

## Convex Model Predictive Control for Down-regulation Strategies in Wind Turbines

Gonzalez Silva, J.; Ferrari, Riccardo M.G.; van Wingerden, J.W.

**DOI**

[10.1109/CDC51059.2022.9993421](https://doi.org/10.1109/CDC51059.2022.9993421)

**Publication date**

2022

**Document Version**

Final published version

**Published in**

Proceedings of the IEEE 61st Conference on Decision and Control (CDC 2022)

**Citation (APA)**

Gonzalez Silva, J., Ferrari, R. M. G., & van Wingerden, J. W. (2022). Convex Model Predictive Control for Down-regulation Strategies in Wind Turbines. In *Proceedings of the IEEE 61st Conference on Decision and Control (CDC 2022)* (pp. 3110-3115). IEEE. <https://doi.org/10.1109/CDC51059.2022.9993421>

**Important note**

To cite this publication, please use the final published version (if applicable). Please check the document version above.

**Copyright**

Other than for strictly personal use, it is not permitted to download, forward or distribute the text or part of it, without the consent of the author(s) and/or copyright holder(s), unless the work is under an open content license such as Creative Commons.

**Takedown policy**

Please contact us and provide details if you believe this document breaches copyrights. We will remove access to the work immediately and investigate your claim.

***Green Open Access added to TU Delft Institutional Repository***

***'You share, we take care!' - Taverne project***

**<https://www.openaccess.nl/en/you-share-we-take-care>**

Otherwise as indicated in the copyright section: the publisher is the copyright holder of this work and the author uses the Dutch legislation to make this work public.

# Convex Model Predictive Control for Down-regulation Strategies in Wind Turbines

Jean Gonzalez Silva, Riccardo Ferrari and Jan-Willem van Wingerden\*

**Abstract**—Wind turbine (WT) controllers are often geared towards maximum power extraction, while suitable operating constraints should be guaranteed such that WT components are protected from failures. Control strategies can be also devised to reduce the generated power, for instance to track a power reference provided by the grid operator. They are called down-regulation strategies and allow to balance power generation and grid loads, as well as to provide ancillary grid services, such as frequency regulation. Although this balance is limited by the wind availability and grid demand, the quality of wind energy can be improved by introducing down-regulation strategies that make use of the kinetic energy of the turbine dynamics. This paper shows how the kinetic energy in the rotating components of turbines can be used as an additional degree-of-freedom by different down-regulation strategies. In particular we explore the power tracking problem based on convex model predictive control (MPC) at a single wind turbine. The use of MPC allows us to introduce a further constraint that guarantees flow stability and avoids stall conditions. Simulation results are used to illustrate the performance of the developed down-regulation strategies. Notably, by maximizing rotor speeds, and thus kinetic energy, the turbine can still temporarily guarantee tracking of a given power reference even when occasional saturation of the available wind power occurs. In the study case we proved that our approach can guarantee power tracking in saturated conditions for 10 times longer than with traditional down-regulation strategies.

## I. INTRODUCTION

In the transition to renewable energy sources, several countries reached a penetration level of renewable generation of more than 15% of their overall power-generation mix. Many of them (e.g., Spain, Portugal, Ireland, Germany, Denmark and the United States) have already crossed this threshold significantly, and experienced instantaneous penetration levels higher than 50% [1]. Due to such significant contribution of renewable energy sources, including wind power, grid operators are increasing their demand for ancillary services to be provided by wind turbines (WTs).

In particular, grid operators can make use of so-called *Active Power Control* (APC) to request turbines to provide a given reference power output [2]. The power reference command sent to all generators will guarantee that, at grid level, supply and demand are balanced and grid frequency is stabilized. As the power that a WT can generate is upper bounded by the available power in the incoming wind flow, WTs can only be *down-regulated*, that is operate in a way to track a power reference that is lower than the theoretical available maximum. Due to the nonlinearities present in the

dynamics of WTs, several down-regulation methodologies that achieve power tracking are possible [3]–[5].

Still, existing down-regulation strategies were developed for steady state conditions only, and cannot directly take into account available information on changing wind conditions, such as those provided by short time weather forecasts or LIDAR measurements. Furthermore, they do not accommodate directly the need to minimize structural loads on the WT, which on the long period can lead to premature failures.

In order to address these challenges, in this paper we propose a down-regulation approach based on convex *Model Predictive Control* (MPC). We will show how all the major down-regulation strategies present in the literature can be implemented with the proposed MPC approach. Furthermore, we will introduce a novel down-regulation strategy based on the maximization of kinetic energy, and show its benefits in guaranteeing power tracking also during occasional periods of saturation, when the reference power from the grid is larger than the available power in the wind flow.

