Delft University of Technology

Full Probabilistic Analysis with FEM for the Retaining Wall of a Deep Excavation

Ene, A.; Schweckendiek, T.; Popa, H.

Important note
To cite this publication, please use the final published version (if applicable).
Please check the document version above.

ISRERM

Full Probabilistic Analysis with FEM for the Retaining Wall of a Deep Excavation

A. Ene[*,1,2], T. Schweckendiek[,3], and H. Popa[1]

[1] *Technical University of Civil Engineering Bucharest, Bucharest, Romania*
[2] *Popp & Asociatii Inginerie Geotehnica Co., Bucharest, Romania*
[3] *Deltares & Delft University of Technology, Delft, Netherlands*

Abstract

Complex probabilistic approaches for the analysis of structures have been increasing in interest and application both in the area of research and in practice in order to provide a more uniform level of reliability. Also, the design codes, such as the Eurocodes, are developing provisions to address reliability-based design methods in more detail.

Moreover, FEM is very popular nowadays using advanced constitutive laws and modelling staged construction. This is especially important for geotechnical structures, such as retaining walls for deep excavations, for which soil-structure interaction is a complex mechanism.

The paper will present a full probabilistic analysis performed for a real case temporary retaining wall of a deep excavation. The probabilistic analysis was combined with FEM using advanced constitutive models. The objective is to demonstrate the feasibility of a full probabilistic analysis, using the current advanced design methods, and to assess the reliability produced by Eurocode designs. The main uncertainties were modelled as random variables and the limit state verification was expressed in terms of reliability index or, equivalently, the probability of failure for SLS and ULS verification.

The procedure of coupling probabilistic analysis using Probabilistic Toolkit reliability software with FEM Plaxis 2D commercial software for geotechnical analysis is given in detail while emphasizing some critical aspects in relation with these types of combined analyses and reliability concepts.

At last, some issues related to the provisions of the current design codes are presented and discussed in order to facilitate the implementation of reliability-based design, either through probabilistic methods or by partial factors.

Keywords: reliability-based design, full probabilistic analysis, FEM, deep excavation, temporary structures

1. Introduction

Deep excavations in urban areas imply significant risk and costs for the investment and, for these reasons, there is a great interest for optimization of such design in a rational quantifiable manner.

In terms of tools and methods for performing the calculations, the Finite Element Method (FEM) is very popular and accessible nowadays and can provide more realistic model for complex soil-structure interaction models as in the case of the retaining structures for deep excavations.

At the same time, reliability-based design is also increasing in application in order to account for the uncertainties in a quantitative manner and to provide reliable and cost-effective designs. Combining the two design methods offers both abovementioned advantages. However, coupling probabilistic analysis with FEM is challenging and research is still ongoing to facilitate application for practical designs.

One of the first attempts to evaluate structural and geotechnical reliability using FEM was performed by Waarts [1] using adaptive response surface with FORM and an adapted directional sampling method (DARS). Later, Schweckendiek coupled geotechnical FEM software Plaxis 2D with probabilistic tool ProBox developed by TNO in his master thesis [2]. Other similar works are those of Rippi, Nuttall, Teixeira, & Schweckendiek [3] in a research project at Deltares research institute. More details on this project and procedure of coupling FEM software Plaxis 2D and probabilistic tool OpenTurns are given Rippi's master thesis [4].

The present paper presents a case study of applied full probabilistic analysis of a retaining wall for a real project deep excavation in Bucharest city, Romania, by FEM

calculation in Plaxis 2D software [5] coupled with Probabilistic Toolkit (PTK) software [6] for reliability calculation.

First, the case study of a deep excavation in Bucharest, Romania is introduced, including the statistics of the geotechnical parameters and the random variables chosen for the probabilistic analysis based on a sensitivity analysis. Then, the main reliability-based analysis is presented, with the main requirements relevant for the present study, the limit states functions (LSF) for the verification in accordance with the Eurocode 7 principles [7], the FEM and the probabilistic models and the procedure of coupling these, together with the main results obtained. Subsequently, some discussions are made for the results obtained, in relation with to the input data and requirements available in the literature and codes. Lastly, some conclusions are drawn based on the case study analyses presented.

This case study has been previously presented in more detail for Serviceability Limit State (SLS) verification, using different statistics for the soil parameters [8]. The present paper presents further analysis including Ultimate limit State (ULS) verification, on a chosen set of statistics for the soil parameters, based on the findings of the previous study.

2. Case study description

2.1 Project information

The case study presented in this paper refers to a temporary retaining wall for a deep excavation in Bucharest, Romania.

The excavation pit for the two basement levels was 7.7 m deep for the marginal area and 8.3 m deep in the central area. On most of the area, a sloped excavation was considered, and on a side where the excavation was led

---

[*] E-Mail: alexandra.ene@p-a.ro

near the property limit, a self-supporting embedded wall was provided. The retaining wall consisted of drilled reinforced concrete piles, 80 cm diameter at 85 cm inter-axes distance, 16 m length.

