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# Categorization of formulas for calculation of crack width and spacing in reinforced concrete elements

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**Abstract**

Over the last century, over one hundred crack width formulas have been developed to calculate the width and spacing of cracks in reinforced and prestressed concrete elements. It is unclear which formulas are the most accurate. An extensive comparison study is required to determine which formulas accurately describe the crack patterns, consisting of the crack width and spacing. To make such a study possible, this paper proposes categorizing formulas. The categorization of the formulas is based on their applicability, crack pattern representation, and background. The categorization presents an overview of the different assumptions and application areas for describing crack patterns.

**KEYWORDS**

categorization, crack spacing, crack width, prestressed concrete, reinforced concrete

## 1 | INTRODUCTION

In the Netherlands, civil engineering structures, such as bridges, tunnels, and underpasses, are frequently constructed out of reinforced or prestressed concrete. Their replacement value is estimated at €28 billion and is growing as new concrete structures are being designed.[1,2] In these structures, loading due to external forces or restrained imposed strains may lead to cracks.[3] Although current studies do not all agree if corrosion increases with increasing crack widths,[4] the impact of cracking on durability performance is indicated by various studies,[5–7] and led to the definition of crack width limits for ingress rates. Cracking can jeopardize the service life of these civil engineering structures,[8] and thus, an accurate description of crack patterns in existing and new concrete structures is essential, considering both the crack width $w$ and spacing between the cracks $s_r$.

The description of crack patterns due to loads is based on formulas such as described in EN 1992-1-1.[9] Over one hundred formulas for crack calculation have been developed in the last century for numerous applications in reinforced and prestressed concrete structures. In 1936, Saliger derived formulas to calculate the crack width and spacing in uniaxial loaded elements using bond stress–slip relationships,[10] and an approach for elements loaded in bending in 1950.[11] Empirical formulas were developed in the 60s, such as those developed by Gergely and Lutz.[12] Also, in the 60s, parallel to the introduction of prestressed concrete in civil structures, the first formulas targeted the description of crack width in prestressed structures.[13] Formulas that consider cracking due to the restraint of imposed strains were introduced in the 70s and 80s and are still being developed today.[14,15]

Due to the numerous developments and differences between formulas for calculating crack width and spacing in concrete elements, it is unclear which formulas are the most accurate. To assess their accuracy, a comparative study of the performance of the formulas is required.

Comparisons between crack widths or spacings calculated with formulas and measured crack widths or spacings in experiments are presented by Allam et al.,[16] Pérez Caldentey et al.,[17] CROW-CUR,[15] Scholz,[18] Dawood and Marzouk,[19] and Oh and Kang.[20] However, these studies generally compare only a limited number of formulas for calculating crack widths and spacings. In 2018, Lapi et al.[21] presented a study where 30 formulas were thoroughly described and distinguished if it was based on experiments, fracture mechanics, bond stress–slip relationships, or semi-analytical approaches.[21,22] Lapi et al. used the formulas to compare the outcomes with crack width measurements from experiments described in the literature. It was outside the scope of the cited comparative studies to extensively address the formulas' different assumptions and application areas for describing crack patterns. Hence, categories for formulas calculating the crack width and spacing in reinforced concrete are missing.

This paper categorizes formulas to calculate crack width and spacing in concrete elements loaded in tension and bending. The paper first identifies categories based on the applicability, the representation of crack patterns, and the background used for deriving the formulas for crack width and spacing. Those main categories are then further detailed based on their respective subcategories. Finally, the proposed categories were applied to 130 formulas to describe crack patterns, presented in Appendix A. Conclusions on the application and background of the categories are presented, which provides the basis for recommendations for future research.

## 2 | IDENTIFICATION OF CATEGORIES OF FORMULAS FOR CALCULATING CRACK WIDTH AND SPACING

Three main categories have been identified and applied to 130 formulas from 94 individual publications for calculating the crack width and spacing in reinforced and prestressed concrete structural elements. The main categories are application (A), representation (R), and background (B). These categories are then subdivided to cover the relevant characteristics of the cracking process in concrete structures, which enables all of the selected formulas to be classified. An overview of the categories is presented in Figure 1.

The (A) of each formula concerns different structural types (ST), the state of loading (SL), and the resulting internal strain distribution (SD).

The (R) indicates how the formulas describe the crack patterns, which is essential since various formulas generally do not describe crack patterns similarly. This concerns the applicable cracking stage (CS), the position where the cracks are described (P), and a definition of the crack width (W) and spacing (S).

Each formula's (B) can be used to determine the extent of applicability for the structural and loading application or how the crack patterns are represented. It allows for assessing the assumptions behind each formula and identifies application areas not explicitly described in the corresponding literature. The four subcategories were chosen as fully (B1) empirical models, models based on fracture mechanics (B2), models based on bond stress-slip relationships (B3), and semi-analytical models (B4). The choice of the subcategories was partially inspired by Lapi et al. and Borosnyói and Balázs.[21,22]

## 3 | APPLICATIONS OF FORMULAS FOR CALCULATING CRACK WIDTH AND SPACING

### 3.1 | Structural type

Concrete structures typically contain reinforcing steel (ST1), prestressing steel (ST2), or a combination of the two (ST3). The reinforcement in these structures can be non-metallic,[23] for example, fiber-reinforced polymers, but most reinforced structures contain steel reinforcing bars,[24,25] which are either plain or deformed, often incorporated in the formulas through a bond factor. The formulas for calculating crack widths and spacings usually consider only the longitudinal reinforcement, while, for example, Rizkalla and Hwang,[26] also consider transverse reinforcement.

Contrary to structures with reinforcing steel, prestressed structures use pre or posttensioned bonded and unbonded tendons. Formulas for crack width and spacing are applied to bonded tendons in partially prestressed structures or structures with unbonded tendons combined with reinforcing steel since cracks can develop and be controlled in these structures. Examples include EN 1992-1-1[9] or *fib* Model Code (MC) 2010,[27] where a bond factor is used to convert the bond properties of a prestressed structure to an equivalent reinforced structure, allowing for a straightforward calculation in the case of prestressed steel or a combination of reinforcing and prestressing steel.

### 3.2 | State of loading

The state of loading of a concrete structure is crucial since it mainly determines the crack width.[28] In the formulas