MPCs approaches have already demonstrated their potential in several works on wind turbine and wind farm [6] control. An MPC formulation based on power flow and energy was presented in [7], while [8] and [9] have extended it by including the tower flexural model and by considering the presence of faults, respectively. By assuming the knowledge of future demanded power and wind variations, in [10] the authors are able to damp grid frequency oscillations by storing and releasing the WT kinetic energy. The use of kinetic energy as an energy reserve for grid stabilization is also explored in [11], where power generation can be increased by temporarily supplying kinetic energy from the rotor. To the best of the authors' knowledge, anyway, no work did consider the problem of promoting flow stability on the WT blades during down regulation. Indeed operating in low flow stability regions can lead to rotor speed oscillations, undesirable turbine responses and ultimately cause stall conditions [4].

The contribution of the present paper is three fold.

- We develop a general convex MPC framework for power tracking on wind turbines which includes the kinetic energy as a degree of freedom;
  - We extend the cost function in order to minimize aerodynamic loads, and add a constraint that enforces flow stability;
  - We present a simulation study based on the NREL 5 MW WT [12] and compare different down-regulation strategies under saturated conditions in OpenFAST [13].
- A key ingredient for obtaining a convex MPC formulation

\* Delft University of Technology, Delft, 2628CD The Netherlands  
{J.GonzalezSilva, R.Ferrari, J.W.vanWingerden}@tudelft.nl

in the present case is to use a linear model of the WT dynamics, expressed in energy form. Such form allows to remove the non-linear relationship among rotor speed, blade pitch angles, and wind speed from the optimization problem. The aerodynamic rotor power is then chosen as an optimization variable, which is constrained by a piecewise affine approximation of the available wind power. This formulation allows naturally to include the kinetic energy as a degree of freedom and leads to a linear optimization problem. Due to this freedom, an extra objective can be added to the optimization problem and thus recover the different existing down-regulation strategies. Finally, we show how to avoid stall conditions by implementing a further linearized constraint. As will be seen in the simulation results, the time period during which the demanded power is tracked, by maximizing kinetic energy, can be up to ten times longer with respect to other down regulation strategies.

The structure of this paper is as follows. First, the wind turbine model is described in Section II. Next, down-regulation strategies are analyzed in terms of kinetic energy and flow stability in Section III. Section IV introduces our proposed convex MPC for different down-regulation methods and their constraints. A simulation study under saturated conditions is presented in Section V. Finally, the paper is concluded in Section VI.

## II. WIND TURBINE MODEL

The non-linear wind turbine dynamics can be modelled using the rotor torque balance equation. By considering a rigid shaft and neglecting losses, this leads to the following model.

$$\dot{\omega}_g(t) = \frac{1}{J} \left[ \frac{1}{G_B} T_r(t) - T_g(t) \right], \quad (1)$$

where  $J$  is the equivalent moment of inertia of the rotor-generator-drive-train assembly referred to the high speed shaft,  $\dot{\omega}_g(t)$  the generator acceleration,  $T_r(t)$  the aerodynamic torque,  $G_B$  the gearbox ratio and  $T_g(t)$  the generator torque.

The non-linearity due to the aerodynamic torque relation can be expressed as

$$T_r(t) = \frac{0.5}{\omega_r(t)} \rho A_r v^3(t) C_P(\lambda(t), \theta(t)) \quad (2a)$$

$$= \Phi(v(t), \omega_r(t), \theta(t)) v^3(t) / \omega_r(t) \quad (2b)$$

$$= 0.5 \rho A_r R v^2(t) C_Q(\lambda(t), \theta(t)), \quad (2c)$$

where  $\rho$  the air density,  $A_r$  the rotor area,  $\theta(t)$  the collective blade-pitch angle, and  $\lambda(t) = R\omega_r(t)/v(t)$  is the tip-speed ratio, being  $R$  the rotor radius and  $\omega_r(t) = \omega_g(t)/G_B$  the rotor speed. The representation in (2a) is as function of the power coefficient  $C_P(\lambda(t), \theta(t))$ . In (2b), instead the function  $\Phi(v(t), \omega_r(t), \theta(t)) = 0.5 \rho A_r C_P(\lambda(t), \theta(t))$  is defined. Finally, the rotor torque in (2c) is a function of the torque coefficient  $C_Q(\lambda(t), \theta(t))$ , where it holds  $C_Q(\lambda(t), \theta(t)) = C_P(\lambda(t), \theta(t)) / \lambda(t)$ .

