# *FEM analysis of the cracking behavior of a beam subjected to bending: A discrete crack width calculation using DIANA*

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# PREFACE

In order to obtain the Master degree at the faculty of Civil Engineering and Geoscience one is obliged to write a Master Thesis regarding a subject of own interest. This reports covers my interest regarding the cracking behavior in reinforced concrete structures subjected to bending. The main focus of this research lies upon the influence of the concrete cover and the crack spacing on the cracking behavior in a beam. This research has been carried out in collaboration with ARCADIS.

I would like to express my deepest gratitude to everyone who helped me during the completion of this thesis. My outmost appreciation goes out to my mentors Dr. Ir. C. R. Braam and Ir. C van der Vliet, whose guidance and patience was very important during this research. I would also like to thank Dr. Ir. M.A.N. Hendriks for making the remote facilities available for the use of the Finite element program DIANA.

Sincerely,

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# ABSTRACT

During the calculation for the crack width control in reinforced elements subjected to bending as stated in the NEN-EN 1992-1-1 (Eurocode 2) the engineers at ARCADIS found that the outcome of these calculations lead to the application of larger amounts of steel reinforcement in order to limit the crack width in the structure, compared to calculations which were carried out according to the VBC 1995 (NEN-6720). It was also clear that with increasing cover the amount of steel needed for crack width control in the Eurocode 2 calculations increased substantially compared to the VBC 1995 calculations. Due to these differences, it was necessary to have a better look at the cracking behavior in thick- walled reinforced concrete elements proposed by Eurocode 2. To ensure a safe, durable and economical design for thick-walled reinforced concrete elements, the cracking behavior is analyzed with the help of the following codes: National European Standards NEN-EN-1992-1-1, the NEN 6720 (VBC 1995), the NEN 3880 (VB 1974/1985) and also with a numerical analysis of the finite element program DIANA. The influence of the concrete cover and the crack spacing on the cracking behavior are also taken into account. This research provides more insight in which regulation can be used for a safe and durable structure when it comes to the crack width control in thick-walled reinforced structures subjected to bending.

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| | |
|---|---|
| $L$: | Length |
| $b$: | Width |
| $h$: | Height |
| $A_b$: | Cross section |
| $d$: | Position of the main reinforcement from the top of the beam |
| $d_s$: | Effective depth |
| $c_{min}$: | Minimum amount of the concrete cover required |
| $c$: | Applied concrete cover |
| $k_c$: | Factor taking into account the applied concrete cover |
| $h_{ceff}$: | Height of the effective concrete area |
| $A_{ceff}$: | Effective concrete area |
| $f_{ccm}$: | Mean value of the cube compressive strength at the day of testing |
| $f_{ck}$: | Characteristic value of the cube compressive strength at the day of testing |
| $f_{ctm}$: | Mean value of the cube tensile strength |
| $f_{ctk}$: | Characteristic value of the tensile strength |
| $f_{ct,eff}$: | Effective value of the tensile strength ($= f_{ctm}$) |
| $E_c$: | Young's modulus concrete |
| $E_s$: | Young's modulus steel |
| $\alpha_e$: | Ratio: $\frac{E_s}{E_c}$ |
| $\emptyset_{km}$: | Diameter of the reinforcement bar applied |
| $\emptyset_{stirrups}$: | Diameter of the stirrups applied |
| $n$: | Number of steel bars applied |
| $s$: | Bar spacing |
| $A_s$: | Amount of reinforcement applied |
| $\rho_{peff}$: | Effective reinforcement ratio |
| $P$: | External force applied |
| $M_{applied}$: | Bending moment due to an external force |

| | |
|---|---|
| $\sigma_s$: | Steel stress in the tensioned reinforcement assuming a cracked cross section |
| $\tau_{bm}$: | Bond stresses |
| $M_{cr}$: | Cracking moment |
| $\kappa_{cr}$: | Curvature at which the beam cracks |
| $\kappa_{fdc}$: | Curvature at which the crack pattern has fully initiated |
| $\Delta\kappa$: | Tension stiffening |
| $\varepsilon_{cr}$: | Rupture strain |
| $\varepsilon_{sm}$: | Mean value of the elongation in the reinforcing steel |
| $\varepsilon_{cm}$: | Mean value of the elongation in the concrete element |
| $\Delta l$: | Mean value of the crack spacing |
| $s_{r,max}$: | Maximum value of the crack spacing |
| $w_{mean}$: | Mean value of the crack width |
| $w_{k,max}$: | Maximum value of the crack width |

# 1 INTRODUCTION

In this chapter a brief introduction is given of the subject, which will be followed by the problem statement and the project procedure.

## 1.1 GENERAL

Concrete is one of the most frequent used building materials for the construction of buildings, bridges and other facilities. Its freedom of shape and high durability is one of the reasons why concrete is widely used. But concrete has a low tensile strength and is therefore mostly used in combination with steel bars, this is also known as reinforced concrete. One of the main problems we come across in the building industry is the cracking behavior of reinforced concrete. In concrete structures, cracks should be able to develop prior to failure. This is necessary to indicate that measurements need to be taken to prevent failure of the structure. If this doesn't occur the structure will fail without any warning which can be very dangerous to the environment and people. There are several reasons which cause cracking in concrete, for example: cracks can occur during the hydration process or they can occur after hardening of the concrete due to external loading or due to an imposed deformation [1]. Cracking in a structure can occur up to a certain limit and when this limit is reached the cracking behavior can have an effect on the durability of the structure and, as a result, on structural strength. This is the primary reason why the cracking behavior needs to be controlled.

The engineers at ARCADIS come across a certain problem during the calculation for crack width control in thick-walled reinforced elements subjected to bending according to the Eurocode 2. The problem is that the outcome of these calculations lead to the application of larger amounts of steel reinforcement in order to control the crack width in the structure, compared to calculations which were carried out according to the VBC-1995 (NEN-6720). In the following tables an example of the problem is given. The value of the steel stress ($\sigma_{s,rep}$) and the permissible moment capacity ($M_{rep}$) calculated according to the VBC-1995 and according to NEN-1992-1-1 are given in the tables below. These calculations were based on a maximum allowable crack width of: $w_{\max} = 0.2\ mm$. Both calculations were carried out for a beam loaded in pure bending with a thickness of $800\ mm$ and a width of $1000\ mm$. Table 1-1 represents the calculations carried out according to the VBC-1995 and Table 1-2 represents the calculations carried out according to the Eurocode 2 (NEN-1992-1-1). More information about these calculations can be found in Appendix 1.

*Table 1-1: Calculation of the allowable steel stress and permissible maximum moment according to VBC-1995*

| **Strength Class: C-35/45** | | | | | **Aggressive environment:** | |
|---|---|---|---|---|---|---|
| **Applied cover on the outer reinforcement : c = 40 mm** | | | | | **Crack width: w = 0,2 mm** | |
| **Minimum applied Reinforcement: Inside = 0; Outside $O_k$ = 20 mm** | | | | | | |
| **Reference period T =100 years.** | | **Cover 10 mm extra.** | | | **$c_{min}$ =40** | |
| **At a chosen bar diameter ($O_k$) & bar spacing (s) related values** | | | | | **$k_c$ =1.00** | |
| **$Ø_k$ =** | **s =** | **d =** | **$A_s$ =** | **$M_u$ =** | **$\sigma_{s;rep}$ <** | **max. $M_{rep}$ =** |
| 20 | 100 | 730 | 3142 | 962 | 217 | 481 |
| 20 | 125 | 730 | 2513 | 775 | 196 | 349 |
| 20 | 150 | 730 | 2094 | 649 | 179 | 266 |
| 20 | 200 | 730 | 1571 | 490 | 152 | 171 |

*Table 1-2: Calculation of the allowable steel stress and permissible maximum moment according to NEN-1992-1-1*

| **Strength class concrete:C-35/45** | | | | **Allowable crack width: $w = k_c * w_k = 0.20\ mm$** | | |
|---|---|---|---|---|---|---|
| **$Ø_k$ =** | **s =** | **$A_s$ =** | **d =** | **$s_{r;max}$ =** | **$\sigma_{s;rep}$ =** | **$M_{freq}$ <** |
| **[ mm ]** | **[ mm ]** | **[ mm²/m ]** | **[ mm ]** | **[ mm ]** | **[ N/mm² ]** | **[kNm/m]** |
| **20** | 100 | 3142 | 730 | 393 | 169 | 363 |
| **20** | 125 | 2513 | 730 | 441 | 151 | 261 |
| **20** | 150 | 2094 | 730 | 488 | 137 | 197 |
| **20** | 200 | 1571 | 730 | 583 | 114 | 125 |

Looking at the values in Table 1-1 it can be seen that the allowable steel stress calculated according to VBC-1995 for a bar diameter of 20 mm is equal to $\sigma_{s,rep} = 217\ N/mm^2$. In Table 1-2 the amount of the steel stress calculated according to NEN-1992-1-1 is equal to $\sigma_{s,rep} = 169\ N/mm^2$. So this means that the allowable steel stress calculated with Eurocode 2 is almost 23 % smaller than the allowable steel stress calculated with VBC-1995, which implies that a lot more reinforcement is needed to control the crack width in thick-walled concrete elements according to Eurocode 2. It should be noted that these calculations were based on a concrete cover of $c = 40\ mm$. When these calculations were carried out in Eurocode 2 for a cover of $c = 60\ mm$ the values for the allowable steel stress $(\sigma_{s,rep})$ decreased to a value of $\sigma_{s,rep} = 131\ N/mm^2$ (see Table 1-4). In the VBC-1995 the value of the allowable steel stress increased with about 11% for a concrete cover of $c = 60\ mm$ (see Table 1-3). This means that the difference between the Eurocode 2 calculations and the VBC-1995 calculations increased with about 80% for a larger applied concrete cover.

*Table 1-3: Calculation of the allowable steel stress and the maximum moment according to VBC-1995 for a concrete cover of 80 mm*

| **Strength Class: C-35/45** | | | | | **Aggressive environment:** | |
|---|---|---|---|---|---|---|
| **Applied cover on the outer reinforcement : c = 60 mm** | | | | | **Crack width: w = 0.2 mm** | |
| **Minimum applied Reinforcement: Inside = 0; Outside $O_k$ = 20 mm** | | | | | | |
| **Reference period T =100 years. Cover 10 mm extra.** | | | | | **$c_{min}$ =40** | |
| **At a chosen bar diameter ($O_k$) & bar spacing (s) related values** | | | | | **$k_c$ =1.00** | |
| **$Ø_k$ =** | **s =** | **d =** | **$A_s$ =** | **$M_u$ =** | **$\sigma_{s;rep}$ <** | **max. $M_{rep}$ =** |
| 20 | 100 | 730 | 3142 | 934 | 236 | 507 |
| 20 | 125 | 730 | 2513 | 753 | 215 | 373 |
| 20 | 150 | 730 | 2094 | 631 | 198 | 287 |
| 20 | 200 | 730 | 1571 | 476 | 188 | 205 |

*Table 1-4: Calculation of the allowable steel stress and permissible maximum moment according to NEN-1992-1-1 for a concrete cover of 80 mm*

| **Strength class concrete:C-35/45** | | | | **Allowable crack width: $w = k_c * w_k = 0.20\ mm$** | | |
|---|---|---|---|---|---|---|
| **$Ø_k$ =** | **s =** | **$A_s$ =** | **d =** | **$s_{r;max}$ =** | $\sigma_{s;rep}$ = | $M_{freq}$ < |
| **[ mm ]** | [ mm ] | [ mm²/m ] | [ mm ] | [ mm ] | [ N/mm² ] | [kNm/m] |
| **20** | 100 | 3142 | 710 | 509 | 131 | 273 |
| **20** | 125 | 2513 | 710 | 574 | 116 | 195 |
| **20** | 150 | 2094 | 710 | 637 | 105 | 147 |
| **20** | 200 | 1571 | 710 | 759 | 88 | 93 |

Due to these differences, it was necessary to have a better look at the cracking behavior in thick-walled reinforced concrete elements proposed by Eurocode 2. So this is the basic reason why this research has been performed. In this research the focus lies upon the crack formation in thick-walled reinforced due to an increased concrete cover in concrete elements subjected to bending. The reinforcement is concentrated mainly at the edges of the element. This research is also performed to bring more clarity in the use of the NEN-EN-1992-1-1 for crack width control in thick-walled reinforced concrete elements, whether it does comply with the reality or not.

To ensure a safe, durable and economical design for thick-walled reinforced concrete elements, the cracking behavior is analyzed with the help of the following codes: National European Standards NEN-EN-1992-1-1, the NEN 6720 (VBC 1995), the NEN 3880 (VB 1974/1985) and also with a numerical analysis of the finite element program DIANA. The influence of the concrete cover and the maximum crack spacing on the allowable steel stress are also taken into account. The outcome of this research will provide more insight in which regulations can be used for a safe and durable structure when it comes to the crack width control in thick-walled reinforced structures subjected to bending.

## 1.2 METHODOLOGY OF CRACK WIDTH CONTROL

Since the 1950's crack width calculations have been widely covered by different standards regarding concrete structures. In 2010 the National European Standards NEN-EN-1992-1-1 have been incorporated in the Netherlands included with their own national annex. Before 2010 the NEN 6720 regulations (VBC-1995) were used for the calculation of crack width in reinforced concrete structures. However the criteria for crack width calculations in the NEN6720 were based on the older code: VB 1974/1984. In the following sections a short description is given about these codes, followed by their requirements regarding cracking behavior.

### 1.2.1 VB 1974/1984

The VB 1974/1984 represent a series of previous used codes for concrete designs. In the years from 1974 to 1978 there were separate regulations published for the design of concrete structures. After some assessment it was concluded to combine and revise all the norms into one standard: VB 1974/1984 (NEN 3880). The requirements regarding crack width control can be found in article E-508 of this code. In this article it is stated that the mean crack width ($w_m$) depends on the mean value of the elongation in the steel bars ($\varepsilon_{sm}$), the elongation of the concrete element itself ($\varepsilon_{cm}$) and the mean value of the crack spacing ($\Delta l$). The following equation is presented for the mean value of the crack width:

1. $w_m = (\varepsilon_{sm} - \varepsilon_{cm})\Delta l$
   In Which:
   $w_m$: mean value of the crack width
   $\varepsilon_{sm}$: mean value of the elongation in the reinforcing steel
   $\varepsilon_{cm}$: mean value of the elongation in the concrete element
   $\Delta l$ : mean value of the crack spacing calculated with: $\Delta l = \xi_2(2c + \xi_3 \frac{\emptyset_{km}}{\rho_{p,eff}})$ (for more information see section 1.4.1).

The maximum value of the crack width can be calculated with the following equation:

2. $w_{max} = 2.1 * w_m$ with a reduction factor of 0.8 to account for the fact that not all the loads in the serviceability state (sls) are always present:

$$w_{max} = 0.8 * \sigma_s * \Delta l * 10^{-5}$$

In this equation $\sigma_s$ is defined as the value of the tensile strength of the steel acting on the structure which is coupled to the limit state value of the crack width requirements according to article E-401.4 [2]. It should be noted that the mean value of the crack spacing ($\Delta l$) is limited to $\Delta l \leq 10\emptyset_{km}$ for ribbed bars. Further information on crack width control according to VB 1974/1984 can be found in [2].

### 1.2.2 NEN 6720 Regulations for concrete – Structural requirements and calculation methods (VBC 1995)

The NEN 6720 Regulations for concrete, also known as the TGB 1990 - Voorschriften Beton – Constructieve eisen en rekenmethoden (VBC-1995), is part of the set of regulations for the building industry. The establishment started in 1980 where the deterministic consideration was replaced by a probabilistic design consideration. In this manner these regulations reflect the internationally accepted insight concerning the assessment of the reliability of structures [3]. In the VBC 1995 the crack width requirements are specified by means of the bar diameter ($\emptyset_{km}$) and the allowable bar spacing ($s$) in a structure. According to section 8.7.2 the following criteria must be met in case of a fully developed crack pattern (stabilized cracking stage):

1. The average bar diameter in the considered tensile zone must be equal to:
   $\emptyset_{\mathrm{km}} \leq \frac{\mathrm{k}_1 * \xi}{\sigma_{\mathrm{s}}} * \mathrm{k}_{\mathrm{c}}$ (in mm); for: $\mathrm{k}_{\mathrm{c}} = \frac{\mathrm{c}}{\mathrm{c}_{\mathrm{min}}} \ngtr 2$
2. The center to center distance (s) between the reinforcing bars in the considered zone must be equal to:

   $\mathrm{s} \leq 100 * \left(\frac{\mathrm{k}_2 * \xi}{\sigma_{\mathrm{s}}} - 1.3\right) * \mathrm{k}_{\mathrm{c}}$ ( in mm) ; for: $\mathrm{k}_{\mathrm{c}} = \sqrt{\frac{\mathrm{c}}{\mathrm{c}_{\mathrm{min}}}} \ngtr \sqrt{2}$

   In which:
   c: the applied cover on the outer layer of the reinforcement
   $\mathrm{c}_{\mathrm{min}}$: cover prescribed by art. 9.2
   $k_1 and\ k_2$ : are factors depending on the environment according to table 38 of NEN 6720
   $\xi$ : is the bond factor according to table 39.
   $\sigma_s$: largest calculated value of the steel stress in the cracked cross section

From the equations above it can be seen that these requirements differ from the previous mentioned regulations (VBC 1974/1984). It should be mentioned however, that the crack width requirements in the VBC 1995 are actually based on the requirements in the VBC 1974/1984. In the NEN 6720 the equations for crack width control are simply replaced by tables which specify the demands on the combination of the bar diameter and the steel stress (equation 1) or on the combination of the bar distance (s) and the steel stress (equation 2) [4]. For more extensive information on the cracking behavior in the stabilized cracking stage according to VBC1995 the reader is referred to [3].

### 1.2.3 NEN-EN 1992-1 (Eurocode 2: Design of concrete Structures-Part 1-1: General rules and rules for buildings)

The principles and requirements for safety, serviceability and durability of concrete structures, together with specific provisions for buildings are described in EN 1992-1-1. These requirements are based on the limit state concept concurrent with a partial factor method. In section 7.3.2 and section 7.3.4 of NEN-EN 1992-1 some general considerations that need to be taken account for crack width control are covered. The main equation for crack width control is [5]:

1. $w_k = s_{r,max}(\varepsilon_{sm} - \varepsilon_{cm})$, in which:

$w_k$: the design value for the crack width;
$s_{r,max}$: the maximum value of the crack spacing and can be calculated with the following equation:

2. $s_{r,max} = k_3 * c + k_1 * k_2 * k_4 * \frac{\emptyset}{\rho_{p,eff}}$ (for more information regarding the parameters see section 1.4.1.)

And $(\varepsilon_{sm} - \varepsilon_{cm})$: is the difference between the mean strain in the reinforcement ($\varepsilon_{sm}$) under relevant combination of loads and the mean strain in concrete between cracks ($\varepsilon_{cm}$). This strain difference can be calculated with the following equation:

3. $(\varepsilon_{sm} - \varepsilon_{cm}) = \frac{\sigma_s - k_t * \frac{f_{ct,eff}}{\rho_{p,eff}} * (1 + \alpha_e * \rho_{p,eff})}{E_s} \geq 0.6 * \frac{\sigma_s}{E_s}$, In which:

$\sigma_s$: the stress in the tension reinforcement assuming a cracked cross section
$f_{ct,eff}$: the mean value of the tensile strength of the concrete effective at the time when the cracks may first be expected to occur
$f_{ct,eff} := f_{ctm}$
$\alpha_e$: the ratio between the modulus of elasticity of steel and that of concrete : $\frac{E_s}{E_{cm}}$
$\rho_{p,eff}$: the effective area of the applied reinforcement: $\rho_{p,eff} = (\frac{A_s}{A_{c,eff}})$
$A_{c,eff}$: the effective concrete area as defined in chapter 1.4.2
$k_t$: factor depended on the duration of the load: $k_t = \begin{cases} 0.6 \ (short\ term\ loading) \\ 0.4 \ (long\ term\ loading) \end{cases}$

Looking at equation 1 it can be seen that it is almost equal to the equation for crack width control according to VBC 1974/1984, the only difference is that in the VBC 1974/1984 the mean crack spacing is limited to a value of $\Delta l \leq 10\emptyset_{km}$. This requirement is not taken up in the NEN-EN 1992-1-1. In Appendix 2 more extensive information on the cracking behavior according to Eurocode 2 can be found [5].

## 1.3 CRACKING BEHAVIOR

The cracking behavior in a structure can be explained with the help of the tensile member model. In this model, the first crack in the member will occur in the stage when the concrete tensile stress ($\sigma_c$) is equal to the concrete tensile strength ($f_{ctm}$). Just before the first crack occurs the strain in the reinforcement ($\varepsilon_s$) is equal to the strain in the concrete member ($\varepsilon_c$). In the crack itself, right after the first crack occurs, the tensile force is carried by the steel at a certain stress level ($\sigma_{sr}$). At both sides of the crack, the steel strength is transferred to the concrete by bond stresses ($\tau_{bm}$). Bond stresses occur because the occurring strain differences lead to displacement differences between the tensile bar and the concrete around it. The bond between concrete and steel determines the crack width, the distance between the cracks and the load deformation diagram of a concrete element under tension or bending [1].

The transfer of forces between steel and concrete goes on until a new crack develops in the tensile member. In Figure 1-1, the transfer of forces between steel and concrete is shown. After the first crack has occurred, new cracks will gradually develop due to an increasing external load or an imposed deformation.

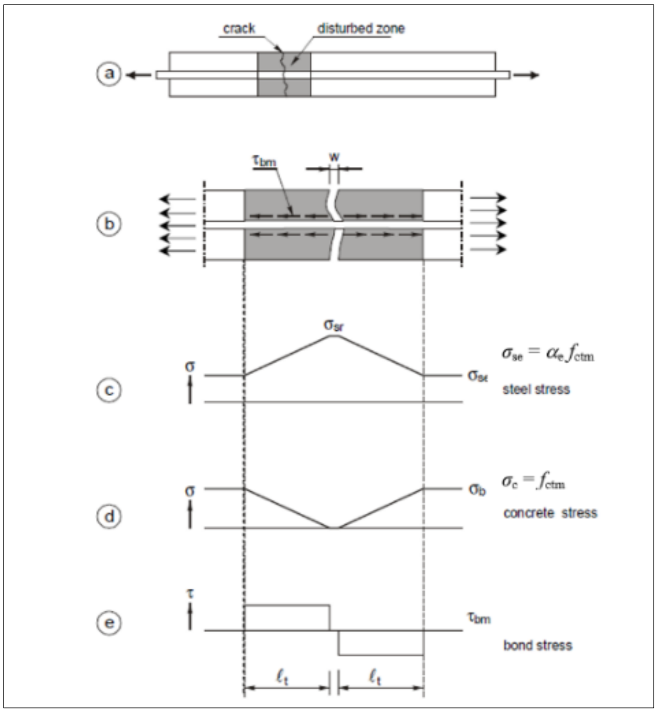



*Figure 1-1: Tensile member model [12]*

a. Prismatic reinforced concrete member with crack
b. Force transfer/transmission ("disturbed") area
c. Steel stress
d. Concrete tensile stress
e. Bond stress $\tau_{bm}$ and transmission length $l_t$

### 1.3.1 Characteristic behavior of a reinforced beam subjected to bending

When we look at the load-deformation diagram of a member subjected to bending (Figure 1-2), we see that it can be branched into three parts [1]. The first part, which is also known as the uncracked part goes from the origin of the diagram to the point determined by the cracking moment ($M_{cr}$) and the curvature ($\kappa_{cr}$) at which the beam cracks. This curvature is determined by the rupture strain of concrete ($\varepsilon_{cr}$). The cracking moment ($M_{cr}$) is determined by the concrete tensile strength ($f_{ctm}$). This is the point where the first crack occurs. The second part, which is also known as the cracking phase or the crack formations stage is determined by the average curvature at the moment when the first crack occurs ($\kappa_{cr}$) and the average curvature at which the crack pattern has fully initiated ($\kappa_{fdc}$). In this phase the moment is assumed to be constant ($M = M_{cr}$). Here the crack distances are not the same. The third and the final part of the diagram is characterized as the fully developed crack pattern or the stabilized cracking stage (phase 3). In this stage no new cracks develop and the number of cracks are considered to be constant. The crack distance ($\Delta l_m$) and the tension stiffening effect ($\Delta\kappa$) are also considered to be constant. For extensive information about the reinforced flexural beam the reader is referred to [1]. The calculations in this research are based on the stabilized cracking stage (phase 3).

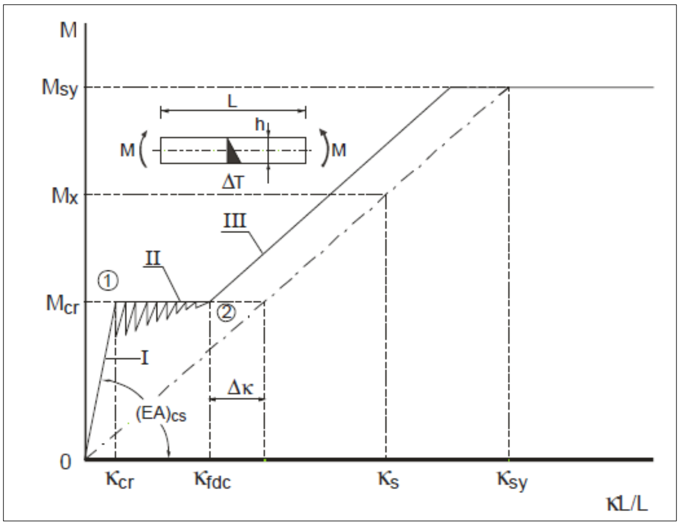



*Figure 1-2: Load deformation diagram of a flexural reinforced beam [1].*

## 1.4 IMPORTANT PARAMETERS REGARDING CRACKING BEHAVIOR

In this chapter some relevant parameters which are essential for the calculation of the crack width will be discussed. The difference between the calculations of the parameters regarding the codes will also be presented. First the crack spacing will be discussed, followed by the effective concrete area and finally the influence of the concrete cover will be taken into account.

### 1.4.1 Crack spacing

When calculating the crack width, the crack spacing ($\Delta l$) is one of the essential parameters. For a tensile member model, the crack distance ($\Delta l$) can be determined by the transmission length ($l_t$) (see Figure 1-1). For a fully developed crack pattern the crack spacing ($\Delta l$) varies between $l_t$ and $2l_t$. At this point there are no sections left in the concrete where the tensile stress is equal to the concrete tensile strength and so the crack spacing ($\Delta l$) remains constant at about $1.5l_t$. This is also the case for an increasing load.

The calculations for the crack spacing according to the codes is presented below:

1. **VB 1974/1984**

The mean crack spacing is defined in Article E-508.2 and can be calculated with the following equation: $\Delta l = \xi_2(2c + \xi_3 \frac{\emptyset_{km}}{\rho_{p,eff}})$ with an upper limit value of: $\Delta l = 10\xi_2 \emptyset k_m$

In which:

$\xi_2 = 1$(for ribbed steel bars); $\xi_2 = 1.25$( for smooth steel bars)

$\xi_3 = 4$ (beams subjected to bending); $\xi_3 = 8$ (beams subjected to tension)

$c$: concrete cover on the main reinforcement

$\emptyset_{km} = \frac{\sum \emptyset_{km}}{n}$: mean value of the applied bar diameters; $n$: number of bars applied

$\rho_{p,eff} = \frac{A_s}{A_{c,eff}} * 100$: effective reinforcement ratio; in which $A_s$ is the area of the reinforcement applied and $A_{c,eff}$ is the effective concrete area according to Figure 1-5.

### 2. NEN 6720 (VBC 1995)

The VBC 1995 does not supply any calculations regarding the maximum crack spacing. Since the cracking behavior is based on the bar diameter ($\emptyset_{km}$) and the allowable bar spacing ($s$) all the equations and tables are formulated to comply to these requirements. These equations however, are all based on the requirements regarding crack width control specified in the VB 1974/1984.

### 3. NEN 1992-1-1 (Eurocode 2)

In the Eurocode 2, the calculation of the crack spacing is a bit different [5]. The following equation is used:

$s_{r,max} = k_3 * c + k_1 * k_2 * k_4 * \frac{\emptyset}{\rho_{p,eff}}$, in which:

$s_{r,max}$: maximum value of the crack spacing

$\emptyset$: the bar diameter, when several bar diameters are used in a section an equivalent bar diameter should be used: $\emptyset_{eq} = \frac{n1\emptyset1^2 + n2\emptyset2^2}{n1\emptyset1 + n2\emptyset2}$

$c$: the cover to the longitudinal reinforcement

$k_1$: coefficient which takes into account the bond properties of the bonded reinforcement $k_1 = 0.8$ for high bond bars.

$k_2$: takes into account the distribution of the strain: $k_2 = 0.5$ for bending and $k_2 = 1.0$ for pure tension.

Recommended values for $k_3 = 3.4$ and $k_4 = 0.425$

In areas where the spacing of bonded reinforcement exceeds: $5(c + \frac{\emptyset}{2})$ or where there is no bonded reinforcement within the tension zone, an upper bound to the crack width may be found by using the following equation for the crack spacing:

$s_{r,max} = 1.3 * (h - x)$, where $h$ is the height of the element and $x$ the area of the compressive zone of the concrete.

$\rho_{p,eff} = \frac{A_s}{A_{c,eff}} * 100$: effective reinforcement ratio; in which $A_s$ is the area of the reinforcement applied and $A_{c,eff}$ is the effective concrete area according to Figure 1-4.

### 1.4.2 Effective cross section

For a flexural beam the mean crack distance also depends on the effective cross section of the hidden tensile member. The effective cross section can be characterized as the zone in which the crack pattern is determined by the reinforcing steel. This can be illustrated with the help of the crack pattern in a T-beam (Figure 1-3) [1].

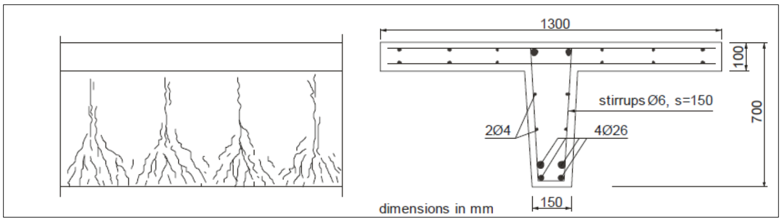



*Figure 1-3: Crack pattern in a T-beam [1]*

In Figure 1-3 we see that the reinforcement influences the crack spacing and the crack width at the bottom part of the T-beam. Looking at the web it is clear that the effect of the reinforcement is not present and we get a lot of gathering cracks ("verzamelscheuren") [1].There were several methods proposed for the calculation of the effective concrete area.

In 1976 the researcher Leonhardt defined the effective concrete area with the help of the bar diameter. In his method he derived an approach to consider the concrete beam as a tensile member by introducing "the effective concrete area" around the main reinforcement (Figure 1-4a):

$$\rho_{p,eff} = \frac{A_s}{b * h_{c,eff}} \qquad (1)$$

A few years later (in 1986) a different approach was presented by the researchers Schiessl and Wölfel, their approach was based on the effective beam depth (Figure 1-4b) [6]:

$$\rho_{p,eff} = \frac{A_s}{\alpha b * (h-d)} \qquad (2)$$

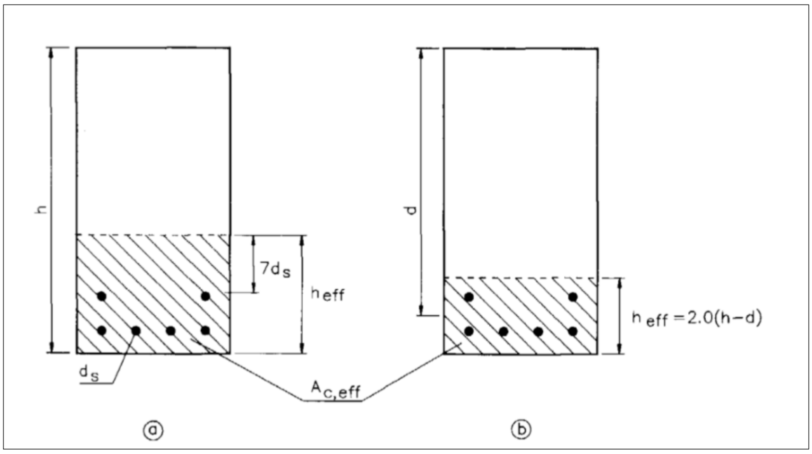



*Figure 1-4: Definitions for the "effective concrete cross section" by Leonhardt (a) and Schiessl and Wölfel (b)*

## VB 1974/1984

In the VB 1974/1984 the following definitions were provided for the effective concrete area *[2]*:

$$A_{c,eff} = b * h_{c,eff} \qquad (3)$$

in which $b$ is width of the effective concrete area and $h_{c,eff}$ is equal to the height of the effective concrete area, see Figure 1-5.

$h_{c,eff} = 0.8 * \emptyset_{km} + \bar{c} \leq h_t - x$ ( 1 layer of reinforcement).

$h_t$: total height of the beam

$x$: height of the compression zone

For two layers of reinforcement see Figure 1-5 (third figure).

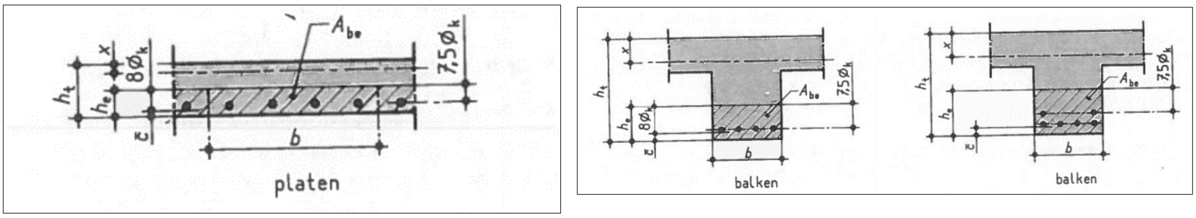



*Figure 1-5: Effective concrete area (figures E-73 a to c from VB 74/84)*

## NEN-EN 1992-1-1

The NEN-EN 1992-1-1 provides the following equation for the calculation of the effective concrete area:

$A_{c,eff} = h_{c,eff} * b$  (4) , in which

$$h_{ceff}: \leq \begin{cases} 1: & 2.5 * \left(c + \frac{\emptyset_k}{2}\right) \\ 2: & \frac{h-x}{3}\ (bending) \\ 3: & \frac{h}{2}\ (tension) \end{cases}$$

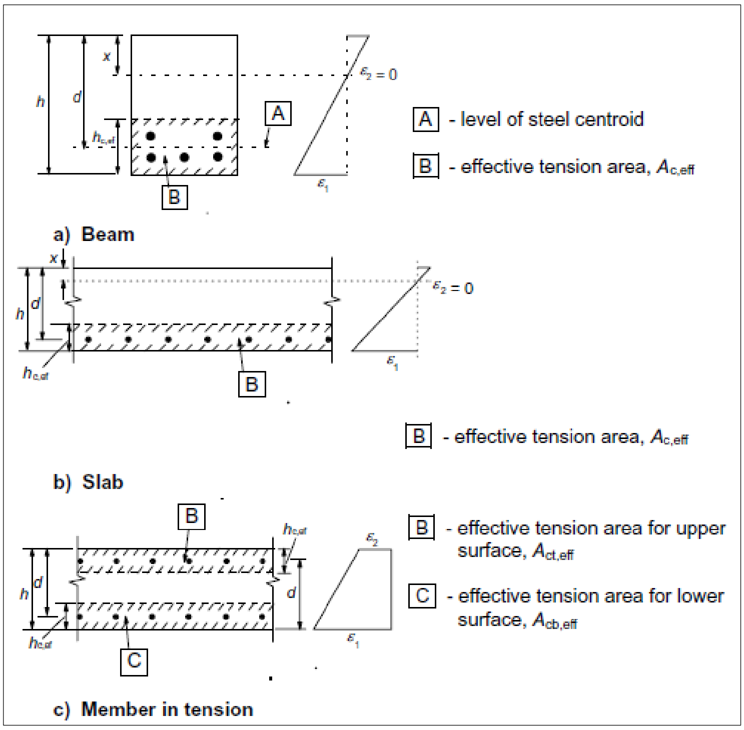



*Figure 1-6: Effective concrete area for a beam according to the Eurocode 2 fig. 7.1(NEN 1992-1-1) [5]*

So as mentioned above there are several ways to calculate the effective concrete area. There are more definitions for the effective concrete area, but these will not be discussed. In this research the effective concrete area will be calculated according to the equations specified in the NEN-EN-1992-1-1. For more information, the reader is referred to [6].

### 1.4.3 Concrete cover

The concrete cover is the distance between the area of the reinforcement and the outer edge of the concrete. The thickness of the concrete cover is very important for the durability of the structure. If the cover is too small the reinforcement can be effected by the environment when cracking occurs e.g. corrosion. When we look at the equations for the effective concrete area presented in chapter 1.4.2, it is clear the concrete cover ($c$) also plays a role in the crack width calculations. There are minimum requirements for the concrete cover taken up in the Eurocode 2 section 4.4.1. These requirements depend on environmental conditions and exposure classes such as [5]:

1. XO: no risk of corrosion attack,
2. XC1 to Xc4: corrosion attack induced by carbonation,
3. XD1 to XD3: corrosion induced by chlorides,
4. XS1 to XS3: corrosion induced by chlorides from sea water,
5. XF1 to XF4: freeze/thaw attack and
6. XA1 to XA3: chemical attack

In section 4.4.1 of the Eurocode 2 the requirements for the minimum concrete cover can be found.

## 1.5 MAIN ASPECT

The main problem of the engineers at ARCADIS lies within the calculations of the amount of reinforcement considering crack width control for thick-walled reinforced elements with a relatively large concrete cover. This research is performed in order to investigate whether the crack width calculations according to the European Standards (NEN-EN 1992-1-1) are in agreement with the actual behavior when applied to thick-walled structures subjected to pure bending.

### 1.5.1 Problem statement

Current European standards demand a lot more reinforcement for thick-walled reinforced structures. When these regulations are applied in practice they result in large amounts of reinforcements for crack width control. So in knowing this, the following problem statement can be formulated:

***Does the NEN-EN 1992-1-1 comply with the actual cracking behavior of a thick-walled reinforced concrete element consisting of a relatively large concrete cover subjected to bending in the stabilized cracking stage?***

### 1.5.2 Research questions

The following research questions regarding the problem statement can be formulated:

a. *What influence does the cover and the crack spacing have on the amount of steel needed for crack width control in thick-walled reinforced elements subjected to bending according to the Eurocode 2 and the VBC 1995?*
b. *What influence does the limitation of the maximum crack spacing have on the amount of steel needed for crack width control in thick-walled reinforced elements subjected to bending according to the Eurocode 2?*
c. *What influence does the concrete cover and the crack spacing have on the crack width calculated in thick-walled reinforced elements subjected to bending according to the Eurocode 2 and the VB 1974/1984 ?*
d. *What influence does the limitation of the crack spacing have on the crack width calculated in thick-walled reinforced elements subjected to bending according to the Eurocode 2 and the VB 1974/1984?*
e. *Is the crack width calculated according to the Eurocode 2 in agreement with the actual behavior of crack width for thick –walled reinforced structures in the stabilized cracking stage?*

### 1.5.3 Objectives

With this research the following goals will be reached:

1. A better understanding of the influence of concrete cover and crack spacing on the crack width calculations for thick-walled elements subjected to bending.
2. Insight in the difference between the crack width development according to the European Standards (NEN-EN-1992-1-1) and the actual behavior of the crack width development in thick-walled reinforced structures.

### 1.5.4 Procedure

The following steps are carried out for obtaining the answers to the research questions formulated in paragraph 1.5.2.

1. Perform a literature study on the cracking behavior of concrete in thick-walled reinforced elements. During this step previous experimental research which have been performed on this subject will be studied and briefly reported in section 2.

   After the literature study the analysis of the problem will be conducted analytically and also numerically. This is described in the following steps.

2. The analytical analysis will be carried out on a few practical cases from step 1. During this step the crack width will be calculated in Excel according to NEN-EN 1992-1-1 section 7.3 and compared to the experimental data found in step 1. The parameters which were varied in step 1 (Experimental cases) will also be varied. Some parameters which were varied during the experiments are: the cross section of the concrete structure, reinforcement ratio and the concrete cover.

An excel sheet for the calculation of the maximum steel stress and the permissible moment will be analyzed and modified in order to gain more insight in the problem mentioned in section 1.1. During this step the maximum allowable steel stress will be calculated according to the NEN-EN-1992-1-1 and the NEN-6720. These calculations will also be used to analyze the influence of several parameters on the amount of steel needed for crack width control in elements subjected to bending, such as the maximum crack spacing and the concrete cover. The results of the analytical analysis can be found in section 3 of this report.

After the analytical analysis of the experimental research is finished, the numerical analysis will be performed.

3. Model some cases mentioned in steps 2 and run a non-linear analysis using the finite element program DIANA. During this step the development of the crack pattern will be looked at and the crack width will be analyzed using the smeared-crack concept.

4. Compare the results from the numerical analysis of step 3 with the experimental results from step 1. This comparison will be done in order to validate whether the real behavior of crack width development can be simulated with the finite element program DIANA.

5. A numerical and analytical analysis will be carried out for the cases modelled in step 3. First the cases which were modelled numerically in step 3 will also be modelled with an increasing concrete cover of $c = 50\ mm$ and $c = 70\ mm$. This will be followed by an analytical analysis of the same cases for which the cover is increased. During this step the influence of an increasing cover and the limitation of the crack spacing on the cracking behavior on a beam subjected to bending according to the codes (Eurocode 2 and VB74/84) will be analyzed.

6. Evaluation: The cracking behavior in thick–walled reinforced elements will be evaluated and with that a better insight in the cracking behavior proposed by the different codes will be provided.

# 2 LITERATURE SURVEY

Across the years several researchers have investigated and studied the cracking behavior of concrete structures with the help of experiments and empirical relations. In this chapter a short description of the research project and articles which were of essence for the completion of this thesis is presented. These research projects were:

- Control of crack width in deep reinforced concrete beams by Braam.

The experiments which were carried out during this research were used for my analytical and numerical analysis.

- Further analysis of the influence of the concrete cover on crack width control (Nadere analyse invloed dekking op scheurbeheersing) by Vosslamber. This is an article from the concrete journal: Cement

## 2.1 CONTROL OF CRACK WIDTH IN DEEP REINFORCED CONCRETE BEAMS [6]

In 1990 a research program was set up by Braam to present a cracking theory which described the cracking behavior over the entire height of deep beams. His experimental research was used to provide information for the verification of existing theoretical models. During his research various analytical and semi-empirical relations of the cracking behavior were studied and compared to each other. On the basis of experimental results conducted by the German researcher Helmus a model was initiated for the calculation of the steel stress and crack spacing at the side faces of tensile members and beams. Extensive research on deep beams was also carried out which was followed by a description of the formulae for their cracking behavior. Observations showed that those formulae had low credibility and so an experimental research program was introduced. This experimental research provided the important information that was needed for practical design rules for the web reinforcement in deep beams [6]. Below a brief description will be given of the cases which were used for the analytical and numerical analysis of this research.

### 2.1.1 Experimental research

During the experimental research there were 15 beams tested in total, namely 12 T-beams and 3 rectangular beams. The beams were casted with a concrete mix composed of Portland A and C cement and glacial river gravel aggregates with a particle size of 16 mm maximum. Each beam was casted in four or five layers and during casting internal vibrators were used to compact the concrete. For standard tests 12 cubes and 6 cylinders were casted together with each beam. Demolding and storage of the beams and standard specimens took place after about two or three days. Reinforcement bars of 10, 12, 16 and 20 mm diameter were used and 6 mm diameter bars were used for stirrups [6]. The stress-strain curves of the various bars are presented in Figure 2-1.

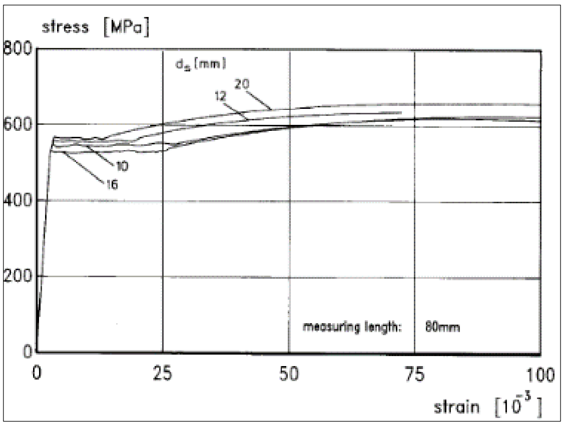



*Figure 2-1: Stress-strain curves of the bars used for longitudinal reinforcement [6]*

The dimensions and cross section of the beams are presented in Figure 2-2.

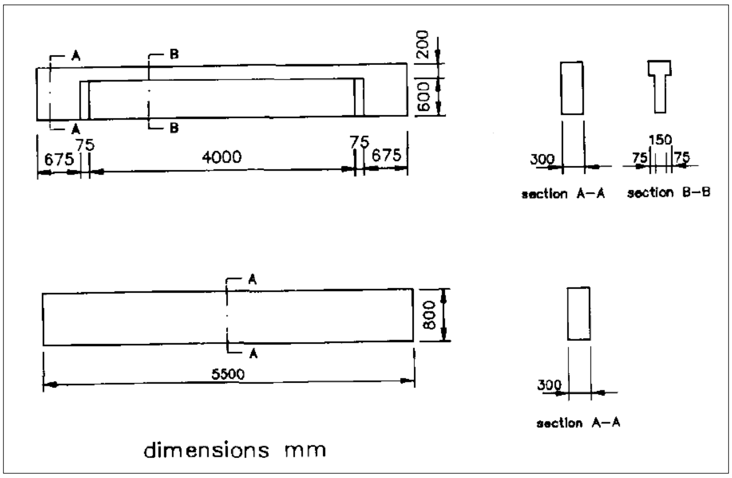



*Figure 2-2: Dimensions and cross sections of beams 1-12 (a) and 13-15 (b) [6]*

The beams were distinguished with numbers: The T- beams are numbered from beam 1 to beam 12 and the rectangular beams are numbered as beam 13, 14 and 15 respectively. Of the 15 specimens an analytical analysis was carried out for 2 of them: 1 T-beam (beam 3) and 1 rectangular beam (beam 13). A numerical analysis was also carried out for T-beam 3 and rectangular beam 13. The position of the main reinforcement is given in Figure 2-3.

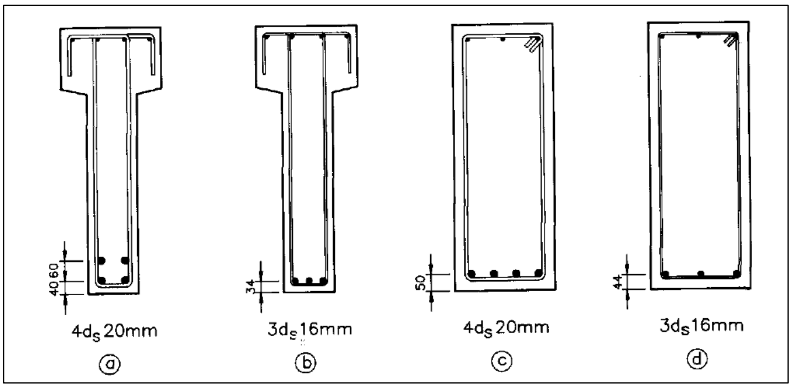



*Figure 2-3: Position of the main reinforcement of the beams 1-6 (a), 7-12 (b), 13 (c) and 14-15 [6]*

The details of the beams that were used for this research are presented in Table 2-1. The length of the beams was equal to 5.5 m and were loaded in four-point bending with a span of 5 m. The loading scheme is presented in Figure 2-5. The crack width and the deflection measurements were restricted to the pure-bending zone in between the load activators. The strains were measured with the help of extensometers. The influence of the dead weight was not incorporated in the measurement results of the deflection and the strains. After applying the loading frame on the beam, the actual load was then applied in four to five increments by a hand-operated hydraulic jack. These measurements are given in an extensive report in which all the experimental results were presented [7]. For my analytical and numerical analysis, the load- deflection diagrams and the crack width measurements at the level of the main reinforcement were used.

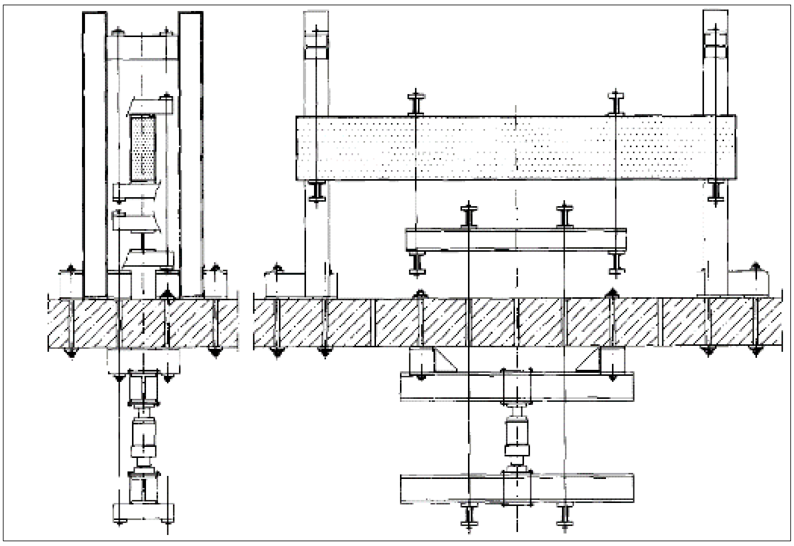


*Figure 2-4: Loading scheme*

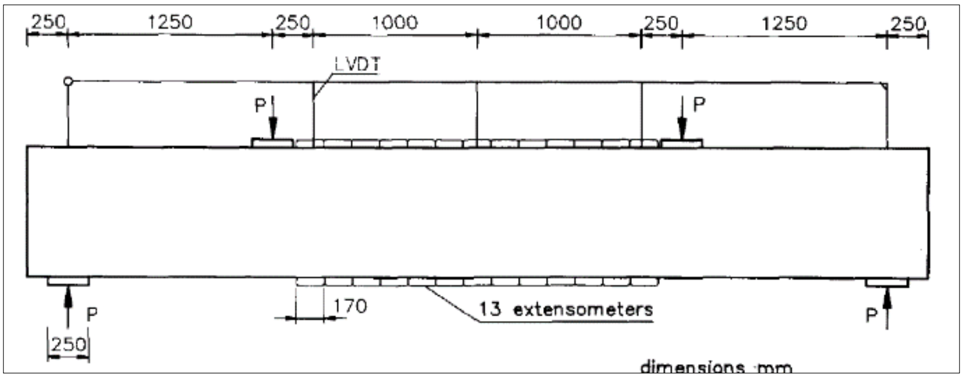



*Figure 2-5: Position of the measuring devices*

In the following table the geometrical properties and material properties of the 2 beams which were used for the analytical analysis are presented.

*Table 2-1: Geometrical and material properties of T-beams 3 and Rectangular beam 13*

| Beam no. | Dimensions [mm] | | | Material Properties [MPa] | | Main reinforcement [mm] | Web reinforcement (per side) [mm] | | | Stirrups [mm] | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | h | d | Web width | $f_{ccm}$ | $f_{cspl}$ | $d_s$ | $d_s$ | # Layers | Bar Spacing | $d_{ss}$ | c |
| 3 | 800 | 730 | 150 | 51.4 | 3.79 | 4Φ20 (2 layers) | 12 | 2 | 200 | 10 | 20 |
| 13 | 800 | 750 | 300 | 51 | 3.72 | 4Φ20 (1 layer) | 12 | 1 | 100 | 10 | 30 |

## 2.2 Further analysis of the concrete cover on crack width control [8]

Concrete structures built in extreme environmental conditions require a large concrete cover for the protection of the reinforcement. For these structures the permissible steel stress in the serviceability limit state, calculated with the expressions in NEN-EN 1992-1-1 (Eurocode 2) for crack width control results in quite low values, compared to calculations with the previous regulations NEN 6720 (VBC). In order to investigate the influence of the concrete cover on the permissible steel stress it was proposed to limit the maximum crack spacing ($s_{r,max}$), since the maximum crack spacing depends on the applied concrete cover. This influence was investigated analytically by Vosslamber of Ballast Nedam Engineering by limiting the maximum crack spacing and modifying the expression of the permissible steel stress in Eurocode 2. By rewriting the expressions for crack width control found in the Eurocode 2 and the VBC 1995 the permissible steel stress could be calculated. With the data shown in Table 2-2 and the equations presented in Table 2-3 the steel stress was estimated and compared. In this table the equations for the steel stress are presented as provided by the regulations. By comparing the values, it was clear that the steel stress calculated with the help of the Eurocode 2 was 35 % lower than the steel stress calculated according to the VBC 1995. After modifying the expressions for steel stress in the Eurocode 2 by limiting the crack spacing ($s_{r,max}$), the same calculations were carried out, only with a larger cover. These results are presented in Table 2-4 and show that the difference between the calculations of the steel stress according to VBC and the Eurocode 2 was much smaller (10%). So with this article it could be concluded that the maximum crack spacing and the concrete cover has a large influence on the crack width calculations according to the Eurocode 2. It can also be concluded that re-arranging the parameters in the expressions for crack width control in the Eurocode 2 can provide better results. Since certain assumptions were made to obtain the modified equation it can only be applied to structures with heights of 600 mm and larger [8].

*Table 2-2: General data used for the calculation of the permissible steel stress ($\sigma_s$)*

| | NEN 6720 | NEN-EN 1992-1 |
|---|---|---|
| **Longitudinal reinforcement** ($\emptyset$) | $\emptyset 20 - 150$ | $\emptyset 20 - 150$ |
| **Stirrups** ($\emptyset_{bgl}$) | $\emptyset 16$ | $\emptyset 16$ |
| **Cover** ($c_{toeg}$) | 45 | 45 |
| **Nominal cover** ($c_{nom}$) | 40 | 45 |
| **Lifespan** | 50 | 50 |
| **Environmental Class** | XD3 | XD3 |
| **Strength class** ($f_{ck}$) | C35/45 | C35/45 |

*Table 2-3: Equations used in the calculations before limiting $Sr, max$*

| | NEN 6720 (VBC) | NEN-EN 1992-1 |
|---|---|---|
| $\sigma_s[\frac{N}{mm^2}]$ | $\leq k_2\left[0.01\frac{s}{\sqrt{k_c}}+1.3\right]^{-1}$ | $\leq\left(\frac{w_{max}k_cE_s}{0.6}\right)*\left[0.54\left(\frac{c_{applied}+\emptyset_{stirrups}+\frac{1}{2}\emptyset}{\emptyset}\right)* s+3.4(c_{applied}+\emptyset_{stirrups})\right]^{-1}$ |
| **Calculated value** | 184 | 135 |
| **Difference of 35%** | | |

*Table 2-4: Equations used in the calculations after limiting $Sr, max$*

| | NEN 6720 (VBC) | NEN-EN 1992-1 |
|---|---|---|
| $\sigma_{s,mod}$ | | $=\left(\frac{w_{max}k_cE_s}{0.6\emptyset}\right)*\left[max[(50-0.8f_{ck});15]\right]^{-1}$ |
| **Cover ($c_{toeg}$)** | 45 | 50 |
| **Calculated value** | 184 | 168 |
| **Difference of 10%** | | |

As mentioned earlier this research shows that by rewriting the equations presented in the codes and restraining them to some extent, good results can be obtained. During my research the influence of the maximum crack spacing on the crack width calculations will also be analyzed in EXCEL. This will be done by reviewing the influence of the concrete cover on the amount of steel needed for crack width control in concrete elements subjected to bending. After this a numerical analysis will be carried out in the finite element program DIANA to investigate what influence an increasing cover has on the cracking behavior of a beam subjected to bending.

# 3 ANALYTICAL ANALYSIS

As mentioned in section 1.5.4 an analytical analysis was performed in Excel. There were two analysis preformed namely:

1. Analysis 1: The influence of the concrete cover and maximum crack spacing on the allowable steel stress calculated according to NEN-EN 1992-1-1 and NEN 6720
2. Analysis 2: Calculation of the crack with according to NEN-EN 1992-1-1. In this analysis the crack width was calculated and compared with the experimental results mentioned in section 2.1.

In the following chapters the results of the above mentioned analysis is presented.

## 3.1 INFLUENCE OF CONCRETE COVER ON THE MAXIMUM ALLOWABLE STEEL STRESS

In order to investigate the influence of concrete cover on the maximum allowable steel stress an analytical analysis was carried out. The calculations were based on an existing EXCEL-sheet for the calculation of the steel stress and maximum allowable bending moment in concrete slabs subjected to pure bending. Expressions from different codes were used namely: the VBC (NEN6720) and Eurocode 2 (1992-1-1). This sheet was composed by ARCADIS employee Kees van der Veen.

### 3.1.1 Procedure

First the existing Excel-sheet was studied and the several equations were looked at. This was followed by calculating the steel stress when varying the concrete cover. The influence of this variation on the maximum allowable steel stress was analyzed by comparing the EUROCODE 2 (crack width expressions) and the VBC calculations (detailing rules: bar diameter/bar spacing/steel stress).

### 3.1.2 Results: Variation of the concrete cover

The calculations were carried out for a slab with a thickness of 800 mm and a width of 1000 mm. The concrete cover was varied with the following values: $c = 40\ mm\ ; c = 50\ mm\ and\ c = 60\ mm$. All calculations were carried out for three different bar diameters: $\emptyset 16; \emptyset 20\ and\ \emptyset 25$ based on a crack width limit of $w_{\max} = 0.2\ mm$. In Appendix I: section 1.2.1 an example calculation based on the VBC 1995 equations regarding crack width control is presented. The example calculation based on the Eurocode equation regarding crack width control can be found in Appendix I section 1.2.2. In the following chapters the results are presented.

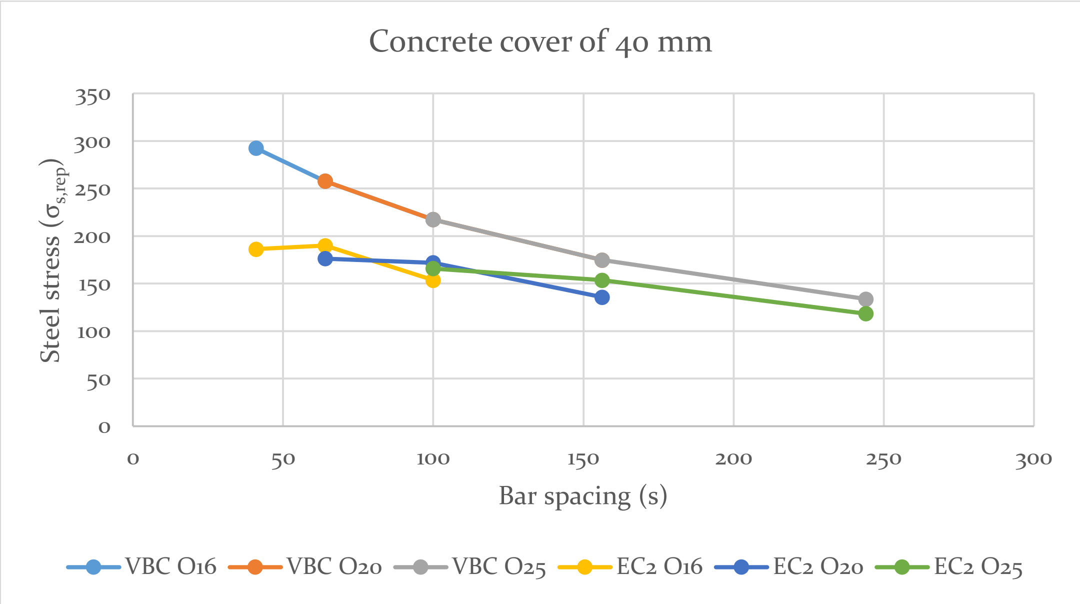
**Applied cover:** $c = 40\ mm$:

*Table 3-1: Values of the maximum steel stress for a concrete cover of c=40 mm according to NEN 6720*

| NEN 6720<br>c=60 mm | | Aggressive environment: Crack width: w = 0.2 mm<br>$c_{min} = 40\ mm$<br>$k_c = 1$ | |
|---|---|---|---|
| $\phi_k$ = | $A_s$ = | s = | $\sigma_{s,rep}$ < |
| [ mm ] | [ mm²/m ] | [ mm ] | [ N/mm² ] |
| 16 | 2011 | 100 | 217 |
| 16 | 3142 | 64 | 258 |
| 16 | 4909 | 41 | 292 |
| 20 | 2011 | 156 | 175 |
| 20 | 3142 | 100 | 217 |
| 20 | 4909 | 64 | 258 |
| 25 | 2011 | 244 | 134 |
| 25 | 3142 | 156 | 175 |
| 25 | 4909 | 100 | 217 |

*Table 3-2: Values of the maximum steel stress for a concrete cover of c=40 mm according to NEN-EN 1992-1-1*

| NEN-EN-1992<br>$c = 40\ mm$ | | Allowable crack width:<br>$w = k_c * w_k = 0.2$ mm | | | |
|---|---|---|---|---|---|
| $\Phi_k$ = | $A_s$ = | s = | $\rho_{s;eff}$ = | $s_{r;max}$ = | $\sigma_{s;rep}$ = |
| [ mm ] | [ mm²/m ] | [ mm ] | [ % ] | [mm] | [ N/mm² ] |
| 16 | 2011 | 100 | 0.0118 | 434 | 154 |
| 16 | 3142 | 64 | 0.0185 | 351 | 190 |
| 16 | 4909 | 41 | 0.0289 | 298 | 186 |
| 20 | 2011 | 156 | 0.0118 | 491 | 136 |
| 20 | 3142 | 100 | 0.0185 | 388 | 172 |
| 20 | 4909 | 64 | 0.0289 | 322 | 176 |
| 25 | 2011 | 244 | 0.0118 | 563 | 118 |
| 25 | 3142 | 156 | 0.0185 | 434 | 154 |
| 25 | 4909 | 100 | 0.0289 | 351 | 166 |




*Graph 3-1: Maximum allowable steel stress for a cover of 40 mm according to VBC- and Eurocode 2 calculations*

By comparing Table 3-1 and Table 3-2 it can be seen that the value of the steel stress decreases with about $\frac{217-154}{154} * 100 = 41\%$ for a bar diameter of $16\ mm$. With increasing bar diameter the difference between the two calculated steel stresses decreases. In Graph 3-1 the steel stress is depicted as a function of bar spacing. This graph shows that at smaller values of the bar spacing ($s < 150\ mm$) the steel stress calculated according to the NEN-EN-1992-1 is much smaller than the steel stress calculated according to the NEN-6720.

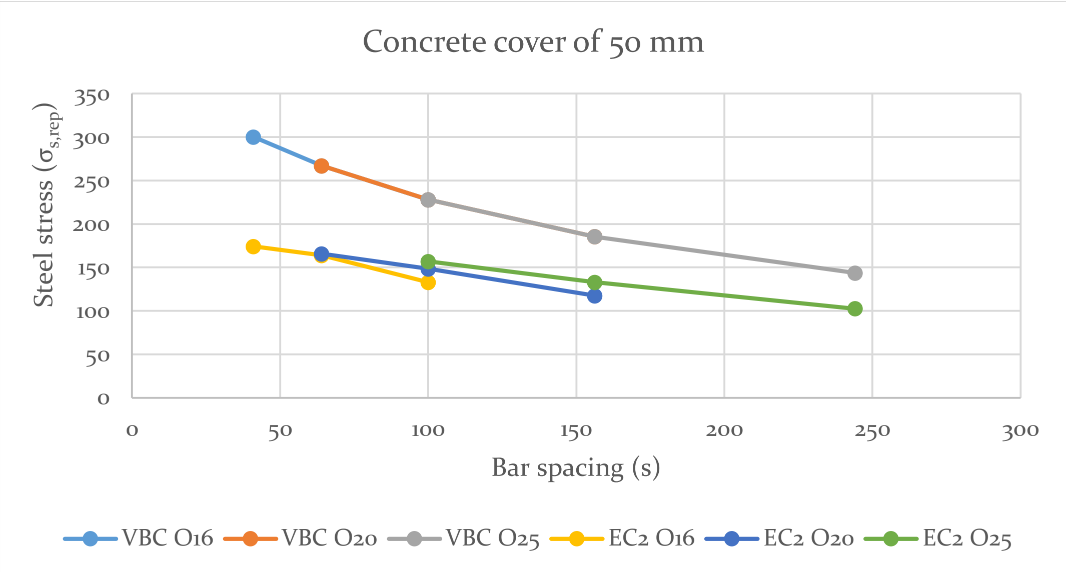
### Applied cover: $c = 50\ mm$:

*Table 3-3: Values of the maximum steel stress for a concrete cover of c=50 mm according to NEN 6720*

| NEN6720 c=50 mm | | Aggressive environment: Crack width: w = 0.2 mm $c_{min} = 40\ mm$ $k_c = 1.25$ | |
|---|---|---|---|
| $\phi_k$ = | $A_s$ = | s = | $\sigma_{s;rep}$ < |
| [ mm ] | [ mm²/m ] | [ mm ] | [ N/mm² ] |
| 16 | 2011 | 100 | 228 |
| 16 | 3142 | 64 | 267 |
| 16 | 4909 | 41 | 300 |
| 20 | 2011 | 156 | 185 |
| 20 | 3142 | 100 | 228 |
| 20 | 4909 | 64 | 267 |
| 25 | 2011 | 244 | 144 |
| 25 | 3142 | 156 | 185 |
| 25 | 4909 | 100 | 228 |

*Table 3-4: Values of the maximum steel stress for a concrete cover of c=50 mm according to NEN-EN 1992-1-1*

| **NEN-EN-1992** $c = 50\ mm$ | | **Allowable crack width:** $w = k_c * w_k = 0.2$ **mm** | | | |
|---|---|---|---|---|---|
| $\Phi_k =$ | $A_s =$ | s = | $\rho_{s;eff} =$ | $S_{r;max} =$ | $\sigma_{s;rep} =$ |
| [ mm ] | [ mm²/m ] | [ mm ] | [ % ] | [mm] | [ N/mm² ] |
| **16** | 2011 | 100 | 0.0103 | 502 | 133 |
| **16** | 3142 | 64 | 0.0161 | 407 | 164 |
| **16** | 4909 | 41 | 0.0251 | 346 | 174 |
| **20** | 2011 | 156 | 0.0103 | 568 | 117 |
| **20** | 3142 | 100 | 0.0161 | 449 | 148 |
| **20** | 4909 | 64 | 0.0251 | 373 | 166 |
| **25** | 2011 | 244 | 0.0103 | 650 | 103 |
| **25** | 3142 | 156 | 0.0161 | 502 | 133 |
| **25** | 4909 | 100 | 0.0251 | 407 | 157 |




*Graph 3-2: Maximum allowable steel stress for a cover of 50 mm according to VBC- and Eurocode 2 calculations*

In Table 3-3 it can be seen that the value of the steel stress ($\sigma_{s;rep}$) according to VBC increases between 3% and 7% compared to Table 3-1. This occurs because the factor $k_c = \frac{c}{c_{min}}$ also increases when the concrete cover increases. Table 3-4 shows that the value of the steel stress according to Eurocode 2 decreases further between 5% and 14% compared to Table 3-2. Graph 3-2 shows that maximum allowable steel stress calculated with Eurocode 2 is much smaller than the maximum allowable steel stress calculated with the VBC, it is clear that the difference in the steel stress between the two applied codes increases for a larger cover.

