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SOME OPTICAL DEVICES FOR TESTING ALIGNMENT AND FLATNESS

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PROEFSCHRIFT

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SUMMARY

In this thesis a description is given of some optical instruments for testing the straightness of lines or the flatness of surfaces. In technical applications not only the checking, but also the adjustment is important. All possibilities are not investigated.

In the second chapter some of the traditional instruments are mentioned; their use will not be discussed in detail. Even though for many purposes the accuracy of these instruments is sufficient, they are not always easy to use. Measuring transverse displacements at a large distance for example, often requires a very precise angular measurement, so that an expensive instrument should be employed. But then it often cannot be mounted with sufficient stability with respect to the point to be measured. In spite of the possible disadvantages, one should of course always use the instrument which leads to the desired results as quickly as possible. One of the most accurate instruments is the automatic optical level. It is not necessary to mount it as stably as a theodolite or a telescopic sight (especially if a horizontal plane is to be generated this instrument may be used).

Some methods where interference and diffraction of the light are utilized are also mentioned in the second chapter; in traditional instruments these phenomena are only inconvenient and limit the accuracy. The double-slit-method, the zoneplate, the Fresnel's mirrors, and the like, discussed in this chapter, function only thanks to the wave properties of light. Spherical aberration also appears to be usable for alignment devices.

It will be shown that the precision with which the straightness of e.g. the bed of a lathe can be measured, is limited only by inhomogeneities in the atmosphere which may be present. One of the great advantages of most of the non-conventional instruments mentioned is, that measurements often can be done locally at the measuring-point itself instead of from a remote point.

The properties of the pentaprism are also discussed, especially the inaccuracies which may be introduced by injudicious use of such a prism. Examples of measurements are mentioned.

In the third chapter a survey is given of several possibilities of testing the flatness of surfaces, or of adjusting more than two points in one plane. Interference and spherical aberration also appear to be useful in this case. Different possibilities are examined.

We have not yet succeeded in designing a device which offers the same advantages in alignment as the zoneplate (here a straight line is defined by two points, an illuminated small aperture and the center of the zoneplate; the distance between the two points may be large. One could speak of a very long collimator; a small shift of one of the points does not have so much influence on the generated line as in the case where a telescopic sight is used). The instruments for generating a plane, designed up to now, are all afflicted with the same disadvantage as the telescopic sight namely, these devices must be mounted with very great care. Finally some remarks are made on the adjustment of the instruments mentioned, about some applications, and about possibilities for automatic alignment.

CHAPTER 1

INTRODUCTION

1.1 The alignment of a straight line

In engineering practice the mathematical straight line never occurs; a line is always formed by the intersection of two more or less flat surfaces, a straight edge, a collection of two or more points, the centers of bearings, and the like. Alignment is concerned with the checking of the straightness, the adjustment or the manufacture of such a line. It often consists of the establishment of a reference line from which measurements can be made.

1.2 The alignment of a plane surface

The "alignment of a plane surface" is the control of the flatness, the adjustment or the manufacture of such a surface. It is not necessary that there be a real surface; it can also consist of three or more technical points, or one or more lines plus one or more points which need to be brought into one plane. Alignment of a plane often involves the establishment of a reference plane or flat from which measurements can be made.

1.3 Applications

The techniques for alignment of straight lines and the flatness control are much desired in scientific and industrial metrology: the bed of a lathe must be straight, in order that the motion of the carriage be rectilinear, and parallel to the axis of rotation; the shaft of a machine often has more than two bearings, the center-lines of which must coincide; the straightness of an optical bench or the flatness of a carpenter's bench must be measured; the changes in inclination of a tower or in general, the changes with time in position of a point with respect to other points must be measured; the ends of segments of a tunnel under a canal must be plane in order that they fit exactly onto each other; the flatness of large surface plates must be known.

CHAPTER 2

STRAIGHT LINES

2.1 The use of traditional instruments

In this section we shall deal with optical tools and instruments of the ordinary kind, namely the instruments that make use of image formation by the aid of lens systems corrected for aberrations as far as possible or at least, to the extent necessary. These instruments are the telescopic sight, the optical-tooling level, the jig transit, the vertical collimator, the alignment telescope, the autocollimator, the pentaprism, the flat mirror, and the like. [van Heel 1964, Kissam 1962, Hume 1965, de Bruin 1961, Fultz 1949].

A telescopic sight is a telescope which generates a straight line. When focused the telescope images the crosshairs in the object space; these images form a line which is straight only if the optical axis of the movable focusing lens coincides with the optical axis of the telescope.

The optical-tooling level consists of a spirit level attached to a telescopic sight. If the instrument is correctly adjusted the line of sight is horizontal when the level bubble is centered. The line of sight is the straight line provided by the telescope. Modern automatic leveling instruments yield a straight horizontal line, even if the telescope has been set nearly horizontal.

The jig transit is a telescopic sight which may be turned in any direction; it rotates about two axes, a horizontal axis which is perpendicular to the optical axis and a vertical axis which is perpendicular to both the optical and the horizontal axes. These three axes intersect at one point. The transit is provided with two plate levels. If the telescope of the instrument is rotated about, for instance, the horizontal axis of the instrument, then a vertical plane is generated by the line of sight. In order to make this plane a flat plane instead of a cone the crosshairs of the telescopic sight are adjustable so that the line of sight is perpendicular to the elevation axis.

At the end of the horizontal axis of most jig transits a flat mirror is mounted. It can be adjusted so that this axis is perpendicular to the mirror and thus, so that the mirror is parallel to the plane generated by the line of sight. If the jig transit is moved over a certain distance along a line perpendicular to the mirror – this can be checked with the aid of this mirror and a fixed autocollimator – then the telescopic sight generates a second plane which is parallel to the plane generated in the first position.

In addition to the optical-tooling level there exists the so-called vertical collimator. It consists of a telescopic sight mounted vertically. Long vertical reference lines can be established with this instrument.

The alignment telescope consists of a telescopic sight built in a heavy steel cylindrical barrel whose outside surface is held to extremely close tolerances; the line of sight coincides with the axis of the cylinder. Internal optical micrometers are provided for measurements in two directions. Often alignment telescopes are arranged for autocollimation.

The autocollimator is a telescopic sight that can be used either as a collimator or as an alignment telescope. A half-silvered mirror is placed between the crosshairs and the eyepiece, at an angle of 45 degrees with the optical axis. Light from a small light source is reflected by the mirror towards the crosshairs; thus a silhouette of the crosshairs is projected through the objective lens and a line of sight is generated. This line can be reflected by a flat mirror perpendicular to the line of sight; the light then returns through the objective lens, the half-silvered mirror and the eyepiece into the eye of the observer. If the mirror is adjusted to bring the reflection of the crosshairs upon the actual crosshairs, the mirror is then perpendicular to the line of sight.

A pentaprism turns the line of sight through an invariant angle of 270°, invariant because the number of reflections in the pentaprism is even (see sect. 2.5). If a pentaprism is placed in front of the objective lens of a collimator and the pentaprism is rotated about the line of sight, the twice reflected line of sight then generates a flat plane perpendicular to the original line of sight.

All instruments mentioned above are rather small; a small displacement in vertical direction of one of the leveling screws thus causes an extrapolated error: if one of the three leveling screws sags one micron and the distance between the screws is about five centimeters the error is about 4 seconds of arc, or 1 mm at a distance of 50 meters. The focusing at different distances also introduces an error.

In the sections hereafter we will describe some simple alignment devices where extrapolation is avoided while offering advantages of higher accuracy and greater convenience and versatility, as well as some devices where extrapolation is not avoidable. In all methods described, the necessity of focusing is obviated, which is an important practical advantage.

2.2 The use of diffraction and interference

2.2.1 *Sighting*

One of the oldest alignment methods making use of diffraction is "sighting" by means of three small circular holes a, b and c (fig. 2.2.1a). A, B and C are the centers of these apertures. The hole a is illuminated and the light arriving at C is diffracted by b. If the light path by way of the edge of b to the center C is not to exceed the direct path ABC by more than $\lambda/2$, then the diameter of the hole b must not be larger than $\sqrt{\lambda l}$ (λ is the wavelength of the light used) and the light waves arriving at C are more or less in phase.

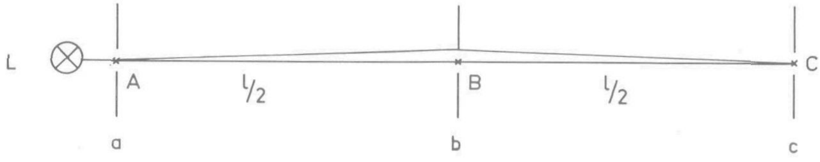


Fig. 2.2.1a "Sighting" with three holes

If AB = BC = $l/2$ and $\lambda = 0.56\,\mu$m the accuracy of putting C on the line AB is about $40/\sqrt{l}$ seconds of arc, l being measured in meters [van Heel, 1961a]. This accuracy is generally unsatisfactory. Also the amount of light arriving at C is very small, often necessitating that observations be done during the night. Another disadvantage is that as long as A, B and C are not aligned, no light reaches the eye of the observer. Sighting is described by Bonaffé [1930], van Heel [1946, 1949, 1950, 1961a] and Liem [1961].

2.2.2 *The double slit*

One of the first improvements of this method was the double slit method. (A two-hole arrangement was first set up by Young in 1807, Fresnel used in 1816 an arrangement with two slits for demonstrating the wave properties of light). A narrow illuminated slit sends light to two other slits in a screen, the three slits being parallel. Behind the screen a series of interference fringes is produced, generating a straight line or in other words, the locus of the zero-order maxima is a straight line; in fact only one coordinate of this line can be determined. This is sufficient for many technical applications, especially where small unidirectional displacements of a construction are of interest. The distance between the principal maxima (or minima) in the interference pattern is $\delta = l'\lambda/h$ (fig. 2.2.2a).

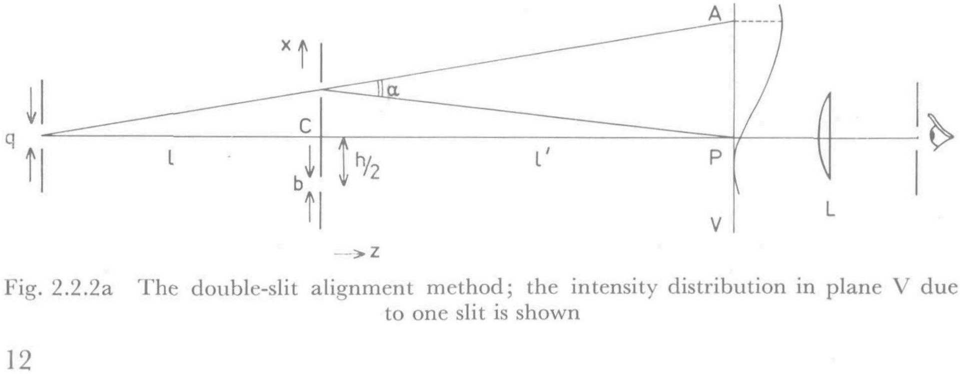


Fig. 2.2.2a The double-slit alignment method; the intensity distribution in plane V due to one slit is shown

The accuracy attainable is some 1/100 or 1/200 of this distance. For instance, when the distance between the two slits is $h = 6$ mm, the distance between the double slit and the observed interference pattern is $l' = 60$ meters and the wavelength of the light used is $\lambda = 0.55\,\mu$m, the precision of a setting is some 30 to 60 μm. In practice the precision of straightness measurements at this large distance is limited by the inhomogeneities in the air. When the position of the zero-order maximum is measured by calculating the mean value of two settings at two sharp color transitions symmetrical with respect to the maximum, a precision of 1/1000 of the distance between the minima can be attained. This can only be done, of course, if the light is not monochromatic.

In fig. 2.2.2a the first slit and the double slit are shown. In plane V the interference pattern is observed with the help of a lens L (e.g. a magnifying-glass or an ordinary spectacle-glass), with a magnification depending upon the fineness and the accuracy desired: when the distance between the two dark fringes next to the central white fringe or zero-order maximum is δ mm an accuracy of $\delta/100$ mm or $^1/_{100} l'\lambda/h$ is attainable. The magnification of the lens used should be about $1/\delta$, which means that the focal length is 250δ mm and that the angle between the minima mentioned as seen by the observer's eye is 0.004 radians or about 15 minutes of arc. A cylindrical lens is sometimes more suitable for the purpose than a spherical lens.

The entrance pupil of the optical system is the double slit. Because this slit is imaged near the back focal plane of the magnifying-glass the pupil of the eye must coincide with this image. It is often necessary to place an extra diaphragm in the image of the double slit to cut out stray light. The edge of the lens limits the field of view, or the image field stop is formed by the edge of the lens.

The fringes will only appear if the light reaching the two slits is sufficiently coherent; this is the case if the width of the first slit is not more than

$$q = \frac{l\lambda}{2h} \qquad (2.2.2a)$$

where l is the distance between the first and the double slit. In practice the slit width often can be somewhat larger. For the width mentioned the contrast of the fringes is about 0.64.

The disadvantage of the double slit method is that the illuminance is rather poor, because the light reaching the observer's eye has been diffracted; if there were no diffraction the observer would see nothing at all. Besides, when the width of the two slits is not equal, the centers of the fringe patterns do not lie on a straight line. The center of a fringe pattern of a poorly manufactured double slit is not even defined because the pattern is not symmetrical.

The illuminance on plane V at P, due to the diffracted light of one slit only, should be sufficiently large with respect to the illuminance at A (fig. 2.2.2a). This implies that the angle α must not be smaller than about $0.75\lambda/b$: If $h = 20$ mm, $b = 0.4$ mm and $l = l'$, then the minimum distance is about 20 meters; the distance between the minima $\delta = 500\,\mu$m if $l' = 20$ meters and the precision is about 5 μm or 0.05 seconds of arc. This last precision means that the uncertainty in direction of P as seen from the point C is 0.″05 (the symbol ″ is used to mean seconds of arc and not inches).

The amplitude of the diffraction pattern at P can be calculated from the diffraction integral [e.g. van Heel 1964]:

$$\bar{a}_P = \frac{C}{\lambda}\left(\frac{q}{ll'}\right)^{1/2} \int \exp\left\{-\frac{2\pi i}{\lambda}\left(\frac{x^2}{2l} + \frac{x^2}{2l'}\right)\right\} \mathrm{d}x =$$

$$= \frac{2C}{\lambda}\left(\frac{q}{ll'}\right)^{1/2} \left\{\frac{\lambda ll'}{2(l+l')}\right\}^{1/2} \int_{\alpha(h-b)/2}^{\alpha(h+b)/2} \exp\{-{}^1\!/_2 \pi i t^2\}\mathrm{d}t$$

where

$$\alpha = \left(\frac{2l+2l'}{\lambda ll'}\right)^{1/2} \tag{2.2.2b}$$

(C is a constant, and the edges of the slits are the limits of integration.) If for a certain application the distances l and l' and the desired angular precision are known, then h and q can be calculated. The only variable left is the quantity b, the width of the slits; variation of this quantity gives us the possibility of calculating the value of b for which the intensity at P has a maximum value; so for which b is $\partial \bar{a}_P/\partial b = 0$? This is the case when

$$\frac{1}{2}\pi\left\{\frac{\alpha}{2}(h-b)\right\}^2 = \frac{1}{2}\pi\left\{\frac{\alpha}{2}(h+b)\right\}^2 - \pi \qquad \text{or}$$

$$b = \frac{2}{h\alpha^2} = \frac{\lambda ll'}{h(l+l')} \tag{2.2.2c}$$

This means that the slit covers exactly one "Fresnel zone"; other maxima appear if the slit covers an odd-number of Fresnel zones. With (2.2.2b) the intensity at P is proportional to

$$I_P \sim \left[\left(\frac{l\lambda_0}{2h}\right)^{1/2} \frac{1}{\lambda}\left(\frac{1}{ll'}\right)^{1/2} \left\{\frac{\lambda ll'}{2(l+l')}\right\}^{1/2} |A|\right]^2 = \frac{C'\lambda_0 l}{\lambda h(l+l')}\,|A|^2 \tag{2.2.2d}$$

the limits of integration are

$$\left(\frac{l+l'}{2\lambda ll'}\right)^{1/2}(h \pm b)$$

Thus, large values of h only decrease the intensity of the interference pattern, not only because the intensity is proportional to h^{-1}, but the square of the integral, $|A|^2$, also decreases when h is increased [van Wijngaarden 1949]. Therefore one should not make h larger than necessary in order to achieve the desired accuracy.

One more question arises: should the screen with the double slit be perpendicular to the generated line, and if not, how much may it be tilted? When the screen is rotated about its center there will be a path difference between the two "rays of light" arriving at the point P (fig. 2.2.2b).

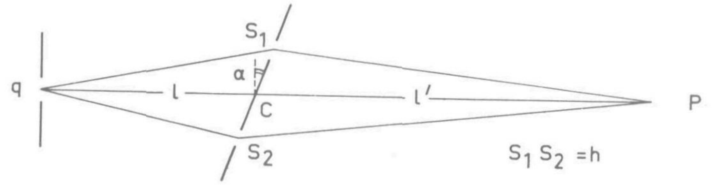


Fig. 2.2.2b Screen with double slit inclined with respect to the axis

A series development of this difference shows that in first approximation

$$\Delta E = qS_1P - qS_2P = -\frac{1}{8}h^3\left(\frac{1}{l^2} - \frac{1}{l'^2}\right)\sin\alpha \qquad (2.2.2e)$$

Only if $l' \gg l$ or $l \gg l'$ can this path difference result in a systematic error. Let the smallest distance from the center C of the double slit to the slit q or the target P be L, then:

$$\Delta E = -\frac{1}{8}\left(\frac{h}{L}\right)^2 h \sin\alpha \qquad (2.2.2f)$$

We want this path difference to be smaller than say $\lambda/1000$, so that the error is never greater than the accuracy of setting the target at the zero-order maximum of the diffraction pattern. If we assume h to be never more than 20 mm and $\sin\alpha < 0.1$ ($\alpha < 5°$) the smallest distance must not be smaller than about 700 times the distance h between the two slits:

$$L \geqslant 700h \qquad (2.2.2g)$$

In a given case the quantity L may be smaller. The screen with the double slit must then be set perpendicular to the alignment line with smaller tolerance in the angle α:

$$\sin\alpha < 8\frac{L^2}{h^3}\cdot\frac{\lambda}{1000} \qquad (2.2.2h)$$

When only small displacements are to be measured (thus l and l' do not change), the angle α is of little importance. Because it is not necessary in this case that the line qCP be straight, the two slits may be scratches in an aluminized glassplate, which need not be very plane or parallel.

The double-slit method and its applications are described by van Heel [1946, 1948, 1949, 1950, 1951, 1955, 1961a], Franx [1951], de Haas [1953], Harrison [1954, 1956, 1960], Doekes [1955] and van Herk [1958].

2.2.3 *Multi-slits*

An improvement of the double-slit method is the multi-slit method: it only differs from the double-slit method in the number of slits used in either the first or the second screen. The double slit can be illuminated by one single slit, but also by three five or an odd-number of slits. An even number of slits is not recommended because then the center of the diffraction pattern can be a minimum, and the accuracy of a setting is correspondingly smaller. The distance between the slits must be chosen in such a way that the maxima of the diffraction patterns caused by each of the slits together with the double slit coincide. This implies that if a screen with say five slits is used, this slit system can only be used at a few specified distances (l) from the screen with the double slit. Therefore it seems better to use a single slit together with a screen with 2, 4, etc. slits. For generating a straight line it is necessary that these slits be placed symmetrically. The advantage of this method is that the maxima are more pronounced and that more maxima and minima appear than in the diffraction pattern produced by two slits. The intensity is also increased [van Heel 1949, 1950, 1961a, and Lohmann 1962].

