

Curving Concrete – A Method for Manufacturing Double Curved Precast Concrete Panels using a Flexible Mould

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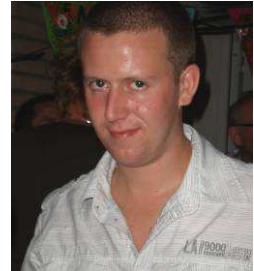
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Summary

Free form architecture with complex geometry brings along new challenges for manufacturers of building components. This paper describes the application of structural mechanics to predict the behaviour of an elastic mould surface, used as formwork for the manufacturing of double curved panels in precast concrete. Results are presented of laboratory experiments with a formwork to validate the model. The authors demonstrate that the model together with the mould prototype enable a flexible yet straightforward production method for curved concrete products that is applicable in many free form architecture projects nowadays.

Keywords: *precast concrete; shells; panels; formwork; freeform; curved; flexible mould; splines.*

1. Introduction

Double curved building elements, of which some examples are shown in Fig. 1, are more difficult to produce against acceptable prices than orthogonal and flat concrete panels. Realisation of freeform architecture requires economically feasible manufacturing methods for these elements [1]. For several building materials, solutions have already been found. The authors have, in their research, concentrated on precast concrete elements. Mass production of double curved concrete elements has often been regarded only possible after the realisation of a *flexible mould*: an adjustable formwork consisting of an elastic material that can be formed into the desired curved surface by the use of pistons, actuators, gravity, pin beds or other means.

Several concepts for a flexible mould have been designed over the years, starting with the sketch of Renzo Piano (Fig. 2a), already made in the 1960's. Several others have worked on the concept, among which Lars Spuybroek [2] (Fig. 2b) and Florian-Peter Kosche [3] (Fig. 2c). Although not all researchers use an elastic material as intermediate layer between actuators and concrete, the casting of concrete almost inevitably requires some kind of formwork layer. Without this intermediate layer, it is difficult to



Fig. 1: double curved panels made by TU Delft students

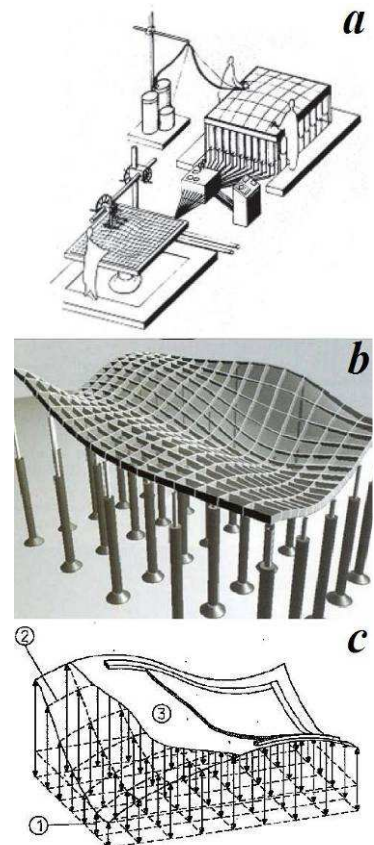


Fig. 2: Several sketches of a flexible formwork

realize a sufficiently smooth concrete surface. Apart from this required smoothness, one also does not want to pour concrete directly over the mechanism due to the vulnerability of the moving parts.

A prototype built under guidance of Rietbergen and Vollers [4] (Fig. 3) demonstrated that the proper choice of the elasticity of this intermediate layer is one of the crucial factors to success: using a too stiff material leads to difficulties in adjusting the formwork into the desired curvature, using a too flexible material leads to a uneven surface which is not acceptable from an aesthetic point of view (Fig. 4): As far as the authors have been able to observe, a theoretical structural mechanics model of the behaviour of the elastic material under forced displacement in the specific situation of a flexible mould was never published until now.

The authors believe it is necessary to understand what is happening in the intermediate layer in terms of elasticity to be able to successfully choose the material specifications and adjustment method. In this paper, the process of deformation of the flexible mould in terms of structural mechanics therefore is examined more closely.



