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Pavel, Marilena D.; den Ouden, Wessel; Bertolani, Giulia; Giulietti, Fabrizio

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UNDERSTANDING THE EFFECTS OF ROTOR DYNAMICS ON HELICOPTER INCREMENTAL NON-LINEAR CONTROLLERS

Marilena D. Pavel†, Wessel den Ouden†

Giulia Bertolini‡, Fabrizio Giulietti‡

†Faculty of Aerospace Engineering, Delft University of Technology, The Netherlands

‡ Flight Mechanics Laboratory, University of Bologna, Italy

Abstract
With an increasing trend towards automatic flight control system applied to rotorcraft, the goal of the present paper is to understand the effects of rotor dynamics on the design of robust incremental non-linear controllers such as INDI (Incremental nonlinear Dynamic Inversion) and IBS (Incremental Backstepping Control). Nonlinear dynamic controllers are a desirable solution to helicopter flight control as it can solve its highly nonlinear dynamic behavior. However, conventional nonlinear controllers heavily rely on the availability of accurate model knowledge and this can be problematic for rotorcraft. Therefore, incremental control theory can solve the modelling errors sensitivity by relying on the information obtained from the sensors instead. The paper will demonstrate that for helicopters the incremental nonlinear controllers depend on the delays introduced in the controller by rotor dynamics. The paper will show how the residualization and synchronization methods need to be applied to an IBS controller in order to remove the effects of the flapping (disc-tilt) dynamics from the controller. This indicates that the incremental nonlinear controllers can have relatively small stability robustness margin when subjected to rotorcraft time delays and unmodelled dynamics that influence the feedback path and should be therefore carefully applied.

1. INTRODUCTION

While in the helicopter world the search for a more stable helicopter was directed towards rotor design, the fixed-wing world was evolving towards automatic flight control systems. However, with the evolution of modern rotorcraft, advanced control models are increasingly used to modify the aircraft dynamics and improve rotorcraft control characteristics. Thinking at the design of a flight control system (FCS) for helicopters, this task is rather complex due to some main deficiencies that complicate the attaining of a proper rotorcraft model, e.g.:

1. impurity of the primary response in all axes (roll rate p , pitch rate q and yaw rate r), with fast transitions between attitudes and rates control, in a degree depending on the flight speed.
2. strong cross-couplings in all axes.
3. unstable characteristics in hover. Poor stability especially at low speed with longitudinal and lateral body eigenmodes characterized by low damping and low frequency. Moving into forward flight, depending on the rotor hub configuration, the body eigenmodes frequency and damping are increasing for some helicopters while for others the damping can reduce and stability can worsen (particularly for the highly responsive hingeless rotors).
4. degradation of helicopter's response and handling qualities at the limits of the flight envelope; "Carefree handlings" allowing the flight control system to deal with any pilot inputs while it takes care of the structural and aerodynamic limits of the aircraft and simultaneously maximizes aircraft performance difficult to achieve throughout the whole flight envelope.
5. the rotor acts like an actuator, filtering the high bandwidth required to fly high precision tasks. The rotor degrees of freedom (DoFs), i.e. the flap, lag, and torsional motions, are significantly more complex than a simple servo system and have low enough damping to compromise stability for high pilot gain control tasks.

These deficiencies make the helicopter synonymous with "dangerous", even though this is not all inclusive. Indeed, from flight control point of view helicopters are:

1) unstable systems: such systems are fundamentally, and quantifiably, more difficult to control than stable ones.
2) their controllers as they are related to unstable systems are operationally critical.
3) helicopters' closed-loop systems with unstable components are only locally stable.

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Control difficulties associated with the helicopter unstable poles arise not merely because the body eigenmodes are characterized by low damping and low frequency. Rather, they arise if the ratio of an unstable mode to available bandwidth (BW) is large (Stein, 2003) or in other words, the eigenmode pole is large compared with available bandwidth. A controller's goal would be to keep this ratio as small as possible over the considered range of control as this will keep small the sensitivity of the controller in the feedback loop (the so-called minimum broomstick problem). A large sensitivity of the controller will put the feedback loop close to the critical unstable point of the system), and even minor imperfections in the controller implementation will cause instability. However, in the case of a helicopter, vehicle reduced bandwidth due to rotor dynamics results in a larger ratio of unstable mode to BW and therefore in larger sensitivity of a controller in the feedback loop. Not only has the plant (helicopter) itself become difficult to control but the compensator as well through the pilot actions. Indeed, the stick operations by a human operator introduces many complex limitations associated with perception, computation, and actuation of limbs. Many years of study and experimentation have gone into the characterization of these limitations. Such limitations are usually cured in the FCS design, but often at the expense of introducing even further lags into the control loops. With typical helicopter actuator and rotor time constants, the total effective time delay between pilot control input and rotor control demand can be of more than 200 ms (see Table 1 from Tischler (1990) showing the typical equivalent time delays that are the result of implementing a digital FCS in a helicopter). Such a time delay can be a strong cause of mental mismatch for the pilot with vehicles' dynamics and can halve again the response bandwidth capability of an 'instantaneous' rotor. It follows that a vicious loop is created between unstable poles- reduced bandwidth- time delays that will create a high sensitivity in the feedback design.

Table 1 Equivalent time delays for rotorcraft (Tischler, 1990)

Element	Delay (ms)	% of total
Rotor	66	30
Actuators	31	14
Control laws	17	8
Computations	22	10
Notch filter	11	5
Stick dynamics and filtering	76	34
Total delay	223	

The helicopter as a real physical systems has therefore a multitude of limitations on available bandwidth, not just one or two that can be easily pushed out or removed. Some of the major hardware elements in the control loop that limit this bandwidth include the following (Stein, 2003):

1) sensors: rate gyros and accelerometers, used for innerloop stabilization. Their bandwidths, measured in the usual 3-dB-gain sense, are typically 120 rad/s or more (Stein, 2003).
2) actuators: high-pressure hydraulic systems with servos to position each aerodynamic surface of the rotor blade. The available bandwidth of actuators is approximately 70 rad/s, at least for small signals (Stein 2003)
3) aerodynamics: flow conditions around the aircraft that map its geometric configuration.
4) Airframe: the mechanical structure linking the attachment points of actuators to the attachment points of sensors.
5) control processors: digital computer systems sampling sensor data and computing surface commands at 80 Hz. This update rate can pass signals with good fidelity up to 30-40 rad/s (two to three samples per radian).