All methods mentioned up to now make use of diffraction and interference. The intensity of the diffraction patterns is never very high. On the other hand the manufacture of the different parts necessary for these methods is very simple and therefore inexpensive and quick; the accuracy can be high.

One practical remark should be made: The first slit is illuminated by a lightsource. This can be done directly only if the source is large enough to fill the single and the double slit with light; if not, a condenser lens must be used, the position and the focal length being such that both the single and the double slit are filled with light.

2.2.4 *Fresnel's biprism and Fresnel's mirrors*

There are some methods where the light arrives at the region of interference by uninterrupted rectilinear paths. These are all arrangements for observing interference with two (coherent) sources side by side: a. The Fresnel's mirrors and b. the Fresnel's biprism: Two images of a slit source are formed by respectively two plane mirrors set at a small angle, and by refraction in two prisms of small angle.

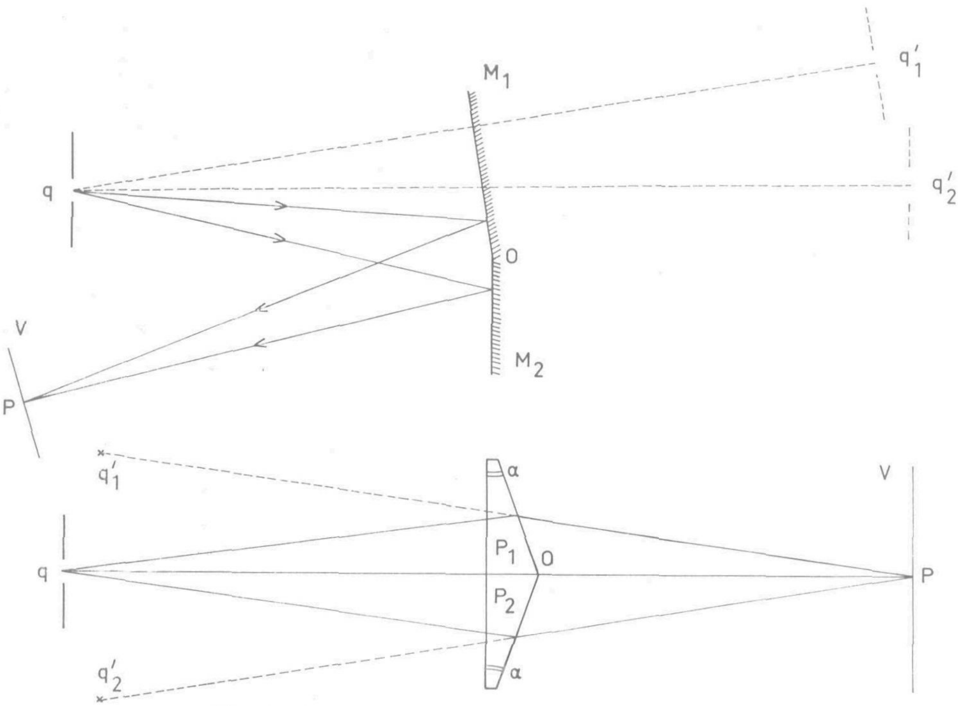


Fig. 2.2.4a Fresnel's mirrors and Fresnel's biprism

a. The light from the source q (fig. 2.2.4a) arrives at the plane V by two paths, via mirror M_1 and via mirror M_2. q_1' and q_2' are the virtual coherent images of q. When the mirror system is rotated about, for instance, the line of intersection O of M_1 and M_2 the center P of the fringe pattern also rotates about O. The direction of the straight line generated by the system (qM_1M_2) *thus depends upon the angular stability of the mirror system*; the line OP is an extrapolation from a small system, therefore we will reject this method for the moment as being not better than the traditional instruments.

b. The light from the source q (fig. 2.2.4a) arrives at the plane V via prism P_1 and via prism P_2. When the angle α of the prisms is small, the distance between the two slit images q_1' and q_2' is

$$h = 2(n-1)\alpha l \qquad (2.2.4a)$$

The distance between the maxima of the interference pattern at V is

$$\delta = \frac{\lambda(l+l')}{2(n-1)\alpha l} \qquad (2.2.4b)$$

The zero-order maximum lies on the line qO so the point P does not change when P_1P_2 is rotated about O. (In fact there will be a small shift of P due to the axial thickness of the prism system). This is only true if the prism angles α_1 and α_2 are exactly equal; if not the system can be considered as an ideal prism system ($\alpha_1' = \alpha_2' = \alpha_1/2 + \alpha_2/2$) attached to a prism of angle $(\alpha_1 - \alpha_2)/2$ which gives a deviation of the line OP with respect to the line qO of $(n-1)(\alpha_1 - \alpha_2)/2$.

From (2.2.4b) can be seen that the accuracy (in length units) of a setting on the fringes does not depend upon l' if $l \gg l'$ [van Heel 1964, Ditchburn 1963].

A slight modification of the Fresnel's mirrors gives a system where the angle between the two mirrors is not somewhat smaller than 180°, but somewhat smaller than 90°. In this case the light arriving at V has been reflected twice. The distance between the two virtual coherent slit images is

$$h = 4\alpha l \tag{2.2.4c}$$

when the angle between the two mirrors is $\beta = 90° - \alpha$. The position of q_{12}' and q_{21}' (fig. 2.2.4b) does not change when the mirror system is rotated about an axis O because the number of reflections is even.

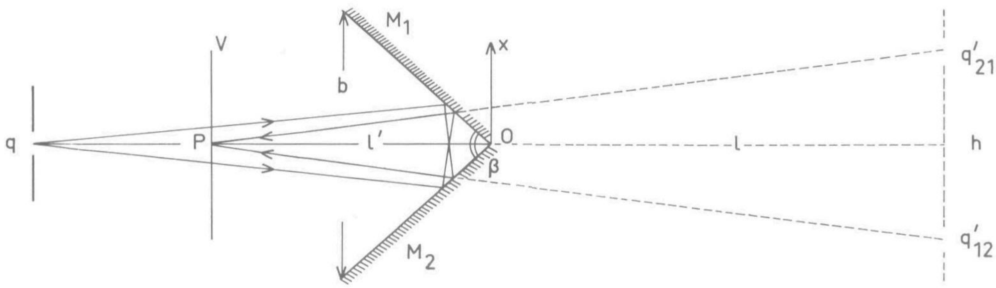


Fig. 2.2.4b Modified Fresnel's mirrors

The distance between the maxima of the interference pattern at V is

$$\delta = \frac{\lambda(l+l')}{4\alpha l} \tag{2.2.4d}$$

The mirror system can also be made as a prism; then the distance between the minima is $1/n$ times the distance δ from (2.2.4d); n being the refractive index of the prism. The point P changes a little when the system is rotated about O.

The system of two mirrors is very useful when the angle α can be varied in order to adapt it to a real problem where a minimum and maximum value of l' and also the desired accuracy are given. A disadvantage with respect to the double-slit method is that there exists a maximum distance l' if the distance between the slit images is larger than the diameter of the system (or if $4\alpha l > b$): points P outside this region do not receive light from the mirrors.

The intensity of the fringes, however, is much larger than that of the fringes caused by the double slit because direct light and not diffracted light is used, and also because the allowed width of the slit (in connection with coherence) is greater. If for the two systems (the double slit and the double mirror system) the accuracy of a setting or the fineness of the fringe pattern is taken to be the same and the width of the illuminating slit in the two-slit system is $q = \lambda l/2h$, independent of l', then the width in the two-mirror system is

$$q = \tfrac{1}{2}\delta \frac{l}{l'} = \frac{\lambda(l+l')}{8\alpha l'} = \frac{\lambda(l+l')l}{2hl'} \tag{2.2.4e}$$

which is $(l+l')/l'$ times the width allowed in the double-slit system.

The amplitude of the diffraction pattern at P is (fig. 2.2.4b):

$$\bar{a}_P = \frac{2C}{\lambda}\left(\frac{q}{ll'}\right)^{1/2} \int_0^\infty \exp\left[-\frac{2\pi i}{\lambda}\left\{\frac{(x-h/2)^2}{2l} + \frac{x^2}{2l'}\right\}\right] \mathrm{d}x =$$

(omitting a phase factor)

$$= \frac{2C}{\lambda}\left(\frac{q}{ll'}\right)^{1/2} \left\{\frac{\lambda ll'}{2(l+l')}\right\}^{1/2} \int_{-\frac{h}{4l}\left\{\frac{8ll'}{\lambda(l+l')}\right\}^{1/2}}^{\infty} \exp(-\tfrac{1}{2}\pi i t^2)\mathrm{d}t \tag{2.2.4f}$$

For the upper limit of integration ∞ is chosen instead of $b/2$ because for large values of t the integrand fluctuates so rapidly as not to appreciably contribute to the value of the integral. With (2.2.4e) the intensity at P is proportional to

$$I_P \sim \frac{C'\lambda_0 l}{\lambda h l'} |A|^2 \tag{2.2.4g}$$

(if q is continuously varied with l'; A is the integral from (2.2.4f).) Comparing (2.2.2d) with this equation it is obvious that the intensity in the latter case is greater: $|A|^2$ from eqn. (2.2.2d) is always much smaller than $|A|^2$ from (2.2.4g). The latter is larger than 0.5, because

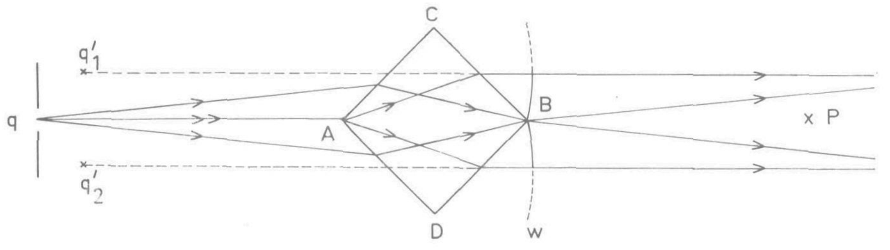
$$\left|\int_0^\infty \exp(-\tfrac{1}{2}\pi i t^2)\mathrm{d}t\right| = \tfrac{1}{2}\sqrt{2}$$

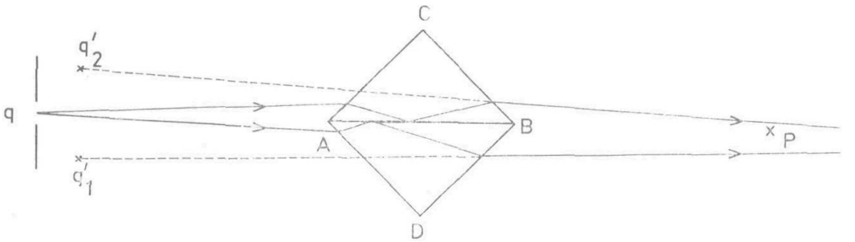
Also in this case, the intensity decreases as the accuracy increases (greater value of h).

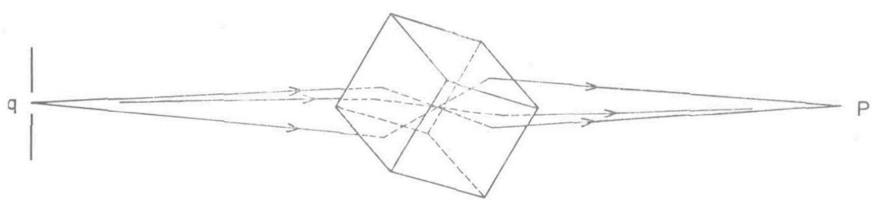
Remarks: It is necessary to rotate the system a little about an axis x, otherwise the slit q forms an obstacle for the light arriving at P. If the mechanical quality of this axis x is very good and the angle between the axis and the line qO is exactly 90°, then a plane is generated by the system when rotated about x. The flatness of a surface can be determined with this method.

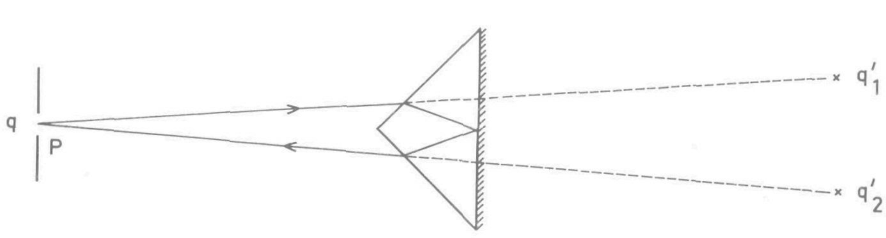
2.2.5 *Modifications of the Fresnel's biprism*

In the previous section two kinds of Fresnel's mirrors were mentioned. The traditional set-up is suitable for determining small angles, the new design is suitable for measuring either displacements or straightness because it is not sensitive to tilt. The Fresnel's biprism is also insensitive to small rotations of the biprism as far as the direction of the generated line is concerned. A small shift of the line of sight, however, is produced by a rotation of the biprism. This disadvantage also applies to the devices which we will now discuss in brief.









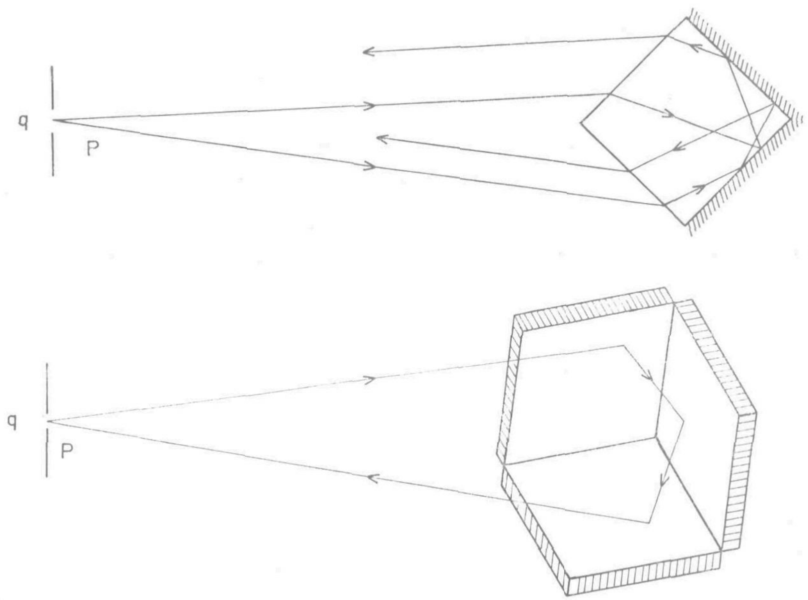


Fig. 2.2.5a . . . f Modifications of the Fresnel's biprism (fig. a . . . e) and a "bad" corner cube (fig. f)

The first device is simply a cube. Two images of an illuminated slit are produced; the great difference between the cube and the biprism is, that the distance *h* between the two coherent images does not depend upon the distance *l*. If all angles are ninety degrees and the refractive index of the cube is about 1.5, then the distance between the images is about two-thirds of the length of the edge. This means that a cube of $14 \times 14 \times 14$ mm can be used to obtain an accuracy better than 0″.2. A disadvantage is perhaps that the distances *l* and *l′* must not be too large, since otherwise the diffraction at the two edges A and B will disturb the interference pattern. In fig. 2.2.5a several rays of light are shown as well as the wavefront *w*. Remark: it is not necessary that a cube be used; it is not even necessary to make the angles at A and B equal. In the latter case *h* does depend upon *l*. However, the device must be symmetrical with respect to the line AB, for if not, then the lines qA and AP do not lie on a straight line.

When an angle instead of a displacement must be measured [van Heel 1955, van Herk 1958] this device may be altered slightly. By manufacturing the cube in two parts, namely ABC and ABD, and by aluminizing the two surfaces AB before cementing them together, a rotation of this new device produces a rotation of the line BP twice as great (fig. 2.2.5b). When two aluminized Dove-prisms are cemented to each other, exactly the same device is obtained.

The prisms must be of equal size and the cementing must be done in such a way, that the faces AC and DB are parallel to each other.

Both devices generate interference fringes at which settings can be done in one coordinate only. If all six surfaces of a cube are used, three coherent images of an illuminated small hole are produced, and both coordinates of the line of sight are determined (fig. 2.2.5c). Fig. 2.2.5g shows a reproduction of the produced interference fringes.

Two other modifications exist, which can, for example, be used for autocollimation measurements. The first generates a straight line for which the place and the position are defined. It consists simply of a roof-prism or a half cube, the hypotenuse of which is aluminized (fig. 2.2.5d). Only one coordinate can be determined. The second device is designed so as not to rotate the line of sight when the device itself is rotated. It is a cube, of which two surfaces are aluminized; only one coordinate is defined (fig. 2.2.5e).

A device which is very similar to the modified Fresnel's mirror system is a "bad" corner cube. "Bad" here means that the angles between the three planes are somewhat smaller than ninety degrees. Three flat mirrors can also be used (fig. 2.2.5f). Here the distance between the images does depend upon the distance l between the hole q and the three mirrors. The admissable slit width q in all these cases is the same as for the Fresnel's biprism, and the like (eqn. 2.2.4e).

2.2.6 *Zoneplates*

As said before only one coordinate of the straight lines produced by the instruments mentioned before the last section can be determined. The second coordinate is in fact often wanted in practice.

