Inelastic scattering time for conductance fluctuations

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(Received 30 May 2001; revised manuscript received 19 September 2001; published 28 February 2002)

We revisit the problem of inelastic times governing the temperature behavior of the weak localization correction and mesoscopic fluctuations in one- and two-dimensional systems. It is shown that, for dephasing by the electron electron interaction, not only are those times identical but the scaling functions are also the same.

DOI: 10.1103/PhysRevB.65.115317 PACS number(s): 73.23.−b, 73.63.Nm, 72.15.Rn, 73.20.Fz

I. INTRODUCTION

In 1982, Altshuler, Aronov, and Khmelnitsky (AAK) established[1] that electron-electron scattering in metals is characterized by *three* (generally, distinct) time scales. These scales are phase-relaxation time τ_ϕ, energy-relaxation time τ_E, and out-scattering time τ_e (see Table I). The former one is quantum-mechanical and has no classical analog, while the two latter have a semiclassical interpretation in terms of Boltzmann equation. The three scales differ in the case when the energy transferred between electrons in one collision is small as compared to the temperature of the system T.

One can understand the difference between τ_E and τ_e by considering the inelastic collision integral in Boltzmann equation

$$\begin{aligned}\mathrm{St}\{f(\epsilon)\} &= \int d\epsilon_1 d\omega K(\omega) \\ &\times\{-f(\epsilon)[1-f(\epsilon-\omega)]f(\epsilon_1)[1-f(\epsilon_1+\omega)] \quad \text{(out)} \\ &+[1-f(\epsilon)]f(\epsilon-\omega)[1-f(\epsilon_1)]f(\epsilon_1+\omega)\} \quad \text{(in)}, \quad (1)\end{aligned}$$

where $f(\epsilon)$ is the electron distribution function, and the kernel $K(\omega)$ characterizes matrix elements of the interaction, with the energy transfer ω. In clean two-dimensional (2D) and 3D systems, $K(\omega)$ is independent on the transmitted energy ω, $K(\omega)\simeq 1/\epsilon_F$. This results in a Fermi liquid behavior of the inelastic rate $1/\tau_{in}\simeq \max(\epsilon,T)^2/\epsilon_F$. The situation in disordered systems, however, is different,[2–5] the kernel $K(\omega)$ grows with the decrease of the transmitted frequency ω,

$$K(\omega)\simeq \frac{1}{|\omega| g(L_\omega)} \propto \omega^{d/2-2}, \quad (2)$$

where $g(L_\omega)$ is the dimensionless conductance (in units if $e^2/\pi\hbar$) of the d-dimensional disordered sample of the size $L_\omega=(D/\omega)^{1/2}$, where D is the diffusion constant of the metallic sample. For more details on origin of Eq. (2), see, e.g., Refs. 6,7.

Substituting Eq. (2) into Eq. (1), one estimates[5]

$$\mathrm{St}\{f(\epsilon)\}\approx -\frac{\delta f(\epsilon)}{\tau_E}, \quad \frac{\hbar}{\tau_E}\simeq \frac{\epsilon^*}{g(L_{\epsilon^*})}, \quad (3)$$

where $\epsilon^*=\max(\epsilon,T)$. Equations (2) and (3) are applicable for systems in the metallic regime, $g(L)\gg 1$. In this regime $\epsilon^*\tau_E\gg\hbar$, i.e., quasiparticles are well defined. Notice that, even though the kernel K is divergent, the energy relaxation rate (3) is finite because of the two energy integrations in Eq. (1). Therefore, for the study of the phenomena governed by the Boltzmann equation, the infrared divergence of the matrix elements (2) does not cause any problems. These phenomena include, for instance, electron distribution function measured via tunneling spectroscopy[8] or crossover from 1/3 to $\sqrt{3}/4$ shot noise in metallic wires.[9]

It is not the end of the story, though. If we estimate only one (''out'') term from the collision integral (1), we encounter an infrared divergence in two- and one-dimensional cases

$$\mathrm{St}_{\mathrm{out}}\{f(\epsilon)\}\approx -\frac{\delta f(\epsilon)}{\tau_e}, \quad \frac{\hbar}{\tau_e}=\frac{T}{g(L_T)}\int_{\omega^*}^{T}\frac{d\omega}{\omega}\left(\frac{T}{\omega}\right)^{(2-d)/2}, \quad (4)$$

where ω^* is the low energy cutoff to be found, and $L_T=\sqrt{D/T}$ is the temperature length. (The same result may be obtained from the calculation of the first loop correction to the self-energy.[4]) This divergence of only one contribution to the collision integral is a simple consequence of the fact that each term in collision integral is not a gauge invariant quantity, and only both terms taken together have a physical meaning (3), which is not cut-off dependent. One can argue, however, that τ_e has its own observable consequences for the quantum interference processes. Indeed, naive argument is that the ''out'' processes completely suppress the interference, whereas ''in'' processes are incoherent. Inclusion of some of the higher order processes[4,6] cures the divergence and makes the expression for $1/\tau_e$ finite. One may naively expect that τ_e found from such procedure is, indeed, responsible for the temperature behavior of quantum corrections.[10]

AAK showed[1] that it is not correct for the temperature behavior of weak localization correction, because the inelastic excitations with energy transfer smaller than decoherence

TABLE I. Temperature dependence of electron-electron scattering time scales for different dimensions d. We assume $T\gg\epsilon$.

d	$\hbar/\tau_E$	$\hbar/\tau_\phi$	$\hbar/\tau_e$
1	$(T/D\hbar)^{1/2}\nu^{-1}$	$(T^2/D\nu^2\hbar^4)^{1/3}$	$(T^2/D\nu^2\hbar^6)^{1/3}(p_F l)^{2/3}$
2	$(T/D\hbar)\nu^{-1}$	$(T/D\nu\hbar)\ln(p_F l/\hbar)$	$(T/D\nu\hbar)\ln[(p_F l/\hbar)^3\epsilon_F/T]$
3	$(T/D\hbar)^{3/2}\nu^{-1}$	$(T/D\hbar)^{3/2}\nu^{-1}$	$(T/D\hbar)^{3/2}\nu^{-1}$

rate itself do not suppress this correction, see Sec. 2.2.2 of Ref. 7 and our Sec. II for the corresponding physical argument. This leads to the infrared cutoff $\omega^* \simeq 1/\tau_\phi$ in Eq. (4) and to the self-consistency equation for the dephasing rate

$$\frac{\hbar}{\tau_\phi} \simeq \frac{T}{g(L_\phi)}, \quad L_\phi = \sqrt{D\tau_\phi}. \tag{5}$$

However, there is a prejudice, see, e.g., Ref. 6, that the inelastic time governing the magnitude of the conductance fluctuation is given by $\tau_e \ll \tau_\phi$, so that τ_e has its own observable effect.

