Merging active and passive surface wave data with interferometry by multidimensional deconvolution

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Merging active and passive surface wave data with interferometry by multidimensional deconvolution

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For the degree of Master of Science in Applied Geophysics at Delft University of Technology

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Faculty of Faculty of Civil Engineering and Geosciences (CiTG) \cdot Delft University of Technology



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Merging active and passive surface wave data with interferometry by multidimensional deconvolution

by

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in partial fulfillment of the requirements for the degree of MASTER OF SCIENCE APPLIED GEOPHYSICS

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Abstract

Seismic interferometry is a technique by which the Green's function (or impulse response) between two receivers can be acquired from the crosscorrelations of wavefield responses at these receivers. Recent developments of this method has led researchers to exploit active as well as passive seismic wavefields to retrieve surface wave Green's functions by crosscorrelation. The primary objective of these applications has been to gain near surface resolution from the high frequency content of the active data while gaining greater depth resolution from the low frequency content of the passive data. In these applications however, a Green's function is retrieved for each data type and therefore a matching filter or a form of joint inversion is required to benefit from the additional bandwidth of both data types.

Interferometry by multidimensional deconvolution (MDD) is a relatively new method of Green's function retrieval that provides several advantages over interferometry by crosscorrelation. This thesis proposes a new method of merging active and passive data during the process of MDD. A primary advantage of this method over the alternatives is that the source signatures are disregarded and only a single Green's function with the combined characteristics of both the active and passive data is retrieved.

Using numerical modelling it is demonstrated that a broadband Green's function response can be retrieved from combined active and passive data without the need to compensate for the differences in source signatures or variations in amplitude. Merging active and passive data prior to deconvolution may in fact improve the retrieved response due to the additional illumination provided by the supplementary data. In addition to expanding the bandwidth of the retrieved response, this method is shown to be capable of using data from one source type to spatially infill gaps in illumination in another source type when the bandwidth of the two are comparable.

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Delft, University of Technology August 8, 2011 Jonathan P. Wall

"Without encroaching upon grounds appertaining to the theologian and the philosopher, the domain of natural sciences is surely broad enough to satisfy the wildest ambition of its devotees. In other departments of human life and interest, true progress is rather an article of faith than a rational belief; but in science a retrograde movements is, from the nature of the case, almost impossible. Increasing knowledge brings with it increasing power, and great as are the triumphs of the present century, we may well believe that they are but a foretaste of what discovery and invention have yet in store for mankind."

— John William Strutt, 3rd Baron Rayleigh (1884) Address to the British Association in Montreal

Chapter 1

Introduction

1-1 Background and motivation

The discipline of geophysics is essential to our ability to gain valuable insight into the complex processes and intricate details of the earth's structure. The interactions of earth's subsurface materials with an adequately energetic wavefield produces quantifiable responses which are proportional to a particular combination of the physical properties of the lithology. Sources of energetic wavefields with a known finite and impulsive form are generally favored due, in part, to the appreciably higher interpretability of the measured response. For this reason active (or controlled) sources are regularly employed to investigate the subsurface, but often with limitations in regards to their bandwidth, magnitude and positioning. As an alternative several branches of geophysics exploit passive ambient wavefields that may have more desirable qualities for attaining responses from targets that are difficult to effectively illuminate using controlled sources.

While the physics of seismic wave propagation is generally well understood (Aki and Richards, 2002), seismology is a particularly exemplifying discipline where researchers must resort to using varied sources, active and passive, to resolve subsurface details due to associated wave-field limitations of one type over the other. Active sources, such as air-guns, dynamite, weight drops, et cetera, can be deployed in an organized and repeatable way, but are typically inefficient at producing desirable low frequencies and are usually confined to the surface or very near surface. Passive sources on the other hand, such as earthquakes, oceanic microseisms, or urban noise, primarily generate lower frequencies, but have the trade-off of being spatially and temporally sporadic and typically produce wavefields that are of a much more complicated nature.

Fortunately, mathematical physics provides a way in which wavefields with even the most complicated signature can be expressed in a functional form. From the 1830's research of the originally unknown and self taught mathematician, George Green, a type of function used in solving inhomogeneous differential equations was produced. 'Green's function,' as it is regularly referred to nowadays is a profoundly important concept in modern science, especially geophysics. The function itself can be considered a system's response to an impulse, or in the context of seismology, an impulse response of the earth. An actual seismic response can be represented by the convolution of a source function with an appropriate Green's function.

1-1-1 Interferometry by cross-correlation

Retrieval of the Green's function is the basis for seismic interferometry. In general terms, seismic interferometry is a technique by which the interference pattern resulting from the superposition of wavefields recorded at two receivers is used to construct a new impulse response at one receiver as though the other was acting as a source. The concept of retrieving the Green's function from passive seismic data originated from the well-known paper by Claerbout (1968) in which it was shown for a one-dimensional medium that the seismic reflection response can be synthesized from the autocorrelation of a transmission response due to a source at depth.

Since its conception the mathematical framework of seismic interferometry has been considerably expanded, acquiring its most momentous contributions in recent years. Notably, the theory has been generalized for application to three-dimensional inhomogeneous media based on physical arguments using the time-reversal invariance of the acoustic wave equation (Derode et al., 2003; Wapenaar et al., 2005) and alternatively by derivation from Rayleigh's reciprocity theorem (Wapenaar, 2004). The theoretical configuration for latter approach is shown in Figure 1-1 in which a surface of monopole (e.g., pressure) and dipole (e.g., particle velocity/displacement) sources S bounds a lossless inhomogeneous volume V. Receiver \mathbf{x}_A and \mathbf{x}_B are contained within V and are uniformly illuminated by all sources at \mathbf{x}_S on S. The correlation-type representation for the causal and acausal acoustic Green's function between \mathbf{x}_A and \mathbf{x}_B for this configuration can be expressed in the frequency domain as (van Manen et al., 2005; Wapenaar et al., 2005; Wapenaar and Fokkema, 2006)

$$\bar{G}(\mathbf{x}_B, \mathbf{x}_A, \omega) + \hat{G}^*(\mathbf{x}_B, \mathbf{x}_A, \omega) = \oint_{\mathbb{S}} \frac{-1}{j\omega\rho(\mathbf{x}_S)} \Big(\partial_i \hat{\bar{G}}^*(\mathbf{x}_B, \mathbf{x}_S, \omega) \hat{G}(\mathbf{x}_A, \mathbf{x}_S, \omega) - (\hat{\bar{G}}(\mathbf{x}_B, \mathbf{x}_S, \omega) \partial_i \hat{G}^*(\mathbf{x}_A, \mathbf{x}_S, \omega) \Big) n_i \, d\mathbf{x}_S \,, \quad (1-1)$$

where j is an imaginary unit, $\rho(\mathbf{x}_S)$ is the density of the medium, n_i is the normal to the surface \mathbb{S} at \mathbf{x}_S , the hat over \hat{G} denotes the frequency domain, and there is an implicit Einstein summation over i. The bar over \bar{G} is used to denote a reference state which has an identical volume \mathbb{V} , but may differ at and/or outside of \mathbb{S} . The partial derivatives on the right hand side of the equation correspond to dipole sources; hence the multiplication between the Green's functions represents a crosscorrelation in the time domain (due to the complex conjugate of one term) between responses from monopole and dipole impulse sources. The retrieved causal and acausal Green's function on the left hand side represents a volume injection rate source at \mathbf{x}_A received as a pressure response at \mathbf{x}_B .

Seismic interferometry by crosscorrelation makes several assumptions about the configuration used in describing Eq. (1-1). The first assumption is that Eq. (1-1) is utilized within the high frequency regime in which case the normal derivative of the Green's function in the integrand on the right hand side can be approximated by $\pm j \frac{\omega}{c(\mathbf{x})} |\cos \alpha(\mathbf{x})|$ where $c(\mathbf{x})$ is the velocity at S and $\alpha(\mathbf{x})$ is the angle between the ray of the wave and the normal on S. Separating the Green's function into ingoing and outgoing parts on S allows for the two terms within



Figure 1-1: Configuration for seismic interferometry by crosscorrelation. Receivers \mathbf{x}_A and \mathbf{x}_B are contained within an anisotropic lossless volume \mathbb{V} and bounded by sources \mathbf{x} on \mathbb{S} with normal n_i . Outside of \mathbb{S} the medium assumed to be is homogeneous. (Wapenaar et al., 2011b)

the integrand to be reduced to the crosscorrelation of a dipole sources Green's function with a monopole source Green's function. The further assumption that the medium outside of S is homogeneous is also used to avoid additional terms that correspond to the retrieval of "ghost" artifacts. Following this the dipole source Green's function in the integrand can be substituted by a monopole source Green's function by assuming that the rays of all wavefields are orthogonal to S at \mathbf{x}_S , i.e., $\alpha = 0$. With these assumptions the integrand of Eq. (1-1) can be simplified to give a concise practical expression for Green's function retrieval, stated in the time domain as (Wapenaar and Fokkema, 2006)

$$G(\mathbf{x}_B, \mathbf{x}_A, t) + G(\mathbf{x}_B, \mathbf{x}_A, -t) \approx \frac{2}{\rho c} \oint_{\mathbb{S}} G(\mathbf{x}_B, \mathbf{x}_S, t) * G(\mathbf{x}_A, \mathbf{x}_S, -t) \, d\mathbf{x}_S \,, \tag{1-2}$$

where * denotes convolution. $G(\mathbf{x}_B, \mathbf{x}_S, t)$ and $G(\mathbf{x}_A, \mathbf{x}_S, t)$ are the Green's functions at receivers \mathbf{x}_B and \mathbf{x}_A due to impulsive sources at \mathbf{x}_S , respectively. As a consequence of the time reversal of the latter term the integrand is in fact a crosscorrelation which integrated over all \mathbf{x}_S results in the causal and acausal Green's function from a virtual impulsive source at \mathbf{x}_A recorded at \mathbf{x}_B , namely, $G(\mathbf{x}_B, \mathbf{x}_A, t)$ and $G(\mathbf{x}_B, \mathbf{x}_A, -t)$.

There are a few practical drawbacks of Eq. (1-2) due to the aforementioned simplifying assumptions. Since the medium outside of S is assumed to be homogeneous, Eq. (1-2) does not account for scatterers outside of S and their presence will result in the retrieval of the desired Green's function accompanied by artifacts. Finally the offset versus angle behaviour of the illuminating wavefields is not preserved because not all rays are orthogonal to S. The practical implications of the latter assumption is likely to be erroneous amplitudes in the retrieved Green's functions but the phase of the response will be unaffected.

The capabilities of seismic interferometry by crosscorrelation to accurately retrieve the Green's function from records of a diffuse wavefield at two receivers has been demonstrated on controlled source and passive seismic data. For example, Shapiro and Campillo (2004) recover fundamental mode Rayleigh waves from correlations of ambient seismic noise generated from ocean microseisms and atmospheric perturbations and subsequently demonstrate the possibility to measure Rayleigh wave dispersion curves between regionally spaced pairs of receivers.

In another application, Ruigrok et al. (2010) apply interferometry to several phase responses from a distribution of earthquakes which illuminate a regional surface array of receivers to retrieve lithospheric reflections. From the retrieved reflections a 2-D velocity model was determined and used in migrating the data to obtain high resolution lithospheric images.

While the robustness of seismic interferometry by crosscorrelation has been demonstrated in several applications, in the majority of practical implementations the underlying assumptions of the Green's function representations are unavoidably violated to some degree. The most common partially fulfilled requirements are that medium is lossless and the wavefield is equipartitioned. The latter can be considered in practical terms as uncorrelated sources illuminating the receivers from all directions with identical power spectra. This is particularly challenging assumption for applications to noise wavefields where sources are likely to be irregularly distributed and have strongly varying amplitudes. Not entirely satisfying these assumptions can result in unreliable amplitudes, nonphysical arrivals and blurring (Froment et al., 2010; Kimman and Trampert, 2010; Snieder et al., 2006; Thorbecke and Wapenaar, 2008).

Since only sources in the regions of constructive interference, or stationary phase, contribute to the reconstruction of the Green's function (Snieder, 2004), in practical applications of interferometry based on Eq. (1-2) partially invalidated assumptions have generally been tolerated and additional strategies have been implemented to mitigate the most severe artifacts (Thorbecke and Wapenaar, 2008). The aforementioned examples are no exception. Shapiro and Campillo (2004) acknowledge in their study that the amplitude of seismic noise varies by many orders of magnitude. Consequently, the crosscorrelation of the noise records can be overwhelmed by the most energetic noise sources. They are able to recover fundamental mode Rayleigh waves from correlations of ambient seismic noise and avoid significant data contamination from artifacts by disregarding the amplitudes and only using the phase components of the noise records. In the lithospheric imaging study by Ruigrok et al. (2010), while taking advantage of different earthquake generated phase responses was adequate to fill the illumination gaps, the irregularity in phase distribution introduced under- and overillumination artifacts in the retrieved reflection response. Using a ray parameter dependent weighing scheme they were able to sufficiently attenuate the artifacts to successfully retrieve the high resolution lithospheric reflections.

1-1-2 Interferometry by multi-dimensional deconvolution

Recognition of the aforementioned limitations of interferometry by crosscorrelation has prompted several researchers to study alternative approaches to Green's function retrieval where the most limiting assumptions are reduced or can be circumvented in an appropriate manner as a means to improve its implementation and expand its employment to a broader range of applications. In particular, recent studies have focused on strategies to deal with biased radiation patterns of the illuminating wavefield from dominant and/or irregularly distributed sources.

As an example, van der Neut and Bakulin (2009) demonstrate for the closely related 'virtual source method' (Bakulin and Calvert, 2006) that the radiation pattern can be estimated from the f-k spectrum of the multidimensional autocorrelation of the incoming wavefield at receivers within a homogeneous layer. Exploiting this they improve the redatumed source by

spatially and temporally deconvolving the virtual source response with the estimated radiation pattern. Their method corrects for amplitude variations while the phase of the signal remains unchanged.

In a 'directional-balancing' algorithm based approach proposed by Curtis and Halliday (2010), the crosscorrelation of the full wavefield is deconvolved with an estimated correction factor based on the differences in radiation patterns between modelled and real data. This method is facilitated by a local array of receivers surrounding the virtual source. Duguid (2010) demonstrates the successful retrieval of surface waves in an engineering application setting using active as well as passive sources. In this process however, additional steps are necessary to predict and remove non-physical arrivals that may be introduced due to heterogeneity in the medium (Curtis and Halliday, 2010).