The collective blade pitch angle, generator speed and torque are limited by their upper and lower bounds as

follows:

$$\theta_{\min} \leq \theta(t) \leq \theta_{\max}; \quad (3a)$$

$$\omega_{g,\min} \leq \omega_g(t) \leq \omega_{g,\max}; \quad (3b)$$

$$0 \leq T_g(t) \leq T_{g,\max}. \quad (3c)$$

The aerodynamic power extracted from the wind by the rotor is given as

$$P_r(t) = T_r(t) \omega_r(t) = 0.5 \rho A_r v^3(t) C_P(\lambda(t), \theta(t)), \quad (4)$$

while the electrical generator power is given by

$$P_g(t) = \eta_g T_g(t) \omega_g(t), \quad (5)$$

where  $\eta_g$  is the generator efficiency.

The electrical generator power is constrained by

$$0 \leq P_g(t) \leq P_{g,\text{rated}}, \quad (6)$$

where  $P_{g,\text{rated}}$  is the rated generator power.

In terms of power flow and energy, the dynamics in (1) is rewritten as

$$\dot{K}(t) = P_r(t) - \frac{1}{\eta_g} P_g(t), \quad (7)$$

where  $K(t)$  is the kinetic energy stored in the rotating components and relates to the generator speed as

$$K(t) = \frac{J \omega_g^2(t)}{2}. \quad (8)$$

In the proposed convex MPC formulation,  $P_r$  and  $P_g$  in (7) are chosen to be the decision variables. The set of constraints in (3) needs then to be rewritten as a function of  $P_r$  and  $P_g$  and  $K$  as well, as the latter depends on the former via the system dynamics (7). Note that the transformed constraints should be also convex to lead to a convex problem [14]. Using (8) and (5), the rotor speed and generator torque constraints from (3b) and (3c) can be expressed respectively as

$$(J/2) \omega_{g,\min}^2 \leq K(t) \leq (J/2) \omega_{g,\max}^2, \quad (9)$$

$$0 \leq P_g(t) \leq \eta_g \sqrt{(2/J)K(t)} T_{g,\max}. \quad (10)$$

Since  $\sqrt{(2/J)K(t)}$  is a concave function, (10) is a convex constraint on  $P_g(t)$  and  $K(t)$ . Defining the available power as

$$P_{\text{av}}(v, K) = \max_{\theta_{\min} \leq \theta \leq \theta_{\max}} \Phi(v, (1/G_B) \sqrt{(2/J)K}, \theta) v^3,$$

the aerodynamic rotor power constraint is set as

$$P_r(t) \leq P_{\text{av}}(v(t), K(t)). \quad (11)$$

This includes the constraint (3a) in the formulation.

For a range of wind speeds and blade pitch angles and realistic  $\Phi$  functions, the available power turns out to be a concave function of  $K$ . Therefore, by fitting  $k$  piecewise affine functions [15], the available power can be approximated as

$$\hat{P}_{\text{av},v_i}(K(t)) = \min\{a_1 K(t) + b_1, \dots, a_k K(t) + b_k\} v_i^3,$$

where a linear interpolation is done between different wind speeds to obtain  $\hat{P}_{av}(v(t), K(t))$ .

$$\hat{P}_{av}(v(t), K(t)) = (1 - \Theta)\hat{P}_{av,v_1}(K(t)) + \Theta\hat{P}_{av,v_2}(K(t)), \quad (12)$$

with  $\Theta = \frac{v(t) - v_1}{v_2 - v_1}$ . The linear interpolation of concave functions results in a concave function.

The thrust force, which presents another non-linear behavior, is modelled as

$$F_T(t) = 0.5\rho A_r v^2(t) C_T(\lambda(t), \theta(t)), \quad (13)$$

where  $C_T$  is the thrust coefficient.

In order to develop the down-regulation strategy that minimizes thrust force, we approximate the thrust force through a linearization with respect to the aerodynamic rotor power and kinetic energy by assuming the knowledge of the wind speed at each time-step as follows.

First, the power and thrust coefficients are expressed with a first-order Taylor series expression around the current kinetic energy  $K^*$  and blade pitch angle  $\theta^*$ ,

$$\begin{aligned} C_P(K(t), \theta(t)) &\approx C_P(K^*, \theta^*) + \left. \frac{\partial C_P}{\partial K} \right|_{K^*, \theta^*} (K(t) - K^*) \\ &+ \left. \frac{\partial C_P}{\partial \theta} \right|_{K^*, \theta^*} (\theta(t) - \theta^*) = q_P(t)\theta(t) + r_P(t)K(t) + s_P(t), \end{aligned} \quad (14)$$

$$\begin{aligned} C_T(K(t), \theta(t)) &\approx C_T(K^*, \theta^*) + \left. \frac{\partial C_T}{\partial K} \right|_{K^*, \theta^*} (K(t) - K^*) \\ &+ \left. \frac{\partial C_T}{\partial \theta} \right|_{K^*, \theta^*} (\theta(t) - \theta^*) = q_T(t)\theta(t) + r_T(t)K(t) + s_T(t), \end{aligned} \quad (15)$$

where  $q_P(t)$ ,  $r_P(t)$ ,  $s_P(t)$ ,  $q_T(t)$ ,  $r_T(t)$  and  $s_T(t)$  are the corresponding time-varying parameters.