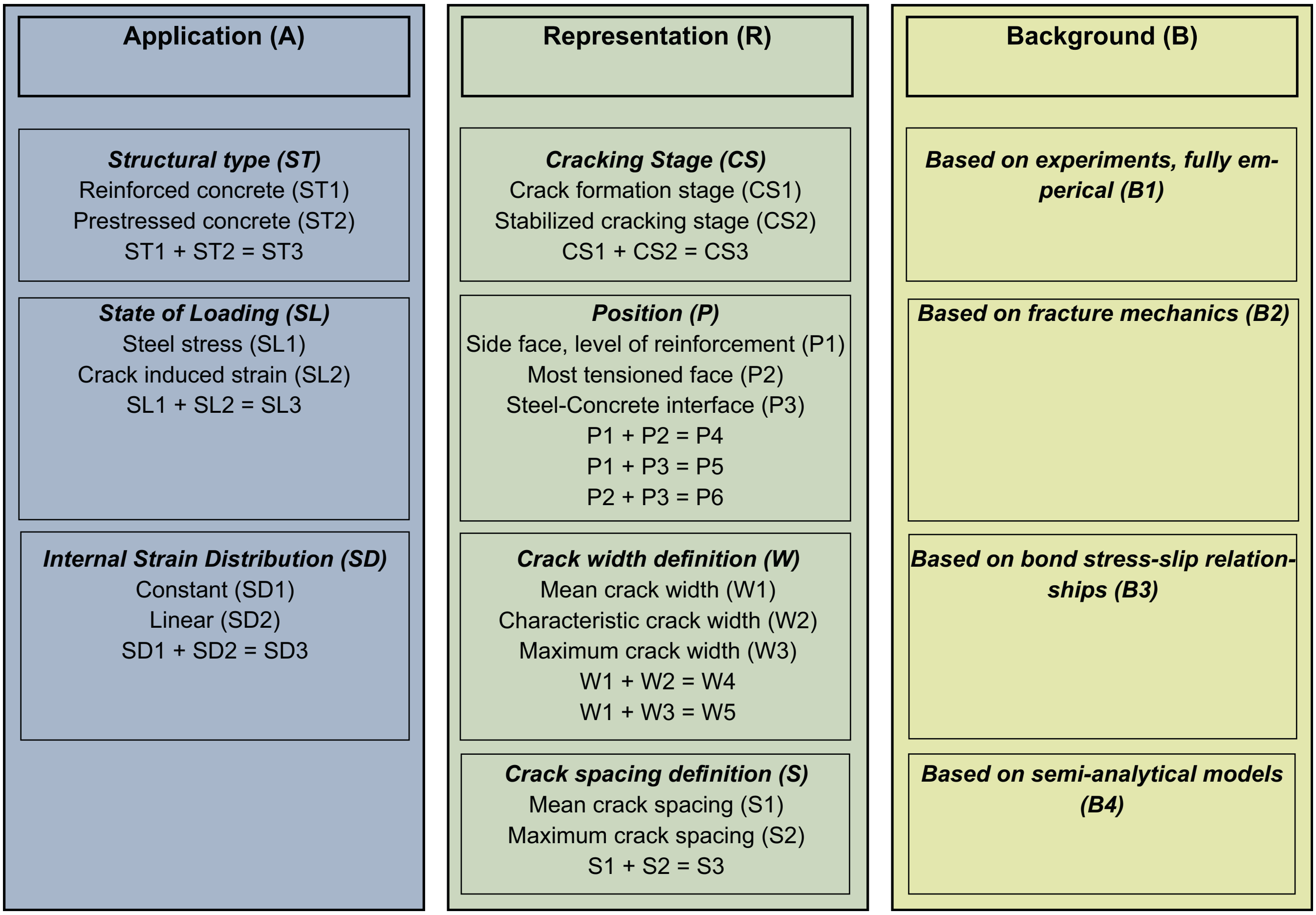


FIGURE 1 Categorization of formulas based on Application (A), Representation (R), Background (B), and their respective subcategories. The abbreviation of the categories is denoted between brackets.

for crack width and spacing, it is represented in various ways.

(SL1) First, a specific steel stress $\sigma_s$ can be used, depending on the source of the applied load. In case of an external force, the resulting steel stress is implemented in various formulas, like EN 1992-1-1,[9] *fib* MC 2010,[27] and many others. For structures that are restrained at the ends (Figure 2a), the stress in the reinforcement, just after a crack occurred and denoted as $\sigma_{\text{sr}}$, is frequently used for $\sigma_s$. This stress is obtained from the axial force $N_{\text{cr}}$ that is required for cracking and depends on the concrete's tensile strength $f_{\text{ct}}$.

(SL2) Second, the load effect can be represented by a crack-inducing strain $\varepsilon_{\text{cr}}$. The formulas can then be applied to structures subjected to an edge restraint (Figure 2b) or an internal restraint. In EN 1992-3,[29] the crack-inducing strain is defined as the restrained component of the free strain $\varepsilon_{\text{free}}$, resulting from drying or autogenous shrinkage, temperature loading, or creep.[30] The crack-inducing strain in a structural element results from an axial imposed strain or an imposed curvature. Formulas for calculation of the crack width that use a crack-inducing strain are CIRIA C766,[30] ICE/0706/012,[31] or eq. (M.3) of EN 1992-3.[29] In addition, some formulas using $\varepsilon_{\text{cr}}$ implement a strain release,[30] which is frequently assumed to be equal to half of the ultimate strain capacity of concrete in tension and allows for examining the cracking behavior after the first or subsequent cracks occur.[31,32]

(SL3) Third, some formulas require an input of both the steel stress and a crack-inducing strain. These formulas are used if crack-inducing strains are combined with external forces, called a load and deformation combination.[32] These formulas use a steel stress, calculated by the user of the formula, based on the magnitude of the external forces, and a calculated steel stress, representing the effect of crack-inducing strains. Finally, the stresses are added together, considering the sequence of the different applied loads, and used as input in the specific formula.[32] Examples of those formulas are described in NEN 6720[33] and ICE/0706/012.[31] Other literature[28,34] indicates how load and deformation combinations might be addressed

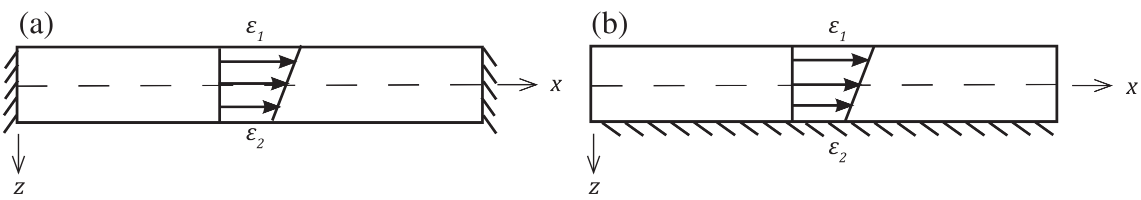


**FIGURE 2** Strain conditions in structures subjected to (a) End restraints, (b) Edge restraints. The largest and smallest imposed strains are indicated with $\varepsilon_1$ and $\varepsilon_2$, respectively.

but does not explicitly present formulas for calculating crack widths and spacings for this case.

## 3.3 | Internal strain distribution

The described loads (SL) lead to an internal strain distribution in the considered structural element, influencing the crack pattern. Constant strain over the height of the cross-section, that is, an evenly distributed strain in an element (SD1), represents an axial load, while a linear varying strain distribution (SD2) represents a bending moment. In most formulas, the strain distribution is incorporated by a reduction factor for the transfer length, which is based on the reasoning that in the case of bending, a smaller load is needed to generate a new crack with respect to the case of pure tension.[17] Therefore, no reduction is applied in the case of a constant strain. The case of constant strain is discussed by Saliger,[10] Broms and Lutz,[35] and Ouyang and Shah.[36] Unlike tension, bending leads to a curvature $\kappa$ of the cross-section. Thus, the strains vary linearly over the cross-section height, and a reduction factor of 0.5 is applied in the case of pure bending. Cracking in structures subjected to pure bending can be calculated by the formulas developed by Base et al.,[37] Oh and Kang,[20] and Gergely and Lutz.[12] For the combination of axial and bending loads (SD3), the resulting strain distribution is calculated based on the strain at the most tensioned side $\varepsilon_1$ and the least tensioned side $\varepsilon_2$. The way how those strains need to be combined to calculate the reduction factor is often prescribed in the documentation of the specific formula, like EN 1992-1-1,[9] *fib* MC 2010,[27] and Noakowski.[14,38]

# 4 | REPRESENTATION OF FORMULAS FOR CALCULATING CRACK WIDTH AND SPACING

## 4.1 | Cracking stage

The cracking stages of reinforced and prestressed concrete structures can be divided into the crack formation

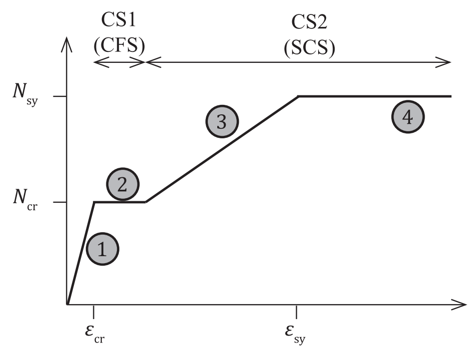


**FIGURE 3** $N$, $\varepsilon$-relation for a concrete tie. Branch (1) indicates the uncracked behavior, (2) the crack formation stage, (3) the stabilized cracking stage before yielding of the reinforcement, and (4) the stabilized cracking stage at yielding. $N_{cr}$: internal force at cracking. $\varepsilon_{cr}$: strain at cracking. $N_{sy}$: internal force at yielding. $\varepsilon_{sy}$: strain at yielding.

stage (CS1) and the stabilized cracking stage (CS2),[3] see Figure 3. Following the uncracked behavior, indicated by branch (1), the crack formation stage often applies for cracking due to restrained imposed strains and is characterized by a constant branch (2). In contrast, the stabilized cracking stage often represents cracking in the case of external forces and is represented by a linear branch (3) and eventually the horizontal branch at yielding (4).[33]

Although most formulas for crack calculation do not explicitly mention for which cracking stage they apply, frequently, their applicability can be derived from the context or the background of the formulas; the latter is further described in Section 5.