Fig. 3: prototype of a flexible mould built by Rietbergen and Vollers

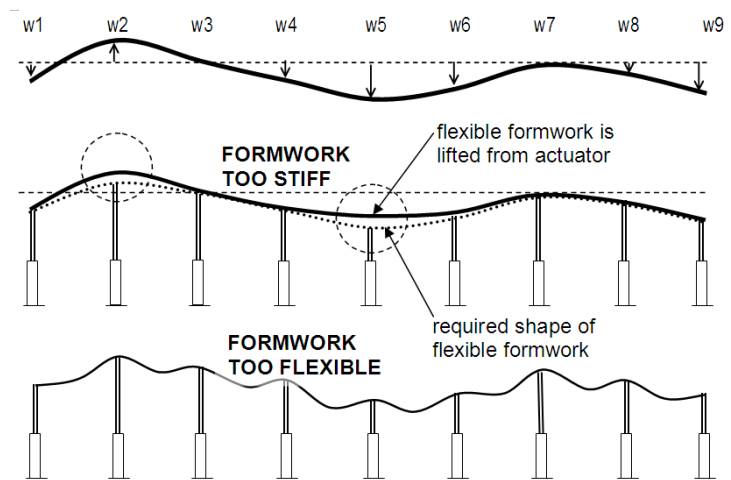


Fig. 4: difficulties in finding the required flexibility

2. Theoretical research

2.1 Splines

Shape description of curves and curved surfaces in CAD drawings is usually realized through the definition of several variants of *splines*. Although nowadays the word is almost directly associated with computer drawing, it is illustrative to look back at the use of splines in their original context: the making of hand drawings in e.g. ship, airplane and automotive engineering.

Splines are thin flexible rods, fixed in position on the drawing board by small metal weights connected to these rods. The splines were used by draftsmen for drawing smooth and freely formed curves by hand. It is easy to understand that the spline was also introduced as a tool in CAD software: some smooth shapes cannot be defined by a circle, ellipse or hyperbola, and require some other sort of numerically constructed curve. In 1959 and 1962 the Frenchmen De Casteljaud and Bézier worked on a mathematical algorithm [5] to construct a curve from a number of control points. The term *control points* refers to a number of predefined points that is followed by the curve, for example, points that define the contour of a road, ship or car. By moving the points, the curve follows. The control points can be compared to the metal weights in the traditional 'analogue' splines. A flexible formwork is quite similar to the thin flexible rod that is used in splines. The control points also show close resemblance to the actuators in the flexible mould concept. Can this analogy be used to solve the flexible mould problem?



Fig. 5: Splines used e.g. for ship design

2.2 From splines to NURBS and back again

In CAD software nowadays curves are drawn with a more enhanced version of splines: Non-uniform rational B-splines (NURBS). These primitives can describe virtually any curvature by interpolation and weighing of a number of control points. An extensive overview of their characteristics and mathematical background is given in [5]. Some theory is briefly discussed here as a stepping stone towards the research done by the authors. NURBS offer far more user control than the traditional spline, since they have the following properties:

1. NURBS are *subdivided* in several shorter curves to allow more complex shapes. The subdivision in smaller segments enables the user to make complex curves with curvature into different directions and radii.
2. The *degree* of the curve defines the extent up to which interpolation between control points takes place: the higher the degree, the more control points are included in the calculation of the coordinates of a certain point on the curve, the more complex and smooth the shape can be. In terms of mathematics: a curve with a degree 2 is defined by a quadratic polynomial (parabola); a curve with a degree 3 is defined by a cubic polynomial, and so on.
3. A *knot vector* is defined to collect the knots where the different curves are joined, allowing for a smooth connection from one curve segment to the following.
4. NURBS offer the possibility to add *weight* to each control point, making some control points “more important” than others.