Then, combining (14) with (4) and (15) with (13), and eliminating the collective blade pitch angle, an affine relationship can be derived at each time-step as

$$\hat{F}_T(t) = Q_{F_T}(t)P_r(t) + R_{F_T}(t)K(t) + S_{F_T}(t) \quad (16)$$

$$\begin{aligned} \text{with } Q_{F_T}(t) &= \left( \frac{q_T(t)}{q_P(t)v(t)} \right), & R_{F_T}(t) &= \\ 0.5\rho A_r v(t)^2 \left( r_T(t) - r_P(t) \frac{q_T(t)}{q_P(t)} \right), & \text{and } S_{F_T}(t) &= \\ 0.5\rho A_r v(t)^2 \left( s_T(t) - s_P(t) \frac{q_T(t)}{q_P(t)} \right). \end{aligned}$$

### III. DOWN-REGULATION

As discussed earlier, there are several practical benefits in being able to down-regulate turbines to track a specific demanded power from the grid. However, there are multiple control solutions to down-regulate wind turbines. In Table I, the main strategies from the literature are summarized and qualitatively compared in terms of their capabilities to track a power reference, reduce structural loads and guarantee flow stability.

TABLE I

QUALITATIVE CATEGORIZATION OF DOWN-REGULATION STRATEGIES

Strategies	Power tracking	Structure Loads	Flow stability
Maximum $\omega_r$	High	High	Low
Minimum $C_T$	Low	Low	Low
Constant $\lambda$	Medium	Medium	High
Constant $\omega_r$	Depending on the $\omega_r$ value	Depending on the $\omega_r$ value	Low

#### A. The degree of freedom on kinetic energy

The possibility of having multiple down-regulation strategies is a consequence of the following:

*Proposition 1:* There exist an non-unique steady state operating condition when a power demand is below the maximum available power.

*Proof:* Lets then assume a steady state condition with power demand and inflow wind as being  $P_{dem,ss}$  and  $v_{ss}$ , respectively. Also, consider the down-regulation of a turbine to be asymptotic stable by feedback [2], [16], meaning that as  $t \rightarrow \infty$  the demanded power flow tends to be reached by the generator, and the derivative of the kinetic energy from Eq. (7) tends to zero in a steady state condition. Then, the following equation would hold near to an equilibrium.

$$0 \approx P_r(t) - \frac{1}{\eta_g} P_{dem,ss}(t), \quad (17)$$

From combining (17) with (4)

$$\frac{1}{\eta_g} P_{dem,ss} \approx 0.5\rho A_r v_{ss}^3 C_P \left( \frac{\omega_r(t)R}{v_{ss}}, \theta(t) \right). \quad (18)$$

Therefore, as depicted by the  $C_P$  contours in Fig 1, a desired  $C_P$  value lower than the maximum  $C_P$  can be reached by different combinations of  $\lambda$  and  $\theta$ . ■

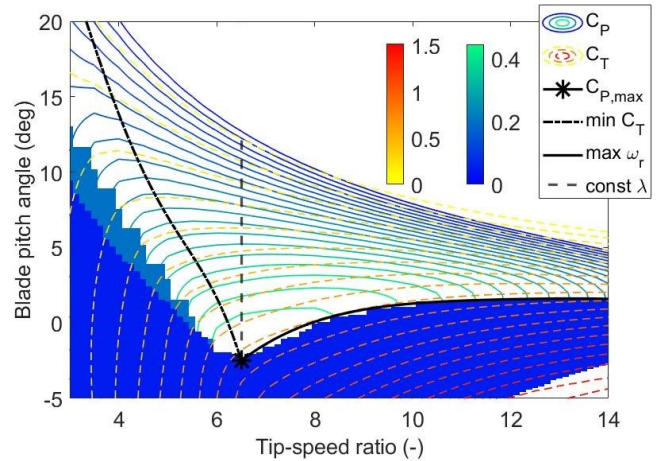


Fig. 1. Operation curves of down-regulation methods and stall regions of an NREL 5MW turbine. The shades of bright and dim blue colors correspond to  $\partial C_Q(\lambda, \theta)/\partial \theta > 0$  and  $\partial C_Q(\lambda, \theta)/\partial \omega_r > 0$ , respectively, characterizing the stall regions.

