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DOI
10.1016/j.commtr.2025.100221

Publication date
2025

Document Version
Final published version

Published in
Communications in Transportation Research

Communications in Transportation Research 5 (2025) 100221

Contents lists available at ScienceDirect

Communications in Transportation Research

journal homepage: www.journals.elsevier.com/communications-in-transportation-research

Full Length Article

Optimal speed limit under multi-class user equilibrium: A prescriptive approach using mathematical programming

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ARTICLE INFO

Keywords:
Traffic management
Speed limit
Optimization
Multi-class user equilibrium
Mixed-integer quadratic programming
Value of time

ABSTRACT

In practice, speed limits on road networks are often determined pragmatically, which can give suboptimal solutions for traffic performance and unfair results for the underlying user classes. This study presents an elegant approach to determine optimal speed limits on a traffic network with asymmetric user classes under congested conditions, that minimizes individual user travel time and does justice to differences in economic importance. Existing prescriptive approaches typically lack one or more of these features, cannot guarantee optimality or are difficult to solve. We formulate a new prescriptive method using mixed-integer quadratic programming. The model can be solved with well-established operation research approaches and commercial solvers such as Cplex or Gurobi. To demonstrate the approach, we apply it to a regional network in the Netherlands. The result shows a reduction of travel time of passenger cars by 6% and of trucks by 13%, with mild changes in speed limits compared to the base situation, of between −20% and +10%. The speed limit changes and impacts are in line with the relatively high economic importance of freight traffic. Also we find in this case that the speed limit changes are ordered by major routes through the network, which makes implementation relatively straightforward.

1. Introduction

Traffic management is an efficient tool that can steer traffic in a way so that the road infrastructures are used more efficiently (Wismans, 2012) in an urban area or with motorways (Hegyi, 2004; Papageorgiou et al., 2007). Research provides theoretical approaches that assess the network performances of newly proposed traffic management measures before implementing. Different measures are applied in different scenarios such as in a city or in free way traffic management. These include policy-based measures (e.g., perimeter control by Ingole et al. (2020), tolling by Gonzales and Daganzo (2012)), and infrastructure-based measures (e.g., ramp metering (Bellemans et al., 2006), signal control (Zhang et al., 2013), speed limit (Popov et al., 2008)), and participant-based measures (e.g., connectivity of autonomous vehicles (Guériau et al., 2016)). These measures can be used to achieve different objectives: investigating minimizing total travel time (Bahrami and Roorda, 2020), reducing environmental impact (Osorio and Nanduri, 2015), designing best road pricing strategies (Hassan et al., 2013), and reducing congestion (Chen and Yang, 2012).

Setting speed limits for freeways and urban roads is an important tool for regional and network level traffic management. It is also a complex matter as it can be viewed from different perspectives and has impacts on various aspects. Elvik (2002) summarizes the involved perspectives as societal, road user, taxpayer, residential. Van Benthem (2015) discusses the effects of speed limits on different aspects, namely travel time, pollution, the likelihood and severity of accidents. The authors point out that determining speed limits on motorways involves considering aspects such as transportation network performance, road deterioration, and societal/environmental impacts.

We do not exhaust all the above mentioned aspects but aim to providing an effective analytical framework and formulation for optimizing speed limits considering the network performance. To achieve this, traffic models or simulations under the so-called user equilibrium (UE) condition are often used to consider these traffic management measures and their effect on road networks (Yang et al., 2015), which provide a baseline for evaluation of traffic management policies. However, descriptive approaches often do not guarantee optimality, while analytical models are usually formulated in a (non-linear & non-convex)

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https://doi.org/10.1016/j.commtr.2025.100221
Received 30 May 2025; Accepted 5 June 2025
Available online xxxx

bi-level programming fashion and difficult to solve (we elaborate these points in Section 2). Based on our recent research (Lin et al., 2022), this study proposes an approach for optimal design of speed limit in traffic networks under multi-class UE. The problem is formulated as mixed-integer quadratic programming with a quadratic objective function and linear constraints. The proposed approach uses a formulation of optimization and can be easily solved using existing operations research tools (such as solvers developed by Gurobi).

The remainder of this paper is organized as follows. Section 2 reviews literature and identifies the research gap and the contribution of this study. Section 3 explains the mathematical formulation of our proposed approach. Section 4 implements the traffic management optimization in a real-world network with numerical experiments and discusses the results. Section 5 concludes the study and points out future research.

2. Literature review

There are in general two approaches to design traffic management strategies. One is to pre-define the scheme/policies (which usually involve combinations of traffic management measures and KPIs to select the best combination) and use models or simulations to evaluate their effectiveness (Fig. 1), usually under equilibrium given certain network and travel demand. The performances of these pre-defined combinations are then compared and the best is selected for implementation. These approaches are descriptive tools that computes system performance given certain inputs but do not give any suggestions on improving decisions. Another approach for traffic management is prescriptive tools that involve analytical modeling and optimization (Fig. 2). Different from descriptive methods, they give suggestions to assist decision-making towards desired performance.

Abundant research addresses both descriptive and prescriptive methods in traffic management scheme design and optimization. On the descriptive side, Guériau et al. (2016) design different scenarios in traffic management to assess the benefit of connected vehicles. Monteil et al. (2013) use simulations to evaluate the stability of traffic flow with communication and control methods. Descriptive methods are also widely used by practitioners (in de Vegte et al., 2005), where several options for local freight traffic management schemes are proposed and simulations are used to determine the best option for implementation. Wismans (2012) points out that the enumeration of strategies relies only on experts' experiences rather than thoroughly searching for all alternatives, as a result, optimality of the solutions is not guaranteed.