In this paper we revisit this problem. We will show that the inelastic rate governing the mesoscopic fluctuations is precisely the same as for the weak localization, see Eq. (5). Moreover, the scaling functions governing the magnetic field and the temperature behavior of conductance fluctuations are found to be identical to their weak localization counterparts, see Secs. III, IV.

The remainder of the paper is arranged as follows. Section II is devoted to the qualitative discussion of the role of the effect of the real electron-hole pair excitations on the weak localization and mesoscopic conductance fluctuations. The main point of this section is to explain why the singlet excitations with transmitted frequency smaller than $1/\tau_\phi$ affect neither weak localization nor mesoscopic fluctuations. In Sec. III we explicitly calculate the effect of interactions on mesoscopic fluctuations of conductance in one dimension, using the same approach as AAK.[1] We will also identify the diagrammatic contributions which are missed in the arguments for the role of $\tau_e \ll \tau_\phi$ in the conductance fluctuations. Section IV generalizes the calculation to two dimensions. Our findings are summarized in Sec. V.

II. QUALITATIVE DISCUSSION

The purpose of this section is to explain interference processes, taking into account possibility of excitations of real electron-hole pairs, see also Ref. 12. For the weak localization correction, similar arguments were used in Ref. 7.

A qualitative physical interpretation of quantum corrections is usually based on the following arguments, see, e.g., Ref. 13. Consider an electron diffusing in a good conductor $p_F l \gg \hbar$. Probability w for the electron to reach, say, point i starting from point f, see Figs. 1(a), 2(a), can be obtained by first finding the semiclassical amplitudes A_α for different paths connecting the points, and then, calculating the absolute value of their sum

$$w = \left| \sum_\alpha A_\alpha \right|^2 = \sum_\alpha |A_\alpha|^2 + \sum_{\alpha \neq \beta} A_\alpha A_\beta^* . \tag{6}$$

The first term in Eq. (6) is nothing but the sum of the classical probabilities of the different paths, and it may be found from the classical Boltzmann equation. The second term is the quantum mechanical interference of the different paths. In what follows, we will discuss the contribution of this term to transport and how it is affected by the electron-electron interaction.

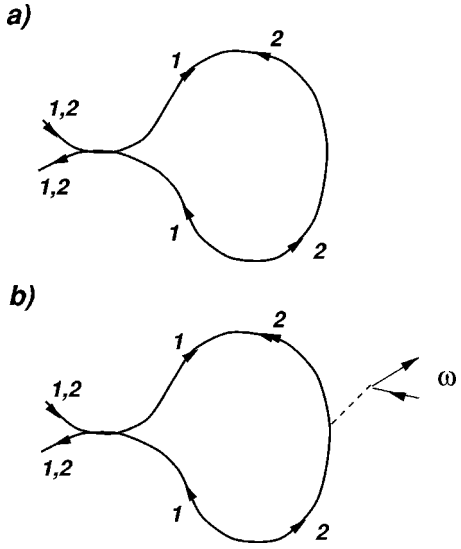


FIG. 1. (a) Example of classical paths between points i and f contributing to the weak localization. (b) The same paths with the excitation of one electron-hole pair with energy ω.

A. Weak localization correction

For generic pairs α, β, the product $A_\alpha A_\beta^*$ oscillates as the function of impurity configurations, see Fig. 2(a). This is because the lengths of paths α and β are substantially different. As the result, contribution of such paths is not relevant for disorder averaged quantities but contributes to the mesoscopic fluctuations of the conductance.

There are pairs of paths, however, which preserve the same phase, with the change of the disorder configuration. An example of such paths is shown in Fig. 1(a). These paths almost coincide everywhere except the loop segment BEB [see Fig. 1(a)] which is traversed by trajectories 1 and 2 in the opposite directions. In the absence of the magnetic field and spin-orbit interactions, the phases of the trajectories 1 and 2 are equal. Therefore, the contribution of these paths to the probability w becomes

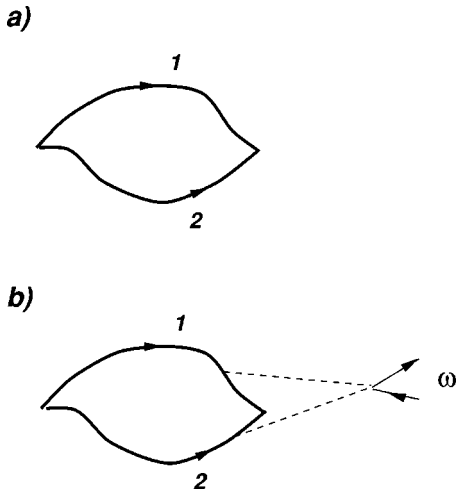


FIG. 2. (a) Example of classical paths between points i and f contributing to the mesoscopic conductance fluctuations. (b) The same paths with the excitation of one electron-hole pair with energy ω.

$$|A_1+A_2|^2=|A_1|^2+|A_2|^2+2\mathrm{Re}A_1A_2^*=4|A_1|^2, \quad (7)$$

i.e., twice larger than the classical probability. Thus, in order to evaluate the weak localization correction to the conductivity, one has to determine the classical probability to find such a self-intersecting trajectory.

Let us now consider the main effect of electron-electron interactions on the weak localization—excitation of soft electron-hole pairs. We consider processes involving either one excitation (probability P_1) or no excitations (probability $P_0=1-P_1$), see Fig. 1(b). Allowing for the excitation of an electron-hole pair, one obtains

$$A_\alpha \to A_\alpha^0 + A_\alpha^1, \quad (8)$$

where the superscripts 0 and 1 correspond to the amplitudes involving emission of no electron-hole pairs or one electron-hole pair, respectively.

Because the states with different number of excitations are orthogonal to each other, we obtain, instead of Eq. (7),

$$|A_1^0+A_1^1+A_2^0+A_2^1|^2=|A_1^0|^2+|A_2^0|^2+|A_1^1|^2+|A_2^1|^2 + 2\mathrm{Re}A_1^0[A_2^0]^*+2\mathrm{Re}A_1^1[A_2^1]^*, \quad (9)$$

where the last two terms correspond to the interference correction. It is important to emphasize that the interference persists even if the final state contains an electron-hole excitation (last term).

We now notice that the emission of a soft electron-hole pair does not alter the geometrical form of the trajectory, thus, it does not change the classical probability corresponding to path α. As the result, we have

$$|A_\alpha^0|^2=P_0|A_\alpha|^2, \quad |A_\alpha^1|^2=P_1|A_\alpha|^2, \quad (10)$$

where amplitudes without superscript correspond to those in the absence of the interaction. What the emission of the electron-hole pair may change, however, is the phase of the quantum amplitude.

