

Fatigue Crack Propagation Behaviour of Steel Recovered from the Old Botlek Bridge

Master Thesis
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Fatigue Crack Propagation Behaviour of Steel Recovered from the Old Botlek Bridge

by

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Preface

This thesis acts as the final chapter of my two years at Delft University of Technology and presents the graduation research that concludes my MSc studies in Civil Engineering.

This research investigates the fatigue crack propagation behaviour of structural steel originating from the Old Botlek Bridge in Rotterdam. The specimens examined were extracted from dismantled structural elements of the bridge and analyzed using experimental testing, numerical modeling, and data-driven post-processing. The project aims to contribute to a better understanding of the remaining fatigue performance of these elements and to support future studies on their safe reuse.

In 2024, I arrived in the Netherlands, far from my friends and family, yet eager to participate in an educational programme at one of the world's best academic institutions in the field of civil engineering. The path I have walked since then has been full of challenges and difficulties, however, the efforts invested along the way have not gone unrewarded. TU Delft has changed my perspective on engineering as a whole, helped me make the most of my abilities and equipped me with skills that I can proudly apply anywhere in the world.

Throughout this journey, I encountered many inspiring minds among professors, industry professionals and fellow students alike. Each of them contributed in some way to my development, not only as an engineer but also as a person. I am grateful to everyone I met along this path and to all those who helped me become a better engineer than I was before enrolling in this programme.

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Last but not least, my deepest gratitude belongs to my whole family. Without their support I would not have been able to overcome the challenges. I am especially thankful to my mother, Dr. E. Nagy, whose encouragement and belief in me have always been a source of motivation and inspiration. I am equally grateful to my girlfriend, Zs. Pauza, who stood beside me during the most difficult moments and supported me in my studies.

Use of generative AI: During this thesis project, generative AI was used to support brainstorming, debugging and documenting Python code, formatting the layout of figures, and correcting linguistic errors and inconsistencies. All methodological choices, calculations, interpretation of the results, and final decisions remained the responsibility of the author. All AI-assisted content was reviewed and verified by the author.

*Daniel Agocs
Delft, September 2026*

Abstract

The reuse of recovered materials plays an increasingly significant role in improving sustainability in the construction industry. For safe reuse, a detailed understanding of material behaviour is essential. The Old Botlek Bridge in Rotterdam was dismantled in 2017, and this study investigated the fatigue properties of the steel recovered from its main girders. The objective was to determine the fatigue crack growth parameters of the recovered steel using linear elastic fracture mechanics and the Paris–Erdogan equation.

In order to derive these parameters, experimental fatigue testing and numerical analyses were carried out. Eleven SENB specimens were tested under high-cycle fatigue loading at a nominal stress ratio of $R = 0.1$. Crack propagation during the applied cyclic loading was monitored using three separate measurement systems: a crack gauge on the side, a crack opening displacement gauge at the bottom, and a strain gauge on the top of the specimens. In the case of the crack gauge, crack propagation was measured by recording the breakage of discrete conductive wires. A semi-automatic algorithm was developed to post-process the measured voltage signal and extract the wire-breakage points. Since the crack-opening displacement and strain measurements did not provide crack length directly, it was inferred from the measured datasets using calibrations obtained from two-dimensional finite element models. Crack growth rates were calculated using seven-point local polynomial differentiation for the crack-gauge data and neural-network regression with automatic differentiation for the COD- and strain-gauge-based data. The finite element models used linear-elastic material behaviour and mainly a geometrically linear formulation, while a geometrically nonlinear formulation was also applied as a sensitivity study. Stress intensity factors were extracted from the same numerical models using contour integrals. The experimental crack growth rates were then combined with the specimen-specific stress intensity factor ranges to determine the Paris-law parameters.

All three measurement methods captured the same general crack growth trend. The calculated crack growth rates ranged from approximately 10^{-5} to 10^{-3} mm/cycle, but the three methods gave different results for the same specimen during the same experiment. Based on the specimen-balanced pooled results from the geometrically linear formulation, the crack-gauge-based evaluation gave $m = 4.062$ and $C = 3.50 \times 10^{-10}$, the COD-based evaluation gave $m = 4.109$ and $C = 1.71 \times 10^{-10}$, and the strain-gauge-based evaluation gave $m = 3.618$ and $C = 8.93 \times 10^{-10}$. These values apply within the method-specific fitting ranges, with da/dN expressed in mm/cycle and ΔK in $\text{MPa}\sqrt{\text{m}}$. The geometrically linear stress intensity factor results showed close agreement with the Baratta and Fett reference solution, with a mean absolute difference of 0.21% and a maximum absolute difference of 0.44%. The difference between the geometrically linear and nonlinear results increased towards larger crack lengths, showing that geometrical nonlinearity can affect the stress intensity factor in the final stage of crack propagation.

These differences show why the complete measurement and post-processing procedure must be reported alongside fatigue crack growth results and considered when comparing them. Over the common range of $\Delta K = 20\text{--}40 \text{ MPa}\sqrt{\text{m}}$, all three Old Botlek Bridge Paris-law curves were positioned above the selected literature curves for S235 J2, S355 J2, and old crane steel. The evaluated specimens therefore showed higher crack growth rates at the same ΔK and lower apparent fatigue crack-growth resistance. However, this difference cannot be attributed exclusively to the previous service history of the bridge steel because unused reference specimens from the same steel were not available.

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Nomenclature

Abbreviations

Abbreviation	Definition
AE	Acoustic Emission
ASTM	American Society for Testing and Materials
BF	Blast Furnace
CCD	Charge-Coupled Device
COD	Crack Opening Displacement
CSV	Comma-Separated Values
CT	Compact Tension
CZM	Cohesive Zone Model
DC	Direct Current
DIC	Digital Image Correlation
DOF	Degree of Freedom
DRI	Direct Reduced Iron
EAF	Electric Arc Furnace
EN	European Standard
EPD	Environmental Product Declaration
EU	European Union
FEM	Finite Element Method
LCA	Life Cycle Assessment
LEFM	Linear Elastic Fracture Mechanics
MAD	Median Absolute Deviation
MLP	Multilayer Perceptron
MSE	Mean Squared Error
NN	Neural Network
RMSE	Root Mean Square Error
SENB	Single-Edge Notched Bending
SGD	Stochastic Gradient Descent
SIF	Stress Intensity Factor
TSL	Traction-Separation Law
XFEM	Extended Finite Element Method

Symbols

Symbol	Definition	Unit
<i>Latin symbols</i>		
a	Crack length	[mm]
a_0, a_{init}	Initial notch length	[mm]
a_i	Crack length at data point i	[mm]
B	Specimen thickness	[mm]
b	Intercept of the linear regression in log–log space, $b = \log_{10}(C)$	[-]
$c(N)$	Cumulative number of detected crack-gauge wire breakages	[-]
C	Paris-law coefficient	[mm/cycle (MPa $\sqrt{\text{m}}$) $^{-m}$]

Symbol	Definition	Unit
da/dN	Fatigue crack growth rate	[mm/cycle]
dV	Threshold used for crack-gauge jump detection	[-]
E	Young's modulus	[GPa]
E'	Effective elastic modulus in the J - K relationship	[GPa]
E_{11}	Longitudinal strain component extracted from the FEM model	[-]
F	Total applied vertical load	[kN]
G	Energy release rate	[N/mm]
I	Second moment of area of the specimen cross-section	[mm ⁴]
J	J -integral	[N/mm]
K	Stress intensity factor	[MPa√m]
K_I, K_{II}, K_{III}	Stress intensity factors for Modes I, II, and III	[MPa√m]
K_{IC}	Mode-I plane-strain fracture toughness	[MPa√m]
L	Support span	[mm]
l	Lever arm between a support and the loading point	[mm]
m	Paris-law exponent	[-]
M	Bending moment in the constant-moment region	[kN m]
N	Number of loading cycles	[-]
$N_{\max}$	Maximum recorded number of loading cycles	[-]
$n(N)$	Number of broken crack-gauge wires	[-]
p	Crack-gauge wire spacing	[mm]
$P_5(\alpha)$	Fifth-order polynomial used in the geometry correction function	[-]
r	Radial distance from the crack tip	[mm]
R	Load ratio, $R = F_{\min}/F_{\max}$	[-]
S	Support span used in the geometry sensitivity study	[mm]
s_i	Value of the log-transformed crack-gauge signal at data point i	[-]
t	Specimen thickness, equivalent to B	[mm]
U	Measured crack opening displacement used in the COD calibration	[mm]
U_i	Crack-gauge voltage at measurement point i	[V]
U_i^{env}	Upper-envelope value of the crack-gauge voltage signal	[V]
$\bar{U}$	Strain energy density	[MPa]
W	Specimen height	[mm]
x_0	Signed offset between the initial notch tip and the first relevant crack-gauge wire	[mm]
x_i	Log-transformed stress intensity factor range	[-]
y_i	Log-transformed crack growth rate	[-]
Y	Dimensionless geometry correction factor	[-]
Y_{base}	Geometry correction factor for the reference specimen geometry	[-]
Y_{case}	Geometry correction factor for the investigated geometry	[-]
z	Distance from the neutral axis to the outer fibre	[mm]
Greek symbols		
α	Relative crack length, $\alpha = a/W$	[-]
δ_c	Critical cohesive separation	[mm]
δ_K	Relative difference between numerical and literature stress intensity factors	[%]
δ_q	Relative difference of a mesh-dependent result with respect to the reference mesh	[%]

Symbol	Definition	Unit
Δc_i	Increment in the cumulative number of detected wire breakages	[-]
ΔF	Applied load range, $\Delta F = F_{\max} - F_{\min}$	[kN]
ΔK	Stress intensity factor range, $\Delta K = K_{\max} - K_{\min}$	[MPa $\sqrt{\text{m}}$]
$\Delta K_{\min,j}$	Lower stress intensity factor limit used for the Paris-law regression of measurement method j	[MPa $\sqrt{\text{m}}$]
$\Delta K_{\max,j}$	Upper stress intensity factor limit used for the Paris-law regression of measurement method j	[MPa $\sqrt{\text{m}}$]
ΔK_{th}	Threshold stress intensity factor range	[MPa $\sqrt{\text{m}}$]
ΔY_{rel}	Relative difference of the geometry correction factor	[%]
ε	Measured strain	[-]
ε_{ij}	Strain tensor	[-]
Γ	Integration contour surrounding the crack tip	[-]
Γ_0	Cohesive fracture energy	[N/mm]
ν	Poisson's ratio	[-]
σ	Characteristic stress	[MPa]
σ_c	Critical cohesive traction	[MPa]
σ_{ij}	Stress tensor	[MPa]
σ_{nom}	Nominal bending stress at the outer fibre	[MPa]
θ	Polar angle measured around the crack tip	[rad]

Introduction

This thesis examines the fatigue properties of structural steel using material obtained from a bridge, namely the Old Botlek Bridge, after 60 years of service. In the project, experimental crack growth measurements are combined with finite element calculations of the stress intensity factor range to determine the Paris-law parameters. Three measurement and post-processing workflows are compared to examine their influence on the fitted Paris-law curves. This chapter presents the concept of fatigue in steel, the importance of the fatigue assessment of steel structures, and the bridge from which the investigated material was obtained. In addition to these, the research objectives and scope of the research are presented.

1.1. Sustainability aspects

Today, sustainability is a crucial topic as the global average temperature continues to rise. The increase of atmospheric greenhouse-gas concentrations is considered the main cause of global warming. Therefore, greenhouse-gas emissions must be reduced to limit the consequences.

The construction industry plays a significant role in total emissions, including those associated with steel production. Until now, a linear economic model has been followed; however, this approach results in the generation of more waste than necessary. A transition to a circular economy is needed in which, ideally, no raw materials are wasted and the system relies on renewable resources.

The European Union defined goals to support this transition. Within a circular model, the reuse of existing products and structures is essential. However, the reuse of structural members raises advanced engineering-related questions. Currently, there are still significant gaps in the available knowledge regarding the reuse of engineering structures.

1.2. Fatigue in steel and its importance for bridges

Fatigue is a progressive, localised, and permanent structural change that occurs in a material subjected to repeated or fluctuating strains at nominal stresses with maximum values less than the tensile strength of the material. The aforementioned structural change is manifested by the formation of cracks or complete fracture after a certain number of fluctuations. Fracture is the separation of a solid body into two or more pieces under the action of stress. [12]

Fatigue develops in three stages: crack initiation, crack propagation, and final fracture. Initiation strongly depends on microscopic imperfections in the material. It can occur even in carefully prepared steel elements that appear smooth at the macroscopic level. In the propagation phase, the crack is no longer governed by micro-scale properties. Under cyclic loading, as the crack length increases, the load-bearing capacity of the structure decreases until a sudden final failure. Fatigue is particularly critical because fatigue cracks can lead to sudden fracture. Before the final fracture, little or no visible deformation occurs, therefore, it is challenging to detect such small cracks.

Starting from the 1950s, many steel bridges have been built in the Netherlands as a result of the increase in traffic. Since then, traffic loading has continuously increased, including both traffic density and the weight of vehicles. This increase presumably affects the expected service life of these structures. Therefore, regular inspection and assessment are essential to ensure their structural safety. [25] [49] In numerous structures, severe fatigue cracks have already occurred in recent years that must be treated as soon as possible. However, in some cases, repair or strengthening is not feasible for economic, functional, or safety reasons. In such cases, demolition is necessary. For steel structures, dismantling may allow individual elements to be reused. After a long service life, the behaviour of the steel ma-

material is uncertain and should be further investigated before the structural elements are reused. This requires material-level fatigue testing combined with fracture-mechanics-based numerical assessment to characterise the crack propagation behaviour of the steel.

1.3. The Botlek Bridge

The material investigated in this thesis originates from the Old Botlek Bridge, which was located in the port of Rotterdam. The structure was constructed in 1953. The bridge consisted of two separate structural systems. One was a steel truss structure that formed three spans on the west side of the bridge, where the middle span included a vertical-lift section. On the east side, six spans were formed by a steel girder structure with an approximate 2 m girder height. The bridge had one railway track and two road lanes, as well as bicycle and pedestrian paths on both sides.

Demolition was carried out in 2017, and the structural elements were stored in Maasdijk at a storage yard of Rijkswaterstaat. Specimens were extracted from the structural elements for experimental testing to support decisions regarding their reuse. This project involves experiments with the material from the girder flanges.



Figure 1.1: The Old Botlek Bridge in Rotterdam [40]

1.4. Research objectives and questions

This study focuses on fatigue crack propagation based on tests performed on single-edge notched bend (SENB) specimens. The goal of the research is to determine the Paris-law parameters of steel obtained from the Old Botlek Bridge after its service life. To process and evaluate the experimental data, a Python workflow was developed for crack length determination, crack growth rate calculation, and Paris-law parameter evaluation.

Based on the experimental results, the fatigue crack growth behaviour of the specimens is characterised, and potential differences between the investigated specimen groups are evaluated. The experimental crack length histories are post-processed to determine the corresponding crack growth rates. In parallel, the stress intensity factor ranges, ΔK , are determined numerically in Abaqus as a function of crack length, considering the specimen-specific geometries and loading conditions. Finally, the experimental crack growth rates are combined with the corresponding numerical ΔK values to

derive the Paris-law parameters for each specimen.

Main research question of this study:

What are the fatigue crack propagation parameters of steel recovered from the Old Botlek Bridge based on SENB specimen experiments?

In order to better understand the topic, the following sub-questions are investigated:

- How can crack length measurements from different experimental techniques be combined with numerically determined stress intensity factor ranges and the analytical Paris-law formulation to determine fatigue crack growth parameters?
- How can the stress intensity factor range, ΔK , required for the Paris-law evaluation, be determined numerically for the tested SENB specimens, and how sensitive is the corresponding geometry correction factor to the investigated SENB specimen dimensions and four-point bending configuration?
- How do these specimens obtained from a previously used structure behave compared with raw steel materials?
- Do the specimen-specific Paris-law parameters provide evidence of two distinct clusters that could be consistent with the different prior stress histories of the top and bottom flanges?

1.5. Scope of research

The study is limited to a material- and specimen-level investigation of the fatigue crack propagation behaviour of SENB steel specimens originating from the Old Botlek Bridge. The experimental programme focuses on high-cycle, constant-amplitude fatigue testing in a four-point bending configuration. Crack gauges, COD devices, and strain gauges are used to monitor crack propagation. The magnetometer measurements are not evaluated in the present study.

A Python-based post-processing workflow is developed to determine the crack length, crack growth rate, and Paris-law parameters from the experimental data. Due to the use of different measurement systems, method-specific procedures are applied. The COD and strain gauge measurements are converted into crack length using relationships obtained from finite element analyses, while the crack growth rates are determined using numerical differentiation and a neural-network-based regression method.

The numerical part of the study includes two-dimensional finite element analyses in Abaqus. The models are used to determine the crack length–COD and crack length–strain relationships and to calculate the Mode I stress intensity factor. The numerical model is assessed through a mesh sensitivity analysis and a comparison with available stress intensity factor solutions from the literature. Both geometrically linear and geometrically nonlinear formulations are considered.

Only one nominal specimen size, with a nominal specimen height of $W \approx 65 \text{ mm}$ and a nominal thickness of $B \approx 32 \text{ mm}$, is included in the experimental programme. The measured specimen dimensions are presented in Section 3.2 and Table 3.1. Therefore, experimental size effects cannot be directly validated. The influence of the specimen dimensions and the four-point bending span is considered through a geometrically linear sensitivity study of the SIF geometry correction factor. This is important because the geometry correction factor directly influences the calculated ΔK values and the determined Paris-law parameters. The geometries included in the sensitivity study are defined in Section 4.3 and Table 4.2.

Possible differences between the tested specimens and measurement methods are investigated. However, the original positions of the extracted materials within the bridge girders are unknown, and therefore a direct comparison between materials originating from the top and bottom flanges cannot be made. Comparison with unused steel is limited to data available in the literature, since unused reference specimens are not included in the experimental programme.

Other degradation mechanisms, such as corrosion, detailed metallurgical and microstructural analyses, variable-amplitude loading, crack initiation, and structural-scale remaining-life assessment are outside the scope of the research. Although the project is related to sustainability, a detailed environmental or life-cycle assessment is not included.

Theoretical background and literature review

2.1. Production of new steel or reuse of old steel

Thanks to technological progress, the widespread use of steel in structural systems began in the late 19th and early 20th centuries. Bessemer's steel processing method was developed in the 1850s and revolutionised the production of steel. The process made it possible to reduce the carbon content, the quality of steel production improved, and the method provided large quantities of steel commercially available. Since then, steel has been used for all kinds of load bearing structures, including building structures and bridges. Steel has become one of the most widely used materials in the construction industry due to its high strength and durability. Despite the long history of this material, its long-term behaviour in reused or long-term operated structures has not yet been fully characterised. However, today a significant number of steel structures have been in operation for several decades, making it both possible and necessary to investigate their long-term performance. [23] [28]

Many existing structures are coming to the end of their service life or are becoming unsafe due to material degradation, such as corrosion and fatigue. This is especially relevant for bridges, which are subjected to cyclic loading throughout their service life. The design of these structures was generally carried out based on standards and traffic conditions that are now outdated. This also applies to structures in the Netherlands. Since the end of the Second World War, traffic density and vehicle weights have gradually increased, especially over the last 40 years, making infrastructure engineering structures more vulnerable. [13] [9] The number of safety issues is rising due to the ageing of such structural systems, and these issues must be handled appropriately. As a consequence of the widespread use of steel, a large number of structures are currently facing these problems. [9]

The Old Botlek Bridge was constructed in 1953 and its steel therefore represents the material and production conditions of the mid-20th century. Based on the production methods commonly used for European structural steels during this period, the investigated material was most likely a mild steel produced by either the Thomas or the Siemens–Martin process and cast into ingots before hot rolling. However, data is not available about the original production methods. Therefore, the exact production method cannot be stated with certainty and would require chemical and metallographic investigations for confirmation. Nevertheless, the production period remains relevant because the chemical composition, solidification conditions, and rolling process determine the microstructure and mechanical properties of the resulting steel. [29] [53]

Historical mild steels generally had a ferritic–pearlitic microstructure. During ingot solidification, carbon, phosphorus, and sulphur could segregate towards the core of the ingot. Oxide and sulphide inclusions and a non-uniform grain-size distribution could also remain in the material. Carbon increases the strength of steel through an increased pearlite content, while local concentrations of carbon, phosphorus, and sulphur may reduce ductility and fracture toughness. A coarse or non-uniform grain structure may permit the formation of longer slip bands and promote local cyclic plasticity, thereby increasing the variability of the fatigue behaviour. [29] [45]

The influence of these metallurgical features on fatigue behaviour depends on the size, shape, location, and orientation of the inclusions and segregated regions. Oxide and sulphide inclusions have different mechanical properties from the surrounding ferritic–pearlitic matrix. As a result, cyclic stresses and strains may concentrate at the interface between an inclusion and the steel matrix. Repeated

loading can cause separation at this interface or fracture of a brittle inclusion, resulting in the formation of a small fatigue crack. In this way, a sufficiently large inclusion can reduce the number of cycles required for crack initiation and decrease the fatigue strength of the material. Surface and near-surface inclusions are generally more critical because a crack originating from them can propagate more easily than a crack completely surrounded by material. [36]

In comparison, modern continuously cast and controlled-rolled structural steels generally have a more uniform chemical composition, a lower and more controlled inclusion content, and a finer and more homogeneous grain structure. Grain refinement generally improves strength and fracture toughness and restricts the development of local cyclic slip. These differences do not necessarily mean that historical steel always has a lower fatigue resistance than modern steel. However, they provide a reason to expect that its fatigue behaviour may be different and may show greater scatter.

The effects of inclusions and material inhomogeneity are generally most significant during fatigue crack initiation, short-crack propagation, and near-threshold crack growth. When a sufficiently long crack has developed, the stress intensity factor range is the main parameter of crack propagation. Nevertheless, interactions between the crack tip and inclusions, hard and soft segregated regions, and grain boundaries may locally change the crack path, crack-tip plasticity, and crack-closure behaviour. This can affect the derived Paris-law parameters and increase the scatter of the fatigue crack growth data. An inclusion or a locally brittle region may accelerate crack propagation. Therefore, historical steel is not necessarily expected to show consistently faster crack growth than modern steel, but its behaviour may be less uniform and less predictable. Published comparisons have also demonstrated differences between the fatigue crack propagation behaviour of historical and modern structural steels. The properties of the Old Botlek Bridge steel should therefore be determined experimentally instead of assuming that fatigue crack propagation data obtained for modern steels are directly applicable. [45] [44]

Although production technology has improved, steel production is still one of the most environmentally harmful activities in the industrial sector. Steel production is responsible for approximately 9% of the total CO_2 emissions, and these emissions must be reduced in the near future. The pressure arises from an urgent matter, namely global warming. [26] Reusing steel recovered from existing structures could reduce the demand for newly produced steel. However, safe reuse requires reliable knowledge of the present mechanical and fatigue properties of the historical material. The investigation of the fatigue crack propagation behaviour of the Old Botlek Bridge steel therefore contributes both to the assessment of ageing structures and to the evaluation of the potential reuse of historical structural steel.

2.1.1. Regulations and approaches for climate neutrality

In 2019, the European Commission launched a policy strategy called The European Green Deal. Its aim is to significantly reduce emissions in Europe through legal regulations and make Europe the first climate-neutral continent. The plan includes several action plans, each addressing one of the key challenges. [22] According to the Climate Law, emission rates must be reduced by at least 55% by 2030, and total climate neutrality by 2050. [20]

The Net-Zero Industry Act supports environmentally clean net-zero technologies and attaches great importance to hydrogen. By scaling up the production of hydrogen, fossil fuel consumption is expected to decrease. Using hydrogen makes it possible to achieve net-zero production, which would make steel production more sustainable. However, this methodology is still under research and development and is not yet widely applied. In the future, the relevance of traditional blast furnace and purely electric arc furnace steel production is expected to decline, with direct reduced iron combined with electric arc furnace (DRI+EAF) taking over, because the former blast furnace (BF) method is highly polluting, while the technology using purely electric arc furnace (EAF) is limited by its dependence on scrap availability and contamination of the scrap material. [32][21] [31]

European Commission follows another strategy closely related to the construction industry in the form of Circular Economy Act. The goals of the Act are to create a single market for secondary raw materials, increase the supply of high-quality recycled materials, and encourage their use within the EU. Today, the rate of circularity is solely 12%, the EU aims to reach 24% by 2030.[19]

2.1.2. Differences between linear and circular economy

The conventional linear model relies on raw material extraction. As a consequence, even though efficiency has increased, there is still a demand for raw, non-renewable materials. Over the previous decades, it is evident that the demand for production has been monotonically increasing. Until now, the focus has been on improving efficiency by reducing resource and fossil energy use per unit of manufacturing output. However, improving efficiency alone does not lead to a long-term sustainable economy, as a certain level of consumption still exists despite finite resources.

The linear economy follows the 'take-make-dispose' model. After the extraction of the raw materials, they are transformed into products. At the end of the product life they are disposed, resulting in large quantities of waste that must be treated accordingly. If the need for the product is still present, the process starts again at the phase of extraction.

On the other hand, the circular economy has a different perspective. This model is based on a few simple principles. One of them is "Design out waste". If, during the design phase, the components of a product are designed for disassembly and refurbishment to be used again with minimal energy and the highest conserved quality, the amount of waste can theoretically be reduced to zero. [16] Unfortunately, it is not possible in every case to start a new service life after the end of the first service life according to the cradle-to-cradle material flow system described by McDonough and Braungart. [34]

For structural applications reuse is only feasible if the safety and reliability of the existing material can be demonstrated. Therefore, the transition towards a circular economy also requires engineering methods that can assess the remaining structural performance of reused elements.



Figure 2.1: Comparison of linear and circular economy [48]

2.1.3. R-hierarchy

When waste at the end-of-life phase cannot be completely avoided, there are still options to limit its generation. The framework in Figure 2.2 helps to categorise different use and waste management approaches according to their environmental impact. There are 10 procedures from R0 to R9, where R0 is the most environmentally friendly option.

Considering the Botlek bridge, there are more possibilities within the R-hierarchy. As the bridge

reached its service life, the conventional approach would be demolition and melting down the elements, using them as scrap for raw steel production. This solution already has some advantages, as it can be considered recycling.

However, if the structural members are used, for instance, in a new bridge structure, it is considered reuse, which is significantly better. The situation is less clear when the elements are used in a building as load-bearing elements. The building has a different function, but the element itself still carries loads in a similar way to the bridge. Therefore, if load-bearing elements are reused again as load-bearing elements, it can be considered reuse.

In some cases, it may be possible to use a rod originating from a truss that was loaded exclusively axially as a beam in bending. In that case, strictly speaking, the element has a different function, and that would be repurposing instead of reusing.

Of course, the design of a structure built from reused elements requires a higher level of expertise and additional effort to ensure a proper and safe design. This is mainly due to the uncertainties related to the material properties, previous loading history, and possible damage or deterioration. Therefore, detailed assessment, testing, and verification of the elements are often necessary before reuse. In addition, the design process may require more complex checks compared to conventional structures using new materials.

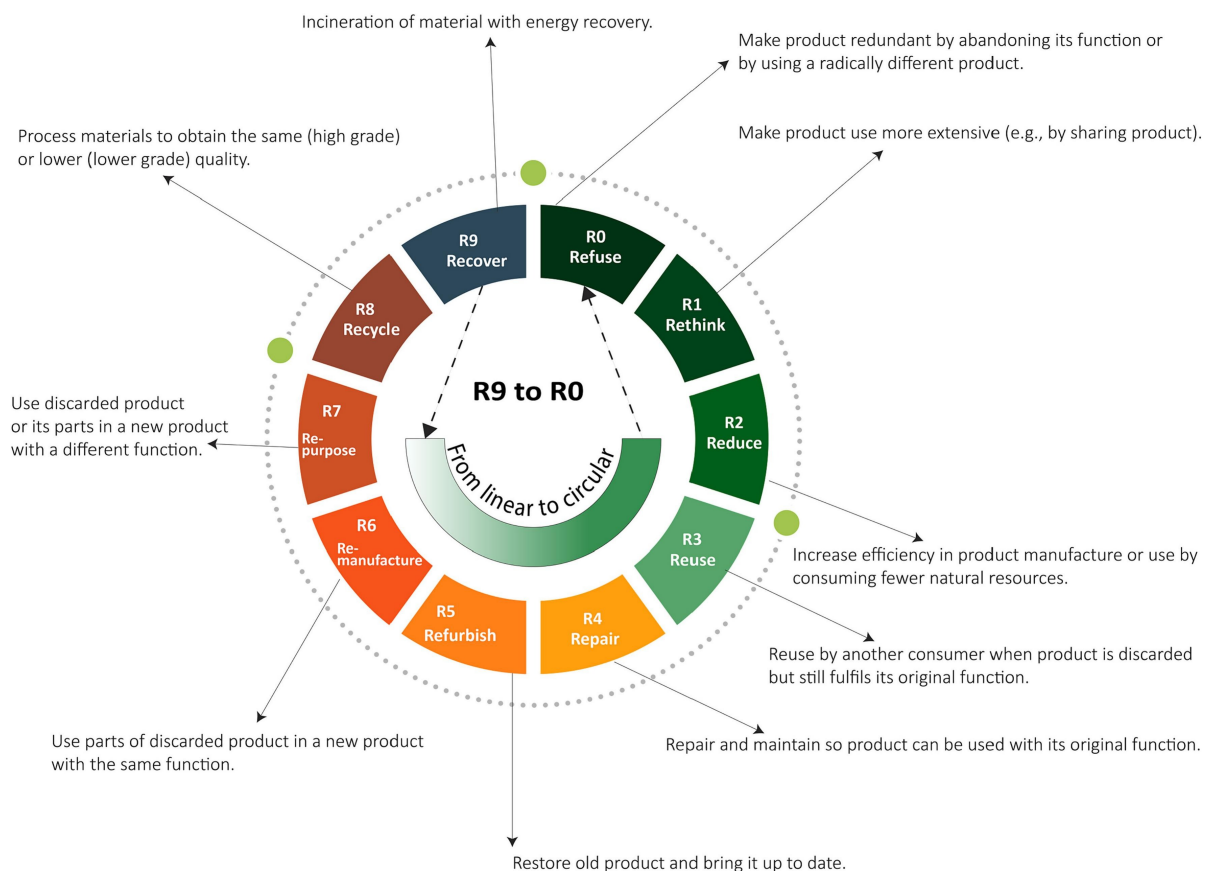


Figure 2.2: Overview of circularity strategies [35]

2.1.4. Life Cycle Assessment

As discussed in the previous section, relying solely on the R-hierarchy is not sufficient for making decisions. Quantification is necessary to compare different solutions and projects. LCA is a method that can be used to quantify the environmental impact of products. Based on the EN 15804 standard, all producers can prepare Environmental Product Declarations (EPDs) in a uniform manner to make life cycle assessments simpler. LCA includes several stages of a product's life. It is possible to calculate the so-called CO_2 equivalent emissions for each and summarize the environmental impact of a product. Not every greenhouse gas contributes equally to global warming. Therefore, CO_2 equivalent emissions

represent all greenhouse gases as a common unit.

Furthermore, it is possible to use this metric to determine external costs. These are the costs required to compensate for the harm caused by pollution.

In EN 15804, the stages of a life cycle are split into A, B, C and D stages, including substages for more detailed analysis. From A1 to A5, the steps are related to construction. These steps typically cause the most environmental effects and are associated with the highest CO_2 emissions.

The B stage describes the longest period during the service life, as it is the use phase, which can last 100 years or even more. Maintenance and operational activities can also contain a remarkable amount of CO_2 equivalent emissions.

At the end-of-life phase in stage C, there are steps responsible for deconstruction and disposal. The linear economy model ends here, and for a new product the A stage begins again.

In the case of a circular concept, the service life does not end at this point, because there is stage D for the utilisation of recycling or reuse possibilities. These steps usually balance the previous stages by CO_2 equivalent quantities with a negative sign.

The question of whether it is more beneficial to produce new steel elements or to reuse existing ones has been addressed in previous studies. This question is particularly relevant for steel bridges that are reaching the end of their service life, such as the Old Botlek Bridge. Pongiglione and Calderini found that structural steel reuse could reduce the required amount of steel by up to 30%. [38] A comparative life-cycle assessment by Buzatu et al. showed an emission reduction of approximately 29–35% when reclaimed steel elements were used instead of new ones. The main benefit resulted from avoiding the production of new steel. [11]

However, reuse is not always the optimal solution. Additional transportation, deconstruction, testing, processing, refurbishment, or strengthening may reduce its environmental benefits. The availability of suitable elements, uncertain material properties, and the limited availability of design regulations may also create difficulties. [8] Therefore, a project-specific assessment is required to determine whether reuse or the production of new elements results in a lower overall environmental impact.

Life-cycle assessment can be used to evaluate the environmental effects of reuse, but it cannot demonstrate the structural suitability of the recovered material. Structural suitability must be assessed separately based on the condition and mechanical properties of the steel. The present study does not include an LCA but investigates the fatigue crack propagation behaviour of the Old Botlek Bridge steel as one of the material properties relevant to its possible reuse.

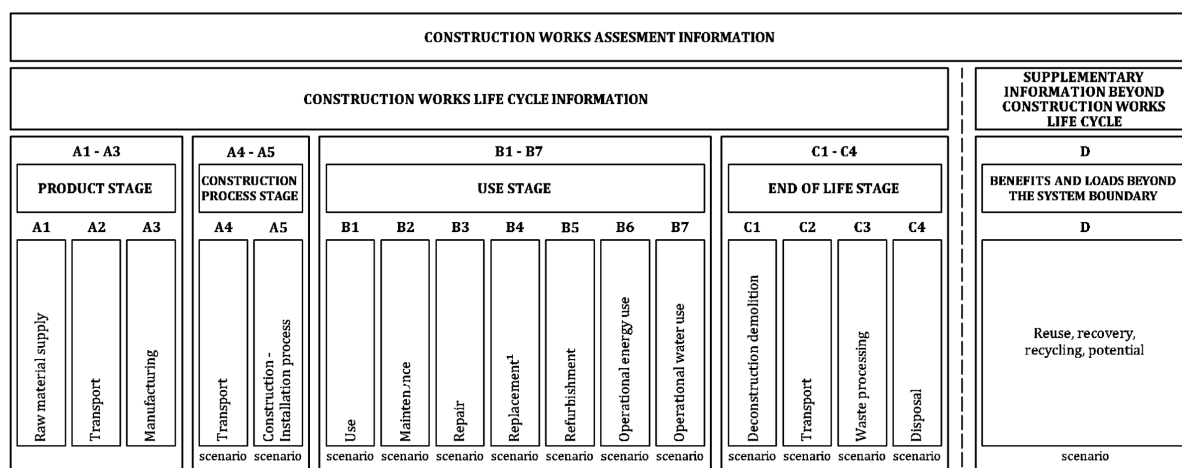


Figure 2.3: LCA stages [46]

2.2. Fatigue of steel and fracture mechanics

In the 1800s, it was observed that bridges were cracking when subjected to cyclic loading. In the 1850s, A. Wöhler presented a method to treat this issue in design and he proposed the so-called stress-life method which applies S-N curves also known as Wöhler-curves. This method is still widely used in assessment of fatigue. These curves can be obtained by experimental results and one curve presents the fatigue properties of one structural detail. Generally, steel structures consist of numerous

elements and connections resulting in many different load paths and stress distributions, causing high level of fatigue complexity. In Eurocode 3 several constructional detail categories are included for design purposes. These should be analysed thoroughly to identify the critical details considering fatigue. In addition, these diagrams provide information only about the cycle numbers and the corresponding fracture stress levels, but do not describe crack initiation and propagation explicitly. Therefore, S-N curves are useful for design purposes, but cannot be used for every specific case. An example curve can be seen in Figure 2.4 .[39]

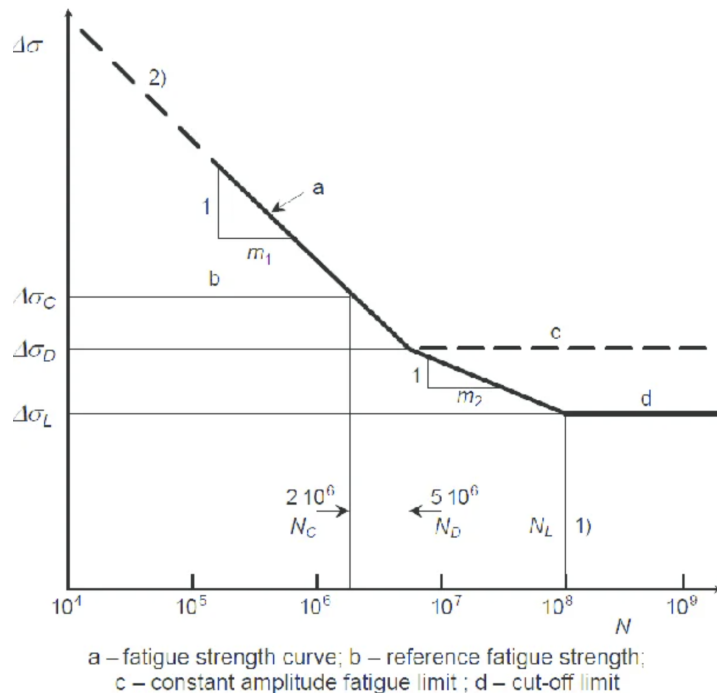


Figure 2.4: General S-N curve from EN 1993-1-9 [18]

2.2.1. Fatigue behaviour of steel

Crack formation and development can be divided into three distinct phases: crack initiation, crack propagation, and final fracture. In the first stage, cracks start to form on the free surface, highly depending on the microstructure and defects of the material. Even properly surface-treated elements that appear perfectly smooth have microscopic defects that serve as origins for fatigue cracks. Crack initiation is a shear driven phenomenon that occurs between the steel grains. After a certain point, which is challenging to determine precisely, the fracture goes further and starts to propagate, driven by tension. At this stage, crack propagation is no longer sensitive to the microstructure. When the crack sufficiently weakens the element, the final fracture occurs and the load bearing capacity of the structure is lost.

In general, fatigue is considered a dangerous phenomenon, as the final fracture occurs in a brittle manner. As can be seen in Figure 2.5 the fast fracture includes more than half of the cross sectional area. Two types of fatigue can be distinguished, low-cycle fatigue and high-cycle fatigue. In the case of low-cycle fatigue, the fracture occurs before approximately 10000 cycles, and due to the loading, higher strains and plastic deformation can be observed. On the other hand, high-cycle fatigue occurs after a larger number of cycles and the behavior can be explained with elastic theory. In case of bridges, the second option can be seen mainly. With a well-manufactured steel surface, the crack initiation may represent a large part of the fatigue life. In conclusion, recognisable macro-level cracks typically appear only in a relatively advanced stage of fatigue, when the damage has already become severe and structural safety can no longer be ensured.

2.2.2. Crack initiation in steel

Through microscopic investigations, it has been recognized that the nucleation of microcracks occurs early in fatigue life. Despite this early crack nucleation, microcracks remain invisible, and crack growth

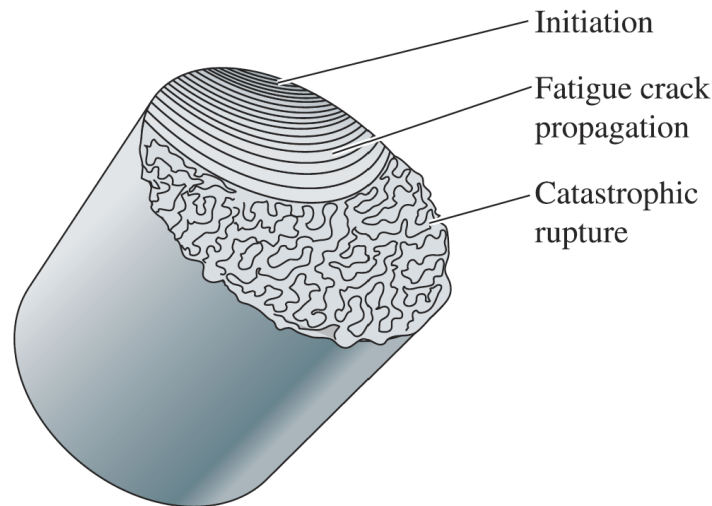


Figure 2.5: Schematic fatigue fracture surface [4]

remains slow and uncertain, as it is governed by the microstructure, such as grain boundaries. Crack initiation occurs due to cyclic microplasticity in the grains on the material surface. Since the material at the surface is usually not continuously supported by solid material, the constraint for slip between surface grains is lower. Therefore, cyclic shear stress can create relative deformations between slip planes. When this occurs, strain hardening can also be observed due to the increase in load. This results in shear stress in the reversed direction within the same slip band, but reverse shear stress can also occur in neighboring grains. With the repetition of cycles, the strains along the slip bands become larger until a microcrack evolves to macroscopic size. [43]

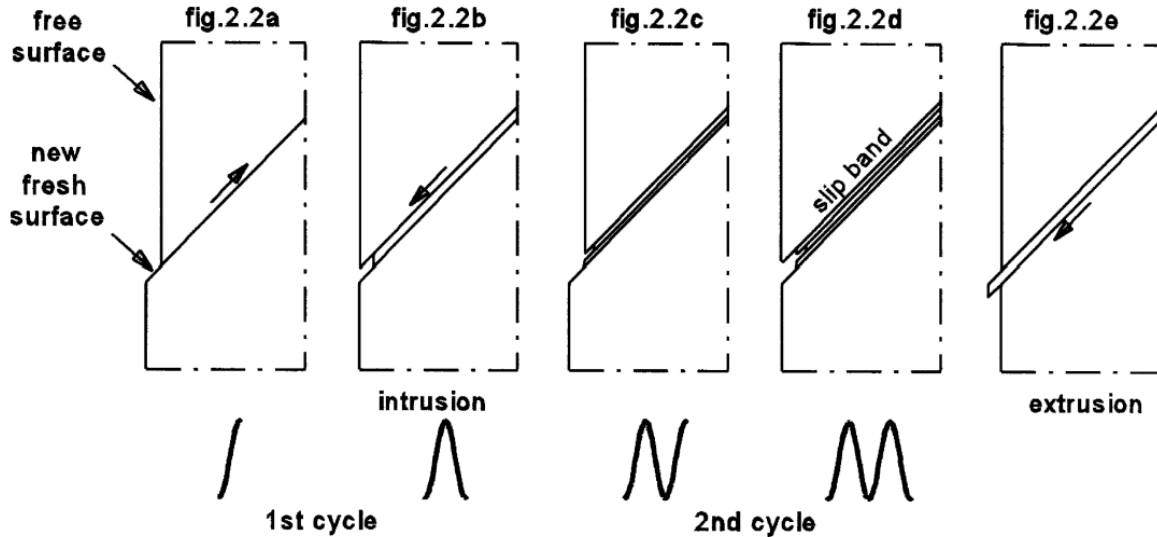


Figure 2.6: Crack nucleation by cycle slip [43]

2.2.3. Fatigue crack propagation and linear elastic fracture mechanics

When the crack growth does not depend on the grain and slip bands anymore, fatigue has reached the crack propagation phase. In the initiation phase, the microcrack forms at a 45-degree angle to the surface, but during crack propagation, the crack evolves perpendicular to the direction of the maximum principal stress. At the point the crack starts to penetrate into the material crack growth is a bulk material phenomenon. The simplest way to represent crack growth is a graph with the crack length data against

the number of cycles. The slope of these curves is da/dN which is the crack growth rate in most cases in mm/cycle.

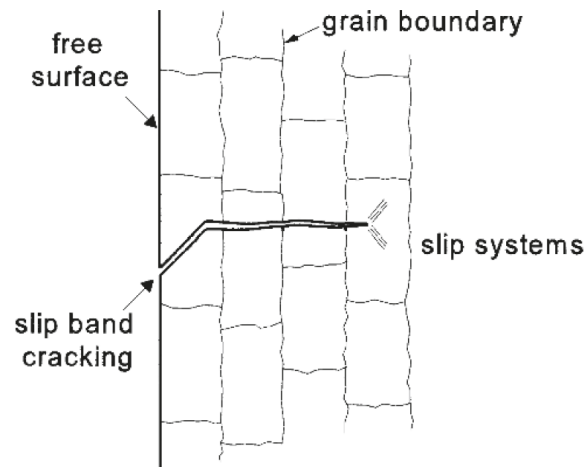


Figure 2.7: Cross section of microcrack [43]

The analysis of crack propagation builds on linear elastic fracture mechanics. Fracture mechanics is the branch of mechanics that uses analytical methods from solid mechanics to determine the force driving a crack and to describe how resistant a material is to breaking. In fracture mechanics, 3 different crack modes can be identified which can be seen in Figure 2.8. By the superposition of these, mixed mode cracking is also possible.

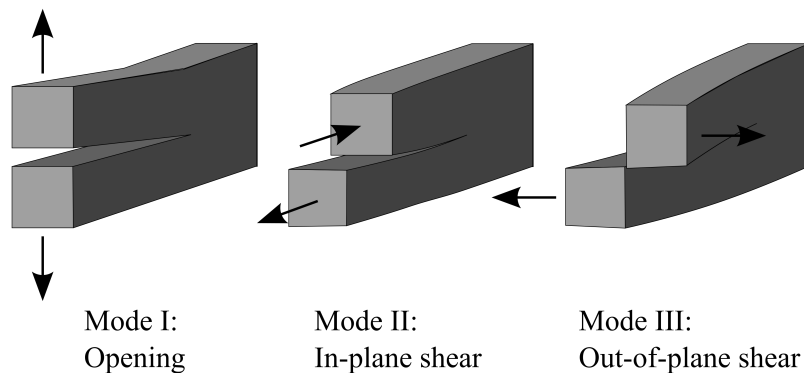


Figure 2.8: The three basic crack loading modes [52]

Cracks redirect forces sharply and create a high-stress zone at the crack tip. Stress becomes highly concentrated, theoretically reaching infinitely large values. Due to the fact that the crack length is significantly larger than the radius of the crack tip ($r \ll a$), it shows a singularity, as all stress components go to infinity if $r \rightarrow 0$.

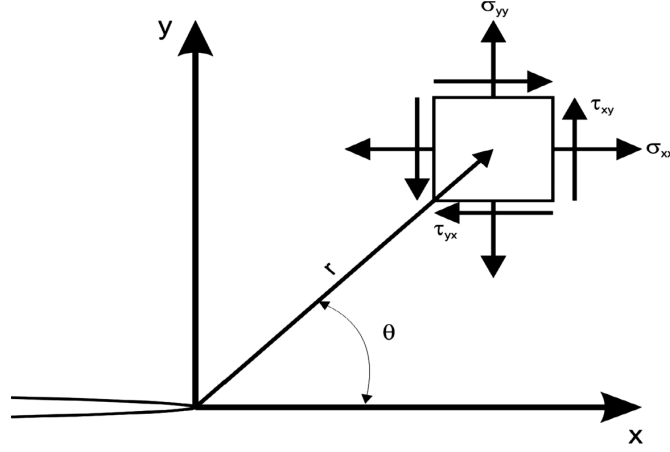


Figure 2.9: Definition of the coordinate axis around the crack tip [37]

To be able to quantify the stress field singularity at such cracks a new quantity has to be defined, namely the stress intensity factor K . For certain crack configurations subjected to external forces it is possible to derive closed-form expressions for the stresses in the body assuming isotropic linear elastic material.

$$\sigma_{ij} = \left(\frac{k}{\sqrt{r}} \right) f_{ij}(\theta) + \sum_{m=0}^{\infty} A_m r^{\frac{m}{2}} g_{ij}^{(m)}(\theta) \quad (2.1)$$

The stress near the crack tip varies with $1/\sqrt{r}$ regardless of the configuration of the cracked body. Equation (2.1) describes a stress singularity, since the stress is asymptotic as $r \rightarrow 0$. Each mode of loading produces the $1/\sqrt{r}$ singularity at the crack tip, but the proportionality constants k and f_{ij} depend on the mode.

It is convenient to replace k with K , where $K = k\sqrt{2\pi}$. The stress intensity factor (SIF) defines the amplitude of the crack tip singularity, meaning that stresses near the crack tip increase in proportion to K . This parameter makes it possible to quantify crack tip conditions and to determine all components of stress, strain, and displacement as functions of r and θ .

For every basic mode, it is possible to determine the SIF for a given configuration, accounting for different external forces and geometries. In the case of simple setups, analytical solutions can be found in the literature, while for more complex cases, experimental or numerical approaches must be used. A general formulation of SIF can be seen below in Equation (2.2)

$$K_{(I,II,III)} = Y\sigma\sqrt{\pi a} \quad (2.2)$$

where σ is a characteristic stress, a is a characteristic crack dimension, and Y is a dimensionless constant that depends on geometry and mode of loading. Based on the assumptions made, the material fails at certain stress and strain levels. Since there is a direct relationship between the SIF and stresses and strains, it is possible to define a critical SIF level that aligns with the failure criterion. This K_{IC} is a measure of fracture toughness, which is a material constant that is independent of the size and geometry of the cracked body.

Using the SIFs corresponding to the maximum and minimum load levels, resulting in the SIF range, ΔK , and $\frac{da}{dN}$, it is possible to formulate the widely used Paris–Erdogan equation, which is an empirical relationship describing the fatigue crack growth rate in metals:

$$\frac{da}{dN} = C(\Delta K)^m \quad (2.3)$$

where $\frac{da}{dN}$ is the crack growth per cycle, ΔK is the stress intensity range, and C and m are material constants.

The Paris–Erdogan law is typically valid only within an intermediate crack growth regime, commonly referred to as Region II of the fatigue crack growth curve. At low values of ΔK , crack growth is very

slow or may even stop due to crack closure effects and the presence of a threshold value, ΔK_{th} . In contrast, at high ΔK levels approaching the fracture toughness K_{IC} , rapid and unstable crack growth occurs, leading to final failure.

The parameters C and m can be determined experimentally from a log-log plot of $\frac{da}{dN}$ versus ΔK . Understanding these regimes is crucial for accurate remaining lifetime prediction and damage tolerance analysis of structural components.

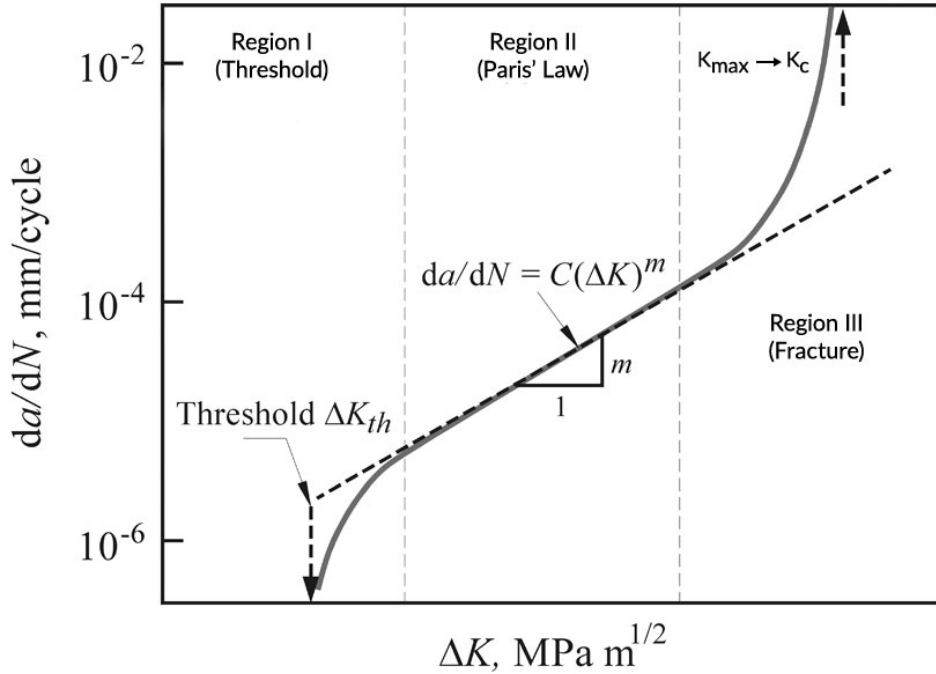


Figure 2.10: Fatigue crack growth rate stages [33]

2.3. Numerical analysis approaches for steel fatigue

Using the conventional finite element method (FEM) makes it possible to determine the SIFs when analytical solutions are not available. For calculating the SIF values, so-called contour integrals are used.

To interpret these integrals, the concepts of energy-based formulation must be introduced. These concepts originated from Alan Arnold Griffith, who developed a criterion for crack propagation. His theory relies on the strain energy stored in the material. Creating new crack surfaces requires energy, as atomic bonds must be broken. On the other hand, crack extension leads to a reduction in the stored elastic strain energy. Therefore, crack propagation is energetically favourable when creating new surfaces requires less energy than the decrease in elastic energy. Originally, Griffith's theory was limited to perfectly brittle materials; however, it provides the foundation for modern fracture mechanics.[3]

In 1956, George Rankin Irwin proposed his approach and defined Griffith's criterion as the energy release rate, where the energy dU released during a crack elongation of da covers the necessary surface energy:

$$G = -\frac{dU}{da} \quad (2.4)$$

The J-integral represents a path-independent measure of the energy release rate and is widely used in both linear and nonlinear fracture mechanics. The integration is independent of the path, which makes it particularly suitable for numerical methods. The J-integral physically represents the energy release during crack propagation.[3]

$$J = \int_{\Gamma} \left(\bar{U} dy - \sigma \frac{\partial \mathbf{u}}{\partial x} ds \right) \quad (2.5)$$

where $\bar{U}$ is the strain energy density defined as $\bar{U} = \int_0^{\varepsilon_{ij}} \sigma_{ij} d\varepsilon_{ij}$, σ_{ij} is the stress tensor, ε_{ij} is the strain tensor, $\vec{\sigma}$ is the stress vector, $\vec{u}$ is the displacement vector along the integration path Γ , and ds is the differential path length.

In the case of a linear elastic material, $J = G$, and the stress intensity factor can be directly calculated.

The relation between J and SIF for a linear elastic material (LEFM):

$$J = \frac{K_I^2}{E'} \quad (2.6)$$

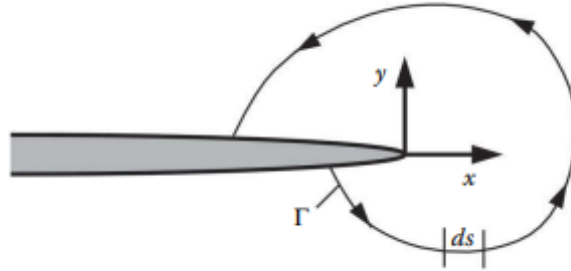


Figure 2.11: Contour around the crack tip [3]

The use of the J-integral provides a solution for the calculation of SIFs within the FEM framework. However, in the classical FEM approach, the calculation applies to a specific crack geometry that must be followed by the mesh. The propagation criterion must be evaluated, and the FEM mesh has to be adjusted accordingly. The analysis must then be repeated as many times as necessary which can be cumbersome. Using more advanced methods makes it possible to automate the analysis procedure. Automated re-meshing can be applied using plug-ins for commercial FEM software such as Abaqus; however, it may lead to numerical instability and high computational costs. [51]

Different crack criterias and models exist for the analysis of crack propagation. In linear-elastic regime, the most widely used is the SIF which quantifies the stress field. [2] An alternative approach is to use cohesive zone models. In CZM, the fracture process is modelled by a traction-separation law. Several different TSLs exist that try to model the separation of crack faces as accurately as possible. [42]

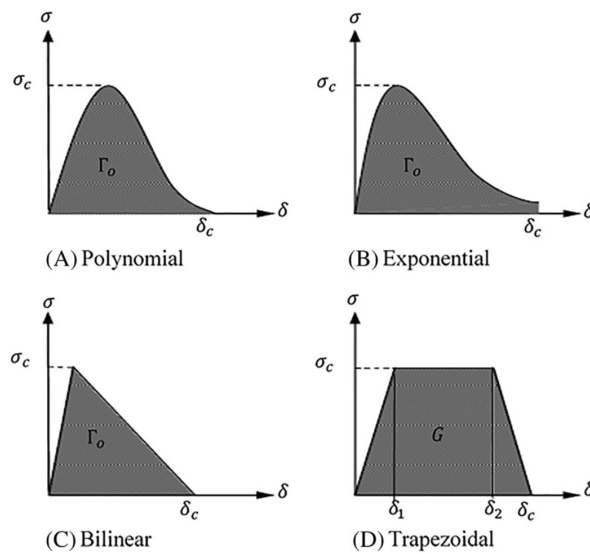


Figure 2.12: Widely applied traction separation laws [42]

The cohesive model tries to capture the phenomenon of separation rather than quantify singularity. Traction-separation laws introduce a relationship between stresses and strains; furthermore, they allow modelling progressive damage and material degradation. For a pre-defined TSL, two of the cohesive parameters (Γ_0 , σ_c , and δ_c) are usually sufficient for simulating the fracture process, where Γ_0 is the energy dissipated during the formation of new surfaces, σ_c is the critical cohesive traction, and δ_c is the critical cohesive separation at which the cohesive element fails. [42] The CZM criterion assumes that crack growth occurs when the cohesive forces at the crack tip are no longer able to hold the two crack faces together. [2] Unlike traditional fracture mechanics methods, CZM is less sensitive to the mesh configuration, making it easier to implement in finite element models. [1]

On the contrary, the cohesive elements must be placed *a priori* in the FEM scheme, which may limit applicability where the crack path is not known in advance. [51]

To deal with scenarios where the crack path is not pre-defined, the extended finite element method (XFEM) can be used. This method is an extension of traditional FEM; it uses the enrichment of nodes to define discontinuities in the displacement field. There are two enrichment functions incorporated in XFEM. The first includes a Heaviside function to represent the gap caused by the crack in the material. The second includes crack tip functions, which account for the singularity at the crack tip.