Wapenaar et al. (2008) propose an alternative strategy for retrieving the Green's function that replaces the crosscorrelation by multidimensional deconvolution. In this method the correlation function, which is derived from the crosscorrelation of the responses from a pair of recievers, is deconvolved with the crosscorrelation of the incoming wavefield response at an orthogonal receiver array in which one of the pair of receivers is contained. The crosscorrelated response at this array is appropriately termed the point-spread function (PSF) as it provides a quantitative measure of the virtual source imperfections due to issues such as correlating noise sources and directionally biased illumination. This approach benefits from the inherit validity of the convolution-type representation to lossy media on which it is based, and additionally, the deconvolution provides compensation for irregular source distributions as shown by Wapenaar et al. (2011a).

The formulation for interferometry by MDD varies only slightly for application to active sources in comparison to noise sources, i.e., responses from (simultaneous) noise sources are combined prior to correlation and (sequential) active seismic responses are combine after correlation. However, the two formulations represent similar inverse problems and are expressed in identical terms (i.e., correlation function and point-spread function) prior to multidimensional deconvolution. Since MDD removes the source signature from the correlation function a natural and advantageous opportunity is provided to exploit the beneficial characteristics of active and passive responses by merging the two data types to retrieve a single broadband Green's function.

1-2 Research objective

Seismic interferometry has been applied by a number of researchers on both active and noise source wavefields for the purpose of retrieving surface waves (Halliday, 2011; Malovichko et al., 2005; de Ridder and Biondi, 2010; Shapiro and Campillo, 2004). In the majority of cases, only either active or passive sources are utilized for reasons such as preference of bandwidth, desired investigation depth, and source-type availability. While one particular source type may offer advantages over the other in each of these regards, the combined benefits of using both source types has been recognized in several studies in which active and noise data were both used for retrieving surface waves (Halliday et al., 2008; Park et al., 2005). In these examples however, the Green's function has been acquired from each source type separately, and in some cases researchers have had to rely on an additional method of joint inversion of the two retrieved responses for the intended application (Foti et al., 2007). A method of combining active and passive data in an interferometry process has not yet been demonstrated, likely due to the aforementioned limitations of the crosscorrelation method (i.e., sources are equally distributed and have identical autocorrelations). This thesis proposes a method of retrieving a single Green's function from merged active and noise surface wave data using interferometry by MDD. The capabilities of this method as well as some of the benefits of exploiting mixed sources are demonstrated and assessed using numerical examples.

Surface waves are used in this thesis to exemplify the concepts and functionality of the method and because practical applications of this approach are likely to primarily focus on surface wave retrieval. However when illumination conditions are adequately satisfied this approach is equally applicable to other direct wave applications and these demonstrations may provide insight into its use with alternative configurations (such as those discussed in Section 5-3).

1-3 Thesis structure

The first chapter introduces seismic interferometry and discusses applications based on the crosscorrelation method. Particular challenges and recent advancements in interferometry are discussed which leads to the introduction of multidimensional deconvolution. Following Wapenaar et al. (2011b), the derivation of interferometry by multidimensional deconvolution from the convolution-type Green's function representation is briefly reviewed in **Chapter 2**. Separate formulation of the method for sequential active sources and simultaneous noise sources are discussed after which a process for combining data types during interferometry is proposed.

Several applications of seismic interferometry for surface wave retrieval from active and passive data are reviewed in the introduction to **Chapter 3**. Active and passive surface wave records are modelled based on the two-dimensional acoustic wave equation in homogenous lossless fluids. After demonstrating the application of interferometry by MDD for active and passive data individually, the proposed method of combining the data is shown using numerically modelled examples. The first example exploits the differing bandwidth of the two data-types to retrieve a broadband surface wave response. In the second example the noise data is used to infill illumination gaps in the active source data. In both examples the correlation functions and point spread functions are reviewed in order to demonstrate the effects of the process.

In interferometry applications to noise data artifacts may arise in the retrieved response as a consequence of the correlations of noise sources signatures, but also from noise sources that repeat within the correlation window. The implications of repeating sources of noise in seismic interferometry are considered in **Chapter 4**. Numerical models are used exemplify how artifacts arising from source periodicity are addressed by multidimensional deconvolution.

Conclusions of the proposed method are discussed in **Chapter 5**. Possible practical applications as well as limitations of the method are described. Additionally, future work including a proposition for variations of source placements are outlined.

Chapter 2

Multidimensional deconvolution

The crosscorrelation type Green's function representation expressed as Eq. (1-1) is the fundamental representation in the majority of interferometry applications (Draganov et al., 2009; de Ridder and Biondi, 2010; Shapiro and Campillo, 2004). In particular interferometric investigations, such as when the amplitudes of the illuminating wavefields are significantly irregular, it can be beneficial to replace crosscorrelation by deconvolution that addresses the spatial and temporal nature of the interferometric problem, namely, interferometry by multi-dimensional deconvolution.

Multidimensional deconvolution can be formulated from the Green's function representation of the convolution-type using the configuration shown in Figure 2-1. This configuration consists of a receiver \mathbf{x}_B enclosed by a boundary S containing receivers \mathbf{x} , with illumination provided by a source \mathbf{x}_S outside of S. The convolution-type representation for this configuration is given by (Wapenaar et al., 2011b)

$$\hat{G}(\mathbf{x}_B, \mathbf{x}_S, \omega) = \oint_{\mathbb{S}} \frac{-1}{j\omega\rho(\mathbf{x})} \Big(\partial_i \hat{\bar{G}}(\mathbf{x}_B, \mathbf{x}, \omega) \hat{G}(\mathbf{x}, \mathbf{x}_S, \omega) - (\hat{\bar{G}}(\mathbf{x}_B, \mathbf{x}, \omega) \partial_i \hat{G}(\mathbf{x}, \mathbf{x}_S, \omega)) \Big) n_i \, d\mathbf{x} \,,$$
(2-1)

where $\hat{G}(\mathbf{x}, \mathbf{x}_S, \omega)$ and $\hat{G}(\mathbf{x}_B, \mathbf{x}_S, \omega)$ are the Green's functions received at \mathbf{x} and \mathbf{x}_B from \mathbf{x}_S , respectively, and $\hat{G}(\mathbf{x}_B, \mathbf{x}, \omega)$ is the desired Green's function between \mathbf{x}_B and resulting virtual source \mathbf{x} . The bar over \hat{G} denotes a reference state in which the medium outside of \mathbb{S} is homogenous but is identical to the actual state in the volume \mathbb{V} .

While Eq. (2-1) is similar in form to Eq. (1-1), there are a number of notable differences that fundamentally alter its application and capabilities in interferometry. Most clearly, in the latter equation both the causal and acausal parts of the desired Green's function, namely $\hat{G}(\mathbf{x}_B, \mathbf{x}_A, \omega)$ and $\hat{G}^*(\mathbf{x}_B, \mathbf{x}_A, \omega)$, are retrieved between the enclosed receivers \mathbf{x}_A and \mathbf{x}_B . In Eq. (2-1) however, since receivers \mathbf{x} and \mathbf{x}_B are illuminated by source \mathbf{x}_S from outside of S there are no acausal Green's function contributions to the integral. Additionally, as a consequence of the absence of complex conjugation in the Green's function representation



Figure 2-1: Configuration for the convolution type Green's function representation. Receivers are contained within an anisotropic dissipative volume \mathbb{V} and bounded by receivers \mathbf{x} on \mathbb{S} with normal n_i . For the reference state, denoted by the bar over \overline{G} , the medium outside of \mathbb{S} is homogeneous. Receivers \mathbf{x} and \mathbf{x}_B record the Green's functions from source \mathbf{x}_S . (Wapenaar et al., 2011b)

of the convolution type, unlike interferometry by crosscorrelation, interferometry by multidimensional deconvolution remains valid for lossy media. Finally, the sought after Green's function term is conveniently located on the left-hand side of correlation-type representation in Eq. (1-1) whereas it is contained with the integrand on the right-hand side of Eq. (2-1). The retrieval of the Green's function by multidimensional deconvolution interformetry based on the latter representation is therefore an inverse problem.

Multidimensional deconvolution of the correlation function can provide several benefits to the virtual source response, i.e., the retrieved Green's function. The primary advantages of MDD over crosscorrelation based interferometry are its ability to correct for an irregular source distribution, suppress spurious multiples, improve the radiation pattern of the virtual source, and account for dissipation (Wapenaar et al., 2010). The improved functionality of MMD has been demonstrated for passive and controlled source applications. van der Neut et al. (2011) show on synthetic data the redatuming of surface sources to receiver locations within a well below a complex overburden. Using a crustal scale numerical model, Wapenaar et al. (2011a) compare the Green's function retrieval between distant receivers from irregularly distributed oceanic sources using a crosscorrelation approach, and alternatively, MDD interferometry. In their example, clusters of noise sources produce noticeable over-illumination in particular directions and results in blurring and artifacts in the reconstructed response in the correlation method. These issues are shown to be addressed during multidimensional deconvolution which provides in an appreciably improved Green's function.

Chapter 2 begins with a brief review of the formulation of active and passive direct wave seismic interferometry by MDD following Wapenaar et al. (2011b). After reviewing the method for both source-types independently, an approach to combine the data in a convenient and potentially advantageous way during the MDD process is proposed.

2-1 Interferometry by MDD

The desired Green's function, $\bar{G}(\mathbf{x}_B, \mathbf{x}, \omega)$, which represents an impulse from the virtual source at \mathbf{x} and recorded at receiver \mathbf{x}_B (Figure 2-1), appears in both of the two convolution products in the integrand on the right-hand side of Eq. (2-1). Wapenaar et al. (2011b) show using pseudo-differential operator theory that these terms can be simplified to a single term by rewriting $\hat{G}(\mathbf{x}, \mathbf{x}_S, \omega)$ as the superposition of inward and outward propagating parts at \mathbf{x} on \mathbb{S} , according to $\hat{G}(\mathbf{x}, \mathbf{x}_S, \omega) = \hat{G}^{in}(\mathbf{x}, \mathbf{x}_S, \omega) + \hat{G}^{out}(\mathbf{x}, \mathbf{x}_S, \omega)$. This simplification can also be achieved by using the high-frequency approximation for the derivatives in the integrand as equal to $\pm j(\omega/c)|\cos \alpha|$, where c is the wave propagation velocity and α is the angle between the wave ray and n_i on \mathbb{S} (Wapenaar et al., 2005). In doing either, the contributions from \hat{G}^{in} simplify to $2(n_i\partial_i\hat{G})\hat{G}^{in}$ while contributions from \hat{G}^{out} cancel out entirely. Following the simplification of the integrand, Eq. (2-1) can be rewritten as

$$\hat{G}(\mathbf{x}_B, \mathbf{x}_S, \omega) = -\oint_{\mathbb{S}} \frac{2}{j\omega\rho(\mathbf{x})} (n_i \partial_i \hat{\bar{G}}(\mathbf{x}_B, \mathbf{x}, \omega)) \hat{G}^{in}(\mathbf{x}, \mathbf{x}_S, \omega) \, d\mathbf{x} \,.$$
(2-2)

To be applicable within the practical confines of source and receiver deployment the bounding integration surface S in Eq. (2-1) can be considered as an open boundary of receivers S_{rec} and an enclosing surface S_{enc} , as depicted in Figure 2-2. By extending S_{enc} to infinity, as a consequence of the Sommerfeld radiation condition the contribution to the integral along this portion of S vanishes entirely. Hence, from hereon the integral in Eq. (2-2) will be considered an open boundary and expressed solely as the contributions along S_{rec} .

The integrand in Eq. (2-2) can be expressed more concisely by replacing the monopole response between \mathbf{x} and \mathbf{x}_B by a dipole response, defined as $\hat{G}_d(\mathbf{x}_B, \mathbf{x}, \omega) = \frac{-2}{j\omega\rho(\mathbf{x})}\partial_i\hat{G}(\mathbf{x}_B, \mathbf{x}, \omega)$ where the subscript 'd' is used to denote a dipole source. Following this source substitution Eq. (2-2) can be expressed for an open receiver boundary \mathbb{S}_{rec} as

$$\hat{G}(\mathbf{x}_B, \mathbf{x}_S, \omega) = \int_{\mathbb{S}_{rec}} \hat{\bar{G}}_d(\mathbf{x}_B, \mathbf{x}, \omega) \hat{G}^{in}(\mathbf{x}, \mathbf{x}_S, \omega) \, d\mathbf{x} \,.$$
(2-3)

After Fourier transformation, Eq. (2-3) is stated in the time domain as,

$$G(\mathbf{x}_B, \mathbf{x}_S, t) = \int_{\mathbb{S}_{rec}} \bar{G}_d(\mathbf{x}_B, \mathbf{x}, t) * G^{in}(\mathbf{x}, \mathbf{x}_S, t) \, d\mathbf{x} \,.$$
(2-4)

Recall that the bar over $\bar{G}_d(\mathbf{x}_B, \mathbf{x}, t)$ represents the reference medium. In this case the reference state has the identical medium properties as the actual state in volume \mathbb{V} and is homogenous outside of the bounding surface \mathbb{S} . Hence, this desired impulse response between a virtual source at \mathbf{x} and recorded at \mathbf{x}_B is not influenced by wavefield scattering from the medium outside of \mathbb{V} . As mentioned in the previous section, retrieving $\bar{G}_d(\mathbf{x}_B, \mathbf{x}, t)$ from the integrand of Eq. (2-4) is an inverse problem. For a single source this is an ill-posed inverse problem because there is insufficient illumination to adequately construct an interference pattern between \mathbf{x} and \mathbf{x}_B . However by increasing the number of sources outside of \mathbb{S}_{rec} as depicted in Figure 2-2, Eq. (2-4) can be stated for each source $\mathbf{x}_S^{(i)}$ individually, where *i* denotes the source number, and therefore becomes a well posed inverse problem.



Figure 2-2: MDD configuration for interferometry. Bounding surface of receivers S consists of S_{rec} and S_{enc} , where the latter is extended to infinity. As a consequence of the Sommerfield radiation condition the contributions from S_{enc} vanishes. Sources $\mathbf{x}_{S}^{(i)}$, to the left of S_{rec} , propagate waves $u^{in}(\mathbf{x}, \mathbf{x}_{S}^{(i)}, t)$ to receivers \mathbf{x} along S_{rec} as well as $u(\mathbf{x}_{B}, \mathbf{x}_{S}^{(i)}, t)$ to receiver \mathbf{x}_{B} . The cross-correlation of $u^{in}(\mathbf{x}_{A}, \mathbf{x}_{S}^{(i)}, t)$, where \mathbf{x}_{A} is on S_{rec} , with $u^{in}(\mathbf{x}, \mathbf{x}_{S}^{(i)}, t)$ gives the correlation function Eq. (2-8). The point-spread function in Eq. (2-10) is derived from the cross-correlation of $u^{in}(\mathbf{x}, \mathbf{x}_{S}^{(i)}, t)$.