An elastic material controlled by a number of actuators physically interpolates, as it were, the position of its control points through its elastic behaviour. It is important to notice that the many control possibilities for NURBS curves cause some difficulties as soon as one tries to translate digital shapes back to the analogue shape of a flexible mould:

- a) The first simple observation that can be made is that, for example, the control points of the NURBS-curve in Fig. 6 are only laying on the curve itself in the endpoints and one intermediate point. While the profile of the traditional analogue spline is controlled by a number of small weights that hold it in place on the drawing board, simply translating control points of the curve into position of the actuators of the flexible mould does not work.
- b) Furthermore sharp turns (“bumps”, “loops” or “cusps”) will be hard to realize using a flexible formwork: the radius can become so small that an average elastic formwork material would probably break under the resulting high bending moment. For the drawing of very sharp curves (equivalent to a small radius), it is obvious that the rod of the spline has to be more flexible than for curves with a big radius. This relation is also apparent in the flexible mould: the choice of the stiffness or elasticity depends on the amount of curvature one needs to reach. Shapes with a small radius require a thin and very flexible formwork, whereas large radii can be formed with thicker and less flexible materials.
- c) Finally, the natural bending behaviour of an elastic material is not necessarily described by the same polynomial that describes the interpolation of control points in its digital counterpart.

We will look at the consequences of these observations in the next subsections.

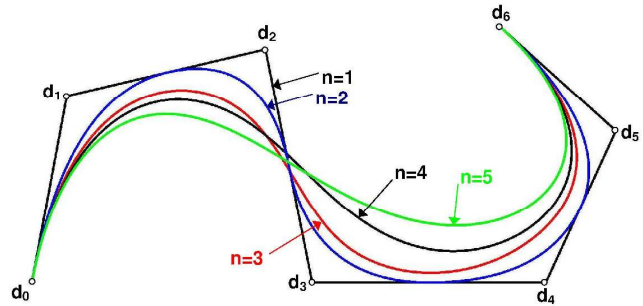


Fig. 6: Influence of the degree n in a NURBS curve

2.3 Elasticity and the adjustable formwork

The concept of the traditional spline in terms of structural mechanics actually is very similar to the basic idea of an adjustable formwork: a flexible material fixed in a curved position by control points. The control points can be moved by pistons, actuators, worm-screws, or any other adjusting method. In Fig. 3 the prototype built by Rietbergen and Volders was already shown, which in this setup was finished using a thin intermediate layer of timber. Other materials such as rubber with varying specifications have also been tested by MSc students Schoofs and Huyghe [4]. As discussed in the previous subsection, in the flexible mould concept, different from NURBS-curves, the control points are per definition laying *on* the curve. The elastic material is forced to 'follow' the position of the actuators, causing the problems sketched earlier schematically in Fig. 4. In the tests by Schoofs and Huyghe this appeared to cause some unpredictable effects disturbing the smooth adjustment of the actuators and eventually also hindering the accuracy of the resulting panel shape. Let us take a closer look at the elastic effects in the moulds intermediate layer.

2.4 How does one part of an elastic beam influence another part?

Fig. 7 illustrates in a very simplified manner how an elastic layer is supported by three actuators (or in terms of structural mechanics: support points) A, B and C. By moving support point B up and down, the shape can be adjusted. In Fig. 7, support point D is not adjusted by an actuator, but is freely following the bending stiffness of the elastic material. If point B is adjusted downwards (negative) point D -not surprisingly- moves up, and vice versa. It is clear that the position of point D is ruled by point B. If one would add an extra support in point D (as in Fig. 8), somehow the forces in support points B and D become 'interconnected' by the elasticity of the material.

Fig. 8 shows the resulting deflection of the elastic layer after addition of an extra support in point D. Not only will the force necessary to move actuator D be influenced by actuator B, but it is also clear that the shape of the mould between point C and D is influenced by the position of actuator B. Actually, also the position of actuators A and C have influence on the situation between C and D. This is exactly what happens in a flexible mould setup with multiple support points. The consequences of this elastic behaviour are affecting the shape of the flexible mould, both in a 2D setup (beams) and in a 3D setup (panels). Which consequences these are and how they can be predicted is elaborated now.

2.5 Beam theory

This effect, which is quite trivial in structural engineering, was already described by the Euler-Bernoulli differential equation that describes the bending behaviour of an elastic beam:

$$EI \frac{\partial^4 w(x)}{\partial x^4} = q \quad (1)$$

where x is the coordinate on the beam axis, EI is the bending stiffness of the beam, $w(x)$ is the deflection of the beam in coordinate x , and q is the load on the beam. This equation relates the bending stiffness and deflection on the left hand side of the equation to vertical forces on the beam, such as external loads, self weight and support reaction forces on the right hand side of the equation. The fourth order visible in the equation is responsible for the form of many well-known design equations used in structural engineering.

