Reducing the Peak to Average Power problem for OFDM.

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Abstract.

A major disadvantage of Orthogonal Frequency Division Multiplexing (OFDM) is that it results in a large Peak to Average Power Ratio (PAPR). This significantly decreases the efficiency of the transmitter power amplifier.

First of all in this report the basics of OFDM are given and the size of the problem of having a large PAPR is examined. The cdf's for the PAPR's are given for different amounts of carriers and it is seen that the PAPR increases with the amount of subcarriers used.

Also this report treats multiple methods -Clipping, Selective Backoff, Windowing and Symbol Substraction- to decrease the PAPR problem. This is done by applying some kind of predistortion after making the OFDM symbol (before amplifying). A scrambling scheme is also treated.
It is then seen that it is possible to decrease the PAPR when using symbol subtraction to about 6 dB for a different amount of subcarriers without distorting the frequency spectrum. Some increase in error rate has to be accepted.

Contents

1. Introduction

OFDM (Orthogonal Frequency Division Multiplexing) is a multi carrier modulation scheme which produces a time domain signal containing a summation of many independent modulated sinewaves. This could result in a signal with very large amplitudes. Typically amplitudes as large as the amount of subcarriers can be generated.
Therefore the transmitter amplifier has to have a large back-off, meaning it has to amplify all OFDM symbols linearly even though symbols with very high amplitudes occur very infrequently. The transmitted average power will therefore be much less than the peak power, significantly reducing the efficiency of the power stage. This is a large disadvantage for portable equipment, which has to run from a battery.

We therefore have to find a way to decrease the Peak to Average Power Ratio (PAPR) so as to increase the power efficiency. This reduction could cause distortion and an increase in error rate. These effects should be minimized.

In this report first the basics of OFDM will be given so it can be seen how large peaks in OFDM symbols are generated. Then the probability of these peaks occurring will be approximated through mathematical derivations and simulations in chapter 2.
Chapter 3 then deals with just clipping the signal, meaning we do nothing to reduce these peaks and let the amplifier saturate. This clipping generates significant in-band and out-of-band radiation. Too much out-of-band radiation makes the system impractical due to spectral regulations. We want to make an OFDM system with variable amounts of subcarriers so filtering won't be an acceptable solution.
The chapters 5 till 7 describe methods which can decrease the problem of having these large peaks and lowering the PAPR (Peak to Average Power Ratio) with a minimum effect on spectral distortion and error rate.

2. The basics of OFDM (Orthogonal Frequency Division Multiplexing)

OFDM is a multi-carrier modulation scheme, this means that data -in contrary to conventional systems- is transmitted in parallel. This is done by sending the data over multiple independent carriers which are mutually orthogonal.

Each subcarrier is modulated by some form of M-quadrature amplitude modulation scheme (M-QAM) with M being the constellation size. Therefore the time signal for one subcarrier will be a sine-wave of finite duration that is phase and/or amplitude modulated according to the chosen M-QAM modulation scheme. The time and frequency signal for one carrier are then given by figure 1.

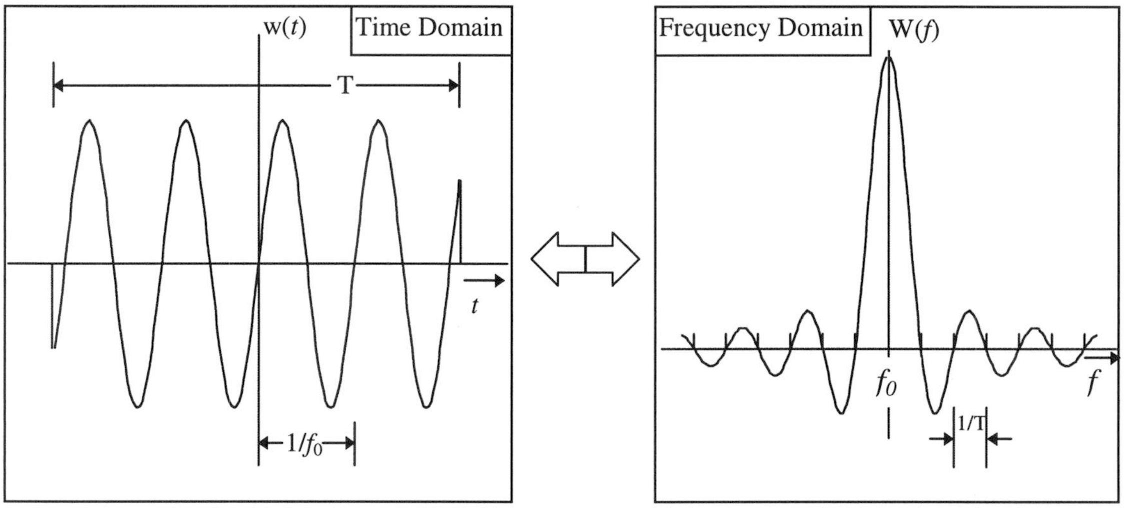

Figure 1: One OFDM-carrier

From figure 1 it is seen that a sine-wave of finite duration results in a sinc pulse in the frequency domain. From that frequency domain it can be seen that the sinc-pulse has zero crossings spaced at 1/T with T being the symbol duration time in the time domain.
When using a subcarrier spacing of 1/T between the sub-carriers, the peaks will be located on all the other subcarriers' spectra zero crossings. Although there will be spectral overlaps among the sub-carriers, they won't interfere with each other. In other word, they maintain spectral orthogonality.

This is show in figure 2. The sine-waves in the time domain are chosen in frequency such that they maintain orthogonality in the frequency domain (the peaks fall together with all the zero crossing of the other sub-carriers in the frequency domain).

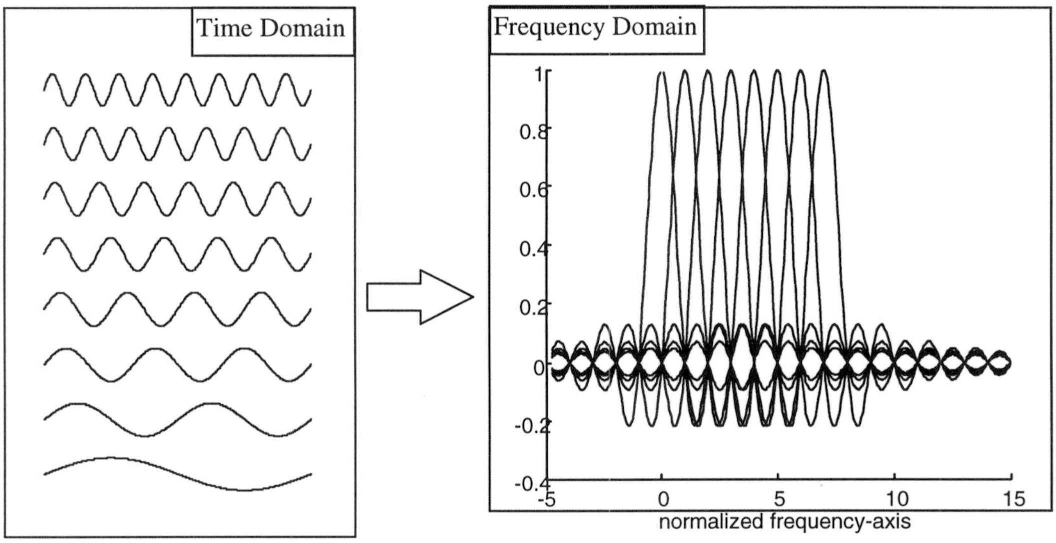

Figure 2: Eight carriers, spaced orthogonal (OFDM N=8)

Therefore OFDM is basically frequency division multiplexing with the frequency band divided in N smaller frequency bands each spaced at the symbol (baud) rate making the subcarriers orthogonal. That is, if the symbol duration is T, the subcarrier frequencies are at 1/T, 2/T , 3/T,.... resulting in an OFDM bandwidth of approximately N/T (see figure 3).

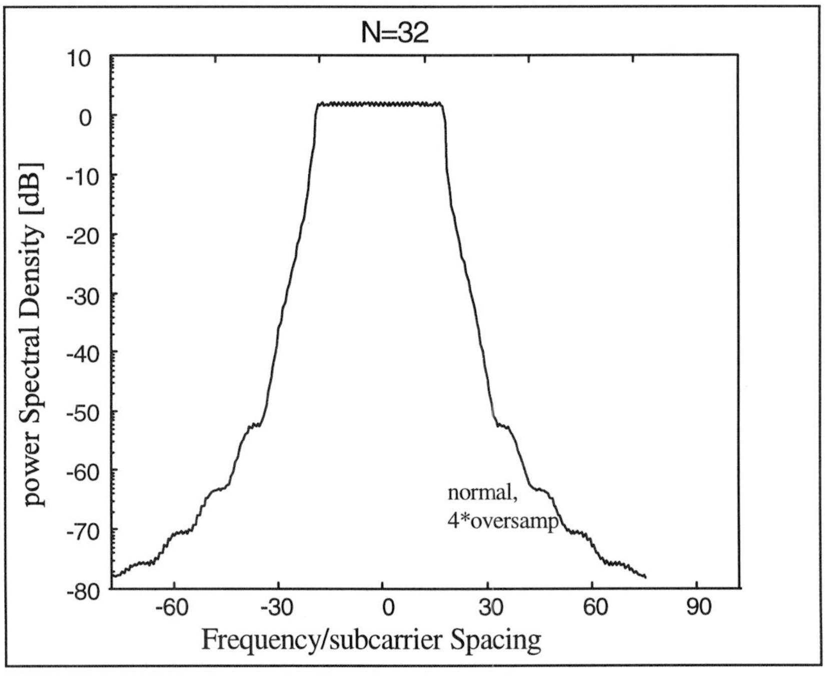

Figure 3: normalized OFDM-signal spectrum (N=32).

An OFDM model block design is shown in figure 4. At the input of the OFDM transmitter, first a binary serial data stream is encoded using a M-QAM encoder into a multilevel data stream.

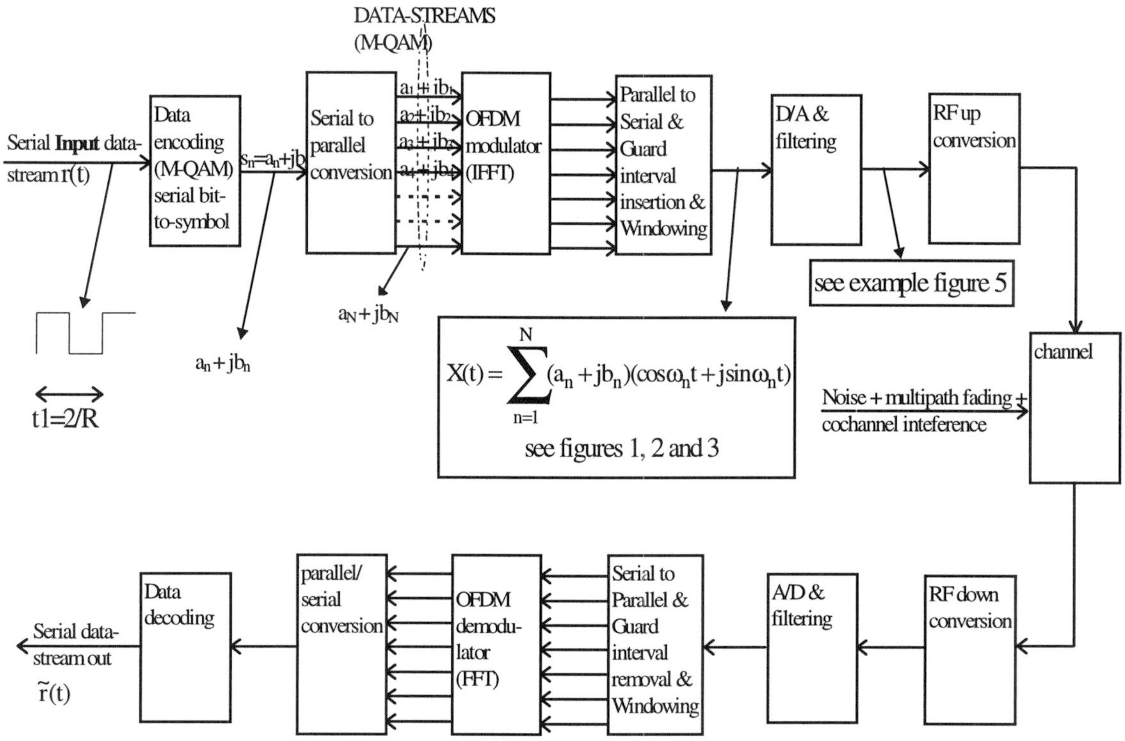

Figure 4: OFDM model block design.

The binary input will be transformed into a multilevel signal reducing the symbol rate to

$$D = \frac{R}{\log_2 M}(\text{symbol / sec}) \tag{1}$$

with R being the bitrate of the data stream (bits/sec).
The serial M-QAM signal will then be converted to a parallel data-stream. This parallel data stream will then be OFDM modulated. Basically OFDM modulation is summation of all the N M-QAM modulated signals resulting in summation of all the sine-waves (figures 1 and 2).

$$X(t) = \sum_{n=1}^{N}(a_n + jb_n)(\cos\omega_n t + j\sin\omega_n t) \tag{2}$$

When we observe this equation it is seen that this is the complex IFFT of the complex M-QAM data. Therefore the OFDM-modulator block can be ‘replaced’ with an IFFT-block.