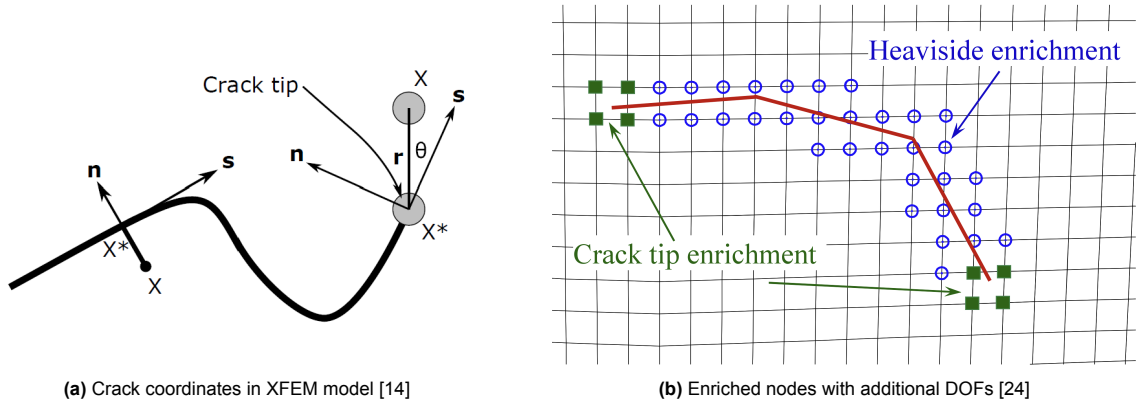


Figure 2.13: XFEM enrichment and crack coordinate system

By the nodal degree-of-freedom enrichment, the crack can propagate through the elements, in contrast to the classical FEM approach, where displacement jumps can only take place along the element boundaries.

$$\vec{u} = \sum_{i=1}^N N_i(x) \left[\vec{u}_i + H(x) \vec{a}_i + \sum_{\alpha=1}^4 F_{\alpha}(x) \vec{b}_i^{\alpha} \right] \quad (2.7)$$

where $N_i(x)$ are the usual nodal shape functions; the first term on the right-hand side of the above equation, $\vec{u}_i$, is the usual nodal displacement vector associated with the continuous part of the finite element solution; the second term is the product of the nodal enriched degree of freedom vector, $\vec{a}_i$, and the associated discontinuous jump function $H(x)$ across the crack surfaces; and the third term is the product of the nodal enriched degree of freedom vector, $\vec{b}_i^{\alpha}$, and the associated crack-tip functions, $F_{\alpha}(x)$. The first term on the right-hand side is applicable to all nodes in the model; the second term is valid for nodes whose shape function support is cut by the crack interior; and the third term is used only for nodes whose shape function support is cut by the crack tip. [14]

$$H(\mathbf{x}) = \begin{cases} 1, & \text{if } (\mathbf{x} - \mathbf{x}^*) \cdot \mathbf{n} \geq 0, \\ -1, & \text{otherwise.} \end{cases} \quad (2.8)$$

Enhancing the entire mesh would result in significant computational costs. Therefore, nodal enrichment zones can be defined in regions where crack propagation is expected, allowing the majority of nodes to retain the standard FEM formulation and thereby mitigating the increase in computational demand. Furthermore, more advanced methods can be found in the literature to reduce computational costs and improve accuracy, such as adaptive meshing techniques. [2]

Although several numerical approaches can be used to model fatigue crack initiation and propagation, the present thesis does not aim to simulate the full crack propagation process. Instead, the FEM model is used to determine the stress intensity factor range, ΔK , for different crack lengths, which is required for the evaluation of the Paris-law parameters.

2.4. Experimental measurement techniques

The $\frac{da}{dN}$ crack growth rate of a specific material must be determined through standardised experimental procedures. These methods enable reproducibility and allow comparison between different materials. The most widely used standards in this field are published by ASTM International.

Several standardised specimen geometries exist; the most important ones are compact tension (CT) and single edge notched bending (SENB) specimens. These tests must follow strict regulations and be carried out carefully, considering several factors, including load ratio, environment, specimen geometry, and measurement accuracy. To obtain reliable results, well-designed testing procedures and proper data interpretation are essential. The tests can be either force-controlled or strain-controlled. In the case of high-cycle fatigue, force-controlled testing is typically used. During the test, the loading and the number of cycles are governed by the loading system, but crack monitoring must be performed using additional measurement techniques.

The CT and SENB configurations apply Mode I loading in different ways. In a CT specimen, the cyclic force is transferred through two loading holes, and the crack propagates through the remaining ligament. CT specimens are commonly used for determining fatigue crack growth rates according to ASTM E647 because their geometry and stress intensity factor solutions are standardised. Crack length can be monitored visually or by indirect techniques such as compliance and electrical potential-drop measurements. [6]

In a SENB specimen, the notched beam is subjected to three-point or four-point bending, and the crack propagates from the tensile side of the specimen. The same basic configuration is commonly denoted as SE(B) in fracture toughness standards. In four-point bending, the region between the two loading points is subjected to a constant bending moment. This configuration provides space for measurement devices on the top, bottom, and side surfaces of the specimen. However, the stress intensity factor depends on the specimen dimensions, crack length, and loading span, and therefore an appropriate geometry correction function is required. [5]

A relevant example of a CT experiment was presented by Seitzl et al., who compared the fatigue crack propagation behaviour of S235 J2, S355 J2, and steel extracted from an old crane structure. CT specimens with a width of $W = 50$ mm and a thickness of $B = 10$ mm were tested at a stress ratio of $R = 0.1$. The crack length was measured on both sides of each specimen, and the average value was used to determine the crack growth rate and the Paris-law parameters. This experimental arrangement allowed the historical and modern steels to be compared using the same specimen geometry and loading conditions. [44]

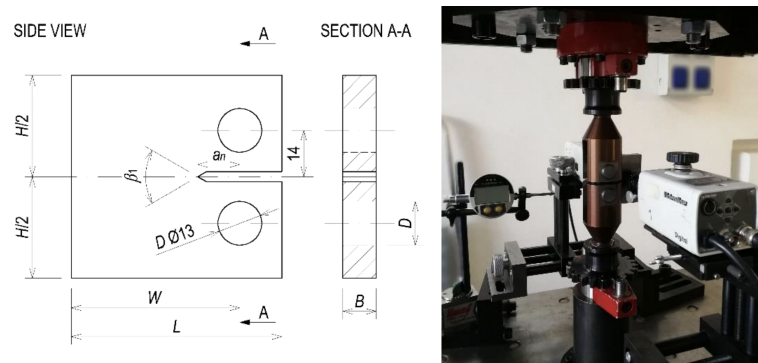


Figure 2.14: CT specimen geometry and experimental setup for fatigue crack propagation tests [44]

A relevant SENB experiment was presented by Saeed et al., who investigated short- and long-crack propagation in welded S355NL structural steel. The specimens were subjected to force-controlled four-point bending at a stress ratio of $R = 0.1$. The tests performed in air used a frequency of 10 Hz. Crack propagation was monitored using strain gauges positioned near the notch on the front face and at

the centre of the back face of the specimens. The measured strain values were converted into crack length using compliance relationships. This arrangement allowed the short- and long-crack regimes to be monitored using the same specimen geometry. The experimental setup is shown in Figure 2.15. [41]

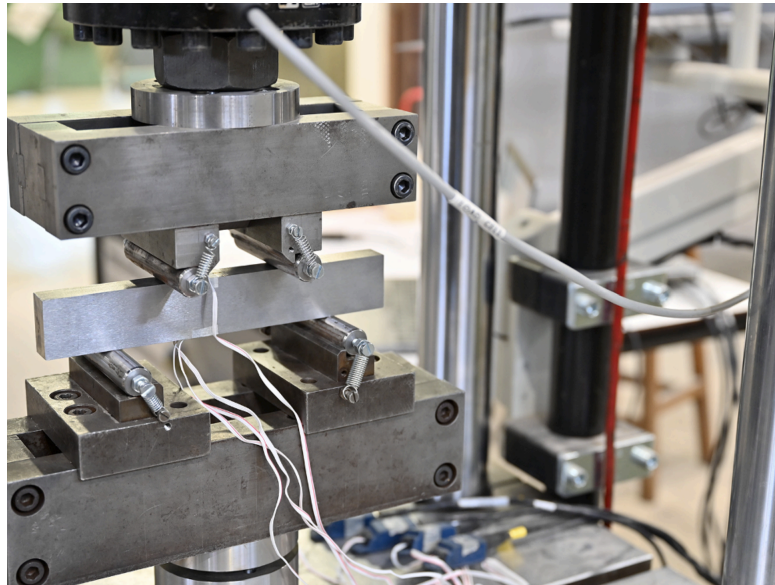


Figure 2.15: Four-point-bending fatigue test setup reported by Saeed et al. for SENB specimens made of welded S355NL structural steel [41].

Another possible method to capture crack propagation is to attach crack gauges to the specimen. In such gauges, conductive wires are embedded in a polymer matrix. As the crack propagates in the specimen, it also propagates through the gauge, breaking the wires due to the elongation associated with crack opening. The gauge must be connected to an electrical circuit, and the change in electrical resistance is measured. The main drawback of this technique is that it provides discrete data points regarding crack growth, corresponding to the sequential failure of individual wires. This means the exact crack tip location is unknown at the moment of the detection of wire breakage. Furthermore, the output data requires careful processing and interpretation. [47]

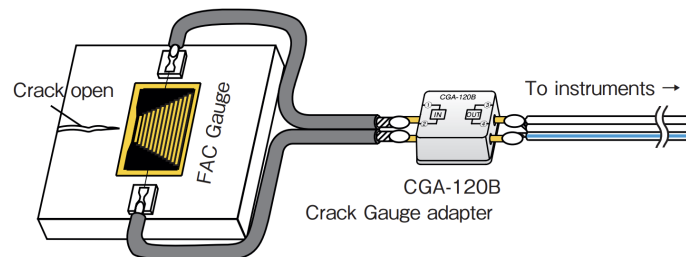


Figure 2.16: Crack detection scheme with gauge from FAC series produced by TML [47]

Another frequently used method is based on the precise measurement of crack opening displacement, which can be correlated with crack length. ASTM standards define specific requirements for specimen preparation.[5] For accurate crack length determination, precisely machined reference edges are required. So-called Crack Opening Displacement gauges are available commercially; one example is the Instron 2670 Series COD clip-on device. However, while this method provides precise and continuous output data, it is indirect. The crack length at each cycle must be determined from the measured displacement using specimen-specific compliance relationships. Therefore, inaccuracies in specimen geometry or its measurement can introduce additional errors into the evaluation.

Thus, crack gauges provide more direct but discrete measurements, whereas COD-based methods

offer continuous but indirect crack length estimation. The information on crack length evolution obtained from these measurement techniques makes it possible to determine key crack growth parameters, such as the crack growth rate $\frac{da}{dN}$.

Furthermore, several additional techniques can be applied to monitor crack propagation, including CCD cameras [44], acoustic emission (AE) measurements [27], Digital Image Correlation (DIC), and the detection of magnetic field discrepancies. AE technology enables real-time monitoring of microcrack initiation and growth by capturing transient elastic waveforms generated within the material. DIC provides full-field displacement and strain maps, allowing for a detailed analysis of surface crack propagation. The detection of magnetic field discrepancies can also provide valuable information on crack presence and progression in ferromagnetic materials, as changes in the magnetic response may indicate damage evolution.

In this thesis, the crack length information obtained from the applied measurement techniques is used to determine the crack growth rate, da/dN , which is then combined with the numerically determined ΔK values for the evaluation of the Paris-law parameters.

Methodology for fatigue crack growth evaluation

3.1. Overview of the methodology

This chapter presents the workflows and methods used to determine the fatigue crack propagation parameters of the tested SENB specimens. The test setup, specimen geometry, loading conditions and the applied measurement systems are included. Furthermore, the developed post-processing methodology is described, which was used to evaluate the experimental data. The main objective was to obtain the Paris-law parameters, therefore, the most important datasets are the crack length curves. The crack propagation rate is calculated by differentiating the crack length data, while the stress intensity factor range is directly related to the crack length. These two quantities are used for the determination of the Paris-law parameters. In the calculation of the stress intensity factor range, a geometry correction function had to be obtained; for this purpose, a numerical analysis was carried out, which is also presented. Since different measurement systems were applied, the post-processing procedures had to be adjusted to determine the crack length evolution. These different procedures are also presented in detail in the following sections.

3.2. Experimental programme

In order to determine the fatigue crack growth properties accurately, laboratory tests were performed at the Stevin Laboratory of TU Delft. The specimens used in this study were extracted by Nebest B.V. from the flanges of the main girders of the Old Botlek Bridge.

The location of the main girders within the bridge is shown in Figure 3.1.

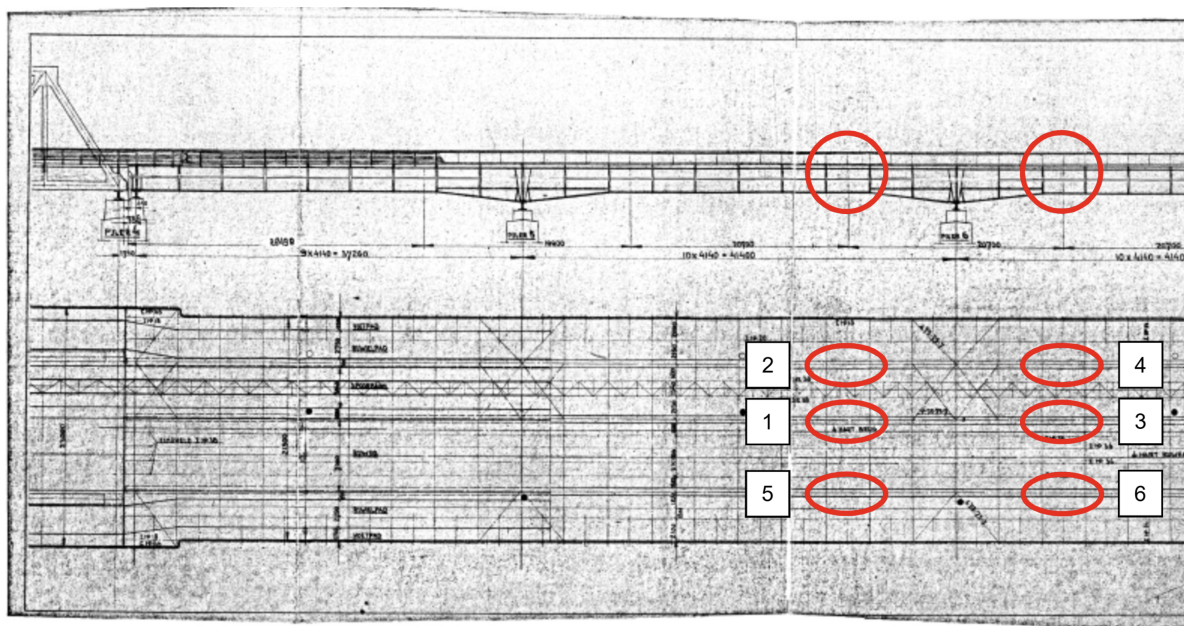


Figure 3.1: Elevation (top) and plan view (bottom) of part of the Old Botlek Bridge, with the locations of the recovered main-girder sections highlighted in red. Source: TNO report [17].

The steel I-girders are currently stored in the Rijkswaterstaat storage yard. The original positions of the extracted flange sections within the main girders are unknown. Therefore, it cannot be determined whether the specimens were taken from flanges mainly subjected to tension or compression during service.

A total of twelve experiments were performed in two testing rounds. Six specimens were tested in September 2025, and the remaining six were tested in February 2026.

The origin and preparation of the tested specimens are shown in Figure 3.2.



(a) Dismantled steel members from the Old Botlek Bridge at the storage site.



(b) Marked locations for specimen extraction.



(c) Steel section after extraction from the original member.



(d) Prepared SENB specimens before fatigue testing.

Figure 3.2: Origin and preparation of the SENB specimens extracted from dismantled steel members of the Old Botlek Bridge. Photographs provided by Nebest B.V.

3.2.1. Specimen geometry and loading conditions

In fracture mechanical testing, different specimen types are used. In this study, a single-edge notched bending specimen was selected due to its widespread application. The specimen was loaded in four-point bending with two concentrated forces and two simple supports. Steel cylindrical rollers were used for both loading and support. This setup allowed stable crack growth under Mode I crack-opening conditions. The SENB specimen consists of a beam with a single-edge notch, which guides the propagation of the crack. The role of the pre-manufactured notch is to create a sharp crack tip and to minimise the influence of crack initiation. The crack length is denoted by a , while the initial notch length before fatigue loading is denoted by a_{init} .

In Figure 3.3, the nominal geometry is shown. Because of the manufacturing of the specimens, some discrepancies occurred due to machining tolerances. The detailed specimen geometries are listed in Table 3.1.

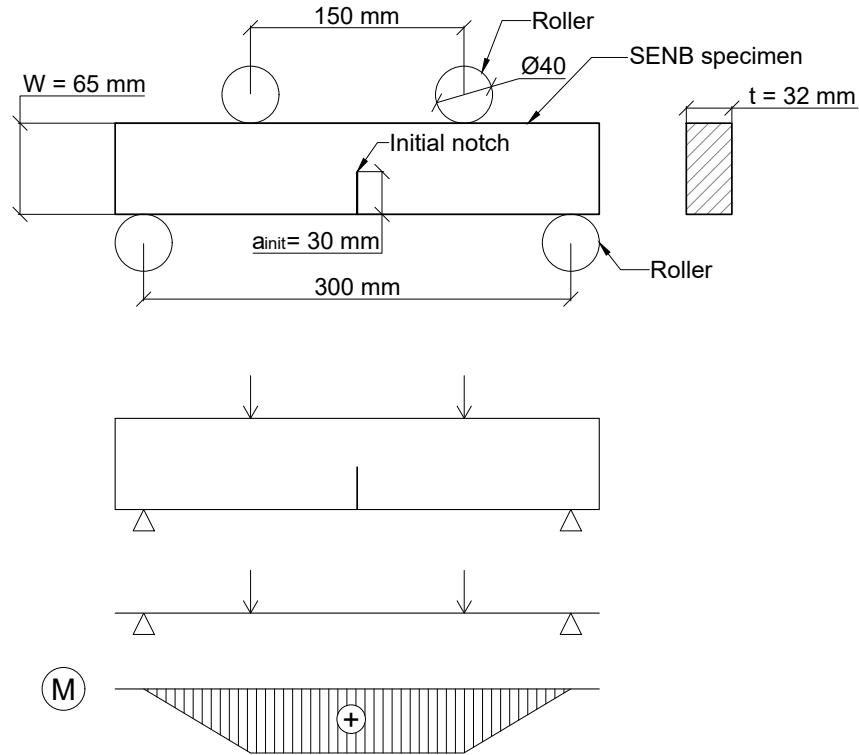


Figure 3.3: SENB specimen geometry and basic mechanical scheme in four-point bending

Table 3.1: Specimen geometries

Specimen name	W (mm)	B (mm)	a_{init} (mm)
S111	63.54	31.85	29.20
S112	63.5	31.73	29.11
S112str	63.5	31.75	29.36
S131	66.64	33.32	30.99
S132	66.97	33.56	30.83
S132str	67.08	33.64	30.30
S211	65.55	32.73	30.0
S212	64.47	32.17	30.0
S212str	64.58	32.60	30.0
S232	66.10	32.97	30.0
S233	65.99	32.94	30.0
S233str	65.96	32.97	30.0

Loading protocol

During the tests, a specified loading protocol was applied. High-cycle fatigue tests were conducted, allowing for the assumption of linear-elastic material behaviour. Therefore, force-controlled loading was used rather than strain-controlled loading. Sinusoidal cyclic loading was applied at a frequency of 10 Hz.

For the testing, a servo-hydraulic testing machine was used to apply cyclic loading to the specimens. In principle, a stress ratio of $R = 0$ would have been satisfactory, however, in the case of a hydraulic system, it is challenging to control forces near zero load. In every cycle, the machine would have to be completely unloaded and loaded again. This leads to practical difficulties and could negatively affect the accuracy of the test. Therefore, the stress ratio was chosen as $R = 0.1$. Additionally, the contact between the machine and the specimen would be limited for a moment in each cycle, which could lead to small relative displacements of the specimens in the form of slipping, introducing possible errors in the measurements. Furthermore, at $R = 0$, crack closure occurs, making the comparison of results

less reliable. [15]

At the beginning of each experiment, the load was gradually increased in two steps to the mean load value over 25 seconds. Then, the oscillation was started with the amplitude load. The stop criterion was a 13 mm vertical displacement. Once this was reached, the test stopped.

Table 3.2 shows the number of cycles recorded until the end of the crack propagation tests. Specimen S111 lasted considerably longer during testing than the other specimens. The reason for this difference was the different loading protocol applied during the test. Therefore, specimen S111 was excluded from the evaluation. The remaining specimens showed a fatigue life of approximately 400,000 cycles, with some scatter.

Table 3.2: Crack propagation life of the tested SENB specimens

Specimen name	Number of cycles
S111*	772 975
S112	409 082
S112str	368 443
S131	420 057
S132	419 955
S132str	336 903
S211	430 344
S212	586 130
S212str	427 759
S232	453 460
S233	456 975
S233str	410 904

* A different loading protocol was applied to S111.

3.2.2. Applied measurement systems

During the tests, different measurement techniques were applied in order to monitor crack growth. Crack gauges, strain gauges, and a COD device were used simultaneously to improve the reliability of the measurements. The principles of these measurement techniques were introduced in Chapter 2. The COD device was positioned at the crack mouth on the bottom side of the specimens using plastic clips glued to the specimen to fix the device. The crack gauge was glued to the side of the specimen, starting at the crack tip, to measure the propagation of the crack tip directly. Finally, the strain gauge was attached to the top surface of the specimen using adhesive. An overview of the installed measurement systems is shown in Figure 3.4. A close-up of the strain gauge and the COD device is presented in Figure 3.5.

In addition, magnetometers were used to record the magnetic stray field, as it carries information about the magnetisation of the specimens. However, this measurement technique is outside the scope of the present study.

Table 3.3: Used measurement products

Gauge type	Used product
Crack gauge	TML FAC-20
Strain gauge	Unknown
COD	Instron 2670-132

3.3. Processing of experimental data

By conducting the experiments, large datasets were obtained through the measurements. These datasets had to be processed for interpretation. Due to the use of different measurement methods, different post-processing workflows were used. In the following section, these workflows are presented in detail. The general workflow can be seen in Figure 3.6.

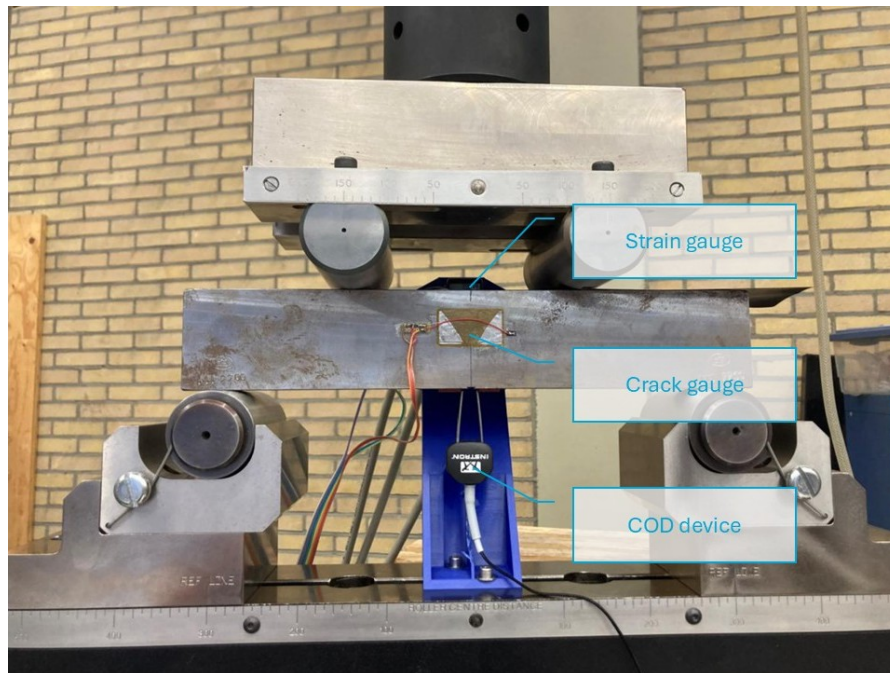


Figure 3.4: Overview of the measurement systems applied during the fatigue tests

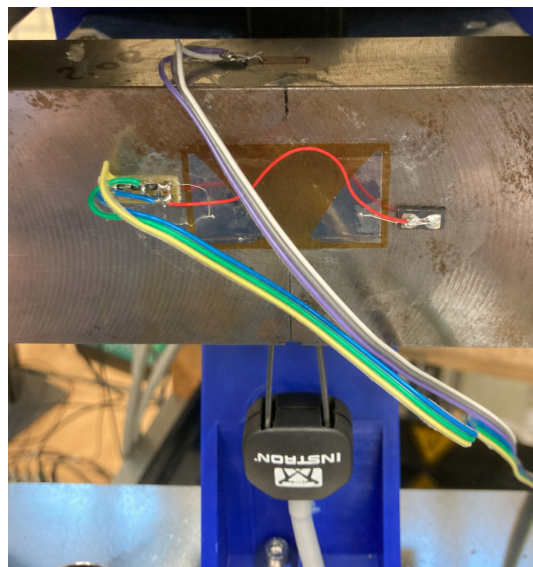


Figure 3.5: Close-up photo of the measurement devices

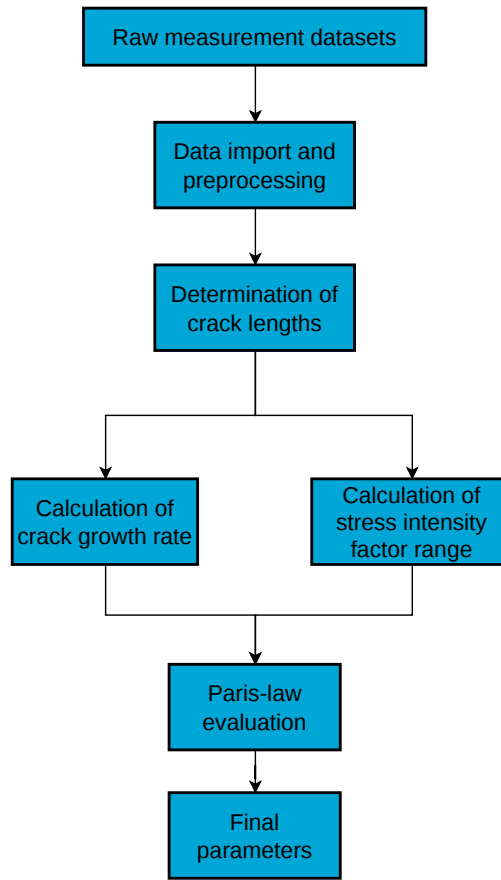


Figure 3.6: General post-processing workflow

3.3.1. Crack length determination from crack gauge data

In the case of a crack gauge, a DC circuit must be set up with the crack gauge acting as a resistor. The individual conductive wires in the gauge behave as multiple resistors connected in parallel. The aim is to record the voltage while maintaining a constant current. As the test progresses, the crack propagates through the specimen and the bonded crack gauge, causing the wires to break one after another. As a result, the total resistance of the gauge increases. This produces a quasi-stepwise voltage signal with a certain amount of noise. From the stepwise signal, the cycle numbers corresponding to the individual wire breakages had to be extracted. These positions on the cycle-number axis were then used to determine the sequence of wire failures and calculate the corresponding crack length.

The crack length can be written as

$$a(N) = a_{\text{init}} + x_0 + n(N) \cdot p \quad (3.1)$$

Here, $a(N)$ is the crack length at a given number of cycles N . The term a_{init} is the initial notch, which was measured in advance of the experiments. The term x_0 is the signed offset between the initial notch tip and the first relevant wire of the crack gauge. It accounts for the fact that the crack gauge was not glued exactly at the notch tip. The term $n(N)$ is the number of broken wires at cycle number N , which increases stepwise as the crack propagates through the gauge. Finally, p is the wire spacing of the FAC-20 crack gauge; in this case, $p = 0.5 \text{ mm}$. The sign of x_0 depends on the position of the first gauge wire relative to the notch tip. If the first wire is located ahead of the notch tip in the crack propagation direction, x_0 should be positive. If it is located behind the notch tip, x_0 should be negative.

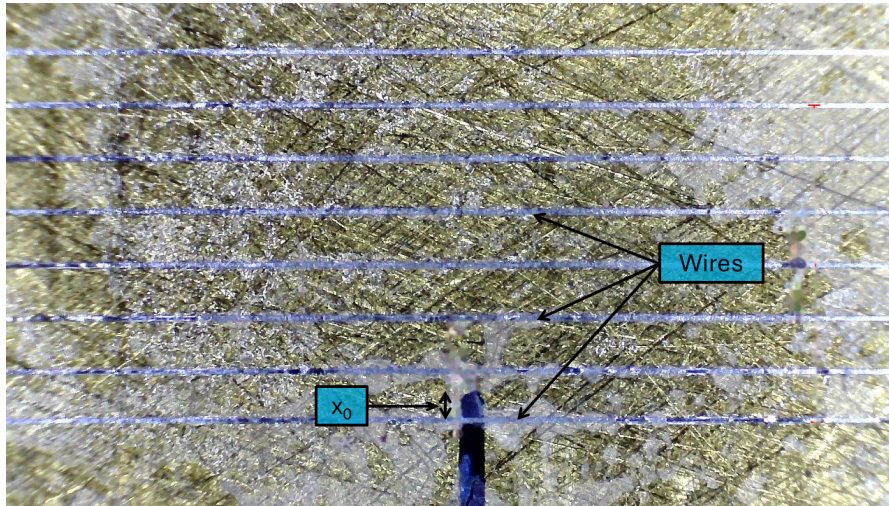


Figure 3.7: Distance between the crack tip and the first wire in a microscopic image

The most complex task in data processing was detecting the wire breakage points on the cycle counter axis to determine $n(N)$ vectors for all specimens.

The noise in the raw data was not critical and could be handled. Occasional spikes appeared in some cases due to external disturbances, and these had to be filtered out to make the wire crack detection possible considered as preprocessing.

In the preprocessing phase, a rolling median filter was applied in order to eliminate the outlier data points. In this method, a loop was applied over all data points, and each point was evaluated within a certain window. The median of the window was calculated for each point, and the difference between the current data point and the median was evaluated. For the assessment of the data point, the MAD, or Median Absolute Deviation, was used. If the difference was too large compared to the local median, the data point was considered an outlier and was removed from the signal. The purpose of this filtering was not to smoothen the entire signal, but only to eliminate the spikes. This step was done for all of the examined specimens.

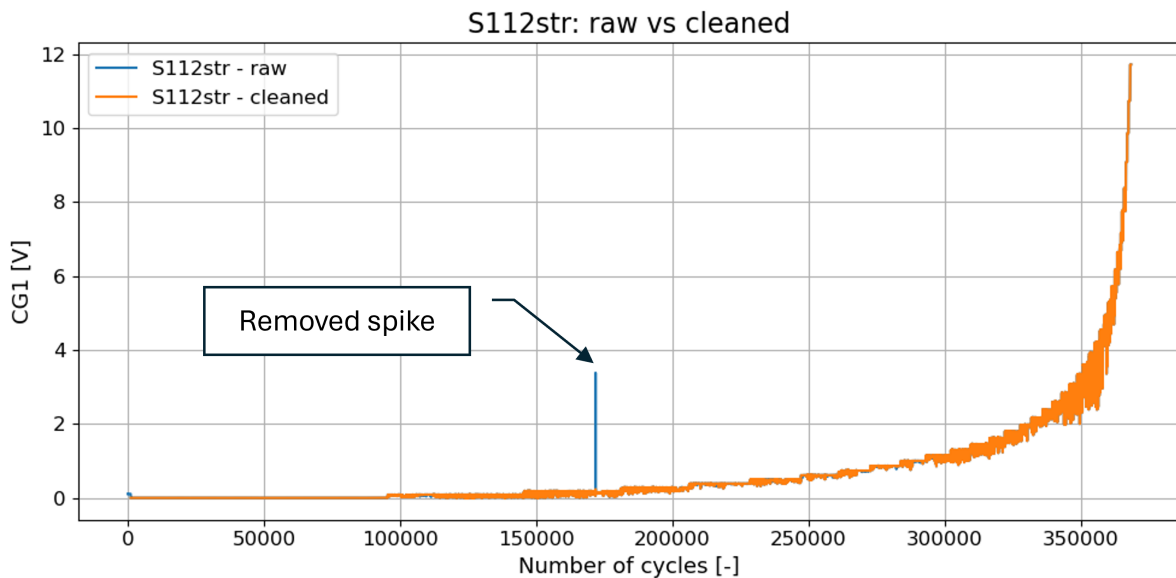


Figure 3.8: Example of preprocessing

At this stage, the data had too high fluctuation for precise wire breakage detection. An upper envelope was generated point by point as a running maximum of the measured voltage signal. At each data point, the current voltage value was compared with the previous envelope value, and the larger

value was kept as the new envelope value:

$$U_i^{\text{upper env}} = \max(U_i, U_{i-1}^{\text{upper env}}), \quad U_0^{\text{upper env}} = U_0 \quad (3.2)$$

As a consequence, monotonically increasing signals were obtained. An example can be seen in Figure 3.9.

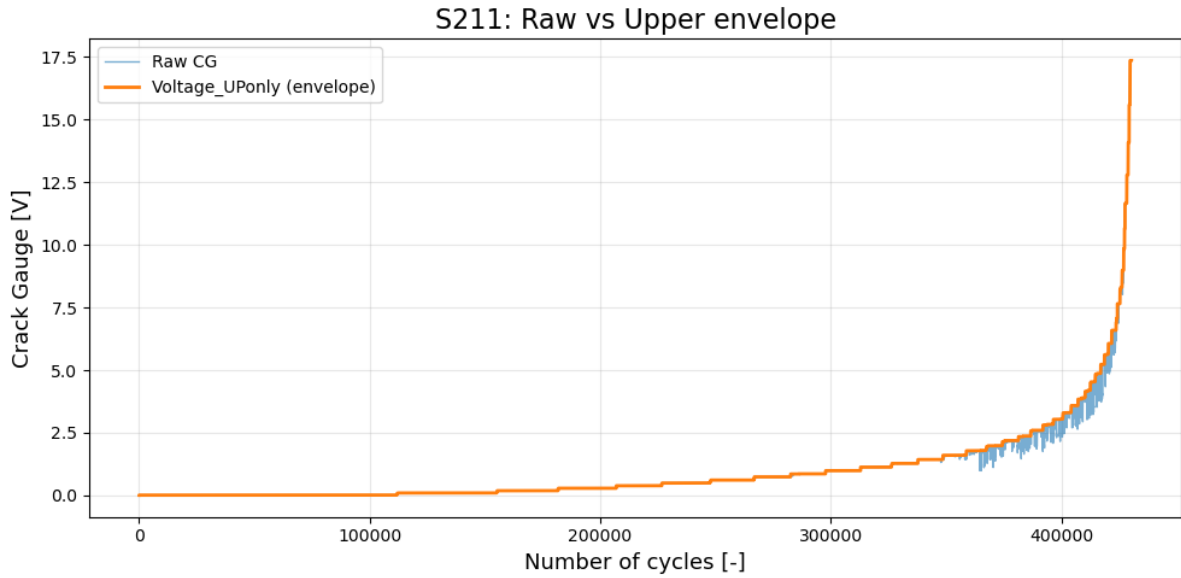


Figure 3.9: Example of an upper envelope

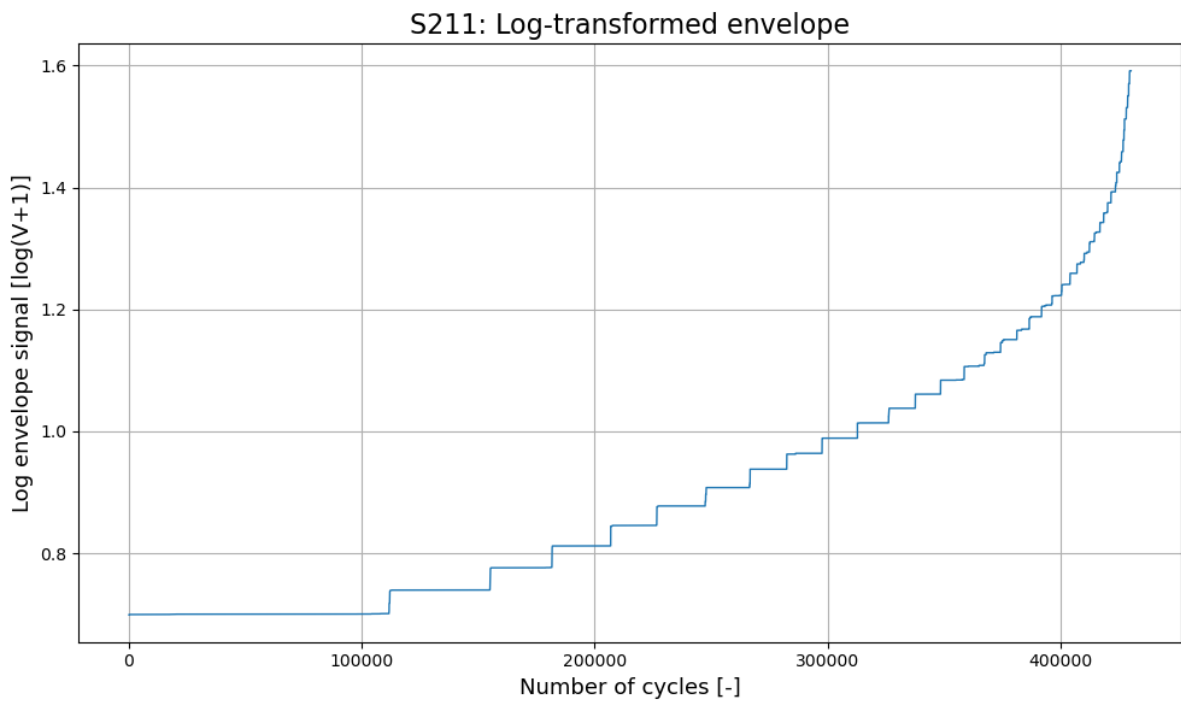


Figure 3.10: Example of a log-transformed envelope

Based on the envelopes, it was already possible to determine the discrete jumps in the signal, which were considered as wire breakages. In the initial stage, the magnitude of the voltage jumps was significantly lower than in the final stage due to the exponentially increasing behaviour of the signal.

Thus, it was difficult to define a basic logic that could precisely capture the data points of interest. To reduce the large difference in the magnitude of the jumps, the log-transformed voltage signal was used to make the jumps more even. The basic assumption of this method is that the breakage of a wire results in a positive jump in the voltage signal. The algorithm checks whether the difference between two consecutive measurement points exceeds a certain threshold.

$$s_i > s_{i-1} + dV \quad (3.3)$$

where dV is an a priori defined threshold, s_i is the i th measurement point, and s_{i-1} is the $(i - 1)$ th measurement point. When the condition is met, the cumulative number of cracks is increased by one in the list $c(N)$. By this $c(N)$ becomes a step function. At every cycle it shows how many cracks are detected. The indices of crack events can be found by the following rule:

$$\Delta c_i = c_i - c_{i-1} > 0 \quad (3.4)$$

These indices define the points where the algorithm identified a new crack. After collecting all the data points where a new crack occurred, $n(N)$ is obtained, and the crack length can be calculated.

If the dV value is too low, the check is too sensitive and false detections may occur. On the other hand, if dV is too high, it cannot capture all wire cracks. To solve this issue, a binary search was conducted for all specimens to determine the best dV value within a predefined interval.

$$dV \in [dV_{\text{low}}, dV_{\text{high}}] \quad (3.5)$$

During the binary search, the aim was to minimize the difference between the detected number of cracks and a target number. In the FAC-20 crack gauge, there are 41 wires in total. Therefore, the required number of crack detections is 41. In the binary search, the target wire crack number was first set to 41, which means that the algorithm iterated until it found the dV value that detected exactly 41 cracks. However, this did not always provide the desired result, since in some cases additional unnecessary points were detected, while in other cases some actual wire cracks were missed. Therefore, the target number was increased to 49 to generate an intentionally over-detected set of candidates. The eight candidates with the smallest cycle spacing to neighbouring detections were treated as likely duplicates and removed, leaving 41 detected events. A retained event was accepted only when its location coincided with a visible positive step in the log-transformed voltage signal. The final wire-breakage detections were visually checked against the voltage signal. If necessary, the detection settings were adjusted and the semi-automatic procedure was repeated. In some cases, fewer than 41 wire-breakage events were visible in the signal because the crack did not propagate through all the conductive wires in the gauge.

As illustrated in Figures 3.11 and 3.12, the initial detection procedure identified several potential wire breakage points, which were subsequently filtered to obtain the final set of crack detections. The resulting points were then converted into crack length values using Equation 3.1. An example of the resulting crack length evolution is shown in Figure 3.13. This curve represents the crack length history, $a(N)$, obtained from the crack gauge data and serves as one of the input datasets for the crack growth rate calculation.

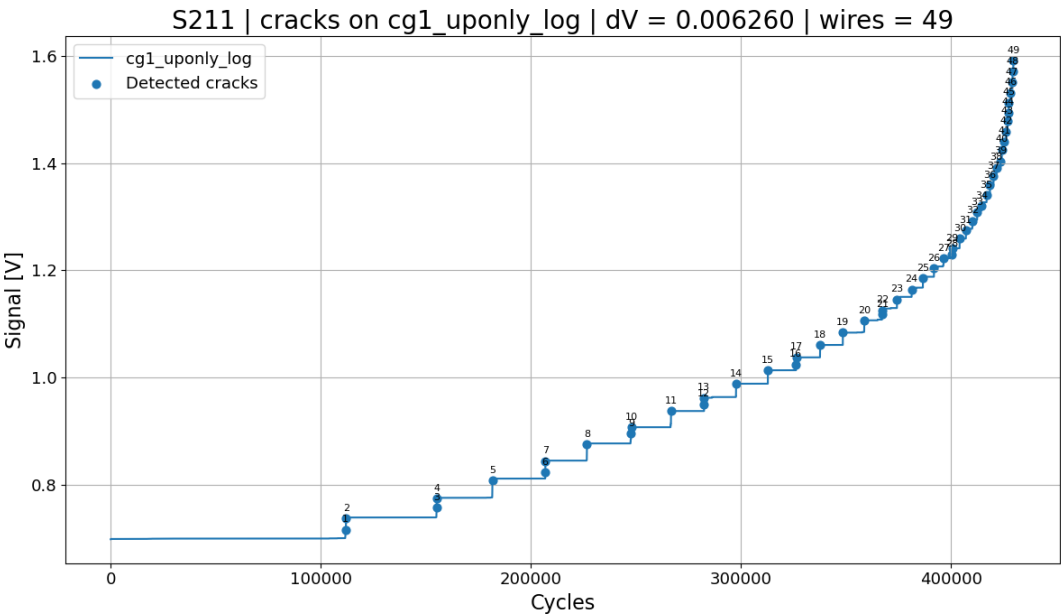


Figure 3.11: All detected potential cracks on the log-transformed signal

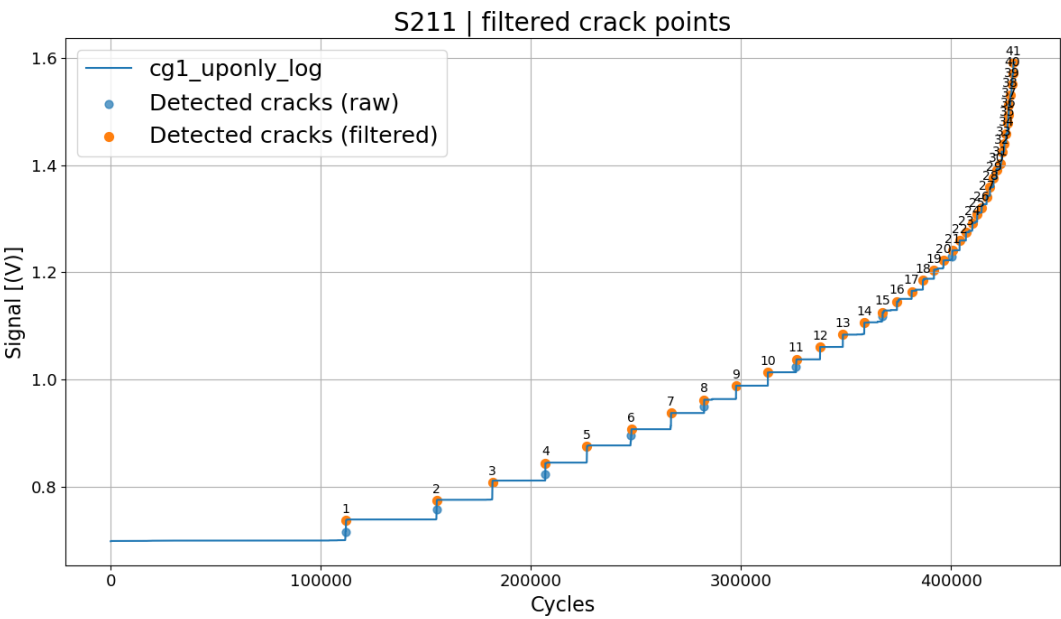


Figure 3.12: Filtered crack detections on the log-transformed signal

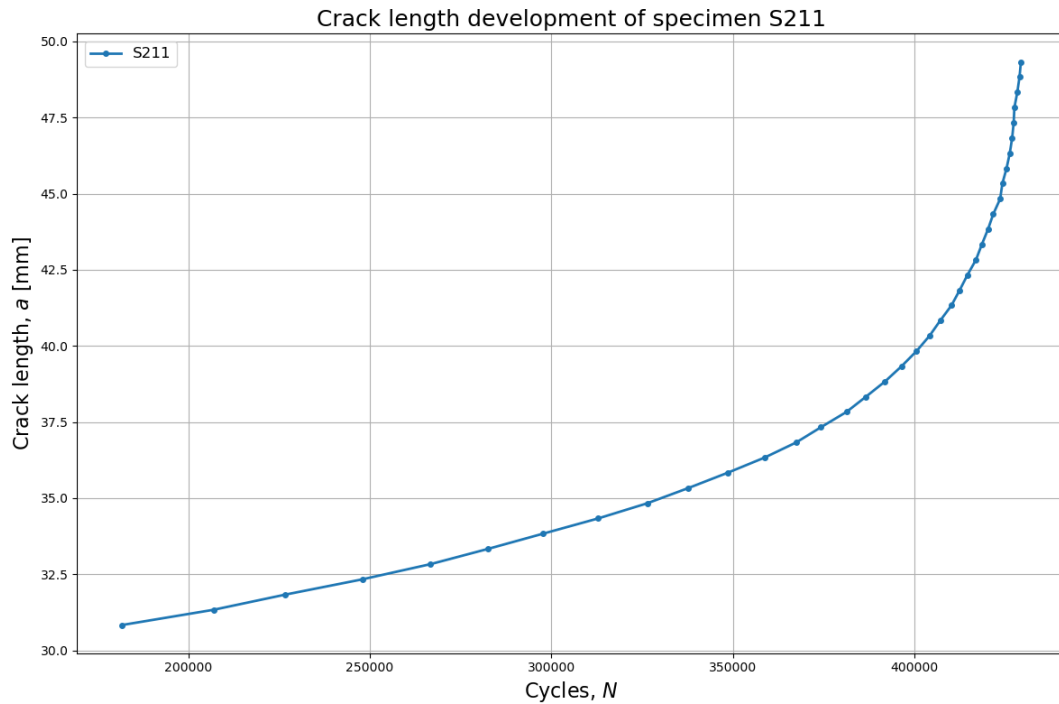


Figure 3.13: Example of a crack length curve using crack gauge measurements

3.3.2. Crack length calculation from COD data

In the case of the crack gauge, the crack length can be calculated directly from the measurements by counting the broken wires in the gauge. Using COD measurements, the calculation is indirect, and the crack length must be inferred from the measured crack opening. Therefore, the relationship between the crack opening and the crack length had to be determined in order to evaluate crack propagation. This requires precise calibration, which is not straightforward analytically for the present test configuration; therefore, a FEM model was applied for this purpose.

In Abaqus, a 2D in-plane model was set up to simulate the deformation of the specimen under the nominal loading, including the notch geometry. Several models were created with predefined crack lengths ranging from 30 mm to 55 mm in 1 mm increments. Static analyses were carried out using linear-elastic material properties for steel. Namely, a Young's modulus of 200 GPa was used. The model was formulated under a plane stress assumption and discretised mainly using CPS8R elements, which are 8-node biquadratic plane stress quadrilateral continuum elements with reduced integration. Consequently, the model does not capture through-thickness effects, such as crack-front curvature or variations in crack-tip constraint through the specimen thickness. Two point supports were applied to represent the simply supported boundary conditions, and two concentrated forces were applied at the top of the model to represent the cylindrical loading rollers. The applied nominal load was 25 kN in total, corresponding to 12.5 kN from each roller.

The exact positions of the COD knife edges were measured on all specimens in order to determine the two points between which the crack opening had to be evaluated in the numerical model. In general, the distance between these points was 13 mm; therefore, this value was implemented in the model. All calculations were performed using a Python script based on the nodes located at the measured positions. The existence of these nodes was ensured by predefined datum planes, which forced the automated meshing procedure to include nodes along these lines. The absolute coordinates of the selected nodes were extracted from the deformed model using the Python script. The distance between the two nodes was then calculated for each model, and the relative crack opening was determined from the change in this distance with respect to the undeformed configuration. By extracting the crack openings for all predefined crack lengths, it was possible to fit a curve to these data points and determine the $a - \text{COD}$ relationship.

The numerical procedure therefore provided a calibration between the crack length and the corresponding COD value for the present four-point bending specimen configuration. Since the analyses

were linear elastic, the calculated COD values were assumed to scale linearly with the applied load. Consequently, the COD values obtained from the model with a total load of 25 kN could be scaled to the actual load level of each specimen. The resulting calibration curve was then used in inverse form to determine the crack length history from the experimentally measured COD data. This calibration was only applied within the crack length range covered by the numerical models.

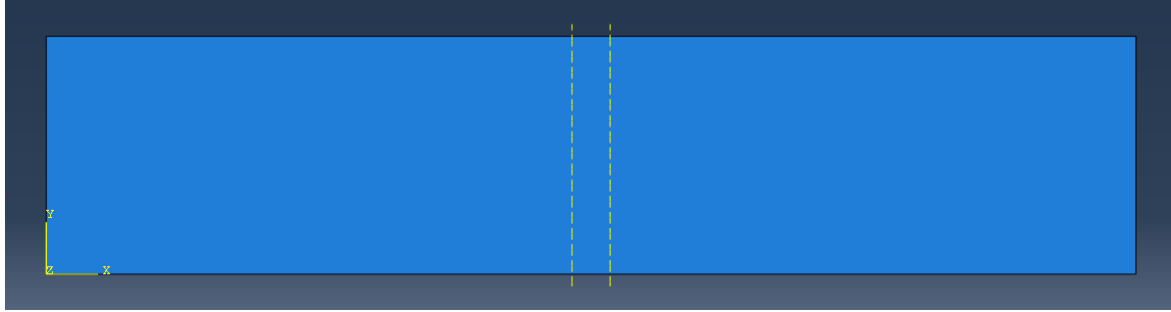


Figure 3.14: The defined datum planes for COD value extraction

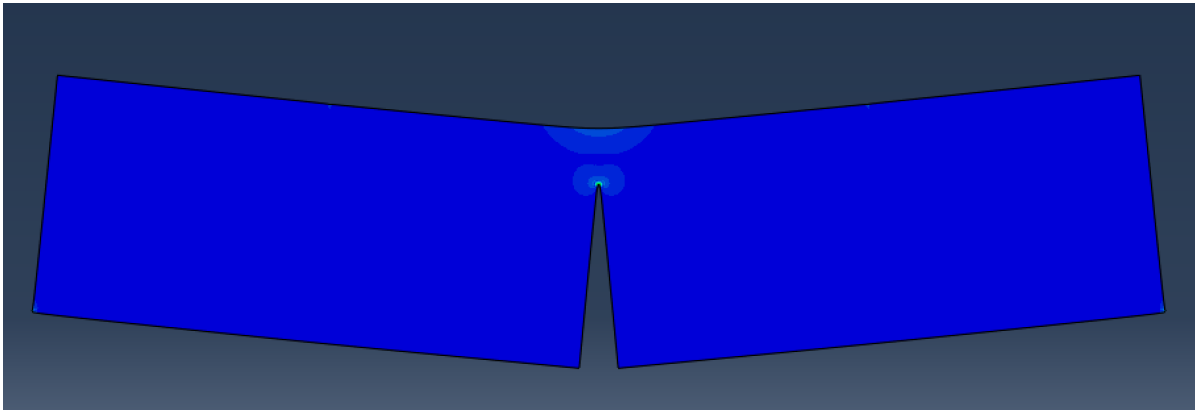


Figure 3.15: Crack opening in Abaqus (scaled for better visual representation)

A power-law function was fitted to the extracted numerical points using the least-squares method. The fitted function describes the crack length as a function of the measured COD value and has the following form:

$$a(U) = A + B \cdot U^C \quad (3.6)$$

where a is the crack length, U is the measured crack opening displacement, and A , B and C are the fitted parameters. The obtained parameters were:

$$a(U) = 71.02 - 17.13 \cdot U^{-0.3242} \quad (3.7)$$

The quality of the fit was assessed by calculating the root-mean-square error and the maximum absolute error between the Abaqus data points and the fitted curve. The resulting RMSE was 0.0742 mm, while the maximum absolute error was 0.1606 mm. These small differences show that the fitted function represents the numerical $a - \text{COD}$ relationship with sufficient accuracy and can be used to convert the measured COD values into crack lengths.