*Remark 1:* The fact that different combinations of  $\lambda$  are possible means that there is an extra degree of freedom on choosing a desired rotational speed and, thus, a kinetic

energy. This will indeed be leveraged to reduce aerodynamic loads and the risk of stall conditions.

### B. Flow stability

A current problem occurring when down-regulating turbines is due to the risk of loss of flow stability along the blades. This phenomenon, which can occur in low and high tip speed ratios, can lead to undesirable oscillations [17] and stall and is thus problematic for the design of turbine control.

Stall is characterized by the decrease of lift force on turbine blades as function of their angle of attack [18]. When the turbine is operating in a region where the derivatives of the aerodynamic torque with respect to rotor speed and pitch angle are positive, then it will eventually reach stall conditions [19]–[21]. These regions are shown by the blue shades in Fig. 1 when the partial derivatives of the torque coefficient  $C_Q$  are positive, thus indicating the possible onset of stall.

In particular, each down-regulation method can be analyzed in terms of flow stability from their operation distance with respect to stall regions. First, the constant  $\omega_r$  strategy can easily reach stall regions with low or high tip speed ratios depending on wind speed. From the same point of view, the maximum  $\omega_r$  strategy operates on the boundary of the stall pitch region. The minimum  $C_T$  strategy is categorized as low flow stability as its operation is close to stall regions at low tip-speed ratios. Finally, the constant  $\lambda$  always remains far from stall regions, being the strategy that operates under the most stable flow conditions.

*Remark 2:* The flow stability analysis herein is based on a steady state model. Dynamic flow effects and model-plant mismatches introduce a considerable uncertainty on the stall conditions. Conservative stall constraints should therefore be considered to avoid such regions.

### C. The use of kinetic energy

On one hand, the high kinetic energy is beneficial for power tracking, for instance, in the case where the demanded power exceeds the current maximum available power in the wind. In that case, the stored kinetic energy on the rotor speed is released so power tracking can be maintained longer. On the other hand, high rotor speeds may lead to operation conditions close to stall regions and to higher aerodynamic loads as seen by the max  $\omega_r$  curve and the  $C_T$  contours in Fig. 1. In this regard, the different down-regulation strategies are further explored in Section V in the convex model predictive control framework.

## IV. CONVEX MODEL PREDICTIVE CONTROL

Convex MPC is based on solving a convex optimization problem and is supported by a fairly complete body of research. Convex MPC can be solved numerically very efficiently, making it suitable to several applications.

The down-regulation in wind turbines is here formulated as an optimization problem based on the linear dynamics and convex constraints defined in Section II. Different from previous works, such as [7], the turbine herein is set to track a

demanded power instead of maximizing power extraction. As consequence of Proposition 1, a down-regulation operation is not uniquely defined, so an extra objective is added corresponding to the chosen down-regulation methodology.

First, we define the extra objectives - in Table II- and corresponding additional constraints in terms of energy and power flows. Then, further the flow stability constrain is derived. In the end, the general optimization problem is defined for all down-regulation strategies.

TABLE II  
DOWN-REGULATION STRATEGIES AND EQUIVALENT OBJECTIVES

Down-regulation Strategy	Equivalent Objective	Weights for (27) $[\alpha_5, \alpha_6, \alpha_7]$
Maximum $\omega_r$	Maximizing $K(t)$	$[\alpha_5, 0, 0]$
Minimum $C_T$	Minimizing $F_T(t)$ from (16)	$[0, 0, \alpha_7]$
Constant $\lambda^{\text{opt}}$	Tracking $K^{\text{ref}}(t)$	$[0, \alpha_6, 0]$
Constant $\omega_r$	Tracking constant $K^{\text{ref}}$	$[0, \alpha_6, 0]$

The minimum  $C_T$  strategy is equivalent to minimize the thrust force  $F_T(t)$ , which is also equivalent to minimize the term  $R_{F_T}K$  from Eq. (16) while the rotor power  $P_T$  would match its associated power reference.  $R_{F_T}$  is a time-varying parameter, which depends on the current operation point, so an extra variable  $F_{T,\text{extra}}$  is instead minimized by including the following constraints.

$$(R_{F_T}K)'R_{F_T}K \leq F_{T,\text{extra}}, \quad (19)$$

$$F_{T,\text{extra}} \geq 0, \quad (20)$$

where the term  $R_{F_T}K$  is therefore indirectly minimized based on the robust linear program [14]. This is done because the term  $R_{F_T}K$  is composed by a time-varying parameter, instead of a constant, and a decision variable.

