Summary of existing bilevel models for intermodal Service Network Design and Pricing problems.

Reference	Fleet constraints	Deterministic or Stochastic	Demand function $F(\cdot)$[a]	Hetero-geneity	Cross-level variables
Tsai et al. (1994)	✓	D	$F(c)$		Price
Brotcorne et al. (2008)		D	$F(c)$		Price
Crevier et al. (2012)	✓	D	$F(c, LoS)$	✓	Price
Ypsilantis and Zuidwijk (2013)	✓	D	$F(c)$		Price
Lee et al. (2014b)	✓	D	$F(c, VoT)$	✓	Price
Tawfik and Limbourg (2019)		D	$F(c, VoT)$		Price
Zhang and Li (2019)	✓	D	$F(c, VoT, LoS)$		Price
Qiu et al. (2021)	✓	D	$F(c)$		Price
Wang et al. (2023)		D	$F(c, VoT, VoR)$	✓	Price & Freq.
Proposed CD-SNDP	✓	S	$F(c, f, u)$	✓	Price & Freq.

[a] c = costs, LoS = level of service, VoT = value of time, VoR = value of reliability, f = frequency, u = unobserved attributes.

2.2. Choice-based optimization in transportation

The term "choice-based (or choice-driven) optimization" refers to optimization problems that explicitly include a discrete choice model into their formulation (Pacheco Paneque, , 2020). That is why works decoupling the optimization from the demand, using iterative procedures such as simulation–optimization, are not considered here (e.g., Liu et al. (2019)).

Although not for freight, choice-based optimization has been used in a few works to model passenger SND problems. Wang and Lo (2008) propose a profit maximization problem to support the design of ferry services, where the operator decides on the itineraries and schedules of the ferries. They assume that the passenger demand is split according to a logit model including two attributes: a given price, and the travel time, which is dependent on the decision variables. Huang et al. (2018) also include a logit model into a profit maximization problem to design a car-sharing network. Among other things, the operator decides on the number of car-sharing stations to open. The utility function of car-sharing is composed of given rental costs and walking access costs. The latter are directly dependent on the number of opened stations. A drawback of these two models is that they are non-linear due to the exponential terms inherent to the logit model. A Mixed-Integer Linear Programming (MILP) including a logit mode choice model is proposed by Hartleb et al. (2021) to design passenger rail services. The main decision is the selection of lines to open. To get rid of the exponential terms of the logit model, the authors precompute the modal shares of rail for each possible solution. This precomputation technique is useful when only binary or integer variables are included in the choice model. However, as mentioned by the authors, the model can become intractable when the instance size increases.

Choice-based optimization has also been applied to facility location and pricing problems. It is used by Lüer-Villagra and Marianov (2013) to set up hubs and prices for an airline company. The demand is split between companies using a logit model with price as a unique attribute. A similar modeling approach is adopted by Zhang (2015) to locate retail stores and set selling prices. Zhang et al. (2018) study an intermodal dry port location and pricing problem where the route choice of shippers is determined using a logit model including six attributes, where only transport cost depends on the decision variables. The common point of these three models is that they are all non-linear: therefore, heuristics are required to solve them.

In most of the aforementioned models, the inclusion of discrete choice into an optimization problem results in a non-linear model. In their work, Paneque et al. (2021) propose a general framework to deal with more advanced choice models. In particular, the authors rely on the Sample Average Approximation (SAA) principle to deal with the non-linearities of the choice model and, therefore, come up with a MILP model. The proposed model is then applied to the pricing of parking services using a Mixed logit to represent the demand. The latter comprises price as an endogenous attribute and other exogenous attributes. Bortolomiol et al. (2021) develop this framework further to model oligopolistic competition, whereas Schlicher and Lurkin (2022) present a non-linear cooperative game to model collaborative pricing of urban mobility.

The present CD-SNDP is inspired by the work of Paneque et al. (2021) to integrate an existing Mixed logit model within a bilevel setting. Specifically, error terms are included in the utilities to account for the attributes that are not captured by the model but still play a role in the mode choice. Moreover, the coefficient representing cost sensitivity is considered randomly distributed to consider the heterogeneous preferences of shippers. It is assumed that the probability distributions of the error terms and the cost coefficient are known and the resulting CD-SNDP problem is solved using stochastic optimization. The addition of these more detailed behavioral attributes within SND models aims at providing a more realistic representation of shippers' reaction to the proposed services, ultimately helping intermodal operators to make more informed design and pricing decisions.

The following section provides a recap of the main characteristics of the previously reviewed bilevel SNDP models and sums up the contributions of our work.

2.3. Contributions

The existing bilevel models for SNDP presented in Section 2.1 are sorted in Table 1. In particular, it shows whether some constraints regarding the fleet are included (e.g., size limit). It also indicates how the transport demand is modeled: most works assume that shippers are deterministic cost minimizers. Still regarding demand, it can be noticed that no existing model considers unobserved attributes that play a role in the choices of shippers. In addition, only three studies embed shippers' heterogeneity through distinct values of time (or reliability). Finally, only one work considers that frequencies also influence the demand alongside the prices endogenously in the optimization model.

The proposed CD-SNDP is a generalization of the model by Tawfik and Limbourg (2019). Firstly, it generalizes the network structure as cycles and services with multiple stops are now allowed. Secondly, the shippers' objective is also generalized as they do not only proceed to a minimization of their costs but instead maximize their utilities. These utilities contain other attributes besides the costs, such as frequency, accessibility, etc. Thirdly, our formulation generalizes the representation of shippers as it can accommodate some unobserved attributes (via randomly distributed error terms) and shippers' heterogeneity (through the Mixed logit formulation). Because of these features, the proposed CD-SNDP becomes a stochastic problem, as opposed to the previous works that all assumed a deterministic setting. Finally, the service frequency is made endogenous to the optimization model along with the price. A summary of the aforementioned features can be found on the last row of Table 1.

The contributions of this paper are summarized as follows:

1. Inclusion of shippers' heterogeneity and unobserved attributes within an SNDP model, which leads to a stochastic optimization model;
2. Consideration of realistic features (service frequency as a cross-level variable alongside the price, fleet constraints, cycle-based formulation), which increases the problem's complexity;
3. Application to a real logistics network, whose data have also been used to estimate and validate the choice model integrated into the SNDP.

3. Methodology

The service network design and pricing problem is represented by a bilevel formulation. The upper level represents the decisions of the transport operator aiming at maximizing their profit and the lower level corresponds to the utility maximization of the shippers. The latter brings additional complexity to the problem as the two decision variables of the upper level (price and frequency) are now included in the lower level, unlike the JDP of Tawfik and Limbourg (2019) where only the price is included but not the frequency.

Concerning the upper level, it differs from the JDP in two aspects. Firstly, the arc-based formulation of services is replaced by a cycle-based formulation. The latter is deemed more accurate to represent realistic decision-making. Indeed, most intermodal transport services go back and forth on an itinerary with a defined schedule. The cycle-based representation also enables a more elaborate representation of services as multiple intermediary stops can be added in both directions. In addition, it simplifies the asset management of the operators. In an arc-based formulation, they may need to re-balance the vehicles at the end of the planning horizon; whereas a cycle-based representation ensures that each vehicle ends up at its starting point. It is noteworthy that the arc-based pricing representation is not changed compared to the JDP. Indeed, shippers will pay only for the transport of their cargo from its origin to its destination, and not for the whole cycle. The second difference is the addition of fleet size and cycle time feasibility constraints. The former restricts the actions of the transport operator as they do not have an infinite number of vehicles at their disposal to satisfy the demand. The latter determines, for each service, the number of cycles that can be performed by one vehicle during the planning horizon given the cycle's duration. Moreover, a heterogeneous fleet is considered.

In the remainder of this paper, the JDP with fleet constraints will be used as *Benchmark*. The benchmark with cycle-based formulation (instead of path-based) will be further referred to as *SNDP*. Finally, the proposed choice-driven model, which considers utility maximization of shippers, is denoted *CD-SNDP*. The notations for the CD-SNDP are described in the following paragraphs.

3.1. Problem formulation

We consider a transport operator as the decision-maker: they have a potential demand made of multiple shippers and face the competition of several other carriers/transport modes. The operator and their competitors operate on a multimodal network. The competition's services are assumed known by the operator, who has to decide which terminals to serve and at which frequency. In addition, the operator has to set a single price for each Origin–Destination (OD) pair that they will charge the shippers. The transport network is represented as a directed graph $\mathcal{G} = (\mathcal{N}, \mathcal{A})$, where $\mathcal{N}$ is the set of terminals and $\mathcal{A} = \{(i,j) : i,j \in \mathcal{N}, i \neq j\}$ the set of arcs (OD pairs) between these terminals.

3.1.1. Upper level

The operator's fleet is heterogeneous, therefore the different vehicle types are denoted by set $\mathcal{K}$. The number of available vehicles per type is V_k and their capacity is Q_k.

The set $\mathcal{S}$ includes all transport services that can be run by the operator. Unlike the benchmark, where each service corresponds to a single arc of $\mathcal{A}$, a service is composed of a sequence of arcs. Each arc in this sequence is called a leg and the whole sequence of legs for a given service s is denoted $\mathcal{L}_s$. The cycle-based formulation of the problem implies that the sequence starts and ends at the same node. This set of services $\mathcal{S}$ has to be generated prior to the optimization. For small networks, it can contain all possible services; but for large networks, decision rules have to be implemented to restrict the size of this set (e.g.: enforce minimal/maximal number of stops or travel time per service).

The maximum number of cycles of service s that can be performed by vehicle type k is named W_{sk}: it typically consists of the maximum operating time divided by the cycle time (sum of travel time and time at terminals). Each service s has a fixed cost c_{sk}^{FIX}

Table 2
Notation.

Sets:	
$\mathcal{N}$	Set of terminals (indices: i, j)
$\mathcal{A}$	Set of arcs (i, j)
$\mathcal{K}$	Set of vehicle types (index: k)
$\mathcal{S}$	Set of potential services (index: s)
$\mathcal{L}_s$	Set of legs of service $s \in \mathcal{S}$ (index: l_s)
$\mathcal{H}$	Set of competing alternatives (index: h)
Parameters:	
V_k	Number of vehicles of type k in the operator's fleet
Q_k	Capacity of vehicle type k [TEUs]
W_{sk}	Maximum number of cycles of service s that can be performed by vehicle type k
c_{sk}^{FIX}	Fixed cost of operating service s with vehicle type k [€]
c_{ijsk}^{VAR}	Variable cost of transport between i and j with service s and vehicle type k [€/TEU]
δ_{ijl_s}	Dummy parameter equal to 1 if container traveling from i to j uses service leg l_s, 0 otherwise
D_{ij}	Aggregated transport demand of shippers between i and j [TEUs]
U_{ij}^{O}	Utility of using the operator's services between i and j
U_{ij}^{h}	Utility of using competing alternative h between i and j
Variables:	
v_{sk}	Number of vehicles of type k assigned to service s by the operator
f_{sk}	Frequency of service s operated with vehicle type k
p_{ij}	Price charged by the operator to shippers wanting to transport goods from i to j [€/TEU]
x_{ijsk}	Cargo volume using service s operated with vehicle type k between i and j [TEUs]
z_{ij}^{h}	Cargo volume using competing alternative h between i and j [TEUs]

of operating it with vehicle type k and a variable cost c_{ijsk}^{VAR} per container transported between terminals i and j. Moreover, we introduce the parameter δ_{ijl_s}, which equals one if a container traveling from i to j uses the service leg l_s and zero otherwise.