The double-slit method can be extended to a four-slit method; the screen has two pairs of slits, being perpendicular to each other. The illuminated slit must be replaced by an illuminated hole. This is a rather clumsy method. It is better to make a screen with a circular slit instead of four straight slits. The diffraction pattern then becomes circular and this fact, together with the great sense of symmetry of the observer, make it possible to aim a circular target at the diffraction pattern with high accuracy. The intensity of the pattern, however, is not great; it is even smaller than the intensity of the double-slit-fringes. In the case of a circular slit, a slit covering an odd number of Fresnel zones also yields maximum intensity of the pattern: $b = (2k+1)\lambda ll'/h(l+l')$. The illuminated slit must be replaced by a more or less circular hole. For example the intensity on the axis at a distance l' from a zoneplate with only one circular slit, with diameter h and width b is

$$I_P = \frac{C^2\lambda_0{}^2l^2}{h^2(l+l')^2}\sin^2\left\{\frac{\pi hb(l+l')}{2\lambda ll'}\right\} \tag{2.2.6a}$$

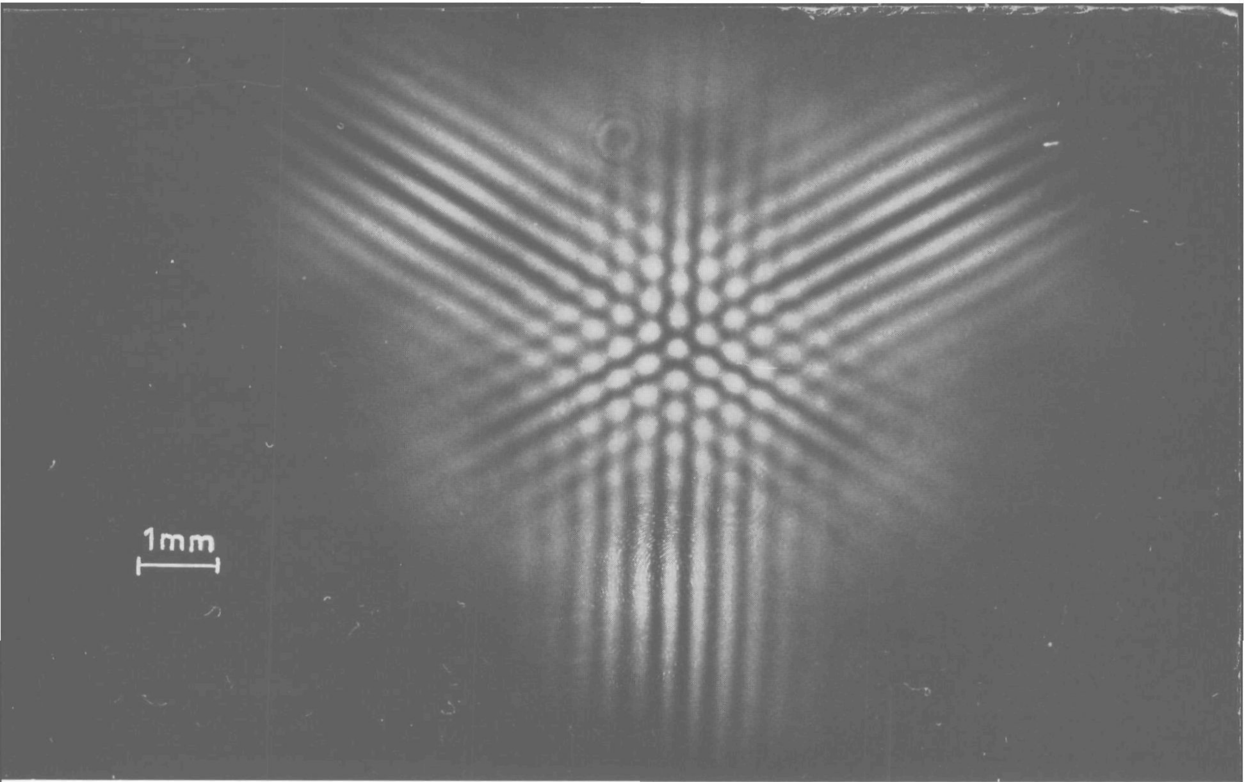


Fig. 2.2.5g Interference pattern produced by an illuminated pinhole and a cube of $7 \times 7 \times 7$ mm ($q = 0.1$ mm $l = 1.2$ m $l' = 2.4$ m)

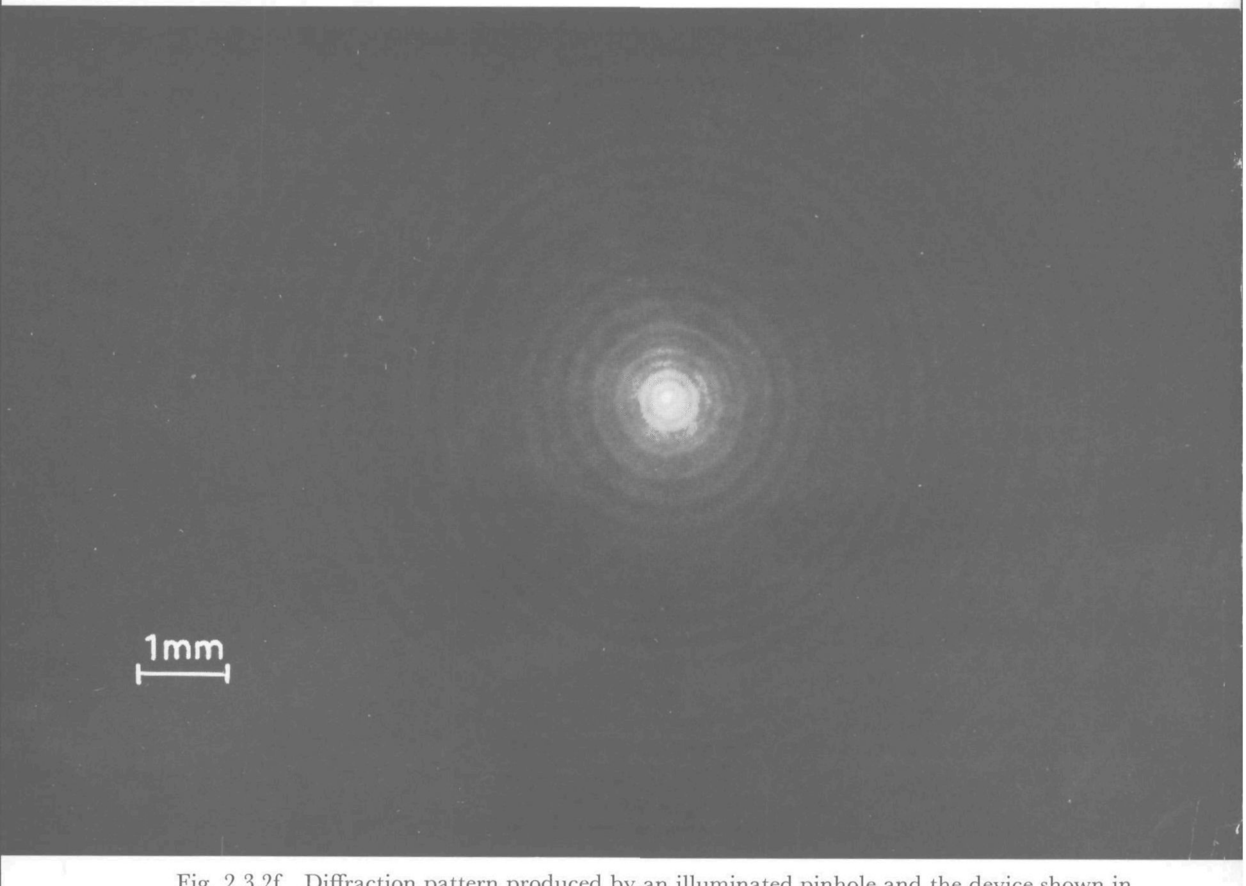


Fig. 2.3.2f Diffraction pattern produced by an illuminated pinhole and the device shown in fig. 2.3.2e ($q = 0.1$ mm $l = 1.5$ m $l' = 2.1$ m)

When more concentric slits are used the intensity improves. A screen with concentric slits will be called a zoneplate. This device has proved its advantages in practice for many years. Applications have been described in several published papers: van Heel [1949, 1950, 1951, 1955, 1960a, 1960b, 1961a, 1961b, 1962a, 1962b], Franx [1951, 1953], Richardus [1954], Doekes [1955], Moonen [1955], van Herk [1958], Ukita [1960], Liem [1960, 1961], de Bruin [1961], Beranek [1962], Reitz [1963], van Milaan [1964, 65] and Raterink [1965].

The great advantages are 1) the method is inexpensive and easy to manage, 2) the line generated by the hole and the zoneplate is not an extrapolation from a short instrument as is true with the alignment telescope for instance, the distance hole – zoneplate can be made as large as desired.

At very small distances – say 0.2 m – and at very large ones measurements can be made. The latter measurements are now feasible since the advent of the laser. A laser, however, is not always necessary for large distances, because white light can be used. The precision of a setting on a colored diffraction pattern can be even higher than a setting done on a monochromatic pattern. Therefore, it is often better to use e.g. a high pressure mercury source instead of a laser, whenever the brightness of an ordinary lamp proves insufficient for a special case.

For example, when (small) displacements of a construction are of interest, a zoneplate can be attached to the construction. The illuminated hole and the crosswires, scale or other kind of target, are attached to fixed points in the neighborhood of the zoneplate and set in one line with it.

In many cases the distance between zoneplate and crosswires is large, so that the intensity of the pattern is not great when an ordinary electric lamp is used. So when the movements of the construction are rapid, they must be registrated with the aid of a high-speed camera; since the exposure times are necessarily short the intensity of the pattern must be great.

Remark: the displacement is not immediately measured but the extrapolated displacement with respect to the fixed target can be determined from the film. [van Milaan 1964, 65, Raterink 1965].

A precision of $0''.2$ is attainable (see cited lit.) and this is sufficient in many cases. In theory a zoneplate diameter of about 6 mm is sufficient to obtain this accuracy. In practice zoneplates of 20 or even 30 mm are used; the diameter q of the illuminated hole is generally larger than would be expected. The coherence decreases near the edge of the zoneplate. The incoherent part of the light makes it easier to find, roughly, the position of the point of observation P.

Remark: If the diameter of the zoneplate is kept constant, say 7 mm, then it turns out that for large distances $l = l' \geqslant 90$ m, the width of one Fresnel zone is more than the radius of the zoneplate. Therefore it is better to employ a circular aperture than a zoneplate ("sighting"). For a precision of 0.″2 the diameter of the aperture must be about $h+b = 14$ mm.

The rings between the circular slits of a zoneplate must be kept together with the aid of one, or better yet two, bars (b_1 and b_2, fig. 2.2.6a), their intersection being at the axis of the zoneplate. The bars are made on a milling machine; the slits are cut on a precision lathe.

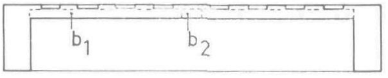


Fig. 2.2.6a Cross-section of a metal zoneplate

When the distance between the hole and the zoneplate remains constant during the measurements, which is often the case when only small displacements must be measured, a so-called phase-zoneplate may be used. It can be made by evaporating for instance magnesium fluoride on a plane (parallel) glass plate, using a metal zoneplate as a mask. If the glass is rotated about the axis of the zoneplate during the evaporation a phase-zoneplate is made without "shadows" of the bars of the mask. The optical thickness of the layer must give a phase difference π between rays passing the coated part and rays passing the uncoated part of the zoneplate. For MgF_2 this means that the thickness t of the layer is 7.2 times the thickness t' necessary for a non-reflecting coating: $t = \lambda/2(n-1) = 2nt'/(n-1) = 7.2t'$ because the refractive index of MgF_2 is about 1.38.

If the glassplate is not exactly parallel or not set perpendicular to the line of alignment, then the line through the point source, the center of the zoneplate and the center of the diffraction pattern is not straight, but small displacements can still be measured with high accuracy. An advantage of the phase-zoneplate is, that the intensity of the diffraction pattern is larger than when produced by a metal zoneplate: A zoneplate is a circular grating producing zero-order, first-order, etc. maxima, which, in their turn, produce the observed diffraction pattern. The zero-order maximum produced by a phase-zoneplate is zero in amplitude, the energy flowing to the other maxima is greater in that case.

Another possibility of generating a straight line is the reflection-zoneplate: with the aid of a stamp of suitable form [van Heel 1961a] a zoneplate can be ground (with emery) in a flat piece of glass. The glass is then coated with aluminum. The ground part scatters the light, so if this device is illuminated by an illuminated hole (placed near or on the axis), then the reflected light forms the same circular diffraction patterns as the light transmitted by a metal zoneplate. The axis is the line through the center of the zoneplate perpendicular to its surface. The straight line produced by this device can also be aimed at in autocollimation, which doubles the accuracy. Autocollimation means here that the hole and the graticule are attached to each other. The illuminated hole and graticule may lie on opposite sides of the axis, or both on the axis. In the latter case the graticule coincides with the image of the hole formed by a beamsplitter. The beamsplitter must be a cube with a semi-reflecting diagonal and not a diagonal mirror, since an oblique plate of glass introduces astigmatism of the diffraction pattern, lowering the accuracy of the settings.

The zoneplate described above has reflecting and non-reflecting zones. One can also make a zoneplate where all zones reflect, but where the phase difference between the light waves reflected by two neighboring zones equals π. This reflection-phase-zoneplate can be made in the same way as the phase-zoneplate used in transmitted light. MgF_2 is evaporated on a flat piece of glass, using a metal zoneplate as a mask. The thickness t of the thin layer must be $\lambda/4$ or 1.38 times the thickness t' of an anti-reflecting coating ($t = nt'$).

These reflecting zoneplates must not be used to measure small displacements. If the hole has a fixed position the generated line is very sensitive to a slight tilt of the zoneplate during the measurements. For that very reason it is better to use these zoneplates for measuring small changes of tilt of the zoneplate itself [Beranek 1962].

An important application is the measurement of small electric currents with a mirror galvanometer. In this case, however, only one coordinate of the rotation of the galvanometer-mirror is of interest; two strips of evaporated aluminum represent two "slits". If the diameter of the mirror is 5 mm an accuracy of $0''.2$ is attainable. This means that if the sensitivity of the galvanometer is e.g. 10^{-9} A m mm^{-1}, a current of 10^{-12} A can be detected [Gorter 1959]. If the traditional method is used the accuracy is not better than $2''$ or $3''$ (10^{-11} A). Another advantage is in this case that the flatness of the mirror is not very important.

In Britain another type of zoneplate has been used [Dyson 1958, 1960, 1961, Greenland 1962]. The most significant difference between the rather coarse circular gratings mentioned above and the British "Rodolite" (rod of light) is the number of circular slits per mm of radius. The coarse gratings have 0.5 to 2 "lines" per mm, the Rodolite has about 5 to 15 lines per mm. The accuracy of manufacture must be very high and it is very difficult to make a metal zoneplate with so many rings. A metal zoneplate is necessary if the line produced by the device must be straight. For the measurements of small displacements this is not always necessary. These Rodolites are made on glass by a photographic-chemical process; both the transmitting and the reflecting type exist. The Rodolite produces a concentric diffraction pattern which can be considered as the "image" of the illuminating pinhole. Though the "limit of resolution" (not equal to the setting accuracy) is a little bit better than that of a conventional optical system [Dyson 1958], the contrast is rather poor. Instead of a pinhole another circular grating is used in some cases; the width of each "slit" must be small enough in order to obtain a coherent illumination of the Rodolite. The different slits, however, can not be considered as coherent light sources, so the contrast in the image of the grating produced by the Rodolite is not high. The intensity of the image of such a grating is naturally higher than that of the image of an illuminated hole. The accuracy of settings made on the centers of the images of a small hole differs from the accuracy of the coarse zoneplate: Rodolite: $\pm 1/30$ of the distance between the maxima, or $d(l+l')/60l$; d is the distance between the successive "slits" of the Rodolite [Dyson 1958]. Coarse zoneplate: $\pm\lambda l'/100h$. The difference is caused by the fact that the diameter h of the narrow zone which effectively yields the diffraction pattern in the case of the Rodolite, varies with the distance l' between the Rodolite and the observed pattern. In the case of the coarse zoneplate one must always try to make the zone at the edge contribute importantly to the diffraction phenomenon. The maximum value of l' mentioned in the literature is about seven meters; for larger distances, say fifty meters, the intensity of the image produced by these fine gratings is far too low for practical use.

2.3 The use of spherical aberration and diffraction

This section will deal with some devices yielding a straight line by making use of spherical aberration. Strictly speaking the Fresnel's biprism and the Fresnel's mirrors (sect. 2.2.4) belong in this chapter. Both devices transform an incident spherical wavefront into two spherical wavefronts which do not have the same center. This can be looked at as one wavefront which is not spherical, i.e. it is afflicted with spherical aberration. The aberration must be large enough to be useful for alignment purposes. When an illuminated hole is placed on the axis of one of the devices mentioned above, all points on the axis receive light by rectilinear paths. If the distribution of the phase and amplitude of the light in the entrance pupil of the system is known the light distribution on any plane perpendicular to the axis can be calculated. In the systems which shall be described, the entrance pupil will be in the plane of the device which produces aberration. If the device is axisymmetric, the diffraction pattern consists of dark and light circle-shaped fringes with their center on the axis. The locus of the centers P is a straight line.

2.3.1 *The axicon*

McLeod [1954] describes different kinds of axicons of which the most useful one seems to be a device that can be considered as a Fresnel's prism rotated about its axis. The surface of revolution is thus a cone. The intensity distribution in a meridional plane is about the same as in the case of a biprism, while the fineness of the fringes and therefore the accuracy is also nearly the same. The intensity of the diffraction pattern is much less because a hole instead of a slit is used. There are two disadvantages. The first one is the same as in the case of the biprism, namely, a small rotation of the axicon about an axis perpendicular to the optical axis causes a small shift of the generated line. Secondly, it seems to be very difficult if not impossible to manufacture a cone with a precision of say one-tenth of a fringe. However, irregularities in the surface can be detected by rotating the axicon 180° about its axis. Not only must the surfaces be perfect, but the "parallelism" of the two surfaces should be within say 0.″1 if an accuracy of 0.″05 is desired. In spite of these disadvantages the axicons seem to give satisfying results in practice. For measuring small displacements they are excellent instruments [McLeod 1954, 1960, Leete 1961].

A device which is easier to manufacture, is a pair of Fresnel's biprisms, cemented to each other in a crossed position. Such a device can also be made out of one piece of glass (fig. 2.3.1a).

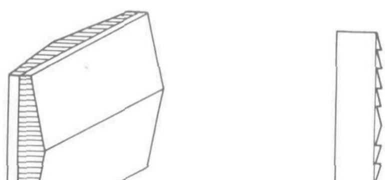


Fig. 2.3.1a, b Two biprisms; a "Fresnel conic mirror"

Fujiwara [1961] manufactured a special type of axicon, a "Fresnel conic mirror". A rotating mask of suitable form is placed in front of a plane and parallel glass plate during evaporation of magnesium fluoride, thus producing a zoneplate with a cross section as shown in fig. 2.3.1b. This seems to be an easier way of producing a device with very good axial symmetry.

These axicons can also be used for measuring small rotations. If one of the surfaces is aluminized the device can be used in autocollimation. A small tilt of a reflection axicon causes a tilt of the same magnitude in the generated line. In order to make the range large or to increase the intensity of the interference pattern, the plane (aluminized) side of an axicon can be made either concave or convex.

2.3.2 *Lenses*

Steel [1960] described a device which he called an "axicon with spherical surfaces". This is not more than a single lens having so much spherical aberration that points on the axis (P_1, P_2, P_3 . . .) receive light from a small hole by uninterrupted paths (fig. 2.3.2a). The manufacture of these lenses can be done with great precision and they are therefore preferable to axicons with non-spherical surfaces. The intensity of the fringes produced by the latter, however, is often higher.

The disadvantages of Fresnel's biprism, McLeod's axicon and Steel's lens-axicon is that they are all afflicted with third order coma (and astigmatism). The result of this is that the generated line is not straight if the illuminated hole is not placed on the axis of the device.

With the "bad" lens used by Steel a collimator, a telescope or an autocollimator can be made.