Indeed, denote the point of emission of electron-hole pair of energy ω on a classical trajectory by t_α^{em}—time it takes for the electron moving along the trajectory α with energy ϵ_F to reach the emission point, see Fig. 1. Denote the total time along the path α as t_α. Then the electron moves time t_α^{em} with the energy ϵ and time $t_\alpha-t_\alpha^{em}$ with the energy $\epsilon-\omega$. As a result, the geometrical phase, accumulated by electron, changes as

$$\arg A_\alpha^1=\arg A_\alpha^0-\omega(t_\alpha-t_\alpha^{em}).$$

Thus,

$$A_\alpha^0[A_\beta^0]^*=P_0A_\alpha A_\beta^*,$$

$$A_\alpha^1[A_\beta^1]^*=P_1A_\alpha A_\beta^* e^{i\omega(t_\beta-t_\alpha-t_\beta^{em}+t_\alpha^{em})}. \quad (11)$$

Substituting Eqs. (10) and (11) into Eq. (9), we obtain [instead of Eq. (7)] for paths contributing to the weak localization correction

$$|A_1^0+A_1^1+A_2^0+A_2^1|^2 = 2|A_1|^2+2|A_1|^2[P_0+P_1\cos\omega(t_1^{em}-t_2^{em})]. \quad (12)$$

The last term in Eq. (12) describes the effect of the excitation of an electron-hole pair in the system on the weak localization correction. One can readily see that not each inelastic process destroys the interference. For instance, for $\omega\to 0$, Eq. (12) reproduces Eq. (7) exactly. On the other hand, the time t_α^{em} is shorter than τ_ϕ. Thus, we may conclude that inelastic processes with energy transfer $\omega\lesssim 1/\tau_\phi$ do not destroy the interference (see, e.g., Refs. 14,7), which gives the physical reason for the low energy cutoff $\omega^*\simeq 1/\tau_\phi$ in Eq. (4).

B. Mesoscopic conductance fluctuations

Effect of inelastic processes. The arguments of the previous subsection are easily generalized for the effect of inelastic processes on mesoscopic conductance fluctuations. We can still talk about a pair of two paths, but now we will take those paths to be generic, see Fig. 2. The interference contribution from those paths

$$\delta G\sim 2\mathrm{Re}A_1A_2^* \quad (13)$$

does not affect the average conductance because of random phases of those amplitudes, but it gives rise to the mesoscopic fluctuations of the conductance

$$\langle \delta G^2\rangle \sim 2\langle |A_1|^2\rangle\langle |A_2|^2\rangle. \quad (14)$$

Let us now consider the effect of the excitation of an electron-hole pair of energy ω. To do so, we use the qualitative argument of previous subsection [starting from Eq. (8)] and substitute Eq. (11) into Eq. (13). It yields

$$\delta G\sim 2\mathrm{Re}[A_1A_2^*(P_0+P_1e^{i\omega(t_2-t_1-t_2^{em}+t_1^{em})})]. \quad (15)$$

Once again, we arrive to the conclusion that the excitations of frequencies smaller than the inverse times to traverse the trajectories $1/t_{1,2}$, do not change the interference correction. Similarly to the weak localization the lengths of paths are limited by τ_ϕ. Thus, we may conclude that inelastic processes with energy transfer $\omega\lesssim 1/\tau_\phi$ do not affect mesoscopic fluctuations, which gives the physical reason for the low energy cutoff $\omega^*\simeq 1/\tau_\phi$ in Eq. (4). Thus, inelastic time entering the weak localization and mesoscopic fluctuations should be approximately the same. The exact equality of those times will be proven in the next section by a direct calculation, however, this result is definitely model dependent. Namely, it implies that the contribution of the quasistatic fluctuations in the systems does not overwhelm the role of the inelastic processes, and we discuss such fluctuations now.

Effect of quasistatic fluctuations. In the linear response theory, a many-body system in its stationary state is excited at some time t_1 and then the behavior of some observable quantity is studied at times $t>t_1$. If the temperature is finite,

the initial stationary state of the system can be not only its ground state E_0, but also any of many-body eigenstates, E_α; the probability that the system is initially in such a state is $\propto e^{-E_\alpha/T}$. If there were no interaction, it would result only in the thermal average of the mesoscopic fluctuations. However, electron-electron interaction leads to the effective dependence of the disordered potential for electrons. The simplest, and the most effective example of this mechanism is the dependence of the Hartree potential of the electrons on the electron configuration. Since the measurable conductance is the result of the large number of measurements, each time the initial state may be different.

In principle, one could expect that the averaging over different configurations of the self-consistent potential may lead to an effect stronger than the excitations of the electron-hole pairs. This is possible, when there is an additional slow degree of freedom such as magnetic impurities,[15] moving defects,[11] or slow fluctuations of the gauge field.[16] However, this is not the case for the Coulomb electron-electron interaction, as we explain below.

To find the magnitude of the effect, we first have to estimate the characteristic value of possible fluctuations, then evaluate the effect of such fluctuations on the mesoscopic fluctuations of conductance, and then compare this effect with effect of τ_ϕ coming from the inelastic processes. According to Nyquist noise formula, the amplitude of the electric field $\delta E(L)$ fluctuating on the spatial scale L is given by

$$\delta E^2(L) \simeq \frac{T}{\sigma_d}\frac{\overline{\omega}}{L^d}, \tag{16}$$

where σ_d is the conductivity of the system in d dimensions, and $\overline{\omega} \ll 1/\tau_\phi$ is the high-energy cutoff above which fluctuations cannot be considered as quasistatic.

To have a strong effect on mesoscopic fluctuations, the electric field should change significantly the wave functions of the electrons on the scale L_ϕ, which translates into the condition[17]

$$e|\delta E|L_\phi \gtrsim \frac{\hbar D}{L_\phi^2} = \frac{\hbar}{\tau_\phi}. \tag{17}$$

On the other hand, we estimate from Eq. (16)

$$e^2|\delta E(L_\phi)|^2 L_\phi^2 = T\overline{\omega}\frac{e^2 L_\phi^{2-d}}{\sigma_d} = \frac{T\hbar\overline{\omega}}{g(L_\phi)},$$

where $g(L)$ is the dimensionless conductance on the linear scale L. Taking into account Eq. (5) and the condition $\overline{\omega}\tau_\phi \ll 1$ we conclude that for the dephasing by the Coulomb interaction the condition (17) can be never satisfied, and therefore the quasistatic fluctuations are negligible in comparison with the inelastic processes. We reiterate that this result does not hold for the scattering on the collective modes, which have peak in their spectral density on frequencies much smaller than $1/\tau_\phi$.

