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Optimising public transport passenger transfer waiting time comparing the impacts of alternative objectives

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Optimising public transport passenger transfer waiting time: comparing the impacts of alternative objectives

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ABSTRACT

This study proposes three mathematical programming models with distinct optimization objectives for transfer optimization in a bi-modal public transport network. To improve the applicability of the models and expedite the solution process, some acceleration techniques, including eliminating redundant constraints and incorporating valid inequalities, are suggested. The models and solution methods are applied to a small toy network and a real-life bi-modal public transport network. The results indicate that compared to the third model, the second model can reduce the total transfer waiting time by 12.29% to 30.31%, while the longest transfer waiting time may increase by 4.35% to 22.22%. Furthermore, the third model, which prioritizes minimizing the longest transfer waiting time, may increase the total or average transfer waiting time. The results suggest that decision-makers need to make a trade-off between reducing total passenger transfer waiting time (for efficiency) and reducing the longest passenger transfer waiting time (for fairness).

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Public transport; seamless transfer; multimodal transport; transfer coordination; transfer waiting time; integer programming

1. Introduction

1.1. *Background and motivation*

In many cities worldwide, public transport (PT) systems serve as the backbone of urban mobility, playing a critical role in alleviating traffic congestion, reducing environmental pollution, and enhancing the quality of life for citizens. An effective PT network relies on passengers seamlessly transferring between service lines whereas passengers encountering long transfer waiting time or failed transfer connections present major challenges to the attractiveness and convenience of a PT system (Chen et al. 2009; Daganzo and Anderson 2016; Nuzzolo and Lam 2016; Vuchic 2005). A survey on bus transport in Beijing, China,

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reveals that transfer time and access time together make up an average of 35% of the total journey duration (The State Council of the People's Republic of China 2016). Some studies show that extended transfer waiting time is a critical factor influencing passengers' mode choice (Gu et al. 2024; Yap, Wong, and Cats 2024) and their travel satisfaction (Chowdhury and Ceder 2016; Kim et al. 2018; Susilo and Cats 2014). Therefore, an efficient transfer system is essential for enhancing the attractiveness and usability of an integrated PT system (Abdolmaleki, Masoud, and Yin 2020; Ceder 2016; Gnecco, Hadas, and Sanguineti 2021; Yang et al. 2025).

Optimising PT transfers can be achieved through timetable coordination at the tactical planning level (Liu, Cats, and Gkiotsalitis 2021), or through transfer synchronisation at the operational control level (Gavriilidou and Cats 2019; Gkiotsalitis, Cats, and Liu 2023; Zhang et al. 2024). This study addresses the tactical-level timetable coordination problem. Timetable coordination is a cost-effective method for improving transfer connections and overall PT service without requiring significant additional investments. By ensuring that timetables are coordinated, passengers can benefit from reduced waiting time and more reliable transfer connections, thus increasing the efficiency and appeal of the PT system (Ansarilari et al. 2023; Cortés et al. 2023; Lee et al. 2022; Liu et al. 2023; Wu et al. 2015). This study contributes to the field of PT timetable coordination by formulating three integer programming models with distinct optimisation objectives for optimising transfers in a bi-modal PT network. These models aim at addressing the challenges of optimising the number of successfully coordinated transferred passengers and long individual transfer waiting time, and thereby improve the overall efficiency and attractiveness of a PT system. By leveraging advanced optimisation techniques, we seek to provide practical solutions for enhancing the transfer coordination of multimodal PT networks, thereby contributing to the development of more efficient, seamless, and user-friendly PT systems.

1.2. Literature review

Various solution approaches have been developed to address the transfer coordination problem, including the analytical modelling approach, mathematical programming approach, heuristic rule-based approach and simulation approach (Liu, Cats, and Gkiotsalitis 2021). Among them, mathematical programming is widely used in optimising PT timetable coordination (Liu, Cats, and Gkiotsalitis 2021; Gkiotsalitis et al. 2023; Chai et al. 2024). Prior studies have explored various optimisation objectives, such as maximising the number of simultaneous arrivals of vehicles at transfer stations, maximising the number of successfully coordinated transferred passengers, and minimising total passenger transfer waiting time.

The objective of maximising the number of simultaneous arrivals of vehicles at transfer stations has been extensively studied. A transfer is considered successful as long as passengers do not miss their last possible connecting trips at the transfer station. Successful coordinated transfers are often defined as either the simultaneous arrivals of vehicles at transfer stations or arrivals within a predefined time window (Cao et al. 2019; Ceder, Golany, and Tal 2001; Fleurent, Lessard, and Séguin 2004; Ibarra-Rojas, López-Irarragorri, and Rios-Solis 2016). For instance, Ibarra-Rojas and Rios-Solis (2012) examined the synchronisation of bus timetabling by developing a model that maximises the number of simultaneous arrivals of vehicles at transfer stations and also considering preventing bus bunching. Dou, Meng, and Guo (2015) introduced a bus schedule coordination model aimed at reducing transfer failures, particularly for last train services, by adjusting bus timetables in an intermodal bus-and-train transport network. Similarly, Guo et al. (2017) formulated a multi-period timetable optimisation model for metro networks, which adjusts train schedules to match varying passenger travel demands and maximise transfer synchronisation events. More recently, Liu et al. (2023) presented a bi-objective optimisation model for integrated PT transfer optimisation and vehicle scheduling, generating Pareto-optimal solutions using an ϵ-constraint method. Lai et al. (2023) designed an artificial bee colony algorithm combined with simulated annealing to maximise the number of synchronisations in a metro-bus bi-modal PT network.

The primary drawback of the approach undertaken by the abovementioned studies is that they employ a binary variable to indicate whether a transfer connection is successful or not. This method does not provide detailed transfer waiting time information and fails to incorporate the number of transferring passengers, potentially optimising transfers which are not undertaken by any passengers. Consequently, the optimisation results may lack practical relevance.

With advancements in information and telecommunication technologies, as well as smart payment systems, more detailed passenger information can now be incorporated into optimisation objectives (Nuzzolo and Lam 2016; Yap et al. 2019). A second group of studies has focused on maximising the number of successfully coordinated transferred passengers by including passenger numbers as a weighting factor in the objective function (Fouilhoux et al. 2016; Wu, Liu, and Jin 2016). For example, Wu et al. (2016) addressed the bus timetabling synchronisation problem, aiming to maximise the number of passengers benefiting from successful transfers while minimising deviations from a reference timetable. Nasirian and Ranjbar (2017) proposed a scatter search algorithm to minimise total passenger transfer waiting time, demonstrating significant reductions in passenger transfer waiting time. Chen et al. (2019a) considered heterogeneous transfer walking time to optimise successful transfers for last train service network. Nesmachnow, Muraa, and Risso (2020) introduced a new mixed-integer programming model for bus timetable synchronisation, employing evolutionary algorithms to enhance planning efficiency. Yang et al. (2021) proposed a distributionally robust last-train coordination planning problem model that aims at maximising the flow of successful transfer passengers in a subway system. Massobrio et al. (2022) presented a learning-based optimisation method for solving the bus synchronisation problem in PT systems, aiming to synchronise bus timetables to optimise passenger transfers between bus lines.

This second modelling approach shares the limitation of using a binary variable to indicate transfer success, which still does not provide detailed transfer waiting time information, leading to potential fairness issues among different transfers. That is, the optimisation model may result in some transfers having very long waiting time, while others experience relatively short waiting time. To address this, some studies have focused on optimising passenger transfer waiting time (Abdolmaleki, Masoud, and Yin 2020; Kang and Meng 2017; Lai et al. 2024; Shafahi and Khani 2010). For example, Jansen, Pedersen, and Nielsen (2002) developed a model to minimise transfer waiting time by weighting different types of passengers. Saharidis, Dimitropoulos, and Skordilis (2014) proposed an optimisation model, aiming to minimise the waiting time of passengers at transfer situations. Poorjafari, Long Yue, and Holyoak (2014) proposed a simulated annealing-based approach for timetable coordination, aiming at minimising the total waiting time for transferring passengers, and demonstrated the applicability of the algorithm for a small-scale network. Gkiotsalitis and Maslekar (2018) introduced a heuristic method based on sequential hill-climbing for coordinating regularity-based bus services, with the goal of reducing passenger waiting time at transfer stations and improve service regularity while satisfying operational constraints. Liu et al. (2018) developed a method integrating simulated annealing and parallel computing to optimise train departure time from terminals in an urban rail network, aiming to minimise total transfer waiting time. Gkiotsalitis, Eikenbroek, and Cats (2019) proposed to consider the potential variability in inter-station vehicle travel time and dwell time as well as service regularity when optimising bus transfer synchronisation. Sadrani, Tirachini, and Antoniou (2022) developed a mixed-integer nonlinear programming (MINLP) model to optimise transit vehicle dispatching with the objective of minimising passenger waiting time in a travel corridor while considering mixed-fleet operations. The model was efficiently solved using a simulated annealing algorithm. Zhou et al. (2023) presented an optimisation model for heterogeneous passenger subway transfer timetables considering social equity, aiming at minimising subway operating costs and transfer waiting time for all passenger groups, while assigning higher priority weights to vulnerable passenger groups. Ansarilari, Bodur, and Shalaby (2024) introduced a novel mixed-integer linear programming model for transfer synchronisation in PT networks, aiming at minimising passenger transfer waiting time, and developed a Lagrangian relaxation-based heuristic solution method for efficiently solving large problem instances.

Several studies considered multiple optimisation objectives. For example, Wu et al. (2016) considered maximising the total number of passengers benefiting from successful transfers and minimising the departure time deviation. Ibarra-Rojas and Muñoz (2016), and Silva-Soto and Ibarra-Rojas (2021) examined the optimisation of transit transfer synchronisation at common stops, aiming to improve service level and reduce operational costs. Liu, Ceder, and Chowdhury (2017) focused on maximising the number of successful transfer connections while minimising the required vehicle fleet size. Ansarilari et al. (2023) conducted detailed comparisons of different optimisation objectives in PT transfer coordination optimisation, such as maximising the number of successful transfers, minimising total transfer waiting time. Tan et al. (2023) proposed a bi-objective optimisation model for electric vehicle charging scheduling to balance fairness and efficiency, considering both the maximum individual waiting time and the operating cost of charging stations. Wen et al. (2024) formulated a multi-objective optimisation model to weigh passenger accessibility and operation cost for the end-of-service period operation, and used a benders decomposition algorithm to solve the model. They conducted a comprehensive assessment of different transfer coordination optimisation models at the network level as well as for individual transfer stations.

Table 1 summarises related work on PT timetable coordination optimisation, comparing network characteristics, model features, and solution methods. Past research has primarily focused on optimising either the number of simultaneous vehicle arrivals, the number of successfully coordinated transferred passengers, or the total transfer waiting time. However, to the best of our knowledge, there is lack of knowledge on how to optimise timetable coordination so as to minimise passenger transfer waiting time while guaranteeing the number of successfully coordinated transferred passengers. To bridge this gap, this study aims to optimise the total, average, or longest transfer waiting time while ensuring the maximum number of successfully coordinated transferred passengers in a multimodal PT network.