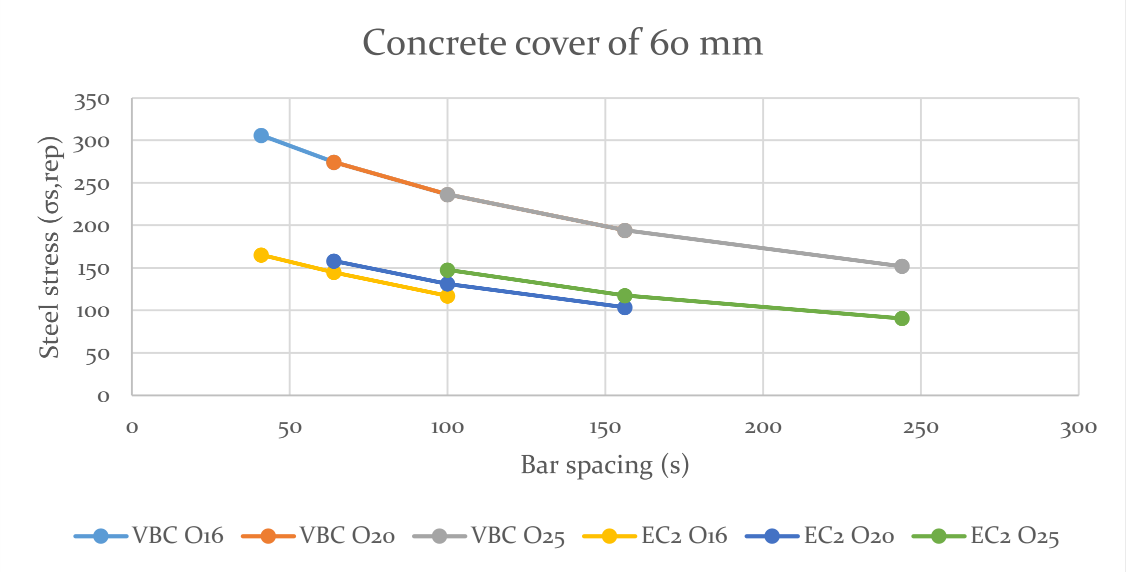
### Applied cover: $c = 60\ mm$:

*Table 3-5: Values of the maximum steel stress for a concrete cover of c=60 mm according to NEN 6720*

| NEN6720<br>c=60 mm | | Aggressive environment: Crack width: w = 0.2 mm<br>$c_{min} = 40\ mm$<br>$k_c = 1.5$ | |
|---|---|---|---|
| **$\Phi_k$ =** | $A_s$ = | **s =** | **$\sigma_{s;rep}$ <** |
| [ **mm** ] | [ mm²/m ] | [ mm ] | [ N/mm² ] |
| **16** | 2011 | 100 | 236 |
| **16** | 3142 | 64 | 274 |
| **16** | 4909 | 41 | 306 |
| **20** | 2011 | 156 | 194 |
| **20** | 3142 | 100 | 236 |
| **20** | 4909 | 64 | 274 |
| **25** | 2011 | 244 | 152 |
| **25** | 3142 | 156 | 194 |
| **25** | 4909 | 100 | 236 |

*Table 3-6: Values of the maximum steel stress for a concrete cover of c=60 mm according to NEN-EN 1992-1-1*

| NEN-EN-1992<br>$c = 60\ mm$ | | Allowable crack width:<br>$w = k_c * w_k = 0.2$ mm | | | |
|---|---|---|---|---|---|
| **$Ø_k$ =** | $A_s$ = | **s =** | $\rho_{s;eff}$ = | $s_{r;max}$ = | $\sigma_{s;rep}$ = |
| [ **mm** ] | [ mm²/m ] | [ mm ] | [ % ] | [mm] | [ N/mm² ] |
| **16** | 2011 | 100 | 0.009 | 570 | 117 |
| **16** | 3142 | 64 | 0.014 | 461 | 145 |
| **16** | 4909 | 41 | 0.024 | 387 | 165 |
| **20** | 2011 | 156 | 0.009 | 644 | 104 |
| **20** | 3142 | 100 | 0.014 | 508 | 131 |
| **20** | 4909 | 64 | 0.024 | 416 | 158 |
| **25** | 2011 | 244 | 0.009 | 737 | 90 |
| **25** | 3142 | 156 | 0.014 | 567 | 117 |
| **25** | 4909 | 100 | 0.024 | 452 | 147 |




*Graph 3-3: Maximum allowable steel stress for a cover of 60 mm according to VBC- and Eurocode 2 calculations*

The variation of the concrete cover does have an influence on the maximum allowable steel stress $\sigma_{s,rep}$ calculated according to the NEN-EN 1992-1-1. In Table 3-2,Table 3-4 and Table 3-6 it can clearly be seen that $\sigma_{s,rep}$ decreases when a larger cover is applied. We also see that the maximum crack spacing ($s_{r,max}$) increases when the concrete cover is larger. This can be explained by looking at the equation for $s_{r,max}$ : $s_{r,max} = k_3 * c + k_1 * k_2 * k_4 * \frac{\emptyset}{\rho_{p,eff}}$. We see that when increasing the cover, the effective depth ($h_{c,eff} = 2.5(h-d)$) increases and the effective concrete area ($\rho_{p,eff}$) decreases. This causes the maximum crack spacing ($s_{r,max}$) to increase. Since the maximum allowable steel stress also depends on the maximum crack spacing and the effective concrete area, which can be seen in the equation below, the steel stress will decrease when the $s_{r,max}$ increases and $\rho_{p,eff}$ decreases.

$$\sigma_{s,rep} < \frac{w_k}{s_{r,max}} * E_s + k_t * \frac{f_{ct;eff}}{\rho_{p;eff}} * \left(1 + \alpha_e * \rho_{p;eff}\right)$$

When comparing the results of the NEN 6720 calculations with the NEN-EN 1992-1-1 it is clear that allowed the steel stress calculated in the NEN-EN 1992-1-1 is much smaller than that of NEN 6720. This was also mentioned in section 1.1. The amount of reinforcement that is needed to control the cracking behavior calculated with the NEN-EN 1992-1-1 regulations is larger than the amount of reinforcement calculated with the NEN 6720 regulations.

## 3.2 INFLUENCE OF THE LIMITATION OF THE MAXIMUM CRACK SPACING ON THE MAXIMUM ALLOWABLE STEEL STRESS

In the previous chapter it was clear that the maximum allowable steel stress decreased when the concrete cover increased. In this chapter the influence of the concrete cover on the maximum allowable steel stress is further analyzed by limiting the maximum crack spacing in the NEN-EN 1992-1-1.

#### NEN 6720

In the NEN6720 the crack spacing is not limited since the crack width calculations are based on the bar diameter ($\emptyset_{km}$) and the allowable bar spacing ($s$). These equations are based on the requirements regarding crack width control as presented in the VB 1974/1984. The equations from the VB 1974/1984 are rewritten in terms of bar diameter and bar spacing to meet the conditions concerning the cracking behavior [4]. The values for the maximum allowable steel stress remain the same for the NEN 6720 calculations.

#### NEN-EN 1992-1-1

Looking at the results of the Eurocode 2 calculations in section 3.1.2 it can be seen that the value of the maximum crack spacing ($s_{r,max}$) increases when a larger concrete cover is applied. To further analyze this, the conditions regarding the cracking behavior in VB 1974/1984 are also studied. When doing this it is clear that the crack width calculations in the Eurocode are almost the same as the calculations in the VB 1974/1984. There is one difference however, since in VB 1974/1984 the mean crack spacing ($\Delta l_m$) has an upper limited value of $\Delta l = 10\emptyset k_m$. This limitation is not found in the Eurocode 2. So for further analysis this upper limit value will be applied in the Eurocode 2 equation for the maximum crack spacing ($s_{r,\max}$).

The following upper boundaries are also applied in the calculation for $s_{r,max}$: $s_{r,max} \leq Max\,\{(50 - 0.8f_{ck})\emptyset; 15\emptyset\}$. This upper boundary is taken from VARCE (Vraag en antwoord rubriek in CEMENT :NEN-EN 1992-1-1 +C2: 2011/NB:2011), which was obtained at ARCADIS.

Thus the calculation of the maximum crack spacing is modified in excel twice with the following equations:

1) $s_{r,max} = k_3 * c + k_1 * k_2 * k_4 * \frac{\emptyset}{\rho_{p,eff}}; s_{r,max} \leq 10\emptyset$
2) $s_{r,max} = k_3 * c + k_1 k_2 k_4 * \frac{\emptyset}{\rho_{p,eff}} \leq Max\{(50 - 0.8 * f_{ck})\emptyset\,; 15\emptyset\}$

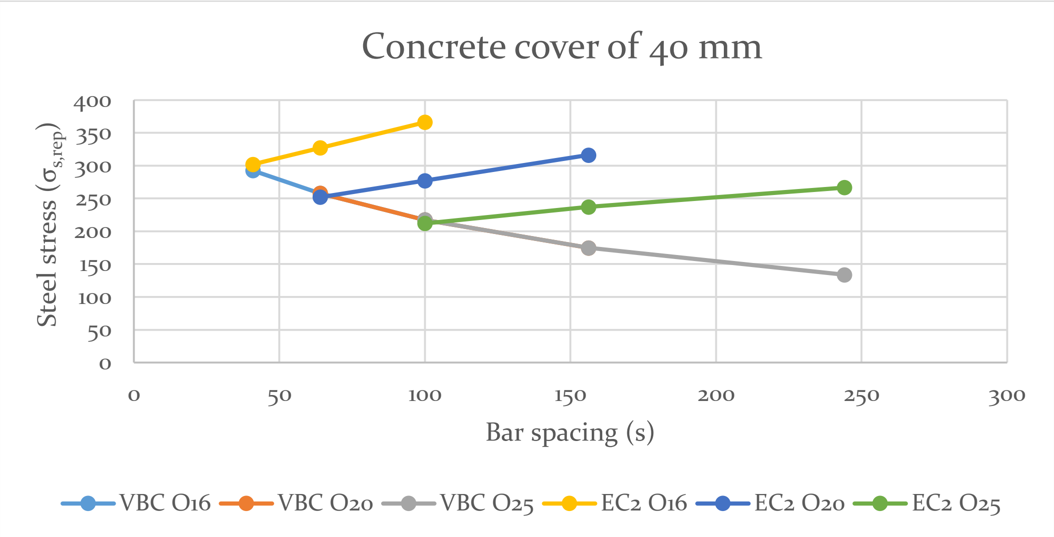
### 3.2.1 Results: Comparison of $\sigma_{s,rep}$ after limiting $s_{r,max} \leq 10\emptyset$ with the value of $\sigma_{s,rep}$ in VBC1990 (NEN 6720)

The calculations were carried out for a slab with a thickness of 800 mm and a width of 1000 mm. The concrete cover was varied with the following values: $c = 40\,mm\,; c = 50\,mm\ and\ c = 60\,mm$. All calculations were carried out for three different bar diameters: $\emptyset 16; \emptyset 20\ and\ \emptyset 25$ based on a crack width limit of $w_{\max} = 0.2\,mm$. An example calculation of the limitation of maximum crack spacing according to equation 1 can be found in Appendix I section 2.2.1. The example calculation of the limitation of maximum crack spacing according to equation 2 can be found in Appendix I section 2.2.2. In the following chapters the results are presented.

**Applied cover:** $c = 40\ mm$

*Table 3-7: Values of the maximum steel stress for a concrete cover of c=40 mm according to NEN-EN 1992-1-1 after limitation of $s_{r,max}$ with $s_{r,max} \leq 10\emptyset$*

| NEN-EN-1992 c=40 | | Allowable crack width: $w = k_c * w_k = 0.2$ | | | |
|---|---|---|---|---|---|
| $\Phi_k$ = | $A_s$ = | s = | $\rho_{s,eff}$ = | $s_{r;max}$ = | $\sigma_{s;rep}$ = |
| [ mm ] | [ mm²/m ] | [ mm ] | [ % ] | [mm] | [ N/mm² ] |
| 16 | 2011 | 100 | 0.012 | 160 | 366 |
| 16 | 3142 | 64 | 0.018 | 160 | 327 |
| 16 | 4909 | 41 | 0.029 | 160 | 302 |
| 20 | 2011 | 156 | 0.012 | 200 | 316 |
| 20 | 3142 | 100 | 0.018 | 200 | 277 |
| 20 | 4909 | 64 | 0.029 | 200 | 252 |
| 25 | 2011 | 244 | 0.012 | 250 | 267 |
| 25 | 3142 | 156 | 0.018 | 250 | 237 |
| 25 | 4909 | 100 | 0.029 | 250 | 212 |



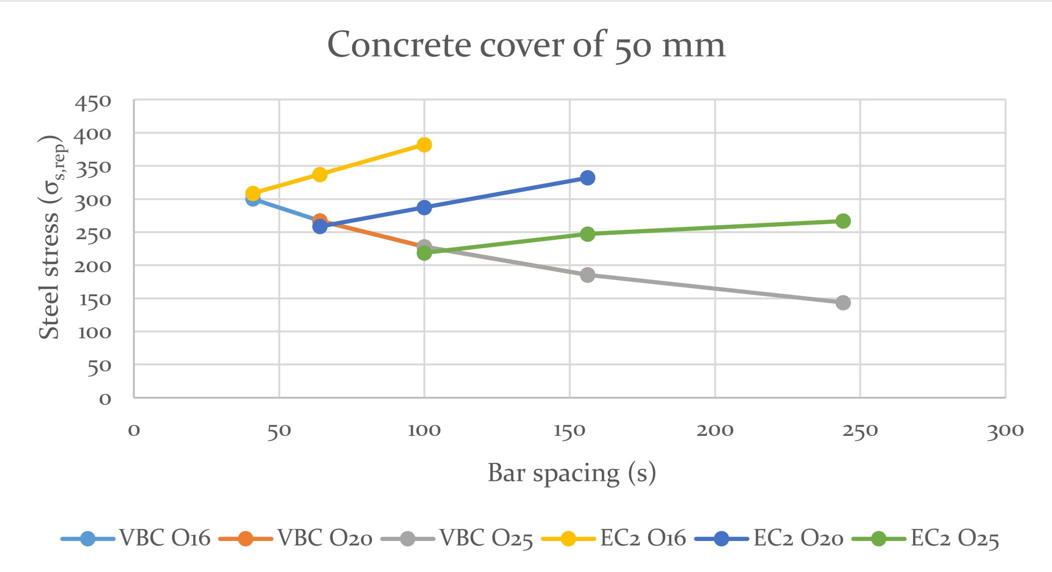

*Graph 3-4: Maximum allowable steel stress for a cover of 40 mm according to VBC- and Eurocode 2 calculations after limitation of the maximum crack spacing with $s_{r,max} \leq 10\emptyset$*

When we look at the values for the maximum allowable steel stress it can be seen that the $\sigma_{s;rep}$ increases substantially. Before limitation the steel stress was equal to $\sigma_{s;rep} = 154\ N/mm^2$ and after limitation it was equal to: $\sigma_{s;rep} = 366\ N/mm^2$. This means that the maximum steel stress increases with : $\frac{366-154}{154} * 100\% = 138\ \%$ compared to the calculation without limitation of the maximum crack spacing for a cover of $c = 40\ mm$. In Graph 3-1 the difference between the steel stress calculated with the limited value of the maximum crack spacing according to the Eurocode 2 is compared to the maximum steel stress calculated according to the VBC.

### Applied cover: $c = 50\ mm$

*Table 3-8: Values of the maximum steel stress for a concrete cover of c=50 mm according to NEN-EN 1992-1-1 after limitation of $s_{r,max}$ with $s_{r,max} \leq 10\emptyset$*

| NEN-EN-1992 c=50 mm | | Allowable crack width: $w = k_c * w_k = 0.2$ | | | |
|---|---|---|---|---|---|
| $Ø_k$ = | $A_s$ = | s = | $\rho_{s,eff}$ = | $s_{r;max}$ = | $\sigma_{s;rep}$ = |
| [ mm ] | [ mm²/m ] | [ mm ] | [ % ] | [mm] | [ N/mm² ] |
| 16 | 2011 | 100 | 0.010 | 160 | 382 |
| 16 | 3142 | 64 | 0.016 | 160 | 337 |
| 16 | 4909 | 41 | 0.025 | 160 | 309 |
| 20 | 2011 | 156 | 0.010 | 200 | 332 |
| 20 | 3142 | 100 | 0.016 | 200 | 287 |
| 20 | 4909 | 64 | 0.025 | 200 | 259 |
| 25 | 2011 | 244 | 0.010 | 250 | 267 |
| 25 | 3142 | 156 | 0.016 | 250 | 247 |
| 25 | 4909 | 100 | 0.025 | 250 | 219 |



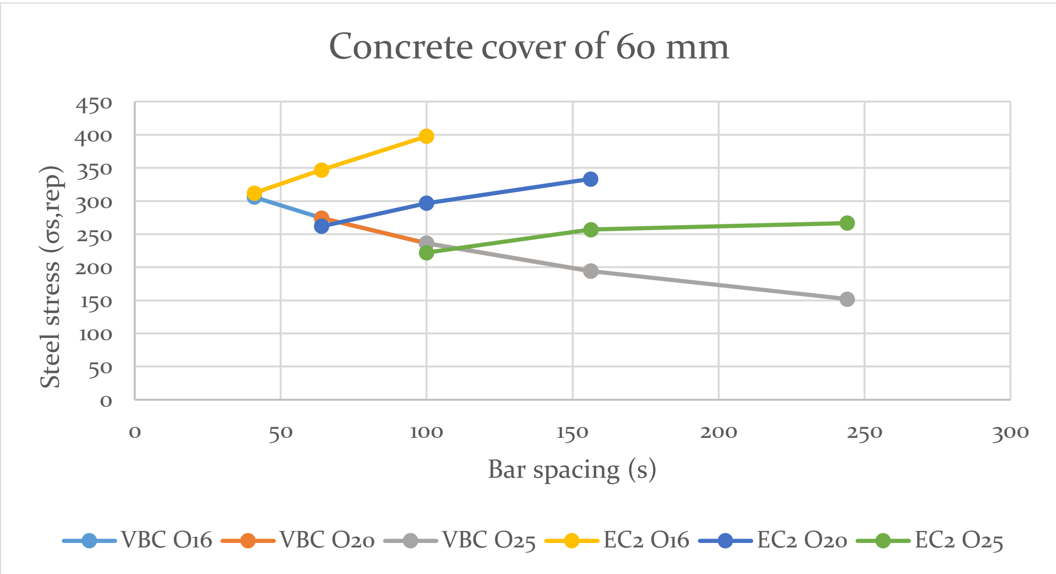

*Graph 3-5: Maximum allowable steel stress for a cover of 50 mm according to VBC- and Eurocode 2 calculations after limitation of the maximum crack spacing with $s_{r,max} \leq 10\emptyset$*

When we look at the values for the maximum allowable steel stress in Table 3-8 it can be seen that the $\sigma_{s;rep}$ increases again. Before limitation the steel stress was equal to $\sigma_{s;rep} = 133\ N/mm^2$ and after limitation it was equal to: $\sigma_{s;rep} = 382\ N/mm^2$. This means that the maximum steel stress increases with : $\frac{382-133}{133} * 100\% = 187\ \%$ compared to the calculation without limitation of the maximum crack spacing for a cover of $c = 50\ mm$.

### Applied cover: $c = 60\ mm$

*Table 3-9: Values of the maximum steel stress for a concrete cover of c=60 mm according to NEN-EN 1992-1-1 after limitation of $s_{r,max}$ with $s_{r,max} \leq 10\emptyset$*

| NEN-EN-1992 c=60 mm | | Allowable crack width: $w = k_c * w_k = 0.2$ | | | |
|---|---|---|---|---|---|
| $\emptyset_k$ = | $A_s$ = | s = | $\rho_{s,eff}$ = | $s_{r;max}$ = | $\sigma_{s;rep}$ = |
| [ mm ] | [ mm²/m ] | [ mm ] | [ % ] | [mm] | [ N/mm² ] |
| **16** | 2011 | 100 | 0.009 | 160 | 398 |
| **16** | 3142 | 64 | 0.014 | 160 | 347 |
| **16** | 4909 | 41 | 0.024 | 160 | 312 |
| **20** | 2011 | 156 | 0.009 | 200 | 333 |
| **20** | 3142 | 100 | 0.014 | 200 | 297 |
| **20** | 4909 | 64 | 0.024 | 200 | 262 |
| **25** | 2011 | 244 | 0.009 | 250 | 267 |
| **25** | 3142 | 156 | 0.014 | 250 | 257 |
| **25** | 4909 | 100 | 0.024 | 250 | 222 |




*Graph 3-6: Maximum allowable steel stress for a cover of 60 mm according to VBC- and Eurocode 2 calculations after limitation of the maximum crack spacing with $s_{r,max} \leq 10\emptyset$*

When we look at the results for a cover of 60 mm presented in Table 3-9 we see a further increase in the value of the steel stress. The value maximum steel stress increases with : $\frac{366-154}{154} * 100\% =$ $240\ \%$ compared to the calculation without limitation of the maximum crack spacing for a cover of $c = 60\ mm$. So with increasing cover the value of the steel stress increases with more than $130\ \%$ in the cases where the maximum crack spacing is limited with $s_{r,max} \leq 10\emptyset$.

In Graph 3-4 to Graph 3-6 it can clearly be seen that the maximum allowable steel stress increases far above the values calculated in the NEN 6720. Since the maximum crack spacing is very small due to the limitation, we can see that the steel stress increases dramatically when a larger concrete cover is applied. These values seem very unrealistic since the steel stress is expected to be about $\sigma_s = 300\ N/mm^2$ in the S.L.S.

With this calculation a better insight has been obtained in the influence of the limitation of the maximum crack spacing according to VB 1974/1984 on the maximum allowable steel stress. It is clear that the maximum crack spacing has an influence on the calculation of the maximum allowable steel stress.

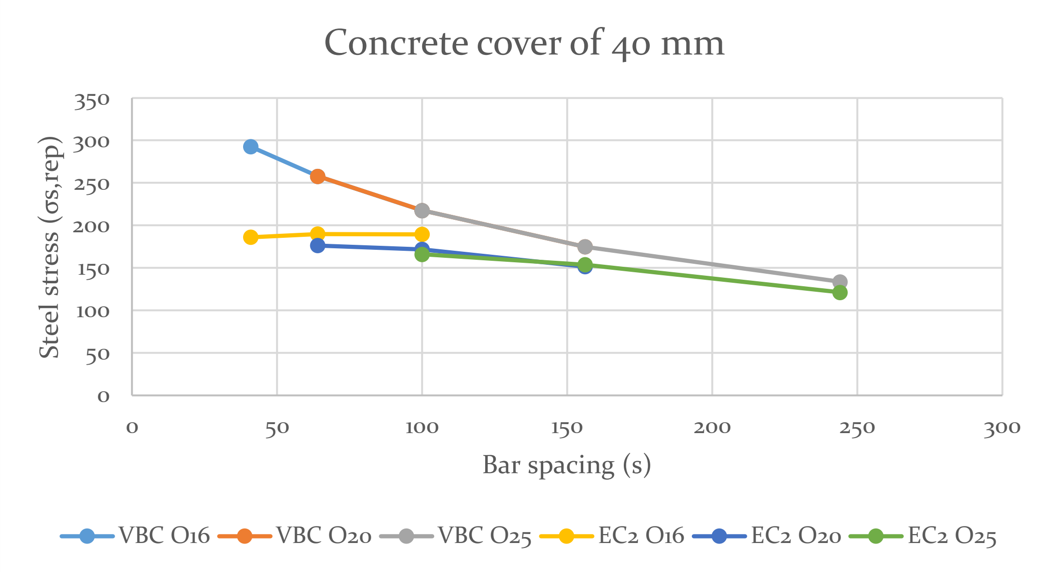
### 3.2.2 Results: Comparison of VBC with the limitation fo $s_{r,max} \leq Max\{(50 - 0.8 * f_{ck})\emptyset\ ; 150\emptyset\}$

When the maximum crack spacing is limited with $s_{r,max} \leq Max\{(50 - 0.8 * f_{ck})\emptyset\ ; 150\emptyset\}$ we see that there is an increase in the value of the steel stress allowed ($\sigma_{s,rep} = 189\ N/mm^2$) compared to the value of the steel stress without limitation of the maximum crack spacing ($\sigma_{s,rep} = 154\ N/mm^2$). At an increasing concrete cover the value of $\sigma_{s,rep}$ reaches a constant value of $\sigma_{s,rep} = 189\ N/mm^2$ for a bar diameter of $16\ mm$. The results which were obtained are presented below.

**Applied cover:** $c = 40\ mm$

*Table 3-10: Values of the maximum steel stress for a concrete cover of c=40 mm according to NEN-EN 1992-1-1 after limitation of $s_{r,max}$ with $s_{r,max} \leq Max\{(50 - 0.8 * f_{ck})\emptyset\ ; 150\emptyset\}$*

| NEN-EN-1992 c=40 mm | | Allowable crack width: $w = k_c * w_k = 0.2$ | | | |
|---|---|---|---|---|---|
| $Ø_k$ = | $A_s$ = | s = | $\rho_{s,eff}$ = | $s_{r;max}$ = | $\sigma_{s;rep}$ = |
| [ mm ] | [ mm²/m ] | [ mm ] | [ % ] | [mm] | [ N/mm² ] |
| **16** | 2011 | 100 | 0.0118 | 352 | 189 |
| **16** | 3142 | 64 | 0.0185 | 351 | 190 |
| **16** | 4909 | 41 | 0.0289 | 298 | 186 |
| **20** | 2011 | 156 | 0.0118 | 440 | 152 |
| **20** | 3142 | 100 | 0.0185 | 388 | 172 |
| **20** | 4909 | 64 | 0.0289 | 322 | 176 |
| **25** | 2011 | 244 | 0.0118 | 550 | 121 |
| **25** | 3142 | 156 | 0.0185 | 434 | 154 |
| **25** | 4909 | 100 | 0.0289 | 351 | 166 |

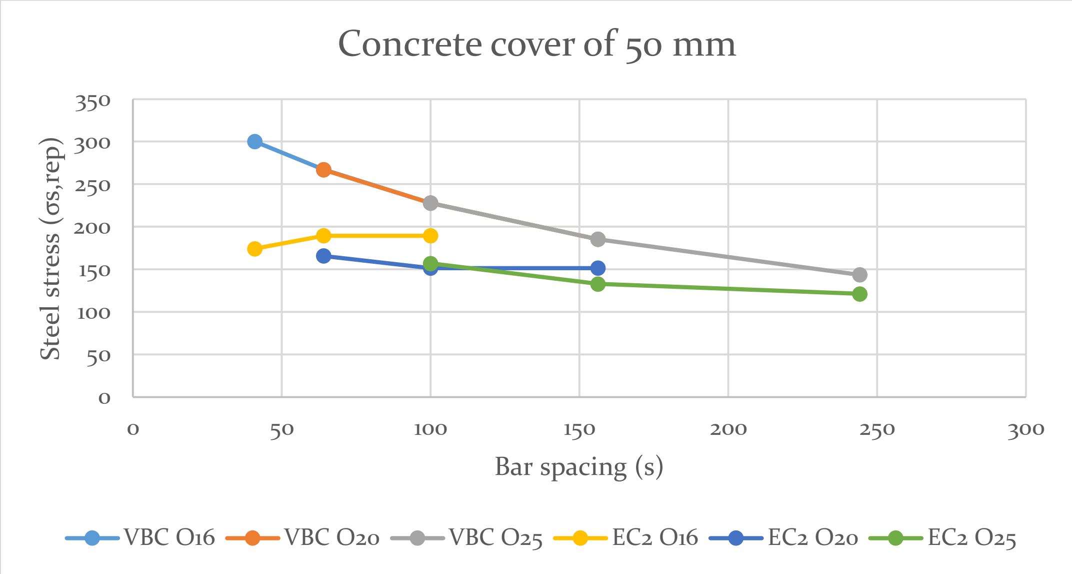



*Graph 3-7: Maximum allowable steel stress for a cover of 40 mm according to VBC- and Eurocode 2 calculations after limitation of the maximum crack spacing with* $s_{r,max} \leq Max\{(50 - 0.8 * f_{ck})\emptyset\,;150\emptyset\}$

**Applied cover:** $c = 50\ mm$

*Table 3-11: Values of the maximum steel stress for a concrete cover of c=50 mm according to NEN-EN 1992-1-1 after limitation of* $s_{r,max}$ *with* $s_{r,max} \leq Max\{(50 - 0.8 * f_{ck})\emptyset\,;150\emptyset\}$

| **NEN-EN-1992 c=50 mm** | | **Allowable crack width:** $w = k_c * w_k = 0.2$ | | | |
|---|---|---|---|---|---|
| **$Ø_k$ =** | $A_s$ = | **s =** | $\rho_{s,eff}$ = | $s_{r;max}$ = | $\sigma_{s;rep}$ = |
| [ **mm** ] | [ mm²/m ] | [ mm ] | [ % ] | [mm] | [ N/mm² ] |
| **16** | 2011 | 100 | 0.010 | 352 | 189 |
| **16** | 3142 | 64 | 0.016 | 352 | 189 |
| **16** | 4909 | 41 | 0.025 | 346 | 174 |
| **20** | 2011 | 156 | 0.010 | 440 | 152 |
| **20** | 3142 | 100 | 0.016 | 440 | 152 |
| **20** | 4909 | 64 | 0.025 | 373 | 166 |
| **25** | 2011 | 244 | 0.010 | 550 | 121 |
| **25** | 3142 | 156 | 0.016 | 502 | 133 |
| **25** | 4909 | 100 | 0.025 | 407 | 157 |

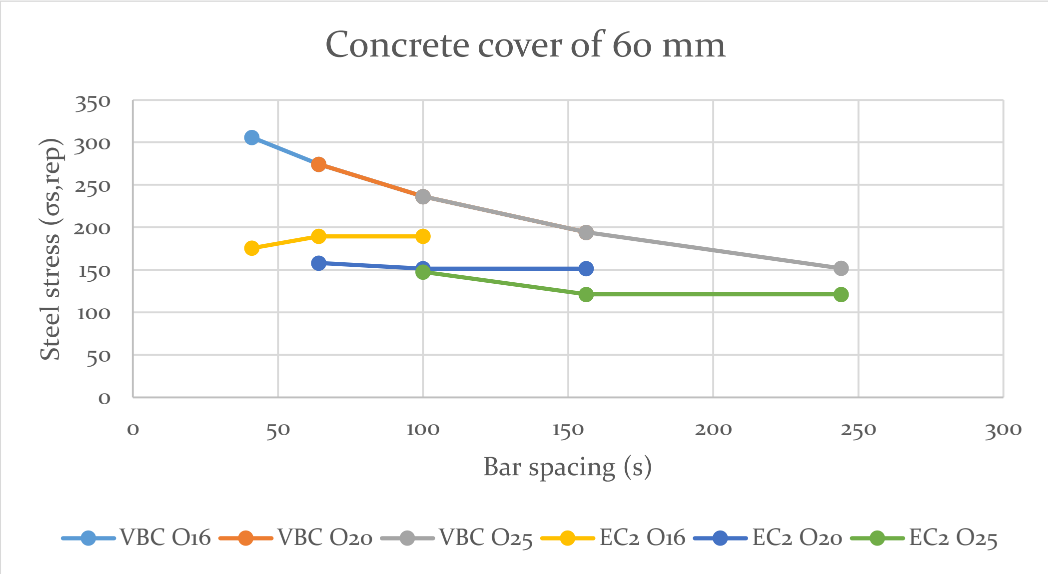



*Graph 3-8: Maximum allowable steel stress for a cover of 50 mm according to VBC- and Eurocode 2 calculations after limitation of the maximum crack spacing with* $s_{r,max} \leq Max\{(50 - 0.8 * f_{ck})\emptyset\,;150\emptyset\}$

**Applied cover:** $c = 60\ mm$

*Table 3-12: Values of the maximum steel stress for a concrete cover of c=50 mm according to NEN-EN 1992-1-1 after limitation of* $s_{r,max}$ *with* $s_{r,max} \leq Max\{(50 - 0.8 * f_{ck})\emptyset\,;150\emptyset\}$

| **NEN-EN-1992 c=60 mm** | | **Allowable crack width:** $w = k_c * w_k = 0.2$ | | | |
|---|---|---|---|---|---|
| $\emptyset_k$ = | $A_s$ = | s = | $\rho_{s,eff}$ = | $s_{r;max}$ = | $\sigma_{s;rep}$ = |
| [ mm ] | [ mm²/m ] | [ mm ] | [ % ] | [mm] | [ N/mm² ] |
| **16** | 2011 | 100 | 0.009 | 352 | 189 |
| **16** | 3142 | 64 | 0.014 | 352 | 189 |
| **16** | 4909 | 41 | 0.024 | 352 | 176 |
| **20** | 2011 | 156 | 0.009 | 440 | 152 |
| **20** | 3142 | 100 | 0.014 | 440 | 152 |
| **20** | 4909 | 64 | 0.024 | 416 | 158 |
| **25** | 2011 | 244 | 0.009 | 550 | 121 |
| **25** | 3142 | 156 | 0.014 | 550 | 121 |
| **25** | 4909 | 100 | 0.024 | 452 | 147 |




*Graph 3-9: Maximum allowable steel stress for a cover of 60 mm according to VBC- and Eurocode 2 calculations after limitation of the maximum crack spacing with* $s_{r,max} \leq Max\{(50 - 0.8 * f_{ck})\emptyset\,; 15\emptyset\}$

In the second limitation of the maximum crack spacing ($s_{r,max}$) we see that the steel stress also increases, but not as much as the previous limitation of $s_{r,max}$. When we look at the results presented earlier we see that the difference in the allowable steel stress calculated with the NEN 6720 is equal to : $\frac{217-189}{189} * 100\% = 15\%$. Before the upper limit value for the maximum crack spacing was applied ($\sigma_{s,rep} = 154\ N/mm^2$), the difference between the NEN 6720 calculations ($\sigma_{s,rep} = 217\ N/mm^2$) and the Eurocode calculations was about 41% for a concrete cover of $40\ mm$. This means that with the limitation of the crack spacing: $s_{r,max} \leq Max\{(50 - 0.8 * f_{ck})\emptyset\,; 15\emptyset\}$ rather favorable results are obtained. It should be mentioned however, that the values of the maximum allowable steel stress calculated with the Eurocode 2 equations are still smaller than the values of the steel stress calculated with VBC. The amount of reinforcement that is needed to control the cracking behavior calculated with the NEN-EN 1992-1-1 regulations is still larger than the amount of reinforcement calculated with the NEN6720 regulations.

From the calculations which were carried out it is clear that the large differences in the steel stress occur when the bar spacing is smaller than $150\ mm$ (about 41%). When the bar spacing is larger than 150 mm the difference between the calculated steel stress smaller than 15%.

The limitation taken from the VB 1974/ 1984 (NEN 3880) leads to values which are too optimistic and are way above the expected value of the maximum steel stress in the S.L.S. ( $\sigma_s = 300\ N/mm^2$) (section 3.2.1).The alternative limitation of $s_{r,max}$ as proposed in the VARCE leads to the increase of the maximum allowable steel stress, but this increase still is not close to the NEN 6720 calculations presented in section 3.1.2.Therefore it should be considered to refine the limitation of $s_{r,max}$ provided by VARCE where the main interest should lie upon the smaller bar distances, thus in beams and not in the plates.

## 3.3 Crack with calculation according to NEN-EN 1992-1-1

In order to investigate whether the crack width calculations according to EUROCODE 1992-1-1 for a beam subjected to bending are in agreement with reality, an analytical analysis was carried out. The cases which were calculated came from the experimental research performed by Dr. Ir. Braam [6]. Of the 15 beams that were tested during the experiment, two beams were calculated with the help of the EUROCODE 2 equations. The two beams were: T-beam 3 and Rectangular beam 13. Both beams were subjected to bending. In the following chapter the procedure of this analysis will be briefly explained and the results of the crack width calculations will be presented. An example calculation of Rectangular beam 13 can be found in Appendix II. The calculations for T-beam 3 can be found in the Excel sheet: Test Data 2 C.R. Braam: fully developed crack pattern.

### 3.3.1 Procedure

With the help of EXCEL, a calculation sheet was setup in which the calculations for the crack width were performed. First the data for the beams were collected followed by the crack width calculations according to Eurocode 2.These equations are mentioned in section 1.2.3. After the calculations were completed they were compared to the results from the experimental research of Braam.

**Mean value of the crack width**

After the crack width was calculated according to the Eurocode 2 equations (equations 1,2 and 3 mentioned in section 1.2.3) the following procedure was followed for the calculation of the mean crack width:

In a fully developed crack pattern the following criterion holds for the mean value of the crack width [1]:

$w_m * \gamma_s * \gamma_\infty \leq w_{serv}$

In which:

$w_{serv}$: the prevailing crack width criterion

$w_m$: the mean value of the crack width

$\gamma_s$: Factor for scatter:

$\gamma_s = 1.7$ (fully developed crack pattern for a beam subjected to bending)

$\gamma_\infty$: factor considering sustained load/alternating load:

$\sigma_s \leq 295 : \gamma_\infty = 1.3$

$\sigma_s \geq 295 : \gamma_\infty = \frac{1}{1-9*{\sigma_s}^3*10^{-9}}$

And so the mean value of the crack width is calculated with the following equation:

$$w_m = \frac{w_{k,max}}{1.7 * 1.3} = \frac{w_{k,max}}{2.2}$$

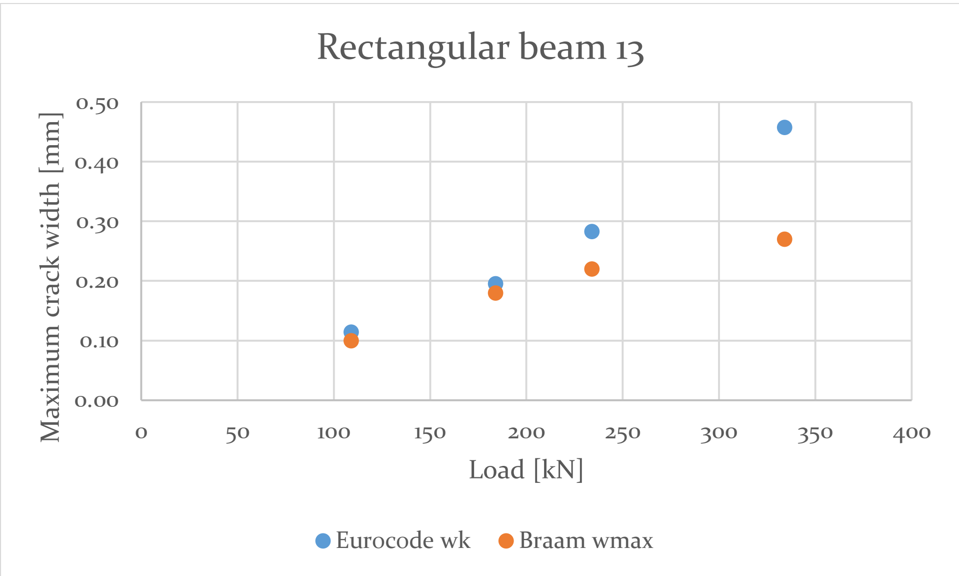
### 3.3.2 Results: Comparison theory with practice for Rectangular beam 13

**Maximum crack width** ($w_{k,max}$)

The calculated value of the maximum crack width ($w_{k,max}$) was compared to the tested value of the maximum crack width.

*Table 3-13: Comparison calculated value of $w_{k,max}$ with the tested value of $w_{max}$ for Rectangular beam 13*

| $F_{applied}$ | Eurocode $w_{k,max}$ | Experiments Braam: $w_{k,max}$ | Difference (%) |
|---|---|---|---|
| [kN] | [mm] | [mm] | [%] |
| **109** | 0.11 | 0.10 | 14 |
| **184** | 0.20 | 0.18 | 9 |
| **234** | 0.28 | 0.22 | 29 |
| **334** | 0.46 | 0.27 | 70 |




*Graph 3-10: Comparison of the calculated value of the crack width with the tested value of Rectangular beam 13*

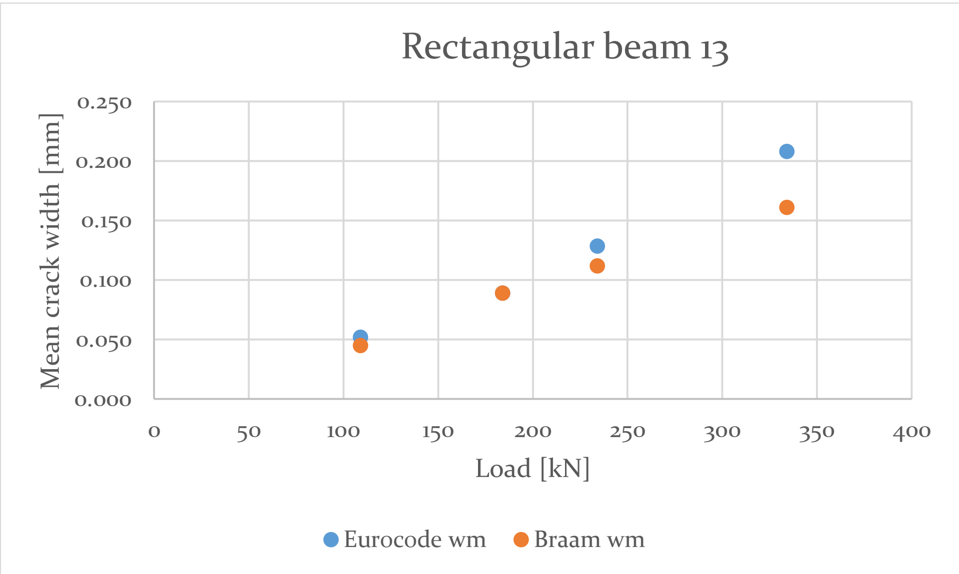
Looking at the results of rectangular beam 13 in Table 3-13 we see that the difference between the calculated values of the crack width and the tested values reaches 70% in the last loading stage. In Graph 3-10 we see that the values in the first two loading stages are overlapping each other. In the last two loading stages we see that the Eurocode 2 calculations is about with a 70% larger than the tested value. So also in this case we see that the difference between the calculated and the tested value of the maximum crack width increases with an increasing load.

## Mean value of the crack width ($w_m$)

The calculated value of the mean crack width ($w_m$) was compared to the tested value of the mean crack width

*Table 3-14: Comparison calculated value of $w_m$ with the tested value of $w_m$ for Rectangular beam 13*

| $F_{applied}$ | Eurocode $w_m$ | Experiments Braam: $w_m$ | Difference (%) |
|---|---|---|---|
| [kN] | [mm] | [mm] | [%] |
| **109** | 0.052 | 0.045 | 15 |
| **184** | 0.089 | 0.089 | 0 |
| **234** | 0.129 | 0.112 | 15 |
| **334** | 0.208 | 0.161 | 29 |




*Graph 3-11: Comparison of the mean value of the crack width with the tested value of Rectangular beam 13*

When we look at the mean value of the crack width in Table 3-14 we see that the maximum difference between the calculated and the tested value is about 29% ,which is at the last loading stage: $P = 334\ kN$. In all the other loading stages the difference is smaller than 15%. The results are also presented in Graph 3-11.

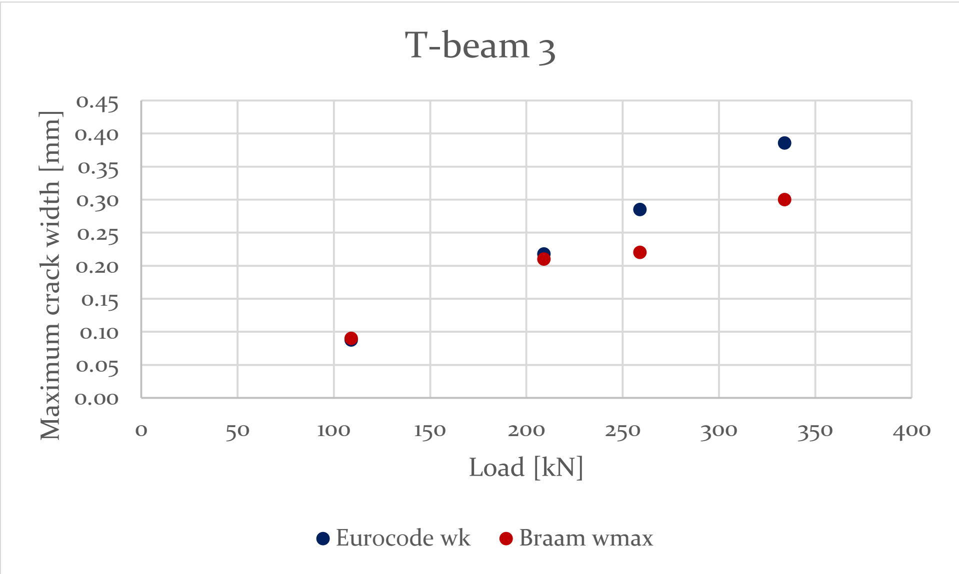
### 3.3.3 Results: Comparison theory with practice for T-beam 3

**Maximum crack width** ($w_{k,max}$)

The calculated value of the maximum crack width ($w_{k,max}$) was compared to the tested value of the maximum crack width.

*Table 3-15: Comparison calculated value of $w_{k,max}$ with the tested value of $w_{max}$ for T-beam*

| $F_{applied}$ | Eurocode $w_{k,max}$ | Experiments Braam: $w_{k,max}$ | Difference (%) |
|---|---|---|---|
| [kN] | [mm] | [mm] | [%] |
| **109** | 0.09 | 0.09 | 2 |
| **209** | 0.22 | 0.21 | 4 |
| **259** | 0.28 | 0.22 | 30 |
| **334** | 0.39 | 0.30 | 29 |




*Graph 3-12: Comparison of the calculated value of the crack width with the tested value of T-beam 3*

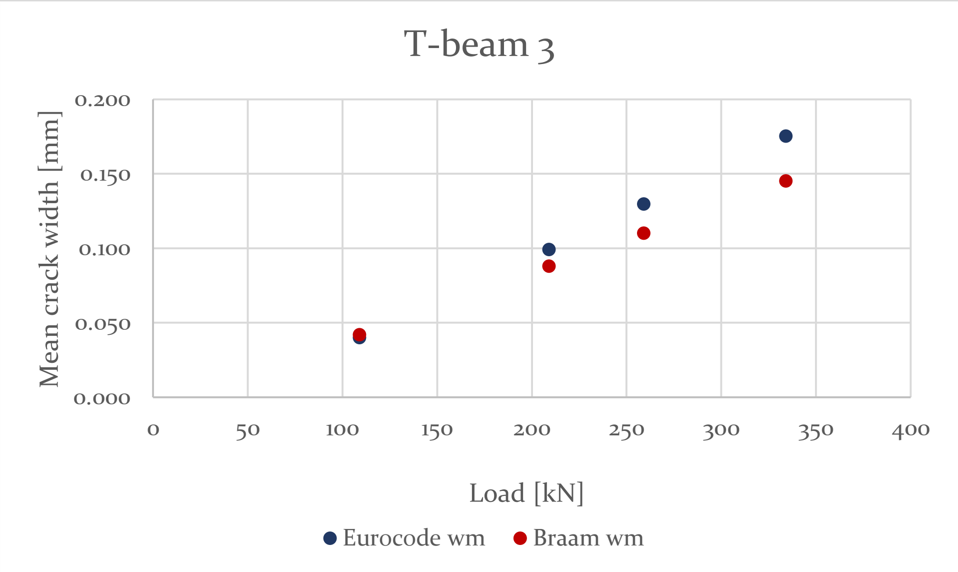
Looking at the results of T-beam 3 it can be seen that the calculated values of the crack width are larger than the tested values of the crack width. In Table 3-15 it can be seen that with increasing load the calculated crack width increases with 30%. The results are also presented in Graph 3-12.

**Mean value of the crack width** ($w_m$)

The calculated value of the mean crack width ($w_m$) was compared to the tested value of the mean crack width.

*Table 3-16: Comparison calculated value of $w_m$ with the tested value of $w_m$ for T-beam 3*

| $F_{applied}$ | Eurocode $w_m$ | Experiments Braam: $w_m$ | Difference (%) |
|---|---|---|---|
| [kN] | [mm] | [mm] | [%] |
| **109** | 0.040 | 0.042 | 5 |
| **209** | 0.099 | 0.088 | 13 |
| **259** | 0.130 | 0.11 | 18 |
| **334** | 0.175 | 0.145 | 21 |




*Graph 3-13: Comparison of the mean value of the crack width with the tested value of T-beam 3*

In Table 3-16 it can be seen that the mean value of the crack width calculated in excel is also larger than the tested value of T-beam 3. The maximum difference is about 21% in the last loading stage. Graph 3-13 shows that with increasing load the mean crack width according to Eurocode 2 increases which also leads to a larger difference between the tested value of the mean crack width. Overall we see that the calculated value of the mean crack width is close to the tested value of the mean crack width. And so we can conclude that the cracking behavior according to the NEN-EN 1992-1-1 is in agreement with the actual cracking behavior of a beam subjected to bending.

# 4 NUMERICAL ANALYSIS

The numerical analysis was performed to validate whether the real cracking behavior could be simulated with the finite element program DIANA. From the experiments of Braam two cases were modelled: Rectangular beam 13 and T-beam 3.

In the following chapter a brief explanation of the finite element program DIANA will be given, this will be followed by an explanation of how the beam was modelled. After this the results of the numerical analysis of Rectangular beam 13 and T-beam 3 are presented which will be followed by a comparison of the experimental results with the numerical results.

## 4.1 DIANA

DIANA stands for Displacement Method Analyzer and has been under development at the Dutch organization for applied scientific research (TNO) since 1972. The developer and distributer of the Diana finite element code is TNO DIANA BV. DIANA is a multi-purpose finite element software package that is dedicated to a wide range of applications in Civil engineering including structural, geotechnical, tunneling, earthquake, and oil & gas engineering. Several engineering problems can be solved with this program. It can be applied in the design and assessment of reinforced concrete, composite and steel structures, simulations of the process of excavations, tunneling, construction of buildings and structures, prediction and quantification of force transmission and deformation of all kind of systems. Material aspects such as cracking of concrete, plastic yielding of steel, creep and shrinkage, aging and ambient influences, can also be taken into account in DIANA [9].

### 4.1.1 Smeared cracking model

In DIANA cracking in concrete can be modelled with the help of the smeared cracking concept. This concept consists of 2 cracking models namely:

1. **Multi-directional fixed cracking model**

The most important feature of the multi-directional fixed crack model is the decomposition of the total strain ($\varepsilon$) into an elastic strain ($\varepsilon^{e}$) and a crack strain($\varepsilon^{cr}$). The modelling of cracks that simultaneously occur is made possible by the sub-decomposition of the cracking strain($\varepsilon^{cr}$). In this model the main focus lies on how the cracks initiate and rotate simultaneously with the stresses. Figure 4-1 shows the decomposition of the cracking strain [9].

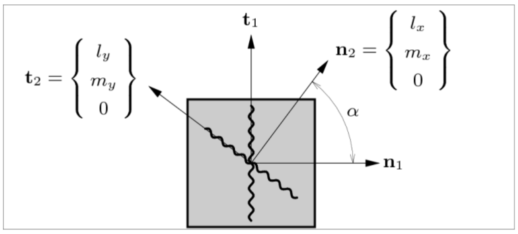



*Figure 4-1: Multi-directional Fixed crack model*

The multi-directional fixed crack model can be specified as a combination of tension cut-off, tension softening and shear retention.

### 1. Total strain cracking models

The stress-strain relationship is described by a constitutive model that is based on the total strain. There are various approaches possible within the strain-stress relationships. The coaxial stress-strain concept is often used. In this approach, which is also known as the Rotating crack model, the stress-strain relationships are evaluated in the principal directions of the strain vector. The Rotating crack model is eligible for reinforced concrete structures. A better approach to the cracking behavior is the total strain fixed crack model. In this model the stress-strain relationships are evaluated in a coordinate system which is fixed during cracking. The main idea behind the total strain cracking models is that the stress is assessed in the directions which are given by the crack directions. For the numerical analysis in this research the total strain concept is used.

### 4.1.2 Tensile behavior

In DIANA the tensile behavior of concrete can be modelled with the help of a predefined tension softening function. Diana contains several tension softening models which describe the behavior of the concrete in the cracked state. The tension softening curves which are available for the total strain crack model are shown in Figure 4-2. The tension softening model of Hordijk et al (model g in Figure 4-2) is used for the numerical analysis of this research.

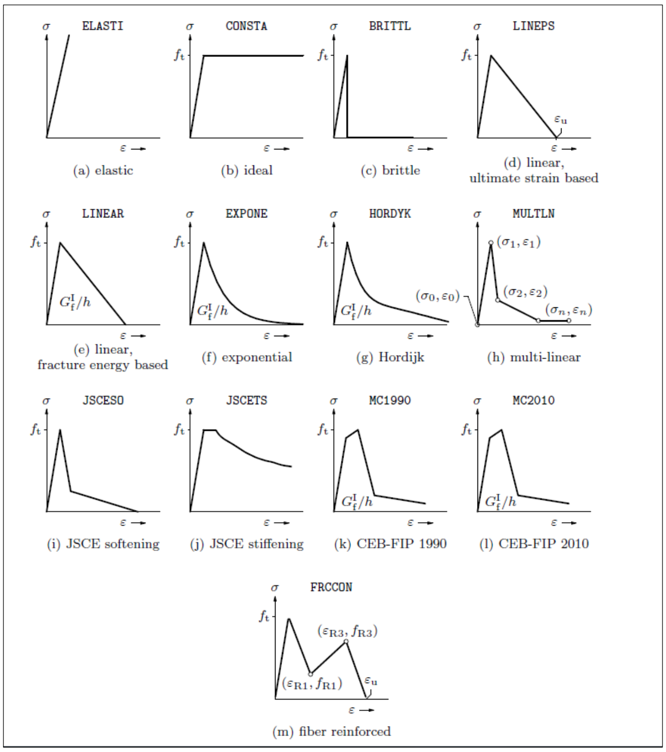



*Figure 4-2: Predefined tension softening for Total Strain crack model*

### 4.1.3 Reinforcement

Reinforcement adds stiffness to the finite element model. Reinforced concrete structures can be modelled by plain concrete elements and steel reinforcement bars. The reinforcement can be modelled as embedded reinforcement, where the reinforcements are fully embedded in the elements in which they are located and are fully coupled. In this type of reinforcement relative slip is not permitted. Other possibilities to model reinforcement in DIANA is the type of reinforcement for which the deformation of the reinforcements may be different than for the elements in which they are located. In this type of reinforcement relative slip is allowed. These reinforcements are often used to model bond-slip reinforcements and pile foundations [9]. For the non-linear analysis bond slip reinforcement will be used. The main reinforcement is modelled with the help of a bar reinforcement in plane stress elements (Figure 4-3) and the stirrups are modelled with the help of a grid reinforcement in plane stress elements (Figure 4-4). For more extensive information on the modeling of reinforcement the reader is referred to [9].

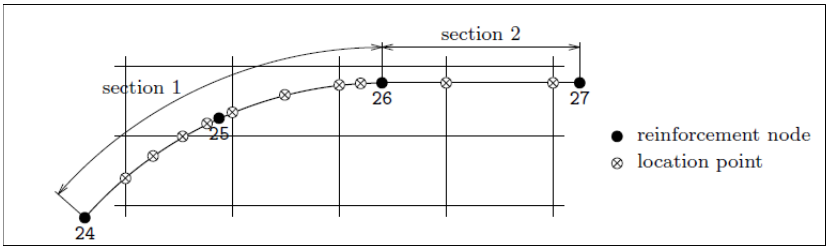



*Figure 4-3: Bar Reinforcement in plane stress element*

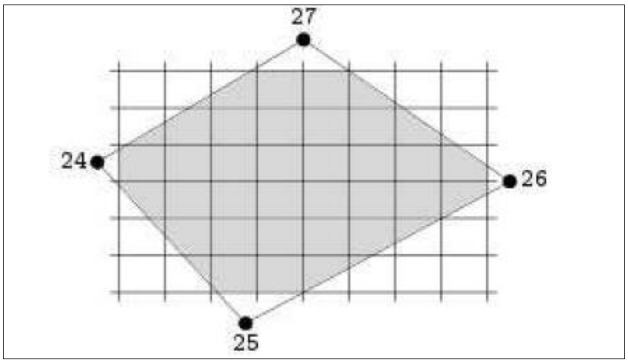



*Figure 4-4: Grid reinforcement in plane stress elements*

### 4.1.4 Bond-slip behavior

The bond between concrete and steel is very important since it determines the crack width, distance between the cracks and the load-deformation diagram of an element under bending. With the help of a bond-slip mechanism the relationship between concrete and steel in reinforced concrete can be modelled. In this mechanism the relative slip of the reinforcement and the concrete is described. The bond-slip mechanism is based on a total deformation theory, in which the stresses are expressed as a function of the total relative displacements [9]. The power law of Noakowski (Figure 4-5 graph b) is used to model the bond-slip behavior between the concrete and reinforcement during the non-linear analysis. In DIANA the power law of Noakowski is specified with the help of the following equation: $\tau = a(\Delta u_t)^b\ [N/mm^2]$. In which the input parameters are:

$a = 0.38 f_{ccm}\ [N/mm^2]$ ; $\Delta u_t = 0.0001\ [mm]$ ; $b = 0.18\ [-]$

where: $f_{ccm}$ (compressive strength of concrete on the day of testing)

During the non-linear analysis in DIANA the power law of Noakowski was scaled based on the Model code 2010 due to the unit dependency. The values of $a$ and $b$ were modified (see section 4.2.1). Thus the following modified parameters were used in the finite element analysis:

$a = 5.3 f_{ccm}\ [N/m^2]\ ; \Delta u_t = 0.0000001\ [m]; b = 0.4\ [-]$

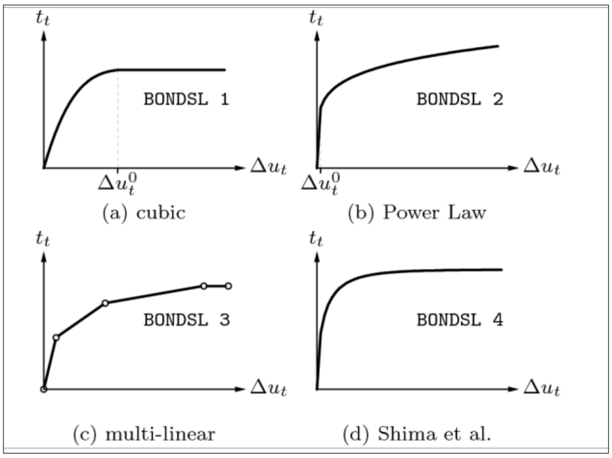



*Figure 4-5: Bond-shear traction curve*

## 4.2 Finite element model of rectangular Beam 13

For the non-linear analysis in Diana a 2-D finite element model was developed. The input parameters governing the geometrical properties, finite element mesh, element type, material type, boundary conditions and interface properties will be provided below. Since the loading scheme of the real experiments was applied in four-point bending, the beam was also modelled in this way in Diana (Figure 4-6).

### 4.2.1 Input Material and geometrical properties

The material and geometrical properties which were specified for the finite element model of beam 13 are presented in the tables below.

**Material- and Geometrical properties of concrete**

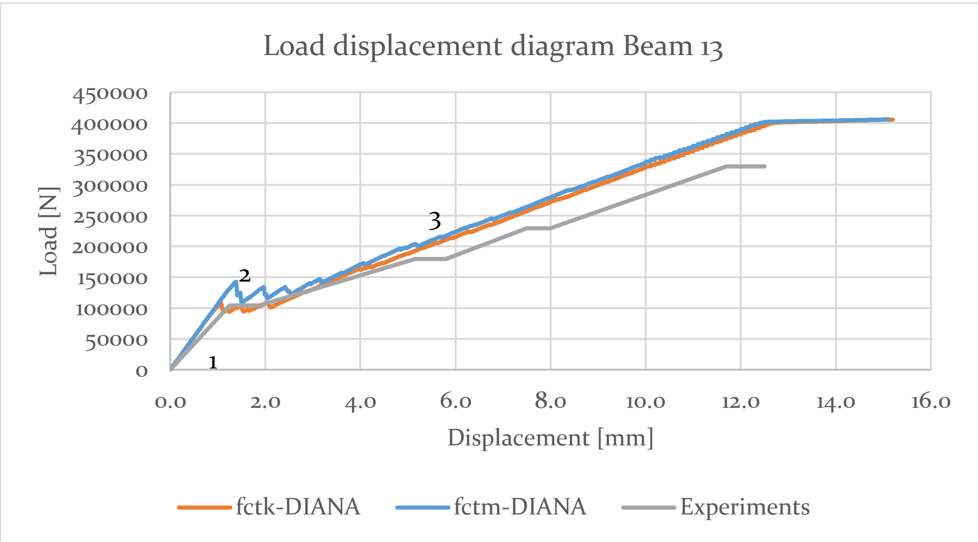
*Table 4-1: Properties rectangular beam 13*

| | | | |
|---|---|---|---|
| L (length) | 5.500 | [m] | **[Experimental data of C.R. Braam table 6.3]** |
| B (width) [m] | 0.3 | [m] | |
| H (height) [m] | 0.8 | [m] | |
| $A_c$ (cross section) | 0.24 | [$m^2$] | |
| $f_{ccm}$( Mean value of the cube compressive strength at the day of testing) | $55.9 * 10^6$ | [$N/m^2$] | **[Experimental data of C.R. Braam table 6.4]** |
| $f_{ck}$(Characteristic value of the cube compressive strength at the day of testing) | $47.9 * 10^6$ | [$N/m^2$] | **[Calculated: EUROCODE 2 table 3.1]** |

| | | | |
|---|---|---|---|
| $f_{cspl}$ (mean value of the cube tensile splitting strength at the day of testing) | $4.08 * 10^6$ | $[N/m^2]$ | **[Experimental data of C.R. Braam table 6.4]** |
| $f_{ctm}$ (mean value of the cube tensile strength) | $3.95 * 10^6$ | $[N/m^2]$ | **[Calculated: EUROCODE 2 table 3.1]** |
| $f_{ctk}$ (characteristic value of the cube tensile strength) | $2.765 * 10^6$ | $[N/m^2]$ | |
| $E_c$ (Modulus of elasticity) | $3.18 * 10^{10}$ | $[N/m^2]$ | **[Experimental data of C.R. Braam table 6.4]** |
| $\upsilon$ (poisson ratio) | 0.2 | | |
| $G_f$ (fracture energy) $[N/m]$ | 150.6 | | **[ Calculated: Model Code]** |

## Strength properties

Because of the fact that failure due to cracking occurs at the position of the cracking force and all other cracks develop at the local weaker spots in the beam, it is reasonable to apply the characteristic value of the tensile strength ($f_{ctk}$) for the brittle behavior (cracking) of concrete. When the mean value of the tensile strength ($f_{ctm}$) was applied in the non-linear analysis we saw that the first crack occurred at a higher level compared to the experiments. By applying the characteristic value of the tensile strength we saw that the cracking force decreased, since $f_{ctk} < f_{ctm}$. In Graph 4-1 the load displacement diagrams of Rectangular beam 13 are presented. In this graph the load displacement diagram of beam 13 calculated in both cases (mean value- and maximum value of the tensile strength) is compared to the experiments. When we look at the cracking force in the crack formation stage (stage 2) we see that the analysis using $f_{ctm}$ generated a higher cracking force. And when $f_{ctk}$ was applied the results were closer to the experiments.




*Graph 4-1: Load displacement diagrams Rectangular beam 13*

***Note*:** For the ductile behavior of the reinforcement (the bond-slip behavior) the mean value of the tensile strength ($f_{ctm}$) is applied, since there is a certain failure zone in which the mean values are smeared out along the beam.*

## Fracture energy

The fracture energy is the linear stiffness which is essential for the initiation of a new crack during the unloading process.

The fracture energy was calculated with the help of the following equation taken from the model code [10]:

$$G_f = 73 * f_{cm}^{\ 0.18} = 73 * 55.9^{0.18} = 150.6 \ N/m$$

## Total Strain Crack Model

The tensile and compressive behavior of reinforced concrete can be modelled with total strain cracking models in DIANA. The basic concept of the Total Strain crack models is that the stress is evaluated in the directions which are given by the crack directions. The fixed stress-strain suits the physical nature of cracking better. In this concept the stress-strain relationships are evaluated in a coordinate system which is fixed upon cracking.

The input parameter for the total strain crack model in which the crack-directions switches from rotating to fixed is:

$$Epsfix = \frac{f_{ctk}}{E_c} * 2 = \frac{2.765}{31800} * 2 = 1.74 * 10^{-4}$$

## Material- and Geometrical properties of steel plates

For the top and bottom supports steel plates were used. The material and geometrical properties of the steel are given in Table 4-2.

*Table 4-2: Properties steel plates*

| L | 0.25 | [m] |
|---|---|---|
| B (width) | 0.1 | [m] |
| H (height) | 0.1 | [m] |
| $\mathbf{A_{plate}}$ | 0.01 | [m²] |
| $\mathbf{E_s}$ (Modulus of elasticity steel) | $2 * 10^{15}$ | [N/m²] |
| $\boldsymbol{\upsilon}$ (poisson ratio) | 0.3 | |

When we zoom in on the model the steel plates can be seen in Figure 4-6.

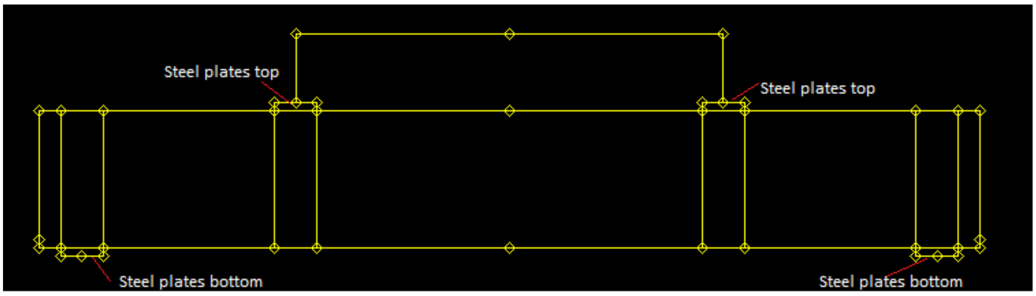



*Figure 4-6: Steel plates at top and bottom of the beam*

## Geometrical properties of the interface

For the benefit of the modelling a rubber interface was applied  at the supports between steel and concrete with a thickness of 0.05 m (Figure 4-7). During the laboratory tests rubber was not used.

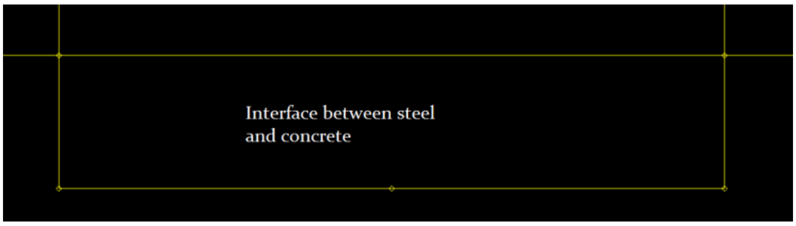



*Figure 4-7: Rubber Interface*

$D_{stiff}: \frac{E_{rubber}}{h_{int}} = 60\ N/mm^2$  (normal stiffness)

With $E_{rubber} = 3\ N/mm^2$

## Geometrical properties of the Loading scheme

The loading scheme was modelled as an equator construction to ensure displacement controlled calculation. The loading scheme is built up out of 2 truss elements and a steel beam (Figure 4-8).