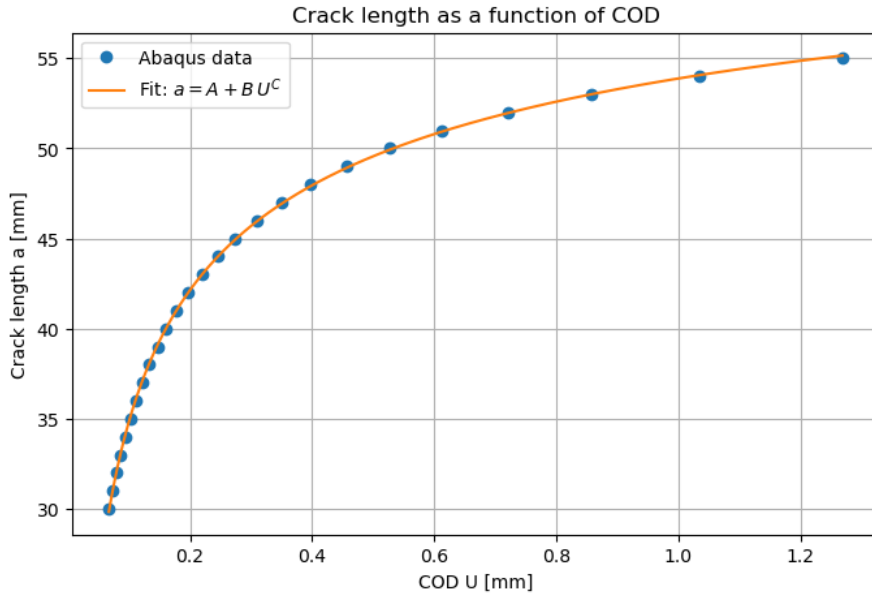


Figure 3.16: Crack length as a function of COD measurements

3.3.3. Crack length calculation from strain gauge data

Exactly the same method was used for the post-processing of the strain gauge data. In the Abaqus model, the used datum plane was positioned in the middle and the node at the top edge was evaluated for E_{11} strains. Similarly, a power-law function was also fitted to the strain-based numerical data using the least-squares method. In this case, the fitted relationship describes the crack length as a function of the measured strain, denoted by ε :

$$a(\varepsilon) = 65.60 - 0.91 \cdot |\varepsilon|^{-0.4894} \quad (3.8)$$

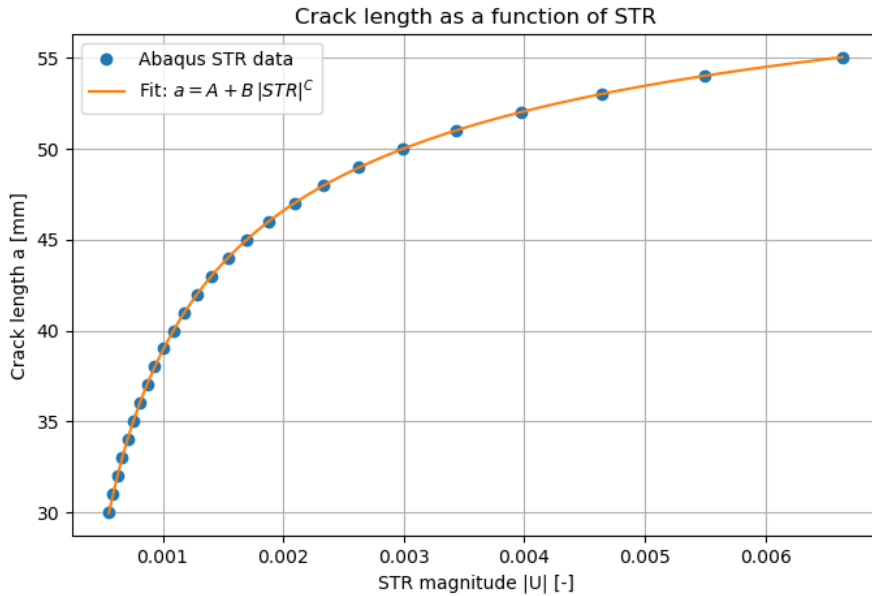


Figure 3.17: Crack length as a function of strain measurements

The accuracy of the fit was again evaluated by calculating the RMSE and the maximum absolute error between the numerical data points and the fitted curve. The resulting RMSE was 0.0171 mm,

while the maximum absolute error was 0.0418 mm, which indicates that the strain-based fitted function also represents the numerical relationship with very high accuracy.

3.3.4. Final post-processing of COD- and strain-gauge-based crack-length data
After converting the COD and strain-gauge measurements into crack length, the data were filtered before the crack growth rate calculation. Constant crack-length offsets of -5 mm and -2 mm were applied to the COD-based histories of S211 and S233str, respectively, to account for discrete discrepancies.

After the offset correction, data points satisfying $a < a_{\text{init},i} + 1$ mm were removed for each specimen i to exclude the initial crack-initiation region. Points satisfying $a > 55$ mm were also removed because the numerical calibration did not cover crack lengths above this value. The remaining crack-length histories were used for the subsequent crack growth rate and Paris-law calculations.

3.4. Crack growth rate calculation

3.4.1. Differentiation of crack length data

After obtaining the wire breakage points, the crack length $a(N)$ on the specimens can be calculated using the previously given formulation. For setting up the Paris-law equation (2.3) it is necessary to calculate the fatigue crack growth rate curves $\frac{da}{dN}$. To do this, there are two recommended methods in ASTM E647 for numerical differentiation applied to limited number of points. [6] The first is the simpler secant method, which estimates the average crack growth rate between two adjacent crack length data points by calculating the slope of crack length versus load cycle curve.

$$\left(\frac{da}{dN} \right)_i = \frac{a_{i+1} - a_i}{N_{i+1} - N_i}. \quad (3.9)$$

The crack length in the midpoint can be used for the corresponding ΔK :

$$a_i^* = \frac{a_{i+1} + a_i}{2}. \quad (3.10)$$

Another option is using the polynomial incremental method. This method reduces data scatter by using several consecutive data points from crack length measurements instead of calculating the crack growth rate directly between two adjacent points. Second-order polynomial is fitted to the data points in a moving window, and the crack growth rate is obtained from the derivative of the fitted curve. This results in smoother crack growth curves; however, with this method it is not possible to calculate the derivative close to the beginning and the end of the dataset due to the window size.

Although some data are lost due to the nature of the method, the first data points are often not meaningful due to crack initiation effects. Even though the notch is manufactured to be as sharp-edged as possible, it still does not behave as a real crack tip. Eliminating the first few data points guarantees that these disturbed data points do not affect the results. A similar perspective applies to the last data points, where crack propagation becomes significantly faster and the resolution of the crack gauge is most likely not sufficient to capture the crack growth precisely. Eliminating the last few data points mitigates the errors caused by these effects.

The calculation procedure can be expressed mathematically as follows. For each evaluated point, a moving window of $(2n + 1)$ consecutive data points is selected. The standard recommends using $n = 3$; therefore, in this work, the fitted window contains 7 crack length measurements:

$$(N_{i-n}, a_{i-n}), \dots, (N_i, a_i), \dots, (N_{i+n}, a_{i+n}). \quad (3.11)$$

A second-order polynomial is then fitted to the crack length versus number of cycles data within this window:

$$a(N) = c_2 N^2 + c_1 N + c_0. \quad (3.12)$$

The polynomial coefficients are obtained by minimizing the least-squares error:

$$\sum_{j=i-n}^{i+n} [a_j - (c_2 N_j^2 + c_1 N_j + c_0)]^2. \quad (3.13)$$

The crack growth rate is calculated as the derivative of the fitted polynomial at the central point of the window:

$$\left(\frac{da}{dN}\right)_i = \left.\frac{da}{dN}\right|_{N=N_i} = 2c_2N_i + c_1. \quad (3.14)$$

Thus, each calculated crack growth rate value is assigned to the central crack length measurement of the corresponding window:

$$\left(N_i, a_i, \left(\frac{da}{dN}\right)_i\right). \quad (3.15)$$

Due to the seven-point differentiation method, (da/dN) could not be calculated for the first and last three crack-length points. The corresponding (da/dN) values were therefore left empty. However, these crack-length data were retained throughout the workflow and are included in the crack-length plots.

3.4.2. Neural-network-based crack growth rate calculation

Due to the large number of data points and the local fluctuations in the measurements, the calculated derivative became highly scattered when the previously used polynomial fitting method was applied. The previously described method was suitable during the crack gauge post-processing, but it did not provide sufficiently stable results for the COD-based crack length data. As a result, the $da/dN - N$ curves became noisy and this made it difficult to extract reliable data points for the $da/dN - \Delta K$ relationship.

Increasing the fitting window reduced the local noise, but it also resulted in significant loss of data, especially in the final and most important part of the crack propagation curve. On the other hand, smaller window sizes preserved more data points, but the first part of the calculated da/dN curve remained noisy. To overcome this issue, a parametric regression model was fitted to the crack length data. For this purpose, artificial neural networks were applied, where the measured data points were used for training the model. The fitted model provided a smooth representation of the crack growth curve, from which the crack propagation rate could be evaluated in a more stable way.

For the neural network application, PyTorch was used as an open-source deep learning library. A relatively simple feedforward neural network was sufficient to fit the data properly. The applied architecture was a multilayer perceptron (MLP), and its general scheme is shown in Figure 3.18. A separate model was trained for each specimen using the same architecture and training settings. The model had one input variable and one output variable, which were the number of cycles, N , and the crack length a , respectively.

The size of the architecture was intentionally kept small since the expected $a - N$ relationship is smooth and does not require a model with high complexity. The network consisted of two hidden layers, each containing 32 neurons. This provided sufficient flexibility for fitting the crack growth curves while reducing the risk of unnecessary overfitting. One of the most important characteristics of a neural network is the applied activation function. In this model, the hyperbolic tangent function, $\tanh$, was used. This activation function provides a smooth and differentiable response, which is essential for calculating the crack growth rate from the derivative of the fitted crack growth curve. Therefore, the use of $\tanh$ helped to obtain a smooth regression model and a stable evaluation of da/dN .

During the training process, the model was optimised by minimizing the mean squared error (MSE) loss function, using the Adam optimiser. The Adam optimiser is an adaptive gradient-based optimisation algorithm derived from stochastic gradient descent (SGD). In SGD, the model parameters are updated iteratively in the direction that reduces the loss function, based on gradients calculated from subsets of the training data. Adam improves this basic update by using estimates of the first and second moments of the gradients, which allows an adaptive learning rate for each model parameter.

Before training, the data were split into training and validation datasets with a ratio of 9:1. The input and output data were also normalised using the mean and standard deviation of the training data. This improved the numerical stability of the training process and made the convergence of the neural network more stable.

After training, the neural network represented a smooth function $a = f_\theta(N)$, where θ denotes the trained model parameters. The crack propagation rate was calculated as the derivative of this function with respect to the number of cycles. This was performed using PyTorch automatic differentiation,

where the gradient was evaluated with respect to the input variable N . Since the model was trained using normalised variables, the obtained derivative was transformed back to the physical scale using the corresponding standard deviations of a and N . The main hyperparameters and settings of the neural network model are summarized in Table 3.4.

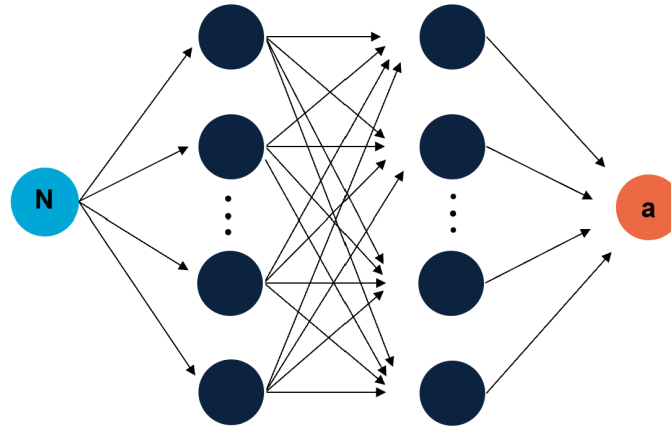


Figure 3.18: Multilayer Perceptron scheme

Table 3.4: Main hyperparameters of the neural network regression model

Hyperparameter	Setting
Model type	Multilayer perceptron (MLP)
Input variable	Number of cycles, N
Output variable	Crack length, a
Network architecture	1 input neuron, 2 hidden layers, 32 neurons per hidden layer, 1 output neuron
Activation function	Hyperbolic tangent, $\tanh$
Loss function	Mean squared error (MSE)
Optimizer	Adam optimizer
Training-validation split	90% training data, 10% validation data
Normalization	Standard normalization
Derivative calculation	PyTorch Autograd
Learning rate	1×10^{-3}
Weight decay	1×10^{-6}
Batch size	32
Maximum training points	Up to 1000 points per specimen
Training epochs	1500

In Figure 3.19, the evolution of the training and validation losses is shown for specimen S112. The training was performed for 1500 epochs. In this context, an epoch means one complete pass through the available training data. During the training process, the model calculates the prediction error, and based on this error, the weights are updated step by step in order to reduce the loss.

As can be seen, both the training and validation losses decrease rapidly at the beginning of the training. This shows that the model is able to capture the main trend of the crack growth curve within the first few epochs. After approximately 100 epochs, the losses become almost constant, which indicates that the training has practically converged. Further training does not result in a significant improvement in the model accuracy.

The validation loss follows the training loss closely during the whole process. This suggests that the model performs similarly on the training and validation data, and no clear signs of overfitting can be observed. Some small fluctuations and occasional peaks appear in the training loss, but these are temporary and do not influence the overall tendency of the convergence.

The relatively fast convergence is most likely related to the limited complexity of the crack length curve of this specimen. The curve is relatively short and smooth, and it follows an almost monotonic

increasing trend rather than showing repeated oscillations, local peaks, and valleys. As a result, the relationship between the number of cycles and the crack length can be learnt efficiently. Therefore, after the initial decrease of the loss, only very small improvements are achieved during the remaining epochs.

Figure 3.20 shows the fitted crack length curve and the corresponding crack growth rate for specimen S112. The measured crack length data are shown together with the fitted ($a(N)$) curve on the left-hand side, while the derived da/dN curve is presented on the right-hand side.

The fitted curve follows the experimentally calculated crack length values well. The resulting function behaves smoothly throughout the whole data range and does not show any fluctuations or noise. This is a favourable property, as irregularities in the fitted crack length curve would affect the calculated crack growth rate curves.

The corresponding crack growth rate curve shows a physically reasonable trend for fatigue crack propagation. At lower cycle numbers, the crack growth rate remains small and increases only gradually. As the crack length increases, the growth rate also increases, followed by a pronounced rise near the final part of the test. This behaviour is consistent with the expected acceleration of crack propagation at larger crack lengths and higher stress intensity factor ranges.

Overall, both curves provide a stable and continuous representation of the crack propagation behaviour. The absence of visible noise or oscillations indicates that the fitted curve is suitable for deriving crack growth rate data for the subsequent Paris-law evaluation.



Figure 3.19: Training and validation loss evolution during neural network training

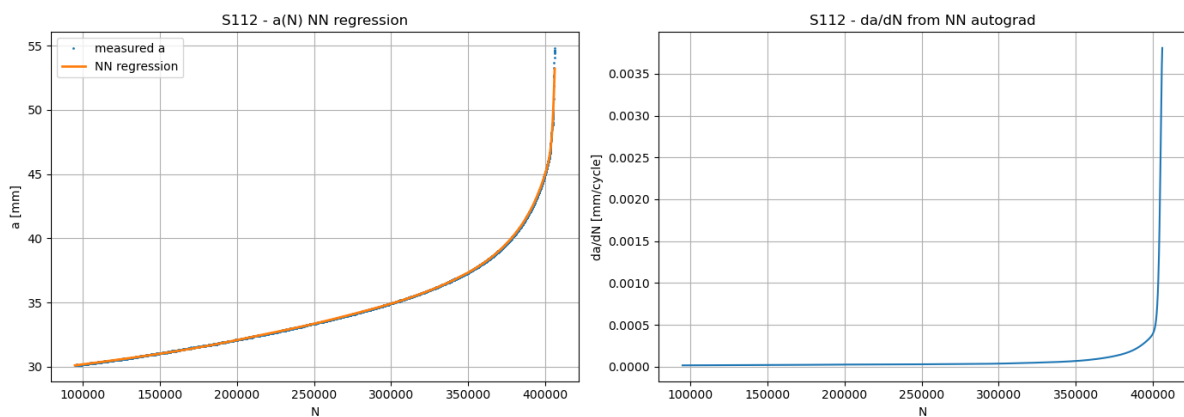


Figure 3.20: Fitted crack length and calculated crack growth curves

3.5. Numerical determination of the stress intensity factor range

In addition to the crack growth rate, the stress intensity factor range was required for the evaluation of the Paris-law parameters, as illustrated in Figure 2.10. For this purpose, the same Abaqus models as those developed for the crack length–COD calibration were also used to determine the relationship between crack length and the Mode I stress intensity factor for the tested SENB geometry. This ensured that the crack length–COD calibration and the stress intensity factor calculation were based on the same geometry, boundary conditions, loading arrangement, and predefined crack lengths. The analyses were performed assuming linear elastic material behaviour, since the subsequent Paris-law evaluation was based on linear elastic fracture mechanics. As the purpose of the model was to obtain the nominal Mode I stress intensity factor, K_I , rather than to resolve the detailed three-dimensional crack-front behaviour, a two-dimensional plane stress idealisation was adopted. The specimens had a thickness of approximately $B = 32$ mm. Consequently, the two-dimensional plane-stress model cannot represent crack-front curvature or variations in crack-tip constraint through the specimen thickness, where the interior may experience a higher constraint than the free surfaces. Furthermore, the linear-elastic formulation does not account for local crack-tip plasticity. The extracted K_I values should therefore be interpreted as nominal values applicable within the LEFM framework and used for the consistent comparative evaluation of the specimens. The influence of the adopted idealisation may become more relevant at larger crack lengths, where the remaining ligament is reduced.

The modelling approach was therefore identical to that described in Section 3.3.2. The specimen was discretised mainly using CPS8R elements, while the four-point bending setup was represented using idealised simply supported boundary conditions and two concentrated forces applied at the loading positions. A total nominal load of 25 kN was applied in the reference models, corresponding to 12.5 kN at each loading point. Owing to the linear-elastic material formulation, the resulting stress intensity factors could subsequently be scaled to the specimen-specific load levels.

The models with predefined crack lengths between 30 and 55 mm, in 1 mm increments, were evaluated for the stress intensity factor extraction. For each crack length, the corresponding value of K_I was obtained. These values were then used to derive a continuous relationship between crack length and stress intensity factor, $K_I(a)$, which formed the basis for calculating the specimen-specific stress intensity factor ranges, ΔK . Both geometrically linear and geometrically nonlinear finite element formulations were evaluated. In both cases, the material behaviour was kept linear elastic, therefore the analyses remained within the framework of LEFM. The geometrically nonlinear formulation was included to assess whether the deformed geometry influences the extracted stress intensity factors, especially at larger crack lengths where the remaining ligament becomes smaller. The same specimen geometry, boundary conditions, loading arrangement, crack lengths, and contour integral procedure were used for both formulations.

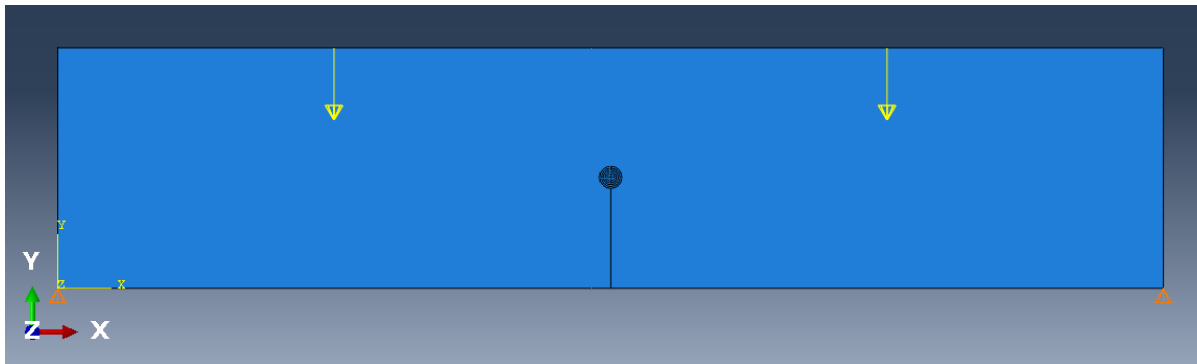


Figure 3.21: Model in Abaqus for SIF calculation with $a = 30$ mm

3.5.1. Stress intensity factor extraction

For each predefined crack length, the Mode I stress intensity factor was extracted from the finite element results using the contour integral method available in Abaqus. Around the crack tip, a structured spiderweb-like mesh was applied, as shown in Figure 3.22. This mesh layout provides a refined and radially organised discretisation in the crack-tip region, which is required to obtain stable contour integral

results.

The stress intensity factor was evaluated using five contours surrounding the crack tip. The representative K_I value was calculated as the arithmetic mean of contours 2–5. Contour 1 was excluded because it is directly adjacent to the crack tip and is therefore more sensitive to the local element formulation and mesh discretisation. The individual contour values were retained for the contour-stability assessment presented in Section 4.1.

The extracted values formed the discrete numerical dataset relating crack length to stress intensity factor. This dataset was subsequently used to determine the geometry correction function described in the following subsection.

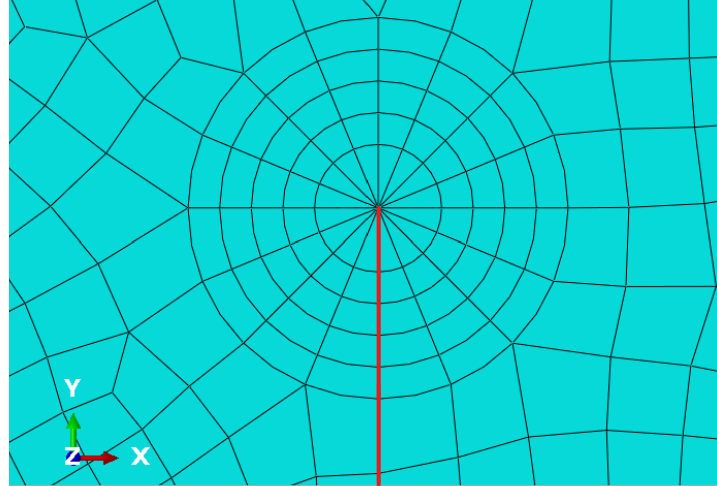


Figure 3.22: Spiderweb-like mesh used at the crack tip (crack shown in red)

3.5.2. Fitted geometry correction function

The finite element analyses provided stress intensity factor values for discrete crack lengths. However, for the Paris-law evaluation, the stress intensity factor range had to be calculated at the crack length values obtained from the experimental post-processing. Therefore, the discrete finite element results were converted into a continuous geometry correction function.

The stress intensity factor was expressed in the standard form

$$K = \sigma_{\text{nom}} \sqrt{\pi a} Y \quad (3.16)$$

where σ_{nom} is the nominal bending stress, a is the crack length, and Y is the geometry correction factor. In the present case, Y was not taken from an analytical solution, but was derived from the numerical stress intensity factor values obtained from Abaqus.

The nominal bending stress at the outer fibre of the cross-section was calculated as

$$\sigma_{\text{nom}} = \frac{M}{I} z. \quad (3.17)$$

For the four-point bending configuration and rectangular cross-section used in this study,

$$M = \frac{F}{2} l, \quad (3.18)$$

$$I = \frac{BW^3}{12}, \quad (3.19)$$

$$z = \frac{W}{2} \quad (3.20)$$

which gives

$$\sigma_{\text{nom}} = \frac{M}{I} z = \frac{3Fl}{BW^2}. \quad (3.21)$$

Here, F is the total applied vertical load, B is the specimen thickness, and W is the specimen height. Since F represents the total force applied by the testing machine, the force acting at each loading point is $F/2$. Therefore, the bending moment in the constant-moment region is $M = (F/2)l$, where l is the distance between a support and the nearest loading point. Substitution into the stress intensity factor expression gives

$$K(a) = \frac{3Fl}{BW^2} \sqrt{\pi a} Y. \quad (3.22)$$

Since the geometry correction factor depends on the relative crack length, it was written as a function of

$$\alpha = \frac{a}{W}. \quad (3.23)$$

For each crack length analysed in Abaqus, the corresponding numerical stress intensity factor $K_{\text{FEM},i}$ was extracted. The associated geometry correction factor was then back-calculated as

$$Y_i = \frac{K_{\text{FEM},i}}{\sigma_{\text{nom}} \sqrt{\pi a_i}}. \quad (3.24)$$

Together with

$$\alpha_i = \frac{a_i}{W},$$

this resulted in a set of discrete data points (α_i, Y_i) . These points describe the geometry correction factor for the investigated SENB geometry over the considered crack length range. The fitting procedure was applied separately to the geometry correction factors obtained from the geometrically linear and geometrically nonlinear finite element formulations. This resulted in two continuous geometry correction functions, which are presented in Section 5.5.1.

To obtain a continuous expression, the geometry correction factor was fitted as a function of the relative crack length. The following functional form was adopted:

$$Y(\alpha) = \frac{P_5(\alpha)}{(1 - \alpha)^{3/2}} \quad (3.25)$$

where $P_5(\alpha)$ is a fifth-order polynomial,

$$P_5(\alpha) = c_0 + c_1\alpha + c_2\alpha^2 + c_3\alpha^3 + c_4\alpha^4 + c_5\alpha^5. \quad (3.26)$$

This form was selected because the denominator accounts for the increasing stress intensity factor as the remaining ligament decreases, while the polynomial numerator provides sufficient flexibility to fit the numerical data over the investigated range.

For the curve fitting procedure, the finite element data were first transformed as

$$\tilde{Y}_i = Y_i(1 - \alpha_i)^{3/2}. \quad (3.27)$$

A fifth-order polynomial was then fitted to the modified values $\tilde{Y}_i$. The fitted coefficients and the resulting continuous geometry correction function are presented in Section 5.5.1.

In addition to the geometry correction function derived for the tested SENB geometry, a separate geometrically linear sensitivity study was carried out to assess how strongly the fitted geometry factor depends on the investigated specimen dimensions and four-point bending span. This verification step is presented in Section 4.3.

3.5.3. Calculation of specimen-specific ΔK values

For the Paris-law evaluation, the stress intensity factor range was required instead of the absolute stress intensity factor. The fitted geometry correction function was therefore combined with the measured dimensions and applied load range of each specimen.

Within the linear-elastic fracture-mechanics evaluation, the stress intensity factor range was calculated from the applied force range. Since ΔF represents the range of the total force applied by the testing machine, the force range acting at each loading point is $\Delta F/2$. Consequently, the bending

moment range in the constant-moment region is $\Delta M = (\Delta F/2)l$, where l is the distance between a support and the nearest loading point.

Using the rectangular specimen cross-section, the specimen-specific stress intensity factor range was calculated as

$$\begin{aligned}\Delta K(\alpha) &= \Delta \sigma_{\text{nom}} \sqrt{\pi W \alpha} Y(\alpha) \\ &= \frac{3 \Delta F l}{B W^2} \sqrt{\pi W \alpha} Y(\alpha),\end{aligned}\tag{3.28}$$

where B and W are the measured specimen thickness and height, respectively, and $\alpha = a/W$ is the relative crack length. The fitted numerical form of $Y(\alpha)$ is presented in Section 5.5.1.

Equation 3.28 was evaluated at each crack length obtained from the crack-gauge, COD-based, and strain-gauge-based histories. This procedure produced a specimen-specific ΔK history for each available measurement method.

The expression was applied only within the crack length range covered by the finite element analyses, namely $a = 30\text{--}55$ mm. For the nominal specimen height of $W = 65$ mm, this corresponds approximately to $\alpha = 0.46\text{--}0.85$. Extrapolation outside this interval was avoided. The calculations were performed using the full precision of the fitted coefficients.

The specimen-specific ΔK histories were calculated separately using the geometry correction functions obtained from the geometrically linear and geometrically nonlinear formulations. The geometrically linear formulation provides the directly verified reference case, while the geometrically nonlinear formulation provides an additional numerical comparison. Both sets of ΔK values were carried forward to the Paris-law evaluation and are reported separately.

3.6. Paris-law parameter evaluation

After the crack length history, the crack growth rate, and the corresponding stress intensity factor range had been determined, the fatigue crack propagation behaviour of each specimen could be evaluated in terms of the Paris law. For each specimen, the post-processing resulted in paired values of crack growth rate, da/dN , and stress intensity factor range, ΔK , at the corresponding crack length and cycle number.

The aim of this step was to determine the Paris-law parameters describing the relationship between ΔK and da/dN in the stable crack propagation regime. Since the Paris law is applicable only to the approximately linear part of the fatigue crack growth curve in log-log representation, an appropriate ΔK range had to be selected for each measurement method. The following subsections describe the logarithmic regression procedure, the selection of the method-specific ΔK ranges, the output parameters determined for each specimen, and the aggregation procedures used to obtain representative method-level parameters.

3.6.1. Log-log regression procedure

The Paris law relates the fatigue crack growth rate to the stress intensity factor range as

$$\frac{da}{dN} = C(\Delta K)^m\tag{3.29}$$

where C and m are the Paris-law parameters. In this equation, m describes the slope of the fatigue crack growth curve in the Paris regime, while C determines its vertical position.

For the regression procedure, the equation was transformed into logarithmic form. Using a base-10 logarithm gives

$$\log_{10} \left(\frac{da}{dN} \right) = \log_{10}(C) + m \log_{10}(\Delta K).\tag{3.30}$$

Thus, the Paris law can be fitted as a linear relation in log-log space. For each valid data point, the transformed variables were defined as

$$x_i = \log_{10}(\Delta K_i)\tag{3.31}$$

and

$$y_i = \log_{10} \left(\frac{da}{dN} \right)_i. \quad (3.32)$$

A first-order polynomial was then fitted to the transformed data using least-squares linear regression:

$$y_i = mx_i + b. \quad (3.33)$$

In this formulation, the slope of the fitted line corresponds directly to the Paris exponent m , while the intercept is

$$b = \log_{10}(C). \quad (3.34)$$

Therefore, the coefficient C was calculated as

$$C = 10^b. \quad (3.35)$$

The regression was carried out separately for each specimen using the selected data points in the Paris regime. The fitted line was therefore obtained in logarithmic space, rather than by fitting a nonlinear power law directly to the original ΔK and da/dN values.

3.6.2. Selection of method-specific ΔK range

The Paris-law regression was restricted to the part of the fatigue crack growth relation that exhibited an approximately linear trend in $\log_{10}(da/dN)$ – $\log_{10}(\Delta K)$ space. The complete fatigue crack growth curves were not represented adequately by a single linear relation over the entire investigated range. Data at the lower and upper ends of the curves were therefore excluded from the regression when they showed a systematic departure from the central approximately log-linear region.

The fitting intervals were selected by visual inspection of the combined fatigue crack growth curves obtained from all available specimens, as shown in Figure 3.23. Since the crack gauge, COD, and strain gauge data were obtained using different measurement principles and post-processing procedures, the resulting curves did not exhibit identical approximately log-linear ranges. A separate fitting interval was therefore defined for each measurement method.

Within a given measurement method, the same lower and upper limits were applied to all specimens. The intervals were not adjusted individually for each specimen and were not selected by maximising the coefficient of determination of the individual regressions. This approach was used to maintain a consistent evaluation procedure within each measurement method.

The selected fitting intervals are summarised in Table 3.5.

Table 3.5: Method-specific ΔK intervals used for the Paris-law regressions.

Measurement method	$\Delta K_{\min}$ [MPa $\sqrt{\text{m}}$]	$\Delta K_{\max}$ [MPa $\sqrt{\text{m}}$]
Crack gauge	17	43
COD	20	40
Strain gauge	18	45

For measurement method j , a data point i was included in the Paris-law regression when

$$\Delta K_{\min,j} \leq \Delta K_i \leq \Delta K_{\max,j}. \quad (3.36)$$

In addition, only finite and strictly positive values of ΔK and da/dN were retained, since the logarithmic transformation used for the regression is not defined for zero or negative values.

For the crack-gauge results, the interval from 17 to 43 MPa $\sqrt{\text{m}}$ covered the central approximately log-linear region common to the available specimens. The crack-gauge datasets generally contained only limited common data coverage above the selected upper boundary.

For the COD-based results, curvature was visible at the lower end of the investigated range, while several curves departed progressively from the central approximately linear trend at higher values of ΔK . Consequently, the narrower interval from 20 to 40 MPa $\sqrt{\text{m}}$ was retained.

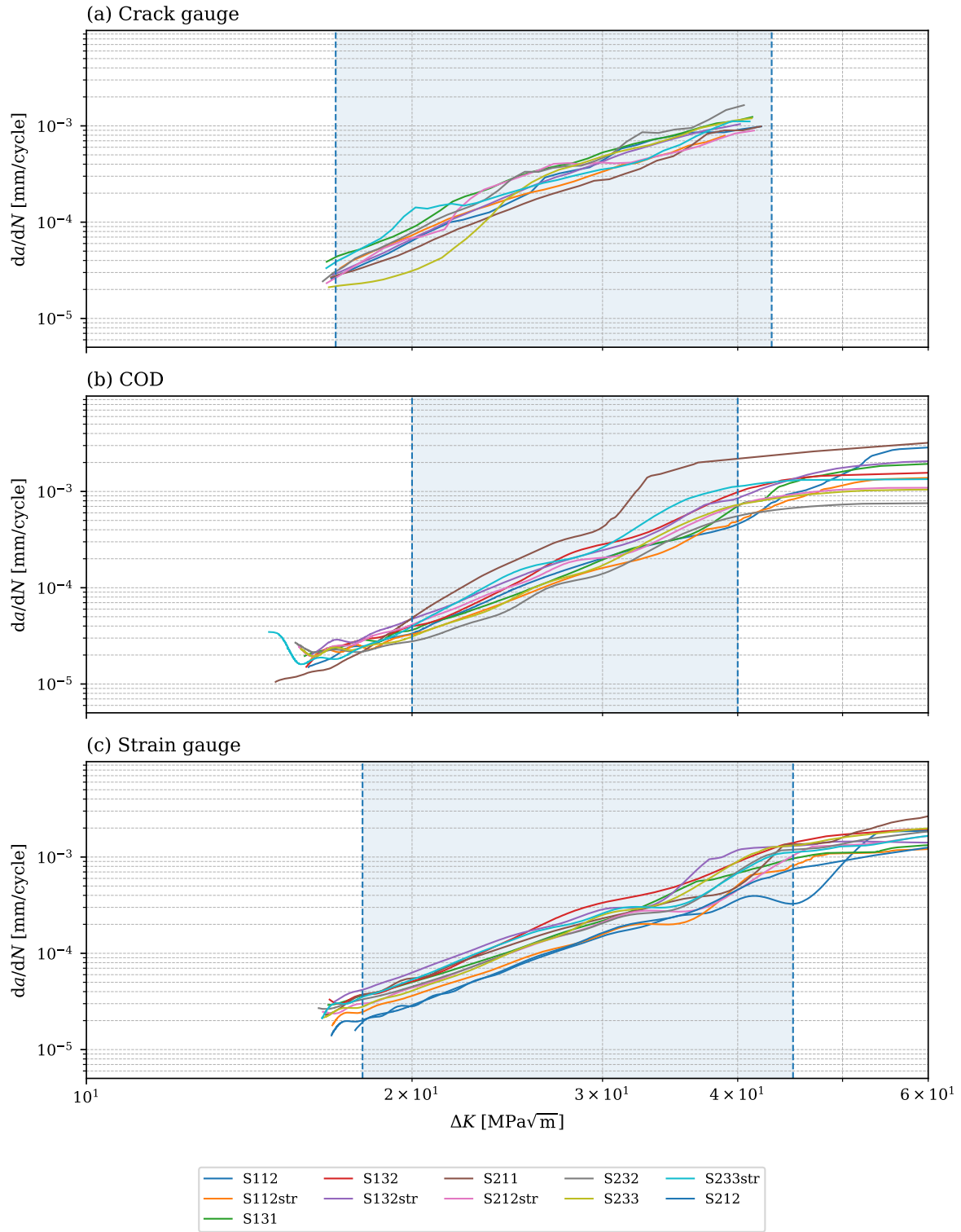


Figure 3.23: Combined fatigue crack growth curves obtained from the crack-gauge, COD, and strain-gauge measurements. The dashed vertical lines and shaded regions indicate the method-specific ΔK intervals retained for the Paris-law regressions.

For the strain-gauge-based results, the lower limit of $18 \text{ MPa}\sqrt{\text{m}}$ excluded the initial curved portion of the curves. The upper limit of $45 \text{ MPa}\sqrt{\text{m}}$ was selected because the differences between the curves increased and several datasets began to depart from the central log-linear trend above this value.

The selected limits should be interpreted as operational boundaries used for the Paris-law regression. They do not represent exact physical transition points between fatigue crack growth regimes and should not be interpreted as estimates of the fatigue threshold or fracture toughness. No formal cutoff sensitivity analysis was performed.

3.6.3. Output parameters

The log-log regression resulted in one fitted Paris-law relation for each specimen. From each fitted line, the slope m and the intercept $b = \log_{10}(C)$ were extracted. The Paris coefficient was then calculated as $C = 10^b$. Thus, the final output parameters for each specimen were m , $\log_{10}(C)$, and C .

These parameters describe the fatigue crack propagation behaviour of the tested material within the selected Paris regime. The exponent m characterises the sensitivity of the crack growth rate to changes in the stress intensity factor range, while C defines the vertical position of the fitted curve in the da/dN - ΔK diagram. The obtained parameters were then used for the comparison between specimens and for the subsequent statistical evaluation.

3.6.4. Aggregation of the Paris-law parameters

The specimen-specific regressions resulted in several Paris-law parameter pairs for each measurement method. To obtain representative parameters for the crack-gauge, COD-based, and strain-gauge-based results, two aggregation procedures were applied. Both procedures were evaluated separately using the geometrically linear and geometrically nonlinear stress intensity factor ranges. The same method-specific ΔK intervals as those defined in Section 3.6.2 were used, and all valid specimen results were retained.

The first procedure was based on the mean of the specimen-specific fits. Each specimen was first fitted separately, resulting in a slope m_i and an intercept $b_i = \log_{10}(C_i)$. The representative parameters were then calculated as

$$\begin{aligned} m_{\text{mean}} &= \frac{1}{n_s} \sum_{i=1}^{n_s} m_i, \\ b_{\text{mean}} &= \frac{1}{n_s} \sum_{i=1}^{n_s} b_i, \\ C_{\text{mean}} &= 10^{b_{\text{mean}}} = \left(\prod_{i=1}^{n_s} C_i \right)^{1/n_s}, \end{aligned} \quad (3.37)$$

where n_s is the number of specimens with a valid result for the investigated measurement method. Since the values of C cover several orders of magnitude and the regression is performed in logarithmic space, the intercepts $b_i = \log_{10}(C_i)$ were averaged instead of calculating the arithmetic mean of the untransformed C_i values. Consequently, C_{mean} corresponds to the geometric mean of the specimen-specific coefficients.

This procedure can be described as “fit first, then average”. Every specimen contributes one pair of parameters to the calculation, independently of the number of data points available for that specimen. However, after the individual regressions have been performed, the distribution of the original data points within each specimen is no longer included directly in the aggregation.

The second procedure was a specimen-balanced pooled regression. In this approach, the logarithmically transformed data points from all specimens belonging to one measurement method were combined and one common Paris-law line was fitted. Let i denote the specimen and j denote a data point within that specimen. Furthermore, let n_i be the number of valid data points for specimen i , $x_{ij} = \log_{10}(\Delta K_{ij})$, and $y_{ij} = \log_{10}[(da/dN)_{ij}]$. The pooled parameters were obtained from the weighted least-squares problem

$$\begin{aligned}
(m_{\text{pool}}, b_{\text{pool}}) &= \arg \min_{m, b} \sum_{i=1}^{n_s} \sum_{j=1}^{n_i} w_{ij} [y_{ij} - (mx_{ij} + b)]^2, \\
w_{ij} &= \frac{1}{n_i}, \quad \sum_{j=1}^{n_i} w_{ij} = 1, \\
C_{\text{pool}} &= 10^{b_{\text{pool}}}.
\end{aligned} \tag{3.38}$$

The weight $w_{ij} = 1/n_i$ ensures that the data points belonging to each specimen have a total weight of one. Therefore, a specimen containing a large number of recorded points does not dominate the regression over a specimen containing fewer points. This procedure can be described as “balance the specimens, then fit”. In contrast to the mean of the specimen-specific fits, the point-level distribution and the shape of each dataset within the selected ΔK interval remain part of the regression.

The two aggregation procedures therefore give equal total weight to the specimens at different stages of the evaluation. The first method averages parameters that have already been determined separately, while the second method determines one line directly from the combined, specimen-balanced data. The two procedures are not expected to give identical results when the specimens have different slopes, intercepts, data distributions, or available ΔK ranges. Their comparison indicates how sensitive the representative Paris-law parameters are to the selected aggregation procedure.

Because the data points belonging to the same specimen are not statistically independent observations, the specimen-balanced pooled regression was used as a descriptive aggregate fit. It was not used as a basis for point-level confidence intervals. All calculations used ΔK in $\text{MPa}\sqrt{\text{m}}$ and da/dN in mm/cycle .

3.6.5. Exploratory specimen comparison

An exploratory comparison was performed to assess whether the specimen-specific Paris-law results indicated a repeatable ordering or grouping of the specimens. Since m and C are coupled parameters and the three measurement methods produced different absolute parameter levels, the comparison was based on the processed Paris-law data at a common stress intensity factor range.

A target value of $\Delta K = 30 \text{ MPa}\sqrt{\text{m}}$ was selected because it lies within the fitting intervals used for all three measurement methods. For each specimen–measurement–method combination, the actual processed data point with the value of ΔK closest to the target was selected. No interpolation or additional curve fitting was applied.

Only the nine specimens with results available from all three measurement methods were included in the direct comparison. The specimens were ranked separately within each method according to their selected da/dN value. This procedure was used as a descriptive exploratory comparison and not as a formal clustering or classification method.

Verification of the numerical models

4.1. Mesh sensitivity analysis

To ensure sufficient reliability of the numerical model, a mesh sensitivity analysis was performed. The original setup applied a general mesh size of 1 mm. Therefore, to evaluate the mesh dependency, additional models with 2 mm and 0.5 mm general mesh sizes were analysed using the same model geometry, boundary conditions, loading arrangement, and crack lengths. The mesh size modifications also affected the spider-web mesh at the crack tip, and its radius was adjusted according to the general mesh size. Mesh sizes larger than 2 mm were not considered, because for larger crack lengths, the remaining ligament becomes limited, and a coarser discretisation would not leave sufficient space for a properly defined crack-tip mesh required for the contour integral evaluation.

The mesh sensitivity analysis was performed for both the geometrically linear and geometrically nonlinear formulations. In the geometrically linear case, all three mesh sizes, 0.5 mm, 1 mm, and 2 mm, provided stable results. In the geometrically nonlinear case, the 1 mm and 2 mm meshes also provided usable results. However, the 0.5 mm mesh led to an unstable stress intensity factor extraction for the longer crack length. This case was therefore used to illustrate the sensitivity of the contour integral evaluation, but it was not considered reliable for the final quantitative evaluation of K_I .

The comparison was carried out for two representative crack lengths, $a = 35$ mm and $a = 50$ mm. These values were selected in order to check the mesh sensitivity both at an intermediate crack length and at a longer crack length, where the crack-tip field and the remaining ligament may become more sensitive to the discretisation. For each mesh size and crack length, the COD value, the longitudinal strain component E_{11} , and the Mode I stress intensity factor K_I were extracted.

The extracted quantities were compared with the values obtained using the original 1 mm mesh. The relative difference was calculated as

$$\delta_q = \frac{q_h - q_{1\text{ mm}}}{q_{1\text{ mm}}} \cdot 100\%. \quad (4.1)$$

where q_h is the value obtained with either the 0.5 mm or 2 mm mesh, and $q_{1\text{ mm}}$ is the corresponding value obtained with the 1 mm mesh. This comparison was performed separately for the COD value, E_{11} , and K_I .

Figure 4.2 shows the relative differences of the extracted COD, E_{11} , and K_I values with respect to the 1 mm mesh. The rows correspond to the investigated quantities, while the columns show the two selected crack lengths. The solid lines represent the geometrically linear formulation, whereas the dashed lines represent the geometrically nonlinear formulation.

The COD values were found to be unaffected by the mesh size. Crack opening represents the global deformation response of the element. The maximum relative difference in the calculated COD values was approximately 0.003%, indicating that the results are mesh-independent within the investigated range.

The longitudinal strain component E_{11} was more sensitive to the mesh size. This was especially visible for the 2 mm mesh, where the relative difference reached approximately 18% for the longer crack length. This larger deviation can be explained by the fact that strain is a local quantity and is therefore more sensitive to the local discretisation at the strain gauge position. In contrast, the 0.5 mm mesh remained much closer to the 1 mm mesh, with differences of approximately 1–2%.

For the geometrically linear formulation, the extracted K_I values were practically independent of the mesh size. The maximum relative difference with respect to the 1 mm mesh was below 0.001%.

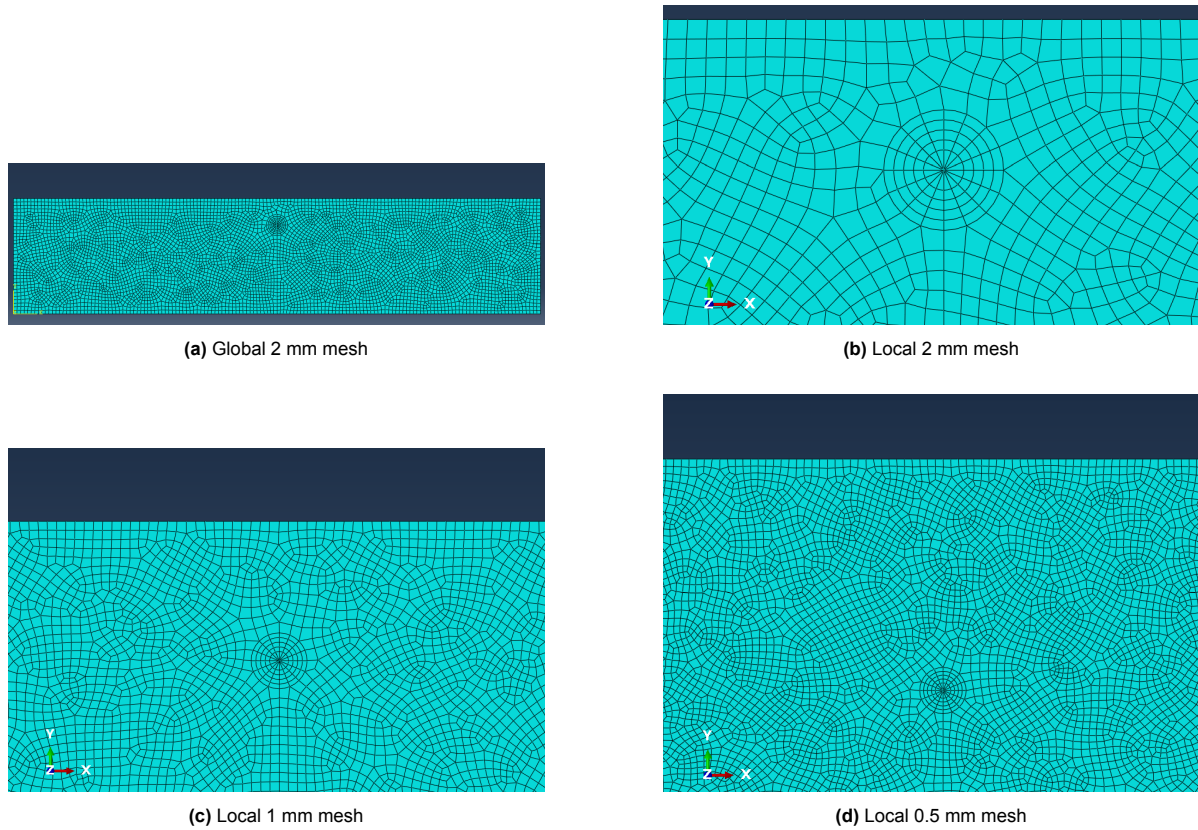


Figure 4.1: Mesh configurations used for the mesh sensitivity analysis.

This confirms that, for the geometrically linear formulation, the stress intensity factor extraction was not significantly influenced by the investigated general mesh sizes.

In the case of the geometrically nonlinear formulation, the 1 mm and 2 mm meshes gave comparable stress intensity factor values. The maximum relative difference between the two results did not exceed 1.6%. However, the 0.5 mm mesh showed a large deviation for $a = 50$ mm, where the obtained K_I value differed by 77% from the 1 mm result. Therefore, the corresponding contour integral calculation was not considered reliable when using nonlinear geometry settings, although the global COD response remained stable.

To further assess the reliability of the stress intensity factor extraction, the contour-to-contour variation of K_I was evaluated. For each model, the extracted contour values were compared with the mean value of contours 2–5. The first contour was included in the comparison, but it was not used for the representative K_I value because it is closest to the crack tip and can be more sensitive to the local element formulation and numerical discretisation.

Figure 4.3 shows the normalised deviation of each contour value from the mean value of contours 2–5. This representation makes it easier to identify whether the first contour or a specific mesh configuration gives unstable stress intensity factor results.

The geometrically linear models showed very stable contour integral results. The scatter between contours 2–5 remained below 0.005%, and the first contour differed by less than 0.03% from the mean value of the remaining contours. This indicates that the contour integral evaluation was highly stable for the geometrically linear formulation.

For the geometrically nonlinear formulation, the first contour showed a much larger deviation from the remaining contours. This was especially visible for the longer crack length. For $a = 50$ mm, the 0.5 mm mesh showed a strong contour-to-contour variation, with a scatter of approximately 38% between contours 2–5. Therefore, this case was not considered reliable for the quantitative evaluation of K_I . The 1 mm geometrically nonlinear model also showed a larger contour variation than the geometrically linear models, but the later contours remained considerably more stable than in the 0.5 mm case. The 2 mm geometrically nonlinear model showed the smallest contour scatter among the nonlinear

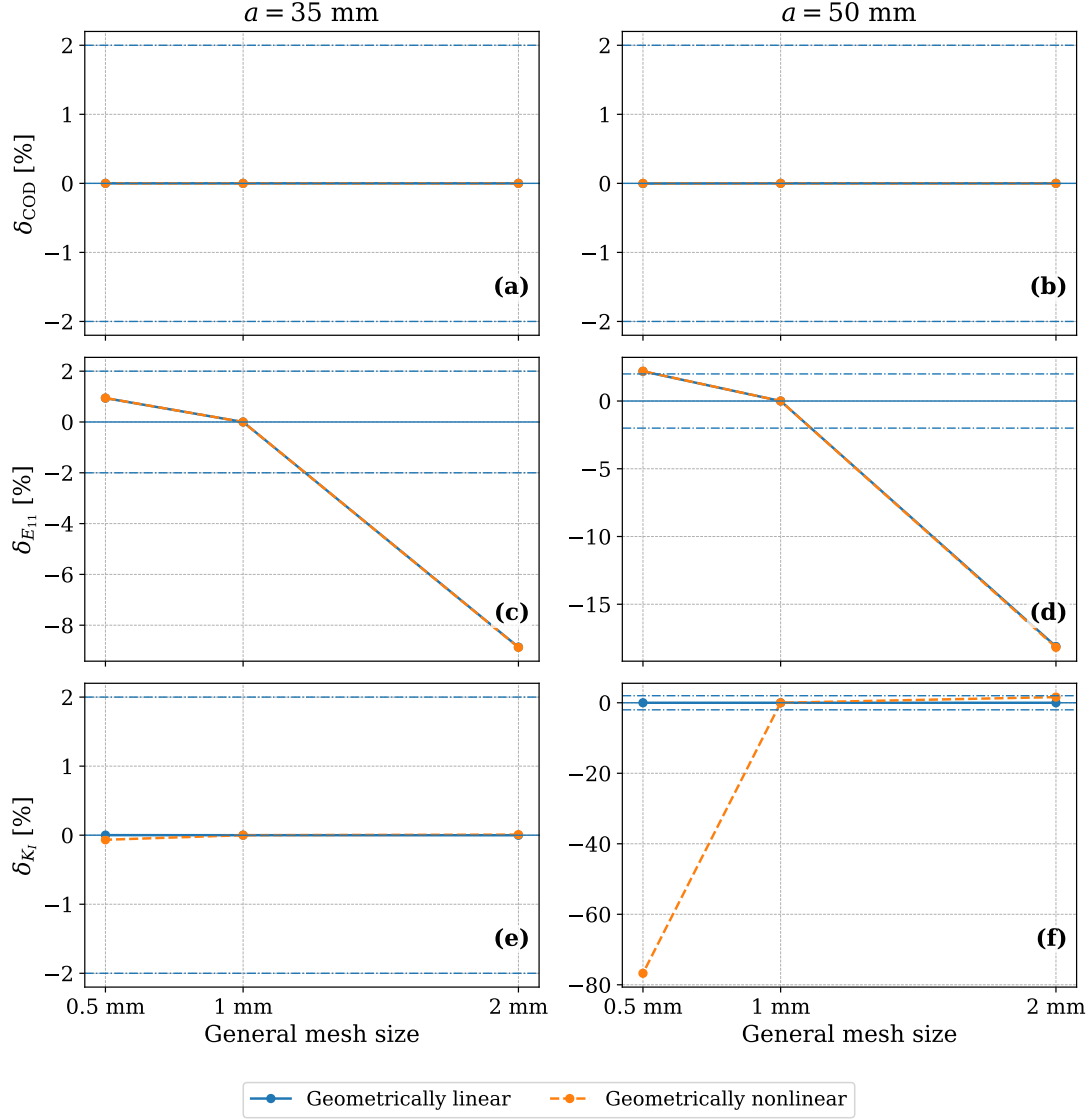


Figure 4.2: Relative mesh sensitivity of COD, E_{11} , and K_I .

cases for $a = 50$ mm, but the coarser mesh was less suitable for the local strain extraction.

Overall, the mesh sensitivity analysis showed that the COD response was insensitive to the investigated mesh sizes, while the local strain value E_{11} was more affected by mesh coarsening. The geometrically linear stress intensity factor results were highly stable. In the geometrically nonlinear formulation, the 0.5 mm mesh did not provide reliable contour integral results for the longer crack length, whereas the 1 mm and 2 mm meshes gave comparable K_I values. Considering both the stability of the extracted quantities and the need for sufficient local resolution, the 1 mm mesh was retained for the final numerical evaluation.

4.2. Verification against literature SIF solutions

To assess whether the numerical model provides reasonable and reliable stress intensity factor values, the calculated results were compared with available literature solutions. In this comparison, the geometrically linear Abaqus model was used as the primary validation case, since the literature solutions are based on linear elastic fracture mechanics assumptions. The purpose of this comparison was therefore to verify whether the finite element model reproduces established linear-elastic SIF solutions.

The geometrically linear Abaqus results were compared with the solutions of Brown [10], Baratta

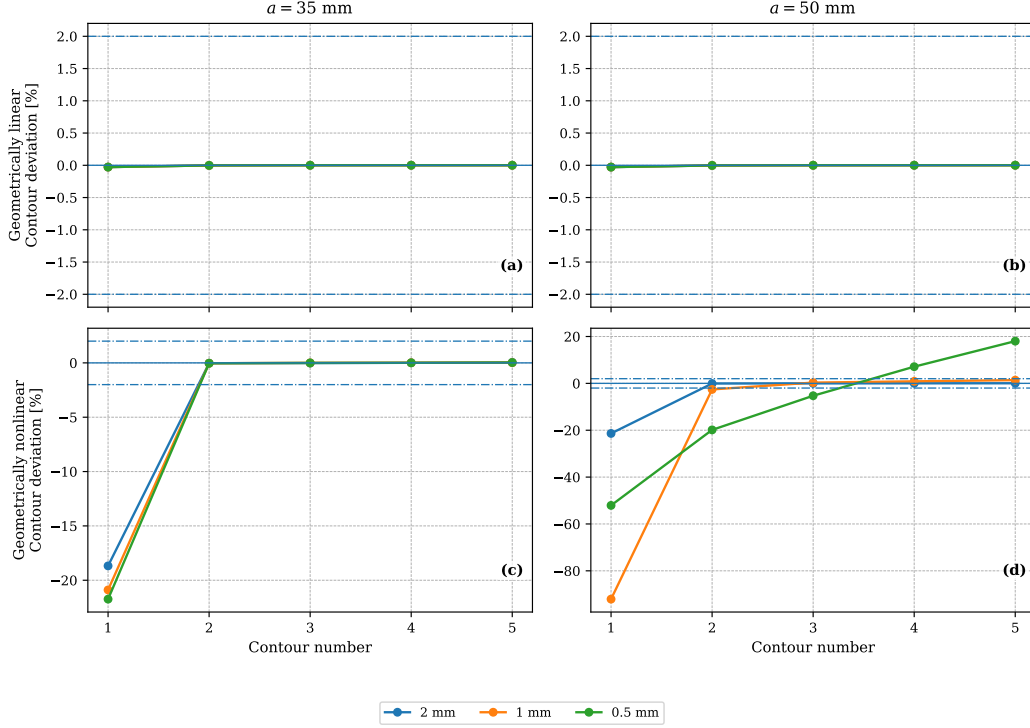


Figure 4.3: Normalised contour deviations of the extracted K_I values.

and Fett [7], and C. H. Wang [50]. Among these references, the Baratta and Fett solution is especially relevant for the present comparison, because it represents a published SIF solution for bend-type fracture toughness specimens and was derived within a linear elastic fracture mechanics framework. The solution is based on a weight function approach, which provides stress intensity factors for a given crack geometry and loading condition. Since the geometrically linear Abaqus model is based on the same linear-elastic assumptions, agreement with this reference provides a direct validation of the numerical geometry, boundary conditions, loading arrangement, and contour-integral-based K_I extraction.