Regarding the context, the crack formation stage often occurs when imposed strains are restrained.[32] This can be observed in the corresponding formula if $\sigma_{sr}$ is used as the steel stress directly after a crack occurs, or $\varepsilon_{cr}$ as the crack-inducing strain. The related internal force is indicated with $N_{cr}$. For the stabilized cracking stage, the load effect is considered by a general value of $\sigma_s$, which is larger than $\sigma_{sr}$. In some cases, two distinct formulas for both the crack formation stage and stabilized cracking stage are provided, such as Van Breugel et al.,[32] FprEN 1992-1-1,[28] EN 1992-3,[29] and *fib* MC 2020.[39] However, frequently, only one formula is given, specified for the

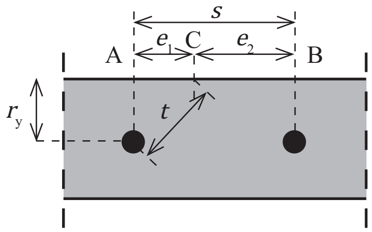


**FIGURE 4** Schematic representation of a cross-section to determine the distance $t$. $t$ is the distance from the reinforcing bar to the considered location on the concrete surface, indicated with $C$. $t$ can be calculated based on Pythagoras' theorem, $t = \sqrt{e^2 + r_y^2}$, with $e = \frac{e_1 e_2}{s}$, $r_y$ is the vertical distance from the center of the reinforcing bar to the most tensioned face and $s$ is the spacing between the reinforcing bars.

stabilized cracking stage, which can be modified to the crack formation stage by substitution of $\sigma_{sr}$ into $\sigma_s$. This is, amongst others, implemented in *fib* MC 2010[27] and NVN-ENV 1997-1-1.[40]

## 4.2 | Position of the measured crack width

Formulas for calculating crack widths evaluate the crack width at the level of the reinforcement (P1), the most tensioned face (P2), or the steel-concrete interface (P3). Model codes, like *fib* MC 1990,[41] 2010,[27] 2020,[39] and codes such as EN 1992-1-1[9] and FprEN 1992-1-1,[28] consider the crack width at the most tensioned face since the calculated crack widths are compared to crack width limits, following from the exposure class of the structure.[27]

In the case of bending, the crack width varies over the height of the crack. *fib* MC 2020[39] and FprEN 1992-1-1[28] extrapolate the crack widths from the reinforcement level to the most tensioned face, using a curvature factor $k_{1/r}$,[17,39] based on an analysis of the cracked concrete element. $k_{1/r}$ is defined as[42]:

$$k_{1/r} = \frac{h - x}{d - x} \tag{1}$$

where $h$ represents the height of the cross-section, $x$ represents the depth of the concrete compression zone, and $d$ represents the effective depth. The curvature factor only affects elements loaded in bending; for elements loaded in pure tension, the factor equals one. The inverse of Equation (1) is sometimes implemented in empirical-based formulas to convert the description of the calculated crack width from the most tensioned face to the side face at the level of the reinforcement.[22,42]

Broms and Lutz[35] and Frosch[43] allow calculating the crack width at an arbitrary place on the most tensioned face. They implement the distance $t$ from the center of the reinforcing bar to the location on the tension face where the crack width is calculated (Figure 4).

In addition, BS 8007[44] and BS 8110[45] allow for the calculation of crack width at an arbitrary place on the concrete surface, though they define $t$ as the distance between the steel–concrete interface and the most tensioned face.

Finally, crack widths are evaluated at the steel–concrete interface, following the analytical derivation from the bond stress-slip relationship.

## 4.3 | Crack width definition

Due to the stochastic material properties of concrete,[22] crack width and spacing vary in a structural element. Hence, formulas for calculating the crack width and spacing frequently express a representative value of the calculated crack width, which might deviate from actual crack widths observed on a structure.[39] Most used are the calculated mean crack width $w_m$ and the calculated characteristic crack width $w_k$.

The calculated mean crack width (W1) is addressed by the formulas of Saliger,[10,11] Broms,[46] Ferry-Borges,[47] and *fib* MC 2020.[39] However, comparing calculated values with values obtained from crack width measurements is not straightforward. For instance, there is no consensus in the literature on the definition of $w_m$ based on measured crack widths in a structure. *fib* MC 2020[39] defines $w_m$ as the mean value of the measurements of at least 10 individual points, which can be individual cracks, denoted to the closest 0.05 mm, having the same restraint at randomly selected locations. Other codes do not specifically state which cracks should be taken into account and define the average of all individually measured crack widths.

The calculated characteristic crack width (W2) is regularly defined as the 95% quantile for the statistical variation when assuming a lognormal distribution.[48] This can be interpreted as the crack width at the 95% fractile of the crack widths, which can be expected in a reinforced or prestressed member.[49] Beeby defined $w_k$ using an 80% quantile, reasoning that the reduced quantile was appropriate since the 5% probability of exceedance is too small to pose a risk of corrosion or seriously impair appearance.[21,50] This was later implemented in CP 110 as part of the British Standards.[21] The calculated design crack width $w_d$ is frequently considered equal to the calculated characteristic crack width.[21] For formulas allowing to calculate both the mean and characteristic crack width, a conversion between the two is usually obtained with a factor $\beta_w = w_k / w_m$.[28,32]

The formulas developed by Gergely and Lutz,[12] Oh and Kang,[20] and König and Tue,[51] use the maximum crack width $w_{\max}$ (W3). The maximum calculated crack width is regularly considered to be equal to the calculated characteristic crack width.[21]

### 4.4 | Crack spacing definition

Formulas describing crack patterns might provide a crack spacing besides the crack width. Crack spacing is often relevant for the stabilized cracking stage only since, in that stage, the number of developed cracks and, thus, the crack spacing is constant. Nonetheless, formulas applicable for end restraints like EN 1992-3,[29] CIRIA C766[30] and ICE/0706/012[31] present a crack spacing, even though these formulas frequently describe the crack formation stage.

Crack spacing in formulas is defined as the mean crack spacing $s_{r,m}$ (S1) and the maximum crack spacing $s_{r,\max}$ (S2). The relation between mean and maximum crack spacing is reported in some cases by the factor $\beta_w$, however, based on the ratio $s_{r,\max}/s_{r,m}$ instead of $w_k/w_m$.[26,39,40] These ratios are not equivalent.[52]

## 5 | BACKGROUND OF FORMULAS FOR CALCULATING CRACK WIDTH AND SPACING

### 5.1 | Based on experiments, fully empirical (B1)

Empirical formulas, fully based on experiments, are derived by fitting formulas with unknown regression constants. The regression constants are obtained by comparing the formulas with the measured crack widths.

