Discrete elements in structural concrete design

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In the sixties Prof. J. Witteveen introduced a discrete model for the elastic analysis of slabs (Heron 1966). This article presents a similar approach for the design of reinforced concrete walls and deep beams, with holes or otherwise. The model – which is called stringer-panel model – combines the advantages of the popular strut-and-tie method and the standard finite element method. It has the same geometry as the plastic model proposed by Prof. M.P. Nielsen in the seventies (Kærn 1979). However, the stringer-panel model accurately uses the non-linear behaviour of reinforced concrete. So, it encompasses both the elastic and plastic states. The method provides crack width information in the serviceability limit state and allows for redistribution of forces in the ultimate limit state. A design example shows the usability of the model in engineering practice.

Key words: Wall, structural concrete, stringer-panel, strut-and-tie, computer aided design

1 Introduction

To date two approaches are commonly used to design structural concrete: The strut-and-tie method and the linear finite element method. The strut-and-tie method is simple, economical and safe. However, since it is essentially a plasticity approach it gives no information on crack widths in serviceability conditions. As a consequence it is difficult to convince the responsible building authorities of the durability of a design. The finite element method on the other hand is very suitable for designing for serviceability conditions but leads to an uneconomical reinforcement layout since redistributions of the flow of forces at ultimate limit states or before are not taken into account.

For an accurate description of cracks and redistribution of forces a non-linear model is necessary. Non-linear finite element analysis is still very time consuming and it requires an expert to operate a finite element package. This is obviously why non-linear analysis is not common in a normal consultancy company. In order to introduce non-linear techniques in practise we need a simplified model that can be evaluated in about one minute on a modern desktop computer. This is the subject of this paper.

2 Stringer-Panel Model

As can be observed in every day practise, the resulting reinforcement of walls and deep beams is often concentrated in bundles along the edges and around holes. Starting from this geometry we developed a discrete element model in which some elements called *stringers* contain main reinforcing bundles and others called *panels* contain a distributed reinforcing mesh (see figure 1).

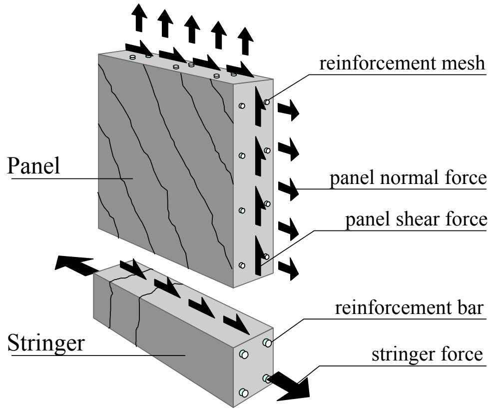

Fig. 1. Stringers and panels are the building blocks of a concrete wall model.

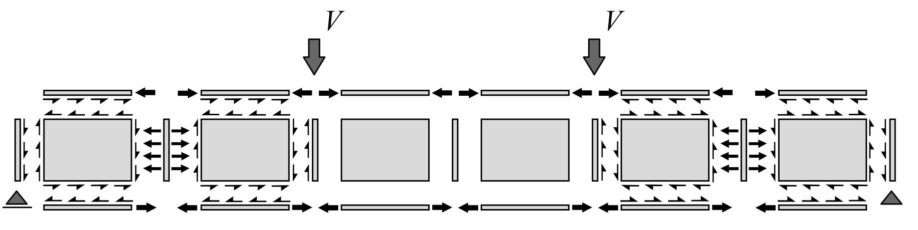

Fig. 2. An exploded view of a stringer-panel model of a beam shows that all elements are perfectly in equilibrium.

The beam in figure 2 is modelled as an assembly of stringers and panels. Panels and stringers are in perfect equilibrium and as such the method is similar to the strut-and-tie method (Schlaich et al 1987). However, a stringer-panel model takes compatibility conditions into account because the stringers and panels have grips that are connected with nodes. As figure 3 shows, a panel has four grips with which it can be connected to adjacent stringers. A stringer has three grips with which it can be connected to other stringers, panels, supports or forces.