The Brown–Srawley and Wang solutions were retained as additional reference curves in order to show the range of available literature estimates. Although the agreement with these two solutions was less close than with the Baratta and Fett solution, the calculated stress intensity factors remained in the same order of magnitude and followed the same increasing tendency with crack length. This further supports the validity of the numerical model, since the finite element results are consistent with the general level and trend of established literature solutions.

Figure 4.4 shows the stress intensity factor values obtained from the Abaqus models together with the literature solutions. The geometrically linear Abaqus curve follows the same trend and magnitude as the literature curves over the investigated crack length range. In particular, the agreement with the Baratta and Fett solution is very close. This indicates that, under purely linear assumptions, the finite element model provides stress intensity factor values consistent with an established weight-function-based solution.

To quantify the agreement, the relative difference between the geometrically linear Abaqus result and each literature solution was calculated as

$$\delta_K = \frac{K_{I,\text{FEM}} - K_{I,\text{lit}}}{K_{I,\text{lit}}} \cdot 100\%. \quad (4.2)$$

Here, $K_{I,\text{FEM}}$ is the stress intensity factor obtained from Abaqus, and $K_{I,\text{lit}}$ is the corresponding literature value. For literature data given at different crack length values, the Abaqus results were linearly interpolated to the same crack lengths before calculating the relative difference.

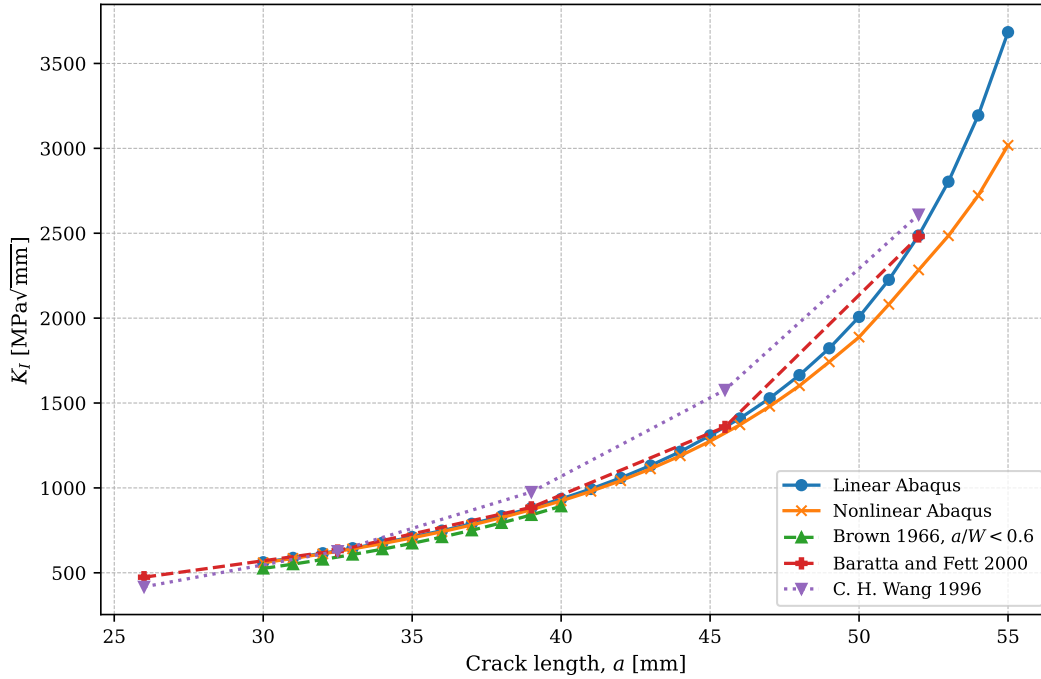


Figure 4.4: Comparison with literature SIF solutions.

Table 4.1: Quantitative comparison of the geometrically linear Abaqus results with literature SIF solutions.

Reference solution	Crack length range [mm]	Points [-]	Mean abs. diff. [%]	Max. abs. diff. [%]
Brown [10]	30–40	11	5.68	6.94
Baratta and Fett [7]	32.5–52	4	0.21	0.44
C. H. Wang [50]	32.5–52	4	7.14	13.81

The quantitative comparison confirms the visual agreement shown in Figure 4.4. The best agreement was obtained with the Baratta and Fett solution. The mean absolute relative difference was only 0.21%, and the maximum absolute relative difference was 0.44%. This very small difference shows that the geometrically linear finite element model reproduces the established weight-function-based solution with high accuracy.

Figure 4.5 shows the comparison with the Baratta and Fett solution in more detail. The close agreement confirms that the numerical geometry, loading arrangement, boundary conditions, and contour-integral-based extraction of K_I are correctly implemented for the linear elastic case.

The comparison with Brown also showed reasonable agreement over the range $a = 30\text{--}40$ mm, with a mean absolute relative difference of 5.68%. The difference with the C. H. Wang solution was larger, with a mean absolute relative difference of 7.14% and a maximum difference of 13.81%. These differences are considered acceptable, since the literature solutions are based on different idealisations, geometry correction formulations, and crack length ranges. More importantly, both solutions still resulted in stress intensity factor values of the same order of magnitude as the finite element results and showed the same increasing tendency with increasing crack length. Therefore, they provide additional support that the numerical model gives physically reasonable K_I values, even though the closest quantitative agreement was obtained with the Baratta and Fett solution.

The geometrically nonlinear Abaqus curve is also shown in Figure 4.4, but it should not be interpreted as the primary verification curve. The literature solutions are based on linear elastic assumptions, whereas the geometrically nonlinear model additionally accounts for the influence of the deformed geometry. Therefore, the nonlinear curve is used to assess the influence of geometrical nonlinearity after the linear finite element model has been validated.

Overall, the comparison demonstrates that the finite element model gives correct stress intensity

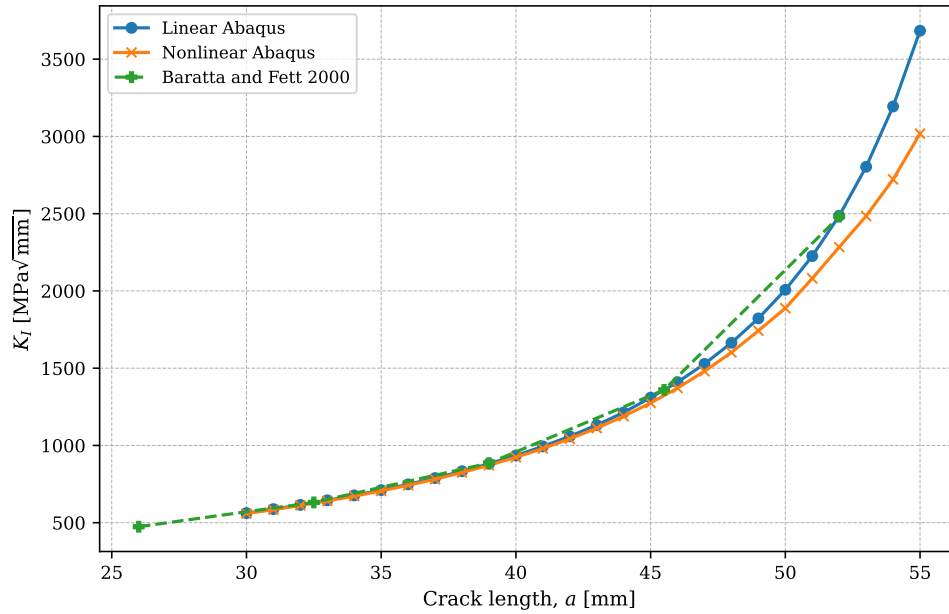


Figure 4.5: Validation against the Baratta and Fett solution.

factor values when the same linear assumptions as the literature solutions are used. The excellent agreement between the geometrically linear Abaqus model and the Baratta and Fett solution validates the numerical geometry, loading, boundary conditions, and contour-integral-based extraction of K_I . The additional comparison with Brown–Srawley and Wang further confirms that the calculated stress intensity factors are within the expected range of literature values and follow the correct physical trend. After this validation step, the geometrically nonlinear formulation can be considered as an additional modelling refinement, rather than as the baseline validation case.

4.3. Geometry-factor sensitivity to specimen geometry

After the verification against literature SIF solutions, an additional numerical sensitivity study was carried out to examine how general the geometry correction function is for the investigated SENB configuration. The aim was not to validate experimental size effects, since only one nominal specimen size was tested experimentally. Instead, the purpose was to check whether the geometry correction factor obtained for the base specimen remains representative when the specimen dimensions or the four-point bending span are modified.

The comparison was performed using geometrically linear finite element analyses. This was chosen because, in the linear case, the stress intensity factor scales linearly with the applied load. Therefore, after normalisation by the nominal bending stress, the resulting geometry correction factor can be compared directly between different geometries.

The base model corresponded to the tested SENB specimen geometry. Three additional geometries were analysed: two models with modified four-point bending spans and one geometrically scaled model. The investigated cases are summarised in Table 4.2.

Table 4.2: Geometries used for the numerical geometry-factor sensitivity study.

Case	S [mm]	W [mm]	B [mm]	S/W [-]	l [mm]	a range [mm]
Base geometry	300	65	32	4.62	75.0	30–55
Modified span	260	65	32	4.00	65.0	30–54
Short span	130	65	32	2.00	32.5	30–54
Scaled geometry	600	130	65	4.62	150.0	60–110

Here, S is the support span, W is the specimen height, B is the specimen thickness, and l is the lever arm between the support and the nearest loading point. The modified-span models were used to investigate the influence of the four-point bending configuration while keeping the specimen cross-section unchanged. The scaled model was used to investigate whether a geometrically similar specimen gives a similar normalised geometry correction factor.

For each geometry, the geometry correction factor Y was calculated using Equation 3.24, while the normalised crack length α was determined according to Equation 3.23. For the present sensitivity study, K_I was extracted from Abaqus, and the nominal bending stress was calculated from the bending moment in the constant-moment region of the four-point bending setup.

Since the comparison was performed using $Y(\alpha)$, the effects of the absolute load level and nominal stress were removed. The relative difference with respect to the base geometry was calculated as

$$\Delta Y_{\text{rel}} = \frac{Y_{\text{case}} - Y_{\text{base}}}{Y_{\text{base}}} \cdot 100\%. \quad (4.3)$$

When the analysed crack length ratios did not coincide exactly between the different models, the base $Y(a/W)$ curve was linearly interpolated to the corresponding a/W values of the compared case.

Figure 4.6 shows the geometry correction factor curves obtained for the investigated geometries. The relative differences with respect to the base geometry are shown in Figure 4.7. The quantitative comparison is summarised in Table 4.3.

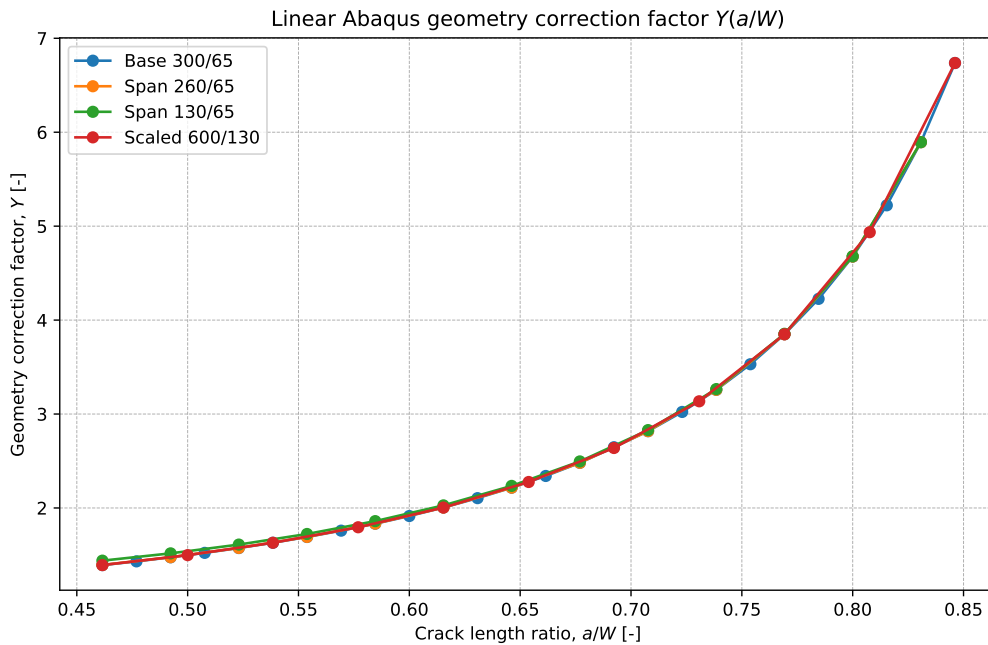


Figure 4.6: Geometry correction factor $Y(a/W)$ obtained from geometrically linear Abaqus analyses for the investigated SENB geometries.

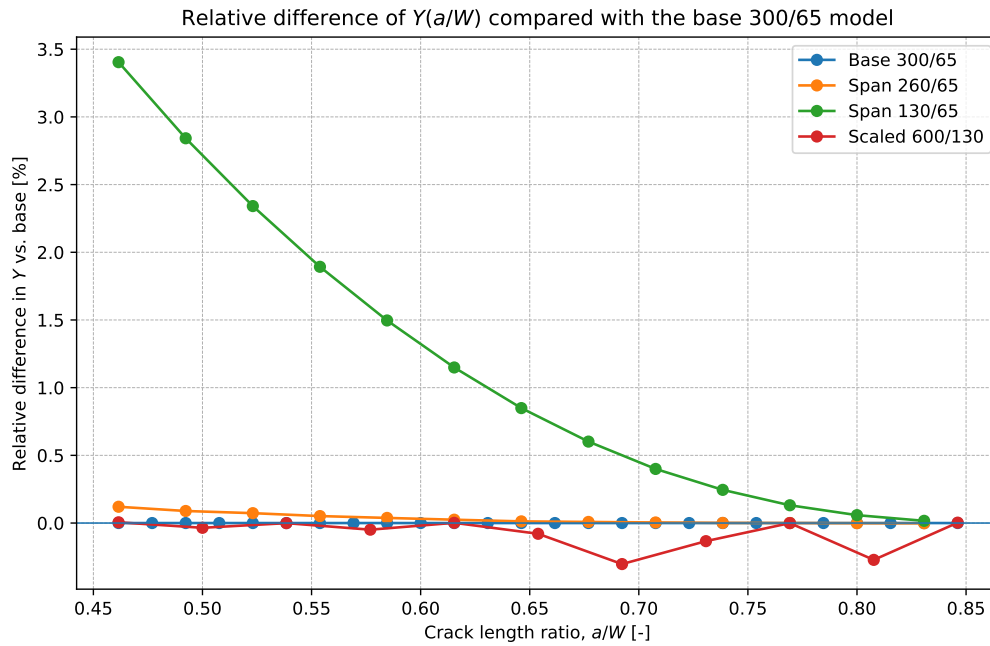


Figure 4.7: Relative difference of the geometry correction factor compared with the base SENB geometry.

Table 4.3: Relative difference of the linear geometry correction factor compared with the base SENB geometry.

Case	a/W range [-]	Mean abs. diff. [%]	Max. abs. diff. [%]
Modified span	0.462–0.831	0.03	0.12
Short span	0.462–0.831	1.19	3.40
Scaled geometry	0.462–0.846	0.08	0.30

The comparison shows that the geometry correction factor is only weakly affected by the investigated geometric variations. The geometrically scaled specimen gives almost the same $Y(a/W)$ curve as the base geometry. The largest difference is obtained for the shortest span configuration, where the four-point bending span is reduced significantly.

Based on these results, the geometry correction function obtained for the base specimen can be considered representative for the investigated SENB-type four-point bending configurations. However, this conclusion is limited to the numerical SIF geometry factor in the geometrically linear case. It should not be interpreted as an experimental validation of size effects, since the experimental programme included only one nominal specimen size.

The verification presented in this chapter applies specifically to the numerical SIF calculation within the investigated geometry and crack-length range. The COD- and strain-based crack-length calibration relationships were also derived from finite element results, but no independent experimental crack-length measurements were available for their separate validation. Consequently, the corresponding crack-length histories remain dependent on the numerical calibration assumptions.

Fatigue crack growth results and comparison of evaluation methods

5.1. Overview of the analysed dataset

This chapter presents the results obtained from the post-processing of the SENB fatigue tests. The experimental programme included twelve specimens. Their main geometrical parameters, loading conditions, and maximum recorded cycle numbers are summarised in Table 5.1. The specimens had similar dimensions, with thicknesses between 31.73 mm and 33.64 mm and heights between 63.50 mm and 67.07 mm. The initial notch length was approximately 30 mm for all specimens. The applied force range varied between 20.64 kN and 27.93 kN, while the load ratio remained between 0.092 and 0.102.

Specimen S111 is included in the experimental overview for completeness but was excluded from the final evaluation because it was subjected to a different loading protocol. For the eleven retained specimens, the availability of the post-processed results depended on the measurement method. Table 5.2 therefore summarises the crack length histories available from the crack-gauge, COD-based, and strain-gauge-based evaluations.

The availability indicated in Table 5.2 refers to the final post-processed histories used in this chapter. A missing result does not necessarily indicate that no raw signal was recorded; rather, it indicates that the available raw data could not be reliably processed into a crack length history using the developed method-specific tools.

Table 5.1: Overview of the tested SENB specimens and loading conditions

Specimen	B [mm]	W [mm]	a_0 [mm]	ΔF [kN]	R [-]	$N_{\max}$ [-]
S111	31.85	63.54	29.2	27.93	0.097	772 975
S112	31.73	63.50	29.1	20.94	0.092	409 082
S112str	31.75	63.50	29.4	20.64	0.092	368 443
S131	33.32	66.64	31.0	23.04	0.094	420 057
S132	33.56	66.97	30.8	23.82	0.094	419 955
S132str	33.64	67.08	30.3	24.79	0.096	336 903
S211	32.73	65.55	30.0	22.43	0.102	430 344
S212	32.17	64.47	30.0	21.23	0.099	586 130
S212str	32.60	64.58	30.0	21.23	0.099	427 759
S232	32.97	66.10	30.0	22.80	0.100	453 460
S233	32.94	65.99	30.0	22.94	0.101	456 975
S233str	32.97	65.96	30.0	22.83	0.100	410 904

Table 5.2: Availability of post-processed crack length histories for the different measurement methods

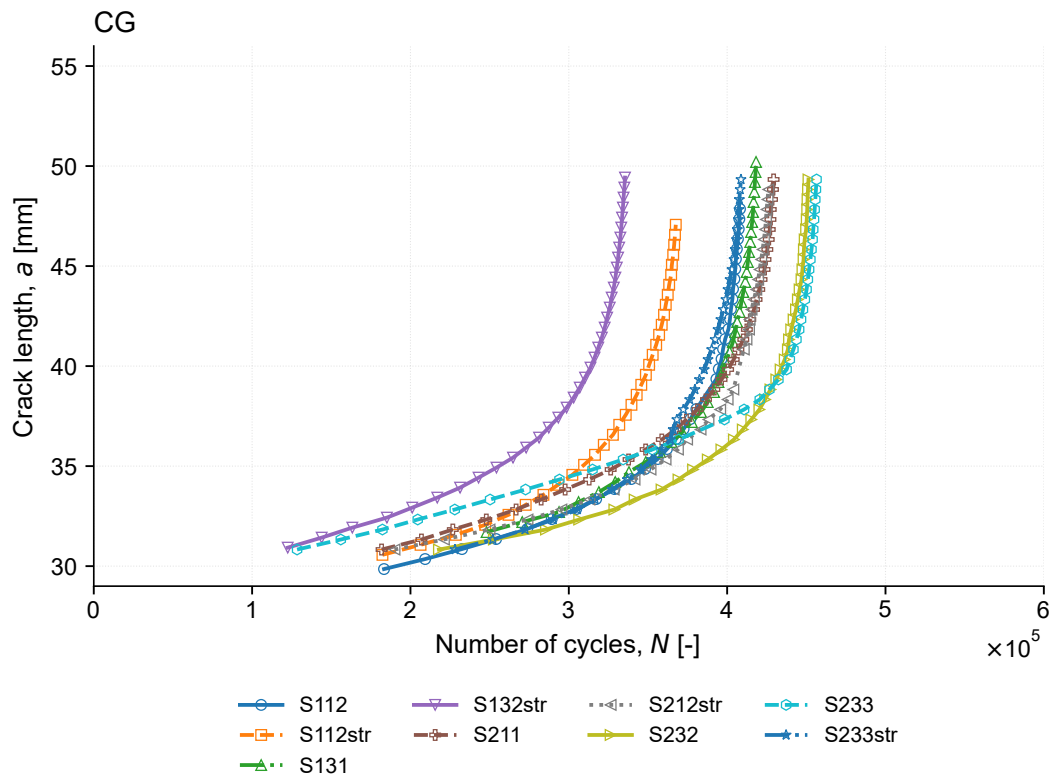
Specimen	Crack gauge	COD-based	Strain-gauge-based	Comment
S111	No	No	No	Excluded from final evaluation
S112	Yes	Yes	Yes	Complete dataset
S112str	Yes	Yes	Yes	Complete dataset
S131	Yes	Yes	Yes	Complete dataset
S132	No	Yes	Yes	No retained crack-gauge result
S132str	Yes	Yes	Yes	Complete dataset
S211	Yes	Yes	Yes	Complete dataset
S212	No	No	Yes	Only strain-gauge-based result retained
S212str	Yes	Yes	Yes	Complete dataset
S232	Yes	Yes	Yes	Complete dataset
S233	Yes	Yes	Yes	Complete dataset
S233str	Yes	Yes	Yes	Complete dataset

Nine specimens provided useful results from all three measurement methods. Specimen S112 was selected as a representative case for the direct method comparison in Section 5.4.

5.2. Crack length results

The post-processed crack length histories obtained from the three measurement methods are presented separately in Figures 5.1–5.3.

The crack-gauge histories in Figure 5.1 contain discrete crack length values corresponding to consecutive wire breakages. Therefore they begin at the first retained wire-breakage event rather than at the initial notch length and cover a shorter crack length interval than the two continuous methods. All available histories show a progressive increase in crack length followed by accelerated propagation. Specimen S132str reached this stage after the lowest number of cycles in the crack-gauge dataset.

**Figure 5.1:** Crack length histories obtained from the crack-gauge measurements

The COD-based histories in Figure 5.2 provide densely sampled, indirect crack-length estimates. The experimental COD response was recorded at the data-acquisition sampling frequency and subsequently converted into crack length using the finite-element-based calibration relationship. Thus, unlike the crack-gauge histories, these results are not restricted to discrete crack-length increments associated with individual wire breakages. After post-processing, only values satisfying $a_{\text{init},i} + 1 \text{ mm} \leq a \leq 55 \text{ mm}$ were retained. Consequently, the curves begin at specimen-specific lower limits rather than at a common nominal crack length of 30 mm. The crack length increases gradually during most of each test and considerably more rapidly during the final stage. Differences between the specimens are visible in both the cycle count required to reach a given crack length and the begin of rapid propagation.

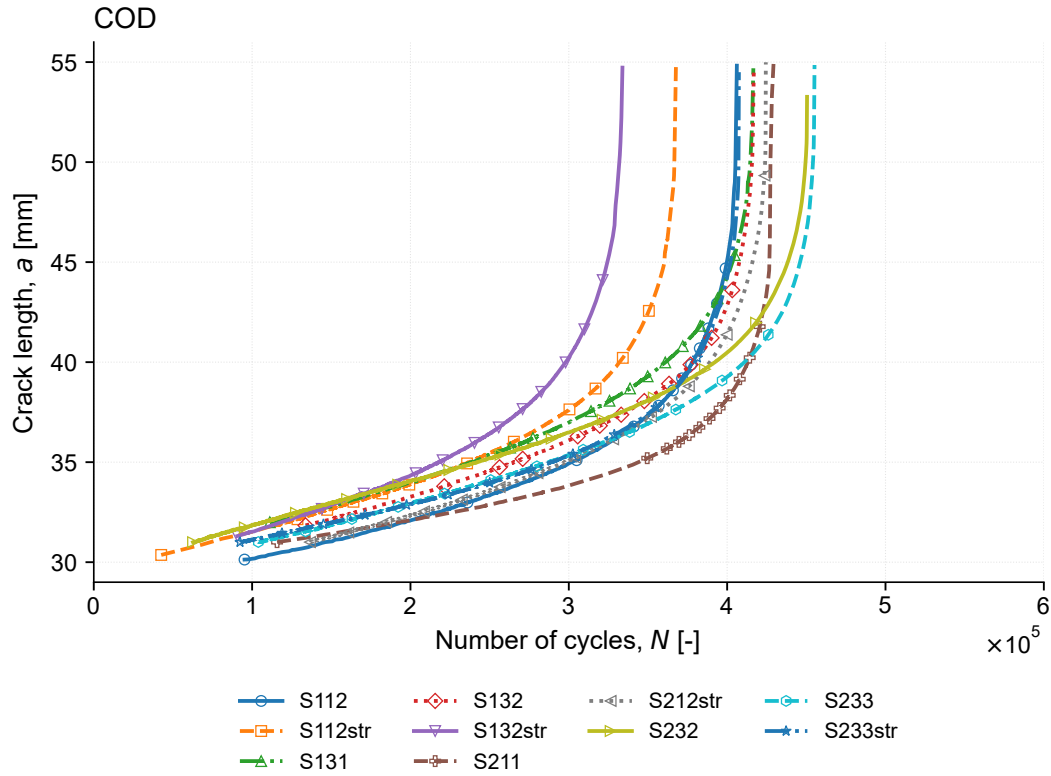


Figure 5.2: Crack length histories obtained from the COD-based evaluation

The strain-gauge-based histories in Figure 5.3 show the same general development as the COD-based histories. Specimen S132str reached the rapid propagation stage comparatively early, whereas S212 showed the longest crack propagation history and reached similar crack lengths only after approximately 5.8×10^5 cycles.

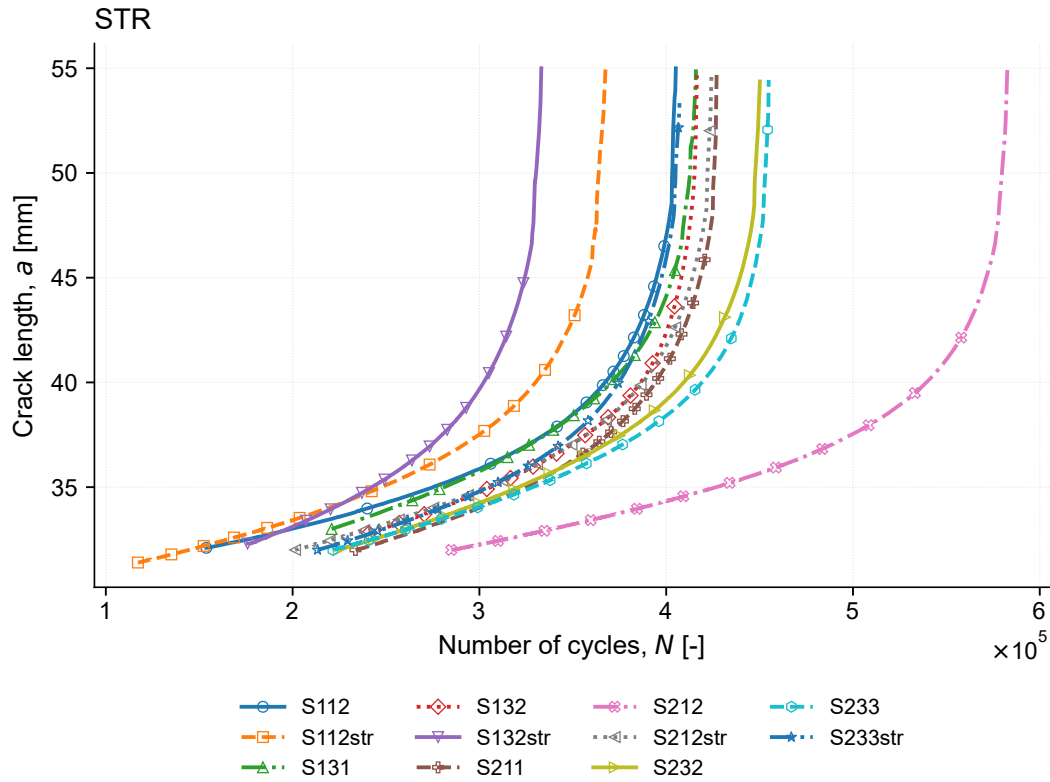


Figure 5.3: Crack length histories obtained from the strain-gauge-based evaluation

Overall, all three methods capture a progressive increase in crack length followed by accelerated propagation. Since the specimen dimensions and applied force ranges were not identical, the cycle counts alone are not used as a direct measure of the material fatigue crack growth resistance. The measurement methods are compared directly for specimen S112 in Section 5.4.

5.3. Crack growth rate results

The crack growth rate histories obtained from the three measurement methods are presented separately in Figures 5.4–5.6. A logarithmic vertical axis is used because the calculated values cover several orders of magnitude. Identical axis limits are applied to support visual comparison.

The crack-gauge results shown in Figure 5.4 cover only the portion of the propagation process monitored after the first retained wire break. The first and last three data points are omitted because they are lost during the polynomial differentiation procedure. For all available specimens, the crack growth rate increases with the cycle count and rises more rapidly during the final stage. The calculated values generally increase from the order of 10^{-5} to approximately 10^{-3} mm/cycle.

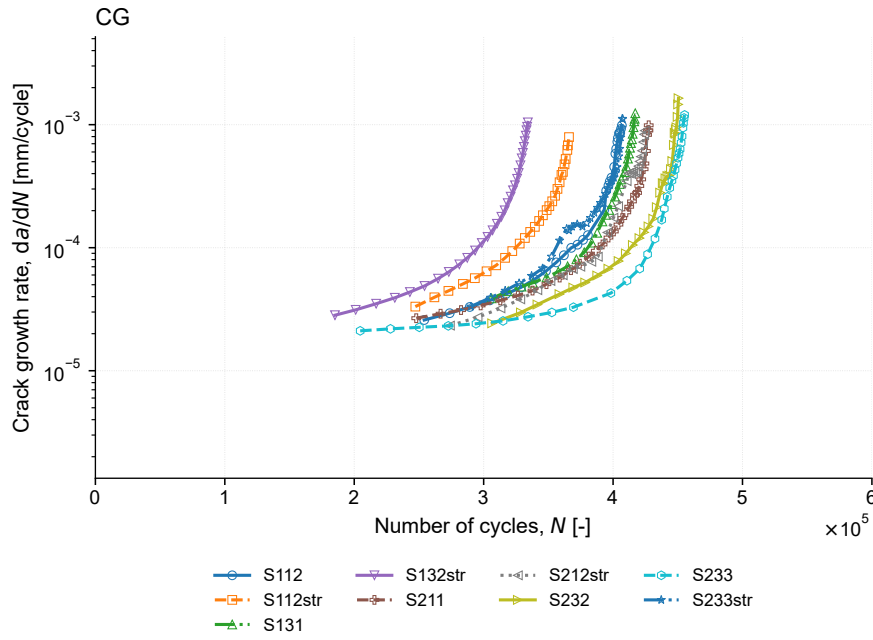


Figure 5.4: Crack growth rate histories obtained from the crack-gauge measurements

The COD-based histories in Figure 5.5 cover a larger part of the tests because continuous crack length estimates were available from approximately the initial notch length. The crack growth rate remains comparatively low during the initial part of each test and subsequently increases as the crack propagates.

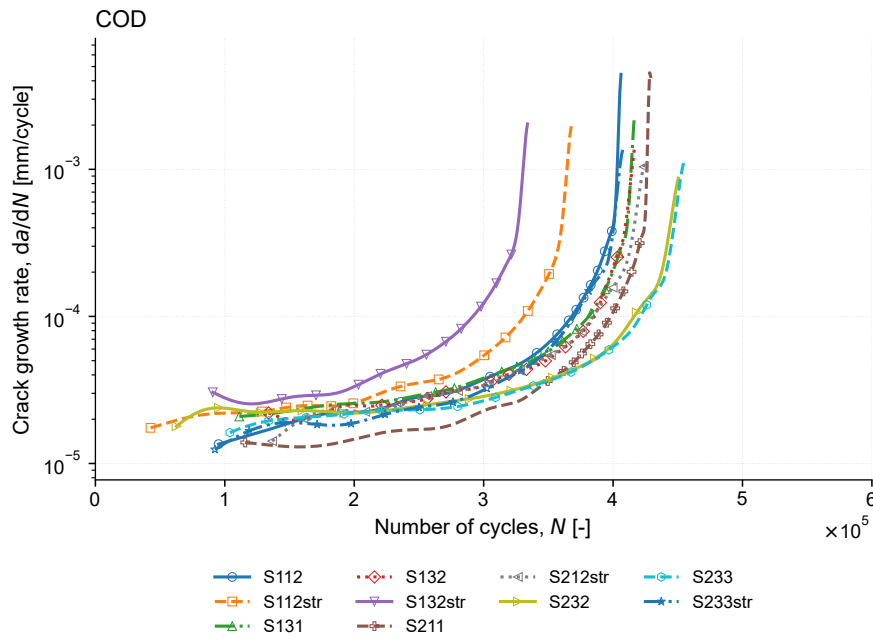


Figure 5.5: Crack growth rate histories obtained from the COD-based evaluation

The strain-gauge-based results in Figure 5.6 show a similar overall development. Small local variations occur in the initial parts of several curves, while the increasing trend becomes dominant during the later stage.

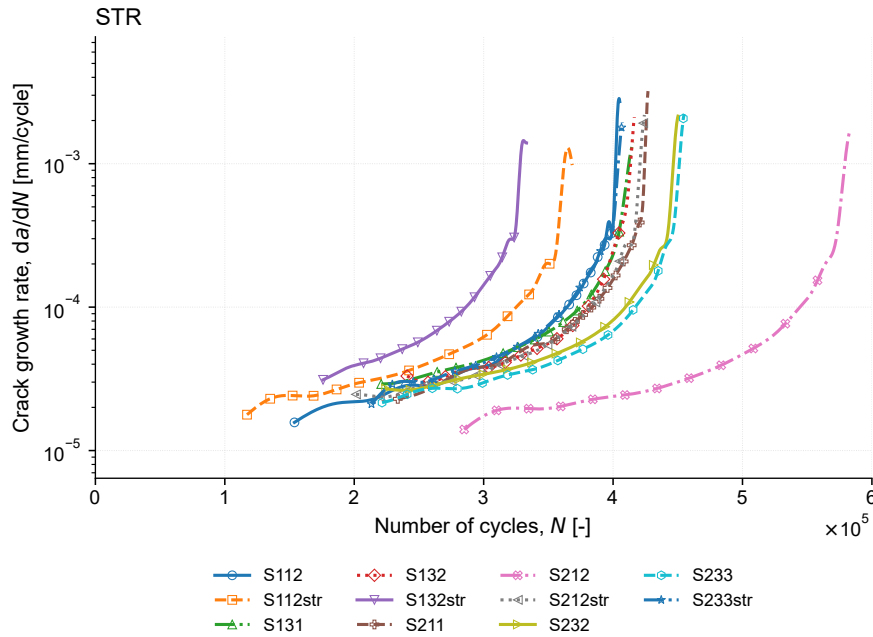


Figure 5.6: Crack growth rate histories obtained from the strain-gauge-based evaluation

All three methods show the same general development: a comparatively low crack growth rate during the earlier part of the test followed by an accelerating increase. The calculated values and the cycle count at which this increase occurs nevertheless differ between methods and specimens. The relationship between the crack growth rate and the stress intensity factor range is evaluated later using the Paris-law plots.

5.4. Comparison of the measurement methods

Specimen S112 was selected as the representative case because post-processed crack length and crack growth rate results were available from all three measurement methods. Since all measurements were obtained during the same fatigue test, the methods can be compared without differences in specimen geometry or loading conditions between tests.

5.4.1. Crack length comparison for S112

Figure 5.7 compares the crack length histories obtained from the three methods. The COD- and strain-gauge-based histories agree closely over almost the complete test. Both begin near the initial notch length and show a gradual increase followed by rapid propagation near the end of the test.

Within the overlapping cycle range, crack gauge gives systematically lower crack length values than the two continuous methods. The difference amounts to several millimetres and increases towards the final stage. The crack-gauge history ends at approximately $a = 46$ mm, whereas the COD- and strain-gauge-based histories extend to approximately 54–55 mm.

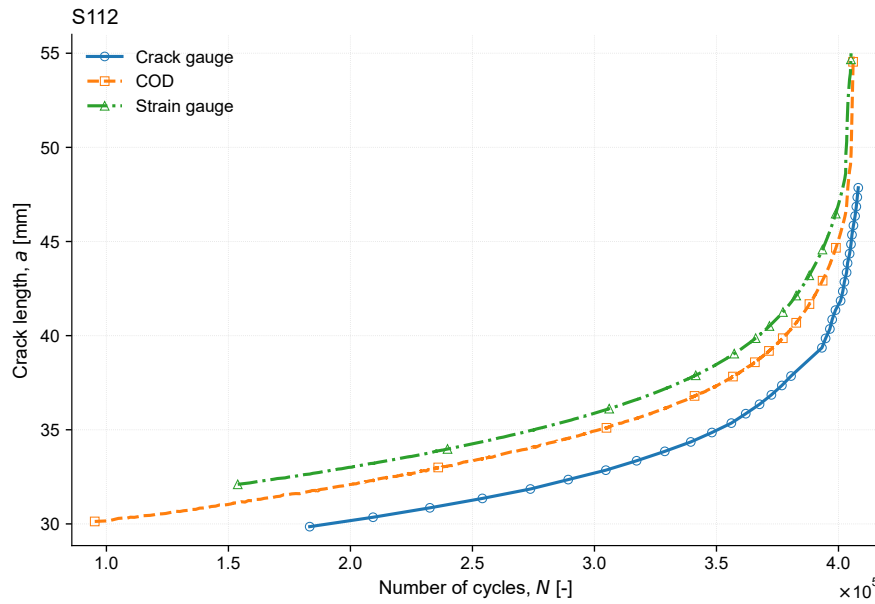


Figure 5.7: Comparison of the crack length histories obtained from the three measurement methods for specimen S112

5.4.2. Crack growth rate comparison for S112

The corresponding crack growth rate histories are compared in Figure 5.8. Despite the differences between the crack length curves, the three methods provide crack growth rates of a similar order of magnitude within their overlapping range.

The COD- and strain-gauge-based results agree particularly closely during most of the test. The crack-gauge result follows the same general trend, although the differences become more visible during the final stage of rapid propagation. At the final available points, the COD-based method gives the highest crack growth rate, followed by the strain-gauge- and crack-gauge-based results.

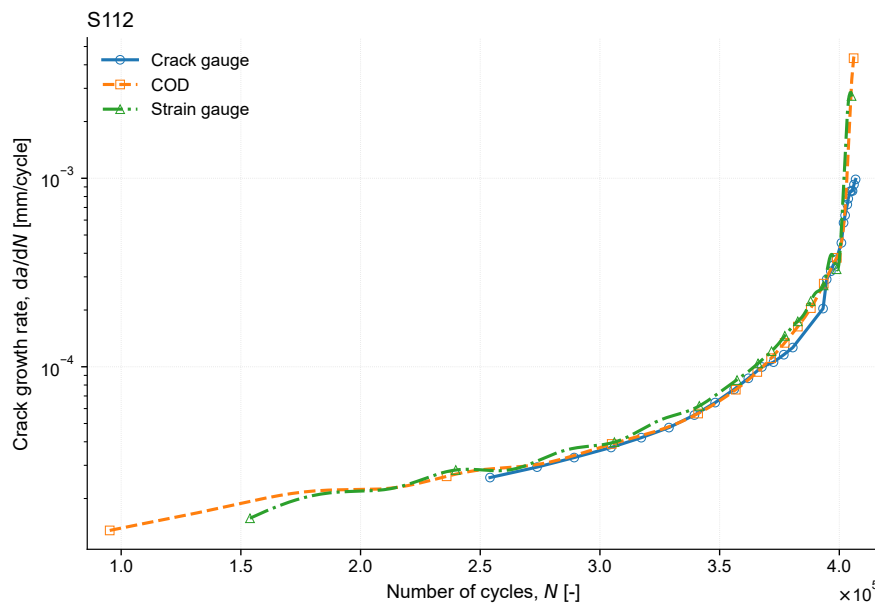


Figure 5.8: Comparison of the crack growth rate histories obtained from the three measurement methods for specimen S112

5.5. Stress intensity factor and geometry correction functions

The numerical analyses provided the relationship between crack length and the Mode I stress intensity factors for the tested SENB geometry. These results were converted into a dimensionless geometry correction factor and were subsequently used to calculate the specimen-specific stress intensity factor ranges required for the Paris-law evaluation. This section presents the obtained geometry correction functions and compares the results of the geometrically linear and geometrically nonlinear analyses.

5.5.1. Geometry correction functions for the tested SENB geometry

The geometry correction factors obtained from the geometrically linear and geometrically nonlinear finite element analyses were fitted separately. The geometrically linear formulation is used as the reference case because its stress intensity factor results were directly verified against solutions available in the literature. The geometrically nonlinear formulation is included as an additional numerical case.

For the geometrically linear formulation, the fitted geometry correction function is

$$Y_{\text{lin}}(\alpha) = \frac{P_{\text{lin}}(\alpha)}{(1 - \alpha)^{3/2}}, \quad (5.1)$$

$$P_{\text{lin}}(\alpha) = 0.6008 + 1.7288\alpha - 9.0249\alpha^2 + 16.1640\alpha^3 - 13.3171\alpha^4 + 4.2388\alpha^5.$$

For the geometrically nonlinear formulation, the fitted geometry correction function is

$$Y_{\text{nl}}(\alpha) = \frac{P_{\text{nl}}(\alpha)}{(1 - \alpha)^{3/2}}, \quad (5.2)$$

$$P_{\text{nl}}(\alpha) = 4.7986 - 34.2667\alpha + 113.6548\alpha^2 - 191.7449\alpha^3 + 161.9933\alpha^4 - 54.6924\alpha^5.$$

Here, $\alpha = a/W$ denotes the relative crack length. Because the polynomial representation is sensitive to coefficient rounding, the full-precision values are included in the Appendix, while the coefficients are presented here to four significant figures. Both functions are valid only within the fitted interval, approximately $\alpha = 0.46\text{--}0.85$.

The fitted functions are compared in Figure 5.9.

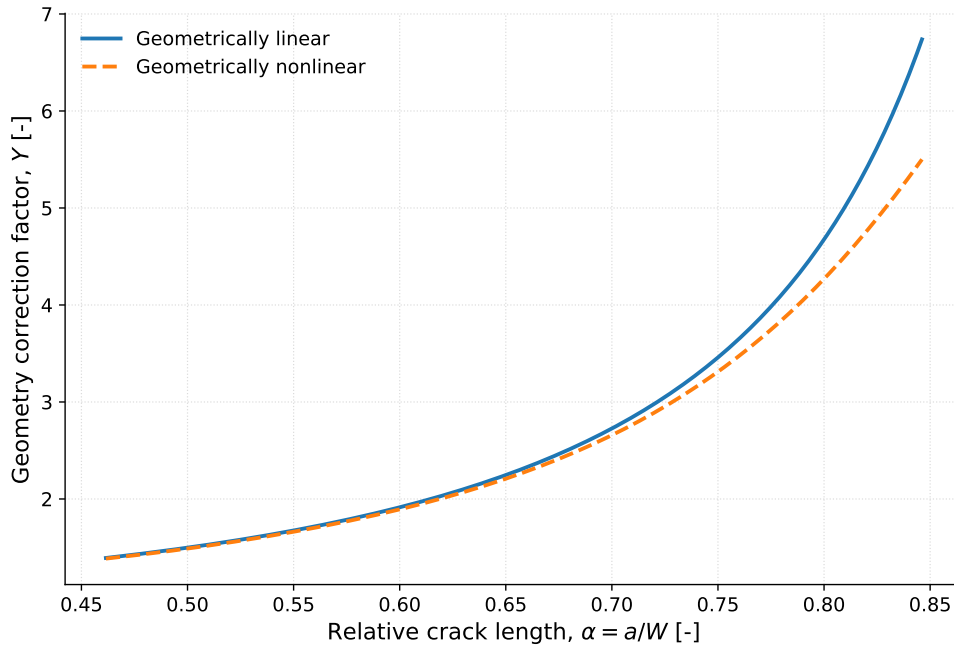


Figure 5.9: Fitted geometry correction functions obtained from the geometrically linear and geometrically nonlinear finite element analyses

The two functions give similar geometry correction factors at smaller relative crack lengths. Their difference increases with increasing α , with the geometrically nonlinear formulation resulting in lower Y values towards the upper end of the investigated interval. The influence of this difference on the calculated stress intensity factor ranges and Paris-law parameters is evaluated in the following sections.

5.5.2. Comparison of geometrically linear and nonlinear analyses

The Mode I stress intensity factors obtained from the geometrically linear and geometrically nonlinear analyses are compared in Figure 5.10. The same reference geometry, applied load, crack lengths, boundary conditions, and contour extraction procedure were used in both formulations. Therefore, the observed differences result from the inclusion of geometrical nonlinearity.

The two formulations give similar stress intensity factors at smaller crack lengths. At $a = 30$ mm, the geometrically nonlinear result is approximately 0.47% lower than the geometrically linear result. The difference increases to approximately 1.24% at $a = 40$ mm, 5.92% at $a = 50$ mm, and 18.09% at $a = 55$ mm. The increasing difference is consistent with the increasing influence of the deformed geometry as the crack grows and the remaining ligament becomes smaller. The left and right vertical axes show equivalent values in $\text{MPa}\sqrt{\text{mm}}$ and $\text{MPa}\sqrt{\text{m}}$, respectively.

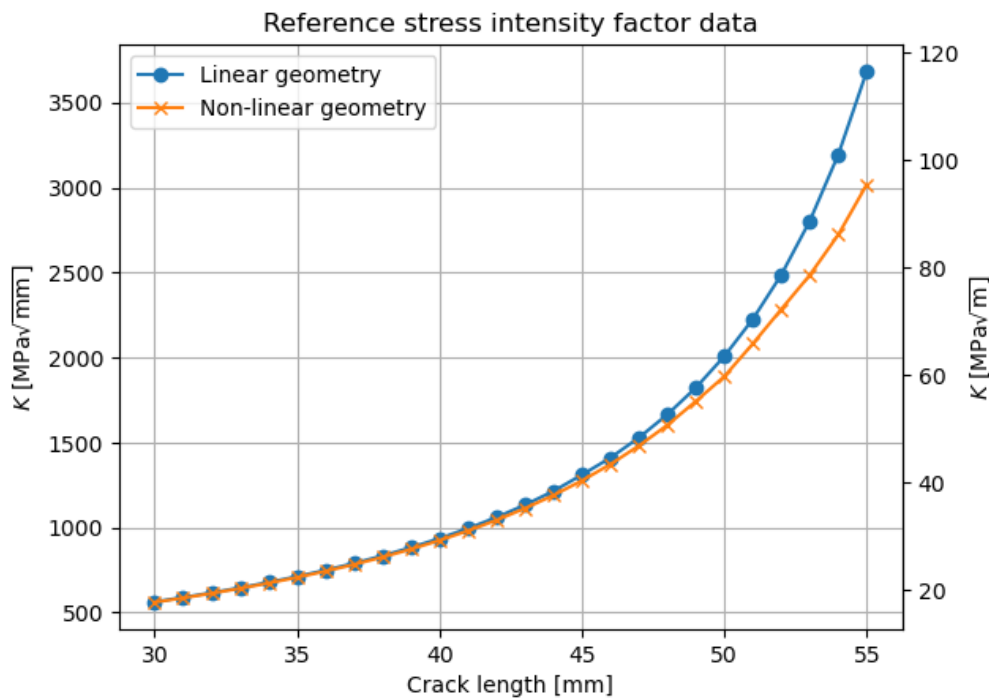


Figure 5.10: Comparison of the geometrically linear and nonlinear stress intensity factors

5.6. Paris-law parameter results

The Paris-law parameters were determined by combining the crack growth rates with the corresponding specimen-specific stress intensity factor ranges. The evaluation was performed separately using the geometrically linear and geometrically nonlinear geometry correction functions presented in Section 5.5.1. The geometrically linear formulation is used as the verified reference case, while the geometrically nonlinear formulation is retained to evaluate the sensitivity of the obtained Paris-law parameters to the numerical modelling approach.

The results are presented separately for the crack-gauge, COD-based, and strain-gauge-based evaluations. First, the specimen-specific Paris-law fits are presented, followed by a comparison of the resulting parameters.

5.6.1. Individual Paris-law fits

The Paris-law regression was performed separately for each specimen and each available measurement method. The representative comparison presented in this subsection was calculated using the geometrically linear geometry correction function. The method-specific data-selection rules described in the methodology were applied consistently to all specimens.

Specimen S112 was selected for the direct comparison because crack-gauge, COD-based, and strain-gauge-based results were available from the same fatigue test. Figure 5.11 shows the selected crack growth rate data and the corresponding Paris-law fits. For the continuous COD-based and strain-gauge-based datasets, the number of displayed points was reduced only for visual clarity. The regressions were performed using all valid data points within the selected fitting intervals. All valid discrete crack-gauge points were retained.

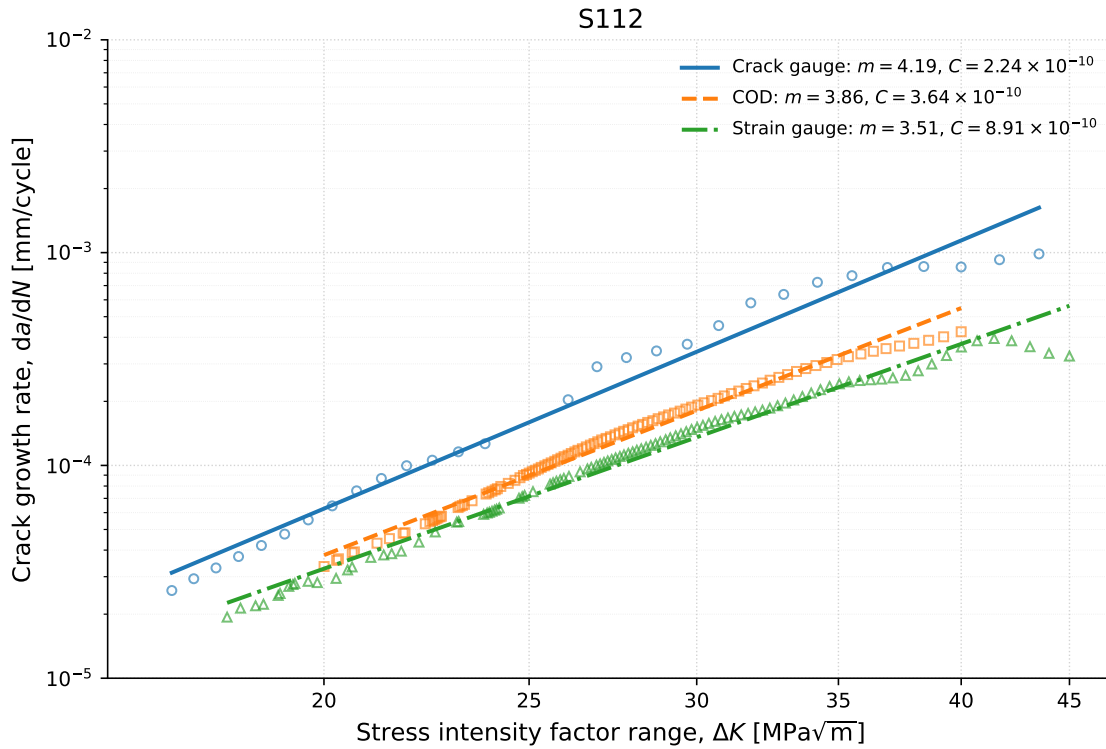


Figure 5.11: Paris-law fits for specimen S112.

For specimen S112, the crack-gauge data resulted in $m = 4.19$ and $C = 2.24 \times 10^{-10}$. The COD-based evaluation gave $m = 3.86$ and $C = 3.64 \times 10^{-10}$, while the strain-gauge-based evaluation resulted in $m = 3.51$ and $C = 8.91 \times 10^{-10}$. All three measurement methods show an approximately linear relationship in the log-log representation within their selected fitting ranges. However, the fitted slopes and vertical positions are not identical.

The complete set of individual Paris-law fits is provided in Appendix D.2. The parameters obtained for all specimens are summarised in the following subsection.

5.6.2. Aggregated Paris-law parameters

The specimen-specific Paris-law parameters obtained from the available measurement methods are summarised in Tables 5.3 and D.1. The method-specific data-selection rules described in the methodology were applied consistently to all specimens. The reported numerical values of C correspond to a crack growth rate expressed in mm/cycle and a stress intensity factor range expressed in $\text{MPa}\sqrt{\text{m}}$. Missing entries indicate that no usable result was available for the corresponding specimen and measurement method.

The relationship between the Paris exponent m and the regression intercept $b = \log_{10}(C)$ is shown in Figures 5.12 and D.22. Each point represents one specimen evaluated using one measurement

The specimen-specific parameters were subsequently summarised separately for each measurement method. Since the values of C cover several orders of magnitude, the statistical evaluation was performed using $b = \log_{10}(C)$ rather than the untransformed values of C . Table 5.4 presents the sample means and sample standard deviations. All valid parameter pairs were retained, and no statistical outlier exclusion was applied.

Table 5.4: Descriptive statistics of the Paris-law parameters.

Formulation	Method	n	$\bar{m}$	s_m	$\bar{b}$	s_b
Linear	Crack gauge	9	4.070	0.500	-9.467	0.732
	COD	10	4.349	0.575	-10.093	0.723
	Strain gauge	11	3.753	0.287	-9.228	0.390
Nonlinear	Crack gauge	9	4.188	0.513	-9.607	0.747
	COD	10	4.466	0.575	-10.233	0.724
	Strain gauge	11	3.853	0.298	-9.345	0.403

where n is the number of specimens, $\bar{m}$ is the mean Paris-law exponent, s_m is its sample standard deviation, $\bar{b}$ is the mean logarithmic intercept, with $b = \log_{10}(C)$, and s_b is its sample standard deviation.

For a visual comparison of the parameter scatter, Figure 5.13 shows one-dimensional normal curves constructed using the method-specific sample means and sample standard deviations. The short markers along the horizontal axes show the individual specimen values. These curves provide a descriptive representation of the observed scatter and are not used as a formal test of normality.

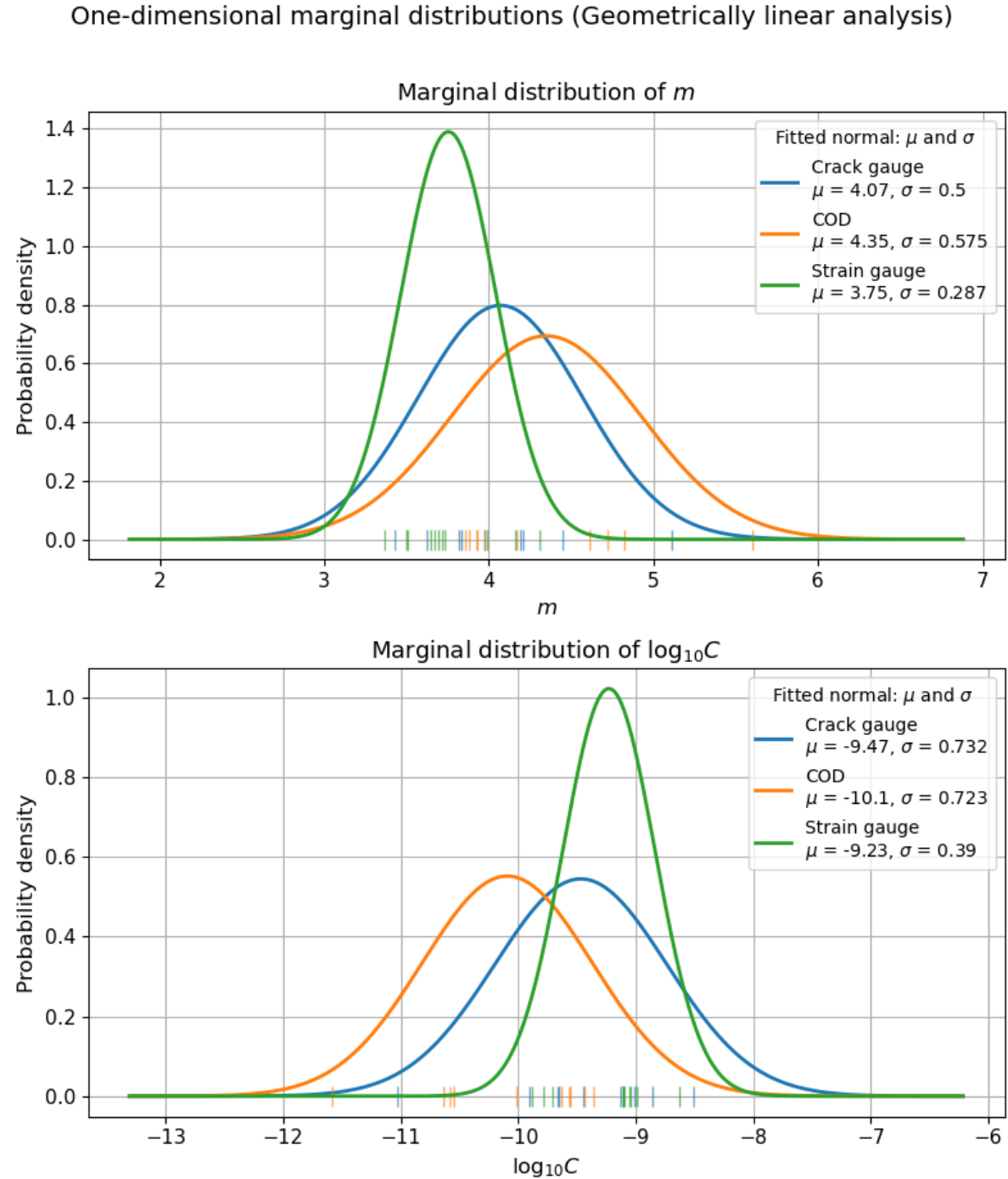


Figure 5.13: One-dimensional parameter distributions: linear formulation.