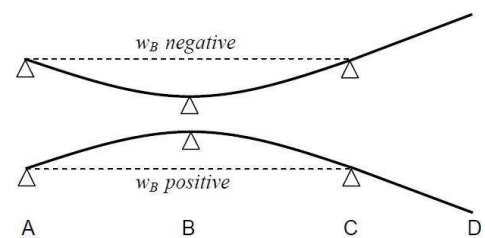


Fig. 7: elasticity 'connects' different parts of the curve

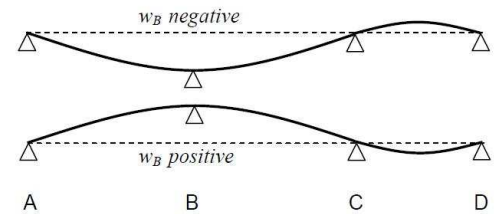


Fig. 8: forces in support points are interdependent, as is the shape in different parts of the beam

For many practical cases the solution of the differential equation is a fourth order polynomial, such as:

$$w(x) = \frac{qx}{24EI}(L^3 - 2Lx^2 + x^3) \quad (2)$$

for a beam on two supports with a distributed load q . In the situation of the flexible mould, the deflection is not only caused by a vertical load q , but also by deliberate adjustment of the support points, as illustrated in Fig. 9, to achieve the desired curved element shape. In order to understand what parameters influence the shape of the mould and the forces on the supports we will investigate this most simple example first. To lower the support point B in Fig. 9 over a distance w_B , a force F_B is needed, due to the bending stiffness of the mould EI . Both force F_B and the shape of the mould are governed by equation (1). The solution (4) can be found in most structural engineering handbooks, e.g. [6]:

$$F_B = \frac{48EI}{L^3} \cdot w_B \quad (3)$$

$$w(x) = \frac{F_B L^3}{48EI} \left(-4 \frac{x^3}{L^3} + 3 \frac{x}{L} \right) = w_B \left(-4 \frac{x^3}{L^3} + 3 \frac{x}{L} \right) \quad (4)$$

Equation (4), valid for $0 < x < L/2$, shows that the function for the shape of the curved beam under a forced displacement w_B in $x = L/2$ is a third-order polynomial. In each case where $q = 0$ equation (1) solves into a third-order polynomial. If support point B is not the centre of the beam, but on a random other position $x = a$ on the beam, the equations for the displacements become:

$$w(x)_{left} = \frac{Fb x}{6EI L} (2L(L-x) - b^2 - (L-x)^2) \quad (5) \quad \text{and} \quad w(x)_{right} = \frac{Fa(l-x)}{6EI L} ((2Lb) - b^2 - (l-x)^2) \quad (6)$$

in which equations (5) is valid for $0 < x < a$ and equation (6) for $a < x < L$ and $L = a + b$.

2.6 Beam with multiple support points

Using these equations, the situation shown in Fig.4 can be modelled as a beam with 9 supports on constant intervals that undergo forced displacements on 7 positions. This results in a 7-foldly statically indeterminate system, with the support reactions as unknown variables and the displacements w_i of the support points, the total length L , the interval lengths between supports and the bending stiffness EI as known variables. For the equation in each support point the equations (5) and (6) have to be filled in seven times, for each unknown reaction force. In the research [7] Maple was used for solving the system of equations.

The interesting result of solving the model in Maple is that one is able to 'play' with the bending stiffness of the flexible mould and that the model exactly defines the freedom of choice in the interval between the two undesired extremes shown in Fig. 10. This was used for the setup of several tests.