Another apporach for traffic management is prescriptive tools that involve analytical modeling and/or optimizaiton. Different from descriptive methods, they give suggestions to assist decision-making towards the desired system performance. The best/optimal policy is not obtained by selecting from several pre-defined scenarios. Instead, the prescriptive model internalizes the selection process to a pure mathematical form, often by formulating an optimization problem that explores possible measure combinations as decision variables (Fig. 2). Designers or traffic managers could benefit largely from this prescriptive approach since they only need to define the KPIs (i.e., objective function) and let computer find the optimal policy. Literature with prescriptive approaches usually makes use of mathematical programming with equilibrium constraint (MPEC), leading to a bi-level programming method. In such bi-level formulation, the upper-level is an objective function to maximize or minimize one or more network performance indicators, and the lower-level is traffic assignment under UE conditions. To solve such bi-level programming problems, different algorithm are developed: artificial bee colony (Szeto et al., 2015), particle swarm optimization (Wang et al., 2015), genetic algorithm (Liu and Luo, 2012; Mathew and Sharma, 2009), descent based algorithm (Patriksson and Rockafellar, 2002), etc. Dempe (2020) points out that bi-level programming problems are non-convex and non-differentiable, even if their defining functions are convex and smooth. This makes bi-level programming problems difficult to solve and in many cases only sub-optimal solutions can be obtained.

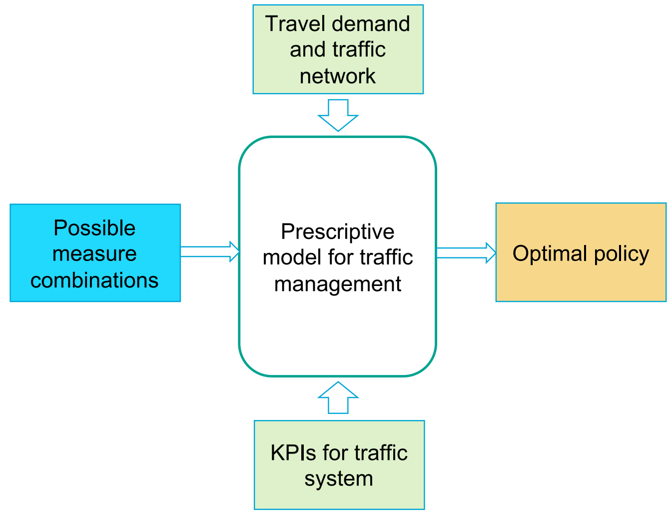

Fig. 2. Prescriptive approach for traffic management measure optimization.

In all the traffic management measures, we focus on speed limit in this paper. Speed limit design can improve both local and network level traffic. Csikós et al. (2018) design a model predictive controller for variable speed limits (VSLs) to suppress moving jamwaves and reduce pollution locally. For the network level speed limit design or optimization, complex formulations are often involved. Yang et al. (2013) develop a bi-level programming approach to optimize speed limit in a road network under UE flows. Wang (2013) transfers a bi-level

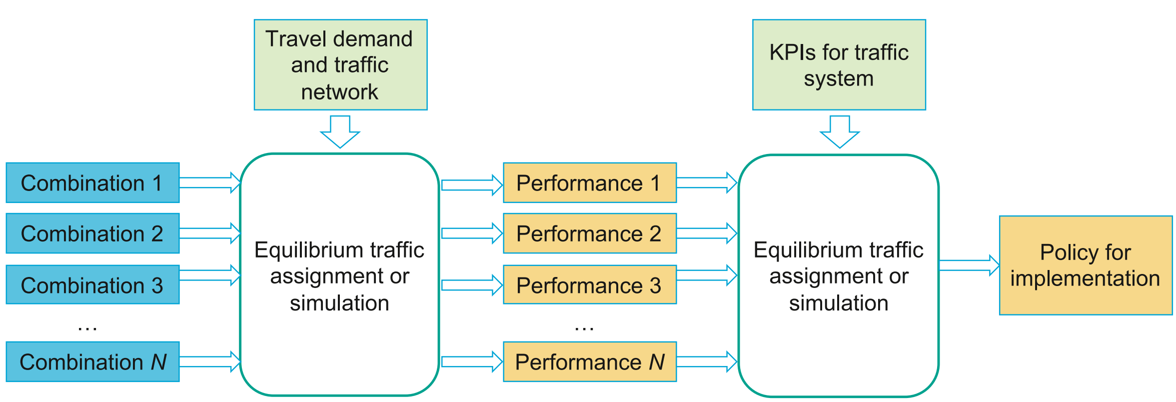

Fig. 1. Descriptive approach for traffic management measure design.

programming problem to a conic quadratic mixed-integer programming problem, which can be solved by off-the-shelf solvers. However, the proof for the convexity of this formulation is not given, and non-convex formulations can lead to local optimal solutions. Yang et al. (2012) give explicit formulation for UE traffic assignment with speed limit and analyze the impacts of altering link speed limits on the network equilibrium and conclude that it is possible to design appropriate link-specific speed limits that can improve network performance (in this case, reducing both emissions and travel time for the whole network). Both Yang et al. (2012, 2015) admit that the optimal design of speed limit scheme using single or multi-objective optimization remain an interesting research direction.

Since traffic usually consists of more than passenger cars, it is necessary for traffic managers to consider multiple vehicle classes in traffic modeling and management. Ezaki et al. (2022) show that by considering logistics operations, researchers and practitioners (e.g., traffic managers) may acquire more complete insights on traffic network dynamics. Freight traffic takes up an important part in the traffic composition. According to Statistics Netherlands (CBS) (2020), road transport within the Netherlands carried 766 million of goods in the year of 2019, which was 41% of the total freight being transported. Freight traffic has very different characteristics from passenger traffic. This includes different origin/destinations and vehicle characteristics such as length and speed (Schreiter, 2013). In particular, the value of time (VOT) is different for different vehicle (user) classes. According to SP surveys (De Jong et al., 2014; Kouwenhoven et al., 2014), the VOT for freight is 38 EUR/h in the Netherlands, and the VOT for passenger car is 9 EUR/h. Despite the differences, freight and passenger are both active participants taking a share in the public traffic infrastructures.

The calculation of UE with multiple vehicle class capabilities are rarely performed in forms of mathematical programming (MP), despite MP is seen as one of the most practical approach for traffic assignment because of the widely available solving methods (Bliemer and Bovy, 2003). This is mainly because the previous approaches (e.g., Beckmann transformation) require an symmetric assumption that makes the procedure non-realistic.

Lin et al. (2022) recently propose an approach for multi-class UE assignment with mixed-integer linear programming (MILP). This approach does not require the symmetry assumption mentioned above. It provides new opportunities to formulate traffic management problems differently. In this study, we build upon this and propose a mixed-integer quadratic programming approach to optimize speed limit design with UE conditions. Solving the MIQP gives the traffic management measures that optimize the network performances (e.g., the total travel time, emission, or value loss by congestion) under the multi-class UE condition.