1.3. Contributions and organisation

Based on the comprehensive literature review, it becomes evident that studies simultaneously considering the number of successfully coordinated transferred passengers and optimising transfer waiting time are scarce. This research area holds significant practical importance as it enables PT systems to meet the transfer needs of a large number of passengers while maintaining a high level of service. The contributions of this study are three-fold. First, we introduce three mathematical programming models to address the optimisation of transfers in multimodal PT networks. The first model focuses on maximising the number of successfully coordinated transferred passengers. Building upon this, the second and third models optimise the total and longest transfer waiting time, respectively, while ensuring the maximum number of successfully coordinated transferred passengers. Second, to enhance the applicability of the proposed models to large-scale, real-life PT networks, we propose several techniques for accelerating their solution. These include eliminating redundant variables and incorporating valid inequalities to reduce the solution space and expedite the solution process. These techniques are crucial for making the models practical and efficient in real-world implementations. Third, the effectiveness and efficiency of the proposed models and solution methods are demonstrated by means of a numerical example and a real-life bi-modal PT network in Yibin, China. The network consists of an Autonomous Rail Rapid Transit (ART) (also known as Trackless Tram) and regular bus services. The computational results confirm the practical applicability of the models and highlights their potential to significantly improve transfer efficiency in bi-modal PT systems.

This paper comprises seven sections including this introductory section. Section 2 provides a formal description of the PT timetable coordination problem, along with an illustrative example. Section 3 presents the formulations of the three mathematical programming models. The solution method, together with the eliminating redundant variables and incorporating valid inequalities techniques, is described in Section 4. Section 5 presents a numerical example to demonstrate the model and solution method. A case study of the Yibin ART-bus bimodal PT network is detailed in Section 6. Finally, Section 7 concludes our work and discusses limitations, as well as possible directions for future research.

2. Problem description

Consider a PT network comprising a set of service lines $L = \{1, 2, 3 \dots l\}$, a set of terminal stations D, and a set of transfer stops/stations N. Let P represent the total number of transfer passengers. The average vehicle running time between terminal stations and transfer stations, as well as between transfer stations, are given and can take the values of historical averages because the problem is solved at the tactical level. Within a specified planning period T, the PT timetable coordination problem aims to design a timetable to achieve specific objectives, which may include maximising the number of successfully coordinated transferred passengers, minimising the total transfer waiting time, or minimising the longest transfer waiting time.

Table 1. Comparisons of previous and our studies on public transport timetable coordination optimisation.

	PT network			Optimisation model			
Authors (years)	PT mode	Transfer passengers	Headway	Objective function	Decision variable	Valid inequalities	Solution method
Ceder, Golany, and Tal (2001)	Single	No	Uneven	Max the number of simultaneous arrivals of vehicles	Offset time	No	Heuristic algorithm
Shafahi and Khani (2010)	Single	Yes	Even	Min average transfer waiting time	Offset time, dwell time	No	Genetic algorithm, optimisation solver CPLEX
Ibarra-Rojas and Rios-Solis (2012)	Single	No	Uneven	Max number of synchronisations	Departure time	Yes	Multi-start iterated local search
Poorjafari, Long Yue, and Holyoak (2014)	Single	No	Even	Min total transfer waiting time	Offset time	No	Simulated annealing
Fouilhoux et al. (2016)	Single	No	Uneven	Max weighted sum of synchronised transfers	Departure time	Yes	Optimisation solver CPLEX
Guo et al. (2017)	Single	No	Uneven	Max number of synchronisations	Departure/ arrival time, running/dwell time, headway	No	Particle swarm optimisation and simulated annealing algorithm
Nasirian and Ranjbar (2017)	Single	Yes	Uneven	Min total transfer waiting time	Headway	No	Scatter search algorithm, optimisation solver CPLEX
Nesmachnow, Muraa, and Risso (2020)	Single	Yes	Uneven	Max number of successfully coordinated transferred passengers	Offset time	No	Evolutionary algorithm
Massobrio et al. (2022)	Single	Yes	Uneven	Max number of successfully coordinated transferred passengers	Offset time	No	Virtual savant
Lai et al. (2023)	Multiple	No	Uneven	Max number of synchronisations	Offset time	No	Artificial bee colony algorithm
Liu et al. (2023)	Single	No	Even	Max number of synchronisations, Min fleet size	Offset time	No	ϵ-constraint method and Gurobi
Ansarilari, Bodur, and Shalaby (2024)	Single	Yes	Uneven	Min weighted sum of total transfer waiting time	Departure/ arrival time, running/dwell time, headway	No	Lagrangian relaxation-based heuristic algotithm
This study	Multiple	Yes	Even	Max number of successfully coordinated transferred passengers, Min total or longest transfer waiting time with guaranteed share of successfully coordinated transferred passengers	Offset time	Yes	Optimisation solver Gurobi, valid inequalities

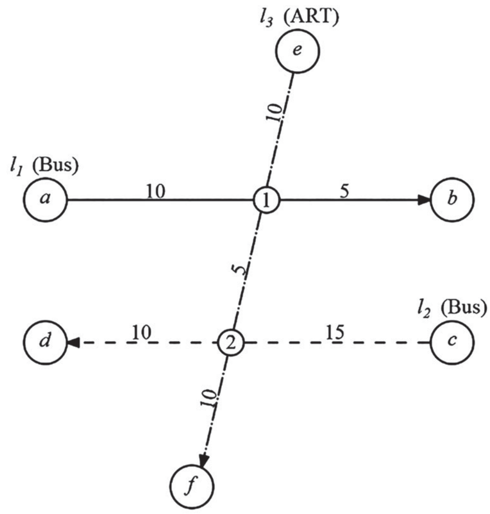

Figure 1. An illustrative bi-modal PT network.

Our aim is to optimise PT transfer waiting time while also maximising the number of successfully coordinated transferred passengers. Initially, an optimisation model is employed to maximise the number of successfully coordinated transferred passengers. Building on this, a second and a third model are proposed: the second model minimises the total transfer waiting time, and the third model minimises the longest transfer waiting time, using the number of successfully coordinated transferred passengers as a constraint in both cases. The optimisation results from the second and third models provide PT decision-makers with a basis to balance between minimising the total transfer waiting time and minimising the longest transfer waiting time.

2.1. An illustrative example

A small toy network, as shown in Figure 1, is employed to illustrate the transfer coordination optimisation in a multimodal PT network. The network includes two transit modes: bus and Autonomous Rail Rapid Transit (ART). It comprises three lines (Lines l_1, l_2, and l_3), six terminal stations (Stations *a*, *b*, *c, d, e* and *f*), and two transfer stations (Stations 1 and 2). The numbers next to the line segments in Figure 1 indicate the average vehicle running time, in minutes. The headways of Lines l_1, l_2, and l_3 are 10, 10 and 15 min, respectively, and the considered planning period *T* is [7:00, 7:30]. The numbers of transfer passengers between these three lines are presented in Table 2. It shows that the total number of transfer passengers is 44. Following Shafahi and Khani (2010), transfer passengers are assumed to be uniformly distributed among transfers during the planning period; that is, each transfer between the same lines has the same number of transfer passengers. Once the departure time of the first vehicle trip is determined, the arrival time of all vehicles at transfer stations can be obtained. Thus, it is easy to know whether a transfer connection is successful or not.

Table 2. Number of transfer passengers within the illustrative network.

Transfer station	From line	To line		
		l_1	l_2	l_3
1	l_1	–	–	15
1	l_3	12	–	–
2	l_2	–	–	9
2	l_3	–	8	–

Table 3. An initial timetable for the toy transit network.

Line	Trip number		
	1	2	3
l_1	7:05	7:15	7:25
l_2	7:10	7:20	7:30
l_3	7:15	7:30	–

Table 4. Arrival time of all vehicle trips at the transfer stations with the initial timetable.

Transfer station	Vehicle trip arrival time		
	l_1	l_2	l_3
1	7:15	–	7:25
1	7:25	–	7:40
1	7:35	–	–
2	–	7:25	7:30
2	–	7:35	7:45
2	–	7:45	–

2.1.1. Optimising the number of successfully coordinated transferred passengers

An initial timetable for the toy transit network is presented in Table 3. The timetable describes vehicle trip departure time at the terminal stations, within the planning period. Based on it and the line segment running time, the arrival time of all vehicle trips at the transfer stations can be obtained, which are shown in Table 4. Based on the arrival time of all vehicle trips, one can directly find out whether a transfer connection is successful or not. The results are summarised in Table 5. A transfer is considered successful if the departure time of the connecting trip is no earlier than the arrival time of the preceding arrival trip. However, if there are still subsequent vehicles arriving from the connecting route, passengers can still complete the transfer, making the transfer successful as well. A transfer is only considered unsuccessful when passengers miss the last vehicle trip of the connecting route.

According to Table 4, we can see that the arrival time of the second vehicle trip of Line l_3 at transfer Station 1 is 7:40, and the arrival time of the last vehicle trip of Line l_1 at transfer Station 1 is 7:35. Thus, the passengers taking the second vehicle trip of Line l_3 cannot successfully transfer to Line l_1 at transfer Station 1. However, all the other transfer passengers can have a successful transfer. Thus, for the initial timetable, the number of successfully coordinated transferred passengers is 38.

If we adjust the departure time of the first vehicle trip of Line l_1 to be 7:10 and keep the departure time of the first trips of the other two lines unchanged, a revised timetable can be obtained, as shown in Table 6. The arrival time of all vehicle trips at transfer stations with the

Table 5. Transfer success or failure with the initial timetable.

Transfer station	Transfer					
1	From Line l_1 to Line l_3			From Line l_3 to Line l_1		
	Vehicle trip ID of Line l_1	Number of transfer passengers	Success or Failure	Vehicle trip ID of Line l_3	Number of transfer passengers	Success or Failure
	1	5	Success	1	6	Success
	2	5	Success	2	6	Failure
	3	5	Success			
2	From Line l_2 to Line l_3			From Line l_3 to Line l_2		
	Vehicle trip ID of Line l_2	Number of transfer passengers	Success or Failure	Vehicle trip ID of Line l_3	Number of transfer passengers	Success or Failure
	1	3	Success	1	4	Success
	2	3	Success	2	4	Success
	3	3	Success			

Table 6. The revised trip departure timetable for the toy transit network.

Line	Trip number		
	1	2	3
l_1	7:10	7:20	7:30
l_2	7:10	7:20	7:30
l_3	7:15	7:30	–

Table 7. The arrival time of all trips at transfer stations with the revised timetable.

