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Publication date
2019

Document Version
Accepted author manuscript

Published in
Proceedings of the FIB Symposium 2019

Citation (APA)
Abad, B. F., Lantsoght, E., & Yang, Y. (2019). Shear capacity of steel fibre reinforced concrete beams. In W. Derkowski, P. Krajewski, P. Gwozdziewicz, M. Pantak, & L. Hojdys (Eds.), *Proceedings of the FIB Symposium 2019: Concrete - Innovations in Materials, Design and Structures* (pp. 1710-1717). International Federation for Structural Concrete (fib).

SHEAR CAPACITY OF STEEL FIBRE REINFORCED CONCRETE BEAMS

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Abstract

The Critical Shear Displacement Theory (CSDT) was developed to determine the shear capacity of reinforced concrete beams based on the different shear-carrying mechanisms (concrete in the compression zone, aggregate interlock, and dowel action). This research aims at extending the CSDT to Steel Fibre Reinforced Concrete (SFRC) by adding the contribution of steel fibres. The model extension was developed based on formulations for the contribution of steel fibres to the shear capacity from the literature. With this extension to the CSDT, the shear strength of steel fibre reinforced concrete beams without stirrups could be estimated. An extensive database is developed from the literature in order to evaluate, compare, and analyse the shear capacity of SFRC beams. The analysis indicates that two models are capable of predicting the shear strength of SFRC beams with reasonable accuracy. The mean, standard deviation, and coefficient of variation are 0.9, 0.28, 0.31 and 1.1, 0.33 and 0.30 respectively. The main geometric variables of the steel fibres that influenced the shear strength are the length, diameter, and fibre type (hooked, crimped, and straight). From the comparison between the results in the database and the proposed extensions to the CSDT it is found that the critical shear displacement of Δ_{cr} = 0.025 mm, gives reasonable results for SFRC. As such, this proposed method can be used to estimate the shear strength of SDRC based on a mechanical model.

Keywords: Beam; Critical Shear Displacement; Database; Shear; Steel Fibre Reinforced Concrete.

1. Introduction

The incorporation of fibres randomly and discontinuously in the concrete matrix changes the brittle response of concrete in tension-driven failure modes to a more ductile response. Adding small proportions of fibres modifies mechanical responses such as the modulus of rupture, shear strength, ductility, and energy absorption (Narayanam & Darwish 1985), resulting in increased toughness and ductility. Since shear failure is a tension-driven failure mode, fibres can be used to increase the tensile strength and shear strength. The effectiveness of fibres depends on variables such as the fibre/matrix interface, maximum aggregate size, geometry of the fibres, tensile strength, size and percentage of steel fibre (Hameed, et al. 2009). Concrete with steel fibres has been used in different civil engineering applications; for instance, pavements, tunnels, foundations, slabs and prefabricated panels used in bridges (Wang, et al. 1987; Meda, et al. 2005; Bentur 2007; Aoude, et al. 2012; Swamy & Bahia 1989; Sahoo et al., 2012). However, the use of steel fibres in critical structural elements such as beams has been limited; the reluctance of the building industry towards the use of fibres in structural applications is attributed to the fact that not all codes include provisions for steel fibre concrete, as well as to the large scatter on test results when compared to the existing equations, especially for the shear capacity (Lee, et al. 2013). To have a better understanding of the shear capacity of steel fibre reinforced concrete

(SFRC) beams, mechanical models are necessary. For reinforced concrete beams, the Critical Shear Displacement Theory (CSDT) (Yang 2014) describes the shear capacity based on the mechanisms of shear resistance. In this paper, the CSDT is extended with the participation of the steel fibres as an additional shear-resisting mechanism. As such, the proposed method gives a mechanical model that explains how SFRC beams carry shear.