To sum up the literature review: Speed limit optimization under UE condition is usually formulated in bi-level programming. This formulation is non-convex and non-differentiable and therefore, difficult to solve. Despite facing such issues, researchers keep searching for new prescriptive approaches that benefit from mathematical programming formulations. On the other hand, calculating multi-class UE with MP usually comes with an unrealistic assumption of symmetry. For these reasons, to our best knowledge, few approaches optimize speed limit under multi-class UE condition with easy-to-solve formulations.

3. Methodology

This section starts from the recent research of analytical multi-class UE assignment (Lin et al., 2022) with MILP and builds up to the traffic management optimization approach. In this study, we briefly recap the MILP approach for UE assignment. Subsequently, we formulate the traffic management problem in an MIQP formulation with a quadratic objective function and (mixed-integer) linear constraints, in which techniques are applied to linearize some of them.

3.1. MILP formulation to solve UE

Let $\mathcal{G} = \{\mathcal{P}, \mathcal{E}\}$ be a graph with a collection of nodes ($\mathcal{P}$) and directed links ($\mathcal{E}$). Between some of the nodes are OD pairs $w \in \mathcal{W}$. Between each OD pair, vehicles of classes $m \in \mathcal{M}$ can travel via several paths $p \in \mathcal{P}_w$. Let x_{mwp} be the number of vehicles of class m that travel between OD pair w via path p; and c_{mwp} the travel time experienced by vehicles with class m travelling via path p between OD pair w; let c^*_{mw} be the lowest cost for class m to travel between OD pair w. For the traffic network defined above, a UE condition is described as Eqs. (1)–(3):

$$x_{mwp} \geq 0 \tag{1}$$

$$c_{mwp} \geq c^*_{mw} \tag{2}$$

$$x_{mwp}(c_{mwp} - c^*_{mw}) = 0 \tag{3}$$

The above conditions represent UE as a variational inequality principle (Van Vliet, 1987). In the these conditions, Eq. (1) specifies that for each vehicle type m, between each OD pair w, on each path p, the flow volume is non-negative. Equation (2) ensures that all traversed and untraversed paths between the OD pair w has a travel cost no less than a minimal travel cost between this OD pair. Equation (3) indicates that if vehicle type m has flow on path p between OD pair w, the travel cost for the vehicle type on this path should be the lowest. The general formulation of multi-class UE assignment in MP is listed below:

$$\min J|_{x_{mwp}} = \sum_m \sum_w \sum_{p_w} a_{mwp}(c_{mwp} - c^*_{mw}) \tag{4}$$

s.t.

$$a_{mwp} = \begin{cases} 1, & \text{if} \quad x_{mwp} > 0 \\ 0, & \text{if} \quad x_{mwp} = 0 \end{cases} \tag{5}$$

$$\sum_p x_{mwp} = d_{mw} \tag{6}$$

$$x_{mwp} \geq 0 \tag{7}$$

$$c_{mwp} - c^*_{mw} \geq 0 \tag{8}$$

$$x^{\text{link}}_{me} = \sum_w \sum_p \delta_{pe} x_{mwp} \tag{9}$$

$$c_{mwp} = \sum_e \delta_{pe} c^{\text{link}}_{me} \tag{10}$$

$$c^{\text{link}}_{me} = c_{me,0}\left(1 + \alpha\left(\frac{\sum_m x^{\text{link}}_{me}\pi_m}{K_e}\right)^{\beta}\right) \tag{11}$$

Note that Lin et al. (2022) provide approaches to linearize the above MP model so that it can be solved in MILP form. The objective function (4) is non-negative and when it reaches 0, the solution x gives the UE state. Condition (5) uses a_{mwp} to flag whether path p between OD pair w is used by vehicle class m. Equation (6) specifies that the total volume of vehicle class m between OD pair w should be equal to the demand d_{mw}. Equation (7) ensures that path flow volumes are non-negative. Equation (8) introduces c^*_{mw} which takes the lowest value of travel time in all paths for vehicle class m between OD pair w. The relationships between link flow/cost and path flow cost are specified using Eqs. (9) and (10). The link travel time (c^{link}_{me}) is associated to the link vehicle flows (x^{link}_{me}). A volume-delay function such as BPR can be used Eq. (11), with π_m the passenger car equivalent (PCE) value for class m (we use $\pi_{\text{car}} = 1$; $\pi_{\text{truck}} = 2.5$). Parameters α and β usually take value of 0.15 and 4, respectively. Readers are referred to Lin et al. (2022) for the details on the complete MILP formulation of the UE assignment.

Traditionally, to consider the modeling of traffic management measures, the above formulation is included in a bi-level programming with equilibrium constraints. In this study, we formulate the same problem in a different way so that it can be easily solved using mature operation research techniques.

3.2. Formulating traffic management policy optimization

This section extends the MILP UE formulation to include traffic management policy optimization. Firstly we rewrite the previous objective function Eq. (4) as a constraint, ensuring the UE condition in the solution of flows:

$$a_{mwp}\left(c_{mwp} - c^*_{mw}\right) = 0 \ \forall m, w, p \tag{12}$$

Similar to Eq. (4), in Eq. (12), a_{mwp} is a binary (decision) variable denoting whether path p is used by class m between OD pair w and follows Eq. (5). If $a_{mwp} = 0$, the path is not used, and the path travel cost c_{mwp} can be larger than the minimum travel cost c^*_{mw} among the available paths; otherwise c_{mwp} should be the minimal, i.e., $c_{mwp} - c^*_{mw} = 0$. This constraint is equivalent to the equilibrium constraint (Eq. (3)) in MPEC (reviewed in Section 2) formulations. The advantage of using Eq. (12) is that it can be easily linearized. As a result, the MP formulation is convex since it only contains linear constraints and can be easily solved with global optimality.