*Table 4-3: Properties steel beam*

| | | |
|---|---|---|
| L (length) | 2.5 | [m] |
| B (width) | 0.1 | [m] |
| H (height) | 0.1 | [m] |
| $\mathbf{A_{beam}}$ | 0.01 | [m²] |
| $\mathbf{E_s}$ (Modulus of elasticity steel) | $2 * 10^{15}$ | [N/m²] |
| $\boldsymbol{\upsilon}$ (poisson ratio) | 0.3 | |

***Note*:*** *In order to prevent additional deformation a higher value for the young's modulus of the steel beam was used ($E_s = 2 * 10^{15}\ N/m^2$) compared to the young's modulus of the trusses ($E_s = 2 * 10^{11} N/m^2$).*

*Table 4-4: Properties trusses*

| | | |
|---|---|---|
| L (length) | 0.4 | [m] |
| B (width) | 0.1 | [m] |
| H (height) | 0.1 | [m] |
| $\mathbf{A}_{\mathbf{truss}}$ | 0.01 | [m²] |
| $\mathbf{E_s}$ (Modulus of elasticity steel ) | $2 * 10^{11}$ | [N/m²] |
| $\boldsymbol{\upsilon}$ (poisson ratio) | 0.3 | |

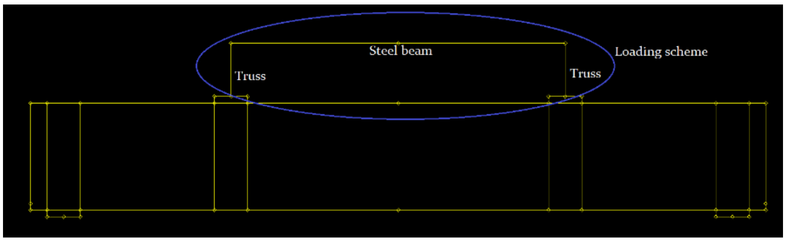



*Figure 4-8: Loading scheme*

## Boundary conditions

For the benefit of the results the supports were modelled as hinges (Figure 4-9).

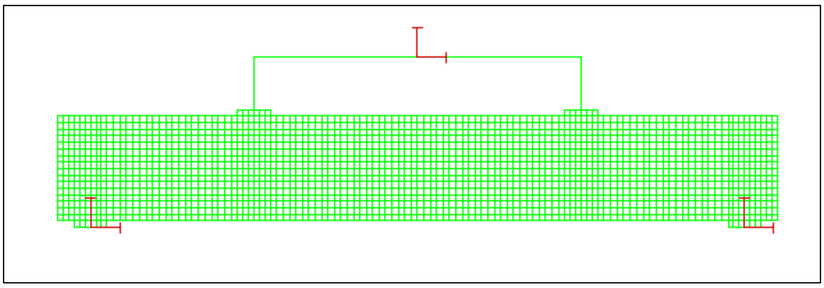


*Figure 4-9: Boundary conditions*

## Properties of the Reinforcement

The properties of the reinforcement are presented in the tables below.

## Main Reinforcement

*Table 4-5: Properties of the main reinforcement*

| Main reinforcement | | |
|---|---|---|
| Ø (Bar diameter) | 0.02 | [m] |
| # bars | 4 | |
| # layers | 1 | |
| $\mathbf{A_s}$ (Cross section) | $1.256 * 10^{-3}$ | [m²] |
| Perimeter | $251 * 10^{-3}$ | [m] |
| $\mathbf{E_s}$ (Youngs modulus) | $2 * 10^{11}$ | [N/m²] |
| $\boldsymbol{f_y}$ (yield strength) | $5.7 * 10^{8}$ | [N/m²] |

| | | |
|---|---|---|
| $\boldsymbol{\upsilon}$ (poisson ratio) | 0.3 | |

## Bond-slip

The bond-slip behavior was modelled with the power law of Noakowski (Figure 4-10). In DIANA this was specified with the following equation: $\tau = a(\Delta u_t)^b$. The input parameters were: $a = 0.38 f_{ccm}\ [N/m^2]$ ; $\Delta u_t = 0.0001\ [mm]$ ; $b = 0.18\ [-]$.

where: $f_{ccm} = 55.9 * 10^6\ N/m^2$ (Mean value of the compressive strength of concrete on the day of testing).

Due to unit dependency the power law of Noakowski was scaled to the Model Code 2010. The values of $a$ and $b$ were fitted to match the bond-slip behavior as specified in the Model Code 2010 (Section 6.1.1.1: Local bond stress-slip model, ribbed bars).The modified graph is presented in Graph 4-2. The following modified parameters were finally used in the finite element analysis:

$a = 5.3 f_{ccm} [N/m^2]$ ; $\Delta u_t = 0.0000001 [\ m]$ ; $b = 0.4\ [-]$

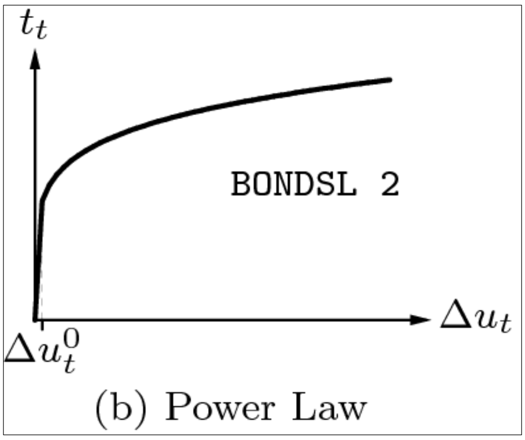
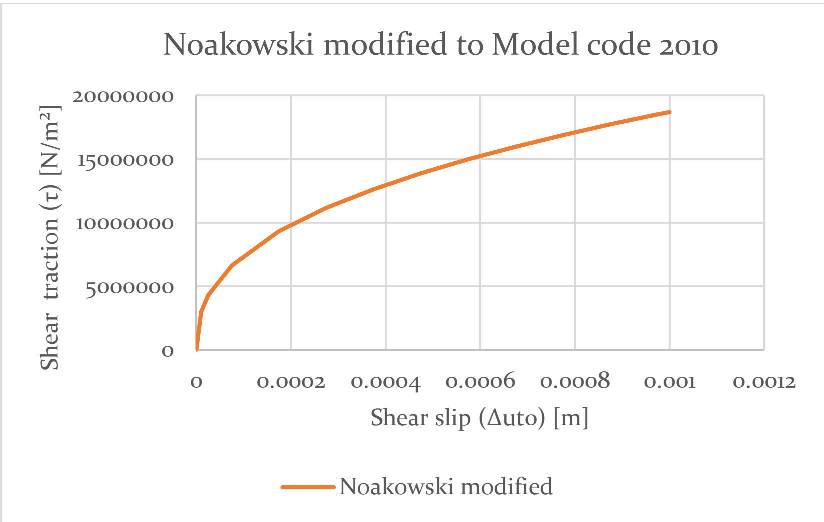



*Figure 4-10: Power law of Noakowski*




*Graph 4-2: Bond-slip model Noakowski fitted to the Model Code 2010*

## Linear stiffness

The interface in bond-slip elements require input of the linear stiffness DSTIFF : $D_{11}$ $D_{22}$ in which :

$D_{11}$ : sets the relation between the normal traction ($t_n$)and the normal relative displacement ($u_n$)

$D_{22}$ : sets the relation between the shear traction ($t_t$) and the shear relative displacement ($u_t$)

The value of the linear stiffness was calculated with the use of equation 6.1-16 from Model code 2010 : $s_s = 6.0 * \tau_{bmax} = 6.0 * 0.38 * 55.9 = 127.45 \frac{N}{mm^3} = 1 * 10^{11}\ N/m^3$.

And so the input parameters for the linear stiffness were:

$D_{stiff}$: $1 * 10^{11}$ $1 * 10^{11}$

## Stirrups

*Table 4-6: Properties of the stirrups*

| Stirrups | |
|---|---|
| ∅ (Bar diameter) [m] | 0.01 |
| Bar spacing [m] | 0.1 |
| $\mathbf{E_s}$ (Youngs modulus) [$\mathbf{N/m^2}$] | $2 * 10^{11}$ |
| $\boldsymbol{f_y}$ (yield strength) [$\mathbf{N/m^2}$] | $5.7 * 10^{8}$ |
| $\boldsymbol{\upsilon}$ (poisson ratio) | 0.3 |

In DIANA the grid reinforcement for the stirrups was specified by the thickness of the total applied reinforcement. The following equation was used:

$$t = \frac{2 * A_{st}}{c.tc.\ distance} = \frac{2 * \frac{1}{4}\pi * 10^2}{200} = 0.785\ mm$$

## Bond slip

The same bond-slip properties of the main reinforcement were specified for the stirrups.

## Linear stiffness

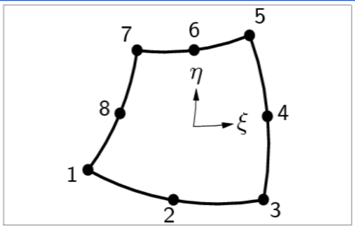
The same value for the linear stiffness of the main reinforcement was specified for the interface of the stirrups

### 4.2.2 Input Element type and Material type

For the computation in the finite element model, a specific element and material type needed to be specified to the structure in DIANA. The specified types for each part of the structures is given below.

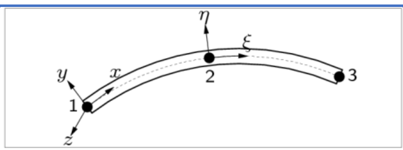
## Concrete beam

*Table 4-7: Element type Concrete Beam*

| **Element type: CQ16M** |  |
|---|---|
| **Number of nodes** | 8 |
| **Total degrees of freedom** | 16 |
| **Material type** | Isotropic |
| **Interpolation** | Quadratic |
| **Dimension** | 2D |

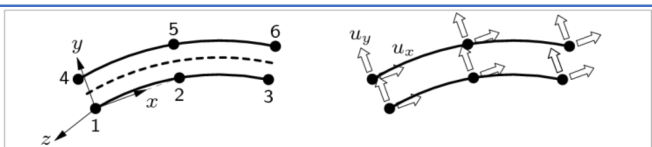
## Steel Plates and Steel beam

*Table 4-8: Element type of the steel plates and steel beam*

| **Element type: CL9BE Class III Beam** |  |
|---|---|
| **Number of nodes** | 3 |
| **Total degrees of freedom** | 9 |
| **Material type** | Isotropic |
| **Interpolation** | Quadratic |
| **Dimension** | 2D |

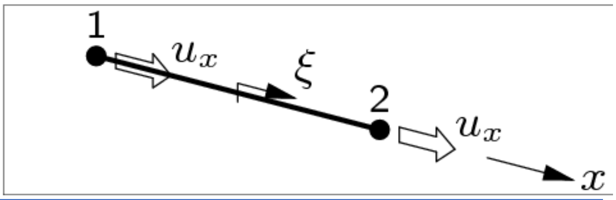
## Interface: Rubber

*Table 4-9: Element type of the rubber interface*

| **Element type: CL12I Line** |  (a) topology (b) displacements |
|---|---|
| **Number of nodes** | 3 +3 |
| **Total degrees of freedom** | 12 |
| **Material type** | Isotropic |
| **Interpolation** | Quadratic |
| **Dimension** | 2D |

## Loading scheme: Trusses

*Table 4-10: Element type truss*

| **Element type: L2TRU** |  |
|---|---|
| **Number of nodes** | 2 |
| **Total degrees of freedom** | 2 |
| **Material type** | Isotropic |
| **Interpolation** | Quadratic |
| **Dimension** | 2D |

## Main reinforcement

The main reinforcement was represented by a truss element in which only tensile forces can be generated.

*Table 4-11: Element type main reinforcement*

| **Element type:** | **Bar Reinforcement** |
|---|---|
| **Number of nodes** | 2 |
| **Total degrees of freedom** | 2 |
| **Material type** | Embedded bar reinforcement |
| **Interpolation** | Quadratic |
| **Dimension** | 1-D |

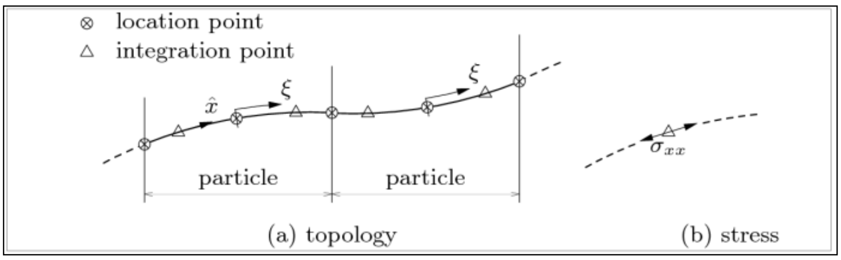



*Figure 4-11: General reinforcement bar*

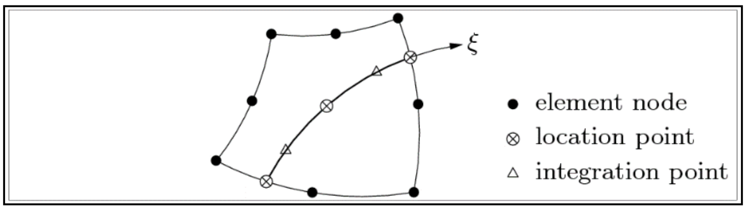



*Figure 4-12: Reinforcement bar in particle plain stress element*

**Stirrups:**

*Table 4-12: Element type Stirrups*

| Element type | Grid Reinforcement |
|---|---|
| **Number of nodes** | 4 |
| **Total degrees of freedom** | 8 |
| **Material type** | Embedded grid reinforcement |
| **Interpolation** | Quadratic |
| **Dimension** | 1-D |

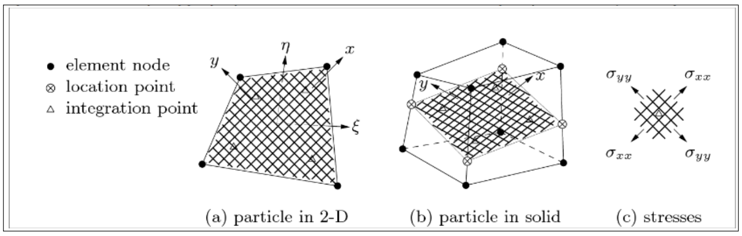



*Figure 4-13: General reinforcement grid*

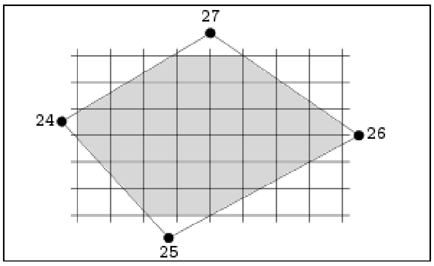



*Figure 4-14: Example : Grid section in Plane stress element*

After the material properties, geometrical properties and the element types were inserted in the model, the mesh could be generated (Figure 4-15). By generating the mesh, the structure was divided into small elements. So after the mesh had been generated the analysis could finally start.

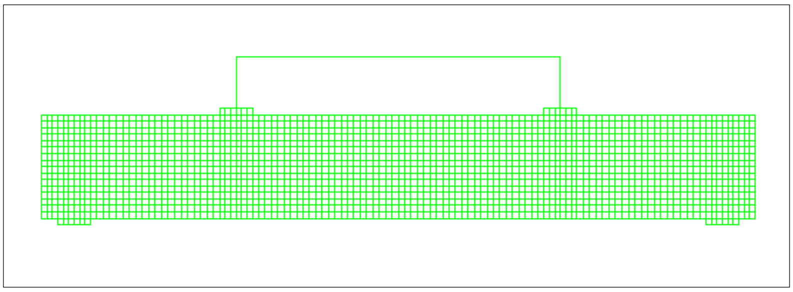


*Figure 4-15: Mesh finite element model beam 13*

## 4.3 Comparison Results Rectangular Beam 13

During the laboratory tests the actual load for beam 13 was applied in four loading stages with a hand-operated hydraulic jack. Looking at the loadings scheme of the experiments it can be seen that the total load was applied in 2 points (Figure 4-16). The results that were registered by Dr. C. R. Braam [7] were based on one loading point only. In this point the total applied load during each loading stage was equal to half of the load applied by the hydraulic jack and half the weight of the loading frame (Table 4-13 column 4) .

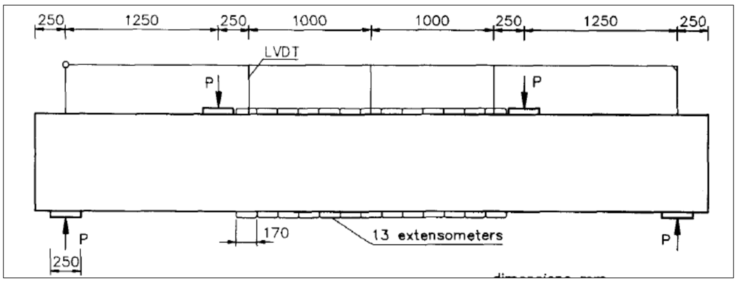



*Figure 4-16: Loading scheme experimetal research*

In the following table the loading stages are presented.

*Table 4-13: Loading stages Beam 13*

| Loading stage: Beam 13 | Weight of the loading frame [kN] | Total load Hydraulic jack [kN] | Applied load per loading point [considered in the experiments] [kN] | Total Load [considered in DIANA] [kN] |
|---|---|---|---|---|
| 1 | 18 | 200 | 109 | 218 |
| 2 | 18 | 350 | 184 | 368 |
| 3 | 18 | 450 | 234 | 468 |
| 4 | 18 | 650 | 334 | 668 |

In the numerical model however the loading scheme was a bit different. There was only one loading point. Because of this the results in DIANA were registered at the total applied load at each loading stage. This load was equal to the total load applied by the hydraulic jack and the total weight of the loading frame (Table 4-13 column 5).

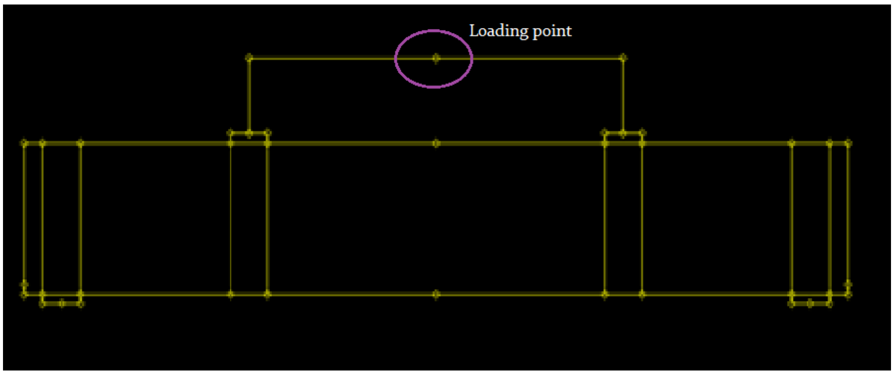



*Figure 4-17: Loading scheme numerical analysis*

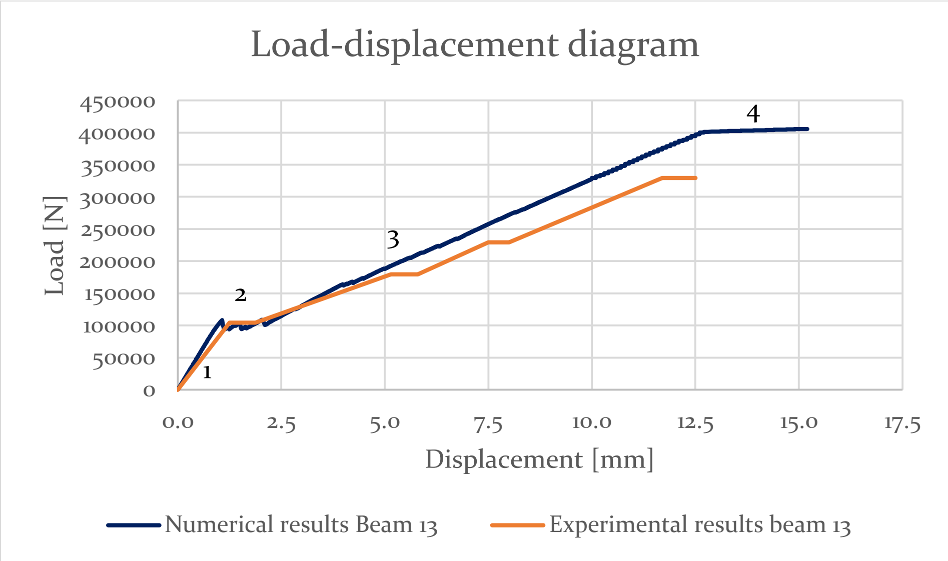
These loading stages are essential for the comparison of the crack pattern and for the crack width measurements.

In this chapter the following aspects ,which were needed to obtain a good comparison between the numerical- and the experimental results of Beam 13 were covered:

1. The load-displacement diagram
2. The magnitude of the force when the first crack occurs
3. The crack pattern
4. The mean value of the crack width at each loading stage at the level of the main reinforcement

### 4.3.1 The load- displacement diagram

In Graph 4-3 the load-displacement diagram of the experimental results is presented together with the load–displacement diagram of the numerical analysis of beam 13.




*Graph 4-3: Load displacement diagrams of the Experimental Results and Numerical Results of beam 13*

The load displacement diagram represents the average structural behavior of a beam subjected to an external load. By comparing the load displacement diagram of the experiments with the numerical results it can be seen whether the structural behavior of the beam in the numerical analysis corresponds with the structural behavior of the experiments. The load-displacement diagram can be subdivided in four stages [1]. These stages are:

- Stage 1: Uncracked stage: in this stage the applied force ($F_{applied}$) is smaller than the cracking force ($F_{crack}$).
- Stage 2: Crack formation stage (in this stage the beam starts to crack, this occurs when the tensile force($F_{applied}$) is larger than the cracking force ($F_{crack}$) in the cross section.
- Stage 3: Stabilized cracking stage: in this stage the applied load increases which results in the widening of the already existing cracks. The number of cracks in this stage remains unchanged.
- Stage 4: The stage in which the yield strength is reached.

In the numerical analysis the beam was loaded until the yield strength of the reinforcement was reached (until stage 4). This was not the case in the laboratory tests. The horizontal branches in the experimental results of beam 13 were caused by the increasing deformation which occurred during the crack width registration.

In Graph 4-3 it can be seen that the forces obtained in  in stage 1 (uncracked stage) and stage 2 (crack development stage) are overlapping each other. In stage 3 (stabilized cracking stage) however the forces are higher than that of the experiments. During the measurements of the crack width a certain amount of creep may have occurred. This could have an effect on the load-displacement diagram, however the effect of creep was not taken into account during the numerical analysis. Overall it can be said that the average structural behavior of a beam subjected to an external load can be simulated with a fem-analysis in DIANA.

### 4.3.2 Magnitude of the force when the first crack occurs

The magnitude of the force during the first crack in the finite element analysis is equal to :$P = 105\, kN$ and the displacement $u = 1.08\, mm$. When we looked at the cracking force of the experimental results we saw that the first crack occurred at force of $P = 104.5\, kN$ and a displacement of $u = 1.25\, mm$. Given the scatter in the properties of concrete these values are in good agreement with each other.

### 4.3.3 Crack pattern

The best way to compare the crack pattern of the FEM-analysis with the experimental results is to look at the crack pattern at the last loading stage in DIANA ($P = 334\, kN$). The number of cracks were also compared. The figures below represent the crack pattern of beam 13 from the experimental results and the crack pattern from the numerical analysis respectively.

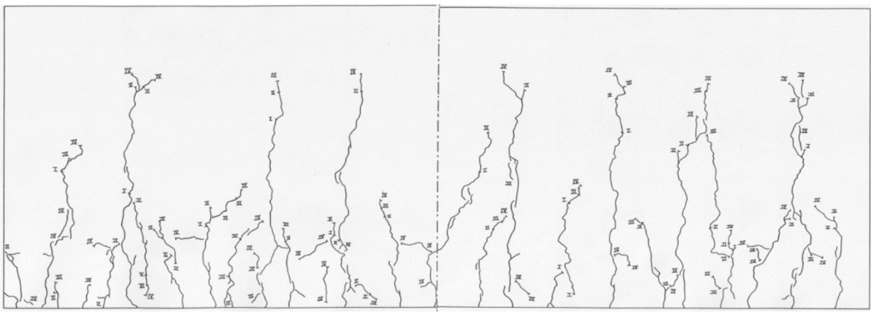


*Figure 4-18: Crack Pattern experimental results Beam 13 ( side I) [7]*

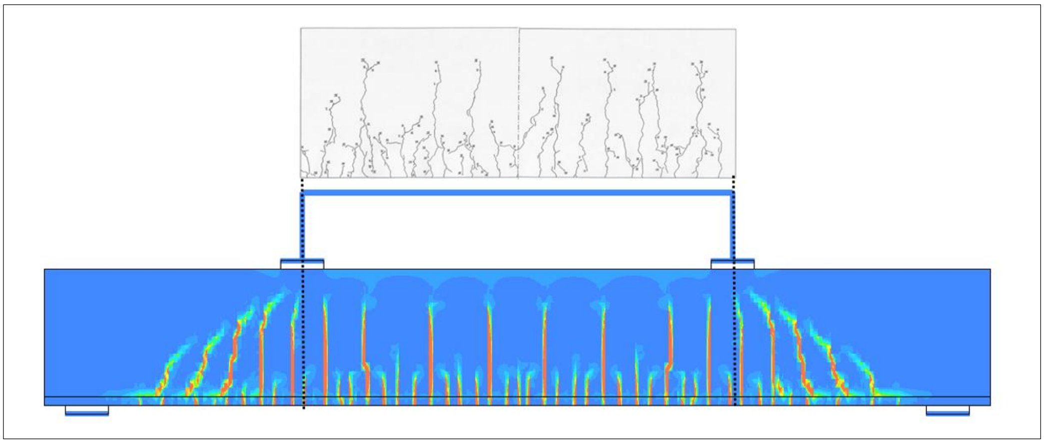


*Figure 4-19: Crack pattern experimental results beam 13 and crack pattern FEM analysis*

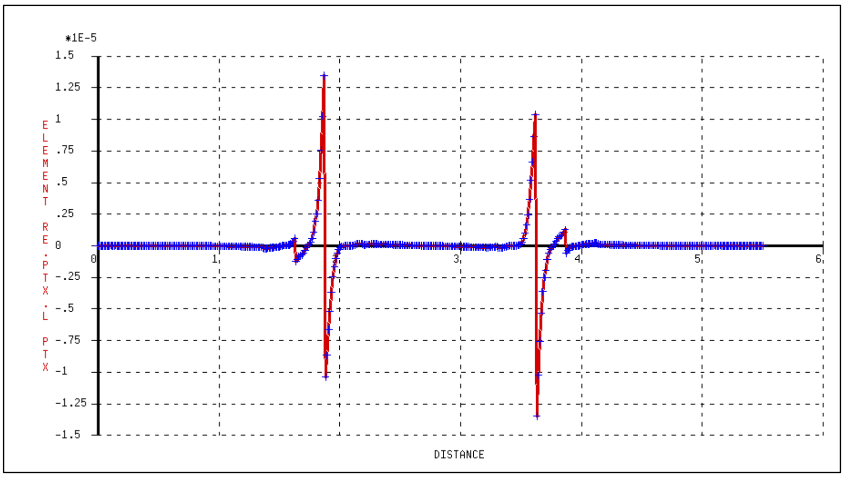
In the experiments the crack pattern was registered only in the middle of the beam between the top supports. So the number of primary cracks in the numerical analysis between these supports at the last loading stage was is equal to 27 cracks. In the experiment there were about 29 cracks at side I of the rectangular Beam. In Figure 4-19 we can also see small cracks occurring at the level of the main reinforcement. Since the number of cracks are very close to each other, it can be concluded that the numerical analysis in DIANA gives a good indication of the real cracking behavior.

### 4.3.4 The mean value of the crack width at each loading stage at the level of the main reinforcement

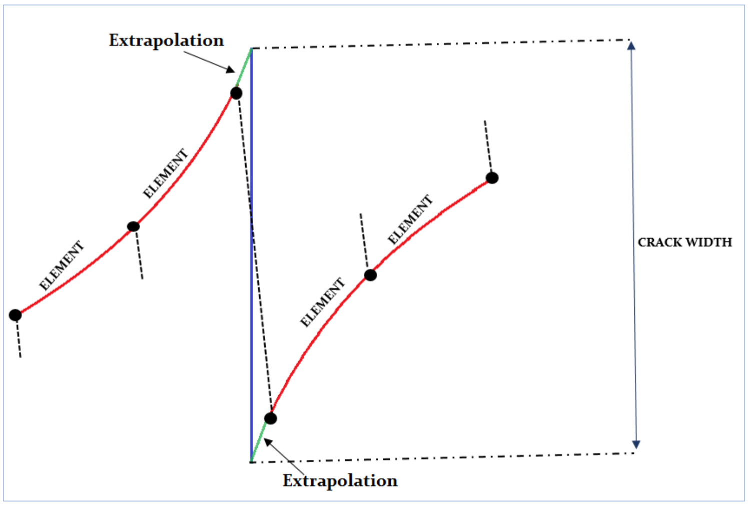
After the analysis had been carried out in DIANA the mean value of the crack width ($w_{mean}$) was calculated in Excel and compared to the experiments. In this chapter the numerical results of the mean value of the crack width was compared to the experimental results at each loading stage. The number of cracks and the mean value of the crack spacing ($l_{mean}$) were also compared.

**Crack width analysis**

The final comparison that was carried out was based on the mean value of the crack width. In DIANA there were several ways to estimate the mean value of the crack width. During this research the mean value of the crack width was calculated by generating the relative displacement between the reinforcement bar and the surrounding concrete in DIANA. In Graph 4-4 the relative displacement (slip) of the concrete beam and the reinforcement bar is presented. By importing these values in Excel and by estimating where the slopes were located in the graph the position of the cracks could be estimated. After this the crack width was calculated by taking the maximum value of the slip. An example of the location of the crack width is presented in Graph 4-5. For the calculation of the mean value of the crack width ($w_{mean}$) only the cracks which were larger than $w_{min} = 0.01\ mm$ were considered.




*Graph 4-4: Slip value of the reinforcement bar surrounding the concrete when the first crack occurs*




*Graph 4-5: Crack width analysis in Excel*

In Graph 4-5 we zoom in on the relative displacement for the explanation of the crack width calculation in DIANA. We see that by extrapolating the maximum value of the slip to the top of the graph, the crack width can be calculated. This method is carried out in Excel for the crack width calculation.

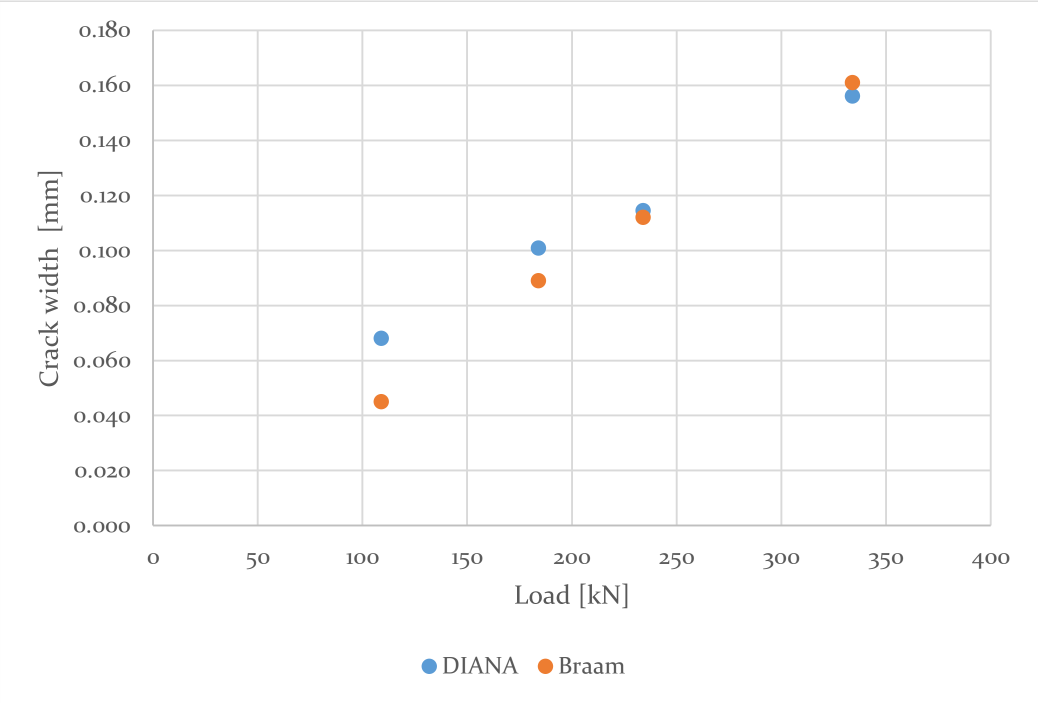
## Mean value of the crack width

In the following table the mean value of the crack width ($w_{mean}$) at each loading stage is presented at the level of the main reinforcement.

*Table 4-14: Comparison of the mean value of the crack width ($w_{mean}$) of Beam 13*

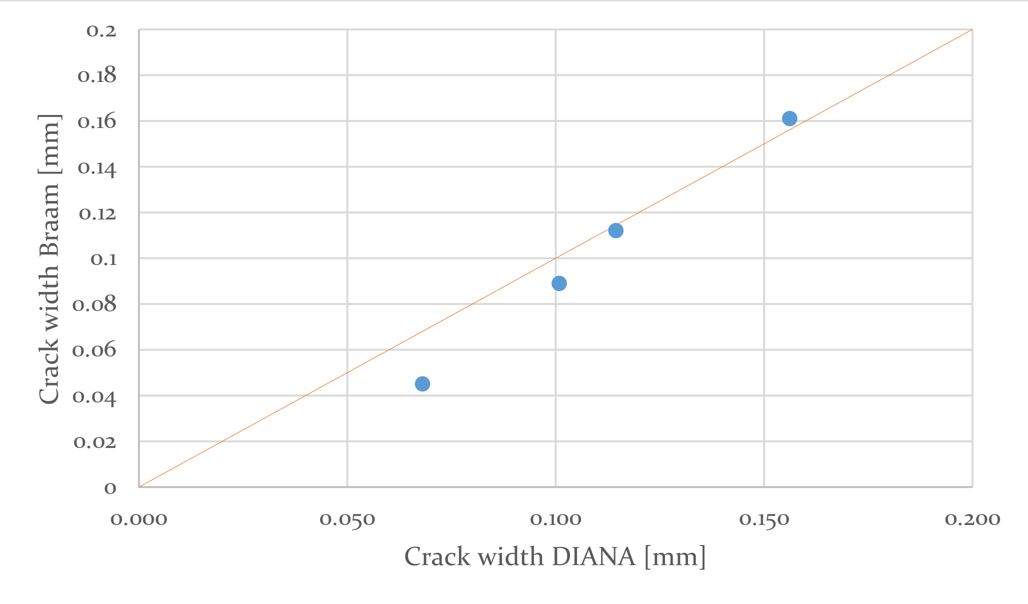
| Loading stage | Numerical results (DIANA) | Experimental results (Braam) | Difference |
|---|---|---|---|
| P | $w_{mean}$ | $w_{mean}$ | |
| [kN] | [mm] | [mm] | % |
| **109** | 0.068 | 0.045 | 51 |
| **184** | 0.101 | 0.089 | 13 |
| **234** | 0.114 | 0.112 | 2 |
| **334** | 0.156 | 0.161 | -3 |

In Table 4-14 it can be seen that the mean value of the crack width at the first loading stage from the numerical analysis ($P = 109\ kN$) is 51% larger than that of the experiments. This could be explained by the fact that in DIANA the beam is modelled as to be homogeneous and in reality concrete is an inhomogeneous material, which leads to different strength properties at different places in the beam. This could have an influence on the mean value of the crack width. However when we look at the other loading stages ($P = 184\ kN; P = 234\ kN\ and\ P = 334\ kN$) we can see that the difference become smaller than 15%. In the two last loading stages which are in the stabilized cracking stage, the values of the numerical results and the experiments are smaller than 3%. So with increasing load the numerical results come closer to the experimental results. This can be seen in Graph 4-6. In this graph the mean values of the crack width of both the numerical- and the experimental results are depicted as a function of the loading stages.




*Graph 4-6: Comparison of the numerical analysis with the experimental results of Beam 13*

In Graph 4-7 it can be seen that the experimental values of the crack width are smaller than the numerical results. Only in the last loading stage ($P = 334\ kN$) the mean value of the crack width of the numerical results is smaller ($w_{mean} = 0.156\ mm$) than the mean crack width of the experiments ($w_{mean} = 0.161\ mm$).




*Graph 4-7: Comparison of the numerical results with the experimental results*

## Number of cracks

The number of cracks at each loading stage were also registered. These results are presented in the table below. In Table 4-15 it can be seen that the number of cracks registered in DIANA are smaller than the number of cracks in the experiments, except for the first loading stage. It should be mentioned however that in the experimental results the cracks were measured on both sides of the beam, but in the numerical results number of cracks at only 1 side of the beam was calculated in Excel.

*Table 4-15: Average number of cracks measured in the numerical analysis at each loading stage compared to the average number of cracks in the experiments*

| Loading stage | Number of cracks | | |
|---|---|---|---|
| P | Experiments: (Braam: 2 -sides) | Experiments: (Braam: Average number of cracks per side) | Numerical analysis: (Diana) |
| 109 | 24 | 12 | 14 |
| 184 | 44 | 22 | 20 |
| 234 | 52 | 26 | 25 |
| 334 | 58 | 29 | 27 |

## Mean value of the crack spacing

In the experiments all the crack width measurements were restricted to the part between the loading plates of the beam. The mean value of the crack spacing was calculated by dividing the crack width measuring zone ($l = 2300\ mm$) by the number of cracks calculated in this zone. In the experiment the cracks were registered on both sides of the beam and so the crack width measuring zone was equal to: $l = 4600\ mm$.

In Table 4-16 the mean crack spacing found in the finite element analysis is compared to the mean crack spacing of the experiments. At the first loading stage ($P = 109\ kN$) we see that the mean crack spacing of the experimental analysis is about : $\frac{192-164}{164} * 100\% = 17\ \%$ larger than the mean crack spacing of the numerical results. In the second loading stage we see the mean crack spacing of the numerical analysis is larger, because the number of cracks in this stage is smaller than that of the experiments. We see that with increasing load the mean value of the cracks spacing decreases since the number of cracks increases. However, after the crack initiation phase (first loading stage), the difference between the experimental and numerical crack spacing is less than 10%, which indicates that the actual cracking behavior of a beam subjected to bending can be simulated with the finite element program DIANA.

*Table 4-16: Comparison of the mean value of the cracks spacing for beam 13 ($l_m$)*

| Mean value of the crack spacing ($l_m$) | | | |
|---|---|---|---|
| **P** | **Numerical Results** | **Experimental Results** | **Difference** |
| **[kN]** | **[mm]** | **[mm]** | **%** |
| **109** | 164 | 192 | 17 |
| **184** | 115 | 105 | -8 |
| **234** | 92 | 98 | 7 |
| **334** | 85 | 92 | 8 |

## 4.4 Finite element model of T-Beam 3

The second finite element model was that of T-beam 3. For this model the geometrical properties were different, but the element types were the same as that of Rectangular beam 13. In the following chapter the input for the geometrical – and material properties will be provided. After this the comparison of the FEM- analysis with the experimental results will be presented.

### 4.4.1 Input material and geometrical properties

The geometrical and material properties which were specified for the T-beam in DIANA are specified in the tables below.

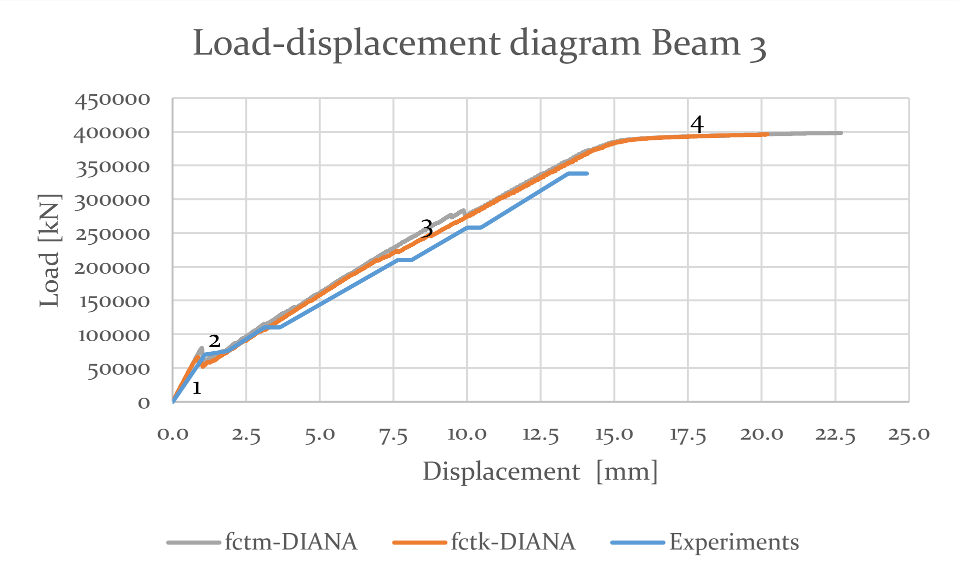
**Material- and Geometrical properties of concrete**

*Table 4-17: Geometrical properties Beam 3*

| | | | |
|---|---|---|---|
| L (length) [m] | 5.500 | [m] | **[Taken from experimental data of C.R. Braam table 6.3]** |
| $B_{flange}$ (width) [m] | 0.3 | [m] | |
| $H_{flange}$ (height) [m] | 0.2 | [m] | |
| $B_{web}$ (width) [m] | 0.15 | [m] | |
| $H_{web}$ (height) | 0.6 | [m] | |
| $\mathbf{A_c}$ (cross section) | 0.15 | $[\mathrm{m}^2]$ | |
| $\boldsymbol{f_{ccm}}$(mean value of cube compressive strength at the day of testing) | $52.2 * 10^6$ | $[\mathrm{N/m^2}]$ | **[Taken from experimental data of C.R. Braam table 6.4]** |
| $\boldsymbol{f_{ck}}$ (Characteristic value of cube compressive strength at the day of testing) | $47.9 * 10^6$ | $[\mathrm{N/m^2}]$ | **[Calculated with the help of EUROCODE 2]** |
| $\boldsymbol{f_{cspl}}$ (mean value of the cube tensile splitting strength at the day of testing) | $3.72 * 10^6$ | $[\mathrm{N/m^2}]$ | |
| $\boldsymbol{f_{ctm}}$ (mean value of the tensile strength) | $3.75 * 10^6$ | $[\mathrm{N/m^2}]$ | **[Calculated with the help of EUROCODE 2]** |
| $\boldsymbol{f_{ctk}}$ (characteristic value of the cube tensile strength) | $2.625 * 10^6$ | $[\mathrm{N/m^2}]$ | |
| $\boldsymbol{E_c}$ (Modulus of elasticity) | $3.12 * 10^{10}$ | $[\mathrm{N/m^2}]$ | **[Taken from experimental data of C.R. Braam table 6.4]** |
| $\boldsymbol{\upsilon}$ (poisson ratio) | 0.2 | | |
| $\boldsymbol{G_f}$ (fracture energy) | 74.35 | $[\boldsymbol{N/m}]$ | **Calculated: Model code** |

**Strength properties**

Because of the fact that failure due to cracking occurs at the position where the first crack initiates and all other cracks develop at the local weaker spots in the beam, it is reasonable to apply the characteristic value of the tensile strength ($f_{ctk}$) for the brittle behavior (cracking) of concrete. When the mean value of the tensile strength ($f_{ctm}$) was applied in the non-linear analysis we saw that the first crack occurred at a higher level compared to the experiments. By applying the characteristic value of the tensile strength we saw that the cracking force decreased, since $f_{ctk} < f_{ctm}$. In Graph 4-8 the load displacement diagrams of T-beam 3 are presented. In this graph the load displacement diagram of beam 3 calculated in both cases (mean value- and maximum value of the tensile strength) is compared to the experiments. When we look at the cracking force in the crack formation stage (stage 2) we see that the analysis using $f_{ctm}$ generated a higher cracking force. And when $f_{ctk}$ was applied the results were closer to the experiments.




*Graph 4-8: Load displacement diagram T-beam 3*

***Note*:** For the ductile behavior of the reinforcement (the bond-slip behavior) the mean value of the tensile strength ($f_{ctm}$) is applied, since there is a certain failure zone in which the mean values are smeared out along the beam.*

## Fracture energy

The fracture energy of concrete ($G_f$) is a material characteristic which describes the resistance of concrete subjected to tensile stresses. The fracture energy depends on several aspects such as : water/cement ratio, maximum aggregate size, the age of concrete, curing conditions and the size of the structural member [10]. The fracture energy, which is defined as the energy which is required to propagate a tensile crack of a unit area, can be estimated with the following equation according to Model code 2010 [10]: $G_f = 73 * f_{ccm}{}^{0.18}$

In which $f_{ccm}$ is the mean compressive strength in $N/mm^2$

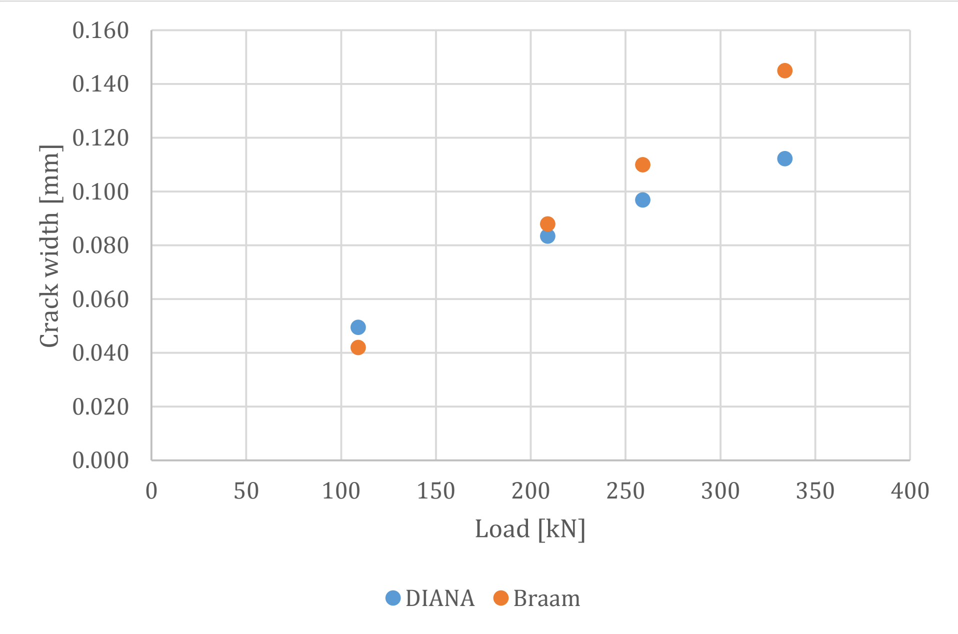
The fracture energy used for beam 3 was:

$$G_f = 73 * f_{ccm}{}^{0.18} = 73 * 52.2^{0.18} = 150.6\ N/m$$

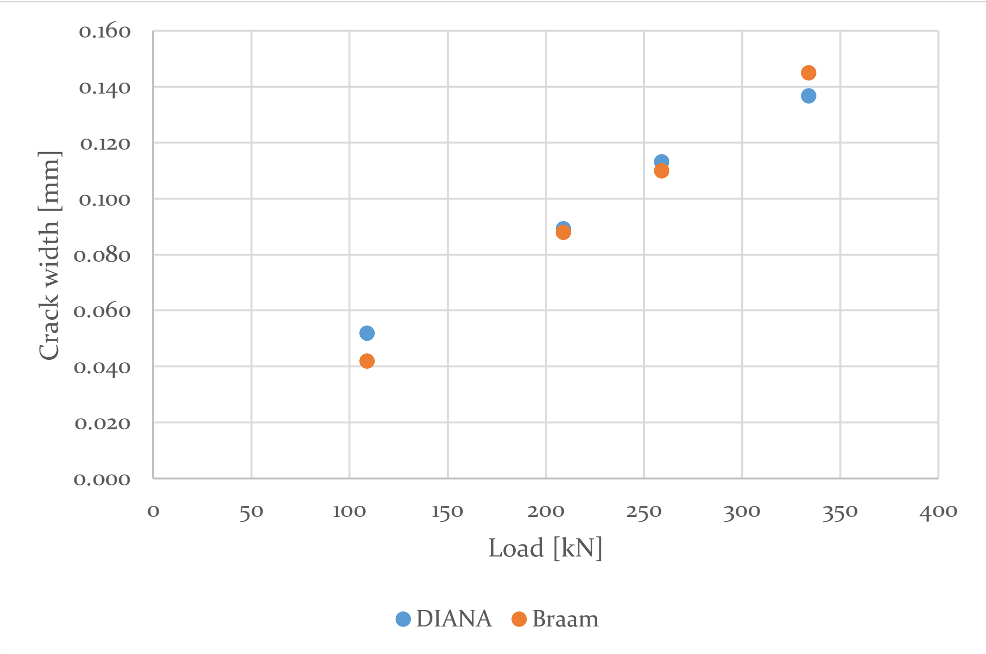
When applying this value in the numerical analysis the mean value of the crack width were much smaller than those of the experiments. This can be seen in Graph 4-9. By dividing the fracture energy with a factor 2 the results for the mean value of the crack width were much closer to that of the experiments (Graph 4-10). So the fracture energy calculated with the model code equation seemed to be a bit large for the model. This aspect has not been investigated during this thesis, but further research on this matter is necessary in order to validate whether the fracture energy according to the Model Code 2010 is actually on the large side. This may be done in combination with spatial stochastic properties.

The final input parameter for the fracture energy was:

$$G_f = 73 * f_{ccm}{}^{0.18} = \frac{73 * 52.2^{0.18}}{2} = \frac{150.6}{2} = 74.35\ N/m$$




*Graph 4-9: Comparison of the numerical analysis with the experimental results of Beam 3 for* $G_f = 150.6\ N/mm^2$




*Graph 4-10: Comparison of the numerical analysis with the experimental results of Beam 3 for* $G_f = 74.35\ N/mm^2$

## Total Strain Crack Model

The tensile and compressive behavior of reinforced concrete can be modelled with total strain cracking models in DIANA. The basic concept of the Total Strain crack models is that the stress is evaluated in the directions which are given by the crack directions. The fixed stress-strain suits the physical nature of cracking better. In this concept the stress-strain relationships are evaluated in a coordinate system which is fixed upon cracking.

The input parameter for the total strain crack model in which the crack-directions switches from rotating to fixed is:

$$Epsfix = \frac{f_{ctk}}{E_c} * 2 = \frac{2.625}{31200} * 2 = 1.68 * 10^{-4}$$

## Geometry

In Diana the outer edges of the T-beam were modelled in the same way as in Figure 4-20 ( taken from fig 6.2 of [6] ).

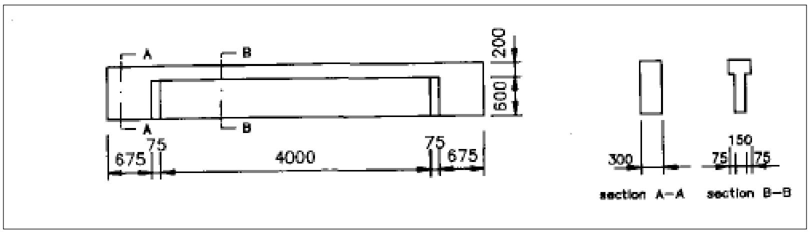



*Figure 4-20: Cross section T-Beam 3 [6]*

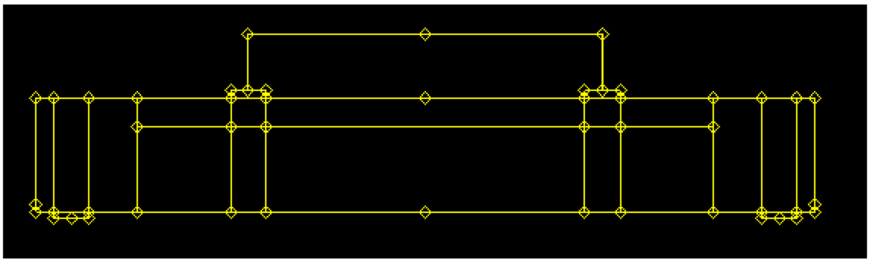


*Figure 4-21: Finite element model T-Beam 3*

The material- and geometrical properties of the steel plates, interface, loading scheme are the same as that of Rectangular Beam 13 (Section 4.2.1).

## Properties of the Reinforcement

In the following tables the properties of the reinforcement are presented.

*Table 4-18: Properties of the main Reinforcement*

| Main reinforcement | | |
|---|---|---|
| Ø (Bar diameter) | 0.02 | [m] |
| # bars | 4 | |
| # layers | 2 | |

| | | |
|---|---|---|
| $\mathbf{A_s}$ (Cross section) | $1.256 * 10^{-3}$ | **[m²]** |
| Perimeter | $251 * 10^{-3}$ | **[m]** |
| $\mathbf{E_s}$ (Youngs modulus) | $2 * 10^{11}$ | [N/m²] |
| $\boldsymbol{f_y}$ (yield strength) | $5.7 * 10^{8}$ | [N/m²] |
| $\boldsymbol{\upsilon}$ (poisson ratio) | 0.3 | - |

## Bond slip

The same bond-slip properties of Beam 13 were specified for the main reinforcement of beam 3 (section 4.2.1).

The input parameters were:

$a = 5.3 f_{ccm}$ ; $\Delta u_t = 0.0000001\ m$ ; $b = 0.4$

With: : $f_{ccm} = 52.2 * 10^6\ N/m^2$ (Mean value of the compressive strength of concrete on the day of testing).

## Linear stiffness

The same equation of beam 13 was used for the calculation of the linear stiffness for interface of the main reinforcement (section 4.2.1):

$$s_s = 6.0 * \tau_{bmax} = 6.0 * 0.38 * 52.2 = 119 \frac{N}{mm^3} = 1 * 10^{11}\ N/m^3$$

And so the input parameters for the linear stiffness were:

$D_{stiff}$: $1 * 10^{11}$ $1 * 10^{11}$

## Stirrups

*Table 4-19: Properties of the stirrups*

| Stirrups | | |
|---|---|---|
| $\emptyset$ (Bar diameter) | 0.01 | **[m]** |
| Bar spacing | 0.2 | **[m]** |
| $\mathbf{E_s}$ (Youngs modulus) | $2 * 10^{11}$ | [N/m²] |
| $\boldsymbol{f_y}$ (yield strength) | $5.7 * 10^{8}$ | [N/m²] |
| $\boldsymbol{\upsilon}$ (poisson ratio) | 0.3 | |

In DIANA the grid reinforcement for the stirrups was specified by the thickness of the total applied reinforcement. The following equation was used:

$$t = \frac{2 * A_{st}}{c.tc.\ distance} = \frac{2 * \frac{1}{4}\pi * 10^2}{200} = 0.785\ mm$$

## Bond slip

The same bond-slip properties of the main reinforcement were specified for the stirrups.

**Linear stiffness**

The same value for the linear stiffness of the main reinforcement was specified for interface of the stirrups.

## 4.5 INPUT ELEMENT TYPE AND MATERIAL TYPE

The input for the element and material type are the same as that of Rectangular beam 13 (Section 4.2.2).

## 4.6 COMPARISON RESULTS T- BEAM 3

During the experimental analyses the actual load for beam 3 was applied in four loading stages with a hand-operated hydraulic jack. Looking at the loadings scheme of the experiments it can be seen that the total load was applied in 2 points (Figure 4-22). The results that were registered by Dr. C. R. Braam [7] were based on one loading point only. In this point the total applied load during each loading stage was equal to half of the load applied by the hydraulic jack and half the weight of the loading frame (Table 4-20: column 4) .

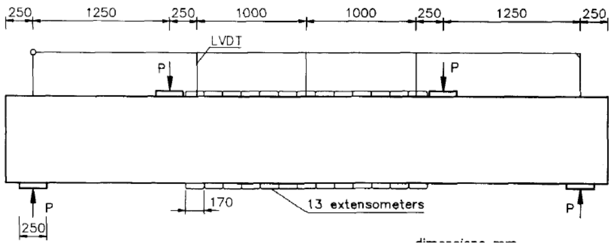



*Figure 4-22: Loading scheme experimental research*

*Table 4-20: Loading stages*

| Loading stage: Beam 3 | Weight of the loading frame [kN] | Total load Hydraulic jack [kN] | Applied load per loading point [considered in the experiments] [kN] | Total Load [considered in DIANA] [kN] |
|---|---|---|---|---|
| 1 | 18 | 200 | 109 | 218 |
| 2 | 18 | 400 | 209 | 418 |
| 3 | 18 | 500 | 259 | 518 |
| 4 | 18 | 650 | 334 | 668 |

In the numerical model however the loading scheme was a bit different. There was only one loading point. Because of this the results in DIANA were registered at the total applied load at each loading stage. This load was equal to the total load applied by the hydraulic jack and the total weight of the loading frame (Table 4-20: column 5).

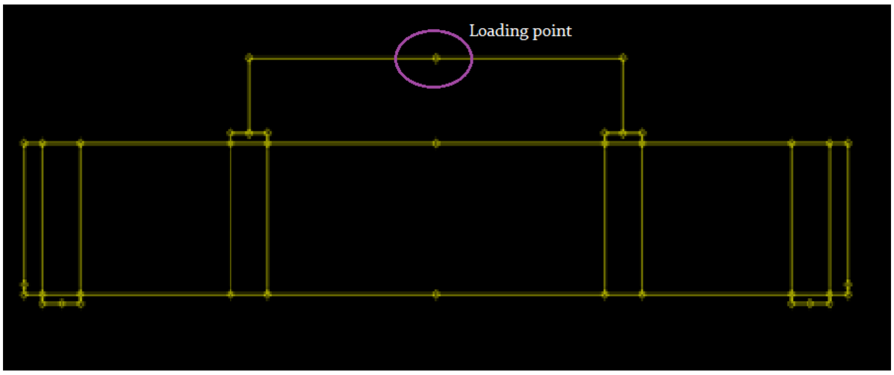



*Figure 4-23: Loading scheme numerical analysis*

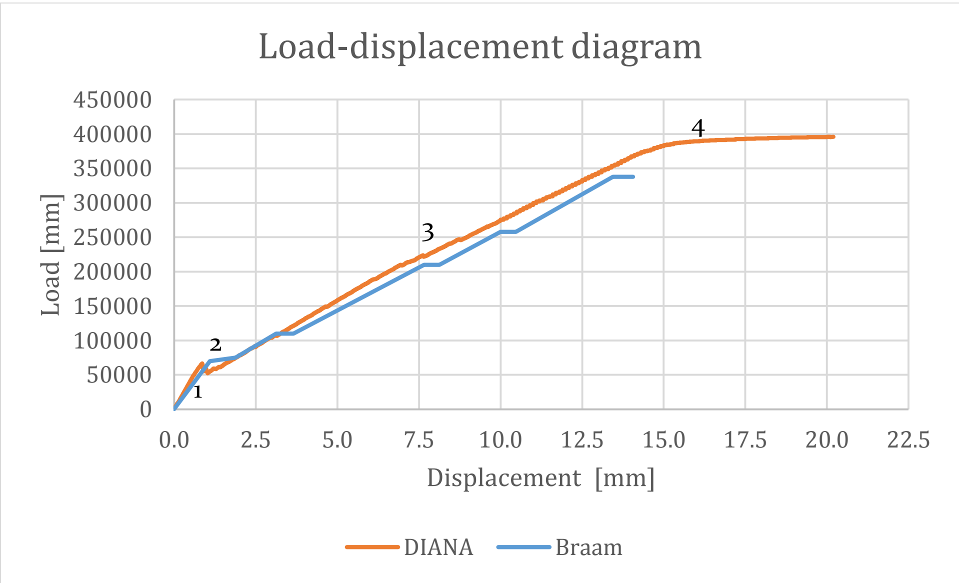
These loading stages are essential for the comparison of the crack pattern and the crack width measurements.

In this chapter all the aspects which were compared with the experimental results of Beam 3 to obtain a good numerical analysis are covered. The aspects were:

1. The load-displacement diagram
2. The magnitude of the force when the first crack occurs
3. The crack pattern
4. The mean value of the crack width at each loading stage at the level of the main reinforcement

### 4.6.1 The load-displacement diagram

When comparing the load-displacement diagram to that of the experimental results of Beam 3 fairly good results were obtained.




*Graph 4-11: Load displacement diagrams of the Experimental Results and Numerical Results of beam 3*

The load displacement diagram represents the average structural behavior of a beam subjected to an external load. By comparing the load displacement diagram of the experiments with the numerical results it can be seen whether the structural behavior of the beam in the numerical analysis coincides with the structural behavior of the experiments. The load-displacement diagram can be subdivided in four stages [1]. These stages are:

- Stage 1: Uncracked stage: in this stage the applied force ($F_{applied}$) is smaller than the cracking force ($F_{crack}$).
- Stage 2: Crack formation stage (in this stage the beam starts to crack, this occurs when the tensile force($F_{applied}$) is larger than the cracking force ($F_{crack}$) in the cross section.
- Stage 3: Stabilized cracking stage: in this stage the applied load increases which results in the widening of the already existing cracks. The number of cracks in this stage remains unchanged.
- Stage 4: The stage in which the yield strength is reached.

In the numerical analysis the beam was loaded until the yield strength of the reinforcement was reached (until stage 4). This was not the case in the laboratory tests. The horizontal branches in the experimental results of beam 3 were caused by the increasing deformation which occurred during the crack width registration.

Looking at Graph 4-11 it can be seen that the numerical results overlap the experimental results in stage 1 (uncracked stage) and stage 2 (crack formation stage). In stage 3 however the numerical forces are a bit higher than the experimental forces, so it is clear that the Forces obtained in DIANA are a bit higher than that of the experiments. This could result in a higher value of the mean crack width. During the measurements of the crack width a certain amount of creep may occur. This could have an effect on the load-displacement diagram, however the effect of creep was not taken into account during the numerical analysis. Overall it can be said that the average structural behavior of a beam subjected to an external load can be simulated with a fem-analysis in DIANA.

### 4.6.2 The magnitude of the force when the first crack occurs

The magnitude of the force during the first crack in the finite element analysis is equal to : $P = 65\ kN$ and the displacement $u = 0.87\ mm$. And when we look at the cracking force of the experimental results we see that the first crack occurs at force of $P = 70\ kN$ and a displacement of $w = 1.1\ mm$. Given the scatter in properties of concrete these values are in good agreement with each other.

### 4.6.3 The crack pattern

The best way to compare the crack pattern of the FEM-analysis with the experimental results is to look at the crack pattern at the last loading stage in DIANA ($P = 334\ kN$). The number of cracks were also be compared. The figures below represent the crack pattern of beam 3 from the experimental results and the crack pattern from the numerical analysis respectively.

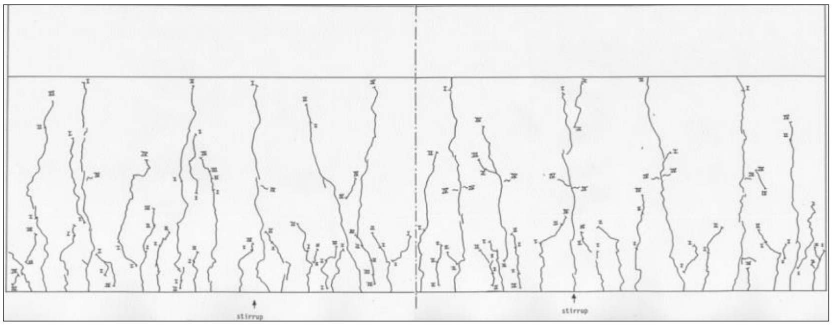


*Figure 4-24: Crack Pattern experimental results Beam 3 ( side I) [7]*

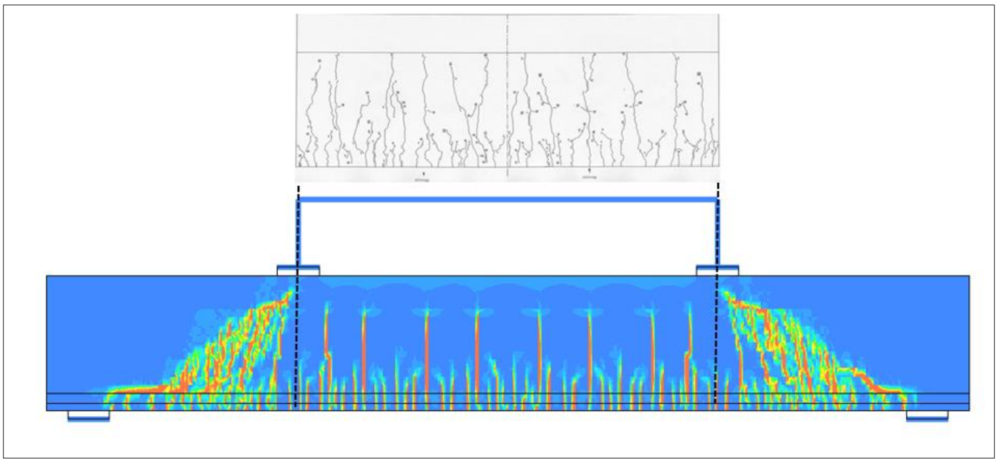


*Figure 4-25: Crack pattern experimental results beam 3 and crack pattern FEM analysis*

In the experiments the crack pattern was registered only in the middle of the beam between the top supports (Figure 4-25). So the number of cracks in the numerical analysis between these supports was equal to 37. In the experiment the number of cracks in the last loading stage at side I of the rectangular Beam was equal to 31.

### 4.6.4 The mean value of the crack width at each loading stage at the level of the main reinforcement

The mean value of the crack width was calculated in the same way as mentioned in section 4.3.4. Besides the mean value of the crack spacing ($w_{mean}$), the number of cracks and the mean value of the crack spacing ($l_m$) were also compared at each loading stage.

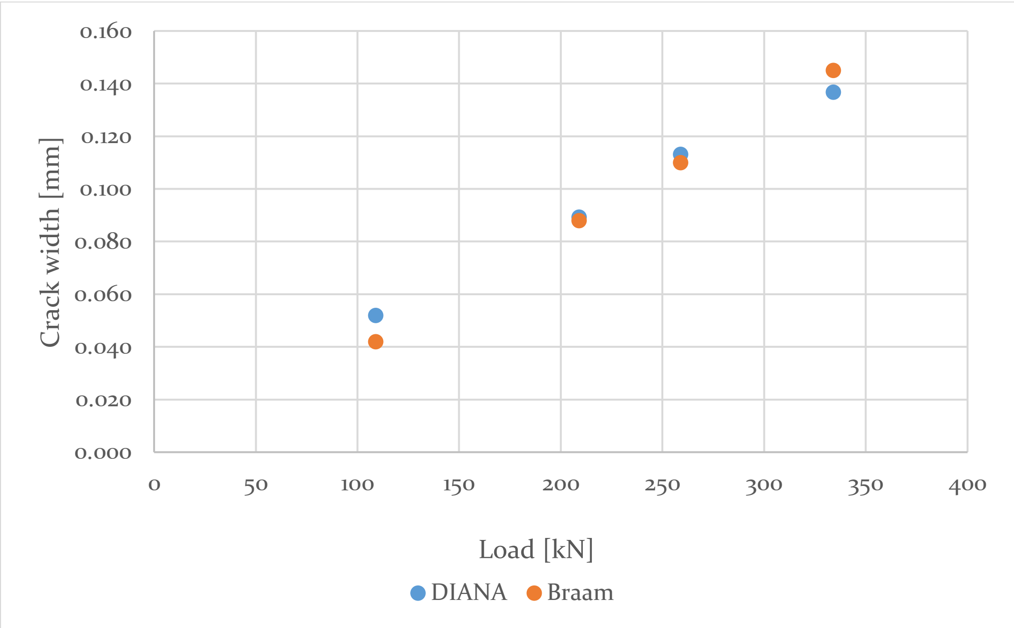
**Mean value of the crack width**

In the following table the mean value of the crack width ($w_{mean}$) at each loading stage is presented at the level of the main reinforcement.