2-2 Active sources

In the practical application of seismic interferometry using active sources, referring to controlled sequential transient sources, wavefields $u(\mathbf{x}, \mathbf{x}_S, t)$ and $u(\mathbf{x}_B, \mathbf{x}_S, t)$ recorded at receivers \mathbf{x} and \mathbf{x}_B , respectively, can be expressed as the Green's function (impulse response) convolved with a source wavelet $s^{(i)}(t)$ according to

$$u_{active}^{in}(\mathbf{x}, \mathbf{x}_{S}^{(i)}, t) = G^{in}(\mathbf{x}, \mathbf{x}_{S}^{(i)}, t) * s^{(i)}(t)$$
(2-5)

and

$$u_{active}(\mathbf{x}_B, \mathbf{x}_S^{(i)}, t) = G(\mathbf{x}_B, \mathbf{x}_S^{(i)}, t) * s^{(i)}(t).$$
(2-6)

The superscript "*in*" is used to denote that only incoming waves are recorded at S_{rec} , as discussed in Section 2-1 and illustrated in Figure 2-2, and the subscript "*active*" refers to the sequential active source type.

Since the Green's functions contained within Eq. (2-5) and Eq. (2-6) are the same as those on the right- and left-hand side of Eq. (2-4), respectively, convolving both sides of the latter equation with $s^{(i)}(t)$ gives

$$u_{active}(\mathbf{x}_B, \mathbf{x}_S^{(i)}, t) = \int_{\mathbb{S}_{rec}} \bar{G}_d(\mathbf{x}_B, \mathbf{x}, t) * u_{active}^{in}(\mathbf{x}, \mathbf{x}_S^{(i)}, t) \, d\mathbf{x} \,.$$
(2-7)

Crosscorrelating the term on the left-hand side of Eq. (2-7) with $u_{active}^{in}(\mathbf{x}_A, \mathbf{x}_S^{(i)}, t)$, where \mathbf{x}_A is on \mathbb{S}_{rec} , and by summing over all sources the correlation function for controlled sources as

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can be defined as

$$C_{active}(\mathbf{x}_B, \mathbf{x}_A, t) = \sum_{i} u_{active}(\mathbf{x}_B, \mathbf{x}_S^{(i)}, t) * u_{active}^{in}(\mathbf{x}_A, \mathbf{x}_S^{(i)}, -t)$$

$$= \sum_{i} G(\mathbf{x}_B, \mathbf{x}_S^{(i)}, t) * G^{in}(\mathbf{x}_A, \mathbf{x}_S^{(i)}, -t) * S_{active}^{(i)}(t),$$
(2-8)

where $S^{(i)}(t)$ is the autocorrelation according to

$$S^{(i)}(t) = s^{(i)}(t) * s^{(i)}(-t).$$
(2-9)

The general source receiver configuration for MDD in Figure 2-2 illustrates this concept. Each controlled source $\mathbf{x}_{S}^{(i)}$ on the left-hand side of the figure produces a wavefield that is recorded at all receivers on \mathbb{S}_{rec} as well as \mathbf{x}_{B} . Crosscorrelating the response at \mathbf{x}_{A} and \mathbf{x}_{B} from each $\mathbf{x}_{S}^{(i)}$, namely, $u_{active}(\mathbf{x}_{A}, \mathbf{x}_{S}^{(i)}, t)$ and $u_{active}(\mathbf{x}_{B}, \mathbf{x}_{S}^{(i)}, t)$, and summing over all available sources provides the correlation function between \mathbf{x}_{A} and \mathbf{x}_{B} .

Additionally, by also crosscorrelating $u_{active}^{in}(\mathbf{x}, \mathbf{x}_S^{(i)}, t)$ from the right-hand side of Eq. (2-7) with $u_{active}^{in}(\mathbf{x}_A, \mathbf{x}_S^{(i)}, t)$ and summing over all sources the point-spread function (PSF) is defined as

$$\Gamma_{active}(\mathbf{x}, \mathbf{x}_{A}, t) = \sum_{i} u_{active}^{in}(\mathbf{x}, \mathbf{x}_{S}^{(i)}, t) * u_{active}^{in}(\mathbf{x}_{A}, \mathbf{x}_{S}^{(i)}, -t)$$

= $\sum_{i} G^{in}(\mathbf{x}, \mathbf{x}_{S}^{(i)}, t) * G^{in}(\mathbf{x}_{A}, \mathbf{x}_{S}^{(i)}, -t) * S^{(i)}(t).$ (2-10)

The PSF contains the multidimensional autocorrelation of the sources and is a quantitative representation of the spatial and temporal smearing of the virtual source. When the illumination from $\mathbf{x}_{S}^{(i)}$ is ideal the PSF approaches a spatially and temporally band-limited delta function (Wapenaar et al. (2011b), Appendix A).

From Eq. (2-8) and Eq. (2-10) the expression for Eq. (2-7) following crosscorrelation with $u_{active}^{in}(\mathbf{x}_A, \mathbf{x}_S^{(i)}, t)$ can be written as

$$C_{active}(\mathbf{x}_B, \mathbf{x}_A, t) = \int_{\mathbb{S}_{rec}} \bar{G}_d(\mathbf{x}_B, \mathbf{x}, t) * \Gamma_{active}(\mathbf{x}, \mathbf{x}_A, t) \, d\mathbf{x} \,.$$
(2-11)

Eq. (2-11) demonstrates that the correlation function contains a Green's function convolved with the point spread function; hence, the correlation can be considered as a spatially and temporally smeared Green's function. Since the autocorrelation of the sources is contained within both the correlation and point spread functions, by deconvolving the latter from the former a source signature free Green's function can be retrieved without any knowledge of the particular source signature. As mentioned however, the desired Green's function is contained within the integral and therefore requires that the correlation function is inverted using the point spread function. In can be shown that the inversion result is equivalent to the leastsquares solution of Eq. (2-7) (van der Neut et al. (2011), Appendix C).

For actual computational inversion of Eq. (2-11), the equation is transformed to the frequency domain and the integration over \mathbf{x} is replaced by summation for all \mathbf{x} on \mathbb{S}_{rec} . Since in practice

the terms in Eq. (2-11) are derived from discrete measurements the equation can be expressed in matrix notation (Berkhout, 1982) as

$$\hat{\mathbf{C}}_{active} = \hat{\bar{\mathbf{G}}}_d \hat{\boldsymbol{\Gamma}}_{active}, \tag{2-12}$$

where $\hat{\mathbf{C}}_{active}$ is a monochromatic matrix in which the entries of the *l*th row and the *m*th column correspond to $\hat{C}_{active}(\mathbf{x}_B^{(l)}, \mathbf{x}_A^{(m)}, \omega)$, and matrices $\hat{\mathbf{G}}_d$ and $\hat{\mathbf{\Gamma}}_{active}$ are constructed in a likewise manner.

2-3 Noise sources

Similar to the expressions defining the recorded responses from controlled sources (Eq. (2-5) - 2-6), the recorded wavefield from a noise source can also be represented by the convolution of a Green's function with a source signature. However in contrast to controlled source records, noise sources may be sporadic and simultaneously acting. Hence a single noise record may include many sources and consequently requires summation of all sources to be expressed in the following way,

$$u_{noise}^{in}(\mathbf{x},t) = \sum_{i} G^{in}(\mathbf{x}, \mathbf{x}_{S}^{(i)}, t) * N^{(i)}(t),$$
(2-13)

$$u_{noise}^{in}(\mathbf{x}_A, t) = \sum_j G^{in}(\mathbf{x}_A, \mathbf{x}_S^{(j)}, t) * N^{(j)}(t),$$
(2-14)

and

$$u_{noise}(\mathbf{x}_B, t) = \sum_{j} G(\mathbf{x}_B, \mathbf{x}_S^{(j)}, t) * N^{(j)}(t),$$
(2-15)

where the subscript "*noise*" is used to refer to the noise source-type. Note that no assumptions are made about the correlatedness of the noise sources and so their crosscorrelation can be defined as

$$\langle N^{(i)}(t) * N^{(j)}(-t) \rangle = S^{(ij)}_{noise}(t).$$
 (2-16)

Following the same process as used to define the correlation and point spread functions for active sources, using Eq. (2-14), Eq. (2-15) and Eq. (2-16), the noise source correlation function is expressed as

$$C_{noise}(\mathbf{x}_B, \mathbf{x}_A, t) = u_{noise}(\mathbf{x}_B, t) * u_{noise}^{in}(\mathbf{x}_A, -t)$$

= $\sum_i \sum_j G(\mathbf{x}_B, \mathbf{x}_S^{(i)}, t) * G^{in}(\mathbf{x}_A, \mathbf{x}_S^{(j)}, -t) * S_{noise}^{(ij)}(t),$ (2-17)

and likewise, using Eq. (2-13), Eq. (2-14) and Eq. (2-16), the noise point spread function is written as

$$\Gamma_{noise}(\mathbf{x}, \mathbf{x}_A, t) = u_{noise}^{in}(\mathbf{x}, t) * u_{noise}^{in}(\mathbf{x}_A, -t)$$

= $\sum_i \sum_j G^{in}(\mathbf{x}, \mathbf{x}_S^{(i)}, t) * G^{in}(\mathbf{x}_A, \mathbf{x}_S^{(j)}, -t) * S_{noise}^{(ij)}(t).$ (2-18)

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With these new definitions the basic expression for MDD for simultaneously acting noise sources can be written in identical in form to Eq. (2-11) as

$$C_{noise}(\mathbf{x}_B, \mathbf{x}_A, t) = \int_{\mathbb{S}_{rec}} \bar{G}_d(\mathbf{x}_B, \mathbf{x}, t) * \Gamma_{noise}(\mathbf{x}, \mathbf{x}_A, t) \, d\mathbf{x} \,.$$
(2-19)

In the same manner as described for active sources, inversion of Eq. (2-19) is carried out in the frequency domain using matrix notation according to

$$\hat{\mathbf{C}}_{noise} = \hat{\bar{\mathbf{G}}}_d \hat{\boldsymbol{\Gamma}}_{noise}.$$
(2-20)

Note while comparing the terms that contribute to the MDD expressions for active source responses (Eq. (2-11)) and noise reponses (Eq. (2-19)) that for temporally non-overlapping active sources there is only a single source summation within the correlation function in Eq. (2-8) and point-spread function in Eq. (2-10). This is in contrast to MDD for simultaneously acting noise sources where the source summation within each noise record (Eq. (2-13) - (2-15)) results in a double summation in the corresponding correlation and point spread functions. The implications of this double summation can be appreciated by restating Eq. (2-17) and Eq. (2-18) as the following,

$$C_{noise}(\mathbf{x}_{B}, \mathbf{x}_{A}, t) = \sum_{i} \sum_{j} G(\mathbf{x}_{B}, \mathbf{x}_{S}^{(i)}, t) * G^{in}(\mathbf{x}_{A}, \mathbf{x}_{S}^{(i)}, -t) * S_{noise}^{(i)}(t)\delta_{ij}$$
(2-21)
+ $\sum_{i} \sum_{j} G(\mathbf{x}_{B}, \mathbf{x}_{S}^{(i)}, t) * G^{in}(\mathbf{x}_{A}, \mathbf{x}_{S}^{(j)}, -t) * S_{noise}^{(ij)}(t)(1 - \delta_{ij})$
= $\sum_{i} \sum_{j} G(\mathbf{x}_{B}, \mathbf{x}_{S}^{(i)}, t) * G^{in}(\mathbf{x}_{A}, \mathbf{x}_{S}^{(i)}, -t) * S_{noise}^{(i)}(t)\delta_{ij}$
+ "crosstalk"

and

$$\Gamma_{noise}(\mathbf{x}, \mathbf{x}_{A}, t) = \sum_{i} \sum_{j} G^{in}(\mathbf{x}, \mathbf{x}_{S}^{(i)}, t) * G^{in}(\mathbf{x}_{A}, \mathbf{x}_{S}^{(i)}, -t) * S_{noise}^{(i)}(t)\delta_{ij}$$

$$+ \sum_{i} \sum_{j} G^{in}(\mathbf{x}, \mathbf{x}_{S}^{(i)}, t) * G^{in}(\mathbf{x}_{A}, \mathbf{x}_{S}^{(j)}, -t) * S_{noise}^{(ij)}(t)(1 - \delta_{ij})$$

$$= \sum_{i} \sum_{j} G^{in}(\mathbf{x}, \mathbf{x}_{S}^{(i)}, t) * G^{in}(\mathbf{x}_{A}, \mathbf{x}_{S}^{(i)}, -t) * S_{noise}^{(i)}(t)\delta_{ij}$$

$$+ "crosstalk"$$
(2-22)

respectively. The first term in the two equations results from when i=j and consequently the superscripts containing (j) or (ij) are replaced by (i). Evidently the first term of Eq. (2-21) and Eq. (2-22) for noise source data are akin to Eq. (2-8) and Eq. (2-10) for active source data; hence, this is the essential term for the reconstruction of the response. The second term in the two equations , on the other hand, arises from the correlation of noise sources when $i\neq j$. Hence, these terms are equivalent to the "crosstalk" between correlated noises sources.

If, however, noise sources are uncorrelated such that their crosscorrelation can be expressed according to

$$\langle N^{(i)}(t) * N^{(j)}(-t) \rangle = \delta_{ij} S^{(i)}_{noise}(t),$$
 (2-23)

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Eq. (2-17) and Eq. (2-18) simplify to

$$C_{noise}(\mathbf{x}_B, \mathbf{x}_A, t) = \sum_{i} G(\mathbf{x}_B, \mathbf{x}_S^{(i)}, t) * G^{in}(\mathbf{x}_A, \mathbf{x}_S^{(i)}, -t) * S_{noise}^{(i)}(t),$$
(2-24)

and

$$\Gamma_{noise}(\mathbf{x}, \mathbf{x}_A, t) = \sum_{i} G^{in}(\mathbf{x}, \mathbf{x}_S^{(i)}, t) * G^{in}(\mathbf{x}_A, \mathbf{x}_S^{(i)}, -t) * S_{noise}^{(i)}(t).$$
(2-25)

Hence, when noise sources are mutually uncorrelated the expression for the associated correlation function (Eq. (2-24)) and point spread function (Eq. (2-25)) are identical to those for active sources (Eq. (2-8) and Eq. (2-10)) and only differ according to autocorrelation of the respective source signatures. In this case, when noise sources provide equal one-sided illumination to the receivers on S_{rec} , the point spread function for noise (Eq. (2-25)) approaches a spatially and temporally band-limited delta function akin to the PSF for active sources in Eq. (2-10).