Closing this section, we discuss in more detail the role of the magnetic impurities. We restrict ourselves only to the temperatures larger than the Kondo temperature T_K. It is well known that even the elastic scattering of the electrons on magnetic impurities lifts time-reversal symmetry, thus suppressing the weak localization correction. Naively, one could think that such scattering only gives rise to the unitary symmetry, thus suppressing the mesoscopic fluctuations by a factor of 2. However, the spins are a dynamical system, and during the time between the measurements the spin configuration changes completely, thus suppressing the mesoscopic fluctuations similarly to weak localization.[15,18] The rate of recently considered impurity mediated electron-electron interaction[19] is small in comparison with the one of elastic scattering as $1/\ln^2(T/T_K)$, and this mechanism cannot be independently revealed from studies of either weak localization or mesoscopic fluctuations.

III. CONDUCTANCE FLUCTUATIONS IN QUASI-ONE-DIMENSIONAL SYSTEMS

In this section, we consider a quasi-one-dimensional wire of length L and the number of transverse channels $N_\perp$. The static conductance of the wire G is expressed through the nonlocal conductivity $\sigma(x_1,x_2)$ as follows:

$$G = \frac{1}{L^2}\int dx_1 dx_2 \sigma_{xx}(x_1,x_2), \tag{18}$$

where x_1 and x_2 label the coordinates along the wire. To simplify the expressions, we disregard first inelastic processes and include them later on. We express the symmetric part of the conductivity in terms of Green's functions and substitute it in Eq. (18). We find[20]

$$G = \int \frac{d\boldsymbol{r}_1 d\boldsymbol{r}_2}{L^2}\int \frac{d\epsilon}{\pi}\frac{df}{d\epsilon}\hat{j}_{x_1} G^R(\boldsymbol{r}_1,\boldsymbol{r}_2;\epsilon)\hat{j}_{x_2} G^A(\boldsymbol{r}_2,\boldsymbol{r}_1;\epsilon), \tag{19}$$

where the integration is performed over all the sample, the spin degeneracy is taken into account, f is the Fermi distribution function, and the current operator $\hat{j}_x$ is defined as follows:

$$g_1\hat{j}_x g_2 = \frac{ie}{2m}(g_2\partial_x g_1 - g_1\partial_x g_2).$$

For the rest of the article, we employ the system of units with $\hbar = 1$, and restore $\hbar$ in the final results.

In the following, we consider only high-temperature limit, $T \gg D/L^2$, because it is the only case when the inelastic processes (rather than sample size, L) are controlling the magnitude of the fluctuations. In this case the main contribution to the conductance fluctuations is given by two-diffuson and two-cooperon diagrams, Fig. 3. The resulting correlation function for the conductance fluctuations at different magnetic fields H_1, H_2 is expressed in the time domain as

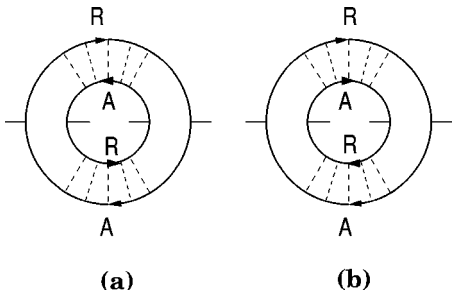


FIG. 3. Diagrams with two diffusons (a) and two cooperons (b) contributing to the conductance fluctuations. Dashed lines represent impurity scattering, interaction is not yet taken into account.

$$\overline{\delta G(H_1)\,\delta G(H_2)} = \frac{(2e^2D)^2}{3\pi TL^4}\int dx_1 dx_2 \int dt \times[|\mathcal{P}_D^{12}(x_1,x_2,t)|^2+|\mathcal{P}_C^{12}(x_1,x_2;t)|^2], \tag{20}$$

where the overbar stands for the disorder averaging. Deriving Eq. (20), one makes use of the approximation

$$\int \frac{d\epsilon_1}{2\pi}\frac{d\epsilon_2}{2\pi}\partial_{\epsilon_1} f \partial_{\epsilon_2} f e^{i(\epsilon_1-\epsilon_2)(t-t')} \approx \frac{1}{12\pi T}\delta(t-t'),$$

justified at time scale larger than $1/T$.

Semiclassical retarded diffuson and cooperon propagators entering into Eq. (20) are solutions of the equations

$$\left(\partial_t - D\partial_x^2 + \begin{Bmatrix} \frac{1}{\tau_D^{12}} \\ \frac{1}{\tau_C^{12}} \end{Bmatrix}\right)\begin{Bmatrix} \mathcal{P}_D^{12}(x,x';t) \\ \mathcal{P}_C^{12}(x,x';t) \end{Bmatrix} = \delta(x-x')\,\delta(t), \tag{21}$$

where D is the diffusion coefficient, and the symmetry breaking parameters $\tau_{C,D}^{12}$ are defined as (see Ref. 7)

$$\frac{1}{\tau_D^{12}} = \frac{e^2a^2D(H_1-H_2)^2}{12\hbar^2c^2}, \quad \frac{1}{\tau_C^{12}} = \frac{e^2a^2D(H_1+H_2)^2}{12\hbar^2c^2}, \tag{22}$$

with a being the transverse dimension of the sample. It is worth mentioning that the numerical coefficient here is geometry dependent.

So far, we merely followed a standard avenue (see, e.g., Ref. 20). Now we are prepared to introduce electron-electron interactions. On the language of diagrams, we must add to Fig. 3 all of the possible interaction lines. Since inner and outer rings represent the measurement at significantly different times, the interaction lines *do not* connect these two rings, and only may be drawn within the same ring, connecting G^R with G^R, G^R with G^A, and G^A with G^A for the same impurity configuration. Following Ref. 1, these lines are conveniently represented by external time-dependent random fields $\varphi^\alpha(x,t)$, where the index α assumes values $\alpha=1$ (outer ring) and $\alpha=2$ (inner ring). These fields are assumed to be Gaussian distributed with zero average. The correlation

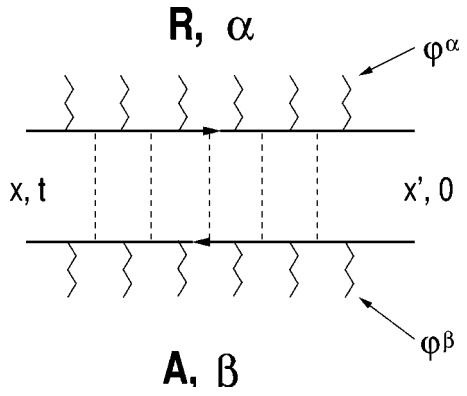


FIG. 4. CF diffuson $\mathcal{P}_D^{\alpha\beta}$. Zigzag lines represent random fields $\varphi^{\alpha,\beta}$.