Transfer station	Vehicle trip arrival time		
	l_1	l_2	l_3
1	7:20	–	7:25
1	7:30	–	7:40
1	7:40	–	–
2	–	7:25	7:30
2	–	7:35	7:45
2	–	7:45	

revised timetable are shown in Table 7. Based on it, we can again directly find out whether a transfer is successful or not. The results are summarised in Table 8, which shows that all 44 transfer passengers experience a successful transfer. It clearly shows that by modifying the timetable, i.e. vehicle trip departure time, one can optimise the number of passengers experiencing successful transfers.

2.1.2. Trade-off between total transfer waiting time and longest transfer waiting time

PT schedulers are also interested in minimising total transfer passenger waiting time (efficiency) or minimising the longest transfer passenger waiting time (fairness). Table 9 shows an example timetable for the toy transit network. The arrival time of all vehicle trips at the transfer stations are shown in Table 10. The results of transfer success or failure, and transfer waiting time are summarised in Table 11. It shows that the longest transfer waiting time

Table 8. Transfer success or failure for the revised timetable.

Transfer station	Transfers					
1	From Line l_1 to Line l_3			From Line l_3 to Line l_1		
	Vehicle trip ID of Line l_1	Number of transfer passengers	Success or Failure	Vehicle trip ID of Line l_3	Number of transfer passengers	Success or Failure
	1	5	Success	1	6	Success
	2	5	Success	2	6	Success
	3	5	Success			
2	From Line l_2 to Line l_3			From Line l_3 to Line l_2		
	Vehicle trip ID of Line l_2	Number of transfer passengers	Success or Failure	Vehicle trip ID of Line l_3	Number of transfer passengers	Success or Failure
	1	3	Success	1	4	Success
	2	3	Success	2	4	Success
	3	3	Success			

Table 9. An example timetable for the toy PT network.

Line	Trip number		
	1	2	3
l_1	7:04	7:14	7:24
l_2	7:04	7:14	7:24
l_3	7:10	7:25	–

Table 10. Arrival time of all vehicle trips at the transfer stations with the example timetable.

Transfer station	Vehicle trip arrival time		
	l_1	l_2	l_3
1	7:14	–	7:20
1	7:24	–	7:35
1	7:34	–	–
2	–	7:19	7:25
2	–	7:29	7:40
2	–	7:39	–

experienced in these settings is 11 min, occurring at the transfer between the second vehicle trip of Line l_1 and the second vehicle trip of Line $l_3$at transfer Station 1, and also at the transfer between the second vehicle trip of Line l_2 and the second vehicle trip of Line $l_3$at transfer Station 2. The total transfer waiting time is 184 min. The number of successfully coordinated transferred passengers is 34.

While ensuring that we maintain the same number of successfully coordinated transferred passengers, we adjust the departure time of the first trip of Line l_1 to be 7:03, the departure time of the first trip of Line l_3 to be 7:13, and keep the departure time of the first trip of Line l_2 the same. By doing so, a new timetable is obtained, as shown in Table 12. The arrival time of all vehicle trips at transfer stations are shown in Table 13. The results of transfer success or failure, and transfer waiting time are summarised in Table 14.

As shown in Table 14, the number of successfully coordinated transferred passengers is indeed the same as the results of the example timetable, shown in Table 11. The longest transfer waiting time is 14 min, occurring at the transfer between the second trip of Line l_2

Table 11. Transfer success or failure, and transfer waiting time with the example timetable.

Transfer station	Transfer					
1	From Line l_1 to Line l_3			From Line l_3 to Line l_1		
	Vehicle trip ID of Line l_1	Number of transfer passengers	Waiting time	Vehicle trip ID of Line l_3	Number of transfer passengers	Waiting time
	1	5	6 min	1	6	4 min
	2	5	11 min	2	6	Transfer failed
	3	5	1 min			
2	From Line l_2 to Line l_3			From Line l_3 to Line l_2		
	Vehicle trip ID of Line l_2	Number of transfer passengers	Waiting time	Vehicle trip ID of Line l_3	Number of transfer passengers	Waiting time
	1	3	6 min	1	4	4 min
	2	3	11 min	2	4	Transfer failed
	3	3	1 min			

Table 12. The new timetable for the toy PT network.

Line	Trip number		
	1	2	3
l_1	7:03	7:13	7:23
l_2	7:04	7:14	7:24
l_3	7:13	7:28	–

Table 13. Arrival time of all vehicle trips at transfer stations with the new timetable.

Transfer station	Vehicle trip arrival time		
	l_1	l_2	l_3
1	7:13	–	7:23
1	7:23	–	7:38
1	7:33	–	–
2	–	7:19	7:28
2	–	7:29	7:43
2	–	7:39	

and the second trip of Line l_3 at transfer Station 2. The total transfer waiting time is 160 min. Although the total transfer waiting time of the new timetable is reduced by 24 min, the longest transfer waiting time increases by 3 min. Conversely, if we switch the new timetable with the example timetable, although the longest transfer waiting time is reduced by 3 min, the total transfer waiting time increases by 24 min. It indicates that when optimising timetable coordination to reduce total transfer waiting time, it may lead to an increase in the longest transfer waiting time, which increases the unfairness of the timetable. Therefore, PT schedulers may need to make a trade-off between minimising total passenger transfer waiting time (efficiency) and minimising the longest transfer waiting time (fairness) when creating coordinated timetables. For example, during peak hours, PT schedulers may prioritise minimising total passenger transfer waiting time to ensure overall system efficiency, potentially disregarding fairness for some passengers. However, during off-peak

Table 14. Transfer success or failure, and transfer waiting time with the new timetable.

Transfer station	Transfer					
1	From Line l_1 to Line l_3			From Line l_3 to Line l_1		
	Vehicle trip ID of Line l_1	Number of transfer passengers	Waiting time	Vehicle trip ID of Line l_3	Number of transfer passengers	Waiting time
	1	5	10 min	1	6	0 min
	2	5	0 min	2	6	Transfer failed
	3	5	5 min			
2	From Line l_2 to Line l_3			From Line l_3 to Line l_2		
	Vehicle trip ID of Line l_2	Number of transfer passengers	Waiting time	Vehicle trip ID of Line l_3	Number of transfer passengers	Waiting time
	1	3	9 min	1	4	1 min
	2	3	14 min	2	4	Transfer failed
	3	3	4 min			

hours when the PT network is less congested, schedulers may prioritise fairness in transfers, choosing to minimise the longest transfer waiting time. Thus, the decision on which waiting time to minimise depends on the specific operational needs.

3. Model formulation

3.1. Notations and assumptions

3.1.1. Notations

We use the notations listed in Table 15 in formulating the optimisation models.

Remark 3.1: The upper limit of the transfer waiting time, *W,* can take on two possible values. If passengers transfer to the first vehicle trip of line *k*, then *W* is equal to the vehicle running time from the terminal station of line *k* to the transfer station, denoted as t_{kn}. Otherwise, *W* is equal to the line headway of line *k* minus one minute, denoted as $h_k - 1$, as it is assumed that passengers can always board the first available vehicle.

The two scenarios are illustrated in Figure 2. In the first scenario, transfer passengers from line *l* transfer to the first vehicle trip of line *k*, as depicted in Figure 2(a). Here, the upper limit of the transfer waiting time, *W,* is equal to the vehicle running time from the terminal station of line *k* to the transfer station, denoted as t_{kn} . In the second scenario, where transfer passengers from line *l* do not transfer to the first vehicle trip of line *k*, shown in Figure 2(b), the upper limit of the transfer waiting time, *W,* is equal to the line headway of line *k* minus one, represented as $h_k - 1$. For example, let's assume that the headway h_k is 10 min. If passengers miss the transfer vehicle, they will need to wait for up to 9 min to board the next vehicle. This means that the maximum transfer waiting time is $h_k - 1$. This is because if the waiting time is exactly equal to the headway, passengers would be able to catch the previous vehicle, resulting in a transfer waiting time of 0 min.

Table 15. Notations.

Indexes	
k, l	Indexes of PT lines, specifically, k is the PT line that passengers are transferring to, and l is the PT line that transfer passengers are coming from;
i, j	Indexes of vehicle departure trips from terminal stations;
n	Index of transfer stops/stations;
Sets	
K, L	Set of PT lines, $k \in K, l \in L$;
I_k, I_l	Set of vehicle departure trips from terminal stations of lines k and l;
N	Set of transfer stops/stations;
Auxiliary variables	
y_{kiljn}	$=$ 1 if the depature time of the i–th trip of line k minus the arrival time of the j–th trip of line l at transfer station n is within the time window $[0, W]$, otherwise $= 0$;
w_{kiljn}	The transfer waiting time for passengers transferring from the j–th trip of line l to the i–th trip of line k at transfer station n, is calculated from the time the passengers arrive at the transfer platform until they board the vehicle of the i–th trip on line k and depart from transfer station n;
P	The maximum number of successfully coordinated transferred passengers;
w_M	The longest transfer waiting time;
x_{ki}	Vehicle departure time of the i–th trip ($i \geq 2$) of line k;
x_{lj}	Vehicle departure time of the j–th trip ($j \geq 2$) of line l;
Decision variables	
x_{k1}, x_{l1}	departure time of the first trip (offset time) of lines k and l, $k \in K, l \in L$;
Parameters	
h_k, h_l	Headways of lines k and l, defined as the time intervals between consecutive vehicle trips on each respective line;
t_{kn}	Vehicle running time from the terminal station of line k to transfer station n;
t_{ln}	Vehicle running time from the terminal station of line l to transfer station n;
t_n	Vehicle dwell time at transfer station n;
w_n	Passenger transfer walking time at transfer station n;
T	Planning period, in minutes;
M	A very large positive constant;
p_{kiljn}	The number of passengers who transfer from the j–th trip of line l to the i–th trip of line $k$at transfer station n;
W	The upper limit of the transfer waiting time; it is t_{kn} if the i–th trip is the first trip of a PT line, otherwise it is h_k–1.

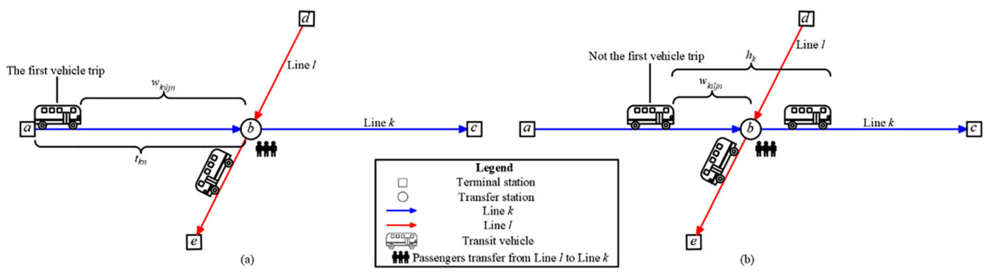

Figure 2. Illustration of the possible longest passenger transfer waiting time: (a) transfer to the first vehicle trip, (b) not transfer to the first vehicle trip.