To obtain the constant  $\lambda^{\text{opt}}$  strategy, the kinetic energy is set to track the following reference as an objective.

$$K^{\text{ref}}(t) = \frac{J(\omega_g^{\text{ref}}(t))^2}{2}, \quad (21)$$

where

$$\omega_g^{\text{ref}}(t) = \frac{\lambda^{\text{opt}v}(t)}{R}G_B. \quad (22)$$

Now, a time-varying inequality that includes a positive tuning parameter  $\delta$  is introduced to constrain the turbine operation out of the stall region as

$$\frac{\partial T_r(t)}{\partial \theta(t)} \leq -\delta, \quad (23)$$

where, using (2c),

$$\frac{\partial T_r(t)}{\partial \theta(t)} = 0.5\rho A_r R v^2(t) \frac{\partial C_Q(\lambda(t), \theta(t))}{\partial \theta(t)}. \quad (24)$$

The value of  $\partial T_r(t)/\partial \theta(t)$  expresses how safe the current WT operation is from stall conditions and it should be always negative as previously discussed in Subsection III-B. The parameter  $\delta > 0$  is recommended to be added to increase robustness as result of Remark 2, therefore conservatively preventing stall to happen.

Similar with the derivation of (16), the affine time-varying constraint that avoids stall regions can be obtained by the linearization of the power coefficient  $C_P(K(t), \theta(t))$  and the derivative of the aerodynamic torque coefficient with respect to the blade pitch  $\partial C_Q(K(t), \theta(t))/\partial \theta(t)$ . Such that the following constraint is obtained.

$$\frac{\partial T_r(t)}{\partial \theta(t)} \approx Q_{T_r'}(t)P_r(t) + R_{T_r'}(t)K(t) + S_{T_r'}(t) \leq -\delta, \quad (25)$$

where  $Q_{T_r'}(t)$ ,  $R_{T_r'}(t)$  and  $S_{T_r'}$  are the corresponding time-varying parameters. An affine function is always convex, so also is this constraint.

Finally, the cost function is defined as the integral of the objective function  $F$  over the time horizon  $T$  while considering the linear dynamic model from (7) and the defined convex constraints from (9), (10), (11) and (25) over the receding horizon.

$$\begin{aligned} \max_{U(t)} \int_t^{T+t} F(x(\tau), u(\tau)) d\tau, \quad \forall t \geq 0, \quad (26) \\ \text{s.t. (7), (9), (10), (11), (25),} \end{aligned}$$

where the objective function  $F$  is defined as

$$\begin{aligned} F(x(t), u(t)) = \\ -\alpha_1 \left[ P_g(t) - P^{\text{ref}}(t) \right]^2 - \alpha_2 \dot{P}_g^2(t) - \alpha_3 \dot{P}_r^2(t) \\ -\alpha_4 \left[ \max\{K(t) - \frac{J}{2} \omega_{g,\text{rated}}^2, 0\} \right] + \alpha_5 K(t) \\ -\alpha_6 \left[ K(t) - K^{\text{ref}}(t) \right]^2 - \alpha_7 F_{T,\text{extra}}(t). \quad (27) \end{aligned}$$

In the previous equations, it holds  $x(t) = [K(t)]$ ,  $u(t) = [P_r(t), P_g(t)]^\top$ , and  $U(t) = [u^\top(t), \dots, u^\top(t+T)]^\top$ . For each down-regulation strategy, the corresponding weights for  $F$  are listed in Table II, where  $\alpha_5, \alpha_6, \alpha_7 \in \mathbb{R}_{>0}$ . The additional constraint (20) for the minimum  $C_T$  strategy is set only when this strategy is chosen.

*Remark 3:* Stability is usually guaranteed by including a terminal cost and terminal constraints. However, the formulation in (26) does not include them. Therefore, we are aware that this choice does not yield a closed-loop stability guarantee.

In the proposed formulation, the optimization problem can be solved globally using efficient algorithms [14]. At a defined sampling time  $T_s$ , the optimal solution for  $U(t)$  as a vector sequence along the time horizon  $T$  is obtained whereby the first input of the sequence is used to compute the current turbine command. Then, the prediction horizon moves ahead to the next step time, from which the optimization is repeated.