function is described by the Keldysh component of the propagator of the screened Coulomb interaction

$$\langle\varphi^\alpha(x,t)\varphi^\beta(x',t')\rangle = \delta_{\alpha\beta}\delta(t-t')\frac{2T}{D\nu_1}\int\frac{dq}{2\pi}\frac{1}{q^2}e^{iq(x-x')}, \tag{23}$$

where ν_1 is the thermodynamic density of states per unit length. Equation (23) is nothing but a space-time version of

$$\langle\varphi\varphi\rangle(q,\omega) = -\mathrm{Im}\frac{2T}{\omega}\frac{Dq^2-i\omega}{Dq^2\nu_1},$$

and we assumed $T\gtrsim\omega$. This assumption is justified, because the main contribution to the dephasing rate is coming from the energy transfer ω much smaller than T. (The diagrams explicitly showing cancellation of all the processes with $\omega > T$ can be found, e.g., in Refs. 20,6.) Because we also disregard all effects due to finite size of the sample, this implies the following hierarchy of energy scales

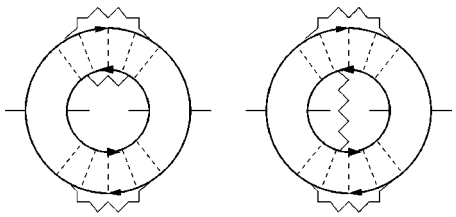
$$T \gg \tau_\phi^{-1} \gg E_c \equiv D/L^2 \tag{24}$$

(here τ_ϕ stands not only for the phase-relaxation time, but for all time scales due to electron-electron scattering). In the following, we assume that the conditions (24) are satisfied.

The factor $\delta_{\alpha\beta}$ in the right-hand side of Eq. (23) explicitly indicates that the fields attached to outer and inner rings of the diagram Fig. 4 are uncorrelated, i.e., no interaction lines, indeed, can be drawn between the rings. The momentum integral in Eq. (23) diverges, but our final result will contain well-defined differences of integrals of this type.

Introduction of the fluctuating fields modifies the equations for the diffuson and cooperon (21), see Fig. 4 and Ref. 1, which now become the functionals of the fluctuating fields

$$\left[\partial_t - D\partial_x^2 + i(\varphi^\alpha(x,t)-\varphi^\beta(x,t)) + \begin{Bmatrix} \frac{1}{\tau_D^{\alpha\beta}} \\ \frac{1}{\tau_C^{\alpha\beta}} \end{Bmatrix}\right] \times \begin{Bmatrix} \mathcal{P}_D^{\alpha\beta}(x,x';t;\{\varphi^\alpha(x,t),\varphi^\beta(x,t)\}) \\ \mathcal{P}_C^{\alpha\beta}(x,x';t;\{\varphi^\alpha(x,t),\varphi^\beta(x,t)\}) \end{Bmatrix} = \delta(x-x')\,\delta(t). \tag{25}$$



(a) **(b)**

FIG. 5. Examples of diagrams with interaction (shown as zigzag lines) contributing to conductance fluctuations. The diagram (a) is reduced to the CF diffuson, while the diagram (b) is not. Conclusion about the differences of inelastic rates for weak localization and conductance fluctuation is a consequence of missing the diagram (b).

The correlation function of conductances is given by the equation similar to Eq. (20), but all the interaction lines in Eq. (25) are connected by the propagator (23)

$$\overline{\delta G(H_1)\,\delta G(H_2)} = \frac{(2e^2D)^2}{3\pi TL^4}\int dx_1 dx_2 \int dt \times[\langle|\mathcal{P}_D^{12}(x_1,x_2,t)|^2\rangle_\varphi + \langle|\mathcal{P}_C^{12}(x_1,x_2;t)|^2\rangle_\varphi], \tag{26}$$

where $\langle\cdots\rangle_\varphi$ stand for the averaging over the fluctuating field φ[1,2].

Before we perform actual calculation in Eq. (26), we pause for a moment to discuss a relation of this formula with the other theoretical work.[6] We observe that the propagator $\langle\mathcal{P}_D^{12}\rangle_\varphi$ contains all possible interaction lines drawn between G^R and G^R, and also between G^A and G^A, but not between G^R and G^A. This is exactly an object (let us call it *CF diffuson*), which determines the out-scattering term in the collision integral in the Boltzmann equation, and it was studied in detail in Ref. 6. In contrast to the "ordinary" diffuson, which is insensitive to electron-electron interaction due to Ward's identity (charge conservation), the CF diffusion $\langle\mathcal{P}_D^{12}\rangle_\varphi$ acquires a massive pole, real part of which is identified with the out-scattering time τ_e. One can thus imagine (and this was, indeed, conjectured in Ref. 6) that the temperature dependence of conductance fluctuations is governed by the time τ_e, which is parametrically different from τ_ϕ. The calculation presented below shows that this conjecture is not correct. The resolution of this fallacy is that the averaging in Eq. (26), which is essentially coupling of all random fields φ^α according to the rules (23), produces not only a contribution which contains averages $|\langle\mathcal{P}^{12}\rangle_\varphi|^2$ [Fig. 5(a)], but also diagrams where interaction lines connect upper and lower Green's functions within the same ring [Fig. 5(b)]. Both contributions diverge in the infrared limit (and have to be regularized in order to extract sensible results[6]), but their sum is well behaved.

To proceed with the evaluation of Eq. (26), we write $\mathcal{P}^{\alpha\beta}$ as a functional integral,[21,1]

$$\mathcal{P}_{D,C}^{\alpha\beta}(x,x';t;\{\varphi(x,t)\}) = \frac{\theta(t)}{\mathcal{Z}} e^{-t/\tau_{D,C}^{\alpha\beta}} \int_{y(0)=x'}^{y(t)=x} \mathcal{D}y(\tau) \exp\left(\int_0^t d\tau \left\{ -\frac{\dot{y}^2(\tau)}{4D} + i\varphi^\alpha[y(\tau),\tau] - i\varphi^\beta[y(\tau),\tau] \right\} \right), \tag{27}$$

where $\theta(t)$ is the step function, $\mathcal{Z}$ is the normalization factor, that will be included in the measure of the functional integration in all of the subsequent formulas. Substituting this expression into Eq. (26), and averaging over Gaussian random fields ($\langle e^{i\varphi}\rangle = e^{-\langle\varphi^2\rangle/2}$), we obtain with the help of Eq. (23)

$$\overline{\delta G(H_1)\,\delta G(H_2)} = \frac{(2e^2D)^2}{3\pi TL^4}\int dx_1 dx_2 \int_0^\infty dt (e^{-2t/\tau_D^{12}} + e^{-2t/\tau_C^{12}}) \times \int_{y_1(0)=x_2}^{y_1(t)=x_1} \mathcal{D}y_1(t) \int_{y_2(0)=x_2}^{y_2(t)=x_1} \mathcal{D}y_2(t) \times \exp\left\{ -\int_0^t dt' \left[\frac{\dot{y}_1^2}{4D} + \frac{\dot{y}_2^2}{4D} + \frac{2T}{D\nu}\left| y_1(t') - y_2(t') \right| \right] \right\}. \tag{28}$$