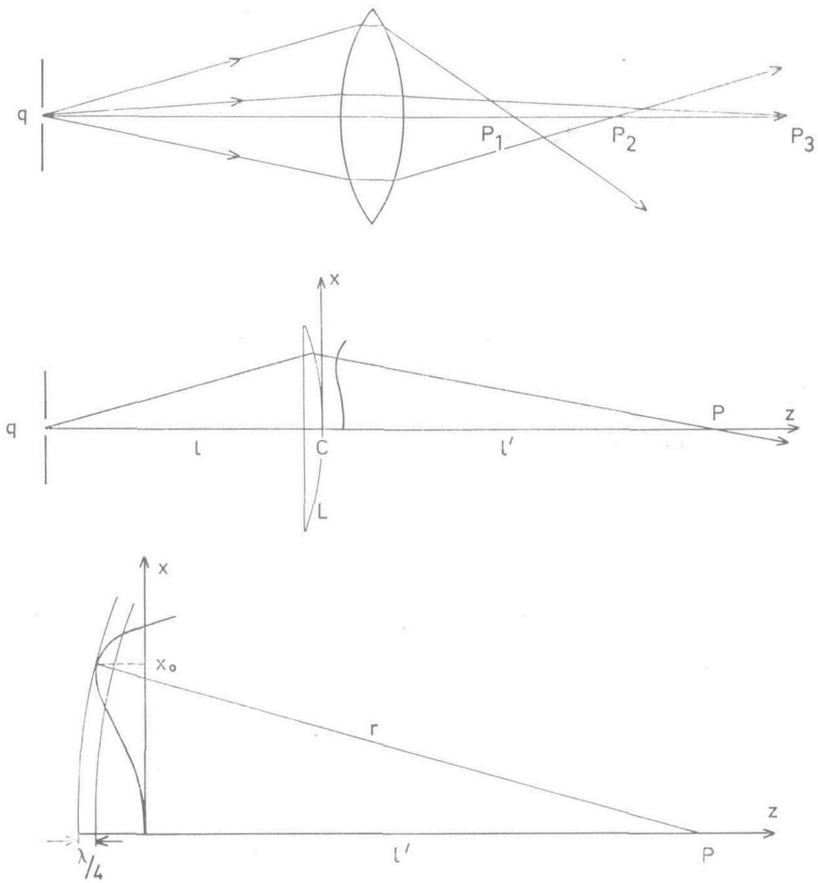


Fig. 2.3.2a, b, c Some lightrays refracted by a lens (fig. a); the shape of the wavefront is shown (fig. b); aspherical wavefront produced by a single lens (fig. c)

We will now discuss the properties of a collimator, where an objective lens uncorrected for spherical aberration is used. In fig. 2.3.2b q is an illuminated hole in a screen, l is the distance from q to the lens L and P is a point on the axis z. The x-axis cuts the z-axis in the (thin) lens at C. If we assume that l is a bit smaller than the absolute value of the first focal length f of the lens, then the wavefront near C will be convex towards P. Because L is a positive single thin lens the spherical aberration will be negative and the wavefront will be concave for large values of x. In fig. 2.3.2c the curvature of the wavefront is greatly exaggerated. Suppose that the higher-than-third-order spherical aberrations be neglected; the intersection of the wavefront and the x–z-plane can then be written as:

$$z = -ax^2+bx^4 \tag{2.3.2a}$$

In this equation a is a paraxial quantity only depending upon l, f and f', and b is a quantity related to the spherical aberration. The amplitude at P is given by:

$$\bar{a}_P = \frac{Cq}{\lambda l l'}\int_0^\infty \exp\left[\frac{2\pi i}{\lambda}\left\{-ax^2+bx^4-\frac{x^2}{2l'}\right\}\right] 2\pi x \mathrm{d}x =$$

$$= \frac{\pi C q}{\lambda l l'}\exp(-i\delta)\int_0^\infty \exp\left[\frac{2\pi i}{\lambda}\left\{x^2\sqrt{b}-\frac{1}{2\sqrt{b}}\left(a+\frac{1}{2l'}\right)\right\}^2\right]\mathrm{d}x^2 \tag{2.3.2b}$$

If a new variable t is introduced,

$$\frac{2\pi i}{\lambda}\left\{x^2\sqrt{b}-\frac{1}{2\sqrt{b}}\left(a+\frac{1}{2l'}\right)\right\}^2 = \tfrac{1}{2}\pi i t^2, \qquad \text{then}$$

$$\bar{a}_P = \frac{\pi C q}{\lambda l l'}\left(\frac{\lambda}{4b}\right)^{1/2}\int_{-k}^{\infty}\exp\{\tfrac{1}{2}\pi i t^2\}\mathrm{d}t$$

or

$$\bar{a}_P = \frac{C'q}{l l'(\lambda b)^{1/2}}\left[\int_{-\infty}^{+\infty} - \int_{+k}^{+\infty}\right] \tag{2.3.2c}$$

where

$$k = (\lambda b)^{-1/2}\left(a+\frac{1}{2l'}\right)$$

C is a quantity only related to the brightness of the lightsource used.

How large is q? When we want to see interference fringes at P the light arriving there must be coherent. It is sufficient that only a part of the diameter of the lens receives coherent light: the first integral of (2.3.2c) is due to the whole wavefront, of course, but that part of the amplitude can also be described as being generated by one "Fresnel-zone" (fig. 2.3.2c). This Fresnel-zone is a zone cut off from the wavefront by two spheres having a radius r and

$(r-\lambda/4)$ with their center at P. (Frequently a Fresnel-zone is defined by taking $\lambda/2$ instead of $\lambda/4$ in this expression; in sect. 2.2 the traditional definition is used.) The radius r is chosen such that the sphere $r(\mathrm{P})$ touches the wavefront. The path difference ΔE between a point on the wavefront and the point P is

$$\Delta E = -ax^2+bx^4 - \frac{x^2}{2l'} = b\left\{x^2 - \left(\frac{a}{2b} + \frac{1}{4bl'}\right)\right\}^2 - b\left(\frac{a}{2b} + \frac{1}{4bl'}\right)^2 \qquad (2.3.2\mathrm{d})$$

This difference is a minimum at a distance x_0 from the axis given by:

$$x_0 = \left(\frac{1+2al'}{4bl'}\right)^{1/2} \qquad (2.3.2\mathrm{e})$$

If l' is not too small, the light must be coherent over a zone having a radius of the order of magnitude $(a/2b)^{1/2}$. In this case the diameter of the hole q must not be larger than

$$q \leqslant \frac{\lambda l}{4}\left(\frac{2b}{a}\right)^{1/2} \qquad (2.3.2\mathrm{f})$$

By putting the variable part of (2.3.2d) equal to $\lambda/4$ the outer and inner radius of the Fresnel-zone can be found, as well as its area A:

$$A = \pi\sqrt{\frac{\lambda}{b}} \qquad (2.3.2\mathrm{g})$$

The amplitude at P will thus only depend upon q, l', l and A:

$$\bar{a}_P \sim \frac{q}{ll'\sqrt{b}} \qquad (2.3.2\mathrm{h})$$

This expression for the amplitude is comparable with (2.3.2c) or, in other words, it seems that the light arriving at P is generated by a circular zone of the wavefront.

From (2.3.2c) we see that if k is not too small, and when (2.3.2f) is used, the intensity at P is then proportional to (eqn. 2.3.2e)

$$I_P \sim \frac{1}{a\lambda l'^2} \qquad (2.3.2\mathrm{i})$$

If a is very small this expression is not correct because (2.3.2f) is no longer correct. A more precise expression for q is (eqn. 2.3.2e)

$$q \leqslant \frac{\lambda_0 l}{4}\left(\frac{4bl_0'}{1+2al_0'}\right)^{1/2} \qquad (2.3.2\mathrm{j})$$

where λ_0 is a fixed wavelength and l_0' is the smallest distance from the lens to the point P. It is obvious that if l_0' is chosen very small, the diameter q and therefore the intensity of the pattern will then be very small. With (2.3.2c) and (2.3.2j) the intensity at P is proportional to:

$$I_P \sim \frac{l_0'}{\lambda l'^2(1+2al_0')} \tag{2.3.2k}$$

It seems that the quantity l has vanished, but in fact a still depends upon l in the following way [van Heel 1964]:

$$a = n\,\frac{|f|-l}{|f|l} \tag{2.3.2l}$$

where n is the refractive index of the object space and $|f|$ the absolute value of the first focal length. The refractive index of the image space is $+1$. We assumed that l is measured from q to the first principal point of the lens.

What will be the accuracy of setting a cross-hair or other mark at the diffraction pattern? The light arriving at P seems to come from a circular zone with radius x_0 (2.3.2e). The distance between the maxima will be about $\delta \approx \lambda l'/2x_0$ and the accuracy will again be about 1/100 of this value:

$$\frac{\lambda l'}{200}\left(\frac{4bl'}{1+2al'}\right)^{1/2} \tag{2.3.2m}$$

For large values of l', or when $2al' \gg 1$, the accuracy is

$$\frac{\lambda l'}{200}\sqrt{\frac{2b}{a}} \tag{2.3.2n}$$

There are, however, some disadvantages to this alignment device. First of all the difference between a zoneplate and the lens-system: a zoneplate can be considered as a bad lens with a variable power, so the distances l and l' can be chosen more ore less arbitrarily. When a lens is used, the distance l must be about equal to the focal length of the lens, which is in many cases rather short. Therefore the hole-lens-system can only be used as an (auto)collimator or as a telescope. (An exception on this rule will be discussed in the last part of this section). Here again the direction of the generated line is defined by a relatively short instrument or, a short instrument extrapolates a long straight line. One should always keep in mind that a small deviation from the original position of the instrument is disastrous for the accuracy. The emission of heat radiation by a lamp or by a person can easily yield a detectable change in the direction of the generated line or the position of the points P [van Herk 1958]. In spite of this, the system has proved its usefulness in practice. Not only because the intensity of the diffraction pattern is great enough for observations in daylight at distances of about thirty meters, but also because it is quite easy to find the approximate location of the central maximum. This is due to the fact that the total diameter of the diffraction pattern is rather large and the intensity quickly increases towards the center.

The second disadvantage is the coma of the lens. The hole q must be put on the line going through the two centers of curvature of the surfaces of the lens. There are several possibilities of avoiding this difficulty.

The first one is that the first surface of the lens has its center at q. A small hole can be positioned with high accuracy at the center of curvature of a spherical surface.

The second possibility is to give the lens only one surface and fill up the room between the hole and the surface with glass [van Heel 1961b]. Fig. 2.3.2d shows such a system; the hole q can be a small aperture in the aluminized first surface. Some rays of light and the wavefront (curvature is exaggerated) are shown.

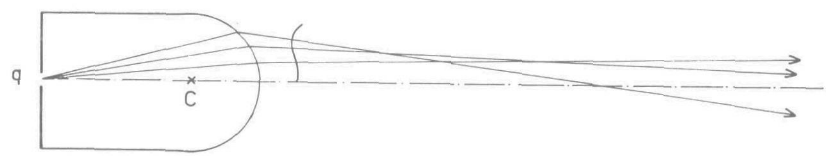


Fig. 2.3.2d A device generating a straight "line of light"

A third possibility is to use a spherical miror [van Heel 1961b]. In this case also only one center of curvature exists.

A fourth method is making the lens concentric. The best system is of course a sphere because this special concentric system is easy to manufacture [Walther 1959, van Heel 1961a, de Bruin 1961].

A system like the one of fig. 2.3.2d has been made. The data are as follows: the total thickness of the piece of glass is $l = 58.01$ mm; the radius of curvature is $r = 20.000$ mm; the refractive index for the Hg–e-line is $n_e = 1.518570$. With the aid of (2.3.2l) we find $a = 1.25 \times 10^{-4}$ mm^{-1}. From ray tracing the value of b is found: $b = 3.6 \times 10^{-6}$ mm^{-3}. Thus for this color the wavefront can be written as $z = -1.25 \times 10^{-4} x^2 + 3.6 \times 10^{-6} x^4$ mm (x in mm).

If for the smallest distance l_0' a value of 700 mm is chosen we see from (2.3.2e) that the radius of the zone x_0 corresponding to this distance is then about 10 mm. So the diameter q of the tiny hole must be smaller than 0.5 μm (in eqn. (2.3.2j) the wavelength must be divided in this case by the refractive index n).

If l_0' is large, then from (2.3.2f) and (2.3.2l) can be calculated how q depends upon the wavelength: since the quantity b – the third order aberration – hardly depends upon λ, the variation of the smallest admissable diameter of the hole is given by the variation of the quantity $\lambda/\sqrt{a}$. The fineness of the diffraction pattern is also directly related to this quantity (2.3.2n). In the red and green portion of the spectrum $\lambda/\sqrt{a}$ is nearly constant, while in the blue portion it increases slightly: C-line, $\lambda_C = 0.656$ μm: 0.47; e-line, $\lambda_e = 0.546$ μm: 0.490 and F-line, $\lambda_F = 0.486$ μm: 0.60. This means that the first few maxima are nearly white and the minima grey or black. The higher order maxima are strongly colored.

The diameter of the zone corresponding to very large distances is $2x_0 = 8.3$ mm, thus the angle between the maxima as seen from the device is $\lambda/2x_0$ or about 14″. The expected accuracy will therefore be $\pm 0\overset{''}{.}14$. In some cases even a higher accuracy is obtainable.

The admissable diameter of the small hole q is very small in this case, which means that the intensity of the pattern is not very high. If the diameter is made about 2 μm, then the contrast of the pattern at distances of more than about two meters is satisfactory. The accuracy at small distances, about $^1/_2$ meter, is not as high as mentioned, but only about one-half as good. The accuracy expressed in length units instead of seconds of arc is more or less constant at small distances: about one micron.

There is a possibility of remarkably increasing the intensity of the diffraction pattern. If instead of one tiny hole two crossed narrow slits are used, then a beautiful diffraction pattern is yielded by the spherical aberration. Besides the gain in intensity, the ease of finding the pattern is a great advantage: one moves the head up and down until the light of one of the slits (scratches) is seen, finding the other slit image is equally simple.

When a device such as described above is only used for small distances (l'), the thickness (l) of the lens can be made somewhat larger. When for instance the largest distance is $l_0' = 2$ meters, the length l can be made about 60 mm instead of 58 mm, the quantity a is then about -2.3×10^{-4}mm^{-1} and the paraxial image of the hole lies two meters behind the "lens".

The straightness of the generated line depends on the homogeneity of the glass and on the sphericity of the spherical refracting surface. If there is a gradient of the refractive index in the direction of the z-coordinate only, the line is still straight. If a small gradient in the x-direction exists, the generated line is only tilted a little bit, which means that the "images" of the illuminated hole are not stigmatic any more, this does not yield a curvature of the line. A large gradient also produces coma, especially if the variation of the refractive index is not linear. Coma does give rise to a curved line. For rather small pieces of glass, as used for the system mentioned above, these variations can be kept small when very homogeneous glass is used (BK7 from Schott-Mainz). If there exist local inhomogeneities in that part of the glassbody where the rays of light penetrate, the straightness of the line is no longer guaranteed. Suppose that somewhere in the light path a small local inhomogeneity is present, with a diameter of say five millimeters; if the refractive index is one unit in the sixth decimal place greater inside the inhomogeneity than outside, then the optical path-difference produced is about one-hundredth of a wavelength, which is just detectable. The polishing of the spherically surface must be done very carefully, in order to obtain a very smooth surface: a lump on the glass surface of one-twentyfifth fringe gives rise to an optical path difference of one-hundredth of a wavelength. The surface smoothness thus must be better than one-twentyfifth of a fringe; the whole surface must be sphericaly, better than say one-tenth of a fringe. Even when the glass is very homogeneous and when the surface is polished with great care, the straightness of the generated line must always be tested.

We have tried to design a lens system which does not have the disadvantage that the lightsource must be placed near the focus of the system. Such a lens system must be able to send rays of light to all points of the axis wherever the lightsource is positioned. We have designed a system with positive power and overcorrected spherical aberration. The focal distance is about +140 mm. When an illuminated hole is placed before the lens at a distance of 500 mm or more, the paraxial image lies about 150 mm behind the lens. The spherical aberration is so large, that the ray of light, coming from an object at infinity, emerging from the system parallel to the axis, has a height above the axis of 4.65 mm for the green mercuryline. This height does not change very much with color. The wavefront at the vertex of the last surface can be written as $z = -ax^2+bx^4$. For this lens $a = -4.2\times10^{-3}$ mm^{-1} and $b = -96\times10^{-6}$ mm^{-3}. When we compare the intensity at a point on the axis with the intensity of the system from fig. 2.3.2d we see that this new system has a "b" which is about 27 times as large as the b of the other system. This means that the intensity is 1/27 as large (eqn. 2.3.2h); the diameter h (or $2x_0$) of the effective zone is about equal in both cases ($\pm$ 9 mm). The disadvantage is not so great, since one can use a bright light source in the new system; most bright light sources produce too much heat, so they can only be used to illuminate the former system if the light is filtered. If not, the change in temperature of the glass-body is disastrous for the stability and the accuracy. In the new system

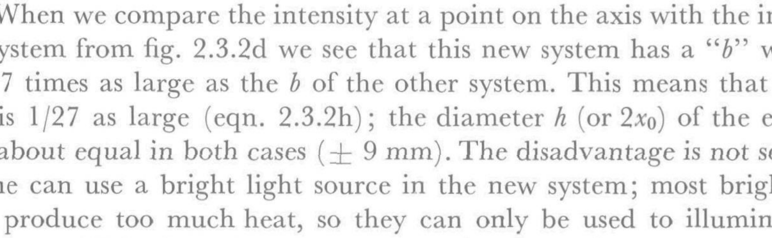


Fig. 2.3.2e A concentric lens system; some rays of light and the generated wavefront

the light source is positioned at a relatively large distance from the lens. We will now give the data of the new lens system (fig. 2.3.2e):

surface	radius (mm)	refractive index (e-line)	thickness (mm)
1	+20.000	1.70340	14.988
2	+ 5.012	1.48675	10.024
3	− 5.012	1.70340	14.988
4	−20.000	–	–

This system is concentric; only one center of curvature exists so the lens system is not sensitive to rotations about this center! So as far as the coma is concerned, this is a better system than the zoneplate. However, the manufacture is rather difficult. It has the additional disadvantage that the line joining the illuminated hole, the center and the center of the diffraction image, is not straight generally, because of errors in manufacture. The diameter $h = 9$ mm gives rise to an accuracy of $0\overset{''}{.}14$. This accuracy is obtainable up to distances $l' = 20$ meters.

In fig. 2.3.2f a photograph of the diffraction pattern at a distance of 2.1 m is shown. (The lack of contrast in this reproduction is only due to the fact that the difference between the intensity of higher order fringes and the intensity of the central maximum is very great. If the pattern is observed with an eyepiece, the contrast is high, especially at the center.)

2.4 Applications and remarks

With the instruments mentioned in the previous sections, most alignment problems can be solved. Very accurate measurements, such as measuring the errors of machine tools, and the more obvious uses in large civil constructional work and industries can be carried out. In many cases the instrument used provides an optical datum plane (double-slit method) or line (zoneplate) to which e.g. the bed of a lathe is referred.

The straightness of a lathe is tested by mounting a zoneplate, axicon or other device on the saddle. The lightsource and illuminated hole can be attached to the spindle. This procedure implies that the crosswires or other fiducial mark is placed in a fixed position on the end of the lathe bed. Even when the bed is straight the position of the diffraction or interference pattern with respect to the mark is generally not constant, because the height of hole q and of the fiducial mark above the bed are not the same generally. Therefore it is better to give the hole q and the zoneplate a fixed position, and attach the crosswires to the saddle. The variation of the pattern with respect to the crosswires is a linear function of the saddle movement in this case. When the straightness of the bed is measured, a reflecting zoneplate, or other device that can be used in autocollimation, is attached to the spindle. The hole q, beamsplitter and cross-wires are mounted on the saddle and can be moved with respect to the saddle in two directions by means of two micrometers. The lack of parallelism between the saddle movement and the spindle axis and the straightness of the axis are measured by rotating the spindle and moving the saddle [Dyson 1961]. The accuracy is in many cases limited by the accuracy of the micrometers, e.g. 1 μm.

This is only one example of an application where zoneplates, axicons, "Rodolites", cubes or other alignment devices can be used. The accuracy is higher than that obtained with traditional instruments.

One other example of an application of reflecting zoneplates for the measurement of small rotations will be discussed briefly; the testing of models of buildings or constructions. The zoneplates are attached to the model. A small light-source is "imaged" by the zoneplates on a screen of frosted glass, the displacements of the centers of the images give a measure of the magnitude and the directions of the rotations when the model is loaded [Beranek 1962].

Many other applications are described in the literature. We will for a few cases mention the accuracy obtained and the device used.

De Haas [1953] used the double-slit method for measuring the movements of among other things glaciers, with respect to two fixed points. The dimensions of the double slit used were $h = 8$ mm and $b = 1.6$ mm, while the distances l and l' were about 60 meters. The accuracy at this distance is about 0.05 mm, which is the same as would be expected (see first par. of sect. 2.2.2). When eqn. (2.2.2c) is applied, it can be seen that each of the slits covers about one Fresnel zone. The width of the illuminating slit de Haas used is about 0.8 mm, while a slit of 2 mm would be allowed (eqn. 2.2.2a). From the fact that, even at a distance of 60 meters, as used in this case, the interference pattern can still be observed, we may conclude again that this method is of great utility in practice.