2.7 Results of theoretical research

What is the practical meaning for the design of the flexible mould? Some first conclusions:

1. The position of the actuators is not the only thing governing the shape of the intermediate layer: the elasticity of the formwork material itself is as important. Although this could already have been concluded intuitively, predicting the behaviour numerically is more complex than one

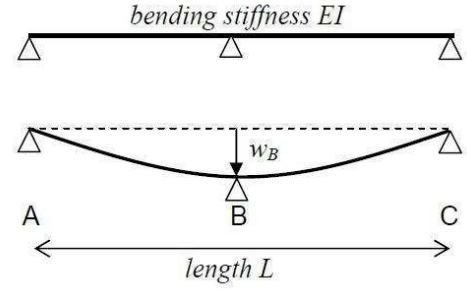


Fig. 9: Beam on 3 support points with forced displacement of middle support

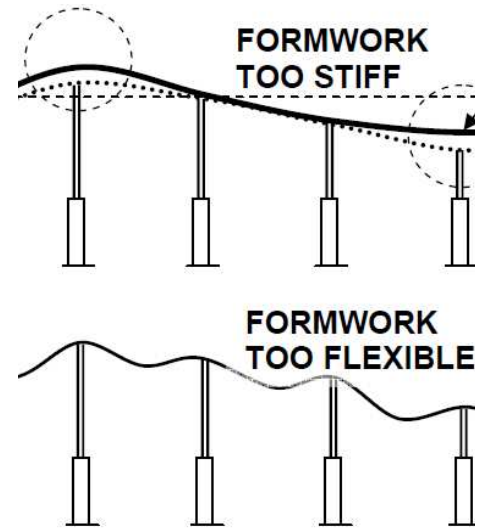


Fig. 10: Maple was used to optimize the bending stiffness EI of the flexible mould for each situation

might think at first glance, due to the fourth order differential equation and the necessary solution of a system of equations.

2. If the shape of the curved element one is going to materialize with the flexible mould is defined by NURBS, the curvature radius of the surface has direct influence on the maximum bending stiffness of the formwork. The sharper the curvature, the less stiff the formwork should be to be able to follow the desired shape. At some point, it might become necessary to modify the architectural shape or find an alternative production method that allows for sharp curvature.
3. Apart from the position of the actuators, the weight of the material (e.g. concrete) and the formwork itself also influence the final shape of the formwork (q in the Euler-Bernoulli equation).
4. Since the developed length of a curved object is larger than the length of the flat original object, normal forces can develop in the curved formwork, resisting the deformation. It was for this reason that all support points were modelled (and built) as rolls, not hindering any horizontal movement of the formwork. Otherwise, the tension stiffness EA of the formwork would lead to axial forces in the formwork, a more complex differential equation, and large horizontal forces on the support points. This problematic effect was indeed found in earlier tests of others.

3. Laboratory experiments

3.1 Experiment setup

To check the validity of the structural mechanics models and find out the effects of deforming concrete after casting several tests were carried out. In the Stevin Lab facilities are available to mix most concrete recipes. In this stage of the research, we mainly used an adapted E2 mixture ($f'_{ck} = 75$ MPa). For this specific mixture it was determined that deformation is best performed after circa 45 minutes of initial binding. For the mould sub layer a variety of materials was applied: 3.8 mm plywood plate, 1 mm steel plate or 3.8 mm plywood strips with a 10 mm soft foam cover with silicone finishing layer to obtain water tightness. For the mould edges a flexible foam polyethylene SG 40 (extra firm) was used, with a silicone finishing layer.

For reinforcement a single mesh of thin rebar $4\phi 3$ mm was used in the concrete elements of 200 mm width and $10\phi 3$ in elements of 1 m width, just enough to de-mould and lift the elements without damage. In this stage no fibres were added; this will be done in future tests.

3.2 Single-curved elements

In Fig. 11 the test setup and resulting single-curved concrete element are shown. A simple timber lattice was used for initial support during casting and initial hardening in horizontal position. By lowering the lattice, the curvature was formed. The height of the curve after deformation was controlled by vertical stands of different height. The reinforcement in the element was bending along with the mould and the still plastic concrete. Using this setup, elements with a size $2,00 \times 0,20 \times 0,05$ m³ have been manufactured, with a minimal radius of 2,5 m. The structural mechanics model calculates the reaction forces, based on EI and q of the mould and concrete and the prescribed displacement. Although the reaction forces could not be measured in this simple setup, it could be observed that the mechanics model gave a good prediction of where positive and negative reaction forces occurred.