Remark 3.2: The calculation of the number of vehicle trips for each line, specifically the determination of sets I_k and I_l, is based on the line headway and the planning period T. That is, $I_k = \left\{1, 2, \ldots, \lceil \frac{T}{h_k} \rceil\right\}, I_l = \left\{1, 2, \ldots, \lceil \frac{T}{h_l} \rceil\right\}$, where $\lceil\rceil$ represents the ceiling function.

3.1.2. Assumptions

To introduce the basic concepts while maintaining generality, the following assumptions are made.

A1. It is assumed that each PT line operates with even headways during the planning period. This assumption is based on the widely accepted formula for average passenger waiting time, $E(w) = \frac{E(h)}{2}\left(1 + \frac{\mathrm{Var}(h)}{E^2(h)}\right)$, where h represents the line headway, w represents the passenger waiting time, $E(h)$ is the expectation value of headways, and $Var(h)$ is the variance of headways. The benefit of using even headways is that for passengers arriving randomly, the expected value of waiting time at the initial boarding station can be approximated to equal half the headway, i.e. $\frac{E(h)}{2}$ (Ceder 2016; Daganzo and Anderson 2016; Liu et al. 2023).

A2. The travel time of vehicles between terminal stations and transfer stations, as well as between transfer stations, are assumed to be fixed and not time-variant during the planning period. Travel times are treated as integer values after rounding them to their closest integer number. This assumption is appropriate for addressing a tactical-level timetable coordination design problem rather than an operational-level control problem. This assumption is conventional in the stream of studies on PT timetable design (Ansarilari, Bodur, and Shalaby 2024; Cao et al. 2019; Ceder, Golany, and Tal 2001; Gkiotsalitis et al. 2023; Ibarra-Rojas, López-Irarragorri, and Rios-Solis 2016).

A3. It is assumed that the capacity of the transfer station is sufficiently large to accommodate coordinated arriving vehicles as well as passengers waiting to transfer.

A4. The planning period is discretised into minutes with the vehicle departure time represented as a discrete integer variable in minutes. Because vehicle travel time is treated as an integer, transfer waiting time is also expressed as an integer value. This approach is consistent with standard practice in daily PT planning and operations.

A5. It is assumed that transfer passengers between two lines are evenly distributed throughout the planning period (Shafahi and Khani 2010). Specifically, the number of passengers transferring from the j-th trip of line l to the i-th trip of line k at the transfer station n is calculated by dividing the total number of transfer passengers from line l to line k at the transfer station n by the total number of vehicle trips on line l.

A6. It is assumed that the capacity of PT vehicles is sufficient to meet passenger demand, meaning that no passengers will be left behind (Ansarilari, Bodur, and Shalaby 2024; Shafahi and Khani 2010).

A7. It is assumed that all vehicles have the same dwell time at a transfer station, and that all passengers have the same transfer walking time at a transfer station.

3.2. Optimisation models

Three optimisation models with different objectives are developed to optimise transfers through timetable coordination. The first model (Model 1) aims to maximise the number of successfully coordinated transferred passengers. Building on the optimisation results of Model 1, two additional models are formulated to optimise transfer waiting time. Specifically, the second model (Model 2) seeks to minimise the total transfer waiting time while ensuring the maximum number of successfully coordinated transferred passengers. The third model (Model 3) focuses on minimising the longest transfer waiting time, also ensuring the maximum number of successfully coordinated transferred passengers. The optimisation objectives of the three models are all passenger-centred, aiming to maximise the fulfilment of passenger transfers while considering total or longest transfer waiting times. The detailed formulations of these three models are provided below.

3.2.1. Model 1: maximising the number of successfully coordinated transferred passengers

Adopting the modelling approach of Ceder, Golany, and Tal (2001), a binary variable y_{kiljn} is employed to indicate the success of a transfer. Specifically, if the departure time of the i-th trip of line k minus the arrival time of passengers from the j-th trip of line l at transfer station n falls within a predefined transfer waiting time window $[0, W]$, then y_{kiljn} is set to 1, indicating a successful transfer where passengers from the j-th trip of line l can transfer to the i-th trip of line k at transfer station n. Otherwise, y_{kiljn} is set to 0. Consequently, when $y_{kiljn} = 1$, the transfer is deemed successful, and passengers are considered to have successfully coordinated transferred. The detailed mathematical formulations of Model 1 are provided below.

$$\max z_1 = \sum_{k\in K, k\neq l} \sum_{i\in I_k} \sum_{l\in L} \sum_{j\in I_l} \sum_{n\in N} p_{kiljn} y_{kiljn} \tag{1}$$

s.t.

$$(x_{ki} + t_{kn} + t_n) - (x_{lj} + t_{ln} + w_n) \geq 0 - M(1 - y_{kiljn}), \quad \forall k \in K,\ i \in I_k,\ l \in L,\ j \in I_l,\ n \in N \tag{2}$$

$$(x_{ki} + t_{kn} + t_n) - (x_{lj} + t_{ln} + w_n) \leq W + M(1 - y_{kiljn}), \quad \forall k \in K,\ i \in I_k,\ l \in L,\ j \in I_l,\ n \in N \tag{3}$$

$$\sum_{i\in I_k} y_{kiljn} \leq 1, \quad \forall k \in K,\ l \in L,\ j \in I_l,\ n \in N \tag{4}$$

$$x_{ki} - x_{k(i-1)} = h_k, \quad \forall k \in K,\ i \in I_k,\ i \geq 2 \tag{5}$$

$$x_{lj} - x_{l(j-1)} = h_l, \ \ \forall l \in L,\ j \in I_l,\ j \geq 2 \tag{6}$$

$$x_{k|I_k|} \leq T, \quad k \in K \tag{7}$$

$$x_{l|I_l|} \leq T, \quad l \in L \tag{8}$$

$$x_{k1} \in \{0, 1, 2, \ldots, h_k\}, \quad \forall k \in K \tag{9}$$

$$x_{l1} \in \{0, 1, 2, \ldots, h_l\}, \quad \forall l \in L \tag{10}$$

$$y_{kiljn} \in \{0, 1\}, \quad \forall k \in K,\ i \in I_k,\ l \in L,\ j \in I_l,\ n \in N \tag{11}$$

where x_{k1} and x_{l1} are the decision variables representing the departure time of the first vehicle trip of lines k and l. Eq. (1) serves as the objective function, designed to maximise the number of successfully coordinated transferred passengers within the network. Constraints (2) and (3) are transfer synchronisation constraints, describing that if the departure time of the i-th trip of line k minus the arrival time of passengers from the j-th trip of line l at transfer station n is within the predefined transfer waiting time range $[0, W]$, then the transfer is feasible and $y_{kiljn} = 1$; otherwise, the transfer is deemed infeasible and $y_{kiljn} = 0$. Constraint (4) ensures that each vehicle trip can connect to at most one connecting trip for transfer. Constraints (5) and (6) maintain an even headway for each line. Constraints (7) and

(8) ensure that the departure time of the last vehicle on each line is within the scheduling horizon T. Constraints (9) and (10) specify the permissible values for the decision variables. Finally, Constraint (11) defines the binary nature of y_{kiljn}.

3.2.2. Model 2: minimising the total passenger transfer waiting time

Upon solving Model 1, we can further refine the optimisation by considering passenger transfer waiting time while maintaining the number of successfully coordinated transferred passengers. Model 2 is designed to minimise the total passenger transfer waiting time while ensuring that the number of successfully coordinated transferred passengers remains unchanged. We introduce the notation w_{kiljn} to represent the transfer waiting time from the j-th trip of line l to the i-th trip of line k at transfer station n. Consequently, the total passenger transfer waiting time for this transfer is given by $p_{kiljn} \cdot w_{kiljn}$. The detailed mathematical formulations for Model 2 are provided below.

$$\min z_2 = \sum_{k \in K, k \neq l} \sum_{i \in I_k} \sum_{l \in L} \sum_{j \in I_l} \sum_{n \in N} p_{kiljn} w_{kiljn} y_{kiljn} \tag{12}$$

s.t.

$$\sum_{k \in K, k \neq l} \sum_{i \in I_k} \sum_{l \in L} \sum_{j \in I_l} \sum_{n \in N} p_{kiljn} y_{kiljn} = P \tag{13}$$

$$\begin{aligned} y_{kiljn}[w_{kiljn} - (x_{ki} + t_{kn} + t_n) + (x_{lj} + t_{ln} + w_n)] \\ = 0, \quad \forall k \in K, \ i \in I_k, \ l \in L, \ j \in I_l, \ n \in N \end{aligned} \tag{14}$$

Equations (2)–(11)

Equation (12) is the objective function, which minimises the total passenger transfer waiting time. Constraint (13) ensures that when optimising total transfer waiting time, the number of successfully coordinated transferred passengers remains maximum, which is also the objective function of Model 1. Constraint (14) calculates the transfer waiting time: if the transfer is successful, indicated by $y_{kiljn} = 1$, then the transfer waiting time w_{kiljn} is given by $(x_{ki} + t_{kn} + t_n) - (x_{lj} + t_{ln} + w_n)$. The remaining constraints of Model 2 are identical to the Constraints (2)-(11) of Model 1.

3.2.3. Model 3: minimising the longest transfer waiting time

Instead of minimising the total passenger transfer waiting time, Model 3 focuses on optimising the longest transfer waiting time. The goal is to improve the fairness of transfers by minimising the maximum waiting time experienced by passengers. A parameter w_M is introduced to represent the longest transfer waiting time. The detailed mathematical formulations for Model 3 are provided below.

$$\min z_3 = w_M \tag{15}$$

s.t.

$$y_{kiljn}(w_{kiljn} - w_M) \leq 0, \quad \forall k \in K, \ i \in I_k, \ l \in L, \ j \in I_l, \ n \in N \tag{16}$$

Equations (2)–(11), (13)–(14)

where w_M represents the longest transfer waiting time. Eq. (15) defines the objective function, which minimises the longest transfer waiting time. Constraint (16) ensures that the transfer waiting time does not exceed the longest transfer waiting time when the transfer is feasible, as indicated by $y_{kiljn} = 1$. The remaining constraints of Model 3 are identical to the Constraints (2)-(11) of Model 1 and Constraints (13)-(14) of Model 2.