We now involve optimal speed limit design by extending the MILP formulation. In this study, we consider the possibility of varying speed limit of passenger vehicles on all road links. Consider link e, vehicles' free flow time through this link is related to (class-specific) free flow speed $v_{me,0}$ and length of the link L_e, i.e., $c_{me,0}v_{me,0} = L_e$. Now we include the free flow speed limit on each link as a decision variable, and $v_{me,0} \in [v^-_{me,0}, v^+_{me,0}]$. Note that in this paper we assume that the speed limit determines the free flow speed $v_{me,0}$. In some other research (Yang et al., 2012), imposing a speed limit constraint changes the shape of the BPR function with an extra non-monotonic segment, leading to an additional source for non-uniqueness solutions. We argue that since piecewise linearization is employed in this study, applying the approach of Yang et al. (2012) does not impose extra challenge to our model. However, in order to observe the effectiveness of our approach, we keep the BPR function monotonic. As a result, the volume-delay function Eq. (11) can be re-written as Eq. (13):

$$c^{\text{link}}_{me} v_{me,0} = L_e\left(1 + \alpha\left(\frac{\sum_m x^{\text{link}}_{me}\pi_m}{K_e}\right)^{\beta}\right) \tag{13}$$

In this study, we design the optimizing problem to minimize the monetary value of the total travel time (ξ_m is the VOT of class m). Having in mind that the value of time for different vehicle classes according to De Jong et al. (2014) and Kouwenhoven et al. (2014) ($\xi_{\text{car}} = 9$ and $\xi_{\text{truck}} = 38$), we have

$$\min J|_{x,v_0} = \sum_{m,w,p} x_{mwp} c_{mwp} \xi_m \tag{14}$$

The objective function (14) is quadratic, leading to the formulation an MIQP. Solving this problem we have the UE distribution of traffic flow (x) as well as the combination of v_0 that minimizes the total cost for travel time. Next we linearize the constraints so that the MIQP is in a convex formulation.

3.3. Linearizing and formulating MIQP

The above formulation for searching for the optimal speed limit combination is, nevertheless, non-linear and non-convex. To simplify the calculation, this section explains how the MP can be reformulated into a MIQP, which can be solved more effectively.

3.3.1. Linearizing UE condition

The UE condition Eq. (12) is included in the model as a constraint and can be easily linearized and re-written as the following condition:

$$c_{mwp}\begin{cases} \geq c^*_{mw}, & \text{if} \quad a_{mwp} = 0 \\ = c^*_{mw}, & \text{if} \quad a_{mwp} = 1 \end{cases} \quad \forall m \in \mathcal{M}, w \in \mathcal{W}, p \in \mathcal{P} \tag{15}$$

We can then formulate the above constraint in linear form with a sufficiently large number M:

$$c_{mwp} - c^*_{mw} \leq M(1 - a_{mwp}) \ m \in \mathcal{M}, w \in \mathcal{W}, p \in \mathcal{P} \tag{16}$$

In addition, the linearization of Eq. (5) is performed using the similar technique:

$$Mx_{mwp} \geq a_{mwp} \ \forall \ m \in \mathcal{M}, w \in \mathcal{W}, p \in \mathcal{P}_w \tag{17}$$

$$x_{mwp} \leq Ma_{mwp} \ \forall \ m \in \mathcal{M}, w \in \mathcal{W}, p \in \mathcal{P}_w \tag{18}$$

3.3.2. Linearizing BPR function

Involving Eq. (13) as a constraint in the MP makes the model non-linear and non-convex, since c^{link}_{me} is related to x_{mwp} and $v_{me,0}$ is considered as decision variable. This section linearizes the BPR function and include decisions on speed limits. We take an approach to inexplicitly include $v_{me,0}$ as decisions, by re-writing Eq. (13) using the following conditions:

$$c^{\text{link}}_{me} v^-_{me,0} \leq L_e\left(1 + \alpha\left(\frac{\sum_m x^{\text{link}}_{me}\pi_m}{K_e}\right)^{\beta}\right) \tag{19}$$

$$c^{\text{link}}_{me} v^+_{me,0} \geq L_e\left(1 + \alpha\left(\frac{\sum_m x^{\text{link}}_{me}\pi_m}{K_e}\right)^{\beta}\right) \tag{20}$$

Inequalities (19) and (20) specify that the actual travel time falls between a range regulated by $v^-_{me,0}$ and $v^+_{me,0}$. Let $f_e(x)$ represent the right-hand-side of Inequalities (19) and (20), and let $x^{\text{link}}_e = \sum_m x^{\text{link}}_{me}\pi_m$ we have

$$c^{\text{link}}_{me} v^-_{me,0} \leq f_e(x_e) \tag{21}$$

$$c^{\text{link}}_{me} v^+_{me,0} \geq f_e(x_e) \tag{22}$$

The decision variable $v_{me,0}$ is excluded from the formulation and can be calculated from the following (after optimization):

$$v_{me,0} = \frac{L_e}{c^{\text{link}}_{me}}\left(1 + \alpha\left(\frac{\sum_m x^{\text{link}}_{me}\pi_m}{K_e}\right)^{\beta}\right) = \frac{f_e(x^{\text{link}}_e)}{c^{\text{link}}_{me}} \tag{23}$$

Inequalities (19) and (20) can be linearized using piecewise approximation. In this study, we make use of a special order set (SOS) feature for piecewise approximation. This feature is found in many off-the-shelf MP solvers such as Cplex and Gurobi. For each link $e \in \mathcal{E}$, the aggregated flow $x^{\text{link}}_e = \sum_m \sum_w \sum_p \delta_{pe} x_{mwp}\pi_m$ monotonically determines the travel time of each vehicle class via this link according to the BPR function. The piecewise BPR function is represented by type 2 SOS constraints. Denote cost function with linearized BPR as $\bar{c}^{\text{link}}_{me}(x)$, the number of segments $L = L^{\text{left}} + L^{\text{right}}$, a type 2 SOS constraint specifies that a set of continuous variables $[b_1, b_2, \ldots, b_l, \ldots, b_L]$, $b_l \in [0, 1]$, only 2 (consecutively) of them can take values other than 0. We then have the following constraints:

$$\left[(b_1, 0), \left(b_2, \frac{K_e}{L^{\text{left}}}\right), \ldots, \left(b_l, \frac{K_e}{L^{\text{left}}} \times (l-1)\right), \ldots, \left(b_{L+1}, \frac{K_e}{L^{\text{left}}} \times L\right)\right] \ \forall m, e \tag{24}$$

$$\sum_{l=1}^{L+1} b_{me,l} = 1 \ \forall m, e \tag{25}$$

$$\sum_{l=1}^{L+1} b_{me,l} \frac{K_e}{L^{\text{left}}} \times (l-1) = x_e^{\text{link}} \ \forall m, e \tag{26}$$

$$\sum_{l=1}^{L+1} b_{me,l} f_e\left(\frac{K_e}{L^{\text{left}}} \times (l-1)\right) \geq \bar{c}_{me}^{\text{link}} v_{me,0}^{-} \ \forall m, e \tag{27}$$

$$\sum_{l=1}^{L+1} b_{me,l} f_e\left(\frac{K_e}{L^{\text{left}}} \times (l-1)\right) \leq \bar{c}_{me}^{\text{link}} v_{me,0}^{+} \ \forall m, e \tag{28}$$