This list above imposes severe limitations on helicopters' available bandwidth (approximately 30 rad/s (Padfield, 2007)). This comes from the helicopter characteristic mechanical structure and from the sampling rate of control computers. "*These limitations cannot simply be ignored, as we often do in formal (control theories). Instead, we should fashion theories to clearly expose the consequences of these limitations, and we should honestly tell ourselves and our employers what the consequences are.*" (Stein, 2003). The goal of the present paper is to exemplify such limitations for the case of a helicopter nonlinear control.

2. NONLINEAR DYNAMIC CONTROL METHODS

Up to the 1980's aircraft flight control had progressed from a simple fixed-gain PID feedback control applied to a linearized structure, to multivariable feedback laws, designed with modern control tools that optimally select command responses according to the task performed, disturbances encountered, and robustness characteristics of the closed-loop airframe/controller system.

During the 1980s, control engineers have commenced to apply alternate methodologies to linear control law methods, which deal directly with nonlinear models rather than with their linear approximations. Such an approach is more advantageous for helicopters: these vehicles are highly nonlinear and unstable systems as described in the Introduction. Indeed, with the introduction of fly-by-wire control systems (Stiles et al. 2004), digital control algorithms could be used; these systems are capable of providing an improved stability and predictable control for helicopter. Next, the theoretical foundations behind non-linear control methods based on Nonlinear Dynamic Inversion (NDI) and Backstepping (BS) are described.

Nonlinear Dynamic Inversion: Nonlinear dynamic inversion (NDI) (also called dynamic inversion (DI) or feedback linearization) is a model-based controller wherein the nonlinear plant dynamics are cancelled out by effectively multiplying state feedback signals with the inverse of the dynamic equations.Theoretically, this method aims at modeling all of the system's nonlinearities in order to remove them by using direct state feedback linearization. The full derivation of NDI method was presented in multiple studies (eg. Meyer, Hunt and Su, 1982; Landis and Glusman, 1987; Enns et. al., 1994). The method has been successfully applied to highly nonlinear systems such as aircraft high angle of attack maneuvering flight (eg. Lane and Stengel (1988), Bugajski and Enns (1992), Reinier, Balas and Garrard, 1996; Hovakimyan et. al., 2001; Lee et. al., 2007).

Backstepping: Backstepping (BS) control uses the Lyapunov stability functions (Acquatella, 2020; Van Kampen, and Chu, 2013) for the plant so that the global stability can be guaranteed. In BS control, each state with slower dynamics is a virtual input control for the states with faster dynamics; this is done until the state controlled by the input control is reached. The method has been succesfuly applied to aircraft control. In this case, the pitch rate will serve as a virtual control for the pitch angle while the pitch rate itself will be controlled by the elevator deflection.

2.1 Incremental nonlinear control

The NDI and BS control as model-based controllers of nonlinear systems have some common drawbacks. A first drawback is that they rely on the availability of accurate models of the controlled vehicle. This means that they are very sensitive to model inaccuracies (e.g. da Costa, Chu & Mulder, 2003). If the model is inaccurate or the dynamic characteristics of the controlled element change, for instance due to a failure, the controller may become unstable.Obtaining an accurate model is often expensive or impossible with the constraints of the sensors that need to be carried onboard of the vehicle. For rotorcraft this will often be the case as the **helicopter simulation models are often non accurate and have various states which cannot be measured**, which is problematic as feedback linearization requires full state feedback. A second drawback of conventional nonlinear control method such as NDI/BS is that it requires the plant model to be affine in control and minimum phase (Horn, 2019). Non-minimum phase systems have zeros in the right half plane of the complex plane which become, after inversion, unstable poles. These can become a problem if present in the closed loop system. However, **rotorcraft is known to be a non-minimum phase** system especially in low speed envelope and therefore the nonlinear controllers are problematic. Also helicopter models can be **not-affine in the control inputs**. Actuator dynamics and rotor dynamics (most likely flapping dynamics) can cause the designed INDI controller to overcontrol the helicopter as proved by (Pavel et. al., 2020) for the Apache's helicopter with the classic ±20% actuator authority given to the flight control system.

In order to eliminate the poor robustness of such nonlinear controllers, successful attempts for helicopters have been made on:

1) dynamically compensating the error in the plant inversion to yield the desired response of selected control variables using an on-line neural net or disturbance observer (e.g. Prasad and Lipp, 1993),
2) designing an additional robust control loop using H_∞ controller (Walker, Turner and Gubbels, 2001) or mu-synthesis based controller.

All these methods are based on the regular explicit NDI method, and this implies that still not all uncertainties are taken into account or they are covered by lumped uncertainties. Also, usually, the control system structure is rather complex.

One alternative solution is to design a controller that is less sensitive to model mismatches and that requires minimal a priori knowledge of the vehicle model. This can be ensured by relying on sensor measurements of the controlled states instead of relying on system dynamics modeling and the approach belongs to 'incremental controllers'. Such controllers are not sensitive to modeling errors, something which is often the case for nonlinear systems such as helicopters.

The first step when applying incremental controllers to a system is to create an incremental system description. Consider the aircraft rotational dynamics described using the nonlinear dynamic system as following:

$$\underline{\dot{x}} = f\left(\underline{x}\right) + g\left(\underline{x},\underline{u}\right) \quad (1)$$

where $\underline{x}$ denotes the aircraft state vector, $\underline{u}$ is the control input vector, f is the system dependent dynamics and g is the control dependent dynamics. To this system one can applying a Taylor series expansion on the dynamics that need to be controlled. Consider $[\underline{x}_0,\underline{u}_0]$ the state at the current time point assume that its state derivative $\underline{\dot{x}}_0 = \underline{f}\left(\underline{x}_0,\underline{u}_0\right)$ can be measured. Using a standard Taylor series expansion one can obtain the first-order approximation of the state derivative for $\underline{x}$ and $\underline{u}$ in the neighborhood of $[x_0,u_0]$ as:

$$\begin{aligned}\underline{\dot{x}} = {} & \underline{f}\left(\underline{x}_0,\underline{u}_0\right) + \underline{g}\left(\underline{x}_0,\underline{u}_0\right) + \frac{\partial}{\partial \underline{x}}\left[\underline{f}\left(\underline{x}\right) + \underline{g}\left(\underline{x},\underline{u}\right)\right]\Bigg|_{\substack{\underline{x}=\underline{x}_0\\ \underline{u}=\underline{u}_0}} \left(\underline{x}-\underline{x}_0\right) \quad + \\ & \frac{\partial}{\partial \underline{u}}\left[\underline{f}\left(\underline{x}\right) + \underline{g}\left(\underline{x},\underline{u}\right)\right]\Bigg|_{\substack{\underline{x}=\underline{x}_0\\ \underline{u}=\underline{u}_0}} \left(\underline{u}-\underline{u}_0\right) + \mathrm{O}\left(\left(x-x_0\right)^2,\left(u-u_0\right)^2\right)\end{aligned} \quad (2)$$

$$\underline{\dot{x}} = \underline{\dot{x}}_0 + F\left(\underline{x}_0,\underline{u}_0\right)\Delta\underline{x} + G\left(\underline{x}_0,\underline{u}_0\right)\Delta\underline{u} + \mathrm{O}\left(\left(x-x_0\right)^2,\left(u-u_0\right)^2\right) \quad (3)$$

Equation (3) can be simplified when some assumptions are made. First, the system sample rate should be high enough, i.e. the sensors and controller operate at a sufficiently high frequency. Second, the actuators are assumed to react instantly to command signals. Finally, it is assumed that, for very small time increments (high sampling frequencies of the controller), the changes in the states x are slow compared to the changes in control input u, in other words the u can change significantly faster than x (so that $x \approx x_0$ even if $u \neq u_0$). This is so-called "time scale separation". By assuming time scale separation, it follows that the assumption $x - x_0 = 0$ can be made. This means that as $\underline{x}$ approaches $\underline{x}_0$ the term in $F\left(\underline{x}-\underline{x}_0\right)$ vanishes so that eq. (3) can be simplified as:

$$\Delta\underline{\dot{x}} \approx G\left(\underline{x}_0,\underline{u}_0\right)\Delta\underline{u} \quad (4)$$

where *G* represents the so-called 'control effectiveness' matrix and $\Delta\underline{u} = \underline{u} - \underline{u}_0$ an incremental input control. Therefore the system coefficients in *F* do not need to be estimated and only control effectiveness *G* remains. The resulting system of Equation (4) is a simplified description of the system which, assuming that all of the controlled states can be measured and the sampling rate is sufficiently high, it can be used to construct an incremental controller controlling the system using increments of control input $\Delta\underline{u}$. Incremental control methods are usually used only for the dynamics part of a system, since the kinematics part is well known and can be dealt with using classic control methods.

2.1.1 Incremental nonlinear dynamic inversion INDI

With the incremental system description given by eq. (4) one can implement several incremental control algorithms. The first algorithm discussed in this paper is the Incremental Nonlinear Dynamic Inversion (INDI). The incremental technique of nonlinear dynamic inversion (INDI) is less model-dependent and more robust. It has been described in the literature since the late 1990s (eg.Bacon and Ostroff, 2000; Ostroff and Bacon, 2002; Bugajski and Enns,1992; Chen and Zhang, 2008; Lee, Ham and Kim, 2005; Cox and Cotting, 2005), sometimes referred to as simplified (Smith, 1998) or enhanced (Ostroff and Bacon, 2002) NDI. In simplified NDI, the aircraft is controlled via body rotational velocities (pitch, roll, yaw) instead of pilot control inputs. Introducing in the controller the sensed and demanded rotational accelerations and control surface deflections and feeding back continuously these values from rotational accelerometers mounted on the vehicle structure, one can compute the control inputs required to reach the target vehicle rotational velocities (pitch, roll yaw). In enhanced INDI, instead of modeling the angular acceleration based on the state and inverting the actuator model to get the control input as in NDI, the angular acceleration is measured by onboard sensors, and an increment of the control input is calculated based on a desired increment in angular acceleration. For rotorcraft, Howitt(2006) explored the potential of INDI approach to control a model-scale rotor rig facility in order to counter air resonance and allow carefree handling protection of hub moment limits. Di Francesco and Mattei (2016) explored the application of INDI to a tiltrotor configuration. At Delft University of Technology, research has been carried out on the robustness of INDI applied to fixed wings (Siebeling, Chu and Mulder, 2010, Simplicio et. al., 2013), MAVs (Smeur et. al., 2016), helicopters (Simplicio et. al, 2013; Pavel et. al., 2020).

Since INDI makes use of sensor measurements, it is considered a sensor-based approach. This way, any unmodeled dynamics, including wind gust disturbances, are measured and compensated. Amongst INDI's advantages one can mention:

1) the control structure is simple;
2) re-allocation of control power and the reconfiguration of control law can be achieved conveniently and rapidly;
3) it is a semi-model free approach;
4) no model identification is needed;
5) the controller is inherently robust.

However, INDI faces also major challenges:

1) the measurement of angular acceleration is often noisy and requires filtering. This filtering introduces a delay in the measurement, which needs to be compensated and;
2) the method relies on inversion and therefore needs a control effectiveness model.

The INDI control law is constructed by inverting the system description of eq. (4), see eq. (5). Thereafter, the state derivative variable $\dot{x}$ is replaced by virtual control input *v*, see eq. (6).

$$\begin{aligned}\Delta\underline{u} &= G^{-1}\left(\underline{x}_0,\underline{u}_0\right)\Delta\underline{\dot{x}} \\ \Delta\underline{u} &= G^{-1}\left(\underline{x}_0,\underline{u}_0\right)\left(\underline{\dot{x}}-\underline{\dot{x}}_0\right)\end{aligned} \tag{5}$$

$$\Delta\underline{u} = G^{-1}\left(\underline{x}_0,\underline{u}_0\right)\left(\underline{v}-\underline{\dot{x}}_0\right) \tag{6}$$

Usually $\underline{v}$ is generated by linear controllers of aircraft rotational velocities (p roll rate, q pitch rate ,r yaw rate). At this point, the main advantage of the INDI can already be identified: the control law (6) for u does not depend anymore on the direct nonlinear feedback term $f(x)$ needed in the regular explicit NDI control. This means that the INDI controller is insensitive to the part of the model that only depends on the system states x. In other words, changes in $f(x)$ are reflected in the measurement of state varying rate $\underline{\dot{x}}_0$, so the controller sensitivity to aircraft aerodynamic model, uncertainty and perturbation is decreased. In the outer control loop of the plant, the system that determines virtual control $\underline{v}$, will have a linear behaviour of the inner loop system. Virtual control $\underline{v}$ can be governed by a PID controller that minimizes the error between a reference signal and a certain state, for instance pitch attitude or rate (Note: A downside of INDI is that stability cannot be guaranteed and the outer-loop PID controller still needs tuning.). However, the disadvantage of the INDI controller is that it does need the vehicle's control derivatives G_0, as well as the online measurements (or estimation) of the state derivative $\underline{\dot{x}}_0$ and the control position $\underline{u}_0$. The effectiveness of the controller is therefore dictated by the accuracy of the sensors (or filtering processes).