4. Solution method

The three optimisation models are integer programming (IP) models, which can be solved by using heuristic and meta-heuristic algorithms, such as simulated annealing and genetic algorithms, to obtain efficient approximate solutions. This approach is particularly advantageous for large-scale, complex problems where exact solution methods may become computationally prohibitive. However, to guarantee the optimality of solutions, exact solution methods can be employed. Commercial optimisation solvers utilise exact algorithms to find optimal solutions. In this study, we utilise the exact solution method (branch-and-cut) implemented in the Gurobi optimiser to solve the three models. Additionally, the proposed Models 2 and 3 include a nonlinear constraint, specifically Eq. (12). This constraint can be handled in Gurobi by incorporating an indicator constraint of the form:

$$f = y \rightarrow a^T x \leq b$$

which indicates that if the binary indicator variable f is equal to y, where $y \in \{0, 1\}$, then the linear constraint $a^T x \leq b$ should be satisfied (Gurobi Optimization 2024). By leveraging Gurobi's capability to manage such indicator constraints, we ensure the nonlinear constraints can be linearised in the optimisation process, maintaining the rigour and accuracy of our models. Constraint (14) can be linearised as follows:

$$w_{kiljn} \leq (x_{ki} + t_{kn} + t_n) - (x_{lj} + t_{ln} + w_n) + M(1 - y_{kiljn}), \quad \forall k \in K,\ i \in I_k,\ l \in L,\ j \in I_l,\ n \in N \tag{17}$$

$$w_{kiljn} \geq (x_{ki} + t_{kn} + t_n) - (x_{lj} + t_{ln} + w_n) - M(1 - y_{kiljn}), \quad \forall k \in K,\ i \in I_k,\ l \in L,\ j \in I_l,\ n \in N \tag{18}$$

Given that the timetable coordination problem is proven to be NP-hard (Ibarra-Rojas and Rios-Solis 2012), additional variable pre-processing and solution space reduction techniques are employed to expedite the solution process. These include the incorporation of valid inequalities, which help to constrain and refine the solution space as detailed in the subsequent sub-sections.

4.1. Eliminating redundant variables

Due to the inherent structure of PT networks, the timetable coordination optimisation models often contain numerous redundant variables and constraints. These can be removed to streamline the model and accelerate the solution process (Liu et al. 2023). For instance, consider the small network depicted in Figure 3. At transfer station 1, transfers occur only between lines l_1 and l_2; there are no transfers between lines l_1 and l_3 or between lines

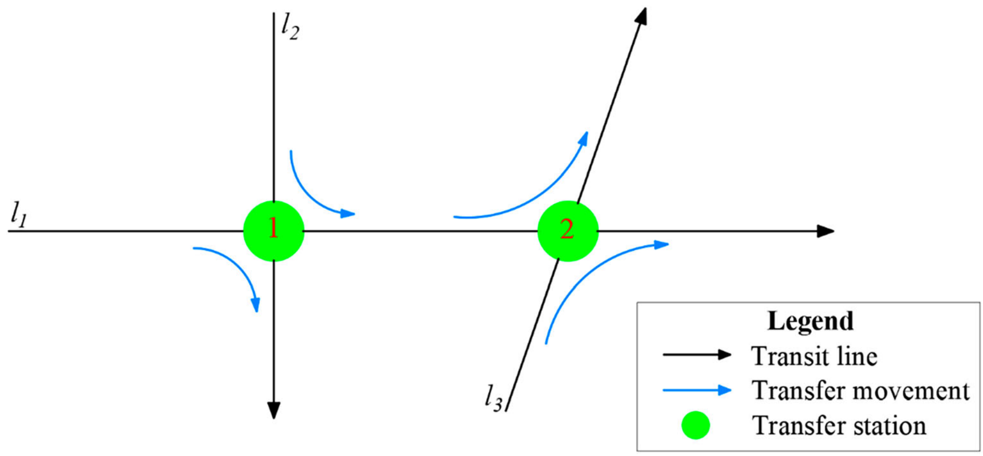

Figure 3. A small PT network with three lines and two transfer stations.

l_2 and l_3. Consequently, for the transfer coordination binary variables y_{kiljn} at $n = 1$, only $k, l \in \{1, 2\}$ are considered; variables y_{kilj1} associated with line l_3 are all zero and can be eliminated from the models, along with their corresponding constraints. Similarly, at transfer station 2, transfers occur only between lines l_1 and l_3; there are no transfers between lines l_2 and l_3 or between lines l_1 and l_2 . Thus, for y_{kiljn} at $n = 2$, only $k, l \in \{1, 3\}$ are considered; variables y_{kilj2} associated with lines l_2 are all zero and can be removed from the models, along with their related constraints.

The removal of redundant variables and constraints is directly implemented in the solver. For each transfer station, only the binary variables and related constraints pertinent to actual transfer movements are considered. By doing so, a substantial number of variables and constraints can be eliminated, thereby simplifying the model and expediting the solution process.

4.2. *Incorporating valid inequalities*

Valid inequalities can help reduce the search space of the optimisation problem without excluding any feasible solutions. By tightening the formulation, the solver can focus on a smaller, more relevant subset of potential solutions, thereby expediting the search for the optimal solution. By cutting off infeasible or suboptimal regions of the solution space early, valid inequalities can lead to faster convergence to the optimal solution. This is particularly beneficial for problems that are computationally intensive and time-consuming (Fouilhoux et al. 2016; Wolsey and Nemhauser 1999). In practice, adding valid inequalities is a common strategy adopted in commercial solvers like Gurobi and CPLEX. Some previous studies on timetable optimisation have demonstrated the advantages of incorporating valid inequalities to expedite the solution process of optimisation models (Cortés et al. 2023; Fouilhoux et al. 2016; Wang, Zhou, and Yan 2022). In this study, a set of valid inequalities, Eq. (19), is added into the original optimisation models to enhance solver efficiency, reduce computational time, and improve solution quality.

Remark 4.1: If passengers from the j-th trip of line l can transfer to the i-th trip of line k at transfer station n, then the following inequalities (17) must be satisfied, vice versa.

$$t_{ln} + w_n + (j-1)h_l \le t_{kn} + t_n + ih_k, \quad \forall k \in K, \quad i \in I_k, \quad l \in L, \quad j \in I_l, \quad n \in N, \quad i,j \ge 1 \tag{19}$$

Let us prove inequalities (19) are valid. Since the departure time of the first trip of line k, denoted as $x_{k1} \in [0, 1, \ldots, h_k]$, the latest departure time of the i-th trip of line k at the terminal station is given by $t_n + h_k + (i-1)h_k = t_n + ih_k$. Thus, the latest departure time of the i-th trip of line k at transfer station n is $t_{kn} + t_n + ih_k$. Similarly, the earliest arrival time of passengers from the j-th trip of line l at transfer station n is $t_{ln} + w_n + (j-1)h_l$. For passengers from the j-th trip of line l to transfer to the i-th trip of line k at transfer station n, the earliest arrival time of passengers from the j-th trip of line l at transfer station n must not be later than the latest departure time of the i-th trip of line k at transfer station n, i.e. $t_{ln} + w_n + (j-1)h_l \le t_{kn} + t_n + ih_k$. Similarly, it can be proven that if inequalities (19) hold, then it is possible for passengers from the j-th trip of line l to transfer to the i-th trip of line k at transfer station$n$. Hence, inequalities (19) are valid. This completes the proof.

4.3. Overall solution process

By eliminating redundant variables and incorporating valid inequalities, the solution space of the optimisation models can be significantly reduced. This streamlined solution space enables the optimisation solver Gurobi to more efficiently identify the optimal solution. The detailed procedures for reducing the solution space are outlined in Algorithm 1.

Algorithm 1: Solution space reduction procedures

Input: K, L, N, t_{kn}, h_k, I_k, t_{ln}, h_l, I_l, t_n, w_n
Output: k, i, l, j, n

for $k \in K$, $l \in L$, $n \in N$ **do**
  if *There is no transfer movement between line k and line l at transfer station n* **then**
    The corresponding variables y_{kiljn} and constraints (2)-(4),(14),(16) will not be generated;
  else
    for $i \in I_k$, $j \in I_l$ **do**
      if $t_{ln} + w_n + (j-1)h_l \le t_{kn} + t_n + ih_k$ **then**
        The corresponding variables y_{kiljn} and constraints (2)-(3),(14),(16) will be generated, variables y_{kiljn} will be included in constraint (4);
      end
    end
  end
end

The overall solution procedure for the three optimisation models is outlined in Algorithm 2.

Algorithm 2: The overall solution procedure

Input: K, L, N, t_{kn}, t_{ln}, h_k, h_l, p_{kiljn}, I_k, I_l, t_n, w_n
Output: P, all x_{ki}, x_{lj} of all models, total transfer waiting time (TTWT), and longest transfer waiting time (LTWT) with each model

for $k \in K$, $i \in I_k$, $l \in L$, $j \in I_j$, $n \in N$ **do**
  if *If the two solution space reduction procedures do not apply* **then**
    continue;
  else
    The corresponding variables y_{kiljn} and constraints (2)-(3),(14),(16) will be generated, variables y_{kiljn} will be included in constraint (4);
  end
end
for *Model 1 (Eqs.(1)-(11))* **do**
  Compute all x_{k1}, x_{l1}, and P by Gurobi;
  Compute all x_{ki}, x_{lj} by $x_{ki} = x_{k1} + (i-1)h_k$ and $x_{lj} = x_{l1} + (j-1)h_l$, $i, j \geq 2$;
  Compute transfer waiting time by $w_{kiljn} = (x_{ki} + t_{kn} + t_n) - (x_{lj} + t_{ln} + w_n)$;
  Obtain LTWT by $\max\{w_{kiljn}; k \in K, k \neq l, i \in I_k, l \in L, j \in I_l, n \in N\}$;
  Compute TTWT by

$$\sum_{k \in K, k \neq l} \sum_{i \in I_k} \sum_{l \in L} \sum_{j \in I_l} \sum_{n \in N} p_{kiljn} w_{kiljn} y_{kiljn}$$

end
for *Model 2 (Eqs.(2)-(14))* **do**
  Compute all x_{k1}, x_{l1}, and minimal TTWT by Gurobi and P from $Model$ 1;
  Compute all x_{ki}, x_{lj} by $x_{ki} = x_{k1} + (i-1)h_k$ and $x_{lj} = x_{l1} + (j-1)h_l$, $i, j \geq 2$;
  Compute transfer waiting time by constraint (14);
  Obtain LTWT by $\max\{w_{kiljn}; k \in K, k \neq l, i \in I_k, l \in L, j \in I_l, n \in N\}$;
end
for *Model 3 (Eqs.(2)-(11), (13)-(16))* **do**
  Compute all x_{k1}, x_{l1}, and minimal LTWT by Gurobi and P from $Model$ 1;
  Compute all x_{ki}, x_{lj} by $x_{ki} = x_{k1} + (i-1)h_k$ and $x_{lj} = x_{l1} + (j-1)h_l$, $i, j \geq 2$;
  Compute transfer waiting time by constraint (14);
  Compute TTWT by

$$\sum_{k \in K, k \neq l} \sum_{i \in I_k} \sum_{l \in L} \sum_{j \in I_l} \sum_{n \in N} p_{kiljn} w_{kiljn} y_{kiljn}$$