Following Ref. 1, we introduce new variables

$$z_{1,2}(t) = \frac{y_1(t) \pm y_2(t)}{\sqrt{2}}.$$

This yields

$$\overline{\delta G(H_1)\,\delta G(H_2)} = \frac{(2e^2D)^2}{3\pi TL^4}\int dx_1 dx_2 \int_0^\infty dt \times J_1(\sqrt{2}x_1, \sqrt{2}x_2; t) J_2(0,0;t) \times (e^{-2t/\tau_D^{12}} + e^{2t/\tau_C^{12}})$$

$$J_1(x_1,x_2;t) = \int_{z_1(0)=x_2}^{z_1(t)=x_1} Dz_1(t) \exp\left(-\int_0^t dt' \frac{\dot{z}_1^2}{4D} \right)$$

$$J_2(x_1,x_2;t) = \int_{z_2(0)=x_2}^{z_2(t)=x_1} Dz_2(t) \times \exp\left\{ -\int_0^t dt' \left[\frac{\dot{z}_2^2}{4D} + \frac{2\sqrt{2}Te^2}{\sigma_1}|z_2(t')| \right] \right\}, \tag{29}$$

where σ_1 is the one-dimensional conductivity, and we used Einstein relation $\sigma_1 = e^2\nu_1 D$.

Now we represent these functional integrals $J_{1,2}$ as solutions of differential equations. The integral J_1 solves

$$(\partial_t - D\partial_{x_1}^2) J_1 = \delta(t)\,\delta(x_1 - x_2). \tag{30}$$

Integrating both sides of Eq. (30) over x_1 and x_2, and neglecting the boundary term at $t \ll L^2/D$, we obtain

$$\int dx_1 dx_2 J_1(\sqrt{2}x_1, \sqrt{2}x_2, t) = \frac{L}{\sqrt{2}}\theta(t). \tag{31}$$

Similarly, J_2 obeys the equation

$$\left(\partial_t - D\partial_{x_1}^2 + \frac{2\sqrt{2}Te^2}{\sigma_1}|x_1|\right) J_2 = \delta(t)\delta(x_1 - x_2). \tag{32}$$

Substituting Eq. (31) into Eq. (29), and using Eq. (32), we find

$$\overline{\delta G(H_1)\delta G(H_2)} = \frac{(2e^2D)^2}{3\sqrt{2}\pi TL^3}[Q_D(x=0) + Q_C(x=0)], \tag{33}$$

and $Q_{D,C}(x)$ obeys the equation

$$\left(\frac{2}{\tau_{C,D}^{12}} - D\partial_x^2 + \frac{2\sqrt{2}Te^2}{\sigma_1}|x|\right) Q_{C,D}(x) = \delta(x). \tag{34}$$

Equation (34) has been previously considered in Ref. 1, and it has the solution in terms of the Airy function $\mathrm{Ai}(x)$,

$$Q_{C,D}(x) = -\frac{L_\phi}{2\sqrt{2}D} \frac{\mathrm{Ai}\left(\frac{\tau_\phi}{\tau_{C,D}^{12}} + \frac{\sqrt{2}|x|}{DL_\phi}\right)}{Ai'\left(\frac{\tau_\phi}{\tau_{C,D}^{12}}\right)}, \tag{35}$$

where the dephasing time τ_ϕ and the dephasing length L_ϕ have exactly the same form as for the weak localization correction[1,5] (numerical coefficient is corrected in Ref. 7),

$$\frac{1}{\tau_\phi} = \left(\frac{e^2T\sqrt{D}}{\hbar^2\sigma_1}\right)^{2/3}, \quad L_\phi = \sqrt{D\tau_\phi}. \tag{36}$$

Substituting Eq. (35) into Eq. (33), one finally obtains

$$\overline{\delta G(H_1)\delta G(H_2)} = \left(\frac{e^2}{\hbar}\right)^2 \frac{\hbar D}{3\pi L^2 T}\frac{L_\phi}{L} \times\left[\eta\left(\frac{\tau_\phi}{\tau_D^{12}}\right) + \eta\left(\frac{\tau_\phi}{\tau_C^{12}}\right)\right],$$

$$\eta(x) = -\frac{1}{[\ln \mathrm{Ai}(x)]'}. \tag{37}$$

Equation (37) with entries (22) and (36) is the main quantitative result of the present section. It shows that the dephasing rate governing temperature and magnetic field dependence of the mesoscopic fluctuations is *exactly* the same as in weak localization. Moreover, this result can be combined with the expression for the weak localization correction

$$\delta G_{\mathrm{WL}}(H_1) = \frac{\delta\sigma_{\mathrm{WL}}(H_1)}{L}; \quad \delta\sigma_{\mathrm{WL}}(H_1) = -\frac{e^2L_\phi}{\pi\hbar}\eta\left(\frac{\tau_\phi}{\tau_C^{11}}\right)$$

to the form free of geometrical uncertainties (as well as uncertainties in the value of the diffusion coefficient)

$$\overline{\delta G(H_1)\delta G(H_2)} = \left(\frac{e^2}{\hbar}\right)\frac{\hbar D}{3L^2T}\left|\delta G_{\mathrm{WL}}\left(\frac{H_1 - H_2}{2}\right) + \delta G_{\mathrm{WL}}\left(\frac{H_1 + H_2}{2}\right)\right|. \tag{38}$$

This result gives the relation between two measurable quantities, and thus may serve as a test for the dephasing mechanism. Equations (37) and (38) are valid provided $\hbar/\tau_{D,C}^{12} \ll T$. It is also assumes that there is no spin-orbit interaction. It may be shown that in the case of strong spin-orbit (SO) interaction, the result (38) still holds up to a numerical factor of 1/2:

$$\overline{\delta G(H_1)\delta G(H_2)} = \left(\frac{e^2}{\hbar}\right)\frac{\hbar D}{6L^2T} \times\left|\delta G_{\mathrm{WL}}\left(\frac{H_1 - H_2}{2}\right) + \delta G_{\mathrm{WL}}\left(\frac{H_1 + H_2}{2}\right)\right|.$$

In the case of the crossover between strong and weak SO interaction one has to identify the singlet δG_s and triplet δG_t contributions to the weak localization correction $\delta G_{\mathrm{WL}} = 3\delta G_t - \delta G_s$ by corresponding fits and replace δG_{WL} in Eq. (38) with $[\delta G_{WL} + 2\delta G_s]/2 = [3\delta G_t + \delta G_s]/2$.

Now, for conceptual clarity, we employ the result (37) to extract the relaxation time associated with conductance fluctuations. It is important that this time is unphysical by itself, and only has a meaning when explicitly linked to Eq. (37).