The strain-gauge-based results show the smallest scatter in both m and $b = \log_{10}(C)$. The COD-based results have the largest sample standard deviation of m . For b , the crack-gauge results show a marginally larger sample standard deviation than the COD-based results in both numerical formulations. The same general pattern is therefore observed for the geometrically linear and nonlinear evaluations.

The two aggregation procedures defined in Section 3.6.4 were applied separately to each measurement method and numerical formulation. The first procedure averages the specimen-specific slopes and logarithmic intercepts, whereas the second procedure fits one line directly to the combined log-log data using specimen-balanced weights. The resulting representative parameters are compared in Table 5.5.

Figure 5.14 illustrates the specimen-balanced pooled regression for the geometrically linear formulation. Each panel contains the retained data from all specimens available for the corresponding measurement method and one common Paris-law fit. For visual clarity, the number of displayed points was reduced deterministically for the continuous measurement methods. However, the regression was performed using all retained data points with the specimen-balanced weighting defined in Equation 3.38.

Geometrically linear analysis

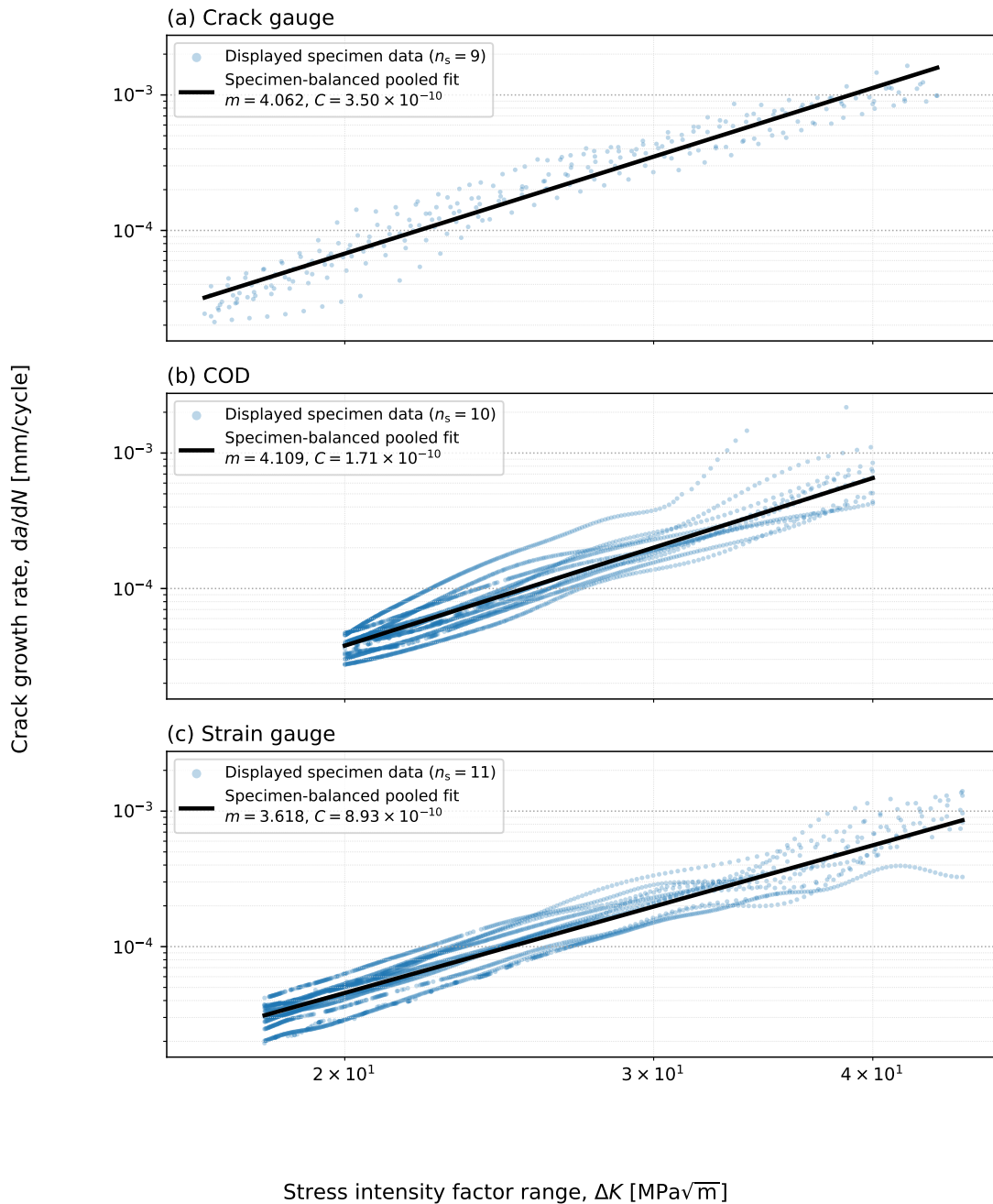


Figure 5.14: Specimen-balanced pooled Paris-law fits: linear formulation.

For the crack-gauge results, the two aggregation procedures produce nearly identical Paris-law parameters. The largest difference is obtained for the COD-based results, while the strain-gauge-based results show an intermediate difference. The same pattern is observed for the geometrically linear

Table 5.5: Comparison of the Paris-law aggregation procedures.

Formulation	Method	Mean of specimen-specific fits		Specimen-balanced pooled fit	
		m_{mean}	C_{mean}	m_{pool}	C_{pool}
Linear	Crack gauge	4.070	3.41×10^{-10}	4.062	3.50×10^{-10}
	COD	4.349	8.07×10^{-11}	4.109	1.71×10^{-10}
	Strain gauge	3.753	5.91×10^{-10}	3.618	8.93×10^{-10}
Nonlinear	Crack gauge	4.188	2.47×10^{-10}	4.180	2.54×10^{-10}
	COD	4.466	5.86×10^{-11}	4.212	1.29×10^{-10}
	Strain gauge	3.853	4.52×10^{-10}	3.702	7.16×10^{-10}

and geometrically nonlinear formulations. Therefore, the aggregated crack-gauge parameters are only weakly affected by the aggregation procedure, whereas the parameters obtained from the continuous measurement methods are more sensitive to how the specimen-level datasets are combined.

5.6.3. Exploratory specimen comparison

The procedure described in Section 3.6.5 was applied to the processed Paris-law datasets.

Figure 5.15 presents the selected crack growth rates. The points belonging to the same specimen are connected to support the visual comparison of the specimen ordering between the methods.

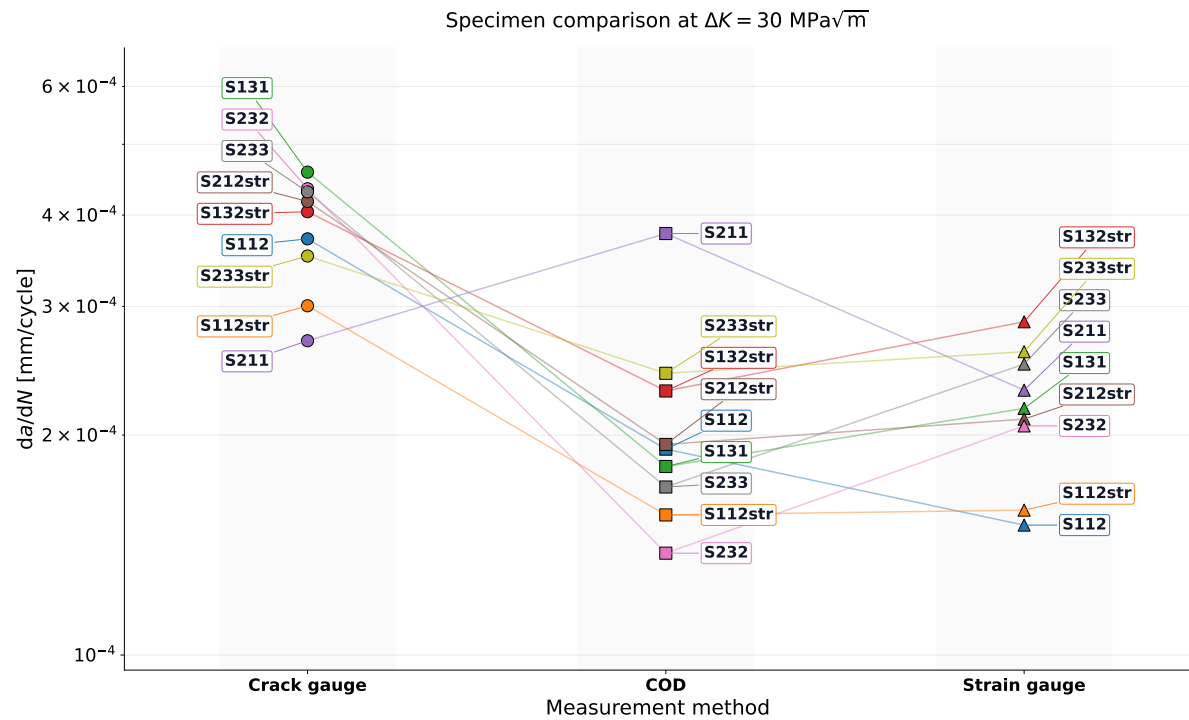


Figure 5.15: Specimen comparison using the processed Paris-law data points closest to $\Delta K = 30 \text{ MPa}\sqrt{\text{m}}$. Lines connect the results belonging to the same specimen.

The corresponding numerical values are listed in Table 5.6. Within each measurement method, the specimens are ordered from the highest to the lowest crack growth rate.

The specimen associated with the highest crack growth rate was different for each method: S131 for the crack gauge, S211 for COD, and S132str for the strain gauge. No specimen occurred among the three highest-ranked results for all three methods. S112str, however, was ranked eighth according to all three methods, indicating that some specimen-level agreement was present.

Table 5.6: Specimen-wise crack growth rates at the processed Paris-law data points closest to $\Delta K = 30 \text{ MPa}\sqrt{\text{m}}$. Values are given in mm/cycle.

Crack gauge		COD		Strain gauge	
Specimen	da/dN	Specimen	da/dN	Specimen	da/dN
S131	4.580×10^{-4}	S211	3.774×10^{-4}	S132str	2.858×10^{-4}
S232	4.347×10^{-4}	S233str	2.430×10^{-4}	S233str	2.601×10^{-4}
S233	4.303×10^{-4}	S132str	2.298×10^{-4}	S233	2.501×10^{-4}
S212str	4.176×10^{-4}	S212str	1.942×10^{-4}	S211	2.304×10^{-4}
S132str	4.043×10^{-4}	S112	1.913×10^{-4}	S131	2.175×10^{-4}
S112	3.712×10^{-4}	S131	1.812×10^{-4}	S212str	2.103×10^{-4}
S233str	3.516×10^{-4}	S233	1.698×10^{-4}	S232	2.059×10^{-4}
S112str	3.006×10^{-4}	S112str	1.555×10^{-4}	S112str	1.579×10^{-4}
S211	2.692×10^{-4}	S232	1.379×10^{-4}	S112	1.506×10^{-4}

5.6.4. Comparison with literature values

Since reference specimens were not included in the experimental programme, the obtained fatigue crack propagation behaviour was compared with published data. Seitzl et al. [44] investigated the long-crack propagation behaviour of modern S235 J2 and S355 J2 structural steels and steel extracted from an old crane structure. Their experiments were performed using CT specimens at the same stress ratio of $R = 0.1$ as applied in the present study. The published crack growth rates were expressed in mm/cycle, while ΔK was expressed in $\text{MPa}\sqrt{\text{m}}$. Therefore, the reported Paris-law parameters can be compared numerically with the present results.

The comparison was performed using the specimen-balanced pooled parameters obtained from the geometrically linear formulation. This aggregation method provides one representative Paris-law relation for each measurement method and is therefore comparable with the single relations reported for the literature materials. The corresponding results obtained from the geometrically nonlinear formulation are retained for the Appendix.

Since m and C are determined simultaneously, comparing either parameter separately may be misleading. Therefore, the tabulated parameters are complemented by a comparison of the complete Paris-law curves over a common ΔK interval.

Table 5.7: Paris-law parameters used for the literature comparison.

Dataset	m	C
Old Botlek Bridge – crack gauge	4.062	3.50×10^{-10}
Old Botlek Bridge – COD	4.109	1.71×10^{-10}
Old Botlek Bridge – strain gauge	3.618	8.93×10^{-10}
S235 J2	3.26	2.0×10^{-9}
S355 J2	2.71	1.0×10^{-8}
Old crane steel	3.59	5.0×10^{-10}

The corresponding Paris-law curves are compared over the common interval of $\Delta K = 20\text{--}40 \text{ MPa}\sqrt{\text{m}}$ in Figure 5.16.

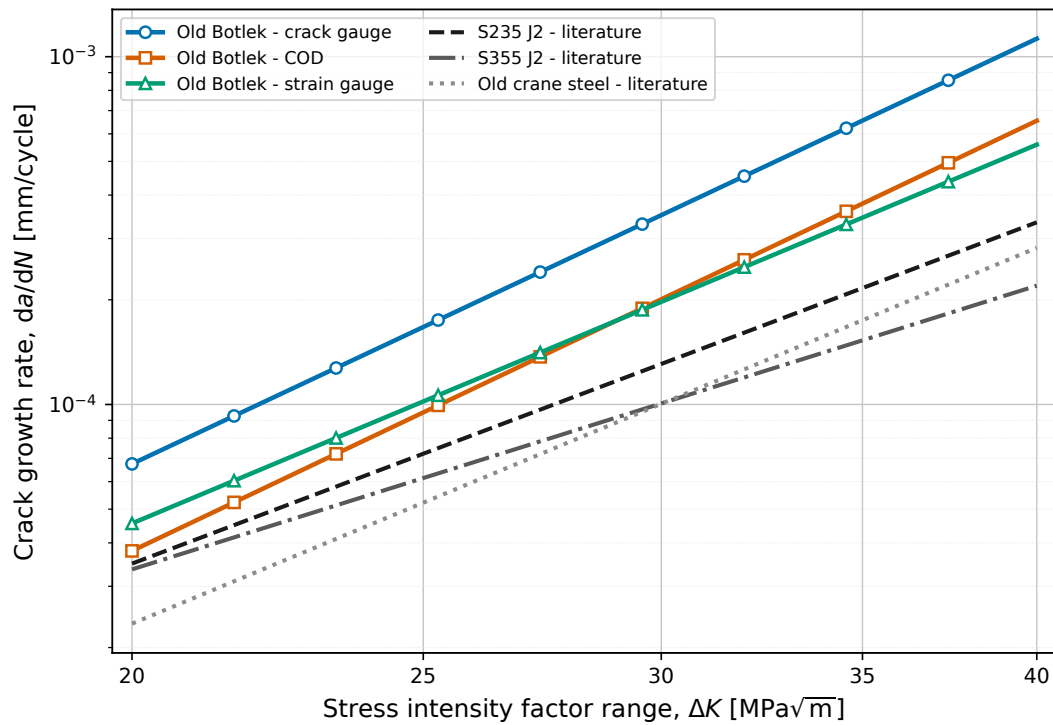


Figure 5.16: Comparison with literature Paris-law curves

At the lower end of the compared ΔK interval, the COD- and strain-gauge-based pooled curves are close to the published curves for S235 J2 and S355 J2. The crack-gauge-based curve gives a higher crack growth rate.

The strain-gauge-based parameter pair is the closest to the values reported for the old crane steel. The Old Botlek Bridge results generally have higher Paris exponents than the literature values. Consequently, the differences between the fitted curves increase towards the upper end of the investigated ΔK interval.

The comparison should not be interpreted as a direct validation against the literature materials. The literature results were obtained using CT specimens and optical crack-length measurements, whereas the present study used SENB specimens and three different measurement systems. Nevertheless, the comparison provides a reference for the position and slope of the experimentally obtained fatigue crack growth curves. The possible reasons for the observed differences are considered in the Discussion chapter.

Discussion of evaluation methods and material behaviour

The aim of this study was to determine the fatigue crack propagation parameters of steel recovered from the Old Botlek Bridge. During the experiments, crack gauges, COD measurements, and strain gauges were used to monitor crack propagation. The resulting datasets required method-specific post-processing before they could be used for the fatigue crack growth evaluation. A Python-based workflow, implemented in a series of Jupyter notebooks, was therefore developed to convert the measured signals into crack-length histories, $a(N)$, and subsequently into crack growth rates, da/dN . In parallel, finite element analyses were used to determine the specimen-specific stress intensity factor ranges, ΔK , as a function of crack length. Combining the experimentally derived crack growth rates with the numerically determined ΔK values enabled the Paris-law parameters to be calculated.

The results show that the three measurement methods capture the same general development of fatigue crack propagation, but the quantitative results are influenced by the measurement principle, the post-processing procedure, and the numerical formulation used to determine ΔK . The differences obtained from three measurement and post-processing workflows applied to the same specimen during the same experiment highlight the importance of understanding how fatigue crack growth results are derived. The complete measurement and post-processing workflow should therefore be explicitly documented and considered when interpreting and comparing reported crack growth rates and Paris-law parameters. This chapter, therefore, focuses on the interpretation and significance of these differences. The measurement and post-processing methods are evaluated first, followed by the numerical determination of ΔK and the interpretation of the Paris-law parameters. The specimen-to-specimen variation, the comparison with conventional structural steels, and the implications for the assessment and potential reuse of recovered steel are then discussed.

6.1. Evaluation of the measurement and post-processing methods

The obtained results indicate that all three measurement methods were able to capture the overall trends in fatigue crack propagation. Usable post-processed histories were obtained for nine specimens using the crack gauge, ten using the COD measurements, and all eleven retained specimens using the strain gauges, as summarised in Table 5.2. The crack-length histories consistently showed slow and gradual growth during the earlier part of the retained data, followed by increasingly rapid propagation towards the final stage, as shown in Figures 5.1–5.3. The corresponding crack growth rates generally covered the same order-of-magnitude range, approximately from 10^{-5} to 10^{-3} mm/cycle, across the three methods. Nevertheless, the agreement was primarily qualitative, as noticeable differences remained in the quantitative crack length and crack growth rate results.

Among the three measurement techniques, the crack gauge provided the most direct indication of crack extension, since the breakage of conductive wires produced discrete jumps in the measured voltage signal. The nature of this method allowed a relatively straightforward inference from the voltage signal to crack length. However, post-processing was more cumbersome because the wire-breakage events had to be identified in the signal. The developed algorithm provided a semi-automatic workflow that reduced the required manual effort compared with identifying every breakage point manually. The algorithm still required specimen-specific tuning, and every detection was manually verified by visual inspection. Nevertheless, this procedure was less labour-intensive than fully manual data processing.

Due to the polynomial differentiation procedure, fewer da/dN values were available than crack-length values. For a polynomial fitting window containing n points, the first and last $(n - 1)/2$ crack-length points were left without a corresponding derivative because a centred derivative could not be evaluated at these positions. With the seven-point window used in this study, this corresponded to the loss of three derivative values at each beginning and end of the crack-length histories. Since the number of available crack-length points was already limited by the crack-gauge architecture, this loss of derivative points can have a recognizable effect on the results.

Because the resulting datasets contained a limited number of detected crack events, individual data points could have a relatively large influence on the final Paris-law fit. The fitted parameters may therefore be sensitive to the detection and processing of individual points. The crack gauge also provided a local measurement, as it recorded crack extension only at the position of the gauge rather than along the entire crack front. In contrast to the discrete crack-gauge measurements, COD and strain gauges produce densely sampled specimen-response datasets, providing a higher sampling resolution. Neither method measures crack length directly, so it must be inferred through a calibration process, which was based on finite element analysis in this study. The densely sampled data allowed smoother $a(N)$ representations to be obtained and enabled most of the retained crack propagation process to be tracked. However, the inferred crack lengths remained dependent on the assumptions and limitations of the numerical model, including the two-dimensional plane-stress formulation, the nominal specimen geometry, and the applied linear elastic material model. Manufacturing deviations and specimen-specific differences in stiffness relative to the nominal model could therefore introduce systematic discrepancies into the inferred crack lengths.

Following the method-specific preprocessing, only data satisfying $a_{\text{init},i} + 1 \text{ mm} \leq a \leq 55 \text{ mm}$ were retained for the COD- and strain-gauge-based evaluations. The lower limit reduced the influence of the notch and crack-initiation region, while the upper limit prevented the calibration functions from being used beyond the crack-length range covered by the finite element analyses.

For several specimens, the initial portions of the COD- and strain-gauge-based $a(N)$ curves indicated a steeper apparent increase than the corresponding crack-gauge histories. Furthermore, the uncorrected numerical conversions occasionally produced crack lengths below the measured initial crack length $a_{\text{init},i}$, which was nominally approximately 30 mm. In the COD-based evaluation, this occurred because the measured crack opening at the applied maximum load was smaller than that predicted by the numerical calibration. This behaviour is consistent with a lower experimental compliance and, therefore, a stiffer response of the specimen–measurement system than represented by the numerical model. A similar model-to-test mismatch may have influenced the strain-based evaluation.

The discrepancy cannot be attributed solely to the assumed elastic modulus of $E = 200 \text{ GPa}$, since the calculated response also depended on the idealised crack and notch geometry, the two-dimensional plane-stress assumption, the nominal specimen dimensions, and the boundary and measurement conditions.

The initial model-to-test discrepancy is illustrated for specimen S131 in Figure 6.1. The curves are shown before applying the initiation-exclusion criterion and the optional vertical corrections.

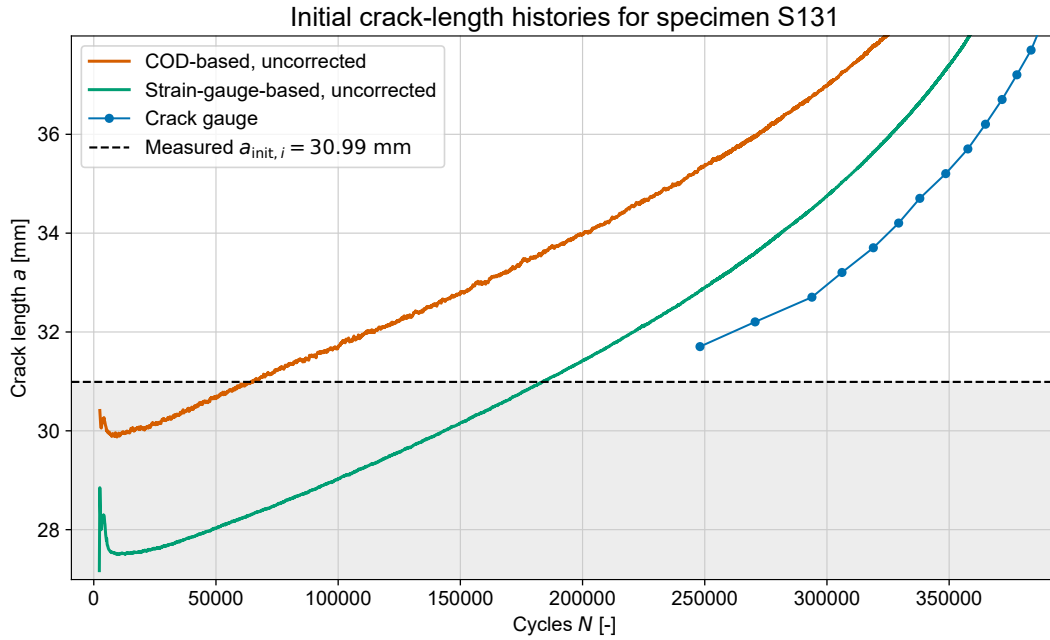


Figure 6.1: Initial crack-length histories for specimen S131

Both indirect methods initially produced crack-length estimates below $a_{\text{init},i}$ and showed a steeper apparent increase than the crack-gauge measurements. The discrepancy decreased as crack propagation progressed. This supports the interpretation that the difference was primarily associated with the initial model-to-test compliance relationship rather than with a physical crack length below the manufactured notch.

The sensitivity of COD measurements to the clip-gauge installation represents an additional source of uncertainty. Small differences in the position, alignment, or zero setting of the gauge can introduce systematic offsets into the recorded COD signal. When COD is converted into crack length, these offsets may propagate into the resulting $a(N)$ histories, the calculated da/dN values, and the Paris-law parameters.

The different sampling characteristics and post-processing requirements of the measurement methods also required different procedures for calculating the crack growth rate, which was not measured directly by any of the techniques. For the crack-gauge data, the previously described seven-point local polynomial differentiation was applied instead of the basic secant differentiation method, resulting in smoother crack-growth-rate curves.

The densely sampled COD- and strain-gauge-based crack-length histories contained small local fluctuations, as can also be observed in Figure 6.1. Since numerical differentiation amplifies such fluctuations, direct local polynomial differentiation produced excessively noisy and unstable crack-growth-rate curves. Different polynomial fitting-window lengths were examined, but changing the window size only altered the balance between noise sensitivity and excessive smoothing, without providing consistently suitable results. This motivated the use of a smooth parametric representation of the crack-length histories before differentiation.

For the COD and strain-gauge measurements, a separate neural-network-based regression model was therefore trained for each specimen. The main objective was to fit a parametric model that was sufficiently flexible to closely represent the measured crack-length curve. The neural network was selected because the automatic-differentiation functionality available in PyTorch allowed the derivative to be evaluated directly and the procedure to be automated. This resulted in smoother derivative functions. However, the smoothness of the calculated derivatives partly reflects the fitted regression model and does not by itself demonstrate the accuracy of the obtained da/dN values.

The direct comparison for specimen S112 is particularly informative because all three measurement methods were applied during the same experiment. The specimen geometry, loading conditions, and material were therefore identical for the three datasets. The methods captured the same general crack propagation tendency, while differences remained in the corresponding crack-length and crack-growth-

rate values. These differences cannot be explained by variations in specimen geometry or loading and are therefore primarily associated with the measurement principles and post-processing procedures. Since the measurement principle and the post-processing procedure varied together, their individual contributions to these differences could not be separated. However, since no independent crack-length reference measurement was available, the comparison cannot establish which method provided the closest representation of the actual crack length or crack growth rate.

The three methods should therefore be regarded as complementary rather than interchangeable. The crack gauge provided a more direct but discrete and local indication of crack extension, whereas the COD- and strain-gauge-based methods provided densely sampled but indirect crack-length histories that depended on numerical calibration and regression-based differentiation. Their combined use strengthened the evaluation by revealing the sensitivity of the derived fatigue crack growth parameters to the measurement and post-processing procedure. Consequently, the method-specific Paris-law results and the differences between them are more informative than selecting one measurement technique as an absolute reference.

The method-related differences were also reflected in the final Paris-law curves. As shown in Figure 5.11, the crack-gauge-based curve lies above the COD- and strain-gauge-based curves over the investigated ΔK range. This systematic difference can be traced back to the crack-length results. For specimen S112, Figure 5.7 shows that the crack-gauge-based evaluation produced smaller crack lengths at the same cycle numbers than the COD- and strain-gauge-based evaluations. At the same time, Figure 5.8 shows that the calculated crack growth rates remained within a similar order of magnitude. Since the calculated ΔK increases with crack length under the same loading conditions, the smaller crack-gauge-based crack lengths resulted in smaller ΔK values. Consequently, comparable values of da/dN were associated with smaller values of ΔK , and this difference was carried through into the fitted Paris-law parameters. At a given value of ΔK , the crack-gauge-based evaluation therefore predicts the largest da/dN and the lowest apparent fatigue crack-growth resistance. Since the three curves were obtained from the same specimen population, this difference should be interpreted as an effect of the measurement and evaluation procedures rather than as poorer material behaviour of a separate specimen group. This supports the earlier conclusion that the agreement between the three methods was stronger at the level of the overall trends than at the level of the exact quantitative results. This comparison highlights the need to report and examine the complete evaluation workflow, including signal-to-crack-length conversion, smoothing and differentiation, ΔK calculation, and selection of the fitting range, when interpreting and comparing fatigue crack growth results.

6.2. Numerical determination of ΔK

The experimental methodology and subsequent post-processing provided the $a(N)$ and da/dN data. For establishing the Paris-law relationship, the determination of ΔK was also necessary. Since ΔK could not be measured directly, it had to be calculated. Using FEA, models were built to extract discrete $K_I(a)$ values using contour integrals. Based on these data points, a geometry correction function $Y(a/W)$ was calculated. This function was combined with the specimen-specific values of B , W , l , and ΔF , allowing the SIF results to be adjusted directly to the individual specimen properties. These numerical ΔK values connected the experimental crack-growth data to the Paris-law regression. Consequently, the reliability of the calculated Paris-law parameters was directly affected by the numerical $Y(a/W)$ geometry correction function.

The geometrically linear SIF formulation showed strong numerical stability. The corresponding mesh- and contour-sensitivity results are shown in Figures 4.2 and 4.3. The relative variation among Contours 2–5 remained below 0.005 %, while the deviation associated with Contour 1 remained below 0.03 %. This indicates that the determined K_I value was not particularly sensitive to the choice of the integration contour. The geometrically nonlinear model produced unstable contour results for the fine 0.5 mm mesh at $a = 50$ mm. However, the results obtained with the 1 mm and 2 mm mesh sizes were more stable and comparable. As a compromise, the 1 mm mesh provided stable and computationally efficient analyses. In general, a finer mesh did not guarantee more stable contour-integral results. This assessment demonstrated numerical stability for the investigated meshes, but should not be interpreted as a formal convergence proof.

The strongest external assessment of the performance of the linear model was provided by comparison with the solution of Baratta and Fett. The mean absolute difference was 0.21 %, while the

maximum absolute difference was 0.44 %. This close agreement supports the correct implementation of the boundary conditions, the K_I extraction, and the consistency of the nominal stress and $Y(a/W)$ calculations. The comparisons with the solutions of Brown and Wang resulted in mean differences of 5.68 % and 7.14 %, respectively. Although these differences were larger, both comparisons supported the correct order of magnitude and the increasing trend with crack length. The quantitative results are summarised in Table 4.1. These comparisons verified the numerical procedure within the investigated geometrical and crack-length range, but did not validate the complete two-dimensional model against the real experimental behaviour.

A sensitivity study was carried out with respect to the geometrical configuration. The study focused on the applicability of the determined $Y(a/W)$ function to the other investigated specimen geometries. The modified span showed only small discrepancies, together with the approximately scaled geometry. The largest difference occurred for the short-span configuration, where the span was smaller relative to the specimen height. The maximum difference was 3.4 %. The corresponding geometry correction functions and relative differences are presented in Figures 4.6 and 4.7. The linear $Y(a/W)$ function therefore behaved relatively robustly within the analysed geometries. This supports the use of the obtained function for the specimen geometries investigated in this study. However, the results do not demonstrate general applicability to other bending configurations or substantially different SENB proportions.

The difference between the geometrically linear and nonlinear results was not uniform over the crack-length range. For relatively short cracks, the difference remained small. However, at $a = 50$ mm, the difference had already increased to 5.92 %, while at $a = 55$ mm it reached 18.09 %. The increasing difference between the two formulations is shown in Figure 5.10. As the crack length increased, the remaining ligament became smaller and the deformation of the specimen increased. Consequently, the deformed body increasingly differed from its initial geometry. Therefore, the effect of geometrical nonlinearity became significant mainly in the large crack-length range. The material model remained linear elastic in both formulations; therefore, the difference originated from geometrical rather than material nonlinearity. Since the nonlinear formulation was not verified to the same extent as the linear formulation, it cannot be stated that it provides a better representation of the real behaviour. However, it shows that the deformed geometry may influence the SIF results. In addition, since the nonlinear geometry correction function was determined at a single reference load and subsequently combined with the specimen-specific load ranges, the nonlinear results should be interpreted as a sensitivity comparison rather than as a fully load-specific nonlinear determination of ΔK .

Based on these findings, the geometrically linear formulation was considered the primary reference for the Paris-law evaluation. This choice was supported by the stable contour results, the mesh-sensitivity assessment, the limited sensitivity to the investigated geometrical variations, and the close agreement with the Baratta and Fett solution. The geometrically nonlinear results were retained as a sensitivity comparison, particularly for the large crack-length range where the remaining ligament became small. This verification does not remove the limitations associated with the two-dimensional plane-stress and linear-elastic idealisations. The calculated linear ΔK values were therefore considered suitable for the comparative Paris-law evaluation performed in this study, provided that they were interpreted within the investigated geometry, crack-length range, and LEFM assumptions.

6.3. Interpretation of the Paris-law parameters

Paris-law describes the second phase of crack growth after crack initiation and before the sudden, final fracture. In log-log scale the Paris-law shows a linear line, therefore fitting Paris-law parameters means fitting a linear line on the calculated datapoints with regression. The intercept of the line is $\log(C)$ and the slope is m . The calculated parameters are valid in the applied range in Table 3.5. The positions and slopes of the fitted Paris-law curves depend on the evaluated $a(N)$, da/dN and ΔK data. The resulting parameter pairs should therefore be interpreted together by comparing the crack growth rates predicted at the same ΔK within the overlapping fitting ranges.

In order to describe the investigated material an aggregation procedure has to be applied from the individual specimen results. In this study two different methods were applied to evaluate the differences and to see how sensitive are these results. The aggregated parameters can be seen in Table 5.5. For the geometrically linear formulation, the two aggregation procedures gave nearly identical parameter pairs for the crack-gauge results, with more noticeable differences for the COD- and strain-gauge-based

results.

The two aggregation procedures represent the dataset in different ways. The mean approach fits a separate Paris-law line for each specimens, then takes the average of the individual m and $\log(C_i)$ parameters. This way all the specimens contributes with one parameter pair to the final result. The pooling approach fits one Paris-law line using all datapoints at once. Specimen balancing provides that the denser datasets do not take more weight than the sparser ones. The result presents the trend of the whole method-level dataset. The two aggregation procedures are not mathematically equivalent, since averaging the parameters obtained from separate regressions does not necessarily produce the same result as fitting a single regression to the combined, specimen-balanced dataset. When the specimen-specific slopes and intercepts are similar, the two procedures are expected to produce comparable parameter values, as observed for the crack-gauge results. In contrast, larger differences between the specimen-specific slopes, intercepts, and covered ΔK ranges can lead to a greater difference between the aggregation procedures. This may explain the more noticeable differences observed for the COD- and strain-gauge-based results. Consequently, the selected aggregation procedure represents an additional source of methodological variability in the resulting method-level Paris-law parameters.

The obtained m and C values should not be interpreted independently. In the log–log representation of the Paris law, m is the slope of the fitted line, while $\log_{10}(C)$ is its intercept. A change in the slope generally requires a corresponding change in the intercept for the fitted line to remain close to the same data points. Therefore, larger m values were generally accompanied by smaller C values. This strong negative correlation is clearly visible in Figure 5.12. Consequently, a large numerical difference in C does not directly indicate the same relative difference in the predicted crack-growth rate. The parameter pairs and the corresponding fitted Paris-law curves should, therefore, be compared together within the applied ΔK ranges.

The influence of the numerical formulation on the fitted Paris-law curves is illustrated in Figure D.24, with the corresponding parameter pairs reported in Table 5.5.

The underlying experimental crack-growth histories remained unchanged; however, the nonlinear formulation changed the calculated ΔK values and, consequently, which points fell within the specified fitting intervals. Since the difference between the linear and nonlinear SIF results increased with crack length, this change was not uniform over the fitted range and therefore affected both the slope and the intercept of the regression. The observed differences represent the sensitivity of the Paris-law parameters to the selected numerical formulation. As the nonlinear formulation was not independently validated, its parameters cannot automatically be considered more accurate than those obtained from the linear formulation.

Overall, the results show that the obtained Paris-law parameters depend not only on the material response but also on the measurement method, aggregation procedure, and numerical determination of ΔK . Therefore, m and C should not be interpreted separately or as unique material constants independent of the applied evaluation procedure. Despite the differences between the individual parameter values, the corresponding Paris-law curves remained comparable within the investigated ΔK ranges. The reported parameter pairs therefore provide a consistent description of the observed crack propagation behaviour within the conditions and assumptions of this study. The remaining specimen-to-specimen variation is discussed in the following section.

6.4. Differences between specimens

The specimen-specific Paris-law parameters showed clear variation for all three measurement methods, as reported in Tables 5.3 and D.1. The corresponding scatter of the parameter pairs is illustrated in Figures 5.12 and D.22. For the geometrically linear formulation, m ranged from 3.43 to 5.11 for the crack-gauge-based evaluation, from 3.86 to 5.60 for the COD-based evaluation, and from 3.37 to 4.31 for the strain-gauge-based evaluation. For the geometrically nonlinear formulation, the corresponding ranges were 3.53–5.26, 3.92–5.69, and 3.46–4.42, respectively. Although the numerical formulation shifted the individual parameter values, the extent and relative pattern of the specimen-to-specimen variation remained similar. The widest range of m was observed for the COD-based results, while the strain-gauge-based results showed the narrowest range.

The positions of the individual specimens were not fully consistent between the methods. For example, specimen S211 had the largest COD-based exponent, $m = 5.60$, but the smallest strain-gauge-based exponent, $m = 3.37$, under the linear formulation. For several specimens, the fitted exponents

were relatively similar between the methods. For S131, the crack-gauge-, COD-, and strain-gauge-based exponents were 3.83, 3.93, and 3.69, respectively, while the corresponding values for S212str were 3.82, 3.97, and 3.65. Similar exponents indicate similar slopes of the Paris-law curves, but do not necessarily imply similar crack growth rates at the same ΔK . Overall, the observed scatter may contain both specimen-related differences and method-specific effects. These contributions could not be separated using the available data, and all valid parameter pairs were retained.

The specimens originated from either the top or bottom flange, but their individual positions are unknown. The two flanges may have experienced different loading histories because of their predominantly compressive and tensile stress states. An exploratory grouping analysis can indicate whether the parameter data contain two separate groups. However, even if two groups are found, they cannot be assigned to the top and bottom flanges or directly attributed to the previous loading history without information about the original specimen locations.

The exploratory comparison did not reveal any clear visual pattern that could support the presence of two specimen groups related to their origins. The ordering of the specimens changed between the different measurement methods, and their distributions did not provide evidence that the specimens could be divided into groups according to their possible top- or bottom-flange origin. For example, as shown in Figure 5.15 and Table 5.6, at the processed data points closest to $\Delta K = 30 \text{ MPa}\sqrt{\text{m}}$, specimen S211 had the lowest crack growth rate according to the crack-gauge evaluation but the highest according to the COD-based evaluation among the nine specimens available for all three methods. This change in ordering indicates that the measurement and post-processing workflow can influence specimen comparisons and should be considered before observed differences are attributed to material variability or prior service loading.

6.5. Comparison with conventional structural steels

To place the obtained results in a broader context, the specimen-balanced pooled parameters from the geometrically linear formulation were compared with the results reported by Seitzl et al. [44] for S235 J2, S355 J2, and old crane steel. The parameter pairs are listed in Table 5.7, while the corresponding Paris-law curves are shown in Figure 5.16. Since m and C are coupled parameters, the comparison was based mainly on the complete fitted curves rather than on either parameter separately. The curves were evaluated over the common range of $\Delta K = 20\text{--}40 \text{ MPa}\sqrt{\text{m}}$ using consistent units.

Within this range, all three Old Botlek Bridge curves lie above the literature curves shown in Figure 5.16. At the same value of ΔK , they therefore predict a larger da/dN and a lower apparent fatigue crack-growth resistance. The separation increases with ΔK because the Old Botlek Bridge exponents range from 3.618 to 4.109, whereas the literature values in Table 5.7 range from 2.71 to 3.59. Among the selected literature datasets, the old crane steel has the exponent closest to the strain-gauge-based Old Botlek Bridge result, with $m = 3.59$ and $m = 3.618$, respectively.

The identical stress ratio of $R = 0.1$ and the consistent units support a meaningful comparison between the datasets. However, the material and the experimental configuration changed simultaneously. Seitzl et al. [44] used 10 mm thick CT specimens, optical crack-length measurements, and loading frequencies between 60 and 100 Hz, whereas the present study used approximately 32 mm thick SENB specimens tested at 10 Hz and three different crack-length evaluation procedures. The higher Old Botlek Bridge curves therefore indicate lower apparent crack-growth resistance relative to the selected literature datasets, but the difference cannot be assigned exclusively to the material. Most importantly, without unused reference specimens from the same Old Botlek Bridge steel, the influence of the previous fatigue-loading history cannot be isolated.

Conclusions and outlook

7.1. Conclusions

Main research question: “What are the fatigue crack propagation parameters of steel recovered from the Old Botlek Bridge based on SENB specimen experiments?”

The fatigue crack propagation parameters were determined using three measurement and post-processing workflows. Their application to the same specimen during the same experiment produced different crack-length and crack-growth-rate estimates and different fitted Paris-law curves. Based on the specimen-balanced pooled results from the geometrically linear formulation, the crack-gauge-based evaluation gave $m = 4.062$ and $C = 3.50 \times 10^{-10}$, the COD-based evaluation gave $m = 4.109$ and $C = 1.71 \times 10^{-10}$, and the strain-gauge-based evaluation gave $m = 3.618$ and $C = 8.93 \times 10^{-10}$. These values apply within the method-specific fitting ranges, with da/dN expressed in mm/cycle and ΔK in $\text{MPa}\sqrt{\text{m}}$. The resulting parameter pairs should therefore be interpreted through their corresponding Paris-law curves and in the context of the measurement and post-processing workflow used to obtain them. These findings highlight the importance of explicitly documenting and examining the complete evaluation workflow when interpreting and comparing fatigue crack growth results. The aggregation procedure had a negligible effect on the crack-gauge parameters but a more noticeable effect on the COD- and strain-gauge-based parameters.

Sub-research question 1: “How can crack length measurements from different experimental techniques be combined with numerically determined stress intensity factor ranges and the analytical Paris-law formulation to determine fatigue crack growth parameters?”

The crack-gauge voltage jumps were converted into discrete crack-length values, while the COD- and strain-gauge measurements were converted into crack lengths using finite-element-based calibration functions. Crack growth rates were calculated using seven-point local polynomial differentiation for the crack-gauge data and neural-network regression with automatic differentiation for the COD- and strain-gauge-based data. The resulting $a(N)$ and da/dN data were combined with the specimen-specific numerical $\Delta K(a)$ values, after which the Paris-law parameters were obtained through linear regression in logarithmic coordinates. Usable results were obtained for nine specimens using the crack gauge, ten using the COD measurements, and eleven using the strain gauges. All three workflows captured the general crack propagation trend, although differences remained in the quantitative results.

Sub-research question 2: “How can the stress intensity factor range, ΔK , required for the Paris-law evaluation, be determined numerically for the tested SENB specimens, and how sensitive is the corresponding geometry correction factor to the investigated SENB specimen dimensions and four-point bending configuration?”

The stress intensity factor range was determined using finite element models from which discrete $K_I(a)$ values were extracted using contour integrals. These results were converted into a geometry correction function, $Y(a/W)$, which was combined with the specimen-specific dimensions, span, and load range to calculate ΔK . For the geometrically linear models, the variation among Contours 2–5 remained below 0.005%, and the deviation associated with Contour 1 remained below 0.03%. Comparison with the Baratta and Fett solution resulted in a mean absolute difference of 0.21% and a maximum absolute difference of 0.44%, while the maximum geometry-related difference within the investigated configurations was approximately 3.4%. The geometrically linear formulation was therefore considered

numerically stable and suitable within the investigated range, whereas the geometrically nonlinear formulation was retained as a sensitivity comparison because its difference from the linear result increased to 5.92% at $a = 50$ mm and 18.09% at $a = 55$ mm.

Sub-research question 3: “How do these specimens obtained from a previously used structure behave compared with raw steel materials?”

The specimen-balanced pooled Paris-law curves from the geometrically linear formulation were compared with literature results for S235 J2, S355 J2, and old crane steel over the common range of $\Delta K = 20\text{--}40 \text{ MPa}\sqrt{\text{m}}$. All three Old Botlek Bridge Paris-law curves were positioned above the selected literature curves within this range. Consequently, the evaluated Old Botlek Bridge specimens showed higher crack growth rates at the same ΔK and, therefore, lower apparent fatigue crack-growth resistance. This direction of the difference was consistent across the three measurement and post-processing workflows. However, differences in material, specimen geometry, loading frequency, and measurement and post-processing procedures between the studies limit the extent to which this difference can be attributed to the material itself. Without unused reference specimens from the same steel tested and evaluated under comparable conditions, including the same stress ratio ($R = 0.1$), loading frequency, specimen geometry, and measurement and post-processing procedures, the effect of prior service loading could not be isolated.

Sub-research question 4: “Do the specimen-specific Paris-law parameters provide evidence of two distinct clusters that could be consistent with the different prior stress histories of the top and bottom flanges?”

No clear evidence of two distinct specimen groups was found. The ordering of the specimens changed between the measurement methods, and no clear visual pattern was observed in the distributions of da/dN . Therefore, the available results do not support dividing the specimens according to their possible top- or bottom-flange origin. Nevertheless, a possible effect of the different prior loading histories cannot be excluded.

7.2. Research limitations and recommendations for further research

Research limitations The absolute accuracy of the three crack-length evaluation methods could not be established because no independent crack-length reference measurement was available. The crack-gauge evaluation required specimen-specific tuning and manual verification, while the COD- and strain-gauge-based results depended on the numerical calibration and regression procedures. Since the measurement principle and the post-processing procedure varied together between the workflows, their individual contributions to the observed differences could not be separated.

The numerical models used two-dimensional plane-stress and linear-elastic idealisations and did not represent the through-thickness crack front. The geometrically nonlinear geometry correction function was determined at a single reference load and was not verified to the same extent as the linear formulation.

Recommendations for further research

1. **Independent optical measurements.** Future experiments should include an independent optical crack-propagation measurement, as in the example of Seitzl et al. [44], who used CCD cameras to measure crack length on both sides of the specimens. Digital image correlation could also be considered as an alternative optical method. This measurement could provide a reference for determining which of the crack-gauge-, COD-, and strain-gauge-based methods most closely represents the observed crack propagation.
2. **Post-processing sensitivity and automation.** The differences observed between the measurement and post-processing workflows in the present study motivate further investigation of the individual evaluation steps. The influence of the differentiation procedure should therefore be investigated by applying different methods to the same crack-length histories while keeping the remaining evaluation steps unchanged.
The semi-automatic crack-gauge wire-break detection procedure could be further developed into a fully automated workflow. A more robust signal-processing algorithm could reduce the need for

specimen-specific tuning and manual verification while improving the reliable distinction between actual wire-break events and signal noise.

3. **Further experimental characterisation and specimen comparison.** Unused reference specimens from steel with comparable chemical composition and manufacturing history would help assess the possible effect of prior service loading, provided that the experimental and evaluation conditions are comparable. Dedicated fatigue experiments on coupon specimens could be used to investigate the crack-initiation phase to complement the crack-propagation results.

Kujawski [30] analysed experimental data from the literature and showed that increasing the load ratio can increase fatigue crack growth rates at the same applied ΔK . Since the present experiments were performed at $R = 0.1$, further testing at different stress ratios should investigate the influence of R on the crack growth rates and fitted Paris-law curves of the recovered steel. A consistent measurement and post-processing workflow should be used to distinguish the effect of stress ratio from differences introduced by the evaluation procedure. This would help establish the applicability of the obtained results to loading conditions relevant to the intended reuse.

Saeed et al. [41] observed higher fatigue crack growth rates in synthetic seawater than in air for the heat-affected zone of welded S355 steel. Further testing under variable-amplitude loading and environmental conditions relevant to the intended reuse would improve the applicability of the obtained results to the assessment and reuse of recovered structural steel.

A more detailed unsupervised machine learning analysis could be performed using the processed datasets to identify possible groups among the specimens, while accounting for measurement and post-processing effects.

4. **Numerical modelling and structural application.** Three-dimensional and elastic-plastic numerical models should be investigated to quantify through-thickness constraint and crack-tip plasticity effects. The nonlinear stress intensity factor should also be evaluated for the actual specimen-specific load levels.

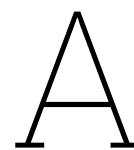
Following further validation, the obtained Paris-law parameters could be implemented in advanced numerical crack-propagation analyses of complex structural details during the redesign of the structure. The results could support the selection of suitable structural details and the development of a life-extension plan.

Bibliography

- [1] Alireza Akhavan-Safar et al. *Cohesive Zone Modelling for Fatigue Life Analysis of Adhesive Joints*. Cham: Springer, 2022. DOI: 10.1007/978-3-030-93142-1.
- [2] Abdalnaser M. Alshoaibi and Yahya Ali Fageehi. "Advances in Finite Element Modeling of Fatigue Crack Propagation". In: *Applied Sciences* 14.20 (2024). ISSN: 2076-3417. DOI: 10.3390/app14209297. URL: <https://www.mdpi.com/2076-3417/14/20/9297>.
- [3] T.L. Anderson. *Fracture Mechanics Fundamentals and Applications*. Fourth Edition. CRC Press, 2017.
- [4] Donald R. Askeland, Pradeep P. Fulay, and Wendelin J. Wright. *The Science and Engineering of Materials*. Sixth Edition. Cengage Learning, 2010.
- [5] ASTM International. *Standard Test Method for Linear-Elastic Plane-Strain Fracture Toughness K_{IC} of Metallic Materials*. Accessed: 2026-04-18. West Conshohocken, PA, USA: ASTM International, 2012. DOI: 10.1520/E0399-12. URL: <https://www.astm.org/e0399-12.html>.
- [6] ASTM International. *Standard Test Method for Measurement of Fatigue Crack Growth Rates*. Accessed: 2026-04-18. West Conshohocken, PA, USA: ASTM International, 2023. URL: <https://www.astm.org/e0647-23.html>.
- [7] F. I. Baratta and T. Fett. "The Effect of Load and Crack Misalignment on Stress Intensity Factors for Bend-Type Fracture Toughness Specimens". In: *Journal of Testing and Evaluation* 28.2 (2000), pp. 96–102. DOI: 10.1520/JTE12080J. URL: https://www.researchgate.net/publication/240757490_The_Effect_of_Load_and_Crack_Misalignment_on_Stress_Intensity_Factors_for_Bend-Type_Fracture_Toughness_Specimens.
- [8] Helen Bartsch. "Reuse of Reclaimed Steel Components in Construction: A Systematic Review of Potential, Challenges and Future Directions". In: *Structures* 80 (2025), p. 110057. DOI: 10.1016/j.istruc.2025.110057.
- [9] A.N. Bleijenberg. "Renewal of civil infrastructure - Dutch national forecast for replacement and renovation". In: *TNO-report* 39 (2021).
- [10] William F. Brown and John E. Srawley. *Plane Strain Crack Toughness Testing of High Strength Metallic Materials*. Tech. rep. 410. Philadelphia, PA: American Society for Testing and Materials, 1966. DOI: 10.1520/STP44663S. URL: https://books.google.com/books/about/Plane_Strain_Crack_Toughness_Testing_of.html?id=RLjdAAAAIAAJ.
- [11] R. Buzatu, V. Ungureanu, and Petr Hradil. "Environmental and Economic Impact of Steel Industrial Buildings Made of Reclaimed Elements". In: *Life-Cycle of Structures and Infrastructure Systems: Proceedings of the 8th International Symposium on Life-Cycle Civil Engineering*. Ed. by Fabio Biondini and Dan M. Frangopol. London: CRC Press, 2023, pp. 1303–1311. DOI: 10.1201/9781003323020-160.
- [12] F.C. Campbell, ed. *Fatigue and Fracture*. ASM international, 2012.
- [13] Alan O'Connor Cathal Leahy Eugene OBrien. "The effect of traffic growth on characteristic bridge load effects". In: *Transportation Research Procedia* 10.14 (2016), pp. 3990–3999.
- [14] Dassault Systèmes Simulia Corp. *ABAQUS v6.14 Documentation*. ABAQUS, Version 6.14. Dassault Systèmes Simulia Corp. Oct. 2016.
- [15] W. Elber. "Fatigue crack closure under cyclic tension". In: *Engineering Fracture Mechanics* 2.1 (1970), pp. 37–45. ISSN: 0013-7944. DOI: 10.1016/0013-7944(70)90028-7.
- [16] Ellen MacArthur Foundation. *Towards the Circular Economy Vol. 2: Opportunities for the Consumer Goods Sector*. Report, with McKinsey & Company. Ellen MacArthur Foundation. 2013. URL: <https://content.ellenmacarthurfoundation.org/m/50c85a620a58955/original/Towards-the-circular-economy-Vol-2.pdf> (visited on 03/22/2026).

- [17] Sjors van Es and Eric Bonet. *Test specimens taken from the Oude Botlekbrug*. TNO report TNO 2018 R10905. Delft, The Netherlands: TNO, Aug. 2018.
- [18] *Eurocode 3: Design of Steel Structures – Part 1-9: Fatigue*.
- [19] European Commission. *Circular Economy*. URL: https://environment.ec.europa.eu/strategy/circular-economy_en (visited on 03/22/2026).
- [20] European Commission. *European Climate Law*. URL: https://climate.ec.europa.eu/eu-action/european-climate-law_en (visited on 03/22/2026).
- [21] European Commission. *Net-zero Industry Act*. URL: https://commission.europa.eu/topics/competitiveness/green-deal-industrial-plan/net-zero-industry-act_en (visited on 03/22/2026).
- [22] European Commission. *The European Green Deal*. URL: https://commission.europa.eu/strategy-and-policy/priorities-2019-2024/european-green-deal_en (visited on 03/22/2026).
- [23] Ahindra Ghosh and Amit Chatterjee. *Ironmaking and Steelmaking Theory and Practice*. Second printing. PHI Learning Private Limited, 2008.
- [24] Eugenio Giner et al. “An Abaqus implementation of the extended finite element method”. In: *Engineering Fracture Mechanics* 76 (Nov. 2008), pp. 347–368. DOI: 10.1016/j.engfracmech.2008.10.015.
- [25] Noah Gooijer et al. “Entity-based System Dynamics for Infrastructure Modeling”. In: *42nd International Conference of the SD Society* (2024), p. 21.
- [26] Carina Harpprecht et al. “Future environmental impacts of global iron and steel production”. In: *Energy and Environmental Science* 20.18 (2025), pp. 8009–8028.
- [27] D. O. Harris and H. L. Dunegan. “Continuous monitoring of fatigue-crack growth by acoustic-emission techniques”. In: *Experimental Mechanics* 14.2 (Feb. 1974), pp. 71–81. ISSN: 1741-2765. DOI: 10.1007/BF02323130. URL: <https://doi.org/10.1007/BF02323130>.
- [28] Xu Kuangdi. *The ECPH Encyclopedia of Mining and Metallurgy*. Second printing. Springer, Encyclopedia of China Publishing House, 2008.
- [29] B. Kühn et al. *Assessment of Existing Steel Structures: Recommendations for Estimation of Remaining Fatigue Life*. Tech. rep. EUR 23252 EN. European Commission, Joint Research Centre, 2008. URL: <https://eurocodes.jrc.ec.europa.eu/sites/default/files/2021-12/EUR23252EN.pdf>.
- [30] Daniel Kujawski. “Load-ratio effects on fatigue crack growth: Analysis and correlation”. In: *International Journal of Fatigue* 210 (2026), p. 109695. DOI: 10.1016/j.ijfatigue.2026.109695.
- [31] Christian Kurrer. “The potential of hydrogen for decarbonising steel production”. In: *Scientific Foresight Unit (STOA)* 8.PE 641.552 (2020).
- [32] Liming Lu. *Iron Ore - Mineralogy, Processing and Environmental Sustainability*. Second edition. Woodhead Publishing Series in Metals and Surface Engineering, 2022.
- [33] Matt Reese. *Fatigue Crack Growth Rate Graph*. APP Knowledge Base. 2017. URL: <https://kb.appinc.co/wp-content/uploads/fatigue-crack-growth-rate-graph.jpg> (visited on 04/11/2026).
- [34] William McDonough and Michael Braungart. *Cradle to Cradle Re-Making the Way We Make Things*. First edition. North Point Press, 2002.
- [35] Santiago Munoz, M. Reza Hosseini, and Robert H. Crawford. “Towards a holistic assessment of circular economy strategies: The 9R circularity index”. In: *Sustainable Production and Consumption* 13.47 (2024), pp. 400–412.
- [36] Yukitaka Murakami and Masahiro Endo. “Effects of Defects, Inclusions and Inhomogeneities on Fatigue Strength”. In: *International Journal of Fatigue* 16.3 (1994), pp. 163–182. DOI: 10.1016/0142-1123(94)90001-9. URL: [https://doi.org/10.1016/0142-1123\(94\)90001-9](https://doi.org/10.1016/0142-1123(94)90001-9).