end

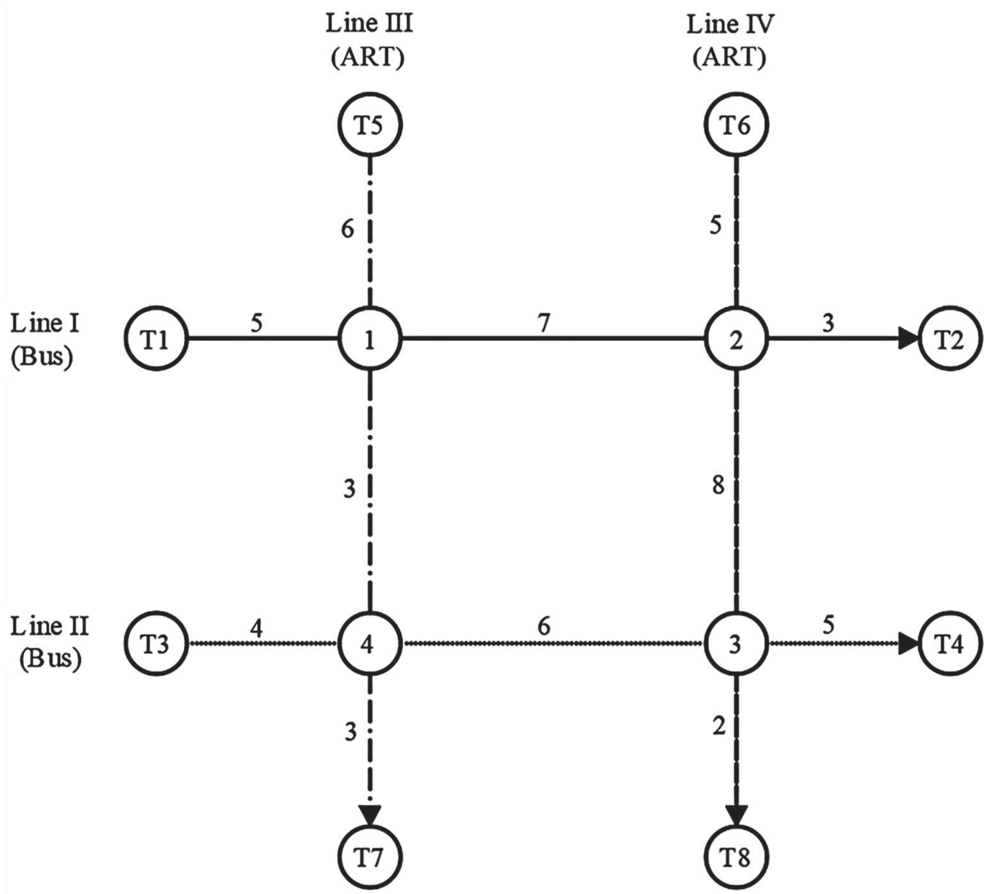

Figure 4. A toy bi-modal PT network.

5. Numerical example

In this section, we present a toy network to illustrate the optimisation models and solution methodology. Detailed computational outcomes are provided to validate the efficiency and feasibility of the proposed models and methodology.

5.1. Toy network

Figure 4 illustrates a toy network, adapted from Liu et al. (2023), comprising four unidirectional transit lines (I, II, III, and IV), four transfer stations (1, 2, 3, and 4), and eight terminal stations (T1 through T8). The numbers displayed adjacent to the route segments denote vehicle running time in minutes. The headways for lines I, II, and III are set to 10 min, while for line IV, it is 20 min. The planning period is defined as [7:00, 7:30], indicating that vehicles are permitted to depart from terminal stations only within this specified time window. Consistent with Liu et al. (2023), the dwell time of vehicles at transfer stations is included in the route segment running time.

Table 16. Number of transfer passengers of the toy network, adapted from Shafahi and Khani (2010).

Transfer station	From line	To line			
		I	II	III	IV
1	I	–	–	36	–
1	III	24	–	–	–
2	I	–	–	–	24
2	IV	16	–	–	–
3	II	–	–	–	30
3	IV	–	16	–	–
4	II	–	–	21	–
4	III	–	33	–	–

The number of passengers transferring from one line to another within the toy network is detailed in Table 16, with a total of 200 transfer passengers, as adapted from Shafahi and Khani (2010). In the numerical computation experiments, it is assumed that transfer passengers are evenly distributed over all transfer connections at the same transfer station. Specifically, for each transfer connection at a transfer station, the number of passengers transferring from the j-th trip of line l to the i-th trip of line k at transfer station n is determined by dividing the total number of transfers from line l to line k at transfer station n by the total number of transfer connections. This method ensures an even distribution of transfer passengers among transfer connections. The approach simplifies the model by eliminating the need to treat the number of transferring passengers as a variable, instead treating it as an input parameter.

5.2. Numerical results

With the input data, which includes vehicle travel time, line headways, the planning period, and the number of transferred passengers, we first solve Model 1. All optimisation models are solved by using Gurobi in conjunction with Python. Gurobi's Python interface allows for an efficient manipulation of individual variables and constraints. It is important to note that the redundant variable removal procedure was directly integrated into Gurobi. Due to the small scale of this numerical example, the addition of valid inequalities was not necessary. Based on the results from Model 1, Models 2 and 3 were subsequently solved by using Gurobi. For this numerical example network, all three models were solved within less than one second.

By solving Model 1, we obtained the maximum number of successfully coordinated transferred passengers, which is $z_1 = 192$, indicating that 8 passengers failed to transfer, specifically those transferring from the second trip of line IV to line II. The longest transfer waiting time is 16 min, occurring during the transfer from the second trip of line II to the second trip of line IV. The total passenger transfer waiting time is 392 min. Given that the number of successfully coordinated transferred passengers is consistent across all three models, the average transfer waiting time is used for comparative purposes. The average passenger transfer waiting time is calculated by dividing the total passenger transfer waiting time by the number of successfully coordinated transferred passengers. For Model 1, the average passenger transfer waiting time is 2.04 min.

Table 17. Results comparisons of the three optimisation models.

Transfer waiting time	Model 1	Model 2	Model 3
Average transfer waiting time (min)	2.04	1.77	2.40
Longest transfer waiting time (min)	16	14	10

Based on the optimal objective function value of Model 1, the maximum number of successfully coordinated transferred passengers is 192. Incorporating this into Constraint (13), we computed the results for Model 2, yielding a minimum total passenger transfer waiting time of $z_2 = 340$ and an average passenger transfer waiting time of 1.77 min. The longest transfer waiting time for Model 2 is 14 min, occurring during the transfer from the first trip of line I to the first trip of line IV.

For Model 3, by setting the maximum number of successfully coordinated transferred passengers to 192 in Constraint (13), we obtained the minimum longest transfer waiting time of $z_3 = 10$ minutes. This occurs for passengers transferring from the second trip of line I to the second trip of line IV or from the second trip of line II to the second trip of line IV. The total transfer waiting time is 460 min, resulting in an average transfer waiting time of 2.40 min for Model 3.

Table 17 summarises and compares the results of the three optimisation models. The results of the three models are further depicted using a two-dimensional graph, as shown in Figure 5, to facilitate decision-makers in balancing the average transfer waiting time (for efficiency) and the longest transfer waiting time (for fairness). Figure 5 shows that the longest transfer waiting time for Model 2 is lower than that of Model 1, indicating that reducing the total passenger transfer waiting time may also reduce the longest transfer waiting time. Conversely, the average transfer waiting time for Model 3 is higher than that of Model 1, suggesting that reducing the longest transfer waiting time may increase the total transfer waiting time. PT operators can select a preferred solution based on their operational priorities for practical implementation.

Figure 6 displays the distribution of successfully coordinated transferred passengers relative to their transfer waiting time for the three models. The results indicate that a substantial proportion of passengers experience zero-transfer waiting time across all three models: 76% in Model 1, 78% in Model 2, and 64% in Model 3. Notably, Model 2 achieves the highest number of passengers with zero-transfer waiting time. In contrast, Model 3, despite having the shortest maximum transfer waiting time of 10 min, shows a significantly higher number of passengers experiencing relatively long transfer waiting time of 4, 6, and 10 min compared to Models 1 and 2. Furthermore, Model 3 has the fewest passengers with zero-transfer waiting time among the three models.

6. Case study

To further validate the effectiveness and efficiency of the proposed models and algorithms, a case study was conducted on a real-world multimodal PT network in Yibin, China. This network encompasses both Autonomous-rail Rapid Transit (ART), also known as trackless trams, and traditional bus lines. The real-world case study provides deeper insights into the performance of the models in practical applications, allowing for a comprehensive assessment of their effectiveness and applicability in managing large-scale PT networks.

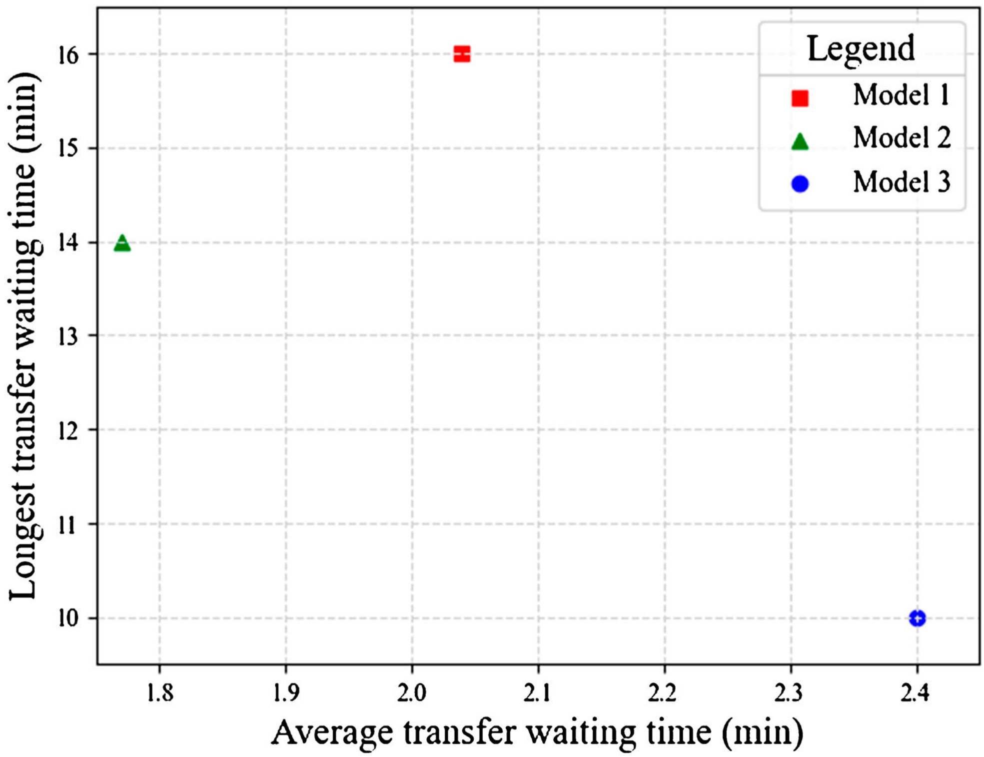

Figure 5. Trade-off between the average transfer waiting time and longest transfer waiting time.