After the OFDM modulation and parallel to serial conversion, a guard interval can be inserted to suppress ISI caused by multipath distortion. The windowing applied after guard time insertion is to reduce the out of band radiation when the OFDM symbol would have a ‘discontinuous’ end or beginning. Normally something like a raised cosine window is used. Then the signal is D/A-converted to produce the analog baseband signal. After which this baseband signal is converted to a bandpass signal to be transmitted. In the channel the signal will be corrupted by noise, multipath fading and cochannel interference. After which demodulation occurs in inverted order.

A major disadvantage of OFDM is its non-constant envelope it produces. Because basically OFDM is adding a number of sine-waves it is possible that all (or most) sine's of all the different frequencies have maxima at the same time, producing a power-spike. This amplitude spike can be as large as the number of sub-carriers used, N (figure 5)!
Figure 5 shows that there are OFDM with large peaks but also OFDM symbols with significantly smaller peaks.
In order to transmit these (large) power-spikes correctly, the transmitter power amplifier needs to have a large back-off, meaning its transmitted average power will be much less than the peak power. This significantly reduces the efficiency of the power stage, which is a large disadvantage for portable equipment, which has to run from a battery. Therefore we have to

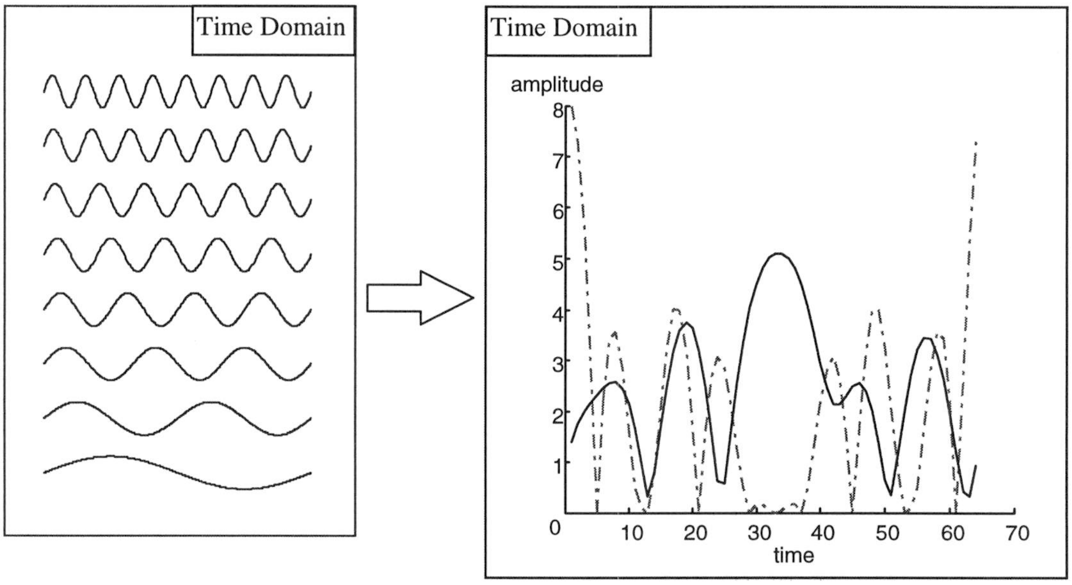

Figure 5: Different PAPR for different OFDM symbols (N=8).

find a way to reduce these peaks. And thus reduce the Peak to Average Power (PAP-ratio) of the OFDM symbols.

3. PAPR distribution, how many symbols are there with a certain PAPR?

In this chapter the distribution of the PAPR is examined, meaning that the probability of an OFDM symbol (of a certain amount of sub-carriers) exceeding a certain PAPR is examined. This will result in a cumulative distribution function.

To determine this PAPR distribution for OFDM modulators, we first have to assume that the data going into the OFDM modulator consists of independent identical distributed (i.i.d.) random numbers. Which is true for random binary data. Then we know that the multilevel data (see figure 4) is also i.i.d.
When we once again give the formula for the OFDM/IFFT block (equation 3), we see that the complex time signal

$$X(t) = \sum_{n=1}^{N} (a_n + jb_n)(\cos \omega_n t + j \sin \omega_n t) \tag{3}$$

consists of a real and an imaginary part which is the summation of N sinewaves with amplitudes a_n and b_n. X(t) can then be written as X(t)=x(t)+jy(t).
From the central limit theorem it follows that when N gets large, x and y become Gaussian random distributed numbers (with zero mean and variance σ^2), assuming that a_n and b_n are i.i.d. random variables. From simulation this is seen to be true for values of N larger than 32 (QPSK).
These I and Q (x and y) values will be set on a carrier frequency as shown in figure 6.

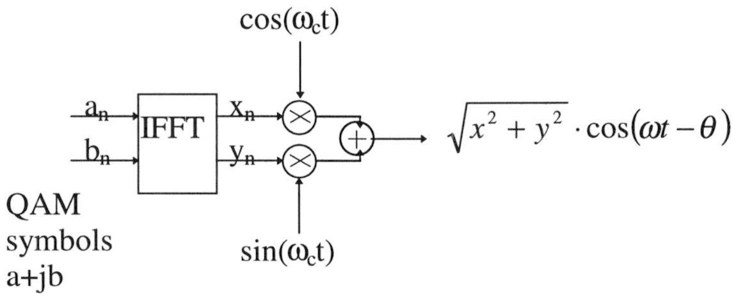

Figure 6: Complex data on a carrier frequency

The amplitude of the OFDM signal $\sqrt{x^2 + y^2}$ will therefore have a Rayleigh distribution with zero mean and a variance of N times the variance of one complex sinusoid (for random input data and large N). The amplitude distribution (pdf) will be given by:

$$f(x) = \frac{x}{\sigma^2} \cdot e^{-x^2/2\sigma^2} \tag{4}$$

From equation 4 it can be seen that there is a finite probability that the amplitude becomes very large.

The power distribution for complex OFDM-symbols having a certain amount of subcarriers is given by $Z = x^2 + y^2$ (see figure 6). Where x^2 denotes the real power, and y^2 the imaginary power of the OFDM-symbols. For large N (N>32, see simulation results) x and y can be treated as Gaussian random variables with zero mean. Therefore Z, the power distribution, becomes a central chi-square distribution with 2 degrees of freedom and zero mean [15], given by (cdf)

$$F_Z(z) = \int_0^z \frac{1}{2\cdot\sigma^2} e^{-\frac{u}{2\sigma^2}} du \qquad \text{for } z \geq 0 \qquad (5)$$

and the pdf

$$P_Z(z) = \frac{1}{2\cdot\sigma^2} e^{-\frac{z}{2\sigma^2}} \qquad \text{for } z \geq 0 \qquad (6)$$

with m=0 and $\sigma = \sqrt{\frac{N}{2}}$ (for complex data $\in \{\frac{1}{\sqrt{2}}(\pm 1 \pm j)\}$ ==> QPSK).

Simulation result given in figure 7 show that for oversampling and non-oversampling with N=64 sub-carriers using QPSK modulation this is seen to be true (bars are simulated, lines are the central chi-square pdf with 2 degrees of freedom).

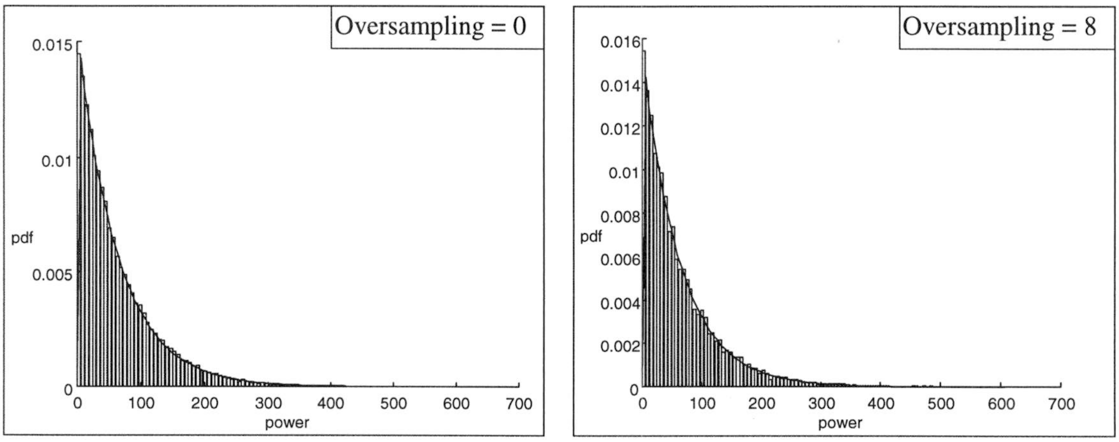

Figure 7:N=64 power pdf, chi-square distribution.

What we want to derive now, is the cdf for the maximum power in an OFDM-symbol. This can be written as, by assuming w being power samples of the OFDM samples:

$$G_Z(N, z) = \Pr\{\max(z) \leq Z\} \qquad (7)$$

Assuming the samples z to be mutually uncorrelated -which it true for non-oversampling- this can be written as (cdf) [15]:

$$G_Z(N,z) = F(z)^N = \left(-\exp\left(-\frac{1}{2}\cdot\frac{z}{\sigma^2}\right)+1\right)^N \tag{8}$$

and the pdf will be

$$g_Z(N,z)=NF(z)^{N-1}\cdot f_Z(z)=N\left(-\exp\left(-\frac{1}{2}\cdot\frac{z^2}{\sigma^2}\right)+1\right)^{(N-1)}\cdot\exp\left(-\frac{1}{2}\cdot\frac{z^2}{\sigma^2}\right)\frac{z^2}{\sigma^2} \tag{9}$$

For non-oversampling this theoretical derivation is plotted against the simulated values in figure 8 for different values of N. Because of the great amount of symbols that can be sent i.e. for N=16 QPSK(4^16), the PAPR in these simulations were estimated via Monto Carlo simulation.

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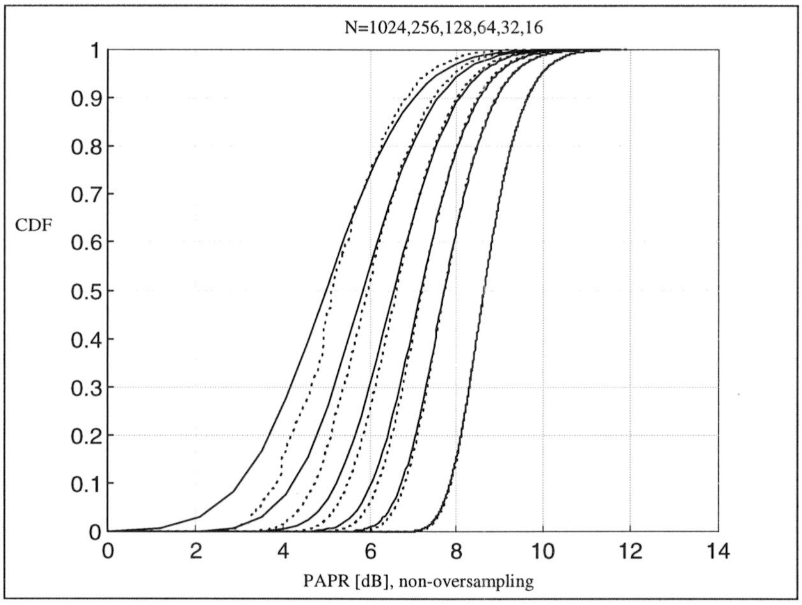


Figure 8:PAPR cdf-distribution (non-oversampling) for (left to right) N=16,32,64,128,256,1024 (solid is simulated)

It is seen from figure 8 that when using more sub-carriers N the assumption that x_n and y_n become Gaussian random distributed becomes more valid.
Note that in practice the PAPR can be as large as the number of sub-carriers used. So i.e. the cdf for N=1024 will end in practice at a PAPR of 1024. It can therefore be seen that those very large PAPR's occur very infrequently, but do happen!!

When oversampling (zeros after data and using FFT or IFFT) is used, the signal will become closer to the continuous OFDM-symbol. In practice an oversampling rate of 2 is usually chosen after which the data is D/A converted to make an analog signal. It can be seen from figure 9 that with oversampling, the amplitude of the symbol can become larger than the amplitude estimated theoretically as described above.

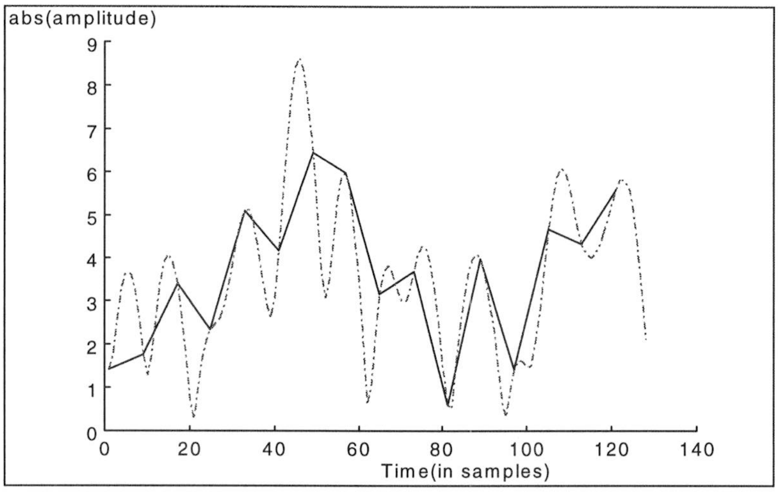


Figure 9: OFDM symbol (N=8), dash-dot 8*oversampled, solid non-oversampling

The assumption made in deriving equation 8 that the samples z should be mutually uncorrelated is not true anymore when oversampling is applied. This can easily be seen from figure 9, when assuming that a couple of consecutive samples are known it is possible to approximate the next. Hence the samples will be correlated.