- [37] Qays Nazarali and Xin Wang. "The effect of T-stress on crack-tip plastic zones under mixed-mode loading conditions". In: *Fatigue and Fracture of Engineering Materials and Structures* 34 (Apr. 2011), pp. 792–803. DOI: 10.1111/j.1460-2695.2011.01573.x.
- [38] Margherita Pongiglione and Chiara Calderini. "Material Savings through Structural Steel Reuse: A Case Study in Genoa". In: *Resources, Conservation and Recycling* 86 (2014), pp. 87–92. DOI: 10.1016/j.resconrec.2014.02.011.
- [39] R.I. Stephens et al. *Metal Fatigue in Engineering*. Second Edition. Wiley Interscience, 2000.
- [40] Rijkswaterstaat. *Rijkswaterstaat Archief*. 2003. URL: www.archieven.nl (visited on 03/15/2026).
- [41] Hasan Saeed, Somsubhro Chaudhuri, and Wim De Waele. "Experimental Evaluation of the Short and Long Fatigue Crack Growth Rate of S355 Structural Steel Offshore Monopile Weldments in Air and Synthetic Seawater". In: *Applied Ocean Research* 149 (2024), p. 104063. ISSN: 0141-1187. DOI: 10.1016/j.apor.2024.104063. URL: <https://doi.org/10.1016/j.apor.2024.104063>.
- [42] Sarmed Salih, Keith Davey, and Zhenmin Zou. "A computationally efficient cohesive zone model for fatigue". In: *Fatigue & Fracture of Engineering Materials & Structures* 42.2 (2019), pp. 518–532. DOI: <https://doi.org/10.1111/ffe.12927>. eprint: <https://onlinelibrary.wiley.com/doi/pdf/10.1111/ffe.12927>. URL: <https://onlinelibrary.wiley.com/doi/abs/10.1111/ffe.12927>.
- [43] Jaap Schijve. *Fatigue of Structures and Materials*. Second Edition. Springer, 2009.
- [44] Seitzl, Stanislav et al. "Comparison of fatigue crack propagation behaviour in two steel grades S235, S355 and a steel from old crane way". In: *MATEC Web Conf.* 310 (2020), p. 00034. DOI: 10.1051/mateconf/202031000034. URL: <https://doi.org/10.1051/mateconf/202031000034>.
- [45] Subra Suresh. *Fatigue of Materials*. 2nd ed. Cambridge: Cambridge University Press, 1998. ISBN: 9780521578479. DOI: 10.1017/CB09780521578479.
- [46] *Sustainability of construction works – Environmental product declarations – Core rules for the product category of construction products*. European standard. Nederlands Normalisatie-instituut, 2019.
- [47] Tokyo Measuring Instruments Laboratory Co., Ltd. *Strain Gauges Catalog 2025–2026*. Accessed: 2026-04-14. Tokyo Measuring Instruments Laboratory Co., Ltd. 2025. URL: <https://tml.jp/eng/documents/Catalog/Straingauge2025-2026.pdf>.
- [48] Elisabeth Unterfrauner et al. "The Maker Movement and the Disruption of the Producer-Consumer Relation". In: *Internet Science*. Ed. by Sotiris Diplaris et al. Cham: Springer International Publishing, 2018, pp. 113–125. ISBN: 978-3-319-77547-0.
- [49] Tim Verlaan. "Mobilization of the Masses: Dutch Planners, Local Politics, and the Threat of the Motor Age 1960-1980". In: *Journal of Urban History* 47.1 (2021), pp. 136–156.
- [50] C. H. Wang. *Introduction to Fracture Mechanics*. Tech. rep. DSTO-GD-0103. Melbourne, Australia: DSTO Aeronautical and Maritime Research Laboratory, 1996. DOI: 10.13140/RG.2.1.1444.2408. URL: https://www.researchgate.net/publication/277137659_Introduction_to_Fracture_Mechanics.
- [51] G. N. Wells and L. J. Sluys. "A new method for modelling cohesive cracks using finite elements". In: *International Journal for Numerical Methods in Engineering* 50 (2001), pp. 2667–2682. DOI: 10.1002/nme.143.
- [52] Wikipedia contributors. *Fracture modes (File: Fracture modes v2.svg)*. Wikipedia. 2024. URL: https://en.wikipedia.org/wiki/Fracture_mechanics#/media/File:Fracture_modes_v2.svg (visited on 04/11/2026).
- [53] William J. Wright and Michael A. Grubb. *Steel Bridge Design Handbook: Bridge Steels and Their Mechanical Properties*. Tech. rep. B901-22. National Steel Bridge Alliance and American Institute of Steel Construction, 2022. URL: https://www.aisc.org/media/b2rltgbl/b901_sbdh_chapter1.pdf.



Computational workflow and key Python routines

A.1. Overview of the post-processing workflow

The analysis was carried out using a set of numbered Jupyter notebooks. The notebooks used the raw experimental measurements, the specimen geometry and loading data, and a set of shared utility functions. The output of each main calculation was used as the input of the next notebook in the corresponding processing branch.

This appendix summarises the input, main steps, and output of each notebook. Selected code blocks are then provided for the main calculations.

Notebook 00: Shared SENB utility functions

Input. No external dataset is read directly.

Main steps. This notebook defines common functions for loading the raw CSV files, selecting repeated measurement columns, cleaning signals, detecting crack-gauge wire breaks, calculating crack-growth values, and preparing plots.

Output. Shared Python functions used by the subsequent data-processing notebooks; no standalone dataset is created.

Notebook 01: Crack-gauge signal to crack length and crack-growth rate

Input. Raw crack-gauge voltage histories and cycle counts, together with the shared signal-processing functions.

Main steps. The CG1 signal is cleaned and changed into a monotonic upper envelope. Wire breaks are detected from voltage jumps. The detections are corrected with specimen-specific settings and checked against the signal plots. The retained wire breaks are converted to crack length. A centred seven-point polynomial fit is used to calculate da/dN .

Output. Specimen-specific CSV files containing the cycle count, all retained crack-length points, and the available crack-growth-rate values.

Notebook 02: Crack-gauge SIF and Paris-law evaluation

Input. The crack-gauge crack-length and crack-growth-rate data, the numerical SIF reference values, and the specimen geometry and loading properties.

Main steps. The reference finite-element values are used to fit $Y(a/W)$. The measured B , W , and $P_{\max}$ values are used to calculate specimen-specific $K_{\max}$ and ΔK . Paris-law fits, fitting-range checks, and specimen-balanced pooled results are then calculated. The final crack-gauge aggregation uses all valid discrete crack-gauge points.

Output. CSV files and figures containing specimen-specific $\Delta K-da/dN$ data, fitted Paris-law parameters, combined plotting data, and specimen-balanced aggregated results.

Notebook 03: COD signal to crack length

Input. Raw COD histories and cycle counts, the finite-element COD calibration, and the specimen geometry and loading properties.

Main steps. Two COD channels are combined and cleaned. The finite-element relation between COD and a/W is fitted. The signal is corrected for the specimen load and height. The optional cycle mask is applied first. The configured vertical shift is then applied, followed by the lower limit $a_{\text{init},i} + 1$ mm and the upper limit of 55 mm.

Output. Specimen-specific and combined CSV files containing the cycle count and final COD-based crack length. The uncorrected initial region is also retained for diagnostic plotting.

Notebook 04: COD crack length to crack-growth rate

Input. The specimen-specific cycle and COD-based crack-length histories obtained from Notebook 03.

Main steps. A separate feedforward neural network is fitted to the $a(N)$ data of each specimen. The selected training settings use a hyperbolic tangent activation, two hidden layers, 32 features, 1500 epochs, and at most 1000 training points. PyTorch automatic differentiation gives da/dN from the fitted function.

Output. Specimen-specific and combined CSV files containing the cycle count, COD-based crack length, neural-network fit, and corresponding da/dN values.

Notebook 05: COD SIF and Paris-law evaluation

Input. The COD-based crack-length and neural-network crack-growth-rate data, together with the numerical geometry correction and specimen geometry and loading properties.

Main steps. Specimen-specific K_{max} and ΔK values are calculated from the COD crack lengths. The Paris-law fit uses $20 \leq \Delta K \leq 40 \text{ MPa}\sqrt{\text{m}}$. Fitting-range checks and specimen-balanced pooled results are also calculated.

Output. CSV files and figures containing specimen-specific $\Delta K-da/dN$ data, fitted Paris-law parameters, fitting-range checks, combined plotting data, and specimen-balanced aggregated results.

Notebook 06: Strain-gauge signal to crack length

Input. Raw strain-gauge histories and cycle counts, the finite-element strain calibration, and the specimen geometry and loading properties.

Main steps. The two strain channels are converted from microstrain to strain, combined, and cleaned. The strain magnitude is converted to a/W and corrected for the specimen load and height. In the final notebook, the lower limit $a_{\text{init},i} + 1$ mm is applied to the unshifted corrected crack length. The optional cycle mask and vertical shift are applied after this limit, followed by the upper limit of 55 mm.

Output. Specimen-specific and combined CSV files containing the cycle count and final strain-gauge-based crack length. The uncorrected initial region is also retained for diagnostic plotting.

Notebook 07: Strain-gauge crack length to crack-growth rate

Input. The specimen-specific cycle and strain-gauge-based crack-length histories obtained from Notebook 06.

Main steps. The same neural-network structure and training settings as in Notebook 04 are used for each strain-gauge-based $a(N)$ curve. Automatic differentiation gives da/dN .

Output. Specimen-specific and combined CSV files containing the cycle count, strain-gauge-based crack length, neural-network fit, and corresponding da/dN values.

Notebook 08: Strain-gauge SIF and Paris-law evaluation

Input. The strain-gauge-based crack-length and neural-network crack-growth-rate data, together with the numerical geometry correction and specimen geometry and loading properties.

Main steps. Specimen-specific K_{max} and ΔK values are calculated from the strain-gauge crack lengths. The Paris-law fit uses $18 \leq \Delta K \leq 45 \text{ MPa}\sqrt{\text{m}}$. Fitting-range checks and specimen-balanced pooled results are also calculated.

Output. CSV files and figures containing specimen-specific $\Delta K-da/dN$ data, fitted Paris-law parameters, fitting-range checks, combined plotting data, and specimen-balanced aggregated results.

Notebook 09: Paris-law fitting-range sensitivity

Input. The method-specific fitting-range sensitivity results and processed $\Delta K-da/dN$ plotting data from the crack-gauge, COD, and strain-gauge workflows.

Main steps. The three methods are combined, the results from the tested fitting ranges are compared, and the common fitting-range figure is prepared.

Output. CSV summary tables and figures comparing the effect of the selected fitting ranges across the three measurement methods.

Notebook 10: Crack-length and crack-growth summary figures

Input. The specimen-specific cycle, crack-length, and crack-growth-rate datasets obtained from all three measurement methods.

Main steps. The three data formats are changed to one common format. The notebook creates the crack-length, crack-extension, and crack-growth-rate figures used in the report. The first and last three crack-gauge crack-length points remain available even though their derivative values are empty.

Output. Report-ready figures presenting crack length, crack extension, and crack growth rate, together with data-availability and quality summaries.

Notebook 11: Per-specimen method comparison

Input. The processed crack-length and crack-growth-rate datasets obtained from the crack-gauge, COD, and strain-gauge methods.

Main steps. Crack length and crack-growth rate are compared for each specimen with common plot settings.

Output. Per-specimen comparison figures for crack length and crack-growth rate, together with method-specific data-availability summaries.

Notebook 12: Specimen-specific Paris-law figures

Input. The specimen-specific $\Delta K-da/dN$ datasets obtained from the three measurement methods.

Main steps. A separate geometrically linear Paris-law figure is fitted and exported for every available specimen. All valid points are used for fitting; only the displayed dense datasets may be reduced for plotting.

Output. Individual and combined PDF figures containing the specimen-specific Paris-law data and fitted lines, together with a CSV summary of the fitted parameters.

Notebook 13: Paris-law aggregation and statistical visualisation

Input. The specimen-specific and specimen-balanced pooled Paris-law parameters obtained from the three measurement methods, together with the literature comparison values.

Main steps. The parameter pairs are converted using $b = \log_{10}(C)$. Descriptive statistics, parameter-space plots, marginal distributions, pooled-fit figures, and the literature comparison are prepared. The outlier filter is disabled in the final settings, so all available parameter pairs are retained.

Output. Tables and figures containing the descriptive statistics, parameter scatter, marginal distributions, aggregation comparison, pooled Paris-law fits, and literature comparison.

A.2. Selected code blocks from the final notebooks

Only the main calculation blocks are shown below. Repeated plotting commands and long validation blocks are not repeated. Every Python line inside the listings is copied directly from the final notebook named in the heading.

Notebook 01: crack detection and polynomial differentiation

The following block contains the final threshold search and wire-break detection routine. The detections were checked visually before the crack-length data were accepted.

```

1 # =====
2 # CRACK DETECTION PARAMETERS - TUNE THESE
3 # =====
4 USE_SIGNAL_KEY = "cg1_uponly_log"
5 TARGET_WIRES = 49
6 DV_LOW = 1e-5
7 DV_HIGH = 0.2
8 MAX_ITER = 200
9 # =====
10
11
12 # -----
13 # Helper: detect cumulative cracked wires from upward jumps
14 # -----
15 def detect_cracked_wires(signal, dV):
16     signal = np.asarray(signal, dtype=float)
17
18     if signal.size == 0:
19         return np.array([], dtype=int)
20
21     cracked_wires = np.zeros(signal.size, dtype=int)
22
23     for i in range(1, signal.size):
24         cracked_wires[i] = cracked_wires[i - 1]
25
26         if signal[i] > signal[i - 1] + dV:
27             cracked_wires[i:] += 1
28
29     return cracked_wires
30
31
32 # -----
33 # Helper: final number of detected cracked wires
34 # -----
35 def cracked_wires_count(signal, dV):
36     cracked_wires = detect_cracked_wires(signal, dV)
37
38     if cracked_wires.size == 0:
39         return 0
40
41     return int(cracked_wires[-1])
42
43
44 # -----
45 # Binary search for specimen-specific optimal dV
46 # -----
47 def find_dV_for_target(signal, target_wires=41, dV_low=1e-4, dV_high=0.5, max_iter=50):
48     signal = np.asarray(signal, dtype=float)
49
50     if signal.size == 0:
51         return np.nan, 0
52
53     c_low = cracked_wires_count(signal, dV_low)
54     c_high = cracked_wires_count(signal, dV_high)
55
56     best_dV = dV_low
57     best_count = c_low
58     best_err = abs(c_low - target_wires)
59
60     err_high = abs(c_high - target_wires)
61     if err_high < best_err:
62         best_dV = dV_high
63         best_count = c_high
64         best_err = err_high
65
66     if target_wires >= c_low or target_wires <= c_high:
67         return float(best_dV), int(best_count)
68
69     lo = dV_low
70     hi = dV_high
71

```

```

72     for _ in range(max_iter):
73         mid = 0.5 * (lo + hi)
74         c_mid = cracked_wires_count(signal, mid)
75         err = abs(c_mid - target_wires)
76
77         if err < best_err:
78             best_err = err
79             best_dV = mid
80             best_count = c_mid
81
82         if err == 0:
83             break
84
85         if c_mid > target_wires:
86             lo = mid
87         else:
88             hi = mid
89
90     return float(best_dV), int(best_count)
91
92
93 # -----
94 # Helper: determine cycle counts at first crack events
95 # -----
96 def first_crack_cycles(counter, cracked_wires):
97     counter = np.asarray(counter, dtype=float)
98     cracked_wires = np.asarray(cracked_wires, dtype=int)
99
100    if cracked_wires.size == 0:
101        return np.array([0.0], dtype=float)
102
103    wires_max = int(np.max(cracked_wires))
104    N_cracks = np.full(wires_max + 1, np.nan, dtype=float)
105    N_cracks[0] = 0.0
106
107    for crack_id in range(1, wires_max + 1):
108        indices = np.where(cracked_wires == crack_id)[0]
109        if indices.size > 0:
110            N_cracks[crack_id] = float(counter[indices[0]])
111
112    return N_cracks
113
114
115 # -----
116 # Dictionary to store final crack analysis results
117 # -----
118 specimens_crack_analysis_np = {}
119 specimen_summary = {}
120
121 for name in specimen_names:
122
123     # -----
124     # Step 1: Load data
125     # -----
126     counter = np.asarray(specimens_uponly_np[name]["counter_cleaned"], dtype=float)
127     signal_used = np.asarray(specimens_uponly_np[name][USE_SIGNAL_KEY], dtype=float)
128
129     # -----
130     # Safety checks
131     # -----
132     if counter.size != signal_used.size:
133         raise ValueError(
134             f"{name}: counter and signal length mismatch: "
135             f"{counter.size} vs {signal_used.size}"
136         )
137
138     if counter.size == 0:
139         raise ValueError(f"{name}: empty signal is not allowed.")
140
141     # -----
142     # Step 2: Find optimal dV

```

```

143 # -----
144 best_dV, best_count = find_dV_for_target(
145     signal=signal_used,
146     target_wires=TARGET_WIRES,
147     dV_low=DV_LOW,
148     dV_high=DV_HIGH,
149     max_iter=MAX_ITER,
150 )
151
152 # -----
153 # Step 3: Detect cracks
154 # -----
155 cracked_wires = detect_cracked_wires(signal_used, best_dV)
156 wires_max = int(np.max(cracked_wires)) if cracked_wires.size > 0 else 0
157
158 # -----
159 # Step 4: Extract crack cycle numbers
160 # -----
161 N_cracks = first_crack_cycles(counter, cracked_wires)
162
163 # -----
164 # Store results
165 # -----
166 specimens_crack_analysis_np[name] = {
167     "counter_cleaned": counter,
168     "signal_used": signal_used,
169     "signal_key": USE_SIGNAL_KEY,
170     "cracked_wires": cracked_wires,
171     "wires_max": wires_max,
172     "N_cracks": N_cracks,
173     "dV_threshold": best_dV,
174     "target_wires": TARGET_WIRES,
175     "count_from_search": best_count,
176 }
177
178 specimen_summary[name] = {
179     "best_dV": best_dV,
180     "count_from_search": best_count,
181     "confirmed_last_row": int(cracked_wires[-1]) if cracked_wires.size > 0 else 0,
182     "n_points": int(signal_used.size),
183     "signal_min": float(np.min(signal_used)),
184     "signal_max": float(np.max(signal_used)),
185 }
186
187 print(
188     f"{name}: crack detection completed | "
189     f"signal = {USE_SIGNAL_KEY} | "
190     f"best dV = {best_dV:.6f} | "
191     f"detected wires = {wires_max}"
192 )

```

The seven-point polynomial derivative is evaluated at the centre of each fitting window.

```

1 poly_data = {}
2
3 window = 7
4 degree = 2
5
6 half = window // 2
7
8 for name, data in crack_length_data.items():
9     N = np.asarray(data["N"], dtype=float)
10    a = np.asarray(data["a"], dtype=float)
11
12    if len(N) < window:
13        raise ValueError(f"Not enough points in dataset {name} for window={window}")
14
15    N_mid = []
16    a_mid = []
17    dadN = []

```

```

18
19     for i in range(half, len(N) - half):
20         N_win = N[i - half : i + half + 1]
21         a_win = a[i - half : i + half + 1]
22
23         # Fit a second-degree polynomial
24         coeffs = np.polyfit(N_win, a_win, degree)
25
26         # polyfit: [c2, c1, c0]
27         dc = np.polyder(coeffs) # Derivative coefficients
28
29         # Evaluate the derivative at the central point
30         rate = np.polyval(dc, N[i])
31
32         N_mid.append(N[i])
33         a_mid.append(a[i])
34         dadN.append(rate)
35
36     poly_data[name] = {
37         "N": np.array(N_mid),
38         "a": np.array(a_mid),
39         "dadN": np.array(dadN)
40     }

```

The final export keeps every crack-length point. The derivative is placed only in the matching central rows, without interpolation.

```

1 from pathlib import Path
2 import numpy as np
3 import pandas as pd
4
5
6 export_dir = Path("csv_export_per_specimen")
7 export_dir.mkdir(exist_ok=True)
8
9 edge_points_without_dadN = 3
10
11 for specimen, crack_data in crack_length_data.items():
12     N = np.asarray(crack_data["N"], dtype=float)
13     a = np.asarray(crack_data["a"], dtype=float)
14
15     if specimen not in poly_data:
16         raise KeyError(f"No polynomial da/dN data found for {specimen}")
17
18     polynomial_data = poly_data[specimen]
19     N_mid = np.asarray(polynomial_data["N"], dtype=float)
20     a_mid = np.asarray(polynomial_data["a"], dtype=float)
21     da_dN_mid = np.asarray(polynomial_data["dadN"], dtype=float)
22
23     if len(N) != len(a):
24         raise ValueError(
25             f"Mismatched N and a lengths for {specimen}: "
26             f"{len(N)} and {len(a)}"
27         )
28
29     expected_dadN_count = len(N) - 2 * edge_points_without_dadN
30     if expected_dadN_count <= 0:
31         raise ValueError(
32             f"Not enough crack-length points for the seven-point "
33             f"polynomial method: {specimen}"
34         )
35
36     if not (
37         len(N_mid) == len(a_mid) == len(da_dN_mid) == expected_dadN_count
38     ):
39         raise ValueError(
40             f"Unexpected polynomial output length for {specimen}. "
41             f"Expected {expected_dadN_count}, obtained "
42             f"N={len(N_mid)}, a={len(a_mid)}, da/dN={len(da_dN_mid)}."
43         )

```



```

44
45 # Confirm that the polynomial results belong to the central points
46 # of the original crack-length series. No interpolation is performed.
47 if not np.array_equal(N_mid, N[3:-3]):
48     raise ValueError(f"Polynomial cycle positions are misaligned for {specimen}")
49
50 if not np.array_equal(a_mid, a[3:-3]):
51     raise ValueError(f"Polynomial crack lengths are misaligned for {specimen}")
52
53 if not np.all(np.isfinite(N)) or not np.all(np.isfinite(a)):
54     raise ValueError(f"Non-finite N or a value found for {specimen}")
55
56 if not np.all(np.isfinite(da_dN_mid)):
57     raise ValueError(f"Non-finite polynomial da/dN value found for {specimen}")
58
59 da_dN_full = np.full(len(N), np.nan, dtype=float)
60 da_dN_full[3:-3] = da_dN_mid
61
62 df_out = pd.DataFrame({
63     "N": N,
64     "a": a,
65     "da_dN_poly": da_dN_full,
66 })
67
68 safe_name = (
69     str(specimen)
70     .replace(" ", "_")
71     .replace("/", "_")
72     .replace("\\", "_")
73 )
74
75 output_path = export_dir / f"{safe_name}.csv"
76 df_out.to_csv(output_path, index=False, na_rep="")
77
78 print(
79     f"{specimen}: exported {len(df_out)} crack-length points; "
80     f"{df_out['da_dN_poly'].notna().sum()} points include da/dN."
81 )
82
83 print(f"Done. CSV files saved to: {export_dir.resolve()}")

```

Notebook 02: crack-gauge SIF and Paris-law calculation

The fitted geometry function uses the transformed finite-element reference values.

```

1
2 # Safety check: alpha must be smaller than 1
3 if np.any(alpha >= 1.0):
4     raise ValueError("All alpha = a/W values must be smaller than 1.")
5
6 # Transform the data:
7 # Y(alpha) = P5(alpha) / (1 - alpha)^(3/2)
8 # Therefore:
9 # Y(alpha) * (1 - alpha)^(3/2) = P5(alpha)
10 Y_transformed = Y * (1.0 - alpha)**(3.0 / 2.0)
11
12 # Fit a 5th-order polynomial to the transformed data
13 coeffs_rational = np.polyfit(alpha, Y_transformed, 5)
14
15 # np.polyfit returns coefficients in descending order:
16 # c5*alpha^5 + c4*alpha^4 + c3*alpha^3 + c2*alpha^2 + c1*alpha + c0
17 c5, c4, c3, c2, c1, c0 = coeffs_rational
18
19
20 def Y_fitted(alpha_value):
21     """Fitted dimensionless geometry correction function."""
22     alpha_value = np.asarray(alpha_value, dtype=float)
23
24     numerator = (
25         c0

```

```

26         + c1 * alpha_value
27         + c2 * alpha_value**2
28         + c3 * alpha_value**3
29         + c4 * alpha_value**4
30         + c5 * alpha_value**5
31     )
32
33     denominator = (1.0 - alpha_value)**(3.0 / 2.0)

```

The next blocks calculate $K_{\max}$ and ΔK for each specimen.

```

1 def add_K_column(df, sigma_nom, w, Y_func):
2     """
3     Add the stress intensity factor K [MPa*sqrt(mm)] to a DataFrame.
4
5     Parameters
6     -----
7     df : pandas.DataFrame
8         DataFrame containing the crack length column 'a' [mm].
9     sigma_nom : float
10        Nominal bending stress [MPa].
11     w : float
12        Specimen height [mm].
13     Y_func : callable
14        Fitted geometry correction function Y(a/W).
15
16     Returns
17     -----
18     pandas.DataFrame
19        Copy of the input DataFrame with an additional column 'K'.
20     """
21     df = df.copy()
22
23     a = df["a"].to_numpy(dtype=float)      # [mm]
24     alpha = a / w                        # [-]
25     Y_values = Y_func(alpha)
26
27     df["K"] = sigma_nom * np.sqrt(np.pi * a) * Y_values
28
29     return df

```

```

1 lever_arm = 75 # [mm]
2 specimen_calc_rows = []
3
4 for name in data_dict:
5     if name not in specimen_props.index:
6         print(f"Warning: {name} is not included in Specimen_result_overview.csv and was skipped.")
7         continue
8
9     B = specimen_props.loc[name, "B [mm]"]
10    W = specimen_props.loc[name, "W [mm]"]
11    F = abs(specimen_props.loc[name, "Pmax [kN]"]) * 1000 # kN -> N
12
13    I = rectang_inertia(B, W)
14    M = bending_moment(F, lever_arm)
15    sigma_nom = sig_nom(M, W, I)
16
17    data_dict[name] = add_K_column(
18        data_dict[name],
19        sigma_nom=sigma_nom,
20        w=W,
21        Y_func=Y_fitted
22    )

```

```

1 # Load ratio used for the SIF range calculation.
2 R = 0.1
3
4 for df in data_dict.values():

```

The Paris-law regression is carried out in logarithmic space.

```

1 def fit_paris_law(dk, dadn):
2     """Fit log10(da/dN) = log10(C) + m log10(DeltaK)."""
3     log_dk = np.log10(dk)
4     log_dadn = np.log10(dadn)
5
6     m, log10C = np.polyfit(log_dk, log_dadn, 1)
7     C = 10.0 ** log10C
8
9     log_pred = log10C + m * log_dk
10    ss_res = np.sum((log_dadn - log_pred) ** 2)
11    ss_tot = np.sum((log_dadn - np.mean(log_dadn)) ** 2)
12    r2 = 1.0 - ss_res / ss_tot if ss_tot > 0 else np.nan
13
14    dadn_at_ref = C * REFERENCE_DELTAK_FOR_DADN ** m
15    log10_dadn_at_ref = np.log10(dadn_at_ref) if dadn_at_ref > 0 else np.nan
16
17    return {
18        "m": m,
19        "log10C": log10C,
20        "C": C,
21        "R2_loglog": r2,
22        f"dadN_at_DeltaK_{REFERENCE_DELTAK_FOR_DADN:g}": dadn_at_ref,
23        f"log10_dadN_at_DeltaK_{REFERENCE_DELTAK_FOR_DADN:g}": log10_dadn_at_ref,
24    }

```

Notebook 03: COD calibration and crack-length export

The two COD channels are first combined into one specimen-response signal.

```

1 S112_cod = 0.5 * S112_cod_1 + S112_cod_2
2 S112str_cod = 0.5 * S112str_cod_1 + S112str_cod_2
3
4 S131_cod = 0.5 * S131_cod_1 + S131_cod_2
5 S132_cod = 0.5 * S132_cod_1 + S132_cod_2
6 S132str_cod = 0.5 * S132str_cod_1 + S132str_cod_2
7
8 S211_cod = 0.5 * S211_cod_1 + S211_cod_2
9 S212str_cod = 0.5 * S212str_cod_1 + S212str_cod_2
10
11 S232_cod = 0.5 * S232_cod_1 + S232_cod_2
12 S233_cod = 0.5 * S233_cod_1 + S233_cod_2
13 S233str_cod = 0.5 * S233str_cod_1 + S233str_cod_2

```

The finite-element calibration is written in terms of $\alpha = a/W$.

```

1 calibration_alpha = calibration_crack_length / reference_width_mm
2
3
4 def power_law_cod_to_alpha(U, A_alpha, B_alpha, C_alpha):
5     return A_alpha + B_alpha * U**C_alpha
6
7
8 popt_alpha, pcov_alpha = curve_fit(
9     power_law_cod_to_alpha,
10    calibration_cod,
11    calibration_alpha,
12    p0=[70.0 / reference_width_mm, -17.0 / reference_width_mm, -0.3],
13    maxfev=10000
14 )
15
16 A_alpha, B_alpha, C_alpha = popt_alpha

```

```

1 # Functions for load correction, alpha calculation, and final crack length calculation
2
3 def convert_cod_to_reference_load_N(cod, pmax_N, reference_load_N=25000.0):
4     cod = np.asarray(cod, dtype=float)
5     pmax_abs_N = abs(float(pmax_N))

```

```

6
7     if not np.isfinite(pmax_abs_N) or pmax_abs_N <= 0:
8         raise ValueError(f"Invalid Pmax value: {pmax_N}")
9
10    return cod * reference_load_N / pmax_abs_N
11
12
13 def calculate_alpha_from_reference_cod(cod_reference):
14     cod_reference = np.asarray(cod_reference, dtype=float)
15
16     alpha = np.full_like(cod_reference, np.nan, dtype=float)
17     valid = np.isfinite(cod_reference) & (cod_reference > 0)
18
19     alpha[valid] = power_law_cod_to_alpha(
20         cod_reference[valid],
21         A_alpha,
22         B_alpha,
23         C_alpha
24     )
25
26     return alpha
27
28
29 def calculate_crack_length_from_cod_load_and_width(cod, pmax_N, width_mm, reference_load_N=25000.0):
30     cod_reference = convert_cod_to_reference_load_N(
31         cod,
32         pmax_N,
33         reference_load_N=reference_load_N
34     )
35
36     alpha = calculate_alpha_from_reference_cod(cod_reference)
37     a = alpha * float(width_mm)
38
39     return a

```

Inside the specimen loop, the optional cycle mask is followed by the vertical shift, the specimen-specific lower limit, and the upper limit.

```

1     # Optional absolute-cycle mask.
2     minimum_cycle_setting = (
3         minimum_cycle_by_specimen.get(
4             specimen_name,
5             None,
6         )
7     )
8
9     if minimum_cycle_setting is None:
10         minimum_cycle = None
11     else:
12         minimum_cycle = float(
13             minimum_cycle_setting
14         )
15
16     if not np.isfinite(minimum_cycle):
17         raise ValueError(
18             f"Non-finite minimum cycle configured "
19             f"for {specimen_name}."
20         )
21
22     df = df.loc[
23         df[counter_col] >= minimum_cycle
24     ].reset_index(drop=True)
25
26     rows_after_optional_cycle_mask = len(df)
27
28     if rows_after_optional_cycle_mask == 0:
29         raise ValueError(
30             f"All rows were removed by the optional "
31             f"cycle mask for {specimen_name}. "
32             f"Configured minimum cycle: "

```

```

33         f"{minimum_cycle}."
34     )
35
36     # 1. Apply the specimen-specific vertical shift.
37     shift_mm = float(
38         vertical_shift_mm_by_specimen.get(
39             specimen_name,
40             0.0,
41         )
42     )
43
44     if not np.isfinite(shift_mm):
45         raise ValueError(
46             f"Non-finite vertical shift configured "
47             f"for {specimen_name}."
48         )
49
50     df[final_a_col] = (
51         df[source_a_col] + shift_mm
52     )
53
54     rows_after_shift = len(df)
55
56     # 2. Determine and apply the specimen-specific
57     # lower limit after shifting.
58     initial_a = float(
59         initial_crack_length_mm_by_specimen[
60             specimen_name
61         ]
62     )
63
64     if not np.isfinite(initial_a):
65         raise ValueError(
66             f"Invalid initial crack length for "
67             f"{specimen_name}: {initial_a}."
68         )
69
70     minimum_final_a = (
71         initial_a + excluded_extension
72     )
73
74     if maximum_final_a < minimum_final_a:
75         raise ValueError(
76             f"Maximum final crack length is below "
77             f"the lower limit for {specimen_name}: "
78             f"{maximum_final_a} < {minimum_final_a} mm."
79         )
80
81     rows_removed_below_minimum = int(
82         (
83             df[final_a_col]
84             < minimum_final_a
85         ).sum()
86     )
87
88     df = df.loc[
89         df[final_a_col] >= minimum_final_a
90     ].reset_index(drop=True)
91
92     rows_after_minimum_limit = len(df)
93
94     if rows_after_minimum_limit == 0:
95         raise ValueError(
96             f"No shifted data with a >= "
97             f"a_init + {excluded_extension:g} mm "
98             f"({minimum_final_a:g} mm) remained "
99             f"for {specimen_name}."
100         )
101
102     # 3. Apply the upper crack-length limit.
103     rows_removed_above_maximum = int(

```

```

104         (
105             df[final_a_col]
106             > maximum_final_a
107         ).sum()
108     )
109
110     df = df.loc[
111         df[final_a_col] <= maximum_final_a
112     ].reset_index(drop=True)
113
114     rows_after_maximum_limit = len(df)
115
116     if rows_after_maximum_limit == 0:
117         raise ValueError(
118             f"No shifted data with a <= "
119             f"{maximum_final_a:g} mm remained "
120             f"for {specimen_name}."
121         )

```

The final COD tables use the common columns Specimen, N, and a.

```

1  # Export the final post-processed COD N-a data.
2  # The downstream interface remains: Specimen, N, a.
3  from pathlib import Path
4  import pandas as pd
5  import numpy as np
6
7  export_dir = Path("nn_COD_csv")
8  export_dir.mkdir(exist_ok=True)
9
10 combined_rows = []
11
12 for specimen_name, df in final_specimen_data.items():
13     out_df = pd.DataFrame({
14         "Specimen": specimen_name,
15         "N": df[counter_column].to_numpy(dtype=float),
16         "a": df[final_a_column].to_numpy(dtype=float),
17     })
18
19     out_df = (
20         out_df.replace([np.inf, -np.inf], np.nan)
21         .dropna(subset=["N", "a"])
22         .sort_values("N")
23         .reset_index(drop=True)
24     )
25
26     specimen_csv = (
27         export_dir / f"{specimen_name}_COD_N_a.csv"
28     )
29     out_df.to_csv(specimen_csv, index=False)
30     combined_rows.append(out_df)
31
32 combined_N_a = pd.concat(combined_rows, ignore_index=True)
33 combined_csv = export_dir / "COD_all_specimens_N_a.csv"
34 combined_N_a.to_csv(combined_csv, index=False)
35
36 print(
37     f"Exported {len(combined_rows)} final specimen CSV files to: "
38     f"{export_dir.resolve()}"
39 )
40 print(f"Combined CSV: {combined_csv.resolve()}")
41 print(combined_N_a.groupby("Specimen").size())
42 combined_N_a.head()

```

Notebook 04: COD neural-network derivative

The network has two hidden layers by default and uses either a hyperbolic tangent or ReLU activation.

```

1 class FNN(nn.Module):

```

```

2  def __init__(self, input_size=1, output_size=1, hidden_features=32, hidden_layers=2, activation_name=
    "tanh"):
3      super().__init__()
4
5      if activation_name.lower() == "tanh":
6          activation = nn.Tanh
7      elif activation_name.lower() == "relu":
8          activation = nn.ReLU
9      else:
10         raise ValueError("activation_name must be 'tanh' or 'relu'")
11
12     layers = []
13     layers.append(nn.Linear(input_size, hidden_features))
14     layers.append(activation())
15
16     for _ in range(hidden_layers - 1):
17         layers.append(nn.Linear(hidden_features, hidden_features))
18         layers.append(activation())
19
20     layers.append(nn.Linear(hidden_features, output_size))
21     self.net = nn.Sequential(*layers)
22
23     def forward(self, x):
24         x = x.reshape(x.shape[0], -1)
25         return self.net(x)

```

Automatic differentiation gives the derivative in normalised coordinates, after which it is returned to physical units.

```

1  def predict_and_autograd_derivative(model, N_values, normalizer_N, normalizer_a, device):
2      model.eval()
3
4      N_values = np.asarray(N_values, dtype=np.float32)
5      N_norm = normalizer_N.normalize(N_values).astype(np.float32)
6
7      x = torch.tensor(N_norm, dtype=torch.float32, device=device).reshape(-1, 1)
8      x.requires_grad_(True)
9
10     a_norm_pred = model(x)
11
12     grad_norm = torch.autograd.grad(
13         outputs=a_norm_pred,
14         inputs=x,
15         grad_outputs=torch.ones_like(a_norm_pred),
16         create_graph=False,
17         retain_graph=False,
18     )[0]
19
20     a_pred = normalizer_a.denormalize(a_norm_pred.detach().cpu().numpy().squeeze())
21     dadN = (normalizer_a.std / normalizer_N.std) * grad_norm.detach().cpu().numpy().squeeze()
22
23     return a_pred, dadN

```

The final training settings and export loop are shown below.

```

1  activation_name = "tanh"
2  hidden_features = 32
3  hidden_layers = 2
4  learning_rate = 1e-3
5  num_epochs = 1500
6  batch_size = 32
7
8  # Set to None to use all points. For first trials, 50k is usually enough and much faster.
9  max_training_points = 1000

```



```

1  all_trained = {}
2  all_results = []
3
4  if train_all_specimens:

```

```

5     for name in specimen_names:
6         print("\n" + "=" * 80)
7         print("Training specimen:", name)
8
9         df_spec = all_data[all_data["Specimen"] == name]
10
11        info = train_nn_for_one_specimen(
12            df_spec=df_spec,
13            specimen_name=name,
14            max_training_points=max_training_points,
15            activation_name=activation_name,
16            hidden_features=hidden_features,
17            hidden_layers=hidden_layers,
18            learning_rate=learning_rate,
19            num_epochs=num_epochs,
20            batch_size=batch_size,
21        )
22
23        result_df = info["result"].copy()
24        history_df = info["history"].copy()
25
26        all_trained[name] = info
27        all_results.append(result_df)
28
29        # Plot each specimen immediately after training
30        plot_training_history(history_df, name)
31        plot_nn_result_for_one_specimen(result_df, name)
32
33        # Export individual specimen result with the main requested columns
34        individual_export = result_df[["N", "a", "dadN"]].copy()
35        individual_csv = individual_out_dir / f"{name}_COD_NN_dadN_N_a.csv"
36        individual_export.to_csv(individual_csv, index=False)
37        print("Saved individual CSV:", individual_csv.resolve())
38
39        all_results_df = pd.concat(all_results, ignore_index=True)
40
41        # Combined export: requested main columns plus specimen name and fitted a for checking
42        combined_export = all_results_df[["Specimen", "N", "a", "dadN", "a_pred_nn"]].copy()
43        combined_csv = out_dir / "COD_all_specimens_N_a_dadN_NN_per_specimen_models.csv"
44        combined_export.to_csv(combined_csv, index=False)
45
46        print("\n" + "=" * 80)
47        print("Finished training, plotting, and exporting all specimens.")
48        print("Saved combined CSV:", combined_csv.resolve())
49    else:
50        print("train_all_specimens is False, so no all-specimen training was run.")

```

Notebook 05: COD SIF and Paris-law calculation

The final COD fitting interval and the columns used in the calculation are set as follows.

```

1 # Input files
2 output_dir = Path("paris_law_COD_results")
3 output_dir.mkdir(parents=True, exist_ok=True)
4 combined_csv_path = Path("nn_COD_results") / "COD_all_specimens_N_a_dadN_NN_per_specimen_models.csv"
5 overview_csv_path = Path("Specimen_result_overview.csv")
6
7 # Crack length column used for K calculation.
8 # Use "a" for measured/smoothed crack length. Change to "a_pred_nn" if you want to use the NN-predicted
9   crack length.
10 CRACK_LENGTH_COLUMN = "a"
11
12 # Column used for the Paris-law crack-growth rate.
13 DADN_COLUMN = "dadN"
14
15 # Plotting/export settings
16 MAX_PLOT_POINTS_PER_SPECIMEN = 5000 # plotting only; fits still use all valid rows
17 SAVE_PROCESSED_CSV = False           # set True if you want to write the large processed CSV
18 SAVE_PARIS_FIT_CSV = True

```



```

19 # SIF / DeltaK interval used for all Paris-law plots and linear fits.
20 # Data with DeltaK below MIN or above MAX MPa sqrt(m) is excluded from every Paris-law-related
    calculation.
21 PARIS_SIF_MIN = 20
22 PARIS_SIF_MAX = 40
23
24 # Literature Paris-law values for comparison plots.

```

The measured geometry, load, and load ratio are used for each specimen.

```

1 processed_parts = []
2 summary_rows = []
3
4 for specimen_name, df_spec in raw_df.groupby("Specimen", sort=True):
5     if specimen_name not in specimen_props.index:
6         print(f"Skipping {specimen_name}: missing from overview.")
7         continue
8
9     props = specimen_props.loc[specimen_name]
10    B = float(props["B [mm]"])
11    W = float(props["W [mm]"])
12    Pmax_N = abs(float(props["Pmax [kN]"])) * 1000.0
13    R = float(props["R"])
14
15    I = rectang_inertia(B, W)
16    M = bending_moment(Pmax_N, lever_arm)
17    sigma_nom = sig_nom(M, W, I)
18
19    out = df_spec.copy()
20    out["B [mm]"] = B
21    out["W [mm]"] = W
22    out["Pmax used [N]"] = Pmax_N
23    out["R used [-]"] = R
24    out["sigma_nom [MPa]"] = sigma_nom
25    out["a_for_K [mm]"] = out[CRACK_LENGTH_COLUMN]
26
27    # Valid alpha range for the fitted reference function is shown for diagnostics.
28    out["alpha [-]"] = out["a_for_K [mm]"] / W
29    out["Y [-]"] = Y_fitted(out["alpha [-]"])
30    out["K_max [MPa sqrt(mm)]"] = calculate_K_from_a(out["a_for_K [mm]"], sigma_nom, W, Y_fitted)
31    out["delta_K [MPa sqrt(mm)]"] = out["K_max [MPa sqrt(mm)]"] * (1.0 - R)
32    out["delta_K [MPa sqrt(m)]"] = out["delta_K [MPa sqrt(mm)]"] / np.sqrt(1000.0)
33
34    processed_parts.append(out)
35
36    valid = (
37        np.isfinite(out["delta_K [MPa sqrt(m)]"])
38        & np.isfinite(out[DADN_COLUMN])
39        & (out["delta_K [MPa sqrt(m)]"] >= PARIS_SIF_MIN)
40        & (out["delta_K [MPa sqrt(m)]"] <= PARIS_SIF_MAX)
41        & (out[DADN_COLUMN] > 0)
42    )
43
44    summary_rows.append({
45        "Specimen": specimen_name,
46        "rows": len(out),
47        "valid Paris rows": int(valid.sum()),
48        "B [mm]": B,
49        "W [mm]": W,
50        "Pmax used [N]": Pmax_N,
51        "R used [-]": R,
52        "sigma_nom [MPa]": sigma_nom,
53        "sigma_nom / sigma_nom_ref [-]": sigma_nom / sigma_nom_ref,
54        "min alpha": out["alpha [-]"].min(),
55        "max alpha": out["alpha [-]"].max(),
56        "min delta_K [MPa sqrt(m)]": out.loc[valid, "delta_K [MPa sqrt(m)]"].min() if valid.any() else np.
nan,
57        "max delta_K [MPa sqrt(m)]": out.loc[valid, "delta_K [MPa sqrt(m)]"].max() if valid.any() else np.
nan,
58    })

```

```

59
60 processed_df = pd.concat(processed_parts, ignore_index=True)
61 specimen_calc_summary = pd.DataFrame(summary_rows).set_index("Specimen")
62
63 data_dict = {
64     name: df.reset_index(drop=True)
65     for name, df in processed_df.groupby("Specimen", sort=True)
66 }
67
68 specimen_calc_summary

```

The selected ΔK interval is applied before the log–log fit.

```

1 def plot_batch(ax, batch, title, colors):
2     local_fit_results = {}
3
4     for idx, name in enumerate(batch):
5         if name not in data_dict:
6             continue
7
8         df = data_dict[name]
9         color = colors[idx % len(colors)]
10        marker = markers[idx % len(markers)]
11
12        dk = df["delta_K [MPa sqrt(m)"]].to_numpy(dtype=float)
13        dadN = df[DADN_COLUMN].to_numpy(dtype=float)
14
15        mask = (
16            (dk >= PARIS_SIF_MIN) &
17            (dk <= PARIS_SIF_MAX) &
18            (dadN > 0) &
19            np.isfinite(dk) &
20            np.isfinite(dadN)
21        )
22
23        dk = dk[mask]
24        dadN = dadN[mask]
25
26        if len(dk) < 2:
27            continue
28
29        order = np.argsort(dk)
30        dk = dk[order]
31        dadN = dadN[order]
32
33        log_dk = np.log10(dk)
34        log_dadN = np.log10(dadN)
35
36        slope, intercept = np.polyfit(log_dk, log_dadN, 1)
37        C = 10**intercept
38
39        local_fit_results[name] = {
40            "slope": slope,
41            "intercept": intercept,

```

Notebook 06: strain calibration and crack-length processing

The strain channels are converted from microstrain to strain and then combined.

```

1
2 # Construct the actual raw STR signal from the two STR channels.
3 # Same structure as in the COD notebook: signal = 0.5 * channel_1 + channel_2
4
5 S112_str    = 0.5 * S112_str_1    + S112_str_2
6 S112str_str = 0.5 * S112str_str_1 + S112str_str_2
7
8 S131_str    = 0.5 * S131_str_1    + S131_str_2
9 S132_str    = 0.5 * S132_str_1    + S132_str_2
10 S132str_str = 0.5 * S132str_str_1 + S132str_str_2
11

```

```

12 S211_str = 0.5 * S211_str_1 + S211_str_2
13 S212_str = 0.5 * S212_str_1 + S212_str_2
14 S212str_str = 0.5 * S212str_str_1 + S212str_str_2
15
16 S232_str = 0.5 * S232_str_1 + S232_str_2
17 S233_str = 0.5 * S233_str_1 + S233_str_2
18 S233str_str = 0.5 * S233str_str_1 + S233str_str_2
19
20 # Optional sign-flipped arrays, kept for inspection/plotting if needed.
21 S112_str_neg = -S112_str
22 S112str_str_neg = -S112str_str
23
24 S131_str_neg = -S131_str
25 S132_str_neg = -S132_str
26 S132str_str_neg = -S132str_str
27
28 S211_str_neg = -S211_str
29 S212_str_neg = -S212_str
30 S212str_str_neg = -S212str_str
31
32 S232_str_neg = -S232_str
33 S233_str_neg = -S233_str
34 S233str_str_neg = -S233str_str

```

The finite-element calibration and the load and height correction use $\alpha = a/W$.

```

1 calibration_alpha = calibration_crack_length / reference_width_mm
2
3
4 def power_law_str_to_alpha(U, A_alpha, B_alpha, C_alpha):
5     U = np.asarray(U, dtype=float)
6     return A_alpha + B_alpha * U**C_alpha
7
8
9 popt_alpha, pcov_alpha = curve_fit(
10     power_law_str_to_alpha,
11     calibration_str,
12     calibration_alpha,
13     p0=[70.0 / reference_width_mm, -5.0 / reference_width_mm, -0.3],
14     maxfev=10000
15 )
16
17 A_alpha, B_alpha, C_alpha = popt_alpha

```

```

1 # Functions for load correction, alpha calculation, and final crack length calculation
2
3 def convert_str_to_reference_load_N(strain_value, pmax_N, reference_load_N=25000.0):
4     strain_value = np.asarray(strain_value, dtype=float)
5     pmax_abs_N = abs(float(pmax_N))
6
7     if not np.isfinite(pmax_abs_N) or pmax_abs_N <= 0:
8         raise ValueError(f"Invalid Pmax value: {pmax_N}")
9
10    return strain_value * reference_load_N / pmax_abs_N
11
12
13 def calculate_alpha_from_reference_str(str_reference):
14     str_reference = np.asarray(str_reference, dtype=float)
15     str_magnitude = np.abs(str_reference)
16
17     alpha = np.full_like(str_magnitude, np.nan, dtype=float)
18     valid = np.isfinite(str_magnitude) & (str_magnitude > 0)
19
20     alpha[valid] = power_law_str_to_alpha(
21         str_magnitude[valid],
22         A_alpha,
23         B_alpha,
24         C_alpha
25     )
26

```

```

27     return alpha
28
29
30 def calculate_crack_length_from_str_load_and_width(strain_value, pmax_N, width_mm, reference_load_N
    =25000.0):
31     str_reference = convert_str_to_reference_load_N(
32         strain_value,
33         pmax_N,
34         reference_load_N=reference_load_N
35     )
36
37     alpha = calculate_alpha_from_reference_str(str_reference)
38     a = alpha * float(width_mm)
39
40     return a

```

In the final strain-gauge notebook, the lower crack-length limit is applied to the unshifted corrected data before the optional cycle mask and vertical shift.

```

1     initial_a = float(
2         initial_crack_length_mm_by_specimen[specimen_name]
3     )
4     if not np.isfinite(initial_a) or initial_a <= 0:
5         raise ValueError(
6             f"Invalid initial crack length for {specimen_name}: "
7             f"{initial_a}."
8         )
9
10    minimum_source_a = initial_a + excluded_extension
11    if (
12        maximum_final_a is not None
13        and maximum_final_a < minimum_source_a
14    ):
15        raise ValueError(
16            f"The maximum final crack length is below the lower "
17            f"threshold for {specimen_name}: "
18            f"{maximum_final_a} < {minimum_source_a} mm."
19        )
20
21    rows_below_specimen_threshold = int(
22        (df[source_a_col] < minimum_source_a).sum()
23    )
24    df = df.loc[
25        df[source_a_col] >= minimum_source_a
26    ].reset_index(drop=True)
27    rows_after_specimen_threshold = len(df)
28    if rows_after_specimen_threshold == 0:
29        raise ValueError(
30            f"No data with original corrected a >= "
31            f"a_init + {excluded_extension:g} mm "
32            f"({minimum_source_a:g} mm) remained for "
33            f"{specimen_name}."
34        )
35
36    minimum_cycle_setting = minimum_cycle_by_specimen.get(
37        specimen_name, None
38    )
39    if minimum_cycle_setting is None:
40        minimum_cycle = None
41    else:
42        minimum_cycle = float(minimum_cycle_setting)
43        if not np.isfinite(minimum_cycle):
44            raise ValueError(
45                f"Non-finite minimum cycle configured for "
46                f"{specimen_name}."
47            )
48    df = df.loc[
49        df[counter_col] >= minimum_cycle
50    ].reset_index(drop=True)
51

```

```

52     rows_after_optional_cycle_mask = len(df)
53     if rows_after_optional_cycle_mask == 0:
54         raise ValueError(
55             f"All rows were removed by the optional cycle mask for "
56             f"{specimen_name}. Configured minimum cycle: "
57             f"{minimum_cycle}."
58         )
59
60     shift_mm = float(
61         vertical_shift_mm_by_specimen.get(specimen_name, 0.0)
62     )
63     if not np.isfinite(shift_mm):
64         raise ValueError(
65             f"Non-finite vertical shift configured for "
66             f"{specimen_name}."
67         )
68
69     df[final_a_col] = df[source_a_col] + shift_mm
70     rows_before_maximum_limit = len(df)
71     if maximum_final_a is not None:
72         df = df.loc[
73             df[final_a_col] <= maximum_final_a
74         ].reset_index(drop=True)
75
76     rows_after_maximum_limit = len(df)
77     if rows_after_maximum_limit == 0:
78         raise ValueError(
79             f"All rows were removed for {specimen_name} after "
80             f"applying the maximum final crack length "
81             f"({maximum_final_a} mm)."
82         )

```

Notebook 07: strain-gauge neural-network derivative

Notebook 07 uses the same network and automatic-differentiation functions as Notebook 04. The following final loop trains and exports the strain-gauge results.

```

1  all_trained = {}
2  all_results = []
3
4  if train_all_specimens:
5      for name in specimen_names:
6          print("\n" + "=" * 80)
7          print("Training specimen:", name)
8
9          df_spec = all_data[all_data["Specimen"] == name]
10
11         info = train_nn_for_one_specimen(
12             df_spec=df_spec,
13             specimen_name=name,
14             max_training_points=max_training_points,
15             activation_name=activation_name,
16             hidden_features=hidden_features,
17             hidden_layers=hidden_layers,
18             learning_rate=learning_rate,
19             num_epochs=num_epochs,
20             batch_size=batch_size,
21         )
22
23         result_df = info["result"].copy()
24         history_df = info["history"].copy()
25
26         all_trained[name] = info
27         all_results.append(result_df)
28
29         # Plot each specimen immediately after training
30         plot_training_history(history_df, name)
31         plot_nn_result_for_one_specimen(result_df, name)
32

```

```

33     # Export individual specimen result with the main requested columns
34     individual_export = result_df[["N", "a", "dadN"]].copy()
35     individual_csv = individual_out_dir / f"{name}_STR_NN_dadN_N_a.csv"
36     individual_export.to_csv(individual_csv, index=False)
37     print("Saved individual CSV:", individual_csv.resolve())
38
39     all_results_df = pd.concat(all_results, ignore_index=True)
40
41     # Combined export: requested main columns plus specimen name and fitted a for checking
42     combined_export = all_results_df[["Specimen", "N", "a", "dadN", "a_pred_nn"]].copy()
43     combined_csv = out_dir / "STR_all_specimens_N_a_dadN_NN_per_specimen_models.csv"
44     combined_export.to_csv(combined_csv, index=False)
45
46     print("\n" + "=" * 80)
47     print("Finished training, plotting, and exporting all specimens.")
48     print("Saved combined CSV:", combined_csv.resolve())
49 else:
50     print("train_all_specimens is False, so no all-specimen training was run.")

```

Notebook 08: strain-gauge SIF and Paris-law calculation

The final strain-gauge fitting interval and analysis columns are set as follows.

```

1 # Input files
2 output_dir = Path("paris_law_STR_results")
3 output_dir.mkdir(parents=True, exist_ok=True)
4 combined_csv_path = Path("nn_STR_results") / "STR_all_specimens_N_a_dadN_NN_per_specimen_models.csv"
5 overview_csv_path = Path("Specimen_result_overview.csv")
6
7 # Crack length column used for K calculation.
8 # Use "a" for measured/smoothed crack length. Change to "a_pred_nn" if you want to use the NN-predicted
9 # crack length.
10 CRACK_LENGTH_COLUMN = "a"
11
12 # Column used for the Paris-law crack-growth rate.
13 DADN_COLUMN = "dadN"
14
15 # Plotting/export settings
16 MAX_PLOT_POINTS_PER_SPECIMEN = 5000 # plotting only; fits still use all valid rows
17 SAVE_PROCESSED_CSV = False # set True if you want to write the large processed CSV
18 SAVE_PARIS_FIT_CSV = True
19
20 # SIF / DeltaK interval used for all Paris-law plots and linear fits.
21 # Data with DeltaK below 15 or above 50 MPa sqrt(m) is excluded from every Paris-law-related calculation.
22 PARIS_SIF_MIN = 18
23 PARIS_SIF_MAX = 45
24
25 # Literature Paris-law values for comparison plots.