The transport operator has three decision variables in the upper level problem:

- v_{sk} is the number of vehicles of type k that the operator allocates to each service s;
- f_{sk} is the frequency of each service s per vehicle type k;
- p_{ij} is the price per container charged to shippers requesting to transport goods from i to j.

3.1.2. Lower level

The shippers are represented as a whole, i.e. their demand is aggregated. The container transport demand between terminals i and j is denoted D_{ij}. Shippers decide to assign demand to the transport operator or their competitors by the maximization of their utility. The utility function of using the services proposed by the transport operator between i and j is denoted U_{ij}^{O} and is dependent on p_{ij} and f_{sk}, whereas the utility of using a competing alternative h is given as U_{ij}^{h}. Finally, the decision variables of the lower level consist of the number of containers that are assigned to the operator's services (x_{ijsk}) and to every competing alternative (z_{ij}^{h}). When some demand from i to j is assigned to the transport operator, it is assumed that the operator will determine the services to which the containers will be assigned. Of course, the chosen services have to pass through both i and j and to have enough remaining capacity.

All the aforementioned sets, parameters, and decision variables are listed in Table 2.

3.1.3. Mathematical model

The proposed SNDP is expressed as a Bilevel Problem (BLP):

$$(\textbf{BLP}) \max_{v,f,p,x,z} \quad \sum_{(i,j)\in\mathcal{A}} \sum_{s\in\mathcal{S}} \sum_{k\in\mathcal{K}} p_{ij} x_{ijsk} - \sum_{s\in\mathcal{S}} \sum_{k\in\mathcal{K}} c_{sk}^{\text{FIX}} f_{sk} - \sum_{(i,j)\in\mathcal{A}} \sum_{s\in\mathcal{S}} \sum_{k\in\mathcal{K}} c_{ijsk}^{\text{VAR}} x_{ijsk} \tag{1}$$

$$\text{s.t.} \quad \sum_{s\in\mathcal{S}} v_{sk} \le V_k \qquad \forall k \in \mathcal{K} \tag{2}$$

$$f_{sk} \le W_{sk} v_{sk} \qquad \forall s \in \mathcal{S}, \ \forall k \in \mathcal{K} \tag{3}$$

$$\sum_{(i,j)\in\mathcal{A}} \delta_{ijl_s} x_{ijsk} \le Q_k f_{sk} \qquad \forall l_s \in \mathcal{L}_s, \ \forall s \in \mathcal{S}, \ \forall k \in \mathcal{K} \tag{4}$$

$$x_{ijsk} \le \sum_{l_s\in\mathcal{L}_s} \delta_{ijl_s} D_{ij} \qquad \forall (i,j) \in \mathcal{A}, \ \forall s \in \mathcal{S}, \ \forall k \in \mathcal{K} \tag{5}$$

$$p_{ij} \ge 0 \qquad \forall (i,j) \in \mathcal{A} \tag{6}$$

$$v_{sk}, f_{sk} \in \mathbb{N} \qquad \forall s \in \mathcal{S},\ \forall k \in \mathcal{K} \tag{7}$$

where x and z solve:

$$\max_{x,z} \quad \sum_{(i,j)\in\mathcal{A}} \left(\sum_{s\in\mathcal{S}} \sum_{k\in\mathcal{K}} U^O_{ij} x_{ijsk} + \sum_{h\in\mathcal{H}} U^h_{ij} z^h_{ij} \right) \tag{8}$$

$$\text{s.t.} \quad \sum_{s\in\mathcal{S}} \sum_{k\in\mathcal{K}} x_{ijsk} + \sum_{h\in\mathcal{H}} z^h_{ij} \ = \ D_{ij} \qquad \forall (i,j) \in \mathcal{A} \tag{9}$$

$$x_{ijsk} \geq 0 \qquad \forall (i,j) \in \mathcal{A},\ \forall s \in \mathcal{S},\ \forall k \in \mathcal{K} \tag{10}$$

$$z^h_{ij} \geq 0 \qquad \forall (i,j) \in \mathcal{A},\ \forall h \in \mathcal{H} \tag{11}$$

At the upper level, the objective function of the transport operator (1) is to maximize their profit. It is computed as the revenues from the transported containers minus the fixed and variable costs of the offered services. Constraint (2) is the fleet size constraint for each vehicle type. Constraint (3) ensures that the service's frequency is inferior to the maximum number of cycles that can be performed by the assigned vehicles. Constraint (4) assures that the total number of containers transported on each leg of every service does not exceed the available capacity of the service, whereas constraint (5) ensures that no container can be assigned to a service that does not go through the origin or destination terminal of the container. The domains of the operator's decision variables are defined by constraints (6)–(7).

Regarding the lower level, shippers seek to maximize their utility (8) by assigning their containers either to the operator's services or to the competition. Moreover, constraint (9) enforces the total transport demand to be met. Finally, constraints (10)–(11) define the domain of the decision variables of the shippers.

3.2. Model transformation

The proposed bilevel problem can be reformulated as a single level problem and then linearized, for more details on these procedures the reader is referred to Tawfik and Limbourg (2019). For the reformulation, additional variables $\lambda_{ij},\ \forall (i,j) \in \mathcal{A}$ are introduced: they represent the dual variables related to constraints (9). The model can then be transformed using the Karush–Kuhn–Tucker conditions. After this process, the following constraints appear:

$$\lambda_{ij} \ \leq \ -U^O_{ij} \qquad \forall (i,j) \in \mathcal{A},\ \forall h \in \mathcal{H} \tag{12}$$

$$\lambda_{ij} \ \leq \ -U^h_{ij} \qquad \forall (i,j) \in \mathcal{A},\ \forall h \in \mathcal{H} \tag{13}$$

$$(-U^O_{ij} - \lambda_{ij}) \sum_{s\in\mathcal{S}} \sum_{k\in\mathcal{K}} x_{ijsk} \ = \ 0 \qquad \forall (i,j) \in \mathcal{A} \tag{14}$$

$$(-U^h_{ij} - \lambda_{ij}) z^h_{ij} \ = \ 0 \qquad \forall (i,j) \in \mathcal{A} \tag{15}$$

Note that constraints (14) and (15) are non-linear. To address these, the big M technique is used and binary variables are introduced: y^{I}_{ij} and y^{II}_{ij} for constraint (14); $y^{\mathrm{I}h}_{ij}$ and $y^{\mathrm{II}h}_{ij}$ for constraint (15).

The only remaining non-linear expression is the first term of the operator's objective function (1). To remedy it, the strong duality theorem can be applied to the lower level problem (8)–(11), as in the work of Tawfik and Limbourg (2019). At optimality, we have:

$$-\sum_{(i,j)\in\mathcal{A}} \left(\sum_{s\in\mathcal{S}} \sum_{k\in\mathcal{K}} U^O_{ij} x_{ijsk} + \sum_{h\in\mathcal{H}} U^h_{ij} z^h_{ij} \right) \ = \ \sum_{(i,j)\in\mathcal{A}} D_{ij} \lambda_{ij} \tag{16}$$

In addition, the following form is considered for the utility function of the transport operator:

$$U^O_{ij} \ = \ \bar{U}^O_{ij} + \beta_c p_{ij} + \beta_f f_{ij} \ = \ \bar{U}^O_{ij} + \beta_c p_{ij} + \beta_f \sum_{s\in\mathcal{S}} \sum_{k\in\mathcal{K}} \phi_{ijs} f_{sk} \tag{17}$$

where $\bar{U}^O_{ij}$ is the part of utility depending on attributes exogenous to the model (e.g., accessibility, presence of a seaport, value of time), β_c and β_f are the coefficients respectively weighting the importance of price and frequency in the utility function, and ϕ_{ijs} is a dummy equal to one if both terminals i and j are contained in service s and zero otherwise. Then, using Eqs. (16) and (17), the first term in (1) can be expressed as:

$$\sum_{s\in\mathcal{S}} \sum_{k\in\mathcal{K}} p_{ij} x_{ijsk} \ = \ -\frac{1}{\beta_c} \left(D_{ij}\lambda_{ij} + \sum_{h\in\mathcal{H}} U^h_{ij} z^h_{ij} + \sum_{s\in\mathcal{S}} \sum_{k\in\mathcal{K}} x_{ijsk} \left(\bar{U}^O_{ij} + \beta_f \sum_{s\in\mathcal{S}} \sum_{k\in\mathcal{K}} \phi_{ijs} f_{sk} \right) \right) \tag{18}$$

Because we now have x_{ijsk} multiplying the sum of f_{sk}, we still did not completely eliminate non-linearity. This new term is nevertheless more convenient as the order of magnitude of the frequency is more limited than that of the price. Let us first define the frequency per OD pair: $f_{ij} = \sum_{s\in\mathcal{S}} \sum_{k\in\mathcal{K}} \phi_{ijs} f_{sk}$. This term can then be represented in base 2 conveniently: $f_{ij} = \sum_{b=0}^{B_{ij}-1} 2^b f_{ijb}$, where f_{ijb} are binary variables and B_{ij} an upper bound of $\log_2 f_{ij}$. The product term in (18) can ultimately be linearized using the well-known technique for the product of binary and continuous variables. The variable representing the product is referred to as a_{ijskb}.