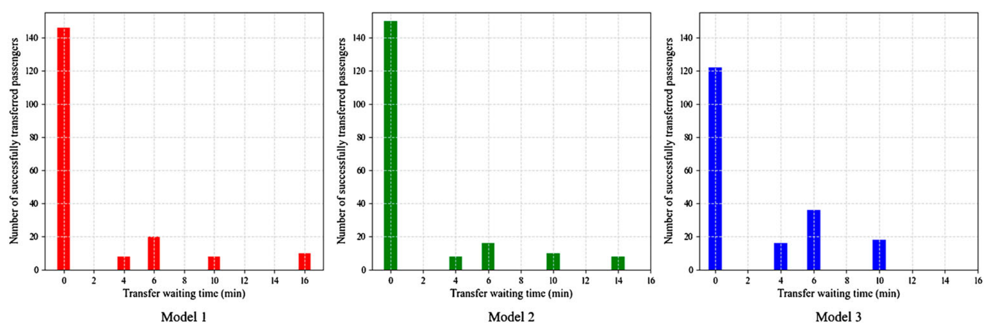

Figure 6. Distribution of transfer passengers with respect to the transfer waiting time of the three models.

6.1. PT network of Yibin

Yibin is located in the southern part of Sichuan Province, China. As of 2023, the city has a permanent resident population of approximately 4.6 million people. ART serves as a medium-capacity PT system, combining the benefits of both rail and bus. Compared to traditional rail systems, ART offers notable advantages, including lower investment costs, shorter construction periods, and flexible operations. Since the trial operation of the first ART line in Yibin in December 2019, the city has expanded to a total of four ART lines by the

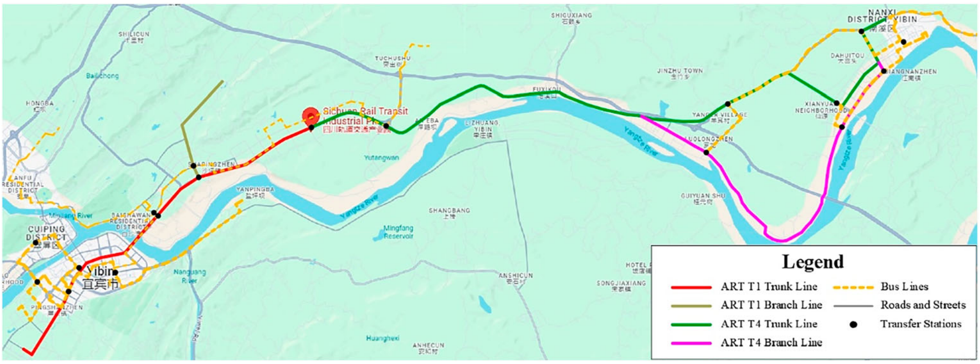

Figure 7. The bi-modal PT network of Yibin, China.

Table 18. The total number of transfer passengers for different planning periods.

Planning period	Total number of transfer passengers
[10:00, 11:00]	231
[10:00, 12:00]	380
[10:00, 14:00]	761

June of 2024 (Fang et al. 2024). With the growing ART network, the coordination between ART and bus systems has become increasingly critical. ART and regular buses both play crucial roles in urban transportation. ART serves as the backbone of urban public transport, connecting different districts of the city and accommodating a larger passenger flow. Regular buses, on the other hand, have a relatively lower passenger flow, but offer higher accessibility. Better timetable coordination between ART and buses can reduce passengers' transfer waiting time between the two transit modes, and improve their overall travel experience. Consequently, several bus routes intersecting with ART lines were selected to create a multimodal PT network for the case study. Figure 7 illustrates the ART-bus bi-modal PT network in Yibin.

6.2. Data collection

The case study PT network comprises seven bus routes and four ART lines, all operating unidirectionally. This multimodal PT network includes a total of eighteen transfer stations, as illustrated in Figure 7. The network and transfer stations are mapped onto the current road layout using AutoCAD 2022, as depicted in Figure 7. Transfer passenger numbers at each station were gathered through manual surveys, with the total transfer passenger numbers for each planning period detailed in Table 18. Headways for each route were obtained from publicly available information provided by the local ART and bus agencies, and are listed in Table 19. Vehicle travel time were collected using the AutoNavi (Gaode) map app and stored in a Microsoft Excel spreadsheet.

PT systems typically operate across multiple time periods, each with varying vehicle travel time and passenger demand. In this case study, we focus on an off-peak period from

Table 19. Line headways for the case study network.

Line ID	A1	A2	A3	A4	B1	B2	B3	B4	B5	B6	B7
Headway (min)	10	10	15	25	20	11	15	10	14	17	24

Note: Ai indicates the ART lines, Bj indicates the bus lines (i = 1,2,3,4; j = 1,2,3,4,5,6,7).

10:00 to 14:00. This period is chosen because the headways of all lines are consistent within this period, and the vehicle travel time are more stable due to reduced traffic congestion.

6.3. Results and analysis

All optimisation models and solution algorithms are implemented in Python 3.11.4 and solved using Gurobi 10.0.3 on a personal computer. The latter has the following specifications: an Intel(R) Core (TM) i5-8300H CPU @ 2.30 GHz, 8GB of RAM, and a 64-bit Windows 11 operating system.

The computational results for the three models under different planning periods – 1 hour, 2 hours, and 4 hours – are presented in Figure 8. For each planning period, all three models achieve the same number of successfully coordinated transferred passengers: 135 for the 1-hour period, 300 for the 2-hour period, and 691 for the 4-hour period. These results indicate that the longer the planning period becomes, the higher the share of successfully coordinated transferred passengers, i.e. reaching 58%, 79%, and 91% for the 1-hour, 2-hour, and 4-hour periods, respectively. One key reason for this result is that when the planning period is short, some lines may not have vehicles arriving at certain transfer stations, resulting in fewer transfer opportunities for passengers. Conversely, when the planning period is longer, more lines will have vehicles arriving at the transfer stations, thus increasing passengers' transfer opportunities. In other words, the share of successfully coordinated transferred passengers becomes higher.

Figure 8 demonstrates that across all three planning periods, Model 2 consistently yields the shortest average transfer waiting time – 5.78 min for the 1-hour period, 4.53 min for the 2-hour period, and 4.99 min for the 4-hour period. Consequently, Model 2 also results in the lowest total passenger transfer waiting time: 780 min for the 1-hour period, 1360 min for the 2-hour period, and 3445 min for the 4-hour period. However, compared to Models 1 and 3, Model 2 produces the longest individual transfer waiting time across all planning periods. For the 2-hour and 4-hour planning periods, Models 1 and 2 result in the same longest transfer waiting time.

For all three planning periods, Model 3 consistently achieves the shortest longest transfer waiting time – 19 min for the 1-hour period, 18 min for the 2-hour period, and 23 min for the 4-hour period. In the 2-hour and 4-hour planning periods, the results of Model 1 serve as compromise solutions, not dominated by the solutions of Models 2 and 3. However, for the 1-hour planning period, Model 1's solution is dominated by that of Model 3. All in all, the results from all three planning periods show that, compared to Model 3, Model 2 reduces total transfer waiting time by 12.29% to 30.31%, while the longest transfer waiting time may increase by 4.35% to 22.22%. Moreover, the number of passengers affected by the longer transfer waiting times in Model 2 is minimal, accounting for less than 3% of the total number of successfully coordinated transferred passengers. Based on these results, Model 2 appears to perform better overall. However, Model 3 still achieves some reduction

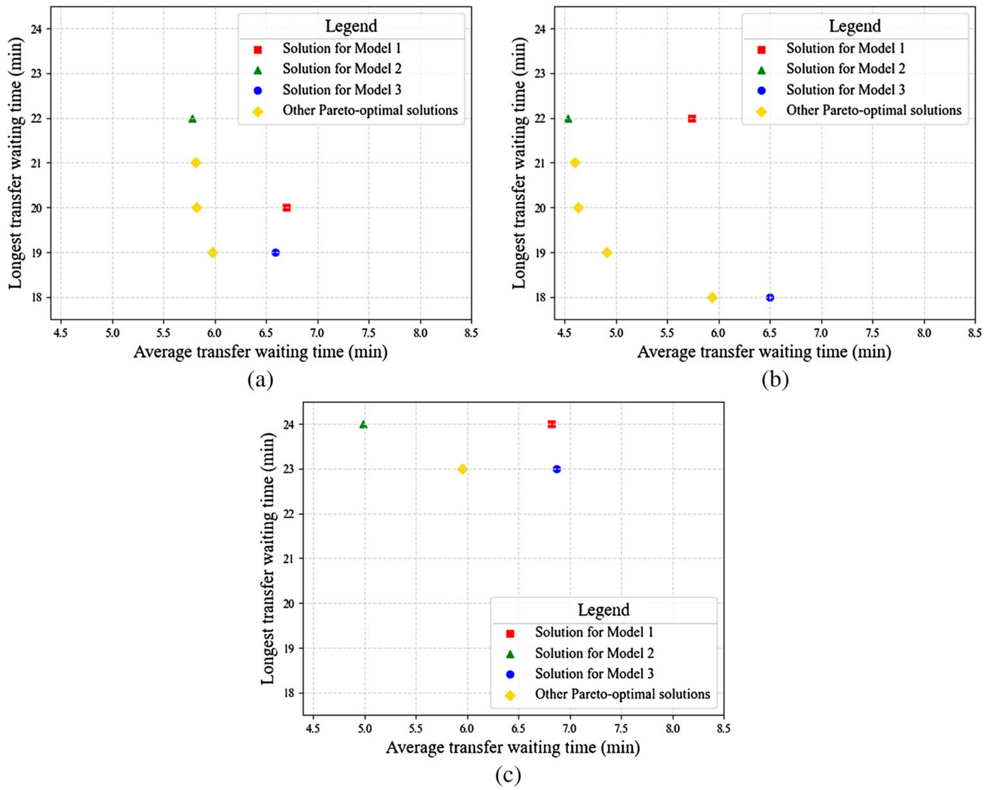

Figure 8. Comparisons of model results for three planning periods: (a) a one-hour planning period, (b) a two-hour planning period, and (c) a four-hour planning period.

in the longest transfer waiting time. Therefore, when optimising PT timetable coordination, a trade-off should be made between average transfer waiting time (or total passenger transfer waiting time) and the longest transfer waiting time.

Additionally, if multi-objective optimisation is applied, i.e. by considering total transfer waiting time and longest transfer waiting time under the condition of maximising the number of successfully coordinated transferred passengers, this may yield different results. Here we briefly describe how to obtain a subset of Pareto-optimal solutions based on the existing models. We treat the objective function of Model 3, Eq. (15), as a constraint of Model 2. Based on the longest transfer waiting time in Model 2, we reduce this value by 1 min in each iteration and use it as a new constraint for optimisation Model 2. The iteration process stops when the value of the longest transfer waiting time equals that of Model 3. This approach yields a subset of Pareto-optimal solutions that optimise the average transfer waiting time while maintaining the longest transfer waiting time. Figure 8 shows the Pareto-optimal solutions.