The excavation pit layout, with the adjacent constructions and site investigations is shown in Fig. 1 and the characteristic section of the retaining system, including the lithology is shown in Fig. 2.
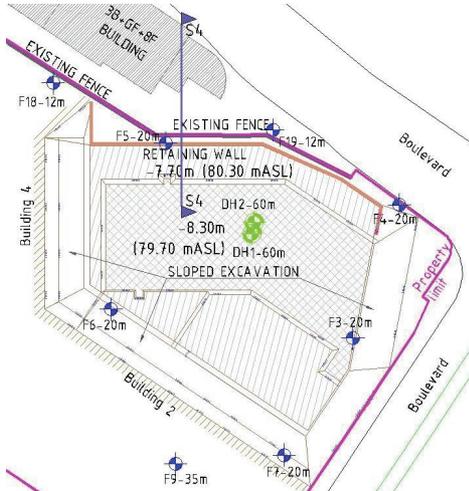

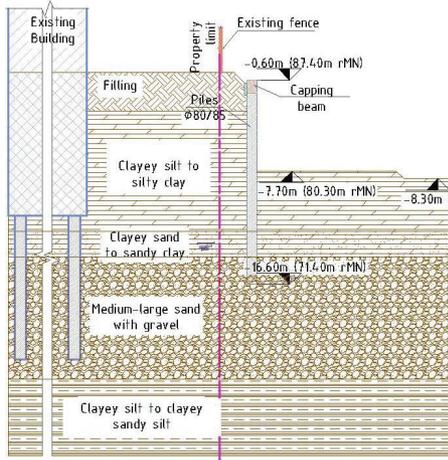**Figure 1.** Layout of the excavation and retaining system.

Figure 2. Characteristic section of the excavation and retaining system.

The ground investigations consisted of seven geotechnical boreholes with sampling of soil specimens and Standard Penetration Tests in cohesionless soil layers. Also, Downhole and Crosshole tests were performed in some of the boreholes to assess the seismic characterization of the site and provide small strain stiffness soil parameters.

The main layers representative for the retaining wall model were:

0. Filling (88÷85 m ASL), consisting of construction waste material, cemented or incorporated in a stiff clay;
1. Silty clay (85÷75.5 m ASL), stiff and with increased consistency in some areas - typical "Bucharest Clay" layer;
2. Sandy clay (75.5÷73.5 m ASL), stiff;
3. Sand with gravel (73.5÷64 m ASL), dense.

The results of the site and laboratory tests were statistically described to determine the geotechnical parameters for the design.

For the "Bucharest Clay" (layer 1, silty clay), which is dominant for the retaining system in this project, the mean and standard deviation for spatial (layer) average properties as described in more detail in section 0, while the ground properties of the other soil layers were taken as characteristic values according to Eurocode 7 [7] and NP 122 [9], assuming normal distribution and spatial averaging, see Tab. 1.

Table 1. Characteristic values for the geotechnical parameters used in the design.

Layer	γ_k^{sup} [kN/m³]	φ'_k^{inf} [°]	c'_k^{inf} [kPa]	E_{50} [MPa]	G_0 [MPa]	p_{ref} [kPa]
0	19	15	5	3	40	40
1			*See Tab. 2*			
2	19.8	12.8	29.3	19.2	240	280
3	20.2	39	-	40	500	470

γ_k^{sup} – superior characteristic values for unit weight; φ'_k^{inf} - inferior characteristic values for the effective angle of friction; c'_k^{inf} - inferior characteristic values for the effective cohesion; E_{50} - the triaxial loading stiffness at the reference pressure; G_0 - small strain shear modulus at the reference pressure; p_{ref} – reference pressure.

2.2 Statistical description of the soil properties

For the scope of the present study, the main geotechnical parameters of the dominant 9÷11 m thick silty clay layer, called "Bucharest Clay", were statistically described as variables for the probabilistic calculations, while for the rest of the layers the parameters were considered as deterministic with characteristic values (see Tab. 1).

An analysis of different statistic distributions for the geotechnical parameters was performed prior to the reliability analysis, using different assumptions: normal distribution without prior knowledge for all parameters, normal distribution with prior knowledge according to NP 122 [9], linear regression correlation with normally distributed parameters for shear resistance parameters (cohesion and friction angle). More details of this analysis is given in a previous study presented by the authors [8].

Table 2. Statistics of the soil parameters considered for the "Bucharest Clay" layer.

