# Combining Complexity and Harmony by the Box-Counting Method

## *A comparison between entrance façades of the Pantheon in Rome and Il Redentore by Palladio*

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**Abstract.** *When Benoît Mandelbrot raised the question about the length of Britain's coastline in 1967, this was a major step towards formulating the theory of fractals, which also led to a new understanding of irregularity in nature. Since then it has become obvious that fractal geometry is more appropriate for describing complex forms than traditional Euclidean geometry (not only with regard to natural systems but also in architecture). This paper provides another view on architectural composition, following the utilization of fractal analysis. The procedure concerning the exploration of a façade design is demonstrated step by step on the Roman temple front of the Pantheon by Appolodorus and its re-interpretation – in the particular case the entrance front of Il Redentore, a Renaissance church by Palladio. Their level of complexity and range of scales that offer coherence are visualized by the specific measurement method of box-counting.*
**Keywords.** *Fractal analysis; box-counting method; Pantheon; Il Redentore; Palladio.*

## INTRODUCTION

This paper has two objectives:

1. The first one concerns the description of harmony defined by the appearance of architectural elements of different sizes and scale.
2. The second one utilizes the first one, introducing an objective comparison method between an architectural design (acting as origin) and its historical followers.

Apart from an analysis concerning the utilization of characteristic architectural elements, the current study focuses on the overall viewpoint specified by a harmonic expression of distributions across different scales. The author uses for the first time a particular fractal analysis method as measurement of reminiscence, applied to the Roman temple front of the Pantheon (built between 110 and 125 AD by Appolodorus) and the Renaissance temple front of Il Redentore in Venice by Palladio (groundbreaking in 1577) – The Pantheon was chosen as Palladio (1984) emphasized the particular importance of that building. As benefit of the quantitative method, similarity between two façades can be proved with regard to visual complexity.

## *Fractal analysis*

Fractals – the term was introduced by Mandelbrot in 1975 – are characterized by specific properties, which include development through iterations, infinite complexity, roughness, irregularity, scale invariance and self-similarity. The latter is a central feature – although not a guarantee that the structure is fractal – and sometimes, if statistically, difficult to describe. In mathematical terms, a self-similar composition exists, if parts look exactly or approximately like the whole. With variations, however, it is difficult to detect the basic connection between the whole and its parts, or, in other words, to decipher the underlying rules. Characterization is then provided by the Hausdorff dimension – Mandelbrot (1982) calls it fractal dimension – which in the case of a fractal structure exceeds its topological dimension. In addition, according to Bovill (1996), visually, fractal dimension is the expression of the degree of roughness – that is how much texture an object has. With regard to architecture, it specifies the relationship between a building unit on a higher level (larger scale) and its components on a lower level (smaller scale). Throughout this paper, in order to measure the fractal dimension, box-counting – whose result is equivalent to the fractal dimension – is used as fractal analyzing method.

As is described elsewhere (Lorenz, 2012), fractal analysis in architecture ostensibly leads to two different groups:

3. The first one includes buildings with rather smooth façades and a few well distinguishable architectural elements. Such a conception indicates closer relationship to Euclidean geometry.
4. In contrast, the second group comprises buildings with elements of many different scales whose number increases while scale decreases and whose smaller parts reflect the whole through a common idea. An object of this category is, in terms of harmony, a consistent whole, which is reflected in all of its parts – a concept that is rather close to Fractal geometry (Mandelbrot, 1981; 1982).

## *Harmony and Box-Counting*

Harmony fulfills the expectations of the observer for a quantity of new architectural elements on smaller scales that reflect – at least in their roughness – the whole (Salingaros 2006). However, parts need not be exact, scaled down copies of the whole, but should reflect the basic motif or the basic idea with variation (Lorenz 2011). Otherwise the result gets monotonous or in the other extreme confusing. In short, a continuing irregularity is the reflection of a harmonious connection between the whole and its parts (as it is true for a theme in music). Nevertheless, due to the process of building, the intention of the architect and material restrictions, fractal characteristics are, in any case, restricted to a certain range of scales.