Figure 9 illustrates the distribution of successfully coordinated transferred passengers in relation to transfer waiting time across the three models for three planning periods. The data reveals that for all three models, at least 86% of passengers experience a transfer waiting time of no more than 10 min across all planning periods. This is largely due to the 10-minute headway of transit lines in the network. Passengers experiencing transfer

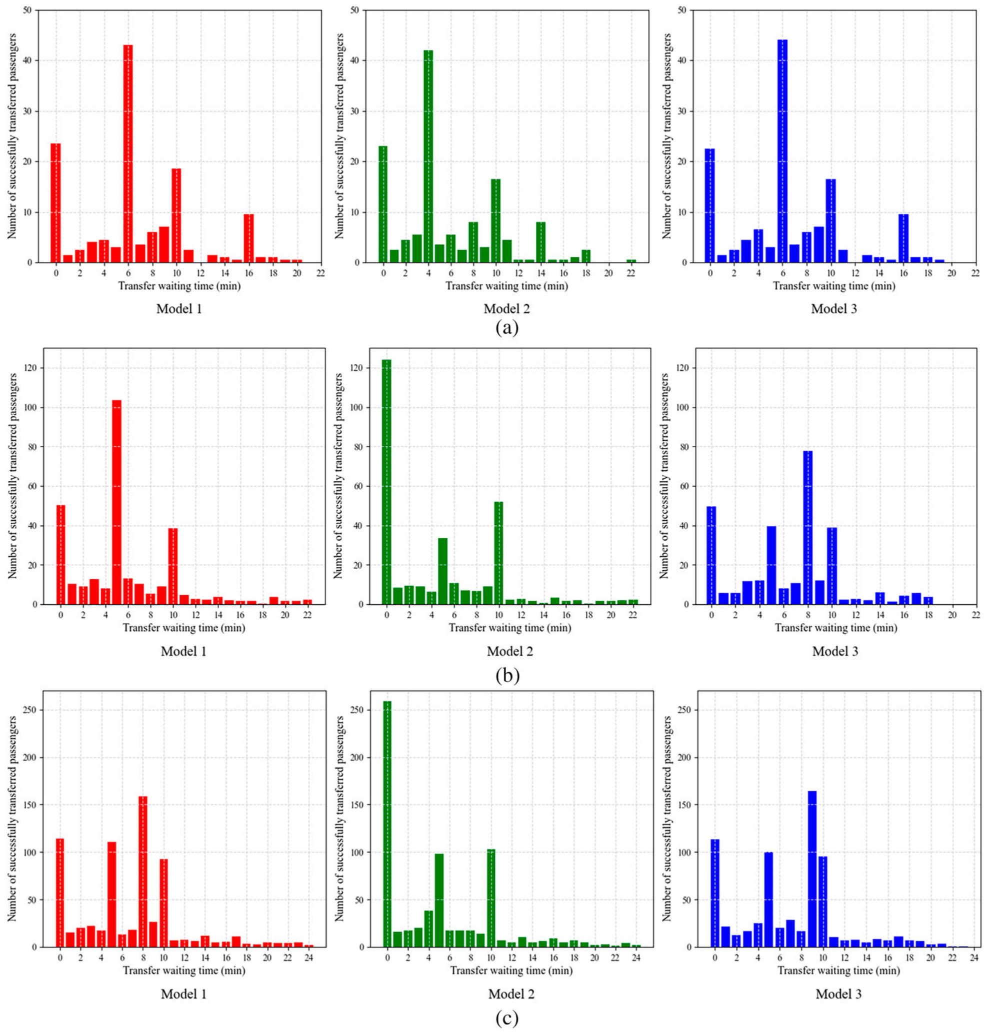

Figure 9. Distribution of transfer passengers in relation to transfer waiting time across the three models for three planning periods: (a) a one-hour planning period, (b) a two-hour planning period, and (c) a four-hour planning period.

waiting time exceeding 10 min are primarily those transferring to the first trip of the connecting transit line, underscoring the importance of optimising the departure time of the first vehicle trip to minimise excessive transfer waiting time.

Figure 9 further shows that, across all three planning periods and models, the majority of passengers either experience zero-waiting-time transfers or transfers with waiting time close to half of the line headways. Model 2 consistently results in the highest number of passengers with zero transfer waiting time and the most passengers experiencing transfers with waiting time of less than half of the minimal line headway (i.e. 5 min). While Model 3 excels in reducing the longest transfer waiting time across the three models and planning periods, it leads to a higher proportion of passengers encountering transfer waiting time exceeding half of the minimal line headway. Specifically, 70% of passengers for the

Table 20. Computation time (s) comparisons without and with incorporating valid inequalities.

Planning periods		Models		
		Model 1	Model 2	Model 3
2 h	Without	5	3657.61	1273.90
	With	2	181.94	50.70
4 h	Without	30.87	$>$ 86,400	$>$ 86,400
	With	27.64	1823.73	7219.52

1-hour period, 59% for the 2-hour period, and 58% for the 4-hour period experience longer transfer waiting time of more than 5 min with Model 3. This suggests that Model 2 is superior not only in reducing total and average transfer waiting time but also in minimising the number of passengers experiencing longer transfer waiting time. However, Model 3 has the advantage of reducing the longest transfer waiting time. Consequently, there is a trade-off between reducing the longest transfer waiting time (for fairness) and reducing the total transfer waiting time (for efficiency).

To further assess solution efficiency, we compare the computation time of the three models. For the 1-hour planning period, due to the smaller problem scale, all three models are solvable in less than 5 s without the need for additional variable pre-processing or solution space reduction techniques. However, for the 2-hour and 4-hour planning periods, the models could not be efficiently solved without incorporating valid inequalities. Table 20 presents the computation time comparisons for the 2-hour and 4-hour planning periods. The data indicate that the elimination of redundant variables and the incorporation of valid inequalities significantly reduced computation time across all models. Specifically, for the 4-hour planning period, Models 2 and 3 could not be solved within 24 h without these optimisation procedures. In all cases where a solution was obtained, the optimality gap is 0%. The results show that the proposed method consistently provides optimal solutions.

To demonstrate the computational complexity of the optimisation model and the impact of incorporating valid inequalities, we further conducted a comparison of the number of variables and constraints both with and without the use of valid inequalities, for planning periods of two and four hours. The results are presented in Table 21, which shows that the number of variables can be reduced by over 40%, demonstrating a significant reduction. Additionally, since transfer opportunities remain consistent across all models within the same planning period, the reduction rates of variables are identical for different models within the same planning period. Furthermore, the reduction in the number of constraints is not as substantial. This is primarily due to Constraint (4), which considers that passengers from each trip on line l can only transfer to at most one trip on line k. As the transfer relationship and the number of vehicles are fixed, the number of constraints (4) does not decrease when valid inequalities are applied. However, by leveraging valid inequalities, redundant variables y_{kiljn} in Constraint (4) can still be eliminated. Finally, after excluding Constraint (4), it shows that the number of constraints directly affected by valid inequalities can be reduced by more than 40%. These results highlight the substantial benefits of eliminating redundant variables and incorporating valid inequalities in enhancing solution efficiency for large-scale timetable coordination problems.

Table 21. Comparing the number of variables and constraints with and without incorporating valid inequalities.

Planning periods	Models	With or without incorporating valid inequalities	Number of variables	Reduction rate	Number of constraints	Reduction rate	Number of constraints (except constraint (4))	Reduction rate
2 h	M1	Without	3427	43.13%	12,470	28.21%	8042	43.75%
		With	1949		8952		4524	
	M2	Without	6854	43.13%	14,802	31.02%	10,374	44.25%
		With	3898		10,211		5783	
	M3	Without	6854	43.13%	18,805	33.98%	14,377	44.45%
		With	3898		12,415		7987	
4 h	M1	Without	13,758	46.33%	40,925	35.98%	32,015	46.00%
		With	7384		26,199		17,289	
	M2	Without	27,516	46.33%	50,460	38.33%	41,550	46.55%
		With	14,768		31,120		22,210	
	M3	Without	27,516	46.33%	65,552	39.50%	56,642	45.71%
		With	14,768		39,659		30,749	

7. Conclusion

This study proposes three mathematical programming models with distinct optimisation objectives to enhance transfers in a multimodal PT network. The first model aims to maximise the total number of passengers benefiting from successful transfers. Building on the first model, the second model focused on minimising the total passenger transfer waiting time while ensuring the number of successfully coordinated transferred passengers. The third model aims to minimise the longest transfer waiting time, also guaranteeing the number of successfully coordinated transferred passengers. These models are solved using the commercial optimisation solver Gurobi. The models and solution methods are demonstrated for a small toy network and a multimodal PT network of Yibin, China. For larger instances, the eliminating redundant variables and incorporating valid inequalities techniques are utilised to expedite the solution process. The outcomes of the second and third models provide public transport decision-makers with the ability to balance timetables that minimise total passenger transfer waiting time against those that minimise the longest transfer waiting time.

Our findings show that Model 2, which optimises the total transfer waiting time based on Model 1, demonstrates the highest effectiveness. A low total transfer waiting time signifies a high service level across the entire network. Although it may result in a longer longest transfer waiting time, the impact on overall model results is minimal due to the small number of passengers experiencing the longer longest transfer waiting time. Model 3, which focuses on minimising the longest transfer waiting time, is less effective. In some cases, Model 3's result may lead to an increased total transfer waiting time compared to Model 1. This issue is likely due to passengers transferring to the first departure trip. Thus, there is a trade-off between transfer fairness and overall efficiency. Furthermore, the results indicate that as the planning period length increases, the share of successfully coordinated transferred passengers also rises.

One limitation of this study is that the results comparison only considers a single feasible solution resulting from Model 1. A more comprehensive analysis could be achieved by exploring and comparing all potential feasible solutions. This would allow for a deeper understanding of the solution space and provide more policy implications into the trade-offs between different optimisation objectives. Future research could focus on identifying and analysing all feasible solutions to offer a more comprehensive comparison. Additionally, future research may consider: (i) optimising the departure time of the first trip of each transit line to further reduce the longest transfer waiting time; (ii) optimising the departure time of the last trip to increase the transfer success rate and accessibility (Chen et al. 2019b; Dou, Meng, and Guo 2015); (iii) including transfer penalties and valuation of interchanges of a multimodal transit network in the optimisation objectives (Yap, Wong, and Cats 2024); (iv) taking into account more accurate transfer passenger demand and uncertainties in vehicle travel time and passenger demand to develop more robustly coordinated timetables (Wu et al. 2015; Yang et al. 2021); (v) designing algorithms with high computational efficiency to solve timetable optimisation problems for more complex scenarios (e.g. a full day, additional lines or transfer stations) to obtain more practical results.

Disclosure statement

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