Variable	$\gamma_{unsat}=\gamma_{unsat}$ [kN/m³]	E_{50}^{ref} [MPa]	$tg(\varphi')$ [-]	c'_{ref} [kPa]
n	42	15	33	
μ	19.42	12 552	0.4251 tg(23.9⁰)	39.5
σ_x	0.06	1 240	0.0369	5.95
v_x	3%	10%	9%	15%
ρ	-	-	-0.8741	
Distribution	Normal	Lognormal	Normal	

n - sample size (number of observations), μ - mean, σ_x - standard deviation, v_x - coefficient of variation, ρ - correlation factor.

The statistics chosen for the geotechnical parameters considered are given in Tab. 2, with the spatial averaging assumption and statistical uncertainty by Student-t factor, as mentioned before.

2.3 Random variables

Other relevant uncertainties are related to the soil model parameters which are estimated empirically or based on literature references. Since these parameters were not determined directly, their variance was estimated conservatively based on engineering judgement within a rage considered as reasonable, as presented in Tab. 3.

Table 3. Other variables of the soil model analysed and their estimated statistics.

Variable	$R_{E50/Eeod}$ [-]	$R_{Eur/E50}$ [-]	p_{ref} [kPa]	m(power) [-]	R_{inter} [-]
μ	1.3	3	100	0.7	0.7
σ_x	0.2	0.6	20	0.105	0.105
ρ	15%	20%	20%	15%	15%
Distribution			Normal		

$R_{E50/Eeod}$ - ratio of linear deformation modulus (E_{50}) and volumetric deformation modulus from oedometer (E_{eod}), $R_{Eur/Eeod}$ - ratio of unloading-reloading deformation modulus (E_{ur}) and linear deformation modulus (E_{50}), p_{ref} - reference pressure for deformation and strength parameters, m - power of the strain hardening hyperbolic law, R_{inter} - interface reduction factor.

Also, a model factor for FEM analysis of the retaining structure (f_m^{FEM}) was considered as a random variable. This was taken as normally distributed with mean 1 and standard deviation 0.1 as per Section 3.7, table 3.7.5.1 for "Stability of retaining (sheetpiled) walls" in Probabilistic Model Code [10].

In order to reduce the calculation time for probabilistic analysis, a sensitivity analysis was performed with the random variables described above using the "extremes" of 5% and 95% fractiles of the variables.

The results of the sensitivity analysis are presented in Fig. 3 and Fig. 4 for each of the output parameters of the reliability analysis.
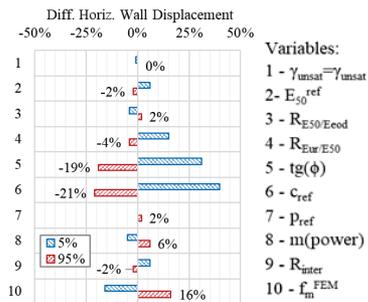

Figure 3. Results of the sensitivity analysis for the maximum horizontal wall displacement
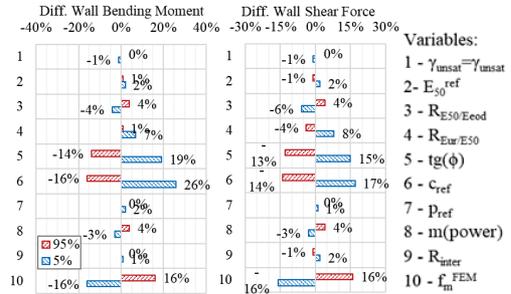

Figure 4. Results of the sensitivity analysis for the maximum bending moment and shear force in the retaining wall

Based on the sensitivity analysis, the "significant" random variables chosen for the reliability analysis are the shear resistance parameters of the "Bucharest Clay" layer (c_{ref} and ϕ), and the model factor (f_m^{FEM}).

3. Reliability Analysis

3.1 Limit state functions

3.1.1 Verification of Serviceability Limit State

The SLS verification concerned the horizontal displacement of the retaining wall towards a target value. In lack of specific provisions, for the reliability analysis, the design value of the limit displacement (d_{wall}^{SLS}) was set at 3.5 cm to account for architectural and structural limitations of the available space.

The limit function was set as follows:

$$Z_{wall}^{SLS} = d_{wall}^{SLS} - \max U_{x,wall} \cdot f_m^{FEM} \qquad (1)$$

Where $U_{x,wall}$ is the value of the maximum horizontal displacement of the retaining wall resulted from the FEM analysis.

3.1.2 Verification of geotechnical Ultimate Limit State

For the reliability analysis, besides from the "safety factor" provided by the safety analysis ($\sum M_{sf}$), it is also important to account that if the structure is instable in a specific deterministic phase – for a certain possible combination of the variables (i.e. the strength parameters), then the calculation does not converge, so the safety analysis is not performed. To cover this scenario and obtain relevant results, this situation is included in the LSF based on the Plaxis parameter $\sum M_{stage}$, which represents the ratio between the load for which failure occurs and the desired ultimate load.