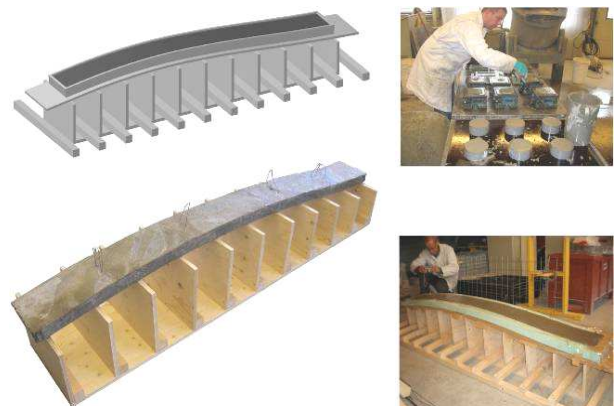


Fig. 11: Test setup for single-curved elements

3.3 Double-curved elements

Based on the same principle, a 3D-setup was built for manufacturing double-curved elements: a pin bed of 6×11 pins, distributed over distances of $0,20 \times 0,20$ m². Each pin has two positions: an initial height for casting the concrete horizontally and a second height corresponding to the CAD model

for the deformed situation. In the first tests, a thin plate was used as sub-layer formwork to cast the concrete on. In later tests, a strip mould was used (Fig. 12 top image). This strip mould uses a setup of perpendicular and crossing single curved splines, vertically adjusted by the pin bed.

Three elements taken from the virtual building envelope in Fig. 13 were chosen as example, one element with positive Gaussian curvature, and two with a negative (saddle-shaped) Gaussian curvature.

The strips accurately followed the required pin height following from the panel geometry in Fig. 13. At some points around the edges the formwork had to be pulled slightly downwards to the pins, because a negative support reaction was needed. This was indeed predicted by the mechanics model. The polyethylene edge profiles hold the concrete in the mould before and also after deformation, even though the concrete is still plastic. Under the horizontal load of the fluid concrete, the edge stays practically perpendicular to the mould surface.

For each element circa 100 litres of the E2 mixture concrete was used. The surface quality of the different elements in some cases was quite uneven, as a result of both inequalities in the finishing layer of the mould and difficulties in smoothening the casting side manually. The thickness of the element, however, appeared not to change significantly as a result of the deformation process.

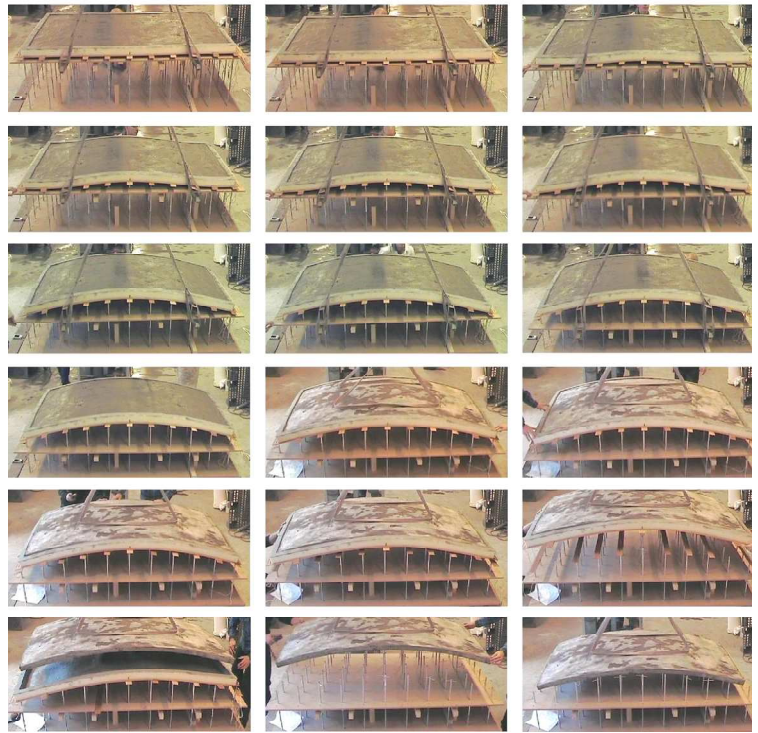
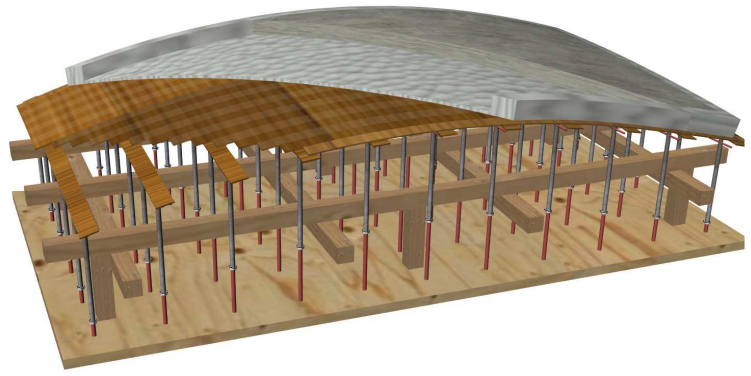