*Table 4-21: Comparison of the mean value of the crack width $(w_{mean})$ of Beam 3*

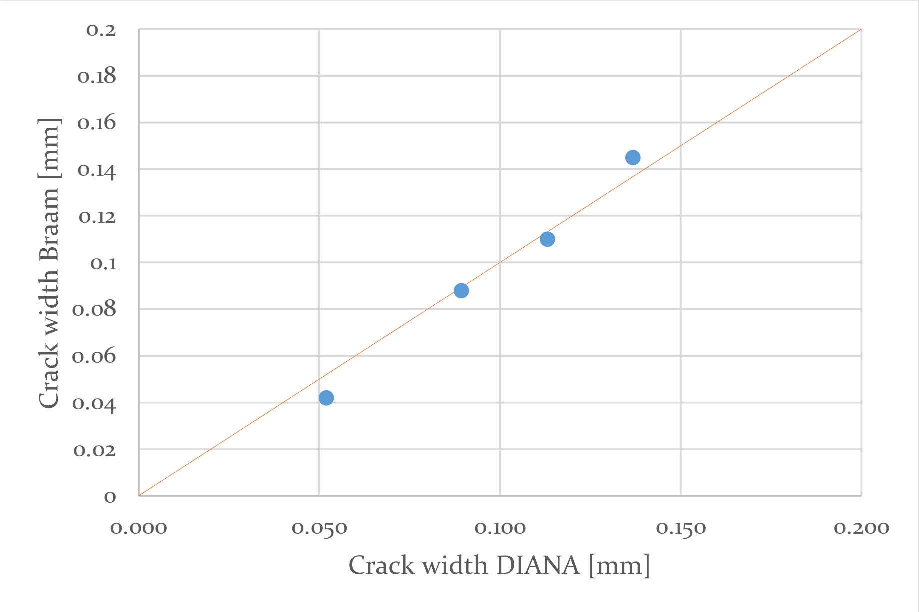
| Loading stage | Numerical results (DIANA) | Experimental results (Braam) | Difference |
|---|---|---|---|
| P | $w_{mean}$ | $w_{mean}$ | |
| [kN] | [mm] | [mm] | % |
| 109 | 0.052 | 0.042 | 24 |
| 209 | 0.089 | 0.088 | 1 |
| 259 | 0.113 | 0.110 | 3 |
| 334 | 0.137 | 0.145 | 6 |

In Table 4-21 it can be seen that the mean value of the crack width at the first loading stage from the numerical analysis $(P = 109\,kN)$ is about 24 % larger than that of the experiments. However when we look at the other loading stages $(P = 209\,kN; P = 259\,kN\ and\ P = 334kN)$ we can see that the difference become smaller than 3 %. So with increasing load the numerical results are closer to the experimental results. This can be seen in Graph 4-12. In this graph the mean values of the crack width of both the numerical- and the experimental results are depicted as a function of the different loading stages.




*Graph 4-12: Comparison of the numerical analysis with the experimental results of Beam 3*

In Graph 4-13 it can be seen that the experimental values of the crack width are smaller than the numerical results. Only in the last loading stage $(P = 334\,kN)$ the mean value of the crack width of the numerical results is smaller $(w_{mean} = 0.137\,mm)$ than the mean crack width of the experiments $(w_{mean} = 0.145\,mm)$.




*Graph 4-13: Comparison of the numerical results with the experimental result*

## Number of cracks

The number of cracks at each loading stage were also registered. In Table 4-22 it can be seen that the number of cracks registered in DIANA are larger than the number of cracks in the experiments, except for the first loading stage. It should be mentioned however that in the experimental results the cracks were measured on both sides of the beam, but in the numerical results the number of cracks at only 1 side of the beam was calculated in Excel.

*Table 4-22: Number of cracks measured in the numerical analysis at each loading stage compared to the number of cracks in the experiments*

| Loading stage | Number of cracks | | |
|---|---|---|---|
| **P [kN]** | **Experiments: (Braam: 2 -sides)** | **Experiments: (Braam: Average number of cracks per side)** | **Numerical analysis: (Diana)** |
| **109** | 56 | 28 | 27 |
| **209** | 61 | 31 | 33 |
| **259** | 61 | 31 | 34 |
| **334** | 62 | 31 | 37 |

## Mean value of the crack spacing

In the experiments all the crack width measurements were restricted to the part between the loading plates of the beam. The mean value of the crack spacing was calculated by dividing the crack width measuring zone ($l = 2300\ mm$) by the number of cracks calculated in this zone. In the experiment the cracks were registered on both sides of the beam and so the crack width measuring zone was equal to: $l = 4600\ mm$. In Table 4-23crack initiation phase (first loading stage), the difference between the experimental and numerical crack spacing is less than 10%, which indicates that the actual cracking behavior of a beam subjected to bending can be simulated with the finite element program DIANA. the mean crack spacing in DIANA is compared to the mean crack spacing of the experiments. In the first loading stage ($P = 109\ kN$) we see that the mean crack spacing of the numerical analysis is about : $\frac{85-82}{82} * 100\% = 3.7\%$ larger than the mean crack spacing of the experimental results. In the other loading stage we see the mean crack spacing of the numerical analysis is smaller, because the number of cracks in this stage is also larger than that of the experiments. We see that with increasing load the mean value of the cracks spacing decreases since the number of cracks increases. Overall we see that after the crack initiation phase, the difference between the experimental and numerical crack spacing is less than 20%, which indicates that the actual cracking behavior of a beam subjected to bending can be simulated with the finite element program DIANA.

*Table 4-23: Comparison of the mean value of the cracks spacing for beam 3 ($l_m$)*

| **Mean value of the crack spacing ($l_m$)** | | | |
|---|---|---|---|
| **P** | **Numerical Results** | **Experimental Results** | **Difference** |
| **[kN]** | **[mm]** | **[mm]** | **%** |
| **109** | 85 | 82 | -4 |
| **209** | 70 | 75 | 8 |
| **259** | 68 | 75 | 11 |
| **334** | 62 | 74 | 19 |

# 5 INFLUENCE OF THE COVER IN NONLINEAR ANALYSIS

In section 3.1 and section 3.2 we saw that the variation of the concrete cover and the limitation of the maximum crack spacing did have an influence on the maximum allowable steel stress that was needed to control the crack width. In section 3.2.2 it was also clear that the limitation according to the VARCE needed to be investigated further for bar distances smaller than 150 mm. And since we could simulate the actual cracking behavior with the finite element program DIANA (Chapter 4) a final analysis was carried out to investigate the influence of the concrete cover and the limitation of the crack spacing on the actual cracking behavior in beams subjected to bending according to the different codes: NEN-EN 1992-1-1 (Eurocode 2) and NEN 3880 (VB 74/84). This analysis was conducted to investigate whether the cracking behavior due to an increased concrete cover proposed by the codes (NEN-EN 1992-1-1 and NEN 3880) is in agreement with the actual cracking behavior.

The final analysis consisted of two parts:

1. <u>Numerical analysis:</u> Analysis of the influence of the concrete cover on the cracking behavior of a beam subjected to bending with Finite element program DIANA
2. <u>Analytical analysis:</u> Analysis of the influence of the concrete cover and the limitation of the crack spacing on the cracking behavior a beam subjected to bending calculated according to the NEN-EN 1992-1-1 and the NEN 3880.

In section 5.1 the procedure and the results of the numerical analysis are presented. The procedure and the results of the analytical analysis are presented in section 5.2. In chapter 6 the results of the numerical and the analytical analysis are compared and evaluated.

## 5.1 NUMERICAL ANALYSIS: INFLUENCE OF THE CONCRETE COVER ON THE CRACKING BEHAVIOR OF A BEAM SUBJECTED TO BENDING

In order to investigate what influence the concrete cover has on the actual cracking behavior of a beam subjected to bending a numerical analysis was carried out in DIANA. The analysis was performed for the same practical cases mentioned in sections 4.2 and 4.4: Rectangular Beam 13 and T-beam 3. During this analysis each beam was modelled with a cover of $c = 50\ mm$ and $c = 70\ mm$. This can be seen in the figures presented in section 5.1.2.

### 5.1.1 Procedure

During the analysis all parameters in the models remained unchanged; only the position of the main reinforcement was modified (with a vertical shift upward), resulting in an increased bottom cover. After the non-linear analysis was finished the results were generated. In these cases the crack width was calculated at the four loadings stages mentioned in sections 4.3 and 4.6. In the following chapter the results for Rectangular beam 13 and T-beam 3 are presented.

*Note: The side cover is not investigated since we are dealing with a 2D analysis.*

### 5.1.2 Results: Variation of the concrete cover

For a good approximation of the actual cracking behavior the following results will be presented at each applied concrete cover:

1. The mean value of the crack width
2. The mean and maximum value of the crack spacing

***Note*:*** *The maximum crack spacing was calculated with the following equation* $\Delta l_{max} = 2.2 * \Delta l_{mean}$ *since the mean value of the crack width is calculated with the following equation:* $w_{mean} = \frac{w_{k,max}}{2.2}$ *in the Eurocode 2.*

#### Results Rectangular Beam 13

**Applied concrete cover: $c = 30\ mm$**

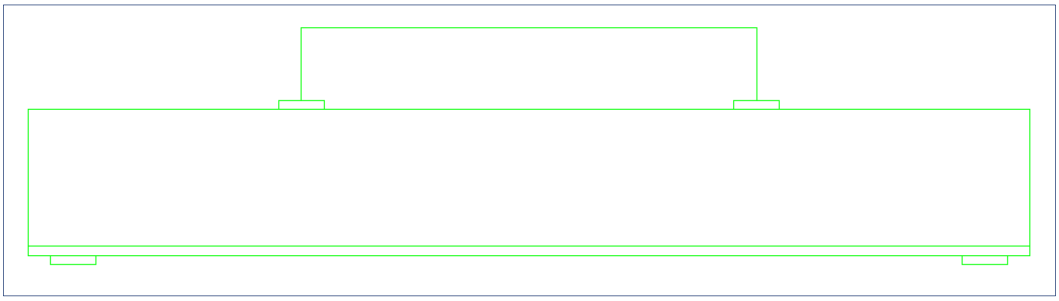
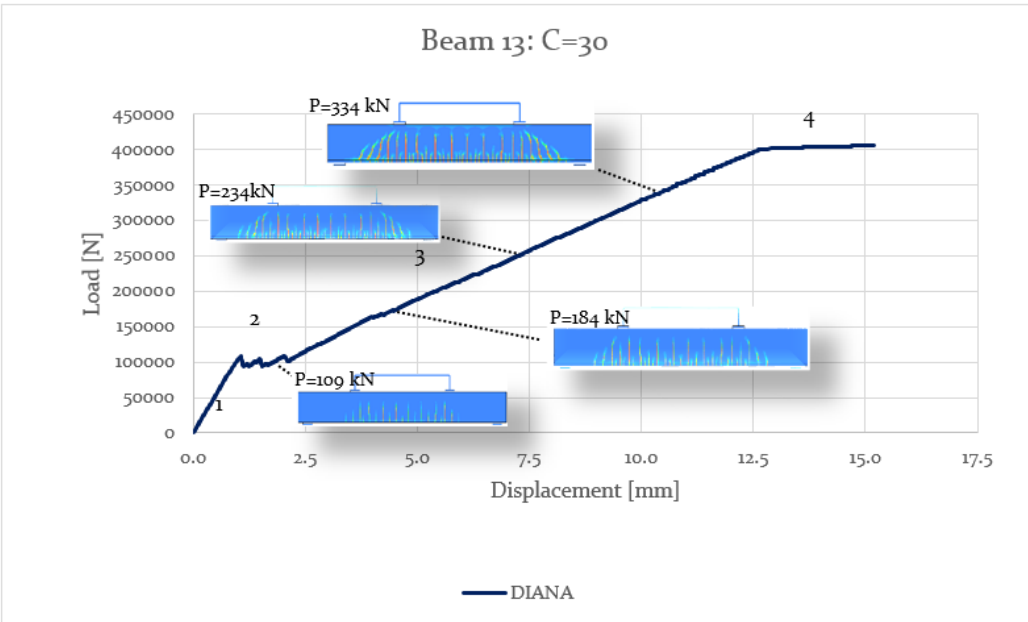


*Figure 5-1: Finite element model Rectangular beam 13 (c=30 mm)*

**Load-displacement diagram**




*Graph 5-1: Load displacement diagram of Rectangular Beam 13 for c=30 mm*

Graph 5-1 presents the load-displacement diagram of beam 13 at an applied cover of 30 mm. In this graph it can also be seen how the cracks develop at each loading stage.

## The mean value of the crack width

*Table 5-1: Mean value of the crack width for $c = 30\ mm$*

| $F_{applied}$ | $\sigma_s$ | $w_{mean}$ |
|---|---|---|
| [kN] | [N/mm²] | [mm] |
| **109** | 148 | 0.068 |
| **184** | 261 | 0.101 |
| **234** | 333 | 0.114 |
| **334** | 478 | 0.156 |

## The mean- and maximum value of the crack spacing

*Table 5-2: Mean- and maximum value of the crack spacing for $c = 30\ mm$*

| $F_{applied}$ | $\Delta l_{mean}$ | $\Delta l_{max}$ |
|---|---|---|
| [kN] | [mm] | [mm] |
| **109** | 164 | 361 |
| **184** | 115 | 253 |
| **234** | 92 | 202 |
| **334** | 85 | 187 |

### Applied concrete cover: $c = 50\ mm$

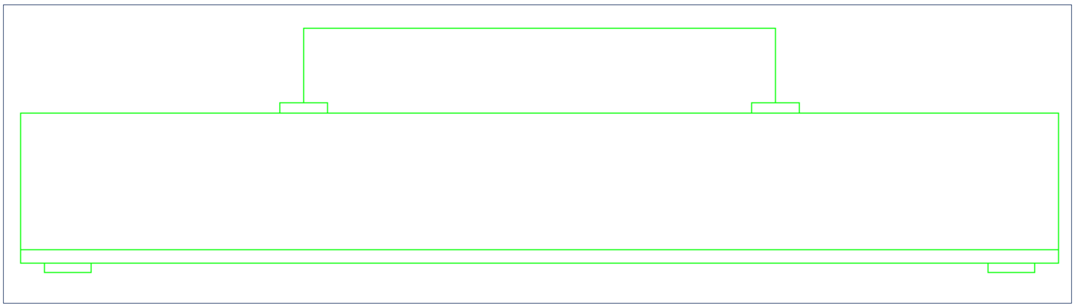
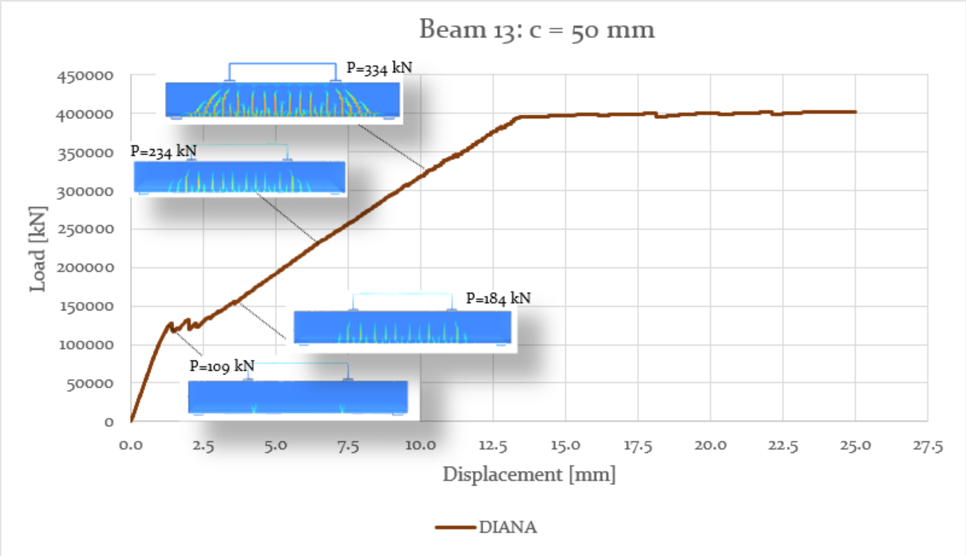


*Figure 5-2: Finite element model Rectangular beam 13 (c=50 mm)*

## Load-displacement diagram




*Graph 5-2: Load displacement diagram Rectangular Beam 13 for c = 50 mm*

## The mean value of the crack width

*Table 5-3: Mean value of the crack width for c = 50 mm*

| $F_{applied}$ | $\sigma_s$ | $w_{mean}$ |
|---|---|---|
| [kN] | [N/mm²] | [mm] |
| **109** | 67.50 | 0.011 |
| **184** | 259 | 0.083 |
| **234** | 337 | 0.131 |
| **334** | 483 | 0.174 |

## The mean- and maximum value of the crack spacing

*Table 5-4: Mean- and maximum value of the crack spacing for c = 50 mm*

| $F_{applied}$ | $\Delta l_{mean}$ | $\Delta l_{max}$ |
|---|---|---|
| [kN] | [mm] | [mm] |
| **109** | 2300 | 5060 |
| **184** | 144 | 316 |
| **234** | 144 | 316 |
| **334** | 121 | 266 |

By Comparing the results of Table 5-1 with Table 5-3 it can be seen that the crack width increases when the cover is increased. The crack spacing also increases when a larger cover is applied.

### Applied concrete cover: $c = 70\ mm$

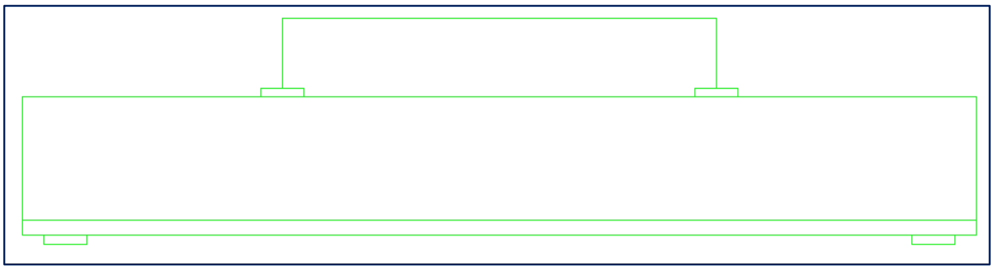
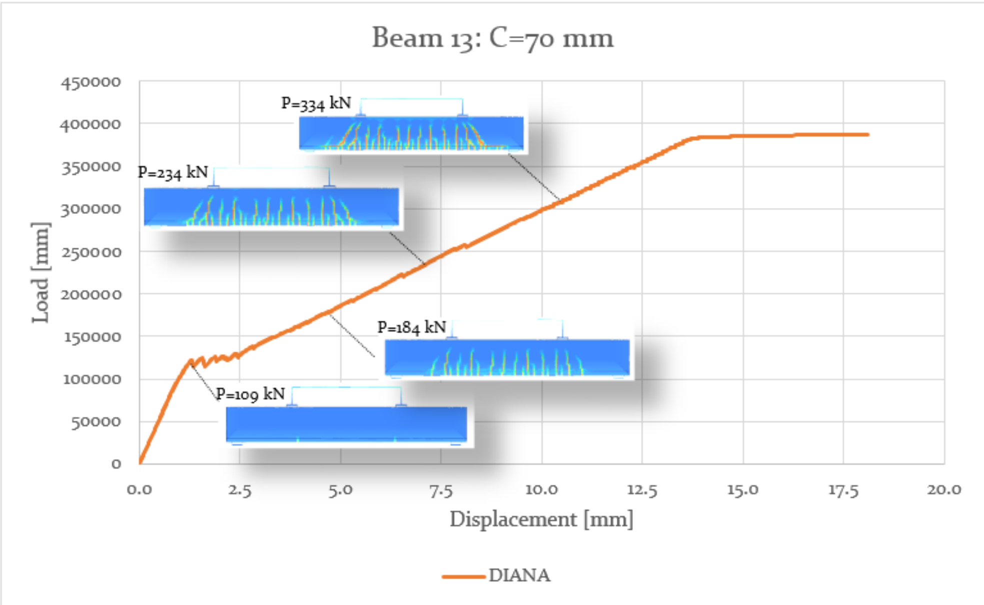


*Figure 5-3: Finite element model Rectangular beam 13 (c=70 mm)*

## Load-displacement diagram




*Graph 5-3: Load displacement diagram Rectangular beam 13 for c=70 mm*

## The mean value of the crack width

*Table 5-5: Mean value of the crack width for $c = 70\ mm$*

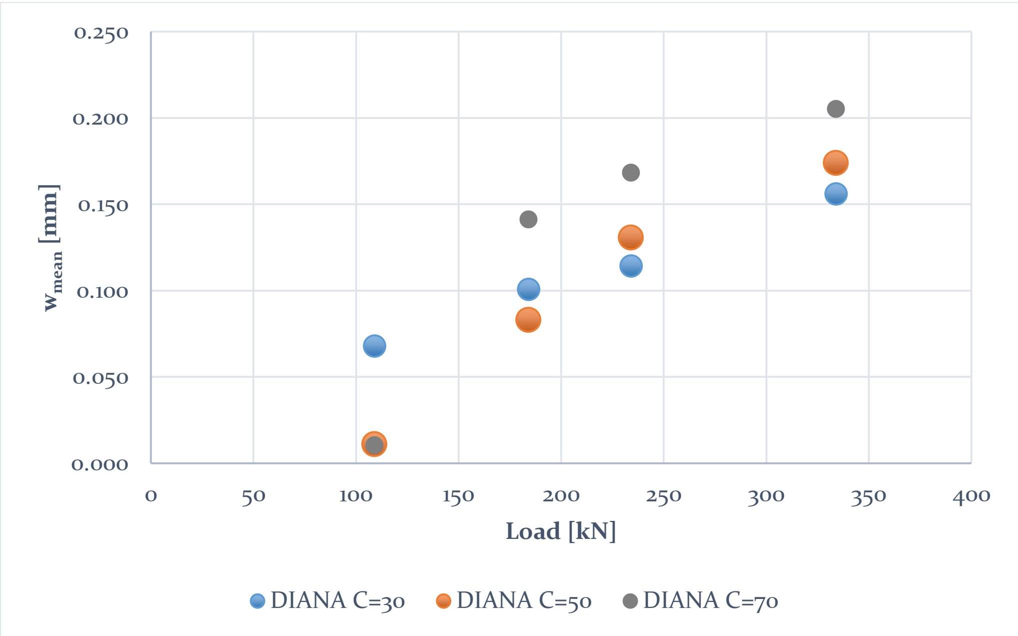
| $F_{applied}$ | $\sigma_s$ | $w_{mean}$ |
|---|---|---|
| [kN] | [N/mm²] | [mm] |
| 109 | 61.7 | 0.010 |
| 184 | 270 | 0.141 |
| 234 | 348 | 0.168 |
| 334 | 499 | 0.205 |

## The mean- and maximum value of the crack spacing

*Table 5-6: Mean- and maximum value of the crack spacing for $c = 70\ mm$*

| $F_{applied}$ | $\Delta l_{mean}$ | $\Delta l_{max}$ |
|---|---|---|
| [kN] | [mm] | [mm] |
| **109** | 2300 | 5060 |
| **184** | 192 | 422 |
| **234** | 153 | 337 |
| **334** | 135 | 298 |

Comparing the results of Table 5-3 with Table 5-5 we also see that the crack width increases when a larger cover is applied. This is also the case for the crack spacing.




*Graph 5-4: Influence of the concrete cover on the mean crack width for beam 13*

In Graph 5-4 it can be seen that the mean value of the crack width ($w_{mean}$) increases when a larger concrete cover is applied. But there is a difference however, in the crack formations stage (Loading stage 1) the mean crack width found at a cover of c=30 mm was larger than the mean crack with which were calculated at the larger applied covers in DIANA. Since there were no cracks registered at the first loading stage ($F = 109\ kN$ ) the crack width was registered at the point where the first crack occurred ( see Graph 5-2 and Graph 5-3). This can be the reason why the crack width were smaller when larger covers were applied in the first loading stage. In Table 5-4 and Table 5-6 it is clear that the mean crack spacing is 2300 $mm$ at the first loading stage. This indicates that only one crack occurred at this point.

### Results T-Beam 3

**Applied concrete cover: $c = 20\ mm$**

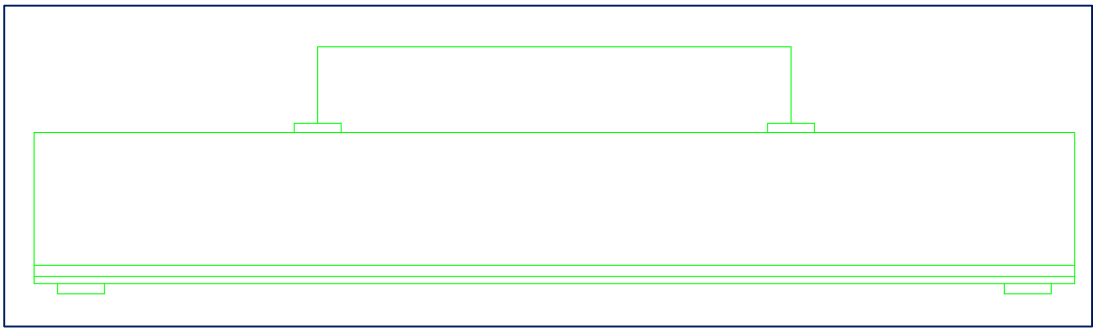
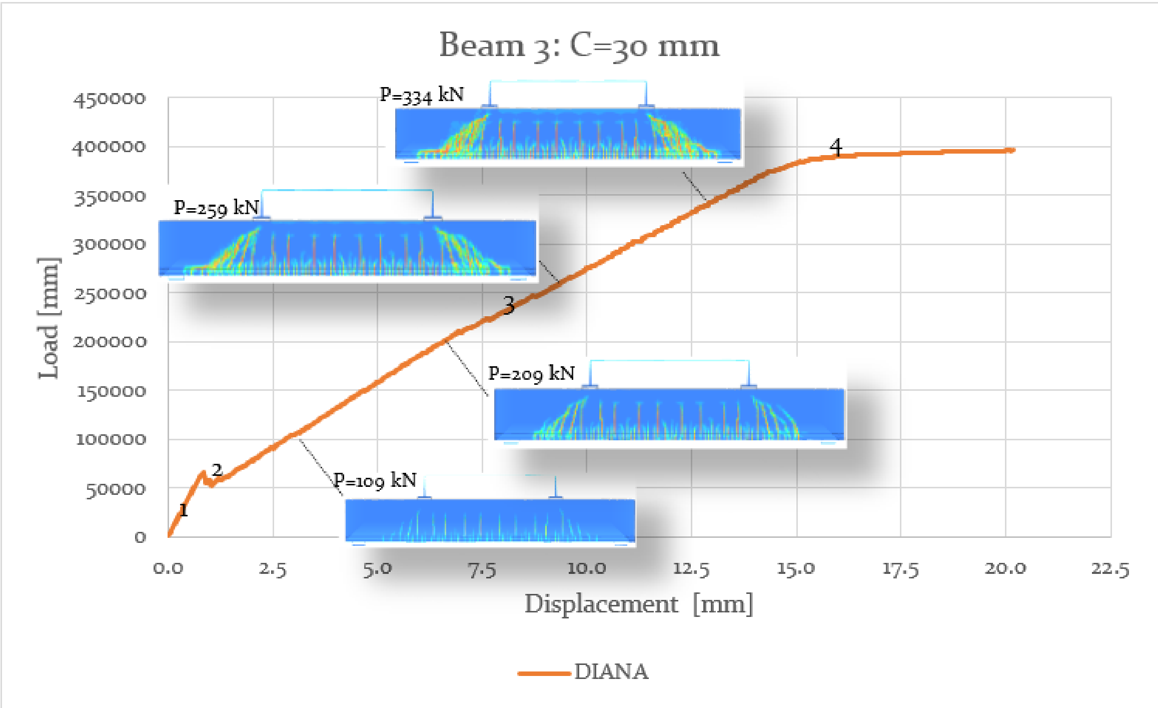


*Figure 5-4: Finite element model T-beam 3 (c=20 mm)*

**Load-displacement diagram**




*Graph 5-5: Load displacement diagram T-beam 3 for c=30 mm*

Graph 5-5 presents the load- displacement diagram of T-beam 3. It can be seen how the crack pattern developes at each loading stage.

## The mean value of the crack width

*Table 5-7: Mean- and maximum value of the crack width for $c = 20\ mm$*

| $F_{applied}$ | $\sigma_s$ | $w_{mean}$ |
|---|---|---|
| [kN] | [N/mm²] | [mm] |
| **109** | 167 | 0.052 |
| **209** | 323 | 0.089 |
| **259** | 402 | 0.113 |
| **334** | 521 | 0.137 |

## The mean- and maximum value of the crack spacing

*Table 5-8: Mean- and maximum value of the crack spacing for $c = 20\ mm$*

| $F_{applied}$ | $\Delta l_{mean}$ | $\Delta l_{max}$ |
|---|---|---|
| [kN] | [mm] | [mm] |
| **109** | 85 | 187 |
| **209** | 70 | 153 |
| **259** | 68 | 149 |
| **334** | 62 | 137 |

### Applied concrete cover: $c = 50\ mm$

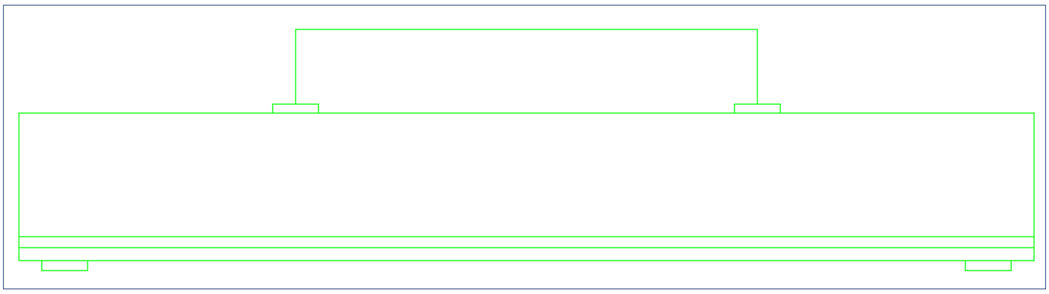
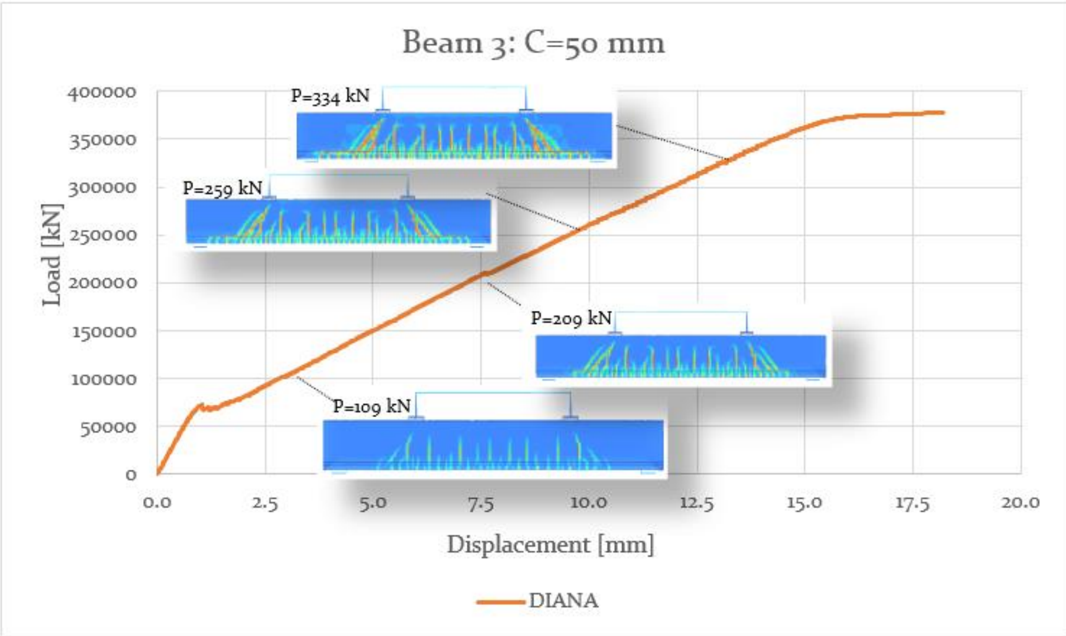


*Figure 5-5: Finite element model T-beam 3 (c=50 mm)*

## Load displacement diagram




*Graph 5-6: Load displacement diagram T-beam 3 for C=50 mm*

## The mean value of the crack width

*Table 5-9: Mean- and maximum value of the crack width for $c = 50\ mm$*

| $F_{applied}$ | $\sigma_s$ | $w_{mean}$ |
|---|---|---|
| [kN] | [N/mm²] | [mm] |
| **109** | 165 | 0.058 |
| **209** | 336 | 0.078 |
| **259** | 419 | 0.082 |
| **334** | 542 | 0.095 |

## The mean- and maximum value of the crack spacing

*Table 5-10: Mean- and maximum value of the crack spacing for $c = 50\ mm$*

| $F_{applied}$ | $\Delta l_{mean}$ | $\Delta l_{max}$ |
|---|---|---|
| [kN] | [mm] | [mm] |
| **109** | 144 | 316 |
| **209** | 88 | 195 |
| **259** | 79 | 174 |
| **334** | 88 | 195 |

### Applied concrete cover: $c = 70\ mm$

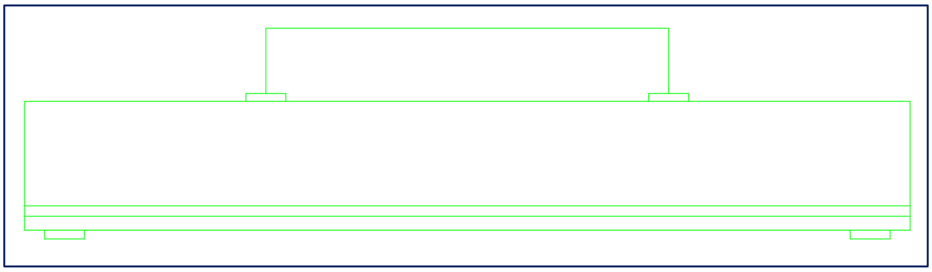


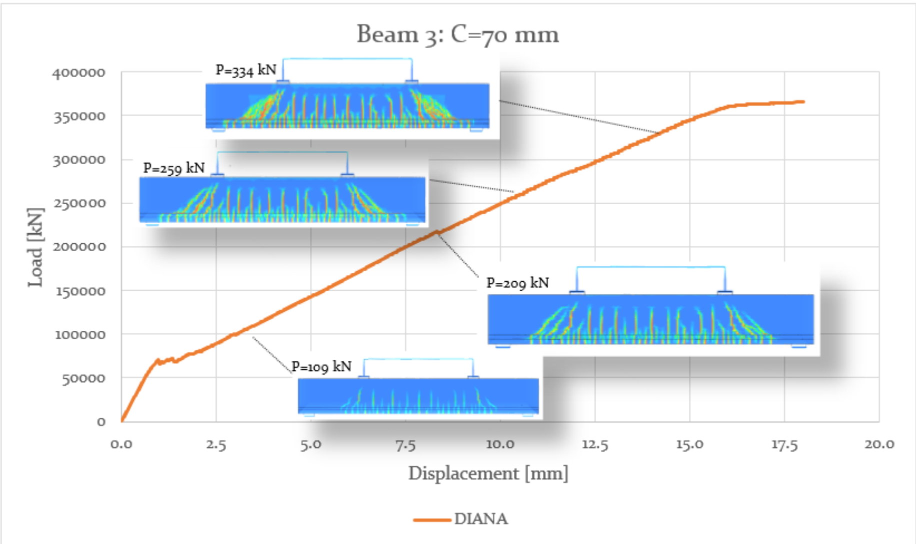
*Figure 5-6: Finite element model T-beam 3 (c=70 mm)*

## The mean value of the crack width

*Table 5-11: Mean- and maximum value of the crack width for $c = 70\ mm$*

| $F_{applied}$ | $\sigma_s$ | $w_{mean}$ |
|---|---|---|
| [kN] | [N/mm²] | [mm] |
| **109** | 175 | 0.059 |
| **209** | 348 | 0.097 |
| **259** | 431 | 0.109 |
| 334 | 557 | 0.114 |

## Load displacement diagram

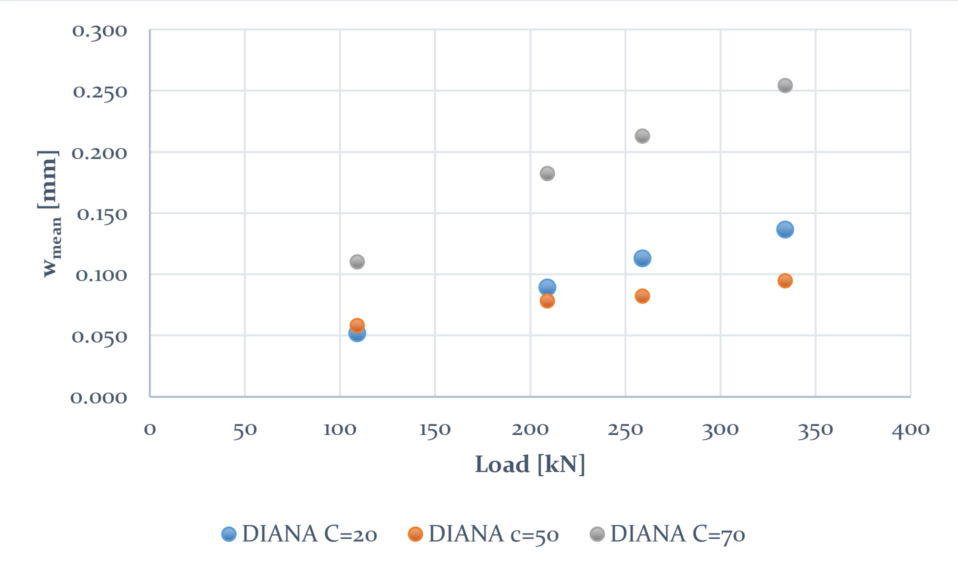



*Graph 5-7: Load displacement diagram T-beam 3 for c=70 mm*

## The mean- and maximum value of the crack spacing

*Table 5-12: Mean- and maximum value of the crack spacing for $c = 70\ mm$*

| $F_{applied}$ | $\Delta l_{mean}$ | $\Delta l_{max}$ |
|---|---|---|
| [kN] | [mm] | [mm] |
| **109** | 115 | 253 |
| **209** | 88 | 195 |
| **259** | 88 | 195 |
| **334** | 96 | 211 |




*Graph 5-8: Influence of the concrete cover on the mean crack width for T-Beam 3*

When we compare the results of Table 5-8 with Table 5-10 we see that the mean crack width decreases when a cover of $c = 50\ mm$ is applied compared to a $c = 30\ mm$. This contradicts the expected behavior: that with increasing cover the crack width also increases. But when we compare the values of Table 5-9 and Table 5-11 we see that the mean value of the crack width increases with 24% in the third loading stage ($P = 259\ kN$) and with less than 20% in the other loading stages. This can also be seen in Graph 5-8.

## 5.2 Analytical analysis: influence of the concrete cover and limitation of the crack spacing on the crack width calculations according to NEN-EN 1992-1-1 and NEN 3880

In order to investigate what influence the concrete cover has on the cracking behavior of a beam subjected to bending according to the different regulations: NEN-EN1992-1-1 and VB74/84 an analytical analysis was carried out. The same practical cases which were calculated in section 5.1 (Beam 13 and Beam 3) were analyzed. These cases came from the experimental research conducted by Braam [6]. In the following section the procedure of this analysis will be briefly explained and the results of the crack width calculations will be presented for both regulations. In Appendix III the calculation details of Rectangular beam 13 according to both regulations (NEN-EN 1992-1-1 and NEN 3880) can be found. The calculations for T-beam 3 can be found in the Excel sheet: Test Data 2 C.R. Braam: fully developed crack pattern

### 5.2.1 Procedure

With the help of EXCEL, a calculation sheet was setup in which the calculations for the crack width was performed. The influence of the concrete cover was investigated by comparing the Eurocode 2 (crack width expressions) with the VB74/84 (regulations regarding crack width control). Additionally, the value of the crack spacing was limited and then the influence of this limitation on mean value- and the maximum value of the crack width was analyzed according to both regulations (Eurocode 2 and VB74/84). During this step the concrete cover was also varied.

### 5.2.2 Results: Variation of the concrete cover

The calculations were carried out for the 2 beams which were analyzed numerically in DIANA: T-beam3 and Rectangular beam 13. Both beams were subjected to bending. The cover was varied for both beams. In the T-beam (Beam 3) the applied concrete cover was varied with the following values: $c = 20\ mm$ ; $c = 50\ mm$ and $c = 70\ mm$. In the rectangular beam (Beam 13) the applied concrete cover was varied with the following values: $c = 30\ mm$ ; $c = 50\ mm$ and $c = 70\ mm$. In Appendix III section 1.3 an example calculation of beam 13 based on the Eurocode 2 equations regarding crack width control is presented. The example calculation of beam 13 based on the VB74/84 equations regarding crack width control can be found in section 1.4 of Appendix III.

### 5.2.3 NEN-EN 1992-1-1 (Eurocode 2)

The equations mentioned in section 1.2.3 were used for the crack width calculations. The mean crack width was calculated in the same way as presented in section 3.3. In the following tables the results of the crack width calculations are presented.

### Results Rectangular beam 13:

$c = 30\ mm$

*Table 5-13: Values of the crack width calculated for a cover of c=30 mm according to NEN-EN 1992-1-1 for Beam 13*

| $F_{applied}$ | $\sigma_s$ | $s_{r,max}$ | $\varepsilon_{sm}-\varepsilon_{cm}$ | $w_{mean}$ |
|---|---|---|---|---|
| [kN] | [N/mm²] | [mm] | | [mm] |
| **109** | 136.57 | 282 | 4.10E-04 | 0.053 |
| **184** | 230.55 | 282 | 6.92E-04 | 0.089 |
| **234** | 293.20 | 282 | 9.99E-04 | 0.128 |
| **334** | 418.49 | 282 | 1.63E-03 | 0.209 |

$c = 50\ mm$

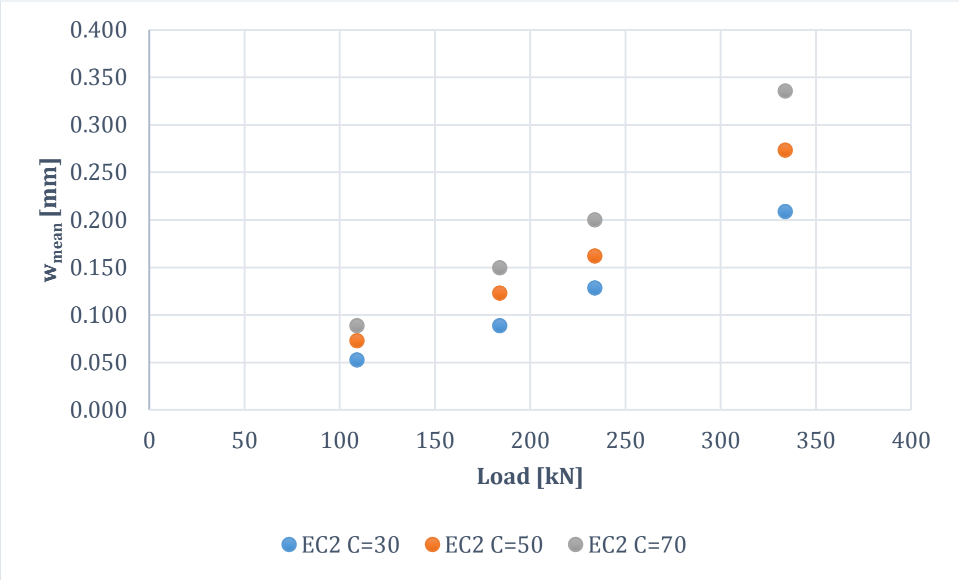
*Table 5-14: Values of the crack width calculated for a cover of c=50 mm according to NEN-EN 1992-1-1 for Beam 13*

| $F_{applied}$ | $\sigma_s$ | $s_{r,max}$ | $\varepsilon_{sm}-\varepsilon_{cm}$ | $w_{mean}$ |
|---|---|---|---|---|
| [kN] | [N/mm²] | [mm] | | [mm] |
| **109** | 140.55 | 380 | 4.22E-04 | 0.073 |
| **184** | 237.26 | 380 | 7.12E-04 | 0.123 |
| **234** | 301.73 | 380 | 9.39E-04 | 0.162 |
| **334** | 430.67 | 380 | 1.58E-03 | 0.273 |

$c = 70\ mm$

*Table 5-15: Values of the crack width calculated for a cover of c=70 mm according to NEN-EN 1992-1-1 for Beam 13*

| $F_{applied}$ | $\sigma_s$ | $s_{r,max}$ | $\varepsilon_{sm}-\varepsilon_{cm}$ | $w_{mean}$ |
|---|---|---|---|---|
| [kN] | [N/mm²] | [mm] | | [mm] |
| **109** | 144.76 | 448 | 4.34E-04 | 0.089 |
| **184** | 244.36 | 448 | 7.33E-04 | 0.149 |
| **234** | 310.76 | 448 | 9.82E-04 | 0.200 |
| **334** | 443.57 | 448 | 1.65E-03 | 0.335 |




*Graph 5-9: Influence of an increasing concrete cover on the mean value of the crack width according to NEN-EN 1992-1-1 for Beam 13*

Looking at the results of beam 13 we see that the mean crack width increases due to increasing concrete cover. It can be seen that when a cover of 50 mm is applied the crack width increases with: $\frac{0.073-0.053}{0.053} * 100\% = 38\%$ in the first loading stage. When a cover of 70 mm is applied the mean value of the crack width increases with: $\frac{0.089-0.073}{0.073} * 100\% = 22\%$. The increase of the crack width is caused by the increase of the maximum crack spacing ($s_{r,max}$) and the decrease of the effective reinforcement ratio ($\rho_{peff}$).

### Results T-beam 3:

### $c = 20\ mm$

*Table 5-16: Values of the crack width calculated for a cover of c=20 mm according to NEN-EN 1992-1-1 for Beam 3*

| $F_{applied}$ | $\sigma_s$ | $s_{r,max}$ | $\varepsilon_{sm}-\varepsilon_{cm}$ | $w_{mean}$ |
|---|---|---|---|---|
| [kN] | [N/mm²] | [mm] | [-] | [mm] |
| **109** | 140.98 | 207 | 4.23E-04 | 0.040 |
| **209** | 270.32 | 207 | 1.04E-03 | 0.098 |
| **259** | 335 | 207 | 1.37E-03 | 0.129 |
| **334** | 432 | 207 | 1.85E-03 | 0.175 |

### $c = 50\ mm$

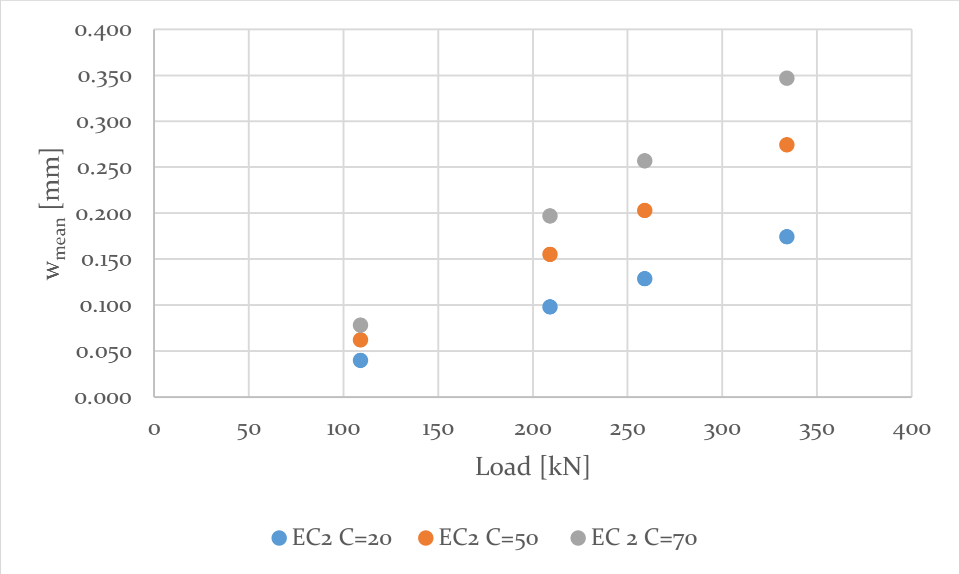
*Table 5-17: Values of the crack width calculated for a cover of c=50 mm according to NEN-EN 1992-1-1 for Beam 3*

| $F_{applied}$ | $\sigma_s$ | $s_{r,max}$ | $\varepsilon_{sm}-\varepsilon_{cm}$ | $w_{mean}$ |
|---|---|---|---|---|
| [kN] | [N/mm²] | [mm] | [-] | [mm] |
| **109** | 147.44 | 310 | 4.42E-04 | 0.062 |
| **209** | 282.71 | 310 | 1.10E-03 | 0.155 |
| **259** | 350.34 | 310 | 1.44E-03 | 0.203 |
| **334** | 451.79 | 310 | 1.95E-03 | 0.275 |

### $c = 70\ mm$

*Table 5-18: Values of the crack width calculated for a cover of c=70 mm according to NEN-EN 1992-1-1 for Beam 3*

| $F_{applied}$ | $\sigma_s$ | $s_{r,max}$ | $\varepsilon_{sm}-\varepsilon_{cm}$ | $w_{mean}$ |
|---|---|---|---|---|
| [kN] | [N/mm²] | [mm] | [-] | [mm] |
| **109** | 152.08 | 378 | 4.56E-04 | 0.078 |
| **209** | 291.61 | 378 | 1.15E-03 | 0.197 |
| **259** | 361.37 | 378 | 1.50E-03 | 0.257 |
| **334** | 466 | 378 | 2.02E-03 | 0.347 |




*Graph 5-10: Influence of an increasing concrete cover on the mean value of the crack width according to NEN-EN 1992-1-1 for Beam 3*

Looking at Graph 5-10 it is clear that the mean value of the crack width increases due to the increasing concrete cover. We see that when a cover of 50 mm is applied the mean value of the crack width increases with: $\frac{0.062-0.04}{0.04} * 100\% = 55\%$ in the first loading stage. When a cover of 70 mm is applied the crack width increases with: $\frac{0.078-0.062}{0.062} * 100\% = 26\%$. The increase of the crack width is caused by the increase of the maximum crack spacing ($s_{r,max}$) and the decrease of the effective reinforcement ratio ($\rho_{peff}$).

### 5.2.4 NEN 3880 (VB 1974/1984)

The equations mentioned in section 1.2.1 were used for the crack width calculations. In the following tables the results of the crack width calculations are presented.

**Results Rectangular beam 13:**

$c = 30\ mm$

*Table 5-19: Values of the crack width calculated for a cover of c=30 mm according to NEN 3880 for Beam 13*

| $F_{applied}$ | $\sigma_s$ | $\Delta l_{mean}$ | $\varepsilon_{sm}$-$\varepsilon_{cm}$ | $w_{mean}$ |
|---|---|---|---|---|
| [kN] | [N/mm²] | [mm] | [-] | [mm] |
| **109** | 136.57 | 134 | 5.58E-04 | 0.075 |
| **184** | 230.55 | 134 | 1.03E-03 | 0.138 |
| **234** | 293.20 | 134 | 1.34E-03 | 0.180 |
| **334** | 418.49 | 134 | 1.97E-03 | 0.264 |

$c = 50\ mm$

*Table 5-20: Values of the crack width calculated for a cover of c=30 mm according to NEN 3880 for Beam 13*

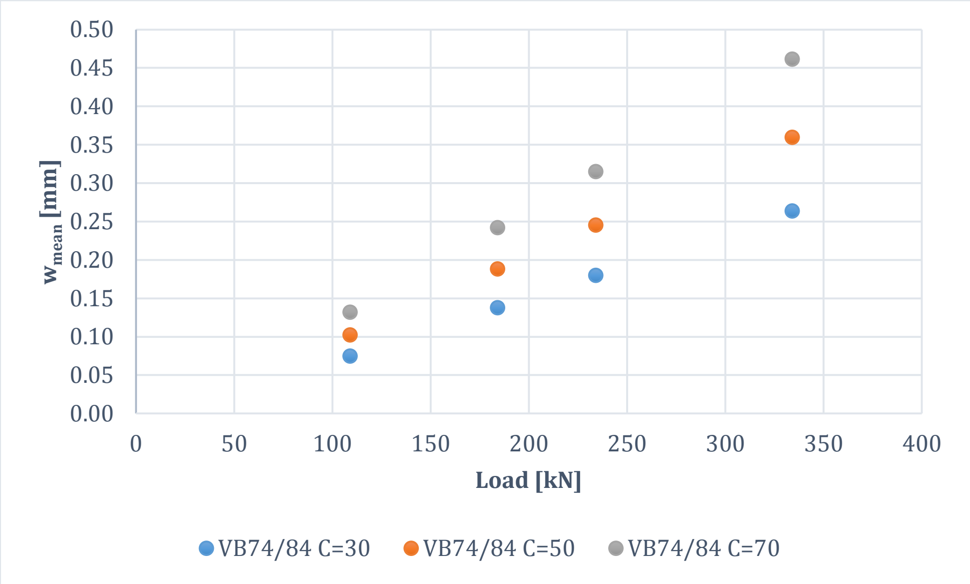
| $F_{applied}$ | $\sigma_s$ | $\Delta l_{mean}$ | $\varepsilon_{sm}$-$\varepsilon_{cm}$ | $w_{mean}$ |
|---|---|---|---|---|
| [kN] | [N/mm²] | [mm] | [-] | [mm] |
| **109** | 140.55 | 177 | 5.78E-04 | 0.103 |
| **184** | 237.26 | 177 | 1.06E-03 | 0.188 |
| **234** | 301.73 | 177 | 1.38E-03 | 0.245 |
| **334** | 430.67 | 177 | 2.03E-03 | 0.360 |

$c = 70\ mm$

*Table 5-21: Values of the crack width calculated for a cover of c=70 mm according to NEN 3880 for Beam 13*

| $F_{applied}$ | $\sigma_s$ | $\Delta l_{mean}$ | $\varepsilon_{sm}$-$\varepsilon_{cm}$ | $w_{mean}$ |
|---|---|---|---|---|
| [kN] | [N/mm²] | [mm] | [-] | [mm] |
| **109** | 144.76 | 220 | 5.99E-04 | 0.132 |
| **184** | 244.36 | 220 | 1.10E-03 | 0.242 |
| **234** | 310.76 | 220 | 1.43E-03 | 0.315 |
| **334** | 443.57 | 220 | 2.09E-03 | 0.462 |

When we look at the results we see that the mean value of the crack width increases when a larger cover is applied. This occurs due to an increase of the mean crack spacing ($\Delta \boldsymbol{l}_{\boldsymbol{mean}}$) and also due to the increasing value of the strain difference ($\varepsilon_{sm} - \varepsilon_{cm}$). The strain difference increases due to the fact that the steel stress ($\sigma_s$) increases. This can also be seen in the tables above. We see that for a cover of $c = 50\ mm$ the mean crack width increases with: $\frac{0.103-0.075}{0.075} * 100\% = 36\%$ in the first loading stage compared to a cover of $c = 30\ mm$. When a cover of $70\ mm$ is applied the mean value of the crack width increases with 28% compared to a cover of $c = 50\ mm$. These differences can be seen in the following graph.




*Graph 5-11: Influence of the concrete cover on the mean value of the crack width according to NEN 3880 for beam 13*

### Results T-beam 3:

### $c = 30\ mm$

*Table 5-22: Values of the crack width calculated for a cover of c=20 mm according to NEN 3880 for Beam 3*

| $F_{applied}$ | $\sigma_s$ | $\Delta l_{mean}$ | $\varepsilon_{sm}$-$\varepsilon_{cm}$ | $w_{mean}$ |
|---|---|---|---|---|
| [kN] | [N/mm²] | [mm] | [-] | [mm] |
| **109** | 140.98 | 100 | 5.85E-04 | 0.06 |
| **209** | 270.32 | 100 | 1.23E-03 | 0.12 |
| **259** | 335 | 100 | 1.55E-03 | 0.16 |
| **334** | 432 | 100 | 2.04E-03 | 0.20 |

### $c = 50\ mm$

*Table 5-23: Values of the crack width calculated for a cover of c=50 mm according to NEN 3880 for Beam 3*

| $F_{applied}$ | $\sigma_s$ | $\Delta l_{mean}$ | $\varepsilon_{sm}$-$\varepsilon_{cm}$ | $w_{mean}$ |
|---|---|---|---|---|
| [kN] | [N/mm²] | [mm] | [-] | [mm] |
| **109** | 147.44 | 163 | 6.17E-04 | 0.10 |
| **209** | 282.71 | 163 | 1.29E-03 | 0.21 |
| **259** | 350.34 | 163 | 1.63E-03 | 0.27 |
| **334** | 451.79 | 163 | 2.14E-03 | 0.35 |

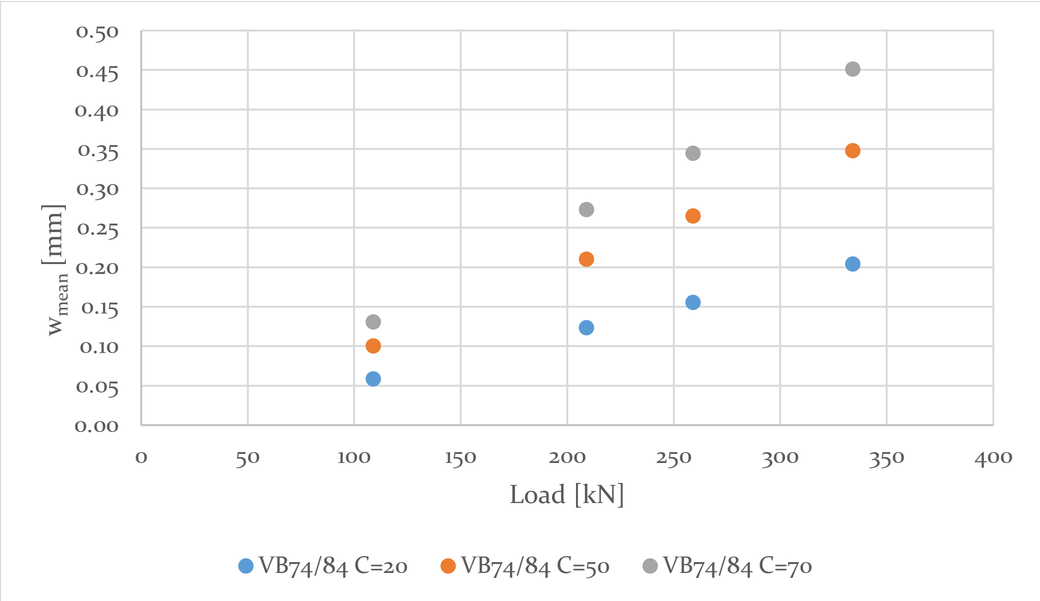
### $c = 70\ mm$

*Table 5-24: Values of the crack width calculated for a cover of c=70 mm according to NEN 3880 for Beam 3*

| $F_{applied}$ | $\sigma_s$ | $\Delta l_{mean}$ | $\varepsilon_{sm}$-$\varepsilon_{cm}$ | $w_{mean}$ |
|---|---|---|---|---|
| [kN] | [N/mm²] | [mm] | [-] | [mm] |
| **109** | 152.08 | 204 | 6.40E-04 | 0.13 |
| **209** | 291.61 | 204 | 1.34E-03 | 0.27 |
| **259** | 361.37 | 204 | 1.69E-03 | 0.34 |
| **334** | 466 | 204 | 2.21E-03 | 0.45 |

When we look at the results in the tables above we see that the mean value of the crack width increases when a larger concrete cover is applied. This could be explained by the increasing value of the mean crack spacing ($\boldsymbol{\Delta l_{mean}}$) and also due to the increasing value of the strain difference ($\varepsilon_{sm} - \varepsilon_{cm}$). The strain difference increases due to the fact that the steel stress ($\sigma_s$)

increases. We see that for a cover of $c = 50\ mm$ the mean crack width increases with: $\frac{0.1-0.06}{0.06} * 100\% = 67\%$ in the first loading stage compared to a cover of $c = 20\ mm$. With increasing load the difference increases up to 75%. When a cover of $70\ mm$ is applied the mean value of the crack width increases with 28% compared to a cover of $c = 50\ mm$. These differences can be seen in the following graph.




*Graph 5-12: Influence of the concrete cover on the mean value of the crack width calculated according to NEN 3880 for T-beam 3*

## 5.3 RESULTS: INFLUENCE OF THE LIMITATION OF THE CRACK SPACING ON THE CRACK WIDTH CALCULATIONS

In the previous chapter it was clear that the mean value of the crack width increased when a larger cover was applied. This was mainly caused by the increase of the crack spacing. In this chapter it will be investigated what the influence limitation of the crack spacing has on the crack width calculations according to the codes (NEN-EN 1992-1-1 and NEN 3880) for T-beam 3 and Rectangular beam 13.

### 5.3.1 NEN-EN 1992-1-1 (Eurocode 2)

Looking at the results of the Eurocode 2 calculations in section 5.2.3 it was clear that the value of the maximum crack spacing ($s_{r,max}$) increased when a larger concrete cover was applied. So by limiting the maximum crack spacing we expect that the values of the crack width would decrease. The VARCE (Vraag en antwoord rubriek in CEMENT :NEN-EN 1992-1-1 +C2: 2011/NB:2011) suggested an upper boundary limit for the maximum crack spacing ($s_{r,max}$ ) of:

$$s_{r,max} \leq Max\,\{(50 - 0.8 f_{ck})\emptyset; 15\emptyset\}$$

This equation was applied for the calculation of $s_{r,max}$ in order to investigate what influence this limitation had on the crack width calculations. The calculations were carried out for a concrete cover of $c = 20\,mm; c = 50\,mm\ and\ c = 70\,mm$ for T-beam 3 and in Rectangular Beam 13 the concrete cover was varied with the following values: $c = 30\,mm; c = 50\,mm$ and $c = 70\,mm$.

The same procedure was followed as the example calculation in Appendix III section 1.3. Only the calculation of $s_{r,max}$ in Excel was modified to:

$$s_{r,max} = k_3 * c + k_1 k_2 k_4 * \frac{\emptyset}{\rho_{p,eff}} \leq Max\{(50 - 0.8 * f_{ck})\emptyset\,; 15\emptyset\}$$

The example calculation for the limitation according to the VARCE can be found in Appendix III section 2.1.

#### Results Rectangular beam 13:

#### $c = 30\,mm$

*Table 5-25: Influence of the limitation of the maximum crack spacing on the crack width according to VARCE for a cover of 30 mm*

| $F_{applied}$ | $\sigma_s$ | $s_{r,max}$ | $\varepsilon_{sm}-\varepsilon_{cm}$ | $w_{mean}$ |
|---|---|---|---|---|
| [kN] | [N/mm²] | [mm] | [-] | [mm] |
| **109** | 136.57 | 282 | 4.10E-04 | 0.053 |
| **184** | 230.55 | 282 | 6.92E-04 | 0.089 |
| **234** | 293.20 | 282 | 9.99E-04 | 0.128 |
| **334** | 418.49 | 282 | 1.63E-03 | 0.209 |

### $c = 50\ mm$

*Table 5-26: Influence of the limitation of the maximum crack spacing on the crack width according to VARCE for a cover of 50 mm*

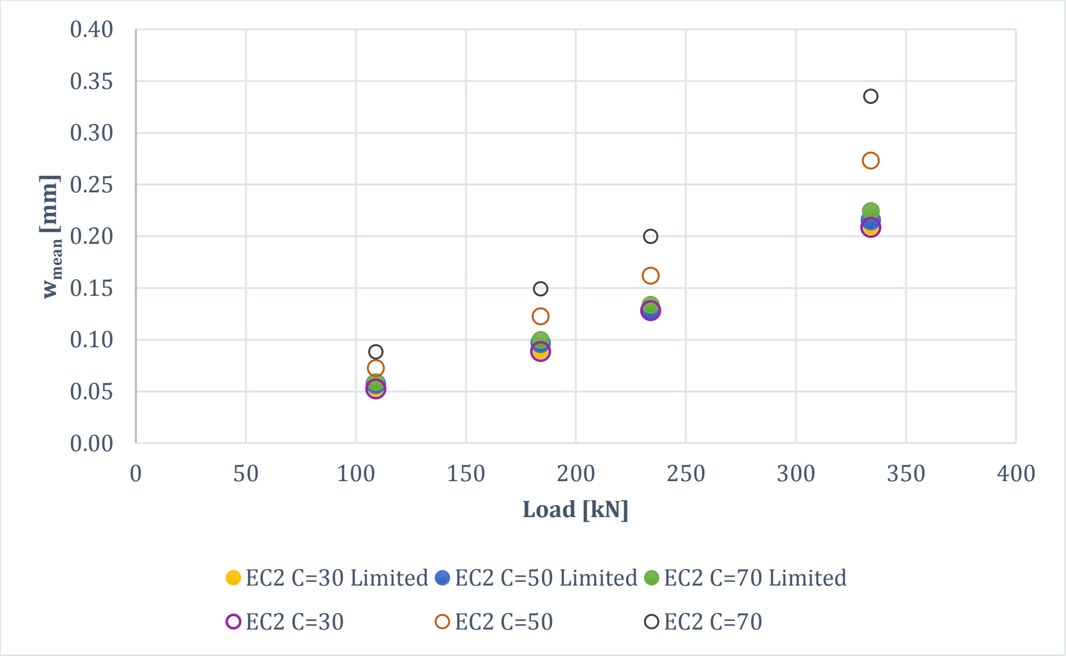
| $F_{applied}$ | $\sigma_s$ | $s_{r,max}$ | $\varepsilon_{sm}-\varepsilon_{cm}$ | $w_{mean}$ |
|---|---|---|---|---|
| [kN] | [N/mm²] | [mm] | [-] | [mm] |
| **109** | 140.55 | 300 | 4.22E-04 | 0.057 |
| **184** | 237.26 | 300 | 7.12E-04 | 0.097 |
| **234** | 301.73 | 300 | 9.39E-04 | 0.128 |
| **334** | 430.67 | 300 | 1.58E-03 | 0.216 |

### $c = 70\ mm$

*Table 5-27: Influence of the limitation of the maximum crack spacing on the crack width according to VARCE for a cover of 70 mm*

| $F_{applied}$ | $\sigma_s$ | $s_{r,max}$ | $\varepsilon_{sm}-\varepsilon_{cm}$ | $w_{mean}$ |
|---|---|---|---|---|
| [kN] | [N/mm²] | [mm] | [-] | [mm] |
| **109** | 144.76 | 300 | 4.34E-04 | 0.059 |
| **184** | 244.36 | 300 | 7.33E-04 | 0.100 |
| **234** | 310.76 | 300 | 9.82E-04 | 0.134 |
| **334** | 443.57 | 300 | 1.65E-03 | 0.224 |

In tables Table 5-25 to Table 5-26 it can be seen what influence the limitation of the maximum crack spacing has on the crack width calculation. We see that at a an applied cover of $c = 30\ mm$ the limitation has no influence on the mean value of the crack width. But when a cover of $c = 50\ mm$ is applied the mean value of the crack width decreases with: $\frac{0.073-0.057}{0.073} * 100\% = 21\%$. And when a cover of $c = 70\ mm$ is applied we see that the mean value of the crack with decreases with $\frac{0.089-0.059}{0.089} * 100\% = 33\%$. So the limitation of the maximum crack spacing according to the VARCE influences the mean value of the crack width at an increasing concrete cover. The results are presented in Graph 5-13.