From figure 7 (right) it was seen that for oversampling (continuous signal) the power distribution will still be a chi-square distribution with two degrees of freedom. The maximum power as derived for equation 8 however is not valid anymore for oversampling.

To derive an equation for the maximum power in an OFDM symbol, we have to know how much the samples are correlated. Because this is very difficult, the PAPR distribution can be approximated by a chi-square distribution to the power N (equation 8) that is shifted to the right. With this distribution it is then possible to approximate how many symbols there are with a PAPR higher than a certain threshold.

From simulations it is seen that using a factor $\alpha \approx 2.8$, equation 10 can be a good approximation (for large N) (see figures 10 and 11) for the oversampled maximum power distribution. The maximum power distribution will than be given by (cdf)

$$G_Z(N,z) = F(N,z)^{\alpha \cdot N} = \left(-\exp\left(-\frac{1}{2} \cdot \frac{z}{\sigma^2} \right) + 1 \right)^{\alpha \cdot N} \tag{10}$$

and the pdf

$$g_z(N,z) = \alpha \cdot N F(N,z)^{\alpha N - 1} \cdot f_Z(N,z) = \alpha \cdot N \left(-\exp\left(-\frac{1}{2} \cdot \frac{y^2}{\sigma^2} \right) + 1 \right)^{(\alpha \cdot N - 1)} \cdot \exp\left(-\frac{1}{2} \cdot \frac{y^2}{\sigma^2} \right) \cdot \frac{y^2}{\sigma^2} \tag{11}$$

In figures 10 and 11, the PAPR-distribution (cdf) for a different amount of carriers is given. The dotted lines are the simulated curves.

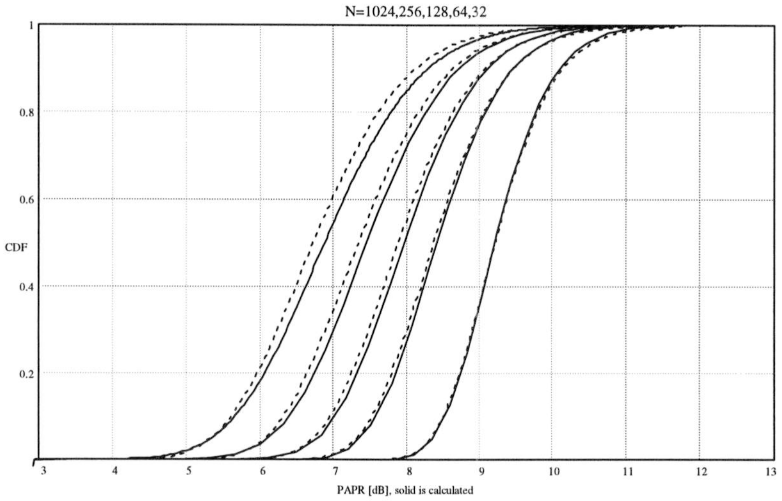
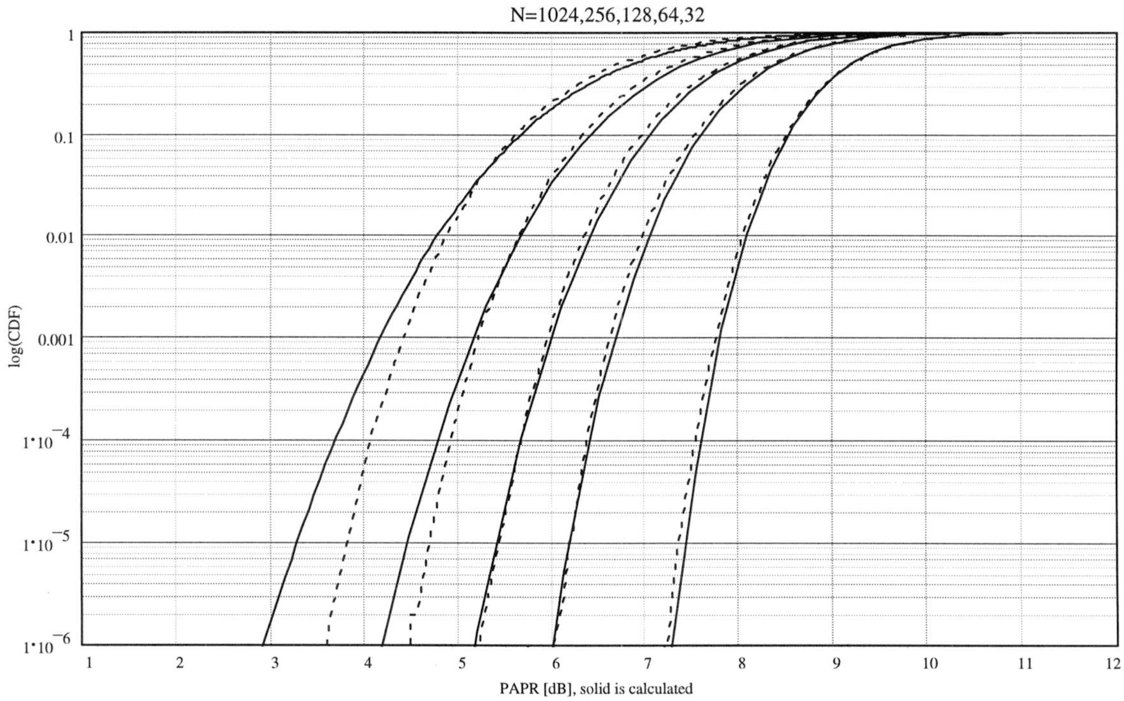


Figure 10:PAPR-cdf for oversampling, solid lines are calculated.

It can be seen in figure 10 that equation 10 is an over-estimation for large PAPR-values. And to show that this is true, in figure 11 (right) 1-cdf is plotted on a log scale to see if the tail of the curves result in an overestimation. Figure 11 (left) shows the cdf on a log scale (figure 10 on a log-scale) and it can be seen that there are minimum PAPR's for the different amounts of carriers.



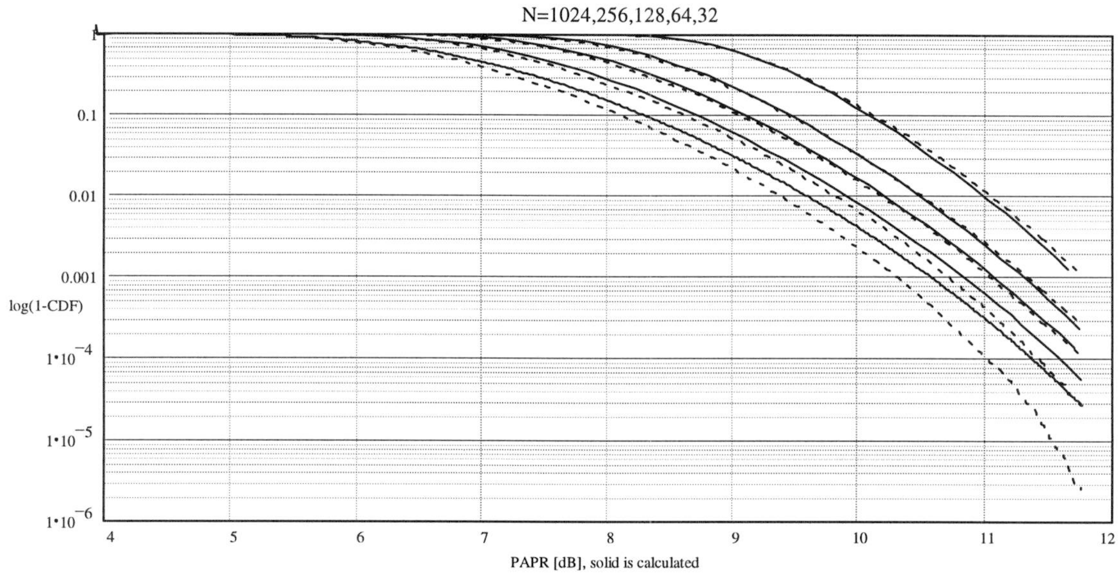


Figure 11: above, the cdf distribution function on a log-scale for the PAPR using different amount of subcarriers. below, (1-cdf) on an log-scale to show the tail of figure 10. Solid lines are calculated.

From figure 11(right) it is shown that the α factor used will result in an overestimation for large values of N, here for N=1024 and N=256. It is seen that the frequency of occurrence of OFDM symbols with very large PAPR's can be approximated using equation 10. When we want to use this approximation for smaller amounts of carriers the α factor should be taken smaller.

When we do not do anything about symbols with a high PAPR, these symbols will run the amplifier in saturation, causing the amplifier to create non-linear distortion. When we assume such symbols to be in error the worst-case symbol error rate can be written as:

$$\text{Worst}_{\text{symbol-error-rate}} = 1 - G_Z(N, \text{Aclip}) \tag{12}$$

With Aclip being the level where the OFDM symbols will be clipped and G_Z given by equation 10 and equation 8 for oversampling and non-oversampling respectively.

In the next chapter it will be assumed that nothing will be done to reduce the PAPR and symbols with large PAPR will occur. These symbols will be clipped and non-linear distortion will be added. This will result in an increased bit error rate and a distorted frequency spectrum which is not acceptable for practical systems.

4. Clipping, driving amplifier in saturation.

In the previous chapter it was seen that the OFDM time signal is a summation of multiple independent modulated sinewaves. Therefore large peaks can occur in the time domain. When a signal with large peaks is passed though a linear amplifier as shown in figure 12, the amplifier can be driven into saturation and will clip the peaks (only amplitude distortion is taken into account, no phase distortion).

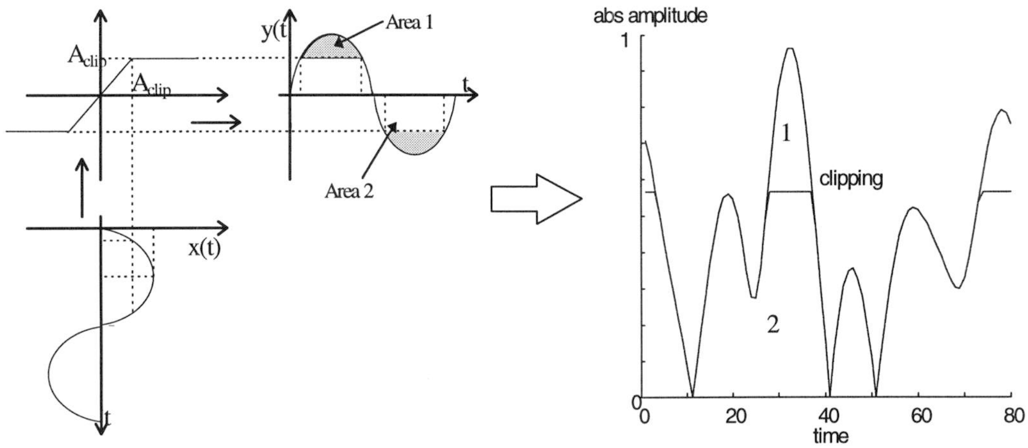


Figure 12: Clipping

From figure 12 it can be seen that the clipped signal is the result of non-linear amplification. The clipping will therefore not only result in a decreased signal power but will also result in an increase of inband and outband distortion. All these effect will result in a larger error rate.

First the decreased signal power will be estimated in par 4.1. Then the introduced inband distortion will be treated (par 4.2.). After which the BER curves can be estimated and simulated in par 4.3. Finally the resulted spectrum distortion due to clipping will be shown in par 4.4.

4.1. Lost signal power caused by clipping

The transfer function for clipping showed in figure 12 can be written as

$$y(t) = \begin{cases} -A_{clip} & \text{if} \quad x(t) \le -A_{clip} \\ x(t) & \text{if} \quad -A_{clip} < x(t) < A_{clip} \\ A_{clip} & \text{if} \quad x(t) \ge A_{clip} \end{cases} \qquad (13)$$

with y(t) being the clipped signal and x(t) being the unclipped signal.
In chapter 3 it was shown that when the amount of sub-carriers N becomes large, the magnitude of the complex OFDM-symbols will be Rayleigh distributed with a standard deviation of N times the standard deviation of one sinewave.

$$\sigma = \sqrt{\frac{N}{2}} \tag{14}$$

The Rayleigh distributed given by

$$p(x) = \frac{x}{\sigma^2} \cdot e^{-\frac{x^2}{2\sigma^2}} \tag{15}$$

The probability of an OFDM signal being clipped will therefore be the probability of the magnitude of the OFDM signal given by equation 15 being larger than a certain Clip-level A_{Clip}. This can be written as:

$$P_{Clip} = \int_{Aclip}^{\infty} p(x)dx \tag{16}$$

The power of a clipped OFDM signal at a clipping level Aclip will be the sent power when the magnitude is smaller than A_{Clip} (first term, equation 17) plus the sent power when the magnitude is larger than A_{Clip} (second term, equation 17).

$$signal\ clipped\ power = \int_{0}^{Aclip} y^2 \cdot p(x)dx + \int_{Aclip}^{\infty} A_{Clip}^2 \cdot p(x)dx \tag{17}$$

This results, when using equation 15, in the signal power after clipping.

$$signal\ clipped\ power = 2\sigma^2 \cdot \left(-1 + e^{\frac{A_{Clip}^2}{2\sigma^2}} \right) \cdot e^{\frac{A_{Clip}^2}{2\sigma^2}} \tag{18}$$

In figure 13, equation 18 is plotted against simulated values.