The robustness evaluation of this controller follows the same procedure applied for the NDI. Assuming ideal sensors, all the model inaccuracies lie in G (uncertainties in f are reflected in $\underline{\dot{x}}_0$). When $\underline{\dot{x}}_0$ is measureable, the only uncertainty within the INDI controller is in the control effective matrix $G\left(\underline{x}_0,\underline{u}_0\right)$. When uncertainties exist in the control effectiveness matrix *G*, the system can be rewritten as:

$$\dot{\underline{x}} = \dot{\underline{x}}_0 + G_0\left(\underline{x}_0, \underline{u}_0\right)\Delta\underline{u} + \Delta G\left(\underline{x}\right)\Delta\underline{u} \tag{7}$$

Applying INDI to the uncertain system (7) and using the nominal control increment, the closed-loop system becomes:

$$\begin{aligned} \dot{\underline{x}} &= \dot{\underline{x}}_0 + G_0\left(\underline{x}_0, \underline{u}_0\right)G_0^{-1}\left(\underline{x}_0, \underline{u}_0\right)\left[\underline{\nu} - \dot{\underline{x}}_0\right] + \Delta G\left(\underline{x}_0, \underline{u}_0\right)G_0^{-1}\left(\underline{x}_0, \underline{u}_0\right)\left[\underline{\nu} - \dot{\underline{x}}_0\right] \\ &= \underline{\nu} + \Delta G\left(\underline{x}_0, \underline{u}_0\right)G_0^{-1}\left(\underline{x}_0, \underline{u}_0\right)\underline{\nu} - \Delta G\left(\underline{x}_0, \underline{u}_0\right)G_0^{-1}\left(\underline{x}_0, \underline{u}_0\right)\dot{\underline{x}}_0 \end{aligned} \tag{8}$$

With the assumption of high sample rate, the difference between two consecutive measurements of the state vector derivative can be neglected, i.e. $\dot{\underline{x}}_0 \approx \dot{\underline{x}}$. The uncertain closed-loop system is further simplified as:

$$\dot{\underline{x}} = \underline{\nu} + \Delta G\left(\underline{x}\right)G_0^{-1}\left(\underline{x}\right)\underline{\nu} - \Delta G\left(\underline{x}\right)G_0^{-1}\left(\underline{x}\right)\dot{\underline{x}} \tag{9}$$

resulting in:

$$\dot{\underline{x}} = A^{-1}A\underline{\nu} = \underline{\nu} \tag{10}$$

where $A = \left[I + \Delta G\left(\underline{x}\right)G_0^{-1}\left(\underline{x}\right)\right]$. Therefore, when the sampling frequency of the controller is high enough, the result $\dot{\underline{x}} = \underline{\nu}$ still holds, meaning that uncertainties in the control effectiveness matrix *G* do not significantly affect the INDI-based control loop and no robust control design is needed in this case. This is a remarkable result under the conditions that the angular acceleration is measured and sample rate is high. The control structure is simple, there is no need for all state accelerations (derivatives), different controlled outputs require different state accelerations. The angular acceleration can be obtained (processed) from angular rate sensors (rate gyros) which are available in most cases in the aircraft.

In the case of a helicopter the control effectiveness matrix *G* is a 3 by 4 matrix and can be written as:

$$\underline{G}(\underline{x}_0, \underline{u}_0) = \begin{bmatrix} \left(\dot{\underline{\omega}}^T\left(\underline{x}_0, \underline{u}_0 + \left[\tau_{\theta_{1s}} \quad 0 \quad 0 \quad 0\right]^T\right) - \dot{\underline{\omega}}^T\left(\underline{x}_0, \underline{u}_0 - \left[\tau_{\theta_{1s}} \quad 0 \quad 0 \quad 0\right]^T\right)\right) \Big/ 2\tau_{\theta_{1s}} \\ \left(\dot{\underline{\omega}}^T\left(\underline{x}_0, \underline{u}_0 + \left[\tau_{\theta_{1c}} \quad 0 \quad 0 \quad 0\right]^T\right) - \dot{\underline{\omega}}^T\left(\underline{x}_0, \underline{u}_0 - \left[\tau_{\theta_{1c}} \quad 0 \quad 0 \quad 0\right]^T\right)\right) \Big/ 2\tau_{\theta_{1c}} \\ \left(\dot{\underline{\omega}}^T\left(\underline{x}_0, \underline{u}_0 + \left[\tau_{\theta_{0tr}} \quad 0 \quad 0 \quad 0\right]^T\right) - \dot{\underline{\omega}}^T\left(\underline{x}_0, \underline{u}_0 - \left[\tau_{\theta_{0tr}} \quad 0 \quad 0 \quad 0\right]^T\right)\right) \Big/ 2\tau_{\theta_{0tr}} \\ \left(\dot{\underline{\omega}}^T\left(\underline{x}_0, \underline{u}_0 + \left[\tau_{\theta_0} \quad 0 \quad 0 \quad 0\right]^T\right) - \dot{\underline{\omega}}^T\left(\underline{x}_0, \underline{u}_0 - \left[\tau_{\theta_0} \quad 0 \quad 0 \quad 0\right]^T\right)\right) \Big/ 2\tau_{\theta_0} \end{bmatrix}^T \tag{11}$$

Relation (11) shows that the elements of G matrix can be obtained by perturbing each helicopter control variable (longitudinal control θ_{1s}, lateral control θ_{1c}, tailrotor collective θ_{0tr} and rotor collective pitch θ_0) separately and determine its effect on the angular accelerations $\dot{\underline{\omega}}$, where $\tau_{\theta_{1s}}$, $\tau_{\theta_{1c}}$, $\tau_{\theta_{0tr}}$, τ_{θ_0} are finite variations of the helicopter control inputs. In practice, the determination of the control effectiveness matrix requires the trimming of the helicopter model around x_0. The so-called pseudo-inverse is applied to solve the control allocation problem in (11). Note that in the case of rotary wing aircraft, the control effectiveness *G* is still influenced by the state vector $\underline{x}$ (e.g. rotor inflow dynamics, flapping dynamics, rotor rpm dynamics). Also, the signs of the entries of matrix *G* have to be correctly known, otherwise, instead of compensating for tracking errors, the controller will tend to increase them and lead to an unstable response.