The final MILP is then formulated as follows:

$$
\begin{aligned}
(\textbf{MILP}) \max_{v,f,p,x,z} \quad & \sum_{(i,j)\in\mathcal{A}} -\frac{1}{\beta_c}\left(D_{ij}\lambda_{ij} + \sum_{h\in\mathcal{H}} U_{ij}^h z_{ij}^h + \sum_{s\in\mathcal{S}}\sum_{k\in\mathcal{K}} \bar{U}_{ij}^O x_{ijsk} + \sum_{s\in\mathcal{S}}\sum_{k\in\mathcal{K}} \beta_f \sum_{b=0}^{B_{ij}-1} 2^b a_{ijskb}\right) \\
& - \sum_{s\in\mathcal{S}}\sum_{k\in\mathcal{K}} c_{sk}^{\text{FIX}} f_{sk} - \sum_{(i,j)\in\mathcal{A}}\sum_{s\in\mathcal{S}}\sum_{k\in\mathcal{K}} c_{ijsk}^{\text{VAR}} x_{ijsk}
\end{aligned} \tag{19}
$$

s.t. constraints (2)–(7) & (9)–(11)

$$a_{ijskb} \le D_{ij} f_{ijb} \qquad \forall (i,j)\in\mathcal{A},\ \forall s\in\mathcal{S},\ \forall k\in\mathcal{K},\ \forall b\in\mathcal{B} \tag{20}$$

$$a_{ijskb} \le x_{ijsk} \qquad \forall (i,j)\in\mathcal{A},\ \forall s\in\mathcal{S},\ \forall k\in\mathcal{K},\ \forall b\in\mathcal{B} \tag{21}$$

$$a_{ijskb} \ge x_{ijsk} - D_{ij}(1 - f_{ijb}) \qquad \forall (i,j)\in\mathcal{A},\ \forall s\in\mathcal{S},\ \forall k\in\mathcal{K},\ \forall b\in\mathcal{B} \tag{22}$$

$$\sum_{s\in\mathcal{S}}\sum_{k\in\mathcal{K}} \phi_{ijs} f_{sk} = \sum_{b=0}^{B_{ij}-1} 2^b f_{ijb} \qquad \forall (i,j)\in\mathcal{A} \tag{23}$$

$$\lambda_{ij} \le -(\bar{U}_{ij}^O + \beta_c p_{ij} + \beta_f \sum_{b=0}^{B_{ij}-1} 2^b f_{ijb}) \qquad \forall (i,j)\in\mathcal{A} \tag{24}$$

$$-(\bar{U}_{ij}^O + \beta_c p_{ij} + \beta_f \sum_{b=0}^{B_{ij}-1} 2^b f_{ijb}) - \lambda_{ij} \le M_{ij}^{\text{I}} y_{ij}^{\text{I}} \qquad \forall (i,j)\in\mathcal{A} \tag{25}$$

$$\sum_{s\in\mathcal{S}}\sum_{k\in\mathcal{K}} x_{ijsk} \le M_{ij}^{\text{II}} y_{ij}^{\text{II}} \qquad \forall (i,j)\in\mathcal{A} \tag{26}$$

$$y_{ij}^{\text{I}} + y_{ij}^{\text{II}} \le 1 \qquad \forall (i,j)\in\mathcal{A} \tag{27}$$

$$\lambda_{ij} \le -U_{ij}^h \qquad \forall (i,j)\in\mathcal{A},\ \forall h\in\mathcal{H} \tag{28}$$

$$-U_{ij}^h - \lambda_{ij} \le M_{ij}^{\text{I}h} y_{ij}^{\text{I}h} \qquad \forall (i,j)\in\mathcal{A},\ \forall h\in\mathcal{H} \tag{29}$$

$$z_{ij}^h \le M_{ij}^{\text{II}h} y_{ij}^{\text{II}h} \qquad \forall (i,j)\in\mathcal{A},\ \forall h\in\mathcal{H} \tag{30}$$

$$y_{ij}^{\text{I}h} + y_{ij}^{\text{II}h} \le 1 \qquad \forall (i,j)\in\mathcal{A},\ \forall h\in\mathcal{H} \tag{31}$$

$$f_{ijb} \in \{0,1\} \qquad \forall (i,j)\in\mathcal{A},\ \forall b\in\mathcal{B} \tag{32}$$

$$a_{ijskb} \in \mathbb{N} \qquad \forall (i,j)\in\mathcal{A},\ \forall s\in\mathcal{S},\ \forall k\in\mathcal{K},\ \forall b\in\mathcal{B} \tag{33}$$

$$y_{ij}^{\text{I}}, y_{ij}^{\text{II}}, y_{ij}^{\text{I}h}, y_{ij}^{\text{II}h} \in \{0,1\} \qquad \forall (i,j)\in\mathcal{A} \tag{34}$$

$$\lambda_{ij} \in \mathbb{R} \qquad \forall (i,j)\in\mathcal{A} \tag{35}$$

3.3. Stochastic formulation

In Eq. (17), we set the generic form of the utility function U_{ij}^O. In particular, it was assumed to be fully deterministic, but this is not the case in reality. Firstly, the utility traditionally contains a random term (also called "error term"), representing the unobserved attributes playing a role in the mode choice. Secondly, one or several β coefficients can be assumed as randomly distributed, to account for heterogeneous preferences. Note that these remarks also hold for U_{ij}^h. The probability distributions of the random terms need to be assumed or estimated a priori: in our setting, we consider that the distributions are given.

With these considerations, the CD-SNDP model becomes a stochastic optimization problem. We then come up with a SAA formulation to solve it. In this formulation, shippers are not represented as a whole anymore; on the contrary, the population is represented as a sample $\mathcal{R}$ composed of R individual shippers. The total demand per OD pair (i,j) is equally divided among the shippers, so that term D_{ij} is replaced by D_{ij}/R in the MILP formulation above. For every shipper r, the utility function of the transport operator becomes:

$$U_{ijr}^O = \bar{U}_{ijr}^O + \beta_c^r p_{ij} + \beta_f^r f_{ij} + \epsilon_r^O \tag{36}$$

where ϵ_r^O is the error term representing unobserved attributes influencing the choice of shipper r toward the transport operator. Similarly, the utility of competing modes also becomes shipper-dependent: $U_{ijr}^h(\beta^{hr}, \epsilon_r^h)$. For each shipper's random β and ϵ, a draw is performed in the parameter's distributions and the corresponding utility functions are computed. Since utility functions now differ per shipper r, this impacts the mode choice such that the decision variables x_{ijskr} and z^h_{ijr} become dependent on the sampling. This is also true for the dual variables λ_{ijr}. It means that all constraints dependent on these variables need to hold for each shipper r. Nevertheless, the decision variables of the transport operator (p_{ij}, v_{sk}, f_{sk}, f_{ijb}) are fixed once and for all, independently of the sampling. Finally, the objective function is modified to maximize the sum of the profits over all shippers r in the sample $\mathcal{R}$.

3.4. Predetermination heuristic

To speed up the solution time of the stochastic formulations, we propose a "predetermination heuristic". As its name suggests, it consists of determining the operator's utility based on given price and frequency values before the optimization. To compute the operator's utility, discrete sets of predefined prices $\mathcal{P}$ and frequencies $\mathcal{F}$ are considered. It is also assumed that the sampling of the shippers' population is already performed so that the utilities of competing alternatives U^h_{ijr} can be computed. Along with the predefined prices p and frequencies ψ, it allows to pre-compute the demand faced by the operator $d^{\psi p}_{ij}$ for each OD pair.

To compute the resulting profit on an OD pair ij, the fixed and variable costs per vehicle type k are needed. However, the available cost parameters are expressed per service s and not directly per OD pair. Therefore, only the direct service between terminals i and j is considered. Since the variable cost is also dependent on i and j, we simply select the variable cost of the direct service for each vehicle type $\hat{c}^{\text{VAR}}_{ijk}$. For the fixed cost $\hat{c}^{\text{FIX}}_{ijk}$, we select the fixed cost of the direct service for each vehicle type and divide it by two (to get the cost for only one service leg). For a given frequency ψ and OD pair ij, we further consider the set Ξ^{ψ}_{ij} of all the possible combinations of frequencies per vehicle type ψ_{ijk}, such that $\sum_{k\in\mathcal{K}}\psi_{ijk} = \psi$ and that the fleet size and cycle time feasibility constraints are respected. Algorithm 1 shows the steps to compute the resulting profit for a given combination of ψ_{ijk} and a specific price p knowing the demand $d^{\psi p}_{ij}$.

Algorithm 1: Profit computation per OD pair ij

Rank the vehicle types in increasing order of $\hat{c}^{\text{VAR}}_{ijk}$ to form the set $\mathcal{K}'$;
for $k' \in \mathcal{K}'$ **do**
    Define the capacity $\Theta_{ijk'} = \psi_{ijk'}Q_{k'}$;
    Define the payload per vehicle type $q_{ijk'} = \min(d^{\psi p}_{ij}, \Theta_{ijk'})$;
    Update the demand left to assign $d^{\psi p}_{ij} = d^{\psi p}_{ij} - q_{ijk'}$
end
Return the profit: $\sum_{k'\in\mathcal{K}'}(pq_{ijk'} - \psi_{ijk'}\hat{c}^{\text{FIX}}_{ijk'} - q_{ijk'}\hat{c}^{\text{VAR}}_{ijk'})$

For each combination $\xi \in \Xi^{\psi}_{ij}$, it is then possible to determine the price $P^{\xi}_{ij\psi}$ generating the most profit. The steps to obtain this value for every OD pair ij and predefined frequency ψ are given in Algorithm 2.

Algorithm 2: Price determination method

for $(i,j) \in \mathcal{A}$, $r \in \mathcal{R}$ **do**
    Determine $U'_{ijr} = \max_{h\in\mathcal{H}} U^h_{ijr}$, and $h'_{ijr} = \text{argmax}_{h\in\mathcal{H}} U^h_{ijr}$;
end
for $(i,j) \in \mathcal{A}$, $\psi \in \mathcal{F}$ **do**
    for $p \in \mathcal{P}$ **do**
        $d^{\psi p}_{ij} = 0$;
        for $r \in \mathcal{R}$ **do**
            Compute $U^O_{ijr}(\psi, p)$ according to (36);
            if $U^O_{ijr}(\psi, p) \geq U'_{ijr}$ **then**
                $d^{\psi p}_{ij} = d^{\psi p}_{ij} + \frac{D_{ij}}{|\mathcal{R}|}$;
            end
        end
        for $\xi \in \Xi^{\psi}_{ij}$ **do**
            Compute the associated profit $\pi^{\xi p}_{ij\psi}$ using Algorithm 1;
        end
    end
    for $\xi \in \Xi^{\psi}_{ij}$ **do**
        $P^{\xi}_{ij\psi} = \text{argmax}_{p\in\mathcal{P}} \pi^{\xi p}_{ij\psi}$;
    end
end