```

Notebook 08 reconstructs its geometry function from the reference SIF values before applying the specimen-specific calculation.

```

1 alpha_ref = crack_length_ref / W_ref
2 Y_ref = Y_geometric_function(K_ref, sigma_nom_ref, crack_length_ref)
3
4
5 # Rational form used in the original notebook:
6 # Y(alpha) = polynomial_4(alpha) / (1 - alpha)^(3/2)
7 Y_transformed = Y_ref * (1.0 - alpha_ref)**(3.0 / 2.0)
8 coeffs_rational = np.polyfit(alpha_ref, Y_transformed, 4)
9 c4, c3, c2, c1, c0 = coeffs_rational
10
11
12 def Y_fitted(alpha):
13     alpha = np.asarray(alpha, dtype=float)
14     return (
15         c0
16         + c1 * alpha
17         + c2 * alpha**2

```

```

18         + c3 * alpha**3
19         + c4 * alpha**4
20         + c5 * alpha**5
21     ) / (1.0 - alpha)**(3.0 / 2.0)
22
23 alpha_fit = np.linspace(alpha_ref.min(), alpha_ref.max(), 300)
24
25 fig, ax = plt.subplots(figsize=(8, 5))
26 ax.plot(alpha_ref, Y_ref, "o", label="Calculated reference Y")

```

```

1 processed_parts = []
2 summary_rows = []
3
4 for specimen_name, df_spec in raw_df.groupby("Specimen", sort=True):
5     if specimen_name not in specimen_props.index:
6         print(f"Skipping {specimen_name}: missing from overview.")
7         continue
8
9     props = specimen_props.loc[specimen_name]
10    B = float(props["B [mm]"])
11    W = float(props["W [mm]"])
12    Pmax_N = abs(float(props["Pmax [kN]"])) * 1000.0
13    R = float(props["R"])
14
15    I = rectang_inertia(B, W)
16    M = bending_moment(Pmax_N, lever_arm)
17    sigma_nom = sig_nom(M, W, I)
18
19    out = df_spec.copy()
20    out["B [mm]"] = B
21    out["W [mm]"] = W
22    out["Pmax used [N]"] = Pmax_N
23    out["R used [-]"] = R
24    out["sigma_nom [MPa]"] = sigma_nom
25    out["a_for_K [mm]"] = out[CRACK_LENGTH_COLUMN]
26
27    # Valid alpha range for the fitted reference function is shown for diagnostics.
28    out["alpha [-]"] = out["a_for_K [mm]"] / W
29    out["Y [-]"] = Y_fitted(out["alpha [-]"])
30    out["K_max [MPa sqrt(mm)]"] = calculate_K_from_a(out["a_for_K [mm]"], sigma_nom, W, Y_fitted)
31    out["delta_K [MPa sqrt(mm)]"] = out["K_max [MPa sqrt(mm)]"] * (1.0 - R)

```

The method-specific interval is then applied before fitting m and C .

```

1 def plot_batch(ax, batch, title, colors):
2     local_fit_results = {}
3
4     for idx, name in enumerate(batch):
5         if name not in data_dict:
6             continue
7
8         df = data_dict[name]
9         color = colors[idx % len(colors)]
10        marker = markers[idx % len(markers)]
11
12        dk = df["delta_K [MPa sqrt(m)]"].to_numpy(dtype=float)
13        dadN = df[DADN_COLUMN].to_numpy(dtype=float)
14
15        mask = (
16            (dk >= PARIS_SIF_MIN) &
17            (dk <= PARIS_SIF_MAX) &
18            (dadN > 0) &
19            np.isfinite(dk) &
20            np.isfinite(dadN)
21        )
22
23        dk = dk[mask]
24        dadN = dadN[mask]
25
26        if len(dk) < 2:

```

```
27         continue
28
29     order = np.argsort(dk)
30     dk = dk[order]
31     dadN = dadN[order]
32
33     log_dk = np.log10(dk)
34     log_dadN = np.log10(dadN)
35
36     slope, intercept = np.polyfit(log_dk, log_dadN, 1)
37     C = 10*intercept
38
39     local_fit_results[name] = {
40         "slope": slope,
41         "intercept": intercept,
```

Use of the remaining notebooks

Notebooks 09–13 read the outputs described above and prepare the sensitivity comparison, report figures, per-specimen comparisons, individual Paris-law plots, and final statistical and pooled results.

B

Crack-gauge detection results

This appendix presents the specimen-specific results of the crack-event detection used in the crack-gauge post-processing. Each figure shows the log-transformed upper-envelope signal, the events initially identified by the threshold-based algorithm, and the points retained after the specimen-specific close-event filtering. The detection settings were selected separately for each specimen, and the resulting points were checked visually against the measured signal.

The figures represent the detection and close-event filtering stage.

Specimens S132 and S212 are not included because no crack-gauge result was retained for these specimens. Specimen S111 was excluded from the final evaluation.

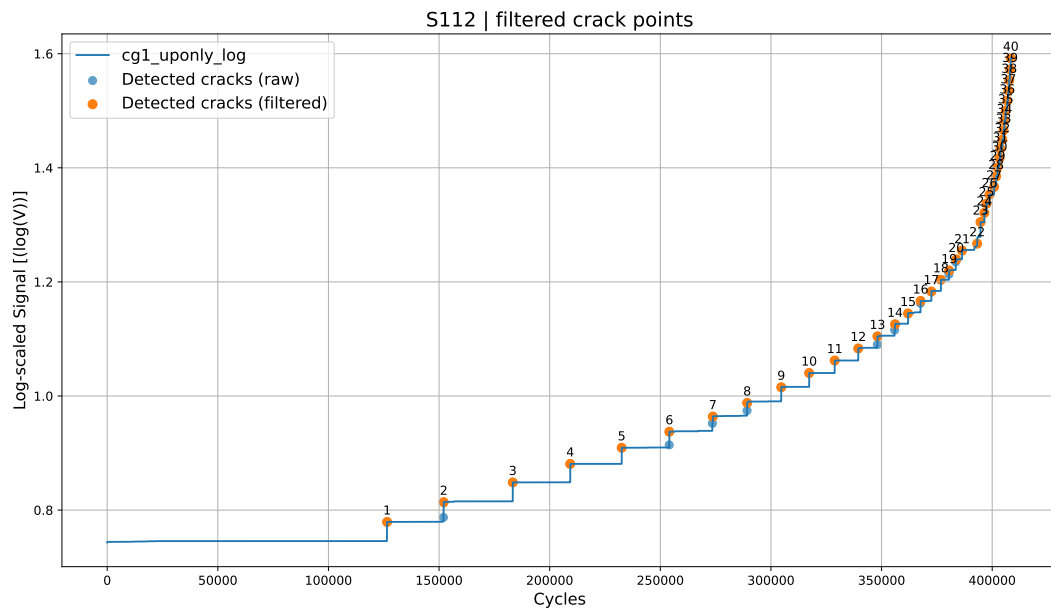


Figure B.1: Crack-gauge event detections before and after the specimen-specific close-event filtering for specimen S112.

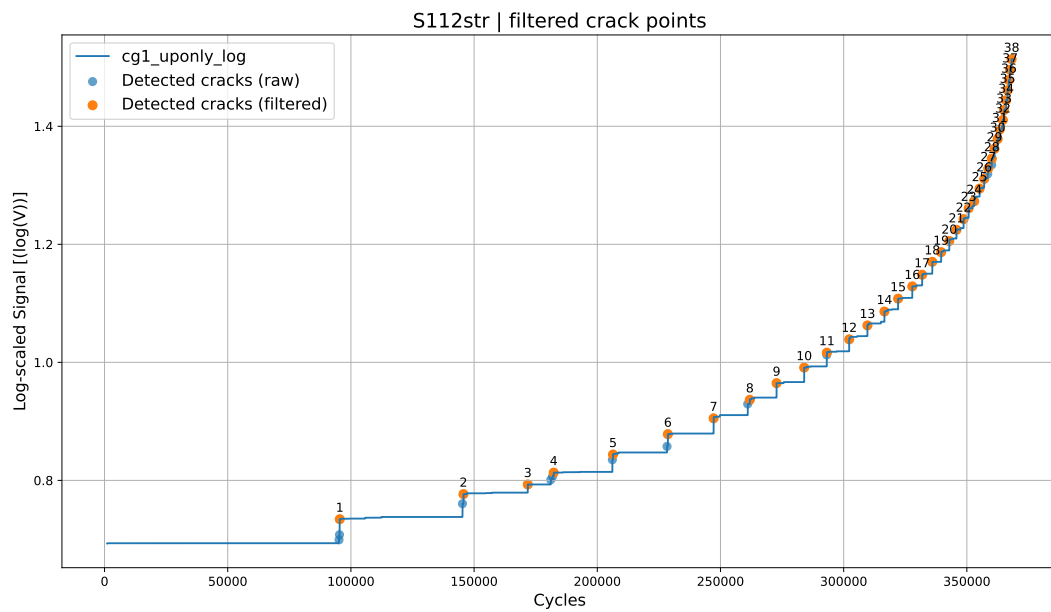


Figure B.2: Crack-gauge event detections before and after the specimen-specific close-event filtering for specimen S112str.

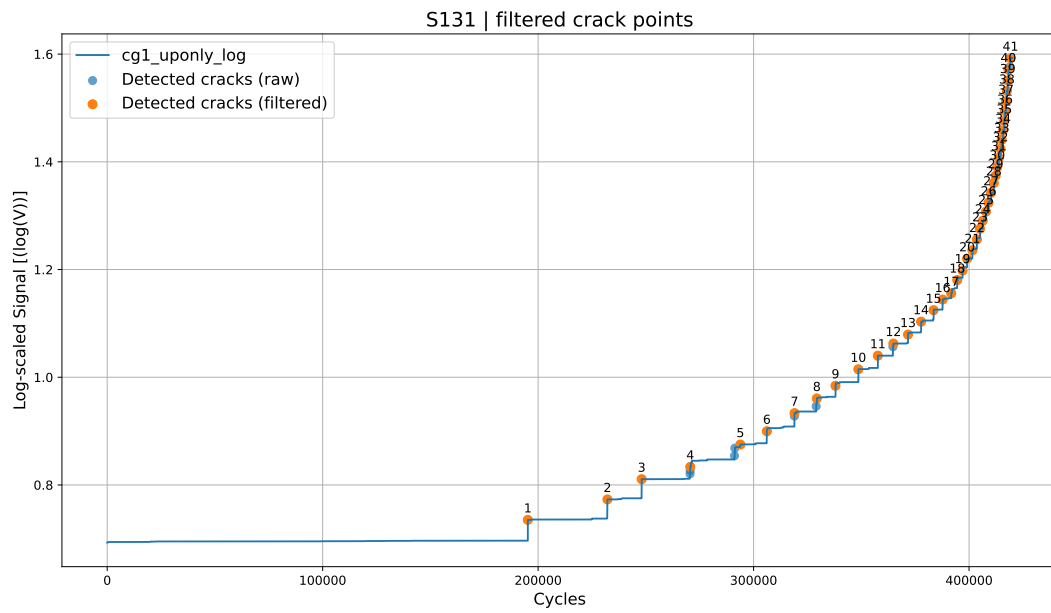


Figure B.3: Crack-gauge event detections before and after the specimen-specific close-event filtering for specimen S131.

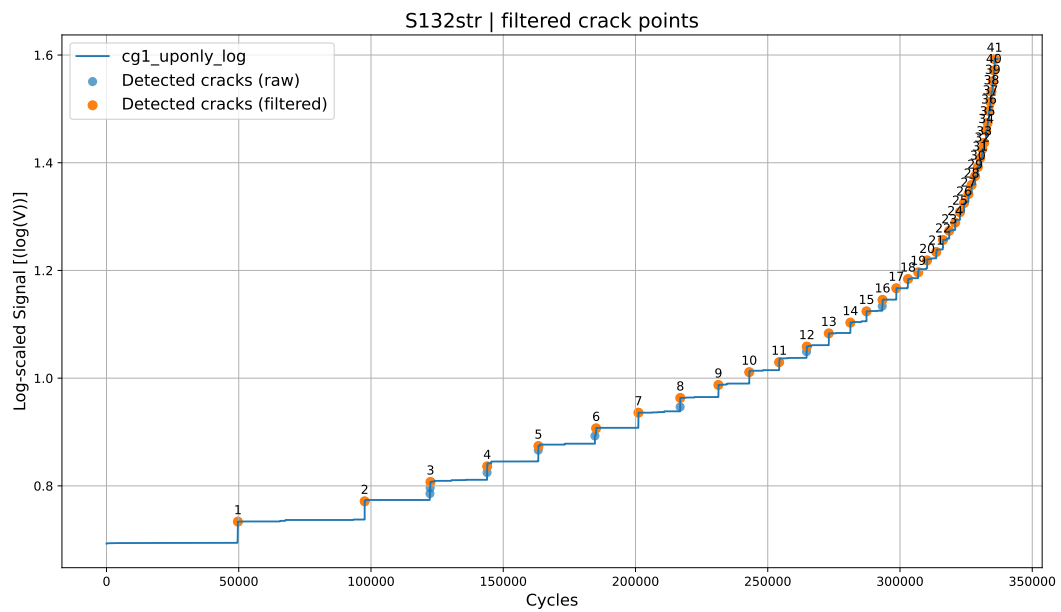


Figure B.4: Crack-gauge event detections before and after the specimen-specific close-event filtering for specimen S132str.

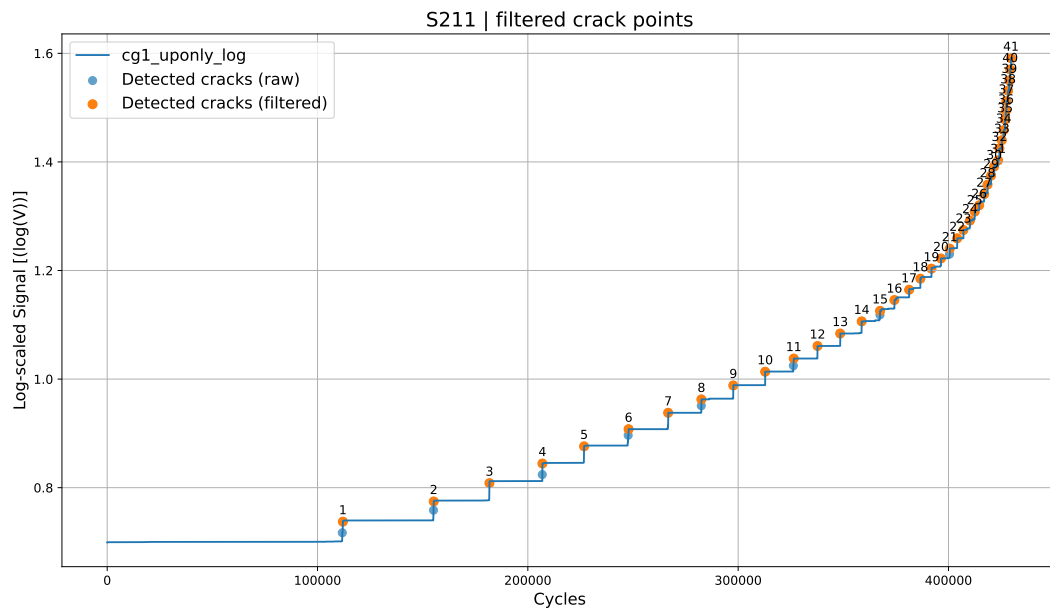


Figure B.5: Crack-gauge event detections before and after the specimen-specific close-event filtering for specimen S211.

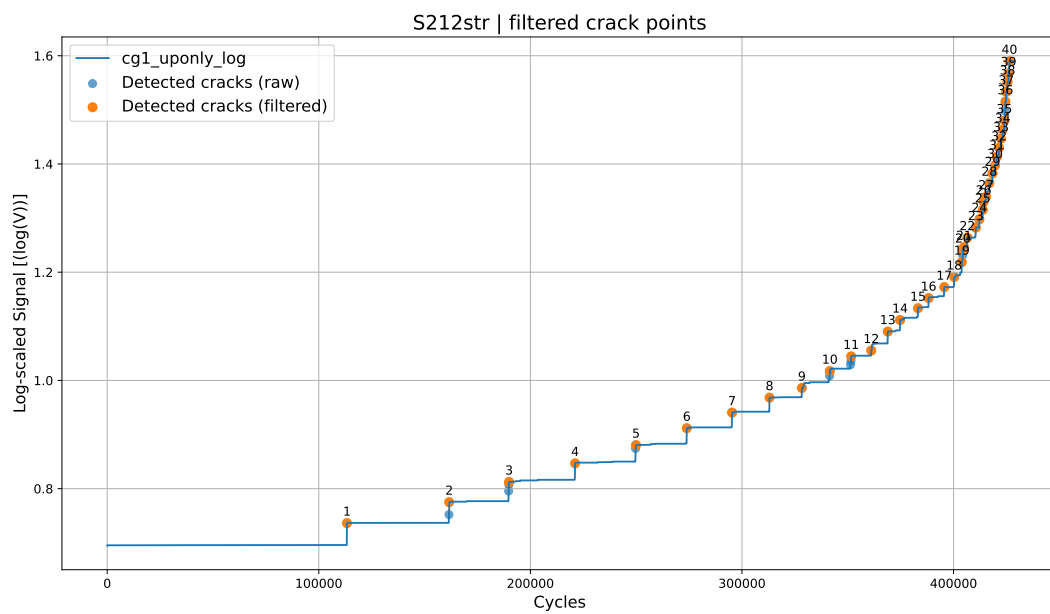


Figure B.6: Crack-gauge event detections before and after the specimen-specific close-event filtering for specimen S212str.

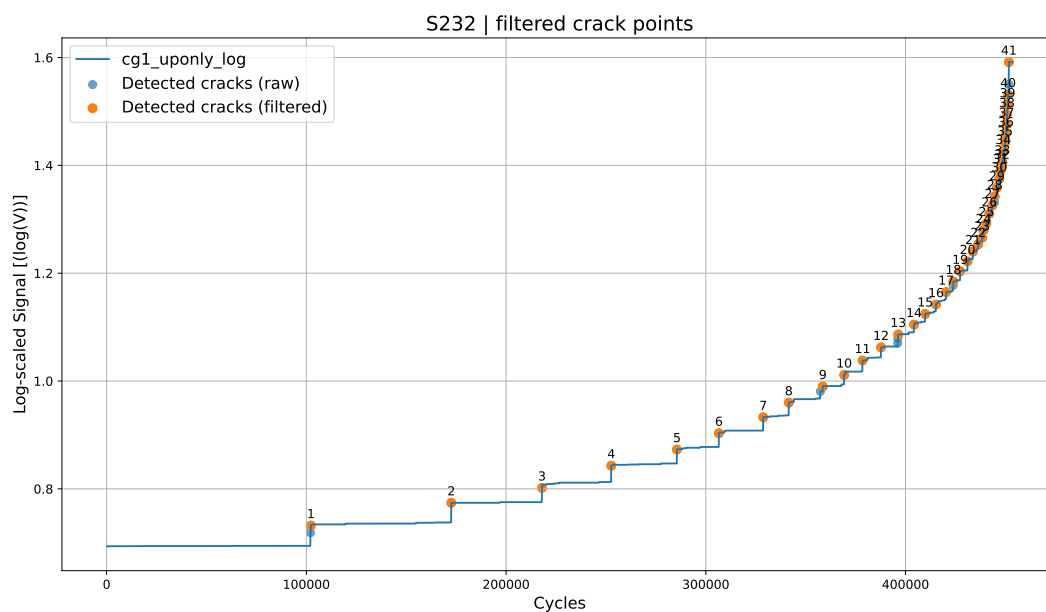


Figure B.7: Crack-gauge event detections before and after the specimen-specific close-event filtering for specimen S232.

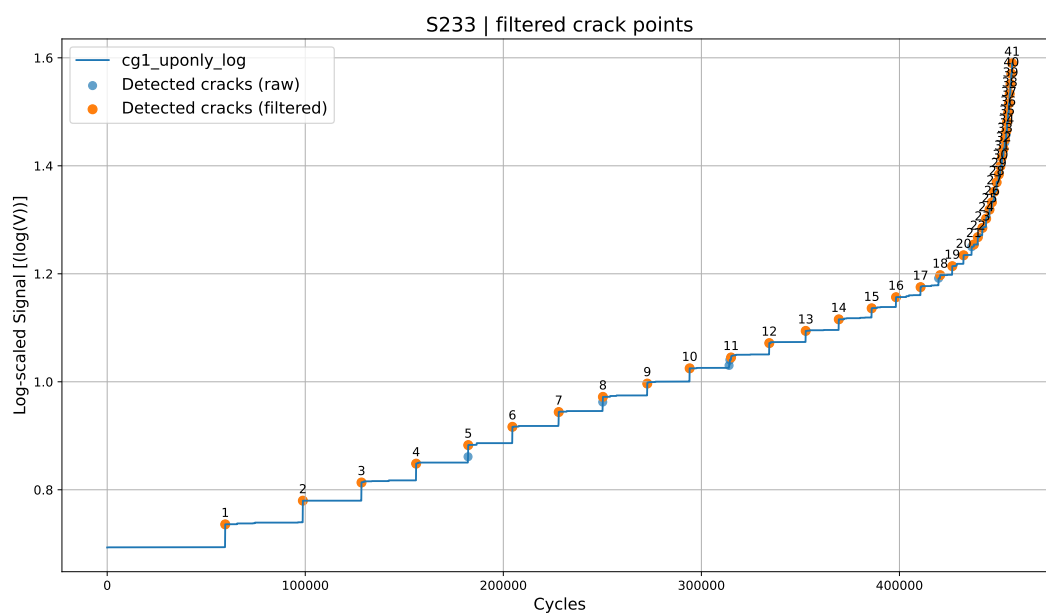


Figure B.8: Crack-gauge event detections before and after the specimen-specific close-event filtering for specimen S233.

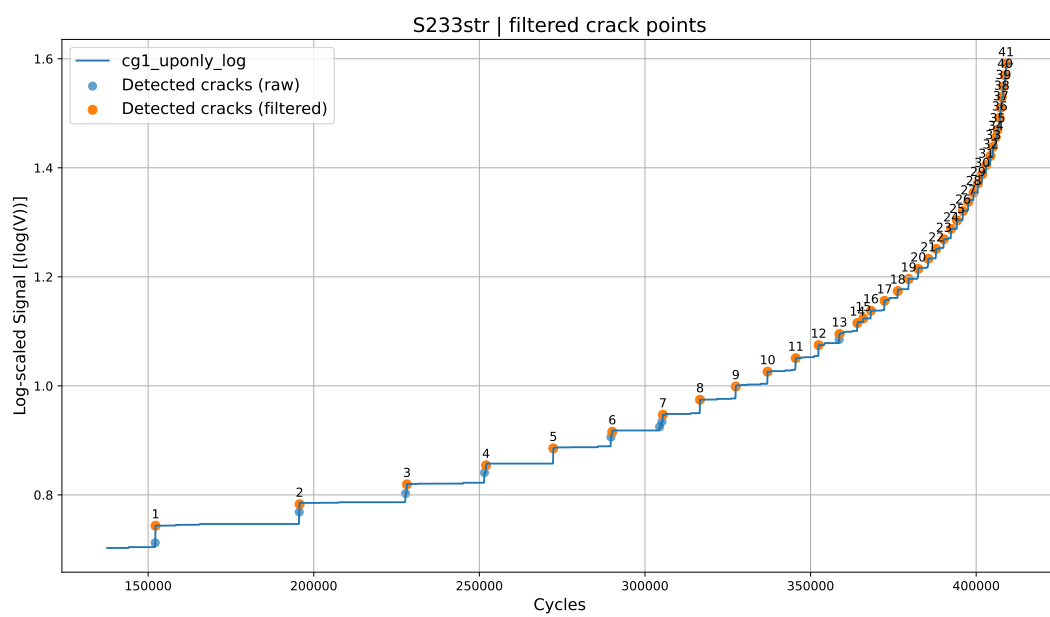


Figure B.9: Crack-gauge event detections before and after the specimen-specific close-event filtering for specimen S233str.



Individual crack length histories

This appendix presents the crack length histories obtained for the individual specimens using the available measurement methods. Crack-gauge, COD-based, and strain-gauge-based results are shown where usable processed data were available. Therefore, not every figure contains all three measurement methods.

The same axis limits are used for all figures to support direct comparison between specimens. Specimen S111 is not included because it was subjected to a different loading protocol and was excluded from the evaluation.

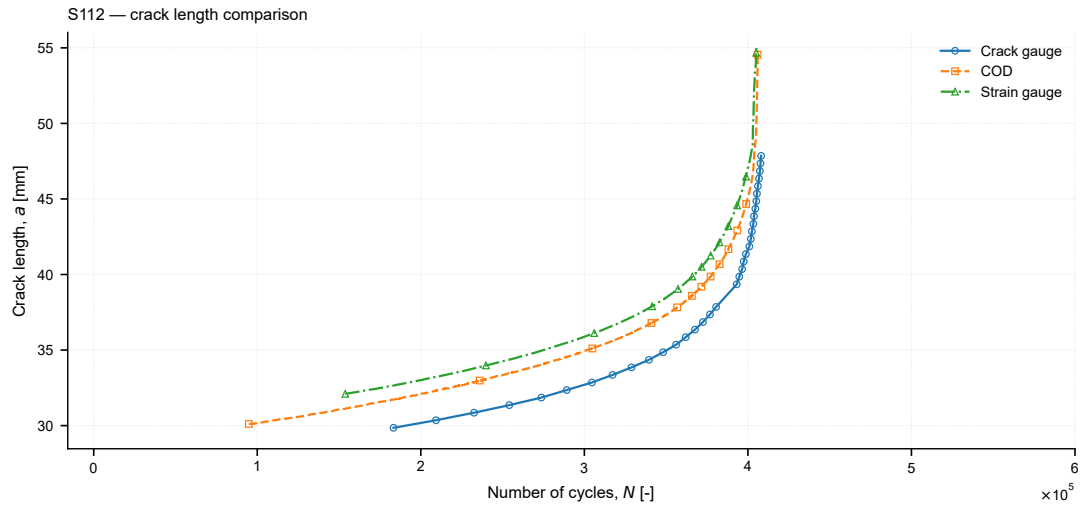


Figure C.1: Crack length histories obtained from the available measurement methods for specimen S112.

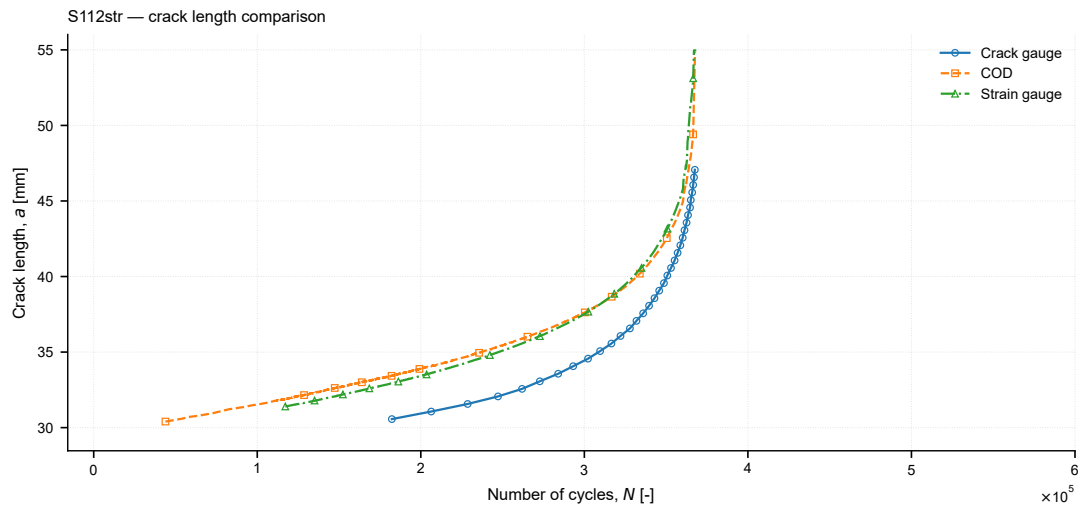


Figure C.2: Crack length histories obtained from the available measurement methods for specimen S112str.

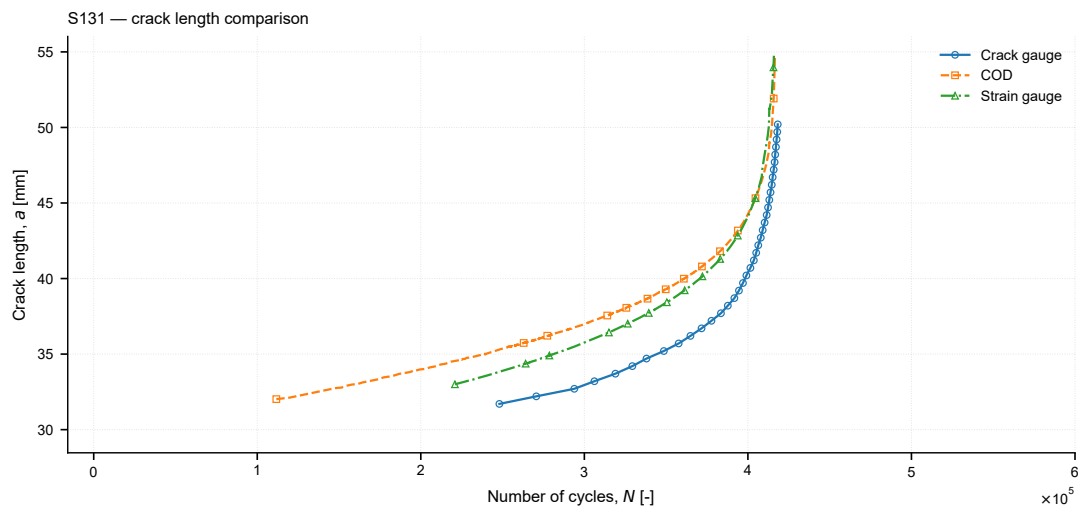


Figure C.3: Crack length histories obtained from the available measurement methods for specimen S131.

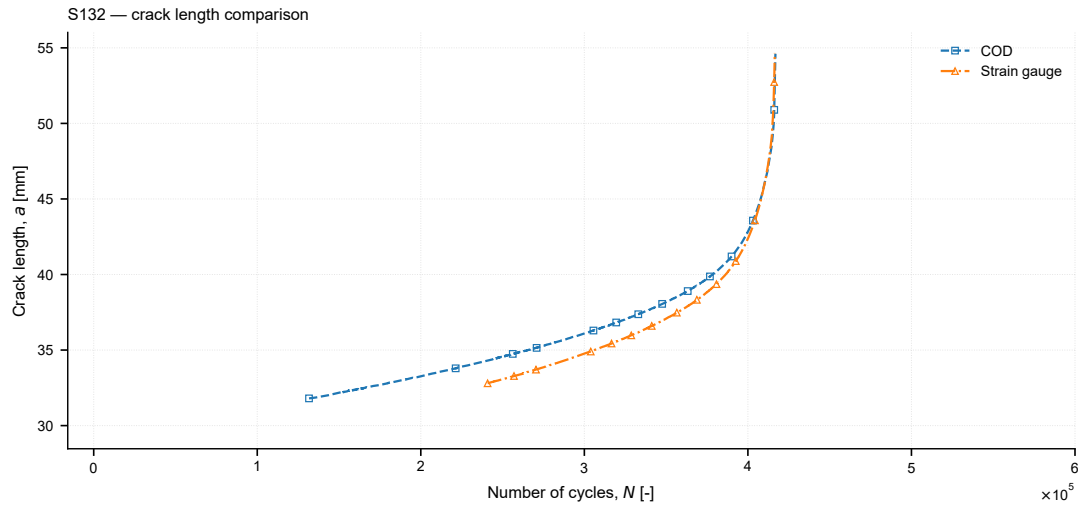


Figure C.4: Crack length histories obtained from the available measurement methods for specimen S132.

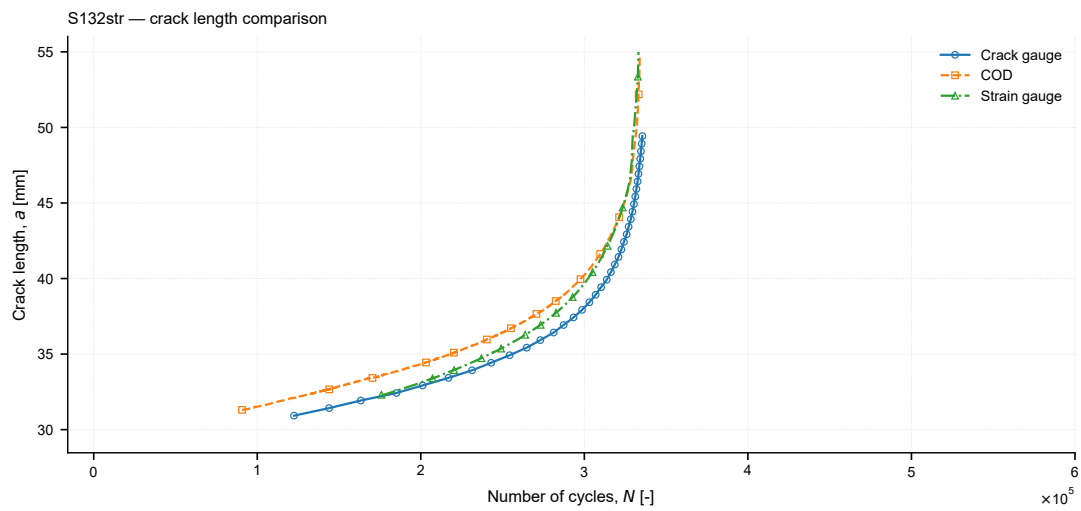


Figure C.5: Crack length histories obtained from the available measurement methods for specimen S132str.

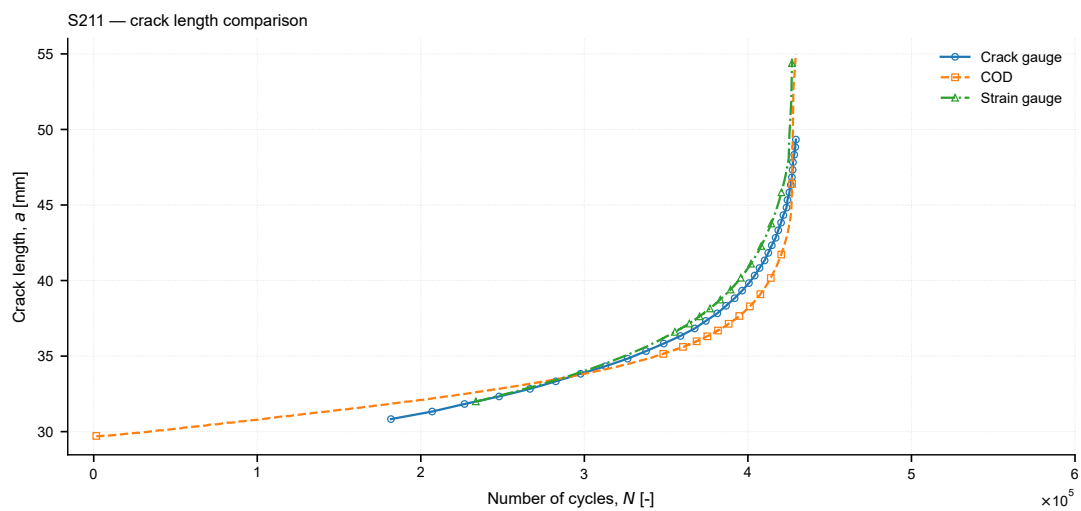


Figure C.6: Crack length histories obtained from the available measurement methods for specimen S211.

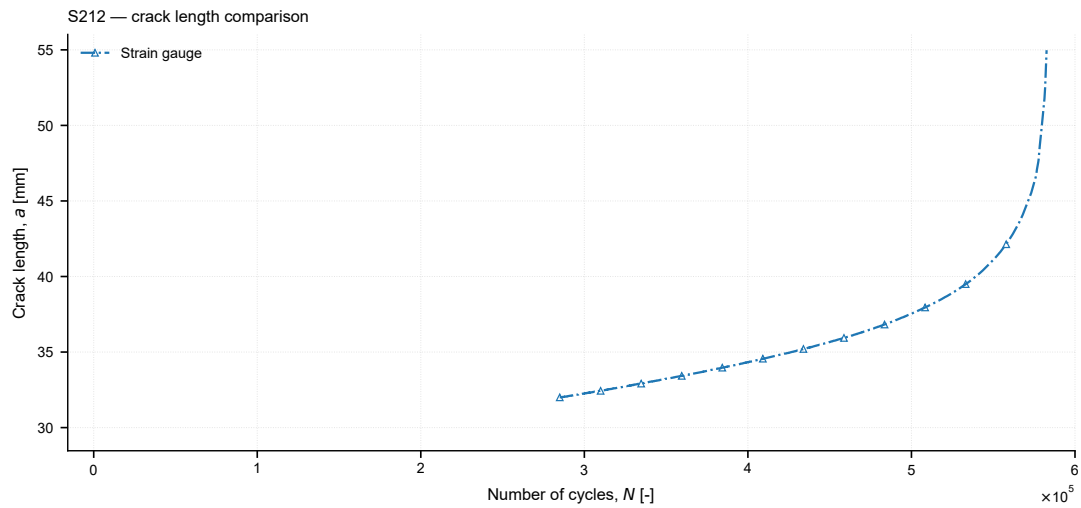


Figure C.7: Crack length histories obtained from the available measurement methods for specimen S212.

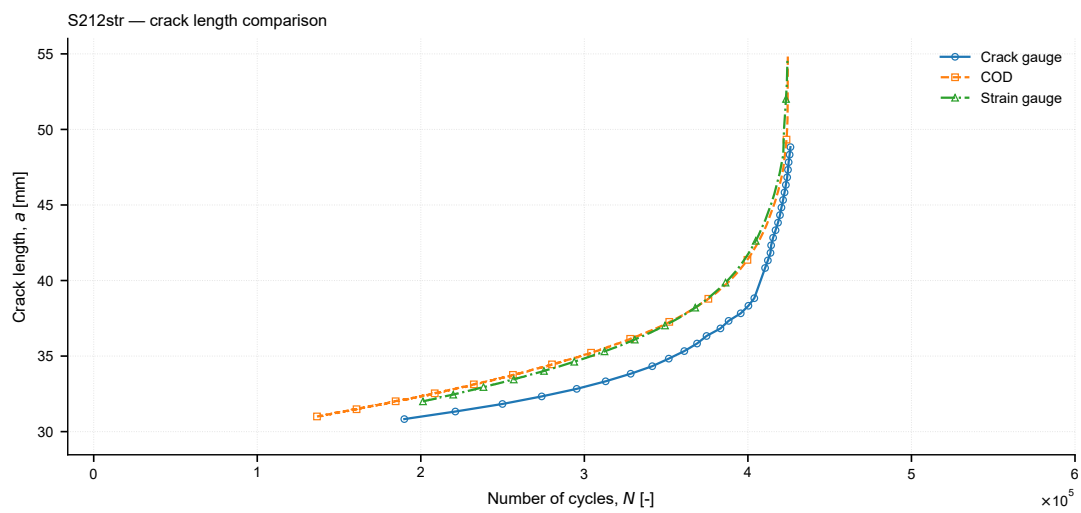


Figure C.8: Crack length histories obtained from the available measurement methods for specimen S212str.

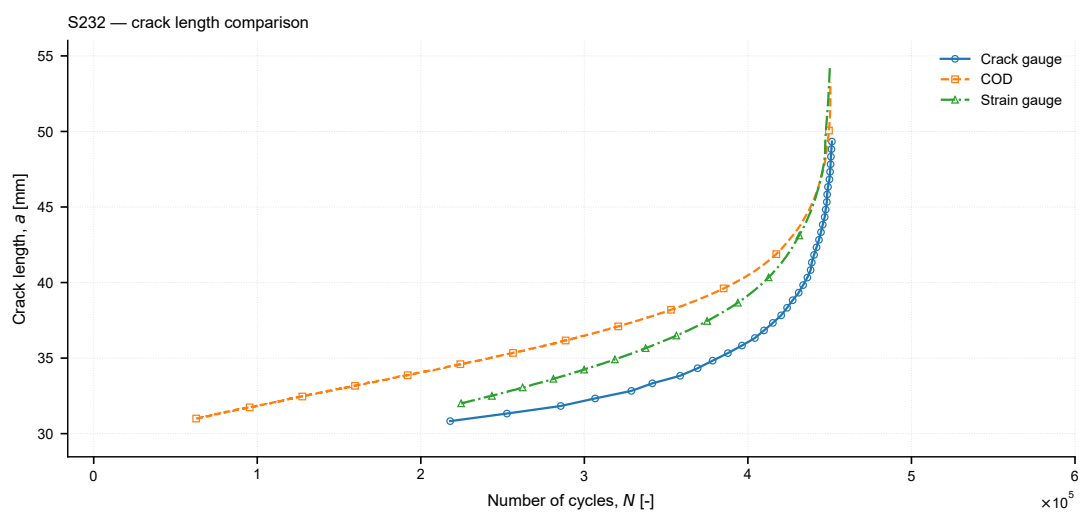


Figure C.9: Crack length histories obtained from the available measurement methods for specimen S232.

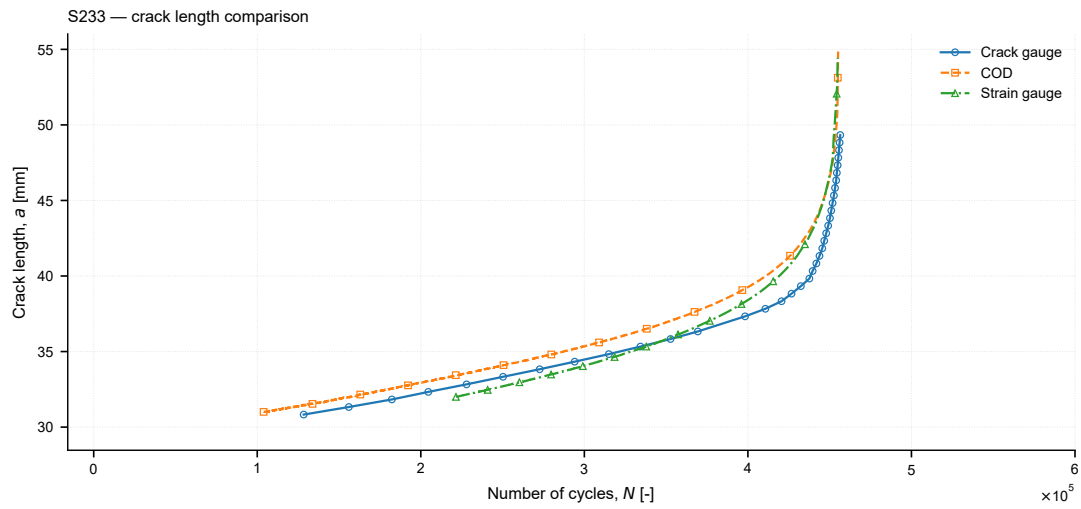


Figure C.10: Crack length histories obtained from the available measurement methods for specimen S233.

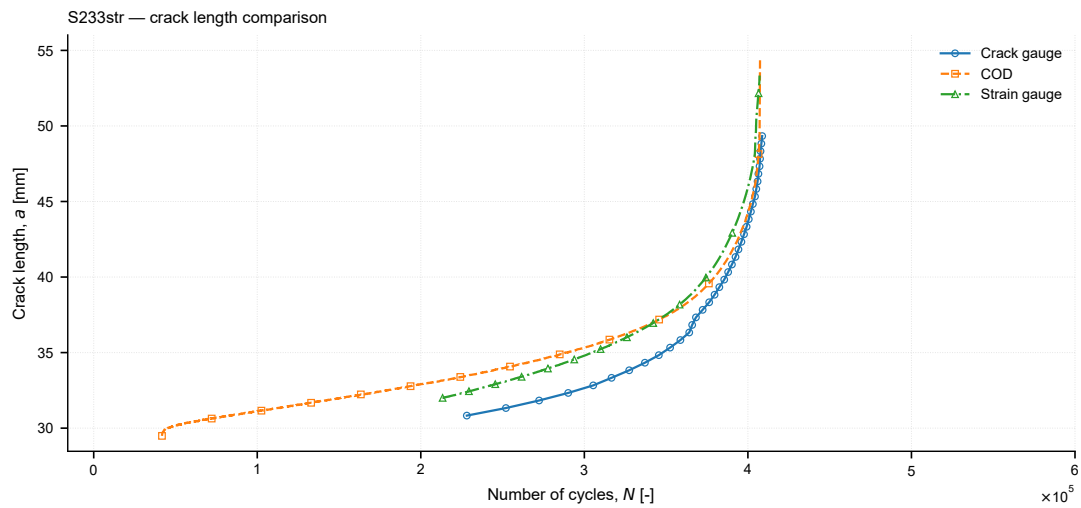


Figure C.11: Crack length histories obtained from the available measurement methods for specimen S233str.

D

Individual crack growth rates and additional Paris-law results

D.1. Individual crack growth rate histories

This section presents the crack growth rate histories obtained for the individual specimens using the available measurement methods. The crack-gauge-based rates were calculated using local polynomial differentiation, while the COD- and strain-gauge-based rates were obtained from the neural-network regressions using automatic differentiation.

Only positive and finite crack growth rate values are shown. A logarithmic vertical axis is used because the crack growth rates cover several orders of magnitude. The same axis limits are applied to all figures to support comparison between specimens. Only measurement methods with usable processed results are included. Specimen S111 is not presented because it was subjected to a different loading protocol and was excluded from the evaluation.

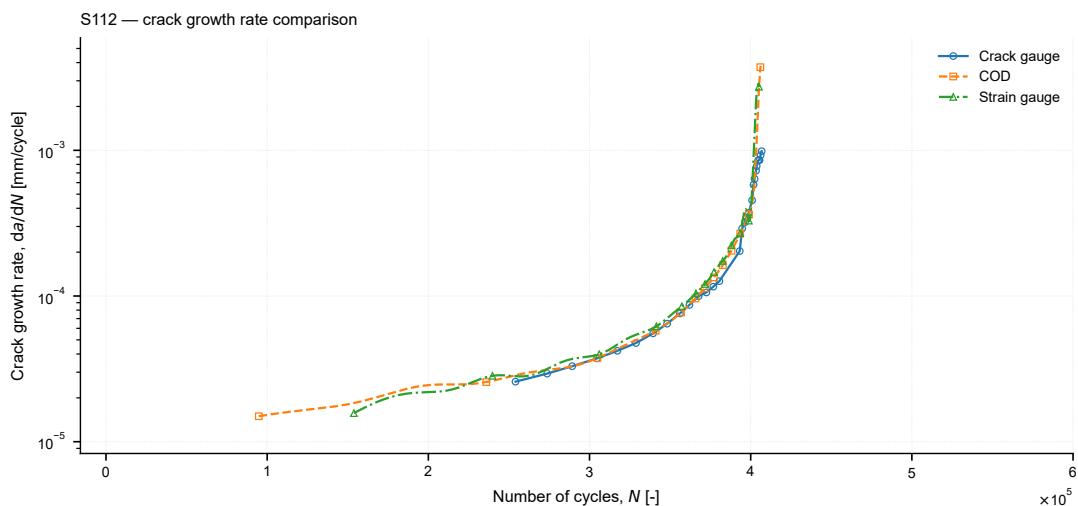


Figure D.1: Crack growth rate histories obtained from the available measurement methods for specimen S112.

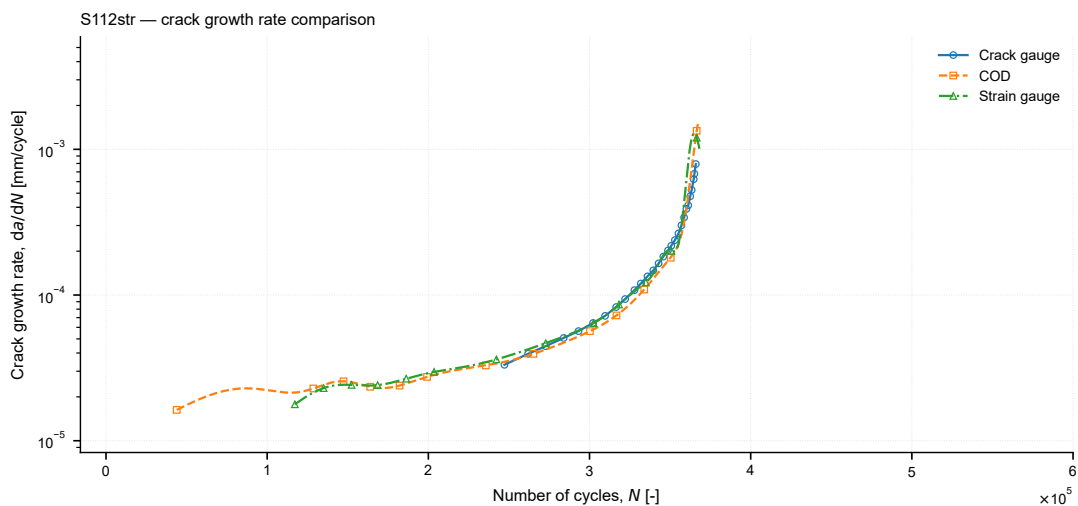


Figure D.2: Crack growth rate histories obtained from the available measurement methods for specimen S112str.

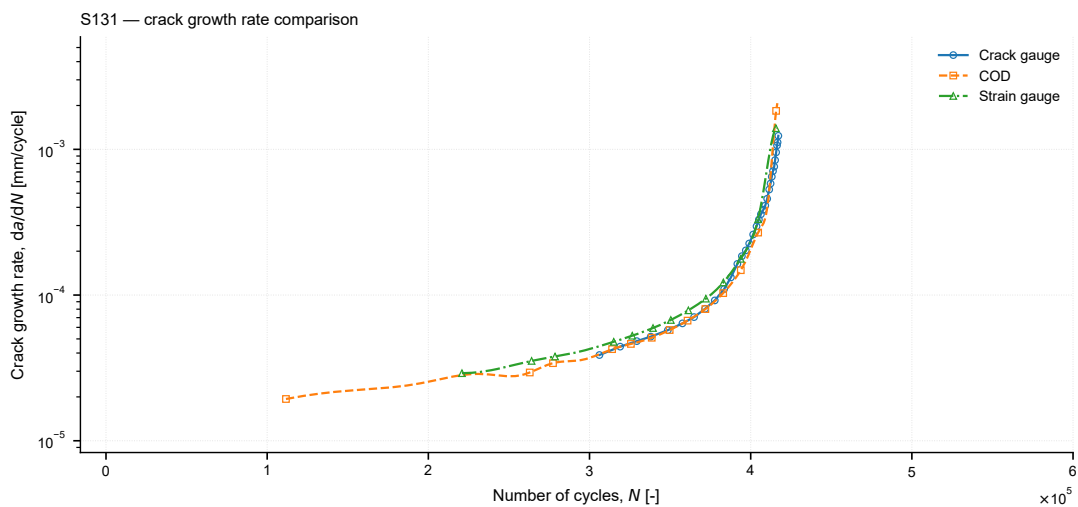


Figure D.3: Crack growth rate histories obtained from the available measurement methods for specimen S131.

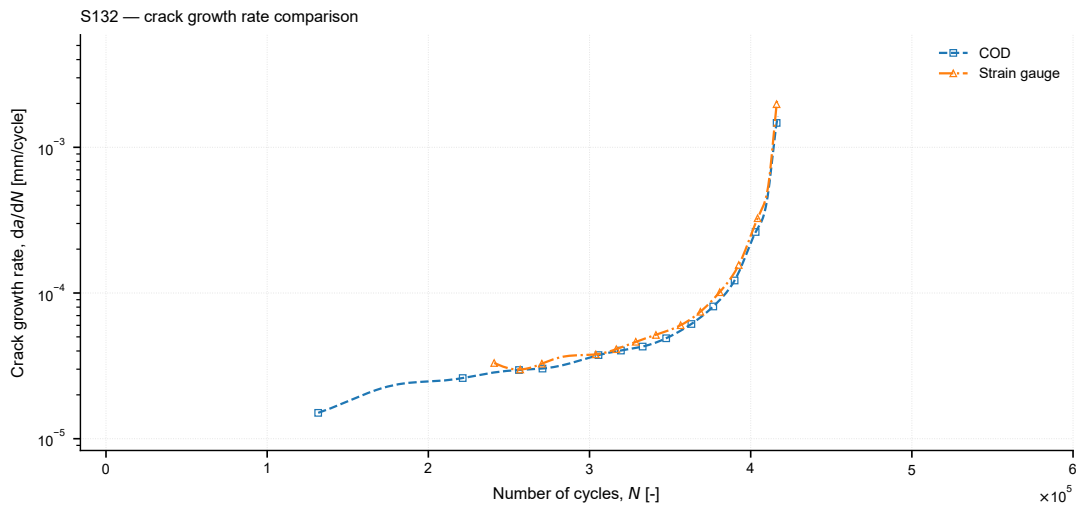


Figure D.4: Crack growth rate histories obtained from the available measurement methods for specimen S132.

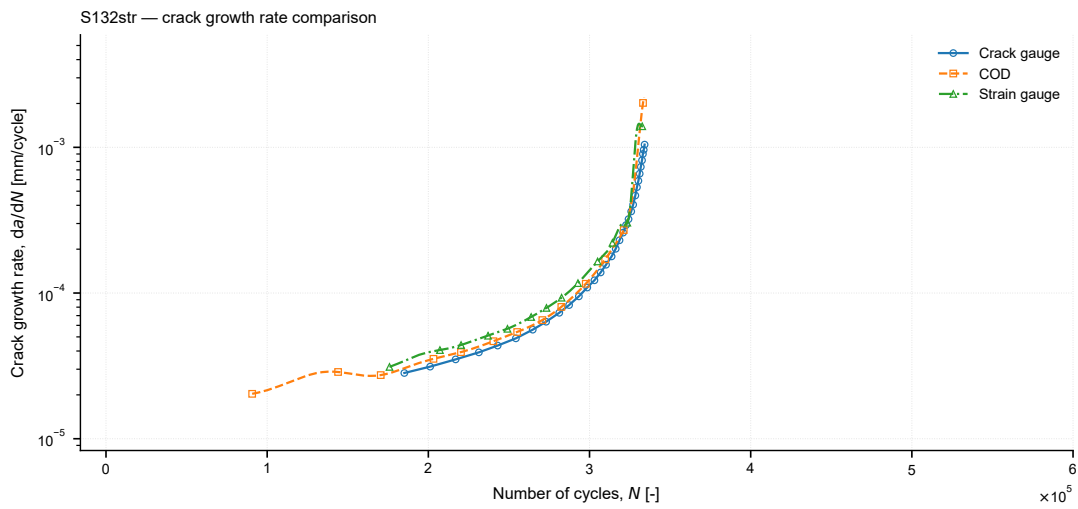


Figure D.5: Crack growth rate histories obtained from the available measurement methods for specimen S132str.

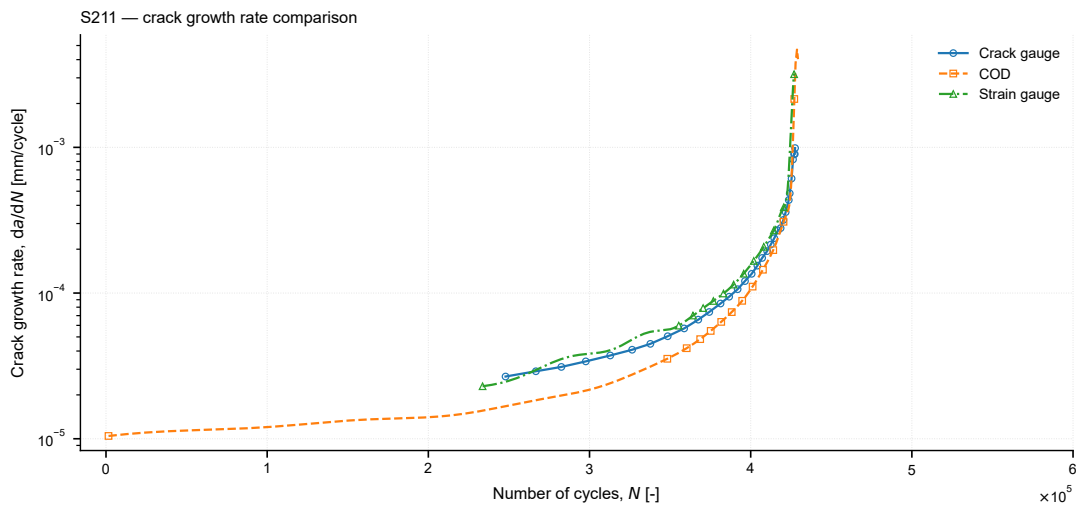


Figure D.6: Crack growth rate histories obtained from the available measurement methods for specimen S211.