2.1.2 Incremental backstepping control

Another promising and more recent control algorithm making use of increments is IBS. It uses the stability characteristics of Control Lyapunov Functions (CLF) to guarantee stability. Applications of the IBS control structure is given in (Koschorke 2012), (Keijzer et al. 2019) and (Acquatella, 2020). It departs from the same system description given in eq. (4). Defining error state ***z*** as the difference between the current state and the reference state, one can assure stability and tracking if the derivative of a chosen CLF is negative definite. For example, assume CLF as:

$$V = \frac{1}{2} z^2 \tag{12}$$

where $z = x - x_{ref}$. It follows that $\dot{z} = \dot{x} - \dot{x}_{ref}$. The derivative if CLF is then:

$$\begin{aligned} \dot{V} &= z\dot{z} \\ \dot{V} &= z\left(\dot{x} - \dot{x}_{ref}\right) \end{aligned} \tag{13}$$

Substituting (4) into (13) gives:

$$\dot{V} = z\left(\dot{x}_0 + G\left(x_0, u_0\right)\Delta u - \dot{x}_{ref}\right) \tag{14}$$

Equation (14) is negative defined when:

$$\Delta u = G^{-1}\left(x_0, u_0\right)\left(-\dot{x}_0 + \dot{x}_{ref} - Cz\right), C > 0 \tag{15}$$

Equation (15) gives the final IBS control law. When choosing gain C>0 the use of a CLF assures stability and tracking, given that the time scale separation principle and sampling frequency assumptions hold. Other than the INDI controller, no additional PID controller is necessary. Note that in some cases the INDI and IBS control laws can become equal. When no outer loop dynamics are considered, eq. (6) equals Equation (15) if the P(ID) controller that drives *v* is designed as $\upsilon = K_p\left(x_{ref} - x\right) + x_{ref}$ where K_p is a proportional gain larger than zero and equal to *C* from the IBS law. This results in the same controller dynamics for both methods. Although INDI and IBS controllers are commonly applied to dynamic systems in literature, only recently time delay margins and robustness tolerances against parameter uncertainties were explicitly quantified (Huang et al. 2022). Until this point sampling rates were always assumed sufficiently high and no systematic theory existed to calculate maximum parameter mismatch. It was found that control effectiveness mismatch could reach up to 50% of its true value when actuator dynamics were not included in the model. Adding actuator dynamics even significantly increased the robustness of the INDI controller against model errors. It was also found that underestimating the control effectiveness lead to better tracking performance than perfect estimation. To estimate properly the control effectiveness one would require knowledge of the analytical form of the time derivatives of each of the control laws. This would quickly become prohibitively difficult to calculate with increasing order of the system and also it requires the system to be in strict feedback form. A solution to both these problems is to apply command filtering on the intermediate control laws resulting in the Command filtering INDI/IBS controller. This will remove both the need for calculation of the derivatives of the control laws themselves and the strict feedback requirement.

3 ADAPTATION OF INCREMENTAL NON-LINEAR CONTROL TO HELICOPTERS FOR SUCCESSFUL APPLICATION

As discussed above, when applying an incremental controller, one relies on sensor measurements instead of a mathematical model to obtain the state of the system. This is an important advantage, since estimation errors in the mathematical model are excluded. However, a rotorcraft has multiple rotor dynamics states that cannot be measured while this should be done for successful control. This can be explained as follows: while in conventional fixed-wing aircraft, control moments are transmitted directly from the control surfaces to the aircraft, in contrast, in rotorcraft, the control inputs are transmitted through the swashplate to the blade pitch, causing the rotor to flap and thence transmitting moments to the aircraft. cyclic inputs are. Thus, low-frequency pilot inputs (applied at 1/rev-frequency through the swashplate mechanism) generate high-frequency blade excitations. Clearly, rotor blade excitations, in the form of flap and lag motion, can be transformed back to the fixed airframe system, influencing thus the plant. Based on flight experience with modern helicopters, for example, excitation of the main rotor flap and lead-lag (inplane) modes by cyclic inputs results in aircraft roll and pitch vibrations or in the excitation of the low frequency pendulum mode of external slung loads by delayed collective and/or cyclic control inputs due to couplings of the load dynamics via elastic cables. Since the rotor dynamics are neglected while assuming time scale separation and because the direct control effectiveness of the control input on body pitch/roll accelerations is negligible, the nonlinear controller will probably be unable to control the helicopter. This situation can be corrected by applying 1) residualization of the higher-order rotor dynamics and 2) synchronization of input signal. Next, some control cases will be shown where these procedures will be exemplifies.

3.1 Effects of flapping dynamics on nonlinear pitch control

In order to get some physical feeling for the problem, a very simple manoeuvre is used as an example, i.e. the first few instants during the transition from hover to forward flight, after a step input of longitudinal cyclic pitch. One may assume that just a pitching motion of the helicopter occurs at the very beginning of this manoeuvre, before forward speed builds up and begins to have an influence. For notations, see Figure 1.