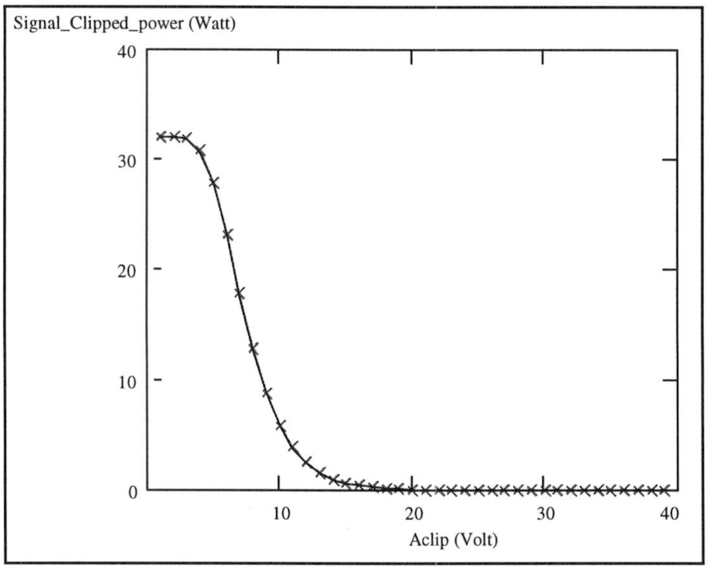


Figure 13: The lost power signal clipped power due to clipping, solid is equation 18, crosses are simulated (N=32).

In this example (figure 13) a N=32, 2 times oversampled FFT is used to generate the OFDM signal. The total signal power in this case for N=32 is 32. Hence clipping the signal to about 20 does not result in a significant signal_clipped_power because there are only a very small amount of symbols with amplitudes larger than 20. For a clipping level of zero, all the signal will be clipped off and the signal_clipped_power will be 32. From figure 13 it is therefore seen that equation 18 is an good estimation of the signal power after clipping.

4.2. Introduced clipping noise

Noise power introduced by clipping is treated well in paper [16]. However in deriving the noise-power they assumed the magnitude of the complex OFDM symbols to be Gaussian distributed whereas here the magnitude of the symbols is assumed to be Rayleigh distributed. Therefore the basic idea in deriving the clipping-noise power is the same and is given in appendix A. The introduced noise power can be written as (see appendix A):

$$\text{Clip noise power} = -(\chi-1)^2 \cdot x^2(t) + y^2(t) \qquad (19)$$

with

$$\chi = \frac{1}{4\sigma} \cdot \left(4\sigma - A_{Clip} \cdot \sqrt{2\pi} \cdot e^{\frac{Aclip^2}{2\sigma^2}} + A_{Clip} \cdot \sqrt{2\pi} \cdot erf\left(\frac{\sqrt{2} \cdot A_{Clip}}{2\sigma} \right) \cdot e^{\frac{Aclip^2}{2\sigma^2}} \right) \cdot e^{-\frac{Aclip^2}{2\sigma^2}} \qquad (20)$$

and with $y^2(t)$ being the power of the clipped signal given by equation 18, and $x^2(t)$ being the power of the unclipped signal.

4.3. Simulation results BER.

Clipping result is less signal power and introduces additional noise which can be treated as white noise [10]. The SNR for clipping OFDM-symbols can therefore be written as:

$$\text{Clipping SNR}=\frac{Signal\ \ Clipped\ \ power}{Clip\ \ noise\ \ power+noise\ \ power} \tag{21}$$

With this equation and the equations 18 and 19 we can draw the BER curves for QPSK using equation 22 [15].

$$P_e = Q\left(\sqrt{2\left(\frac{E_b}{N_o}\right)}\right) \tag{22}$$

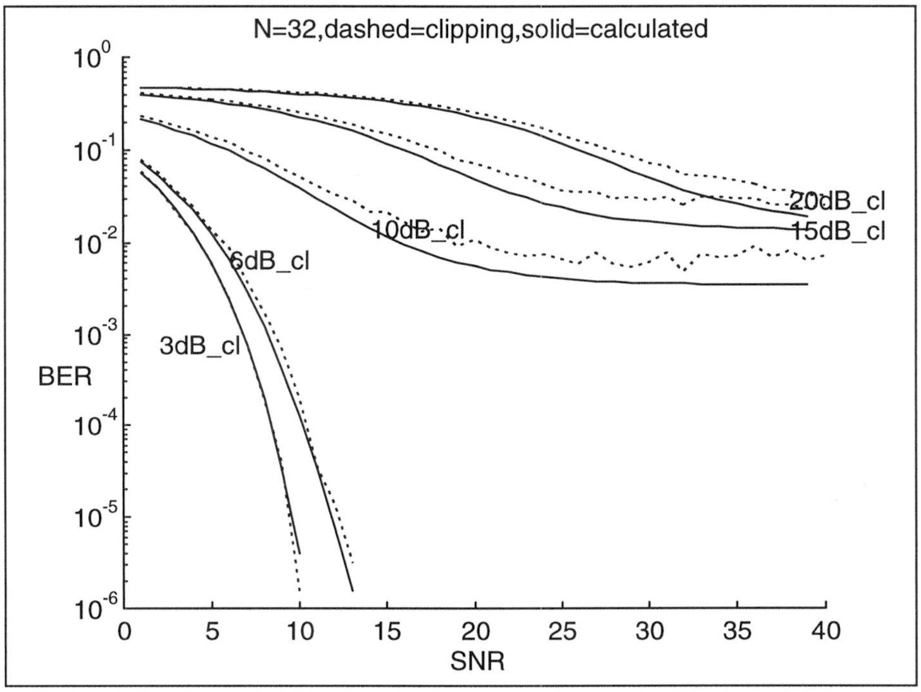


Figure 14:BER curves for clipping the amount of dB below maximum amplitude. (dashed simulated, solid calculated)

From figure 14 it is seen that clipping not only increases the error rate, but it also introduces an irreducible BER. The deviation between what is calculated and simulated in the figure for large clipping values results from assumptions made in appendix A.
When the goal is to reduce the PAPR without a great amount of increase in error rate, clipping could be a good candidate. When clipping at a level for instance of 3dB above the RMS value the PAPR can be reduced here to about 4dB. The BER curve would the same as it is for 6 dB below the maximum amplitude in figure 14. In that case the irreducible error rate can be neglected especially when coding is applied. For larger amounts of carriers it is also possible to clip the PAPR down at the cost of acceptable increase in error rate.

4.4. Spectrum distortion due to clipping.

But Clipping not only introduces inband distortion (as seen in figure 14) but also introduces out-off-band distortion as can be seen in figure 15.

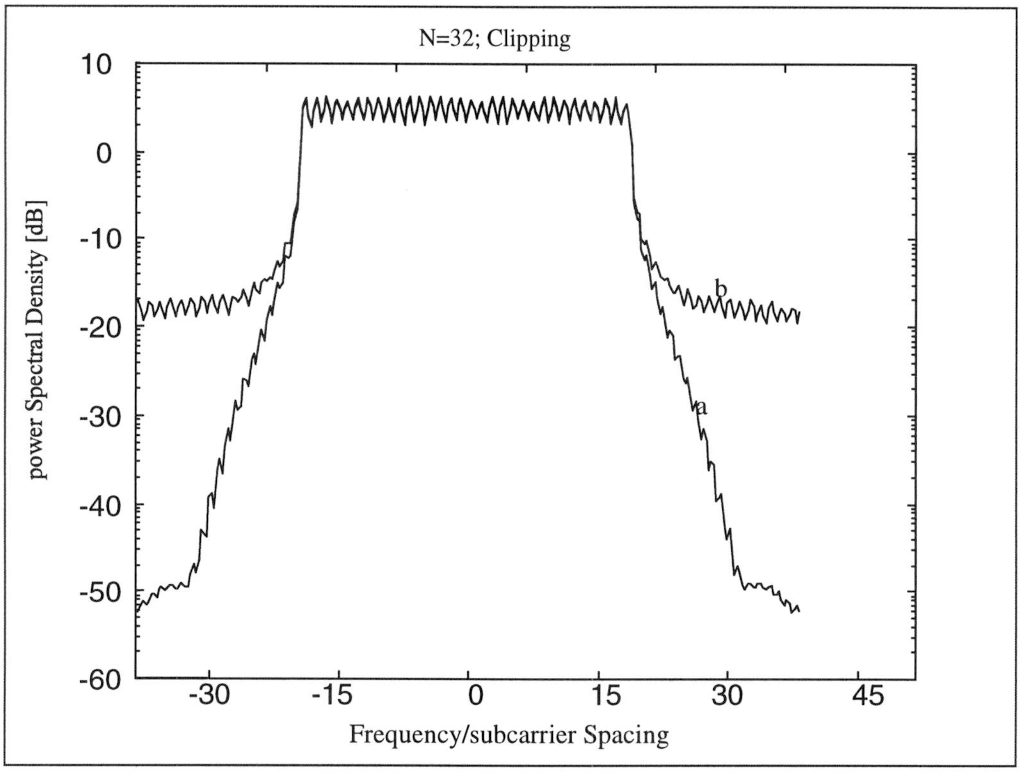


Figure 15: Power Spectra density for a)undistorted spectrum with 32 subcarriers, PAPR=15dB. b)spectrum for clipping to PAPR=4dB.

This out-off-band distortion could become unacceptable in practice because of spectral regulations. Therefore there has to be found a way to reduce the PAPR with a low as possible increase in error rate and spectrum distortion.

5. Selective Backoff.

To prevent the amplifier from saturation and to reduce the effect of out-off-band radiation, the OFDM symbols will now be backed-off digitally just after the IFFT as shown in figure 16. Backoff means that the entire OFDM symbol will be decreased in amplitude.

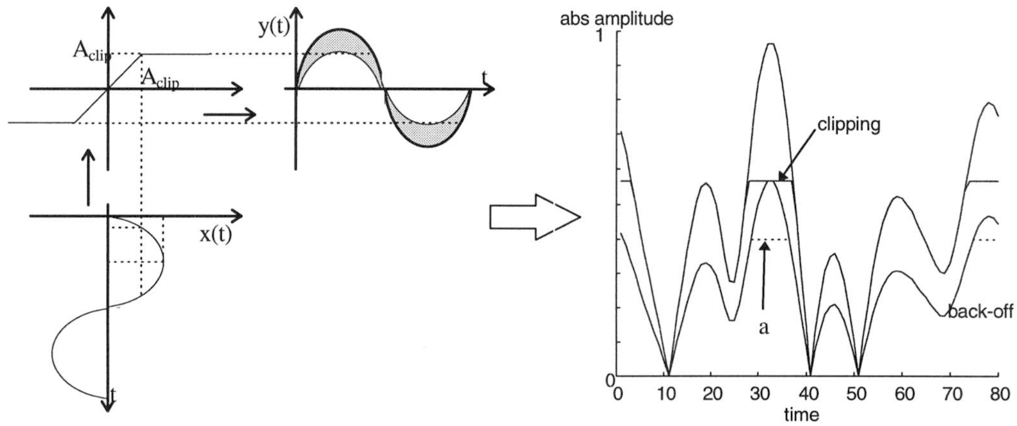


Figure 16:Backoff

When an OFDM symbol has a sample which is larger than a certain clip level (determined by the choice your of amplifier) the entire signal will be 'backed-off'. Resulting in a decrease of the signal power. But introducing -when comparing with clipping- no additional noise and therefore no irreducible error floor.
The main disadvantage is the unchanged PAPR and therefore a decreased SNR for that symbol which is backed-off. The SNR for that symbol can be written as:

$$Backof\ \ SNR = \frac{signal\ \ backed\ \ power}{noise\ \ power} \tag{23}$$

The probability of an OFDM-symbol being backed off will be the probability that an OFDM symbol has a sample with an amplitude larger than the clipping level. This will be a Rayleigh cdf distribution to the power N (see chapter 3). The pdf for backing off will be:

$$g(y) = N \cdot H(y)^{N-1} \cdot h(y) \tag{24}$$

With h(y) and H(y) being the Rayleigh pdf an Rayleigh cdf respectively (see equation 4). The power for backing off OFDM-symbols will be the power when not backing off a symbol plus the power when backing off an entire symbol. The power for backing-off the OFDM-symbols at a certain 'clip'-level Aclip will therefore be:

$$\text{power} = \left(\int_0^{A_{Clip}} g(y)dy + \int_{A_{Clip}}^{\infty} \left(\frac{A_{Clip}}{y} \right)^2 \cdot g(y)dy \right) \cdot N \tag{25}$$

In figure 17 simulation results are plotted against the mathematical approximation of equation 25 and equation 23.

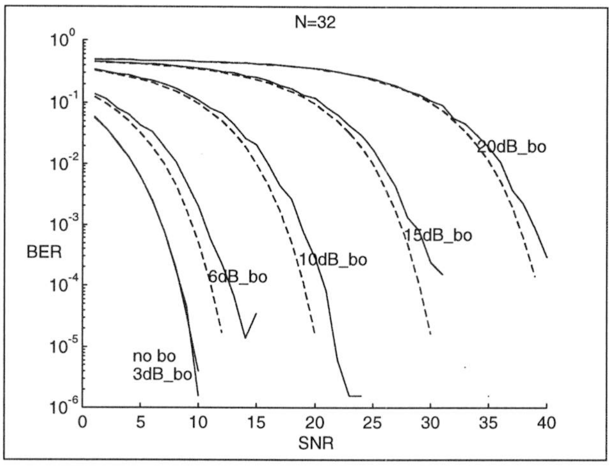


Figure 17: BER curves for backoff in dB below maximum amplitude for N=32, dashed is calculated.