One of the first empirical formulas for reinforced concrete structures was developed by Kaar and Mattock in 1963,[53] investigating cracking in high-strength reinforcing bars for rectangular and T-beams loaded in bending. Flexural cracking was further investigated in 1965 by Kaar and Hognestad.[42] Both authors proposed a crack width formula:

$$w = A\sqrt[4]{\frac{A_{c,\mathrm{eff}}}{n}}\sigma_s, \tag{2}$$

where $A$ is a regression constant, $n$ is the number of reinforcement bars, and $A_{c,\mathrm{eff}}/n$ is the effective area of the concrete $A_{c,\mathrm{eff}}$, related to a single reinforcing bar.

In 1968, Gergely and Lutz[12] used a largely similar formula as Equation (2) but used a cube root[12] and incorporated the distance from the tension face to the center of the reinforcing bar $r_y$, resulting in:

$$w = A\sqrt[3]{\frac{A_{c,\mathrm{eff}}}{n}r_y}\sigma_s. \tag{3}$$

Multiple authors and codes used a similar form for formulas for crack width calculation.[34,54–57]

Besides reinforced concrete structures, fully experimental-based formulas were also developed for prestressed structures based on the fictitious or nominal tensile stress and the net stress methods.[58]

In 1967, Abeles introduced the fictitious stress method.[59] In this method, the fictitious stress is calculated based on the uncracked properties of the cross-section. Similar to Equations (2) and (3), the formulas use regression constants and are then fitted with the measured crack widths obtained with laboratory tests. Finally, the crack widths can be calculated based on formulas considering the calculated fictitious tensile stress; this stress should be lower than an allowable fictitious tensile stress. Raju et al.[60] and Meier and Gergely[55] further implemented fictitious stress methods.

Contrary to the fictitious stress method, the net stress method calculates the steel stress based on the cracked concrete properties. The steel stress in the formulas for prestressed concrete structures is considered a change in the steel stress in the prestressing element, evaluated from the load level at the onset of decompression at the beam's tensile face to the load level at which the crack width is considered.[60] Formulas incorporating the net stress method are presented by Bennet and Veerasubramanian,[61] Nawy and Potyondy,[62] Martino and Nilson,[63] and Suri and Dilger.[54]

Finally, recently, empirically derived formulas for formulas to calculate the crack width and spacing might be derived by artificial neural networks (ANNs). The ANNs are used as a regression technique on collected experimental data to derive new formulas for calculating the crack width and spacing.[64,65]

### 5.2 | Based on fracture mechanics (B2)

Formulas can also be derived based on fracture mechanics.[20–22,36,66] Oh and Kang[20] derived the formula for crack width and spacing for elements loaded in bending, using fracture mechanics based on the energy and strength criterion. The results of the numerical experiments then determined the regression constants in the formula, making it a semi-empirical approach.

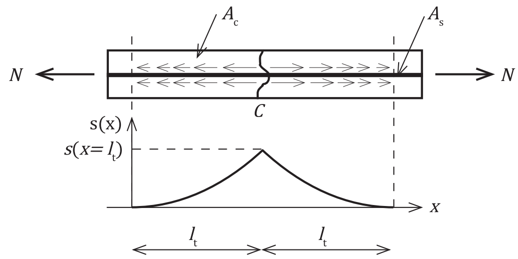


FIGURE 5 Slip distribution along a cracked concrete element. The location of the crack is indicated with point *C*. The figure is based on Lapi et al.[21]

Ouyang and Shah[36,66] derived a formula for elements loaded in pure tension. A fracture resistance curve was used to represent the fracture process zone, and the rates of change of the strain energy from unloading, sliding and debonding were calculated. Then, an energy balancing equation was implemented in the models, and the crack width and spacing were predicted with the resulting formulas.

## 5.3 | Based on bond stress–slip relationships (B3)

Formulas based on bond stress–slip relationships or analytical models are derived using the differential equation for bond slip:

$$\frac{d^2 s_x}{dx^2} - \frac{\tau_b(s_x, x) 4(1+\alpha_e \rho)}{E_s Ø} = 0, \quad (4)$$

where $s_x$ denotes the slip of the reinforcement concerning the concrete, $\rho$ is the reinforcement ratio, $\tau_b(s_x, x)$ is the bond stress as a function of the slip and the position $x$, measured from the start of the transfer length as indicated in Figure 5. Equation (4) can be solved for the slip from which the crack width can be calculated.[67] Due to the assumed symmetry of the slip distribution, the crack width is considered as twice the slip distance, evaluated at the transfer length $l_t$.

This can be presented by the following formula:

$$w = 2 s_x (x = l_t). \quad (5)$$

Equation (4) is often solved for the crack formation stage, and the crack width can then be calculated with Equation (5). Besides the crack formation stage, Equation (4) can theoretically assess the stabilized cracking stage too. However, due to inhomogeneous boundary conditions, the solution procedure and the resulting expression are complex for practical implementation when applying a linear or non-linear bond stress–slip relation.[68,69] Nonetheless, Noakowski used some simplifications in the derivation of the resulting formula on a non-linear bond-stress slip relation, making it applicable in the stabilized cracking stage.[14,38] Theoretically, the solution presented in Equation (5) is valid only in the case of pure tension and presents the crack width at the steel-concrete interface. An extension to bending and a combination of bending and tension was incorporated into the formula developed by Noakowski[14,38] in 1985 and formed the basis of other formulas derived from Equation (4), like.[67] Another adaptation to Equation (5) is the addition of a factor to calculate the crack width at the most tensioned face instead of the steel–concrete interface.[70]

## 5.4 | Based on semi-analytical models (B4)

Formulas based on semi-analytical models are partially based on bond stress–slip relations and calibrated with experimental crack width and spacing measurements. In the formulas, a simple relation between the crack width and spacing is presented as:

$$w = s_r(\varepsilon_{sm} - \varepsilon_{cm}), \quad (6)$$

where $\varepsilon_{sm} - \varepsilon_{cm}$ denotes the difference between the average strain in the reinforcement and the concrete within the transfer length and for cracking due to external forces or end-restraint cracking, often described as:

$$\varepsilon_{sm} - \varepsilon_{cm} = \frac{\sigma_s - \beta_{TS}\sigma_{sr}}{E_s}, \quad (7)$$

where $\beta_{TS}$ is a factor considering the average strain of the steel within the transfer length.[27] In the case of edge restraint cracking, Equation (7) is often written as:

$$\varepsilon_{sm} - \varepsilon_{cm} = R_{ax}\varepsilon_{free}, \quad (8)$$

and used to calculate the strain difference and, subsequently, the crack width, using Equation (6). When formulas like Equation (7) are used, the steel stress $\sigma_{sr}$ is frequently implemented. The development of formulas based on semi-analytical models started in 1936 with Saliger.[10] Saliger proposed an equivalent formula for crack spacing based on mechanical considerations[52]:

$$s_r \sim l_t = \frac{f_{ct,eff} Ø}{4 \rho_{eff} \tau_{bm}}. \quad (9)$$

In Equation (9), where $\rho_{eff}$ is the effective reinforcement ratio, a mean bond stress $\tau_{bm}$ is considered,