2-4 Merging active and passive data

Regardless whether or not noise sources are uncorrelated, interferometry by MDD of active and noise data as stated in Eq. (2-11) and Eq. (2-19), respectively, represent a similar inverse problem as expressed in general by Eq. (2-4) in which $\bar{G}_d(\mathbf{x}_B, \mathbf{x}, t)$ is also the desired Green's function. There is only a slight variation in formulation between the active and passive inverse problems, specifically the difference in source summation, however the correlation and pointspread functions for the two source types have identical form. The matching form of these equations provides a natural and advantageous opportunity to combine both data types prior to deconvolution. In doing so, $\bar{G}_d(\mathbf{x}_B, \mathbf{x}, t)$ can be retrieved in much the same way as in Eq. (2-12) and Eq. (2-20), however it will benefit from the characteristics of both the active and passive wavefields.

Using the frequency domain correlation functions from the left-hand sides of Eq. (2-11) and Eq. (2-19), the mixed source correlation function can be expressed as

$$\hat{\mathbf{C}}_{mixed} = W_{active}(\omega)\hat{\mathbf{C}}_{active} + W_{noise}(\omega)\hat{\mathbf{C}}_{noise}, \qquad (2-26)$$

and likewise, using the PSF from the right-hand sides of Eq. (2-11) and Eq. (2-19), the mixed source point spread function can be expressed as,

$$\hat{\Gamma}_{mixed} = W_{active}(\omega)\hat{\Gamma}_{active} + W_{noise}(\omega)\hat{\Gamma}_{noise}, \qquad (2-27)$$

where $W_{active}(\omega)$ and $W_{noise}(\omega)$ are frequency depended weighting scalars. These functions will vary in application depending on the desired contributions to the merged data from the respective active and noise sources.

Using Eq. (2-26) and Eq. (2-27) in the basic expression for MDD, in identical form to Eq. (2-12) and Eq. (2-20), the inverse problem can be expressed in the frequency domain as

$$\hat{\mathbf{C}}_{mixed} = \hat{\bar{\mathbf{G}}}_d \hat{\boldsymbol{\Gamma}}_{mixed}.$$
(2-28)

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Comparable to Eq. (2-12) and Eq. (2-20), Eq. (2-28) can be solved for $\mathbf{\bar{G}}_d$ using stabilized matrix inversion for each frequency. Note that the retrieved Green's function is void of a source signature; hence in none of the inversions does the exact signal of a particular source need to be known.

The scaling matrices in Eq. (2-26) and Eq. (2-27) allow for biassing of a particular source type at discrete frequencies. As will be shown in Chapter 3, by weighting the favorable source type when merging the data the virtual source resulting from the inversion of Eq. (2-28) can be spatially and temporally improved. It should be noted however that this approach is not suitable for active and passive data that do not share a portion of overlapping bandwidth. In such cases, the mixed source correlation and point-spread functions resulting from Eq. (2-26) and Eq. (2-27) would contain a significant frequency notch that would result in an unstable inversion of Eq. (2-28) (due to division by zero). A more appropriate approach would be to invert Eq. (2-12) and Eq. (2-20) separately and combine the data afterwards. A primary advantage of the proposed method is that the stability of the inversion and bandwidth of $\hat{\mathbf{G}}_d$ may benefit from the merging of the data, particularly where bandwidths overlap.

Chapter 3

Numerical examples

Surface wave phase velocities, group velocities and attenuation characteristics, have become critically important for characterizing of the earth's subsurface (Aki and Richards, 2002). In the field of crustal seismology, the long period component of earthquake and ocean generated surface waves are commonly used to study the structure of the lithosphere and asthenosphere (Debayle and Kennett, 2000; Gerstoft et al., 2006). Near surface geophysical studies, such as those carried out for engineering or environmental purposes, regularly utilized surface waves to characterize the uppermost tens to hundreds of meters of the earth's subsurface (Foti et al., 2007; Hebeler and Rix, 2001). Even in exploration seismology where reflected waves are of primary interest and surface waves are typically regarded as a considerable hinderance, exploiting the unique characteristics of the latter can aid in estimates of shear wave velocities and static corrections (Roy et al., 2010; Xia et al., 2004).

Horizontal attenuation of surface waves which propagate parallel to the earth's surface is significantly lower as a result of geometrical spreading in comparison to that of three-dimensional propagating body waves. Consequently, surface waves are often the most prominent wave type in controlled source seismic records and are generally the most abundant component of surface recorded ambient noise. Given the dominance of these direct arrivals on local to global scales, seismic interferometry is a well suited tool for surface wave Green's function retrieval from both active and noise source generated wavefields. In exploration seismology for example, attenuating surface waves from records without harming primary reflections is particularly challenging due to their significant overlap in the frequency and time domains, especially when the surface waves are back-scattered from out-of-plane. Among the numerous methods that have been developed for this task, interferometry by crosscorrelation can be used to retrieve the in-plane as well as out-of-plane scattered surface waves and subsequently eliminated from the records using adaptive subtraction (Halliday, 2011).

For subsurface characterization and retrieval by interferometry, surface waves generated from active and passive sources each have inherent advantages and disadvantages. Clearly, controlled source surface waves records are much more coherent than records of passive surface waves since the former source-type are usually individually acting and their signatures, onsets and locations, are well known. Additionally, surface waves generated from active sources provide high frequencies which are generally lacking in passive data. In contrast, producing low frequency wavefields requires significantly higher energy sources than are typically available for controlled source acquisition, but are naturally abundant in the form of microseisms or anthropogenic noise (Park et al., 2004). Sources of passive seismic wavefields, which are usually randomly distributed and may be periodic and/or simultaneously acting, are generally not limited in depth, but unlike controlled sources their locations and signatures may not be well known.

The desire for broadband surface wave retrieval has been one of the primary motivations for several researchers to utilize controlled source in addition to passive seismic data. Malovichko et al. (2005) demonstrate the potential advantages of using both data types by obtaining near-surface layer velocity constraints from high frequency Rayleigh waves in controlled source data, while benefiting from the resolution of the longer wavelengths in deeper layers in passive data sourced from vehicle traffic. The resulting vertical shear-wave velocity profile in their study is a product of the inversion of the combined dispersion curves which were acquired separately. Using this method, they significantly expand the bandwidth of the dispersion curve and are consequently able to recover a deeper velocity profile. Similarly, Park et al. (2005) use active data from hammer sources in addition to random traffic noise to acquire multiple dispersion curves to gain insight into the presence of higher mode surface waves. Halliday et al. (2008) also note similar benefits in comparing surface waves retrieved using active and passive interferometric methods in an urban environment. While all of these examples use interferometry to exploit active and passive surface waves as a means to gain broadband data and therefore optimize the investigation depth and near surface resolution, the retrieved surface waves are contained within two separate datasets which are associated with the particular source. In these examples as well as several others (Boulanger et al., 2005; Foti et al., 2007; Yoon and Rix, 2004; Suzuki and Hayashi, 2003), the active and passive surface wave data are combined in a secondary inversion process (e.g., merging dispersion curves prior to inversion for near surface velocities).

Merging active and passive surface wave data using interferometry by multidimensional deconvolution may in fact benefit the retrieval of the surface wave, and furthermore results in only a single response with the shared characteristics of both data types. The retrieval of $\hat{G}_d(\mathbf{x}_B, \mathbf{x}_A, \omega)$ from the inversion of the combined correlation and point spread functions in Eq. (2-28) is void of a source signature, i.e., $s^{(i)}(t)$ or $N^{(i)}(t)$. As a result, the retrieved response represents the Green's function for the the full band of available frequencies which can then be used to derive the dispersion properties, phase velocities, et cetera. Additionally, combining mixed source data may improve wavefield illumination to the receivers and consequently reduce artifacts that may otherwise result from gaps in illumination.

This chapter begins with a description of the methodology used for modelling surface waves. After demonstrating the application of MDD for surface wave retrieval from active and noise source records, the two data types are merged through the process of MDD. The capabilities of the proposed method are demonstrated for broadening the bandwidth of the retrieved surface wave and spatial infilling of illumination gaps on numerically modelled data.


Figure 3-1: Plan view of the general model configuration. Noise sources $\mathbf{x}_{S_noise}^{(i)}$ are randomly positioned at the left of the configuration and act simultaneously. Active sources $\mathbf{x}_{S_{active}}^{(i)}$ are regularly spaced and act sequentially. \mathbb{L}_A contains 51 receivers, including \mathbf{x}_A , with 4m separation and \mathbb{L}_B contains 8 receivers, including \mathbf{x}_B , with 15m separation.

3-1 Surface wave model

Receivers are positioned along two orthogonal lines in the model acquisition configuration shown in Figure 3-1. The north-south oriented receiver line \mathbb{L}_A , which makes up \mathbb{S}_{rec} as illustrated in the previously discussed theoretical configuration of MDD in Figure 2-2, contains 51 receivers with 4 m station spacing and can be considered akin to a broadside-T spread. As stated for Eq. (2-12) and Eq. (2-20), the point-spread function requires a regular sampling of the incoming wavefields along \mathbb{L}_A ; hence the receiver spacing on the line is subject to the same spatial and temporal Nyquist sampling criteria as in any conventional acquisition.

The west-east oriented receiver line \mathbb{L}_B contains 20 receivers with 15m station spacing, each of which can be individually considered as \mathbf{x}_B in Figure 2-2. As such, unlike receiver line \mathbb{L}_A , receivers on \mathbb{L}_B are not subject to a spatial Nyquist sampling criterion relative to each other, but of course must have adequate temporal sampling. For each receiver on \mathbb{L}_B a Green's function can be retrieved from a chosen virtual source on \mathbb{L}_A . Notwithstanding this spatial leeway it is important to note that there are implications to the relative offset between $\mathbf{x}_S^{(i)}$, \mathbf{x}_A and \mathbf{x}_B . This is discussed in detail in Section 3-3-2.

Uniformly spaced active sources as well as randomly positioned noise sources are located to the left of the receiver configuration in Figure 2-2. As well, there are no scatters present within the model and hence the assumption of only one-sided wavefield illumination to \mathbb{S}_{rec} , required by the simplification of the integrand in Eq. (2-1), is validated. In practice, noise sources as considered in this model may represent urban noise such as a construction site, a highway or railway, et cetera. Even more ideally, 'controlled noise sources' such as a monitored passing vehicle or ground compactor maybe used.

Depth (m)	Vp (m/s)	Vs (m/s)	Density (kg/m ³)
0-1	490	120	1420
1-3	520	200	1430
3-5	650	220	1480
5-11	900	280	1550
11-19	1130	380	1620
19-35	1350	410	1900
35+	1420	500	1980

(a) Six layer subsurface model used in forward modelling of dispersion curve.



(b) Dispersive Rayleigh wave velocity curve based on the above model.

Figure 3-2: Dispersion model for numerically modelled fundamental mode Rayleigh waves.

The open source software package, GEOPSY (Wathelet, 2010), which is based the forward modelling scheme proposed by Wathelet (2005), was used to compute the dispersion of the fundamental mode Rayleigh-wave for a horizontally homogenous model consisting of six layers overlying a half space with medium parameters shown in Figure 3-2a and the corresponding dispersion curve in Figure 3-2b.

Representing the model with fundamental mode Rayleigh-waves only, the expression for the Green's function in Eq. (2-5) and Eq. (2-6) is substituted with the monopole far-field solution of the two-dimensional acoustic wave equation in homogenous lossless fluids (Berkhout, 1987), expressed in the frequency domain as

$$G(\mathbf{x}_{A,B}, \mathbf{x}_{S}^{(i)}, \omega) = \sqrt{\frac{c(\omega)}{8\pi\omega |\mathbf{r}(\mathbf{x}_{A,B}, \mathbf{x}_{S}^{(i)})|}} e^{-j(\frac{\omega |\mathbf{r}(\mathbf{x}_{A,B}, \mathbf{x}_{S}^{(i)})|}{c(\omega)} + \frac{\pi}{4})},$$
(3-1)

where $\mathbf{r}(\mathbf{x}_{A,B}, \mathbf{x}_S^{(i)})$ is the path between source $\mathbf{x}_S^{(i)}$ and receiver \mathbf{x}_A or \mathbf{x}_B , and $c(\omega)$ is the velocity of the dispersive wavefield given by the dispersion curve in Figure 3-2b.

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The Green's function retrieved using multidimensional deconvolution is that of a dipole virtual source as expressed in Eq. (2-3). In order to adequately assess the quality of the MDD inversion in the following examples the dipole far-field solution of the two-dimensional acoustic wave equation in homogenous is also required, and is given by (Berkhout, 1987)

$$G_d(\mathbf{x}_B, \mathbf{x}_A, \omega) = \frac{x_j}{|\mathbf{r}(\mathbf{x}_B, \mathbf{x}_A, \omega)|} \frac{\omega}{c(\omega)} \sqrt{\frac{c(\omega)}{8\pi\omega |\mathbf{r}(\mathbf{x}_B, \mathbf{x}_A, \omega)|}} e^{-j(\frac{\omega |\mathbf{r}(\mathbf{x}_B, \mathbf{x}_A, \omega)|}{c(\omega)} - \frac{\pi}{4})}.$$
 (3-2)

In the following examples Eq. (3-2) is used to provide a reference trace for the Green's function retrieved from MDD between the virtual source \mathbf{x}_A and receiver at \mathbf{x}_B , hence $|\mathbf{r}(\mathbf{x}_B, \mathbf{x}_A, \omega)|$. The sole purpose of calculating this reference trace is to use for comparison with the retrieved dipole source Green's function from MDD.

3-1-1 Active sources

Substituting the appropriate Green's functions from Eq. (3-1) into frequency domain expressions for active source responses at \mathbf{x} and \mathbf{x}_B , given by Eq. (2-5) and Eq. (2-6), Rayleigh waves from active sources are expressed as

$$\hat{u}_{active}^{in}(\mathbf{x}, \mathbf{x}_{S}^{(i)}, \omega) = \sqrt{\frac{c(\omega)}{8\pi\omega |\mathbf{r}(\mathbf{x}, \mathbf{x}_{S}^{(i)})|}} e^{-j(\frac{\omega |\mathbf{r}(\mathbf{x}, \mathbf{x}_{S}^{(i)})|}{c(\omega)} + \frac{\pi}{4})} \hat{s}^{(i)}(\omega), \qquad (3-3)$$

and

$$\hat{u}_{active}(\mathbf{x}_B, \mathbf{x}_S^{(i)}, \omega) = \sqrt{\frac{c(\omega)}{8\pi\omega |\mathbf{r}(\mathbf{x}_B, \mathbf{x}_S^{(i)})|}} e^{-j(\frac{\omega |\mathbf{r}(\mathbf{x}_B, \mathbf{x}_S^{(i)})|}{c(\omega)} + \frac{\pi}{4})} \hat{s}^{(i)}(\omega),$$
(3-4)

respectively.