Condition (24) declares the SOSs for the solver. Each of the sets has $L+1$ pairs of continuous decision variables b_l and their weights $\frac{K_e}{L^{\text{left}}} \times (l-1)$. Equations (25) and (26) specify that the weights $b_{me,l}$ are used to select terminal points and represent the values of x_e^{link}. Inequalities (27) and (28) denote that the actual travel time falls between the boundaries specified by $v_{me,0}^{-}$ and $v_{me,0}^{-}$.

3.4. Final MIQP formulation

So far, all constraints are linear and the final MIQP formulation is presented in conditions (29)–(43):

$$\min J|_{x,v_0} = \sum_{m,w,p} x_{mwp} c_{mwp} \xi_m \tag{29}$$

s.t.

$$x_{mwp} \geq 0 \ \forall m \in \mathcal{M}, w \in \mathcal{W}, p \in \mathcal{P}_w \tag{30}$$

$$\sum_{p} x_{mwp} = d_{mw} \ \forall m \in \mathcal{M}, w \in \mathcal{W} \tag{31}$$

$$M x_{mwp} \geq a_{mwp} \ \forall m \in \mathcal{M}, w \in \mathcal{W}, p \in \mathcal{P}_w \tag{32}$$

$$x_{mwp} \leq M a_{mwp} \ \forall m \in \mathcal{M}, w \in \mathcal{W}, p \in \mathcal{P}_w \tag{33}$$

$$c_{mwp} - c_{mw}^{*} \geq 0 \ \forall m \in \mathcal{M}, w \in \mathcal{W}, p \in \mathcal{P}_w \tag{34}$$

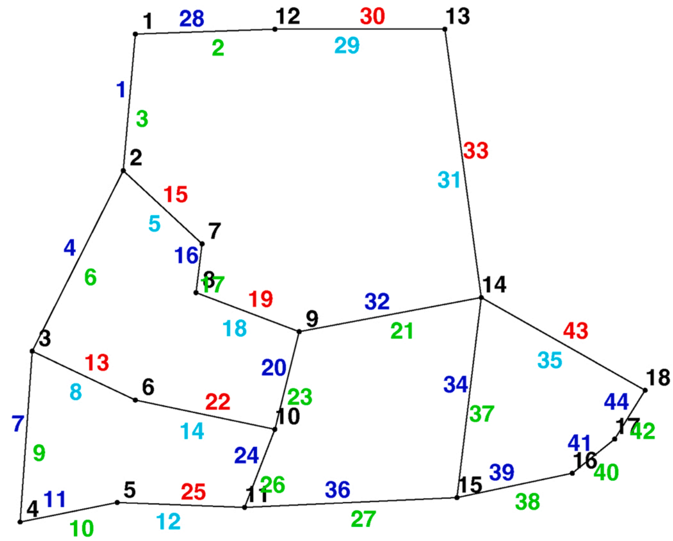

Fig. 4. Link segments considered in the experiments.

$$c_{mwp} - c_{mw}^{*} \leq M(1 - a_{mwp}) \ \forall m \in \mathcal{M}, w \in \mathcal{W}, p \in \mathcal{P}_w \tag{35}$$

$$b_{me,l} \in [0, 1] \ \forall m \in \mathcal{M}, e \in \mathcal{E}, l \in \mathcal{L} \tag{36}$$

$$\sum_{l=1}^{L+1} b_{me,l} = 1 \ \forall m \in \mathcal{M}, e \in \mathcal{E} \tag{37}$$

$$\sum_{l=1}^{L+1} b_{me,l} \frac{K_e}{L^{\text{left}}} \times (l-1) = x_e^{\text{link}} \ \forall m \in \mathcal{M}, e \in \mathcal{E} \tag{38}$$

$$\sum_{l=1}^{L+1} b_{me,l} f_e\left(\frac{K_e}{L^{\text{left}}} \times (l-1)\right) \geq \bar{c}_{me}^{\text{link}} v_{me,0}^{-} \ \forall m \mathcal{M}, e \in \mathcal{E} \tag{39}$$

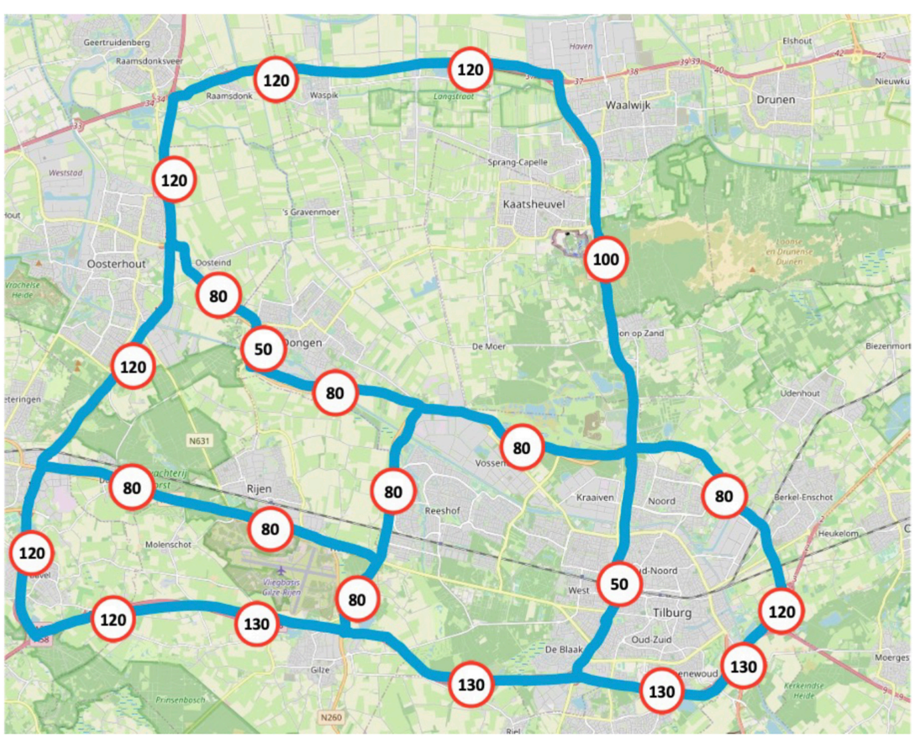

Fig. 3. Speed limits on (two-way) links, adapted from Openstreetmap.