The stringer and panel relations were derived with complementary potential energy and a hybrid method. Both the stringer and the panel have four integration points. The modified compression field theory (Collins et al 1986) was adopted to evaluate the material behaviour. In this simple model for membrane stresses the concrete can both crack and crush and the reinforcement can yield. The stringer-panel model is used for both linear and non-linear analysis. In the linear model the panels carry only shear stresses while the stringers carry all normal stresses. In the non-linear model, however, shear stresses only proved to be not enough to describe the wall behaviour accurately. Therefore, in a non-linear analysis the panels can carry both shear and normal stresses as can be seen in figure 2.

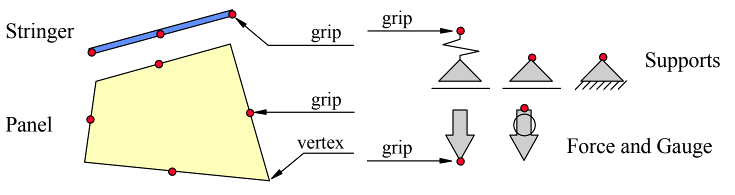

Fig. 3. A stringer-panel model can be assembled from simple components.

3 Validation

Stringer-panel model results have been compared with experiments like bending and shear failure of a slender beam, shear walls and deep beams. So far the results show that the accuracy is satisfactory from a design point of view. Here, the computation results are presented and discussed of shear failure of a slender beam.

Figure 4 shows the results of 178 shear test, reported in the literature (Bræstrup et al 1997) together with the results of a number of stringer-panel computations. Only one panel element was used over the beam height. At the horizontal axis of figure 4 the amount of stirrup

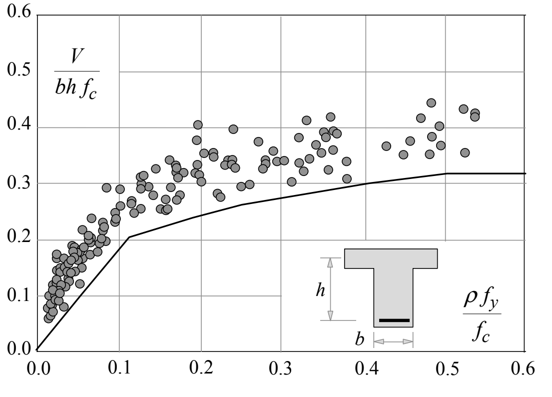

Fig. 4. Results of 178 shear test (dots) compared with stringer-panel computations (line) of slender beams

reinforcement is depicted and at the vertical axis the shear strength. As the graph shows, the model gives a conservative prediction of the beam ultimate load. This result can be understood if one considers that the model is in essence an equilibrium system. According to the plasticity theory this results in an underestimate of the strength. In other words, a real structure somehow finds ways to carry load that are not included in the model. Of course this is only valid as long as proper detailing is provided since a stringer-panel model is too course for an accurate prediction of pull out of reinforcement.
It is emphasised that it is not our intention to add another method for design of stirrups to the many methods that already exists. The shear tests are only used to validate the model.

4 Computer Program

The computer program that is developed to apply the stringer-panel model is called SPanCAD (see figure 5). Its basic philosophy is that the designer is in control and makes decisions as to how to model the structure and where to place reinforcement and how much. The program assists with computations and gives warnings if mistakes are made.