In particular, given the values of  $P_r$ ,  $P_g$  and  $K$  obtained from the MPC solution, we can obtain the blade pitch command from Eq. (4) as

$$\theta(P_r(t), K(t)) = \theta_{\text{table}} \left( \frac{P_r(t)}{0.5 \rho A_r v^3(t)}, \frac{R \sqrt{(2/J) K(t)}}{v(t)} \right), \quad (28)$$

and the generator torque command from Eq. (5) as

$$T_g(P_g(t), K(t)) = \frac{P_g(t)}{\eta_g \sqrt{(2/J) K(t)}}. \quad (29)$$

## V. SIMULATIONS

In this section, the performances of the controller with different down-regulation strategies described in the previous section are compared for the case where a time-varying demanded power exceeds the current maximum available power in the wind. The control parameters used in the OpenFAST simulations are in Table III. The normalized standard test signal from [22] is used for the reference power as in [2], where the time-varying reference power signal is herein set to exceed the maximum available power. The maximum available power is derived from the simulated constant and uniform wind speed of 8 m/s, that is a representative average annual value in a wind plant site.

TABLE III  
CONTROL PARAMETERS

Parameter	Value
Weights, $\alpha_i$	[10 0.1 0.1 0.1 0.1 0.1 0.01]
Time horizon, $T$	20 s
Sampling time, $T_s$	0.2 s
Stall constraint parameter, $\delta$	0

For each down-regulation strategies, the mean values of kinetic energy and aerodynamic loads are computed before the power reference increases at 300 s. The amount of time that the turbine can track the required power after it becomes saturated is presented in Table IV. The turbine's power and reference are depicted in Fig. 2. The reference power in green is set to exceed the available power in orange. The generated powers from each down-regulation strategy present overshoots with respect to the available power when saturation occurs due to the stored kinetic energy. The strategy that maximizes kinetic energy follows longer the

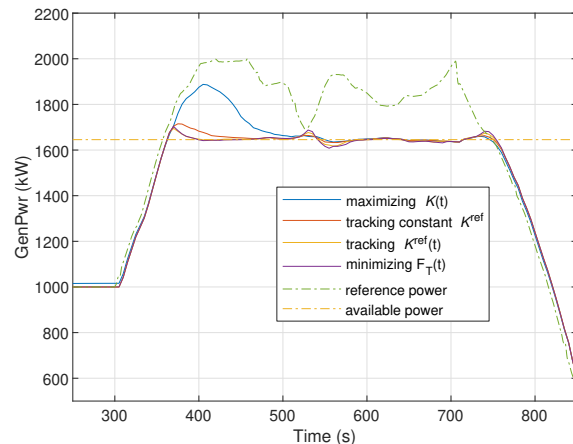


Fig. 2. Active power signals (NREL 5MW).

TABLE IV  
COMPARISON OF CONTROLLER POWER OUTPUT TRACKING AND PREVIOUS AERODYNAMIC LOADING

Down-regulation Strategy	Kinetic Energy (J) [before 300 s]	Mean Load (kN) [before 300 s]	Time of Power Tracking after Saturation (s)
Maximizing $K(t)$	4.8675e+07	306	40
Minimizing $\dot{F}_T(t)$	0.7689e+06	259	4
Tracking $K^{\text{ref}}(t)$	1.3750e+07	264	5
Tracking constant $K^{\text{ref}}$	1.5086e+07	266	9

reference power, although the turbine operates under higher aerodynamic loads.

## VI. CONCLUSIONS

In this paper, we proposed a linear convex model predictive control framework for implementing wind turbine down-regulation. Down-regulating a turbine leads to an additional degree of freedom on the value of the kinetic energy. We leveraged this and shown how different choices of the target kinetic energy can lead to existing down-regulation strategies. We further introduced a novel strategy aimed at reducing aerodynamic loads and reducing the risk of stall conditions. We have demonstrated the results of different down-regulation methodologies in terms of structural loading, flow stability and power tracking capability. Simulation on realistic models reveal the ability to maintain power tracking in saturation conditions, by means of the stored kinetic energy of the rotor.

The shifting paradigm from maximizing to tracking power and the use of kinetic energy as a storage presents a significant economic potential and encourages the research on active power control of wind turbines [16], [23]. As future work, the extension of the proposed MPC approach for the problem of power dispatch in wind farm control, and the effects of wakes, is of particular interest.

## ACKNOWLEDGMENT

The authors acknowledge support from the European Union Horizon 2020 program through the WATEREYE project (grant no. 851207). The authors would like also to thank Atindriyo Pamososuryo from TU-Delft for sharing the baseline simulation code.