The limit function was set as follows:

$$if \sum M_{stage} > 0.995: Z_{GEO}^{ULS} = \sum M_{sf} \cdot f_m^{FEM} - 1$$

$$else: Z_{GEO}^{ULS} = \sum M_{stage} \cdot f_m^{FEM} - 1 \qquad (2)$$

3.1.3 Verification of structural Ultimate Limit State

The structural resistance of retaining structures for deep excavations is usually verified for bending moment capacity and shear capacity. For (in-situ) reinforced concrete elements, the structural resistance is determined by the reinforcement. Thus, it is necessary to estimate the load effects (moments – and axial forces - and shear forces)

for the reinforcement design, in order to subsequently verify the (reliability of) the structural resistance.

The limit functions are set as follows:

$$Z_M^{ULS} = M_{Rd} - max\, M_d \cdot f_m^{FEM} \qquad (3)$$

$$Z_S^{ULS} = S_{Rd} - max\, S_d \cdot f_m^{FEM} \qquad (4)$$

The scope of the present probabilistic calculation is to assess the reliability of the retaining wall at ULS just for the load (effects) part of the general verification. For this, the resistances (bending and shear capacity) are established based on deterministic analysis using characteristic values for the geotechnical parameters and applying the corresponding partial factors, for two combinations, separately [7]:

$M_{Rd,DA1C1}$ / $S_{Rd,DA1C1}$ - design bending moment/ shear force capacity corresponding to the resulting load effect from DA1C1 (i.e. by partial factors for the permanent and unfavourable variable loads $\gamma_G = 1.35$ and $\gamma_Q = 1.5$, respectively);

$M_{Rd,DA1C2}$ / $S_{Rd,DA1C2}$ - design bending moment/ shear force capacity corresponding to the resulting load effect from DA1C2 (i.e. by partial factors for the unfavourable variable loads $\gamma_Q = 1.5$ and for the material properties – soil parameters $\gamma_{\varphi'} = 1.25$ and $\gamma_{c'} = 1.25$).

It was assumed that the reinforcement of the retaining wall led to a capacity exactly equal to design load effect, although in practice this is rarely the case, and the reinforcement will often be slightly over-designed.

3.2 Target reliability

The present study referred to the available target reliability index, as given in Eurocode [11] with the following assumptions:

- Reference target reliability index was established for a design life of the structure of 50 years (i.e. 3.8 target value for ULS and 1.5 for SLS for Reliability Class 2);
- A value of 4.0 for the reliability index should be enough to ensure minimum required reliability for lower reference period (e.g. 2 years);

Note that a beta value of 4 would also be equivalent to transforming the beta value for 1 year reference to 2 years reference through the standard normal cumulative distribution function for time variant problems for the case of "moderate consequences of failure" and "normal relative cost of safety measure" as given in ISO 2394:2015 [12].

- The implications of lowering the reliability index by one class (i.e. 3.3 target value for RC1) and by half a class (i.e. 3.55 target value) for temporary structures – as suggested in the Probabilistic Model Code [10] – is analysed separately.

Also, for the structural ULS verification, since only the load effect is analysed by probabilistic assessment, and the resistance is set to the design value as it would be obtained from deterministic evaluation and the target beta value will be taken at $\alpha_E \cdot \beta$. In lack of other prescriptions, the value of the FORM influence factor for the action effects $\alpha_E = 0.7$ is considered, according to Eurocode [11], although this is provided for general structures and not for geotechnical structures specifically.

Thus, the target reliability index would be:

- $0.7 \cdot 4 = 2.8$ as for permanent structures, but at lower reference period;
- $0.7 \cdot 3.5 = 2.45$ as for temporary structures for which the reliability index is lowered by one class, and at lower reference period.

For Serviceability Limit State, reference is given only for Reliability Class 2 in Eurocode [11] for target reliability index: 1.5 for 50-years reference period and 2.9 for 1-year reference period, respectively.

3.3 Calculation Models

3.3.1 FEM Model

The retaining system of the excavation was modelled using the 2D Finite Element model for plane strain state in Plaxis 2019 software [5], by drained analysis but using total stresses shear resistance parameters.

The finite element model is 35 m deep and 40 m wide and it consists of approximately 1385 triangular 15-node elements for soil cluster, 22 elements of 5-node line type for pile wall and 46 elements of 5-node line for the interface. Also, the boundary conditions of the model consist in fixing its bottom against all directions, its vertical boundaries against horizontal directions and in considering the ground surface free in all directions.

To determine the deformations more realistically, the Hardening Soil with small stiffness behaviour was used with the main parameters given in Table 2 and Table 6.

The execution stages considered in the FEM model were as follows:

0. Stress initialization for the soil volume;
1. Execution of a preliminary excavation of 60 cm deep for the working platform and execution of the pile wall, in a single stage;
2. Excavation to final level (-7.70 m) and activation of the surcharge load.