Fig. 12: Flexible strip mould used for validation of the structural mechanics model and testing of the principle of casting double curved concrete elements

4. Conclusions

From the theoretical and practical work the following conclusions are drawn:

1. The manufacturing of single- and double-curved precast concrete elements is possible through the use of the flexible mould system described in this paper.
2. In order to control the process, it is necessary to predict the support reactions and exact deflection in the deformed shape by using a suitable structural mechanics model. Several model have been developed in this research that describe the behaviour of the flexible mould accurately.

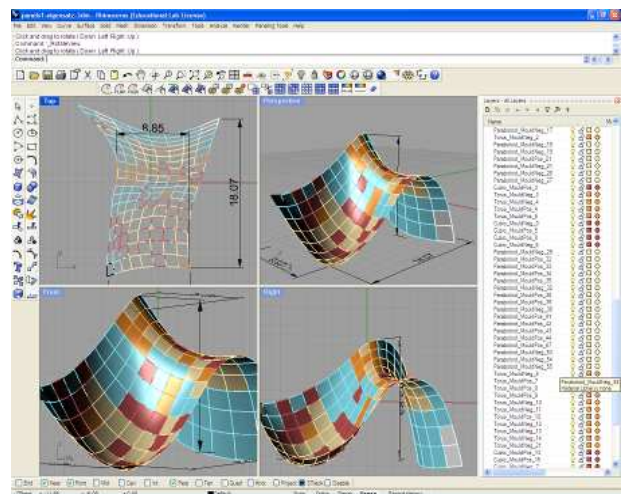


Fig. 13: Example of a NURBS-surface of a virtual building envelope drawn in Rhinoceros

3. The strip mould test setup demonstrated in this paper can be used for the manufacturing of curved elements of 2,00 x 1,00 m² of various thicknesses, typically around 50 mm. Tests were carried out with curvature radii as small as 1.5 meter, which is sufficient for realizing many freely formed building shapes. This kind of radii correspond with a difference in height within one panel of 100 to 200 mm.
4. The thickness of the element itself does not significantly change during the process. The edge profile of soft flexible foam meets the requirements of holding the concrete in the mould before and after deformation. The elements' edges stay practically perpendicular to the mould surface, which makes it possible to fit the precast panels in the building geometry;
5. Using a 3 mm steel reinforcement mesh allows the mesh to deform along with the flexible mould and concrete during the process.
6. The surface quality of the elements ranged from good to rather poor, and has to be further improved in future tests.

In 2011 further work is planned on the following topics:

- experiments with thinner concrete panels;
- apply concrete mixtures with fibre reinforcement;
- experiments with SCC in order to improve surface smoothness and colour;
- cast structural elements applicable as plank floor, e.g. with strands as reinforcement;
- work on joints and interfaces between elements;
- work on fixings.

ACKNOWLEDGEMENTS

The virtual building envelope in Fig. 12 was kindly made available by Evolute, a software firm in Austria specialized in complex architectural geometry.

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