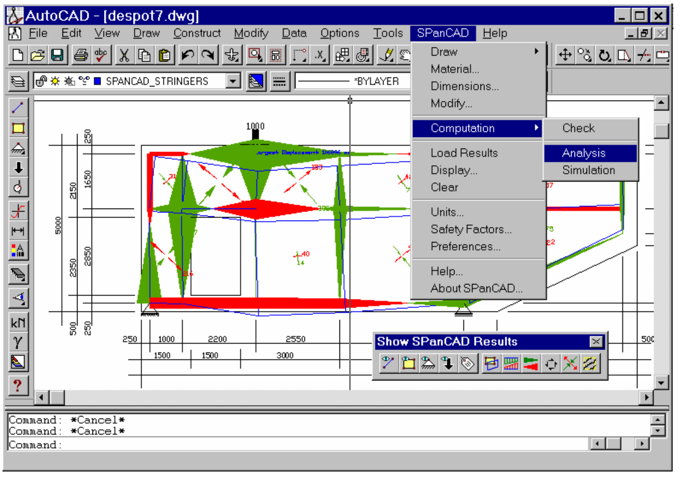

Fig. 5. With the SPanCAD program a model can be easily drawn and modified. The computation results can be displayed in several ways.

SPanCAD is an AutoCAD application for Windows that can be used to draw a simple model of a wall or deep beam including supports and loads. The components of the model (see figure 3) can be inserted with a few mouse clicks and moved or reshaped easily. The program displays linear elastic forces due to multiple load combinations in order to assist in the first dimensioning of the reinforcement. Subsequently, for dominant load combinations, the non-linear behaviour of the structure can be simulated including redistributions, crack widths, collapse load and ductility. The results can be used to check and improve the design interactively. The program can also be used for three dimensional models of for example caissons and box-girder bridges.

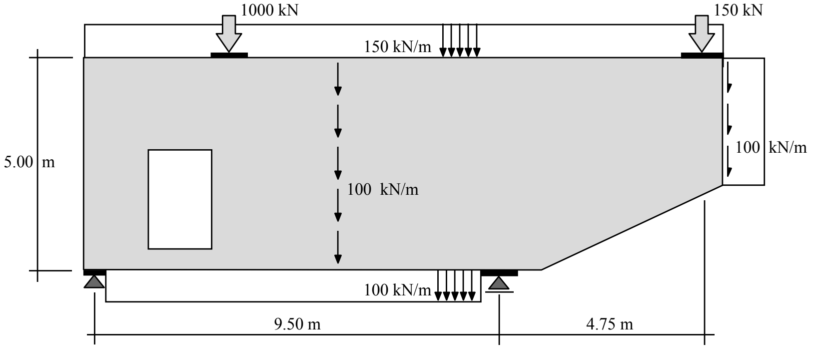

Fig. 6. *The deep beam is loaded with one load combination consisting of concentrated and distributed loads. The beam thickness is 250 mm.*

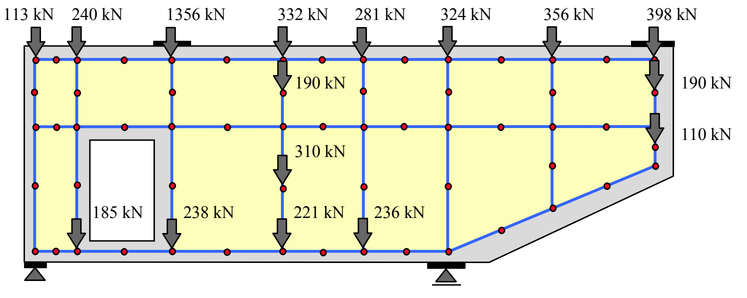

Fig. 7.: *The stringer-panel model is drawn on top of the deep beam. The stringers are depicted blue and the panels yellow.*

5 Design Example

The deep beam of figure 6 was first designed by Dr. Z. Despot at the ETH in Zürich as an example of plastic optimisation (Despot 1995). In this chapter we show our design process for this beam using a stringer-panel model. The design is summarised in table 2.