3.3.2 Probabilistic model

Since full probabilistic procedures are still under research for proper implementation in commercial software, the current available procedure for reliability analysis with FEM is by coupling reliability (probabilistic) evaluations with "deterministic" finite elements calculations, via different interfaces. The reliability analysis tool performs realizations of the random variables, which are given as input for the FEA. Then, the FEM software delivers the output of each deterministic calculation. These outputs are used back by the reliability analysis tool for evaluating the LSF. The entire process is iterative until the reliability analysis reaches the target parameters (usually, the required reliability precision).

The coupling of a reliability tool with the FEM software is actually setting the communication necessary to exchange the data (input and output parameters) between the two. Each of the models (FEM and reliability) is set independently and the calculations are performed in sequences.

The procedure of coupling Probabilistic Toolkit (PTK) with FEM software Plaxis 2D as developed at Deltares is represented in Fig. 5, picture taken from the internal report of Deltares [13].
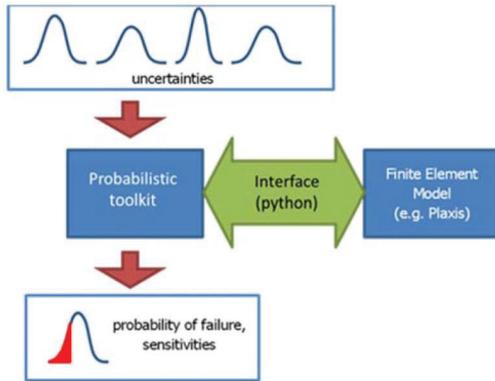
Figure 5. Principle of the coupling PTK and PLAXIS.

The probabilistic model in PTK includes the following settings:

i. The internal model, in which the input and output parameters are defined - which will be set as variables either stochastic or deterministic - and the source code through which the output parameters are calculated from the input parameters, including the definition of the LSF;

ii. The relations with external models, as reference to the communication scripts, executables etc.;

iii. The stochastic/ random variables (uncertainties) and their probability distributions;

iv. The correlation matrix of the random variables;

v. The limit state function (LSF) to be analysed and the chosen reliability method, including the requirements/ setting for the reliability analysis, such as minimum and/or maximum number of iterations, precision, and other settings depending on the method used. The reliability method can be chosen from the ones included in the software or it can be external.

3.4 Selection of reliability methods

First, FORM analysis was attempted, but most of the times this could not reach convergence. This was noticed that the output from Plaxis 2D presented some randomness in the results for the same input values. This led to significant "noise" in the evaluation of the limit state function due to small calculation errors that exact methods such as FORM can not handle.

Then, since crude Monte Carlo requires a very high number of calculations, Importance Sampling (IS) around the (estimated) design point was found as a feasible and accurate alternative. However, in order to obtain a good estimate of the design point, it is necessary to have some prior knowledge of the failure area. This was solved by performing the analysis in two steps: one IS run around the mean values, and a second IS run around the design point obtained from the first run in order to improve precision. For the case study analysed within the present paper, it was necessary to perform about 1000 iterations around mean values and about 500 to 1000 iterations around the design point from the first calculation. Hence, in total about 1500-2000 Plaxis 2D model runs, to reach a precision of 0.1 for the probability of failure (i.e. coefficient of variation).

At last, most of the reliability analyses were performed using Erraga metamodel developed at Delft University of Technology and Deltares [14], which led to very good results in less than 50 iterations most of times. The main results were verified by Importance Sampling performed in two steps, as described above. Also, some supplementary verifications were also performed by Directional Sampling.

3.5 Results of calculations

3.5.1 Verification of Serviceability Limit State

For the SLS verifications of the retaining wall, two models have been analysed:

1. Considering for the soil properties only the shear resistance parameters as stochastic variables;
2. Considering for the soil properties the unit weight of the soil, the linear deformation modulus and the shear resistance parameters as stochastic variables, as given in Tab. 2.

The reliability index resulted from the analyses for vary between 1.17 for the first scenario (which would be unacceptable according to the target value of 1.5 for permanent structures and 50-years reference period provided by Eurocode) and 2.38 for the second scenario (which should be acceptable for lower reference period and for mostly time invariant problems).

3.5.2 Verification of geotechnical Ultimate Limit State

The results provided a reliability index of 4.86, which is significant margin compared to a target beta of 4, as considered for permanent structures with lower reference period for mostly time invariant problem. This also shows that choosing the retaining wall length was overcautious.

Considering the high reliability obtained and the fact that IS was performed around an arbitrary chosen value, which is not actually validated as being the closest point to the failure surface, this should be investigated further through other procedures.

Although this limit state is not expected to dictate the design at least with the given geometrical details of the structure, validating the model can confirm the results obtained so far.