Once Algorithm 2 has been used to compute demand values $d_{ij}^{\psi p}$ and price values $P_{ij\psi}^{\xi}$, they can then be used as parameters to solve an auxiliary optimization problem (AP). This problem consists in determining, for a given sample $\mathcal{R}$, the optimal frequencies for fixed prices $\tilde{p}_{ij}$:

$$(\mathbf{AP})\ \max_{v,f,g,x,z} \quad \frac{1}{|\mathcal{R}|}\sum_{r\in\mathcal{R}}\left(\sum_{(i,j)\in\mathcal{A}}\sum_{s\in\mathcal{S}}\sum_{k\in\mathcal{K}}\tilde{p}_{ij}x_{ijskr} - \sum_{s\in\mathcal{S}}\sum_{k\in\mathcal{K}}c_{sk}^{\text{FIX}}f_{sk} - \sum_{(i,j)\in\mathcal{A}}\sum_{s\in\mathcal{S}}\sum_{k\in\mathcal{K}}c_{ijsk}^{\text{VAR}}x_{ijskr}\right) \tag{37}$$

$$\text{s.t.} \quad \sum_{s\in\mathcal{S}} v_{sk} \leq V_k \qquad \forall k\in\mathcal{K} \tag{38}$$

$$f_{sk} \leq W_{sk}v_{sk} \qquad \forall s\in\mathcal{S},\ \forall k\in\mathcal{K} \tag{39}$$

$$\sum_{r\in\mathcal{R}}\sum_{(i,j)\in\mathcal{A}}\delta_{ijl_s}\frac{x_{ijskr}}{|\mathcal{R}|} \leq Q_k f_{sk} \qquad \forall l_s\in\mathcal{L}_s,\ \forall s\in\mathcal{S},\ \forall k\in\mathcal{K} \tag{40}$$

$$x_{ijskr} \leq \sum_{l_s\in\mathcal{L}_s}\delta_{ijl_s}D_{ij} \qquad \forall (i,j)\in\mathcal{A},\ \forall s\in\mathcal{S},\ \forall k\in\mathcal{K},\ \forall r\in\mathcal{R} \tag{41}$$

$$\sum_{s\in\mathcal{S}}\sum_{k\in\mathcal{K}}x_{ijskr} + z_{ijr} = D_{ij} \qquad \forall (i,j)\in\mathcal{A},\ \forall r\in\mathcal{R} \tag{42}$$

$$\sum_{\psi\in\mathcal{F}} g_{ij\psi} \leq 1 \qquad \forall (i,j)\in\mathcal{A} \tag{43}$$

$$\sum_{s\in\mathcal{S}}\sum_{k\in\mathcal{K}}\phi_{ijs}f_{sk} = \sum_{\psi\in\mathcal{F}}\psi g_{ij\psi} \qquad \forall (i,j)\in\mathcal{A} \tag{44}$$

$$\sum_{r\in\mathcal{R}}\sum_{s\in\mathcal{S}}\sum_{k\in\mathcal{K}}\frac{x_{ijskr}}{|\mathcal{R}|} \leq \sum_{\psi\in\mathcal{F}} g_{ij\psi}d_{ij}^{\psi\tilde{p}} \qquad \forall (i,j)\in\mathcal{A} \tag{45}$$

$$v_{sk}\in\mathbb{N} \qquad \forall s\in\mathcal{S},\ \forall k\in\mathcal{K} \tag{46}$$

$$f_{sk}\in\mathbb{N} \qquad \forall s\in\mathcal{S},\ \forall k\in\mathcal{K} \tag{47}$$

$$g_{ij\psi}\in\{0,1\} \qquad \forall (i,j)\in\mathcal{A},\ \forall s\in\mathcal{S},\ \forall \psi\in\mathcal{F} \tag{48}$$

$$x_{ijskr}\geq 0 \qquad \forall (i,j)\in\mathcal{A},\ \forall s\in\mathcal{S},\ \forall k\in\mathcal{K},\ \forall r\in\mathcal{R} \tag{49}$$

$$z_{ijr}\geq 0 \qquad \forall (i,j)\in\mathcal{A},\ \forall r\in\mathcal{R} \tag{50}$$

This auxiliary problem contains additional elements that deserve some discussion. First, the objective (37) is now formulated as a SAA function and the decision variables x and z are now dependent on r. Constraints (40) to (42) are modified accordingly. A new binary variable $g_{ij\psi}$ is introduced: it is equal to one if the predefined frequency ψ is chosen for OD pair (i,j), and zero otherwise. Constraint (43) ensures that at most one frequency ψ is chosen per OD pair. The value of ψ is then linked to the decision variable of services frequency f through constraint (44). Finally, constraint (45) aggregates the decision variables x_{ijskr} of cargo assigned to the operator over the whole sample and bounds it with the precomputed demand $d_{ij}^{\psi p}$ defined in Algorithm 2. This last constraint allows to keep the utility functions out of the optimization problem. As a result, the variable z_{ijr} is now independent of the competing modes h. Once the optimization is performed, the corresponding value of z_{ijr}^{*} can be assigned to the competing mode h'_{ijr} with the maximum utility as computed in Algorithm 2.

Getting rid of the utilities and the pricing decision in the optimization allows to considerably decrease the solving time. Indeed, the variables p_{ij}, f_{skb}, a_{ijskb}, λ_{ij} and y_{ij}'s are not used anymore, and only the variables $g_{ij\psi}$ are added. The number of constraints is also drastically reduced. The idea of the heuristic is to exploit this advantage to solve the auxiliary problem iteratively, as described in Algorithm 3.

Fig. 3. Network of the case study: the Rhine part of the RA corridor (Rivermap, 2024) together with the considered terminals.

Algorithm 3: Predetermination heuristic

Use Algorithm 2 to pre-compute $d_{ij}^{\psi p}$ and $P_{ij\psi}^{\xi}$;
Define the set of visited solutions $\Omega = \emptyset$;
Set each $\tilde{p}_{ij}$ with arbitrary prices contained in $\mathcal{P}$;
Set each $\tilde{f}_{sk}$ to zero;
while $(\tilde{p}_{ij}, \tilde{f}_{sk}) \notin \Omega$ **do**
  Solve (AP) to get f_{sk}^* and $g_{ij\psi}^*$, i.e. the chosen frequency ψ corresponding to predefined prices $\tilde{p}_{ij}$;
  Update each $\tilde{f}_{sk}$ with f_{sk}^*;
  for $(i,j) \in \mathcal{A}$ **do**
    for $k \in \mathcal{K}$ **do**
      Compute $\psi_{ijk} = \sum_{s \in S} \phi_{ijs} f_{sk}^*$
    end
    Find the combination $\xi \in \Xi_{ij}^{\psi}$ corresponding to the values ψ_{ijk};
    Update $\tilde{p}_{ij}$ with the value $P_{ij\psi}^{\xi}$;
  end
  Add $(\tilde{p}_{ij}, \tilde{f}_{sk})$ to the set Ω;
end
Return the solution $(\tilde{p}_{ij}, \tilde{f}_{sk})$;

The performance of the heuristic is highly dependent on the size of sets $\mathcal{P}$ and $\mathcal{F}$. The more values they contain, the better the approximation at the cost of additional computational resources. These sets should then ensure good coverage of the search space in order for the heuristic to return satisfying solutions.

4. Case study

The proposed CD-SNDP is applied to container transport on a small stretch of the European Rhine-Alpine (RA) corridor consisting of 3 nodes: Rotterdam (RTM), Duisburg (DUI), and Bonn (BON). The network is further extended to 9 nodes, as depicted in Fig. 3. We consider an inland vessel operator competing with two other modes (Road and Rail) and another IWT carrier.

4.1. Overview

The operator's fleet is composed of 2 vessel types: 24 vessels of type M8 and 12 vessels of type M11 with a maximal capacity of 180 TEUs and 300 TEUs, respectively. Each vessel type has a maximal operation time, T^{max}, of 120 h per week. The transport demand inputs are based on the NOVIMOVE project (Majoor et al., 2021), whereas the costs for IWT and the two competing modes as well as the sailing time t^{sail} and the time spent in ports t^{port} are estimated using the model of Shobayo et al. (2021). Based on these inputs, the computation of the maximum number of cycles is straightforward: $W_{sk} = T^{max}/(t_{sk}^{sail} + t_{sk}^{port})$.

Regarding demand modeling, the utility functions for each shipper r are formulated as follows:

$$U^{O}_{ijr} = \alpha^{\mathrm{IWT}} + \beta^{\mathrm{Inter}}_{a} a^{\mathrm{IWT}}_{ij} + \beta^{\mathrm{IWT}}_{q} q_{ij} + \beta^{\mathrm{Inter},r}_{c}(\boldsymbol{p_{ij}} + \mathrm{VoT} t^{\mathrm{IWT}}_{ij}) + \beta^{\mathrm{Inter}}_{f} \boldsymbol{f_{ij}} + \epsilon^{O}_{ijr} \tag{51}$$

$$U^{h=\mathrm{IWT}}_{ijr} = \alpha^{\mathrm{IWT}} + \beta^{\mathrm{Inter}}_{a} a^{\mathrm{IWT}}_{ij} + \beta^{\mathrm{IWT}}_{q} q_{ij} + \beta^{\mathrm{Inter},r}_{c}(p^{\mathrm{IWT}}_{ij} + \mathrm{VoT} t^{\mathrm{IWT}}_{ij}) + \beta^{\mathrm{Inter}}_{f} f^{\mathrm{IWT}}_{ij} + \epsilon^{\mathrm{IWT}}_{ijr} \tag{52}$$

$$U^{h=\mathrm{Rail}}_{ijr} = \alpha^{\mathrm{Rail}} + \beta^{\mathrm{Inter}}_{a} a^{\mathrm{Rail}}_{ij} + \beta^{\mathrm{Inter},r}_{c}(p^{\mathrm{Rail}}_{ij} + \mathrm{VoT} t^{\mathrm{Rail}}_{ij}) + \beta^{\mathrm{Inter}}_{f} f^{\mathrm{Rail}}_{ij} + \epsilon^{\mathrm{Rail}}_{ijr} \tag{53}$$

$$U^{h=\mathrm{Road}}_{ijr} = \alpha^{\mathrm{Road}} + \beta^{\mathrm{Road}}_{a} a^{\mathrm{Road}}_{ij} + \beta^{\mathrm{Road}}_{c}(p^{\mathrm{Road}}_{ij} + \mathrm{VoT} t^{\mathrm{Road}}_{ij}) + \epsilon^{\mathrm{Road}}_{ijr} \tag{54}$$

where, for each mode, α is the alternative specific constant, a is an accessibility metric, and q_{ij} is a dummy equal to one if a seaport is located at i or j (these three attributes were grouped under the term $\bar{U}$ in Eq. (17)). Moreover, p is the cost for shippers in thousands of euros per TEU, f is the weekly frequency for intermodal transports (i.e. IWT and Rail), t is the total travel time in hours, and VoT is the Value of Time expressed in 1000€/TEU/hour. Each attribute is weighted by a coefficient β and each mode has a random error term ϵ. Although they have similarities, it is assumed that the vessel operator and the IWT carrier alternatives are not correlated. The same assumption holds between all alternatives. Therefore, in the remainder of this work, all the error terms ϵ_{ijr} are considered independent and identically distributed (iid).

