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A Pure-Inertia Method for Dynamic Balancing of Symmetric Planar Mechanisms

Jan J. de Jong[1,3](✉), Yuanqing Wu[2], Marco Carricato[2], and Just L. Herder[1,3]

[1] University of Twente, Enschede, The Netherlands
j.j.dejong@utwente.nl
[2] University of Bologna, Bologna, Italy
{yuanqing.wu,marco.carricato}@unibo.it
[3] Delft University of Technology, Delft, The Netherlands
j.l.herder@tudelft.nl

Abstract. In this paper, we present a novel method for the dynamic balance of planar mechanisms, by transforming a mechanism into a dynamically equivalent form where all links have zero mass but non-zero moment of inertia. The dynamic balance of such pure-inertia systems is shown to be governed by mirror symmetry that cancels out the system's total angular momentum. Our method not only covers well-known dynamically balanced 1-DOF mechanisms, such as the slider-crank and four-bar linkages, but also leads to the discovery of a novel dynamically balanced 2-DOF planar mechanism.

Keywords: Dynamic balance · Dynamic equivalence
Equimomental systems · Pure-inertia method · Symmetric space

1 Introduction

For robots and machines operating at high speeds, dynamic balance is desired to eliminate varying reaction forces and moments (also termed shaking forces and moments), which are known to be a major source of wear, noise and accuracy degradation [1]. The necessary and sufficient condition for dynamic balancing is that both the linear and angular momenta of the mechanism are constant. A mechanism is force balanced when only the linear momentum is constant, which in practice comes down to having a fixed center of mass (COM) of the system.

In principle, dynamic balance can be found by symbolically solving the kinematic and balancing equations for unknown kinematic and inertial parameters. A solution is in general not guaranteed, or otherwise very difficult to find, due to high algebraic complexity [1,2]. Alternatively, one may construct dynamically balanced mechanisms from primitive functional modules which, either are dynamically balanced [3] or have the dynamic balance conditions that are easy to obtain symbolically [4–6]. The dynamic balance equations can be simplified by

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reducing the number of parameters through dynamic equivalence [7]. Two mechanisms with equal kinematics, but different mass distributions, are said to be dynamically equivalent or equimomental [8] if their linear and angular momenta are equal when undergoing a common arbitrary motion. Consequently, the two mechanisms will always have the same shaking forces and moments even though their masses, COMs, and moments of inertia are different.

A fruitful source of dynamically balanced mechanisms is the class of linkages, such as the pantograph [6], the slider-crank [4], and the four-bar [1], whose geometric structures exhibit a mirror symmetry about a plane moving with half magnitude (hence also half velocity) of that of the end-effector for full-cycle motion. Recently, the second and third author of this paper made an addendum to this class of mechanisms by systematically investigating plane and line symmetry of a class of parallel mechanisms known as symmetric subspace motion generators [9,10].

The main contribution of this paper is twofold. Firstly, we propose a dynamic balance method via dynamic equivalence to massless systems with only moments of inertia. This approach, called pure-inertia method (PIM), provides a unified understanding of several existing dynamically balanced mechanisms that were previously synthesized by solving complex algebraic equations. Secondly, the PIM is applied to a novel 2-DOF planar symmetric mechanism proposed in [10]. Numerical simulation confirms that the mechanism is dynamically balanced.

2 Pure-Inertia Method

The pure-inertia method relies on the dynamic equivalence of all links in a mechanism to a zero mass with non-zero moment of inertia. A pure inertia may be interpreted as dynamically equivalent to an infinitely large and thin ring.

The principle of dynamic equivalence is illustrated in Fig. 1. A range of dynamically equivalent links can by found via *point mass redistribution* [7]. The initial inertial parameters comprise mass m_o, COM $\boldsymbol{c}_o$, and moment of inertia g_o (Fig. 1(a)), whereas for the dynamic equivalent set we have: m_f, $\boldsymbol{c}_f$, and g_f (Fig. 1(b)). If we start with a link hinged at $\boldsymbol{o}$, we may transfer a point mass of

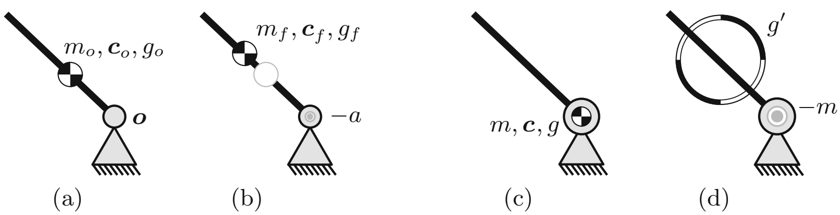

Fig. 1. Dynamic equivalence via mass redistribution. (a) A simple pendulum; (b) a dynamic equivalent of (a) via adding or subtracting point mass at the hinge; (c) a pendulum with COM located at the hinge; (d) a dynamic equivalent of (c) with pure moment of inertia.