The starting-point of our investigation is the definition of a harmonic whole by an appropriate balance between the number of architectural elements of different sizes and the respective scale of consideration. The characteristic values remain the same, irrespective of the considered detail. Box-counting – a fractal analysis method introduced by Mandelbrot (1982) – enables the examination of how characteristics of a structure (details) change with scale. If this method is applied to a façade, this means basically, to translate its two-dimensional representation (the elevation) into a grid-based Pixel image for the purpose of getting the number of boxes that cover the image. This can sufficiently be achieved by placing a grid over the plan in order to count those boxes that contain a significant part of the elevation – represented by lines (Figure 1a). Subsequently, the scaling factor $s_i$, given by the reciprocal number of boxes in the bottom row of the grid, is reduced and covering boxes $N_i$ are counted again. This procedure is repeated depending on the scale of the plan, i.e. until the detail richness corresponding with the distance of the observer is reached. Finally, in a

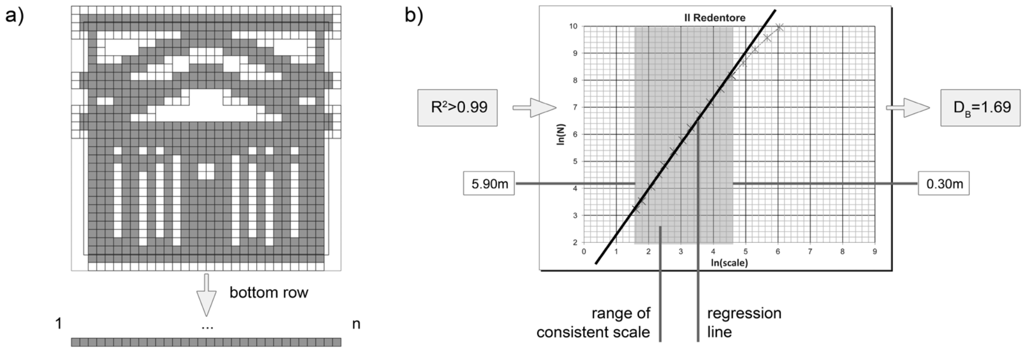



*Figure 1*
*a) Pantheon: A grid is placed over the front view of the Pantheon. Those boxes that cover the composition are colored gray. The reciprocal number of boxes at the bottom row defines the scaling factor.*
*b) Il Redentore: A given coefficient of determination $R^2$ leads to a specific range of scale and finally to the box-counting dimension $D_B$. $D_B$ is equivalent to the slope of the regression line in the graph.*

double-logarithmic graph with the number of boxes $N_i$ versus scaling factor $s_i$, the slope of the regression line defines the box-counting dimension $D_B$ (Figure 1b) for a certain range of scales (Foroutan-pour et al. 1999).

Concerning the box-counting method, a consistent whole across many scales is expressed by a continuing characteristic of complexity, with the characteristic of complexity given by the relation between scale and number of boxes covering the elevation. A small deviation signifies the continuation of a similar irregularity across different scales. Consequently, it is the straight part of the data-curve indicating a harmonious distribution (Figure 1b). The straight part is expressed by a coefficient of determination $R^2$ close to one. Hence, in turn, with a certain single measurement, the smallest and the largest scale act as limits of the specific range of coherence, derived from a given minimum value for the particular characteristic coefficient (Figure 1b).

Bovill (1996) was the first who applied box-counting to architecture as a method for measuring the characteristic visual complexity of buildings. Since then, it has been used by many researchers (Zarnowiecka, 1998; Lorenz, 2003; Ostwald et al., 2008; Vaughan et al., 2010). Advantages of the method are on the one hand its easy usage (hence its simple implementation) and on the other hand its applicability to any object (with and without self-similar characteristics). Nevertheless, in order to use box-counting as a comparison method, several parameters that influence the result in the one or other way have to be taken into consideration (see section *Influences by Parameters*). Some of them, such as line thickness, have been solved by the author's implementation of the algorithm in a CAAD software (Lorenz, 2009; 2012). Other factors are still unsolved, e.g. the definition of what is measured, concerning the selection of relevant parts of a façade and its translation to a plan (elevation). As a consequence, one part of this paper deals with the application of a fractal analysis method for the purpose of figuring out a correct and efficient way of a grid-based representation of an elevation on plan and of testing the box-counting method implemented in AutoCAD. As a word of notice, plans that are used throughout this paper have been prepared in the same manner to guarantee consistence.

## *Box-Counting as Comparison Method*

Throughout history of architecture, one is confronted with buildings that refer to preceding epochs. Descriptions of visual complexity provide a means for comparison, independent of rearranged components or of changes of the purpose that the respective building is used for (church/villa), meaning that the characteristic values of complexity detect connections between two related buildings. In the specific case, a Roman temple front, the Pantheon in Rome, serves as a starting point, while the Renaissance building Il Redentore in Venice represents its successor. Andrea Palladio, the architect of the latter, used the motif of interlocking different combinations and modifications of classical temple fronts deliberately as a harmonic transition from the entrance view to the dome (Wundram et al., 2004).