Within the CD-SNDP context, all the terms contained in the utilities of the competing modes (IWT, Rail, and Road) are exogenous to the optimization model and are thus treated as parameters. Regarding the utility of the operator, only the terms in bold (p_{ij} and f_{ij}) are endogenous while the other terms are also parameters. p_{ij} is the decision variable on pricing and f_{ij} corresponds to the term $\sum_{s\in\mathcal{S}}\sum_{k\in\mathcal{K}} \phi_{ijs} f_{sk}$, as introduced in Eq. (17).

The model's coefficients were estimated with aggregate data using a Weighted Logit methodology. It is named "weighted" because, during the estimation, the log-likelihood function is weighted by the yearly cargo flows on each OD pair (Rich et al., 2009). It thus gives more importance to the OD pairs with high volumes. For more details, the reader is referred to Nicolet et al. (2022). One noteworthy characteristic of the data on which the coefficients were estimated is that the frequency for IWT does not exceed 35 services per week. Therefore, the following constraint is added to our CD-SNDP problem to guarantee consistency between the results and the mode choice model:

$$f_{sk} \leq 35 \qquad \forall s \in \mathcal{S},\ \forall k \in \mathcal{K} \tag{55}$$

We use this case study to compare the results of 3 deterministic and 2 stochastic models. The former consist of the benchmark, the SNDP, and the CD-SNDP, which uses only the deterministic part of the utility functions in Eqs. (51) to (54), i.e. without error terms ϵ_{ijr} and with the same cost coefficients $\beta^{\mathrm{Inter},r}_{c}$ for all shippers. The latter two are stochastic variations of the CD-SNDP:

- Multinomial Logit (MNL): with iid error terms ϵ_{ijr}, following an Extreme Value distribution;
- Mixed Logit: full utility specification as in Eqs. (51) to (54), i.e. with random $\beta^{\mathrm{Inter},r}_{c}$ following a Lognormal distribution of parameters μ^{Inter}_{c} and $\sigma^{\mathrm{Inter}}_{c}$ (representing the heterogeneous cost sensitivity of shippers) together with the iid error terms ϵ_{ijr} (Nicolet et al., 2022).

4.2. Evaluation through out-of-sample simulation

To assess the solutions returned by these models, we simulate the demand response using an out-of-sample population. Indeed, the profit returned by the optimization is the one expected based on the SAA and the model's assumptions, but it does not indicate how well the solution will perform with actual shippers. This out-of-sample simulation also allows to compare the different models with each other. The procedure is as follows:

1. For each OD pair, generate a population of 1000 shippers (i.e. perform 1000 draws of ϵ_{ijr} and $\beta^{\mathrm{Inter},r}_{c}$, note that these draws are different than the ones used in the SAA) and divide the demand D_{ij} equally among the shippers;
2. For each shipper, compute their utilities by plugging the drawn ϵ_{ijr} and $\beta^{\mathrm{Inter},r}_{c}$, as well as the frequencies and prices returned by the model, into Eqs. (51) to (54);
3. Allocate the shipper's containers to the alternative with the maximal utility;
4. When all containers have been allocated, compute the resulting modal shares and the actual profit for the inland vessel operator.

4.3. Coefficients of utility functions

For the out-of-sample simulation, we directly make use of the coefficients of the Weighted Logit Mixture model estimated in Nicolet et al. (2022). However, these true utility functions of the shippers are not known by the operator. The same coefficients cannot, therefore, be used in the CD-SNDP. To alleviate this issue, we use the Weighted Logit Mixture to generate synthetic choice data, from which utility coefficients can be estimated by the operator. This process ensures that the true utility functions remain hidden from the operator, as they only have access to the choice realizations of shippers.

The available inputs are the OD matrices and the attributes related to IWT, Rail, and Road on each OD pair along the RA corridor. To generate a choice instance for a given OD pair using the Weighted Logit Mixture, we first draw the value of $\beta^{\mathrm{Inter},r}_{c}$ and each mode's ϵ_{ijr} in their respective distributions. Then, they are plugged, along with each mode's attributes, into Eqs. (52) to

Table 3
Coefficients of the mode choice models.

Parameter	Actual population	Synthetic data	
	Weighted Logit Mixture	Mixed Logit	MNL
α^{IWT}	0	0	0
α^{Rail}	0.713	0.816	0.338
α^{Road}	2.30	2.35	2.06
β_q^{IWT}	1.63	1.60	1.49
β_f^{Inter}	0.0278	0.0262	0.0229
β_a^{Road}	0.0530	0.0506	0.0469
β_a^{Inter}	0.157	0.173	0.141
β_c^{Road}	−8.68	−8.73	−4.81
β_c^{Inter}			−5.76
μ_c^{Inter}	2.30	2.40	
σ_c^{Inter}	0.690	0.618	
$\bar{\beta}_c^{\text{Inter}}$	−12.65	−13.34	−5.76

Table 4
Solutions of deterministic models with prices of the competing alternatives.

		Benchmark	SNDP	CD-SNDP	Competition		
					IWT	Road	Rail
Prices [€]	RTM-DUI (6500 TEUs)	68	68	120	68	252	203
	DUI-RTM (8400 TEUs)	69	69	133	69	251	203
	RTM-BON (1900 TEUs)	76	76	88	76	317	214
	BON-RTM (1500 TEUs)	74	74	58	74	315	214
	DUI-BON (6700 TEUs)	46	46	–	46	136	152
	BON-DUI (6500 TEUs)	46	46	–	46	136	152
Weekly frequencies [M8 vessels | M11 vessels]	RTM-DUI	16|12	0|13	0|16			
	RTM-BON	0|5	0|5	5|2			
	DUI-BON	32|3	20|0	0|0			
	RTM-DUI-BON	–	19|0	19|0			

(54). Finally, the mode with the highest utility is selected and we get one synthetic choice instance. This process is then repeated for all OD pairs. To remain consistent with the Weighted Logit methodology, the number of generated choice instances per OD pair is set proportional to its cargo volume. In particular, each OD pair gets at least one choice instance and an additional instance is generated per 10’000 TEUs circulating yearly on the OD pair. As a result, we end up with a synthetic dataset composed of 8676 choice instances, from which the MNL and Mixed Logit models can be estimated.

The coefficients of the Weighted Logit Mixture model are presented in Table 3, along with the coefficients of the Mixed Logit and MNL estimated using the synthetic dataset (note that α_{IWT} is normalized to zero). The mean value of β_c^{Inter} is also presented.

4.4. 3-node network results

In this section, we present and discuss the results of these various models applied on the 3-node network, starting with the deterministic ones.

4.4.1. CD-SNDP vs. Benchmark and SNDP

The weekly frequencies for both vessel types and the charged prices together with the total demand on each OD pair are reported in Table 4. In order to better understand the pricing decision, the table also displays the prices of the competing alternatives. For the benchmark and SNDP, the optimal prices are set at the same level as the cheapest competing alternative (in our case, IWT). This is because of the assumption that shippers are purely cost-minimizers and the deterministic nature of the models: if the vessel operator charges just 0.001 € less than the cheapest alternative, then the models will consider that all shippers will choose the services of the vessel operator instead of the competition. In the CD-SNDP, shippers are assumed to consider other attributes besides cost to perform their mode choice: the optimal prices then differ from the cheapest alternative.

Regarding the optimal frequencies, allowing to visit more than 2 terminals per service provides additional flexibility to the SNDP compared to the Benchmark. The SNDP takes advantage of this consolidation opportunity which results in higher expected profits. Fig. 4 displays the expected profits versus the actual ones returned by the out-of-sample simulation: it shows that the SNDP also returns higher simulated profits. The reason is that the vessel operator is able to attract more demand with this 3-stop service. This is seen in Fig. 5, which represents the expected and actual modal shares for each deterministic model.

For the CD-SNDP, the expected profits increase significantly compared to the two other models, although the OD pair DUI-BON is not served anymore by the vessel operator. The distance between these two terminals is indeed relatively short so, as the CD-SNDP takes multiple factors into consideration for the mode choice of shippers, it is evident that Road becomes the preferred option for this OD pair. Nevertheless, the expected profits are higher because the optimal price on the busiest OD pair (RTM-DUI) is twice as

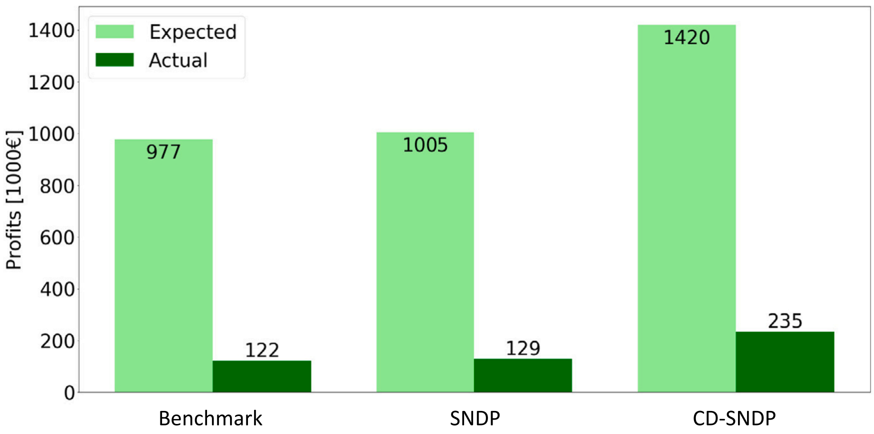

Fig. 4. Comparison of profits obtained with the deterministic models.

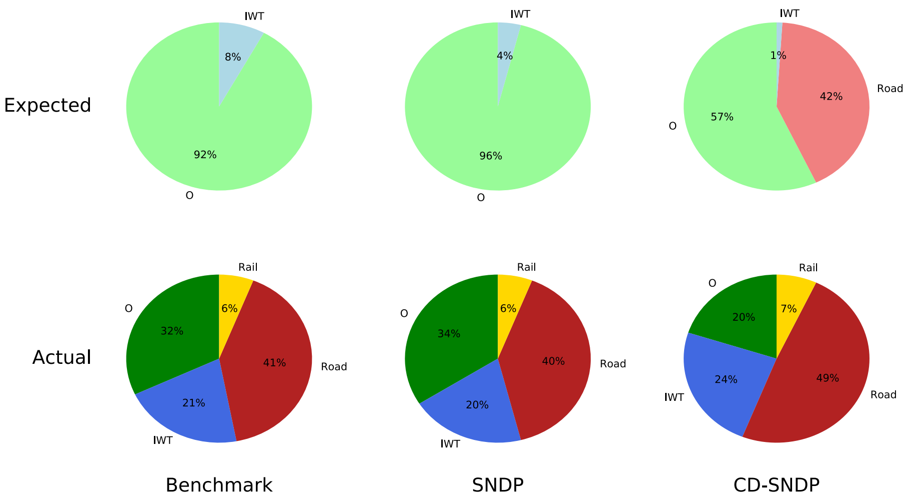

Fig. 5. Comparison of modal shares returned by the deterministic models and simulated ones.

high as in the other two models. Since the choice-driven model also considers frequency in the mode choice, it is able to charge more on this pair. Indeed, the operator's utility remains competitive with the one of the IWT alternative due to the higher proposed frequency (35 services per week) between these two terminals. Although the vessel operator gets smaller market shares than with the Benchmark and SNDP (See Fig. 5), the CD-SNDP returns actual profits that are almost two times higher.