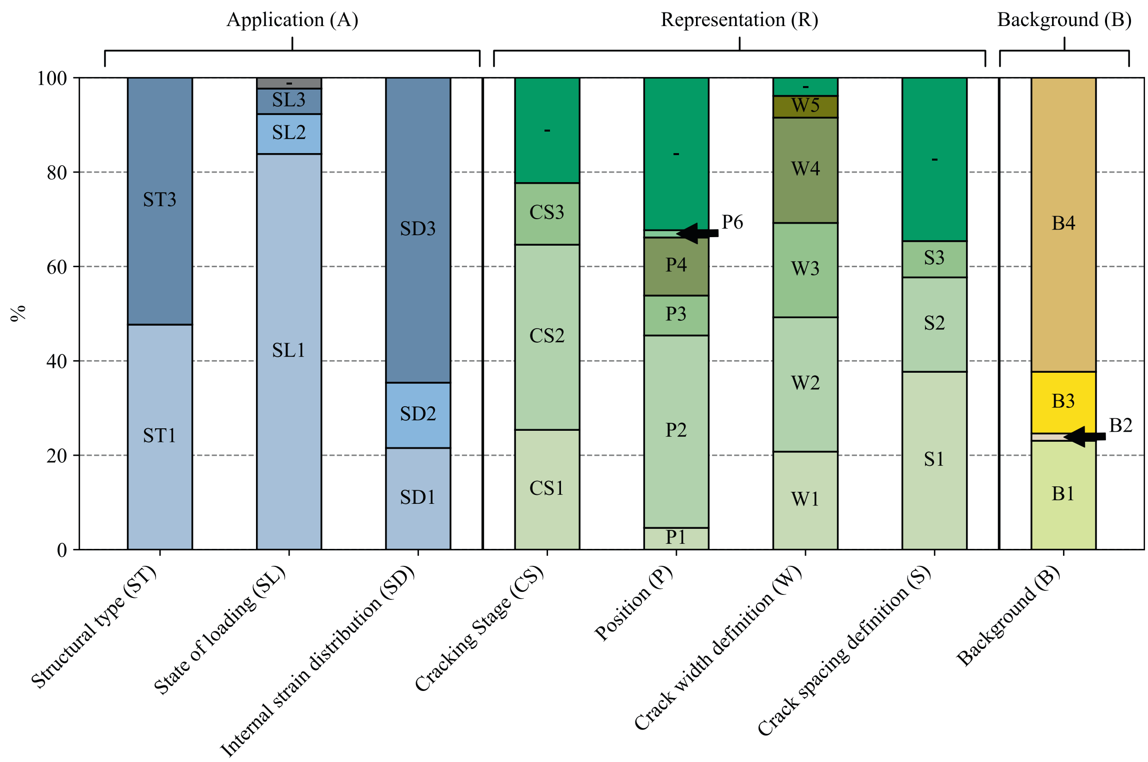


**FIGURE 6** Graphical quantification of Appendix A for the three categories. Each color represents a relevant characteristic of each subcategory, and each column indicates a subcategory, according to Figure 1. The denotation "-" indicates that the corresponding information is not provided or stated in the literature.

simplifying the derivation of the transfer length. Thus, it is assumed that the crack spacing is proportional to the transfer length. In the crack formation stage, the crack width is calculated according to the semi-analytical approach according to Equations (6), (7), and (9), coincides with the bond stress–slip approach. The approach of Saliger was further implemented by the German national annex of EN 1992-1-1,[71] *fib* MC 1990[41] and König and Tue.[51]

In 1965, Broms introduced the concrete cover or no-slip approach.[46] Instead of assuming that the transfer length is entirely dependent on the bond slip, it is assumed that bond failure does not occur and that the distributed zone is proportional to the concrete cover. Hence, the crack spacing is presented by:

$$s_r = k_3 c, \tag{10}$$

where $k_3$ determines the effect of the concrete cover $c$ on the crack spacing. The no-slip approach was further implemented by Broms and Lutz.[35] Base et al.[37] and Beeby[50] proposed a formula based on the elastic analysis of the concrete, for example, incorporated in BS 8007[44] and BS 8110.[45]

In 1966, Ferry-Borges[47] combined the contributions of the concrete cover and the bond stress–slip approach and developed a new formula equivalent to:

$$s_r = k_3 c + k_1 k_2 k_4 \frac{f_{\mathrm{ct,eff}} \varnothing}{\rho_{\mathrm{eff}}\, \tau_{\mathrm{bm}}}. \tag{11}$$

An equivalent form of Equation (11) is implemented in modern (model) codes, such as EN 1992-1-1,[9] *fib* MC 2010,[27] and MC 2020.[39]

## 6 | CATEGORIZATION AND OBSERVATIONS

A categorization of 130 formulas for crack width and crack spacing was performed based on the framework presented in the previous chapters. The results for each formula are presented in Appendix A, and a graphical summary is provided in Figure 6. Based on the complete overview, some observations can be made:

- The formulas do not address ST2. The developed formulas for prestressed concrete structures include the possibility of adding reinforcing steel or were calibrated on experiments on concrete structures having reinforcing and prestressing steel.
- Seven formulas address crack widths in concrete structures subjected to loading and deformation combinations, related to SL3. More literature addresses the importance

of considering these combinations but does not give formulas. Three formulas do not include a loading state.

- The formulas based on empirical models often do not provide a crack spacing but exclusively a crack width. The formulas only allow a steel stress as input and not a crack-inducing strain. Furthermore, little attention is paid to the range of applicability of their input parameters in the literature that describes these empirical formulas.
- Formulas based on fracture mechanics do not address prestressed concrete structures.
- Formulas based on bond stress–slip relationships frequently describe the crack width at the steel–concrete interface. Formulas using fracture mechanics present the crack width at the concrete side face at the reinforcement level, at the steel–concrete interface or the most tensioned face, while formulas based on bond stress–slip relationships frequently use the stress as the state of loading and often describe the crack widths in the crack formation stage at the steel–concrete interface.
- Formulas for calculation of the crack width and spacing only apply to a portion of the suggested categories, thus to a relatively small application area. However, the overview showed that semi-analytical-based formulas do address most categories.

## 7 | CONCLUSIONS AND FUTURE WORK

This paper presents a categorization framework for formulas to calculate the crack width and spacing in concrete elements. The categorization was based on three main categories: application, representation, and background and was further divided into subcategories. In total, 130 formulas for crack calculation have been categorized. The framework can be applied to include new and other existing formulas. The categorized formulas in this paper might also be extended to other application areas. For example, for various formulas that pay attention to the stabilized cracking stage, the crack formation stage can be addressed by implementing the steel stress at the onset of cracking. Also, the extension from the crack width calculation at the reinforcement level to the most tensioned face can be incorporated. The validity of these additional application areas and the potential of these existing formulas is a topic for further research.

The categorization enables researchers to investigate the most accurate formulas inside a category and compare the accuracy between the presented categories. Finally, this paper can guide engineers and designers in selecting an appropriate formula for their crack width calculation.