The study is based, on the one hand, on the assumption that higher complexity leads to a higher box-counting dimension and, on the other hand, that the harmony of a composition is reflected by a trend of the results, i.e. by a straight line of the data-points in a double-logarithmic graph with grid-scale versus number of boxes that cover the composition (see section *Harmony and Box-Counting*). On this basis, the paper describes a further development of the concept with two aspects as indices of complexity: the box-counting dimension (Bovill, 1996) and the interquartile range (Lorenz, 2012) – i.e., the robust estimate of the variability of the data under consideration gives a valuable description of visual complexity and harmony. This suggests that if the harmonic expression (given by the range of scales) and the height of the characteristic box-counting dimension are similar for the ancient temple and Il Redentore, Palladio's interpretation follows its historic inspiration with regard to harmonic expression.

## ANDREA PALLADIO

Palladio's (1508-1580) work is characterized by rediscovering and applying classical Roman architecture – strongly influenced by five travels to Rome conducted in the period between 1541 and 1554, during which he studied classical buildings captured in various drawings. The results of his studies were first published in "L'Antichità di Roma" (Palladio, 2009), a list of preserved and recovered monuments of Rome as they there stand by the mid 16 century. In his book, which is entirely textual, Palladio dedicates more lines to the Pantheon than to any other monument. His views are, however, based solely on existing references. Influences of Palladio's later understanding of form can be deduced from drawings he made of the Pantheon, in which he develops two gables at the same façade (Puppi, 1994). Later, Palladio brought forward the topic of overlapping gables (establishing interlocking architectural orders with a dominant middle order) when commissioned to design the façade of San Francesco della Vigna in Venice in 1562. Finally, both, San Giorgio Maggiore and Il Redentore in Venice, act as results of his continuing development to combine the strict impression of classical temple fronts in a three-aisled church – with Il Redentore providing an obvious relation to the Pantheon (Puppi, 1994).

### *Andrea Palladio and Venice*

Palladio's first assignment in Venice, and moreover, his first practical work on a church was the redesign of the façade of San Pietro in 1558. However, it was not executed before 1594 – presumably because of the commissioner's, the Patriarch Vicenzo Diedo, death and in a modified form (Puppi, 1994). The first design Palladio actually executed in the city of Venice was the Convento della Carità (convent of Santa Maria della Carità), the construction of which began in 1561. The concept is based on a Roman house transformed into monumental scale. While the atrium and a cloister beyond it consist of a Corinthian order, the inner court represents a vertical stacking of three different orders, with the Doric at the base, the Ionic in the middle and the Corinthian at the upper level (Society for the Diffusion of Useful Knowledge 1840).

In the city of Venice, Palladio, well entrusted with designing villas and palazzos, finally could translate classical orders – which he regarded as the embodiment of beauty – to two churches. At first he got the commission for San Giorgio Maggiore situated on San Giorgio di Castello in 1564. The front façade, which is composed by two different reminiscences of classical temple fronts, was finished 30 years after his death (in 1610). The front façade is dominated by its middle part, the entrance, consisting of four three-quarter columns of Composite order on high pedestals, supported by a pediment. The second temple front covers the church aisles by two halves of a pediment. Visually it continues behind the first temple front which is supported by the use of pilasters (of Corinthian order) instead of columns. Both sides are nevertheless held together by the horizontal entablature (especially the cornice), which continues along the main temple front while the upper part of the tympanum is interrupted. Moreover, the pilasters of either side of the entrance belong to the second temple front. Finally, decoration is only found with columns, entablatures and niches.