arbitrary value a - located at the hinge - from the link to the base and vice versa, without changing the link's momentum and shaking forces and moments. In the same manner, the link may exchange a point mass with any body i hinged to it by a revolute joint at a location denoted by $\boldsymbol{o}_i$. This is a valid dynamic equivalent operation, since at a revolute joint the linear velocities of both connecting bodies are equal. Therefore changing the attachment of the a point mass from one body to the other does not affect the mechanism's momentum. We have the following equations:

$$m_f = m_o + \sum_{i=1}^{n} a_i, \quad m_f \boldsymbol{c}_f = m_o \boldsymbol{c}_o + \sum_{i=1}^{n} a_i \boldsymbol{o}_i, \quad g_f = g_o - m_f \|\boldsymbol{c}_f\|^2 + \sum_{i=1}^{n} a_i \|\boldsymbol{o}_i\|^2 \tag{1}$$

To ensure dynamic equivalence, the point mass added to body i should be subtracted from body j, which we refer to as *mass continuity*. If both $m_f = 0$ and $\boldsymbol{c}_f = \boldsymbol{0}$ (Fig. 1(c)), i.e., if $m_o = -\sum_{i=1}^{n} a_i$ and $m_o \boldsymbol{c}_o = -\sum_{i=1}^{n} a_i \boldsymbol{o}_i$, the dynamics of the body is completely determined by a pure moment of inertia (Fig. 1(d)). This pure moment of inertia - or for brevity; pure inertia - shall be denoted with a prime: g'. When this holds for all bodies in the mechanism, the mechanism is said to be a pure-inertia mechanism.

A pure-inertia body is not physically feasible, but it can be transformed back into a feasible one by applying the reverse process of *point mass recomposition* [7], i.e., by adding point masses at the revolute joints. For the purpose of this paper, we shall consider bodies connected with at most two revolute joints, and therefore all vectors in Eq. 1 can be treated as scalars. By assuming that one of the joints is located at the origin and the other at a distance l, the following relations hold for the mass, COM and moment of inertia:

$$m = a_1 + a_2, \qquad mc = a_2 l, \qquad g = g' + a_2 \left(1 - \frac{a_2}{a_1 + a_2}\right) l^2 \tag{2}$$

The links generated from dynamically equivalent pure-inertia bodies, remain physically feasible if the equivalent mass m and moment of inertia g satisfy the following positivity constraints:

$$m > 0 \Leftrightarrow a_1 > -a_2, \qquad g > 0 \Leftrightarrow \frac{g'}{l^2} > a_2 \left(\frac{a_2}{a_1 + a_2} - 1\right) \tag{3}$$

which implies that a pure inertia, denoted g', is allowed to have a negative value. The limits on a_1 and a_2 are carried over to adjacent bodies due to mass continuity. See [7] for more details about the mass redistribution method. It should be noted that a pure-inertia mechanism has a fixed COM with respect to the base and is therefore necessarily force balanced. However, the reverse does not hold in general.

2.1 Dynamic Balance

In the literature, force balance conditions are usually inspected prior to moment balancing. In comparison, for a pure-inertia mechanism, force balance conditions are automatically satisfied and therefore only the angular momentum, denoted ξ, needs to be canceled out to achieve dynamic balance:

$$\xi = \sum_i g_i' \omega_i = 0 \tag{4}$$

where the moments of inertia g_i' are pose independent. The pose dependence of the angular velocities ω_i, on the other hand, imposes conditions on the geometry of the mechanism [1].