From figure 17 it can be seen that the calculated values are smaller than the simulated values. This is the result of not taking into account the oversampling factor in equation 24 as it is used in equation 10 resulting in a underestimation of the simulated values.
What figure 17 shows is a shifted error curve when using back-off. The SNR is different for the different OFDM symbols because the amount of back-off that is applied depends on the maximum amplitude of an OFDM symbol. Therefore selective backoff results in having variable efficiency per symbol. This disadvantage increases when using a larger amount of subcarriers because the number of symbols with a high PAPR increases with a larger amount of subcarriers.

To prevent entire OFDM symbols to be in error when a large backoff is applied, interleaving is used to spread the error over a number of symbols. An example of interleaving is given in figure 18. In this case then amount of subcarriers N is equal to the number of symbols per packet M.

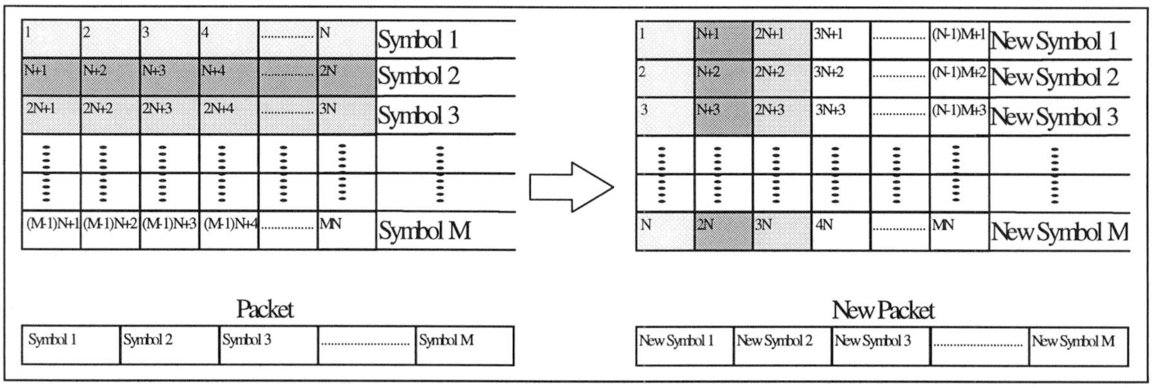


Figure 18: Example of interleaving the data.

Further an 1/2 rate convolution encoder with a viterbi decoder is used to the decrease the error rate. So what is done here is to use coding over multiple symbols. Therefore when a symbol will be lost, the error will be spread over numerous OFDM symbols.
The problem of symbols with a high PAPR will still remain because nothing is done to reduce the PAPR. This means that the amplifier efficiency will be poor. Therefore some kind

of clipping is proposed to clip OFDM symbols after a certain amount of backoff. Hence the reduction of the PAPR is traded off for some spectrum distortion as is shown in figure 19.

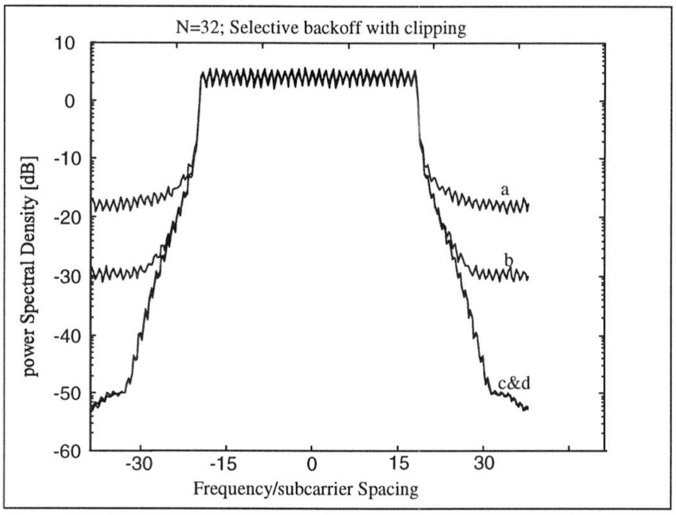


Figure 19:Spectrum when using selective backoff with clipping at levels a)2dB b)3 dB c)4 dB d)no clipping, above RMS value.

The effect on the error rate when using selective backoff with a certain amount of clipping is shown in figure 20.

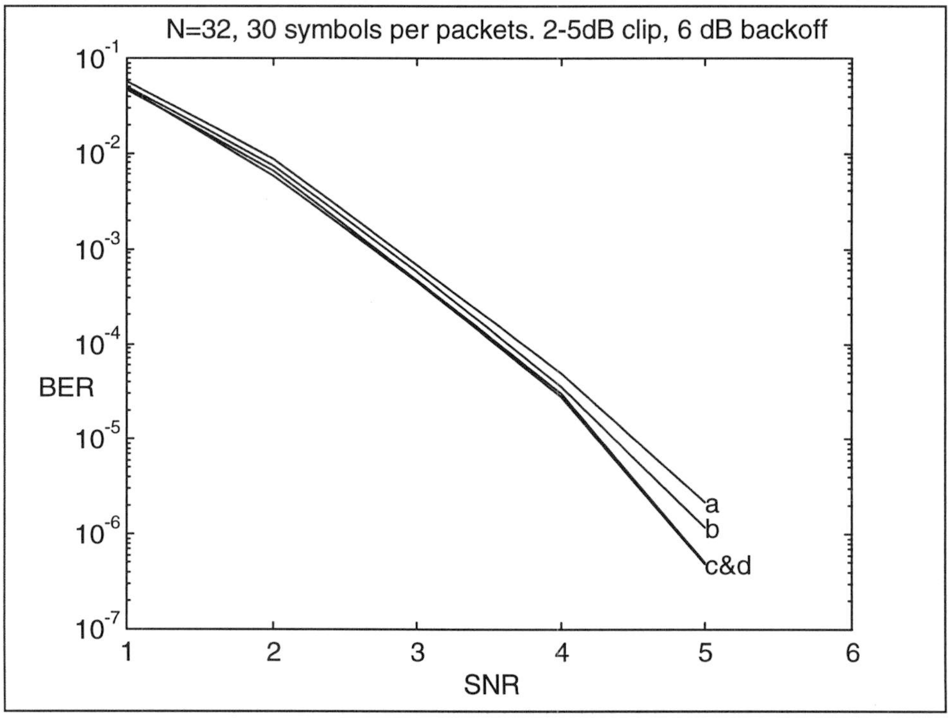


Figure 20: BER curves for using selective backoff and clipping. a)2dB clip highest PAPR=4.8dB,b)3dB PAPR= 5.3dB c)4dB PAPR= 5.9dB d)5dB PAPR= 6.6 dB.

From figure 20 it can be seen that when using this interleaving and coding scheme the error rates are very acceptable. The PAPR ratio can be reduced by accepting a certain amount of clipping which in that case leads to some spectrum distortion. Clipping here is done at a level in dB above the old RMS value of the original OFDM symbol, as shown in figure 16 (shown by a (dotted line), a clipping level such that the signal that is backed-off is clipped). The new PAPR's given above are calculated over one packet. One packet -in this example- contains 30 symbols. When using a smaller amount of symbols per packet the worst PAPR increases.

The only problem exists when all OFDM symbols in a packet are worst-case symbols, the PAPR will be very high and packet will most probably be lost. Therefore a different data scrambling is applied for each retransmission, in order to ensure that the PAPR ratios and error probabilities are uncorrelated for initial packets and retransmissions.

The main conclusions are that Selective backoff (with coding over multiple symbols) is a scheme to ensure the amplifier is not driven into saturation and that the PAPR can be improved when accepting a certain amount of spectrum distortion. Some kind of scrambling technique should be used to prevent the system for a possible dead-lock (see next chapter).

6. Selective Scrambling [19] to exclude symbols with a high PAPR.

Another way to reduce the PAPR when using OFDM, is to use some kind of scrambling scheme to scramble the PAPR down. In practice a bit-sequence of -in case of QPSK- two times the amount of sub-carriers will be scrambled with some sort of Pseudo Noise Sequence (PN-sequence) of the same length. Here we take a certain amount of PN-sequences of length 2N and scramble the original data. All the different data sequences are then passed through the IFFT and their PAPR is determined. The OFDM symbol having the smallest PAPR is then sent. To determine at the receiver which PN-sequence is used some kind of (FEC) coding after scrambling can be used.
When we use multiple PN-sequence the different scrambled-data sequences are uncorrelated. The results of using 2 to 8 scrambling sequences on an OFDM systems with 128 sub-carriers are shown in figure 21.

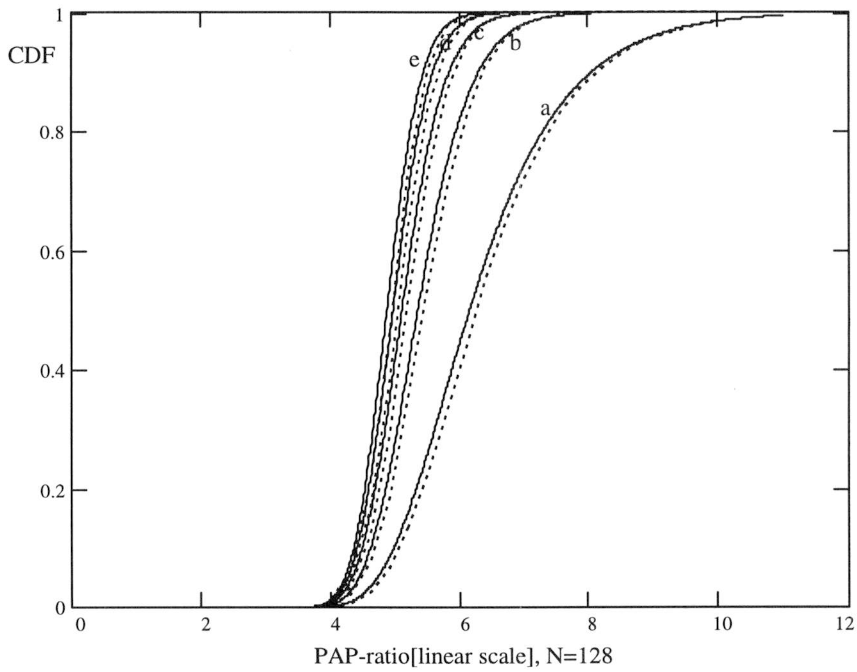


Figure 21: CDF for selective scrambling, a) no scrambling, b)2 OFDM symbols, c)4, d)6, e)8 (solid=simulated).

When we look at the solid lines drawn in figure 21 it can be seen that when using more scrambling sequences OFDM symbols with a high PAPR could be excluded. And that when more scrambling sequences are used, more symbols with a high PAPR can be excluded. The only problem is to determine what the highest PAPR will be after the use of multiple scrambling sequences.
Note that with this technique the lowest PAPR for 128 subcarriers will be around 3 (see figure 11, left) and in practice for selective scrambling around 4 (see figure 21). For a larger amount of sub-carriers the lowest PAPR will therefore be higher as can be seen in figure 10 or 11(left).

The theoretical estimation (figure 21, dashed) for using this scrambling technique follows directly from probability theory. After scrambling the different OFDM symbols are assumed to be uncorrelated. Therefore the different PAPR of the different OFDM symbols are assumed to be uncorrelated. The new CDF for scrambling follows directly from the fact that this is like choosing the smallest number of a certain amount of uncorrelated numbers [15]:

$$\mathrm{CDFcode}(n,y) = \sum_{k=1}^{n} \frac{n!}{(n-k)!\cdot k!} \cdot \left[F(y)^{\alpha \cdot N}\right]^{k} \cdot (-1)^{k+1} \tag{26}$$

with n being the amount of scrambling sequences used plus one and F(y) given by the chi-square distribution with two degrees of freedom (equation 27) as described in chapter 3.

$$F(y) = \int_{0}^{y} \frac{1}{2\cdot\sigma^{2}} \cdot e^{\frac{-u}{2\sigma^{2}}}\, du \tag{27}$$

The PDF follows directly from equation 26 and is given in equation 28.

$$\mathrm{PDFcode}(n,y) = \sum_{k=1}^{n} \frac{n!}{(n-k)!\cdot k!} \cdot \left[F(y)^{\alpha \cdot N \cdot k-1}\right] \cdot (-1)^{k+1} \cdot \frac{\alpha \cdot k \cdot N}{2\cdot\sigma^{2}} \cdot e^{\frac{-y}{2\cdot\sigma^{2}}} \tag{28}$$

The reason that here the theoretical approximated curves deviate from the simulated curves is that -as seen in chapter 3- the α term in equation 27 is an approximation when oversampling the OFDM signal.

Equation 28 is plotted in figure 22 against simulated values. Here 128 QPSK modulated subcarriers are used. Further we used Monto Carlo simulation (10^6 symbols) to determine the pdf functions for the different amount of scrambling codes.