### *Il Redentore – composition and architectural elements*

Towards the end of his life, Palladio was commissioned to plan his second church in Venice, Il Redentore, situated on the island of La Guidecca. The erection of the (procession, monastery and) votive church was decided after Venice had been visited by a plague in 1575, which killed forty thousand of the citizens. The construction work began in 1578, only two years before Palladio's death. Concerning urban planning, the task was similar to San Giorgio Maggiore in setting up a connection between the new church and Piazza di San Marco across the water. The composition of Il Redentore is similar to San Giorgio Maggiore in so far that the dominant middle part of the front view is formed by a large Composite order, while a broader Corinthian order supports the flanking aisles as a transition to the high middle nave. Both façades provide reminiscence of interrelating Roman temple façades and are characterized by simplicity in the ornaments. Differences only become obvious on closer view. Concerning Il Redentore, the middle order is placed on a higher platform and consists of lower pedestals (which look more familiar). Moreover, while the dominant temple front of San Giorgio Maggiore consists of four three-quarter columns of Composite order, the entrance of Il Redentore is flanked by two half-columns of larger intercolumniation followed by one pilaster on each side (both again of Composite order). The middle dominant front does no longer appear to stand free (as the wall behind continues above the gable). Another difference concerns the position of the horizontal cornice of the smaller order which is in the latter case much higher in relation to the columns and pilasters of the middle order. While at San Giorgio Maggiore this architectural element was continued along the dominant temple front (behind the cut off columns), it is now interrupted and only continues in form of the pediment supplementing the entrance. This time, the pilasters of the second order are protruding in the middle part in the form of two half columns flanking the entrance. Moreover, the intercolumns change from broad, narrow, broad, narrow, broad in the case of the earlier church to a more harmonic sequence of narrow, narrow, broad, narrow, narrow in the case of Il Redentore. The frontal view of Il Redentore provides a third temple front formed by the upper part, including the backwards sloping roof as pediment and the side parts sweeping the aisles.

## FRACTAL ANALYSIS

Methodologically, the author follows the box-counting algorithm described in Lorenz (2009; 2012). As noted elsewhere (Lorenz, 2012), results are either influenced by the transformation of the façade into a plan – hence, the preparation of the plan – and by certain factors that are coming along by the method itself (Foroutan-pour et al., 1999). In consequence of the transformation into a plan and to ensure consistency in analysis, the author considers vector-based re-drawings of both façades concerned in this paper.

### *Influences by Parameters*

One of the most crucial aspects influencing the result is the choice of significant parts of the elevation – i.e., translating the façade into a black and white plan (Lorenz, 2003; 2009). In consequence, the choice of represented architectural elements has to be defined unambiguously, referring to the visual perception (Bovill 1996; Lorenz, 2003; 2009), and justified carefully. Bovill (1996) refers to Maertens (1884) when defining the relation between distance and smallest detail (Lorenz, 2009). The smallest detail, for instance, is influenced by the reading field, that is the minimum size of clearness of seeing within an eye angle of 0°1'. In addition, Märtens distinguishes between three distances of observation that correspond to the scale of the façade (in meters). The first one includes the environment (deduced from a viewer's position of 18-20° of building height), the second considers the whole building (27° of building height) and the third one focuses on details (45° of building height). The present measurements correspond to the second and third distance. From a distant view, only main parts of the design are perceived and consequently taken into consideration. Beside the silhouette, this includes columns, the gable, main parts of the entablature (architrave and cornice), but no detail of the capitals. The latter belongs to a closer distance of observation.

The experimental set-up not only includes the selection of lines, but also the definition of the smallest and largest box-size. While the smallest box-size depends on the smallest detail and is reflected by the point where the data curve calculates only the single lines of the elevation ($D_B = 1$), the largest box-size should be one fourth of the smallest side of the measured image. Other influences include the relative position of the grid, the orientation of the grid and the reduction factor of the grid. With a reduction factor of one half, the number of boxes at the bottom row doubles for the next smaller grid-size.

## Implementation

The author's implementation of the grid-based box-counting algorithm into AutoCAD uses vector-based geometries in a tool architects are used to. The script allows various options, which are available in form of tabs:

- *Selecting the area for measurement*. If the area contains the image completely, the bounding box serves for further calculations, otherwise it is cut.
- *Defining specifications*. This includes, for example, the number of iterations (how often the grid-size is reduced), the enlargement factor (percentage of empty space around the selection area), the number of steps between two scales (the reduction factor – which is defined as the ratio of how much the grid-size is reduced from one step to the next – is defined by one half; by inserting a number of steps between two scales the factor is changed to 1/4th, 1/8th and so forth) and the number of boxes at the smaller side (from which the initial grid-size is deduced). Furthermore, for the purpose of accuracy the number of displacement in x- and y- direction can be defined. The number of covered boxes of a certain box-size is then given as the minimum number of all replacements of one and the same grid-size.
- *Changing settings of layout*. This includes drawing a copy of the measured segment and assigning this segment to a corresponding layer.