These deterministic results already suggest that significant gains can be achieved with the Choice-Driven SNDP. More efficient services and pricing can be designed, thus resulting in considerably increased profits.

4.4.2. Stochastic variants with exact method

In this section, the results of the stochastic versions of the CD-SNDP are described. Two random utility formulations are compared: MNL (with random error terms ϵ) and Mixed Logit (with ϵ and distributed cost sensitivity β_c^{Inter}).

The two versions are solved through SAA with a sample size of $R = 500$, i.e. 500 draws are performed in the distributions of ϵ (and of β_c^{Inter}, for the Mixed Logit). For each variant, we run 20 replications with 20 different random seeds, thus generating 20 different samples. The aggregated statistics of the obtained solutions and computation time are presented in Table 5. Note that a time limit of 72 h has been applied, that is why the statistics of the optimality gap are also presented.

The pricing decision is very variable from one replication to another. The variation is slightly more pronounced for the MNL case than for the Mixed Logit, but the main takeaway is that the MNL results in higher prices than the Mixed Logit. Also, both variants find higher prices than the deterministic CD-SNDP. The frequency decision also varies between replications, but the ranges in the MNL case are quite similar to the Mixed Logit.

The influence on the expected and actual profits is depicted in the boxplots of Fig. 6. The higher prices set by the MNL lead to greater expected profits compared to the Mixed Logit. But this difference is canceled out when comparing the simulated profits as the MNL profits fall at a very slightly lower level than the ones of the Mixed Logit. Nevertheless, the actual profits for both variants

Table 5
Solutions of stochastic models with exact method (500 draws).

		MNL			Mixed Logit		
		Min.	Average	Max.	Min.	Average	Max.
Weekly frequencies [M8 vessels | M11 vessels]	RTM-DUI	0|11	0|12	0|14	0|0	0|5	1|13
	RTM-BON	0|0	0|0	0|0	0|0	0|0	0|0
	DUI-BON	0|1	7|2	12|2	0|0	6|2	12|2
	RTM-DUI-BON	21|0	22|0	24|0	12|0	21|0	24|0
Prices [€]	RTM-DUI	188	248	302	129	172	239
	DUI-RTM	188	247	314	135	167	235
	RTM-BON	191	253	316	140	201	284
	BON-RTM	166	235	286	146	189	283
	DUI-BON	139	202	282	106	175	328
	BON-DUI	142	202	282	106	176	328
Computation time [h]		27	58	72	72	72	72
Optimality gap		0%	3%	7%	29%	39%	60%

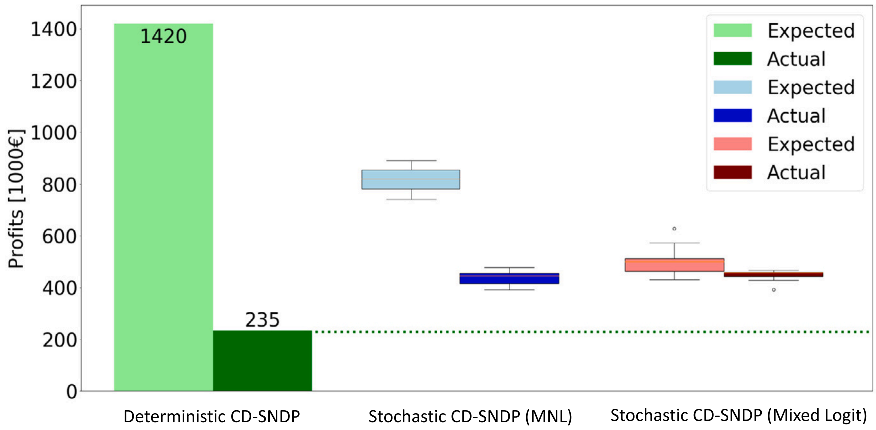

Fig. 6. Comparison of profits by the stochastic models with exact method and the deterministic CD-SNDP.

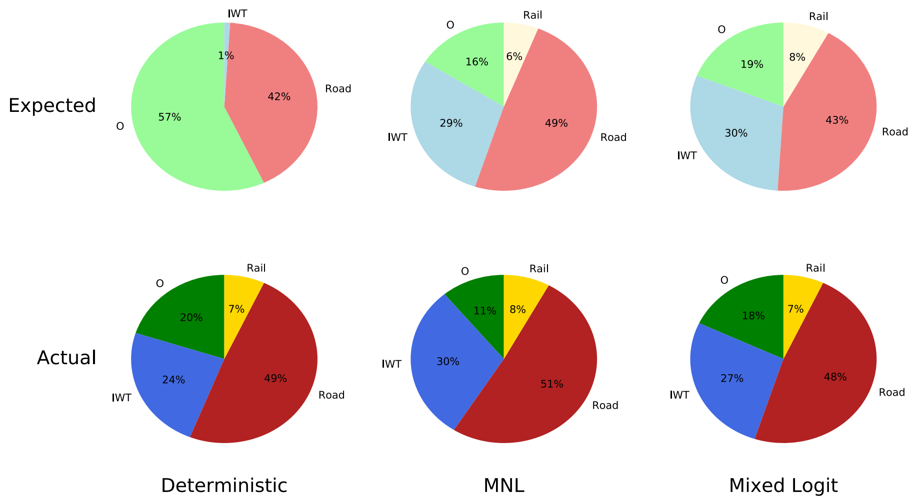

Fig. 7. Comparison of average modal shares returned by the stochastic models with the exact method and simulated ones.

are substantially higher than for the deterministic CD-SNDP. They provide an additional 90% gain compared to the deterministic case. This is because the modal shares can be estimated much better during the optimization due to the more detailed choice models. Indeed, Fig. 7 shows the average expected modal shares against the actual ones. These shares are close to each other for both the MNL and the Mixed Logit, whereas the deterministic model highly overestimates the share of the vessel operator.

Table 6
Solutions of stochastic models with predetermination heuristic (500 draws).

		MNL			Mixed Logit		
		Min.	Average	Max.	Min.	Average	Max.
Weekly frequencies [M8 vessels | M11 vessels]	RTM-DUI	0|11	0|12	0|14	0|0	0|7	0|14
	RTM-BON	0|0	0|0	0|0	0|0	0|0	0|0
	DUI-BON	0|0	4|6	12|2	0|0	8|2	22|0
	RTM-DUI-BON	21|0	22|0	24|0	10|0	21|0	24|0
Prices [€]	RTM-DUI	180	248	320	130	166	230
	DUI-RTM	190	247	310	120	161	230
	RTM-BON	190	251	310	140	199	270
	BON-RTM	160	230	290	140	199	320
	DUI-BON	140	206	320	110	177	330
	BON-DUI	140	207	320	100	181	330
Computation time [h]		0.04	0.05	0.05	0.05	0.05	0.05

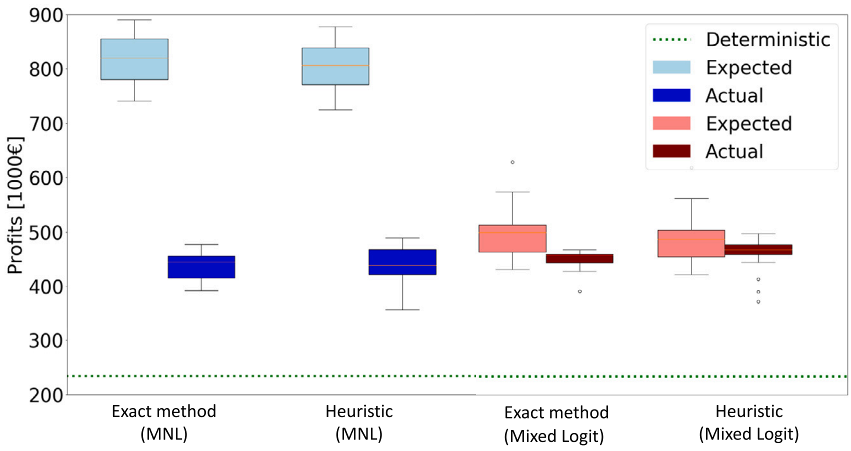

Fig. 8. Comparison of profits by the stochastic models with exact method and the heuristic.

Comparing the MNL with the Mixed Logit, the accuracy of their modal share estimation is nearly equivalent, but the MNL tends to overestimate the operator's share during the optimization process. The expected profits in Fig. 6 are then significantly higher than the actual ones, whereas the expected profits with the Mixed Logit are in line with the actual ones. This is because the MNL used in the CD-SNDP has a much lower cost coefficient β_c^{Inter} in absolute value than in the actual population (see Table 3). On the other hand, the Mixed Logit has coefficients that are more in line with the actual population. The cost sensitivity of the shippers is then underestimated by the MNL, which results in prices that are higher than with the Mixed Logit. The CD-SNDP with MNL then expects that high profits will be realized, whereas in reality there will be less demand than expected due to the higher prices: thus resulting in a decrease in profits.

Nevertheless, the large optimality gaps reported for the Mixed Logit in Table 5 prevent any conclusion at this stage. Even though the addition of stochasticity in the CD-SNDP provides more gains, it is done at the expense of computing time. To remedy this, we make use of the predetermination heuristic presented in Section 3.4.

4.4.3. Stochastic variants with predetermination heuristic

The two stochastic variants are solved using the same samples as for the exact method. We use the following set of predefined prices: $\mathcal{P} = \{10k | k \in \mathbb{N} \cap [0, 50]\}$, and the set of predefined frequencies: $\mathcal{F} = \mathbb{N} \cap [0, 35]$ in accordance with (55). The statistics of the heuristic solutions are reported in Table 6 together with the computation time.