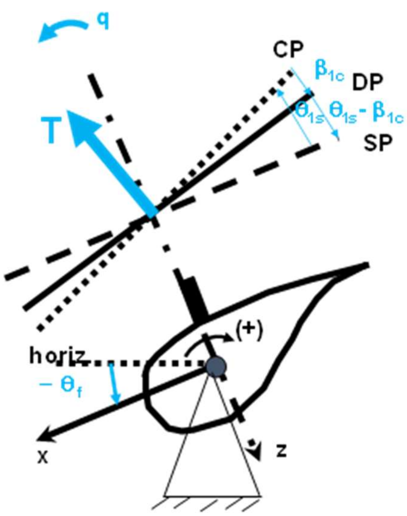

Figure 1 Helicopter pitch motion after a longitudinal cyclic pitch step (CP=control plane, SP = shaft plane, DP = disc plane)

In classical treatments of the subject, the rotordisc tilt is often assumed to respond instantaneously to control inputs, as well as to pitching motion and helicopter velocity. This in fact is equivalent to neglecting the transient flapping motion, which indeed damps out very quickly after a disturbance. Just the quasi-steady response of the rotordisc is taken into account in this classical approach. In the case considered, backward tilt of the rotordisc with respect to the control plane (CP) is given by:

$$\beta_{1c} \cong -\frac{16}{\gamma}\frac{q}{\Omega} \qquad (16)$$

where β_{1c} is the longitudinal disc tilt w.r.t. plane of control CP, γ = Lock number, q = pitching velocity of body, W = angular speed of rotor. Eq. (16) can be combined with the equation describing the pitching rate of the helicopter body q:

$$\dot{q} = -\frac{T}{I_y} h \sin(\theta_{1s} - \beta_{1c}) - \frac{N}{2I_y} K_\beta \left(\theta_{1s} - \beta_{1c}\right) \qquad (17)$$

where Iy = mass moment of inertia around lateral axis, h = distance between body CG rotor hub, N = number of blades, K_β rotor spring constant, θ_{1s} = longitudinal tilt of swashplate (cyclic stick displacement). Assuming small angle approximation, $\sin\left(\beta_{1c} - \theta_{1s}\right) \approx \beta_{1c} - \theta_{1s}$ and substituting (16) into (17) results in:

$$\dot{q} = -K\left(\frac{16}{\gamma}\frac{q}{\Omega} - \theta_{1s}\right) \quad ; \quad K = \frac{Th + \frac{N}{2}K_\beta}{I_y} \qquad (18)$$

where K represent the moment exerted on the body per radian of disc tilt, due to thrust vector offset w.r. to center of gravity, as well as due to direct spring moments. Looking at the pole of this motion described by eq. (18) this motion is always stable and nonoscillatory. $s = -\frac{16}{\gamma}\frac{q}{\Omega}$. The resulting IBS controller for this system will be:

$$\begin{aligned} \theta_{1s} &= \theta_{1s,0} + G_q^{-1}\left(-\dot{q} + \dot{q}_{ref} - C_q z_q\right) \\ z_q &= q - q_{ref} \\ G_q &= K \end{aligned} \qquad (19)$$

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Furthermore, there is a direct relation between the control input θ_{1s} and the pitch acceleration q. This suggests that there is no delay between applying cyclic control input and producing pitch acceleration. While this might be true when the controlled helicopter model is given by eq. (18), as can be seen in Figure 2.

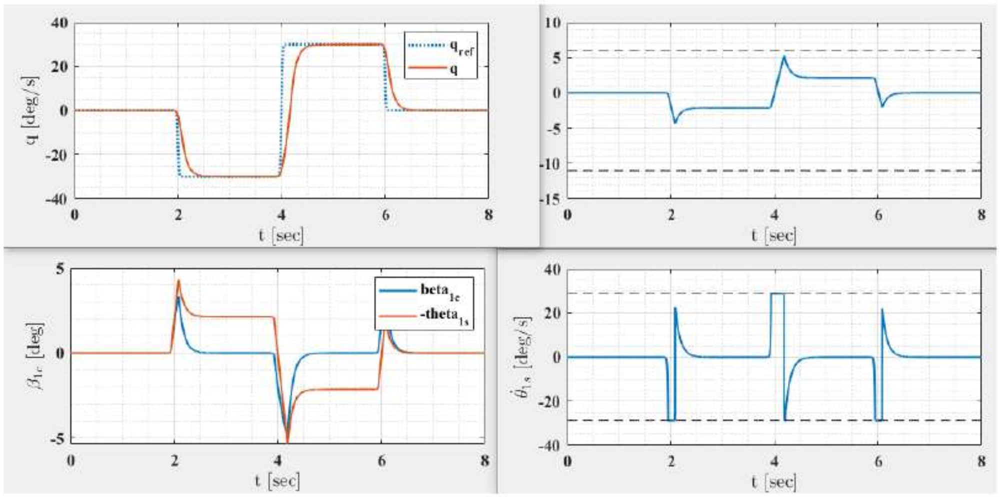


Figure 2 Tracking a helicopter pitch rate doublet with an IBS controller, no flapping dynamics

As discussed in the Introduction, the rotor dynamics affects body dynamics, and therefore a refinement of eq. (17) can introduced. Assume in a first approximation that flapping dynamics affects the tilting of the rotordisc by means of a time constant τ_β:

$$\tau_\beta \dot{\beta}_{1c} + \beta_{1c} \cong -\frac{16}{\gamma}\frac{q}{\Omega} \tag{20}$$

This disc tilt approximation corresponds to taking into account the –low frequency - regressing flapping mode on top of the steady solution. Adding eq. (20) to eq. (18) the IBS controller is again applied to the system for tracking a 25 deg/sec doublet in body pitch rate, see Figure 3.