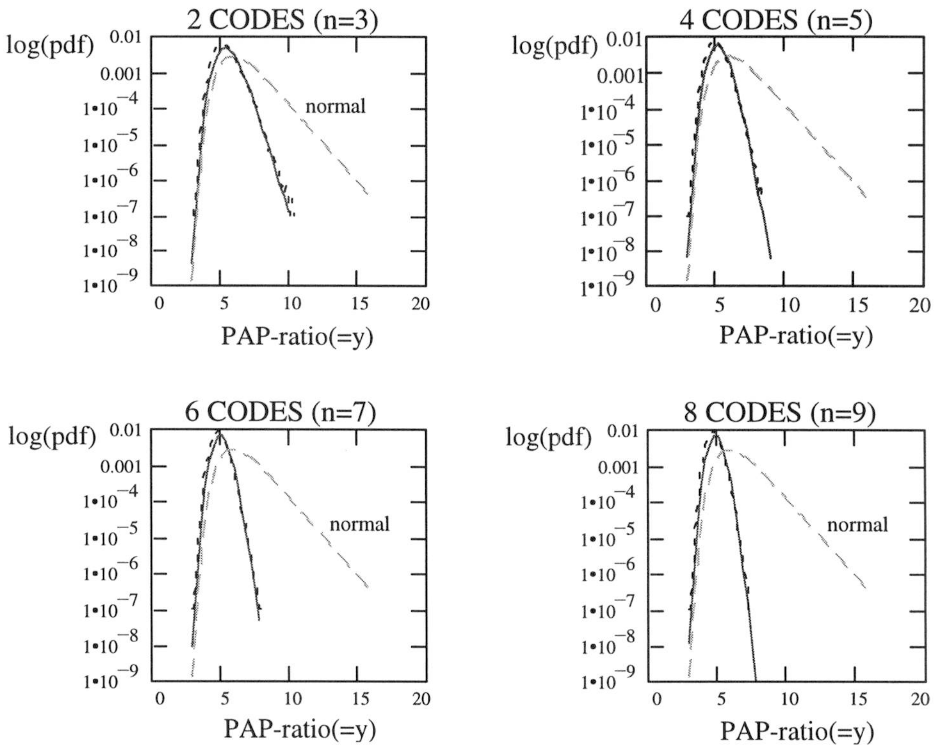


Figure 22: pdf on a log-scale for Selective Scrambling (N=128), solid=simulated.

What is seen in figure 22 is that when using multiple scrambling sequences the probability of OFDM symbols with a high PAPR occurring will be very small. But when we look at -for example- the figure for 8 scrambling codes, the probability of a PAPR of more than 10 occurring will be something of 10^{-20}. This will mean that ones in every 10^{20} symbols there will be a symbol with a PAPR of more than 10. Therefore OFDM symbols with large PAPR will still happen. When we do nothing about this, these symbols will be clipped by the amplifier causing distortion which could be unacceptable in practice. We could therefore use some kind of backoff and interleaving as described in the previous chapter to deal with this problem.

The main disadvantage of this scheme is that multiple FFT's have to be performed and hence high computational effort is needed. Besides that the minimum PAPR will increase with the amount of sub-carriers used. There will still be a need to use some kind of interleaving/back-off or other scheme to deal with the symbols which will still have a large PAPR.

The main advantages will be the unchanged frequency spectrum with a reduced PAPR and no increase in error rate.

7. Windowing to reduce the PAPR.

7.1. Multiplication window.

A different way to reduce the large peaks that occur in the OFDM signal is to not simply clip the peaks but to multiply them with a certain window [22]. This window could be for instance a cosine, Kaiser or Hamming window. An example of reducing the large peaks in OFDM with the use of windowing is given in figure 23.

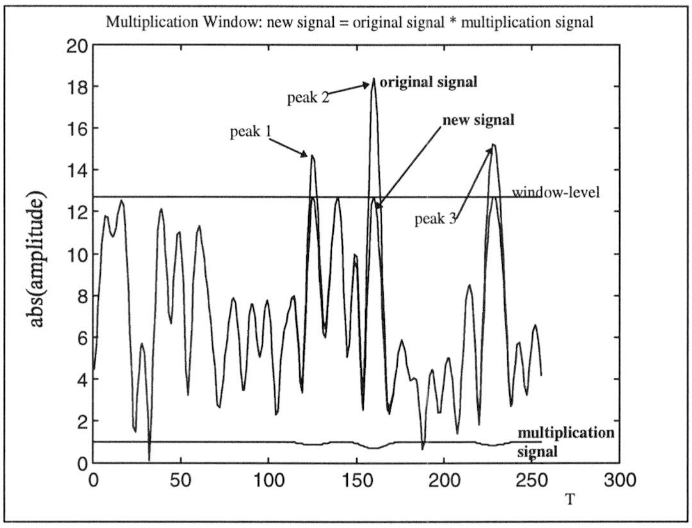


Figure 23:Windowing an OFDM time signal

The samples in the original signal which are larger than a certain level (here the window-level) will be multiplied with a window which is basically a data-row with samples x≤1. The length of the data-row will here be given by 'windowwidth'. The amplitude of the window is such that after multiplying, the peak will be reduced to the window-level (see figure 23). From figure 23 it can be seen that after windowing the signal is amplitude limited resulting in a reduced PAPR. However the BER will increase and the frequency spectrum will be distorted. But figure 23 clearly shows a multiplied signal that looks very linear when comparing it to clipping. Therefore the frequency spectrum will be less distorted. The BER however will be higher than clipping because of the fact that more samples will be mutilated. Some examples of the used windows are given in figure 24.

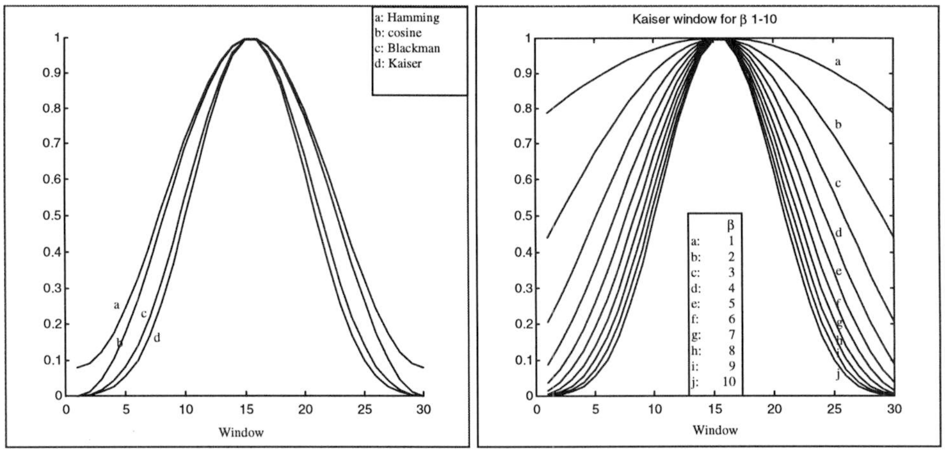


Figure 24: Examples of various window types, here windowwidth=30.

From figure 24 it is seen that the windows shown are more or less the same and that Kaiser windows can be changed using a parameter β. But the main question remains how much these windows distort the frequency spectrum.
Figures 25 and 26 show the frequency response of using those windows (pictures taken from reference[18]).

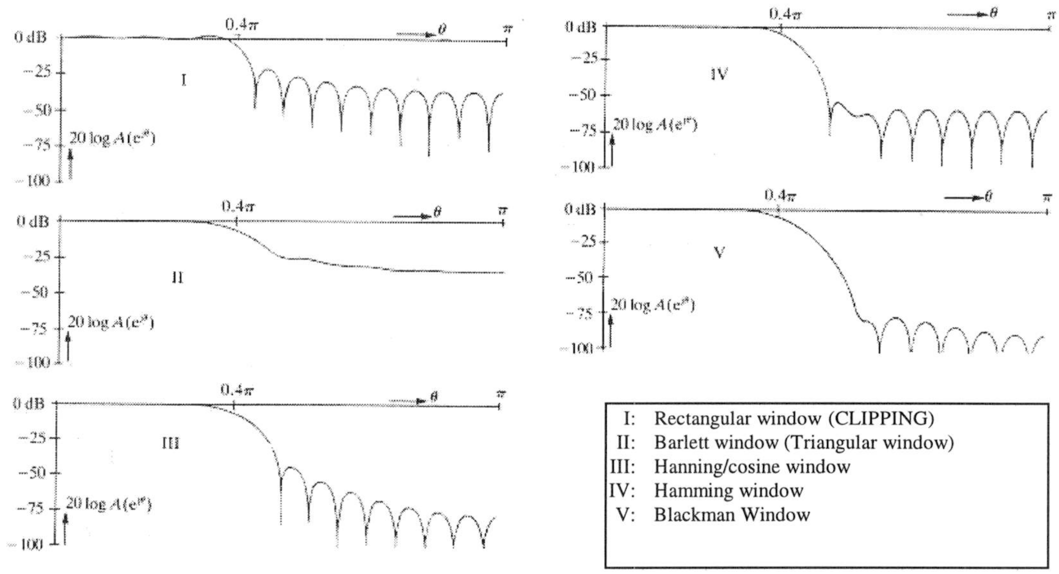


Figure 25: Effects of five windows with windowwidth = 31

Figure 25-I shows the frequency response when using some kind of rectangular windowing such as clipping. The rest of Figure 25 clearly shows that there can be a less distorted frequency spectrum (compared to clipping) when using the other windows. Figure 26 shows the different frequency responses for different β for Kaiser windows. Here some kind of trade off can be made between the sharpness of the filter and its depth.

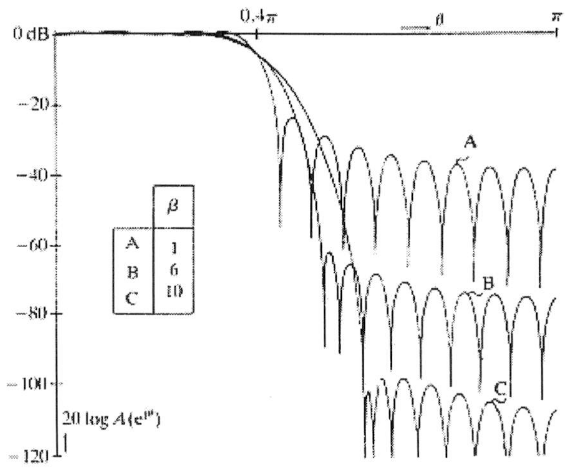


Figure 26: Effect of a Kaiser window for different values of β (windowwidth=31).

Now simulation results for the BER and frequency spectrum will be given when using Hanning/cosine or Kaiser windowing.

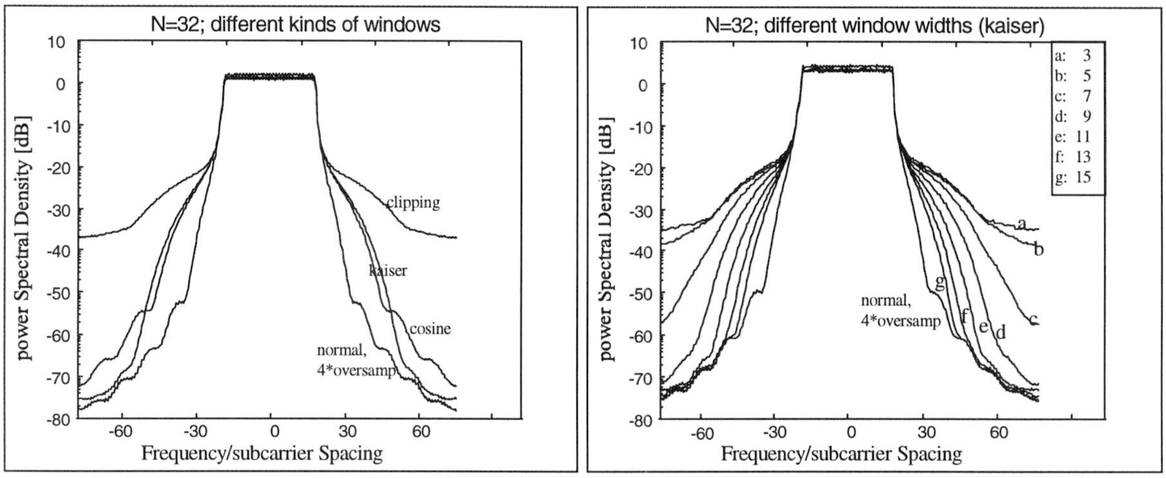


Figure 27: Frequency spectrum as a result of multiplying windowing 3dB above RMS value.

In figure 27 (left) the difference between clipping the signal and windowing the signal can be seen. Note that when using a β factor smaller than used in figure 27 (here windowwidth is 7), the spectrum for Kaiser windowing will fall steeper but will have a larger minimum (see figure 26). For larger values of β the opposite can be expected.
From figure 27 (right) it can be seen that when the windowwidth increases the spectrum distortion decreases. This is due to the fact that the change in the new function will be 'introduced' more gradually and will therefore give smaller out of band radiation. Therefore the use of larger windows is preferred when looking at the frequency spectrum. The computational time and the BER (ICI) will however increase with the use of larger windows. The increase in windowwidth will result in a higher BER because the amount a samples that will be affected will increase. Therefore there will be a trade off between windowing with larger windows at the cost of an increase in error rate.
Figure 28 shows some BER curves (no coding used) for different amount of windowlevels.