3.5.3 Verification of structural Ultimate Limit State

Starting with the partial factors given in the current version of Eurocode - $\gamma_G=1.35$ for permanent loads, and $\gamma_Q=1.5$ for variable loads for DA1C1 – a reliability index (β) of about 3.5 was obtained, which is significantly higher than the reference values chosen for the loads verification: $\beta=0.7\cdot4=2.8$ for permanent structures or $\beta=0.7\cdot3.5=2.45$ for temporary structures, respectively (see sub-section 0). Lower values for the partial factors were used afterwards to calculate the design resistance (bending moment and shear force capacity of the retaining wall), while keeping the ration between the permanent load and the variable load partial factors constant $\gamma_Q/\gamma_G=1.11$.

Using the partial factors for soil shear resistance to determine the design resistance (bending moment and shear force capacity of the retaining wall) according to DA1C2, led to significantly low reliability index for this specific case study of a cantilever retaining wall.

The variation of the reliability index with the corresponding partial factor for the permanent load for DA1C1 is represented in Fig. 6 and Tab. 4.
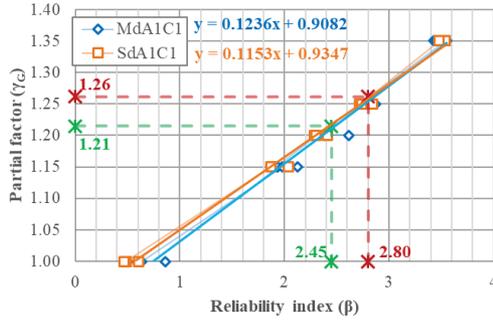

Figure 6. Reliability results for the normal design situation DA1C1.

Table 4. Reliability results for the verification of structural Ultimate Limit State for the normal design situation (Design Approach 1, Combinations 1 and 2).

LSF	Design resistance	γ_G	γ_Q	$\gamma_{\varphi'}$	β	P_f
Z_M^{ULS}	$M_{Rd,A1C1}$=539.1kNm/m	1.35	1.11	1.0	3.45	2.8E-4
	$M_{Rd,A1C1}$=499.1kNm/m	1.25	1.11	1.0	2.74	3.1E-3
	$M_{Rd,A1C1}$=479.2kNm/m	1.2	1.11	1.0	2.31	1.0E-2
	$M_{Rd,A1C1}$=459.2kNm/m	1.15	1.11	1.0	1.98	2.4E-2
	$M_{Rd,A1C1}$=399.3kNm/m	1.0	1.11	1.0	0.63	2.7E-1
Z_S^{ULS}	$S_{Rd,A1C1}$=155.4kN/m	1.35	1.11	1.0	3.54	2.0E-4
	$S_{Rd,A1C1}$=143.9kN/m	1.25	1.11	1.0	2.73	3.2E-3
	$S_{Rd,A1C1}$=138.8kN/m	1.2	1.11	1.0	2.29	1.1E-2
	$S_{Rd,A1C1}$=132.4kN/m	1.15	1.11	1.0	1.87	3.1E-2
	$S_{Rd,A1C1}$=115.1kN/m	1.0	1.11	1.0	0.49	3.1E-1
Z_M^{ULS}	$M_{Rd,A1C2}$=428.7kNm/m	1.0	1.3	1.25	0.92	1.8E-1
	$M_{Rd,A1C2}$= 414.1kNm/m	1.0	1.3	1.0	0.63	2.7E-1
Z_S^{ULS}	$S_{Rd,A1C2}$=118.2kN/m	1.0	1.3	1.25	0.52	3.0E-1
	$S_{Rd,A1C2}$= 117.6kN/m	1.0	1.3	1.0	0.46	3.2E-1

Following the linear regression between the reliability index obtained by probabilistic analysis for the loads verification and the partial factor of the permanent loads to determine the design resistance of the retaining wall, it can be determined that – for this specific case study analysed - a set of partial factors γ_G=1.26 and γ_Q=1.4 would correspond to reliability index of β=2.8 for permanent structures, and a set of partial factors γ_G=1.21 and γ_Q=1.34 would correspond to reliability index of β=2.45 for temporary structures.

3.5.4 Design values

In order to evaluate the design point values for the main variables involved (i.e. for shear resistance parameters of the soil and for the model factor) as an alternative to the load partial factor for DA1C1, FORM analysis is performed based on the alpha values and the reliability index provided by the probabilistic calculation.

Erraga metamodel provides specific design point values, although these are estimated in the uncorrelated space as most of the reliability methods algorithms are developed. For this reason, "corrections" should be applied to provide specific set of values when there is significant correlation between the variables, as the case of the shear resistance parameters.

PTK also provides design point values transforming the variables in correlated space with the alpha values resulted from the reliability analysis, by applying Cholesky decomposition, and this is also given for comparison.

The determination of the design point values obtained for the 1.25 load partial factor (A1C1) model, both for bending moment and shear force verification LSF (Z_M^{ULS} and Z_S^{ULS}), are given in Tab. 5 and Tab. 6 for the following cases:

i. Design point values resulted from Erraga metamodeling (obtained from the reliability analysis);

ii. Design point values given by PTK in the correlated space.