*Graph 5-13: Influence of the limitation of the maximum crack spacing on the mean value of the crack width according to VARCE for Rectangular beam 13*

### Results T-beam 3:

### $c = 20\ mm$

*Table 5-28: Influence of the limitation of the maximum crack spacing on the crack width according to VARCE for a cover of 20 mm*

| $F_{applied}$ | $\sigma_s$ | $s_{r,max}$ | $\varepsilon_{sm}$-$\varepsilon_{cm}$ | $w_{mean}$ |
|---|---|---|---|---|
| [kN] | [N/mm²] | [mm] | [-] | [mm] |
| **109** | 140.98 | 207 | 4.23E-04 | 0.040 |
| **209** | 270.32 | 207 | 1.04E-03 | 0.098 |
| **259** | 334.99 | 207 | 1.37E-03 | 0.129 |
| **334** | 432.00 | 207 | 1.85E-03 | 0.175 |

### $c = 50\ mm$

*Table 5-29: Influence of the limitation of the maximum crack spacing on the crack width according to VARCE for a cover of 50 mm*

| $F_{applied}$ | $\sigma_s$ | $s_{r,max}$ | $\varepsilon_{sm}$-$\varepsilon_{cm}$ | $w_{mean}$ |
|---|---|---|---|---|
| [kN] | [N/mm²] | [mm] | [-] | [mm] |
| **109** | 147.44 | 300 | 4.42E-04 | 0.060 |
| **209** | 282.71 | 300 | 1.10E-03 | 0.150 |
| **259** | 350.34 | 300 | 1.44E-03 | 0.197 |
| **334** | 451.79 | 300 | 1.95E-03 | 0.266 |

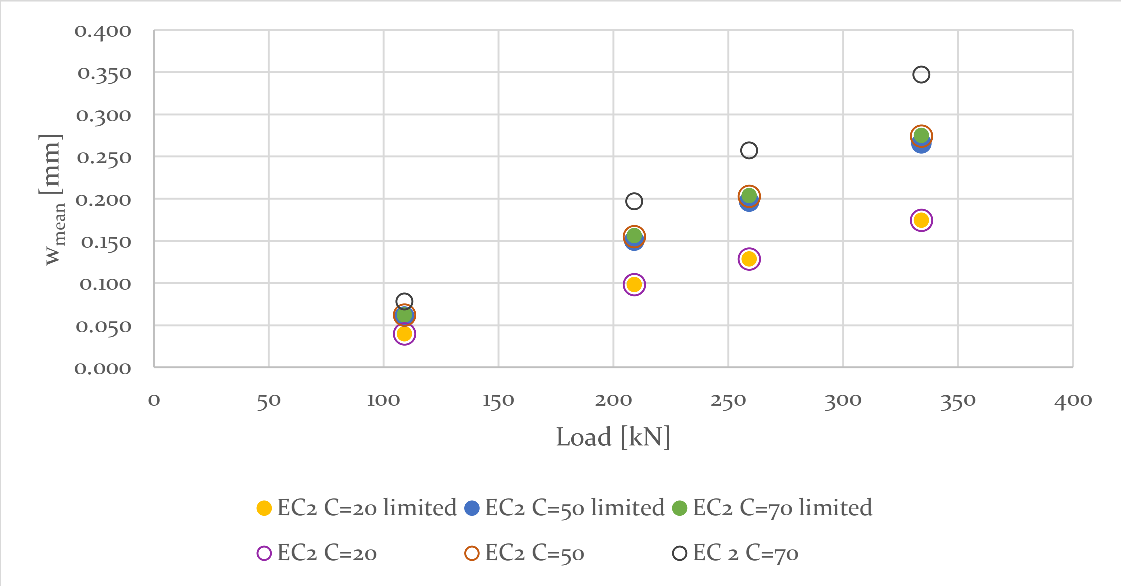
### $c = 70\ mm$

*Table 5-30: Influence of the limitation of the maximum crack spacing on the crack width according to VARCE for a cover of 70 mm*

| $F_{applied}$ | $\sigma_s$ | $s_{r,max}$ | $\varepsilon_{sm}$-$\varepsilon_{cm}$ | $w_{mean}$ |
|---|---|---|---|---|
| [kN] | [N/mm²] | [mm] | [-] | [mm] |
| **109** | 152.08 | 300 | 4.56E-04 | 0.062 |
| **209** | 291.61 | 300 | 1.15E-03 | 0.156 |
| **259** | 361.37 | 300 | 1.50E-03 | 0.204 |
| **334** | 466.01 | 300 | 2.02E-03 | 0.275 |

Looking at the results presented in tables above it can be seen that the limitation only has influence on the larger concrete covers ($c = 50\ mm$ and $c = 70\ mm$). For an applied cover of

$c = 30\ mm$ the mean value of the crack width stays the same. However when a cover of $c = 50\ mm$ is applied the mean value of the crack width decreases with: $\frac{0.062-0.06}{0.06} * 100\% = 3\%$. This difference is very small. But when we look at a cover of $c = 70\ mm$ we see that the mean value of the crack width decreases with $21\%$. So the limitation according to the VARCE influences the crack width calculations when larger covers are applied. The differences can clearly be seen in Graph 5-14.




*Graph 5-14: Influence of the limitation of the maximum crack spacing on the mean value of the crack width according to VARCE for T-beam 3*

### 5.3.2 NEN 3880 (VB 1974/1984)

In section 5.2.4 it was clear that the mean value of the crack spacing had a large influence on the value of the crack width. Also in these cases the mean value of the crack width increased with an increasing value of the cover. We saw that when the cover increased the mean value of the crack spacing also increased, thus resulting in higher values for the crack width. However article E-508.2* of NEN 3880 states that the mean crack spacing should be smaller than $10\emptyset k_m$. This upper boundary was not taken into account during the crack width calculations in section 5.2.4.

So in order to investigate whether this limitation has an influence on the crack width calculations provided by NEN 3880 this upper limit value was applied in Excel.

The calculations were carried out for a concrete cover of $c = 20\,mm; c = 50\,mm\ and\ c = 70\,mm$ for T-beam 3 and in Rectangular Beam 13 the concrete cover was varied with the following values: $c = 30\,mm; c = 50\,mm\ and\ c = 70\,mm$.

The same procedure was followed as the example calculation in Appendix 3 section 1.4. Only the calculation of $s_{r,max}$ in Excel was modified to:

$$\Delta l = \xi_2 \left(2c + \xi_3 \frac{\emptyset_{km}}{\rho_{p,eff}}\right) \leq 10\emptyset_k$$

In Appendix III section 2.2 an example calculation of the limitation according to article E-508.2* of the NEN 3880 can be found.

**Results Rectangular beam 13:**

**$c = 30\,mm$**

*Table 5-31: Values of the crack width calculated for a cover of c=30 mm according to NEN 3880 for Beam 13*

| $F_{applied}$ | $\sigma_s$ | $\Delta l_{mean}$ | $\varepsilon_{sm}-\varepsilon_{cm}$ | $w_{mean}$ |
|---|---|---|---|---|
| [kN] | [N/mm²] | [mm] | [-] | [mm] |
| **109** | 136.57 | 134 | 5.58E-04 | 0.075 |
| **184** | 230.55 | 134 | 1.03E-03 | 0.138 |
| **234** | 293.20 | 134 | 1.34E-03 | 0.180 |
| **334** | 418.49 | 134 | 1.97E-03 | 0.264 |

**$c = 50\,mm$**

*Table 5-32: Values of the crack width calculated for a cover of c=30 mm according to NEN 3880 for Beam 13*

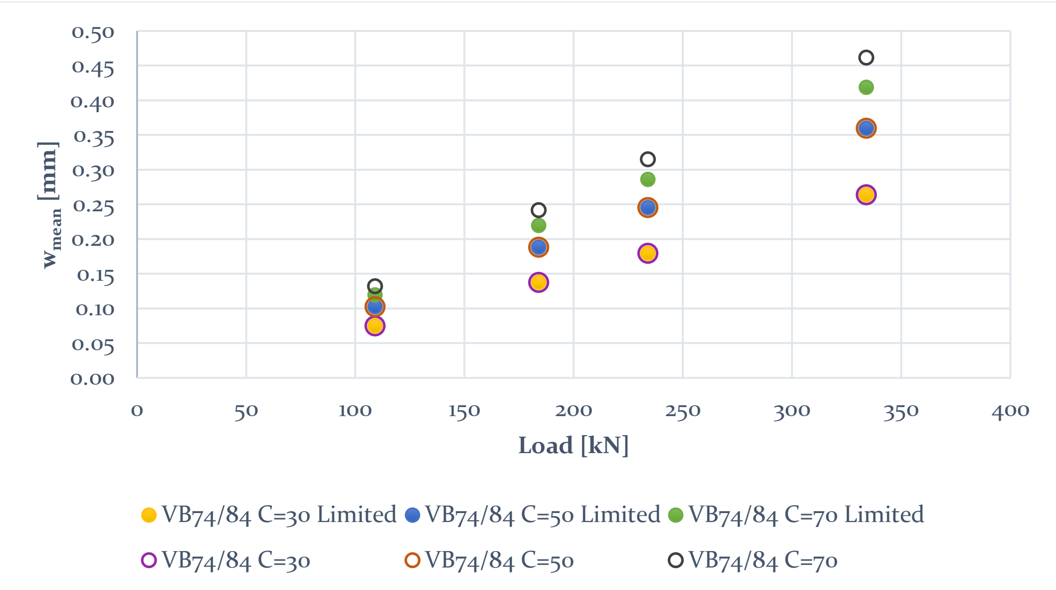
| $F_{applied}$ | $\sigma_s$ | $\Delta l_{mean}$ | $\varepsilon_{sm}-\varepsilon_{cm}$ | $w_{mean}$ |
|---|---|---|---|---|
| [kN] | [N/mm²] | [mm] | [-] | [mm] |
| **109** | 140.55 | 177 | 5.78E-04 | 0.103 |

| | | | | |
|---|---|---|---|---|
| **184** | 237.26 | 177 | 1.06E-03 | 0.188 |
| **234** | 301.73 | 177 | 1.38E-03 | 0.245 |
| **334** | 430.67 | 177 | 2.03E-03 | 0.360 |

**$c = 70\ mm$**

*Table 5-33: Values of the crack width calculated for a cover of c=70 mm according to NEN 3880 for Beam 13*

| $F_{applied}$ | $\sigma_s$ | $\Delta l_{mean}$ | $\varepsilon_{sm}$-$\varepsilon_{cm}$ | $w_{mean}$ |
|---|---|---|---|---|
| [kN] | [N/mm²] | [mm] | [-] | [mm] |
| **109** | 144.76 | 200 | 5.99E-04 | 0.120 |
| **184** | 244.36 | 200 | 1.10E-03 | 0.22 |
| **234** | 310.76 | 200 | 1.43E-03 | 0.29 |
| **334** | 443.57 | 200 | 2.09E-03 | 0.42 |




*Graph 5-15: Influence of the limitation of the mean crack spacing on the mean value of the crack width acc. NEN-3880 for beam 13*

In the tables above it can be seen that the limitation of the mean crack spacing does not have an influence on the mean value of the crack width according to the NEN 3880 calculations, since the mean value of the crack spacing stays the same for an applied cover of $c = 20\ mm$ and $c = 50\ mm$. When a cover of $c = 70\ mm$ is applied we see a slight decrease of the mean crack width (about 9%). This occurs because the mean value of the crack spacing is limited to a value of $200\ mm$. The results are also presented in Graph 5-15.

### Results T-beam 3:

### $c = 30\ mm$

*Table 5-34: Values of the crack width calculated for a cover of c=20 mm according to NEN 3880 for Beam 3*

| $F_{applied}$ | $\sigma_s$ | $\Delta l_{mean}$ | $\varepsilon_{sm}$-$\varepsilon_{cm}$ | $w_{mean}$ |
|---|---|---|---|---|
| [kN] | [N/mm²] | [mm] | [-] | [mm] |
| **109** | 141 | 100 | 5.85E-04 | 0.06 |
| **209** | 270.32 | 100 | 1.23E-03 | 0.12 |
| **259** | 335 | 100 | 1.55E-03 | 0.16 |
| **334** | 432 | 100 | 2.04E-03 | 0.20 |

### $c = 50\ mm$

*Table 5-35: Values of the crack width calculated for a cover of c=50 mm according to NEN 3880 for Beam 3*

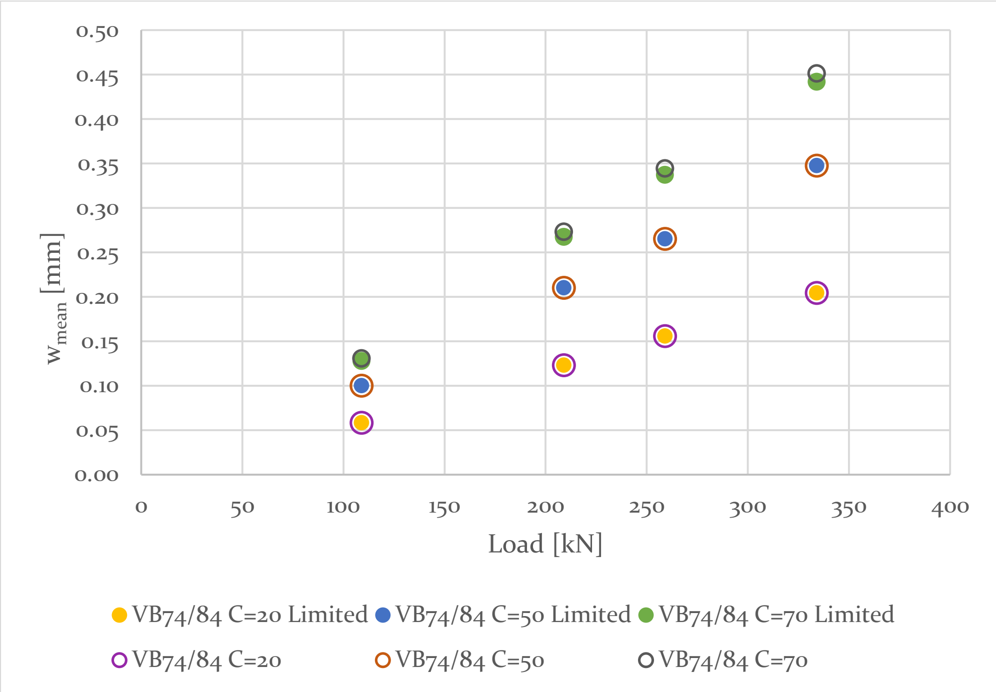
| $F_{applied}$ | $\sigma_s$ | $\Delta l_{mean}$ | $\varepsilon_{sm}$-$\varepsilon_{cm}$ | $w_{mean}$ |
|---|---|---|---|---|
| [kN] | [N/mm²] | [mm] | [-] | [mm] |
| **109** | 147.44 | 163 | 6.17E-04 | 0.10 |
| **209** | 282.71 | 163 | 1.29E-03 | 0.21 |
| **259** | 350.34 | 163 | 1.63E-03 | 0.27 |
| **334** | 451.79 | 163 | 2.14E-03 | 0.35 |

### $c = 70\ mm$

*Table 5-36: Values of the crack width calculated for a cover of c=70 mm according to NEN 3880 for Beam 3*

| $F_{applied}$ | $\sigma_s$ | $\Delta l_{mean}$ | $\varepsilon_{sm}$-$\varepsilon_{cm}$ | $w_{mean}$ |
|---|---|---|---|---|
| [kN] | [N/mm²] | [mm] | [-] | [mm] |
| **109** | 152.08 | 200 | 6.40E-04 | 0.13 |
| **209** | 291.61 | 200 | 1.34E-03 | 0.27 |
| **259** | 361.37 | 200 | 1.69E-03 | 0.34 |
| **334** | 466.01 | 200 | 2.21E-03 | 0.44 |

When we look at the results in the tables above we see that the limitation of the mean crack spacing does not have an influence of the  mean- and maximum value of the crack width in the NEN 3880 calculations, since the mean value of the crack spacing stays the same for an applied cover of $c = 20\ mm$ and $c = 50\ mm$. When a cover of $c = 70\ mm$ is applied we see a slight decrease of the maximum crack width (about 2%). This occurs because the mean value of the crack spacing is limited to a value of 200 $mm$.The results are presented in the following graph.




*Graph 5-16: Influence of the limitation of the mean crack spacing on the mean value of the crack width acc. NEN-3880 for beam 3.*

# 6 COMPARISON RESULTS OF THE FINAL ANALYSIS

In this chapter the results of numerical and analytical analyses of the influence of the variation of the concrete cover will be compared. This will be done to investigate whether the cracking behavior due to an increased concrete cover proposed by the codes (NEN-EN 1992-1-1 and NEN 3880) is in agreement with the actual cracking behavior. During the analytical analysis the crack width was also calculated by limiting the crack spacing. (section 5.3). These results will also be compared to the numerical results obtained in DIANA.

## 6.1 VARIATION OF THE CONCRETE COVER

In this section the results of the influence of the variation of the concrete cover on the cracking behavior is analyzed. The results obtained in Excel from the NEN-EN 1992-1-1 and VB 1974/1984 calculations are compared to the results obtained in DIANA. The results of both beams (Rectangular beam 13 and T-beam 3) are compared. The following aspects were looked at:

- The mean value of the crack width ($w_{mean}$)
- The mean value of the crack spacing ($\Delta l_{mean}$)
- The maximum value of the crack spacing ($s_{r,max}$)

### 6.1.1 Results: Rectangular beam 13

### $c = 30\ mm$

#### The mean value of the crack width ($w_{mean}$)

*Table 6-1: Mean value of the crack width for $c = 30\ mm$*

| Mean crack width ($w_{mean}$) $[mm]$ | | | |
|---|---|---|---|
| P | DIANA | EC2 | VB74/84 |
| 109 | 0.068 | 0.053 | 0.075 |
| 184 | 0.101 | 0.089 | 0.138 |
| 234 | 0.114 | 0.128 | 0.180 |
| 334 | 0.156 | 0.209 | 0.264 |

#### The mean value of the crack spacing ($\Delta l_{mean}$)

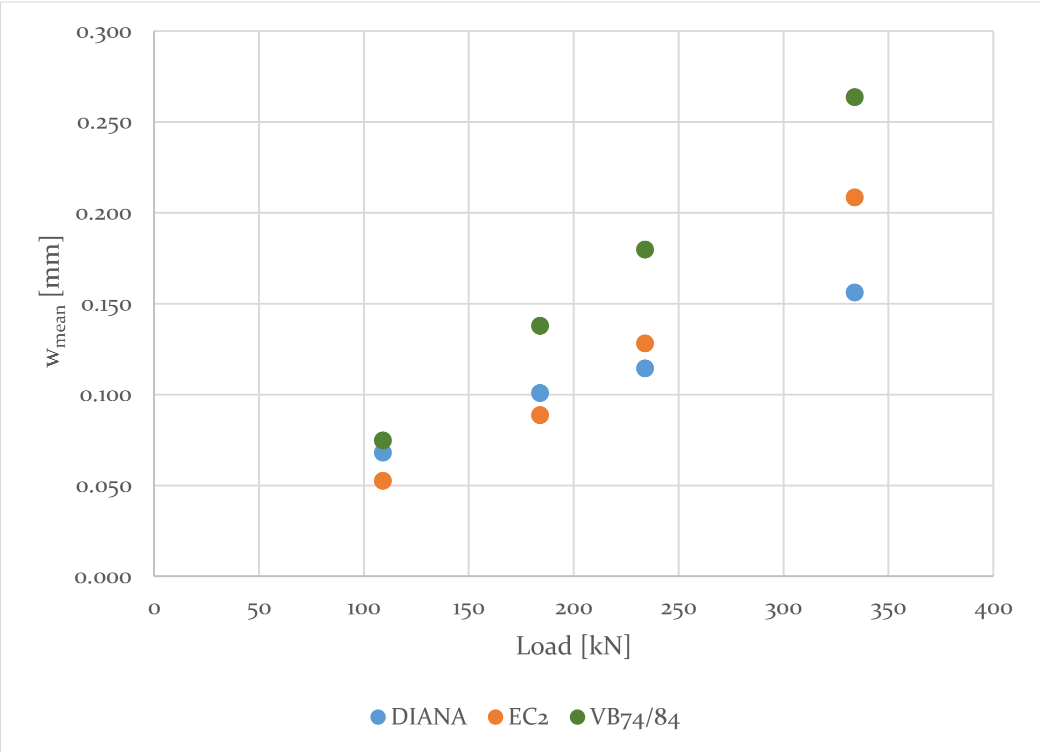
*Table 6-2: Mean crack spacing for $c = 30\ mm$*

| Mean value of the crack spacing $[mm]$ | | | |
|---|---|---|---|
| P | DIANA | VB74/84 | % |
| 109 | 164 | 134 | 18 |
| 184 | 115 | 134 | 17 |
| 234 | 92 | 134 | 46 |
| 334 | 85 | 134 | 57 |

### The maximum value of the crack spacing ($s_{r,max}$)

*Table 6-3: Maximum crack spacing for $c = 30\ mm$*

| Maximum value of the crack spacing [$mm$] | | | |
|---|---|---|---|
| P | DIANA | EC2 | % |
| 109 | 361 | 282 | 22 |
| 184 | 253 | 282 | 12 |
| 234 | 202 | 282 | 40 |
| 334 | 187 | 282 | 51 |




*Graph 6-1: Comparison mean crack width: Numerical results vs Analytical results ($c = 30\ mm$)*

In the results presented for a cover of $c = 30\ mm$ we see that the mean value of the crack width in DIANA is smaller than that of the analytical analysis. It can also be seen that the crack width calculated in VB74/84 is larger than the Eurocode 2 calculation and the numerical analysis in DIANA. In Graph 6-1 it can be seen that the mean crack width calculated in Eurocode 2 is $\frac{0.053-0.068}{0.053} * 100 = 28\%$ smaller than the crack width calculated in the VB74/84. In the other loading stages we see that the mean crack width in the Eurocode calculation increases compared to the mean crack width in the numerical analysis.

### $c = 50\ mm$

### The mean value of the crack width ($w_{mean}$)

*Table 6-4: Mean value of the crack width for $c = 50\ mm$*

| Mean crack width ($w_{mean}$) $[mm]$ | | | |
|---|---|---|---|
| P | DIANA | EC2 | VB74/84 |
| 218 | 0.011 | 0.073 | 0.103 |
| 368 | 0.083 | 0.123 | 0.188 |
| 468 | 0.131 | 0.162 | 0.245 |
| 668 | 0.174 | 0.273 | 0.360 |

### The mean value of the crack spacing ($\Delta l_{mean}$)

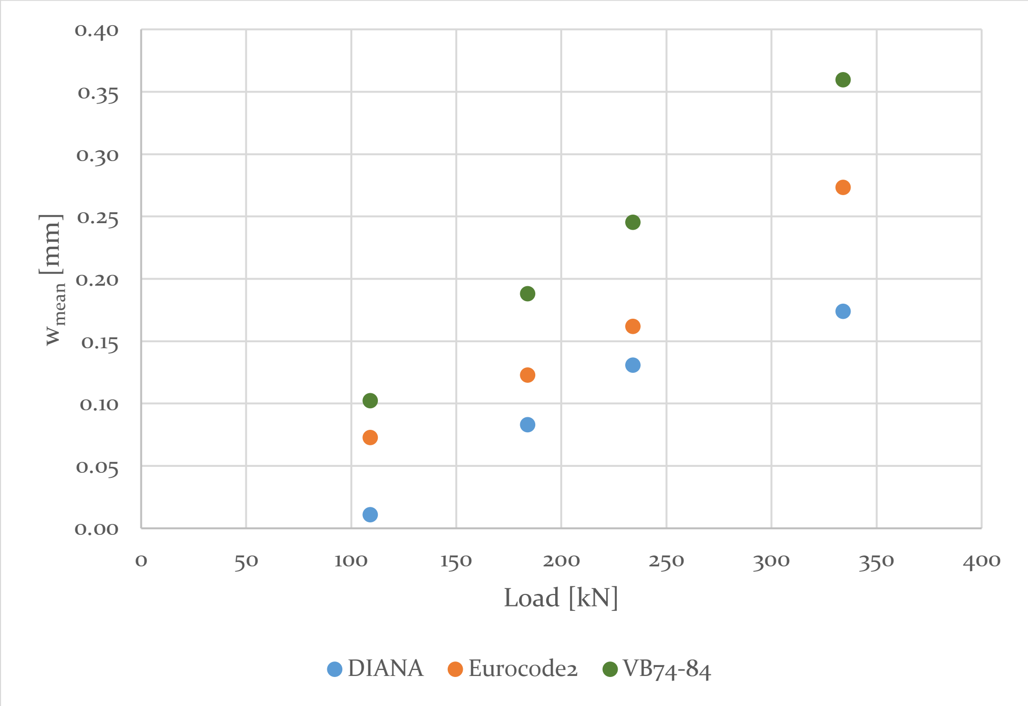
*Table 6-5: Mean crack spacing for $c = 50\ mm$*

| Mean value of the crack spacing $[mm]$ | | | |
|---|---|---|---|
| P | DIANA | VB74/84 | % |
| 218 | 2300 | 177 | 92 |
| 368 | 144 | 177 | 23 |
| 468 | 144 | 177 | 23 |
| 668 | 121 | 177 | 46 |

### The maximum value of the crack spacing ($s_{r,max}$)

*Table 6-6: Maximum crack spacing for $c = 50\ mm$*

| Maximum value of the crack spacing $[mm]$ | | | |
|---|---|---|---|
| P | DIANA | EC2 | % |
| 218 | 5060.00 | 380 | 92 |
| 368 | 316.25 | 380 | 20 |
| 468 | 316.25 | 380 | 20 |
| 668 | 266.32 | 380 | 43 |




*Graph 6-2: Comparison mean crack width: Numerical results vs Analytical results ($c = 50\ mm$)*

Looking at the results above presented for a cover of $c = 50\ mm$ we see a further increase in the difference between the mean value of the crack width calculated with the codes compared to the numerical analysis. The mean value of the crack width in the analytical analyses is 50 % higher than the numerical analysis. Overall we see that the crack width calculated in the VB74/84 is larger than the Eurocode 2 and the numerical analysis.

### $c = 70\ mm$

### The mean value of the crack width ($w_{mean}$)

*Table 6-7: Mean value of the crack width for $c = 70\ mm$*

| Mean crack width ($w_{mean}$) [$mm$] | | | |
|---|---|---|---|
| P | DIANA | EC2 | VB74/84 |
| 218 | 0.010 | 0.089 | 0.132 |
| 368 | 0.141 | 0.149 | 0.242 |
| 468 | 0.168 | 0.200 | 0.315 |
| 668 | 0.205 | 0.335 | 0.462 |

### The mean value of the crack spacing ($\Delta l_{mean}$)

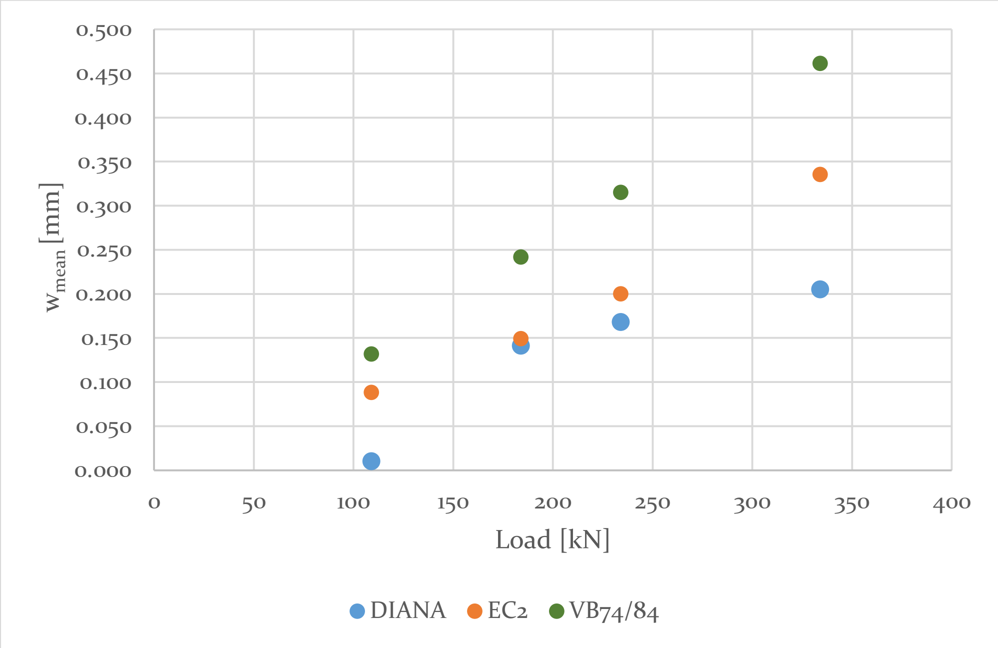
*Table 6-8: Mean crack spacing for $c = 70\ mm$*

| Mean value of the crack spacing [$mm$] | | | |
|---|---|---|---|
| P | DIANA | VB74/84 | % |
| 218 | 2300 | 220 | 90 |
| 368 | 192 | 220 | 15 |
| 468 | 153 | 220 | 44 |
| 668 | 135 | 220 | 63 |

### The maximum value of the crack spacing ($s_{r,max}$)

*Table 6-9: Maximum crack spacing for $c = 70\ mm$*

| Maximum value of the crack spacing [$mm$] | | | |
|---|---|---|---|
| P | DIANA | EC2 | % |
| 218 | 5060 | 448 | 91 |
| 368 | 422 | 448 | 6 |
| 468 | 337 | 448 | 33 |
| 668 | 298 | 448 | 51 |




*Graph 6-3: Comparison mean crack width: Numerical results vs Analytical results ($c = 70\ mm$)*

In the results presented for $c = 70\ mm$ it can be seen that the crack width calculated analytically is larger than the crack width calculated numerically. The crack spacing in the numerical analysis is smaller than that of the analytical analysis. This could explain why we see such large differences in the values of the crack width. In Graph 6-3 we see that in the second loading stage the mean value of the crack width calculated with Eurocode 2 is almost equal to that of the numerical analysis. And when we look at the maximum crack spacing we also see a very small difference between the two values (6%). So we can state that the maximum crack spacing is an important parameter when describing the cracking behavior of a beam subjected to bending.

### 6.1.2 Results: T-Beam 3
### $c = 20\ mm$

#### The mean value of the crack width ($w_{mean}$)

*Table 6-10: Mean value of the crack width for $c = 20\ mm$*

| Mean crack width ($w_{mean}$) $[mm]$ | | | |
|---|---|---|---|
| P | DIANA | EC2 | VB74/84 |
| 109 | 0.052 | 0.040 | 0.06 |
| 209 | 0.089 | 0.098 | 0.12 |
| 259 | 0.113 | 0.129 | 0.16 |
| 334 | 0.137 | 0.175 | 0.20 |

#### The mean value of the crack spacing ($\Delta l_{mean}$)

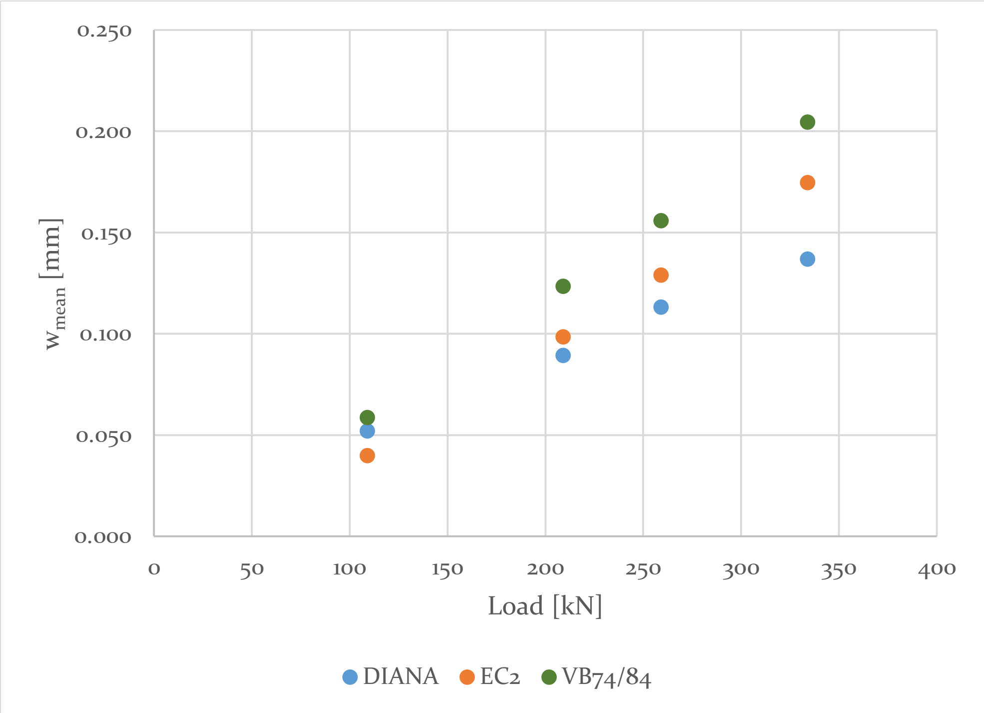
*Table 6-11: Mean crack spacing for $c = 20\ mm$*

| Mean value of the crack spacing $[mm]$ | | | |
|---|---|---|---|
| P | DIANA | VB74/84 | % |
| 109 | 85 | 100 | 18 |
| 209 | 70 | 100 | 44 |
| 259 | 68 | 100 | 48 |
| 334 | 62 | 100 | 61 |

#### The maximum value of the crack spacing ($s_{r,max}$)

*Table 6-12: Maximum crack spacing for $c = 20\ mm$*

| Maximum value of the crack spacing $[mm]$ | | | |
|---|---|---|---|
| P | DIANA | EC2 | % |
| 109 | 187 | 207 | 11 |
| 209 | 153 | 207 | 35 |
| 259 | 149 | 207 | 39 |
| 334 | 137 | 207 | 52 |




*Graph 6-4: Comparison mean crack width: Numerical results vs Analytical results ($c = 20\ mm$)*

In Table 6-10 we can see that the mean value of the crack width calculated in the Eurocode 2 is about 28% larger than the mean value of the crack width in the numerical analysis in the last loading stage ($P = 334\ kN$). In the other two loading stages ($P = 209\ kN$ and $P = 259\ kN$) is less than 15%.

### $c = 50\ mm$

*Table 6-13: Mean value of the crack width for $c = 50\ mm$*

| Mean crack width ($w_{mean}$) [$mm$] | | | |
|---|---|---|---|
| P | DIANA | EC2 | VB74/84 |
| 109 | 0.058 | 0.062 | 0.10 |
| 209 | 0.078 | 0.155 | 0.21 |
| 259 | 0.082 | 0.203 | 0.27 |
| 334 | 0.095 | 0.275 | 0.35 |

### The mean value of the crack spacing ($\Delta l_{mean}$)

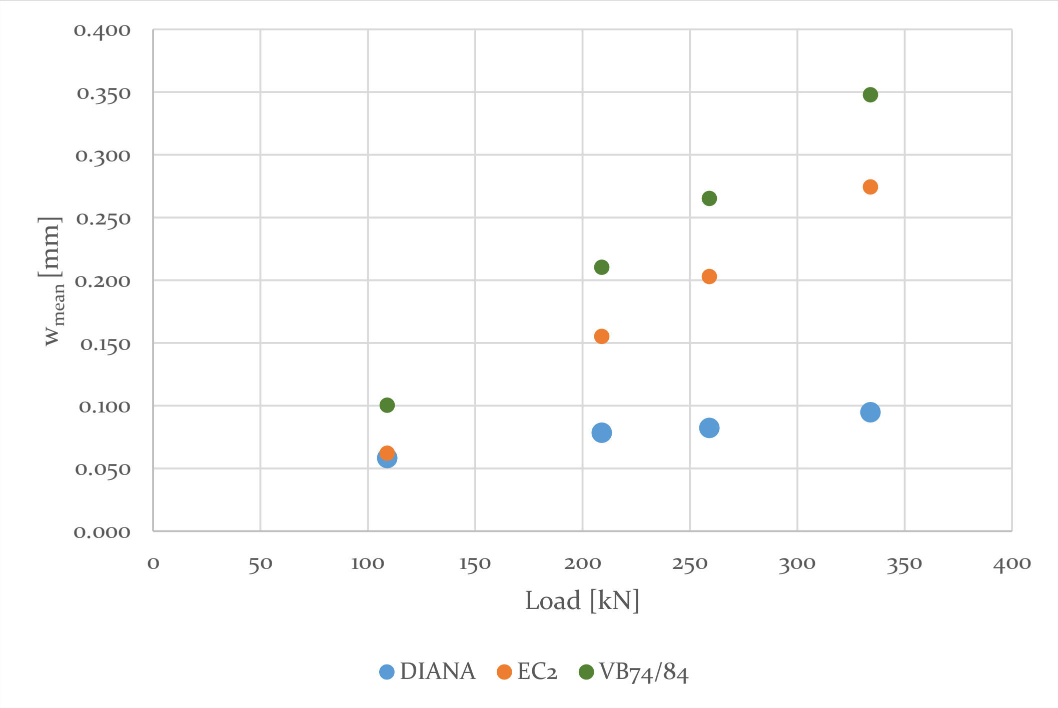
*Table 6-14: Mean crack spacing for $c = 50\ mm$*

| Mean value of the crack spacing [$mm$] | | | |
|---|---|---|---|
| P | DIANA | VB74/84 | % |
| 109 | 144 | 163 | 13 |
| 209 | 88 | 163 | 46 |
| 259 | 79 | 163 | 51 |
| 334 | 88 | 163 | 46 |

### The maximum value of the crack spacing ($s_{r,max}$)

*Table 6-15: Maximum crack spacing for $c = 50\ mm$*

| Maximum value of the crack spacing [$mm$] | | | |
|---|---|---|---|
| P | DIANA | EC2 | % |
| 109 | 316 | 310 | 2 |
| 209 | 195 | 310 | 37 |
| 259 | 174 | 310 | 44 |
| 334 | 195 | 310 | 37 |




*Graph 6-5: Comparison mean crack width: Numerical results vs Analytical results ($c = 50\ mm$)*

Looking at the results for an applied cover of $c = 50\ mm$ we see that the mean value of the crack width is larger in the analytical analysis compared to the numerical analysis. We also see that the crack spacing of the numerical analysis is smaller than the crack spacing calculated with the codes (maximum difference of 51%). Since the crack width is calculated with: $w_k = s_{rmax} * (\varepsilon_{sm} - \varepsilon_{cm})$, we can say that the crack spacing has a large impact on the crack width calculations.

### $c = 70\ mm$

*Table 6-16: Mean value of the crack width for $c = 70\ mm$*

| Mean crack width ($w_{mean}$) [$mm$] | | | |
|---|---|---|---|
| **P** | **DIANA** | **EC2** | **VB74/84** |
| **109** | 0.059 | 0.078 | 0.13 |
| **209** | 0.097 | 0.197 | 0.27 |
| **259** | 0.109 | 0.257 | 0.34 |
| **334** | 0.114 | 0.347 | 0.45 |

### The mean value of the crack spacing ($\Delta l_{mean}$)

*Table 6-17: Mean crack spacing for $c = 70\ mm$*

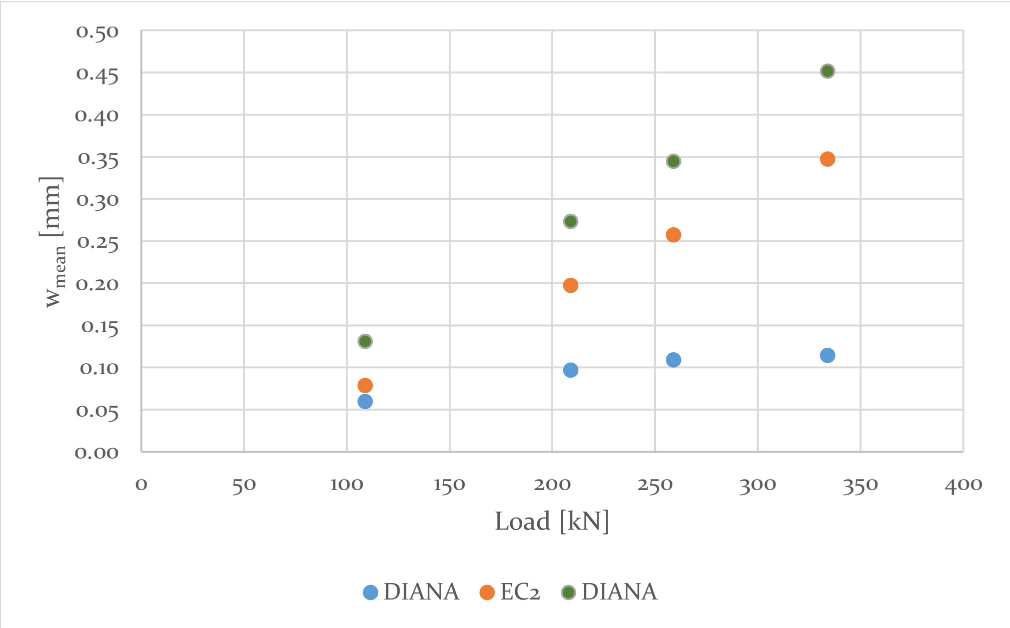
| Mean value of the crack spacing [$mm$] | | | |
|---|---|---|---|
| **P** | **DIANA** | **VB74/84** | **%** |
| **109** | 115 | 204 | 44 |
| **209** | 88 | 204 | 57 |

| | | | |
|---|---|---|---|
| **259** | 88 | 204 | 57 |
| **334** | 96 | 204 | 53 |

### The maximum value of the crack spacing ($s_{r,max}$)

*Table 6-18: Maximum crack spacing for $c = 70\ mm$*

| Maximum value of the crack spacing [$mm$] | | | |
|---|---|---|---|
| P | DIANA | EC2 | % |
| **109** | 253 | 378 | 33 |
| **209** | 195 | 378 | 49 |
| **259** | 195 | 378 | 49 |
| **334** | 211 | 378 | 44 |




*Graph 6-6: Comparison mean crack width: Numerical results vs Analytical results ($c = 70\ mm$)*

The results for a cover of $c = 70\ mm$ show the same behaviour we saw for a cover of $c = 50\ mm$: the mean crack width calculated analytically is larger than the crack width found in the numerical analysis. Also in this case we see that the crack spacing in the analysis is larger than that of the numerical analysis.

## 6.2 Influence of the limitation of the crack spacing on the crack width calculations

When we compared the results of the analytical and the numerical analysis we saw that the crack spacing is an important parameter when describing the cracking behavior in a beam. So in this chapter we will investigate what influence the limitation of the crack spacing has on the actual cracking behavior compared to the codes: NEN-EN 1992-1-1 and NEN 3880. The results obtained in Excel from the NEN-EN 1992-1-1 and VB 1974/1984 calculations will be compared to the results obtained in DIANA. The results of both beams: Rectangular beam 13 and T-beam 3 will be presented. The following aspects will be compared:

- The mean value of the crack width ($w_{mean}$)
- The mean value of the crack spacing ($\Delta l_{mean}$)
- The maximum value of the crack spacing ($s_{r,max}$)

### 6.2.1 Results: Rectangular beam 13

$c = 30\ mm$

#### The mean value of the crack width ($w_{mean}$)

*Table 6-19: Mean value of the crack width for $c = 30\ mm$*

| Mean crack width ($w_{mean}$) [$mm$] | | | |
|---|---|---|---|
| P | DIANA | EC2 | VB74/84 |
| 109 | 0.068 | 0.053 | 0.075 |
| 184 | 0.101 | 0.089 | 0.14 |
| 234 | 0.114 | 0.128 | 0.18 |
| 334 | 0.156 | 0.209 | 0.264 |

#### The mean value of the crack spacing ($\Delta l_{mean}$)

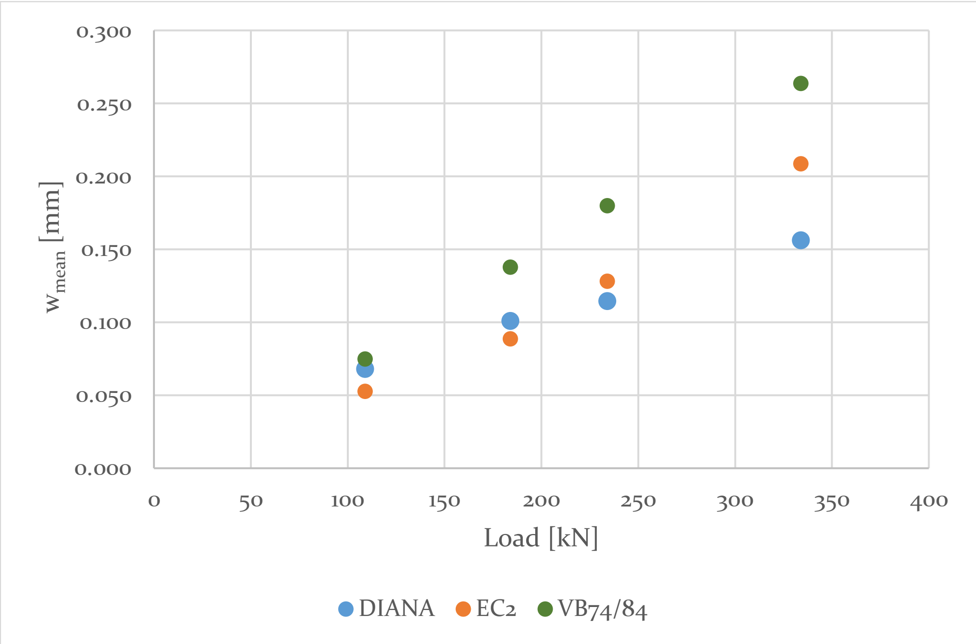
*Table 6-20: Mean crack spacing for $c = 30\ mm$*

| Mean value of the crack spacing [$mm$] | | | |
|---|---|---|---|
| P | DIANA | VB74/84 | % |
| 109 | 164 | 134 | 18 |
| 184 | 115 | 134 | 17 |
| 234 | 92 | 134 | 46 |
| 334 | 85 | 134 | 57 |

### The maximum value of the crack spacing ($s_{r,max}$)

*Table 6-21: Maximum crack spacing for $c = 30\ mm$*

| Maximum value of the crack spacing [$mm$] | | | |
|---|---|---|---|
| P | DIANA | EC2 | % |
| **109** | 361 | 282 | 22 |
| **184** | 253 | 282 | 11 |
| **234** | 202 | 282 | 39 |
| **334** | 187 | 282 | 50 |




*Graph 6-7: Comparison mean crack width: Numerical results vs Analytical results ($c = 30\ mm$)*

When we look at the influence of the limitation of the crack spacing for a cover of $c = 30\ mm$ we see that the crack spacing stays the same in the analytical calculation compared to the calculation of the crack spacing without limitation. And so the crack width does not change in the analytical calculation. We see the same graph as in section 6.1.1.

### $c = 50\ mm$

### The mean value of the crack width ($w_{mean}$)

*Table 6-22: Mean value of the crack width for $c = 50\ mm$*

| Mean crack width ($w_{mean}$) $[mm]$ | | | |
|---|---|---|---|
| P | DIANA | EC2 | VB74/84 |
| 109 | 0.011 | 0.057 | 0.103 |
| 184 | 0.083 | 0.097 | 0.19 |
| 234 | 0.131 | 0.128 | 0.25 |
| 334 | 0.174 | 0.216 | 0.36 |

### The mean value of the crack spacing ($\Delta l_{mean}$)

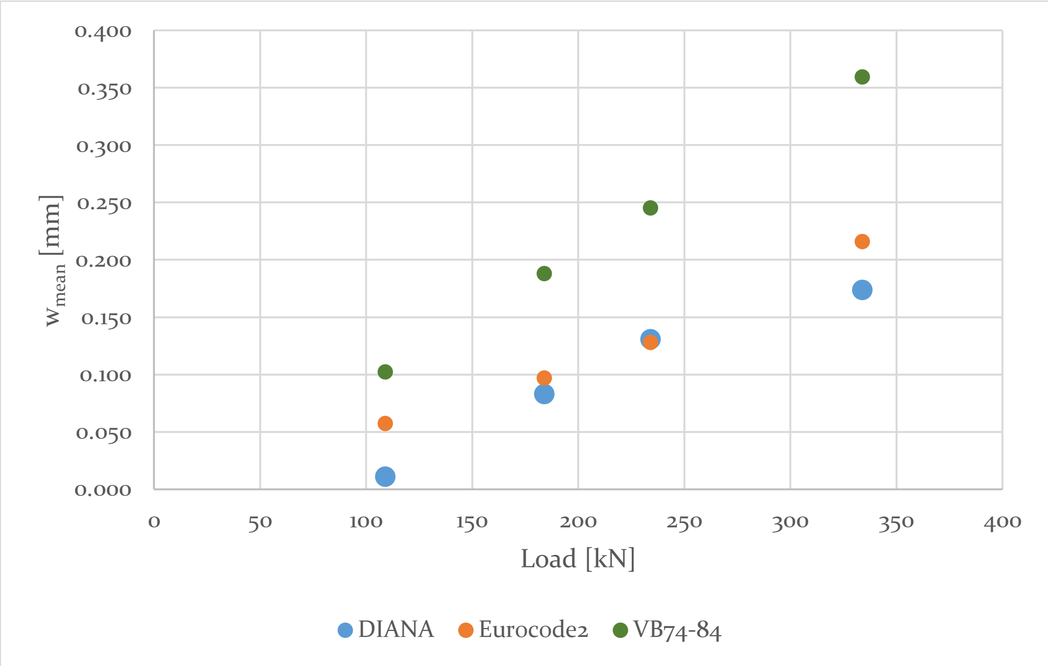
*Table 6-23: Mean crack spacing for $c = 50\ mm$*

| Mean value of the crack spacing $[mm]$ | | | |
|---|---|---|---|
| P | DIANA | VB74/84 | % |
| 109 | 2300 | 177 | 92 |
| 184 | 144 | 177 | 23 |
| 234 | 144 | 177 | 23 |
| 334 | 121 | 177 | 46 |

### The maximum value of the crack spacing ($s_{r,max}$)

*Table 6-24: Maximum crack spacing for $c = 50\ mm$*

| Maximum value of the crack spacing $[mm]$ | | | |
|---|---|---|---|
| P | DIANA | EC2 | % |
| 109 | 5060 | 300 | 94 |
| 184 | 316 | 300 | 5 |
| 234 | 316 | 300 | 5 |
| 334 | 266 | 300 | 13 |




*Graph 6-8: Comparison mean crack width: Numerical results vs Analytical results ($c = 50\ mm$)*

When we look at the results for an applied cover of $c = 50\ mm$ we see that the limitation according to the VARCE in the Eurocode 2 does have an influence on the mean value of the crack width. We see that the mean crack width calculated in Eurocode 2 has decreased almost to the level of the values of the numerical analysis. The difference between the two is less than 20% in the stabilized cracking stage. Looking at the results after the limitation according to the VB74/84 it can be concluded that this limitation has no influence on the crack width calculation since the mean value of the crack spacing has not changed compared to the case without limitation. This could be explained by the fact that the increased concrete cover had no large impact on the crack spacing as we saw in the NEN-EN 1992-1-1 calculations.

**$c = 70\ mm$**

**The mean value of the crack width ($w_{mean}$)**

*Table 6-25: Mean value of the crack width for $c = 70\ mm$*

| Mean crack width ($w_{mean}$) [mm] | | | |
|---|---|---|---|
| P | DIANA | EC2 | VB74/84 |
| 109 | 0.010 | 0.059 | 0.120 |
| 184 | 0.141 | 0.100 | 0.22 |
| 234 | 0.168 | 0.134 | 0.29 |
| 334 | 0.205 | 0.224 | 0.42 |

**The mean value of the crack spacing ($\Delta l_{mean}$)**

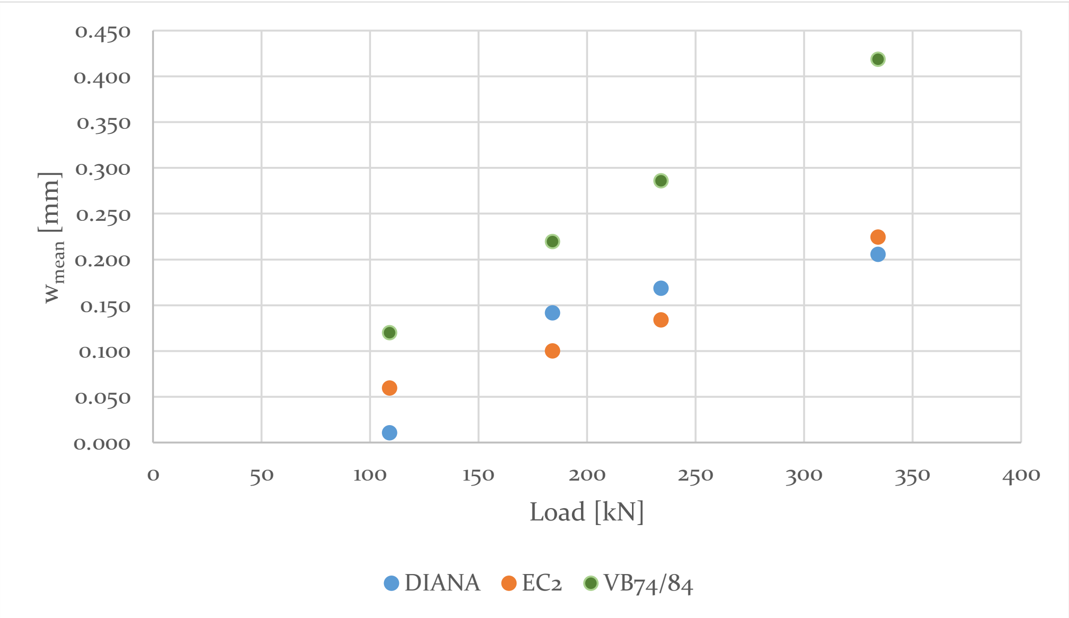
*Table 6-26: Mean crack spacing for $c = 70\ mm$*

| Mean value of the crack spacing $[mm]$ | | | |
|---|---|---|---|
| **P** | **DIANA** | **VB74/84** | **%** |
| **109** | 2300 | 200 | 91 |
| **184** | 192 | 200 | 4 |
| **234** | 153 | 200 | 30 |
| **334** | 135 | 200 | 48 |

### The maximum value of the crack spacing ($s_{r,max}$)

*Table 6-27: Maximum crack spacing for $c = 70\ mm$*

| Maximum value of the crack spacing $[mm]$ | | | |
|---|---|---|---|
| **P** | **DIANA** | **EC2** | **%** |
| **109** | 5060 | 300 | 94 |
| **184** | 422 | 300 | 29 |
| **234** | 337 | 300 | 12 |
| **334** | 298 | 300 | 1 |




*Graph 6-9: Comparison mean crack width: Numerical results vs Analytical results ($c = 70\ mm$)*

Looking at the results for an applied cover of $c = 70\ mm$ we see that the mean crack width calculated with the Eurocode 2 decreases much more due to the limitation proposed by the VARCE. The mean value of the crack width in Eurocode 2 is smaller than that of the numerical analysis. The limitation of the mean crack spacing provided by the VB 1974/1984 has no influence on the comparison between the mean- and maximum value of the crack spacing with the numerical analysis.

### 6.2.2 Results: T-Beam 3

### $c = 20$

#### The mean value of the crack width ($w_{mean}$)

*Table 6-28: Mean value of the crack width for $c = 20\ mm$*

| Mean crack width ($w_{mean}$) [$mm$] | | | |
|---|---|---|---|
| P | DIANA | EC2 | VB74/84 |
| 109 | 0.052 | 0.040 | 0.06 |
| 209 | 0.089 | 0.098 | 0.12 |
| 259 | 0.113 | 0.129 | 0.16 |
| 334 | 0.137 | 0.175 | 0.20 |

#### The mean value of the crack spacing ($\Delta l_{mean}$)

*Table 6-29: Mean crack spacing for $c = 20\ mm$*

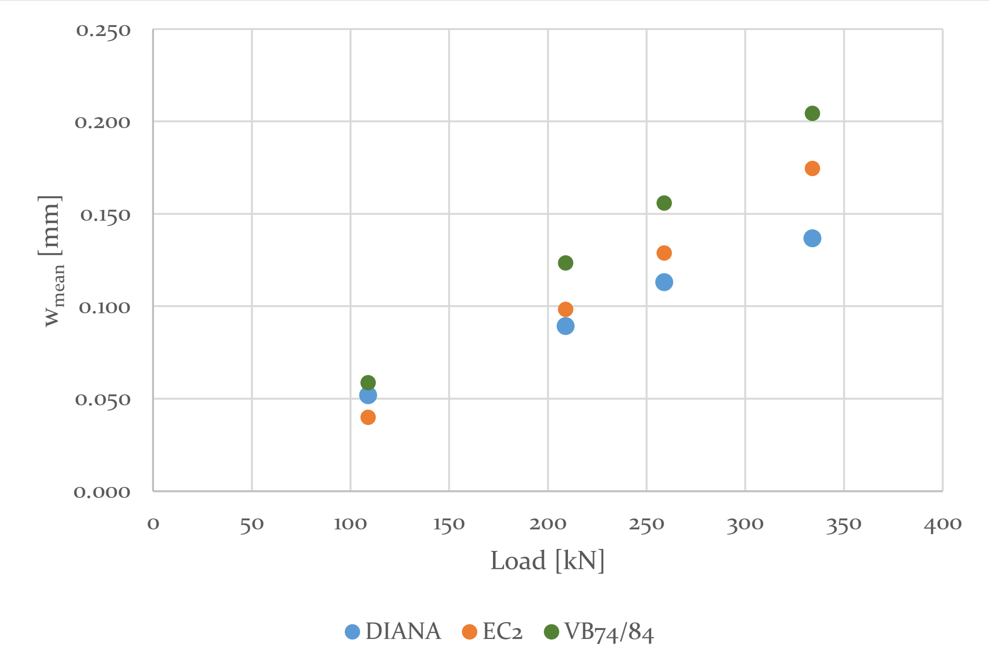
| Mean value of the crack spacing [$mm$] | | | |
|---|---|---|---|
| P | DIANA | VB74/84 | % |
| 109 | 85 | 100 | 18 |
| 209 | 70 | 100 | 44 |
| 259 | 68 | 100 | 48 |
| 334 | 62 | 100 | 61 |

We see that for an applied cover of $c = 20\ mm$ the limitation of the crack spacing had no influence on the cracking behavior of the T-beam. We see that the mean crack width stays the same as presented in section 6.1.2.

#### The maximum value of the crack spacing ($s_{r,max}$)

*Table 6-30: Maximum crack spacing for $c = 20\ mm$*

| Maximum value of the crack spacing [$mm$] | | | |
|---|---|---|---|
| P | DIANA | EC2 | % |
| 109 | 187 | 207 | 60 |
| 209 | 153 | 207 | 96 |
| 259 | 149 | 207 | 102 |
| 334 | 137 | 207 | 119 |




*Graph 6-10: Comparison mean crack width: Numerical results vs Analytical results ($c = 20$ mm)*

### $c = 50\ mm$

*Table 6-31: Mean value of the crack width for $c = 50$ mm*

| Mean crack width ($w_{mean}$) $[mm]$ | | | |
|---|---|---|---|
| **P** | **DIANA** | **EC2** | **VB74/84** |
| **109** | 0.058 | 0.060 | 0.10 |
| **209** | 0.078 | 0.150 | 0.21 |
| **259** | 0.082 | 0.197 | 0.27 |
| **334** | 0.095 | 0.266 | 0.35 |

### The mean value of the crack spacing ($\Delta l_{mean}$)

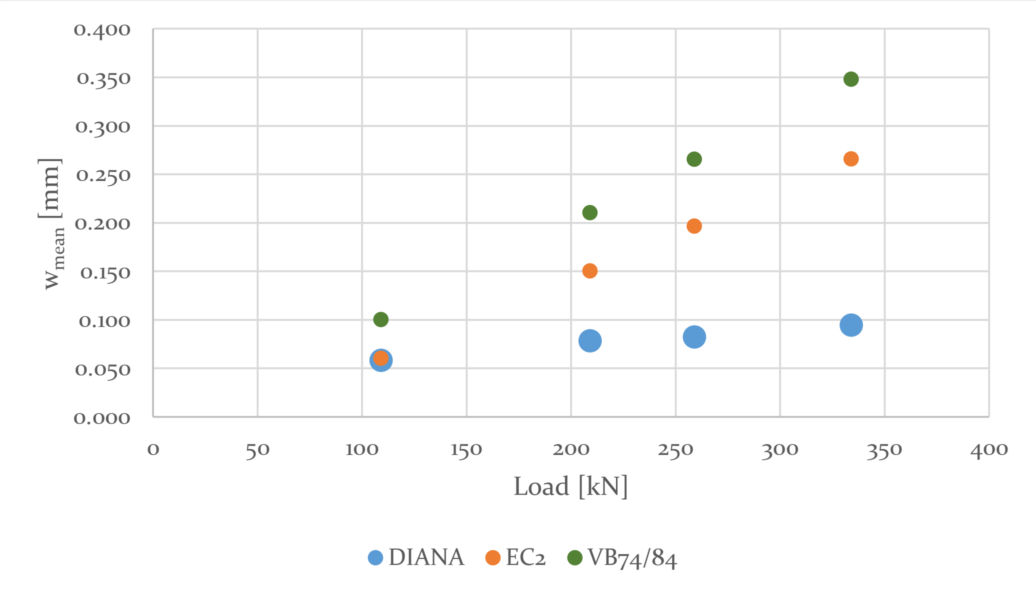
*Table 6-32: Mean crack spacing for $c = 50$ mm*

| Mean value of the crack spacing $[mm]$ | | | |
|---|---|---|---|
| **P** | **DIANA** | **VB74/84** | **%** |
| **109** | 144 | 200 | 39 |
| **209** | 88 | 200 | 56 |
| **259** | 79 | 200 | 60 |
| **334** | 88 | 200 | 56 |

### The maximum value of the crack spacing ($s_{r,max}$)

*Table 6-33: Maximum crack spacing for $c = 50\ mm$*

| Maximum value of the crack spacing $[mm]$ | | | |
|---|---|---|---|
| **P** | **DIANA** | **EC2** | **%** |
| **109** | 316 | 300 | 5 |
| **209** | 195 | 300 | 35 |
| **259** | 174 | 300 | 42 |
| **334** | 195 | 300 | 35 |




*Graph 6-11: Comparison mean crack width: Numerical results vs Analytical results ($c = 50\ mm$)*

When a cover of $c = 50\ mm$ is applied we see a small decrease in the mean value of the crack width calculated in the Eurocode 2. But this decrease doesn't cause the crack width to reach the level of the numerical analysis as can be seen in the graph presented above. Also in this case the VB74/84 limitation had no influence on the crack with calculation.

### $c = 70\ mm$

*Table 6-34: Mean value of the crack width for $c = 70\ mm$*

| Mean crack width ($w_{mean}$) $[mm]$ | | | |
|---|---|---|---|
| **P** | **DIANA** | **EC2** | **VB74/84** |
| **109** | 0.059 | 0.062 | 0.13 |
| **209** | 0.097 | 0.156 | 0.27 |
| **259** | 0.109 | 0.204 | 0.34 |
| **334** | 0.114 | 0.275 | 0.44 |

### The mean value of the crack spacing ($\Delta l_{mean}$)

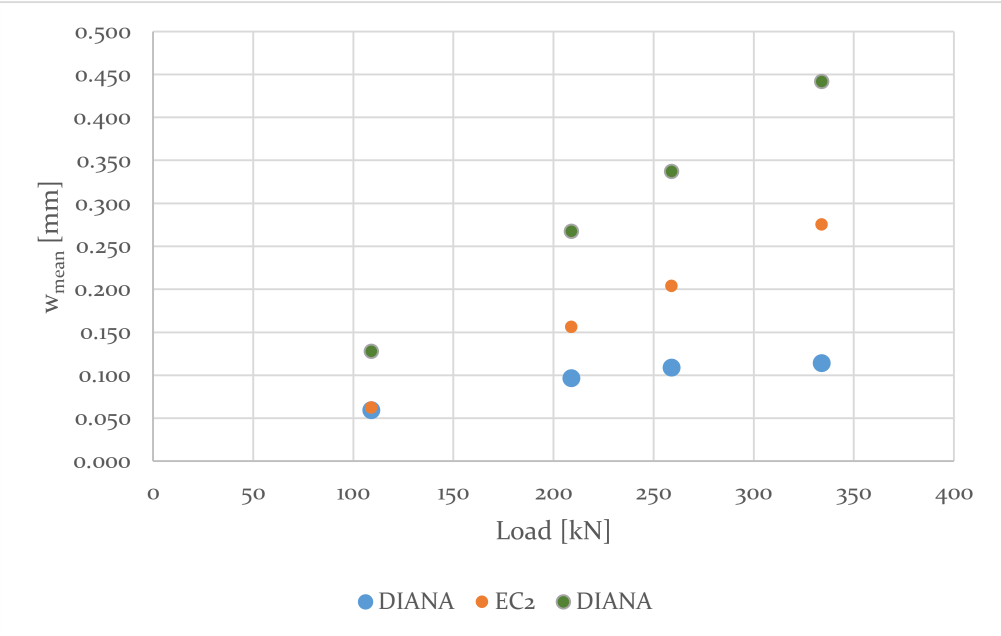
*Table 6-35: Mean crack spacing for $c = 70\ mm$*

| Mean value of the crack spacing $[mm]$ | | | |
|---|---|---|---|
| P | DIANA | VB74/84 | % |
| 109 | 115 | 200 | 43 |
| 209 | 88 | 200 | 56 |
| 259 | 88 | 200 | 56 |
| 334 | 96 | 200 | 52 |

### The maximum value of the crack spacing ($s_{r,max}$)

*Table 6-36: Maximum crack spacing for $c = 70\ mm$*

| Maximum value of the crack spacing $[mm]$ | | | |
|---|---|---|---|
| P | DIANA | EC2 | % |
| 109 | 253 | 300 | 16 |
| 209 | 195 | 300 | 35 |
| 259 | 195 | 300 | 35 |
| 334 | 211 | 300 | 30 |




*Graph 6-12: Comparison mean crack width: Numerical results vs Analytical results ($c = 70\ mm$)*

In the results presented above we see that the limitation of the maximum crack spacing provided by the VARCE does have an influence on the crack width calculations according to the Eurocode 2. And so we see the value of the mean crack width decrease due to the limitation of the maximum crack spacing. However this reduction is not that high to reach the level of the numerical analysis as presented in the graph above. The limitation of the mean crack spacing provided by the VB 74/84 regulations has almost no influence on the comparison of the cracking behavior between the numerical analysis. Overall we see that when the concrete cover increases the crack width calculated with the codes is larger compared to the crack width calculated numerically in DIANA for T-beam 3.

# 7 CONCLUSIONS AND RECOMMENDATIONS

This chapter contains the overall conclusions to the analytical- and numerical analyses preformed during my thesis.

In Section 3.1 and 3.2 we saw that with increasing cover the maximum crack spacing increased, which in turn had an effect on the maximum allowable steel stress. The main differences occurred for smaller bar distances ($s < 150\ mm$). By limiting the maximum crack spacing in the Eurocode 2 using the NEN 3880 equation the value of the maximum allowable steel stress increased way above the expected value of the maximum steel stress in the S.L.S. ($\sigma_s = 300\ N/mm^2$). Thus the limitation provided by the NEN 3880 cannot be used in the Eurocode 2 equation for the maximum crack spacing.

The limitation of the maximum crack spacing as proposed in the VARCE lead to a more realistic increase in the value in the maximum steel stress. However these values were still smaller than the maximum allowable steel stress calculated in the NEN 6720. It should be considered to refine the limitation of the maximum crack spacing provided by the VARCE, where the main interest should lie on bar distances smaller than $150\ mm$.

In Section 3.3 the cracking behavior of a beam subjected to bending was analyzed by using the NEN-EN-1992-1-1 in Excel. We saw that the cracking behavior according to Eurocode 2 is in agreement with the cracking behavior of the tested beams subjected to bending.

In Chapter 4 the cracking behavior was analyzed numerically in the finite element program DIANA. From these results it can be concluded that the actual cracking behavior of a beam subjected to bending can be simulated with DIANA in an accurate manner.

The numerical analysis conducted in section 5.1 provided insight in the influence of concrete cover on the actual cracking behavior in a bending beam. After this analysis it can be concluded that the crack width increased when a larger concrete cover was applied.