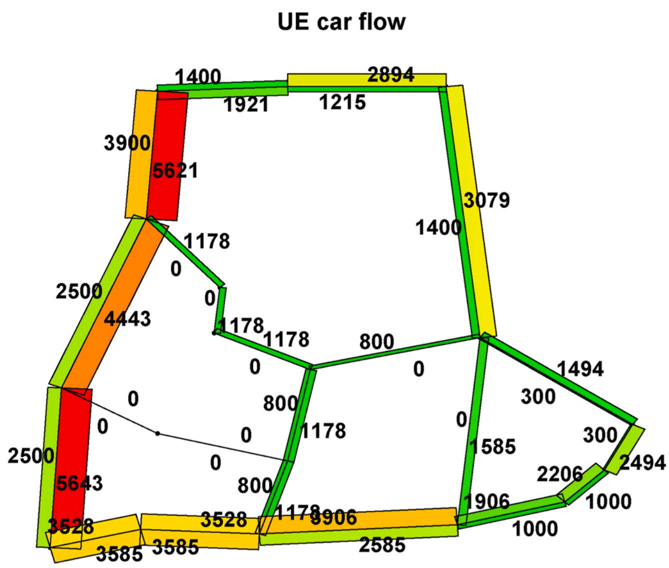

Fig. 5. Passenger car flow distribution without speed limit optimization (Scenario 1).

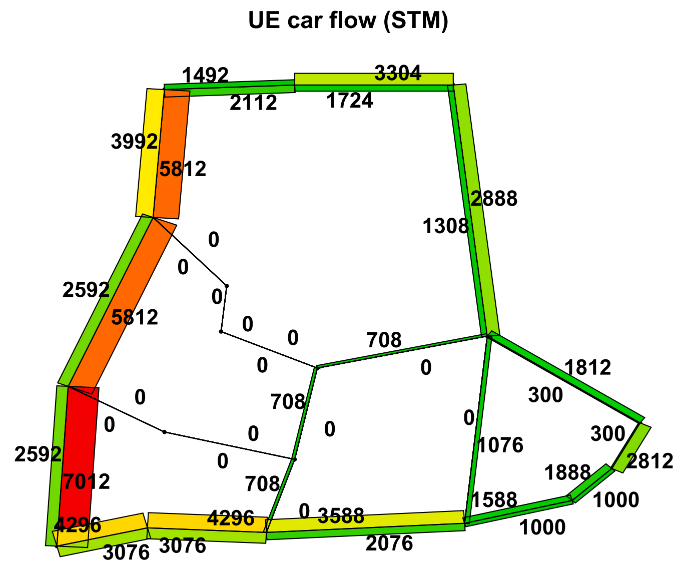

Fig. 7. Passenger car flow distribution with speed limit optimization (Scenario 2).

$$\sum_{l=1}^{L+1} b_{me,l} f_e\left(\frac{K_e}{L^{\text{left}}} \times (l-1)\right) \leq \bar{c}_{me}^{\text{link}} v_{me,0}^{+} \;\; \forall m \in \mathcal{M}, e \in \mathcal{E} \tag{40}$$

$$x_e^{\text{link}} = \sum_{m\in\mathcal{M}} \sum_{w\in\mathcal{W}} \sum_{p\in\mathcal{P}_w} \delta_{pe} x_{mwp} \pi_m \;\; \forall e \in \mathcal{E} \tag{41}$$

$$c_{mwp} = \sum_{e\in\mathcal{E}} \bar{c}_{me}^{\text{link}} \;\; \forall w \in \mathcal{W}, p \in \mathcal{P} \tag{42}$$

In addition, the MIQP includes a type 2 SOS constraint:

$$\left[(b_1, 0), \left(b_2, \frac{K_e}{L^{\text{left}}}\right), \ldots, \left(b_l, \frac{x_e^{\text{cap}}}{L^{\text{left}}} \times (l-1)\right), \ldots, \left(b_{L+1}, \frac{K_e}{L^{\text{left}}} \times L\right)\right] \;\; \forall e \in \mathcal{E} \tag{43}$$

4. Numerical experiments

This section applies the method in a real-world motorway network and reports the results. We consider road links around the city Tilburg, the Netherlands (Figs. 3 and 4). We did two rounds of experiments. The first round presents the current situation in the network without the optimization of speed limit; while the second round includes the optimization of speed limit measurement. The results of these two rounds experiments are compared. The motorway network, the hourly demand of passenger vehicles and freight vehicles information is adapted from the Dutch National Model System (Landelijke Model Systeem, LMS) in Rijkswaterstaat (2012). The Network has 18 nodes and 44 directed links. We selected 12 OD pairs for computing the distribution of traffic. To evaluate the effectiveness of the proposed approach, we designed 3 scenarios in the same network/traffic demand, but with different speed limit design parameters. In Scenario 1, we do not allow any adjustments in speed limits. Scenario 1 is therefore UE traffic assignment presenting the UE distribution of passenger cars and trucks with the current speed limits. In Scenario 2, we involve the possibility of adjusting speed limits of passenger cars within the range of [–20%, +10%] of the original speed limit for each link. In Scenario 3, we only allow a reduction in the speed limits for passenger cars, hence the range for adjustment is [–20%, 0%].