A second modified algorithm does not start from a reduction factor of one half but takes into account that the difference in box-size between two successive grid-sizes is larger as the scale size increases. Consequently, the user can adjust accuracy by a value that defines the addition of boxes from one scale to the next, where the number of added boxes increases with smaller scales.

Finally, the data, coming along as text-file, is estimated by means of statistical methods, specifically by linear regression. With this technology, a regression line is to fit the logarithmically transformed output of grid-scale versus number of covered boxes. In the particular case, the analysis of the data is done in a spreadsheet program, again supported by a special script. For evaluating the relation of the regression line with regard to the measurement points, the coefficient of determination $R^2$ is used. The range of $R^2$ reaches from 0, indicating no relation, to 1, which means highest possible correlation. As more than one measurement is used for analysis (see section *A Set of Measurements*), the range of scales can be adopted for the whole set, resulting in a minimum and maximum coefficient of determination $R^2$. Only if both, the minimum and the maximum are close to one, the result is called 'consistent'. Otherwise, the regression line does not fit for single results, indicating higher diversity.

## A Set of Measurements

It could be demonstrated elsewhere (Lorenz, 2009; 2012) that different measurements lead to different results, due to influences of several parameters coming along with the box-counting method. Therefore, a set of measurements is necessary, rather than a single one. Accuracy is then expressed by the interquartile range of the box-plot (containing 50% of all values). The smaller this range, the smaller is the fluctuation of data-points (single box-counting dimensions) and the more meaningful is the result. The characteristic values are therefore

- the range of scale, given by the smallest and largest box in meter,
- the median, as a characteristic for roughness and
- the interquartile range, as indication of variation.

In turn, a given coefficient of determination leads to a specific range of coherence for a whole set of measurements and, following from that, to a definitive value by the median of the data (Lorenz, 2012).

## ELABORATION

To ensure the required conditions of the author's implementation of the box-counting method, a vectorized representation of the real façade Il Redentore is considered. As statues were added only in the second half of the 17th century (Wundram et al., 2004), they are excluded from measurement, as well as small details including the shaping of capitals. In general, the selection includes main design elements according to a distance from where the building is perceived as a whole (see section *Fractal Analysis*).

In order to minimize potential sources of error, both algorithms are used – dividing the grid by one half (set A) and adding boxes for each step (set B) – with 11 different configurations in each case. The configurations include:

- the factor of enlargement (either one, three or five percent of minimal side length),
- the number of starting boxes in x-direction (either three of four),
- depending on the algorithm, either the number of steps between two grid-sizes (none, one, two or three) or the factor of accuracy (three or four) and
- the number of replacements in x- and y-direction (one by one or three by three).

The interquartile range and the coefficient of determination are the basic instruments of evaluation of the results: While the first value is related to the whole set, single measurements are taken into account by the second criterion. In particular, the latter is specified on the one hand by the minimum $R^2$, which tells us about the most deviating result of a whole set, and, on the other hand, by the average $R^2$, which describes the general fluctuation of data of all measurements.

When discussing the results of measurements it is conspicuous that for any single measurement of Il Redentore, the coefficient of determination exceeds 0.996 (0.997), which is very close to one, proving that each regression line fits the data well (minor deviation). Finally, the spectrum of the resulting box-counting dimensions (slope of the regression line) is shown in a box-plot, separately for set A and B (Figure 2a). The respective small interquartile ranges express high accuracy of all data: For set A it is 1.89 percent (in relation to two as possible results in a two-dimensional space are between 0 and two) while for set B it does not even exceed 1.5 percent. Finally, the median of each set – that is the break line where 50 percent of all values can be found above and below respectively – equals 1.677 and 1.685. From these results, it can be deduced that the façade of Il Redentore is of high complexity, with a consistent use of architectural elements from the whole to a very small scale (Table 1).

As shown in Table 1 the results of both algorithms are very close with slightly higher accuracy of the gradual increase of boxes, i.e. set B (higher min $R^2$ and smaller interquartile range).