Compared to the exact method, the predetermination heuristic is remarkably faster. When the computation was in the order of days for the exact method, it is now reduced to a few minutes. Most of these minutes are spent precomputing the demand and price values with Algorithm 2. With the heuristic, there is also little difference in solving time between the two stochastic variants. Regarding the quality of the solutions, the prices and frequencies found with the heuristic are not identical but they remain consistent with the ones returned by the exact method. The comparison between the profits obtained with the exact method and with the predetermination heuristic is shown in Fig. 8.

The profit ranges found by the heuristic are similar to the ones with the exact method. We still observe a significant gap between the expected and actual profits in the MNL case, whereas these two values are at a more similar level for the Mixed Logit.

These results serve to validate the performance of the heuristic in comparison to the exact method. Therefore, we can now evaluate the CD-SNDP on larger instances which will be presented in the next section.

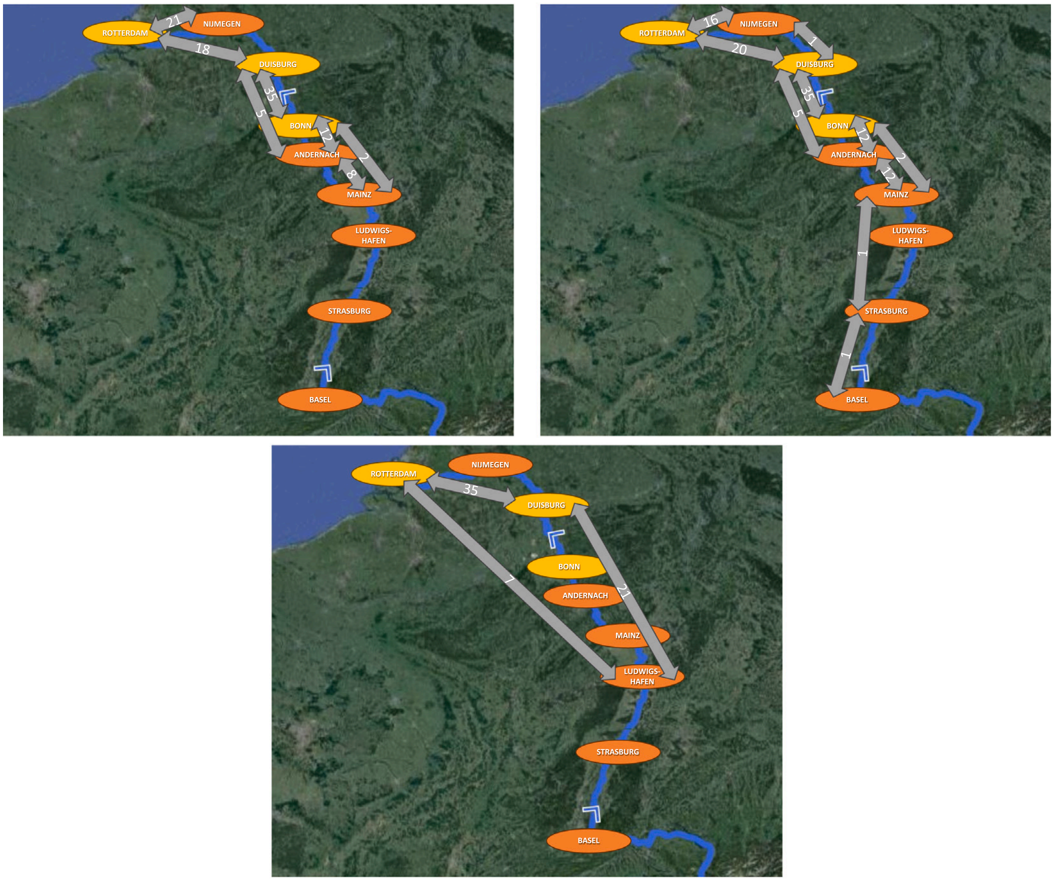

Fig. 9. Optimal weekly frequencies for the Benchmark (top-left), the SNDP (top-right), and the deterministic CD-SNDP (bottom).

Table 7
Ten OD pairs with the most demand.

Rank OD	1st RTM-DUI	2nd DUI-BON	3rd RTM-NIJ	4th RTM-LUH	5th BON-AND	6th AND-MAI	7th RTM-BON	8th RTM-BSL	9th DUI-AND	10th RTM-MAI
Demand [TEUs]	14’900	13’200	10’600	6’400	4’200	3’600	3’400	2’000	1’800	1’200
Distance [km]	230	120	130	590	40	110	360	850	170	520

RTM = Rotterdam, DUI = Duisburg, BON = Bonn, NIJ = Nijmegen, LUH = Ludwigshafen, AND = Andernach, MAI = Mainz, BSL = Basel.

4.5. 9-node network results

In this section, we present and discuss the results of these various models applied on the 9-node network depicted in Fig. 3, starting with the deterministic ones.

4.5.1. CD-SNDP vs. Benchmark and SNDP

The optimal service design for the three deterministic models is shown in Fig. 9. The solution of the benchmark focuses on busy OD pairs, as it allows to serve eight out of the ten pairs with the most demand in the network (see Table 7). Allowing cycles in the SNDP enables to redeploy some vessels: in particular, a service to Strasbourg and Basel is added and the frequency on the OD pairs RTM-DUI and AND-MAI is increased.

While the benchmark and SNDP serve as many high-demand OD pairs as possible, the CD-SNDP only serves two out of the ten pairs with the most demand (RTM-DUI and RTM-LUH). These two pairs are also the ones with the most TEU-kilometers, far ahead of the others: this indicates that the CD-SNDP proceeds to a trade-off between the potential demand that can be attracted and the distance on the OD pair. Indeed, water transport tends to become more attractive to shippers for long-distance transport. Regarding the pricing decisions, the observations remain similar to the 3-node results.

The expected and actual profits resulting from the three deterministic models are illustrated in Fig. 10. Similarly to the 3-node case, the CD-SNDP returns higher profits (both from the optimization and the out-of-sample simulation). This is due to the more

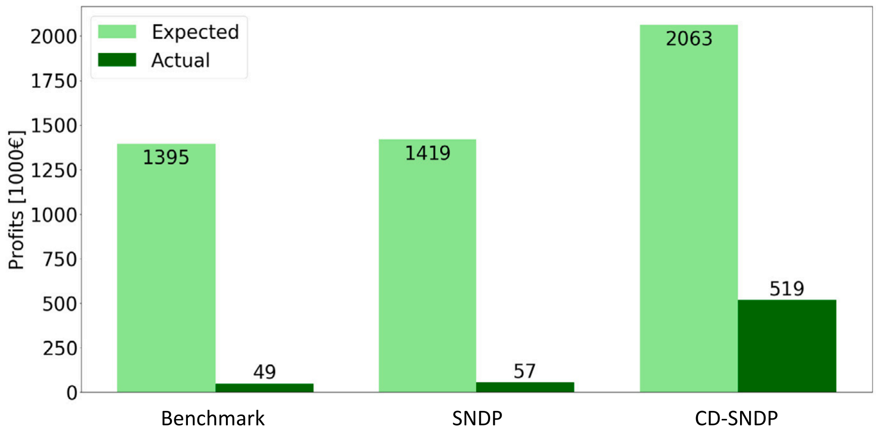

Fig. 10. Comparison of profits obtained with the deterministic models.

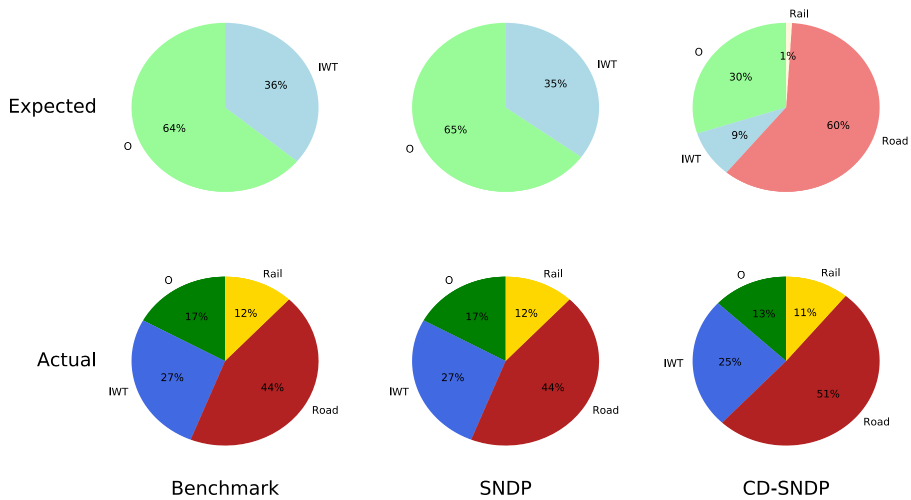

Fig. 11. Comparison of modal shares returned by the deterministic models and simulated ones.

accurate estimation of the demand, as depicted in Fig. 11. Indeed, the benchmark assumes that the whole demand will go to IWT (be it the operator or the competitor), as it is the cheapest mode; but after simulation, it turns out that only 44% of the demand was assigned to IWT. On the other hand, the CD-SNDP estimates that only 39% of the demand will be assigned to IWT, whereas the simulated IWT share is 38%. For the sake of comparison, the observed shares on these OD pairs are 38% for IWT, 55% for Road, and 7% for Rail.

A better demand estimation allows to charge a higher price than the models using the cost-minimization assumption because the other factors influencing the mode choice are also considered. It is also able to target more adequately the OD pairs to serve in priority as it can proceed to a trade-off between the total demand and the attractiveness of water transport on a given OD pair. This allows the operator to make better decisions as the resulting profits are increased by a factor of ten compared to the benchmark.

4.5.2. Stochastic variants

Table 8 presents the minimum, maximum, and average service frequencies on the ten OD pairs with the most demand along with the computation time of the 20 replications. On average, both stochastic models return very similar solutions. The first and third busiest OD pairs have frequencies set at (or close to) the maximum of 35. The rest of the vessels are mostly deployed on the RTM-BON and DUI-BON OD pairs as well as the RTM-LUH OD pair, while a frequency of one service per week is set on the remaining OD pairs. For each model, the differences in maximum and minimum frequencies are explained by the small number of draws (500) relative to the size of the model. This difference is also visible in the computation times, which range from three to fourteen hours. However, the time does not change significantly between the two models.

Regarding the pricing decisions, the observations remain similar to the 3-node results. The resulting profits obtained with the two stochastic models are shown in Fig. 12, together with the profits of the deterministic CD-SNDP. The expected profits (resulting from the optimization) are decreased compared to the deterministic case, whereas the actual ones (obtained through the out-of-sample

Table 8
Frequencies on OD pairs with most demand of stochastic models (500 draws).