Using the configuration in Figure 3-1, the dispersion curve in Figure 3-2b and choosing an Ormsby wavelet with parameters 8-14-18-25Hz for $\hat{s}(\omega)$, the recorded response from a single active source is shown in Figure 3-3. Note that similar responses exist for each $\mathbf{x}_{S_{active}}^{(i)}$ and the choice of wavelet used here for the active sources is deliberately void of low frequencies in order to illustrate the addition of noise data in a later section.

3-1-2 Noise sources

Similarly, noise sources are modelled from substituting Eq. (3-1) into frequency domain expressions for Eq. (2-13), Eq. (2-14) and Eq. (2-15), giving

$$u_{noise}^{in}(\mathbf{x},t) = \sum_{i} \sqrt{\frac{c(\omega)}{8\pi\omega |\mathbf{r}(\mathbf{x},\mathbf{x}_{S}^{(i)})|}} e^{-j(\frac{\omega |\mathbf{r}(\mathbf{x},\mathbf{x}_{S}^{(i)})|}{c(\omega)} + \frac{\pi}{4})} \hat{N}^{(i)}(\omega),$$
(3-5)

$$u_{noise}^{in}(\mathbf{x}_A, t) = \sum_{j} \sqrt{\frac{c(\omega)}{8\pi\omega |\mathbf{r}(\mathbf{x}_A, \mathbf{x}_S^{(i)})|}} e^{-j(\frac{\omega |\mathbf{r}(\mathbf{x}_A, \mathbf{x}_S^{(j)})|}{c(\omega)} + \frac{\pi}{4})} \hat{N}^{(j)}(\omega),$$
(3-6)

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(b) Active source response recorded by receivers on \mathbb{L}_B .

4

4.5

5 ^l

Figure 3-3: Modelled response recorded for a single active source by the receiver configuration in Figure 3-1 using a dispersion curve for the medium given in Figure 3-2b.

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(a) Noise recorded by receivers on \mathbb{L}_A .



(b) Noise recorded by receivers on \mathbb{L}_B .

Figure 3-4: Noise response recorded for simultaneously acting noise sources by the receiver configuration from Figure 3-1 using the dispersion curve for the medium given in Figure 3-2b.

and

$$u_{noise}(\mathbf{x}_B, t) = \sum_{j} \sqrt{\frac{c(\omega)}{8\pi\omega |\mathbf{r}(\mathbf{x}_B, \mathbf{x}_S^{(i)})|}} e^{-j(\frac{\omega |\mathbf{r}(\mathbf{x}_B, \mathbf{x}_S^{(i)})|}{c(\omega)} + \frac{\pi}{4})} \hat{N}^{(i)}(\omega),$$
(3-7)

where $\hat{N}^{(i)}(\omega)$ is a frequency domain expression for a random time series convolved with a wavelet. Figure 3-4 shows a 5 seconds noise panel modelled from random simultaneously acting noise sources with an Ormsby wavelet with parameters 1-3-6-12Hz and the configuration shown in Figure 3-1.

3-2 Multidimensional deconvolution on synthetics

Multidimensional deconvolution for active and passive data was introduced in Section 2-2 and Section 2-3, respectively. Despite the slight difference in formulation due to the sequential (active) versus simultaneous (noise) records, the two applications are similar inverse problems. In this section the retrieval of the Green's function by MDD is demonstrated for the modelled active and passive data separately. In the section following, the proposed method of merging the two data types to retrieve a single Green's function by MDD is shown.

3-2-1 MMD on active source data

The correlation function for active sources given in Eq. (2-8) is acquired from the summed crosscorrelation of a response recorded on a receiver on \mathbb{L}_A (Figure 3-3a) with a response at a receiver on \mathbb{L}_B (Figure 3-3b) from all available active sources in Figure 3-1. As discussed in Section 2-2 the matrix notation of Berkhout (1982) is used to numerically represent the discretely sampled data during MDD. The correlation function for active source in Eq. (2-8) can likewise be expressed in the frequency domain for each frequency as

$$\hat{\mathbf{C}}_{active} = \hat{\mathbf{U}}_{active} \mathbf{W}_S (\hat{\mathbf{U}}_{active}^{in})^{\dagger}, \qquad (3-8)$$

where the columns of matrices $\hat{\mathbf{U}}_{active}$ and $\hat{\mathbf{U}}_{active}^{in}$ correspond to source locations and rows correspond to receiver locations of $u_{active}(\mathbf{x}_B, \mathbf{x}_S^{(i)}, t)$ and $u_{active}^{in}(\mathbf{x}_A, \mathbf{x}_S^{(i)}, t)$, respectively, and † represents the complex conjugate transpose. Additionally, a diagonal matrix \mathbf{W}_S composed of a source weighting factor $W(\mathbf{x}_S^{(i)})$ is included as a means of tapering the amplitudes from edges of the source array.

To exemplify the process of retrieving the correlation function, the crosscorrelation of the records of only the two bold receivers \mathbf{x}_A and \mathbf{x}_B in Figure 3-1 is first demonstrated. Figure 3-5a illustrates the crosscorrelation of the responses at these two receivers for each active source with the relative amplitude determined by the source weighting factor given in Figure 3-6. Summing the crosscorrelations for all source results in constructive interference from sources within the region of stationary phase, while outside of this region the crosscorrelations destructively interfere giving the correlation function shown in Figure 3-5b. This process is inherently carried out in the matrix multiplication in Eq. (3-8). Hence, the elements of the

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sources.



crosscorrelations.

Figure 3-5: Correlation plot.



Figure 3-6: Source weighting function used to taper the edges of the active source line.

correlation function are constructed in an identical manner to Figure 3-5b for every receiver pair combination between lines \mathbb{L}_A and \mathbb{L}_B .

The resulting correlation function for all receivers on \mathbb{L}_B is compared with a reference monopole response¹(Eq. (3-2)) from the virtual source \mathbf{x}_A in Figure 3-7. The correlation function has a very close match to the reference response, however there are very slight differences in amplitude due to non-ideal illumination from the ends of the source distribution.

Similarly, the point spread function in Eq. (2-10) is derived from the crosscorrelation of the

¹Reference responses shown in this and all following figures have been convolved with the autocorrelation of the source signature S(t) to facilitate comparison with the correlation function, unless otherwise stated.

recorded incoming wavefields at receivers on \mathbb{L}_A (Figure 3-3a) and summed over all sources. Likewise, Eq. (2-10) transformed to the frequency domain can be expressed in matrix notation for each frequency as

$$\hat{\mathbf{\Gamma}}_{active} = \hat{\mathbf{U}}_{active}^{in} \mathbf{W}_S (\hat{\mathbf{U}}_{active}^{in})^{\dagger}$$
(3-9)

where \mathbf{W}_S is the same weighting matrix used in Eq. (3-8).

The resulting PSF for the modeled active data shown in Figure 3-8 is focused at t = 0 at the virtual source $\mathbf{x} = \mathbf{x}_A$. Since the active sources are evenly distributed and the source wavelets are non-varying, the PSF is symmetric along space and wavenumber in the x-t and f-k domains, respectively.

The correlation function (Figure 3-7) and point spread function (Figure 3-8) are the required components for the inversion of Eq. (2-12). Inverting Eq. (2-12) in practice requires the addition of a stabilization factor in order avoid possible errors resulting from notches in the frequency spectrum. That factor takes the form of the addition of a small constant to the diagonal of the PSF matrix such that the matrix inversion can be expressed as

$$\hat{\mathbf{G}}_d = \hat{\mathbf{C}}_{active} [\hat{\mathbf{\Gamma}}_{active} + \epsilon^2 \mathbf{I}]^{-1}, \qquad (3-10)$$

where ϵ is the stabilization factor and **I** is the identity matrix. Multidimensional deconvolution in Eq. (3-10) is solved for each frequency using matrix inversion with a stabilization factor of $\epsilon = 0.01 * maximum of |\hat{\Gamma}_{active}|$.

In a same manner the PSF can also be inverted by the following expression,

$$\hat{\Gamma}'_{active} = \hat{\Gamma}_{active} [\hat{\Gamma}_{active} + \epsilon^2 \mathbf{I}]^{-1}.$$
(3-11)

After MDD the PSF from Eq. (3-11) will signify the effects of the inversion. Hence, the inverted PSF is a useful for assessing the effects of applying multidimensional deconvolution as well as evaluating the size of the stabilization factor.

Figure 3-9 demonstrates that the retrieved surface wave Green's function² from MDD is almost exactly matching the reference dipole response. As discussed, the retrieved surface wave only required slight improvement to match the reference response because the correlation function was already considerably well focused. Nevertheless, this improvement can also be noted in the better focusing of the PSF in the x-t domain in Figure 3-10a and is particularly well evidenced by flattening of the f-k spectrum of the PSF shown in Figure 3-10b.

3-2-2 MMD on noise source data

Acquiring the correlation function for each receiver pair from passive sources as expressed by Eq. (2-17) involves the crosscorrelation of the response at each receiver on \mathbb{L}_A (Figure 3-4a) with the response at each receiver on \mathbb{L}_B (Figure 3-4b). The matrix notation expression for Eq. (2-17) in the frequency domain is stated as

$$\hat{\mathbf{C}}_{noise} = \hat{\mathbf{U}}_{noise} (\hat{\mathbf{U}}_{noise}^{in})^{\dagger}, \qquad (3-12)$$

²Along with the reference responses, the Green's function retrieved by MDD in all figures in this thesis has been convolved with the autocorrelation of the source signature S(t) to facilitate comparison with the correlation function, unless otherwise stated.



Figure 3-7: Correlation function from active data for a virtual source at \mathbf{x}_A to all receivers on $\mathbb{L}_B(\text{red})$ overlain by reference response (black). Note that the reference trace has been convolved with autocorrelation of the source signature to facilitate comparison.



(a) PSF in x-t domain.

(b) PSF in f-k domain.

Figure 3-8: Point spread function from modelled data for all active sources shown in Figure 3-1. (Single active source response shown in Figure 3-3a).



Figure 3-9: Retrieved Green's function from active data for a virtual source at \mathbf{x}_A to all receivers on \mathbb{L}_B using MDD (red) overlain by reference dipole source response (black).



(a) PSF in MDD *x*-*t* domain.

(b) PSF after MDD in f-k domain.

Figure 3-10: Active data point spread function in Figure 3-8 after multidimensional deconvolution.

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for each frequency. Again note that since the response for simultaneously excited noise sources are recorded concurrently, $\hat{\mathbf{U}}_{noise}$ and $\hat{\mathbf{U}}_{noise}^{in}$ are therefore vectors with elements corresponding to receiver positions. Consequently, unlike the correlation function for active sources (Eq. (3-8)), sources contributing to the correlation function for noise cannot be tapered by the inclusion of a weighting matrix.

Similarly, the PSF in Eq. (2-18) can be expressed in the frequency domain in matrix notation as

$$\hat{\boldsymbol{\Gamma}}_{active} = \hat{\boldsymbol{U}}_{noise}^{in} (\hat{\boldsymbol{U}}_{noise}^{in})^{\dagger}.$$
(3-13)

Due to the sporadic and overlapping nature of noise sources, retrieving a response from the crosscorrelations of noise records generally requires that the records are long enough to allow adequate convergence of the surface wave response while the "crosstalk" in Eq. (2-21) and Eq. (2-22) does not converge coherently. Rather than crosscorrelate very long records however, it is common practice to separate a noise record into panels which are then correlated and subsequently stacked. This is preferred for two reasons; time windowing can be an effective method of selecting only the incoming wavefields and it can be computationally more efficient then using very long records. All noise examples in this thesis use 20 panels of 40 seconds of modeled noise, each containing 4096 samples.

The correlation function from the modelled noise data is shown with the reference monopole trace³ in Figure 3-11 with the corresponding PSF in Figure 3-12. The low frequency (and fastest velocity) component of the retrieved response in the correlation function has a considerable amplitude mismatch with the reference response due to uncanceled noise correlations from sources at the ends of the noise source distribution. These artifacts contaminate the retrieved response because of the limited north-source extent of the noise sources. As well, due to blurring of the virtual source the high frequency content of the retrieved dispersive surface wave appears to be particularly inconsistent. There are noticeable artifacts throughout the record arising from noise source signature correlations. These issues are also clearly evident in the corresponding point spread function shown in Figure 3-12. In the *x*-*t* domain the poor focusing and additional noise correlations demonstrate the imperfections of the virtual source.

In the same manner as described for active sources in the previous subsection, the PSF (Figure 3-12) is used to invert the noise correlation function (Figure 3-11) using the frequency domain matrix inversion stated in Eq. (3-10). The retrieved surface wave Green's function from MDD of the modelled noise data is shown in Figure 3-13 along with the corresponding PSF in Figure 3-14. Figure 3-13 demonstrates the notable improvement of the retrieved response with a nearly exact match to the virtual source reference response. The slight mismatch at the lower frequencies is related to aforementioned issues regarding the limited north-south extend of the noise sources, but nevertheless in comparison to the correlation function (Figure 3-11) the improvement is substantial. Likewise, after MDD the improved spatial and temporal focusing the point spread function is also evident. Along with improved focusing, the side-lobe energy from noise correlations has been appreciably removed in the x-t domain and the resulting f-k spectrum is considerably flatter. Note that the PSF for noise in this example does not collapse to point comparable to that of the active data (Figure 3-10a) because the noise data lacks higher frequencies and consequently the Fresnel zone of the illuminating noise wavefield is considerably larger than that of the active wavefield.

 $^{^{3}}$ Note that the reference trace used for comparison with the noise has the identical bandwidth as the noise data.

As expected, since the two source types produce wavefields that represent different portions of the dispersion curve (Figure 3-2b) due to their differing frequency content, the retrieved surface waves in Figure 3-9 and Figure 3-13 have noticeably different velocities, therefore demonstrating the value of retrieving surface waves from sources generating different frequency wavefields. Although numerically modelled data has been used in demonstrating the retrieval of surface waves from controlled source and noise data in this section, the frequency content of the model is comparable to that of real data used in similar studies (Boulanger et al., 2005; Halliday et al., 2008; Malovichko et al., 2005; Park et al., 2005; Suzuki and Hayashi, 2003).

3-3 Source mixing using MDD

Section 3-1-1 and Section 3-1-2 demonstrated that using interferometry on wavefields with differing frequency content, which in practice can be achieved using active and passive wavefields, leads to the retrieval of different dispersive components of a surface wave Green's function. The retreived response can then be inverted to acquire the medium's dispersion curve for only that particular range of frequencies. As mentioned, several researchers have demonstrated that data from both source types can be acquired for the same medium and combined in a way as to benefit from the full retreived bandwidth. However, in the majority of these approaches only the dispersion curve is acquired and the manner in which the frequency content of each source type is used varies between different studies. For example, Suzuki and Hayashi (2003) combine the dispersion curve of 2-10Hz passive data with that of 5-30Hz active data and give equal weight to each within the region of overlapping bandwidth to retrieve s-wave velocities. In contrast, Yoon and Rix (2004) give preference to the dispersion curve acquired from 1-10Hz noise data by discarding the overlapping frequencies of a dispersion curve from 4-100Hz active data due to discrepancies from near source effects in the latter. Boulanger et al. (2005) average the shared frequency range of the retrieved dispersion curves from 1-40Hz passive data with 3.5-500Hz active data to account for differences between to the two. These examples suggest that the results of combining active and passive data may benefit from the retrieval of a single response in which the common frequencies of the two data types may aid the retrieval process.