Table 5. Example of design point values for the variables resulted from the reliability analysis.

Cases	Variable (x_d)			$M(X_d)/$
	$tg(\phi)$	c_{ref}	f_m^{FEM}	$S(X_d)$
i.	0.3701	45.3	1.23	$M(X_d)$ = 494 kNm/m ≈ 99% MdA1C1
ii.	0.3701	50.1	1.23	$M(X_d)$ = 475 kNm/m ≈ 95% MdA1C1
i.	0.3592	47.0	1.21	$S(X_d)$ = 144 kN/m ≈100% SdA1C1
ii.	0.3592	52.4	1.21	$S(X_d)$ = 141 kN/m ≈98% SdA1C1

$M(X_d)$ and $S(X_d)$ are the bending moment and shear force, respectively, obtained from the FEM calculation with the design point values of the variables.

It might be concluded that the design point values resulted direct form the reliability analysis (i.e. from Erraga metamodel) leads to the closest LSF verification (i.e. the design loads from the deterministic analysis using the design point values resulted as input are almost equal to the limit resistance imposed). However, this is determined in uncorrelated space and the set of the design point values might not fully compatible mathematically.

Then, the algorithm implemented in Probabilistic Toolkit software accounting for the correlation between the variables leads to a reasonably close verification of the LSF (up to about 5% difference between the design loads from the deterministic analysis using the design point values resulted as input and the limit resistance imposed).

4. Discussions

4.1 General remarks

The present study included only the reliability analysis of the normal design situation, while the accidental and the seismic design situations for a reference period of 1 year should lead to lower reliability.

For the case study presented, the length of the retaining wall was established based on deterministic analysis to limit wall displacement, meaning to ensure sufficient embedment that there is no significant influence for the wall displacement. A more economical approach would be to establish the length of the retaining wall through a full probabilistic analysis for SLS, based on a specific limit value and the associated LSF and a target reliability index, if the geotechnical ULS is ensured. It is expected that this approach could also influence the results obtained for the

structural ULS as well just as well as other case studies could lead to different results and conclusions.

4.2 Target reliability indices

In order to assess the reliability of a structure, we need target (minimum) values for reliability indices, or acceptable probabilities of failure, for the specific design situation and limit state at hand. Although there are some guiding provisions in current Eurocode [11] and in ISO 2394:2015 [12], there are still some important aspects that need further attention, such as:

1. Definition of "base" reference period (1-year or 50-years), to allow more suitable differentiation for mainly time invariant problems;

2. Specific provisions for temporary structures in terms of reliability, including target reliability index, partial factors etc.;

3. More detailed provisions for serviceability limit state (for temporary and permanent structures).

It needs to be emphasized that in lack of more detailed provisions as mentioned before, reliability-based methods might not develop much either for direct design or for code calibration or it would be done based on arbitrary choices, which could lead to higher differentiation of practices.

4.3 Results of the probabilistic analyses

Design Approach 1 - Combination 1 seems overcautious, while Combination 2 is unsafe as currently given in Eurocode 7 [7], for the case study analysed. However, this is a particular case of a relatively flexible structural system, and different results should be obtained in different cases, especially for Combination 2.

The small reliability index obtained for Combination 2 might also be accounted to the uncertainty of the variable load, which was taken with the design value (using 1.3 partial factor) instead of providing relevant statistics for this.

As a preliminary observation, a partial factor of about 1.26 for the permanent load and a corresponding partial factor of 1.4 for the variable load would provide sufficient reliability following the target reliability index provided in Eurocode [11] for permanent structures, slightly adapted to account for a reduced reference period for almost time-invariant problems. If reduced reliability level is accepted for temporary structures by lowering half a class the reliability index according to the provisions in the Probabilistic Model Code [10], then a partial factor of about 1.21 for the permanent load and a corresponding partial factor of 1.34 for the variable load would provide sufficient reliability.

Since the resulted reliability indices are highly similar for both the bending moment verification and the shear force, it can be concluded that at least for DA1C1, the partial factors cover both these uncertainties almost equally.

Following the alpha values resulted from the reliability analysis, it can be noticed that the partial factor used in the deterministic design is mainly covering the model uncertainty (for the assumed the model factor) and less the uncertainty in the applied load.

4.4 Design values

Determining the design values of the main variables would help propose specific reliability-based partial factors both for material properties and for loads or, specifically for this geotechnical application with FEM, the load factor would be ideally replaced by a design model factor.

The FORM-values estimated by PTK in correlated space, based on the alpha values and reliability index resulted from the reliability analysis, led to close design point estimation, with about 1-6% underestimation of the design forces, so an overestimation of design point values, for this case study.

Erraga metamodel gives design point values closer to the failure point, although these are determined in uncorrelated space.