Van Heel [1946] used a double slit where $h = 4$ mm, $b = 2$ mm and $l = l' =$ 35 m. The value of b is again equal to about one Fresnel zone; the accuracy obtained was better than would be expected, namely 1/250 instead of 1/100 of the distance between the minima. With a similar method and a so-called pentaprism the amplitude of oscillations of a church tower could be measured while the bells pealed [van Heel 1949].

In the same article by van Heel [1949] the zoneplate is mentioned; one application is the alignment of the bearings of the propeller-shaft of a ship.

An important application of a zoneplate was described by Franx [1953] and Richardus [1954] and was used by Reitz [1963]. A zoneplate is mounted in front of the objective lens of a theodolite. The theodolite can be aimed at an illuminated hole with a higher accuracy than usual. Other advantages include:

the effect of haze or bright sunshine on the contrast of the diffracted image of the hole is small, the telescope need not be focused for different distances and when measurements are carried out in darkness, no special mirror in the telescope tube is necessary to illuminate the crosswires. Such a mirror even disturbs the diffraction pattern and must therefore be removed.

Moonen [1955] used a zoneplate to find the orientation of a line in a mining-gallery with reference to the meridian line: Two point sources at the bottom of the shaft provide a straight horizontal line in the gallery. A zoneplate is placed midway between the groundlevel and the level of the gallery at the axis of the shaft. The two point sources and the zoneplate provide two straight lines in one plane, the two intersections of these lines with a horizontal plane at groundlevel provide another straight line which is parallel to the line in the gallery. The depth and diameter of the shaft are respectively 150 m and 1.75 m. The accuracy of the orientation of the gallery was about 30″; an accuracy of 18″ is attainable.

Van Heel [1955] and van Herk [1958] measured the flexure of a large telescope, which causes a change in the direction of the axis. An illuminated slit or hole is mounted near the center of the objective lens of the instrument. A Dove-prism is mounted near the axis of rotation in the middle part of the telescope tube and before this prism a double slit or zoneplate is mounted. The diffraction pattern is observed in the eyepiece of the telescope. The shift of the pattern with respect to the crosswires equals the difference between bending of the upper and lower halves of the telescope tube. In this case another device could have been used in order to produce fringes, namely the "two cemented Dove-prisms" shown in fig. 2.2.5b.

In two articles by Liem [1960, 1961] the zoneplate is described as a device that has been used to measure the deformation of experimental runways at Schiphol airport. A truck loaded up to one hundred tons moved across the runway, yielding stresses comparable to what modern airliners produce. An accuracy of 0.02 mm has been obtained. The distances hole-zoneplate and zoneplate-crosswires (plus camera) were twenty meters. This work was carried out by the Technical Physics Department T.N.O.-T.H. at Delft. The zoneplate-method as described above appeared to be of great use for many practical applications. This is perhaps the place to express a few words of thanks to the staff of the Technical Physics Department T.N.O.-T.H. for the many fruitful discussions.

For completeness we will mention here a device, described by Saunders [1963a, b], which also yields a straight line, or rather the intersection of two planes perpendicular to each other. It consists of two Kösters double-image prisms, a semi-reflecting mirror and a lens or mirror system. A light source at a remote point can be positioned on the "axis" of the instrument, the accuracy depending upon the aperture used. If the aperture is 25 cm, the accuracy is 0."05. Saunders does not discuss the demands made upon the homogeneity of the glass and the surfaces of the optical parts used. The instrument can be considered as a pointing telescope or alignment telescope having two eyepieces, one to adjust the light source in the vertical direction and the other for the horizontal direction. We think it is less expensive and easier to use an ordinary alignment telescope or theodolite, to which a zoneplate is attached.

The use of a laser as a lightsource for a zoneplate has already been mentioned; this application is described in articles by van Milaan [1964, 65] and Raterink [1965]. Dynamical measurements of displacements of a bunker under influence of explosions are described. A high-speed camera recorded the movements (one disadvantage is that one must first develop the film before the results are known). Another interesting application is mentioned by van Milaan: in Belgium, near Ronquières an incline is being built, over which ships will be towed in water-filled tanks, for a distance of about 1500 meters, to overcome a difference in height of 68 m between two parts of the canal between Brussels and Charlerois. The movement of the tanks must be rectilinear, in order to avoid unwanted accelerations (the weight of the filled tank being about 5000 tons!). The straightness of the rails was adjusted with the aid of a zoneplate, a laser being used as lightsource. The accuracy was 2 mm, over a distance of 1500 meters.

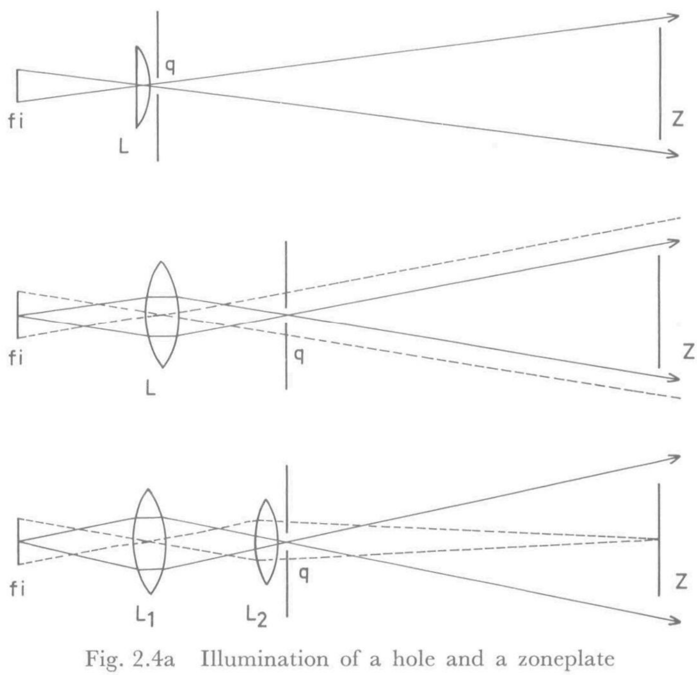


Fig. 2.4a Illumination of a hole and a zoneplate

Some other important applications are mentioned by van Milaan: the adjustment of foundation blocks of a factory building, the adjustment of the end-planes of the parts of an underwater traffic tunnel perpendicular to a given line, and the determination of a horizontal plane for the foundation of a machine. In these cases a pentaprism was used which turns the line, generated by an illuminated hole and a zoneplate, through an angle of 270°.

Some additional remarks must still be made. It is obvious that when the distance between the illuminated hole and the zoneplate (l) is not too large, say 10 meters, the diameter q of the hole must be small (e.g. 0.5 mm). The illumination of both the hole and the zoneplate, must be even. These requirements are easily met if the hole is indeed small. However, when the hole is large, $q \geqslant 5$ mm, which is the case when l is large (100 meters or more), an even illumination of the hole and the zoneplate is somewhat more difficult. The filament of the electric bulb or the dimensions of the discharge of the high pressure mercury arc used, must not be too small. Some possibilities are shown in figure 2.4a; fi is the filament, q is the hole, Z is the zoneplate and L is a lens.

2.5 The pentaprism

The pentaprism turns the line of sight through an angle of 270°; this angle is invariant because, if the prism is properly made, the angle does not change when the prism is rotated about an axis perpendicular to the plane of figure 2.5a. When the prism is rotated about the x-axis, the direction of the emerging light ray remains perpendicular to the incoming ray. If, however, the prism is rotated about the y-axis, then the direction of the emerging light ray changes.

In alignment procedures not only the direction of a line, but also its origin is

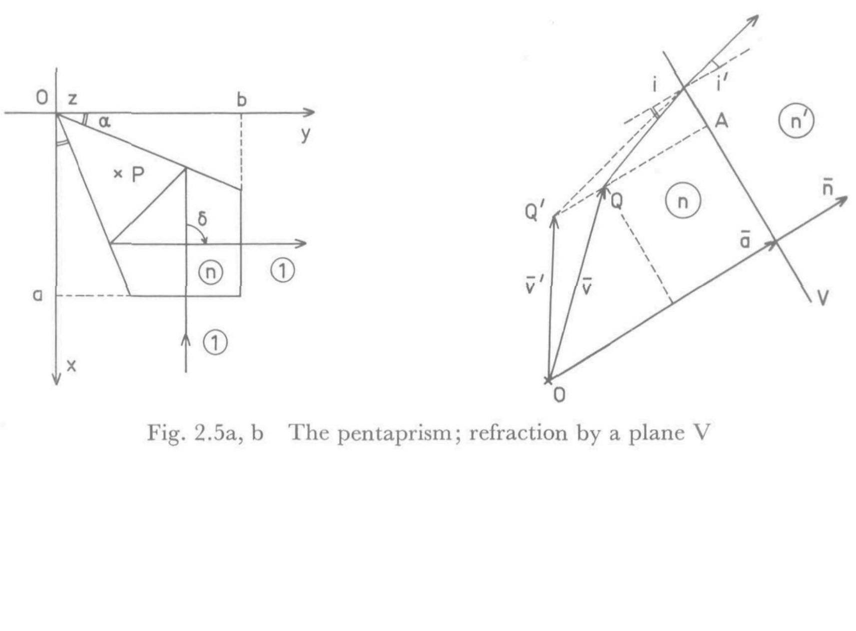


Fig. 2.5a, b The pentaprism; refraction by a plane V

of interest. It can easily be seen that a rotation of the pentaprism about the z-axis causes an unwanted shift. There exists, however, an axis (through the point P) parallel to the z-axis, about which the pentaprism can be rotated without producing a shift (or change of direction) of the emerging light ray.

When one wants to produce an alignment plane by means of an existing alignment line and a pentaprism, the pentaprism must be rotated about the incoming line since a rotation of the pentaprism about any other axis produces something other than a plane. However, the pentaprism can be given such dimensions that it is not necessary that the mechanical quality of this axis be perfect. With correct dimensions we mean such dimensions that the incoming and the emerging light ray intersect at P [van Milaan 1964, 65]; this is not the case in the traditional pentaprism.

We will now calculate how much the prism may be rotated about the y-axis, in order to keep the angle between the two rays within 270° ± say 0.″1. This can be done with a matrix method [T. Smith 1928–29].

In fig. 2.5b a plane V is shown and a point Q. The point Q is situated in a space having the refractive index n. If Q is imaged in the space with refractive index n' by means of a ray of light nearly perpendicular to the plane V, then Snell's law can be written as $ni = n'i'$, i and i' being the angle of incidence and the angle of refraction (if $i = 1°$ and $n = 1.5$, then the angle i' thus calculated differs only 0.″06 from the correct value). So the distance between Q and Q′ is equal to

$$(n'/n-1)\cdot \mathrm{QA} = \mu\cdot \mathrm{QA} \tag{2.5a}$$

Let O be the origin of the coordinate-system, a the distance from O to V, $\bar{n}$ the vector normal to the plane V, $\bar{v}$ the vector OQ and $\bar{v}'$ the vector OQ′. The vector $\bar{v}'$ can be expressed in the other quantities:

$$\bar{v}' = \bar{v} - \left(\frac{n'}{n} - 1\right)\left\{a - \bar{v}\cdot\bar{n}\right\}\bar{n} \tag{2.5b}$$

If x, y and z are the coordinates of the point Q, and L, M and N the direction cosines of the vector $\bar{n}$, the position of the point Q′ is given by:

$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} 1+\mu L^2 & \mu LM & \mu LN & -a\mu L \\ \mu LM & 1+\mu M^2 & \mu MN & -a\mu M \\ \mu LN & \mu MN & 1+\mu N^2 & -a\mu N \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = M \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} \tag{2.5c}$$

The matrix M in the case reflection and not refraction occurs is easy to find by putting $n' = -n$ or $\mu = (n'-n)/n = -2$. The total matrix M_p for the pentaprism can be found by multiplication of a refraction matrix, two reflection matrices, and again a refraction matrix. If only the direction of a light ray OQ is of interest, the fourth row and column of M can be omitted and instead of

x, y and z the direction cosines $\cos\varphi$, $\cos\psi$, and $\cos\vartheta$ introduced. It turns out that when the four planes of the prism are perpendicular to the x–y-plane, and the angle $\alpha = 22\overset{\circ}{.}5$ (fig. 2.5a), the connections between the directions of the incoming ray and the emerging ray are as follows:

$$\begin{pmatrix}\cos\varphi'\\ \cos\psi'\\ \cos\vartheta'\end{pmatrix} = \begin{pmatrix}0 & +1 & 0\\ -1 & 0 & 0\\ 0 & 0 & +1\end{pmatrix} \cdot \begin{pmatrix}\cos\varphi\\ \cos\psi\\ \cos\vartheta\end{pmatrix} \tag{2.6d}$$

or

$$\begin{aligned}\cos\varphi' &= \cos\psi\\ \cos\psi' &= -\cos\varphi\\ \cos\vartheta' &= \cos\vartheta\end{aligned}$$

So the angle δ between the two light rays is given by $\cos\delta = \cos^2\vartheta$ (the directions of the rays are given by two vectors having a magnitude one, so the cosine of the angle is equal to their scalar product). Let β be the angle between the incoming ray and the x-axis, or $\beta = 90° - \vartheta$ and γ be equal to $90° - \delta$. Then $\cos(90° - \gamma) = \cos^2(90° - \beta)$ or $\gamma \approx \beta^2$. This means that when the pentaprism is rotated about the y-axis over an angle β, the angle between the two rays is not 270° anymore, but a little bit larger, namely by an amount $\gamma = \beta^2$. Example: if $\beta = 2.5$ minutes of arc, then $\gamma = 0.1$ seconds of arc.

When the prism is rotated about the y-axis, not only the direction of the emerging ray is incorrect, but also the origin. The pentaprism has the same effect in this case as a thick plane parallel plate of glass. The effective thickness is $2a$ if a and b are equal (fig. 2.5a). Thus a rotation over an angle β produces a shift of $2a\beta(n-1)/n$. If $n = 1.5$ and $a = 60$ mm, then the shift equals 40β mm. $\beta = 2\overset{\prime}{.}5$ causes a shift of 30 μm.

We will now discuss the effects of rotation of the prism about an axis parallel to the z-axis. At the first plane of the prism a refraction occurs; the indices of refraction of the space outside and inside the prism are 1 and n, the distance between the origin O and the plane is a. The next two surfaces cause reflections; the distance from the origin is zero. Finally, the fourth surface; the refractive indices are now n and 1 respectively, the distance from the origin is b. We are only interested in what happens in the x-y-plane, so we omit the third dimension. The normal vectors on the respective planes are given by (1,0), $(\cos\alpha, -\sin\alpha)$, $(\sin\alpha, -\cos\alpha)$ and (0,1). We will take $\alpha = 22\overset{\circ}{.}5$. When we calculate the four matrices from these data, and then calculate the product, we find the complete matrix for the pentaprism:

$$\begin{pmatrix}x'\\ y'\\ 1\end{pmatrix} = \begin{pmatrix}0 & 1 & 0\\ -1 & 0 & (a+b)(n-1)/n\\ 0 & 0 & 1\end{pmatrix} \cdot \begin{pmatrix}x\\ y\\ 1\end{pmatrix} \tag{2.5e}$$

where (x,y) are the coordinates of a point Q, and (x',y') are the coordinates of the image of Q, Q′, formed by the pentaprism.

We now choose a point P about which the pentaprism is rotated. After the rotation we can calculate the displacement of Q′. It is easier to rotate the point Q, then find the image of the rotated point and finally rotate the image in the opposite direction about the same point P. The position of the point Q′ found in this way, can be compared with the position of the point Q″ if no rotation occurs. This can be written schematically as:

$$(Q'') = (Pr)\cdot(Q)$$

$$(Q') = (\beta^-)\cdot(Pr)\cdot(\beta^+)\cdot(Q)$$

(Pr) is the prism-matrix, (β^+) and (β^-) are the two rotation-matrices:

$$(\beta^+) = \begin{pmatrix} \cos\beta & -\sin\beta & x_P(1-\cos\beta)+y_P\sin\beta \\ \sin\beta & \cos\beta & -x_P\sin\beta+y_P(1-\cos\beta) \\ 0 & 0 & 1 \end{pmatrix} \tag{2.5f}$$

where x_P and y_P are the coordinates of the point P. When the three matrices are multiplied we find the position of Q′:

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 & A \\ -1 & 0 & B \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} \tag{2.5g}$$

where

$$A = (x_P - y_P) + \sin\beta\,(x_P + y_P - \tau) + \cos\beta\,(-x_P + y_P)$$

and

$$B = (x_P + y_P) + \sin\beta\,(-x_P + y_P) + \cos\beta\,(-x_P - y_P + \tau)$$

where

$$\tau = (a+b)(n-1)/n \tag{2.5h}$$

If a point $P(x_P,y_P)$ exists, so that a rotation of the pentaprism about P does not produce a change in the position of $Q'(x',y')$, then it must be possible to make the quantity A (and B) independent of the angle of rotation β. This is the case when both $(x_P - y_P)$ and $(x_P + y_P - \tau)$ are zero. The first condition leads to the remarkable fact that the point P must be situated on the bissectrix of the 45-degree angle of the prism, even if the lengths a and b of the prism are not equal. The second condition gives us the precise position of P: $x_P = y_P = \tau/2 = (a+b)(n-1)/2n$ or, if a and b are equal, $x_P = y_P = a(n-1)/n$. The first conclusion is, that if the pentaprism has a refractive index equal to 1, the point P lies at the vertex of the prism, which consists of two plane mirrors set at an angle of 45° [Beggs 1960]. The second conclusion is that this point P generally does not coincide with the point of intersection of the two light rays shown in fig. 2.5a.

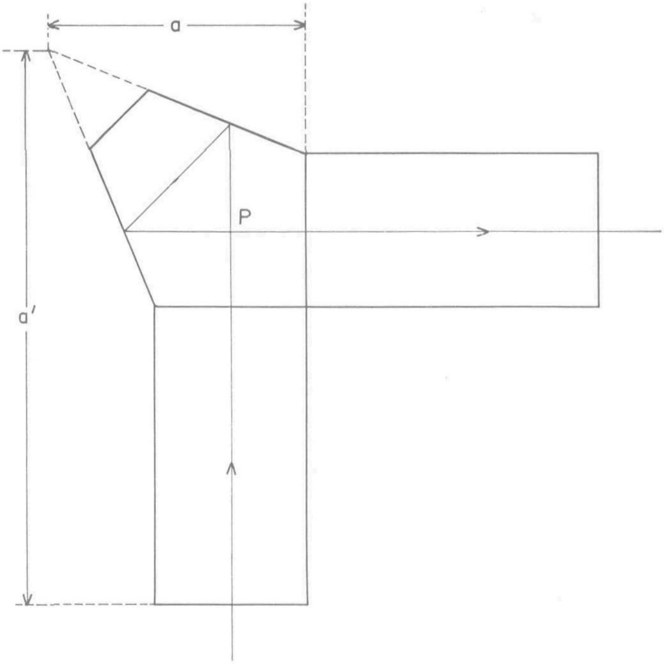


Fig. 2.5c A modified pentaprism

If the traditional pentaprism is modified somewhat, the two points coincide (fig. 2.5c). In this figure the quantity a' is equal to $na\sqrt{2}/2(n-1)$. The two plane and parallel plates must not be cemented to the prism but wrung onto it. When the angles of the prism are not exactly 45°, 112°.5, 112°.5 and 90°, the emerging ray of light generally does not leave the prism at an angle of 270° to the incident ray (only the 45° and 90°-angle are important). The two plates can be given such an angle as to compensate for the errors of the pentaprism.

The disadvantages of this modified prism are: the optical path through glass has become larger, so a rotation β about the y-axis has more influence, and besides, the device is rather clumsy.