2.2 Dynamic Balance of the Slider-Crank Linkage Using the PIM

We shall now illustrate the PIM by considering the dynamic balance of a slider-crank linkage, as shown in Fig. 2. According to [4], a slider-crank linkage can be dynamically balanced under symmetric kinematic conditions ($l_1 = l_2$, $l_3 = 0$), if the total COM is located at the base revolute joint for all configurations, and links' inertial parameters satisfy:

$$c_2 = 0, \qquad m_1 c_1 = -m_2 l_1, \qquad g_1 = g_2 - m_2 \left(\frac{m_2}{m_1} + 1 \right) l_1^2 \tag{5}$$

It is apparent that the symmetry conditions constrain link 1 and 2 to move with equal and opposite angular velocities: $\omega_1 = -\omega_2$. In reference to Eq. 4, the dynamic balance condition for the slider-crank linkage reduces to:

$$g_2' = g_1' \tag{6}$$

We shall show that Eq. 6 is exactly equivalent to the conditions given in [4]. Via the mass recomposition - refer to Eq. 2 - we have:

$$\begin{aligned} m_1 &= a_{10} - a_{21}, & m_1 c_1 &= -a_{21} l_1, & g_1 &= g_1' - a_{21} \left(1 + \tfrac{a_{21}}{a_{10} - a_{21}} \right) l_1^2 \\ m_2 &= a_{21}, & m_2 c_2 &= 0, & g_2 &= g_1' \end{aligned} \tag{7}$$

We remark that the dynamics of the slider can be easily included in the above process. Because it generates no angular momentum (it only translates), it is dynamically equivalent to a point mass which can be included in the dynamic balance conditions via point mass recomposition, as in Eq. 1.

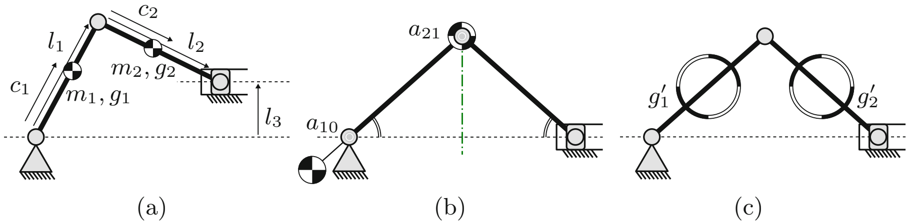

Fig. 2. Dynamic balance conditions for a slider-crank mechanism. (a) Kinematic and inertial parameters. (b) Conventional dynamic balance conditions for a symmetric slider-crank. The symmetry line is represented by the green dash-dotted line. (c) The pure-inertia equivalent of the dynamically balanced slider-crank

2.3 Dynamic Balance of the Four-Bar Linkage Using the PIM

Gosselin and Ricard [1] showed that two types of symmetric four-bar linkages can be dynamically balanced. The first one is the kite type, shown in Fig. 3. It is symmetric about a plane passing through joint q_2 and q_4 (or equivalently q_1 and q_3). The second type, the anti-parallelogram, is symmetric about a plane which bisects the angle formed by the lines through body l_1 and l_3.

We shall focus on the first type here (the same procedure can be applied to the anti-parallelogram). The kinematic symmetry conditions are: $l_1 = l_2$, and $l_0 = l_3$ [3], whereas the dynamic balance conditions are:

$$\begin{aligned} & & g_1 &= g_{c1} + m_1 c_1 (l_1 - c_1) \\ m_2 c_2 &= m_1 c_1 + m_2 l_1, & g_2 &= g_{c1} + m_2 c_2 (l_1 - c_2) \\ m_3 c_3 &= \tfrac{l_3}{l_1}\left(c_2 m_2 + l_1 m_3\right), & g_3 &= -g_{c1} - m_3 c_3 (l_3 - c_3) \end{aligned} \tag{8}$$

in which g_{c1} is a useful collection of inertial parameters introduced in [3]. The corresponding pure-inertia mechanism can be generated by considering the following mass redistribution:

$$\begin{aligned} a_{21} = a_{10} - m_1, \qquad a_{32} = a_{21} - m_2, \qquad a_{03} = a_{32} - m_3 \\ m_1' c_1' = m_2' c_2' = m_1 (c_1 - l_1) + a_{10} l_1, \qquad m_3' c_3' = \frac{l_3}{l_1} m_1' c_1' \end{aligned} \tag{9}$$

The link masses can then be canceled out by setting $a_{10} = -m_1/l_1(c_1 - l_1)$, thus resulting in the following pure-inertia dynamic balance conditions:

$$g_1' = g_2' = -g_3' = g_{c1} \tag{10}$$

which offers a novel interpretation of g_{c1}. It should be noted that these conditions can also be derived from the relation between the angular velocities $\omega_1 + \omega_2 = \omega_3$. Based on these pure-inertia conditions, we can find a range of dynamically equivalent mechanisms, as long as the selection of inertial parameters g_1', a_{10}, a_{21}, a_{32}, and a_{03} respect the positivity constraint $m_i > 0$ and $g_i > 0$.