## LIST OF SYMBOLS

| | |
|---|---|
| $A_{c.eff}$ | Effective concrete area (mm$^2$) |
| $c$ | Concrete cover (mm) |
| $d$ | Effective depth (mm) |
| $E_s$ | Modulus of elasticity of reinforcing steel (MPa) |
| $E_c$ | Modulus of elasticity of concrete (MPa) |
| $f_{ct}$ | Axial tensile strength of concrete (MPa) |
| $f_{ct,eff}$ | Effective axial tensile strength of concrete (MPa) |
| $f_{ctm}$ | Mean value of axial tensile strength of concrete (MPa) |
| $k$ | Coefficient (−) |
| $l_t$ | Transfer length (mm) |
| $n$ | Number of reinforcing bars in the effective tensile zone (−) |
| $N_{cr}$ | Axial cracking force (N) |
| $N_y$ | Axial yielding force of reinforcement (N) |
| $r_y$ | Cover to the center of the bar in the y-direction (mm) |
| $R_{ax}$ | Restrained factor (−) |
| $s$ | Spacing of reinforcing bars (mm) |
| $s_x$ | Slip (mm) |
| $s_r$ | Distance between cracks (mm) |
| $s_{r,m}$ | Mean spacing between cracks (mm) |
| $s_{r,max}$ | Maximum spacing between cracks (mm) |
| $x$ | Depth of concrete compression zone (mm) |
| $w$ | Crack width (mm) |
| $w_m$ | Mean crack width (mm) |
| $w_k$ | Characteristic crack width (mm) |
| $w_{max}$ | Maximum crack width (mm) |
| $w_{lim}$ | Nominal limit value of crack width (mm) |
| $\alpha_e$ | Modular ratio ($=E_s/E_c$) (−) |
| $\beta$ | Ratio ($=w_k/w_m$) (−) |
| $\beta_{TS}$ | Factor considering $\varepsilon_{sm}$ within $l_t$ (−) |
| $\varepsilon_{cm}$ | Average concrete strain within transfer length (−) |
| $\varepsilon_{cr}$ | Crack-inducing strain (−) |
| $\varepsilon_{free}$ | Free strain (−) |
| $\varepsilon_{sm}$ | Average steel strain within transfer length (−) |
| $\varepsilon_r$ | Restraint strain (−) |
| $\sigma_s$ | Steel stress (MPa) |
| $\sigma_{sr}$ | Steel stress in a crack directly after cracking (MPa) |
| $\rho$ | Reinforcement ratio (−) |
| $\rho_{eff}$ | Effective reinforcement ratio (−) |
| $\tau_b$ | Bond stress (MPa) |
| $\tau_{bm}$ | Mean bond stress (MPa) |
| Ø | Nominal diameter of a reinforcing bar (mm) |

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## CONFLICT OF INTEREST STATEMENT

Sonja Fennis and Marco Roosen are co-authors of this paper and are affiliated with Rijkswaterstaat GPO, giving financial support to this research. They declare that their affiliations do not affect their input while writing this paper.

## DATA AVAILABILITY STATEMENT

Data sharing not applicable to this article as no datasets were generated or analysed during the current study.

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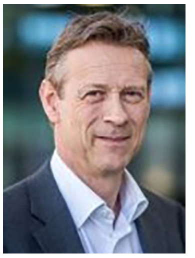

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## APPENDIX A: CATEGORIZATION OF FORMULAS FOR CRACK WIDTH AND SPACING

This appendix presents a categorization of crack width and spacing formulas based on the proposed categories in Figure 1. Please note:

- "-" means that this information is not provided or stated in the corresponding literature or the category cannot be derived or identified.
- If no information on prestressing steel is provided in the literature or the corresponding formula, then it is categorized as only applicable to reinforced concrete structures.
- Prestressed structures will generally lead to a combination of axial and bending loads. Therefore, the internal strain distribution (SD) for formulas describing cracks in prestressed structures is always categorized as "3."

| Info | | | | Categorization | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | A | | | R | | | | |
| ID | Publication | Ref. | Year | ST | SL | SD | CS | P | W | S | B |
| 1 | NEN-EN 1992-1-1, eqs. (7.8, 7.9) | 9 | 2011 | 3 | 1 | 3 | 2 | 2 | 2 | 2 | 4 |
| 2 | FprEN 1992-1-1, eqs. (9.8, 9.11, 9.15) | 28 | 2023 | 3 | 1 | 3 | 3 | 4 | 4 | 1 | 4 |
| | FprEN 1992-1-1, eqs. (9.8, 9.13, 9.15) | | | 3 | 2 | 3 | 1 | 4 | 4 | 1 | 4 |
| 3 | NEN-EN 1992-3, eq. (M.1) | 29 | 2006 | 3 | 1 | 3 | 1 | 2 | 2 | 2 | 4 |
| | NEN-EN 1992-3, eq. (M.3) | | | 3 | 2 | 3 | 1 | 2 | 2 | 2 | 4 |
| 4 | DIN-EN 1992-1-1, eqs. (7.8, 7.9) | 71 | 2013 | 3 | 1 | 3 | 2 | 2 | 2 | 2 | 4 |
| 5 | *fib* MC 1990, eqs. (7.4-2, 7.4-3) | 41 | 1990 | 3 | 1 | 3 | 1 | 2 | 2 | 2 | 4 |
| | *fib* MC 1990, eqs. (7.4-3, 7.6-3) | | | 3 | 1 | 3 | 2 | 2 | 2 | 2 | 4 |
| | *fib* MC 1990, §7.4.3.1.2 | | | 3 | 3 | 3 | 3 | 2 | 2 | 2 | 4 |
| 6 | *fib* MC 2010, eqs. (7.6-2, 7.6-3) | 27 | 2013 | 3 | 1 | 3 | 1 | 2 | 2 | 2 | 4 |
| | *fib* MC 2010, eqs. (7.6-5, 7.6-3) | | | 3 | 1 | 3 | 2 | 2 | 2 | 2 | 4 |
| 7 | *fib* MC 2020 (final draft), eqs. (30.5-2, 30.5-3, 30.5-5, 30.5-13) | 39 | 2023 | 3 | 1 | 3 | 1 | 4 | 4 | 3 | 4 |
| | *fib* MC 2020 (final draft), eqs. (30.5-3, 30.5-5, 30.5-12) | | | 3 | 2 | 3 | 3 | 4 | 4 | 3 | 4 |
| 8 | ACI 224-01, eq. (4–21) | 72 | 2001 | 1 | 1 | 1 | - | 2 | 3 | - | 1 |
| 9 | VB 1974, §508.1 + §508.2 (CS1) | 73 | 1974 | 3 | 2 | 3 | 1 | 2 | 3 | 1 | 4 |
| | VB 1974, §508.1 + §508.2 (CS2) | | | 3 | 1 | 3 | 2 | 2 | 3 | 1 | 4 |
| 10 | NEN 3880, §508.1 + §508.2 (CS1) | 74 | 1984 | 3 | 2 | 3 | 1 | 2 | 3 | 1 | 4 |
| | NEN 3880, §508.1 + §508.2 (CS2) | | | 3 | 1 | 3 | 2 | 2 | 3 | 1 | 4 |
| 11 | NVN-ENV 1997, eqs. (4.80–4.82) | 40 | 1997 | 3 | 1 | 3 | 3 | 2 | 2 | 1 | 4 |
| | NVN-ENV §4.4.2.4(6) | | | 3 | 3 | 3 | 3 | 2 | 2 | 1 | 4 |
| 12 | Noakowski, eq. (17) | 38 | 1985 | 1 | 1 | 3 | 1 | - | 2 | 1 | 3 |
| | Noakowski, eq. (20) | | | 1 | 1 | 3 | 2 | - | 2 | 1 | 3 |
| 13 | Schießl and Wölfel, eq. (10) | 75 | 1986 | 1 | 1 | 3 | 1 | - | 2 | 1 | 4 |
| | Schießl and Wölfel, eq. (2) | | | 1 | 1 | 3 | 2 | - | 2 | 1 | 4 |
| | Schießl and Wölfel, §3.4 | | | 1 | 3 | 1 | 3 | - | 2 | 1 | 4 |
| 14 | Broms and Lutz, eqs. (1–5) | 35 | 1965 | 1 | 1 | 1 | 2 | 2 | 4 | 3 | 4 |
| 15 | Broms, eqs. (1–4) | 46 | 1965 | 1 | 1 | 1 | 2 | 2 | 4 | 3 | 4 |
| 16 | Saliger, eqs. (21, 23) | 11 | 1950 | 1 | 1 | 3 | 2 | - | 4 | 1 | 4 |
| 17 | BS 8110, eq. (12) | 45 | 1997 | 3 | 1 | 3 | 2 | 2 | 2 | - | 4 |
| | BS 8110, eq. (14) | | | 1 | 2 | 3 | 1 | 2 | 2 | - | 4 |