We can also calculate from eqn. (2.5g, h) how much the displacement is, in case the point of rotation P is not the same as the ideal point. Let t be the coordinates of the ideal point, $x_P = t+\lambda$ and $y_P = t+\mu$ the coordinates of the rotation point, x and y are the coordinates of Q. The position of the image of Q is then given by:

$$x' = y+A = y+(\lambda-\mu)+(\lambda+\mu)\sin\beta - (\lambda-\mu)\cos\beta$$

$$y' = -x+B = -x+(\lambda+\mu)-(\lambda-\mu)\sin\beta - (\lambda+\mu)\cos\beta + 2t \qquad (2.5j)$$

When $\beta = 0$, $x_0' = y + (\lambda - \mu) - (\lambda - \mu)$, and we are interested in the change in position of the emerging light ray, so only

$$x' - x_0' = \Delta x' = (\lambda + \mu) \sin \beta - (\lambda - \mu)(\cos \beta - 1) \tag{2.5k}$$

is of interest. When a traditional pentaprism is used, the quantities λ and μ are equal to

$$\lambda = \mu = \left(\tfrac{1}{2}\sqrt{2} - \frac{n-1}{n} \right) a \tag{2.5l}$$

so if β is small, the shift of the ray is given by the following expression:

$$\Delta x' = a \left(\sqrt{2} - 2\,\frac{n-1}{n} \right) \beta$$

Example: $a = 60$ mm, $n = 1.5$ and $\beta = 2\overset{'}{.}5$ ($\beta \approx 73 \times 10^{-5}$ radians): $\Delta x' = 0.033$ mm.

This is about the same shift as produced by a rotation about the y-axis over the same angle. So we think the traditional pentaprism can be used if one takes care that the angle of incidence at the first surface is small. This can be done for instance by evaporating an aluminum zoneplate on this first surface. The zoneplate can be used in transmission together with an illuminated hole to produce a straight line. The hole can be put on the axis of the zoneplate in autocollimation. The pentaprism is mounted so that it can rotate about the axis of the reflecting zoneplate.

Equation (2.5k) calls for comment; first of all, the displacement $\Delta x'$ of the emerging ray does not depend upon the dimension a of the pentaprism. Secondly, the refractive index has no influence upon $\Delta x'$. An important conclusion is that if $\lambda = -\mu$, the displacement of the ray is very small: if $\lambda = -\mu = 20$ mm and $\beta = 2\overset{'}{.}5$ the displacement is then only $\Delta x' = 0.01$ µm.

In several cases a pentaprism is used to establish a plane by rotating the prism about the line of sight of a collimator or telescope, or another alignment line. Rotation about any other axis than the line of sight, will not result in a plane. When the axis e.g. is parallel to the line of sight, at a distance d, two lines at an angle of 180° in the plane produced, will be separated from each other by a distance not greater than $2d$. The distance between two such lines when the rotational axis is at an angle to the line of sight can be calculated from eqn. (2.5k).

2.6 The influence of the atmosphere

When alignment measurements are done over long distances, the inhomogeneities in the air may greatly diminish the accuracy. If, however, the inhomogeneities do not change rapidly with time, then the curvature of the alignment line can be calculated if the gradient of the refractive index of the air is known. It is therefore necessary to know the refractive index of air as function of pressure, temperature and humidity. The refractive index is given by:

$$(n_{t,p}-1) = (n_{15,760}-1)\frac{p}{760}\cdot\frac{1+p(1.049-0.0157t)10^{-6}}{1+760(1.049-0.0157\times 15)10^{-6}}\cdot\frac{1+0.003661\times 15}{1+0.003661t} - \frac{m}{760}41\times 10^{-6} \tag{2.6a}$$

where $n_{15,760}$ is the refractive index at $t = 15\,°C$, $p = 760$ mm Hg and a pressure of the water vapor $m = 0$ mm Hg. The value of $n_{15,760}$ can be found from

$$(n_{15,760}-1)10^8 = 6432.8 + \frac{2949810}{146-\sigma^2} + \frac{25540}{41-\sigma^2} \tag{2.6b}$$

This equation gives the refractive index of dry air containing 0.03% CO_2, σ is the reciprocal value of the wavelength used: $\sigma = \lambda^{-1}\ \mu m^{-1}$. (Am. Inst. of Physics Handbook, VI, McGraw Hill 1963, 96). When $\lambda = 0.56\ \mu m$, $n_{15,760}-1 = 277.6\times 10^{-6}$.

We are interested in small variations of the refractive index. If the index changes rapidly perpendicular to the alignment line, measurements must not

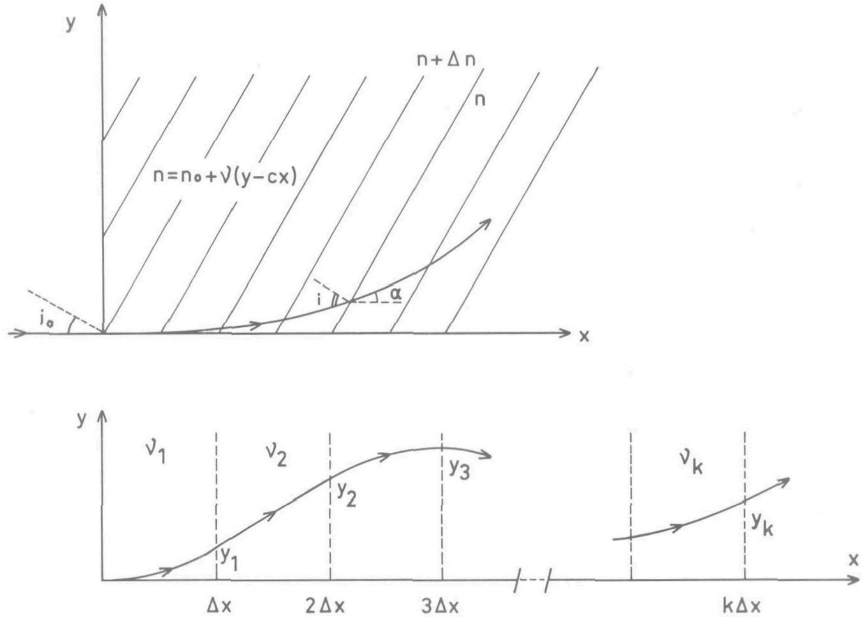


Fig. 2.6a, b The gradient of the refractive index; a ray of light consisting of several parabolas

be attempted. Or, if the corrections for the curvature of the rays of light are much larger than the setting accuracy, then one must wait until the air is more homogeneous. From eqn. (2.6a) the derivatives may be calculated:

$$\frac{\partial n}{\partial t} = -0.97\times 10^{-6}\,{}^{\circ}\mathrm{C}^{-1} \quad \frac{\partial n}{\partial p} = +0.37\times 10^{-6}\mathrm{mm}^{-1} \quad \frac{\partial n}{\partial m} = -0.054\times 10^{-6}\mathrm{mm}^{-1}$$

We will now calculate how a ray of light behaves in a medium where the gradient ν of the refractive index is constant. In fig. 2.6a different surfaces of constant n and a ray of light are shown.
The angle of incidence at a surface is i. Snell's law reads: $n_0 \sin i_0 = n \sin i$. The refractive index is related to the slope y' of the light ray as follows:

$$n = \frac{n_0 \sin i_0}{\sin i} = \frac{n_0 \sin i_0}{\sin(i_0+\alpha)} = \frac{n_0 \sin i_0\,(1+y'^2)^{1/2}}{\sin i_0 + y' \cos i_0} = n_0+\nu(y-cx) \quad (2.6c)$$

where c is equal to cot i_0. So the path of the light ray is given by the differential equation

$$1 + \frac{\nu}{n_0}(y-cx) = \frac{(1+y'^2)^{1/2}}{1+cy'} \qquad y(0)=0 \qquad y'(0)=0 \qquad (2.6d)$$

The solution can be approximated by a parabola

$$y = \frac{\nu}{2n_0}x^2 \qquad (2.6e)$$

The exact solution is a catenary or hyperbolic cosine function. This is easily seen by putting c equal to zero. The solution is then

$$y = \frac{n_0}{\nu}\cosh\frac{\nu x}{n_0} - \frac{n_0}{\nu}$$

If the alignment line is chosen coincident with the x-axis, starting at the origin of the coordinate system of fig. 2.6b, one can measure $\nu = \partial n/\partial y$ for different values of x. The most simple case occurs when ν does not change with x, for then the ray of light is a parabola to a close approximation.

If ν changes with x, but is known at every point, then the y-value of the ray in a point at a distance L from the origin is given by:

$$y(L) = \int_0^L \mathrm{d}x \int_0^x \frac{\nu}{n}\,\mathrm{d}x \qquad \nu = \left(\frac{\partial n}{\partial y}\right)_{y=0} \qquad (2.6f)$$

The quantity $1/n$ in the integrand may be replaced by $1/n_0$ or even by 1 because n_0 is nearly constant and equal to 1.

In most practical applications of eqn. (2.6f) the gradient of the refractive index is measured only at a few places. The light ray can then be approximated by several different parabolas. The different y-values are given by (fig. 2.6b):

$$
\begin{aligned}
y_1 &= \frac{\nu_1}{2n_0}\,\Delta x^2 \\
y_2 - y_1 &= \frac{\nu_2}{2n_0}\,\Delta x^2 + \left(\frac{\nu_1}{n_0}\right)\Delta x^2 \\
y_3 - y_2 &= \frac{\nu_3}{2n_0}\,\Delta x^2 + \left(\frac{\nu_1}{n_0} + \frac{\nu_2}{n_0}\right)\Delta x^2 \\
&\text{etc.} \\
y_k - y_{k-1} &= \frac{\nu_k}{2n_0}\,\Delta x^2 + \frac{\Delta x^2}{n_0}\sum_{1}^{k-1} \nu_i
\end{aligned}
\tag{2.6g}
$$

This equation gives the vertical deviations y_k from the straight line x. A similar equation gives the deviations z_k in a horizontal direction, instead of $\nu = \partial n/\partial y$, the quantity $\partial n/\partial z$ must be used.

In an article by van Milaan [1964, 65] the alignment of a long incline near Ronquières, Belgium is described. (See also sect. 2.4). We will discuss one of the measurements which has been done. The distance between the illuminated hole and the zoneplate was 150 meters, the distance between the hole and the observed diffraction pattern was 350 m. In the table the mean values of $\nu = \partial n/\partial y$ over 50 meters are given as well as the values of $y_k - y_{k-1}$ and y_k as calculated from (2.6g). $\Delta x = 50$ m:

k	$\nu \cdot 10^6$ (m^{-1})	L (m)	$y_k - y_{k-1}$ (mm)	y_k (mm)
1	+0.04	50	0.05	0.05
2	+0.00	100	0.00+0.10	0.15
3	+0.04	150	0.05+0.10	0.30 (0.33; 0.45)
4	+0.04	200	0.05+0.20	0.55
5	+0.04	250	0.05+0.30	0.90
6	+0.04	300	0.05+0.40	1.35
7	+0.00	350	0.00+0.50	1.85 (1.78; 2.45)

(In the case the quantity ν is supposed to be constant along the x-axis, either $0.029 \times 10^{-6} m^{-1}$ or $0.04 \times 10^{-6} m^{-1}$, y_3 and y_7 differ from the values of y_3 and y_7 indicated in the last column of the table. Next to the last column these quantities are given for the two values of ν mentioned). If the straight line through the origin of the coordinate system where the illuminated hole is positioned, and the zoneplate is taken as the reference line, then the deviation from straightness (the distance between the fiducial mark and the straight reference line) is 1.15 mm (1.01 mm or 1.40 mm when the other values of ν as mentioned above are used) or 1.″2.

Because the setting accuracy was about 0.″2 this correction is not too large. The accuracy desired is 1 mm.

Finally some general remarks:

Experiments have shown that the air is rather homogeneous on a cloudy day, when the soil is wet from rain, and the windvelocity is not too small.

Chauvenet [1891] showed that the curvature of a ray of light due to the influence of the earth's gravity on the density of the air only, is about 0.14 times the curvature of the earth's surface. The curvature can be plus or minus 40 times as large, especially on clear days in summer. For overcast days the curvature may be $^1/_{10}$ to $^1/_5$ of the curvature for clear days. In general the curvature is nearly zero at two hours after sunrise and two hours before sunset.

At sea the curvature of a light ray is smaller than at land. Gigas [1960] mentions deviations of $\pm 2'$ (minutes of arc) over a distance of 20 km, or $\pm 6''$ for 1 km distance; the curvature due to the earth's gravity is about 4″ for 1 km distance.

Another effect, which does not necessarily cause a systematic error, is the appearance of relatively small areas of turbulence, which decrease the setting or the aiming accuracy. Washer and Scott [1947] have measured the probable error of a single pointing with a telescope under average weather conditions. With the telescope they used, the accuracy was about 0.″1 in the laboratory, where a collimator was aimed at. In the open air the accuracy from 100 to 1000 meters was about 0.″5, from 1000 to 4500 meters 0.″6, and from 4500 meters to very large distances about 0.″7.

When very precise straight lines must be defined over great distances, it is possible to measure inside a long tube which is evacuated to less than 0.1 mm Hg.

CHAPTER 3

PLANE SURFACES

3.1 Traditional instruments

In section 2.1 some of the traditional instruments were described. A sighting telescope mounted with its axis perpendicular to a rotational axis can generate a flat plane. The accuracy depends upon the adjustment of the telescope axis, the mechanical quality of the axis of rotation and the mechanical precision of the focusing mechanism. In astronomy the focusing error should be zero because all objects observed lie at a nearly infinite distance. Astronomical transit instruments, however, are heavier than the jig transit used in the workshop. The mechanical quality of the axis of rotation must be very good. The errors caused by any deficiencies in the focusing mechanism are rather small in modern instruments, being about one second of arc or even better.

The optical level is used for testing flatness of a horizontal plane. The reproducibility of the automatic optical level is a few tenths of a second of arc.

A pentaprism together with a sighting telescope can also produce a plane. Considerably less demands are made on the mechanical quality of the axis of rotation of the prism than with transit instruments.

The methods for testing the straightness of lines are often used to check the flatness of a surface plate: straightness tests are made along a number of generator lines in the surface. More details are given by e.g. Kissam [1962] and Hume [1965].

3.2 Interference methods

The best known method for testing flats is the use of Newton fringes. A standard flat glass or quartz plate is laid upon the flat to be tested. Light reflected at the lower surface of the standard flat interferes with light reflected at the upper side of the surface to be tested. Along each fringe the thickness of the airspace between the surfaces is constant; the thickness difference between two neighboring fringes is $\lambda/2$ (e.g. $\lambda = 0.55$ μm). The accuracy is about one tenth of "a fringe" (0.03 μm) or even better.

Linnik [1942] and Saunders [1959] describe another interference method, which can be used for testing large areas, such as layout plates. The method of Saunders is relatively simple, with respect to the Linnik method, but the principle is the same. Saunders uses two flat mirrors and a Kösters double image prism; the last device is rather difficult to make.

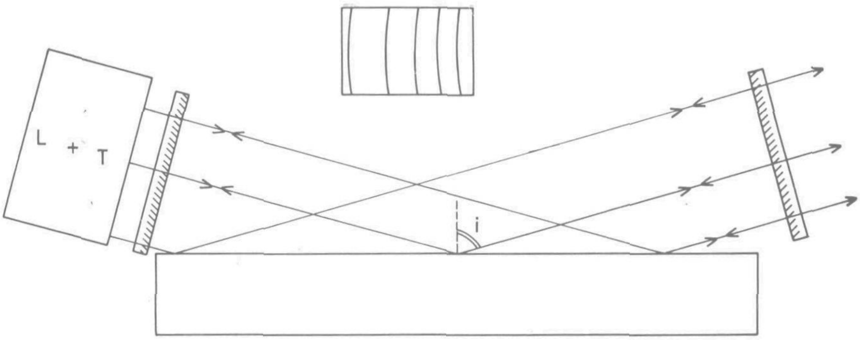


Fig. 3.2a Interference method for testing the flatness of a large surface; a laser L is used as light source; the field of view is shown

In fig. 3.2a a simpler set-up is shown which can only be used when a laser is available. A parallel beam from a He-Ne-laser L is widened with the aid of a telescopic system T. The light then passes a mirror, the surface to be tested, and another mirror. The reflection of the mirrors must be 80 to 90% and the absorption small. The light is reflected several times between the mirrors, and interference fringes are produced which can be photographed e.g. in the plane of the second mirror. It is possible to adjust the fringes so that they run parallel to the long dimension of the test area. One fringe departure from straightness corresponds to a departure from flatness of $\lambda/(4 \cos i)$.

3.3 Diffraction and interference

A zoneplate or double slit can be used if the plane consists of only four points: a carpenter's bench often has two beds which must lie in one plane. By measuring the distance between the beds at different places, the parallelism in one direction can be tested or adjusted; the straightness of each of the beds is checked with the aid of one of the methods described in chapter 2. Doekes and de Bruin [1955] describe a method for testing the parallelism in the other direction. At the end of each bed an illuminated hole or slit is attached, both at the same height above the bed. Between the beds a zoneplate or double slit is attached, in such a way that the center of one of the diffraction patterns lies at the same height above a bed as the hole. The other center is also at the same height above the other bed if the beds are parallel. In this case four points, the two holes and the two centers of the patterns, lie in one plane. The same method was used by Moonen [1955] to measure the direction of a mining gallery with respect to a given line at ground level (sect. 2.4).

Parallelism can also be checked or adjusted with the aid of a pentaprism. This was described by van Milaan [1964, 65].

If a long, straight, narrow slit could be manufactured, then a plane could be generated by this slit and a zoneplate, double-slit or other device as described in chapter 2. This would not be a complete plane, but only that part of it that lies between the sides of an angle of say 90°, which is sufficient for many applications. With a shorter slit and a lens afflicted with aberrations a similar portion of a plane can be generated as will be described in section 3.4.

With the aid of an alignment device, or such a device and a pentaprism, a flat (such as layout plates) can always be tested. If only straight lines are used for checking flatness, the modified Fresnel's mirrors-system (sect. 2.2.4) is in many cases of more practical value than e.g. the double slit. There are three advantages: the intensity of the interference pattern is higher, the accuracy can be varied by changing the angle between the mirrors, and the three points used (the illuminated slit, the intersection of the mirrors and the interference pattern) do not lie on a straight line. In fig. 3.3a a possible set-up is shown for measuring the flatness of a surface plate. q, F and P are respectively the slit, the mirror system and the center of the interference pattern. By changing the position of F straightness can be tested along different lines l.