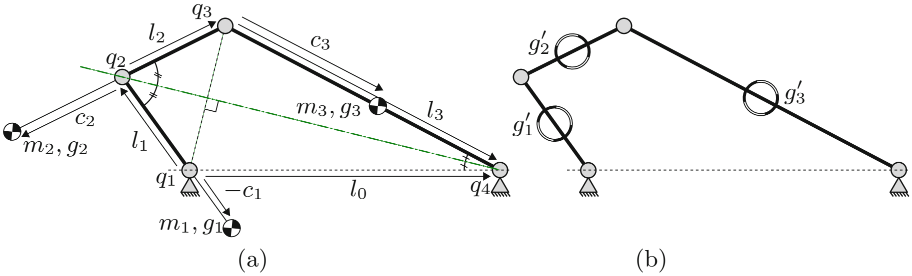

Fig. 3. (a) The dynamically balanced four-bar linkage. (b) A pure-inertia dynamic equivalent of the linkage shown in (a). The green dash-dotted line shows the symmetric plane, bisecting the kite through joint q_2 and q_4.

3 Dynamic Balance of a Novel 2-DOF Symmetric Mechanism Using the PIM

A novel 2-DOF symmetric planar mechanism, capable of generating finite rotations about any axis on its symmetric plane, was recently proposed in [10]. A variant is illustrated in Fig. 4. It comprises two RRR chains acting in-parallel on the end-effector and an angle-bisecting device (gray links in Fig. 4) to ensure a symmetric motion. The angle-bisecting device consists of a pantograph, attached via two revolute joints to two collinear slider joints on the symmetric plane. The symmetric plane bisects both "elbow" angles of the two RRR chains for full-cycle motion. Due to the *half-angle property* [11], the symmetric plane also bisects the angle between the base and the end-effector.

We shall apply the PIM to dynamically balance this novel mechanism in two steps. In the first step, the mass and moment of inertia of the angle-bisecting device are ignored (but assuming that the geometric constraint is still effective). We set base joints (q_1 and q_3) as the input variables. Because of the symmetry conditions, the angular velocity $\omega_{i,j}$ of body i with respect to the actuator $j = \{1, 3\}$ can be written as:

$$\begin{aligned} \omega_{1,1} + \omega_{2,1} = \omega_{4,1} = \omega_{5,1} \\ \omega_{3,3} + \omega_{4,3} = \omega_{2,3} = \omega_{5,3} \end{aligned} \tag{11}$$

Both relations are in fact equivalent to those of the kite-type four-bar linkage: when one of the base joints is locked, the platform and its adjacent bodies move as a rigid body, thus originating a kite-shaped four-bar linkage. Consequently, dynamic balance conditions can be derived from Eq. 10 as:

$$g'_1 = g'_2 = -g'_4 - g'_5, \qquad g'_3 = g'_4 = -g'_2 - g'_5 \tag{12}$$

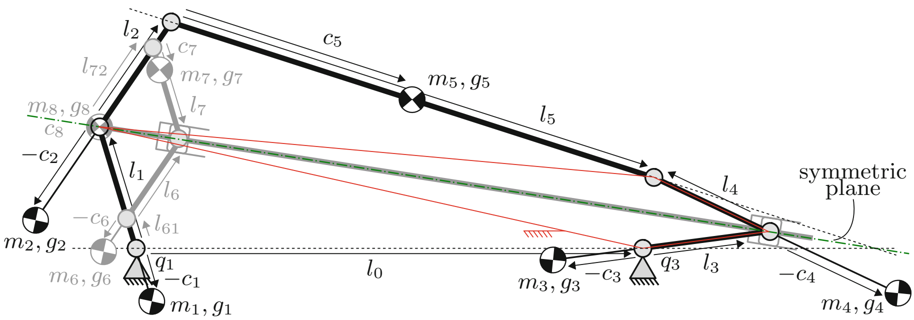

Fig. 4. The dynamically balanced 2-DOF planar symmetric mechanism (a variant of the M_{2A}-type symmetric mechanism proposed in [10]). The angle bisecting pantograph-slide on the symmetry plane is shown in gray. When fixing one of the base joints, in this case q_1, the mechanism acts as a symmetric four-bar mechanism (red).

Table 1. Geometric and inertial parameters for the dynamically balanced 2-DOF symmetric mechanism used in the simulation.