Section 5.2 and 5.3 provided the analytical analysis of the concrete cover and limitation of the maximum crack spacing on the cracking behavior on a beam subjected to bending. The cracking behavior was analyzed analytically with the crack width calculations provided by the different codes: the NEN-EN 1992-1-1 (Eurocode 2) and the NEN 3880 (VB 1974/1984). We saw again that the crack width increased when a larger cover was applied. Additionally the crack width was calculated by limiting the crack spacing. From the obtained results it is clear that the limitation provided by the NEN 3880 had a very small influence on the cracking behavior when a cover of 70 mm was applied. The crack width decreased within the range of 2% to 9% . So it can be concluded that the influence of the limitation provided by the NEN 3880 can be neglected.

We also saw that the VARCE proposal for limiting the maximum crack spacing only affected the cracking behavior when larger covers were applied ($c = 50\ mm$ and $c = 70\ mm$). The crack width decreased with 33% relative to the calculated crack width without limitation of the crack spacing (applied cover of $c = 70\ mm$). Thus it can be concluded that the VARCE proposal for limiting the crack spacing results in a good prediction of the crack width for a single layer of reinforcement with concrete covers in a range of $50\ mm$ to $70\ mm$.

In Chapter 6 the results from the numerical analysis of section 5.1 were compared to the results of the analytical analysis in section 5.2. This was done to investigate whether the cracking behavior due to an increased concrete cover proposed by the codes (NEN-EN 1992-1-1 and NEN 3880) is in agreement with the actual cracking behavior. We saw that the crack width calculated in the analytical analysis was larger than that of the numerical analysis. Specifically in the cases where a larger concrete cover ($c = 50\ mm$ and $c = 70\ mm$) was applied.

The limitation of the mean crack spacing provided by the NEN 3880 had no influence on the cracking behavior compared to DIANA since the concrete cover had no impact on the crack spacing in the NEN 3880 calculations. The results showed that the crack width calculated with the NEN 3880 was much larger than the Eurocode 2 and the DIANA calculations. From these results it was clear that at an increasing cover the mean crack spacing ($\Delta l$) had no effect on the cracking behavior, and so the only parameter that would have an effect on the cracking behavior would be the strain difference ($\varepsilon_{sm} - \varepsilon_{cm}$). By comparing the strain difference calculated in the NEN 3880 with that of the Eurocode 2 we could see that the strain difference in the VB 74/84 was about 50% larger than the strain difference calculated in the Eurocode 2. And since the strain difference mainly depends on the calculated value of the steel stress ($\sigma_s$), it can be concluded that the steel stress is the main parameter which causes the large values of the calculated crack width in the NEN 3880 calculations.

The VARCE proposal for limiting the crack spacing did have an effect on the mean value of the crack width for an applied cover of $c = 50\ mm$ and $c = 70\ mm$. We saw that the mean value of the crack width in the Eurocode 2 calculations came closer to the mean value of the crack with in the numerical analysis of rectangular beam 13. So by applying the VARCE-limitation in the rectangular beam we can conclude that the cracking behavior due to an increased cover proposed by the Eurocode 2 is in agreement with the actual cracking behavior of a beam consisting of a single layer of reinforcement. This is not the case for T-beam 3, since the calculated crack width after the VARCE- limitation remained higher (up to 180% in the last loading stage for c=50 mm) compared to the crack width in the numerical analysis. In T-beam 3 there were two layers of reinforcement applied, this was also simulated in DIANA. But in the analytical analysis the applied cover at the level of the main reinforcement was taken into account for the calculation of the crack spacing, which could have an effect on the effective height of the beam. This may be one of the reasons why there was such a large difference between the cracking behavior of the analytical analysis and the numerical analysis in the T-beam. However further research on this aspect is required.

Overall it can be concluded that at an increasing concrete cover the crack width in the Eurocode 2 calculations is overestimated, due to the increase of the maximum crack spacing. From the results it is clear that the maximum crack spacing should be reduced when a concrete cover larger than 30 mm is applied. This reduction can be obtained by using the VARCE proposal for limiting the maximum crack spacing. But the VARCE-limitation however needs to be refined for bar distances smaller than 150 $mm$. Further research is necessary on the refinement of the VARCE-limitation where the focus lies upon the factor with which the bar diameter is multiplied (the second term in the equation: $15\emptyset$). Also a follow up study is necessary where the cases in DIANA are modelled with different strength properties along the beam. Since concrete is an inhomogeneous material, this study can provide a more accurate simulation of the actual cracking behavior of a beam.

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# Appendix I: The influence of concrete cover and the crack spacing on the maximum allowable steel stress

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# 1 ANALYTICAL ANALYSIS

In order to investigate the influence of concrete cover and maximum crack spacing on the maximum allowable steel stress an analytical analysis was carried out. The calculations which were done are based on an existing EXCEL-sheet for the calculation of the steel stress and maximum allowable bending moment in concrete slabs subjected to pure bending. Expressions from different codes were used namely: the VBC (NEN6720) and Eurocode 2 (1992-1-1). This sheet was composed by Aracadis employee Kees van der Veen.

## 1.1 PROCEDURE

First the existing excel sheet was studied and the several equations were looked at. This was followed by calculating the steel stress when varying the concrete cover. The influence of this variation on the maximum allowable steel stress was analyzed by comparing the EUROCODE 2 (crack width expressions) and the VBC calculations (detailing rules: bar diameter/bar spacing/steel stress). After this the influence of the maximum crack spacing was looked at in the Eurocode 2 calculations. In the final calculations the value of the maximum crack spacing was limited and then the effect of this limitation on the maximum allowable steel stress was analyzed. During this step the concrete cover was also varied.

## 1.2 VARIATION OF CONCRETE COVER

The calculations were carried out for a slab with a thickness of 800 mm and a width of 1000 mm. The concrete cover was varied with the following values: $c = 40\ mm$ ; $c = 50\ mm\ and\ c = 60\ mm$. All calculations were based on a crack width limit of $w_{\max} = 0.2\ mm$. In section 1.2.1 an example calculation based on the VBC equations regarding crack width control is presented. The example calculation based on the Eurocode equation regarding crack width control is presented in section 1.2.2.

### *1.2.1 Example calculation NEN 6720*

**Geometry plate:**

Plate thickness $= 800\ mm$

**Strength properties**

Strength class concrete : $B45\ (C35/45)$

Concrete compressive strength $f_b' = 27\ N/mm^2$

Tensile strength concrete $f_b = 1.65\ N/mm^2$

**Reinforcement detailing:**

Applied cover $c = 40\ mm$

Applied stirrups $\emptyset_b = 20$ mm

**Main reinforcement:** The calculation for the maximum allowable steel stress is carried out for the following reinforcement diameters: $\emptyset 16; \emptyset 20$ *and* $\emptyset 25$. During each calculation a certain reinforcement area $A_s$ is chosen and kept constant. These are: $A_s = \frac{1}{4} * \pi * d^2 * \frac{b}{s} =$ $2011\ mm^2$ /m ; $A_s = 3142\ mm^2/m$ *and* $A_s = 4909\ mm^2/m$

The bar spacing ($s$) is calculated with the help of the following equation:

$s = \frac{\pi * d^2 * b}{4 * A_s}$ ; in which $d$: diameter main reinforcement and $b = 1000\ mm$

This is calculated in excel for the following diameters: $\emptyset 16; \emptyset 20$ *and* $\emptyset 25$

### Environmental class:

Aggressive environment: MK-3,4,5 (Table 1-1)

Crack width: $w_{max} = 0.2\ mm$ (Table 1-2)

Reference period $T = 100\ years$ so $c_{min} = 40\ mm$ and $k_c = \frac{c}{c_{min}} = \frac{40}{40} = 1$

In which $k_c$ is the factor which considers the applied concrete cover.

*Table 1-1: Classification of the environmental classes in NEN 6720*

| milieuklasse | milieu |
|---|---|
| 1 | droog |
| 2 | vochtig |
| 3 | vochtig in combinatie met dooizouten |
| 4 | zeewater |
| 5 (a, b, c of d) | agressief (zwak, matig, sterk of zeer sterk) |

*Table 1-2: Boundary values for the crack width in NEN 6720*

| milieuklasse | $w$ mm | |
|---|---|---|
| | zonder voorspanstaal | met voorspanstaal |
| 1 | 0.4 | 0.3 |
| 2 | 0.3 | 0.2 |
| 3, 4 en 5 | 0.2 | 0.1 |
| Voor omhulde strengen volgens NEN 3868:1991 is de kolom "zonder voorspanstaal" van toepassing. | | |

### Calculation of the maximum allowable steel stress:

The following criteria regarding crack width control ( section 8.7.2 of NEN 6720) are rewritten for the calculation of the maximum allowable steel stress:

1. The average bar diameter in the considered tensile zone must be equal to:
   $\emptyset_{km} \leq \frac{k_1 * \xi}{\sigma_s} * k_c$ (in mm); for: $k_c = \frac{c}{c_{min}} \ngtr 2$

So : $\sigma_{s;rep} < \left(\frac{k_1 * \xi}{\emptyset_{km}}\right) * k_c$

2. The spacing (s) between the reinforcing bars in the considered zone must be equal to:

$s \leq 100 * \left(\frac{k_2 * \xi}{\sigma_s} - 1.3\right) * k_c$ ( in mm) ; for: $k_c = \sqrt{\frac{c}{c_{min}}} \ngtr \sqrt{2} \rightarrow$

$s \leq 100 * \left(\frac{k_2 * \xi}{\sigma_s} - 1.3\right) * k_c$ ( in mm) ; for $\sqrt{k_c} = \frac{c}{c_{min}} \ngtr 2$

So: $\sigma_{s;rep} < k_2 * \xi / \left\{0.01 * \frac{s}{(k_c)^{\frac{1}{2}}} + 1.3\right\}$

In which:

c: the applied cover on the outer layer of the reinforcement

$c_{min}$: cover prescribed by clause 9.2 of NEN 6720

$k_1 and\ k_2$ : are factors depending on the environment according to table 38 of NEN 6720

$\xi = 1$ for ribbed bars : is the bond factor according to table 39.

$\sigma_s$: largest calculated value of the steel stress in the cracked cross section

***Note*:*** *the steel stress $\sigma_s$ should also be smaller than the maximum design value of the yield strength of steel $f_{yd} = 435\ N/mm^2$*

**Calculation:**

Bar diameter: $\emptyset_k = 16\ mm$

Chosen $A_s = 2011\ mm^2/m$

$s = \frac{\pi * d^2 * b}{4 * A_s} = \frac{\pi * 16^2 * 1000}{4 * 2011} = 100\ mm$

$d = h - (c + \emptyset_b + 0.5\emptyset_k) = 800 - (40 + 20 + 8) = 732\ mm$

$x_u = \frac{4}{3} * \frac{f_s * A_s}{b * f'_b} = \frac{4 * 435 * 2011}{3 * 1000 * 27} = 43.19\ mm$

$$M_u = A_s * f_s * z = A_s * f_s * \left(d - \frac{7}{18} x_u\right) * 10^{-6} = 2011 * 435 * \left(732 - \frac{7}{18} * \frac{4 * 435 * 2011}{3 * 1000 * 27}\right) * 10^{-6}$$
$$= 626\ kNm/m$$

$$\sigma_{s,rep} < Min\left\{435, \text{Max}\left(\left(\frac{k_1 * \xi}{\emptyset_{km}}\right) * k_c; k_2 * \xi / \left\{0.01 * \frac{s}{(k_c)^{\frac{1}{2}}} + 1.3\right\}\right)\right\}$$

Factors $k_1$ , $k_2$ can be found in the table below:

*Table 1-3 Values for factors k1 and k2 according to table 38 in NEN 6720*

| milieuklasse | zonder voorspanstaal | | met voorspanstaal | |
|---|---|---|---|---|
| | $k_1$ | $k_2$ | $k_1$ | $k_2$ |
| 1 | 5000 | 1000 | 3750 | 750 |
| 2 | 3750 | 750 | 2500 | 500 |
| 3, 4 en 5 | 2500 | 500 | 1250 | 250 |
| Voor omhulde strengen volgens NEN 3868:1991 is de kolom "zonder voorspanstaal" van toepassing. | | | | |

So :

$$\sigma_{s,rep} < Min\left\{435, \text{Max}\left(\left(\frac{2500*1}{16}\right)*1; 500*\frac{1}{\left\{0.01*\frac{100}{(1)^{\frac{1}{2}}}+1.3\right\}}\right)\right\}$$

$$\sigma_{s,rep} < Min\{435\ N/mm^2, \text{Max}\ (156\ N/mm^2; 217\ N/mm^2)\}$$

And so $\sigma_{s,rep} < 217\ N/mm^2$

The value of the maximum allowable bending moment in S.L.S (max. $M_{rep}$) can also be calculated with the following equation:

$$max. M_{rep} = \frac{\sigma_{s,rep}}{fs} * M_u = \frac{217}{435} * 626 = 313\ kNm/m$$

The above procedure is also used to calculate the maximum allowable steel stress for a concrete cover of $c = 50\ mm$ and $c = 60\ mm$

When calculating the steel stress with a cover of $c = 50\ mm$ and $c = 60\ mm$ with the NEN6720 regulations the values for the maximum allowable steels stress increases between 2% and 7%. This can be seen in the tables below. So it can be concluded that in the NEN 6720 the variation of the cover has a minimal influence on the calculation of $\sigma_{s;rep}$ due to the increase of factor $k_c$.

*Table 1-4: Values of the maximum steel stress for a concrete cover of c=40 mm according to NEN 6720*

| **NEN6720** <br> **c=40 mm** | | **Aggressive environment: Crack width: w = 0.2 mm** <br> $c_{min} = 40\ mm$ <br> $k_c = 1$ | |
|---|---|---|---|
| **$\phi_k$ =** | **$A_s$ =** | **s =** | **$\sigma_{s,rep}$ <** |
| **[ mm ]** | [ mm²/m ] | [ mm ] | [ N/mm² ] |
| **16** | 2011 | 100 | 217 |
| **16** | 3142 | 64 | 258 |
| **16** | 4909 | 41 | 292 |
| **20** | 2011 | 156 | 175 |
| **20** | 3142 | 100 | 217 |
| **20** | 4909 | 64 | 258 |
| **25** | 2011 | 244 | 134 |
| **25** | 3142 | 156 | 175 |
| **25** | 4909 | 100 | 217 |

*Table 1-5: Values of the maximum steel stress for a concrete cover of c=50 mm according to NEN 6720*

| **NEN6720** <br> **c=50 mm** | | **Aggressive environment: Crack width: w = 0.2 mm** <br> $c_{min} = 40\ mm$ <br> $k_c = 1.25$ | |
|---|---|---|---|
| **$\phi_k$ =** | **$A_s$ =** | **s =** | **$\sigma_{s;rep}$ <** |
| **[ mm ]** | [ mm²/m ] | [ mm ] | [ N/mm² ] |
| **16** | 2011 | 100 | 228 |
| **16** | 3142 | 64 | 267 |
| **16** | 4909 | 41 | 300 |
| **20** | 2011 | 156 | 185 |
| **20** | 3142 | 100 | 228 |
| **20** | 4909 | 64 | 267 |
| **25** | 2011 | 244 | 144 |
| **25** | 3142 | 156 | 185 |
| **25** | 4909 | 100 | 228 |

*Table 1-6: Values of the maximum steel stress for a concrete cover of c=60 mm according to NEN 6720*

| **NEN6720** c=60 mm | | **Aggressive environment: Crack width: w = 0.2 mm** $c_{min} = 40\ mm$ $k_c = 1.5$ | |
|---|---|---|---|
| $\Phi_k$ = | $A_s$ = | s = | $\sigma_{s;rep}$ < |
| [ mm ] | [ mm²/m ] | [ mm ] | [ N/mm² ] |
| **16** | 2011 | 100 | 236 |
| **16** | 3142 | 64 | 274 |
| **16** | 4909 | 41 | 306 |
| **20** | 2011 | 156 | 194 |
| **20** | 3142 | 100 | 236 |
| **20** | 4909 | 64 | 274 |
| **25** | 2011 | 244 | 152 |
| **25** | 3142 | 156 | 194 |
| **25** | 4909 | 100 | 236 |

### 1.2.2 *Example calculation NEN-EN1992-1-1*

**Strength properties concrete:**

Strength class concrete: $C35/45$

Characteristic strength of concrete: $f_{ck} = 35\ N/mm^2$

Modulus of elasticity concrete: $E_{cm} = 22 * \left(0.1 * (f_{ck} + 8)\right)^{0.3} = 22 * (0.1 * (35 + 8)^{0.3}) = 34100\ N/mm^2$

Concrete compressive strength $f_{cd} = 23.33\ N/mm^2$

Tensile strength concrete $f_{ctm} = 3.21\ N/mm^2$

Strength Properties of the reinforcing steel:

Reinforcement steel used: $B500B$ (ribbed)

Yield strength of the steel $f_{yd} = 435\ N/mm^2$

Modulus of elasticity steel: $E_s = 200000\ N/mm^2$

$\alpha_e = \frac{E_s}{E_{cm}} = \frac{200000}{34100} = 5.87$

Load duration factor: $k_t = 0.4$ (long term loading)

**Geometry concrete plate:**

Width $b' = 1000\ mm$

Height $h = 800\ mm$

### Reinforcement detailing:

Applied cover: $c = 40\ mm$

Applied stirrups: $\emptyset_b = 20$

Required concrete cover: $c_{min} = 40\ mm$

Ratio of the concrete cover: $k_c = \frac{c}{c_{min}} = 1$

Allowable crack width: $w_{max} = k_c * w_k = 0.2\ mm$

The crack width $(w_k)$ is kept constant

**Main reinforcement**: The calculation for the maximum allowable steel stress is carried out for the following reinforcement diameters: $\emptyset 16; \emptyset 20\ and\ \emptyset 25.$ During each calculation a certain reinforcement area $A_s$ is chosen and kept constant. These are: $A_s = \frac{1}{4} * \pi * d^2 * \frac{b}{s} = 2011\ mm^2$/m

$A_s = 3142\ mm^2/m\ and\ A_s = 4909\ mm^2/m$

The bar distance $(s)$ is calculated with the help of the following equation:

$s = \frac{\pi * d^2 * b}{4 * A_s}$ ; in which $d$: diameter main reinforcement and $b = 1000\ mm$

This is calculated in excel for the following diameters: $\emptyset 16; \emptyset 20\ and\ \emptyset 25$

### Calculation of the maximum allowable steel stress

For the calculation of the maximum allowable steel stress the following requirements regarding crack width according to NEN-EN 1992-1-1 section 7.3.2 are used:

1. $w_k = s_{r,max}(\varepsilon_{sm} - \varepsilon_{cm})$, in which:

$w_k$: the design value for the crack width;
$s_{r,max}$: the maximum value of the crack spacing.
At a certain amount of reinforcement the value of $s_{r,max}$ is constant, or:

2. $(\varepsilon_{sm} - \varepsilon_{cm}) = \frac{w_k}{s_{r,max}}$

The starting point is a bar spacing $(s)$ for which the following criteria holds:

$s < 5 * (c + \frac{1}{2}\emptyset_k)$ and so the following equation holds for $s_{r,max}$:

3. $s_{r,max} = k_3 * c + k_1 * k_2 * k_4 * \frac{\emptyset}{\rho_{p,eff}}$ in which:

c: Cover to the reinforcement

Ø: Bar diameter. Where a mixture of bar diameters is used in a section, the average diameter may be used.
$k_1$: Coefficient which takes account of the bond properties of the bonded reinforcement:

$$k_1 = \begin{cases} 0.8 \text{ (high bond bars)} \\ 1.6 \text{ (bars withwith an effectively plain surface (e. g. prestressing tendons)} \end{cases}$$

$k_2$: Coefficient which takes account of the distribution of strain:

$$k_2 = \begin{cases} 0.5 \text{ (Bending)} \\ 1.0 \text{ (Pure tension)} \end{cases}$$

$k_3 = 3.4$ and $k_4 = 0.425$

*Table 1-7: factors needed for the calculation of $s_{r;max}$*

| k1 | k2 | k3 | k4 |
|---|---|---|---|
| 0.8 | 0.5 | 3.4 | 0.425 |

The reinforcement ratio: $\rho_{p;eff} = \frac{A_s}{A_{c,eff}} = \frac{A_s}{b*h_{c,eff}}$ in which: $h_{c,eff}$ : effective depth which is equal to the minimum value of the following expressions:

$$h_{ceff}: \leq \begin{cases} 1: & 2.5*(h-d) \\ 2: & \frac{h-x}{3} \ (bending) \\ 3: & \frac{h}{2} \ (tension) \end{cases}$$

4. $(\varepsilon_{sm} - \varepsilon_{cm}) = \frac{\sigma_s - k_t * \frac{f_{ct,eff}}{\rho_{p,eff}} * (1 + \alpha_e * \rho_{p,eff})}{E_s} \geq 0.6 * \frac{\sigma_s}{E_s}$, In which:

$(\varepsilon_{sm} - \varepsilon_{cm})$: is the difference between the mean strain in the reinforcement $(\varepsilon_{sm})$ under relevant combination of loads and the mean strain in concrete between cracks $(\varepsilon_{cm})$
$\sigma_s$: the stress in the tension reinforcement assuming a cracked cross section
$\alpha_e$: the ratio between the modulus of elasticity of steel and concrete: $\frac{E_s}{E_{cm}}$

$k_t$: factor depending on the duration of the load: $k_t = \begin{cases} 0.6 \ (short\ term\ loading) \\ 0.4 \ (long\ term\ loading) \end{cases}$

In order to obtain the equation for the maximum permissible steel stress equation the following aspects should be taken into account:

a. By choosing a certain crack width $(w_k)$, the permitted value of the mean strain difference will also be constant: $\Delta\varepsilon_m = (\varepsilon_{sm} - \varepsilon_{cm}) = \frac{w_k}{s_{r,max}}$
b. From the lower limit value of: $\Delta\varepsilon_m = (\varepsilon_{sm} - \varepsilon_{cm}) \geq 0.6 * \frac{\sigma_s}{E_s}$,the minimum allowable stress equals: $\sigma_{s,rep} \leq \frac{(\varepsilon_{sm} - \varepsilon_{cm})}{0.6} * E_s$
c. By rewriting expression for $\Delta\varepsilon_m = (\varepsilon_{sm} - \varepsilon_{cm}) = \frac{\left\{\sigma_s - k_t * \frac{f_{ct;eff}}{\rho_{p;eff}} * (1 + \alpha_e * \rho_{p;eff})\right\}}{E_s}$ the maximum permissible steel stress can be determined:

$\sigma_{s,rep} = \Delta\varepsilon_m * E_s + k_t * \frac{f_{ct;eff}}{\rho_{p;eff}} * (1 + \alpha_e * \rho_{p;eff})$ and by substituting the following expression for $\Delta\varepsilon_m = \frac{w_k}{s_{r,max}}$, the value of the permissible steel stress can be calculated with the following equation:

$$\sigma_{s,rep} = \frac{w_k}{s_{r,max}} * E_s + k_t * \frac{f_{ct;eff}}{\rho_{p;eff}} * (1 + \alpha_e * \rho_{p;eff})$$

***Note[1]*:*** *the steel stress $\sigma_s$ should also be smaller than the maximum design value of the yield strength of steel $f_{yd} = 435\ N/mm^2$ : $\sigma_{s,rep} < f_{yd}$*

***Note[2]*:*** *In the national Annex of NEN-EN 1992-1-1 it is mentioned that in cases where the crack width is calculated according to section 7.3.3. or 7.3.4. of the NEN-EN 1992-1-1 the maximum allowable crack width as provided in table 7.1N (of the national Annex) should be calculated width a factor $k_x$. This factor takes the influence of the concrete cover into account.*

$k_x = \frac{c_{applied}}{c_{nom}}$ , $c_{applied}$: *the applied cover* ($c_{toeg} \geq c_{nom}$) and $c_{nom}$: *the required amount of the nominal concrete cover.*

*For the calculations carried out in this report (according to NEN-EN 1992-1-1) the maximum allowable crack width is kept constant to a value of $w_{max} = k_x * w_k = 0.2$ mm. And so the factor $k_x$ is also kept constant ($k_x = 1$).*

**Calculation:**

Bar diameter: $\emptyset_k = 16\ mm$

Chosen $A_s = 2011\ mm^2/m$

$s = \frac{\pi * d^2 * b}{4 * A_s} = \frac{\pi * 16^2 * 1000}{4 * 2011} = 100\ mm$

$d = h - (c + \emptyset_b + 0.5\emptyset_k) = 800 - (40 + 20 + 8) = 732\ mm$

$x_u = \frac{f_{yd} * A_s}{\alpha * b * f_{cd}} = \frac{435 * 2011}{0.75 * 1000 * 23.33} = 50\ mm$

$Z_u = d - \beta * x_u$ ; in which $\beta = 0.389 \rightarrow Z_u = d - \beta * x_u = 732 - 0.389 * 50 = 713\ mm$

$M_{Rd} = A_s * f_{yd} * z_u * 10^{-6} = 2011 * 435 * 723 * 10^{-6} = 632\ kNm/m$

$x_{rep}$: height of the compression zone:

$$x_{rep} = -\frac{n}{b} * (A_s) + \left\{\left(\frac{n}{b}\right)^2 * (A_s)^2 + 2 * \frac{n}{b} * (d * A_s)\right\}^{\frac{1}{2}} =$$

$$x_{rep} = -\frac{5.87}{1000} * 2011 + \sqrt{\left(\frac{5.87}{1000}\right)^2 * (2011)^2 + 2 * \left(\frac{5.87}{1000}\right) * 732 * 2011} = 120\ mm$$

In which $n = \alpha_e = 5.87$

$$h_{ceff}\text{: Effective depth: } \leq \begin{cases} 1\text{: } 2.5*(h-d) \\ 2\text{: } \frac{h-x_{rep}}{3} \\ 3\text{: } \frac{h}{2} \end{cases} = \min\begin{cases} 1\text{: } 2.5*(800-732)=170 \\ 2\text{: } \frac{800-120}{3}=227 \\ 3\text{: } \frac{800}{2}=400 \end{cases}$$

$h_{ceff} = 170\ mm$

$\rho_{p;eff} = \frac{A_s}{b*h_{ceff}} = \frac{2011}{1000*170} = 0.0118$

$$s_{r,max} = k_3 * c + k_1 * k_2 * k_4 * \frac{\emptyset}{\rho_{p,eff}} = 3.4*60+0.8*0.5*0.425*\frac{16}{0.0118} = 434\ mm$$

$c = 60\ mm$ :the cover on the main reinforcement

And now finally the maximum value of the steel stress can be calculated by taking the smallest value of the following equations:

I. $\sigma_{s,rep} \leq \frac{(\varepsilon_{sm}-\varepsilon_{cm})}{0.6} * E_s = \frac{w_k*E_s}{0.6*s_{r,max}} = \frac{0.2*200000}{0.6*434} =$

$$\sigma_{s,rep} \leq \frac{w_k * E_s}{0.6 * s_{r,max}} \rightarrow \sigma_{s,rep} \leq \frac{0.2*200000}{0.6*434} \leq 154\ N/mm^2$$

II. $\sigma_{s,rep} = \frac{w_k}{s_{r,max}} * E_s + k_t * \frac{f_{ct;eff}}{\rho_{p;eff}} * (1 + \alpha_e * \rho_{p;eff})$

$$\sigma_{s,rep} = \frac{0.2}{434} * 200000 + 0.4 * \frac{3.21}{0.0118} * (1 + 5.87 * 0.0118)$$

$$\sigma_{s,rep} = 208\ N/mm^2$$

III. $\sigma_{s,rep} < f_{yd} \rightarrow \sigma_{s,rep} < 435\ N/mm^2$

So: $\sigma_{s,rep} = 154\ N/mm^2$

The maximum allowable bending moment can be calculated with the following expression:

$$M_{freq} < \sigma_{s;rep} * A_s * \left\{\left(d - \frac{1}{3} * x_{rep}\right)\right\} = 154 * 2011 * \left\{\left(732 - \frac{1}{3} * 120\right)\right\} * 10^{-6} = 214\ kNm/m$$

As mentioned before the above calculations are carried out for 3 different diameters $\emptyset 16; \emptyset 20\ and\ \emptyset 25$ and three different covers $c = 30\ mm$ ; $c = 40\ mm\ and\ c = 50\ mm$. The results of these calculations are presented in the tables below. In section 1.2.3 the results from the NEN 6720 calculations and the NEN-EN1992-1-1 calculations are compared.

*Table 1-8: Values of the maximum steel stress for a concrete cover of c=40 mm according to NEN-EN 1992-1-1*

| NEN-EN-1992 $c = 40\ mm$ | | Allowable crack width: $w = k_c * w_k = 0.2$ mm | | | |
|---|---|---|---|---|---|
| $\Phi_k$ = | $A_s$ = | s = | $\rho_{s;eff}$ = | $s_{r;max}$ = | $\sigma_{s;rep}$ = |
| [ mm ] | [ mm²/m ] | [ mm ] | [ % ] | [mm] | [ N/mm² ] |
| **16** | 2011 | 100 | 0.0118 | 434 | 154 |
| **16** | 3142 | 64 | 0.0185 | 351 | 190 |
| **16** | 4909 | 41 | 0.0289 | 298 | 186 |
| **20** | 2011 | 156 | 0.0118 | 491 | 136 |
| **20** | 3142 | 100 | 0.0185 | 388 | 172 |
| **20** | 4909 | 64 | 0.0289 | 322 | 176 |
| **25** | 2011 | 244 | 0.0118 | 563 | 118 |
| **25** | 3142 | 156 | 0.0185 | 434 | 154 |
| **25** | 4909 | 100 | 0.0289 | 351 | 166 |

*Table 1-9: Values of the maximum steel stress for a concrete cover of c=50 mm according to NEN-EN 1992-1-1*

| NEN-EN-1992 $c = 50\ mm$ | | Allowable crack width: $w = k_c * w_k = 0.2$ mm | | | |
|---|---|---|---|---|---|
| $\Phi_k$ = | $A_s$ = | s = | $\rho_{s;eff}$ = | $s_{r;max}$ = | $\sigma_{s;rep}$ = |
| [ mm ] | [ mm²/m ] | [ mm ] | [ % ] | [mm] | [ N/mm² ] |
| **16** | 2011 | 100 | 0.0103 | 502 | 133 |
| **16** | 3142 | 64 | 0.0161 | 407 | 164 |
| **16** | 4909 | 41 | 0.0251 | 346 | 174 |
| **20** | 2011 | 156 | 0.0103 | 568 | 117 |
| **20** | 3142 | 100 | 0.0161 | 449 | 148 |
| **20** | 4909 | 64 | 0.0251 | 373 | 166 |
| **25** | 2011 | 244 | 0.0103 | 650 | 103 |
| **25** | 3142 | 156 | 0.0161 | 502 | 133 |
| **25** | 4909 | 100 | 0.0251 | 407 | 157 |

*Table 1-10: Values of the maximum steel stress for a concrete cover of c=60 mm according to NEN-EN 1992-1-1*

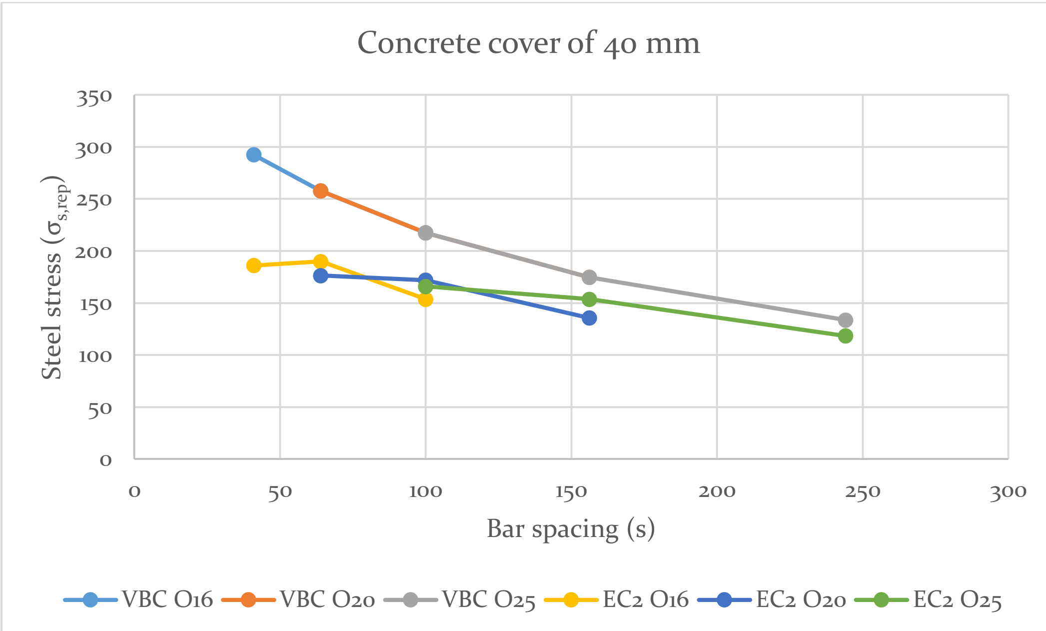
| NEN-EN-1992 $c = 60\ mm$ | | Allowable crack width: $w = k_c * w_k = 0.2$ mm | | | |
|---|---|---|---|---|---|
| $Ø_k$ = | $A_s$ = | s = | $\rho_{s;eff}$ = | $s_{r;max}$ = | $\sigma_{s;rep}$ = |
| [ mm ] | [ mm²/m ] | [ mm ] | [ % ] | [mm] | [ N/mm² ] |
| 16 | 2011 | 100 | 0.009 | 570 | 117 |
| 16 | 3142 | 64 | 0.014 | 461 | 145 |
| 16 | 4909 | 41 | 0.024 | 387 | 165 |
| 20 | 2011 | 156 | 0.009 | 644 | 104 |
| 20 | 3142 | 100 | 0.014 | 508 | 131 |
| 20 | 4909 | 64 | 0.024 | 416 | 158 |
| 25 | 2011 | 244 | 0.009 | 737 | 90 |
| 25 | 3142 | 156 | 0.014 | 567 | 117 |
| 25 | 4909 | 100 | 0.024 | 452 | 147 |

The variation of the concrete cover does have an influence on the maximum allowable steel stress $\sigma_{s,rep}$. In Table 1-8 to Table 1-10 it can clearly be seen that $\sigma_{s,rep}$ decreases when a larger cover is applied. We also see that the maximum crack spacing ($s_{r,max}$) increases when the concrete cover is larger. This can be explained by looking at the equation for $s_{r,max}$ : $s_{r,max} = k_3 * c + k_1 * k_2 * k_4 * \frac{\emptyset}{\rho_{p,eff}}$ . We see that when increasing the cover, the effective depth ($h_{c,eff} = 2.5(h - d)$) increases and the effective concrete area ($\rho_{p,eff}$) decreases. This causes the maximum crack spacing ($s_{r,max}$) to increase. Since the maximum allowable steel stress also depends on the maximum crack spacing and the effective concrete area, which can be seen in the equation below, the steel stress will decrease when the $s_{r,max}$ increases and $\rho_{p,eff}$ decreases.

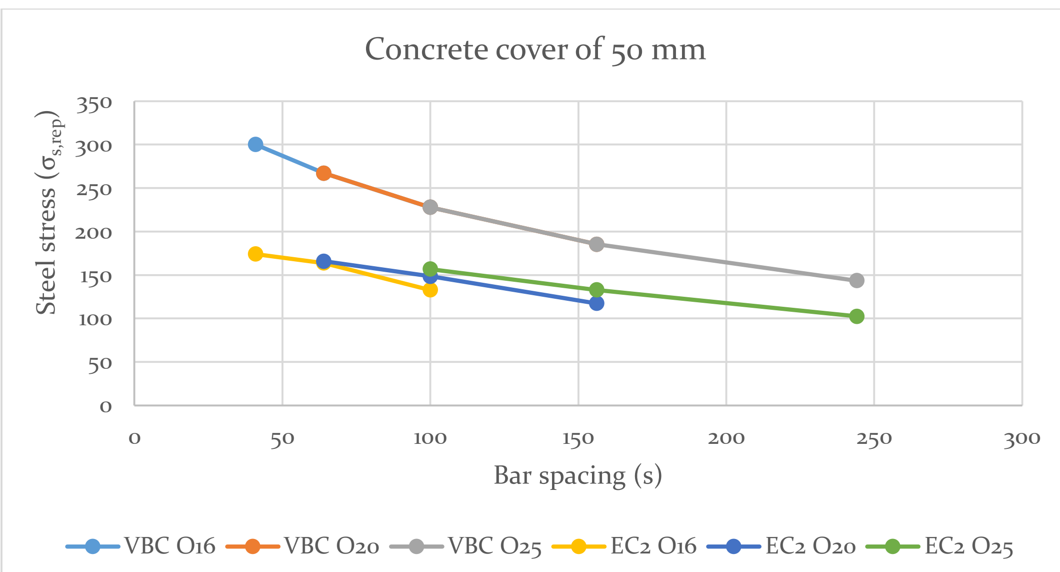
$$\sigma_{s,rep} < \frac{w_k}{s_{r,max}} * E_s + k_t * \frac{f_{ct;eff}}{\rho_{p;eff}} * (1 + \alpha_e * \rho_{p;eff})$$

### 1.2.3 Comparison results

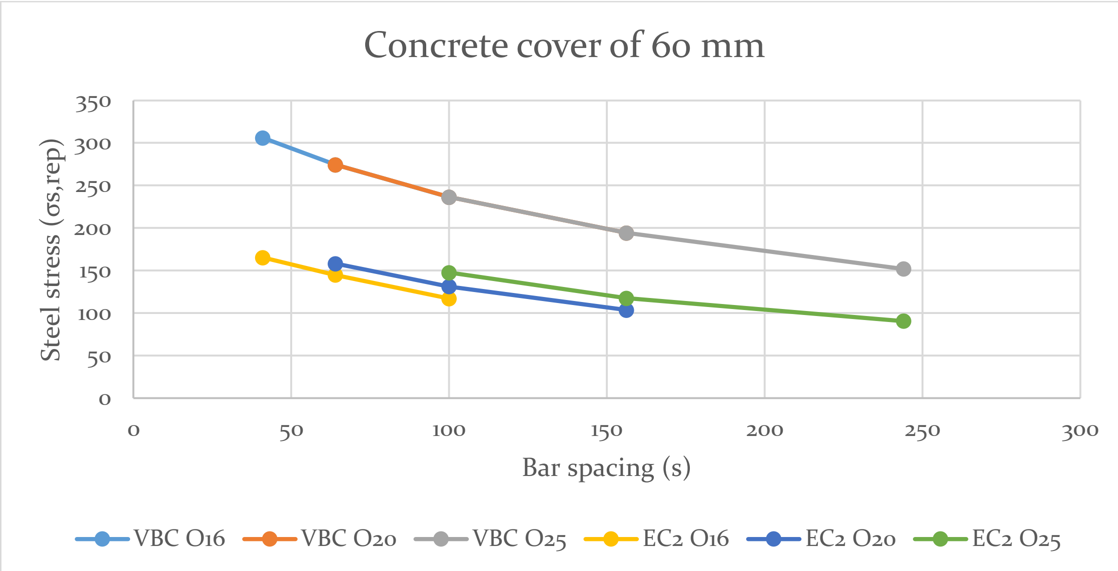
When comparing the results of the NEN6720 calculations with the NEN-EN1992-1-1 it is clear that allowed the steel stress calculated in the NEN-EN1992-1-1 is much smaller than that of NEN 6720. In the following graphs these results are presented. In each graph the steel stress is depicted as a function of the bar spacing. By looking at each graph it is clear that in the NEN-EN 1992 calculations the steel stress decreases when the concrete cover is increased. In the NEN 6720 however the variation of the cover has minimal influence since the factor $k_c$ increases.




*Graph 1-1: Maximum allowable steel stress for a cover of 40 mm according to VBC- and Eurocode 2 calculations*




*Graph 1-2: Maximum allowable steel stress for a cover of 50 mm according to VBC- and Eurocode 2 calculations*




*Graph 1-3: Maximum allowable steel stress for a cover of 50 mm according to VBC- and Eurocode 2 calculations*

# 2 VARIATION OF CONCRETE COVER BY LIMITING THE MAXIMUM CRACK SPACING

In the previous chapter it was clear that the maximum allowable steel stress decreases when the concrete cover increases (Graph 1-1 to Graph 1-3). In this chapter the influence of the concrete cover on the maximum allowable steel stress is further analyzed by limiting the crack spacing.

## 2.1 NEN6720

In the NEN6720 the crack spacing is not limited since the crack width calculations are based on the bar diameter ($\emptyset_{km}$) and the allowable bar spacing ($s$). These equations are based on the requirements regarding crack width control as presented in the VB 1974/1984. The equations from the VB 1974/1984 are rewritten in terms of bar diameter and bar spacing to meet the conditions concerning the cracking behavior [1]. The values for the maximum allowable steel stress remain the same for the NEN 6720 calculations.

## 2.2 NEN-EN1992-1-1

Looking at the results of the Eurocode 2 calculations in chapter 1.2.2 it can be seen that the value of the maximum crack spacing ($s_{r,max}$) increases when a larger concrete cover is applied. To further analyze this, the conditions regarding the cracking behavior in VB 1974/1984 are also studied. When doing this it is clear that the crack width calculations in the Eurocode are almost the same as the calculations in the VB1974/1984. There is one difference however, since in VB 1974/1984 the mean crack spacing ($\Delta l_m$) has an upper limited value of $\Delta l = 10\emptyset k_m$. This limitation is not found in the Eurocode. So for further analysis this upper limit value will be applied in the Eurocode 2 equation for the maximum crack spacing ($s_{r,\max}$).

The following upper boundaries are also applied in the calculation for $s_{r,max}$ : $s_{r,max} \leq Max\,\{(50 - 0.8 f_{ck})\emptyset; 15\emptyset\}$. This upper boundary is taken from VARCE (Vraag en antwoord rubriek in CEMENT :NEN-EN 1992-1-1 +C2: 2011/NB:2011), which was obtained at ARCADIS.

Thus the calculation of the maximum crack spacing is modified in excel twice with the following equations:

1) $s_{r,max} = k_3 * c + k_1 * k_2 * k_4 * \frac{\emptyset}{\rho_{p,eff}}; s_{r,max} \leq 10\emptyset$
2) $s_{r,max} = k_3 * c + k_1 k_2 k_4 * \frac{\emptyset}{\rho_{p,eff}} \leq Max\{(50 - 0.8 * f_{ck})\emptyset\,; 15\emptyset\}$

The calculations are carried out for a concrete cover of $c = 40\ mm; c = 50\ mm\ and\ c = 60\ mm$. The same procedure is followed as the example calculation in chapter 1.2.2. Only the calculation of $s_{r,max}$ is modified as mentioned above. Below the example calculations are presented for both equations.

### 2.2.1 Example calculation for $s_{r,max} \leq 10\emptyset$

**Geometry Plate:** The same plate of chapter 1.2.2 is used: $h = 800\ mm$ and $b = 1000\ mm$

Concrete cover : $c = 40\ mm$ ; stirrups; main reinforcement: $\emptyset_k = 16\ mm$

$$s_{r,max} = k_3 * c + k_1 * k_2 * k_4 * \frac{\emptyset}{\rho_{p,eff}} = 3.4 * 60 + 0.8 * 0.5 * 0.425 * \frac{16}{0.0118} = 434\ mm \text{ (Section 1.2.2)}$$

Upper limit value: $s_{r,max} \leq 10\emptyset \rightarrow s_{r,max} \leq 10 * 16 \leq 160\ mm$

The value for the maximum crack spacing is equal to: $s_{r,max} = min\,(434; 160)$

And so $s_{r,max} = 160\ mm$

The maximum allowable permissible stress is equal to the smallest out of the three values calculated below:

I. $\sigma_{s,rep} \leq \frac{(\varepsilon_{sm}-\varepsilon_{cm})}{0.6} * E_s = \frac{w_k * E_s}{0.6 * s_{r,max}} = \frac{0.2*200000}{0.6*160} =$

$$\sigma_{s,rep} \leq \frac{w_k * E_s}{0.6 * s_{r,max}} \leq \frac{0.2 * 200000}{0.6 * 160} > 417\ N/mm^2$$

II. $\sigma_{s,rep} = \frac{w_k}{s_{r,max}} * E_s + k_t * \frac{f_{ct;eff}}{\rho_{p;eff}} * \left(1 + \alpha_e * \rho_{p;eff}\right)$

$$\sigma_{s,rep} = \frac{0.2}{160} * 200000 + 0.4 * \frac{3.21}{0.0118} * (1 + 5.87 * 0.0118)$$

$$\sigma_{s,rep} = 366 N/mm^2$$

III. $\sigma_{s,rep} < f_{yd} \rightarrow \sigma_{s,rep} < 435\ N/mm^2$

And so $\sigma_{s,rep} = 366\ N/mm^2$

In the tables below the results are presented for each varied concrete cover.

*Table 2-1: Values of the maximum steel stress for a concrete cover of c=40 mm according to NEN-EN 1992-1-1 after limitation of $s_{r,max}$ with $s_{r,max} \leq 10\emptyset$*

| **NEN-EN-1992 c=40** | | **Allowable crack width: $w = k_c * w_k = 0.2$** | | | |
|---|---|---|---|---|---|
| $\Phi_k =$ | $A_s =$ | s = | $\rho_{s,eff} =$ | $s_{r;max} =$ | $\sigma_{s;rep} =$ |
| [ mm ] | [ mm²/m ] | [ mm ] | [ % ] | [mm] | [ N/mm² ] |
| 16 | 2011 | 100 | 0.012 | 160 | 366 |
| 16 | 3142 | 64 | 0.018 | 160 | 327 |
| 16 | 4909 | 41 | 0.029 | 160 | 302 |
| 20 | 2011 | 156 | 0.012 | 200 | 316 |
| 20 | 3142 | 100 | 0.018 | 200 | 277 |
| 20 | 4909 | 64 | 0.029 | 200 | 252 |
| 25 | 2011 | 244 | 0.012 | 250 | 267 |

| | | | | | |
|---|---|---|---|---|---|
| **25** | 3142 | 156 | 0.018 | 250 | 237 |
| **25** | 4909 | 100 | 0.029 | 250 | 212 |

*Table 2-2: Values of the maximum steel stress for a concrete cover of c=50 mm according to NEN-EN 1992-1-1 after limitation of* $s_{r,max}$ *with* $s_{r,max} \leq 10\emptyset$

| **NEN-EN-1992 c=50 mm** | | **Allowable crack width: $w = k_c * w_k = 0.2$** | | | |
|---|---|---|---|---|---|
| $Ø_k$ = | $A_s$ = | s = | $\rho_{s,eff}$ = | $s_{r;max}$ = | $\sigma_{s;rep}$ = |
| [ mm ] | [ mm²/m ] | [ mm ] | [ % ] | [mm] | [ N/mm² ] |
| **16** | 2011 | 100 | 0.010 | 160 | 382 |
| **16** | 3142 | 64 | 0.016 | 160 | 337 |
| **16** | 4909 | 41 | 0.025 | 160 | 309 |
| **20** | 2011 | 156 | 0.010 | 200 | 332 |
| **20** | 3142 | 100 | 0.016 | 200 | 287 |
| **20** | 4909 | 64 | 0.025 | 200 | 259 |
| **25** | 2011 | 244 | 0.010 | 250 | 267 |
| **25** | 3142 | 156 | 0.016 | 250 | 247 |
| **25** | 4909 | 100 | 0.025 | 250 | 219 |

*Table 2-3: Values of the maximum steel stress for a concrete cover of c=60 mm according to NEN-EN 1992-1-1 after limitation of* $s_{r,max}$ *with* $s_{r,max} \leq 10\emptyset$

| **NEN-EN-1992 c=60 mm** | | **Allowable crack width: $w = k_c * w_k = 0.2$** | | | |
|---|---|---|---|---|---|
| $Ø_k$ = | $A_s$ = | s = | $\rho_{s,eff}$ = | $s_{r;max}$ = | $\sigma_{s;rep}$ = |
| [ mm ] | [ mm²/m ] | [ mm ] | [ % ] | [mm] | [ N/mm² ] |
| **16** | 2011 | 100 | 0.009 | 160 | 398 |
| **16** | 3142 | 64 | 0.014 | 160 | 347 |
| **16** | 4909 | 41 | 0.024 | 160 | 312 |
| **20** | 2011 | 156 | 0.009 | 200 | 333 |
| **20** | 3142 | 100 | 0.014 | 200 | 297 |
| **20** | 4909 | 64 | 0.024 | 200 | 262 |
| **25** | 2011 | 244 | 0.009 | 250 | 267 |
| **25** | 3142 | 156 | 0.014 | 250 | 257 |
| **25** | 4909 | 100 | 0.024 | 250 | 222 |

When we look at the values for the maximum allowable steel stress it can be seen that the $\sigma_{s;rep}$ increases substantially. Before limitation the steel stress was equal to $\sigma_{s;rep} = 154\ N/mm^2$ and after limitation it was equal to: $\sigma_{s;rep} = 366\ N/mm^2$. This means that the maximum steel stress increases with : $\frac{366-154}{154} * 100\% = 138\ \%$ compared to the calculation without limitation of the maximum crack spacing for a cover of $c = 40\ mm$. So with increasing cover the value of the steel stress increases with more than 100 % in the cases where the maximum crack spacing is limited with $s_{r,max} \leq 10\emptyset$.

### 2.2.2 Example calculations for $s_{r,max} \leq Max\{(50 - 0.8 * f_{ck})\emptyset\ ; 15\emptyset\}$

**Geometry Plate:** The same slab of section 1.2.2 is used: $h = 800\ mm$ and $b = 1000\ mm$

Concrete cover: $c = 40\ mm$ ; stirrups; main reinforcement: $\emptyset_k = 16\ mm$

$s_{r,max} = k_3 * c + k_1 * k_2 * k_4 * \frac{\emptyset}{\rho_{p,eff}} = 3.4 * 60 + 0.8 * 0.5 * 0.425 * \frac{16}{0.0139} = 434\ mm$ (section 1.2.2)

Upper limit value: $s_{r,max} \leq Max\{(50 - 0.8 * f_{ck})\emptyset\ ; 15\emptyset\} \rightarrow$

$s_{r,max} \leq Max\{(50 - 0.8 * 35)16\ ; 15 * 16\}$

$s_{r,max} \leq Max\{352\ ; 240\}$

The value for the maximum crack spacing is equal to: $s_{r,max} = min\ (434; max\ (352; 240))$

And so $s_{r,max} = 352\ mm$

The maximum allowable permissible stress is equal to the smallest out of the three values calculated below:

I. $\sigma_{s,rep} \leq \frac{(\varepsilon_{sm}-\varepsilon_{cm})}{0.6} * E_s = \frac{w_k * E_s}{0.6 * s_{r,max}} = \frac{0.2*200000}{0.6*352} =$

$$\sigma_{s,rep} \leq \frac{w_k * E_s}{0.6 * s_{r,max}} \leq \frac{0.2 * 200000}{0.6 * 352} > 189\ N/mm^2$$

II. $\sigma_{s,rep} = \frac{w_k}{s_{r,max}} * E_s + k_t * \frac{f_{ct;eff}}{\rho_{p;eff}} * (1 + \alpha_e * \rho_{p;eff})$

$\sigma_{s,rep} = \frac{0.2}{352} * 200000 + 0.4 * \frac{3.21}{0.0118} * (1 + 5.87 * 0.0118)$

$$\sigma_{s,rep} = 230\ N/mm^2$$

III. $\sigma_{s,rep} < f_{yd} \rightarrow \sigma_{s,rep} < 435\ N/mm^2$

And so $\sigma_{s,rep} = 189\ N/mm^2$

In the tables below the results are presented for each varied concrete cover

*Table 2-4: Values of the maximum steel stress for a concrete cover of c=40 mm according to NEN-EN 1992-1-1 after limitation of* $s_{r,max}$ *with* $s_{r,max} \leq Max\{(50 - 0.8 * f_{ck})\emptyset\,;15\emptyset\}$

| **NEN-EN-1992 c=40 mm** | | **Allowable crack width: $w = k_c * w_k = 0.2$** | | | |
|---|---|---|---|---|---|
| **$Ø_k$ =** | $A_s$ = | **s =** | $\rho_{s,eff}$ = | $s_{r;max}$ = | $\sigma_{s;rep}$ = |
| **[ mm ]** | [ mm²/m ] | [ mm ] | [ % ] | [mm] | [ N/mm² ] |
| **16** | 2011 | 100 | 0.0118 | 352 | 189 |
| **16** | 3142 | 64 | 0.0185 | 351 | 190 |
| **16** | 4909 | 41 | 0.0289 | 298 | 186 |
| **20** | 2011 | 156 | 0.0118 | 440 | 152 |
| **20** | 3142 | 100 | 0.0185 | 388 | 172 |
| **20** | 4909 | 64 | 0.0289 | 322 | 176 |
| **25** | 2011 | 244 | 0.0118 | 550 | 121 |
| **25** | 3142 | 156 | 0.0185 | 434 | 154 |
| **25** | 4909 | 100 | 0.0289 | 351 | 166 |

*Table 2-5: Values of the maximum steel stress for a concrete cover of c=50 mm according to NEN-EN 1992-1-1 after limitation of* $s_{r,max}$ *with* $s_{r,max} \leq Max\{(50 - 0.8 * f_{ck})\emptyset\,;15\emptyset\}$

| **NEN-EN-1992 c=50 mm** | | **Allowable crack width: $w = k_c * w_k = 0.2$** | | | |
|---|---|---|---|---|---|
| **$Ø_k$ =** | $A_s$ = | **s =** | $\rho_{s,eff}$ = | $s_{r;max}$ = | $\sigma_{s;rep}$ = |
| **[ mm ]** | [ mm²/m ] | [ mm ] | [ % ] | [mm] | [ N/mm² ] |
| **16** | 2011 | 100 | 0.010 | 352 | 189 |
| **16** | 3142 | 64 | 0.016 | 352 | 189 |
| **16** | 4909 | 41 | 0.025 | 346 | 174 |
| **20** | 2011 | 156 | 0.010 | 440 | 152 |
| **20** | 3142 | 100 | 0.016 | 440 | 152 |
| **20** | 4909 | 64 | 0.025 | 373 | 166 |
| **25** | 2011 | 244 | 0.010 | 550 | 121 |
| **25** | 3142 | 156 | 0.016 | 502 | 133 |
| **25** | 4909 | 100 | 0.025 | 407 | 157 |

*Table 2-6: Values of the maximum steel stress for a concrete cover of c=50 mm according to NEN-EN 1992-1-1 after limitation of $s_{r,max}$ with $s_{r,max} \leq Max\{(50 - 0.8 * f_{ck})\emptyset\, ;150\emptyset\}$*

| **NEN-EN-1992**<br><br>**c=60 mm** | | **Allowable crack width:**<br>$w = k_c * w_k = 0.2$ | | | |
|---|---|---|---|---|---|
| $Ø_k$ = | $A_s$ = | s = | $\rho_{s,eff}$ = | $s_{r;max}$ = | $\sigma_{s;rep}$ = |
| [ mm ] | [ mm²/m ] | [ mm ] | [ % ] | [mm] | [ N/mm² ] |
| **16** | 2011 | 100 | 0.009 | 352 | 189 |
| **16** | 3142 | 64 | 0.014 | 352 | 189 |
| **16** | 4909 | 41 | 0.024 | 352 | 176 |
| **20** | 2011 | 156 | 0.009 | 440 | 152 |
| **20** | 3142 | 100 | 0.014 | 440 | 152 |
| **20** | 4909 | 64 | 0.024 | 416 | 158 |
| **25** | 2011 | 244 | 0.009 | 550 | 121 |
| **25** | 3142 | 156 | 0.014 | 550 | 121 |
| **25** | 4909 | 100 | 0.024 | 452 | 147 |

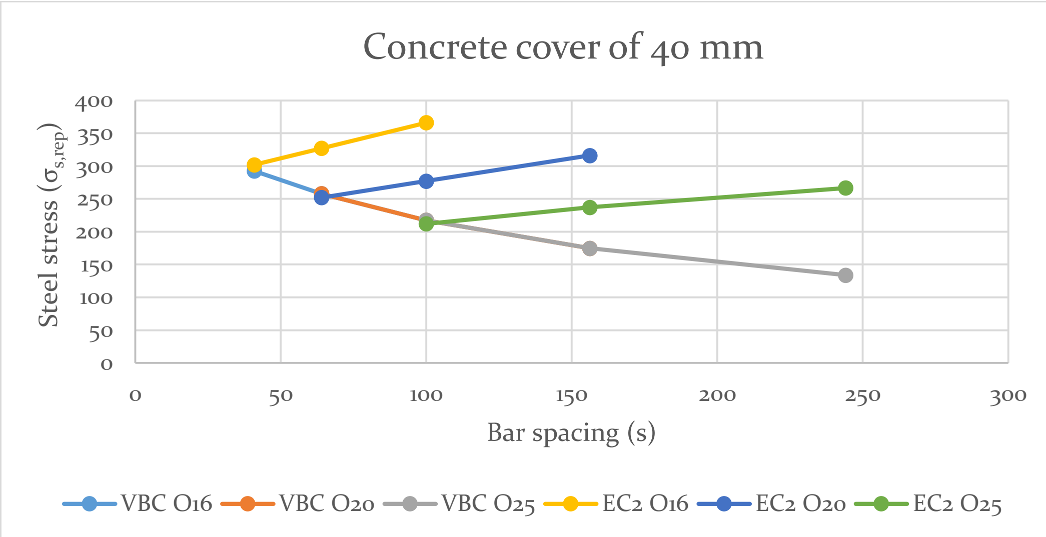
When the maximum crack spacing is limited with $s_{r,max} \leq Max\{(50 - 0.8 * f_{ck})\emptyset\, ;150\emptyset\}$ we see that there is an increase in the value of the steel stress allowed ($\sigma_{s,rep} = 189\ N/mm^2$) compared to the value of the steel stress without limitation of the maximum crack spacing ($\sigma_{s,rep} = 154\ N/mm^2$). At an increasing concrete cover the value of $\sigma_{s,rep}$ reaches a constant value of $\sigma_{s,rep} = 189\ N/mm^2$ for a bar diameter of $16\ mm$.

## 2.3 Comparison results

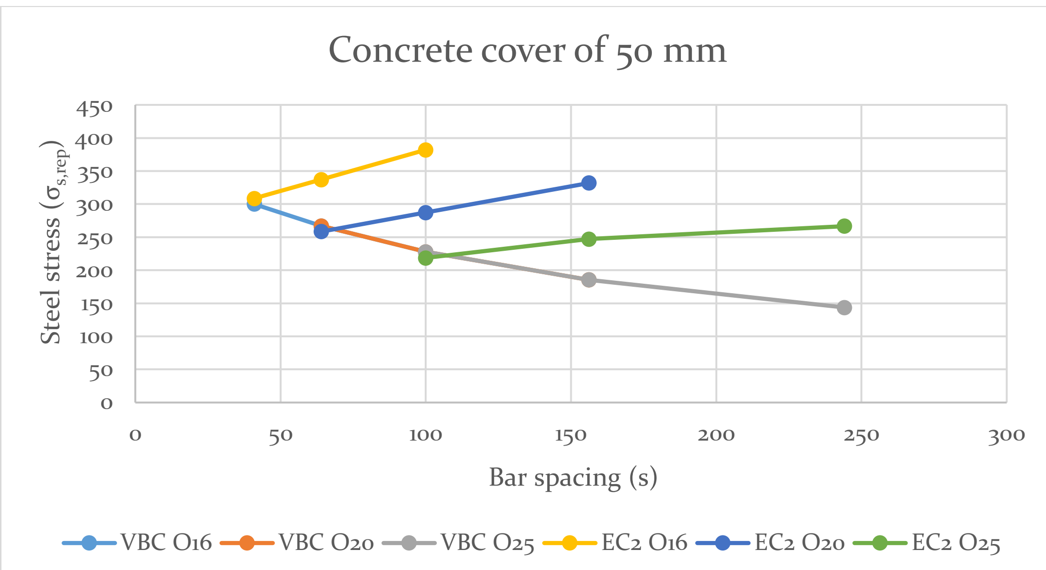
In this chapter the maximum allowable steel stress with limitation of $s_{r,max}$ calculated according to NEN-EN 1992-1-1 is compared to the maximum allowable steel stress calculated according to NEN 6720.

### *2.3.1 Comparison of $\sigma_{s,rep}$ after limiting $s_{r,max} \leq 10\emptyset$ with the value of $\sigma_{s,rep}$ in VBC1990 (NEN 6720)*

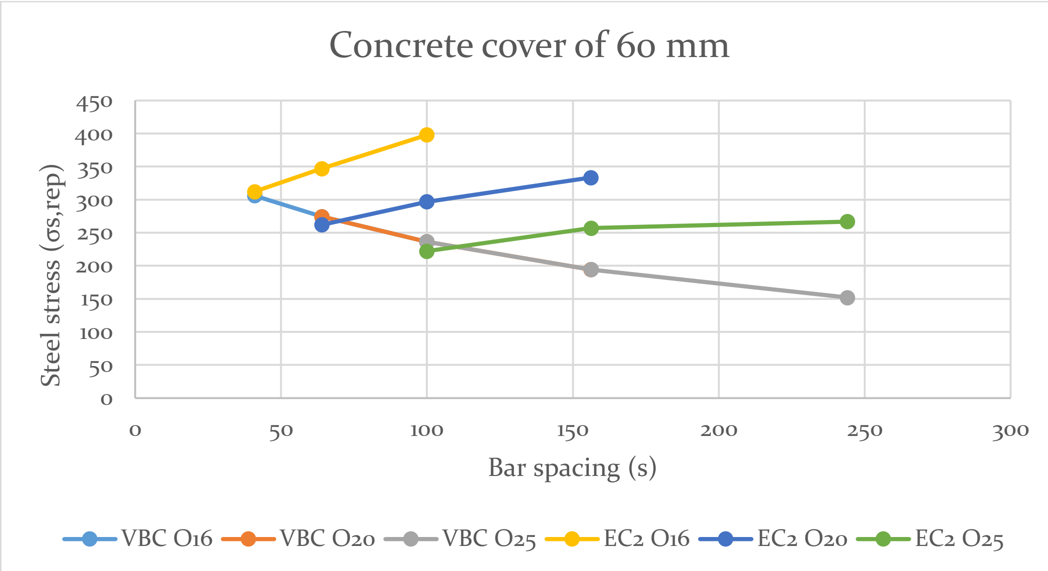
In the following graphs the results of the NEN-EN calculations are compared to the results of the NEN6720 calculations. In each graph the steel stress is depicted against the bar spacing. By looking at each graph it is clear that the steel stress increases substantially when the concrete cover is increased in the NEN-EN 1992 calculations.




*Graph 2-1: Maximum allowable steel stress for a cover of 40 mm according to VBC- and Eurocode 2 calculations after limitation of the maximum crack spacing with* $s_{r,max} \leq 10\emptyset$




*Graph 2-2: Maximum allowable steel stress for a cover of 50 mm according to VBC- and Eurocode 2 calculations after limitation of the maximum crack spacing with* $s_{r,max} \leq 10\emptyset$



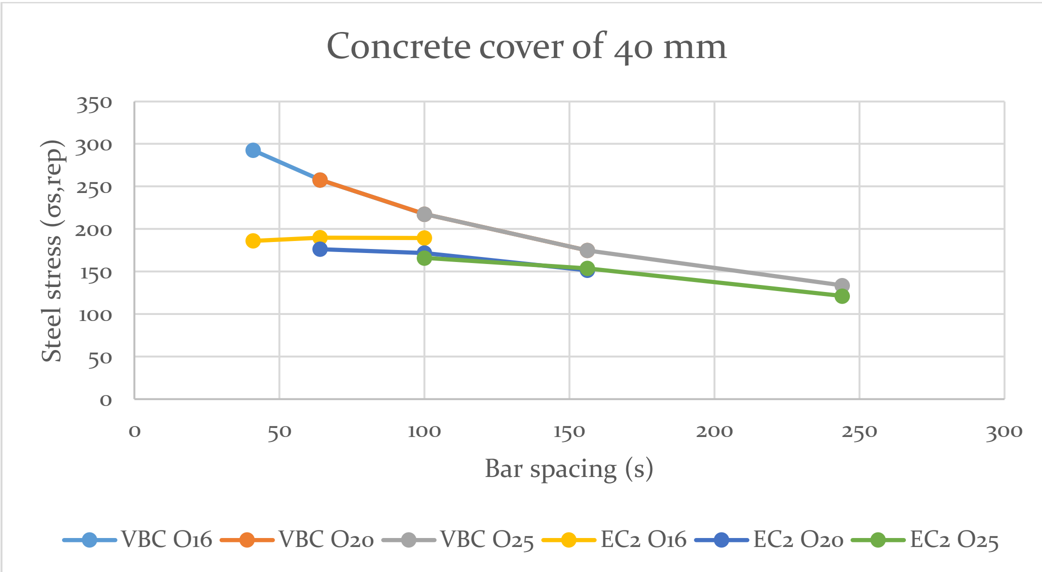

*Graph 2-3: Maximum allowable steel stress for a cover of 60 mm according to VBC- and Eurocode 2 calculations after limitation of the maximum crack spacing with $s_{r,max} \leq 10\emptyset$*

In Graph 2-1 to Graph 2-3 it can clearly be seen that the maximum allowable steel stress increases far above the values calculated in the NEN 6720. Since the maximum crack spacing is very small due to the limitation, we can see that the steel stress increases dramatically when a larger concrete cover is applied. These values seem very unrealistic since the steel stress is expected to be about $\sigma_s = 300\ N/mm^2$ in the S.L.S.

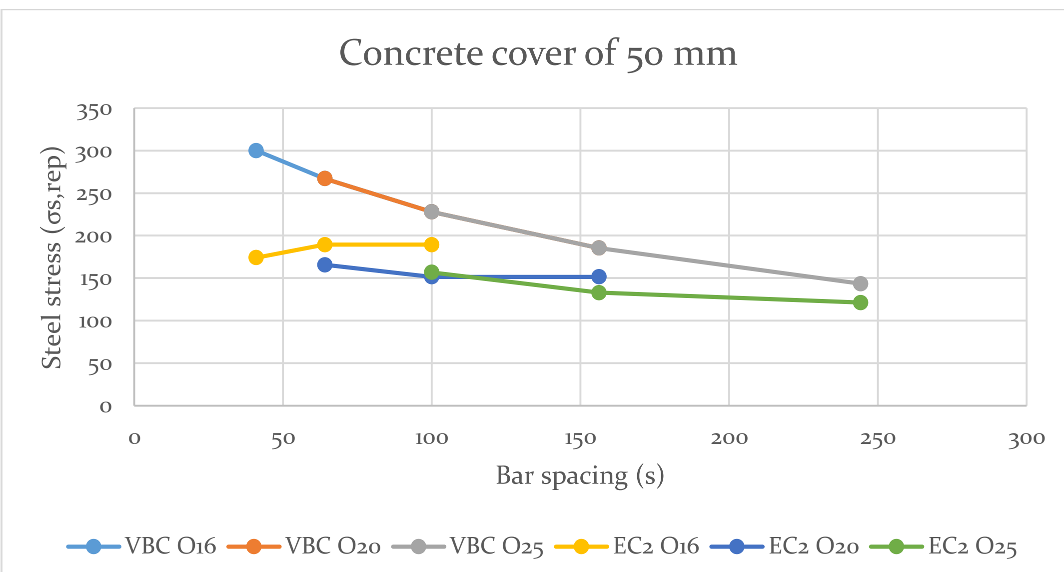
With this calculation a better insight has been obtained in the influence of the concrete cover on the maximum allowable steel stress. It is clear that the maximum crack spacing has an influence on the calculation of the maximum allowable steel stress.