3-3-1 Improving bandwidth

Section 2-4 provided the mathematical framework for merging of active and passive source data through MDD. This is achieved by combining the correlations functions (Figure 3-7 and Figure 3-11) as well as the point-spread functions (Figure 3-8 and Figure 3-12) for the two source types using a weighting scheme prior to MDD inversion. The weighting values for the diagonals of $W_{active}(\omega)$ and $W_{noise}(\omega)$ used in the following data merging examples are shown in Figure 3-15. Recall, an Ormsby wavelet was used in modelling active sources with frequency parameters 8-14-18-25Hz and likewise for noise sources with frequency parameters 1-3-6-12Hz. Herein there is a smooth overlapping transition from the ramping-off of the noise data to the ramping-on of the active data at higher frequencies. In the modelled active and passive data the entire bandwidth contributes to the reconstruction of the surface waves. However in practical applications, such as the aforementioned examples, it may be desirable



Figure 3-11: Correlation function from noise data for a virtual source at \mathbf{x}_A to all receivers on \mathbb{L}_B (red) overlain by reference response (black).



(a) PSF in MDD *x*-*t* domain.

(b) PSF in f-k domain.

Figure 3-12: Point spread function from modelled data for all noise sources shown in Figure 3-1. (5 seconds of noise source response shown in Figure 3-3a).



Figure 3-13: Retrieved Green's function from noise data for a virtual source at \mathbf{x}_A to all receivers on \mathbb{L}_B using MDD (red) overlain by reference response (black)



(a) PSF in x-t domain at MDD. (b) PSF in f-k domain.

Figure 3-14: Noise data point spread function in Figure 3-12 after multidimensional deconvolution.

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to tailor the bandwidth of the data by treating the weighting matrices as bandpass filters such that a particular source type is biased at particular frequencies and only a specific range of frequencies are retained for each source type. In doing so, it is also important note that $W_{active}(\omega)$ and $W_{noise}(\omega)$ must provide at least a portion of overlapping contributions in the frequency mid-range, otherwise the resulting correlation function and point-spread function will have a frequency notch that causes the inversion to become unstable.



Figure 3-15: Weighting function used to bias source type while merging data. Noise sources are biased at low frequencies while active data is excluded. Over the region of shared bandwidth the noise data ramps-off and bias is given to the active data.

The merged correlation function is depicted in Figure 3-16 and corresponding PSF is shown in Figure 3-17. Using these as input to Eq. (2-28) and inverting for $\hat{G}_d(\mathbf{x}_B, \mathbf{x}_A, \omega)$ with the inclusion of a stabilization factor in the same manner as stated in Eq. (3-10), the broadband surface wave response is retrieved and is shown in Figure 3-18. It is shown by the corresponding PSF in Figure 3-19 that the retrieved response contains the full bandwidth of the noise and active data. Moreover, is it clear after comparing the post-inversion f-k spectrums of the PSF for the noise (Figure 3-14b) with the mixed source PSF (Figure 3-19b) that the addition of active data compliments the inversion of the noise data, particularly in the region of common bandwidth. Also note that the broadband surface wave response was retrieved from merged source data without requiring knowledge of the source signatures of either source type or the radiation pattern of the noise data.

3-3-2 Infilling illumination gaps

When the raypaths of the wavefields illuminating \mathbf{x}_A and \mathbf{x}_B coincide, the derivative of their phase are said to be stationary (Snieder, 2004). In the source receiver configuration in Figure 3-1 this region of stationary phase is centered in the middle of the source distribution. Within this region the crosscorrelated source responses between \mathbf{x}_A and \mathbf{x}_B constructively interfere and provide the essential contributions to the correlation function. This is demonstrated by the correlation plot in Figure 3-5a in which each trace corresponds to a source location in Figure 3-1. The rays of the waves from sources near the center of the active source distribution follow very close paths to \mathbf{x}_A and \mathbf{x}_B . Consequently these correlations interfere



Figure 3-16: Merged correlation function from active and noise data (red) overlain by reference source response (black).



Figure 3-17: Merged point-spread function from active and noise data.

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Figure 3-18: Retrieved broadband Green's function from merged active and noise data using MDD (red) overlain by reference dipole source response (black).



(a) PSF in x-t domain at MDD.

Figure 3-19: Merged point-spread function from active and noise data after MDD. Note that this PSF contains the combined bandwidths of the PSF's shown in Figure 3-10 and Figure 3-14 with a flatten spectrum.

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(b) PSF in f-k domain.

constructively and provide the necessary contributions to retrieve the virtual response shown in Figure 3-5b. Outside of this region the crosscorrelated source responses destructively interfere. In order for the crosscorrelations from all neighboring sources to completely interfere either constructively or destructively the time difference between the correlations needs to be smaller than half the minimum period, i.e.,

$$dt < \frac{1}{2f_{max}}.\tag{3-14}$$

While the most valuable contributions to the correlation function come from within the region of stationary phase, it is the outside region of non-stationary phase that the derivative of the phase with respect to the source coordinate has a maximum, and hence has the smallest sampling requirement (Ruigrok, 2011). Based on the geometry of the source receiver configuration this occurs when the derivative of $\cos(\arctan(x_i/x_B))$ reaches its maximum, where x_i is the distance from the stationary phase region (which is the center of the source distribution in Figure 3-1) and x_B is the offset of \mathbf{x}_B from the source line. Ruigrok (2011) show that the largest derivative is obtained for $x_i/x_B = 0.71$.

The implications of this is that as the separation between the source line and \mathbf{x}_B decreases, in order to provide adequate illumination within the region of stationary phase while also relying on complete destructive interference from the region outside, the source spacing must also decrease. When employing controlled sources changing the source spacing is generally not a major difficulty. In fact with just a few correctly positioned sources within the Fresnel zone of the stationary phase a correct correlation function can be retrieved (Halliday et al., 2008). Of course, the frequency content and velocity of the wavefield as well as the source-receiver offset is also influencing the sampling criterion. The sampling criterion for an interferometric configuration where \mathbf{x}_A and \mathbf{x}_B are co-linear orthogonal to the source line direction can be stated as (Ruigrok, 2011)

$$dxs < (\tan(\arccos(0.82 - \frac{c(\omega)}{2\omega dxr})) - 0.71)(x_A + dxr),$$
(3-15)

where 0.82 is equal to $\cos(\arctan(x_i/x_B))$ with $x_i/x_B = 0.71$, dxs is the source spacing, x_A is the offset between the source line and \mathbf{x}_A , and dxr is the separation between \mathbf{x}_A and \mathbf{x}_B .

In the alternative situation when passive waves are used, however, satisfying Eq. (3-15) is not as easily assured. Using random noise sources may result in over-illumination in one direction while source gaps may under-illuminate another direction. Wapenaar et al. (2011a) demonstrate that over-illumination from clusters of sources results in an unbalanced pointspread function, which is shown to be particularly evident in the f-k domain, as well as an inaccurate correlation function. In the same way the PSF can be used to evaluate underillumination.

To show the implications of inadequate illumination from sources in interferometry a new data set is modelled using the configuration in Figure 3-20. In the same manner as described in Section 3-1, the recorded response for sequential controlled sources and simultaneously acting noise sources are modelled at the receiver locations. In this model however, the two source types use a 6Hz Ricker wavelet, but again the noise source is convolved with a random series for each source. Several sources are left out of the active source line, therefore creating a



Figure 3-20: Similar model configuration as shown in Figure 3-1, but with a large portion of the active source line missing. Noise sources in this model are randomly positioned within this general region. The receiver configuration is unchanged.

large gap illumination from active sources. Due to these missing sources, using active sources alone is not adequate to satisfy the sampling criterion of Eq. (3-15).

Consequently the retrieved correlation function from active sources derived using Eq. (3-9) and shown in Figure 3-21 has a considerable mismatch with the reference response. Specifically, there are low frequency early arrivals due in part to the uncanceled correlations from sources outside the stationary phase region. More importantly, the inadequate spatial sampling within the region of stationary phase causes significant blurring of the higher frequencies. The gap in illumination causing this mismatch is particularly well evidenced by the asymmetry of the associated point-spread function in both the x-t and f-k domains show in Figure 3-22.

As outlined in Section 2-4 and demonstrated in Section 3-3, noise records may be merged with the active data during interferometry by MDD. In the configuration in Figure 3-20 this provides the possibility to use sources of noise to spatially infill gaps in illumination from active sources. Herein, W_{active} and W_{noise} in Eq. (2-26) and Eq. (2-27) are allowed equal contributions to the correlation function and point-spread function since the two data types have similar bandwidths . However, as previously discussed, in practice this provides an opportunity to tailor the bandwidths of the two data types such that they only contributed shared frequencies to the merged data. While this may restrict the overall bandwidth, unless a particular frequency is adequately sampled for a particular source type according to Eq. (3-15), including frequencies which are present for only one data type will likely result in blurring of the retrieved surface wave.

The merged correlation function is shown in Figure 3-23. As is expected for this model, the point-spread function of the merged data in Figure 3-24 demonstrates that the addition of the noise contributes within the region of missing illumination in the active data. Note, however, that the f-k spectrum is rather erratic due to the character of the merged noise and as well



Figure 3-21: Correlation function from active sources in Figure 3-20 for a virtual source at \mathbf{x}_A to all receivers on \mathbb{L}_B (red) overlain by reference source response (black).



(a) PSF in x-t domain at MDD.

(b) PSF in f-k domain.





Figure 3-23: Merged correlation function from active and noise data from the configuration shown in Figure 3-20 (red) overlain by reference source response (black).



(a) PSF in x-t domain at MDD.

(b) PSF in f-k domain.

Figure 3-24: Merged point-spread function from active and passive data. Note that the addition of the noise has slightly infilled the gap in f-k spectrum as seen in Figure 3-22b due to missing active sources.



Figure 3-25: Retrieved Green's function from merged active and noise data using MDD (red) overlain by reference dipole source response (black).



(a) PSF in x-t domain at MDD.

(b) PSF in f-k domain.



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there is still a considerable mismatch between correlation function and the reference trace because the radiation pattern of the merged sources is non-ideal.

Similar to the example in previous section, Eq. (3-10) is used to invert the merged data. The retrieved surface wave in Figure 3-25 demonstrates significant improvement following multidimensional deconvolution. This is also noted by the considerable spatial and temporal focusing of the point-spread function in x-t domain in Figure 3-26a and the flattening of the spectrum in the f-k domain in Figure 3-26b.

Chapter 4

Repeating noise sources

With the recent realization of the functionality of noise wavefields due to the advent of seismic interferometry, many researchers have begun to explore the use of different sources of noise to benefit from their associated low frequency content. Wavefields from natural sources such as earthquakes (Ruigrok et al., 2010), atmospheric perturbations, and oceanic microseisms (Shapiro and Campillo, 2004), as well as anthropogenic noise arising from a urban environments (Halliday et al., 2008) have been used in interferometry by crosscorrelation. In the majority of these applications the source signatures of the noise sources have implicitly been assumed to be sufficiently uncorrelated such that crosscorrelation of their long record lengths is adequate to retrieve the virtual response without significant contamination from correlating source signature artifacts.

This chapter demonstrates that when noise sources repeat additional correlation terms that give rise to considerable artifacts are introduced, but which can be removed by multidimensional deconvolution. The implications of these repetitions varies in severity depending on the behaviour of the sources. Although somewhat improbable, when many sources repeat with the same periodicity these additional correlations can become coherent and converge during ensemble averaging of the crosscorrelations. Some of these additional terms contribute to the retrieval of the virtual response, however the majority give rise to coherent spurious multiples. When the periodicity varies for different sources the spurious correlations become much more disperse but can still significantly contaminate the retrieved response. The crosscorrelation of long record lengths may be adequate to reduce artifacts when sources repeat with different periodicity, since the periodicity is also shown to be evident in the point spread function, in particular situations these artifacts may be more effectively removed by MDD.

4-1 Correlations of repeating sources

As discussed in Section 2-3, when the signatures of noise sources are potentially correlated the associated correlation function and point spread function contain a double summation that accounts for additional correlation terms. In Eq. (2-21) and Eq. (2-22) these terms are referred

to as "crosstalk" because they are the undesirable byproduct of crosscorrelating noise records in which the noise source signatures are correlated, and therefore they do not contribute to the reconstruction of the virtual response. Applications of interferometry by crosscorrelation typically rely on very long record lengths to ensemble average the crosscorrelations, in which case the virtual response converges while these additional spurious correlations are reduced.

If, however, sources of noise are repetitive within a record additional "crosstalk" terms to those in Eq. (2-21) and Eq. (2-22) are introduced. When noise sources are periodic the noise source signatures can be restated as $N^{(i)}(t+mT^{(i)})$ in which case the crosscorrelation of the noise signatures yields

$$\left\langle \sum_{m} \sum_{n} N^{(i)}(t + mT^{(i)}) * N^{(j)}(-t - nT^{(j)}) \right\rangle = \sum_{m} \sum_{n} S^{(ij)}(t + mT^{(i)} - nT^{(j)}), \quad (4-1)$$

where m and n are the number of periods of the sources which have period lengths $T^{(i)}$ and $T^{(j)}$, respectively.