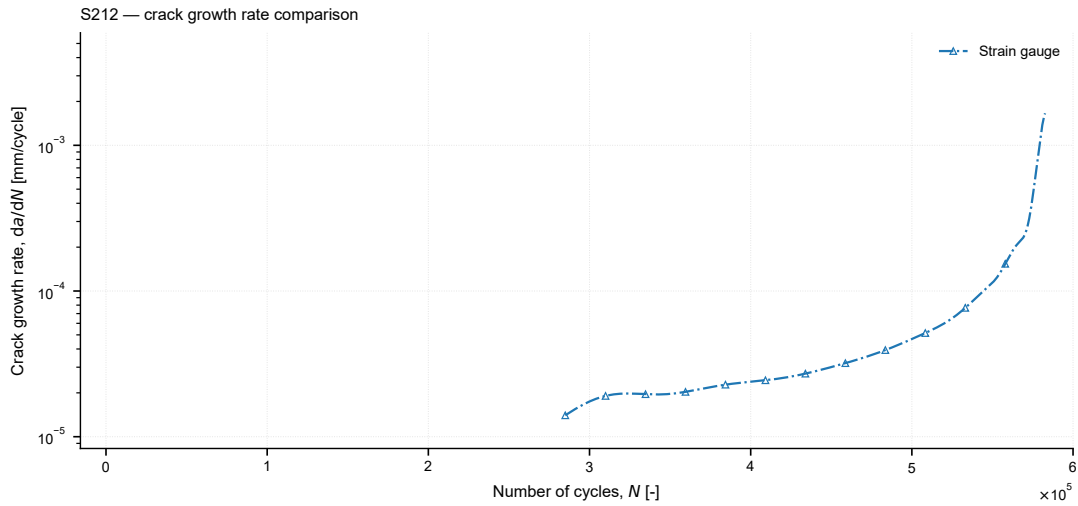


Figure D.7: Crack growth rate histories obtained from the available measurement methods for specimen S212.

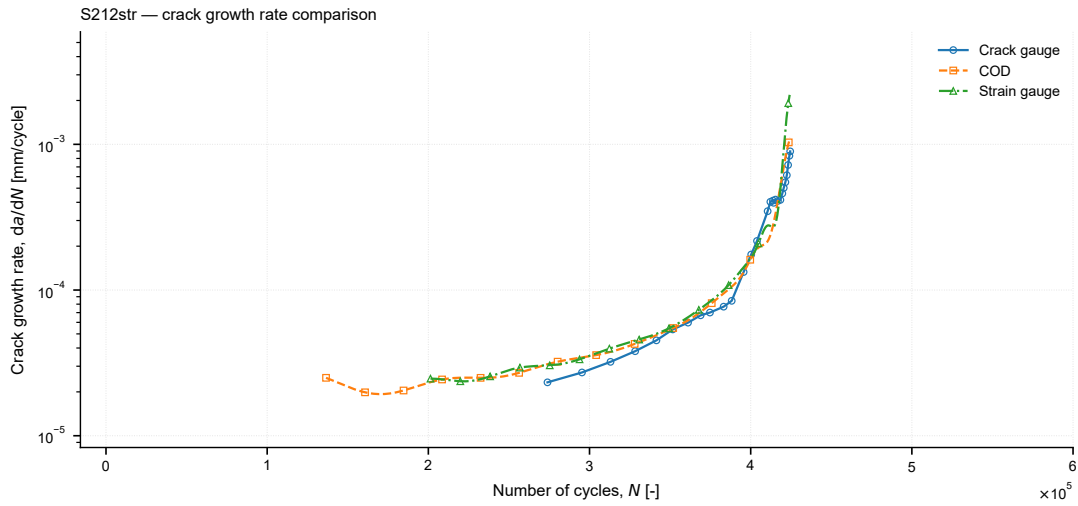


Figure D.8: Crack growth rate histories obtained from the available measurement methods for specimen S212str.

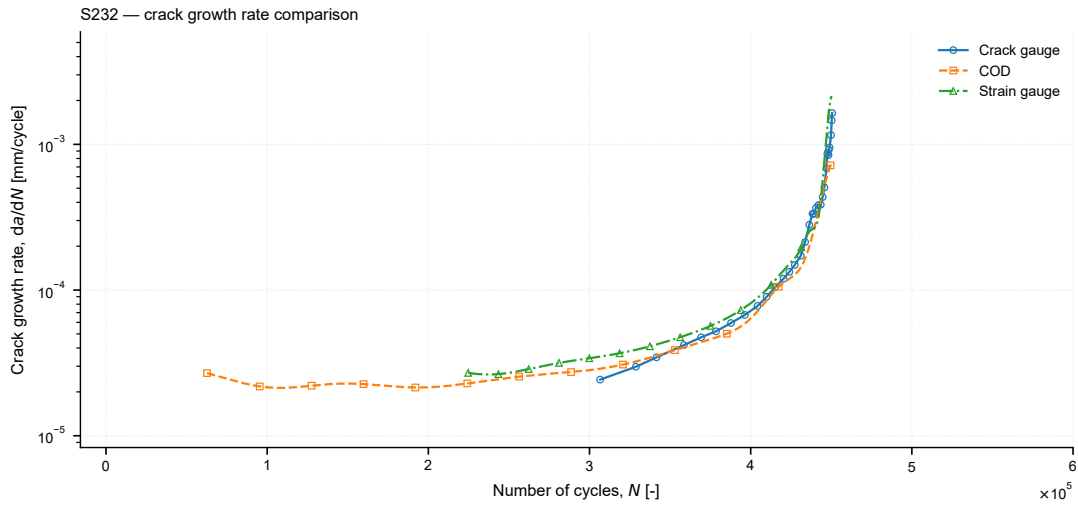


Figure D.9: Crack growth rate histories obtained from the available measurement methods for specimen S232.

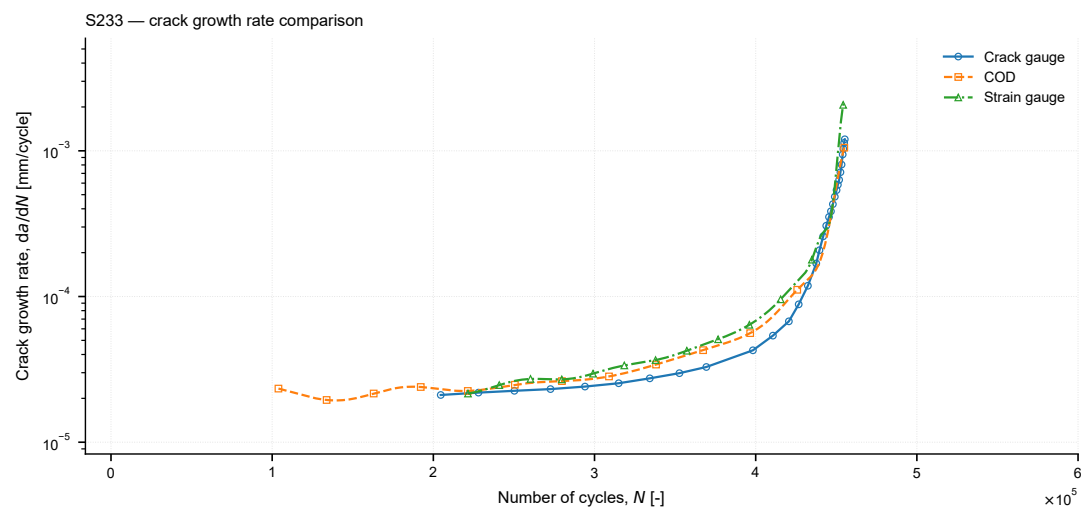


Figure D.10: Crack growth rate histories obtained from the available measurement methods for specimen S233.

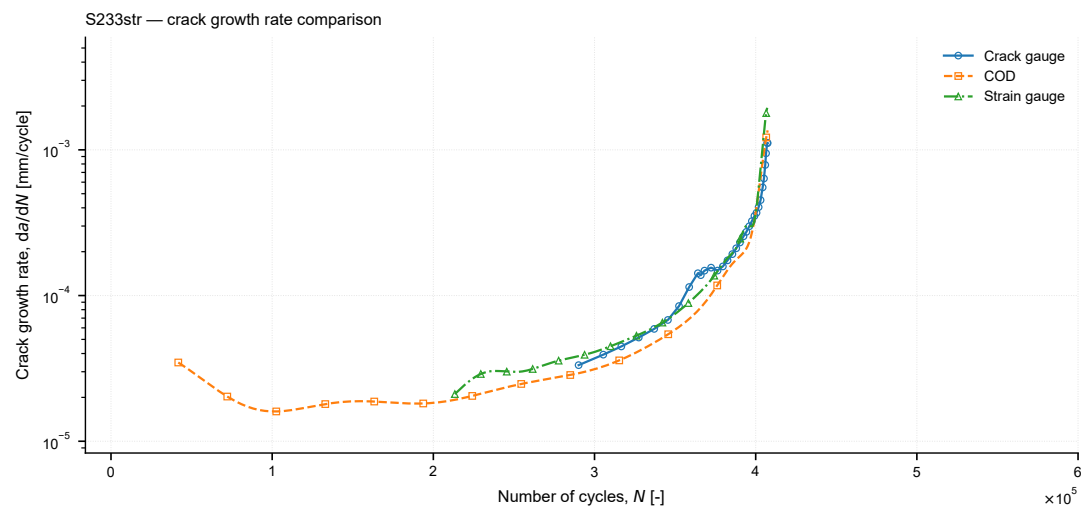


Figure D.11: Crack growth rate histories obtained from the available measurement methods for specimen S233str.

D.2. Individual Paris-law fits

The representative result for specimen S112 is presented in the main text. The following figures present the specimen-specific Paris-law fits for the remaining specimens using the geometrically linear stress intensity factor formulation. The same method-specific fitting ranges and data-selection rules were applied as in the main evaluation.

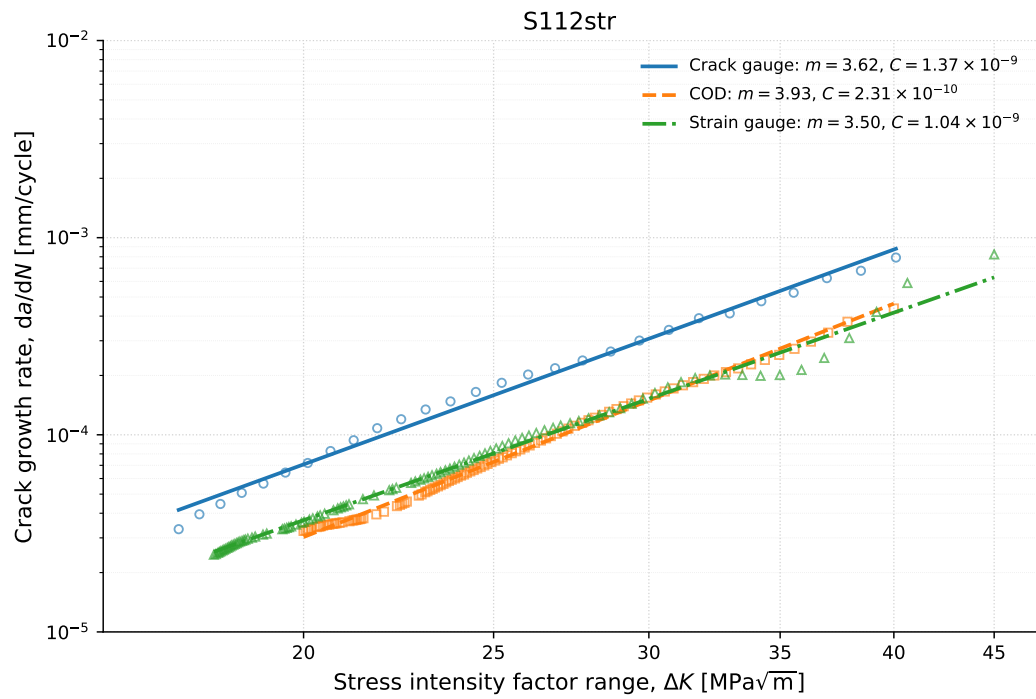


Figure D.12: Specimen-specific Paris-law fits for specimen S112str using the geometrically linear formulation.

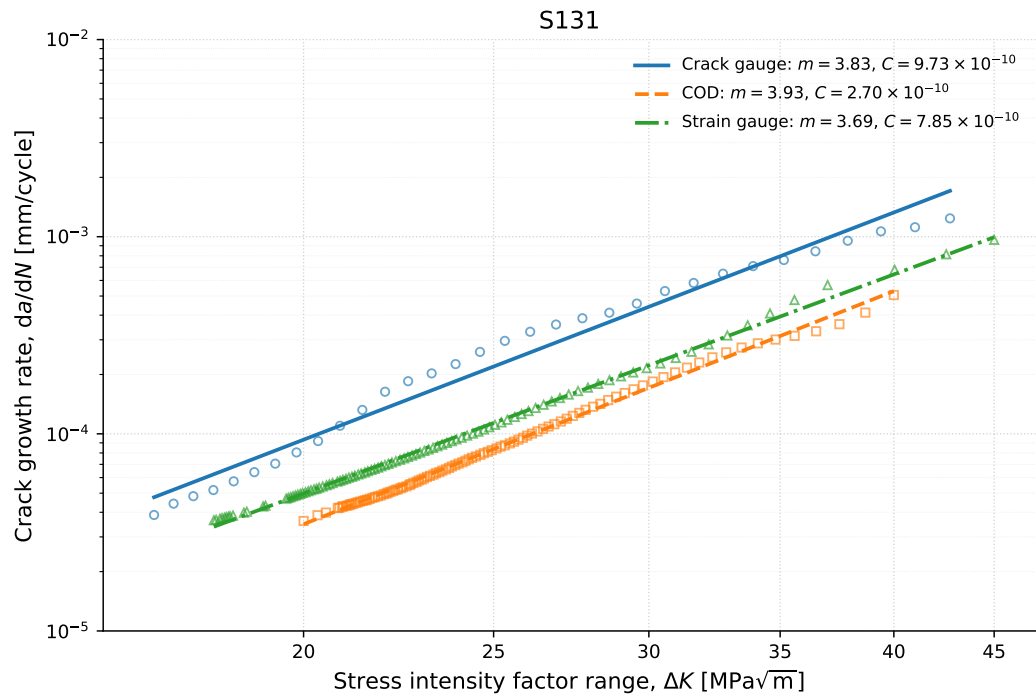


Figure D.13: Specimen-specific Paris-law fits for specimen S131 using the geometrically linear formulation.

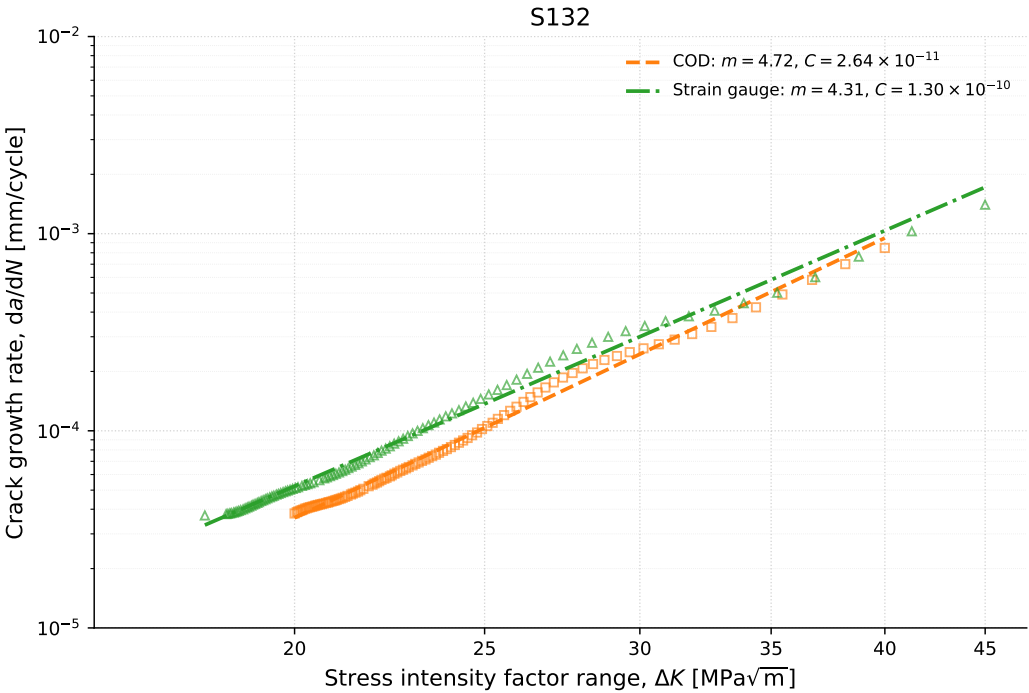


Figure D.14: Specimen-specific Paris-law fits for specimen S132 using the geometrically linear formulation.

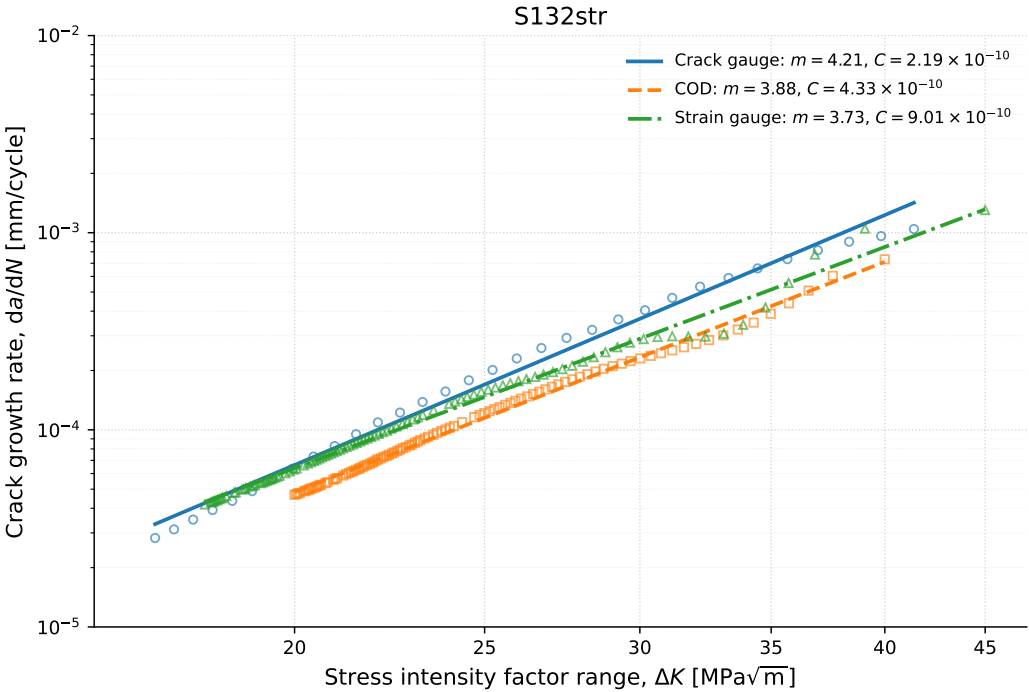


Figure D.15: Specimen-specific Paris-law fits for specimen S132str using the geometrically linear formulation.

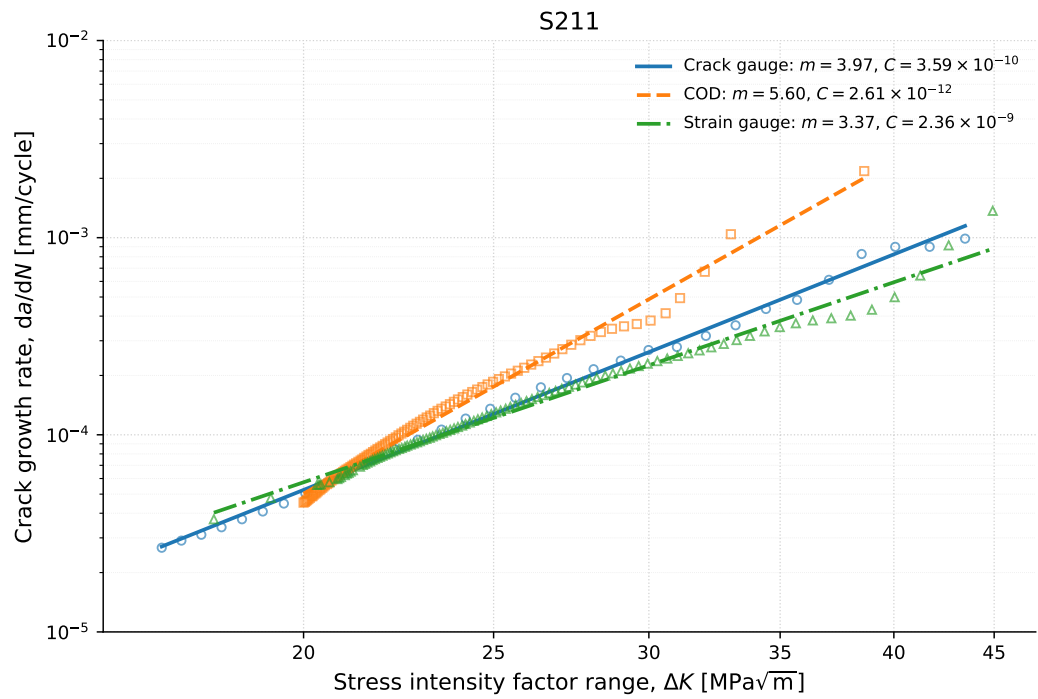


Figure D.16: Specimen-specific Paris-law fits for specimen S211 using the geometrically linear formulation.

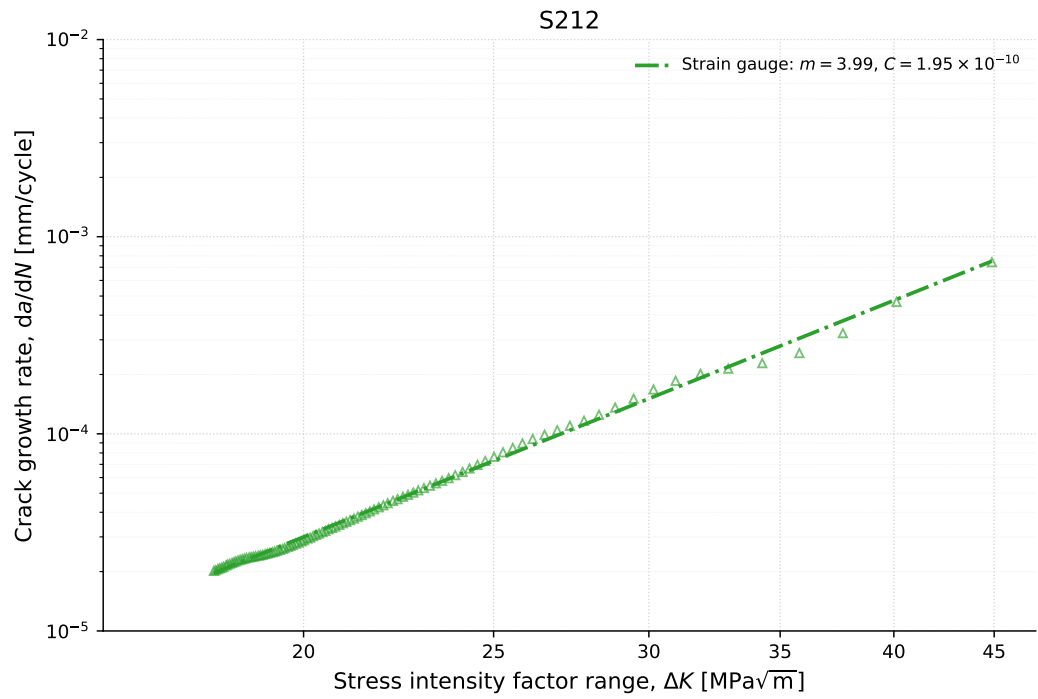


Figure D.17: Specimen-specific Paris-law fits for specimen S212 using the geometrically linear formulation.

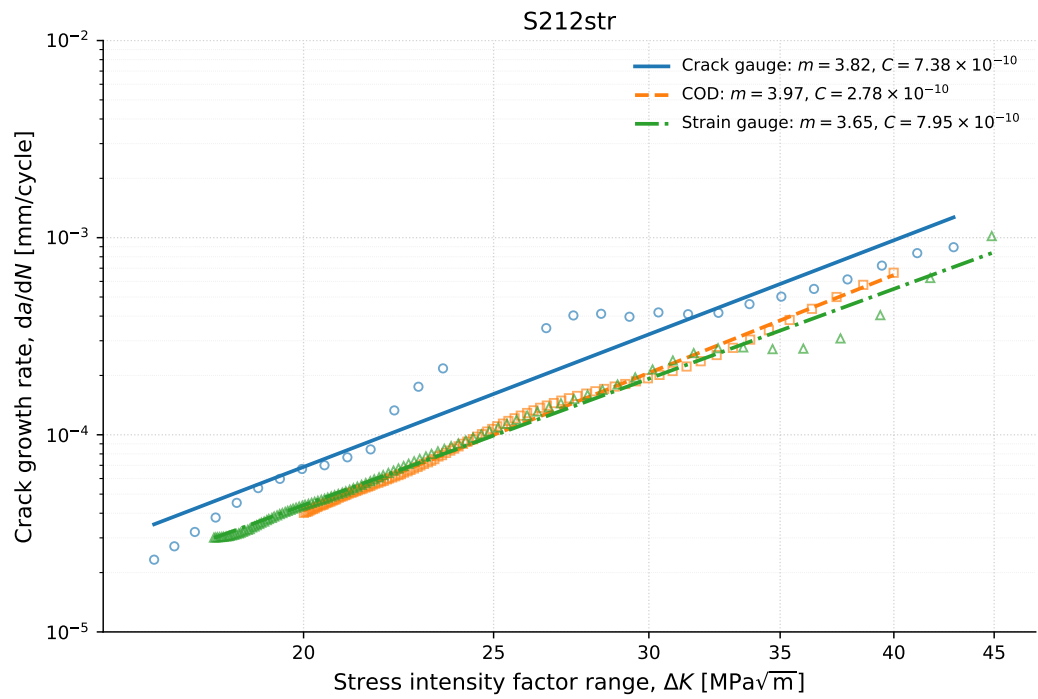


Figure D.18: Specimen-specific Paris-law fits for specimen S212str using the geometrically linear formulation.

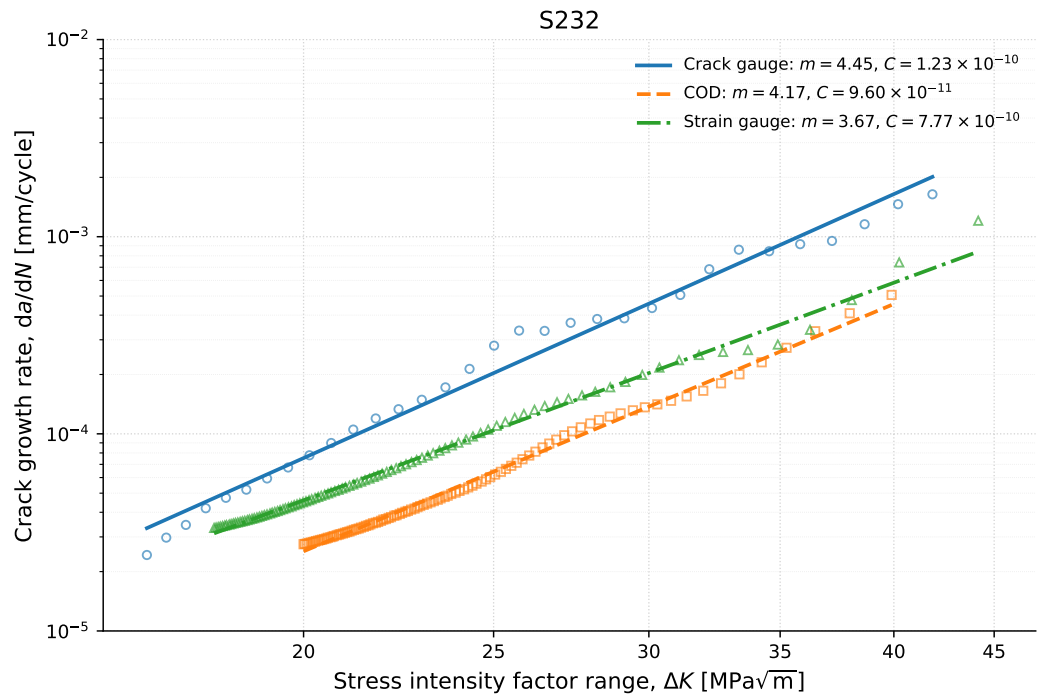


Figure D.19: Specimen-specific Paris-law fits for specimen S232 using the geometrically linear formulation.

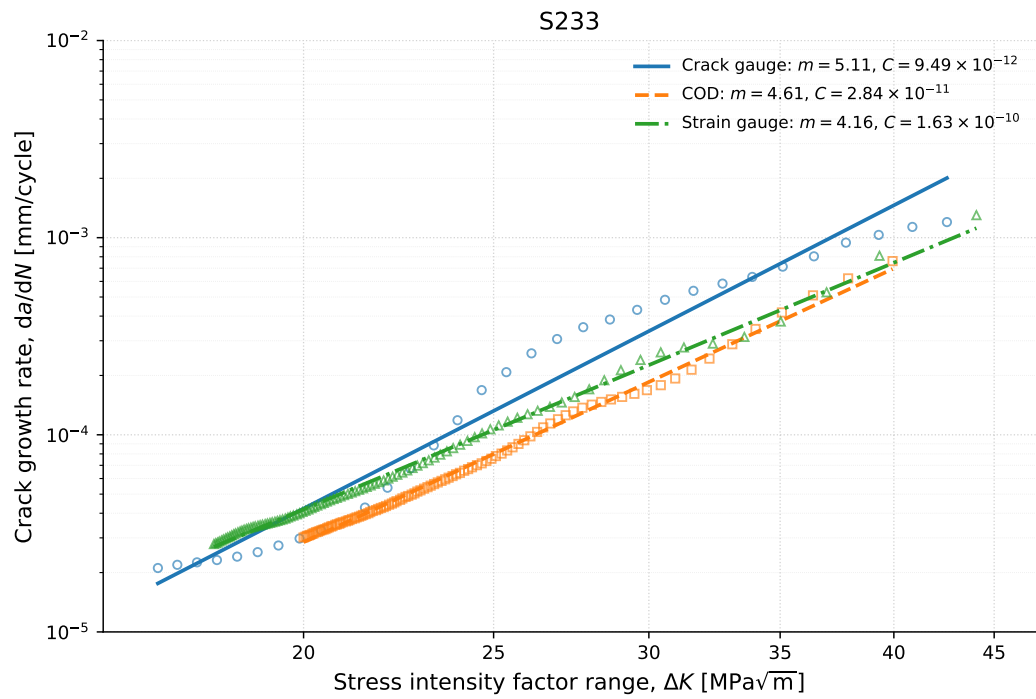


Figure D.20: Specimen-specific Paris-law fits for specimen S233 using the geometrically linear formulation.

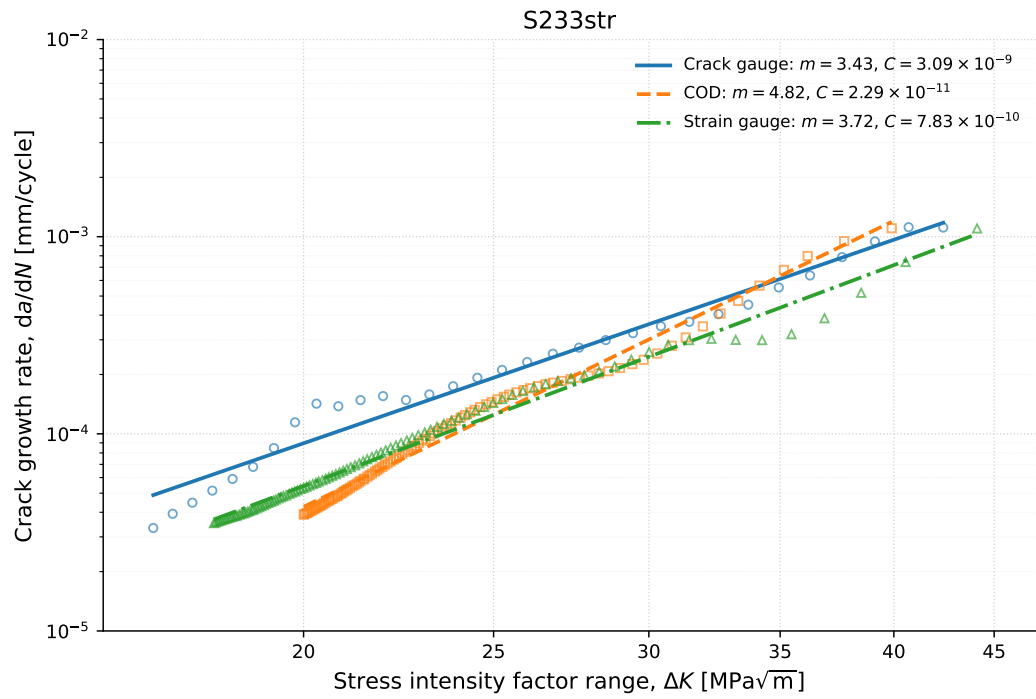


Figure D.21: Specimen-specific Paris-law fits for specimen S233str using the geometrically linear formulation.

D.3. Additional Paris-law parameter results

This section presents additional Paris-law results that complement the main comparison given in Chapter 5. The specimen-specific parameter pairs obtained from the geometrically nonlinear formulation are shown together with the corresponding marginal distributions. A direct comparison of the two aggregation procedures and numerical formulations is also included.

All valid specimen-specific parameter pairs were retained in the statistical evaluation. No statistical outlier removal was applied. In particular, the COD-based result obtained for specimen S211 was retained because no independent evidence of a measurement malfunction, test irregularity, or post-processing error was identified.

Figure D.22 presents the relationship between the Paris exponent m and the intercept $b = \log_{10}(C)$ for the geometrically nonlinear formulation. Each point represents one specimen evaluated using one measurement method. The different marker shapes distinguish the crack-gauge, COD-based, and strain-gauge-based results.

Table D.1: Specimen-specific Paris-law parameters: nonlinear formulation.

Specimen	Crack gauge		COD		Strain gauge	
	m	C	m	C	m	C
S112	4.31	1.59×10^{-10}	3.92	3.15×10^{-10}	3.53	8.74×10^{-10}
S112str	3.73	1.04×10^{-9}	4.07	1.57×10^{-10}	3.61	7.56×10^{-10}
S131	3.95	7.06×10^{-10}	4.05	1.92×10^{-10}	3.80	5.82×10^{-10}
S132	—	—	4.85	1.82×10^{-11}	4.42	9.74×10^{-11}
S132str	4.32	1.62×10^{-10}	3.98	3.32×10^{-10}	3.82	6.96×10^{-10}
S211	4.09	2.61×10^{-10}	5.69	2.07×10^{-12}	3.46	1.85×10^{-9}
S212	—	—	—	—	4.10	1.43×10^{-10}
S212str	3.94	5.31×10^{-10}	4.09	1.99×10^{-10}	3.77	5.59×10^{-10}
S232	4.57	8.73×10^{-11}	4.32	6.12×10^{-11}	3.77	5.80×10^{-10}
S233	5.26	6.37×10^{-12}	4.77	1.84×10^{-11}	4.28	1.18×10^{-10}
S233str	3.53	2.37×10^{-9}	4.92	1.77×10^{-11}	3.82	6.08×10^{-10}

The one-dimensional marginal distributions of m and $\log_{10}(C)$ are shown in Figure D.23. The vertical marks indicate the individual specimen-specific values, while the continuous curves represent normal distributions fitted separately to the results of each measurement method. These distributions are included as a descriptive representation of the location and scatter of the parameters and are not used to exclude individual results.

Figure D.24 compares the mean of the specimen-specific fits with the specimen-balanced pooled fit. The comparison is presented separately for each measurement method and includes both the geometrically linear and geometrically nonlinear formulations. The figure shows the effect of the aggregation procedure and numerical formulation directly in the Paris-law data space.



One-dimensional marginal distributions (Geometrically nonlinear analysis)

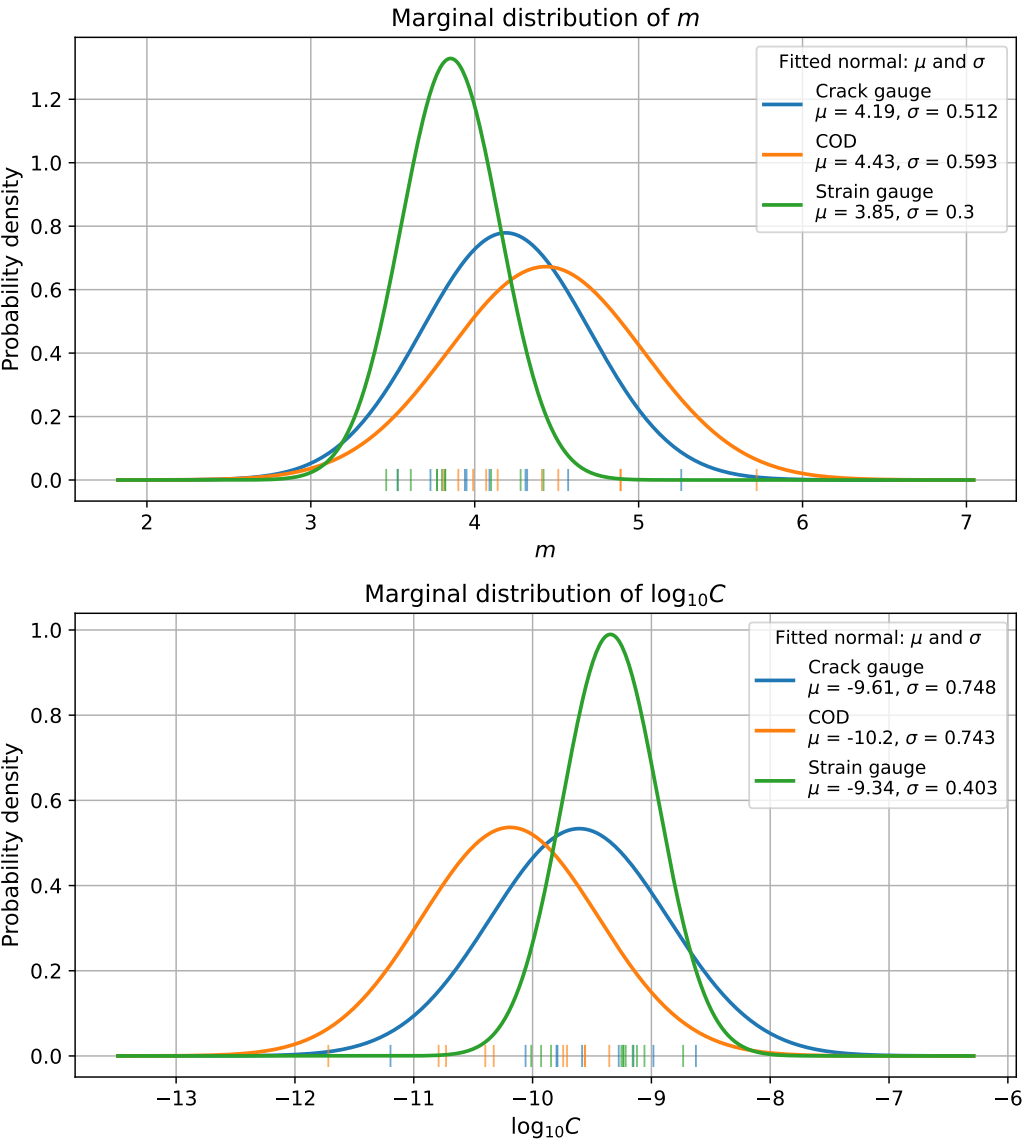


Figure D.23: Marginal distributions of the Paris-law parameters obtained from the geometrically nonlinear formulation.

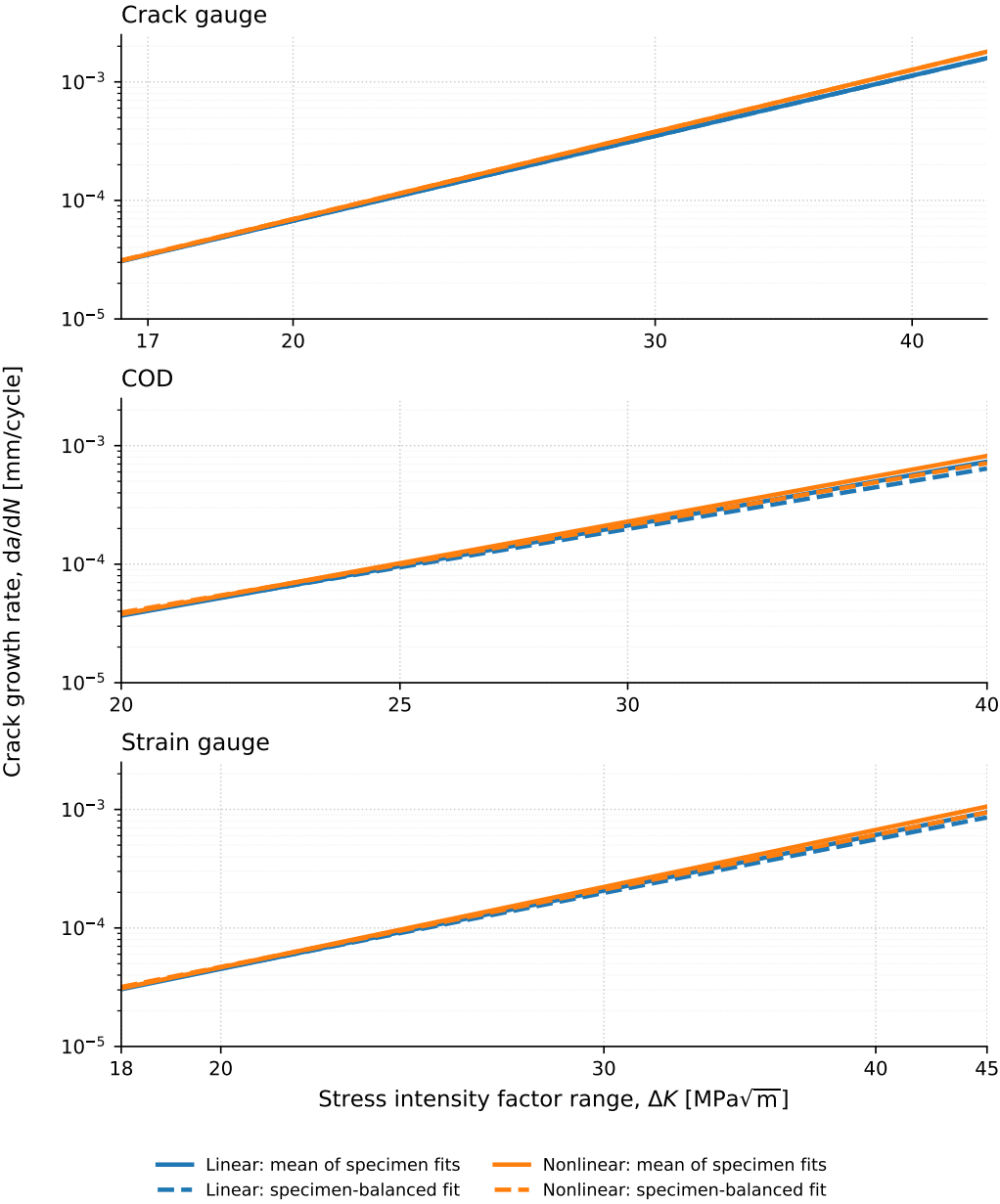


Figure D.24: Comparison of the Paris-law curves obtained using the two aggregation procedures and numerical formulations.

Numerical SIF data and geometry-factor fitting

E.1. Discrete finite element results

The discrete Mode I stress intensity factors used to determine the geometry correction functions are presented in Table E.1. The values were obtained for the reference geometry with $W = 65$ mm, $B = 32$ mm, $F = 25$ kN, and $l = 75$ mm. Here, F denotes the total force applied by the testing machine and l is the distance between a support and the nearest loading point. The corresponding nominal bending stress was $\sigma_{\text{nom}} = 41.605$ MPa.

The geometry correction factors were calculated from the discrete finite element results using Equation 3.24. The geometrically linear and geometrically nonlinear datasets were processed using the same reference geometry and normalisation procedure.

Table E.1: Discrete stress intensity factors and geometry correction factors.

a [mm]	α [–]	$K_{I,\text{lin}}$ [MPa $\sqrt{\text{mm}}$]	Y_{lin} [–]	$K_{I,\text{nl}}$ [MPa $\sqrt{\text{mm}}$]	Y_{nl} [–]
30	0.4615	562.17	1.3918	559.50	1.3852
31	0.4769	587.80	1.4316	584.50	1.4236
32	0.4923	615.26	1.4749	611.80	1.4666
33	0.5077	644.77	1.5220	640.80	1.5127
34	0.5231	676.56	1.5734	672.00	1.5628
35	0.5385	710.95	1.6296	705.60	1.6173
36	0.5538	748.23	1.6911	742.10	1.6772
37	0.5692	788.78	1.7585	781.50	1.7422
38	0.5846	833.05	1.8326	824.80	1.8144
39	0.6000	881.56	1.9143	871.80	1.8931
40	0.6154	934.88	2.0045	923.30	1.9797
41	0.6308	993.77	2.1046	980.10	2.0757
42	0.6462	1059.10	2.2161	1042.50	2.1814
43	0.6615	1131.89	2.3407	1112.00	2.2996
44	0.6769	1213.40	2.4806	1189.00	2.4307
45	0.6923	1309.20	2.6465	1275.00	2.5774
46	0.7077	1409.26	2.8177	1371.40	2.7420
47	0.7231	1527.91	3.0222	1480.20	2.9279
48	0.7385	1664.29	3.2575	1602.00	3.1356
49	0.7538	1822.30	3.5302	1742.30	3.3752
50	0.7692	2007.11	3.8492	1888.20	3.6211
51	0.7846	2225.50	4.2259	2080.81	3.9512
52	0.8000	2486.68	4.6762	2283.66	4.2945
53	0.8154	2803.40	5.2219	2485.00	4.6288
54	0.8308	3193.90	5.8939	2722.00	5.0231
55	0.8462	3684.32	6.7368	3018.00	5.5185

E.2. Full-precision geometry-factor coefficients

For both numerical formulations, the geometry correction factor was represented by

$$Y(\alpha) = \frac{\sum_{j=0}^5 c_j \alpha^j}{(1 - \alpha)^{3/2}}. \quad (\text{E.1})$$

The coefficients reported in the main text are rounded to four decimal places for readability. The higher-precision coefficients used in the post-processing calculations are provided in Table E.2.

Table E.2: Higher-precision coefficients of the geometry correction functions.

Coefficient	Geometrically linear	Geometrically nonlinear
c_0	0.600798524642	4.798599449411
c_1	1.728813758796	-34.266674183813
c_2	-9.024868748630	113.654828452552
c_3	16.163981446918	-191.744876888667
c_4	-13.317052673234	161.993301745554
c_5	4.238781588164	-54.692398977491

E.3. Accuracy of the fitted functions

The accuracy of each fitted function was evaluated at the discrete crack lengths included in Table E.1. The relative fitting error at each data point was calculated as

$$\varepsilon_i = \frac{Y_{\text{fit},i} - Y_{\text{FEM},i}}{Y_{\text{FEM},i}} \cdot 100\%. \quad (\text{E.2})$$

The root-mean-square relative error was calculated as

$$\varepsilon_{\text{RMSE}} = \sqrt{\frac{1}{n} \sum_{i=1}^n \varepsilon_i^2}. \quad (\text{E.3})$$

The resulting fitting errors are summarised in Table E.3. Both functions reproduce the discrete finite element results with a maximum relative error below 0.6% over the investigated interval. The error is lower for the geometrically linear formulation than for the geometrically nonlinear formulation.

Table E.3: Accuracy of the fitted geometry correction functions.

Formulation	Maximum $ \varepsilon_i $ [%]	$\varepsilon_{\text{RMSE}}$ [%]
Geometrically linear	0.255	0.055
Geometrically nonlinear	0.595	0.219

Additional mesh and contour results

This appendix presents supplementary results from the mesh sensitivity analysis described in Section 4.1. The relative differences and normalised contour deviations are presented in the main text. The figures below provide the corresponding absolute response values and the unnormalised stress intensity factors extracted from the individual contours.

F.1. Absolute mesh-sensitivity results

Figure F.1 shows the absolute values of COD, the strain component E_{11} , and the Mode I stress intensity factor K_I obtained with general mesh sizes of 2 mm, 1 mm, and 0.5 mm. Results are presented for crack lengths of $a = 35$ mm and $a = 50$ mm and for both geometrically linear and geometrically nonlinear formulations. These absolute values complement the relative mesh-sensitivity results presented in the main text by showing the numerical scale of the changes caused by mesh refinement.

Mesh sensitivity results for COD, E_{11} , and K_I

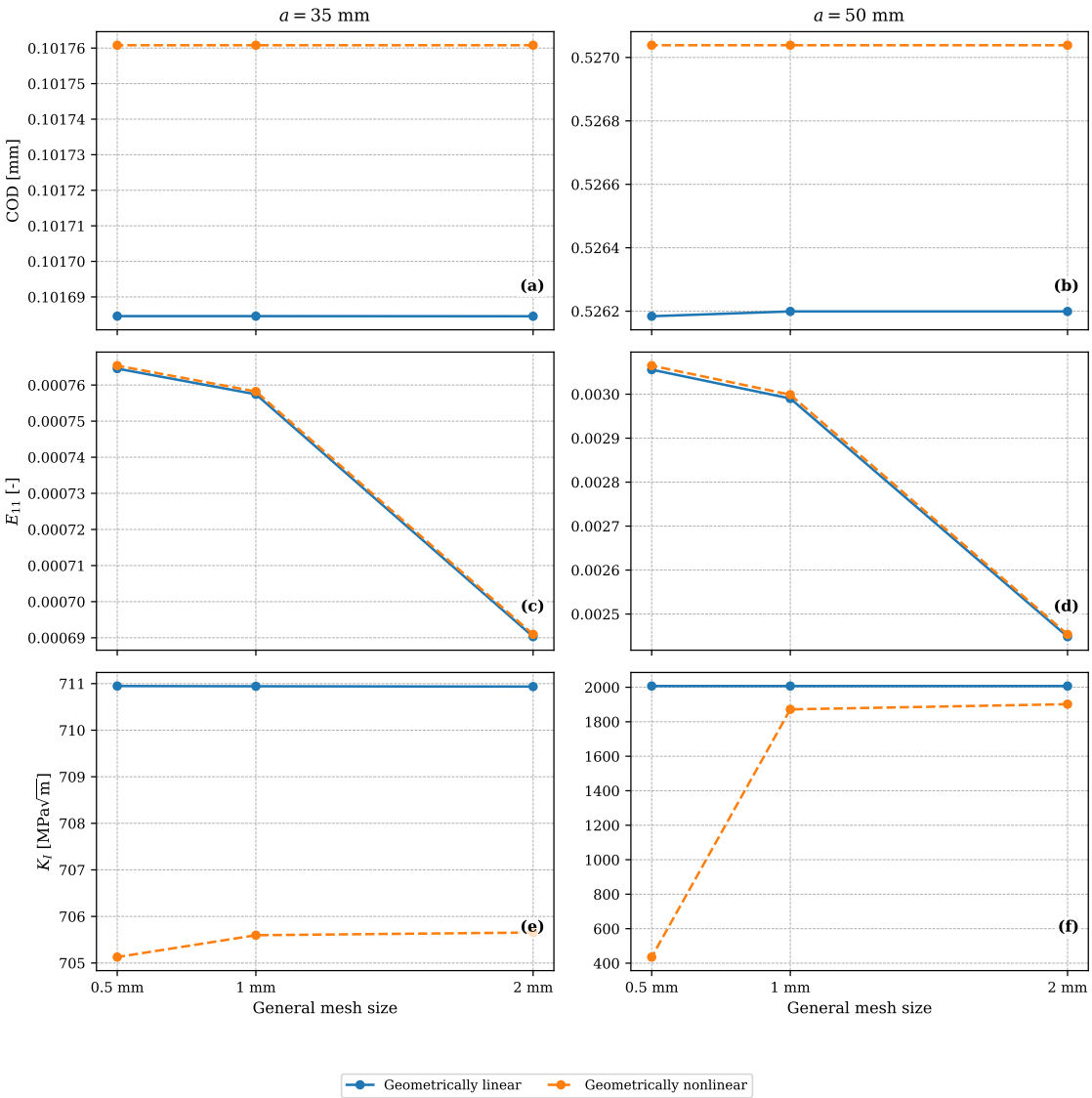


Figure F.1: Absolute mesh-sensitivity results.

F.2. Raw contour results

The stress intensity factor was extracted from five contours surrounding the crack tip. Figure F.2 presents the unnormalised K_I values obtained from each contour for the investigated mesh sizes, crack lengths, and numerical formulations.

The first contour was retained in the figure to show the complete contour-to-contour development. However, the representative stress intensity factor used in the subsequent evaluation was calculated from Contours 2–5, as defined in the numerical methodology. Presenting the raw values makes it possible to assess both the contour stability and the influence of the general mesh size on the extracted stress intensity factor.

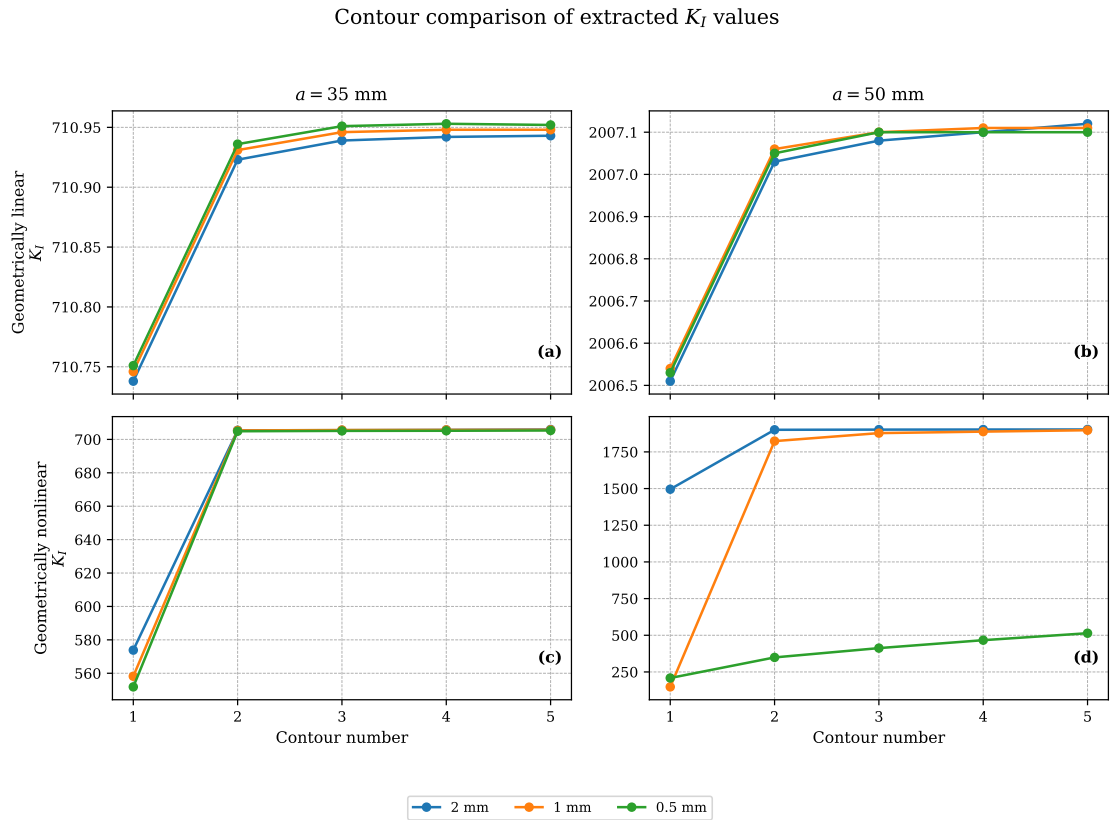


Figure F.2: Raw contour values of the extracted Mode I stress intensity factor.