Often, the geometry of the wall at hand is already available as an AutoCAD drawing. The stringer-panel model is drawn on top of this drawing with commands similar to the standard AutoCAD commands (see figure 7). Stringers are drawn around the hole, at the edges of the wall, at supports and at concentrated loads. Contrary to the elements of a strut-and-tie model the position of the stringers is for a large part determined by the geometry and load of the beam. The

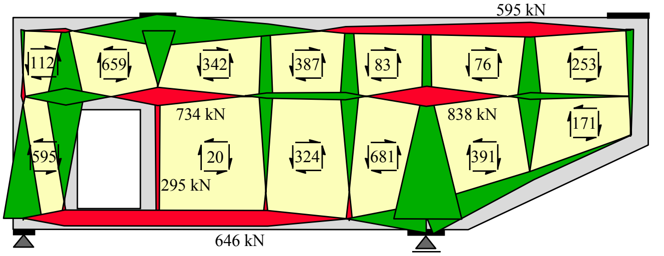

Fig. 8. *The linear stringer forces and panel forces are computed and displayed in only a second. Tensile forces are shown red and compression is green. The width of the colour is proportional to the size of the stringer force. The panel shear forces are shown in kN/m for a convenient selection of the distributed reinforcement.*

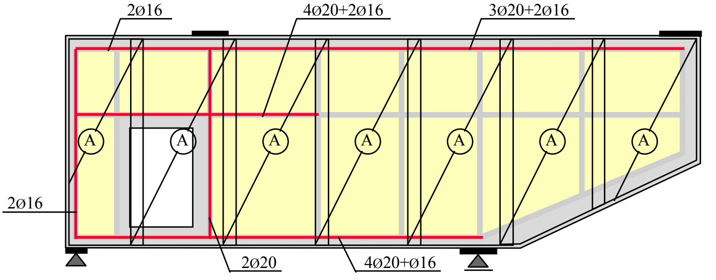

Fig. 9. *The selected reinforcement consists for a large part of meshes and a number of bars. The symbol A refers to two meshes with bars Ø12 - 150 in both directions.*

panels are simply drawn in-between the stringers. Distributed forces are lumped to the stringer ends.

Subsequently, a linear analysis is performed. The program computes the model deformation and the forces in the elements in only a second (see figure 8). If multiple load combinations are present the envelopes of the forces are displayed. Since the panels carry only shear forces and no normal forces, all normal forces are carried by the stringers. The stringers are usually positioned at the edges of the model resulting in a large lever arm for the moment and an efficient reinforcement layout.

The forces are used to select reinforcement in the stringers and panels. Of course, a minimum reinforcement has to be present for crack control in panels that are tensioned. The minimum reinforcement ratio ρ_{min} = 0.0058 is calculated from the concrete and steel tensile strengths using appropriate characteristic values. A standard mesh at both surfaces of Ø12-150 = 1508 mm²/m can fulfil this requirement. The panel with the largest shear force needs 681000 / 460 = 1480 mm2 of reinforcement in both directions (see fig. 8 and table 1). So, the minimum reinforcement is just enough for this panel and more than enough for the other panels.

This wall example has only one load combination, so, substantial redistributions cannot be used to apply less reinforcement. Nevertheless, we choose not to reinforce the horizontal tensioned (red) stringers in the middle of the wall above the right support because it is inconvenient to put these bars in place at the construction site. We just remove the stringers and see what will happen in the subsequent simulation. The resulting reinforcement layout is presented in figure 9. The concrete dimensions of the stringers are 250 mm thick and 140 to 900 mm wide depending on the available wall material and the effective tension area. The panels are each 250 mm thick. The properties of the wall materials are summarised in table 1.

Table 1. The properties of the wall material are design values for both ultimate and serviceability conditions. An average value is used for the concrete tensile strength and a safe value for its compressive strength.