		MNL			Mixed Logit		
		Min.	Average	Max.	Min.	Average	Max.
Weekly frequencies	RTM-DUI	34	35	35	5	33	35
	DUI-BON	0	14	28	1	12	31
	RTM-NIJ	34	35	35	2	31	35
	RTM-LUH	3	5	16	3	5	6
	BON-AND	0	1	3	0	1	1
	AND-MAI	0	1	3	0	1	1
	RTM-BON	0	14	28	1	12	31
	RTM-BSL	1	1	2	0	1	2
	DUI-AND	0	1	3	1	1	1
	RTM-MAI	0	1	3	1	1	2
Computation time [h]		4.0	8.8	13.8	3.3	8.3	13.7

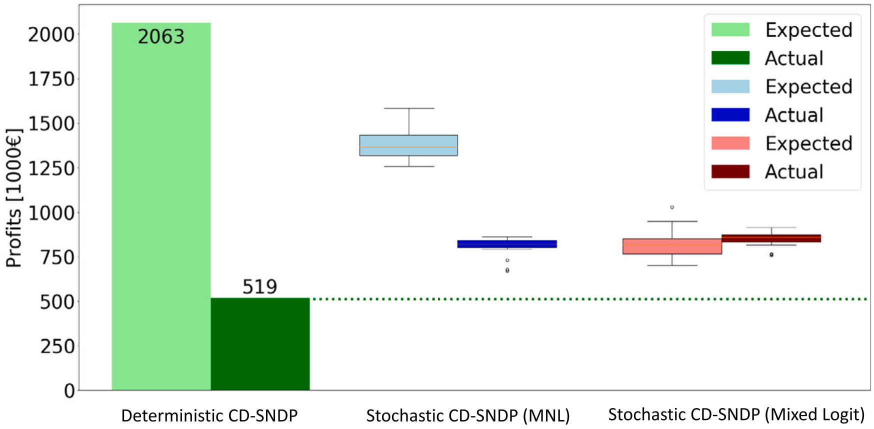

Fig. 12. Comparison of profits obtained with the two stochastic models against the deterministic CD-SNDP.

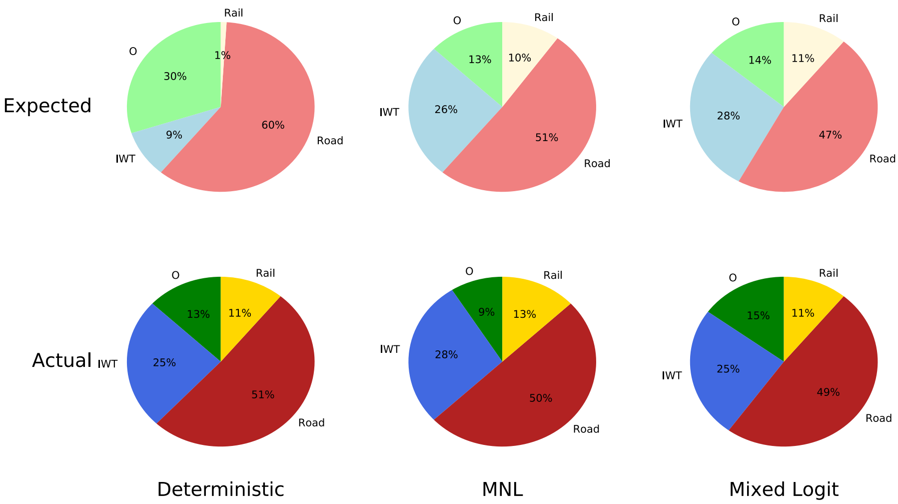

Fig. 13. Comparison of modal shares returned by the two stochastic models and the deterministic CD-SNDP and simulated ones (observed shares are: 38% for IWT+O, 55% for Road and 7% for Rail).

simulation) are 60% higher. This is once again because the stochastic models can estimate better the modal split, hence the potential demand of the transport operator. This is particularly true for the model with Mixed Logit, where the expected profits almost match the actual ones. The modal shares estimated by the models are shown in Fig. 13 together with the ones obtained through the simulation.

4.5.3. Discussion

Compared to the 3-node network, this 9-node instance is more realistic: indeed, most IWT operators do not have more than 10 terminals to serve along the Rhine (Hutchison Ports, 2024; DP World, 2024; Danser, 2024). A notable exception is Contargo (2024) with their 17 terminals along the Rhine. To apply the proposed CD-SNDP to their network, some pre-processing rules have to be implemented to restrict the size of the set of potential services $\mathcal{S}$. For the 9-node network, we still considered all possible services such that $|\mathcal{S}| = 502$. But if the same approach is used on a 17-node network, then we reach $|\mathcal{S}| = 131054$. To reduce the size of this set, the following rules can be applied:

- Consider terminals that are close to each other as a single node (for example, if the network goes down to 13 nodes, then $|\mathcal{S}| = 8178$);
- Only consider services originating from the seaport of Rotterdam (applying this criterion on top of the previous one, we get $|\mathcal{S}| = 4095$);
- Enforce that a service can contain at most one inland terminal, whose demand from/to the seaport of Rotterdam is higher than 2000 TEUs (see Table 7).

Applying these three pre-processing rules together allows to decrease the problem size to $|\mathcal{S}| = 767$. On top of that, the operator may have additional rules, which decrease even more the size of $\mathcal{S}$. In the end, the set of potential services reaches a size of the same order of magnitude as in the 9-node network and the results above show that the proposed method can deal with such instances.

4.6. Key insights

Several takeaways can be gathered from the results presented above. First, a cycle-based formulation (with multiple stops allowed) of the SND problem is more efficient in terms of asset usage as the operator can use consolidation opportunities. This results in both reduced costs and increased demand. The mathematical expression of services is less straightforward than with a path-based formulation due to the addition of service legs, but the improved results justify this effort.

Secondly, it is highly beneficial for the transport operator to include the information they have about the demand during the design of their services. The CD-SNDP results have shown that, even with a simple deterministic model, the solution of the SNDP problem is able to generate actual profits that are nearly three times higher than the benchmark. This is because the benchmark's assumption that shippers are purely cost-minimizers neglects other attributes that still play a role in the decision-making of shippers, such as the service frequencies. The utility functions also include the arbitrage between these attributes through the weighting coefficients. As a result, the estimation of modal shares during the optimization stage is much more accurate. Indeed, the cost-minimizing assumption used in the benchmark overestimates the demand assigned to the operator. This can also be observed in the Rail shares obtained in the paper presenting the benchmark (Tawfik and Limbourg, 2019).

Thirdly, making use of stochastic CD-SNDP exploits further the potential of the model. Indeed, perfect and complete information about the shippers is not available to the operator, so their demand model will miss some aspects that play a role in the shippers' choices. These aspects can indirectly be accounted for by adding random error terms in the model. Including this uncertainty into the model enables gains exceeding 50% compared to the deterministic CD-SNDP. Therefore, the stochastic formulation of the CD-SNDP is one convenient way to account for imperfect information endogeneously to the model.

Finally, quantifying and incorporating the heterogeneous preferences of shippers allows for a more accurate estimation of the profits. Indeed, except for the stochastic CD-SNDP with Mixed Logit, all models presented above substantially overestimate the profits. This can lead to very bad surprises for the operator if they expect a given amount of profit in their budget, but end up realizing much less. On the other hand, the formulation with Mixed Logit expects profits in line with the ones that are realized. Considering heterogeneity then allows to get a better prevision of the profits.

It should be noted that the decisions (pricing and frequency) of the choice-driven SNDP highly depend on the underlying representation of the shipper's behavior. Namely, the range of improvement is closely linked to the elasticity of demand. Therefore, the utility functions need to be carefully studied for the context at hand. In our case, we base them on a study that makes use of real aggregate data on the same network to estimate the parameters and validates the results against real market shares.

5. Conclusion

This work proposes a Service Network Design and Pricing problem that incorporates the mode choice behavior of shippers. Therefore, we develop a so-called Choice-Driven Service Network Design and Pricing problem that directly includes utility-based mode choice models into a bilevel optimization problem, which can then be reformulated as a single level linear problem. The random nature of utility-based models, such as the Multinomial Logit, allows to account for missing information about attributes playing a role in the mode choice. Opting for a Mixed Logit formulation further allows to consider the heterogeneous preferences of shippers, thus getting a more realistic representation of the shippers' population. Due to the randomness, the problem becomes stochastic, which makes it computationally expensive to solve with an exact method. To overcome this issue, we develop a predetermination heuristic that computes utilities prior to the optimization.

The results show that the heuristic can considerably reduce the computational time while finding solutions of similar quality to the exact method. Regarding the proposed model itself, it is compared to a benchmark where shippers are assumed purely cost-minimizers. We show that the profits achieved by our model are substantially higher, even if the embedded mode choice model is simply deterministic. All in all, including more information about the shippers while designing and pricing the services suggests considerable gains for the transport operator. Even if the exact model or parameters are not known, it is still far better than not using the available information.

Nevertheless, several assumptions made in this work deserve to be challenged. Firstly, in the mode choice models, the utilities of the vessel operator and of the competing IWT carrier are considered independent from each other. However, since they are both proposing inland waterway services, these two options are correlated with each other. This could also apply to a lesser extent to the Rail alternative, which also proposes scheduled intermodal services. Further work should consider this correlation between choice alternatives. Secondly, it is assumed that the utilities of the competing alternatives can be computed by the operator, implying that they have full information about their competitors. Some attributes can indeed be found, e.g., the frequency or travel times, but the price that the competitors are applying cannot be known perfectly: at best it can be estimated. The choice-driven model should then be developed further to account for this imperfect information. Thirdly, the competition is assumed exogenous and fixed meaning that they will not react to the operator's new services. But the competitors will also seek to improve their services and profits, even more so if they lose market share to the operator. These dynamics can be covered, for example, through an Agent-Based Model accounting for the reactions of the different parties involved.

Another dynamic aspect that can be included is about the pricing, particularly in the context of inland waterway transport. Indeed, the frequent low water levels on the Rhine reduce the capacity of the vessels, thus increasing the transportation costs per container. The operators then have to increase the price they charge to compensate for the losses. A time dimension could be included in the optimization model to deal with dynamic pricing. Finally, our formulation implies that a single price per OD pair is set for all shippers. However, revenue management techniques can be used to improve the performance of the proposed model. This will provide the operator with additional gains because they can tailor the prices offered to specific customers. A revenue management setting would also develop the full potential of the formulation with Mixed Logit, as the prices can be adapted to the different cost sensitivities of shippers.

CRediT authorship contribution statement

Adrien Nicolet: Writing – review & editing, Writing – original draft, Visualization, Validation, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **Bilge Atasoy:** Writing – review & editing, Validation, Supervision, Methodology, Investigation, Formal analysis, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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