The model is coded and run using MATLAB on Windows 10 (Parallels

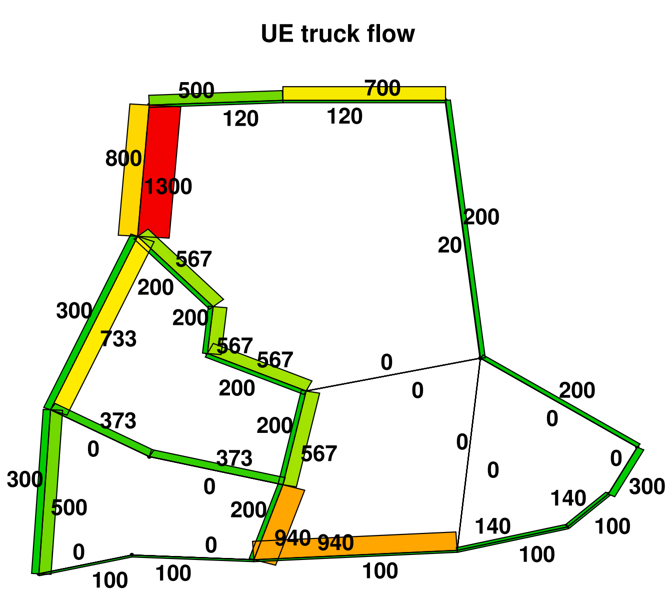

Fig. 6. Truck flow distribution without speed limit optimization (Scenario 1).

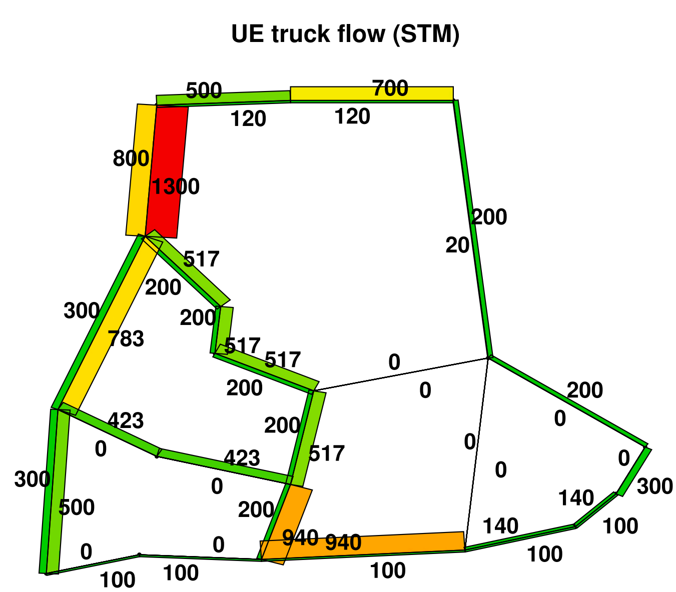

Fig. 8. Truck flow distribution with speed limit optimization (Scenario 2).

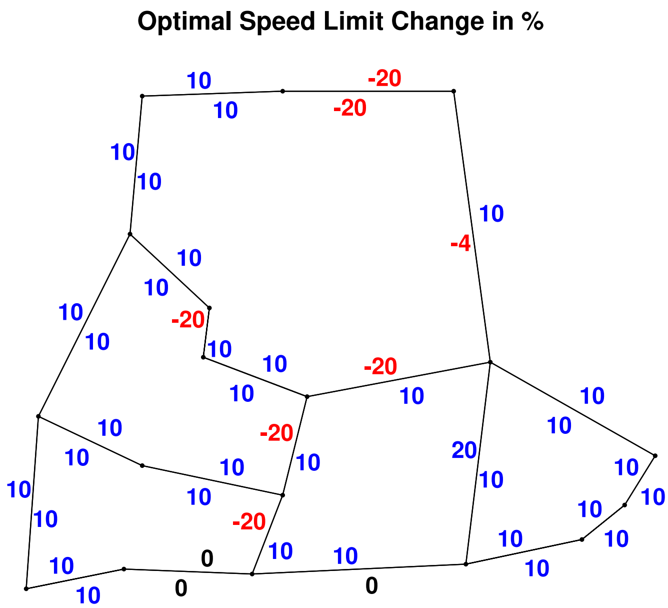

Fig. 9. Speed adjustment (Scenario 2).

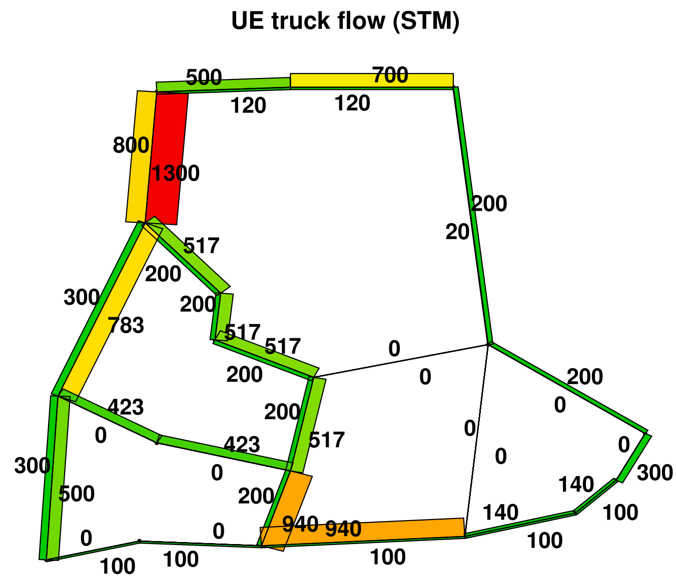

Fig. 11. Truck flow distribution with optimal speed reduction (Scenario 3).

virtual machine) with allocated Intel Core i5-8279 2.4 GHz CPU and 3 GB RAM. The MILP is solved by Cplex v12.10, using 14.91 s; the MIQP is solved by Gurobi v9.5.2, using 17.86 s.

Figs. 5–12 show the resulting traffic distribution of the three scenarios. Figs. 5 and 6 show the passenger car flows and truck flows in Scenario 1. Figs. 7 and 8 show the passenger car flows and truck flows in Scenario 2. Figs. 10 and 11 show the passenger car and truck flows in Scenario 3. Figs. 9 and 12 show the optimal combination of speed limit changes in percentage for each link under Scenarios 2 and 3. Increasing and decreasing of speed limits are represented by blue and red numbers, respectively. Table 1 shows the improvement brought to the traffic network by vehicle hours and monetary values for both vehicle types and total values.