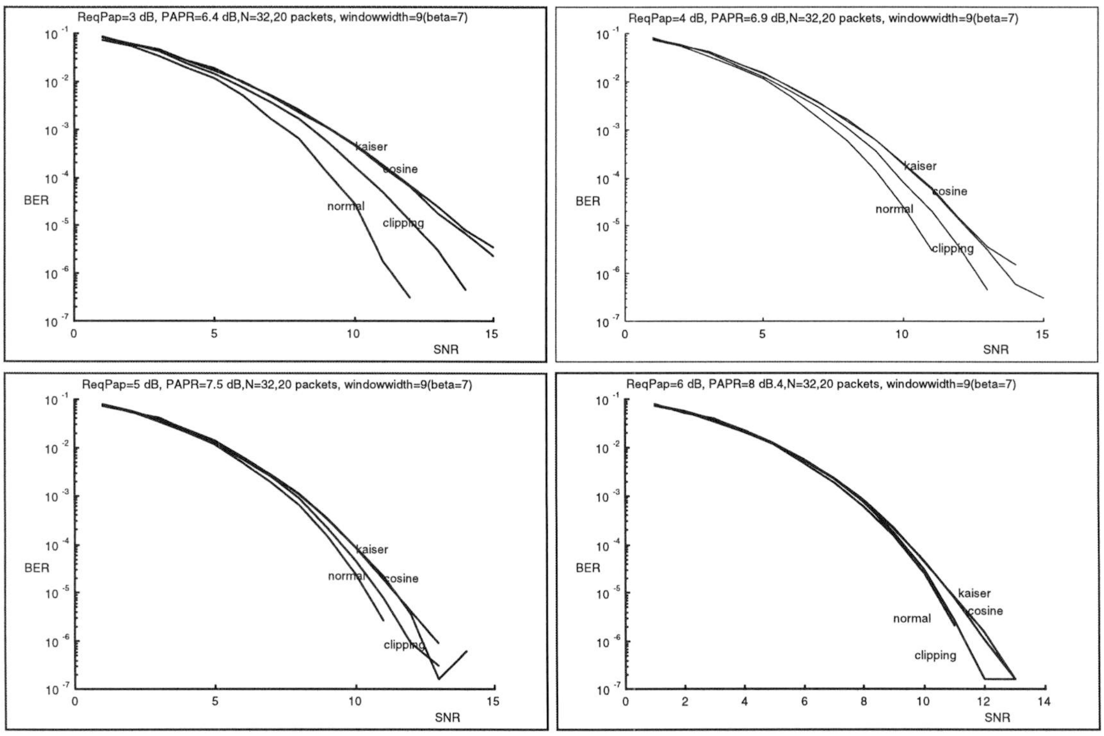


Figure 28: BER curves as a result of multiply-windowing at different levels above the RMS value ,ReqPAP (dB) (SEE PAPR IN FIGURE, IS NEW PAPR AFTER WINDOWING).

It can be seen from figure 28 that the PAPR can be decreased to about 6 to 7 dB with an acceptable error rate. It is possible to reduce the PAPR even further to about 4dB by using a recursive way of windowing. Meaning that after windowing a peak down, the new average power is determined after which the new windowing level is set. The only disadvantage of this method being the very large and unpredictable computational time and the possibility of the process not converging. The BER will also increase.

For a larger amount of carriers the windowwidth has to be increased to ensure that the spectrum distortion won't become too large. With an increasing amount of subcarriers the PAPR will become larger (see chapter 3), therefore the envelope changes can become greater. Therefore the use of larger windowwidths is needed to 'introduce' the change more gradually.

7.2. Subtraction window.

Another way of windowing is not to multiply with a certain window but to make another OFDM symbol which has the same bandwidth and to subtract this symbol from the symbol containing large peaks. The main advantage of this scheme is that no out of band radiation is introduced because the subtraction window has the same bandwidth as the OFDM symbol.
The subtraction window can be made by first detecting those samples in the OFDM symbol which contain amplitudes larger than the windowing level and then making a subtraction symbol with pulses on those same places. These pulses should be as large as the amplitude of the OFDM symbol at that moment minus the windowing level.
This subtraction symbol is then set in the frequency domain by using an FFT after which it is filtered and placed back in the time domain by using an IFFT. In this way a substration symbol is created with peaks at places where the OFDM symbol has its large peaks. Then we subtract this symbol from the OFDM symbol (see figure 29).

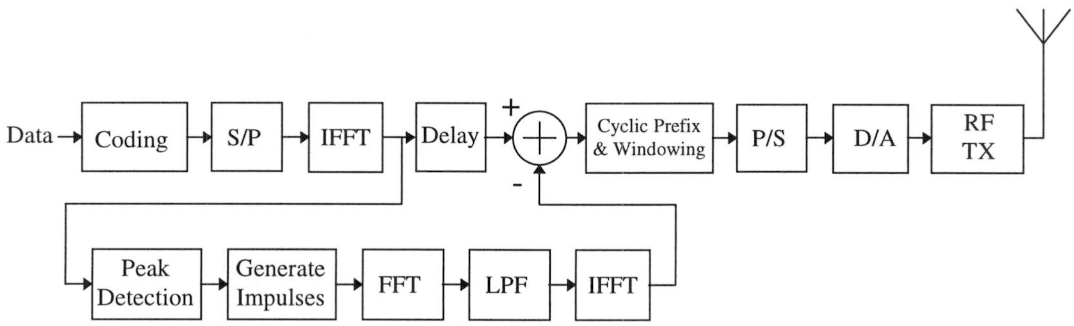


Figure 29: Peak cancellation using FFT/IFFT to generate cancellation signal.

The filtering could be done in the time domain, but we choice this way because it is easier to implement.
An example of subtracting an OFDM symbol with large peaks is shown in figure 30.

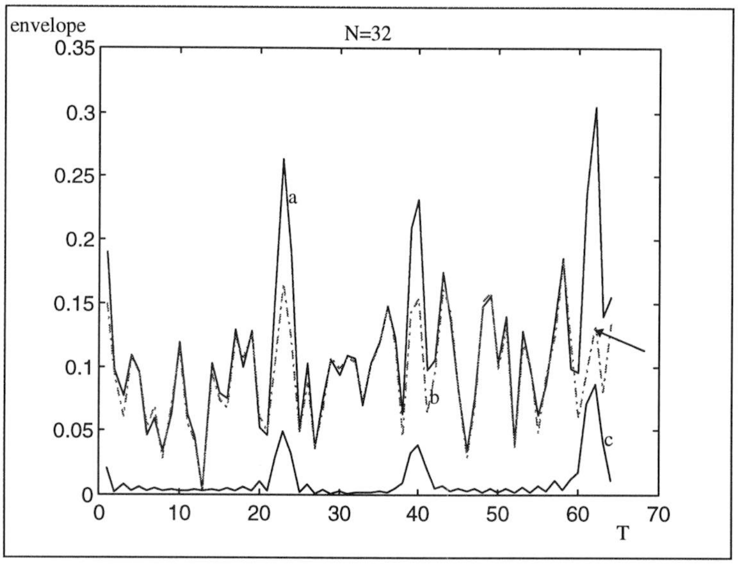


Figure 30: Reducing the PAPR via an OFDM subtraction symbol a)normal signal, b)subtracted signal and c) subtraction signal.

As can be seen in figure 31 the disadvantage of this way of implementing here is that when two consecutive samples lie above the threshold value (see arrow figure 31), two sinc pulses will be subtracted resulting in a smaller amplitude of the signal than it ideally could be (ideally you would like to have an amplitude as large as the windowing level). This will result in a lower average power and therefore a higher PAPR.
In figure 31 several BER curves are given for different PAPR requirements. Here the PAPR requirement is the windowlevel in dB above the average power of the signal.
From figure 31 it is then seen that when using a smaller papreq and therefore a smaller windowing level, more inband distortion is created when using the subtraction windows resulting in a higher BER.

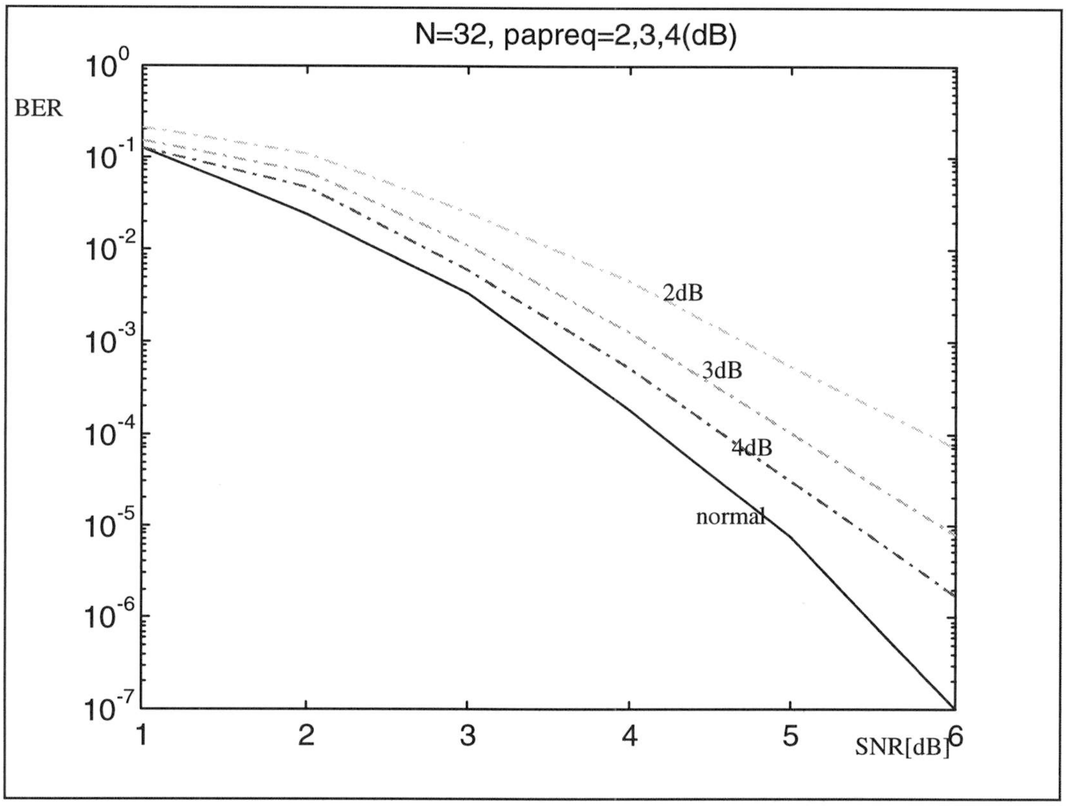


Figure 31: Symbol subtraction for different papreq. 2dB results in 6.2dB PAPR, 3dB in 6dB and 4 dB in 6dB.

To increase the performance of this method of PAPR reduction we used 1/2 rate convolution encoder with Viterbi decoding plus interleaving of the data as described in chapter 5. Note that (see figure 31) the resulted PAPR for a required PAPR of 2dB is less than for 3 or 4dB. This is the result of the decrease average power.
Figure 32 shows the use of a substraction symbol for different amount of subcarriers.

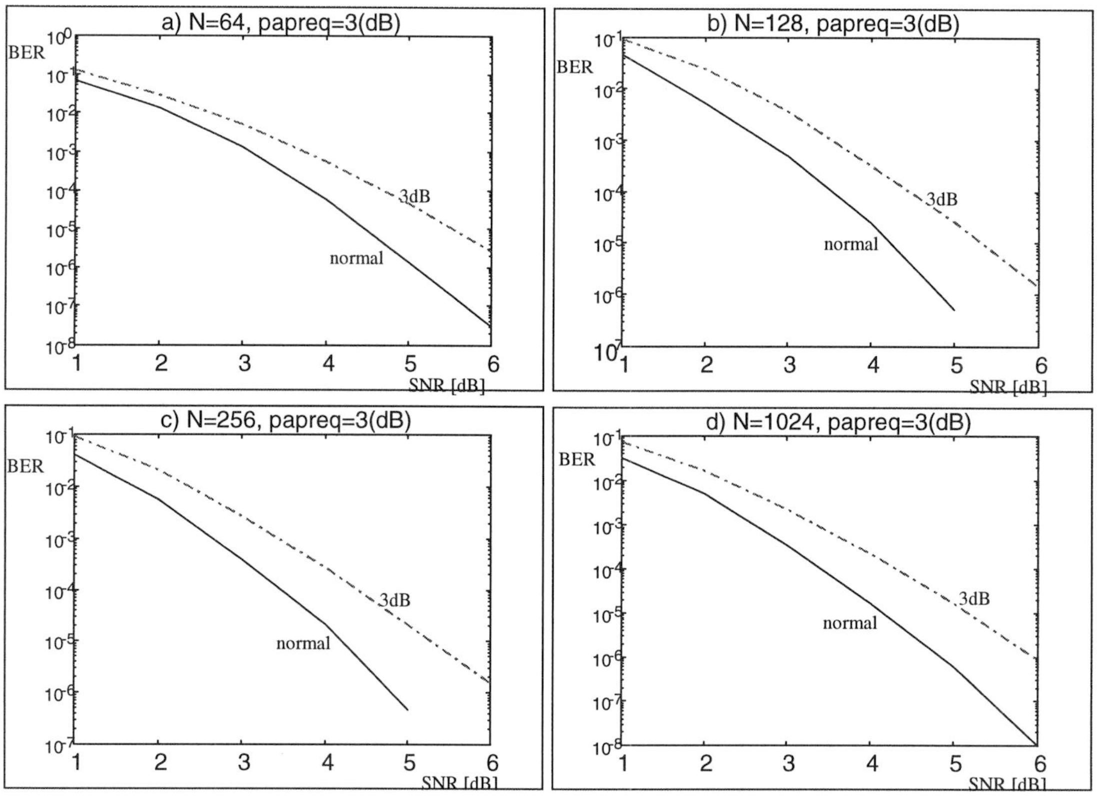


Figure 32: BER-curves for different amount of subcarriers using substation symbol. PAPR after windowing: a)N=64, PAPR=6.3 dB b)N=128, PAPR=6.5 dB c)N=256, PAPR=6.5dB d)N=1024, PAPR=6.5 dB.

From figure 32 it can be seen that this substation scheme not only works for small amount of carriers but can also successfully be applied to higher amounts of carriers. Simulations show new PAPR of about 6-6.5 dB. This PAPR can be reduced by accepting a certain amount of distortion by using for instance clipping.

In order avoid the problem of windowing the large peaks too much and hence reducing the average power as is the case described above, another way of making the OFDM symbol should be used. This could be done by making standard peaks for different widths of peaks so as to not reduce certain peaks too much. This will result in a lower PAPR and a better BER.