### 2.3.2 Comparison of VBC with the limitation for $s_{r,max} \leq Max\{(50 - 0.8 * f_{ck})\emptyset\,; 15\emptyset\}$

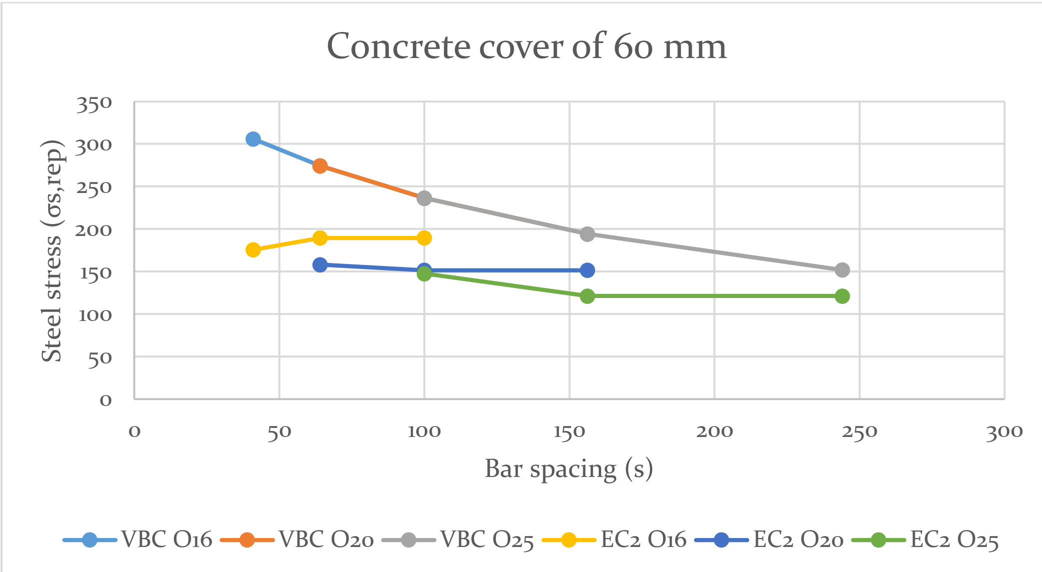
In the second limitation of the maximum crack spacing ($s_{r,max}$) we see that the steel stress also increases, but not as much as the previous limitation of the $s_{r,max}$. When we look at the values in chapter 2.2.2 we see that the difference in the allowable steel stress calculated with the NEN 6720 is equal to 15%. Before the upper limit value for the maximum crack spacing was applied ($\sigma_{s,rep} = 154\ N/mm^2$), the difference between the NEN 6720 calculations ($\sigma_{s,rep} = 217\ N/mm^2$) and the Eurocode calculations was about 41% for a concrete cover of 40 $mm$. This means that with the limitation of the crack spacing: $s_{r,max} \leq Max\{(50 - 0.8 * f_{ck})\emptyset\,; 15\emptyset\}$ rather favorable results are obtained. It should be mentioned however, that the values of the maximum allowable steel stress calculated with the Eurocode 2 equations are still smaller than the values of the steel stress calculated with VBC 1990. The amount of reinforcement that is needed to control the cracking behavior calculated with the NEN-EN 1992-1-1 regulations is still larger than the amount of reinforcement calculated with the NEN6720 regulations.




*Graph 2-4: Maximum allowable steel stress for a cover of 40 mm according to VBC- and Eurocode 2 calculations after limitation of the maximum crack spacing with* $s_{r,max} \leq Max\{(50 - 0.8 * f_{ck})\emptyset\, ; 15\emptyset\}$




*Graph 2-5: Maximum allowable steel stress for a cover of 50 mm according to VBC- and Eurocode 2 calculations after limitation of the maximum crack spacing with* $s_{r,max} \leq Max\{(50 - 0.8 * f_{ck})\emptyset\, ; 15\emptyset\}$




*Graph 2-6: Maximum allowable steel stress for a cover of 60 mm according to VBC- and Eurocode 2 calculations after limitation of the maximum crack spacing with* $s_{r,max} \leq Max\{(50 - 0.8 * f_{ck})\emptyset\ ; 150\emptyset\}$

## CONCLUSION

From the calculations which were carried out it is clear that the large differences in the steel stress occur when the bar spacing is smaller than $150\ mm$ (about 41%). When the bar spacing is larger than 150 mm the difference between the calculated steel stress smaller than 15%.

The limitation taken from the VB 1974/ 1984 (NEN 3880) leads to values which are too optimistic and are way above the expected value of the maximum steel stress in the S.L.S. ( $\sigma_s = 300\ N/mm^2$) (section 2.3.1).The alternative limitation of $s_{r,max}$ as proposed in the VARCE leads to the increase of the maximum allowable steel stress, but this increase still is not close to the NEN 6720 calculations presented in section 2.3.2.

Therefore it should be considered to refine the limitation of $s_{r,max}$ provided by VARCE where the main interest should lie upon the smaller bar distances, thus at the beams and not in the plates.

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# Appendix II: Analytical analysis of the cracking behavior of a beam subjected to bending

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# PREFACE

This report contains information regarding the analytical analysis of the experimental research of Dr. Ir. Braam. The information for the analytical analysis is based on the excel sheets: Test data Braam. The beams which were calculated were taken from the experimental research conducted by Dr. Ir. Braam.

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# 1 Analytical Analysis

In order to investigate whether the crack width calculations according to NEN-EN 1992-1-1 are in agreement with reality an analytical analysis was carried out. The cases which were calculated came from an experimental research done by Dr. Ir. Braam [1]. In this chapter the procedure of this analysis will be explained which will be followed by an example calculation.

## 1.1 Procedure

With the help of EXCEL, a calculation sheet was setup in which the calculations for the crack width was performed. First the data for the beams was collected followed by the crack width calculations according to the Eurocode 2. After the calculations were completed they were compared to the results from the experimental research.

During the laboratory tests 15 beams were tested: 12 T-beams and 3 rectangular beams. For the analytical analysis 2 beams were randomly chosen, namely: 1 T-beam (Beam 3) and 1 rectangular beam 13. The crack width in these beams was calculated using the EUROCODE 2 equations. These beams were subjected to bending. In the following chapter an example calculation of the rectangular beam (Beam-13) is presented. The calculation of T-beam 3 can be found in the EXCEL sheet: Test Data C. R. Braam: fully developed crack pattern (crack width calculation)

**Example: Rectangular Beam # 13:**

In the following tables the data for beam 13 that is used in the calculations are presented.

*Table 1-1: Dimensions and Reinforcement detailing Beam 13*

| **Beam # 13** | | | |
|---|---|---|---|
| Height (h) | 800 | [mm] | **[Experimental data of C.R. Braam table 6.3]** |
| Width (b) | 300 | [mm] | |
| **Main Reinforcement** | | | |
| Bar diameter | 20 | [mm] | |
| # bars | 4 | | |
| # Layers | 1 | | |
| **Web Reinforcement (per side)** | | | |
| Bar diameter | 12 | [mm] | |
| # Layers | 1 | | |
| Diameter stirrups | 10 | [mm] | |
| Bar spacing | 100 | [mm] | |
| Cover (c) | 30 | [mm] | |

*Table 1-2: Material Properties Rectangular Beam 13*

| | | | |
|---|---|---|---|
| L (length) | 5.500 | [m] | **[Experimental data of C.R. Braam table 6.3]** |
| B (width) [m] | 300 | [mm] | |
| H (height) [m] | 800 | [mm] | |
| $\mathbf{A_c}$ (cross section) | 240000 | [mm$^2$] | |
| $f_{ccm}$( Mean value of the cube compressive strength at the day of testing) | 55.9 | [N/mm$^2$] | **[Experimental data of C.R. Braam table 6.4]** |
| $f_{ck}$(Characteristic value of the cube compressive strength at the day of testing) | 47.9 | [N/mm$^2$] | **[Calculated: EUROCODE 2 table 3.1]** |
| $f_{cspl}$ (mean value of the cube tensile splitting strength at the day of testing) | 4.08 | [N/mm$^2$] | **[Experimental data of C.R. Braam table 6.4]** |
| $f_{ctm}$ (mean value of the cube tensile strength) | 3.96 | [N/mm$^2$] | **[Calculated: EUROCODE 2 table 3.1]** |
| $f_{ctk}$ (characteristic value of the cube tensile strength) | 2.77 | [N/mm$^2$] | |
| $E_c$ (Modulus of elasticity) | 31800 | [N/m$^2$] | **[Experimental data of C.R. Braam table 6.4]** |
| $E_{cm}$ (mean value of the young's modulus of concrete) | 36887 | [N/mm$^2$] | **[Calculated: Excel sheet M.T.M. Vlaar]]** |
| $E_c$ (Modulus of elasticity) | 43738 | [N/mm$^2$] | **[Calculated: Excel sheet M.T.M. Vlaar]** |

In Table 1-2 it can be seen that there are two values for the modulus of Elasticity. These values are calculated with the help of an EXCEL sheet that was made available by one of the Engineers at Arcadis: M.T.M. Vlaar. In his sheet it was possible to calculate several properties of the cross section, but only a few values were used for the analytical analysis. These values were: $E_{cm}$ and $E_c$ (modulus of elasticity of concrete); $x$ (height of compression zone); and properties of the cross section: $Z_b$; $Z_o$ and $W_o$ (section Modulus). These values were used in order to get a good approximation between the tested values and the calculated values.

## Crack width calculation according to the NEN-EN-1992-1-1

According to EUROCODE 2 section 7.3.4 the following equations for the calculation of the crack width can be used:

$w_k = s_{r,max}(\varepsilon_{sm} - \varepsilon_{cm})$

Where:

$w_k$: Design crack width

$s_{r,max}$: Maximum value of the crack spacing

$\varepsilon_{sm}$: Mean strain in the reinforcement under the relevant combination of loads, including the effect of imposed deformations and taking into account the effects of tension stiffening. Only the additional tensile strain beyond zero strain in the concrete is considered

$\varepsilon_{cm}$: Mean strain in concrete between cracks

The difference in strains $(\varepsilon_{sm} - \varepsilon_{cm})$ is calculated with the following expression:

$$\varepsilon_{sm} - \varepsilon_{cm} = \frac{\sigma_s - k_t \frac{f_{ct,eff}}{\rho_{peff}}(1 + \alpha_e \rho_{peff})}{E_s} \geq 0.6\frac{\sigma_s}{E_s}$$

In which:

$\sigma_s$: Stress in the tension reinforcement assuming a cracked section

$f_{ct,eff}$: the mean value of the tensile strength of the concrete effective at the time when the cracks may first be expected to occur
$f_{ct,eff} := f_{ctm}$
$\alpha_e$: Ratio of the modulus of elasticity: $\frac{E_s}{E_{cm}}$

$\rho_{peff}$: $\frac{A_s + \xi^2 A_p}{A_{c,eff}}$, where:

$A_{c,eff}$: The effective tension area. $A_{c,eff}$ is the area of concrete surrounding the tensile reinforcement of depth $h_{c,eff}$,where $h_{c,eff}$is the lesser of 2,5(h-d), (h-x)/3 or h/2 (Figure 1-1).

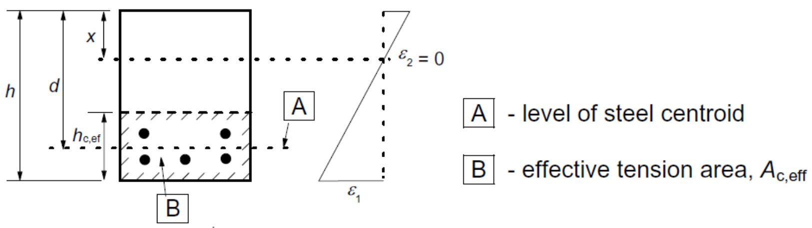



*Figure 1-1: Effective tension area according to EUROCODE 2*

$k_t$: Factor dependent on the duration of the load

$$k_t = 0{,}6 \text{ for short term loading}$$
$$k_t = 0{,}4 \text{ for long term loading}$$

The maximum crack spacing $s_{r,max}$ can be calculated with the following equation :

$s_{r,max} = k_3 c + k_1 k_2 k_4 \frac{\emptyset}{\rho_{peff}}$

Where:

c: Cover applied to the reinforcement

Ø: Bar diameter. Where a mixture of bar diameters is used in a section, the average diameter may be used.

$k_1$: Coefficient which takes account of the bond properties of the bonded reinforcement:

$k_1 = 0.8$ for high bond bars

$= 1.6$ for bars with an effectively plain surface (e.g. prestressing tendons)

$k_2$: Coefficient which takes account of the distribution of strain:

$k_2 = 0.5$ for bending

$k_2 = 1.0$ for pure tension

$k_3 = 3.4$

$k_4 = 0.425$

After the maximum value of the crack width : ($w_{k,max}$) is calculated the mean value of the crack width is calculated with the following equation [2].

In a fully developed crack pattern the following criterion holds for the mean value of the crack width:

$w_m = \gamma_s * \gamma_\infty \leq w_{serv}$ in which:

$w_{serv}$: the prevailing crack width criterion

$w_m$: the mean value of the crack width

$\gamma_s$: Factor for scatter:

$\gamma_s = 1.7$ (fully developed crack pattern for a beam subjected to bending)

$\gamma_\infty$: factor considering sustained load/alternating load:

$\sigma_s \leq 295 : \gamma_\infty = 1.3$

$\sigma_s \geq 295 : \gamma_\infty = \frac{1}{1-9*{\sigma_s}^3*10^{-9}}$

And so the mean value of the crack width is calculated with the following equation:

$$w_m = \frac{w_{k,max}}{1.7 * 1.3} = \frac{w_{k,max}}{2.2}$$

## 1.2 EXAMPLE CALCULATION OF THE CRACK WIDTH AT EACH LOADING STAGE

During the laboratory tests the actual load for all the beams was applied in four or five loading stages with a hand-operated hydraulic jack. Looking at the loadings scheme of the experiments it can be seen that the total load was applied in 2 points (Figure 1-2). The results that were registered by Dr. C. R. Braam [3] were based on one loading point only. In this point the total applied load during each loading stage was equal to half of the load applied by the hydraulic jack and half the weight of the loading frame. The loading stages for beam 13 can be found in Table 1-3 column 4. In the excel sheet : Test data 2: C.R. Braam fully developed crack patternv1.2 the loadings stages for beam 3 can be found.

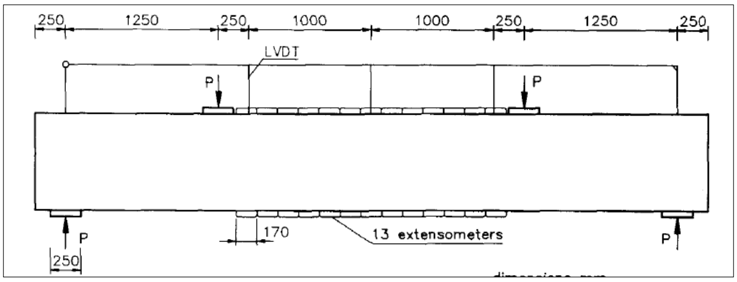



*Figure 1-2: Loading scheme experimental research*

*Table 1-3: Loading stages Beam 13*

| Loading stage: Beam 13 | Weight of the loading frame [kN] | Total load Hydraulic jack [kN] | Applied load per loading point [considered in the experiments] [kN] |
|---|---|---|---|
| 1 | 18 | 200 | 109 |
| 2 | 18 | 350 | 184 |
| 3 | 18 | 450 | 234 |
| 4 | 18 | 650 | 334 |

These loading stages are also used for the crack width calculation Excel.

In the crack width calculations the value of steel stress generated at each loading stage needs to be calculated. Since the beam is subjected to bending, the bending moment was calculated at each loading stage with the following equation:

$M_{applied} = F_{applied} * l$ , In which :

$F_{applied}$: The load applied in each loading stage

$l = 1250\ mm$ (the distance between the top-support and bottom support).

An example calculation is given below for the first loading stage $P = 109\ kN$ moment. The calculation for the other loading stages can be found in the Excel sheet: Test data 2: C.R. Braam fully developed crack patternv1.2.

**The calculation of the crack width for $P = 109\ kN$**

The applied moment is calculated with the following equation:

$$M_{applied} = F_{applied} * l =* 109000\ N * 1250\ mm = 136.25 * 10^6\ Nmm$$

The value for the steel stress is equal to: $\sigma_s = \frac{M_{applied}}{A_s(d_s - \frac{1}{3}x)}$ :

Before we can calculate the maximum value of the steel stress we need to calculate the exact value of the height of the compression zone ($x$).This was also calculated in excel with the help of stress strain diagrams.

The Calculation of the height of the compression zone for beam 13 is explained below:
Total amount of main reinforcement applied: $A_{smain} = \frac{1}{4}\pi\emptyset^2 * \#bars * \#layers = \frac{1}{4} * \pi * 20^2 * 4 * 1 = 1256\ mm^2$

Total amount of web reinforcement applied: $A_{sweb} = \frac{1}{4}\pi\emptyset^2 * \#bars * \#layers = \frac{1}{4} * \pi * 12^2 * 2 * 1 = 226\ mm^2$

Total amount of reinforcement applied : $A_{s,total} = A_{smain} + A_{sweb} = 1256 + 226 = 1482\ mm^2$

Effective depth : $d_s = \frac{A_{s1} * d_1 + A_{s2} * d_2}{A_{s1} + A_{s2}}$ where:

$A_{s1}$: The amount of main reinforcement = 1256 mm²

$A_{s2}$: Amount of web reinforcement = 226 mm²

$d_1$:Postion of the main reinforcement from top of the beam

$$d_1 = h - \left(c + \emptyset_{stirrups} + \frac{\emptyset}{2}\right) = 800 - \left(30 + 10 + \frac{20}{2}\right) = 750\ mm$$

$d_2$:Position of the web reinforcement from the top of the beam

$$d_2 = h - \left(c + \emptyset_{stirrups} + \frac{\emptyset}{2} + 100\right) = 800 - \left(30 + 10 + \frac{20}{2} + 100\right) = 650\ mm$$

So the value for $d_s = \frac{A_{s1} * d_1 + A_{s2} * d_2}{A_{s1} + A_{s2}} = \frac{1250 * 750 + 226 * 650}{1250 + 226} = 734.75\ mm$

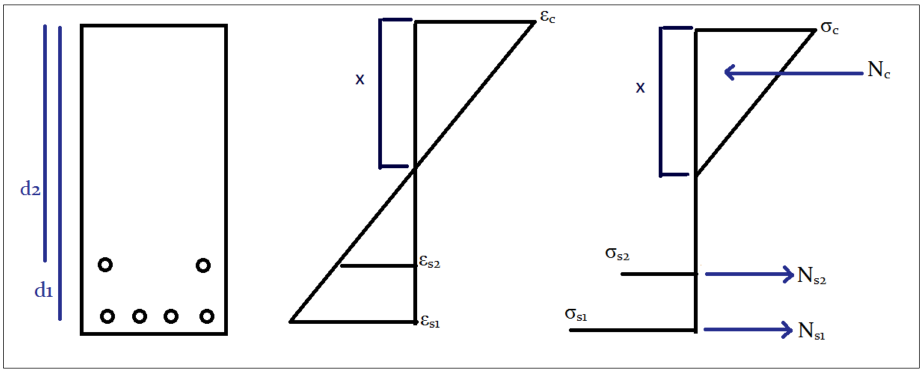



*Figure 1-3: Stress -Strain diagrams beam 13*

The values for the strains in Figure 1-3 are presented below:

$$\varepsilon_{s1} = \frac{d_1 - x}{x} * \varepsilon_c$$

$$\varepsilon_{s2} = \frac{d_2 - x}{x} * \varepsilon_c$$

The values for the Stresses then become:

$\sigma_{s1} = \varepsilon_{s1} * E_s$

$\sigma_{s2} = \varepsilon_{s2} * E_s$

For the calculation of x we take the equilibrium of forces: $-N_c + Ns_1 + N_{s2} = 0$

Where:

$N_c = 0.5 * x * \varepsilon_c * b * E_c$

$N_{s1} = \frac{d_1 - x}{x} * E_s * A_{s1}$

$N_{s2} = \frac{d_2 - x}{x} * E_s * A_{s2}$

Substituting the above into the equilibrium of forces we get:

$-0.5 * x * \varepsilon_c * b * E_c + \frac{d_1 - x}{x} * E_s * A_{s1} + \frac{d_2 - x}{x} * E_s * A_{s2} = 0$

Multiplying the equation above with $\frac{x}{\varepsilon_c}$ we get:

$(d_1 - x) * E_s * A_{s1} + (d_2 - x) * E_s * A_{s2} - x^2 * b * E_c \rightarrow \quad d_1 * E_s * A_{s1} - E_s * A_{s1} * x + d_2 * E_s * A_{s2} - E_s * A_{s2} * x - 0.5 * x^2 * b * E_c$

We can use the $abc\text{-}formula$ for the calculation of the height of the compression zone $x$:

$0.5 * b * E_c * x^2 + (E_s * A_{s1} + E_s * A_{s2})x - (d_1 A_{s1} E_s + d_2 A_{s2} E_s) = 0$

$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4 * a * c}}{2a}$

Where:
$a = 0.5 * b * E_c$

$b = E_s * A_{s1} + E_s * A_{s2}$

$c = d_1 A_{s1} E_s + d_2 A_{s2} E_s$

So this equation is substituted in excel and for beam 13 we get: $x = 184.85\ mm$ (see Excel file: Test Data 2 C.R. Braam fully developed crack pattern c=30 EC2, hoogte drukzone)

After the height of the compression zone is calculated we can find the value of the steel stress with the following equation:

$\sigma_s = \frac{M_{applied}}{A_s(d_s - \frac{1}{3}x)} = \frac{136.25 * 10^6}{1482 * (734.75 - \frac{1}{3} * 184.85)} = 136.57\ N/mm^2$

After the steel stress is calculated the amount of effective reinforcement ($\rho_{peff}$) is necessary for the calculation of the maximum crack spacing ($s_{r,\max} = \mathrm{k}_3 \mathrm{c} + \mathrm{k}_1 \mathrm{k}_2 \mathrm{k}_4 \frac{\emptyset_\mathrm{k}}{\rho_\mathrm{peff}}$).

The amount of effective reinforcement can be calculated with the following equation:

$\rho_{peff} = \frac{A_s}{A_{ceff}}$

In which $A_s$: the total amount off steel applied in the beam and $A_{ceff} = b * h_{ceff}$(the effective concrete area , where $b$ is the width of the beam and

$h_{ceff}$: Effective depth is equal to the smallest value of the following equations:

$$h_{ceff} \leq \begin{cases} 1: 2.5 * (h - d_s) = 2.5 * (800 - 734.75) = 163.15\ mm \\ 2: \frac{h - x_{rep}}{3} = \frac{800 - 184.85}{3} = 205\ mm \\ 3: \frac{h}{2} = \frac{800}{2} = 400\ mm \end{cases}$$

We can see that the minimum value of the above is equal to $h_{ceff} = 163.125\ mm$
Now the value of the effective concrete area can be calculated:

$$A_{ceff} = b * h_{ceff} = 300 * 163.14 = 48941\ mm^2$$

The amount of effective reinforcement is equal to: $\rho_{peff} = \frac{A_s}{A_{ceff}} = \frac{1482}{48941} = 0.0303$

The next step is to calculate the maximum value of the crack spacing ($s_{r,max}$):

$\mathrm{s_{r,max}} = \mathrm{k_3 c + k_1 k_2 k_4 \frac{\emptyset_k}{\rho_{peff}}}$ ; where:

*Table 1-4: factors for $s_{r,max}$*

| k1 | k2 | k3 | k4 |
|---|---|---|---|
| **0.8** | 0.5 | 3.4 | 0.425 |

And: $\emptyset_k = 20\ mm$ (bar diameter of the main reinforcement applied)

So $s_{r,max} = 3.4 * 50 + 0.8 * 0.5 * 0.425 * \frac{20}{0.0303} = 282\ mm$

After the maximum crack spacing is found we need to calculate the value of the strain difference:

The strain difference is equal to the maximum value of :

$$\varepsilon_{sm} - \varepsilon_{cm} = \max\left(\frac{\sigma_s - k_t \frac{f_{ct,eff}}{\rho_{peff}}(1 + \alpha_e \rho_{peff})}{E_s};\ 0.6\frac{\sigma_s}{E_s}\right)$$

Where: $\mathrm{k_t} = 0.6$ (short term loading)

$\mathrm{f_{ct,eff} = f_{ctm} = 3.96\ N/mm^2}$

$\rho_{p,eff} = \frac{A_s}{A_{ceff}} = \frac{1482}{48941} = 0.0303$

$\alpha_e = \frac{E_s}{E_{cm}} = \frac{200000}{31800} = 6.29$

$\mathrm{E_s = 200000\ N/mm^2}$

And so the strain difference ( $\varepsilon_{sm} - \varepsilon_{cm}$) is equal to:

$$\frac{\sigma_s - k_t \frac{f_{ct,eff}}{\rho_{peff}}(1 + \alpha_e \rho_{peff})}{E_s} = \frac{134.98 - 0.6 * \frac{3.95}{0.0303}(1 + 6.29 * 0.0303)}{200000} = 2.29 * 10^{-4}$$

And the lower limit value of $\varepsilon_{sm} - \varepsilon_{cm}$ is equal to:

$0.6 * \frac{\sigma_s}{E_s} = 0.6 * \frac{136.57}{200000} = 4.05 * 10^{-4}$

$$\varepsilon_{sm} - \varepsilon_{cm} = \max(2.29 * 10^{-4}; 4.05 * 10^{-4})$$

The strain difference is equal to: $\varepsilon_{sm} - \varepsilon_{cm} = 4.05 * 10^{-4}$

The crack width can now be calculated:

$w_{k,max} = s_{r,max}(\varepsilon_{sm} - \varepsilon_{cm}) = 282 * 4.05 * 10^{-4} = 0.11\ \text{mm}$

For a fully developed crack pattern the following equation is valid for the calculation of the mean value of the crack with ($w_{mean}$):

$$w_{mean} = \frac{w_{k,max}}{\gamma_s * \gamma_\infty} = \frac{0.11}{2.2} = 0.052\ \text{mm}$$

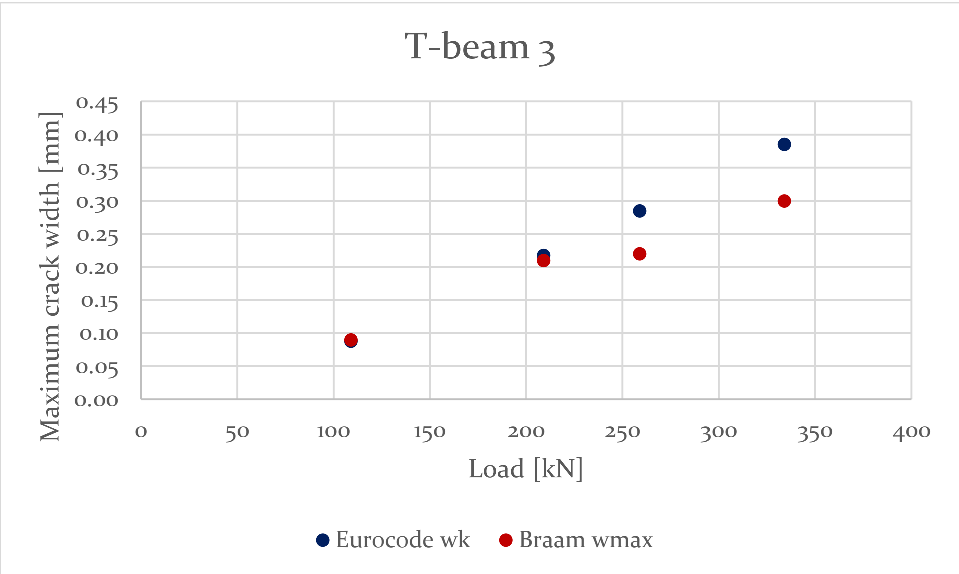
## 1.3 RESULTS: COMPARISON OF THEORY WITH PRACTICE

In this chapter the results of the 2 beams: T-beam 3 Rectangular beam 13 of which the crack width was calculated with the Eurocode equations are compared to the crack width registered in the experiments at each loading stage.

### T-beam 3:
### Maximum Crack width ($w_{k,max}$)

*Table 1-5: Comparison calculated value of $w_{k,max}$ with the tested value of $w_{max}$ for T-beam 3*

| $F_{applied}$ | Eurocode $w_{k,max}$ | Experiments Braam: $w_{max}$ | Difference (%) |
|---|---|---|---|
| [kN] | [mm] | [mm] | [%] |
| **109** | 0.09 | 0.09 | 2 |
| **209** | 0.22 | 0.21 | 4 |
| **259** | 0.28 | 0.22 | 30 |
| **334** | 0.39 | 0.30 | 29 |



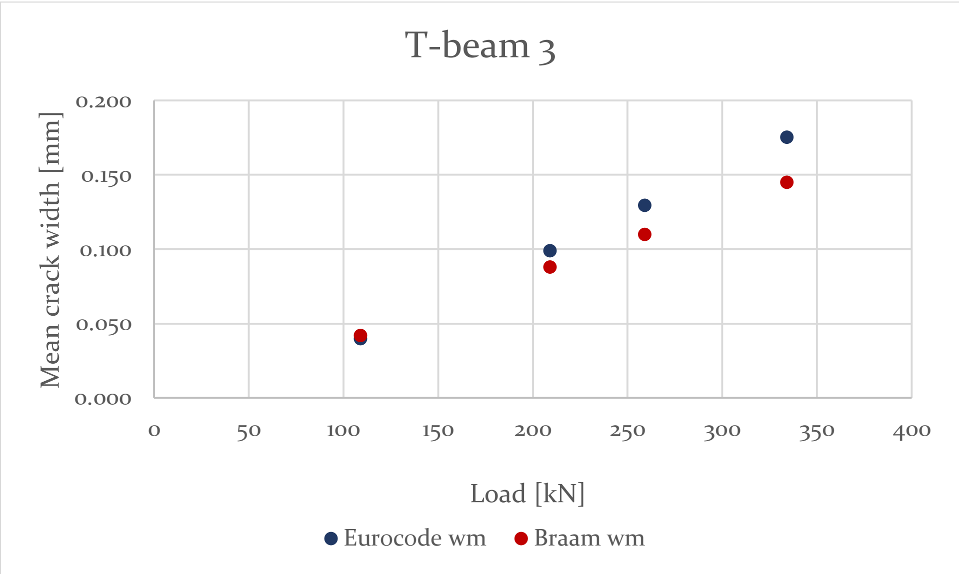

*Graph 1-1: Comparison of the calculated value of the crack width with the tested value of T-beam 3*

Looking at the results of T-beam 3 it can be seen that the calculated values of the crack width are larger than the tested values of the crack width. In Table 1-5 it can be seen that with increasing load the calculated crack width increases with 30%.

### Mean value of the crack width ($w_m$)

*Table 1-6: Comparison calculated value of $w_m$ with the tested value of $w_m$ for T-beam 3*

| $F_{applied}$ | Eurocode $w_m$ | Experiments Braam: $w_m$ | Difference (%) |
|---|---|---|---|
| [kN] | [mm] | [mm] | [%] |
| **109** | 0.040 | 0.042 | 5 |
| **209** | 0.099 | 0.088 | 13 |
| **259** | 0.130 | 0.11 | 18 |
| **334** | 0.175 | 0.145 | 21 |



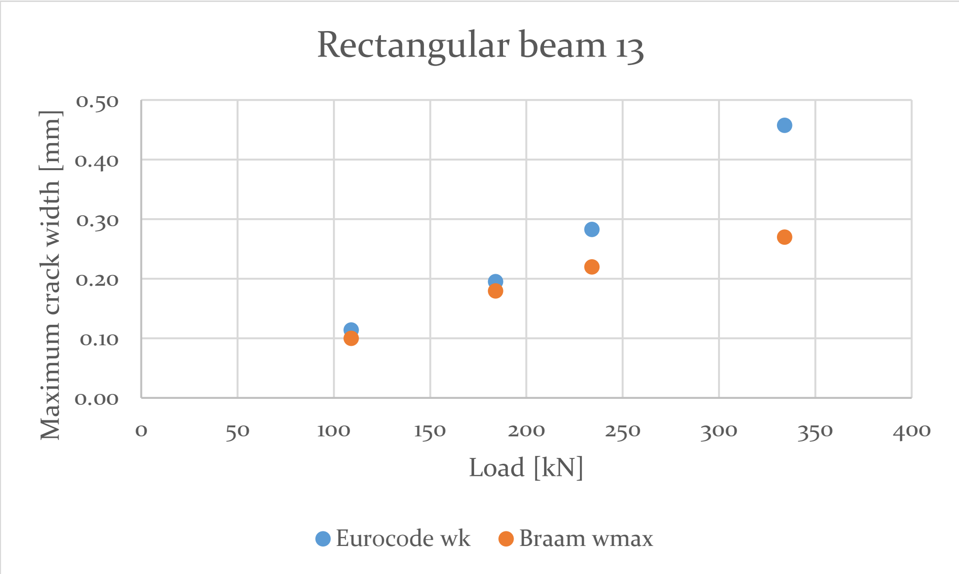

*Graph 1-2: Comparison of the mean value of the  mean crack width with the tested value of T-beam 3*

In Table 1-6 it can be seen that the  mean value of the crack width calculated in excel is also larger than the tested value of T-beam 3. The maximum difference is about 21% in the last loading stage. Graph 1-2 shows that with increasing load the mean crack width according to Eurocode 2 increases which also leads to a larger difference between the tested value of the mean crack width.

### Rectangular beam 13:
### Maximum Crack width ($w_{k,max}$)

*Table 1-7: Comparison calculated value of $w_{k,max}$ with the tested value of $w_{max}$ for Rectangular beam 13*

| $F_{applied}$ | Eurocode $w_k$ | Experiments Braam: $w_k$ | Difference (%) |
|---|---|---|---|
| [kN] | [mm] | [mm] | [%] |
| **109** | 0.11 | 0.10 | 14 |
| **184** | 0.20 | 0.18 | 9 |
| **234** | 0.28 | 0.22 | 29 |
| **334** | 0.46 | 0.27 | 70 |



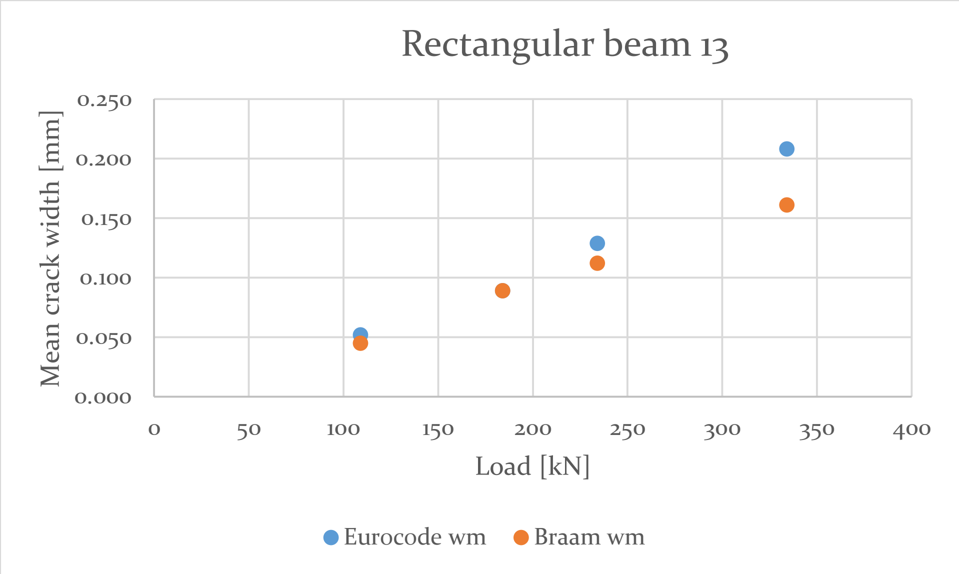

*Graph 1-3: Comparison of the calculated value of the crack width with the tested value of Rectangular beam 13*

Looking at the results of rectangular beam 13 in Table 1-7 we see that the difference between the calculated values of the crack width and the tested values reaches 70% in the last loading stage. In Graph 1-3 we see that the values in the first two loading stages are overlapping each other. In the last two loading stages we see that the Eurocode 2 calculations is about with a 70% larger than the tested value. So also in this case we see that the difference between the calculated and the tested value of the maximum crack width increases with an increasing load.

## Mean value of the crack width ($w_m$)

*Table 1-8: Comparison calculated value of $w_m$ with the tested value of $w_m$ for Rectangular beam 13*

| $F_{applied}$ | Eurocode $w_m$ | Experiments Braam: $w_m$ | Difference (%) |
|---|---|---|---|
| [kN] | [mm] | [mm] | [%] |
| **109** | 0.052 | 0.045 | 15 |
| **184** | 0.089 | 0.089 | 0 |
| **234** | 0.129 | 0.112 | 15 |
| **334** | 0.208 | 0.161 | 29 |




*Graph 1-4: Comparison of the mean value of the crack width with the tested value of Rectangular beam 13*

When we look at the mean value of the crack width in Table 1-8 we see that the maximum difference between the calculated and the tested value is about 29% ,which is at the last loading stage: $P = 334\ kN$. In all the other loading stages the difference is smaller than 15%.

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# Appendix III: Analytical analysis of the influence of the concrete cover on the cracking behavior of a beam subjected to bending

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# PREFACE

This report contains information regarding the analytical analysis of the experimental research of Dr. Ir. Braam. The information for the analytical analysis is based on the excel sheets: Test data Braam invloed dekking EC2 and Test Data Braam invloed dekking Vb74/84. The beams which were calculated were taken from the experimental research conducted by Dr. Ir. Braam.

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# 1 ANALYTICAL ANALYSIS

In order to investigate what influence the concrete cover has on the cracking behavior of a beam subjected to bending according to the different regulations: NEN-EN1992-1-1 and VB74/84 an analytical analysis was carried out. The same cases which were calculated in ANNEX 2: Beam 13 and Beam 3 were analyzed. These cases came from the experimental research which was conducted by C. R. Braam [1]. In this section the procedure of the analysis will be explained which will be followed by an example calculation.

## 1.1 PROCEDURE

With the help of EXCEL, a calculation sheet was setup in which the calculations for the crack width was performed. The influence of the concrete cover was investigated by comparing the Eurocode 2 (crack width expressions) with the VB74/84 (regulations regarding crack width control). In the final calculation the value of the crack spacing was limited and then the influence of this limitation on mean value- and the maximum value of the crack width was analyzed according to both regulations (Eurocode 2 and VB74/84). This analysis has been performed for several valued of the concrete cover.

## 1.2 VARIATION OF CONCRETE COVER

The calculations were carried out for the 2 beams which were analyzed numerically in DIANA: T-beam3 and Rectangular beam 13. Both beams were subjected to bending. The cover was varied for both beams. In the T-beam (Beam 3) the applied concrete cover was varied with the following values: $c = 20\,mm$ ; $c = 50\,mm$ and $c = 70\,mm$. In the rectangular beam (Beam 13) the applied concrete cover was varied with the following values: $c = 30\,mm$ ; $c = 50\,mm$ and $c = 70\,mm$. In section 1.3 an example calculation of beam 13 based on the Eurocode 2 equations regarding crack width control is presented. The example calculation of beam 13 based on the VB74/84 equations regarding crack width control is presented in section 1.4.

## 1.3 EXAMPLE CALCULATION NEN-EN 1992-1-1

The following tables present the data for beam 13 that was used during the calculations.

*Table 1-1: Dimensions and Reinforcement detailing Beam 13*

| Beam # 13 | | | |
|---|---|---|---|
| Height (h) | 800 | [mm] | **[Experimental data of C.R. Braam table 6.3]** |
| Width (b) | 300 | [mm] | |
| **Main Reinforcement** | | | |
| Bar diameter | 20 | [mm] | |
| # bars | 4 | | |
| # Layers | 1 | | |
| **Web Reinforcement (per side)** | | | |
| Bar diameter | 12 | [mm] | |
| # Layers | 1 | | |
| Diameter stirrups | 10 | [mm] | |
| Bar spacing | 100 | [mm] | |
| Cover (c) | 30 | [mm] | |

The material properties of beam 13 are presented in Table 1-2.

*Table 1-2: Material Properties Rectangular Beam 13*

| | | | |
|---|---|---|---|
| L (length) | 5500 | [mm] | **[Experimental data of C.R. Braam table 6.3]** |
| B (width) | 300 | [mm] | |
| H (height) | 800 | [mm] | |
| $\mathbf{A_c}$ (cross section) | 240000 | [mm$^2$] | |
| $\boldsymbol{f_{ccm}}$( Mean value of the cube compressive strength at the day of testing) | 55.9 | [N/mm$^2$] | **[Experimental data of C.R. Braam table 6.4]** |
| $\boldsymbol{f_{ck}}$(Characteristic value of the cube compressive strength at the day of testing) | 47.9 | [N/mm$^2$] | **[Calculated: EUROCODE 2 table 3.1]** |
| $\boldsymbol{f_{cspl}}$ (mean value of the cube tensile splitting strength at the day of testing) | 4.08 | [N/mm$^2$] | **[Experimental data of C.R. Braam table 6.4]** |
| $\boldsymbol{f_{ctm}}$ (mean value of the cube tensile strength) | 3.96 | [N/mm$^2$] | **[Calculated: EUROCODE 2 table 3.1]** |
| $\boldsymbol{f_{ctk}}$ (characteristic value of the cube tensile strength) | 2.77 | [N/mm$^2$] | |
| $\boldsymbol{E_c}$ (Modulus of elasticity) | 31800 | [N/m$^2$] | **[Experimental data of C.R. Braam table 6.4]** |

## Crack width calculation according to the NEN-EN-1992-1-1

According to EUROCODE 2 section 7.3.4 the following equations for the calculation of the crack width can be used:

$w_k = s_{r,max}(\varepsilon_{sm} - \varepsilon_{cm})$

Where:

$w_k$: Design crack width

$s_{r,max}$: Maximum value of the crack spacing

$\varepsilon_{sm}$: Mean strain in the reinforcement under the relevant combination of loads, including the effect of imposed deformations and taking into account the effects of tension stiffening. Only the additional tensile strain beyond zero strain in the concrete is considered.

$\varepsilon_{cm}$: Mean strain in concrete between cracks

The difference in strains $(\varepsilon_{sm} - \varepsilon_{cm})$ is calculated with the following expression:

$$\varepsilon_{sm} - \varepsilon_{cm} = \frac{\sigma_s - k_t \frac{f_{ct,eff}}{\rho_{peff}}(1 + \alpha_e \rho_{peff})}{E_s} \geq 0.6 \frac{\sigma_s}{E_s}$$

In which:

$\sigma_s$: Stress in the tension reinforcement assuming a cracked section

$f_{ct,eff}$: the mean value of the tensile strength of the concrete effective at the time when the cracks may first be expected to occur
$f_{ct,eff} := f_{ctm}$
$\alpha_e$: Ratio of the modulus of elasticity: $\frac{E_s}{E_{cm}}$

$\rho_{peff}$: $\frac{A_s + \xi^2 A_p}{A_{c,eff}}$, where:

$A_{c,eff}$: The effective tension area. $A_{c,eff}$ is the area of concrete surrounding the tensile reinforcement of depth $h_{c,eff}$,where $h_{c,eff}$is the lesser of 2,5(h-d), (h-x)/3 or h/2 (Figure 1-1).

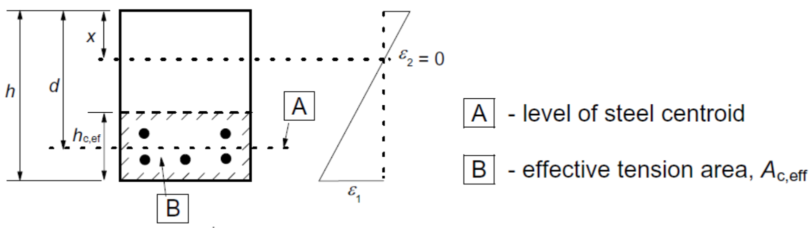



*Figure 1-1: Effective tension area according to EUROCODE 2*

$k_t$: Factor dependent on the duration of the load
$k_t = 0{,}6$ for short term loading
$k_t = 0{,}4$ for long term loading

The maximum crack spacing $s_{r,max}$ can be calculated with the following equation :
$s_{r,max} = k_3 c + k_1 k_2 k_4 \frac{\emptyset_k}{\rho_{peff}}$

Where:

c: Cover applied to the longitudinal reinforcement

$\emptyset_k$: Bar diameter. Where a mixture of bar diameters is used in a section, the average diameter may be used.

$k_1$: Coefficient which takes the bond properties of the bonded reinforcement into account:

$k_1 = 0.8$ for high bond bars

$= 1.6$ for bars with an effectively plain surface (e.g. prestressing tendons)

$k_2$: Coefficient which takes account of the distribution of strain:

$k_2 = 0.5$ for bending

$k_2 = 1.0$ for pure tension

$k_3 = 3.4$

$k_4 = 0.425$

After the maximum value of the crack width : ($w_{k,max}$) is calculated the mean value of the crack width is calculated with the following equation [2].

In a fully developed crack pattern the following criterion holds for the mean value of the crack width:

$w_m * \gamma_s * \gamma_\infty \leq w_{serv}$

in which:

$w_{serv}$: the prevailing crack width criterion

$w_m$: the mean value of the crack width

$\gamma_s$: Factor for scatter:

$\gamma_s = 1.7$ (fully developed crack pattern for a beam subjected to bending)

$\gamma_\infty$: factor considering sustained load/alternating load:

$\sigma_s \leq 295 : \gamma_\infty = 1.3$

$\sigma_s \geq 295 : \gamma_\infty = \frac{1}{1-9*{\sigma_s}^3*10^{-9}}$

And so the mean value of the crack width is calculated with the following equation:

$$w_m = \frac{w_{k,max}}{1.7 * 1.3} = \frac{w_{k,max}}{2.2}$$

**Example calculation of the crack width at each loading stage**

The beams which were calculated (T-Beam 3 and Rectangular Beam 13) in Excel were taken from the experiments which were carried out by C.R. Braam.

During the laboratory tests the actual load for these beams was applied in four loading stages with a hand-operated hydraulic jack. Looking at the loading scheme of the experiments it can be seen that the total load was applied in 2 points (Figure 1-2). The results that were registered in the experiments were based on one loading point only. In this point the total applied load during each loading stage was equal to half of the load applied by the hydraulic jack and half the weight of the loading frame. The loading stages for beam 13 can be found in Table 1-3 column 4. In the excel sheet : Test data 2: C.R. Braam invloed dekking the loading stages of T-beam 3 can be found.

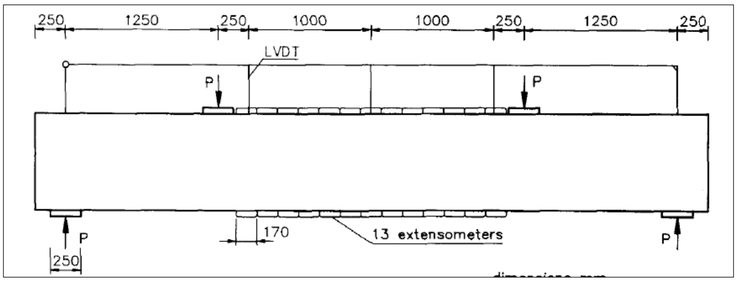



*Figure 1-2: Loading scheme experimental research*

*Table 1-3: Loading stages Beam 13*

| Loading stage: Beam 13 | Weight of the loading frame [kN] | Total load Hydraulic jack [kN] | Applied load per loading point [considered in the experiments] [kN] |
|---|---|---|---|
| 1 | 18 | 200 | 109 |
| 2 | 18 | 350 | 184 |
| 3 | 18 | 450 | 234 |
| 4 | 18 | 650 | 334 |

These loading stages are used for the crack width calculation Excel.

In the crack width calculations the value of steel stress generated at each loading stage needs to be calculated. Since the beam is subjected to bending, the bending moment was calculated at each loading stage with the following equation:

$M_{applied} = F_{applied} * l$ , In which :

$F_{applied}$: The load applied in each loading stage

$l = 1250\ mm$ (the distance between the top-support and bottom support).

An example calculation is presented below for the first loading stage: $P = 109\ kN$. All the other calculations can be found in the Excel sheets: Test data 2: C.R. Braam invloed dekking.

### Calculation of the crack width for $P = 109\ kN$

The applied moment is calculated with the following equation:

$$M_{applied} = F_{applied} * l =* 109000\ N * 1250\ mm = 136.25 * 10^6\ Nmm$$

The value for the steel stress is equal to: $\sigma_s = \frac{M_{applied}}{A_s(d_s - \frac{1}{3}x)}$ :

Before we can calculate the maximum value of the steel stress we need to calculate the exact value of the height of the compression zone ($x$).This was also calculated in excel with the help of stress strain diagrams.

The Calculation of the height of the compression zone for beam 13 is explained below:
Total amount of main reinforcement applied: $A_{smain} = \frac{1}{4}\pi\emptyset^2 * \#bars * \#layers = \frac{1}{4} * \pi * 20^2 * 4 * 1 = 1256\ mm^2$

Total amount of web reinforcement applied: $A_{sweb} = \frac{1}{4}\pi\emptyset^2 * \#bars * \#layers = \frac{1}{4} * \pi * 12^2 * 2 * 1 = 226\ mm^2$

Total amount of reinforcement applied : $A_{s,total} = A_{smain} + A_{sweb} = 1256 + 226 = 1482\ mm^2$

Effective depth : $d_s = \frac{A_{s1} * d_1 + A_{s2} * d_2}{A_{s1} + A_{s2}}$ where:

$A_{s1}$: The amount of main reinforcement = 1256 mm²

$A_{s2}$: Amount of web reinforcement = 226 mm²

$d_1$:Postion of the main reinforcement from top of the beam

$$d_1 = h - \left(c + \emptyset_{stirrups} + \frac{\emptyset}{2}\right) = 800 - \left(30 + 10 + \frac{20}{2}\right) = 750\ mm$$

$d_2$:Position of the web reinforcement from the top of the beam

$$d_2 = h - \left(c + \emptyset_{stirrups} + \frac{\emptyset}{2} + 100\right) = 800 - \left(30 + 10 + \frac{20}{2} + 100\right) = 650\ mm$$

So the value for $d_s = \frac{A_{s1} * d_1 + A_{s2} * d_2}{A_{s1} + A_{s2}} = \frac{1250 * 750 + 226 * 650}{1250 + 226} = 734.75\ mm$

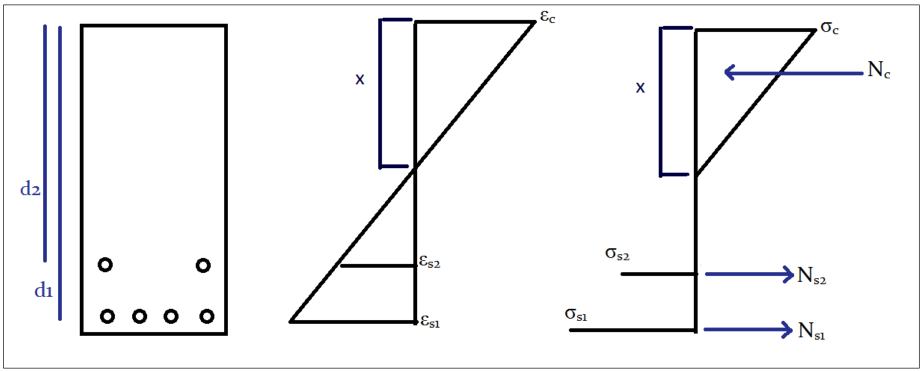



*Figure 1-3: Stress -Strain diagrams beam 13*

The values for the strains in Figure 1-3 are presented below:

$$\varepsilon_{s1} = \frac{d_1 - x}{x} * \varepsilon_c$$

$$\varepsilon_{s2} = \frac{d_2 - x}{x} * \varepsilon_c$$

The values for the Stresses then become:

$\sigma_{s1} = \varepsilon_{s1} * E_s$

$\sigma_{s2} = \varepsilon_{s2} * E_s$

For the calculation of x we take the equilibrium of forces: $-N_c + Ns_1 + N_{s2} = 0$

Where:

$N_c = 0.5 * x * \varepsilon_c * b * E_c$

$N_{s1} = \frac{d_1 - x}{x} * E_s * A_{s1}$

$N_{s2} = \frac{d_2 - x}{x} * E_s * A_{s2}$

Substituting the above into the equilibrium of forces we get:

$-0.5 * x * \varepsilon_c * b * E_c + \frac{d_1 - x}{x} * E_s * A_{s1} + \frac{d_2 - x}{x} * E_s * A_{s2} = 0$

Multiplying the equation above with $\frac{x}{\varepsilon_c}$ we get:

$(d_1 - x) * E_s * A_{s1} + (d_2 - x) * E_s * A_{s2} - x^2 * b * E_c \rightarrow \quad d_1 * E_s * A_{s1} - E_s * A_{s1} * x + d_2 * E_s * A_{s2} - E_s * A_{s2} * x - 0.5 * x^2 * b * E_c$

We can use the $abc\text{-}formula$ for the calculation of the height of the compression zone $x$:

$0.5 * b * E_c * x^2 + (E_s * A_{s1} + E_s * A_{s2})x - (d_1 A_{s1} E_s + d_2 A_{s2} E_s) = 0$

$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4 * a * c}}{2a}$

Where:
$a = 0.5 * b * E_c$

$b = E_s * A_{s1} + E_s * A_{s2}$

$c = d_1 A_{s1} E_s + d_2 A_{s2} E_s$

So this equation is substituted in excel and for beam 13 we get: $x = 184.85\ mm$ (see Excel file: Test Data 2 C.R. Braam invloed dekking c=30 EC2, hoogte drukzone)

After the height of the compression zone is calculated we can find the value of the steel stress with the following equation:

$\sigma_s = \frac{M_{applied}}{A_s(d_s - \frac{1}{3}x)} = \frac{136.25*10^6}{1482*(734.75 - \frac{1}{3}*184.85)} = 136.57\ N/mm^2$

After the steel stress is calculated the amount of effective reinforcement ($\rho_{peff}$) is necessary for the calculation of the maximum crack spacing ($s_{r,\max} = \mathrm{k_3 c + k_1 k_2 k_4} \frac{\phi_k}{\rho_{peff}}$).

The amount of effective reinforcement can be calculated with the following equation:

$\rho_{peff} = \frac{A_s}{A_{ceff}}$

In which $A_s$: the total amount off steel applied in the beam and $A_{ceff} = b * h_{ceff}$ (the effective concrete area , where $b$ is the width of the beam and

$h_{ceff}$: Effective depth is equal to the smallest value of the following equations:

$$h_{ceff} \leq \begin{cases} 1: 2.5 * (h - d_s) = 2.5 * (800 - 734.75) = 163.15\ mm \\ 2: \frac{h - x_{rep}}{3} = \frac{800 - 184.85}{3} = 205\ mm \\ 3: \frac{h}{2} = \frac{800}{2} = 400\ mm \end{cases}$$

We can see that the minimum value of the above is equal to $h_{ceff} = 163.125\ mm$
Now the value of the effective concrete area can be calculated:

$$A_{ceff} = b * h_{ceff} = 300 * 163.14 = 48941\ mm^2$$

The amount of effective reinforcement is equal to: $\rho_{peff} = \frac{A_s}{A_{ceff}} = \frac{1482}{48941} = 0.0303$

The next step is to calculate the maximum value of the crack spacing ($s_{r,max}$):

$\mathrm{s_{r,max}} = \mathrm{k_3 c + k_1 k_2 k_4 \frac{\emptyset_k}{\rho_{peff}}}$ ; where:

*Table 1-4: factors for $s_{r,max}$*

| k1 | k2 | k3 | k4 |
|---|---|---|---|
| **0.8** | 0.5 | 3.4 | 0.425 |

And: $\emptyset_k = 20\ mm$ (bar diameter of the main reinforcement applied)

So $s_{r,max} = 3.4 * 50 + 0.8 * 0.5 * 0.425 * \frac{20}{0.0303} = 282\ mm$

After the maximum crack spacing is found we need to calculate the value of the strain difference:

The strain difference is equal to the maximum value of :

$$\varepsilon_{sm} - \varepsilon_{cm} = \max\left(\frac{\sigma_s - k_t \frac{f_{ct,eff}}{\rho_{peff}}(1 + \alpha_e \rho_{peff})}{E_s};\ 0.6 \frac{\sigma_s}{E_s}\right)$$

Where: $\mathrm{k_t} = 0.6$ (short term loading)

$\mathrm{f_{ct,eff} = f_{ctm} = 3.96\ N/mm^2}$

$\rho_{p,eff} = \frac{A_s}{A_{ceff}} = \frac{1482}{48941} = 0.0303$

$\alpha_e = \frac{E_s}{E_{cm}} = \frac{200000}{31800} = 6.29$

$\mathrm{E_s = 200000\ N/mm^2}$

And so the strain difference ( $\varepsilon_{sm} - \varepsilon_{cm}$) is equal to:

$$\frac{\sigma_s - k_t \frac{f_{ct,eff}}{\rho_{peff}}(1 + \alpha_e \rho_{peff})}{E_s} = \frac{136.57 - 0.6 * \frac{3.95}{0.0303}(1 + 6.29 * 0.0303)}{200000} = 2.16 * 10^{-4}$$

And the lower limit value of $\varepsilon_{sm} - \varepsilon_{cm}$ is equal to:

$0.6 * \frac{\sigma_s}{E_s} = 0.6 * \frac{136.57}{200000} = 4.10 * 10^{-4}$

$$\varepsilon_{sm} - \varepsilon_{cm} = \max(2.16 * 10^{-4}; 4.10 * 10^{-4})$$

The strain difference is equal to: $\varepsilon_{sm} - \varepsilon_{cm} = 4.1 * 10^{-4}$

The crack width can now be calculated:

$w_{k,max} = s_{r,max}(\varepsilon_{sm} - \varepsilon_{cm}) = 282 * 4.1 * 10^{-4} = 0.116 \text{ mm}$

For a fully developed crack pattern the following equation is valid for the calculation of the mean value of the crack with ($w_{mean}$):

$$w_{mean} = \frac{w_{k,max}}{\gamma_s * \gamma_\infty} = \frac{0.116}{2.2} = 0.053 \text{ mm}$$

This procedure is carried out for Beam 13 and Beam 3 at varying concrete covers. The results of the crack width calculation for beam 3 and beam 13 are presented in the following chapter.

### 1.3.1 Results crack width calculation according to NEN-EN 1992-1-1

In the following tables the mean value and the maximum value of the crack width calculated according to Eurocode 2 for beam 3 and beam 13 is presented.

**Results T-beam 3:**

**$c = 20\ mm$**

*Table 1-5: Values of the crack width calculated for a cover of c=20 mm according to NEN-EN 1992-1-1 for Beam 3*

| $F_{applied}$ | $M_{applied}$ | $\sigma_s$ | $s_{r,max}$ | $\varepsilon_{sm}$-$\varepsilon_{cm}$ | $w_{k,max}$ | $w_{mean}$ |
|---|---|---|---|---|---|---|
| [kN] | [kNm] | [N/mm²] | [mm] | [-] | [mm] | [mm] |
| **109** | **136.25** | 140.98 | 207 | 4.23E-04 | 0.09 | 0.040 |
| **209** | **261.25** | 270.32 | 207 | 1.04E-03 | 0.22 | 0.098 |
| **259** | **323.75** | 334.99 | 207 | 1.37E-03 | 0.28 | 0.129 |
| **334** | **417.50** | 432.00 | 207 | 1.85E-03 | 0.38 | 0.175 |

**$c = 50\ mm$**

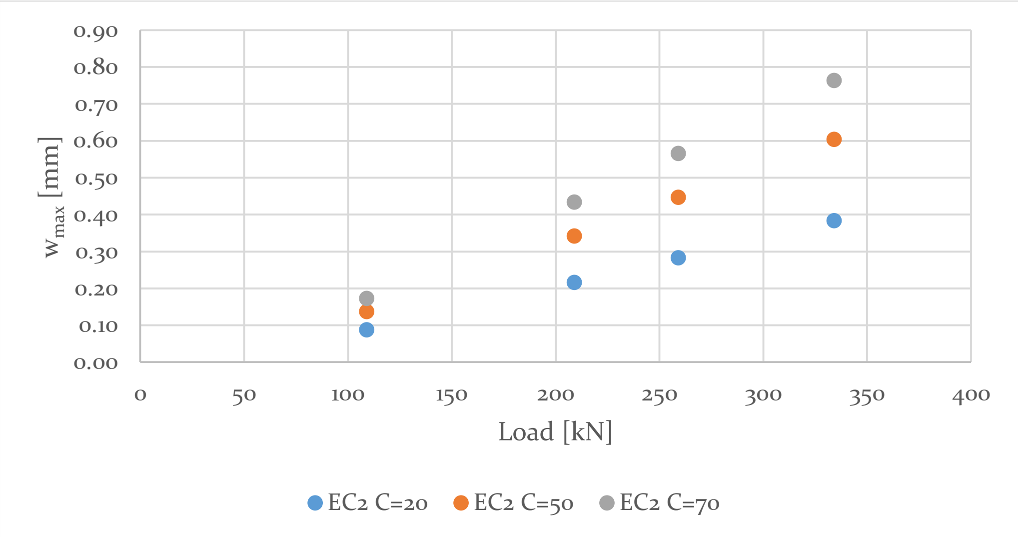
*Table 1-6: Values of the crack width calculated for a cover of c=50 mm according to NEN-EN 1992-1-1 for Beam 3*

| $F_{applied}$ | $M_{applied}$ | $\sigma_s$ | $s_{r,max}$ | $\varepsilon_{sm}$-$\varepsilon_{cm}$ | $w_{k,max}$ | $w_{mean}$ |
|---|---|---|---|---|---|---|
| [kN] | [kNm] | [N/mm²] | [mm] | [-] | [mm] | [mm] |
| **109** | **136.25** | 147.44 | 310 | 4.42E-04 | 0.14 | 0.062 |
| **209** | **261.25** | 282.71 | 310 | 1.10E-03 | 0.34 | 0.155 |
| **259** | **323.75** | 350.34 | 310 | 1.44E-03 | 0.45 | 0.203 |
| **334** | **417.50** | 451.79 | 310 | 1.95E-03 | 0.60 | 0.275 |

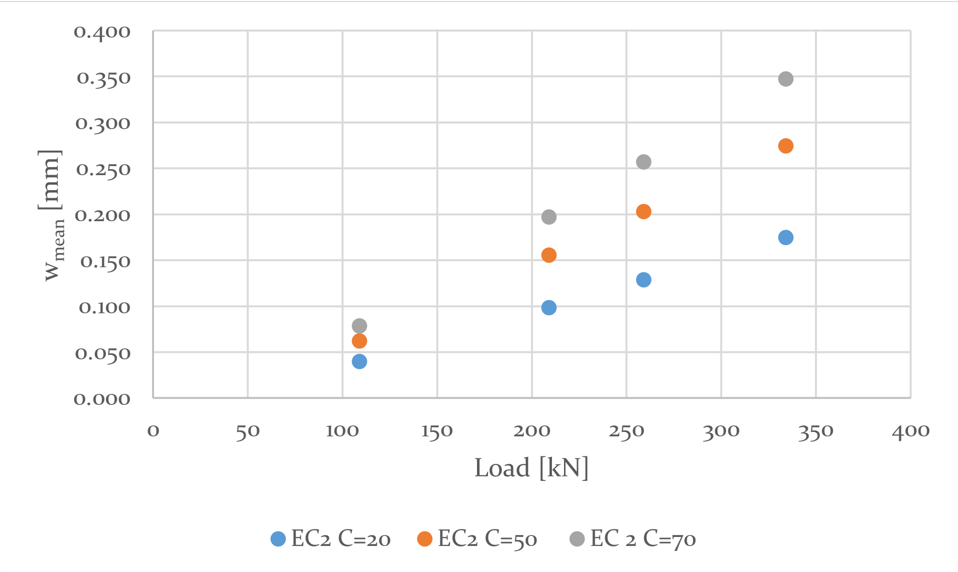
### $c = 70\ mm$

*Table 1-7: Values of the crack width calculated for a cover of c=70 mm according to NEN-EN 1992-1-1 for Beam 3*

| $F_{applied}$ | $M_{applied}$ | $\sigma_s$ | $s_{r,max}$ | $\varepsilon_{sm}-\varepsilon_{cm}$ | $w_{k,max}$ | $w_{mean}$ |
|---|---|---|---|---|---|---|
| [kN] | [kNm] | [N/mm²] | [mm] | [-] | [mm] | [mm] |
| **109** | 136.25 | 152.08 | 378 | 4.56E-04 | 0.17 | 0.078 |
| **209** | 261.25 | 291.61 | 378 | 1.15E-03 | 0.43 | 0.197 |
| **259** | 323.75 | 361.37 | 378 | 1.50E-03 | 0.57 | 0.257 |
| **334** | 417.50 | 466.01 | 378 | 2.02E-03 | 0.76 | 0.347 |




*Graph 1-1: Influence of an increasing concrete cover on the maximum value of the crack width according to NEN-EN 1992-1-1 for Beam 3*




*Graph 1-2: Influence of an increasing concrete cover on the mean value of the crack width according to NEN-EN 1992-1-1 for Beam 3*

Looking at the graphs above it is clear that the mean value and the maximum value of the crack width increases due to increasing concrete cover. It can be seen that when a cover of 50 mm is applied the maximum value of the crack width increases with about: $\frac{0.14-0.09}{0.09} * 100\% = 56\%$ in the first loading stage. When a cover of 70 mm is applied the maximum value of the crack width increases with about $\frac{0.078-0.062}{0.062} * 100\% = 26\%$. This is also the case for the mean value of the crack width. The increase of the crack width is caused by the increase of the maximum crack spacing ($s_{r,max}$) and the decrease of the effective reinforcement ratio ($\rho_{peff}$).