Observing the results in Table 1, with Scenario 2, both passenger cars and trucks can benefit from the speed limit strategy with a class-specific total travel time reduction of 6.41% and 12.77%, respectively. Weighed by VOT, this reduction means a monetary saving of 9.61% for the whole transportation system. In Scenario 3, since passenger cars are more restricted, they have an increase of 20.34% in total travel time, which creates benefit for trucks with a reduction of 24.66% in total travel time. This results in a reduction of 2.64% in the monetary value of total travel time considering both classes. In this scenario, although we only allow reduction of speed limits, the system still performs better. By adjusting the VOT value of passenger cars and trucks, the utility of the two classes can be balanced and justified according to their economic importance.

5. Conclusions

Traffic management, including speed limit design with equilibrium conditions is often approached in descriptive or prescriptive ways. Most of the methods proposed in literature do not guarantee optimality or can be very difficult to solve (especially for bi-level programming problems). This study proposes an MIQP approach that optimizes traffic management measures under the multi-class UE condition. The method is built on a recent developed approach and does not require the symmetry

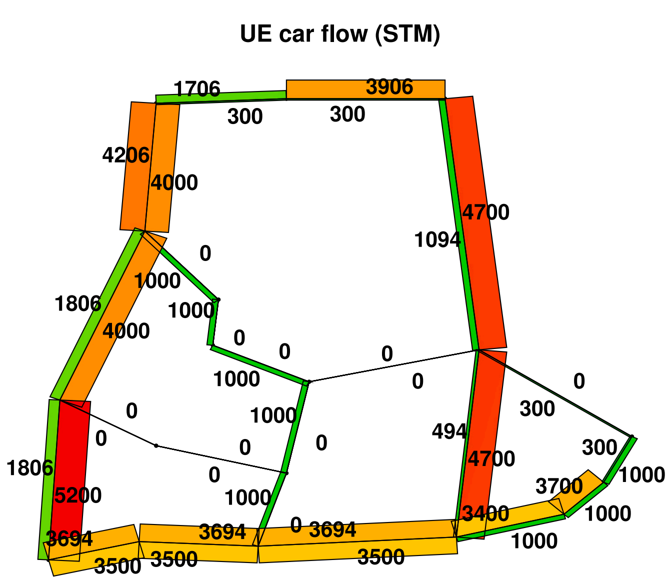

Fig. 10. Passenger car flow distribution with optimal speed reduction (Scenario 3).

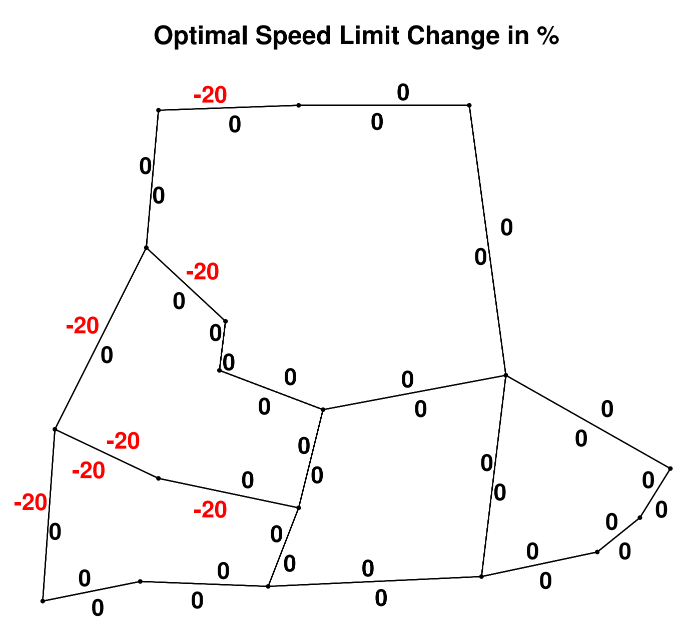

Fig. 12. Speed adjustment (Scenario 3).

Table 1
Total travel time and monetary value reduction per hour.

	Car		Truck		Total value (EUR)
	Vehicle (h)	Car value (EUR)	Vehicle (h)	Truck value (EUR)	
Scenario 1 benchmark	5.3777×10^6	4.8400×10^7	1.3284×10^6	5.0479×10^7	9.8879×10^7
Scenario 2 optimal policy	5.0354×10^6	4.5319×10^7	1.1594×10^6	4.4058×10^7	8.9377×10^7
Scenario 2 savings	6.41%		12.77%		9.61%
Scenario 3 optimal policy	6.4713×10^6	5.8241×10^7	1.0008×10^6	3.8031×10^7	9.6272×10^7
Scenario 3 savings	−20.34%		24.66%		2.64%

assumption for the calculation of multi-class equilibrium. Numerical experiments are conducted in a regional network in the Netherlands. Commercial solvers can solve the problems of this size quite efficiently within 20 s. Optimizing the speed limit of the network can give a 9.61% reduction of total travel time measured in monetary value. Meanwhile, the method considers multi-class values and does justice to the differences of user classes in terms of economic importance.

Our formulation is based on a static traffic model with deterministic UE. The traffic management measure considered is only speed limit. Future research can extend this approach to capture temporal dynamics with stochastic equilibrium settings. It is also interesting to include other traffic management strategies such as lane control and signal control measures.

CRediT authorship contribution statement

Xiao Lin: Data curation, Investigation, Software, Visualization, Writing – review & editing, Conceptualization, Formal analysis, Methodology, Validation, Writing – original draft. **Ludovic Leclercq:** Methodology, Writing – review & editing, Conceptualization, Supervision. **Lóránt Tavasszy:** Funding acquisition, Project administration, Supervision, Conceptualization, Methodology, Resources, Writing – review & editing.

Replication and data sharing

Open source Matlab code for static traffic management can be found at https://github.com/graham-lin/stm_mip.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgements

This work is supported by the Dutch Research Council (NWO) with the project entitled “Freight traffic management as a service” (No. 439.18.456). The authors thank Mr Gaurav Malik for the insightful discussions.

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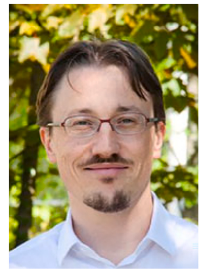

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