2. Determination of shear capacity of SFRC

2.1. Mechanisms of shear transfer in reinforced concrete beams

Four mechanisms of shear resistance are activated after cracking: the action of the concrete in the compression zone, aggregate interlock, dowel action, and residual tension across the crack Figure 1 shows these mechanisms. In SFRC beams, the steel fibres provide additional tensile capacity across the crack. In the compression zone of the cross-section, the uncracked concrete provides shear resistance. The capacity from residual tension comes from the aggregates bridging the crack so that tension can be carried across the crack when its width is small in the tension-softening zone (Pruijssers 1986). Aggregate interlock results from the contact forces between the aggregates bridging the crack (Walraven 1981). Dowel action is the resistance of the flexural steel to shearing (Venizelos and Tassios 1986). The Critical Shear Displacement Theory calculates the shear capacity of a reinforced concrete beam as the sum of the shear resistance of the concrete in the compression zone, the dowel action, and the aggregate interlock capacity (Yang 2014).

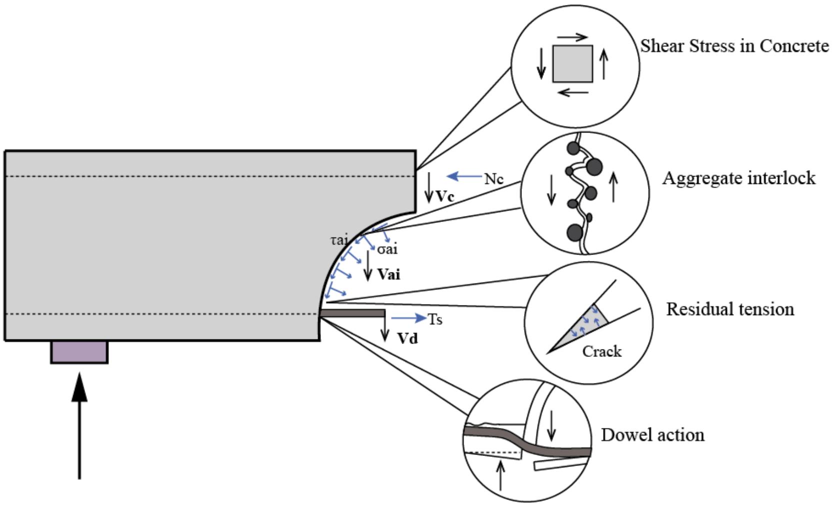

Figure 1: Overview of shear-resisting mechanisms on crack

2.2. Methods to determine the fibre contribution

The effectiveness of steel fibres to resist tension across a diagonal tension crack is a function of the fibre distribution, with fibres perpendicular to the crack as most effective. The distribution is assumed along the length of the critical diagonal cracking. The resistance provided by the fibres depends on the number of fibres per unit area, the orientation and the pull-out force per fibre.

Many equations based on experimentation and theoretical analysis have been proposed to calculate the contribution of the steel fibres to the shear capacity. The equations are usually divided into two categories.

The first category considers that steel fibres directly influence the mechanical properties of concrete. The influence of fibres is determined by the tensile strength or by the modulus of rupture; in other words, the physical characteristics of the fibres are not considered separately.

The second category considers the shear strength properties that steel fibres provide compared to the strength of conventional concrete without stirrups. For these approaches, the contribution of the steel fibres to the shear capacity are generally expressed the fibre factor, F. derived by Narayanan & Palajian (1984).

$$F = \left(\frac{L}{D}\right).V_f.d_f \tag{1}$$

where L is the fibre length, D is the fibre diameter, L/D is the fibre aspect ratio, V_f is percentage of fibre in the volume of mixture and d_f is the bonding factor, which varies from 0.5 to 1.0 depending on the fibre type and concrete matrix. Table 1 summarizes equations from the literature that express the contribution of steel fibres to the shear capacity.

Table 1. Expressions for contribution of steel fibres to shear capacity from the literature.

Reference	**Equation**	**Comment**
Modification of Singh and Jain (2014) of the equation proposed by Dinh, et al. (2011). Model 1	$V_{FRC} = T_f cos\alpha = \left[\sigma_{fu} b(d-c)\cot(\alpha)\right]$ $\sigma_{fu} = N.f$ $N = 0.5\frac{V_f}{\pi r_f^2}$ $f = \tau\pi D_f d_f \frac{l_f}{4}$ $\tau = 0.85\sqrt{f'c}$ (hooked-end) $\tau = 0.75\sqrt{f'c}$ (crimped) $V_{FRC} = \left[0.5\tau D_f V_f \frac{l_f}{d_f} b(d-c)\cot(\alpha)\right]$	For the angle a α the range is between 25° to 36° However, the use of a 30° is recommended. Likewise, the angle value of 29° is used by Dinh (2011).
Mansur (1972). Model 2	$V_{sf} = \sigma_{tu}.b.d$ $\sigma_{tu} = 0.41.\tau.F$ $F = \frac{d_f V_f L_f}{D_f}$ $\tau = 0.68\sqrt{f'c}$	d_f = 1 for hooked steel fibres.
Lee, et al. (2017). Model 3 & Model 4	$V_{sf} = 0.41.V_f.\tau_{max}.\frac{L_f}{D_f}.\rho_w.b_w.(d_s - c).cot\theta$	$cot\theta$ is taken as one (1) since the angle was 45°.