Length $[m]$		PI $[kgm^2]$		Point mass $[kg]$		Mass $[kg]$		COM $[m]$		Inertia $[kgm^2]$	
l_1	1.0	g'_1	1.00	a_{10}	-1.93	m_1	1.3	c_1	-0.43	g_1	0.17
l_2	1.0	g'_2	1.00	a_{21}	-0.570	m_2	0.30	c_2	-0.89	g_2	0.50
l_3	1.0	g'_3	1.01	a_{30}	-1.18	m_3	0.69	c_3	-0.71	g_3	0.17
l_4	1.0	g'_4	1.01	a_{43}	-0.490	m_4	0.23	c_4	-1.1	g_4	0.46
l_5	4.0	g'_5	-2.03	a_{52}	-0.260	m_5	0.52	c_5	2.0	g_5	55×10^{-3}
				a_{54}	-0.260						
l_{61}	0.25	g'_6	10.0×10^{-3}	a_{61}	-10.0×10^{-3}	m_6	7.0×10^{-3}	c_6	-0.32	g_6	7.6×10^{-3}
l_6	0.75	g'_7	10.0×10^{-3}	a_{72}	-10.0×10^{-3}	m_7	13×10^{-3}	c_7	-0.17	g_7	11×10^{-3}
l_{71}	0.25	g'_8	10.0×10^{-3}	a_{76}	-3.00×10^{-3}	m_8	10×10^{-3}	c_8	0.0	g_8	10×10^{-3}
l_7	0.75			a_{81}	-10.0×10^{-3}						

In the second step, the entire mechanism is dynamically balanced. First, note that links l_6 and l_7 of the pantograph (the angle-bisecting device in Fig. 4) have the same angular velocity as that of links l_2 and l_1, respectively. Their moments of inertia can then be easily lumped together in the dynamic balance conditions derived in the first step (Eq. 12). Secondly, due to the half-angle property, the angular velocity of the connecting link l_8, is exactly half the angular velocity of the end-effector. Thus we have:

$$\omega_6 = \omega_2, \qquad \omega_7 = \omega_1, \qquad \omega_8 = 0.5\omega_5 \tag{13}$$

The dynamic balance conditions for the entire mechanism are then given by:

$$g'_1 + g'_7 = g'_2 + g'_6, \qquad g'_3 = g'_4, \qquad g'_5 + 0.5g'_8 = -g'_2 - g'_4 \tag{14}$$

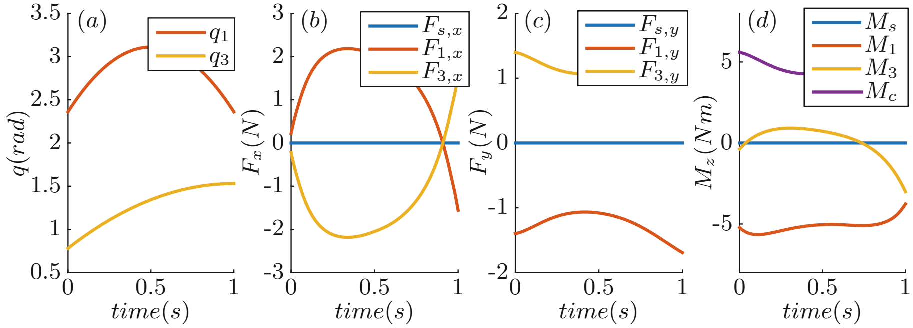

Fig. 5. Simulation results of the novel 2-DOF manipulator. The figure shows the input angles (a), shaking forces in x and y direction (b, and c respectively) and the shaking moment (d). The shaking forces (F_s) are the sum of reaction forces on base joints (F_1, F_3). The shaking moment (M_s) is the sum of motor torques (M_1, M_3) and the couple (M_c) induced by base joint forces.

A simulation of the dynamically balanced 2-DOF mechanism was performed to validate the PIM. The numerical values chosen for the lengths, COM, masses and moments of inertia can be found in Table 1. The simulation results, as shown in Fig. 5, confirm that the sum of the shaking forces and moments are zero for arbitrary motion.

4 Conclusion

All dynamically balanced mechanisms discussed in this paper share the same property of kinematic symmetry and dynamical equivalence to pure-inertia mechanisms. The pure-inertia method provides a simplified approach for achieving dynamic balance for two reasons: (i) Force balance conditions are automatically satisfied; (ii) in the planar case, moments of inertia are pose invariant and can be treated as scalars, reducing the dynamic balance conditions to linear kinematic relations between angular velocities. These relations, in the class of mechanisms shown here, are linear due to kinematic symmetry conditions. Based on this new understanding, a new dynamically balanced 2-DOF mechanism was presented.

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