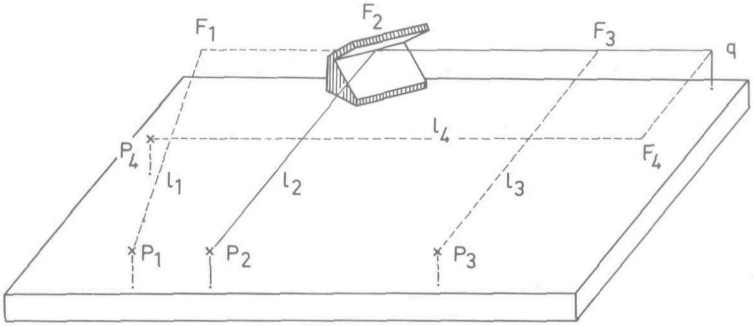


Fig. 3.3a Testing flatness with the aid of a modified Fresnel's mirrors-system

If a traditional Fresnel's mirror-system is manufactured one can check a plane in autocollimation. The length of this plane is large. The breadth, however, is not more than the length of the line of intersection of the two mirrors. An additional mirror M_3 positioned nearly perpendicular to the other mirrors solves the problem (fig. 3.3b). The angle φ must be smaller than 90°; if not, a ray of light through one of the points P would never meet P again. P is an autocollimation device for alignment: an illuminated slit or hole, a dividing mirror and a fiducial mark which must coincide with the image of the slit. The angle α is somewhat smaller than 180° (or smaller than 60°). In this case the distance between the slit and the mirrors and the distance between the mirrors and the cross-hairs are equal ($l = l'$). The equations of sect. 2.2.4 give the

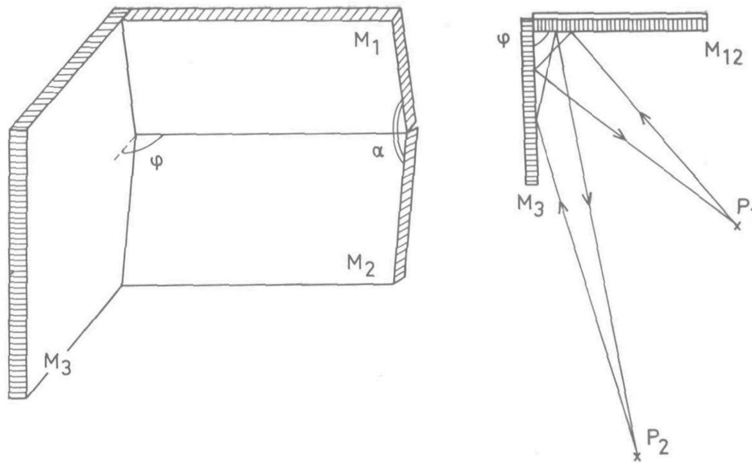


Fig. 3.3b Modified Fresnel's mirrors-system which determines a flat surface

following results: the distance δ between the minima, and therefore the accuracy, does not depend upon l when the angle α between the mirrors is kept constant: $\delta = \lambda/\alpha$. The width of the slit q is half as large as δ and the intensity of the interference pattern is proportional to l^{-1}. If an accuracy is desired of 1 μm, the angle between the mirrors must be about 10 minutes of arc: then the distance between the minima is about $\delta = 0.2$ mm which means that the eyepiece used should magnify the fringes about five times, and the width of the slit must not be larger than $q = 0.1$ mm. The position of this device with respect to the surface to be measured must not change with time, so that again, great care must be taken to avoid any movement of the mirror system.

3.4 Spherical aberration and diffraction

3.4.1 *The use of the rainbow*

Walther [1959] mentioned the possibility of using the rainbow-diffraction pattern for testing flatness. We will only briefly discuss the principles of the method. Some other devices also generating a rainbow will be described.

When a parallel beam of monochromatic light is incident upon a sphere of glass, a part of the beam will not leave the sphere after two refractions, but will first refract, then reflect two times against the inner surface of the sphere and will finally leave the sphere after a second refraction. It can be shown that the deviation of a ray of light, when leaving the sphere with respect to the incoming ray, has a minimum value for a certain height of incidence. This means that the emerging wavefront shows an inflection point which lies upon the "limiting ray of light". By limiting ray we mean the ray with the smallest deviation. The equation that gives the shape of the wavefront is of the third degree. This appears to be a very good approximation, because the next term in the series development is of the seventh degree [Walther 1959]. Such a wavefront gives rise to a very beautiful diffraction pattern which was calculated by Airy (1838). The first maximum of this pattern – at a large distance – does not exactly lie in the direction of the limiting ray, but in a direction with a somewhat larger deviation. The fineness of the fringes depends upon the radius of the sphere, the wavelength of the light used and the refractive index. If the number of internal reflections in the sphere is not two, but one, three, four, etc., then the fineness will be different. It is remarkable and of very great importance for the practical use, that the general structure of the pattern does not change with radius, wavelength, refractive index or number of reflections; only the size of the pattern changes. The diffraction pattern is advantageous because the number of fringes is so very great: the intensity of a maximum in a direction ν with respect to the direction of the limiting ray, is inversely proportional to the square root of ν, or about inversely proportional to the cube root of the order of the maximum.

The refractive index of the sphere can be given such a value ($n = 1.518$) that the minimum deviation is 270 degrees. In this case all limiting rays lie in one plane perpendicular to the direction of the beam incident upon the sphere. If the glass has a somewhat larger refractive index, then the limiting rays may yet lie in one plane, but only if the incident light comes from an illuminated hole at a distance considerably shorter than infinity from the sphere. In practice one can arrange that the deviation of say the second maximum instead of the deviation of the limiting ray is 270 degrees. This offers the advantage that only one measurement need be done to find the position of the plane produced; the direction of the limiting ray can only be found by extrapolation of measurements of several maxima or minima.

There are also some disadvantages to be mentioned. First of all the accuracy of setting crosswires at a maximum of the diffraction pattern is not very high (one or two seconds of arc), at least not with respect to the accuracy obtained with a zoneplate or similar device. The fineness of the pattern can be improved by increasing the radius of the sphere, but the diameter is limited to say 50 mm by practical considerations: the glass of which the sphere is made must be very homogeneous. A second disadvantage is that the intensity of the fringes is rather poor. This is easily understood by remembering that the energy leaving the small illuminated hole is eventually spread in a "plane of light". The maximum distance at which the fringes can be observed in daylight is about ten meters. The third disadvantage is that the refractive index of the sphere, and therefore the direction of the limiting ray, depends upon the temperature of the sphere. A practical set-up for the glass sphere rainbow-system is described by van Heel [1961a].

Some of the difficulties mentioned above can be overcome by using a "white rainbow" produced by a mirror or a mirror-system. De Veer [1961] showed that a spherical mirror can transform a spherical wavefront into a wavefront of exactly the same shape as was mentioned above, namely a third degree curve. Because a mirror is used, the wavefront and the direction of the limiting ray are equal for all colors. This means that the diffraction pattern is nearly white, at least the first few fringes are white and black (if white light is used), the intensity of the pattern is therefore much higher than in the case of the glass sphere.

We will shortly describe the white rainbow in a more mathematical way; for the details we refer to Walther [1959] and de Veer [1961].

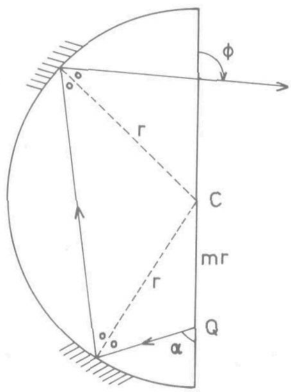


Fig. 3.4.1a Spherical mirror producing a "white rainbow" after two reflections

Fig. 3.4.1a shows a spherical mirror and a ray of light coming from a small illuminated hole Q positioned at a distance mr from the center C. The deviation Φ is a function of m, the angle α and the number of reflections k at the sphere; $k = 2$ in the figure. For a certain value of k and m, the minimum value Φ_0 of the deviation can be found. The angle $(\Phi - \Phi_0)$ can be developed in a series as function of $(\alpha - \alpha_0)$. This gives the equation of the rays of light which lie close to the limiting ray. The wavefront is perpendicular to the light rays, so the wavefront can be calculated. It appears that a good approximation of the equation of the wavefront is

$$z = \frac{h}{3r^2} x^3 \qquad (3.4.1a)$$

The z-axis is taken along the limiting ray and h is defined by

$$h = \frac{(4k^2-1)^2}{8k^2(1-m^2)} \left(\frac{4m^2k^2-1}{1-m^2}\right)^{1/2} \qquad (3.4.1b)$$

The wavefront gives rise to a diffraction pattern of which the direction of the maxima and minima with respect to the direction of the limiting ray is given by a quantity P and a parameter ν. The parameter ν can be read from the table:

	maxima	minima
1.	1.0845	2.4955
2.	3.4669	4.3631
3.	5.1446	5.8922
4.	6.5782	7.2436
	etc.	etc.

The direction of a maximum or minimum is given by

$$\Phi - \Phi_0 = P\nu \qquad (3.4.1c)$$

where P is given by:

$$P = \left(\frac{h}{48}\right)^{1/3} \cdot \left(\frac{\lambda}{r}\right)^{2/3} 2.06265 \times 10^5 \text{ seconds of arc.} \qquad (3.4.1d)$$

This quantity is a measure for the fineness of the pattern; it is proportional to $\lambda^{2/3}$, so the fringes are less colored than the fringes produced by a double-slit where the distance between the maxima is proportional to λ. This is of course only true because Φ_0 does not depend upon the wavelength.

It can also be shown that the minimum deviation is $\Phi_0 = 90°$ if $m = 0.6825$. In that case $\alpha_0 = 73\overset{\circ}{.}95$. A concave mirror was made having a radius $r = 50.2$ mm. This means that $h = 45.7$ and $P = 99.7$ seconds of arc ($\lambda = 0.5461$ μm). For a glass sphere with a refractive index $n = 1.52$, $P = 105.1$ seconds of arc (two internal reflections, $\Phi_0 = 270°$ and $r = 24$ mm). From the table which gives the ν-values and with (3.4.1c) we see that the angular distance between the first and the second minimum is 99.7 (4.3631 − 2.4955) = 186″.

The accuracy of setting a crosswire at the second maximum is somewhat better than one-hundredth of the distance between the neighboring minima, namely 1.3 seconds of arc.

This accuracy is rather disappointing. There is one way to increase the accuracy, namely by increasing the radius of curvature of the mirror. But the accuracy is proportional to $r^{2/3}$ which means that for an accuracy of 0″.5 the radius of the mirror should be 210 mm. This is too large to be of any practical value.

The quantity h can be made smaller by using two mirrors instead of one. This means that the wavefront near the limiting rays is less afflicted with

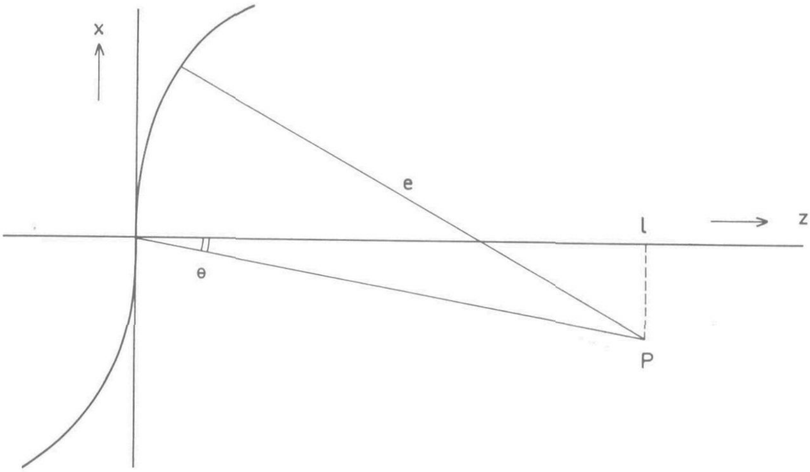


Fig. 3.4.1b Wavefront producing a rainbow-diffraction pattern

aberration. In [de Veer 1961] the design is described of two-mirror systems generating a rainbow in a direction perpendicular to the axis of the system. A setting accuracy of 0.″2 is within reach but there are two disadvantages: the system consists of an illuminated hole and two mirrors. Thus not only the distance between the hole and the two centers must be adjusted with very high accuracy, but these three points must also lie on a straight line. In the case of one mirror and a hole, as described above (fig. 3.4.1a) only one distance has to be adjusted. The second disadvantage is of a more fundamental sort.

The equation of the wavefront is $z = hx^3/3r^2$; this wavefront is not symmetrical with respect to the z-axis (the limiting ray), thus the locus of e.g. the first maxima is not a straight line. The quantities giving the position and the dimensions of the diffraction pattern are only valid if the distance between the mirror and the pattern is very large.

The direction of the pattern at finite distances is different as will be shown below.

In fig. 3.4.1b the wavefront is shown. We will calculate the intensity at a point P at a distance l in a direction θ with respect to the z-axis. The phase φ of the light from a point (x, z) on the wavefront arriving at P is given approximately by

$$\varphi = \frac{2\pi e}{\lambda} = \left(\frac{x^2}{2l} + x\theta - \frac{h}{3r^2}\,x^3 + l\right)\frac{2\pi}{\lambda} \tag{3.4.1e}$$

So, except for a constant phasefactor, the amplitude at P is:

$$\bar{a}_P = \int\limits_{-\infty}^{+\infty} \exp\left[\frac{2\pi i}{\lambda}\left(-\frac{x^2}{2l} + \frac{h}{3r^2}\,x^3 - x\theta\right)\right] \mathrm{d}x \tag{3.4.1f}$$

For x we can write $x = \bar{x}+r^2/2lh$; if we again omit a phasefactor,

$$\bar{a}_P = \int_{-\infty}^{+\infty} \exp\left[\frac{2\pi i}{\lambda}\left\{\frac{h}{3r^2}\,\bar{x}^3 - \left(\theta + \frac{r^2}{4l^2h}\right)\bar{x}\right\}\right] \mathrm{d}\bar{x} \tag{3.4.1g}$$

So the direction of e.g. the first maximum depends upon l. If l is very large this direction is θ; if l is finite this direction is

$$\theta' = \theta+r^2/4l^2h \tag{3.4.1h}$$

Remark: if we take $\theta' = 0$, this means that we find the locus of the points where the relative intensity is constant and equal to the relative intensity on the z-axis at infinity. This locus is given by $\theta = -r^2/4l^2h$ or, if we write z instead of l, and $-x/z$ instead of θ, we get

$$xz = r^2/4h \tag{3.4.1j}$$

This is the equation of the caustic curve! Thus the corresponding points (for which $\nu = 0$) of the diffraction patterns do not lie on a straight line but on a hyperbola which is the evolute of the wavefront.

The difference between θ' and θ is proportional to r^2/h (3.4.1h) or proportional to P^{-3} (3.4.1d). We see that if the accuracy, or the fineness of the pattern (proportional to P^{-1}), is increased, the angular shift of the pattern at finite distances with respect to the pattern at infinity increases very rapidly. Therefore it is of no use to design a mirror system which generates a very fine rainbow-diffraction pattern, at least not for alignment purposes.

For astronomical purposes, however, it can be of use: When a telescope is rotated about the axis of such a system in such a way that the diffraction pattern and a star can be observed at the same time, the system mirrors-telescope can be used as a transit instrument.

Example: The one-mirror system we made has a radius $r = 50.2$ mm and $h = 45.7$. The accuracy of a setting is $1''\!.3$ or $6l$ μm, l being expressed in meters. The shift of the position of the pattern is equal to $r^2/4lh$ (3.4.1h) or $13.8l^{-1}$ μm (l in meters). At a distance $l = 1.5$ m the shift is equal to the accuracy. For a system for which the setting accuracy is five times higher – this means that P is one-fifth as large and the shift 125 times larger – this distance is 25 times larger, or about 37 meters! This means that for measurements at a distance smaller than 37 m, a correction should be made for the shift of the pattern with respect to the direction $\theta'(l = \infty)$.

3.4.2 *The use of lenses*

In sect. 2.3.2 an alignment device which consists of one piece of glass is described. The first surface is flat and is covered with a thin layer of aluminum in which a tiny hole is made. If this hole is illuminated, the system generates a straight line, due to the aberration of the spherical second surface. This line passes through the hole and the center of curvature of the spherical surface. Because the distance between the hole and the center is not very critical, the hole may be positioned at various places on the flat first surface. It is simple to make two holes in the aluminum instead of one: two lines are produced which lie in one plane. With a small flat mirror M a third line in the plane can be produced by adjusting this mirror in a way that e.g. the point P is the center of both diffraction patterns (fig. 3.4.2a). By changing the position of M other lines can be generated, all of them lying in one plane.

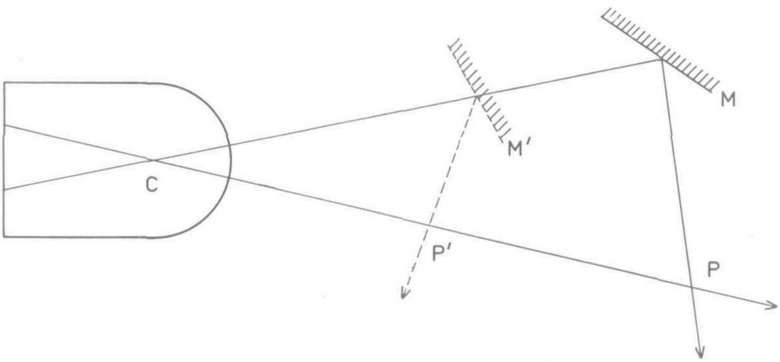


Fig. 3.4.2a Alignment of a plane with the aid of two lines generated by a "dome"

This method is rather clumsy. If it were possible to make a straight scratch from hole to hole, then a flat "plane of light" could be produced.

The straightness of this scratch or slit is very important if the accuracy is to be high, say 0.2 seconds of arc. The distance from the center C to the slit is about 40 mm, so a deviation from straightness of 0.04 μm results in a deviation of 0.2 seconds of arc (10^{-6} radians). The angular dimensions of this plane cannot be much larger than about twenty degrees because the difference in distance between various points of the scratch and the center of curvature must not be too large; the maximum length of the slit is about fifteen millimeters. So one must manufacture a slit of 15 mm length, about 2 μm width and straight within 0.04 μm. We could not make such a slit by scratching with a diamond in aluminum.

Another possibility exists, however. The intersection of two optical flats is a straight line, the straightness is better than 0.03 μm if the surfaces do not deviate from flatness more than one-tenth of an interference fringe. This is a requirement which can be fulfilled: a roofprism was made with a sharp edge. It appeared possible to make the sharpness better than 0.5 μm, but we need only 2 μm. The angle between the two flats can be made smaller than 90° in order to avoid stray light, or the greatest part of the prism can be covered with black paint. The best method is to cover the intersecting faces with aluminum.

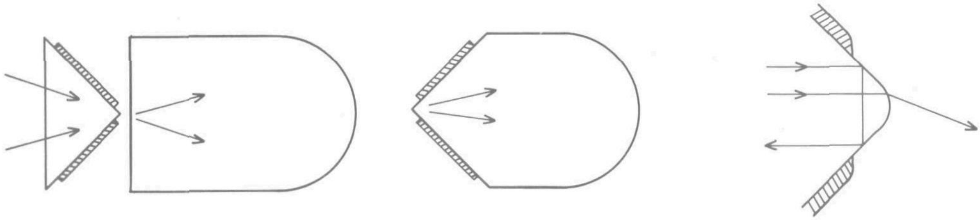


Fig. 3.4.2b The use of the edge of a prism as a straight slit

Fig. 3.4.2b shows two possibilities of the use of a "roof-slit", and a cross-section of the slit greatly enlarged. A test of the first device of fig. 3.4.2b showed that the accuracy of setting the crosswires at the diffracted image of the "slit", and the flatness of the optically produced plane surface can be guaranteed within 1 μm at a distance of 1 meter. The angular spread is about 20°. The great advantage of this system is that it need not be adjusted. If the slit is straight, then the plane through the slit and the center of curvature is flat, assuming that the glass is homogeneous and the surface spherical. A disadvantage is that the "plane of light" is only twenty degrees wide.