### Results Rectangular beam 13:

$c = 30\ mm$

*Table 1-8: Values of the crack width calculated for a cover of c=30 mm according to NEN-EN 1992-1-1 for Beam 13*

| $F_{applied}$ | $M_{applied}$ | $\sigma_s$ | $s_{r,max}$ | $\varepsilon_{sm}$-$\varepsilon_{cm}$ | $w_{k,max}$ | $w_{mean}$ |
|---|---|---|---|---|---|---|
| [kN] | [kNm] | [N/mm²] | [mm] | [-] | [mm] | [mm] |
| 109 | 136.25 | 136.57 | 282 | 4.10E-04 | 0.116 | 0.053 |
| 184 | 230.00 | 230.55 | 282 | 6.92E-04 | 0.20 | 0.089 |
| 234 | 292.50 | 293.20 | 282 | 9.99E-04 | 0.28 | 0.128 |
| 334 | 417.50 | 418.49 | 282 | 1.63E-03 | 0.46 | 0.209 |

$c = 50\ mm$

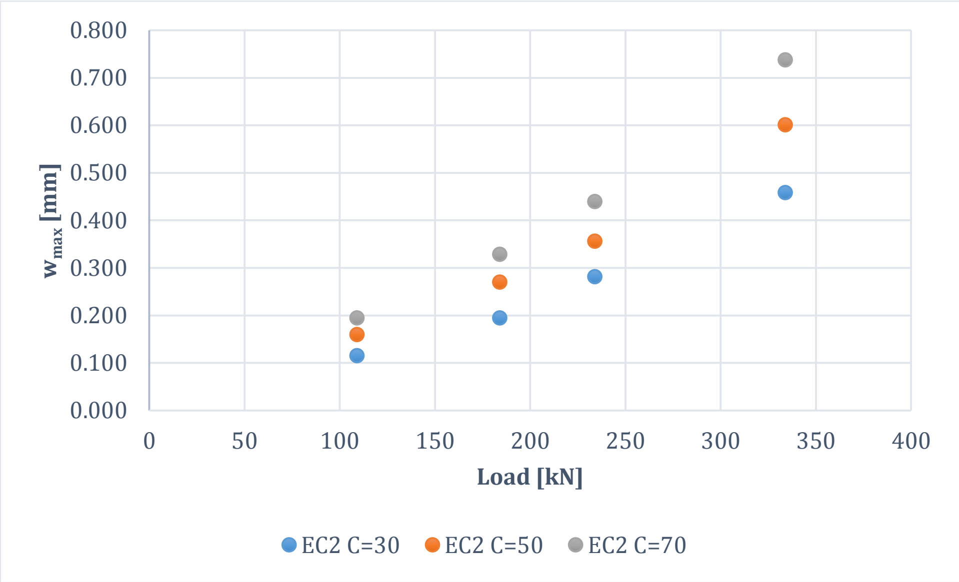
*Table 1-9: Values of the crack width calculated for a cover of c=50 mm according to NEN-EN 1992-1-1 for Beam 13*

| $F_{applied}$ | $M_{applied}$ | $\sigma_s$ | $s_{r,max}$ | $\varepsilon_{sm}$-$\varepsilon_{cm}$ | $w_{k,max}$ | $w_{mean}$ |
|---|---|---|---|---|---|---|
| [kN] | [kNm] | [N/mm²] | [mm] | [-] | [mm] | [mm] |
| 109 | 136.25 | 140.55 | 380 | 4.22E-04 | 0.160 | 0.073 |
| 184 | 230.00 | 237.26 | 380 | 7.12E-04 | 0.27 | 0.123 |
| 234 | 292.50 | 301.73 | 380 | 9.39E-04 | 0.36 | 0.162 |
| 334 | 417.50 | 430.67 | 380 | 1.58E-03 | 0.60 | 0.273 |

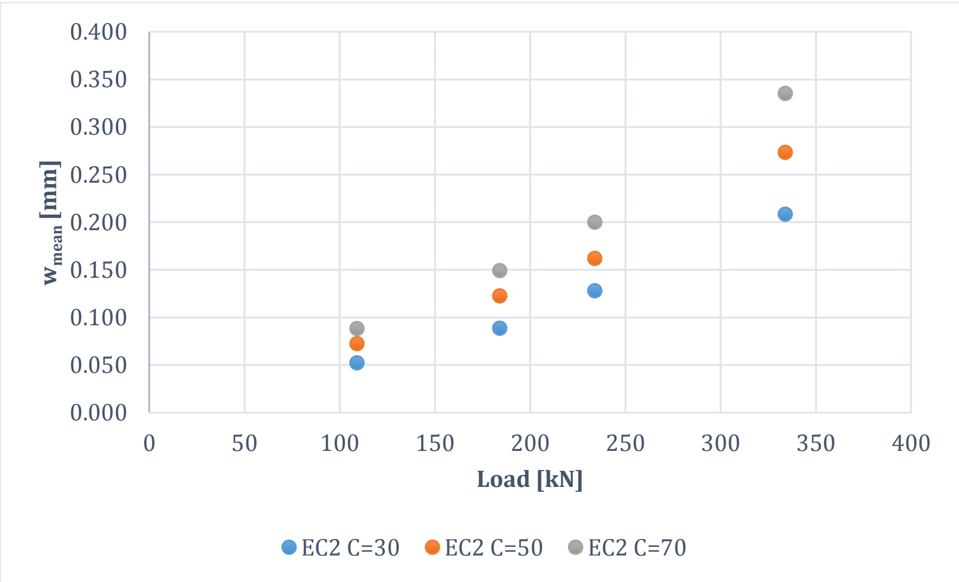
$c = 70\ mm$

*Table 1-10: Values of the crack width calculated for a cover of c=70 mm according to NEN-EN 1992-1-1 for Beam 13*

| $F_{applied}$ | $M_{applied}$ | $\sigma_s$ | $s_{r,max}$ | $\varepsilon_{sm}$-$\varepsilon_{cm}$ | $w_{k,max}$ | $w_{mean}$ |
|---|---|---|---|---|---|---|
| [kN] | [kNm] | [N/mm²] | [mm] | [-] | [mm] | [mm] |
| 109 | 136.25 | 144.76 | 448 | 4.34E-04 | 0.195 | 0.089 |
| 184 | 230.00 | 244.36 | 448 | 7.33E-04 | 0.33 | 0.149 |
| 234 | 292.50 | 310.76 | 448 | 9.82E-04 | 0.44 | 0.200 |
| 334 | 417.50 | 443.57 | 448 | 1.65E-03 | 0.74 | 0.335 |




*Graph 1-3: Influence of concrete cover on the maximum value of the crack width according to NEN-EN 1992-1-1 for Beam 13*




*Graph 1-4: Influence of an increasing concrete cover on the mean value of the crack width according to NEN-EN 1992-1-1 for Beam 13*

Also for beam 13 we see that the mean value and the maximum value of the crack width increases due to increasing concrete cover. It can be seen that when a cover of 50 mm is applied the mean value of the crack width increases with: $\frac{0.073-0.053}{0.053} * 100\% = 38\%$ in the first loading stage. When a cover of 70 mm is applied the mean value of the crack width increases with: $\frac{0.089-0.073}{0.073} * 100\% = 22\%$. This is also the case for the maximum value of the crack width. The increase of the crack width is caused by the increase of the maximum crack spacing ($s_{r,max}$) and the decrease of the effective reinforcement ratio ($\rho_{peff}$).

### 1.4 EXAMPLE CALCULATION NEN3880: VB74/84

In this chapter an example is presented of the crack width calculation carried out according to article E-508 of the NEN3880 (VB74/84) regulations. The crack width calculation has been carried out for Beam 3 and Beam 13 in Excel.

In this example the crack width is also calculated for Beam 13 and so the same geometrical and material properties are used as mentioned in section 1.3. The geometrical and material properties of T-beam 3 can be found in the Excel calculation sheet: Test Data 2 C. R. Braam invloed dekking c=30 mm.

**Crack width calculation according to the NEN 3880 (VB74/84)**

Article E-502 of the VB74/84 provides the following requirements for the crack width calculations:

1. $w_m = (\varepsilon_{sm} - \varepsilon_{cm})\Delta l$
   In Which:
   $w_m$: mean value of the crack width
   $\varepsilon_{sm}$: mean value of the elongation in the reinforcing steel
   $\varepsilon_{cm}$: mean value of the elongation in the concrete element
   $\Delta l$ : mean value of the crack spacing

The mean value of the crack spacing can be calculated with the following equation:

2. $\Delta l = \xi_2(2 * c + \xi_3 \frac{\emptyset_{km}}{\rho_{p,eff}})$ with an upper limit value of : $\Delta l = 10\xi_2 \emptyset k_m$
   In which:
   $\Delta l$ : mean value of the crack spacing

   $\xi_2 = 1$(for ribbed steel bars); $\xi_2 = 1.25$( for smooth steel bars)

   $\xi_3 = 4$ (beams subjected to bending); $\xi_3 = 8$ (beams subjected to tension)

   $c$: concrete cover on the main reinforcement

   $\emptyset_{km} = \frac{\sum \emptyset_{km}}{n}$: mean value of the applied bar diameters; $n$: number of bars applied

   $\rho_{p,eff} = \frac{A_s}{A_{c,eff}} * 100$: effective reinforcement ratio; in which $A_s$ is the area of the reinforcement applied and $A_{c,eff}$ is the effective concrete area according to Figure 1-4.

   $A_{c,eff} = b * h_{ceff}$ :

   $b$: width of the effective concrete area

   $h_{c,eff} = 0.8 * \emptyset_{km} + \bar{c} \leq h_t - x$ (for 1 layer of reinforcement applied)

   $x$: Height of the compression zone

For two layers of reinforcement the height of the compression zone needs to be calculated with the help of Figure 1-4 (second figure).

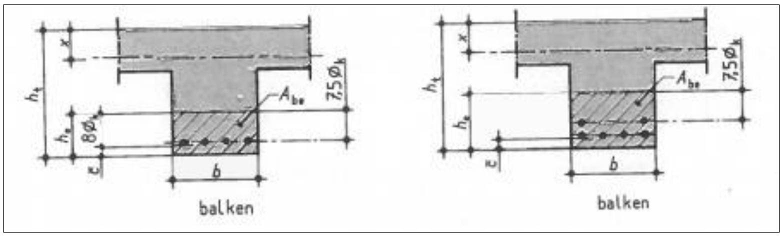



*Figure 1-4: Effective concrete area (From article 508.2* Figures E73-b and E73-c)*

The maximum value of the crack width can be calculated with the following equation:

3. $w_{max} = 2.1 * w_m$ with a reduction factor of 0.8 to account for the fact that not all the loads in the serviceability state (sls) are always present and so:

$$w_{max} = 0.8 * \sigma_s * \Delta l * 10^{-5}$$

In this equation $\sigma_s$ is defined as the value of the tensile strength of the steel acting on the structure which is coupled to the limit state value of the crack width requirements according to article E-401.4 [3]. It should be noted that the mean value of the crack spacing ($\Delta l$) is limited to $\Delta l \leq 10\emptyset_{km}$ for ribbed bars [3].

**Example calculation of the crack width at each loading stage**

The crack width was calculated in the four loading stages as mentioned in section 1.3. For the calculation of the height of the compression zone ($x$) and the value of the steel stress ($\sigma_s$) same procedure was used as in the NEN-EN 1992-1-1- calculations. So these values stay the same (Section 1.3.: Calculation of the crack width for $P = 109\ kN$)

### Calculation of the crack width for $P = 109\ kN$

For the crack width calculation the calculation of the following section properties stay the same:

$P = 109\ kN$

$M_{applied} = F_{applied} * l = 136.25 * 10^6\ Nmm$

The value for the steel stress is equal to: $\sigma_s = \frac{M_{applied}}{A_s(d_s - \frac{1}{3}x)}$

$x = 184.9\ mm$ (see section 1.3: Calculation of the crack width for $P = 109\ kN$)

$\sigma_s = \frac{M_{applied}}{A_s(d_s - \frac{1}{3}x)} = 133.55\ N/mm^2$

Since the value of the steel stress and the height of the compression zone is known the crack width can be calculated according to the NEN 3880 Regulations.

First the effective reinforcement ratio ($\rho_{p,eff}$) needs to be calculated. Before we can calculate this the height of the effective concrete area ($h_{c,eff}$) and the effective concrete area ($A_{c,eff}$) needs to be calculated. This is done with the following equations:

$h_{c,eff} = \min(0.8 * \emptyset_{km} + c\ ; h_t - x)$

$h_{c,eff} = \min(0.8 * 20 + 30 + 10 + 0.5 * 20\ ; 800 - 184.9)$

$h_{c,eff} = min\ (210; 615.1) = 210\ mm$

$A_{c,eff} = b * h_{ceff} = 300 * 210 = 63000\ mm^2$ :

Now the effective reinforcement ratio can be calculated:

$\rho_{p,eff} = \frac{A_s}{A_{c,eff}} * 100 = \frac{1482}{63000} * 100 = 2.3525$

For the crack width calculation the mean value of the crack spacing ($\Delta l$) is an essential parameter. This is calculated with the following equation:

$\Delta l = \xi_2(2c + \xi_3 \frac{\emptyset_{km}}{\rho_{p,eff}})$

$\xi_2 = 1$(ribbed steel bars)

$\xi_3 = 4$ (beams subjected to bending)

$c = 50\ mm$

$\emptyset_{km} = 20\ mm$

$\Delta l = \xi_2 \left(2c + \xi_3 \frac{\emptyset_{km}}{\rho_{p,eff}}\right) = 1 * \left(2 * 50 + 4 * \frac{20}{2.3525}\right) = 134\ mm$

The strain difference($\varepsilon_{sm} - \varepsilon_{cm}$) is also important for the calculation of the mean value of the crack width. The strains are calculated below:

$\varepsilon_{sm} = \frac{\sigma_s}{E_{steel}} = \frac{133.55\ N/mm^2}{200000\ N/mm^2} = 6.68 * 10^{-4}$

$\varepsilon_{cm} = \frac{f_{ctm}}{E_{concrete}} = \frac{3.96\ N/mm^2}{31800\ N/mm^2} = 1.245 * 10^{-4}$

The mean value of the crack width is then:

$w_m = (\varepsilon_{sm} - \varepsilon_{cm})\Delta l = (6.68 * 10^{-4} - 1.245 * 10^{-4}) * 134 = 0.0748\ mm$

The maximum value of the crack width is calculated with:

$w_{max} = 0.8 * \sigma_s * \Delta l * 10^{-5} = 0.8 * 133.55 * 134 * 10^{-5} = 0.146\ mm$

### 1.4.1 Results crack width calculation according to NEN-EN 3880

In the following tables the mean value and the maximum value of the crack width calculated according to VB74/84 for beam 3 and beam 13 is presented.

### Results T-beam 3:

### $c = 30\ mm$

*Table 1-11: Values of the crack width calculated for a cover of c=20 mm according to NEN 3880 for Beam 3*

| $F_{applied}$ | $M_{applied}$ | $\sigma_s$ | $\Delta l_{mean}$ | $\varepsilon_{sm}$-$\varepsilon_{cm}$ | $w_{mean}$ | $w_{max}$ |
|---|---|---|---|---|---|---|
| [kN] | [kNm] | [N/mm²] | [mm] | [-] | [mm] | [mm] |
| **109** | **136.25** | 140.98 | 100 | 5.85E-04 | 0.06 | 0.11 |
| **209** | **261.25** | 270.32 | 100 | 1.23E-03 | 0.12 | 0.22 |
| **259** | **323.75** | 334.99 | 100 | 1.55E-03 | 0.16 | 0.27 |
| **334** | **417.50** | 432.00 | 100 | 2.04E-03 | 0.20 | 0.35 |

### $c = 50\ mm$

*Table 1-12: Values of the crack width calculated for a cover of c=50 mm according to NEN 3880 for Beam 3*

| $F_{applied}$ | $M_{applied}$ | $\sigma_s$ | $\Delta l_{mean}$ | $\varepsilon_{sm}$-$\varepsilon_{cm}$ | $w_{mean}$ | $w_{max}$ |
|---|---|---|---|---|---|---|
| [kN] | [kNm] | [N/mm²] | [mm] | [-] | [mm] | [mm] |
| **109** | **136.25** | 147.44 | 163 | 6.17E-04 | 0.10 | 0.19 |
| **209** | **261.25** | 282.71 | 163 | 1.29E-03 | 0.21 | 0.37 |
| **259** | **323.75** | 350.34 | 163 | 1.63E-03 | 0.27 | 0.46 |
| **334** | **417.50** | 451.79 | 163 | 2.14E-03 | 0.35 | 0.59 |

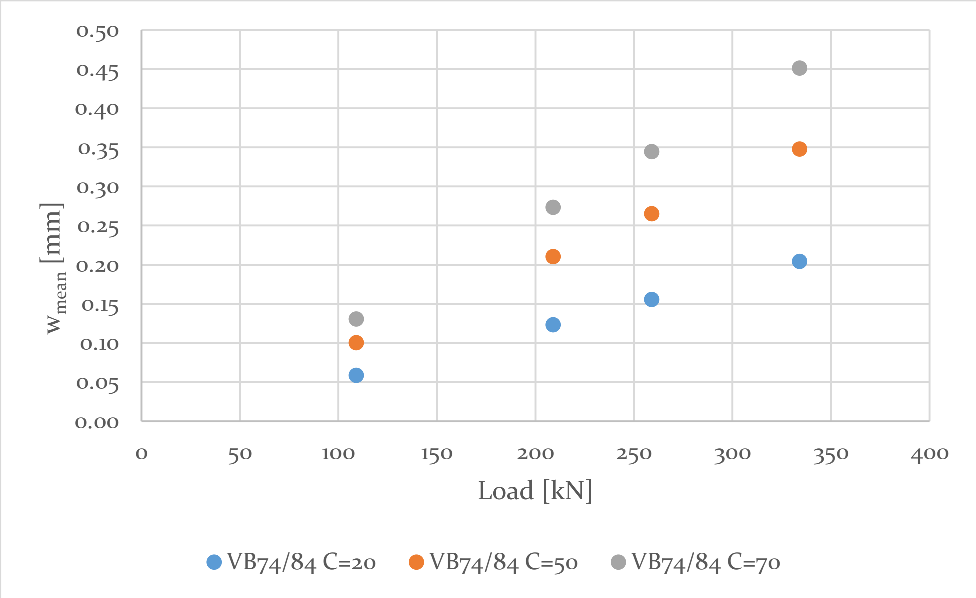
### $c = 70\ mm$

*Table 1-13: Values of the crack width calculated for a cover of c=70 mm according to NEN 3880 for Beam 3*

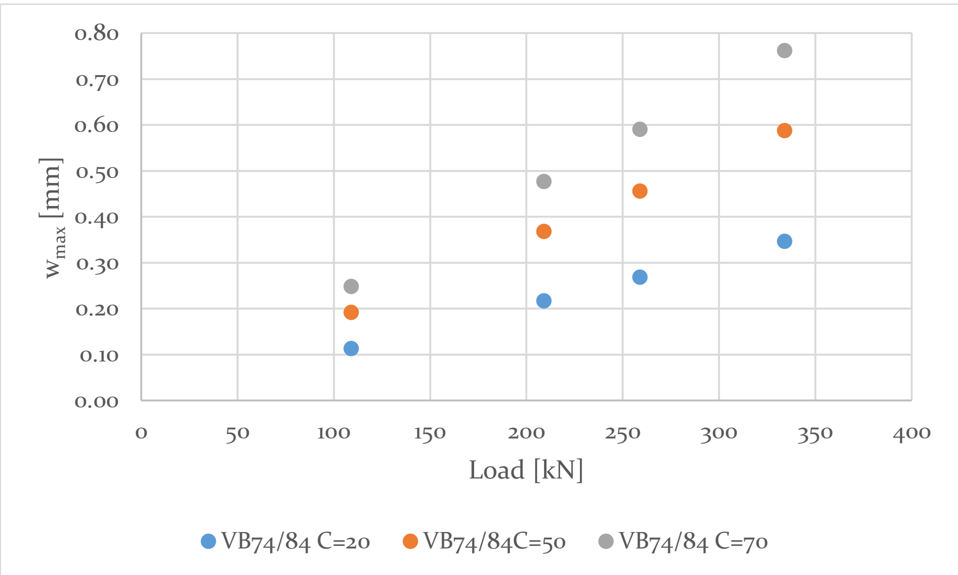
| $F_{applied}$ | $M_{applied}$ | $\sigma_s$ | $\Delta l_{mean}$ | $\varepsilon_{sm}$-$\varepsilon_{cm}$ | $w_{mean}$ | $w_{max}$ |
|---|---|---|---|---|---|---|
| [kN] | [kNm] | [N/mm²] | [mm] | [-] | [mm] | [mm] |
| **109** | 136.25 | 152.08 | 204 | 6.40E-04 | 0.13 | 0.25 |
| **209** | 261.25 | 291.61 | 204 | 1.34E-03 | 0.27 | 0.48 |
| **259** | 323.75 | 361.37 | 204 | 1.69E-03 | 0.34 | 0.59 |
| **334** | 417.50 | 466.01 | 204 | 2.21E-03 | 0.45 | 0.76 |

When we look at the results in the tables above we see that the mean- and maximum value of the crack width increases when a larger concrete cover is applied. This could be explained by the increasing value of the mean crack spacing ($\Delta \boldsymbol{l}_{mean}$) and also due to the increasing value of the strain difference ($\varepsilon_{sm} - \varepsilon_{cm}$). The strain difference increases due to the fact that the steel stress ($\sigma_s$) increases. We see that for a cover of $c = 50\ mm$ the mean crack width increases with: $\frac{0.1-0.06}{0.06} * 100\% = 67\%$ in the first loading stage compared to a cover of $c =$

$20\ mm$. With increasing load the difference increases up to 75%. When a cover of $70\ mm$ is applied the mean value of the crack width increases with 28% compared to a cover of $c = 50\ mm$. These differences can be seen in the following graphs.




*Graph 1-5: Influence of the concrete cover on the mean value of the crack width calculated according to NEN 3880 for T-beam 3*




*Graph 1-6: Influence of the concrete cover on the maximum value of the crack width calculated according to NEN 3880 for T-beam 3*

### Results Rectangular beam 13:

**$c = 30\ mm$**

*Table 1-14: Values of the crack width calculated for a cover of c=30 mm according to NEN 3880 for Beam 13*

| $F_{applied}$ | $M_{applied}$ | $\sigma_s$ | $\Delta l_{mean}$ | $\varepsilon_{sm}-\varepsilon_{cm}$ | $w_{mean}$ | $w_{max}$ |
|---|---|---|---|---|---|---|
| [kN] | [kNm] | [N/mm²] | [mm] | [-] | [mm] | [mm] |
| **109** | **136.25** | 136.57 | 134 | 5.58E-04 | 0.075 | 0.146 |
| **184** | **230.00** | 230.55 | 134 | 1.03E-03 | 0.138 | 0.25 |
| **234** | **292.50** | 293.20 | 134 | 1.34E-03 | 0.180 | 0.31 |
| **334** | **417.50** | 418.49 | 134 | 1.97E-03 | 0.264 | 0.45 |

**$c = 50\ mm$**

*Table 1-15: Values of the crack width calculated for a cover of c=30 mm according to NEN 3880 for Beam 13*

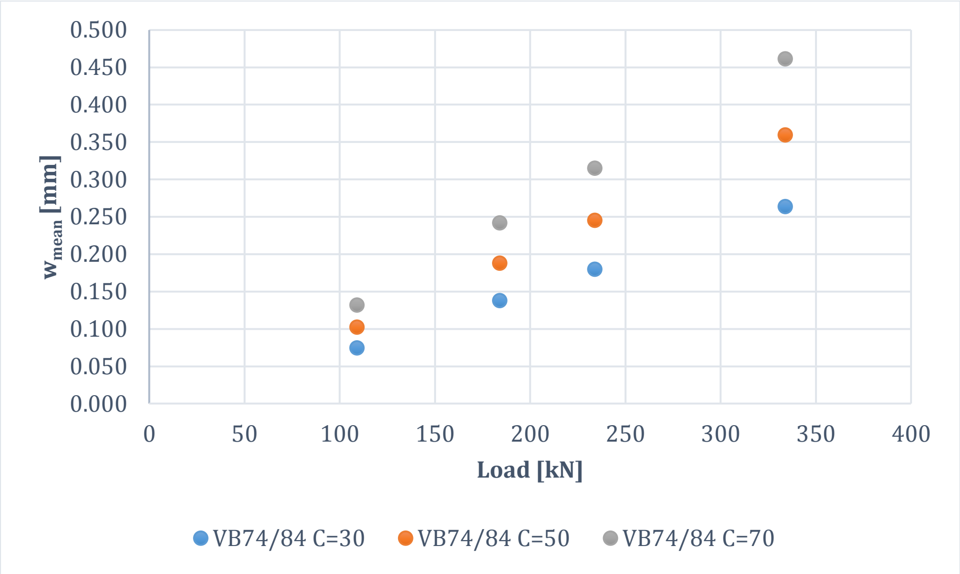
| $F_{applied}$ | $M_{applied}$ | $\sigma_s$ | $\Delta l_{mean}$ | $\varepsilon_{sm}-\varepsilon_{cm}$ | $w_{mean}$ | $w_{max}$ |
|---|---|---|---|---|---|---|
| [kN] | [kNm] | [N/mm²] | [mm] | [-] | [mm] | [mm] |
| **109** | 136.25 | 140.55 | 177 | 5.78E-04 | 0.103 | 0.199 |
| **184** | 230.00 | 237.26 | 177 | 1.06E-03 | 0.188 | 0.34 |
| **234** | 292.50 | 301.73 | 177 | 1.38E-03 | 0.245 | 0.43 |
| **334** | 417.50 | 430.67 | 177 | 2.03E-03 | 0.360 | 0.61 |

**$c = 70\ mm$**

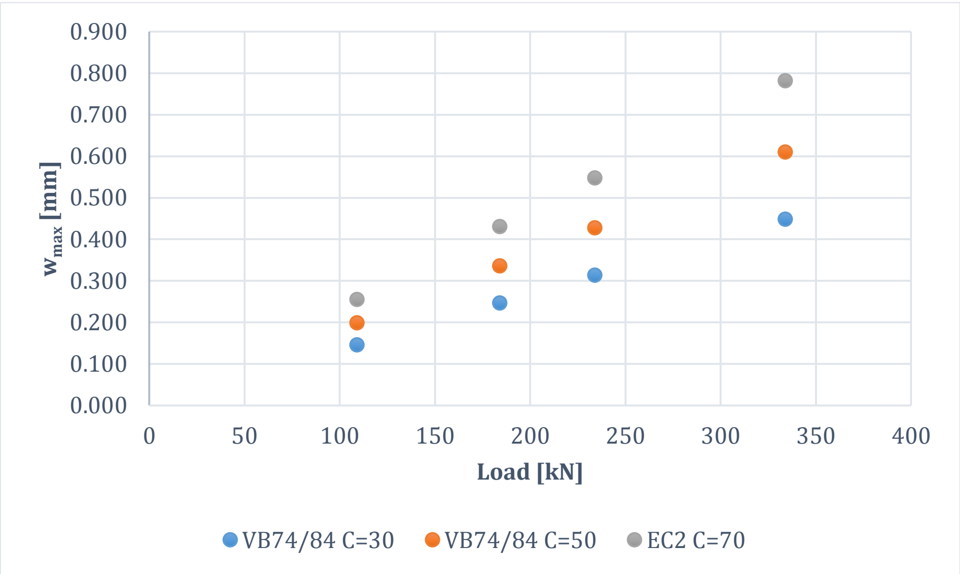
*Table 1-16: Values of the crack width calculated for a cover of c=70 mm according to NEN 3880 for Beam 13*

| $F_{applied}$ | $M_{applied}$ | $\sigma_s$ | $\Delta l_{mean}$ | $\varepsilon_{sm}-\varepsilon_{cm}$ | $w_{mean}$ | $w_{max}$ |
|---|---|---|---|---|---|---|
| [kN] | [kNm] | [N/mm²] | [mm] | [-] | [mm] | [mm] |
| **109** | 136.25 | 144.76 | 220 | 5.99E-04 | 0.132 | 0.255 |
| **184** | 230.00 | 244.36 | 220 | 1.10E-03 | 0.242 | 0.43 |
| **234** | 292.50 | 310.76 | 220 | 1.43E-03 | 0.315 | 0.55 |
| **334** | 417.50 | 443.57 | 220 | 2.09E-03 | 0.462 | 0.78 |

Also in the case of beam 13 we see that the mean- and maximum value of the crack width increases when a larger cover is applied. This occurs due to an increase of the mean crack spacing ($\Delta \boldsymbol{l}_{\boldsymbol{mean}}$) and also due to the increasing value of the strain difference ($\varepsilon_{sn} - \varepsilon_{cm}$). The strain difference increases due to the fact that the steel stress ($\sigma_s$) increases. We see that for a cover of $c = 50\ mm$ the mean crack width increases with about $\frac{0.103-0.075}{0.075} * 100\% = 36\%$ in the first loading stage compared to a cover of $c = 30\ mm$. When a cover of $70\ mm$ is applied the mean value of the crack width increases with 28% compared to a cover of $c = 50\ mm$. These differences can be seen in the following graphs.




*Graph 1-7: Influence of the concrete cover on the mean value of the crack width according to NEN 3880 for beam 13*




*Graph 1-8: Influence of the concrete cover on the maximum value of the crack width according to NEN 3880 for beam 13*

# 2 INFLUENCE OF THE LIMITATION OF THE CRACK SPACING ON THE CRACK WIDTH CALCULATIONS

In the previous chapter it was clear that the mean value and the maximum value of the crack width increases the concrete cover increases. This was mainly caused by the increase of the crack spacing. In this chapter it will be investigated what the influence limitation of the crack spacing has on the crack width calculations according to the codes (NEN-EN 1992-1-1 and NEN 3880) for T-beam 3 and Rectangular beam 13. First an example calculation will be presented for both regulations. In section **Error! Reference source not found.** the results from the codes will be compared to each other.

## 2.1 NEN-EN 1992-1-1 (EUROCODE 2)

Looking at the results of the Eurocode 2 calculations in chapter 1.3.1 it can be seen that the value of the maximum crack spacing ($s_{r,max}$) increases when a larger concrete cover is applied. So if we limit the maximum crack spacing the values of the crack width would decrease. The VARCE (Vraag en antwoord rubriek in CEMENT :NEN-EN 1992-1-1 +C2: 2011/NB:2011) suggested an upper boundary limit for the maximum crack spacing ($s_{r,max}$ ) of:

$$s_{r,max} \leq Max\,\{(50 - 0.8 f_{ck})\emptyset; 15\emptyset\}$$

This equation will be applied for the calculation of $s_{r,max}$ in order to investigate what influence this limitation has on the crack width calculations. The calculations are carried out for a concrete cover of $c = 20\,mm; c = 50\,mm\ and\ c = 70\,mm$ for T-beam 3 and in Rectangular Beam 13 the concrete cover is varied with the following values: $c = 30\,mm; c = 50\,mm\ and\ c = 70\,mm.$

The same procedure is followed as the example calculation in section 1.3. Only the calculation of $s_{r,max}$ is modified in Excel to:

$$s_{r,max} = k_3 * c + k_1 k_2 k_4 * \frac{\emptyset}{\rho_{p,eff}} \leq Max\{(50 - 0.8 * f_{ck})\emptyset\,; 15\emptyset\}$$

Below an example calculation is presented for beam 13.

### Example calculations for $s_{r,max} \leq Max\{(50 - 0.8 * f_{ck})\emptyset\,; 15\emptyset\}$

**Geometry beam 13:** The same properties of section 1.3 are used: $h = 800\,mm$ and $b = 300\,mm$

Applied concrete cover: $c = 30\,mm$

Stirrups $\emptyset_b = 10\,mm$ ; main reinforcement: $\emptyset_k = 20\,mm$

Applied cover to the main reinforcement: $c + \emptyset_b + \frac{\emptyset_k}{2} = 30 + 10 + 10 = 50\,mm$

Before limitation the value of the maximum crack spacing was equal to:

$$s_{r,max} = k_3 * c + k_1 * k_2 * k_4 * \frac{\emptyset}{\rho_{p,eff}} = 3.4 * 50 + 0.8 * 0.5 * 0.425 * \frac{20}{0.0303} = 282\,mm \quad \text{(section 1.3)}$$

Upper limit value: $s_{r,max} \leq\ Max\{(50 - 0.8 * f_{ck})\emptyset\,; 15\emptyset\} \rightarrow$

$s_{r,max} \leq Max\{(50 - 0.8 * 47.9)20\,; 15 * 20\}$

$s_{r,max} \leq Max\{233.6\,; 300\}$

The value for the maximum crack spacing is equal to: $s_{r,max} = min\ (282; max\ (233.6; 300))$

And so $s_{r,max} = 282\ mm$

The limitation of the maximum crack spacing does not have influence on the strain difference and so the crack width can easily be calculated by using the values for the strain difference found in section 1.3:

The strain difference is equal to: $\varepsilon_{\text{sm}} - \varepsilon_{\text{cm}} = 4.1 * 10^{-4}$

The crack width can now be calculated:

$\text{w}_{\text{k,max}} = \text{s}_{\text{r,max}}(\varepsilon_{\text{sm}} - \varepsilon_{\text{cm}}) = 282 * 4.1 * 10^{-4} = 0.116\text{ mm}$

For a fully developed crack pattern the following equation is valid for the calculation of the mean value of the crack with ($\text{w}_{\text{mean}}$):

$\text{w}_{\text{mean}} = \frac{\text{w}_{\text{k,max}}}{\gamma_s * \gamma_\infty} = \frac{0.116}{2.2} = 0.053\text{ mm}$

This procedure is also carried out for $c = 50$ and $c = 70\ mm$. The results are presented in the tables below.

### 2.1.1 Results Limitation maximum crack spacing ($s_{r,max}$) according to VARCE

**Results T-beam 3:**

$c = 30\ mm$

*Table 2-1: Influence of the limitation of the maximum crack spacing on the crack width according to VARCE for a cover of 20 mm*

| $F_{applied}$ | $M_{applied}$ | $\sigma_s$ | $s_{r,max}$ | $\varepsilon_{sm}-\varepsilon_{cm}$ | $w_{k,max}$ | $w_{mean}$ |
|---|---|---|---|---|---|---|
| [kN] | [kNm] | [N/mm²] | [mm] | [-] | [mm] | [mm] |
| **109** | **136.25** | 140.98 | 207 | 4.23E-04 | 0.09 | 0.040 |
| **209** | **261.25** | 270.32 | 207 | 1.04E-03 | 0.22 | 0.098 |
| **259** | **323.75** | 334.99 | 207 | 1.37E-03 | 0.28 | 0.129 |
| **334** | **417.50** | 432.00 | 207 | 1.85E-03 | 0.38 | 0.175 |

$c = 50\ mm$

*Table 2-2: Influence of the limitation of the maximum crack spacing on the crack width according to VARCE for a cover of 50 mm*

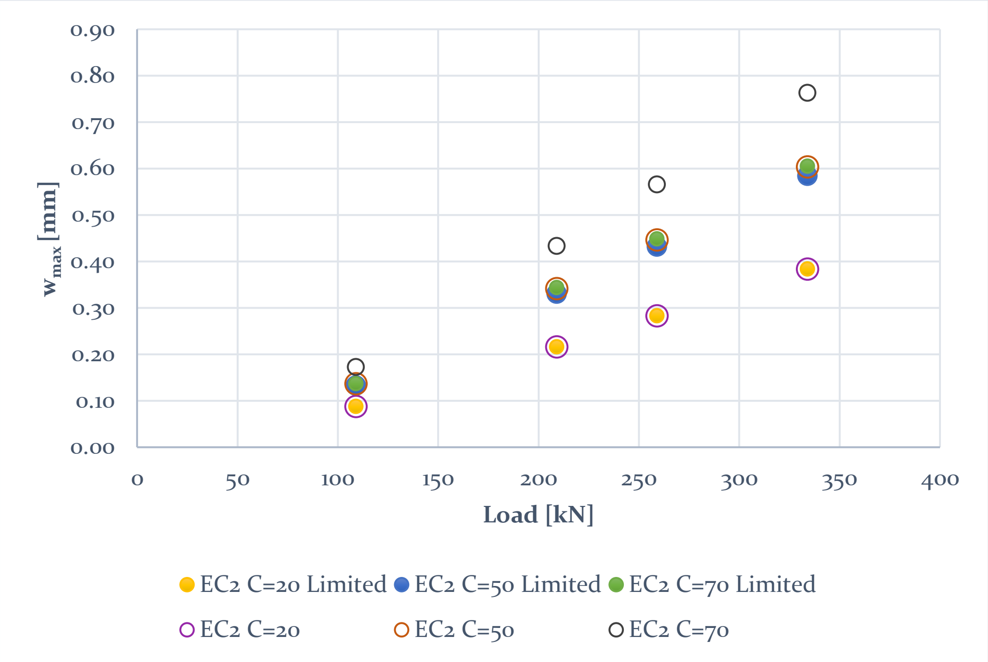
| $F_{applied}$ | $M_{applied}$ | $\sigma_s$ | $s_{r,max}$ | $\varepsilon_{sm}-\varepsilon_{cm}$ | $w_{k,max}$ | $w_{mean}$ |
|---|---|---|---|---|---|---|
| [kN] | [kNm] | [N/mm²] | [mm] | [-] | [mm] | [mm] |
| **109** | **136.25** | 147.44 | 300 | 4.42E-04 | 0.13 | 0.060 |
| **209** | **261.25** | 282.71 | 300 | 1.10E-03 | 0.33 | 0.150 |
| **259** | **323.75** | 350.34 | 300 | 1.44E-03 | 0.43 | 0.197 |
| **334** | **417.50** | 451.79 | 300 | 1.95E-03 | 0.58 | 0.266 |

$c = 70\ mm$

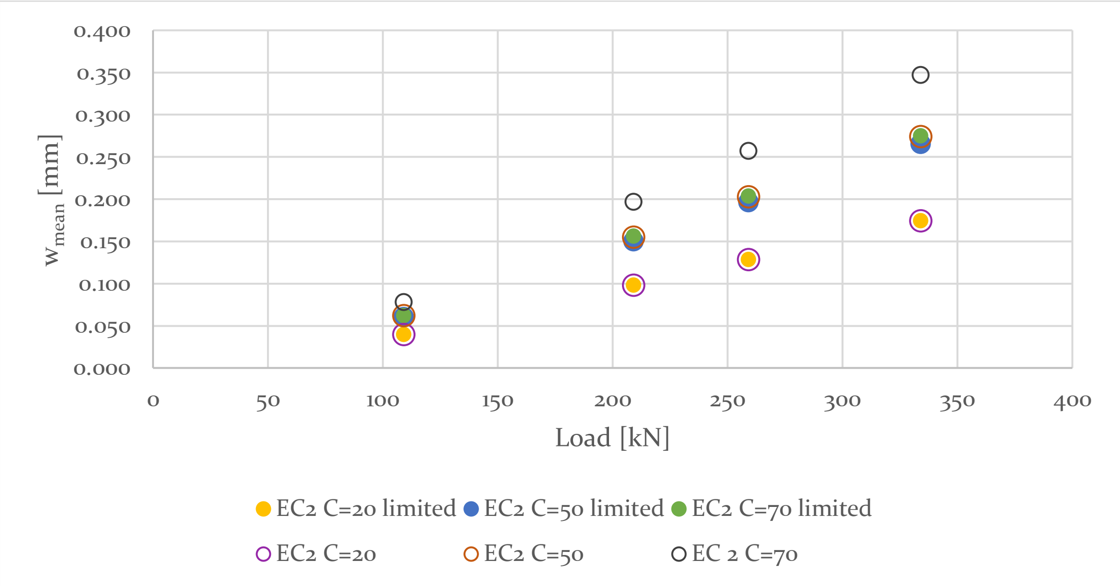
*Table 2-3: Influence of the limitation of the maximum crack spacing on the crack width according to VARCE for a cover of 70 mm*

| $F_{applied}$ | $M_{applied}$ | $\sigma_s$ | $s_{r,max}$ | $\varepsilon_{sm}-\varepsilon_{cm}$ | $w_{k,max}$ | $w_{mean}$ |
|---|---|---|---|---|---|---|
| [kN] | [kNm] | [N/mm²] | [mm] | [-] | [mm] | [mm] |
| **109** | **136.25** | 152.08 | 300 | 4.56E-04 | 0.14 | 0.062 |
| **209** | **261.25** | 291.61 | 300 | 1.15E-03 | 0.34 | 0.156 |
| **259** | **323.75** | 361.37 | 300 | 1.50E-03 | 0.45 | 0.204 |
| **334** | **417.50** | 466.01 | 300 | 2.02E-03 | 0.61 | 0.275 |

Looking at the results presented in tables it can be seen that the limitation only has influence on the larger concrete covers ($c = 50\ mm$ and $c = 70\ mm$). For an applied cover of $c = 30\ mm$ the mean and maximum value of the crack width stays the same. However when a cover of $c = 50\,mm$ is applied the mean value of the crack width decreases with about $\frac{0.062-0.06}{0.06} * 100\% = 3\%$. This difference is very small. But when we look at a cover of $c = 70\,mm$ we see that the mean value of the crack width decreases with about 21%. This is also the case for the maximum crack width. So the limitation according to the VARCE influences the crack width calculations when larger covers are applied. The differences can clearly be seen in Graph 2-1 and Graph 2-2.




*Graph 2-1: Influence of the limitation of the maximum crack spacing on the maximum value of the crack width according to VARCE for T-beam 3*




*Graph 2-2: Influence of the limitation of the maximum crack spacing on the mean value of the crack width according to VARCE for T-beam 3*

### Results Rectangular beam 13:

### $c = 30\ mm$

*Table 2-4: Influence of the limitation of the maximum crack spacing on the crack width according to VARCE for a cover of 30 mm*

| $F_{applied}$ | $M_{applied}$ | $\sigma_s$ | $s_{r,max}$ | $\varepsilon_{sm}$-$\varepsilon_{cm}$ | $w_{k,max}$ | $w_{mean}$ |
|---|---|---|---|---|---|---|
| [kN] | [kNm] | [N/mm²] | [mm] | [-] | [mm] | [mm] |
| **109** | **136.25** | 136.57 | 282 | 4.10E-04 | 0.116 | 0.053 |
| **184** | **230.00** | 230.55 | 282 | 6.92E-04 | 0.20 | 0.089 |
| **234** | **292.50** | 293.20 | 282 | 9.99E-04 | 0.28 | 0.128 |
| **334** | **417.50** | 418.49 | 282 | 1.63E-03 | 0.46 | 0.209 |

### $c = 50\ mm$

*Table 2-5: Influence of the limitation of the maximum crack spacing on the crack width according to VARCE for a cover of 50 mm*

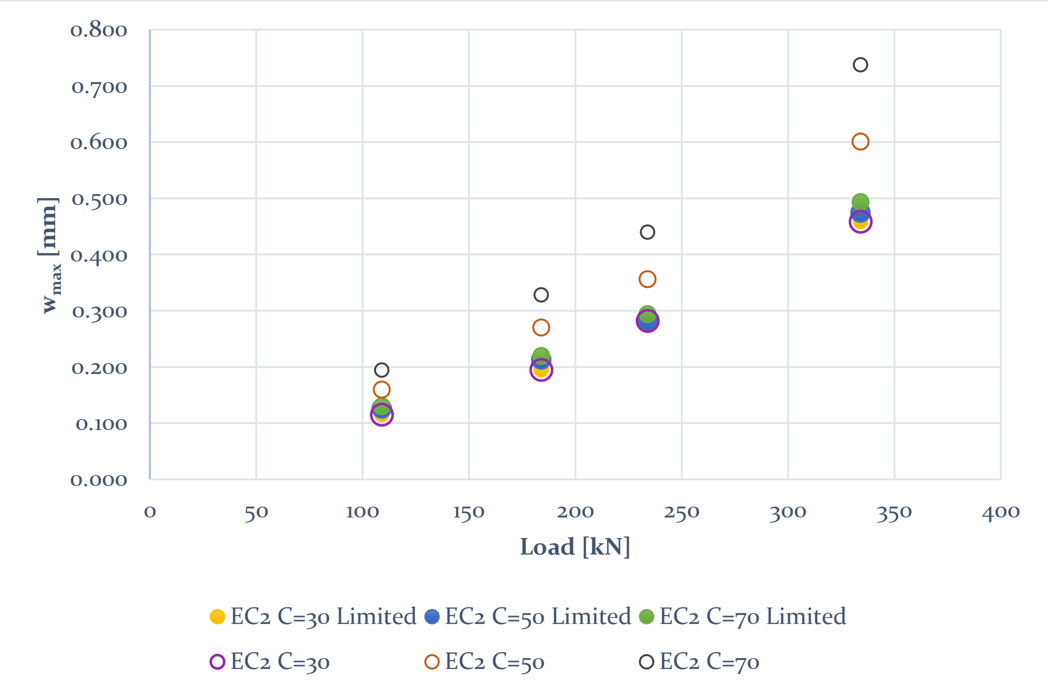
| $F_{applied}$ | $M_{applied}$ | $\sigma_s$ | $s_{r,max}$ | $\varepsilon_{sm}$-$\varepsilon_{cm}$ | $w_{k,max}$ | $w_{mean}$ |
|---|---|---|---|---|---|---|
| [kN] | [kNm] | [N/mm²] | [mm] | [-] | [mm] | [mm] |
| **109** | 136.25 | 140.55 | 300 | 4.22E-04 | 0.13 | 0.057 |
| **184** | 230.00 | 237.26 | 300 | 7.12E-04 | 0.21 | 0.097 |
| **234** | 292.50 | 301.73 | 300 | 9.39E-04 | 0.28 | 0.128 |
| **334** | 417.50 | 430.67 | 300 | 1.58E-03 | 0.48 | 0.216 |

### $c = 70\ mm$

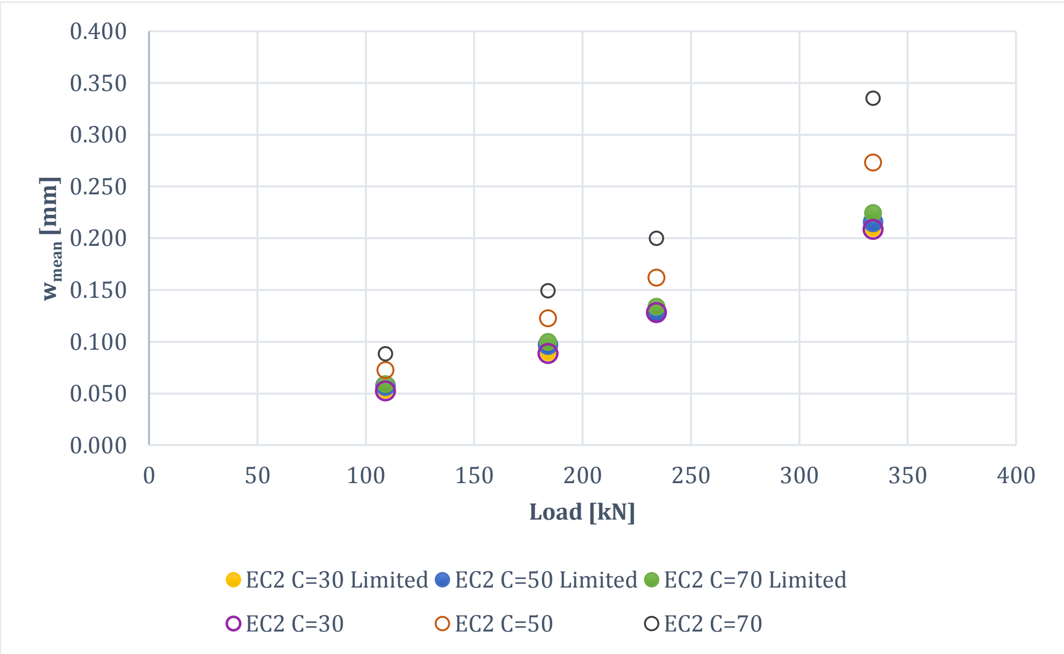
*Table 2-6: Influence of the limitation of the maximum crack spacing on the crack width according to VARCE for a cover of 70 mm*

| $F_{applied}$ | $M_{applied}$ | $\sigma_s$ | $s_{r,max}$ | $\varepsilon_{sm}$-$\varepsilon_{cm}$ | $w_{k,max}$ | $w_{mean}$ |
|---|---|---|---|---|---|---|
| [kN] | [kNm] | [N/mm²] | [mm] | [-] | [mm] | [mm] |
| **109** | 136.25 | 144.76 | 300 | 0.13 | 0.13 | 0.059 |
| **184** | 230.00 | 244.36 | 300 | 0.22 | 0.22 | 0.100 |
| **234** | 292.50 | 310.76 | 300 | 0.29 | 0.29 | 0.134 |
| **334** | 417.50 | 443.57 | 300 | 0.49 | 0.49 | 0.224 |

In tables Table 2-3 to Table 2-6 it can be seen what influence the limitation of the maximum crack spacing has on the crack width calculation. Also for beam 13 we see that at a an applied cover of $c = 30\ mm$ the limitation has no influence on the mean- and maximum value of the crack width. But when a cover of $c = 50\ mm$ is applied the mean value of the crack width decreases with $\frac{0.073-0.057}{0.073} * 100\% = 21\%$. And when a cover of $c = 70\,mm$ is applied we see that the mean value of the crack with decreases with $\frac{0.089-0.059}{0.089} * 100\% = 33\%$. So the limitation of the maximum crack spacing according to the VARCE influences the mean- and maximum value of the crack width at an increasing concrete cover. The results are presented in Graph 2-3 and Graph 2-4.




*Graph 2-3: Influence of the limitation of the maximum crack spacing on the maximum value of the crack width according to VARCE for Rectangular beam 13*




*Graph 2-4: Influence of the limitation of the maximum crack spacing on the mean value of the crack width according to VARCE for Rectangular beam 13*

## 2.2 NEN 3880 (VB74/84)

In section 1.4.1 it was clear that the mean value of the crack spacing had a large influence on the value of the crack width. Also in these cases the mean- and maximum value of the crack width increased with increasing value of the cover. We saw that when the cover increased the mean value of the crack spacing also increased, thus resulting in higher values for the crack width. However article E-508.2* of NEN 3880 states that the mean crack spacing should be smaller than $10\emptyset k_m$. This upper boundary was not taken into account during the crack width calculations in section 1.4.1.

So in order to investigate if this limitation has an influence on the crack width calculations provided by NEN 3880 this upper limit value will be applied in Excel.

The calculations are carried out for a concrete cover of $c = 20\ mm; c = 50\ mm\ and\ c = 70\ mm$ for T-beam 3 and in Rectangular Beam 13 the concrete cover is varied with the following values: $c = 30\ mm; c = 50\ mm\ and\ c = 70\ mm$.

The same procedure is followed as the example calculation in chapter 1.4. Only the calculation of $s_{r,max}$ is modified in Excel to:

$$\Delta l = \xi_2\left(2c + \xi_3\frac{\emptyset_{km}}{\rho_{p,eff}}\right) \leq 10\emptyset_k$$

All the other calculations stay the same.

Below an example calculation is presented for beam 13.

**Example calculations for $\Delta l = \xi_2\left(2\bar{c} + \xi_3\frac{\emptyset_{km}}{\rho_{p,eff}}\right) \leq 10\emptyset_k$**

**Geometry beam 13:** The same properties of section 1.3 are used: $h = 800\ mm$ and $b = 300\ mm$

Applied concrete cover: $c = 30\ mm$

Stirrups $\emptyset_b = 10\ mm$ ; main reinforcement: $\emptyset_k = 20\ mm$

Applied cover to the main reinforcement: $c + \emptyset_b + \frac{\emptyset_k}{2} = 30 + 10 + 10 = 50\ mm$

Before limitation the value of the mean crack spacing was equal to:

$$\Delta l = \xi_2\left(2c + \xi_3\frac{\emptyset_{km}}{\rho_{p,eff}}\right) = 1 * \left(2 * 50 + 4 * \frac{20}{2.3525}\right) = 134\ mm$$

When applying the upper boundary limit of $10\emptyset_k$ the calculation of the mean crack spacing becomes:

$$\Delta l = min\left\{\xi_2\left(2c + \xi_3\frac{\emptyset_{km}}{\rho_{p,eff}}\right); 10\emptyset_k\right\} = min\{134; 10 * 20\}$$

And so $\Delta l = 134\ mm$

The limitation of the mean crack spacing does not have influence on the calculation of the strains and so the crack width can easily be calculated by using the values for the strains found in section 1.4.1.

$\varepsilon_{sm} = 6.68 * 10^{-4}$

$\varepsilon_{cm} = 1.245 * 10^{-4}$

The mean value of the crack width is then:

$w_m = (\varepsilon_{sm} - \varepsilon_{cm})\Delta l = (6.68 * 10^{-4} - 1.245 * 10^{-4}) * 134 = 0.0748\ mm$

The maximum value of the crack width is calculated with:

$w_{max} = 0.8 * \sigma_s * \Delta l * 10^{-5} = 0.8 * 136.57 * 134 * 10^{-5} = 0.146\ mm$

This procedure is also carried out for an applied cover of $c = 50\ mm$ and $c = 70\ mm$. The results are presented in the following chapter.

### 2.2.1 Results Limitation mean value of the crack spacing ($\Delta l$)

**Results T-beam 3:**

$c = 30\ mm$

*Table 2-7: Values of the crack width calculated for a cover of c=20 mm according to NEN 3880 for Beam 3*

| $F_{applied}$ | $M_{applied}$ | $\sigma_s$ | $\Delta l_{mean}$ | $\varepsilon_a$-$\varepsilon_b$ | $w_{mean}$ | $w_{max}$ |
|---|---|---|---|---|---|---|
| [kN] | [kNm] | [N/mm²] | [mm] | | [mm] | [mm] |
| 109 | 136.25 | 141 | 100 | 5.85E-04 | 0.06 | 0.11 |
| 209 | 261.25 | 270.32 | 100 | 1.23E-03 | 0.12 | 0.22 |
| 259 | 323.75 | 335 | 100 | 1.55E-03 | 0.16 | 0.27 |
| 334 | 417.50 | 432 | 100 | 2.04E-03 | 0.20 | 0.35 |

$c = 50\ mm$

*Table 2-8: Values of the crack width calculated for a cover of c=50 mm according to NEN 3880 for Beam 3*

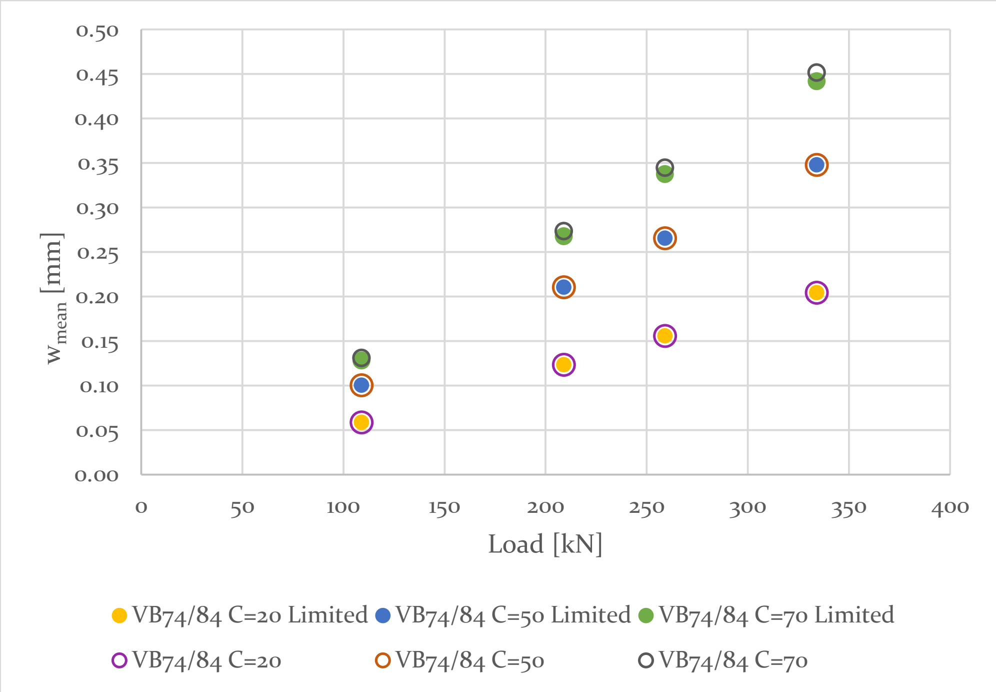
| $F_{applied}$ | $M_{applied}$ | $\sigma_s$ | $\Delta l_{mean}$ | $\varepsilon_a$-$\varepsilon_b$ | $w_{mean}$ | $w_{max}$ |
|---|---|---|---|---|---|---|
| [kN] | [kNm] | [N/mm²] | [mm] | | [mm] | [mm] |
| 109 | 136.25 | 147.44 | 163 | 6.17E-04 | 0.10 | 0.19 |
| 209 | 261.25 | 282.71 | 163 | 1.29E-03 | 0.21 | 0.37 |
| 259 | 323.75 | 350.34 | 163 | 1.63E-03 | 0.27 | 0.46 |
| 334 | 417.50 | 451.79 | 163 | 2.14E-03 | 0.35 | 0.59 |

$c = 70\ mm$

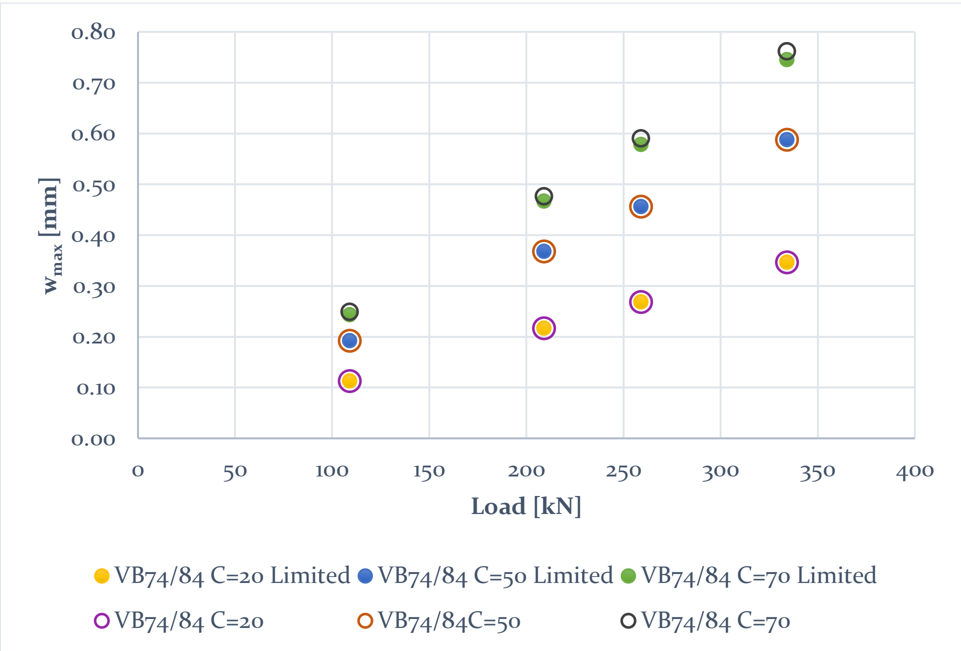
*Table 2-9: Values of the crack width calculated for a cover of c=70 mm according to NEN 3880 for Beam 3*

| $F_{applied}$ | $M_{applied}$ | $\sigma_s$ | $\Delta l_{mean}$ | $\varepsilon_a$-$\varepsilon_b$ | $w_{mean}$ | $w_{max}$ |
|---|---|---|---|---|---|---|
| [kN] | [kNm] | [N/mm²] | [mm] | | [mm] | [mm] |
| 109 | 136.25 | 152.08 | 200 | 6.40E-04 | 0.13 | 0.24 |
| 209 | 261.25 | 291.61 | 200 | 1.34E-03 | 0.27 | 0.47 |
| 259 | 323.75 | 361.37 | 200 | 1.69E-03 | 0.34 | 0.58 |
| 334 | 417.50 | 466.01 | 200 | 2.21E-03 | 0.44 | 0.75 |

When we look at the results in the tables above we see that the limitation of the mean crack spacing does not have an influence of the mean- and maximum value of the crack width in the NEN 3880 calculations, since the mean value of the crack spacing stays the same for an applied cover of $c = 20\ mm$ and $c = 50\ mm$. When a cover of $c = 70\ mm$ is applied we see a slight decrease of the maximum crack width (about 2%). This occurs because the mean value of the crack spacing is limited to a value of 200 $mm$.The results are presented in the following graphs.




*Graph 2-5: Influence of the limitation of the mean crack spacing on the mean value of the crack width acc. NEN-3880 for beam 3.*




*Graph 2-6: Influence of the limitation of the mean crack spacing on the maximum value of the crack width acc. NEN-3880 for beam 3.*

### Results Rectangular beam 13:

$c = 30\ mm$

*Table 2-10: Values of the crack width calculated for a cover of c=30 mm according to NEN 3880 for Beam 13*

| $F_{applied}$ | $M_{applied}$ | $\sigma_s$ | $\Delta l_{mean}$ | $\varepsilon_{sm}-\varepsilon_{cm}$ | $w_{mean}$ | $w_{max}$ |
|---|---|---|---|---|---|---|
| [kN] | [kNm] | [N/mm²] | [mm] | [-] | [mm] | [mm] |
| **109** | **136.25** | 136.57 | 134 | 5.58E-04 | 0.075 | 0.146 |
| **184** | **230.00** | 230.55 | 134 | 1.03E-03 | 0.138 | 0.25 |
| **234** | **292.50** | 293.20 | 134 | 1.34E-03 | 0.180 | 0.31 |
| **334** | **417.50** | 418.49 | 134 | 1.97E-03 | 0.264 | 0.45 |

$c = 50\ mm$

*Table 2-11: Values of the crack width calculated for a cover of c=30 mm according to NEN 3880 for Beam 13*

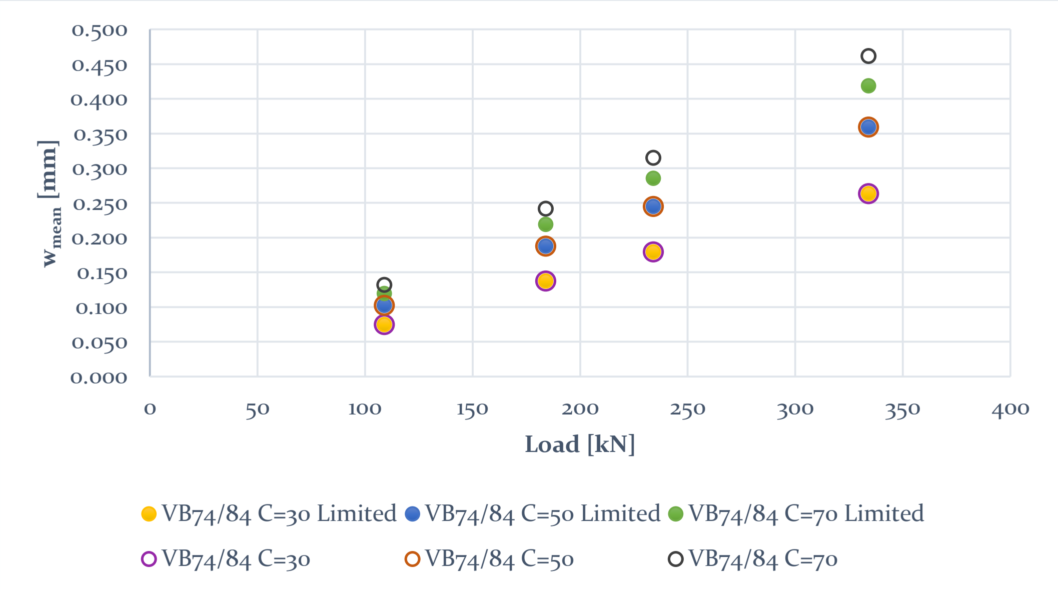
| $F_{applied}$ | $M_{applied}$ | $\sigma_s$ | $\Delta l_{mean}$ | $\varepsilon_{sm}-\varepsilon_{cm}$ | $w_{mean}$ | $w_{max}$ |
|---|---|---|---|---|---|---|
| [kN] | [kNm] | [N/mm²] | [mm] | [-] | [mm] | [mm] |
| **109** | 136.25 | 140.55 | 177 | 5.78E-04 | 0.103 | 0.199 |
| **184** | 230.00 | 237.26 | 177 | 1.06E-03 | 0.188 | 0.34 |
| **234** | 292.50 | 301.73 | 177 | 1.38E-03 | 0.245 | 0.43 |
| **334** | 417.50 | 430.67 | 177 | 2.03E-03 | 0.360 | 0.61 |

$c = 70\ mm$

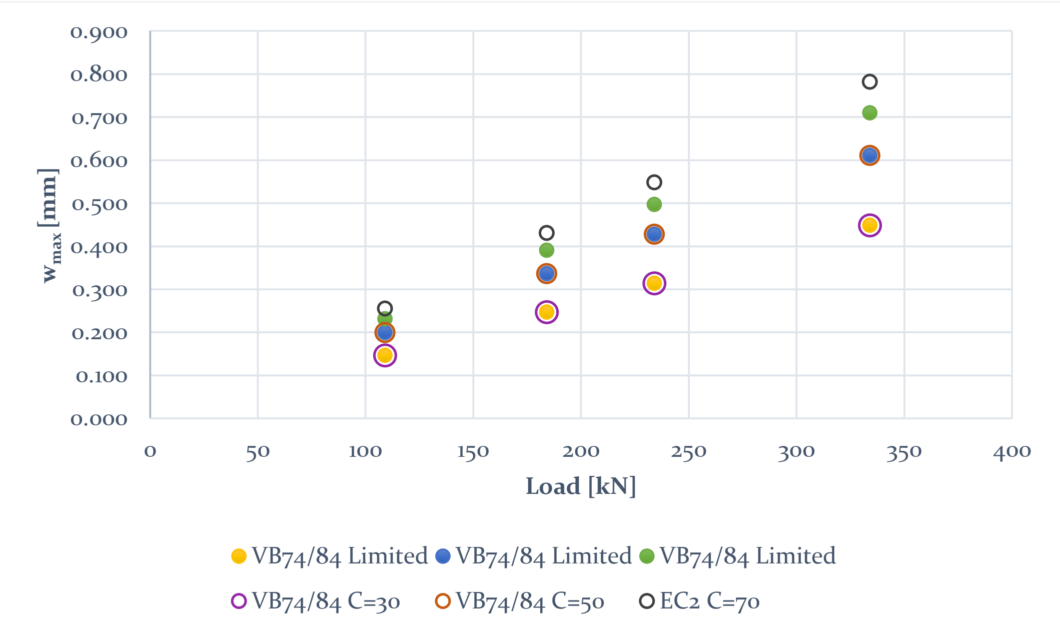
*Table 2-12: Values of the crack width calculated for a cover of c=70 mm according to NEN 3880 for Beam 13*

| $F_{applied}$ | $M_{applied}$ | $\sigma_s$ | $\Delta l_{mean}$ | $\varepsilon_{sm}-\varepsilon_{cm}$ | $w_{mean}$ | $w_{max}$ |
|---|---|---|---|---|---|---|
| [kN] | [kNm] | [N/mm²] | [mm] | [-] | [mm] | [mm] |
| **109** | 136.25 | 144.76 | 200 | 5.99E-04 | 0.120 | 0.232 |
| **184** | 230.00 | 244.36 | 200 | 1.10E-03 | 0.22 | 0.39 |
| **234** | 292.50 | 310.76 | 200 | 1.43E-03 | 0.29 | 0.50 |
| **334** | 417.50 | 443.57 | 200 | 2.09E-03 | 0.42 | 0.71 |

Also for beam 13 we see that the limitation of the mean crack spacing does not have an influence on the mean- and maximum value of the crack width according to the NEN 3880 calculations, since the mean value of the crack spacing stays the same for an applied cover of $c = 20\ mm$ and $c = 50\ mm$. When a cover of $c = 70\ mm$ is applied we see a slight decrease of the mean- and maximum crack width (about 9%). This occurs because the mean value of the crack spacing is limited to a value of $200\ mm$. The results are presented in the following graphs.




*Graph 2-7: Influence of the limitation of the mean crack spacing on the mean value of the crack width acc. NEN-3880 for beam 13*




*Graph 2-8: Influence of the limitation of the mean crack spacing on the maximum value of the crack width acc. NEN-3880 for beam 13*

# 3 CONCLUSION

In this chapter an overall conclusion will be provided regarding the analytical analysis of the influence of the concrete cover and the limitation of the crack spacing on the cracking behavior of a beam subjected to bending.

In section 1.3 it was clear than an increasing cover does have an influence on the cracking behavior of a beam subjected to bending according to the Eurocode 2 calculations. We saw that the mean- and maximum value of the crack width in the Eurocode 2 calculations increased due to the increase of the maximum crack spacing ($s_{r,max}$) and the decrease of the effective reinforcement ratio ($\rho_{peff}$). Since the VARCE suggested a limitation of the maximum crack spacing in the Eurocode 2 calculations, this limitation was applied in the calculations to see what influence it would have on the cracking behavior. This was done in section 2.1. After applying the limitation we saw that it only had an influence on the crack width calculations according to the Eurocode for larger applied covers $c = 50\ mm$ and $c = 70\ mm$. The limitation of the maximum crack spacing according to the VARCE caused the mean- and maximum value of the crack width to decrease, since $s_{r,max}$ also decreased when larger covers were applied.

When the concrete cover was varied in the NEN 3880 calculations we saw that it also had an influence on the cracking behavior of a beam subjected to bending. This was presented in section 1.4. The mean- and maximum value increased due to the increase of the mean crack spacing ($\Delta l$) and due the increase of the strain difference($\varepsilon_a - \varepsilon_b$). In the VB74/84 regulations a limitation of the mean value of the crack spacing was also provided. In order to know whether the limitation of the mean crack spacing had an influence on the crack width calculations, this limitation was also applied in Excel (section 2.2). After the calculations it was clear that the limitation had no influence on the mean- and maximum value of the crack width for a when a cover of $c = 30\ mm$ was applied. When a cover of $c = 50\ mm$ was applied the crack width decreased with less than 3% and for a cover of $c = 70\ mm$ the decrease was less than 10%. And so the influence of the limitation of the mean value of the crack spacing on the cracking behavior according to the VB74/84 can be neglected.

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