| Info | | | | Categorization | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | A | | | R | | | | B |
| ID | Publication | Ref. | Year | ST | SL | SD | CS | P | W | S | B |
| 18 | Van Breugel et al., eq. (4.19) | 32 | 2013 | 1 | 1 | 1 | 1 | - | 4 | - | 3 |
| | Van Breugel et al., eqs. (4.22, 4.23) | | | 1 | 1 | 1 | 2 | - | 4 | 1 | 4 |
| 19 | Menn, §1 | 76 | 1986 | 1 | 1 | 3 | 3 | - | 1 | 1 | 4 |
| 20 | Leonhardt, eqs. (3, 6) | 77 | 1977 | 1 | 1 | 3 | 2 | 2 | 4 | 1 | 4 |
| 21 | Sygula, §2 | 78 | 1981 | 1 | 1 | 3 | - | - | 4 | - | 1 |
| 22 | Wicke, eqs. (18, 19) | 79 | 1991 | 3 | 1 | 3 | 3 | 2 | 3 | 2 | 4 |
| 23 | König and Tue, eq. (21) | 51 | 1996 | 3 | 1 | 3 | 1 | - | 1 | - | 4 |
| | König and Tue, eq. (26) | | | 3 | 1 | 3 | 2 | - | 3 | 2 | 4 |
| | König and Tue, eq. (59) | | | 3 | 1 | 3 | 1 | - | 1 | - | 4 |
| | König and Tue, eq. (62) | | | 3 | 1 | 3 | 2 | - | 3 | 2 | 4 |
| 24 | ECP 203–2007, eq. (4–66) | 80 | 2007 | 3 | 1 | 3 | 3 | 2 | 4 | 1 | 4 |
| 25 | CROW-CUR, §9 | 15 | 2020 | 3 | 1 | 3 | 1 | 2 | 2 | 2 | 4 |
| 26 | Ciria C660, eq. (3.15) | 81 | 2007 | 3 | 2 | 3 | 1 | 2 | 2 | 2 | 4 |
| | Ciria C660, eq. (3.16) | | | 3 | 1 | 3 | 1 | 2 | 2 | 2 | 4 |
| 27 | Empelmann and Krakowski, eq. (34) | 82 | 2015 | 1 | 1 | 3 | 1 | 2 | 4 | 3 | 4 |
| | Empelmann and Krakowski, eq. (35) | | | 1 | 1 | 3 | 2 | 2 | 4 | 3 | 4 |
| 28 | Ciria C766, eqs. (3.11, 3.23) | 30 | 2017 | 3 | 1 | 3 | 1 | 2 | 2 | 2 | 4 |
| | Ciria C766, eqs. (3.10, 2.23) | | | 3 | 2 | 3 | 1 | 2 | 2 | 2 | 4 |
| 29 | AS 3600, eq. (8.6.2.3(1)) | 83 | 2018 | 3 | 1 | 3 | 2 | 2 | 2 | 2 | 4 |
| 30 | CEB-*fib* 1978, eqs. (15.1–15.5) | 84 | 1978 | 3 | 1 | 3 | 3 | 2 | 4 | 1 | 4 |
| 31 | ACI 224.2-R, eq. (3.6) | 57 | 1997 | 1 | 1 | 1 | 2 | 2 | 3 | - | 1 |
| | ACI 224.2-R, eq. (3.7) | | | 1 | 1 | 2 | 2 | 4 | 3 | - | 1 |
| 32 | CUR 85, eq. (9.5) | 85 | 1978 | 1 | 1 | 1 | 1 | - | 1 | 1 | 4 |
| | CUR 85, eq. (9.7) | | | 1 | 1 | 1 | 2 | - | 1 | 1 | 4 |
| 33 | NS 3473, §A.15.6.2.1 | 86 | 2003 | 3 | 1 | 3 | 2 | 2 | 2 | 3 | 4 |
| | NS 3473, §A.15.6.2.2 | | | 3 | 1 | 3 | 1 | 2 | 2 | 1 | 4 |
| 34 | Rizkalla and Hwang, eqs. (9–20) | 26 | 1984 | 1 | 1 | 1 | 3 | - | 5 | 1 | 4 |
| 35 | Ferry-Borges, eqs. (1, 4) | 47 | 1966 | 1 | 1 | 2 | 2 | - | 5 | 1 | 4 |
| 36 | Janovic and Kupfer, eq. (10) | 87 | 1982 | 3 | 1 | 3 | 2 | - | 1 | 1 | 4 |
| 37 | Saliger, §B-b | 10 | 1936 | 1 | 1 | 1 | 2 | - | 1 | 1 | 4 |
| 38 | JSCE, eq. (7.4.4) | 88 | 2007 | 3 | 1 | 3 | 2 | 2 | 2 | 2 | 4 |
| 39 | Nawy, eqs. (4, 8) | 89 | 1985 | 3 | 1 | 3 | - | 4 | 3 | 1 | 1 |
| | Nawy, eqs. (4, 9) | | | 3 | 1 | 3 | - | 4 | 3 | 1 | 1 |
| 40 | Scholz, eq. (7) | 18 | 1991 | 3 | 1 | 3 | - | 2 | - | 1 | 1 |
| 41 | Fëhling and König, eqs. (9, 9a) | 90 | 1988 | 1 | 1 | 3 | 1 | - | 1 | - | 4 |
| | Fëhling and König, eqs. (11, 12) | | | 1 | 1 | 3 | 2 | - | 4 | 3 | 4 |
| | Fëhling and König, §2.6.3 | | | 1 | 3 | 1 | 3 | - | 4 | 3 | 4 |
| 42 | Edwards and Picard, eq. (18) | 91 | 1972 | 3 | 1 | 3 | 1 | 3 | 1 | - | 3 |
| 43 | Krips, eq. (II.13) | 92 | 1985 | 3 | 1 | 1 | 1 | 3 | 1 | - | 3 |
| | Krips, eq. (III.9) | | | 3 | 1 | 1 | 1 | 3 | 1 | - | 3 |
| | Krips, eqs. (IV.15, IV16) | | | 3 | 1 | 1 | 2 | 3 | 1 | - | 3 |
| | Krips, eqs. (V.1, V.6) | | | 3 | 1 | 2 | 3 | - | 1 | 1 | 3 |

(Continues)