2.3. Methods to determine the fibre contribution

To determine the shear capacity of SFRC, the capacity of the steel fibres (as given in Table 1) can be added as an additional shear-carrying mechanism to the procedures from the CSDT. The result is 4 possible models, as indicated in Table 1. The first formulation model describes the contribution of the steel fibres to the shear capacity with the modification of Singh and Jain (2014) of the equation proposed by Dinh, et al. (2011). T_f is the result of the tension of the fibre along the diagonal crack, d is effective height of the beam, b is width of the beam, σ_{fu} is tensile stress resisted by the fibres per unit area in the inclined crack, N is number of fibres along a unit area of the inclined crack (estimated with the Hannant equation (1978)), f is average pull-out force per fibre, τ is the average bond stress d_f (=1 for hooked fibres and 0.75 for crimped fibres, l_f is the fibre length and D_f is the fibre diameter. The second model from Mansur (1972) is a function of td the effective depth, b the width, σ_{f_u} the tensile stress resisted by the fibres per unit area in the inclined crack, l_f the fibre length, D_f the fibre diameter, and V_f the fibre volume fraction. For the third and fourth model the expression of the shear resistance by fibres from Lee et al. (2017) is used. τ_{max} the maximum bond resistance of the steel fibres, b_w is web width, L_f is the steel fibre length, D_f is the fibre diameter, d_s is the effective depth of the beam, c is height of the neutral axis. The third model uses a critical shear displacement of Δ_{cr}=0.025 mm as recommended for reinforced concrete in the CSDT. The fourth model explores the use of a larger value (Δ_{cr} = 0.05 mm), which could be theoretically justified for SFRC in shear.

3. Comparison between models and experimental results

3.1. Development of database of experimental results

To compare the proposed models to experimental results, a database of experimental data from the literature was compiled (Reza & Chao 2017; Rajan & Sharma, 2014; Shoaib et al., 2014; Singh & Jain, 2014; Dinh, et al. 2011; Kwak, et al. 2002; Mansur,etal 1972; Ashour, et al. 1992; Chunxiang & Patnaikuni, 1999; Lim, et al 1987; Casanova & Rossi 1999; Noghabai 2000; Pansuk et al., 2017; Randl et al., 2017; Tan, et al., 1993). The database consists of 152 experiments of SFRC beams failing in shear, without stirrups, and with *M/Vd* ≥ 2 (slender beams). There was not limitation for the aggregate size, concrete compression strength, dimensions of the cross-section, reinforcement ratio, fibre percentage nor length and diameter of the fibre. Table 2 shows the range of the variables of the experiments gathered in the database, with *M/Vd* the generalized form of the shear span to depth ratio, d_a the maximum aggregate size, $f_{c,m}$ the measured concrete compression strength, *h* the height of the cross-section, *d* the effective depth, b_w the web width, ρ the reinforcement ratio, V_f the fibre volume fraction, and *L/D* the fibre aspect ratio.

Table 2. Maximum and minimum values for the variables from the database

	M/Vd (-)	d_a mm	$f_{c,m}$ MPa	*h* mm	*d* mm	b_w mm	ρ %	V_f %	*L/D* (-)
Min. value	2.5	0.4	20.6	150	127	50	0.4	0.5	38
Max. value	6	20	188	1220	1118	610	5.5	2	86

The shear capacity predicted with the 4 proposed models is calculated iteratively by using a Matlab script. Separate programs for beams with a rectangular cross-section as well as for I-beams were used. These 8 scripts are available elsewhere (Filian Abad 2017). Figure 2 shows the procedure used in these scripts, as well as the expressions used to calculate the shear capacity of each of the shear-resisting mechanisms.