Consequently, the expressions for the cross-correlation function and point-spread function for periodic noise sources can be stated as

$$C_{noise}(\mathbf{x}_{B}, \mathbf{x}_{A}, t) = \sum_{i} \sum_{j} \sum_{m} \sum_{n} G(\mathbf{x}_{B}, \mathbf{x}_{S}^{(i)}, t) * G^{in}(\mathbf{x}_{A}, \mathbf{x}_{S}^{(j)}, -t) * S^{(ij)}(t + mT^{(i)} - nT^{(j)})$$

$$= \sum_{i} \sum_{j} G(\mathbf{x}_{B}, \mathbf{x}_{S}^{(i)}, t) * G^{in}(\mathbf{x}_{A}, \mathbf{x}_{S}^{(j)}, -t) * S^{(ij)}(t) \qquad (4-2)$$

$$+ \sum_{i} \sum_{j} \sum_{m \ge 1} \sum_{n \ge 1} G(\mathbf{x}_{B}, \mathbf{x}_{S}^{(i)}, t) * G^{in}(\mathbf{x}_{A}, \mathbf{x}_{S}^{(j)}, -t) * S^{(ij)}(t + m(T^{(i)} - T^{(j)}))\delta_{mn}$$

$$+ \sum_{i} \sum_{j} \sum_{m} \sum_{n} G(\mathbf{x}_{B}, \mathbf{x}_{S}^{(i)}, t) * G^{in}(\mathbf{x}_{A}, \mathbf{x}_{S}^{(j)}, -t) * S^{(ij)}(t + mT^{(i)} - nT^{(j)})(1 - \delta_{mn})$$

and

$$\Gamma_{noise}(\mathbf{x}, \mathbf{x}_{A}, t) = \sum_{i} \sum_{j} \sum_{m} \sum_{n} G(\mathbf{x}, \mathbf{x}_{S}^{(i)}, t) * G^{in}(\mathbf{x}, \mathbf{x}_{S}^{(j)}, -t) * S^{(ij)}(t + mT^{(i)} - nT^{(j)}) \\
= \sum_{i} \sum_{j} G(\mathbf{x}, \mathbf{x}_{S}^{(i)}, t) * G^{in}(\mathbf{x}, \mathbf{x}_{S}^{(j)}, -t) * S^{(ij)}(t) \\
+ \sum_{i} \sum_{j} \sum_{m \ge 1} \sum_{n \ge 1} G(\mathbf{x}, \mathbf{x}_{S}^{(i)}, t) * G^{in}(\mathbf{x}, \mathbf{x}_{S}^{(j)}, -t) * S^{(ij)}(t + m(T^{(i)} - T^{(j)})) \delta_{mn} \\
+ \sum_{i} \sum_{j} \sum_{m} \sum_{n} G(\mathbf{x}, \mathbf{x}_{S}^{(i)}, t) * G^{in}(\mathbf{x}, \mathbf{x}_{S}^{(j)}, -t) * S^{(ij)}(t + mT^{(i)} - nT^{(j)}) (1 - \delta_{mn}),$$

respectively. In Eq. (4-2) and Eq. (4-3), when sources do not repeat, i.e., m = n = 0, the second and third terms are equal to zero and only the first term remains, in which case the resulting equations are identical to Eq. (2-17) and Eq. (2-18), respectively, as should be expected. The second term in the equations results from correlations from the same period of repetition of the sources, i.e., m = n, while the third term arises from correlations from one period of one source with a different period of the another source (i.e., $m \neq n$). The latter term can be always be considered as undesirable "crosstalk". The second term, however, may give rise to "crosstalk" but may also contribute to the reconstruction of the virtual response.

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To provide greater insight into the contributions of the additional terms in Eq. (4-2) and Eq. (4-3) the noise sources are considered to have identical periodicity, i.e., $T^{(i)} = T^{(j)}$, giving

$$C_{noise}(\mathbf{x}_{B}, \mathbf{x}_{A}, t) = \sum_{i} \sum_{j} G(\mathbf{x}_{B}, \mathbf{x}_{S}^{(i)}, t) * G^{in}(\mathbf{x}_{A}, \mathbf{x}_{S}^{(j)}, -t) * S^{(ij)}(t)$$

$$+ \sum_{i} \sum_{j} \sum_{m \ge 1} \sum_{n \ge 1} G(\mathbf{x}_{B}, \mathbf{x}_{S}^{(i)}, t) * G^{in}(\mathbf{x}_{A}, \mathbf{x}_{S}^{(j)}, -t) * S^{(ij)}(t) \delta_{mn}$$

$$+ \sum_{i} \sum_{j} \sum_{m} \sum_{n} G(\mathbf{x}_{B}, \mathbf{x}_{S}^{(i)}, t) * G^{in}(\mathbf{x}_{A}, \mathbf{x}_{S}^{(j)}, -t) * S^{(ij)}(t + T^{(i)}(m - n))(1 - \delta_{mn}),$$

$$(4-4)$$

and

$$\Gamma_{noise}(\mathbf{x}, \mathbf{x}_{A}, t) = \sum_{i} \sum_{j} G(\mathbf{x}, \mathbf{x}_{S}^{(i)}, t) * G^{in}(\mathbf{x}, \mathbf{x}_{S}^{(j)}, -t) * S^{(ij)}(t)$$

$$+ \sum_{i} \sum_{j} \sum_{m \ge 1} \sum_{n \ge 1} G(\mathbf{x}, \mathbf{x}_{S}^{(i)}, t) * G^{in}(\mathbf{x}, \mathbf{x}_{S}^{(j)}, -t) * S^{(ij)}(t) \delta_{mn}$$

$$+ \sum_{i} \sum_{j} \sum_{m} \sum_{n} G(\mathbf{x}, \mathbf{x}_{S}^{(i)}, t) * G^{in}(\mathbf{x}, \mathbf{x}_{S}^{(j)}, -t) * S^{(ij)}(t + T^{(i)}(m - n))(1 - \delta_{mn}),$$
(4-5)

respectively. The noise source correlation $S^{(ij)}(t + mT^{(i)} - nT^{(j)})$ in the second terms of Eq. (4-4) and Eq. (4-5) reduces to $S^{(i)}(t)$ to give an expression which is identical to the first terms in Eq. (4-2) and Eq. (4-3), respectively. Hence, for this situation the same periods of noise (i.e., m = n) contribute to reconstruction of the response but also additional "crosstalk" terms akin to Eq. (2-21) and Eq. (2-22) are introducted. In the third term however, the period length $T^{(i)}$ remains in the source signature, i.e., $S^{(ij)}(t + T^{(i)}(m - n))$, and hence contributes only "crosstalk" which takes the form of multiplies of the desired response with a periodicity dependent on the number of periods and period length of the noise sources.

4-2 Multidimensional deconvolution

To demonstrate the implications of repeating sources of noise in applications of interferometry a new model using only noise sources in Figure 3-1 is created, but with all sources repeating every 1.5 seconds. In the same manner as described in Section 3-2-2 the correlation function and point spread function are derived from the records of the repeating noise sources and are shown in Figure 4-1 and Figure 4-2, respectively. Similar to the example in the aforementioned section the surface wave response is evident in the correlation function but the mismatch to the reference response is considerable. Most noticeably, due to the repetition of the noise sources the retrieved response in the correlation function (Figure 4-1) also repeats with the same period of 1.5 seconds. This coherently repeating "crosstalk" is also quantified within the point-spread function as additional focus points away from t = 0 at intervals of the same period. Additionally, the "crosstalk" due to correlations of the noise source signature is also evident in the corresponding point spread function (Figure 4-2).

Since the "crosstalk" attributed to both the source repetitions and correlating source signatures contaminates the correlation function as well as in point spread function, multidimensional deconvolution may be used to retrieve a Green's function from the correlation function

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without these "crosstalk" artifacts. This is demonstrated by the corresponding surface wave Green's function retrieved from MDD in Figure 4-3 in which the periodicity has been largely removed. Similarly, the associated point spread function is also void of these additional correlations and like the earlier examples the focusing has been improved.

In practice it is unlikely that different sources of noise will have identical periodicity (i.e., $T^{(i)} = T^{(j)}$) as modelled here, however the example is useful for demonstrating the implications of repeating noise sources in this extraordinary case. In the more realistic situation where the periodicity of the noise sources vary, i.e., $T^{(i)} \neq T^{(j)}$ as expressed in Eq. (4-2) and Eq. (4-3), the "crosstalk" does not take the form of coherent repetitions of the primary response in the correlation and point spread function, but the artifacts may still be significant.

Figure 4-5 shows the correlation function from modelled noise data using the same configuration as for the previous model but with period lengths varying between 0.5 to 2 seconds for each of the repeating sources. Evidently the distinct periodicity noted in the previous model is nonexistent in the correlation function (Figure 4-5) and point spread function (Figure 4-6), however the "crosstalk" is still noticeable.

The surface wave Green's function retrieved using multidimensional deconvolution is shown in Figure 4-7. The "crosstalk" has been appreciably suppressed and the focus has been improved as demonstrated by the corresponding point spread function in Figure 4-8. However, in comparison to the to Green's function retrieved from noise sources with identical periodicity (Figure 4-3) the effectiveness of the MDD is slightly degraded. Small amounts of "crosstalk" still litter the retrieved response which may a result of MDD having greater difficulties accounting for spatial correlations whereas the "crosstalk" in previous example were primarily temporal artifacts.

Several similarities exist between the demonstration of repeating noise in interferometry by MDD in this section and the demonstration of simultaneous-controlled-source deblending by MDD given by Wapenaar et al. (2011c). In their derivations of the correlation function and point spread function they identify "crosstalk" between the responses of different sources within a source group. Similar to the periodicity of the noise data posed here, they show that when the ignition times for the controlled sources do not vary the correlation function and point spread function are imprinted by coherent repeating "crosstalk". They subsequently show that when source ignition times are randomized during simultaneous source acquisition the "crosstalk" becomes diffuse. In both of their examples MDD is shown to considerably reduce the "crosstalk", but they note that the effectiveness of the deconvolution is slightly better for the situation where the source ignition times are randomized. This may also suggest difficulties in addressing spurious spatial correlations because, while their point spread function showed repeating focus points within the PSF similar to the example given here, the additional focus points in their PSF are spatially shifted.



Figure 4-1: Correlation function from noise sources that repeat with a period of 1.5 seconds (red) overlain by reference source response (black). Note that the retrieved response repeats with the same periodicity as the noise sources due to the convergence of the "*crosstalk*".



Figure 4-2: Point spread function showing coherent "*crosstalk*" due to the repeating noise sources as also evident in Figure 4-1.



Figure 4-3: Green's function retrieved from noise sources that repeat with a period of 1.5 seconds (red) overlain by reference dipole source response (black). Note that the coherent "*crosstalk*" seen in Figure 4-1 has been appreciably removed by MDD.



Figure 4-4: Point-spread function from Figure 4-2 after MDD. As noted in Figure 4-3 the periodic coherent "*crosstalk*" has been reduced.

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Figure 4-5: Correlation function from noise sources that repeat with varying periodicity (red) overlain by reference response (black). Note that the "crosstalk" is much more dispersed and does not exhibit the same coherency has seen in Figure 4-1.



Figure 4-6: Point spread function resulting showing dispersive "crosstalk" due to repeating sources with varying periodicity.



Figure 4-7: Retrieved Green's function from correlation function in Figure 4-5 using MDD (red) overlain by reference dipole source response (black).



Figure 4-8: Point spread function from Figure 4-6 after MDD.

Chapter 5

Conclusions

A method of merging active- and passive-source data using interferometry by multidimensional deconvolution for the purpose of surface wave Green's function retrieval was proposed. While MDD has been shown to accurately retrieve a surface wave Green's function from active and passive data separately, the similar form of the two inversions provides a natural and advantageous opportunity to merge the two data-types. Owing the the fact that the source signatures of the wavefields contributing to the reconstruction of a surface-wave response are disregarded during the multidimensional deconvolution, by merging data in this process a single Green's function which benefits from the combined characteristics of both data types can be retrieved. Hence, the common processing challenge of designing a matching filter or a joint method of secondary inversion to combine the different data types is not required and is instead replaced with a data-type weighting function which may be used to bias a particular data type for the Green's function retrieval.

Numerical modelling has demonstrated that combining active and passive data using MDD can provide numerous advantages to the reconstructed response. When the two data types have differing bandwidths, which is typical of active and passive wavefields, a single Green's function with the broad bandwidth of the combined data is retrieved. This Green's function can then be convolved with a favorable wavelet and subsequently employed for its intended purpose. Additionally, combining data in this manner has been shown to in fact benefit the retrieval of the Green's function, not only due to the improvements gained from MDD, but also owing to the additional illumination provided by the supplementary sources. Therefore, when one source-type is insufficient to provide adequate spatial illumination, the additional illumination provided from an additional (active or passive) sources may be used to infill the gaps when the frequency content of the two are comparable.

This approach benefits considerably from the inherent functionality of interferometry by multidimensional deconvolution. When combining data associated with different source types using this approach, the relative amplitudes of the active and passive sources imposes no complications because the multidimensional deconvolution compensates for anisotropic illumination; hence, source amplitudes, densities and signatures may vary. The point spread function required for MDD has also been shown to be a useful illumination diagnostic tool. In the numerical examples the inadequacies of the illumination from the source gaps was clearly evident in the PSF and after multidimensional deconvolution the PSF demonstrated that the illumination was sufficiently compensated for by the additional data from the other source-type.

Multidimensional deconvolution was also shown to be capable of removing "crosstalk" due to repeating sources of noise. In the hypothetical situation where all sources of noise repeated with identical periodicity the introduced "crosstalk" was coherent in the form of multiples of the virtual response in the correlation function. Since this "crosstalk" was also accounted for in the point spread function in the form of addition focus points at repeating intervals away from t = 0, they were effectively removed by MDD in the retrieved Green's function. In the situation where sources had variable periodicity the "crosstalk" was much more diffuse in the correlation function, however the effectiveness of the MDD to remove these artifacts was limited. The cause of this inefficiency is likely due to difficulties of the multidimensional deconvolution to adequately resolve the spatial crosscorrelation, since temporal artifacts have been shown to be well resolved.

5-1 Applications

Demonstrations of combining active and passive data using MDD in this thesis have focused on applications to surface wave retrieval. While the vast majority of seismic studies would benefit from broader bandwidth data, given the comparable bandwidths of active and passive surface waves acquired in engineering and/or environmental studies, they are likely to be the primary applications (Halliday et al., 2008; Malovichko et al., 2005; Park et al., 2005). For example, including active source data with noise data acquired from a passing train could be useful for assessing the integrity of the railway embankment and local subsurface (Ditzel, 2003). Alternatively, noise generated from a river may be combined with active source data to characterize the river bed, similar to the investigation by Ivanov et al. (2000). Essentially in all surface wave investigations where the directionally of the wavefield relative to the receiver configuration can be assumed to be incoming this method may potentially prove beneficial.

While the emphasis has been on surface wave retrieval in this thesis, these demonstrations have shown that active and passive data can be combined in a convenient and natural manner using MDD. Since alternative MDD configurations exist for passive and active body-waves (Wapenaar et al., 2011b), this study may provide insight into potential applications in more complex arrangements. (Additional configurations for applications to direct arrivals is proposed in Section 5-3.)