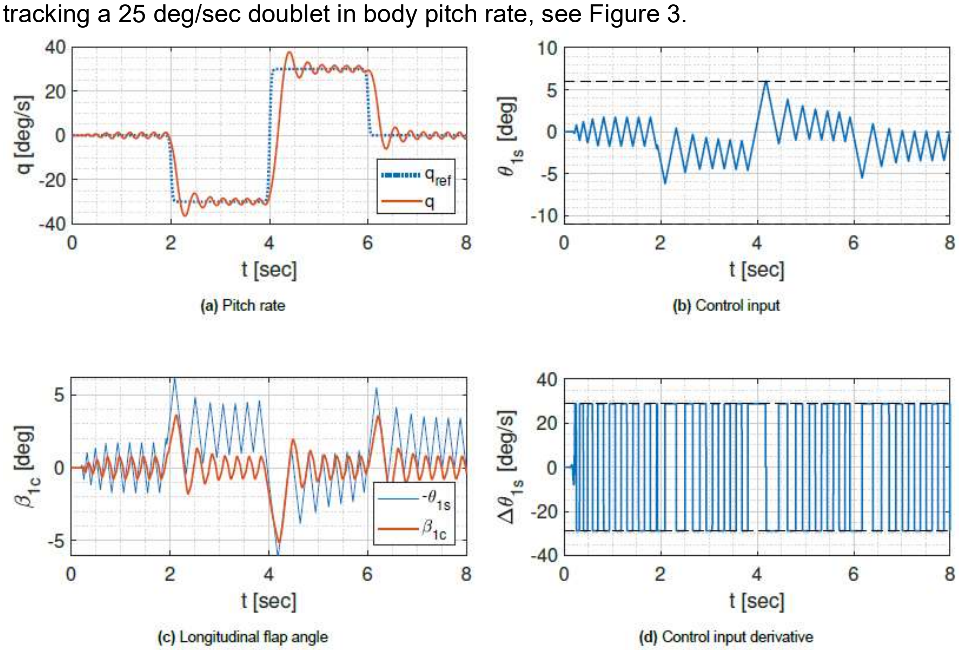


Figure 3 Tracking a helicopter pitch rate doublet with an IBS controller, first order flapping dynamics

Looking at this figure one can see that the IBS controller is unable to track the pitch rate resulting in controller instability. It follows that the delay introduced between the input of cyclic control and the desired pitch acceleration (in this case through flapping dynamics) is responsible for the nonlinear controller instability. As the controller is "unaware" of the flap delay delays, it keeps increasing its control input. The signal is still bounded due to the imposed maximum deflection and the rate limits; otherwise, the helicopter body response would quickly diverge. To correct this instability in an incremental nonlinear control approach, a standard procedure would be to control the pitch acceleration $\dot{q}$ by means of controlling flap angle β_{1c} with θ_{1s}. However, this is not possible with current helicopters since there are no sensors that can be installed on the blades to measure the flap angle of the rotor blades. This would be indeed needed for the incremental control law. Therefore, it is necessary to remove somehow the flap angle from the state vector and increase the control dependency of body pitch rate q on the control input.

Residualization and Synchronization

As demonstrated in Figure 3, the longitudinal control input θ_{1s} indirectly influences the helicopter angular acceleration through the rotor disk tilt angle β_{1c}. Since the system dependent dynamics are neglected through the time scale separation assumption and the direct control effectiveness of the control input on pitch acceleration is negligible, the IBS controller is unable to control the helicopter. Essentially, this boils down to the fact that the time-scale separation assumption is violated. The flap dynamics has non-negligible influence on the body accelerations. Therefore the controller model on which the IBS control law is based has to be adapted. Furthermore, in the simple case analyzed above it was assumed that the actuators and sensors operate at a sufficiently high frequency. While this true for the majority of sensors, actuator delays and dynamics cannot be usually neglected. Furthermore, filters are used to obtain certain states, so the filters induce some kind of delay as well. It follows that the incremental controllers have relatively low robustness when subjected to time delays and unmodelled dynamics that influence the feedback path. Two solutions can be used to correct this problem: residualization and synchronization.

Residualization procedure was applied by Skogestad and Postlethwaite (2001) to separate slow and fast states in a state space system and thereby simplifying the system . The fast states are assumed to be constantly at steady state compared to the slow states, and their dynamics have therefore no effect on the slow states In the case of helicopter, residualization is performed by setting the derivatives of the flapping states equal to zero and fold their dynamics into the remaining states. This will transfer the control dependency of the flapping states to the remaining states, such that the time scale separation principle is less likely to be violated. The residualized state vector for a 6 degree of system will be x_{res} = [$u\ v\ w\ x\ y\ z \mid p\ q\ r\ \phi\ \theta\ \Psi$]. The residualization procedure for flap angle involves writing the flapping dynamics as follows:

$$\ddot{\beta} = F_{\dot{\beta},x_{res}} x_{res} + F_{\dot{\beta},\beta}\beta + F_{\dot{\beta},\dot{\beta}}\dot{\beta} + G_{\dot{\beta}}u \tag{21}$$

Assuming zero flap dynamics transforms equation (21) into:

$$\beta = -F_{\dot{\beta},\beta}^{-1} F_{\dot{\beta},x_{res}} x_{res} - F_{\dot{\beta},\beta}^{-1} G_{\dot{\beta}} u \tag{22}$$

For the body dynamics, the equation of motion is:

$$\dot{x}_{res} = F_{x_{res},x_{res}} x_{res} + F_{x_{res},\beta}\beta \tag{23}$$

Substituting (22) into (23) results in the final residualized system as:

$$\dot{x}_{res} = \left(\underbrace{F_{x_{res},x_{res}} - F_{x_{res},\beta} F_{\dot{\beta},\beta}^{-1} F_{\dot{\beta},x_{res}}}_{F_R} \right) x_{res} + \left(\underbrace{G_{x_{res}} - F_{x_{res},\beta} F_{\dot{\beta},\beta}^{-1} G_{\dot{\beta}}}_{G_R} \right) u \tag{24}$$

After residualizing the state space system for the controller model, the controller dependency on the remaining states in G_R becomes proper for applying an incremental control law. However, now there is a large difference between the controller model and the actual model describing the helicopter dynamics. Namely, the latter model includes dynamics and time delays from flap dynamics. This means that the controller model expects the helicopter to react much faster than it is in reality. Furthermore, sensors, filters and actuator dynamics also have an influence on the control deflection feedback and state measurement feedback. When not accounting for these time differences, instabilities and divergent behavior can occur. Therefore, usually a so-called “synchronization” filter is introduced in the system (Sieberling, Chu and Mulder, 2010), see Figure 4.