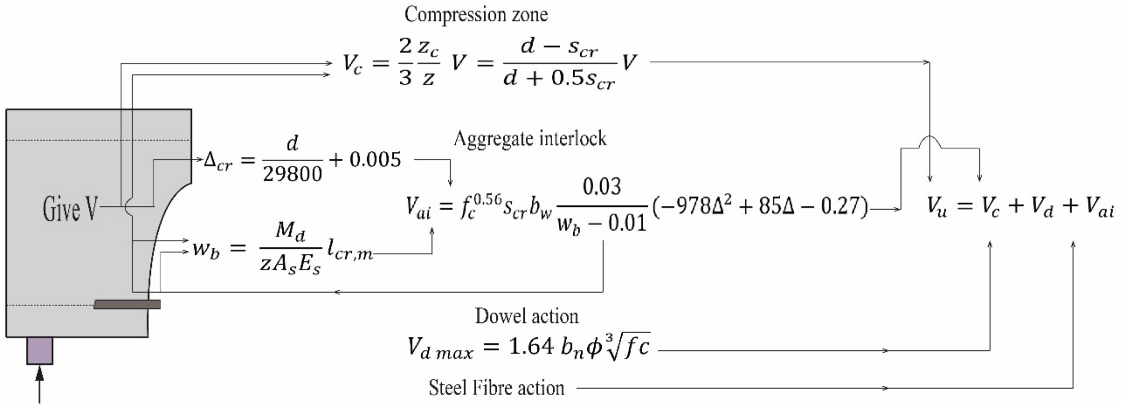

Figure 2: Flowchart of CSDT with steel fibre formulation

In the Figure 2, $V_{d\,max}$ is the resultant shear force component carried by the dowel action, M_d the cracking moment, w_b the crack opening at the level of the tensile reinforcement in the longitudinal direction, Δ_{cr} is the critical shear displacement ,V_{ai} the resultant shear force component carried by aggregate interlock, V_c the resultant shear force component carried in the uncracked concrete compression zone, V_u the maximum shear force in the span before the collapse of the structural member, $l_{cr,m}$ the mean crack spacing of the major cracks (at the mid-height of the beam section), and s_{cr} the height of the crack after it is stabilized.

3.2. Results of comparison

Table 3 shows an overview of the results of the ratio of the tested to predicted shear capacity V_u/V_{cal} according to the four models from Table 1 and the experimental results from the database. The calculated shear capacities are numbered according to the models from Table 1, with V_{cal1} calculated with model 1 from Table 1. The closest predictions are obtained with models 1 and 4. All models present a relative large scatter, with coefficients of variation around 30%. Comparing the results of models 3 and 4 shows that increasing the critical shear displacement for SFRC leads to better predictions.

Table 3: Results of V_u/V_{cal} for different models from Table 1

	$V_u/V_{cal\,1}$	$V_u/V_{cal\,2}$	$V_u/V_{cal\,3}$	$V_u/V_{cal\,4}$
Average	0.90	1.20	1.31	1.10
Standard deviation	0.28	0.34	0.44	0.33
Coefficient of variation	0.31	0.29	0.34	0.30

3.3. Parameter studies

To study the sensitivity of the 4 proposed models from Table 1 to the parameters varied in the experiments, the results of V_u/V_{cal} were plotted as a function of the effective depth, the maximum aggregate size, the concrete compressive strength, the reinforcement ratio, *M/Vd*, the fibre volume fraction, and the fibre aspect ratio (Filian Abad 2017). Figure 3 shows an example of these sensitivity analyses. For this case, the influence of the percentage of steel fibre in the mixture V_f is shown as a function of V_u/V_{cal} for the 4 models. The results show that for all models the predictions become less conservative as the fibre volume fraction increases