Concrete			Steel Reinforcement		
Compressive Strength	-19.5	MPa	Yield Strength	460	MPa
Young's Modules	30000	MPa	Young's Modules	200000	MPa
Tensile Strength	2.2	MPa	Hardening Modules	0	MPa
Ultimate Strain	-0.0035		Ultimate Strain	0.05	

In a simulation of the behaviour of the non-linear model the load factor λ is incremented from 0 until 1 at which the total load combination is carried by the model. A linear constitutive relation was selected for the stringers without reinforcement and a non-yielding relation for the reinforced stringers (see figure 10).

The simulation shows that at the right hand side of the model much of the previous stringer forces now is carried by the panels (see figure 11). Some redistribution of forces occurs from below the hole to above the hole because the bottom stringers crack over their full length and elongate considerably. Striking is that the horizontal stringer force above the hole almost doubles in size. The reason is that the panel above the hole starts to fail and in doing so it dilates substantially. Consequently, the adjacent stringers have to elongate and since they are not

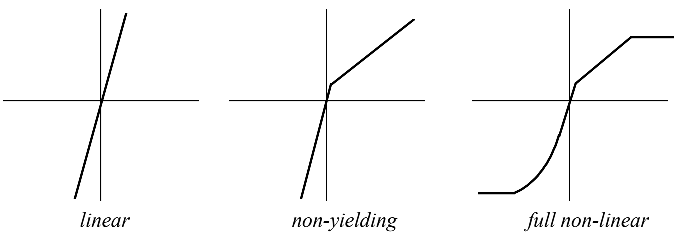

Fig. 10. The reinforced concrete in the stringers can be represented with several constitutive relations.

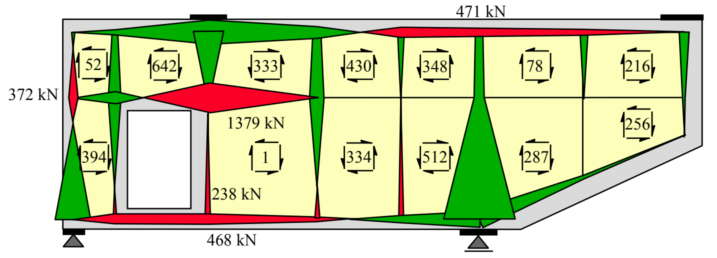

Fig. 11. *The stringer forces and panel forces at ultimate design load (λ = 1.0) show some redistributions. At the right hand side the panel reinforcement carries a substantial part of the forces which were carried by the stringers before. At the left hand side the forces are redistributed from below to above the hole of the deep beam.*

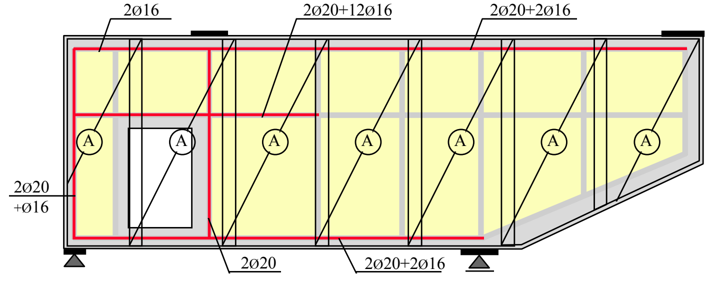

Fig. 12. *The final reinforcement layout consists of 1240 kg steel in the panels and 310 kg steel in the stringers.*

allowed to yield the stringer forces increase, showing exactly how much reinforcement is needed to prevent failure.

The stringer reinforcement is adapted as shown in figure 12. Subsequently, a new simulation was done up to failure using full non-linear behaviour of stringers and panels. The ultimate load occurs at a load factor λ = 1.1 and the displacement at collapse is 16 mm (see figure 13).

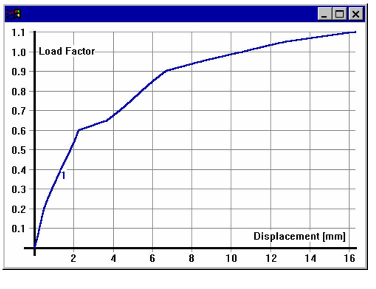

Fig. 13.