## DISCUSSION AND COMPARISON OF RESULTS

Because of its importance for Palladio (see section *Andrea Palladio*), it is the Pantheon in Rome that serves as a reference object. For analysis, two sets (A and B) of 11 measurements each are carried out.

| Il Redentore | | | |
|---|---|---|---|
| Median | | 1.677 | 1.685 |
| Interquartile range | | 0.038 (1.89%) | 0.029 (1.45%) |
| Minimal $R^2$ | | 0.996 | 0.997 |
| Average $R^2$ | | 0.998 | 0.998 |
| Range of coherence | | | |
| | Maximum box-size | 7.93 meters | 8.46 meters |
| | Minimum box-size | 0.32 meters | 0.31 meters |
| Range in % of the height of the front view | | | |
| | Maximum box-size | 29.95 % | 31.95 % |
| | Minimum box-size | 1.20 % | 1.16 % |

*Table 1*
*Il Redentore: Results of measurement; Left: dividing by half; Right: adding boxes.*

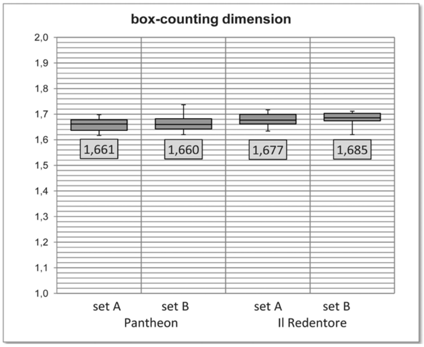
Despite differences of overlapping elements, the results nevertheless display a similar range of coherence in comparison to Il Redentore (Table 2 and Figure 2b). Moreover, the medians of the two sets are similar to Il Redentore: the median of set A equals 1.661 (1.677) and for set B it is 1.660 (1.685) – with slightly higher interquartile ranges of 2.07 and 1.32 percent. This leads to the conclusion that both façades are characterized by a similar development of architectural elements across a similarly broad range of scales (range of coherence: 1-30 percent with Il Redentore and 1.5-28 percent with the Pantheon). In particular, this means that details of a certain size have their correspondence in both façades, although differences in design are obvious. E.g., Il Redentore, for instance, displays not only one but two clearly interrelating Roman temple façades, while the Pantheon consists of two vertically arranged gables. In the case of Il Redentore, niches for statues and the pillars flanking the entrance with own gables display another additional level.

Concerning the different algorithms, both sets of measurement lead, as it is true for Il Redentore, to very similar results (Table 2). The deviation of the data is again low, although this time minimum $R^2$ is slightly lower (0.992 and 0.994) than in the case of Il Redentore (0.996 and 0.997).

## CONCLUSION

The box-counting method provides an objective comparison method between design solutions demonstrated by Il Redentore and the Pantheon. It visualizes the development of roughness across multiple scales and, derived from that, the harmonic relations between the whole and its parts. Both results discussed in this paper show a similar depth of details and a similar level of complexity. Specifically, this means that, even if Palladio changes the com-

a)




b)

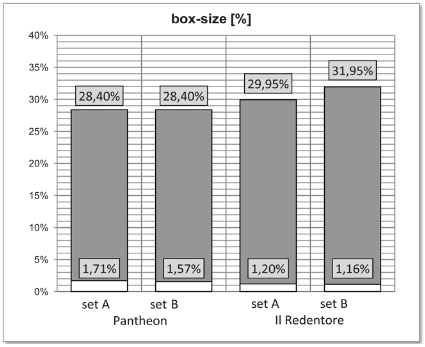



*Figure 2*
*Il Redentore and Pantheon: box plot diagram of box-counting dimensions (a) and box size in percentage of the height of the front view (b).*

*Table 2*
*Pantheon: Results of measurement. Left: dividing by half; Right: adding boxes.*

| **Pantheon** | | | |
|---|---|---|---|
| Median | | 1.661 | 1.660 |
| Interquartile range | | 0.041 (2.07%) | 0.026 (1.32%) |
| Minimal $R^2$ | | 0.992 | 0.994 |
| Average $R^2$ | | 0.994 | 0.995 |
| Range of coherence | | | |
| | Maximum box-size | 9.10 meters | 9.10 meters |
| | Minimum box-size | 0.55 meters | 0.50 meters |
| Range in % of the height of the front view | | | |
| | Maximum box-size | 28.40 % | 28.40 % |
| | Minimum box-size | 1.71 % | 1.57 % |

position of the temple front, the harmonic distribution across all scales is similar to the Pantheon. This proves that, although variations in the reinterpretation occur, Il Redentore nevertheless takes up the same characteristics as its origin of a Roman temple front.

Box-counting reveals similarities and differences between styles with regard to different degrees of roughness and depth of self-similarity. Up to now, the author has analyzed façades, corresponding to a larger distance of the observer. As ornaments are characteristic elements of a building, it would be interesting for future work to deal with a smaller distance as well.

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