For this purpose, we take $H_1 = H_2 = 0$ and define the time τ_T as a mass in the pole in the CF diffuson $\mathcal{P}_D^{12}$ and CF cooperon $\mathcal{P}_C^{12}$ which enter Eq. (20). Writing

$$\mathcal{P}_{C,D}^{12}(x,x',t) = \int \frac{dqd\omega}{(2\pi)^2} e^{iq(x-x') - i\omega t}\frac{1}{Dq^2 - i\omega + \tau_T^{-1}},$$

substituting this expression into Eq. (20) and performing the integration, we obtain for conductance fluctuations

$$\overline{\delta G^2} = \frac{(2e^2D)^2}{6\pi\hbar TL^3}\left(\frac{\tau_T}{D}\right)^{1/2}. \tag{39}$$

Comparing this to the result (37), we identify the inelastic relaxation time τ_T responsible for the temperature dependence of conductance fluctuations

$$\tau_T = \eta^2(0)\tau_\phi \approx 0.53\tau_\phi, \tag{40}$$

where τ_ϕ is defined in Eq. (36), i.e., it is precisely the same time one obtains if one considers weak localization by introducing a finite mass in the pole of the cooperon. Thus, the temperature dependence of conductance fluctuations does not produce a new time scale as compared to Eq. (36) and is

certainly not determined by the out-scattering time τ_e. The numerical coefficient 0.53 reflects the behavior of the scaling function (37) in low magnetic fields.

IV. TWO-DIMENSIONAL CASE

Equation (38) can be readily generalized to the two dimensional sample, and we outline the main steps of the corresponding derivation. Consider a two-dimensional system of the size L. Performing the same steps as in the derivation of Eq. (26) one finds

$$\overline{\delta G(H_1)\delta G(H_2)} = \frac{(2e^2D)^2}{3\pi TL^4}\int d^2\boldsymbol{r}_1 d^2\boldsymbol{r}_2 \int dt \times[\langle|\mathcal{P}_D^{12}(\boldsymbol{r}_1,\boldsymbol{r}_2,t)|^2\rangle_\varphi + \langle|\mathcal{P}_C^{12}(\boldsymbol{r}_1,\boldsymbol{r}_2;t)|^2\rangle_\varphi], \tag{41}$$

where two-dimensional integrations are performed within the sample, $\langle\cdots\rangle_\varphi$ stand for the averaging over the fluctuating field $\varphi^{1,2}$ with correlation function analogous to Eq. (23),

$$\langle\varphi^\alpha(\boldsymbol{r},t)\varphi^\beta(\boldsymbol{r}',t')\rangle = \delta_{\alpha\beta}\delta(t-t')\frac{2T}{D\nu_2}\int\frac{d^2q}{(2\pi)^2}\frac{e^{i\boldsymbol{q}(\boldsymbol{r}-\boldsymbol{r}')}}{q^2}, \tag{42}$$

with ν_2 being the thermodynamic density of states per unit area. In Eq. (42), the integration is limited from above by $|q|\simeq(T/D)^{1/2}$. Such an accuracy of the ultraviolet cutoff is sufficient for the logarithmically divergent integral.

Diffuson and cooperon propagators entering Eq. (41) are the solutions of the two-dimensional analog of Eq. (25)

$$\left[\partial_t - D\begin{Bmatrix}\nabla_D^{\alpha\beta}\\ \nabla_C^{\alpha\beta}\end{Bmatrix}^2 + i[\varphi^\alpha(\boldsymbol{r},t)-\varphi^\beta(\boldsymbol{r},t)] + \begin{Bmatrix}\frac{1}{\tau_D^{\alpha\beta}}\\ \frac{1}{\tau_C^{\alpha\beta}}\end{Bmatrix}\right] \times\begin{Bmatrix}\mathcal{P}_D^{\alpha\beta}[\boldsymbol{r},\boldsymbol{r}';t;\{\varphi^\alpha(\boldsymbol{r},t),\varphi^\beta(\boldsymbol{r},t)\}]\\ \mathcal{P}_C^{\alpha\beta}[\boldsymbol{r},\boldsymbol{r}';t;\{\varphi^\alpha(\boldsymbol{r},t),\varphi^\beta(\boldsymbol{r},t)\}]\end{Bmatrix} = \delta(\boldsymbol{r}-\boldsymbol{r}')\delta(t), \tag{43}$$

where times $1/\tau_{D,C}$, see Eq. (22), describe the effect of the magnetic field component parallel to the film plane. The effect of the magnetic field perpendicular to the plane is described by

$$\nabla_\gamma^{\alpha\beta}\equiv\nabla+\frac{ie}{c}\boldsymbol{A}_\gamma^{\alpha\beta},\quad \alpha,\beta=1,2\quad \gamma=D,C,$$

$$\boldsymbol{A}_D^{\alpha\beta}=\boldsymbol{A}^\alpha-\boldsymbol{A}^\beta,\quad \boldsymbol{A}_C^{\alpha\beta}=\boldsymbol{A}^\alpha+\boldsymbol{A}^\beta, \tag{44}$$

where the vector potentials are such that

$$\nabla\times\boldsymbol{A}^\alpha=\boldsymbol{H}_\perp^\alpha,$$

and $\boldsymbol{H}_\perp^\alpha$ is the component of is the magnetic field perpendicular to the plane.

Transformations leading to Eqs. (26) and (25) are pretty much the same as in 1D provided we make obvious changes $x\to\boldsymbol{r}$, $q\to\boldsymbol{q}$, $\partial_x\to\nabla_{\boldsymbol{r}}$. Writing again the CF diffusons and cooperons $\mathcal{P}_{D,C}^{\alpha\beta}$ as functional integrals (27) and performing an averaging over Gaussian fields φ^α, we obtain a two-dimensional analog of Eq. (28)

$$\overline{\delta G(H_1)\delta G(H_2)} = \frac{(2e^2D)^2}{3\pi TL^4}\int d^2\boldsymbol{r}_1 d^2\boldsymbol{r}_2\int_0^\infty dt\sum_{\gamma=D,C}e^{-2t/\tau_\gamma^{12}} \times\int_{\boldsymbol{y}_1(0)=\boldsymbol{r}_2}^{\boldsymbol{y}_1(t)=\boldsymbol{r}_1}\mathcal{D}\boldsymbol{y}_1(t)\int_{\boldsymbol{y}_2(0)=\boldsymbol{r}_2}^{\boldsymbol{y}_2(t)=\boldsymbol{r}_1}\mathcal{D}\boldsymbol{y}_2(t) \times\exp\left\{-\int_0^t dt'\left[\frac{\dot{\boldsymbol{y}}_1^2}{4D}+\frac{\dot{\boldsymbol{y}}_2^2}{4D}+\frac{ie}{c} \times[\boldsymbol{A}_\gamma^{12}(\boldsymbol{y}_1)\dot{\boldsymbol{y}}_1-\boldsymbol{A}_\gamma^{12}(\boldsymbol{y}_2)\dot{\boldsymbol{y}}_2]+\frac{4T}{D\nu}\int\frac{d\boldsymbol{q}}{(2\pi)^2} \times\frac{1}{q^2}\{1-\cos[\boldsymbol{q}(\boldsymbol{y}_1(t')-\boldsymbol{y}_2(t'))]\}\right]\right\}. \tag{45}$$