So, it would probably be best if, in disfavour of the mathematical correctness of the correlation between the variables, a well-suited set of variables closer to the design (failure) point would be chosen for partial factors calibration.

Another important aspect that needs to be emphasized is that a partial factor for the main variables determined by probabilistic approaches would also have to be applied to the entire model from the beginning (i.e. from stress initialization stage) and to corresponding related parameters. For example, in the case of the present case study, the design value of the friction angle would also be related to the soil pressure coefficient at rest, since these were varied together. The design values obtained by probabilistic methods are physically possible values and would influence the entire model, through all calculation stages. Moreover, the necessity to calculate the parameters related together is justified both to avoid numerical issues of the calculation model, but also because these are physically related. This is in contrast to the present partial factors procedure available in Plaxis (i.e. applied only in specific stages and only to the specific parameters for which they are provided, such as shear soil resistance), and which is also considered as a recommendation (first option) in the current draft for the next Eurocode 7 [7].

4.5 Erraga metamodel

Erraga metamodel, used as an external model in Probabilistic Toolkit software, seems a promising new reliability method for complex analyses. This has shown good estimation of the reliability index in very few iterations, which has been validated by classical methods such as Importance Sampling. Also, in terms of estimation of the design point, Erraga provides trustworthy results.

5. Conclusions

The scope of the study presented was to perform a full probabilistic analysis on a real case study and compare it with the reliability provided by current design standards in Europe, i.e. with the Eurocodes [7], [11]. The case of a deep excavation retaining system was chosen because this is a common application which imply significant cost for the investment, as well as many risks associated with their execution.

This example was simplified, mainly on the safe side, in order to obtain faster results that would indicate at least

in great lines where further directions are needed. Still, it has shown that further optimization can be provided to the Eurocodes to allow for more economical, while reliable enough, design by partial factors.

The case study presented showed that full probabilistic reliability-based design can be implemented in practice to provide economical design, at least for some situations (such as normal design situation). Moreover, this can be performed together with other advanced calculations such as FEM.

For the last decade or more, there has been much development for more efficient reliability methods to allow using these on a larger scale for complex problems. However, a "universal recipe" has not been validated yet and current FEM commercial software do not include such options.

To perform a more uniform risk-based design either through probabilistic methods or through partial factors, more detailed provisions are necessary in terms of target reliability index, including for temporary structures and for SLS verification and for time-invariant problems.

Further research is needed to cover aspects such as surcharge load, accidental and seismic design situations etc. for complete probabilistic design.

Needless to say, a complete reliability analysis should also include the evaluation of the resistance with the associated uncertainties.

Acknowledgment

References

[1]   P. H. Waarts, Structural reliability using Finite Element Analysis, Delft: Delft University Press, 2000.

[2]   T. Schweckendiek, Stuctural Realiability Applied to Deep Excavations, Delft, Netherlands: Delft University of Technology, 2006.

[3]   A. Rippi, J. D. Nuttall, A. Telxeira and T. Schweckendiek, "Reliability analysis of a dike with retaining wall using FEM," in *6th Asia-Pacific Symposium on Structural Realiability and its Applications (APSSRA6)*, Shanghai, China, 2016.

[4]   A. Rippi, Structural reliability analysis of a dike with a sheet pile wall – Coupling Reliability methods with Finite Elements, MSc Thesis, Delft: Delft University of Technology, 2015.

[5]   Plaxis, Reference Manual, Delft: PLAXIS, 2019.

[6]   Deltares, *Probabilistic Toolkit - Manual,* 2019.

[7]   Comité Européen de Normalisation, *EN 1997-1:2004 Eurocode 7: Geotechnical design,* Brussels: CEN/CENELEC, 2004.

[8]   A. Ene, T. Schweckendiek and H. Popa, "Probabilistic FEM Analysis for the Retaining Wall of a Deep Excavation at SLS," in *Proceedings of the 18th International Probabilistic Workshop*, Guimaraes, Portugal, 2020.

[9]   Ministerul Dezvoltării Regionale şi Turismului, *NP 122:2010 Romanian norm on selecting the characteristic and design values of geotechnical parameters,* Monitorul Oficial al României, 2011.

[10]   Joint Committee on Structural Safety, "Probabilistic Model Code," Technical University of Denmark, 2001.

[11]   Comité Européen de Normalisation, *EN 1990:2002 Eurocode: Basis of structural design,* Brussels: CEN/CENELEC, 2001.

[12]   International Organization for Standardization, *ISO 2394:2015 General principles on reliability for structures,* ISO, 2015.

[13]   Deltares, "Probabilistic Tools: Reliability Based Soil-Structure Analysis using FE," 2019.

[14]   A. P. van den Eijnden, T. Schweckwndiek and M. A. Hicks, "Metamodelling for geotechnical reliability analysis with noisy and incomplete models (Manuscript)," *Structural Safety,* pp. -, 2019.