| Info | | | | Categorization | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | A | | | R | | | | B |
| ID | Publication | Ref. | Year | ST | SL | SD | CS | P | W | S | B |
| 44 | Yang and Chen, eq. (26) | 70 | 1988 | 1 | 1 | 1 | 1 | 6 | 4 | - | 3 |
| 45 | Balázs, eq. (20) | 67 | 1993 | 1 | 1 | 3 | 1 | 3 | 1 | - | 3 |
| 46 | Bernardi et al., eqs. (12, 13) | 93 | 1999 | 1 | 1 | 1 | 2 | 2 | - | 1 | 4 |
| 47 | Oh and Kang, eqs. (5, 9) | 20 | 1987 | 1 | 1 | 2 | 2 | 4 | 3 | 1 | 2 |
| 48 | Ouyang and Shah, eqs. (27, 30) | 36 | 1994 | 1 | 1 | 1 | - | 3 | 1 | 1 | 2 |
| 49 | Braam, eqs. (26, 27) | 94 | 1990 | 1 | 1 | 3 | 3 | 3 | 1 | 1 | 3 |
| 50 | Brice, §4 + §5 | 13 | 1964 | 3 | 1 | 3 | 2 | - | 1 | 1 | 4 |
| 51 | Dawood and Marzouk, eq. (18) | 19 | 2010 | 1 | 1 | 1 | 1 | 3 | - | 2 | 3 |
| | Dawood and Marzouk, eq. (19) | | | 1 | 1 | 1 | 2 | 3 | - | 1 | 3 |
| 52 | Kaar and Hognestad, eq. (7) | 42 | 1965 | 3 | 1 | 3 | - | 4 | 3 | - | 1 |
| 53 | Gergely and Lutz, eqs. (19, 20) | 12 | 1968 | 1 | 1 | 2 | - | 4 | 3 | - | 1 |
| 54 | ACI 224.2R-86, eq. (3.6) | 34 | 1986 | 1 | 1 | 1 | - | - | 3 | - | 1 |
| 55 | Suri and Dilger, eq. (3) | 54 | 1986 | 3 | 1 | 3 | - | - | 4 | - | 1 |
| 56 | Rao and Dilger, eq. (2) | 95 | 1992 | 3 | 1 | 3 | - | - | 4 | - | 1 |
| 57 | Ouzaa and Benmansour, eq. (18) | 96 | 2014 | 1 | 2 | 1 | 1 | - | 1 | - | 4 |
| | Ouzaa and Benmansour, eq. (24) | | | 1 | 2 | 1 | 2 | - | 1 | 1 | 4 |
| 58 | Frosch, eqs. (5, 6) | 43 | 1999 | 1 | 1 | 2 | - | 4 | 4 | 3 | 4 |
| 59 | Kaar and Mattock, eq. (11) | 53 | 1963 | 1 | 1 | 2 | - | 1 | 5 | - | 1 |
| 60 | Base et al., §General observations | 37 | 1966 | 1 | 1 | 2 | - | 1 | 5 | - | 4 |
| 61 | Venkateswarlu and Gesund, eqs. (14, 18, 22) | 97 | 1972 | 1 | 1 | 2 | 1 | 6 | 3 | 2 | 3 |
| 62 | Martin et al., eq. (19) | 98 | 1980 | 1 | 1 | 3 | 2 | 1 | 4 | 1 | 4 |
| 63 | Windisch, eqs. (1, 6) | 99 | 2017 | 3 | 1 | 3 | 2 | 2 | 2 | 2 | 4 |
| 64 | NEN 6720, §8.7 | 33 | 1995 | 3 | 3 | 3 | 3 | 2 | 2 | - | 3 |
| 65 | Clark, eq. (2) | 100 | 1956 | 1 | 1 | 2 | - | 2 | 4 | - | 1 |
| 66 | Reignard et al., §CLLM + eq. (77) | 101 | 2019 | 1 | 1 | 1 | 1 | 3 | 1 | - | 3 |
| | Reignard et al., §CHLM + eq. (77) | | | 1 | 1 | 1 | 2 | 3 | 1 | 1 | 3 |
| 67 | Pérez Caldentey et al., eqs. (10–12) | 17 | 2020 | 3 | 1 | 1 | 2 | 2 | 4 | 2 | 4 |
| 68 | ICE/0706/012, eqs. (21, 23, 25) | 31 | 2010 | 3 | 3 | 3 | 3 | 2 | 2 | 2 | 4 |
| 69 | Bennet and Veersubramanian, eq. (4) | 61 | 1972 | 3 | 1 | 3 | - | - | 3 | - | 1 |
| 70 | Meier and Gergely, eqs. (1, 2) | 55 | 1981 | 3 | 1 | 3 | - | - | 3 | - | 1 |
| 71 | Raju et al., §Discussion | 60 | 1973 | 3 | 1 | 3 | - | - | 3 | - | 1 |
| 72 | Martino and Nilson, eq. (5.4) | 63 | 1979 | 3 | 1 | 3 | - | - | 2 | - | 1 |
| 73 | Nawy and Potyondi, eq. (8) | 62 | 1971 | 3 | 1 | 3 | 2 | - | 3 | - | 1 |
| 74 | Giordano and Mancini, eq. (3, 27) | 102 | 2018 | 1 | 1 | 1 | 2 | 2 | 1 | 1 | 4 |
| 75 | Debernardi and Taliano, eqs. (18, 20) | 103 | 2016 | 3 | 1 | 3 | 2 | 2 | 2 | 2 | 4 |
| 76 | Rehm et al., eq. (7) | 104 | 1976 | 1 | 1 | 3 | 2 | - | 4 | 1 | 4 |
| 77 | Chowdhury, eqs. (5–7) | 105 | 2001 | 3 | 1 | 3 | 2 | 4 | 5 | 1 | 4 |
| 78 | NZS 3101, eq. (2.7) | 106 | 2006 | 1 | 1 | 3 | 2 | 4 | 2 | - | 4 |
| 79 | CSA A23.3-04, eq. (10–6) | 107 | 2004 | 1 | 1 | 2 | 2 | 4 | 2 | - | 1 |
| 80 | CEB Bulletin 158, eqs. (2.6.1, 2.6.3) | 49 | 1985 | 3 | 1 | 3 | 2 | 2 | 4 | 1 | 4 |
| | CEB Bulletin 158, eqs. (2.6.1, 2.6.4) | | | 3 | 1 | 3 | 2 | 2 | 4 | 1 | 4 |

| Info | | | | Categorization | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | A | | | R | | | | B |
| ID | Publication | Ref. | Year | ST | SL | SD | CS | P | W | S | |
| 81 | ACI 318-95, eq. (10–5) | 56 | 1995 | 1 | 1 | 1 | 2 | 2 | 3 | - | 1 |
| 82 | ACI 318-19, §24.3 | 108 | 2019 | 3 | 1 | 3 | - | - | 2 | - | 1 |
| 83 | PN-76/B-03264, §Appendix 4 | 109 | 1976 | 1 | 1 | 2 | 2 | - | 4 | 1 | 4 |
| 84 | Sokolov et al., eqs. (3, 5) | 110 | 1975 | 1 | 1 | 3 | - | - | 1 | 1 | 1 |
| 85 | Beeby, eq. (6) | 50 | 1979 | 3 | 1 | 3 | 2 | 2 | 2 | - | 4 |
| 86 | Nawy and Huang, eqs. (4, 7, 8) | 111 | 1977 | 3 | 1 | 3 | 2 | 2 | 3 | 1 | 1 |
| 87 | Nawy and Blair, §Fracture hypothesis | 112 | 1971 | 1 | 1 | 2 | - | 4 | 3 | - | 1 |
| 88 | Abeles, §Working load conditions (serviceability) | 59 | 1967 | 3 | 1 | 3 | - | 2 | 3 | - | 1 |
| 89 | BS 8007, §A.3 | 44 | 1987 | 3 | 2 | 3 | 1 | 2 | 2 | 2 | 4 |
| 90 | Chi and Kirstein, eq. (9) | 113 | 1958 | 1 | 1 | 2 | - | 2 | 1 | - | 4 |
| 91 | CUR 37, eq. (31) | 114 | 1968 | 1 | 1 | 2 | - | 1 | 3 | - | 4 |
| | CUR 37, eq. (20) | | | 1 | 1 | 2 | - | 1 | 1 | - | 4 |
| | CUR 37, §5.1 | | | 3 | 1 | 2 | 3 | 1 | 1 | - | 4 |
| 92 | Holmberg and Lindgren, eq. (9) | 115 | 1970 | 3 | 1 | 3 | - | 4 | 5 | 1 | 1 |
| 93 | Elshafey et al., eq. (13) | 64 | 2013 | 1 | - | 1 | - | - | 1 | - | 1 |
| | Elshafey et al., eq. (14) | | | 1 | - | 2 | - | - | 1 | - | 1 |
| 94 | Elshafey et al., eq. (9) | 65 | 2013 | 1 | - | 3 | 2 | - | - | 1 | 1 |

*Note*: The colours used are in agreement with Figure 6.

Abbreviations: A, application; B, background; CS, cracking stage; P, position of the measured crack width; R, representation; S, crack spacing definition; SD, internal strain distribution; SL, state of loading; ST, structural type; W, crack width definition.