Introducing new variables

$$\boldsymbol{R}(t)=\frac{\boldsymbol{y}_1(t)+\boldsymbol{y}_2(t)}{2},\quad \boldsymbol{r}(t)=\boldsymbol{y}_1(t)-\boldsymbol{y}_2(t),$$

and reducing the functional integrals back to differential equations, we obtain the result

$$\overline{\delta G(H_1)\delta G(H_2)} = \frac{(2e^2D)^2}{3\pi TL^2}[Q_D(|\boldsymbol{r}|=L_T)+Q_C(|\boldsymbol{r}|=L_T)], \tag{46}$$

where $Q_{D,C}(\boldsymbol{r})$ obeys the equation

$$\left[-D(\nabla_{D,C}^{12})^2+U(r)+\frac{1}{\tau_{D,C}^{12}}\right]Q_{C,D}(\boldsymbol{r})=\delta(\boldsymbol{r}), \tag{47}$$

and the potential is given by

$$U(r)=\frac{2T}{D\nu}\int\frac{d\boldsymbol{q}}{(2\pi)^2}\frac{1-\cos(\boldsymbol{q}\cdot\boldsymbol{r})}{q^2}\approx\frac{T}{\pi D\nu}\ln\left(\frac{L_T+r}{L_T}\right), \tag{48}$$

where the last expression and Eq. (46) are written with the logarithmic accuracy and we take into account the high-momentum cutoff at $q\sim L_T^{-1}$, $L_T=(D/T)^{1/2}$.

Equations (46)–(48) should be compared with the corresponding expression for the weak localization correction in two dimensions[1,5]

$$\delta\sigma(H_1)=-\frac{e^2}{\pi\hbar}C(r=l),$$

$$\left[-D(\nabla_C^{11})^2+U(r)+\frac{1}{\tau_C^{11}}\right]C(\boldsymbol{r})=\delta(\boldsymbol{r}), \tag{49}$$

where the logarithmic divergence should be cut at the elastic mean free path l.

Therefore, we conclude that the relation similar to Eq. (38) should hold,

$$\overline{\delta G(H=0)\,\delta G(H=0)} - \overline{\delta G(H_1)\,\delta G(H_2)} = \left(\frac{e^2}{\hbar}\right)\frac{\hbar D}{3L^2T}\left|\delta\sigma_{\rm WL}\left(\frac{H_1-H_2}{2}\right) + \delta\sigma_{\rm WL}\left(\frac{H_1+H_2}{2}\right) - 2\,\delta\sigma_{\rm WL}(0)\right|. \quad (50)$$

It is important to emphasize that the relation (50) holds even before one starts an approximate solution of Eq. (47). Note, however, that the result similar to Eq. (38) does not hold, since both $\overline{\delta G\,\delta G}$ and $\delta\sigma_{\rm WL}$ diverge logarithmically with different cutoffs. This is why in Eq. (50) we had to subtract zero-field contributions, which cancels logarithmic divergences. The effect of the spin orbit interactions on our final result (50) is the same as for one-dimensional geometry, see discussion after Eq. (38).

We write here the explicit expression[1,5] for the weak localization correction in two dimensions for the reference purpose

$$\delta\sigma_{\rm WL}(H,T) = -\frac{e^2}{2\pi^2\hbar}\left[\ln\frac{1}{\tau\Omega_H} - \Psi\left(\frac{1}{2}+\frac{1}{\tau^*\Omega_H}\right)\right],$$

where $\Psi(x)$ is the digamma function, $\Omega_H = 4eDH_\perp/c\hbar$, and τ_* is determined by the equation

$$\frac{1}{\tau^*} = \frac{1}{\tau_H} + \frac{T}{\hbar}\frac{e^2R_\Box}{2\pi\hbar}\ln\frac{T}{\hbar/\tau^* + \hbar\Omega_H}.$$

Similarly to one dimension, we can also extract the inelastic time τ_T, defined as a pole of CF diffuson in zero magnetic field. An explicit calculation gives $\tau_T \approx \tau_\phi$. This relation contains a numerical coefficient of order one, which can only be determined by going beyond the logarithmic accuracy. We do not attempt such a calculation in this paper.

V. DISCUSSION AND CONCLUSIONS

Equations (38) and (50) are the main results of our paper. They give exact relations which must hold between two experimentally observable results for the dephasing by the electron-electron interaction. The only reason for violation of such a relation is that other channels of dephasing with small frequency transfer are present. Thus, the systematic measurements of dependence of conductance fluctuations on temperature and magnetic field and comparing it with the weak localization data obtained on the same sample may give information on the nature of inelastic interactions in disordered metals.

We are not aware of attempts to make such a comparison between inelastic times directly. However, recently Hoadley, McConville, and Birge (HMB)[22,23] presented very careful measurements of the magnetic field dependence of $1/f$ noise in silver films. A standard assumption in the theory of $1/f$ noise in metals (for review, see Ref. 24) is that it is produced by low-frequency motion of impurities. Mathematically, the magnitude of $1/f$ noise in such a model is given by a set of diagrams identical to those for conductance fluctuations (Figs. 3, 5) with the only difference that external and internal rings are described by different impurity configurations.[25,26] As the result the field dependence and the temperature dependence of the noise should be given by the parametric derivative of Eq. (50), i.e., it should be expressed through the derivatives of the parallel field dependence of the weak localization.

HMB compared the time scale defined as a pole in the diffuson (in our notations, τ_T), with the phase relaxation time τ_ϕ, extracted from their own measurements of the weak localization correction on the same films. Their procedure results in $\tau_T \simeq \tau_\phi/2.6$, which was interpreted to be consistent with the theory of Ref. 6. Our results (50) contradict that interpretation.[27]

To our opinion, the only possible reason of this discrepancy is the electron-electron interaction in the triplet channel which we did not take into account. This interaction can be singled out in experiments with the materials with stronger spin-orbit scattering. Other sources of $1/f$ noise seem to be excluded, since the functional form of the experimentally measured by HMB magnetic field dependence perfectly fits theoretical predictions. Dephasing on slow moving impurities itself, see discussion in Sec. II B, would give a temperature dependence different than that in experiment and may be ruled out. We believe that the contradiction between the theory and the experiment revealed in our paper indicates that the quantitative study of inelastic processes in mesoscopic samples remains an interesting topic and deserves future investigation.

ACKNOWLEDGMENTS

One of us (I.A.) was supported by Packard foundation. We are grateful to B.L. Altshuler and N.O. Birge for the discussion of the results.

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