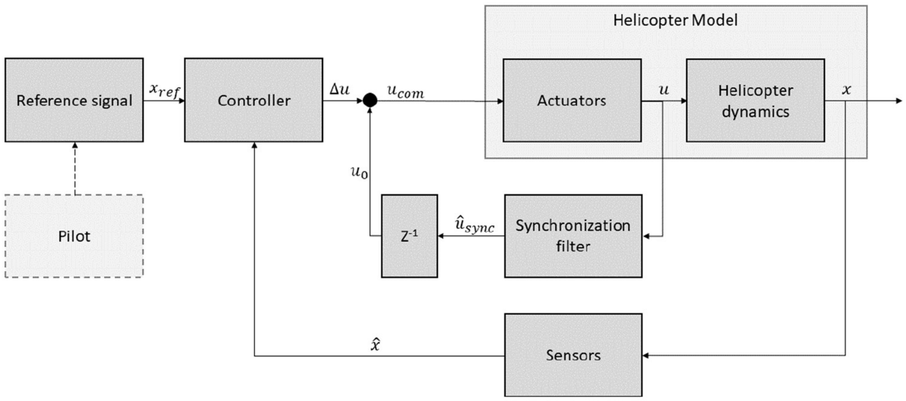


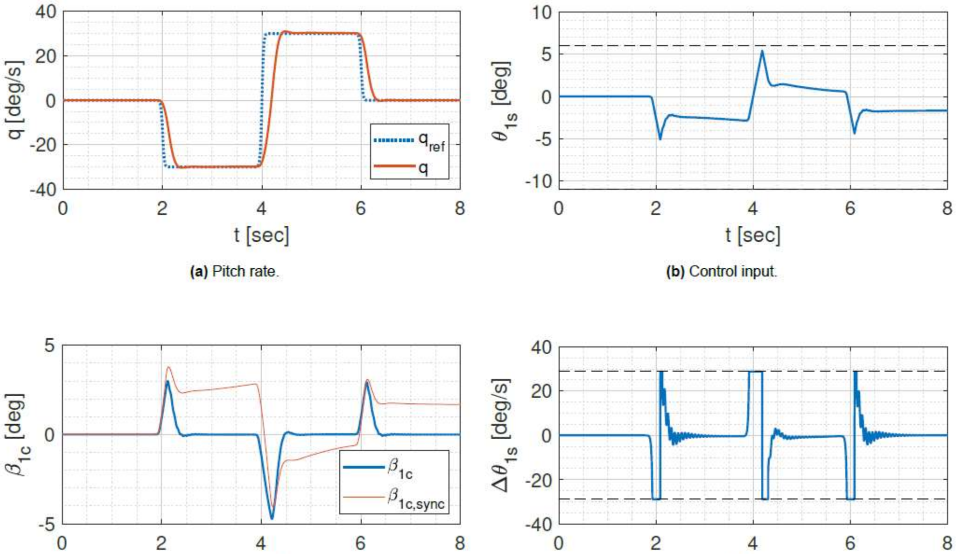
Figure 4 Residualization and Syncronization of the incremental nonlinear controller for helicopter

The synchronization filter delays the feedback measurement of the control input to mimic the delay that the control input otherwise had due to the flap dynamics and other uncontrolled signal manipulations. A downside of this synchronization filter is that some system dynamics coefficients have to be estimated, as it needs to map the expected effect of the controller input by the controller to the real effect of the rotorcraft including the time delay. However, this is just a portion of the total amount of system dynamics coefficients in f(x) that would have been estimated if a non-incremental controller was used. As the flapping dynamics plays an important role in the response of a helicopter, it should be investigated whether this needs to be residualized and included in the synchronization filter. The time delay that is removed during the residualization process can be synchronized using the following equation:

$$\begin{bmatrix} \dot{\beta}_{sync} \\ \theta_{sync} \end{bmatrix} = \begin{bmatrix} F_{\dot{\beta},\beta} \\ G_R^{-1} F_{x_{res},\beta} \end{bmatrix} \beta_{sync} + \begin{bmatrix} G_{\beta} \\ G_R^{-1} G_{x_{res}} \end{bmatrix} \theta_{meas} \tag{25}$$

where β represents the flapping dynamics and θ represents the control vector. Using eq. (25) results in synchronization of the control output of the controller model with the actual control deflection of the relevant actuation system. The filter is placed in the feedback path of the actuator deflection measurement, converting the measured actuator deflections to a synchronized actuator deflection. The sensor dynamics could be accounted for by placing the model of the sensors also on the actuator feedback path. Therefore, in Figure 4, the sensor block is also placed inside the synchronization block. This will cause the possible delay of the sensors to by applied to both the state estimation signal as the actuator feedback, thereby cancelling out any effect of the sensors.

Applying the residualization (to solve for the time scale separation assumption) and synchronization (to solve for the time delay of the flapping dynamics) for the instability of the nonlinear controller as presented in Figure 3 results in an improved tracking performance for the doublet controller as seen in Figure 5. Looking at the figure one ca see that the controller performance is much improved. Some oscillations are still visible in the control input derivative, but they die out as the signal stabilizes.



(a) Pitch rate. **(b)** Control input. **(c)** Longitudinal flap angle. **(d)** Control input derivative.

Figure 5 Tracking a helicopter pitch rate doublet with an IBS controller, first order flapping dynamics, residualization and syncronization filter included in the nonlinear controller

This finding demonstrates the importance of including correct rotor dynamics in the design of incremental nonlinear controllers for helicopters. Especially the value of the time delay in the rotor dynamics should be carefully determined and handled in these controllers.

3. CONCLUSIONS

The paper examined the effects of flapping dynamics on the incremental nonlinear dynamic controllers. In this sense, the paper presented the theoretical were examined. The theoretical foundations for the INDI controller (Incremental Nonlinear Dynamic Inversion) and IBS controller Incremental Backstepping) as modern control laws for future carefree handling rotorcraft were presented and then adapted to the case of a simple helicopter model including first order flapping dynamics. The following conclusions can be drawn:

1) The INDI/IBS control schemes requires using measurements of angular accelerations by sensors-based rotational accelerometers mounted on the vehicle structure instead of use of a complete onboard aerodynamic model. As a result, both the INDI and IBS controllers requires a minimal a priori knowledge of the model, which makes it very robust to model uncertainties.
2) Implementation of the INDI/IBS needs to be carefully performed as the helicopter models can be not-affine in control inputs and can be non-minimum phase systems especially at low speeds.
3) Actuator dynamics and rotor dynamics can cause the designed INDI/IBS controller to overcontrol the helicopter due to additional time delays introduced in the system.
4) The inclusion of a simple flapping dynamics model to the body pitch model created IBS controller instability, which needed to be corrected by using residualization and synchronization. Residualization was used to correct for the violation of the time scale separation assumption and synchronization was used to solve for the time delay of the flapping dynamics
5) The incremental nonlinear controller should be carefully implemented in rotorcraft applications for avoiding controller instability and ensure robustness.

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