In order to increase this angle, another device was made. As a matter of fact, this was done before the experiments with the straight slit were made. The device consists of a half-sphere and a half-cylinder [van Heel 1962a]. The cylinder surface is aluminized: a scratch S is made in the metal coating by means of a diamond. This scratch is again about 2 μm wide. The half-sphere is cemented to the plane surface of the half-cylinder (fig. 3.4.2c). The plane of light produced by this device consists of many rays of light which go through the points of the slit and the center of curvature of the sphere. Two conditions

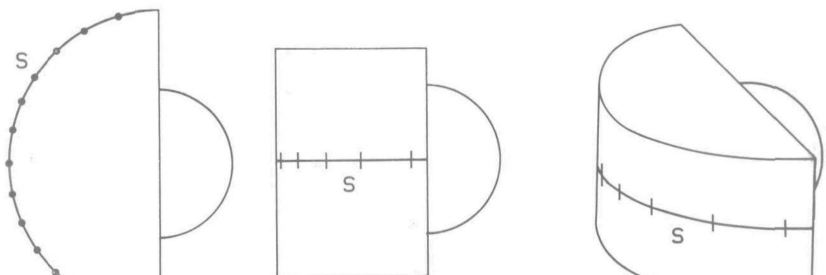


Fig. 3.4.2c Half-sphere – half-cylinder device

must therefore be fulfilled; all points of the slit must lie in one flat plane, and the center of curvature of the half-sphere must lie in that same plane. The slit is manufactured in the following way: two half-cylinders of equal radius are temporarily cemented together. This is done after the polishing of the flat surfaces and after the grinding of the cylindrical surfaces of the two half-cylinders. The polishing of the cylindrical surface is done while the whole cylinder is rotated on a lathe. After evaporating aluminum onto the surface, the cylinder is again positioned in the same lathe, a long circular shaped scratch is then made in the aluminum by slowly rotating the cylinder and gently pressing a diamond tool against the surface. Wandering of the axis of rotation of the lathe must be minimized in order to keep all parts of the slit in one flat plane. Since vibrations during the scratching are disastrous, the rotation of the cylinder was done by hand. It is not necessary that the points of the whole scratch lie in one flat plane within 0.04 μm, because we will only use a ninety-degree-section of this circular slit. If the axis wanders regularly, i.e. without any free-play, then the scratch will have the shape of a sine on the surface. If the amplitude of the sine-curve is 0.27 μm, then the worst deviation from a straight line is about 0.04 μm if only a ninety-degree-section is used; the smallest possible deviation is about 0.004 μm in this case.

In order to check whether the scratch is regular, the device was positioned on a vertical axis of very good mechanical quality. When the plane of light, or the cone of light, produced by the device is about perpendicular to the axis of rotation, the distance d from a fixed point to the plane of light will be a cosine function of the angle of rotation φ. This was checked when the distance between the fixed point and the device was about 7.7 meters, the accuracy of the settings on the diffraction pattern was 6 μm or better. Fig. 3.4.2d shows the results; the short vertical lines represent the values of d measured, and the small crosses lie on a cosine curve.

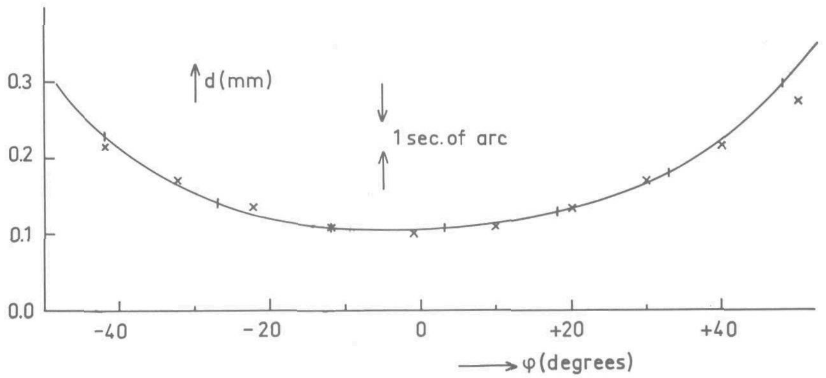


Fig. 3.4.2d Checking the slit; the results of measurements

The device can be used from $-45°$ to $+45°$ about. Between the measured points the diffraction pattern was checked visually. At some points the pattern was disturbed a little due to small local imperfections in the scratch.

In sect. 3.5 the adjustment of the sphere in the plane of the slit in order to produce a plane of light instead of a cone will be described.

The advantages of the half-cylinder–half-sphere device are that the accuracy is high, the intensity is high, and it is very easy to find the position of the pattern by moving the head up and down. However, it must be adjusted before it can be used.

When the long scratch is provided with some transverse scratches, the apparatus can also be used for pure alignment purposes, the diffracted images of one of the "crosses" lie on a straight line.

3.5 Adjustments and remarks

Some remarks must be made on the adjustment of some of the devices described.

One of the apparatus described (sect. 3.4.1) which must be adjusted, is the one-mirror system of fig. 3.4.1a, which produces, after two reflections, a white rainbow. Here the distance between the illuminated hole and the center of the mirror must be adjusted in such a way that the second maximum is seen at an angle of 90° with respect to the axis of the system. Two theodolites are placed with the object glasses facing each other so that the image of one of the crosswires coincides with the other crosswires. Moreover, the telescope axes are adjusted perpendicular to both mechanical axes of the theodolites. If both telescopes are now rotated through a small angle, with the mirror-system placed in between the two telescopes in such a way that in both telescopes a part of the "plane of light" can be seen, then the position of the illuminated pinhole may be changed, until the second maximum of both parts of the rainbow-pattern coincides with the crosswires [de Veer 1961].

This procedure is also followed to adjust roughly the half-cylinder–half-sphere device of fig. 3.4.2c. The half-sphere can be moved up and down along the flat side of the half-cylinder, in order to bring the center of curvature in the plane defined by the scratch. The presence of a very thin layer of slowly hardening cement makes it possible that the movement does not cause scratches on the polished flat glass-surfaces. One of the straight lines from a similar apparatus was used to test the flatness of the plane produced by the device under test. The straight line was moved up and down, till two points (1 and 3 in fig. 3.5a) of the straight line at a distance of 10.5 meters coincided with two points being situated on two lines from the plane. The latter points lie at a distance of 10.3 meters from the center of the device which is to be adjusted. A third line in the plane will generally not cut the straight reference line; the distance between these two lines, which are nearly perpendicular to each other, was made as small as possible by moving the half-sphere up and down. When the cement was hard the distance between the lines at point 2 was measured to be (0.033 ±0.008) mm. The angle between the two lines in the "plane of light" through the points 1 and 3 is about 60°, so if this angle

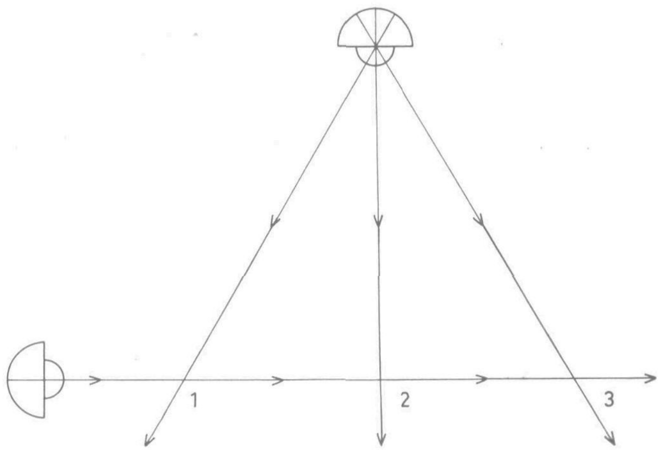


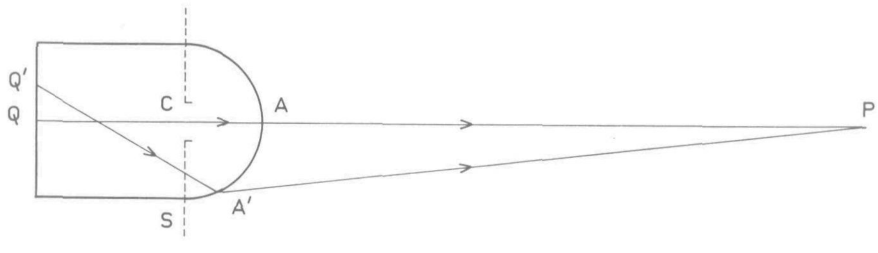
Fig. 3.5a Adjustment of a device generating a "plane of light"

is 90°, the parabolic curve in the plane at a distance of 8.8 m from the device still does not deviate from a straight line by more than 0.033 mm or about 0.8 seconds of arc. The precise curve being known, one can correct the results of measurements of a supposed plane surface.

In many practical applications, where a high accuracy is wanted, it is advisable to measure the surface two times, one time with respect to the reference plane and one time with respect to the reference plane upside down.

Some practical remarks must be made on the illumination of "the dome" (fig. 3.4.2b) or of the half-cylinder–half-sphere device. The difficulties can best be illustrated with the "dome". This is the device which consists of a cylinder of glass, with one face polished flat and the other face spherical. A straight scratch, serving as a slit, is made in the aluminized flat side. The distance between the slit and the spherical surface is a little bit smaller than the first focal length.

When the slit is illuminated from all directions, the contrast of the diffraction pattern is poor: the pattern near P (fig. 3.5b) should be an "image" of the part of the slit near Q, but if every point of the slit sends light in all directions, light also will arrive in P from the part Q′ of the slit. The pattern at P due to light from the part Q′ does not have the same size as the pattern due to light from Q, because the diameter of the Fresnel zone at A is larger than the diameter at A'. Consequently the intensity at P is due to the sum of the intensities of many diffraction patterns of different size; the light emitted by Q is not coherent with the light emitted by Q′, so the contrast of the fringes at P is poor. By illuminating the slit properly the contrast can be made better. A possible,



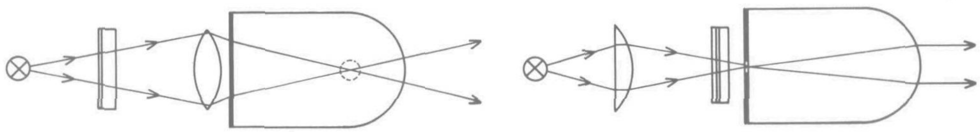


Fig. 3.5b Explanation for the lack of contrast caused by improper illumination; proper illumination of the slit in two planes W and V

but unpractical solution is to place a rather wide slit S near C so that the ray of light Q′A′ is screened off. The proper illumination is as follows: an anamorphotic lens-system forms the image of a light source on the slit in one plane V, while in another plane W, perpendicular to V, the image of the light source is at C. The diameter of this image must be small in order to keep the contrast of the diffraction pattern high. If a "slit" is used as shown in figure 3.4.2b a similar illumination should be used.

3.6 Applications and automatic alignment

Some applications of testing flatness are described by van Milaan [1964, 65]. The adjustment of foundation blocks of a building is carried out with the aid of an illuminated pinhole, a zoneplate and some fiducial marks, a pentaprism and an automatic optical level. Four of the blocks are adjusted with the optical level; the distance from the level to these blocks was the same, so that no focusing of the telescope of the level was necessary. Furthermore, any systematic error in the instrument has no influence. A second requirement was that the foundation blocks be positioned on straight lines perpendicular to each other. The angle of ninety degrees was adjusted with the aid of the pentaprism. Four rows of eighteen blocks were adjusted, the length of one row was 153 m, and the distance between the rows was 18 m; the position of each block was adjusted within 0.5 mm.

In the same article the use of zoneplate and pentaprism is mentioned for the adjustment of the interfaces at the end of each of the parts of an underwater traffic tunnel; the plane must be perpendicular to the axis of the tunnel-part in some cases, or, in other parts, perpendicular to another given base-line. The length of this line was about 80 meters, the dimensions of the interfaces were 8×25 meters. The accuracy was well within the accuracy desired (0.5 mm).

In conclusion we will make some additional remarks on automatic alignment. As we did no experimental work in this field, we will only give some references to the literature.

The article by Raterink [1965] has already been mentioned, in which a method is described for recording fast movements with a high-speed camera. In many cases, however, an electrical signal is wanted which is proportional to the distance between the position of an image or a diffraction pattern and a fixed point. When a position-sensitive photo-electric device is used in an alignment procedure, we will call it "automatic alignment". In space research, automatic alignment is used for star following [Jones and Manns 1947, Thorne and Gillespie 1950 and Manns 1950]. Cook and Marzetta [1961], Guild [1960] and Harrison, Horlock and Hunt [1957] describe methods used in automatic fringe counting interferometers. An important device was invented by Wallmark [1957]. This device can sense the position of a light spot to within 0.025 μm in two directions; the sensitivity is about 2 V $mm^{-1}\,lm^{-1}$, the linear range is 0.5 mm and the stability better than 0.4 μm over several days. Similar devices are described by Williams [1965]. The Wallmark position-sensitive photocell is applied in automatic two-channel recording autocollimators which can detect $0.''1$; [Baker 1961a,b, 1965, Fischbacher 1962 and Hume 1965]. A very interesting application is described by Baker and Williams [1965] for the electronic plotting of wavefronts. Automatic compensation of alignment errors in machine tools is described by Leete [1961].

SAMENVATTING

In dit proefschrift wordt beschreven met welke optische instrumenten de rechtheid van rechte lijnen of de vlakheid van platte oppervlakken kan worden gecontroleerd. Behalve de controle is in de techniek vaak het justeren van belang. Naar volledigheid is niet gestreefd.

In het tweede hoofdstuk worden enkele klassieke instrumenten genoemd; op het gebruik ervan wordt niet in detail ingegaan. Hoewel deze instrumenten in vele gevallen voldoende nauwkeurigheid geven zal toch het gebruik ervan niet altijd bevredigend zijn: het meten van dwarsverplaatsingen op grote afstand bij voorbeeld, kan een zeer nauwkeurige hoekmeting vereisen. Men moet in zo'n geval een duur instrument aanschaffen, dat dan vaak niet voldoende stabiel kan worden opgesteld ten opzichte van het meetpunt. Ondanks eventuele nadelen moet men natuurlijk voor metingen steeds dat instrument gebruiken dat het snelst tot het gewenste resultaat leidt. Een van de nauwkeurigste instrumenten is het automatische waterpas instrument. Dit hoeft niet zo stabiel te worden opgesteld als een theodoliet of een richtkijker, het zal vooral daar worden gebruikt waar horizontale vlakken ter sprake komen.

In het tweede hoofdstuk worden ook enkele methoden genoemd die gebruik maken van interferentie en buiging van het licht, verschijnselen die in de traditionele instrumenten juist hinderlijk zijn en de nauwkeurigheid beperken. De in dit hoofdstuk behandelde dubbele-spleten-methode, de zoneplaat, de Fresnelspiegels enz., functioneren slechts dank zij het golfkarakter van het licht. Ook sferische aberratie of askring blijkt te kunnen worden gebruikt bij het aligneren.

Er wordt aangetoond dat de nauwkeurigheid waarmee de rechtheid van bij voorbeeld het bed van een draaibank kan worden gemeten, in feite slechts begrensd wordt door eventuele inhomogeniteiten in de lucht. Een van de grote voordelen van de meeste der genoemde niet conventionele instrumenten is, dat vaak gemeten kan worden ter plaatse van het meetpunt en niet vanuit een grote afstand.

In dit hoofdstuk wordt ook ingegaan op de eigenschappen van een pentaprisma, vooral wat betreft mogelijke onnauwkeurigheden die kunnen ontstaan door onoordeelkundig gebruik van zo'n prisma.

Voorbeelden van metingen worden vermeld.

Het derde hoofdstuk geeft een overzicht van enkele mogelijkheden om de

vlakheid van platte oppervlakken te bepalen, of het justeren van drie of meer punten in een plat vlak. Ook hier blijkt dat interferentie en askring nuttig kunnen worden gebruikt. Verschillende mogelijkheden werden onderzocht. Het is nog niet gelukt om een toestel te ontwerpen dat dezelfde voordelen biedt bij het aligneren als de zoneplaat. (Hierbij wordt immers een rechte lijn bepaald door twee punten, een verlicht gaatje en het midden van de zoneplaat, die op een grote afstand kunnen liggen; men zou kunnen spreken van een zeer lange collimator; een kleine verschuiving van één van de twee punten is veel minder van invloed op de richting van de lijn als bij een richtkijker het geval is). De tot nu toe ontworpen toestellen die een plat vlak bepalen hebben dus hetzelfde nadeel als de richtkijker: aan het opstellen ervan moet dan ook de grootst mogelijke zorg worden besteed. Het hoofdstuk wordt afgesloten met enkele opmerkingen over het justeren van genoemde toestellen, over toepassingen, en over de mogelijkheid om aligneermetingen te automatiseren.

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STELLINGEN

I.

Bij aligneren over grote afstand kan beter een ronde opening dan een zoneplaat worden gebruikt.

II.

Het octrooi Brit. Pat. 721, 613 (1952) is ten onrechte verleend.

III.

Het meten van kleine verplaatsingen, zoals wordt gedaan bij het propaedeutisch natuurkundig practicum van de T.H. in Delft ter bepaling van de elasticiteitsmodulus van metalen, kan eenvoudiger en nauwkeuriger geschieden door gebruik te maken van de in dit proefschrift genoemde dubbele-spleten methode.

IV.

Met behulp van twee Dove-prisma's kan een richtkijker geconstrueerd worden waarbij geen scherpstelfout optreedt.

Fultz, A. R., Conf. Instr. Soc. Am., paper no. 49-5-1 (1949).

V.

Indien van een lenzenstelsel, waarvan het formaat nog niet vast ligt en waarvan het openingsgetal F niet te klein is, de vijfde orde askringcoëfficiënt voor een sterkte één M_6 genoemd wordt, dan wordt de grootste brandpuntsafstand waarbij de buigingsgrens nog niet wordt overschreden gegeven door $F^6/6M_6$ mm.

VI.

Wanneer een planparallele glasplaat gebruikt wordt als optische micrometer kan de te meten verplaatsing evenredig worden gemaakt met de tangens van de invalshoek.

Leete, D. L., Int. J. Mach. Tool Des. Res. 1 (1961) 312.

VII.

Het gebruik van een fotografische opname van het regenboog-buigingsverschijnsel als voorwerp voor een lens waarvan de contrastoverdrachtsfunctie gemeten moet worden, verdient aanbeveling.

VIII.

De formule die T. Smith geeft voor de verandering van de kromtestralen van een doublet bij het „dikte geven",* kan zodanig veranderd worden, dat ze ook kan worden toegepast in die gevallen waar de dwarsvergroting niet nul is.

* Mededeling van T. Smith aan A. C. S. van Heel.

IX.

Indien twee voorwerpen op bepaalde plaatsen liggen is het in het algemeen niet mogelijk deze voorwerpen met behulp van één optisch stelsel op twee voorgeschreven plaatsen met twee voorgeschreven vergrotingen tegelijk af te beelden.

X.

Indien de vergroting V van een „droog" microscoop-objectief niet groter is dan 100 maal, en de numerieke apertuur $N.A.$ is niet groter dan 0.9, dan gelden de volgende vuistregels: a) De toelaatbare afwijking van de voorgeschreven dekglasdikte is gelijk aan $(0{,}9 - N.A.)/10$ μm. b) De toelaatbare afwijking van de voorgeschreven tubuslengte (160 mm) is gelijk aan $310/V$ mm.

XI.

In de interferometrie kan vaak met succes gebruik worden gemaakt van eenvoudige optische stelsels die alleen voor askring zijn gecorrigeerd.

XII.

Het verdient aanbeveling om kathetometers met een vaste kijker en een verschuifbaar pentaprisma uit te rusten.

XIII.

Bij het testen van prisma's met een hoek van 90°, 60°, 45°, enz., met behulp van een autocollimator, is onmiddellijk te zien of de te maken hoek iets te groot of iets te klein is door de uittreepupil van de autocollimator gedeeltelijk af te dekken.

XIV.

Er moeten onmiddellijk maatregelen genomen worden om het aantal inwoners van Nederland niet nóg groter te laten worden.

XV.

Het bouwen van torenflats moet worden verboden.

XVI.

Met een dubbel-bolle lens kan hypercentrisch perspectief beter worden gedemonstreerd dan met een plat-bolle.