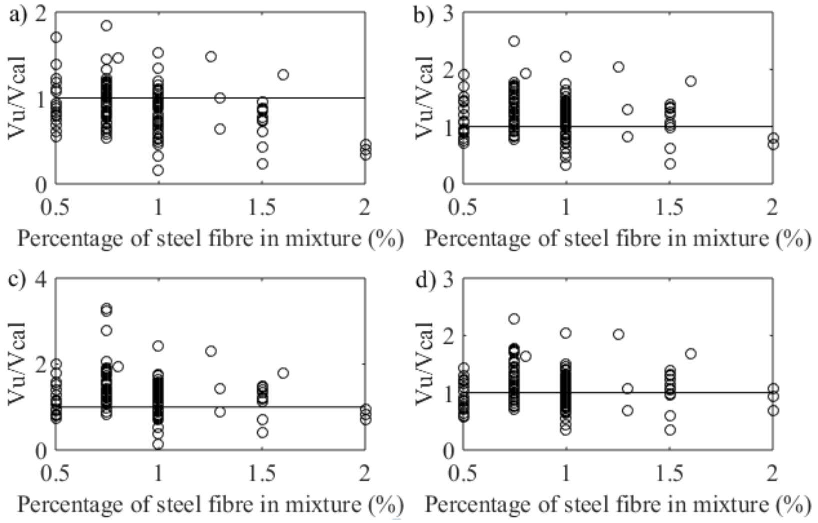

Figure 3: V_u/V_{cal} vs. percentage of fibre in concrete (%) a) Model 1 b) Model 2 c) Model 3 d) Model 4

4. Discussion of results.

According to Kwak; et al. (2002) Dinh; et al. (2011) and Shoaib; et al. (2014) steel fibre reinforced concrete beams show multiple inclined cracks followed by a widening of a diagonal crack prior to shear failure. The spacing between cracks reported in the literature confirms that the spacing for beams with fibre is reduced by 38% as compared to beams without steel fibres (Kwak, et al. 2002). The spacing can range from 0.47*d* to 0.6*d* (Shoaib, et al. 2014). In addition, the size effect occurs in beams with fibres since it has been determined that beams with larger effective depth have a larger crack width (Carnovale & Vecchio 2014). The behaviour between beams with and without steel fibres differs considerably in the pattern of cracking. Since the aggregate interlock capacity according to the proposed procedures is determined based on the crack width calculated for reinforced concrete beams, a future improvement of the model would include a suitable expression of the crack width for SFRC. As such, the influence of the fibres will be coupled with the other shear-resisting mechanisms.

Similarly, the aggregate interlock capacity is affected when steel fibres are added to concrete beams even in moderate fractions; the aggregate interlocking effect is reduced with the presence of steel fibres, since there is an increase in the width of the cracks. The larger opening and slipping of cracks in SFRC can be also represented by a larger value of Δ_{cr}, the critical shear displacement. As shown in Table 3, better results are obtained when the value of Δ_{cr} is increased with respect to the recommended value for reinforced concrete. Further research is necessary to identify the governing critical shear displacement for SFRC.

5. Summary and Conclusions.

This research focused on adding a mathematical expression for the shear-carrying capacity of steel fibres to the Critical Shear Displacement Theory for the estimation of the shear capacity of steel fibre reinforced concrete beams.

The parameters that determine the shear-carrying capacity of the steel fibres are mainly related to the length, diameter, and type of steel fibre. Additionally, other important parameters are the volume of fibre present in the mixture, and the average bond strength between the fibre and the matrix of concrete.

The expressions used in the CSDT for the capacity of the concrete in the compression zone, the shear capacity from aggregate interlock, and the shear capacity from dowel action were not altered. Several expressions from the literature were used to describe the shear-carrying capacity of the steel fibres. The resulting shear capacity was then compared to experimental results from the literature. For this purpose, a database with 152 experiments was developed.

The comparison between the tested and predicted shear capacities showed reasonable results for the four expressions for the shear-carrying capacity of the steel fibres combined with the expressions for the mechanisms of shear resistance of the CSDT. It is expected that these results can be improved when the effect of the fibres will be coupled with the other shear-carrying mechanisms. For this purpose, the crack width of SFRC should be introduced and the critical shear displacement for SFRC should be determined. Exploratory studies showed that using a larger critical shear displacement for SFRC results in a better correspondence with the experimental results.

Acknowledgements

This research was financially supported by the programme of Poligrants of Universidad San Francisco de Quito.

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