Another way is to use a recursive way of subtraction windowing. This can be done by using a sinc reference function windowed with a raised cosine function as can be seen from figure 33. It can be seen that the PAPR can be windowed down to about 4dB.

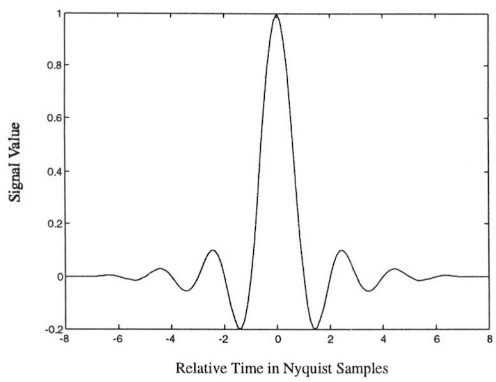


Figure 33: Sinc reference function, windowed with a raised cosine window.

The basic idea is the same as for windowing with the window described in 7.1. But now we subtract sinc pulses. An example of windowing with a sinc pulse is given in figure 34.

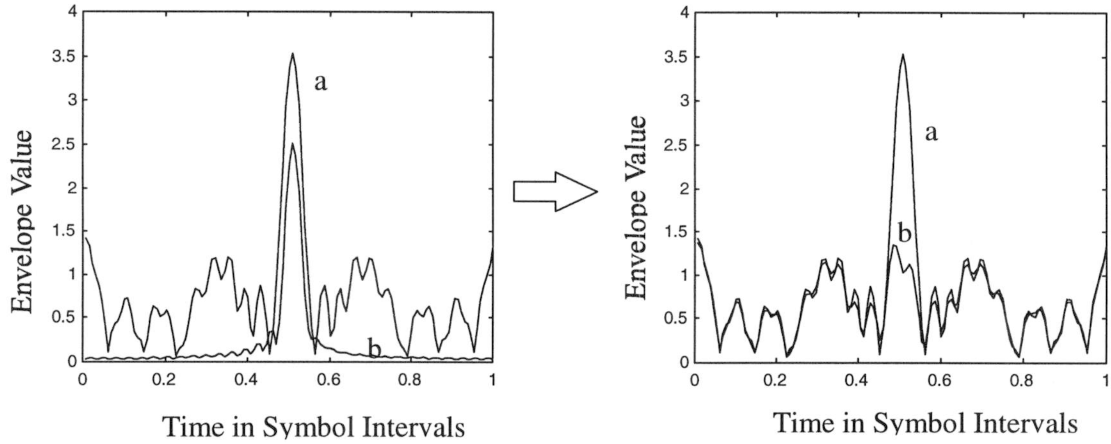


Figure 34: a) OFDM symbol envelope, b) cancellation signal envelope and signal after windowing

The result of windowing this way will be a slightly distorted frequency spectrum (see figure 35) but a better BER than for subtracting a whole OFDM symbol because a smaller amount of sinc pulses will be needed. The spectrum will be slightly distorted because we use a cosine window over the sinc pulse. When we would take a sinc pulse with the same width as the OFDM symbol there wouldn't be any spectrum distortion because a sinc pulse is also an OFDM symbol (a worst case symbol). But when we would window with a sinc pulse with the same width as an OFDM symbol the computational effort would become too large. In figure 35 a frequency spectrum is shown for windowing with a sinc pulse. It can be seen that the spectrum distortion is negligible.

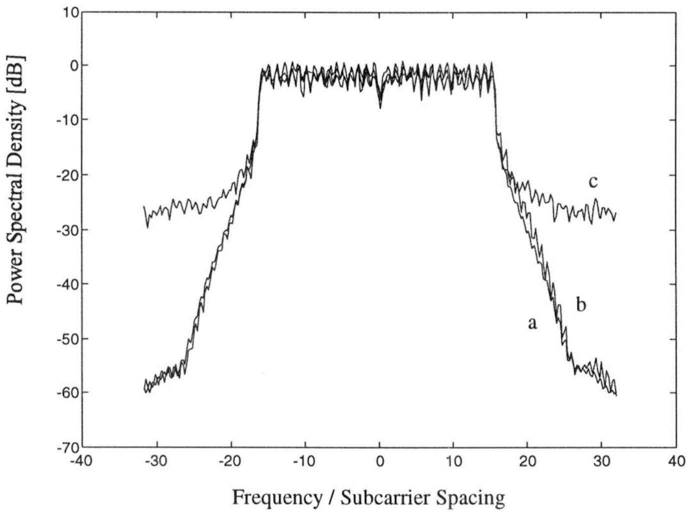


Figure 35: Power spectral density for a) undistorted spectrum with 32 subcarriers, PAPR=15dB, b) spectrum after peak cancellation to PAPR= 4dB, c) clipping to PAPR=4dB. Reference cancellation function has a width of 16 samples, which is about 1/4 of the width of an OFDM symbol.

In figure 36 a BER curve is given for windowing with a sinc-puls (no coding used).

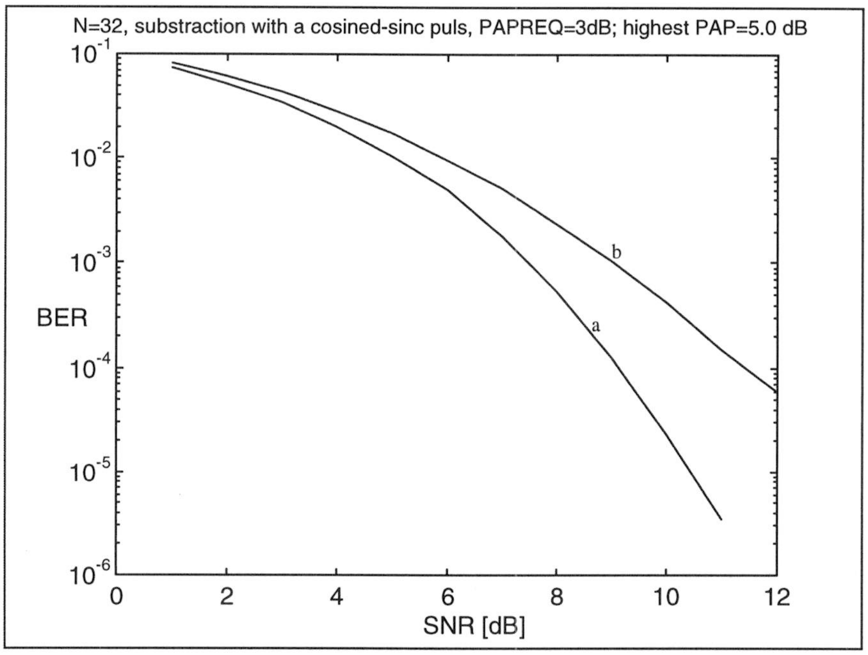


Figure 36: BER curve for windowing with a sinc-pulse at 3dB above RMS value, a)normal (no PAPR reduction) b)windowed.

Note that the results given in figure 36 are comparable in error rate as the results given in figure 28. The highest PAPR is better (thus lower) in the case of substraction windowing.

The main conclusion are that multiplying window is a possible candidate for PAPR reduction however some spectral distortion has to be accepted. Substrating with a windowed sinc puls will have lower spectral distortion and will therefore be a better solution.
The use of a subtraction symbol will result in reduction of the average power and therefore in comparison to the other schemes a larger PAPR. And because of the larger amount of subtraction sinc-pulses an increase in error rate. The spectrum distortion however will be null.

All the methods work for various amounts of carriers. Meaning that for example N=1024 the PAPR can be equally be windowed down as well as for N=32.
The OFDM symbol subtraction having the advantage over the others that the computational time is fixed.

Therefore the choice of windowing-type depends on what kind of OFDM system will be build, how much spectral distortion will be accepted, what error rate is acceptable and what amplifier is chosen.

8. Conclusions & Recommendations.

In this report four different methods for reducing the PAPR problem of OFDM were investigated: Clipping, Selective Backoff, Selective Scrambling and Windowing. First the main conclusions concerning these different methods are given.

The main disadvantage of clipping is that it causes a distorted frequency spectrum which can become very large when clipping hard.
Selective backoff will result in a decrease of the signal power causing the BER to increase. But when applying a convolution code with some interleaving over multiple OFDM symbols this problem can be solved. When using selective backoff the PAPR will be unchanged resulting in an inefficient power stage. This can partially be solved by applying a certain amount of clipping to increase the efficiency and to therefore allow a certain amount of distortion. The amount of distortion that can be tolerated depends on spectral regulations and required error rate.
Selective scrambling has the advantage of neither increasing the BER nor distorting the frequency spectrum. The only disadvantages are the increase of computational time and the increasing PAPR's for a larger amount of sub-carriers. Also some kind of selective backoff or interleaving should be used when scrambling would result in an OFDM symbol with a high PAPR. Therefore this scheme is only interesting for relatively smaller amount of carriers (see figure 10).
Windowing with a subtraction window or multiplying window will result in a minor distorted or no distorted frequency spectrum. The BER however increases even more than for clipping. But this increase of BER can be solved by using the some same kind of convolution encoding as was applied when using selective backoff. Using a subtraction symbol will result in an increase of error rate and an increase in the PAPR compared to the other windowing schemes. The computational effort however will be fixed.

Therefore the choice in practice what scheme should be used depends on what system you are building. If it is a system with a small (<128) amount of carriers, scrambling could be used. But for a higher (lower as well) amount of subcarriers windowing or selective backoff can be used. What kind of windowing or selective backoff should be used depends on how much spectral distortion, how much computational effort, what amplifier is chosen and what error rate is demanded. The PAPR when using windowing will be smaller than for selective backoff with a comparable spectrum distortion. Therefore windowing is preferred over selective backoff when looking at the PAPR and therefore looking at the power efficiency.

Recommendations:

- In this report a simplified model for the amplifier is used. The should be tested using a better amplifier model.
- Other techniques such as coding could be investigated.

Appendix A: Mathematical derivations for Clipping-Noise.

Noise power introduced by clipping is treated well in paper [16]. However in deriving the noise-power they assumed the magnitude of the complex OFDM symbols to be Gaussian distributed whereas here the magnitude of the symbols is assumed to be Rayleigh distributed. Therefore the general idea in deriving the clipping-noise power is the same and given below.

The transfer function given in equation 13 can be written as an error function given by:

$$e(t)=x(t)-y(t) \tag{29}$$

$$e(t)=\begin{cases} x(t)-A_{Clip} & \text{for } |x(t)|>A_{clip} \\ 0 & \text{for } |x(t)|<A_{clip} \end{cases} \tag{30}$$

This function can then be approximated by a Taylor series:

$$e(t)=\chi x(t)+\beta x^3(t)+\gamma x^5(t)+\ldots\ldots \tag{31}$$

For the clipping-noise we are interested in the higher order terms which introduce the noise, we can thus write the error as

$$e(t)=\chi x(t) + z(t)=x(t) - y(t) \tag{32}$$

Because x(t) is a zero mean Gaussian random variable, e(t),y(t) and z(t) are also assumed to have a mean zero (in paper [16] they showed z(t) to be essentially white Gaussian noise). We now have to derive the average noise power $<z^2(t)>$. Therefore we multiply equation 32 with x(t), resulting in

$$\begin{aligned} x^2(t) - y(t)\cdot x(t) &= \chi x^2(t) + z(t)\cdot x(t) \\ x^2(t)\cdot(1-\chi) &= y(t)\cdot x(t) + z(t)\cdot y(t) \end{aligned} \tag{33}$$

In paper [16] they found it true that $<z(t)\cdot x(t)>$ can be considered small compared to $<y(t)\cdot x(t)>$, and can be dropped. This assumption was found to be valid for clip levels above 0.1 times the standard deviation of the normally distributed x(t)'s. For $<y(t)\cdot x(t)>$ we can write:

$$y(t)\cdot x(t)=\begin{cases} |A_{Clip}\cdot x(t)| & x(t)> A_{Clip} \\ |x^2(t)| & -A_{Clip} \le x(t) \le A_{Clip} \\ |A_{Clip}\cdot x(t)| & -A_{Clip}<x(t) \end{cases} \tag{34}$$

When assuming the amplitude of the complex OFDM-symbols being Rayleigh distributed, χ can be written as:

$$\chi = 1-\frac{\int_0^{Aclip} x^2\cdot p(x)dx + A_{Clip}\cdot\int_{Aclip}^{\infty} x\cdot p(x)dx}{\int_0^{Aclip} x^2\cdot p(x)dx} \tag{35}$$

With p(x) being the Rayleigh pdf (see equation 4) and σ given by equation 5.

$$\chi = \frac{1}{4\sigma} \cdot \left(4\sigma - A_{Clip} \cdot \sqrt{2\pi} \cdot e^{\frac{Aclip^2}{2\sigma^2}} + A_{Clip} \cdot \sqrt{2\pi} \cdot erf\left(\frac{\sqrt{2} \cdot A_{Clip}}{2\sigma} \right) \cdot e^{\frac{Aclip^2}{2\sigma^2}} \right) \cdot e^{-\frac{Aclip^2}{2\sigma^2}} \tag{36}$$

With the assumption that $2\chi<z(t)\cdot x(t)>$ is small compared to $<z^2(t)>$ (which is found to be valid for clip levels greater than 0.1 times the standard deviation of x(t) [16]) we can derive an expression for $<z^2(t)>$ from equation 36

$$<z^2(t)>=-(\chi-1)^2\cdot<x^2(t)> + <y^2(t)> \tag{37}$$

With $<y^2(t)>$ being the power of the clipped signal given by equation 18, and $<x^2(t)>$ being the power of the unclipped signal.

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