## REFERENCES

- [1] M. Miller, L. Bird, J. Cochran, M. Milligan, M. Bazilian, E. Denny, J. Dillon, J. Bialek, M. O'Malley, and K. Neuhoff, "Res-e-next: Next generation of res-e policy instruments," IEA-RETD, Tech. Rep., 2013.
- [2] P. Fleming, J. Aho, P. Gebraad, L. Pao, and Y. Zhang, "Computational fluid dynamics simulation study of active power control in wind plants," in *2016 American Control Conf. (ACC)*, 2016, pp. 1413–1420.
- [3] D. van der Hoek, S. Kanev, and W. Engels, "Comparison of down-regulation strategies for wind farm control and their effects on fatigue loads," in *2018 Annual American Control Conference (ACC)*, 2018, pp. 3116–3121.
- [4] W. H. Lio, M. Mirzaei, and G. C. Larsen, "On wind turbine down-regulation control strategies and rotor speed set-point," *Journal of Physics: Conference Series*, vol. 1037, p. 032040, jun 2018.
- [5] D. A. Juangarcia, I. Eguinoa, and T. Knudsen, "Derating a single wind farm turbine for reducing its wake and fatigue," *Journal of Physics: Conference Series*, vol. 1037, p. 032039, jun 2018.
- [6] S. Rivero, S. Mancini, F. Sarzo, and G. Ferrari-Trecate, "Model predictive controllers for reduction of mechanical fatigue in wind farms," *IEEE Transactions on Control Systems Technology*, vol. 25, no. 2, pp. 535–549, 2016.
- [7] T. G. Hovgaard, S. Boyd, and J. B. Jørgensen, "Model predictive control for wind power gradients," *Wind Energy*, vol. 18, no. 6, pp. 991–1006, 2015.
- [8] M. L. Shaltout, M. M. Alhneish, and S. M. Metwalli, "An Economic Model Predictive Control Approach for Wind Power Smoothing and Tower Load Mitigation," *Journal of Dynamic Systems, Measurement, and Control*, vol. 142, no. 6, 03 2020, 061005.
- [9] T. Jain and J. Yamé, "Health-aware fault-tolerant receding horizon control of wind turbines," *Control Engineering Practice*, vol. 95, p. 104236, 2020.
- [10] P. Tielens and D. Van Hertem, "Receding horizon control of wind power to provide frequency regulation," *IEEE Transactions on Power Systems*, vol. 32, no. 4, pp. 2663–2672, 2017.
- [11] P. F. Odgaard, T. G. Hovgaard, and R. Wiesniewski, "Model predictive control for wind turbine power boosting," in *2016 European Control Conference (ECC)*, 2016, pp. 1457–1462.
- [12] J. Jonkman, S. Butterfield, W. Musial, and G. Scott, "Definition of a 5mw reference wind turbine for offshore system development," *National Renewable Energy Laboratory (NREL)*, 2009.
- [13] Openfast documentation. <https://github.com/OpenFAST/openfast>. Accessed: 2022-08-17.
- [14] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge University Press, 2004.
- [15] A. Magnani and S. P. Boyd, "Convex piecewise-linear fitting," *Optimization and Engineering*, vol. 10, no. 1, pp. 1–17, 2009.
- [16] J. Aho, A. Bucksan, J. Laks, P. Fleming, Y. Jeong, F. Dunne, M. Churchfield, L. Pao, and K. Johnson, "A tutorial of wind turbine control for supporting grid frequency through active power control," in *2012 American Control Conf. (ACC)*, 2012, pp. 3120–3131.
- [17] J. C. Heinz, N. N. Sørensen, F. Zahle, and W. Skrzypinski, "Vortex-induced vibrations on a modern wind turbine blade," *Wind Energy*, vol. 19, no. 11, pp. 2041–2051, 2016.
- [18] J. Manwell, J. McGowan, and A. Rogers, *Wind Energy Explained: Theory, Design and Application*. Wiley, 2010.
- [19] P. Novak, T. Ekelund, I. Jovik, and B. Schmidtbauer, "Modeling and control of variable-speed wind-turbine drive-system dynamics," *IEEE Control Systems Magazine*, vol. 15, no. 4, pp. 28–38, 1995.
- [20] A. S. Deshpande and R. R. Peters, "Wind turbine controller design considerations for improved wind farm level curtailment tracking," in *IEEE Power and Energy Soc. General Meeting*, 2012, pp. 1–6.
- [21] A. Choudhry, M. Arjomandi, and R. Kelso, "Methods to control dynamic stall for wind turbine applications," *Renewable Energy*, vol. 86, pp. 26–37, 2016.
- [22] C. Pilong, *PJM Manual 12: Balancing Operations*, 30th ed. Audubon, PA, USA: PJM, 2013.
- [23] E. Ela, V. Gevorgian, P. Fleming, Y. C. Zhang, M. Singh, E. Muljadi, A. Scholbrook, J. Aho, A. Bucksan, L. Pao, V. Singhvi, A. Tuohy, P. Pourbeik, D. Brooks, and N. Bhatt, "Active power controls from wind power: Bridging the gaps."