The simulation of the beam behaviour up to the ultimate limit state. At the vertical axis the load factor λ is displayed and at the horizontal axis the displacement of the top of the beam just left of the hole. The model can carry 10 % more than the load combination requires

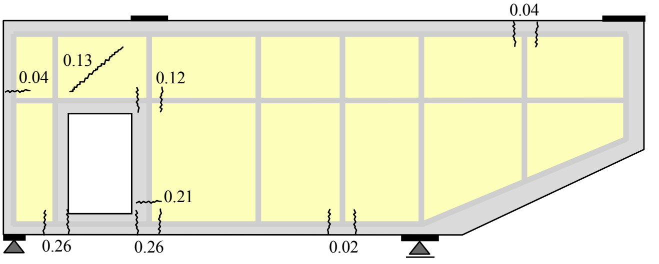

Fig. 14. Only a few elements are cracked at serviceability conditions (λ = 0.8). The cracks are displayed in the panel midst and stringer ends. The largest crack width is 0.26 mm.

Table 2. The design process with a stringer-panel model consists of 9 steps.

1	Draw the concrete shape with dimensions in AutoCAD.
2	Draw a stringer-panel model.
3	Do a linear analysis with SPanCAD for all load combinations.
4	Select main and distributed reinforcement and dimensions of the stringers and panels.
5	Do a simulation with non-yielding stringers of the dominant load combinations.
6	Check the simulation results and improve the design.
7	Do a simulation with full non-linear behaviour and each load combination.
8	Check the simulation results.
9	Detail the reinforcement.

In this example the design load in serviceability conditions is defined as 80 % of the design load for ultimate conditions. The cracks in the stringers and panels at serviceability load are shown in figure 14. The maximum crack width is 0.26 mm which is allowed in normal environmental conditions.

The total mass of steel reinforcement is 1550 kg, which consists of 1240 kg minimum reinforcement in the panels and only 310 kg in the stringers. The plastic optimisation as referred to in the beginning of this chapter gives a reinforcement mass of 1070 kg. The difference is mainly caused by a larger minimum reinforcement used in the stringer-panel model which was considered to be necessary for crack control. A design with the linear finite element method results in 1920 kg reinforcement (Despot 1995). The reinforcement quantities mentioned do not include detailing reinforcement or losses due to cutting.

The design process can be summarised in 9 steps as shown in table 2. Compared to a traditional design, steps 5 trough 8 are added which allow the structural engineer to see how his designed wall or deep beam behaves and to make improvements to the reinforcement.

Finally, it is emphasised that proper detailing of the reinforcement is essential in order to prevent premature failure. For example at the right top corner and the left bottom corner of the hole a localised crack will occur despite the presence of a minimum reinforcement. It is good practice to include a few inclined bars in this corner to disperse the cracks as much as possible. Also, sufficient development length or hooks have to be applied because the stringer-panel model assumes that anchorage is present at all bars.

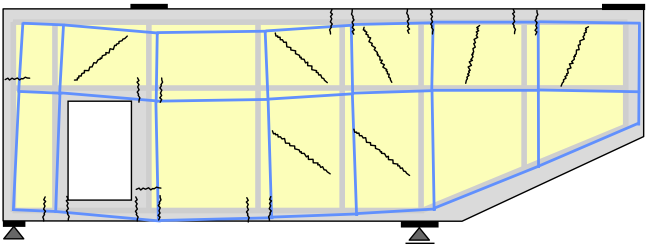

Fig. 15. Deformed model and crack distribution in the ultimate limit state. Some stringer cracks have closed and some panel cracks have rotated.

6 Conclusions

Non-Linear behaviour can be included in a design process of concrete walls and deep beams. This requires a simple model, a new design process and a special graphical user interface. For concrete walls a reduction of the amount of reinforcing steel can be obtained. However, more important is that it results in a reliable design for both ultimate and serviceability conditions.

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