5-2 Limitations of method

The examples presented in this thesis were conceived in order to demonstrate the capabilities of interferometry by MDD to combine active and passive data in a useful way. In practice the feasibility and usefulness of this method is, of course, governed by the physical positioning and characteristics of the available sources. While interferometry by multidimensional deconvolution can compensate for over-illumination from in particular directions, as demonstrated inadequate illumination due to significant spatial gaps in the source distribution will introduce errors in the retrieved Green's function. When combining active and passive data for the purpose of broadening the bandwidth of the virtual response, the spatial sampling criteria must be satisfied for both active and passive source-types over all frequencies. More precisely, since MDD is carried out for each discrete frequency, inadequate illumination at a particular frequency and wavenumber will introduce blurring; hence, the bandwidth of the merged active and passive data may only be used if the illumination for each frequency is adequate. Addressing the same issue, when using one data type to infill spatial gaps in illumination in another data type it is essential that the bandwidths of the two data types are comparable since the gaps are frequency dependent as well.

It is also important to note before combining active and passive data using multidimensional deconvolution that there should be a least a portion of shared bandwidth between to the data types. As discussed, MDD uses the point spread function to invert the correlation function for each discrete frequency. If, however, there is a void range of frequencies between the two data types the stability of the inversion will be compromised.

Fortunately in practice the suitability of the active and passive data in relation to the aforementioned issues can be assessed by comparing the f-k spectra of the point spread functions of the respective data types prior to merging. Additionally, in the examples in this thesis the f-k spectrum of the point spread function after MDD was considerably flatter than before. In numerical modelling this can be achieved fairly easily by using a small stabilization factor. In practice however, real data may have considerably more notches in the frequency spectrum that may produce instabilities, and consequently the spectrum of the retrieved Green's function and point spread function may be more variable.

One of the more challenging limitations of the proposed method is an implication resulting from the underlying assumption which is required to simply the convolution-type Green's function representation (Eq. (2-1)), namely, the assumption that all waves are incoming to the receivers defining \mathbb{S}_{rec} , or conversely that no waves are outgoing. In addition to imposing the restriction that sources must be on one side of the receiver configuration, this limitation also means that scatterers cannot be present on the opposite side unless wavefields can be decomposed. Some researchers have successfully used time-windowing to effectively avoid crosscorrelating records in which outgoing waves are recorded, however this may not be possible if the outgoing field is fairly continuous. A proposed alternative to this approach is to use f-k filtering which is likely to be more promising when the cross-line direction is adequately sampled if a grid of receivers is used.

5-3 Future work

The theoretical configuration for interferometry by multidimensional deconvolution for surface wave retrieval requires that all sources are on one side of the receiver configuration such that all illuminating wavefields are incoming from the virtual source to receiver direction. From this inverse problem the surface wave Green's function is retrieved between a virtual source at \mathbf{x}_A and a receiver at \mathbf{x}_B . As discussed in the previous section this conforms the source locations to a restricted region and demands in and outgoing wavefields are separated with the presence of scatterers on the side opposite the sources. However this problem also raises an important question - **Can sources in the MDD configuration be relocated to retrieve (or benefit the retrieval of) the Green's function from the same receiver configuration?**

To address this problem Joost van der Neut and I propose two additional source locations for the receiver configuration used for MDD. Similar to the original configuration, forward problems are based on the correlation-type Green's function representation while the inverse problems are derived from the Green's function representation of the convolution-type.

To outline this proposal two additional source locations are put forth in the following figures. For completeness the configuration used in this thesis is also included. In each of the three configurations the general forward and inverse problems are stated.

MDD configuration



Figure 5-1: MDD configuration.

Forward

$$G(\mathbf{x}_B, \mathbf{x}_A, t) * S(t) \approx \int_{\mathbb{S}_{src}} u(\mathbf{x}_B, \mathbf{x}_S^{(i)}, t) * u(\mathbf{x}, \mathbf{x}_S^{(i)}, -t) \, d\mathbf{x}_S \,.$$
(5-1)

Inverse

$$u(\mathbf{x}_B, \mathbf{x}_S^{(i)}, t) = \int_{\mathbb{S}_{rec}} \bar{G}_d(\mathbf{x}_B, \mathbf{x}, t) * u^{in}(\mathbf{x}, \mathbf{x}_S^{(i)}, t) \, d\mathbf{x}_A \,.$$
(5-2)
Sources between \mathbf{x}_{A} and \mathbf{x}_{B}



Figure 5-2: Sources located between x_A and x_B .

Forward

$$G(\mathbf{x}_B, \mathbf{x}_A, t) * S(t) \approx \int_{\mathbb{S}_{src}} u(\mathbf{x}_A, \mathbf{x}_S^{(i)}, t) * u^{in}(\mathbf{x}_B, \mathbf{x}_S^{(i)}, t) \, d\mathbf{x}_S \,.$$
(5-3)

Inverse

$$u(\mathbf{x}_B, \mathbf{x}_S^{(i)}, t) = \int_{\mathbb{S}_{rec}} \bar{G}_d(\mathbf{x}_B, \mathbf{x}, t) * u^{in}(\mathbf{x}, \mathbf{x}_S^{(i)}, t) \, d\mathbf{x}_A \,.$$
(5-4)

Sources incoming from \mathbf{x}_{B} and \mathbf{x}_{A}





Forward

$$G(\mathbf{x}_B, \mathbf{x}_A, t) * S(t) \approx \int_{\mathbb{S}_{src}} u(\mathbf{x}_A, \mathbf{x}_S^{(i)}, t) * u^{in}(\mathbf{x}_B, \mathbf{x}_S^{(i)}, -t) \, d\mathbf{x}_S \,.$$
(5-5)

Inverse

$$u(\mathbf{x}_B, \mathbf{x}_S^{(i)}, t) = \int_{\mathbb{S}_{rec}} \bar{G}_d(\mathbf{x}_B, \mathbf{x}, -t) * u^{in}(\mathbf{x}, \mathbf{x}_S^{(i)}, t) \, d\mathbf{x}_A \,.$$
(5-6)

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In this thesis it has been shown that active and passive surface waves can be combined in a practical manner as to potentially improve the over all bandwidth of the surface wave Green's function and/or provide infill to gaps in illumination required for the Green's function retrieval. We hypothesis that these inverse problems for the different source arrangements can be merged in a similar manner to that described in this thesis as way of providing addition illumination (or bandwidth) to the retrieval of the Green's function.

Bibliography

- Aki, K. and Richards, P. G. *Quantitative Seismology*. University Science Books, 2002, 2nd ed.
- Bakulin, A. and Calvert, R. "The virtual source method: Theory and case study." *Geophysics* 71.4 (2006): P.SI139–SI150.
- Berkhout, A. J. Seismic migration: Imaging of acoustic energy by wave field extrapolation. Elsevier Science Publ. Co., Inc., 1982.
- Berkhout, A.J. Applied Seismic Wave Theory, vol. 1. Elsevier Science Publishers B.V., 1987.
- Boulanger, R. W., Dewwolkar, M., Gucunski, N., Juang, C., Kalinski, M., Kramer, S., Manzario, M., and Pauschke, J., eds. Combining Active- and Passive-Source Measurements to Profile Shear Wave Velocities for Seismic Microzonation, 133. American Society of Civil Engineers, Reston, VA: Geotechnical Special Publication, 2005.
- Claerbout, J. "Synthesis of a layered medium from its acoustic transmission response." Geophysics 33.2 (1968): P. 264–269.
- Curtis, A. and Halliday, D. "Directional balancing for seismic and general wavefield interferometry." *Geophysics* 75.1 (2010): SA1–SA14.
- de Ridder, S. and Biondi, B. "Low frequency passive seismic interferometry for land data." SEG Denver 2010 Annual Meeting .
- Debayle, E. and Kennett, B.L.N. "Anisotropy in the Australasian upper mantle from Love and Rayleigh waveform inversion." *Earth and Planetary Science Letters* 184: 339–351.
- Derode, A., Larose, E., Campillo, M., and Fink, M. "How to estimate the Green's function of a heterogeneous medium between two passive sensors? Application to acoustic waves." *Applied Physics Letters* 85.14 (2003): 3054–3056.
- Ditzel, A. Train-induced ground vibrations: modeling and experiments. Ph.D. thesis, Technische Universiteit Delft, 2003.

Master of Science Thesis

- Draganov, D., Campman, X., Thorbecke, J., and Wapenaar, K. "Reflection images from ambient seismic noise." *Geophysics* 75.5 (2009): A63–A67.
- Duguid, C. Applied Seismic Interferometry: Directional Balancing Investigation and a Source-Receiver Data Example. Master's thesis, The University of Edinburgh, 2010.
- Foti, S., Comina, C., and Boiero, D. "Reliability of combined active and passive surface wave methods." *Rivista Italiana Di Geotecnica* 41(2): 39–47.
- Froment, B., Campillo, M., Roux, P., Gouédard, P., Verdel, A., and Weaver, R. L. "Estimation of the effect of nonisotropically distributed energy on the apparent arrival time in correlations." *Geophysics* 74.5 (2010): P.SA85–SA93.
- Gerstoft, P., Sabra, K. G., Roux, P., Kuperman, W. A., and Fehler, M. C. "Green's functions extraction and surface-wave tomography from microseisms in southern California." *Geophysics* 71.4 (2006): P.S123–S131.
- Halliday, D. "Adaptive interferometry for ground-roll suppression." *The Leading Edge* : 532–537.
- Halliday, D., Curtis, A., and Kragh, E. "Seismic surface waves in a suburban environment: Active and passive interferometric methods." *The Leading Edge* 27: 210–218.
- Hebeler, G. L. and Rix, G.J. "Site characterization in Shelby County, Tennessee using advanced surface wave methods." Tech. rep., Georgia Institue of Technology, 2001.
- Ivanov, J., Park, C. B., Miller, R. D., Xia, J., Hunter, J. A., Good, R. L., and Burns, R. A. "Joint analysis of surface-wave and refraction events from river-bottom sediments." SEG Expanded Abstracts.
- Kimman, W.P. and Trampert, J. "Approximations in seismic interferometry and their effects on surface waves." *Geophysical Journal International* .182 (2010): 461–476.
- Malovichko, A. A., Anderson, N. L., Malovichko, D. A., Shylakov, D. Y., and Butirin, P. G. "Active-passive Array Surface Wave Inversion and Comparison to Borehole Logs in Southeast Missouri." *Journal of Environmental and Engineering Geophysics* 10.3 (2005): 243–250.
- Park, C., Miller, R., Laflen, D., Neb, C., Ivanov, J., Bennett, B., and Huggins, R. "Imaging dispersion curves of passive surface waves." SEG Expanded Abstracts 23: 1357–1360.
- Park, C.B., Miller, R.D., Ryder, N., Xia, J., and Ivanov, J. "Combined Use of Active and Passive Surface Waves." *Journal of Environmental and Engineering Geophysics* 10.3 (2005): 323–334.
- Roy, S., Stewart, R. R., and Dulaijan, K. Al. "S-wave velocity and statics from ground-roll inversion." *The Leading Edge* 29.10 (2010): 1250–1257.
- Ruigrok, E. "Derivation of a sampling criterion for surface-wave sources which are on a line perpendicular to the receiver array.", 2011. Unpublished.
- Ruigrok, E., Campman, X., and Wapenaar, K. "High-resolution lithospheric imaging with seismic interferometry." *Geophysical Journal International* 183: 339–357.

- Shapiro, N. M. and Campillo, M. "Emergence of broadband Rayleigh waves from correlation of the ambient seismic noise." *Geophysical Research Letters* 31.L07614 (2004).
- Snieder, R. "Extracting the Green's function from the correlation of coda waves: A derivation based on stationary phase." *Physical Review E* 69.046610 (2004).
- Snieder, R., Wapenaar, K., and Larner, K. "Spurious multiples in seismic interferometry of primaries." *Geophysics* 71.4 (2006): SI111–SI124.
- Suzuki, H. and Hayashi, K. "Shallow S-wave velocity sounding using the microtremors array measurements and the surface wave method." Symposium on Application of Geophysics to Engineering and Environmental Problems (SAGEEP) : (2003) 1371–1376.
- Thorbecke, J. and Wapenaar, K. "Analysis of spurious events in seismic interferometry." SEG Las Vegas 2008 Annual Meeting. SEG, SEG, 2008, 1415–1420.
- van der Neut, J. and Bakulin, A. "Estimating and correcting the amplitude radiation pattern of a virtual source." *Geophysics* 74.2 (2009): P.SI27–SI36.
- van der Neut, J., Thorbecke, J., Mehta, K., Slob, E., and Wapenaar, K. "Controlled-source interferometric redatuming by crosscorrelation and multidimensional deconvolution in elastic media." *Geophysics* 74.4 (2011): P.1–14.
- van Manen, D., Robertson, J.O.A., and Curtis, A. "Modelling of Wave Propagation in Inhomogeneous Media." *Physical Review Letters* 94.164301 (2005).
- Wapenaar, K. "Retrieving the Elastodynamic Green's Function of and Arbitrary Inhomogeneous Medium by Cross Correlation." *Physical Review Letters* 93.254301 (2004).
- Wapenaar, K. and Fokkema, J. "Green's function representation for seismic interferometry." *Geophysics* 71.4 (2006): S133–S146.
- Wapenaar, K., Fokkema, J., and Snieder, R. "Retrieving the Green's function in an open system by cross correlation: A comparison of approaches (L)." *Journal of the Acoustical Society of America* 118: 2783–2786.
- Wapenaar, K., Ruigrok, E., van der Neut, J., and Draganov, D. "Improved surface-wave retrieval from ambient seismic noise by mult-dimensional deconvolution." *Geophysical Re*search Letters 38.L01313 (2011a).
- Wapenaar, K., Slob, E., Snieder, R., and Curtis, A. "Tutorial on seismic interferometry. Part 2: Underlying theory and new advances." *Geophysics* 75.5 (2010): 75A211–75A227.
- Wapenaar, K., van der Neut, J., and Ruigrok, E. "Passive seismic interferometry by multidimensional deconvolution." *Geophysics* 73.6 (2008): A51–A56.
- Wapenaar, K., van der Neut, J., Ruigrok, E., Draganov, D., Hunziker, J., Slob, E., Thorbecke, J., and Snieder, R. "Seismic interferometry by crosscorrelation and by multi-dimensional deconvolution: a systematic comparison." *Geophysical Journal International* 185: 001–024.
- Wapenaar, K., van der Neut, J., and Thorbecke, J. "On the relation between seismic interferometry and the simultaneous-source method." *Geophys. Prosp. (Submitted)*.

Wathelet, M. Array recording of ambient vibrations: surface-wave inversion. Ph.D. thesis, Université de Liège - Faculté des Science Appliquées, 2005.

- Xia, J., Miller, R.D., Ivanov, J., Tian, G., and Chen, C. "Utilization of high-frequency Rayleigh waves in near-surface geophysics." *The Leading Edge* : 753–759.
- Yoon, S. and Rix, G.J. "Combined active-passive surface wave measurements for near surface site characterization." SAGEEP 2004, San Antonio, USA. 2004, pp. 1556–1564.