

Numerical Investigation of Heat Transfer in a Conical, bubbling gas-solid fluidized bed

Master End Thesis

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by

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Preface

This thesis was carried out at the Faculty of Chemical Engineering at Delft University of Technology between September 2025 and April 2026. The work focuses on the numerical investigation of heat transfer in a conical gas–solid fluidized bed using Computational Fluid Dynamics.

I would like to express my sincere gratitude to my supervisor, Dr. Johan Padding, for his guidance and critical feedback throughout the project. I also thank my daily supervisor, Ravi Ramesh, for his continuous support and valuable discussions, particularly during the development of the numerical model.

This project provided an opportunity to deepen my understanding of multiphase flow modelling and heat transfer in fluidized systems, and to engage with the challenges associated with non-isothermal CFD simulations.

Summary

The prediction of heat transfer in gas–solid fluidised beds remains non-trivial, particularly once non-isothermal effects and non-standard geometries are introduced. While the hydrodynamic behaviour of these systems is reasonably well established, the coupling between flow structure and thermal transport is less robustly captured in current modelling approaches. This limitation becomes more apparent in conical geometries, where the superficial gas velocity decreases with height, inherently producing spatial variations in fluidisation regime. In the present configuration, the use of a nozzle-based gas inlet further complicates this behaviour by introducing a localised high-velocity jet near the distributor. This is likely to alter both bubble formation and local heat transfer rates in a way that is not well represented by correlations originally developed for uniformly distributed gas injection.

In this work, heat transfer in a laboratory-scale conical bubbling fluidised bed is investigated numerically using an Euler–Euler two-fluid framework implemented in ANSYS Fluent. The system consists of porous γ -alumina particles fluidised by nitrogen, with gas introduced through a nozzle at the base of the conical section. The model resolves the coupled momentum and energy balances for both phases, with interphase heat transfer described using the Gunn correlation in the dense regions of the bed and the Ranz–Marshall correlation in more dilute zones. This combination was selected to reflect the strong variation in local void fraction, although the transition between regimes introduces some uncertainty, particularly in intermediate regions where neither correlation is strictly valid.

Three heating strategies were considered: wall heating, gas preheating, and a combined configuration. The simulations indicate that the mode of heat input has a clear impact on both the rate and spatial distribution of temperature rise. In the gas preheating case, the bed approaches a relatively uniform temperature profile more rapidly, which suggests that interphase convection dominates under these conditions. In contrast, wall heating produces a more gradual thermal response, with temperature gradients persisting over longer times. This behaviour is consistent with a transport mechanism limited by particle–wall heat exchange and conduction through the dense phase. The combined case exhibits the fastest overall heating, although the improvement appears to be largely additive rather than indicative of a new dominant mechanism.

The conical geometry introduces a noticeable axial dependence in the thermal behaviour. Near the inlet, where gas velocities are highest, heat transfer appears to be convection-driven, likely due to increased slip velocities and more active bubbling. Further up the bed, as the gas velocity decreases, the system transitions towards a denser regime where conductive effects become more significant. This spatial variation is reflected in the temperature fields, which show a clear delay in heating in the upper sections. The influence of superficial gas velocity is not straightforward: increasing velocity enhances local heat transfer coefficients, but at the same time reduces the solids fraction, which limits the overall heat capacity of the bed. The resulting effect is therefore non-monotonic, suggesting the presence of an optimal operating range rather than a simple scaling with velocity.

Overall, the simulations capture the expected qualitative trends in heat transfer behaviour, but some limitations remain. In particular, the results are sensitive to the choice of interphase heat transfer correlation and to the local prediction of void fraction. Despite these uncertainties, the model provides a useful framework for interpreting the interaction between hydrodynamics and heat transfer in conical fluidised beds. The findings highlight the importance of geometry and inlet configuration in determining thermal performance, and suggest that conventional design correlations may require modification when applied to such systems.

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Contents

Preface	i
Summary	ii
Nomenclature	v
1 Introduction	1
1.1 Problem Statement	2
1.2 Research Questions	3
1.3 Thesis Outline	3
2 Literature Background	5
2.1 Gas–Solid Fluidisation: Fundamentals	5
2.1.1 From Packed Bed to Fluidised State	5
2.1.2 Geldart Classification	6
2.1.3 Flow Regimes and Their Thermal Implications	6
2.2 Heat Transfer Mechanisms in Gas–Solid Fluidised Beds	8
2.2.1 Gas–Solid Interphase Heat Transfer: Ranz–Marshall and Gunn Correlations	8
2.2.2 Wall-to-Bed Heat Transfer: The Packet Renewal Mechanism	10
2.2.3 Effective Thermal Conductivity of the Dense Phase	11
2.2.4 Combined Thermal Resistances	11
2.3 Influence of Operating Parameters on Heat Transfer	11
2.3.1 Superficial Gas Velocity	11
2.3.2 Particle Size and Density Effects	12
2.3.3 Void Fraction and Volumetric Heat Transfer Capacity	12
2.4 Reactor Geometry: Cylindrical vs. Conical Beds	13
2.4.1 Cylindrical Beds: The Reference Configuration	13
2.4.2 Conical Beds: Axial Velocity Gradient and Spatial Regime Co-existence	13
2.4.3 Spouted Beds: Hydrodynamic Structure and Heat Transfer	13
2.5 Numerical Modelling of Heat Transfer in Fluidised Beds	14
2.5.1 The Euler–Euler Two-Fluid Framework	14
2.5.2 Phase Energy Conservation Equations	14
2.5.3 Interphase Heat Transfer Coupling	15
2.5.4 Effective Solid-Phase Thermal Conductivity	15
3 Multiphase Modelling in ANSYS Fluent	16
3.1 Description of the System	16
3.1.1 Mesh Description	16
3.1.2 Mesh Dimensions	17
3.1.3 Material Properties	18
3.1.4 Wall Material Properties	20
3.2 Boundary Conditions	20
3.3 Computational Model Configuration	21
3.3.1 Heat Transfer User-Defined Function	21
3.4 Solution Methods	23
3.5 Initialisation	24
3.6 Patching of Initial Conditions	24
4 Results and Discussion	25
4.0.1 Preliminary Note on Simulation Configuration	25
4.1 Gas Heating Operation, Case G, 50 lpm	25

4.1.1	$t = 0.5 \text{ s}$	25
4.1.2	$t = 1.1 \text{ s}$	28
4.1.3	$t = 1.8 \text{ s}$	31
4.1.4	$t = 2.5 \text{ s}$	33
4.1.5	$t = 4.0 \text{ s}$	36
4.1.6	Local and global heat transfer behaviour.	38
4.2	Combined Heating Configuration: Case C, 50 lpm	40
4.2.1	Initialisation Anomaly and Its Consequences for Interpretation	40
4.2.2	$t = 0.5 \text{ s}$	40
4.2.3	$t = 4.0 \text{ s}$	43
4.2.4	Thermal observations throughout the cycle.	45
4.3	Wall Heating Configuration: Case W, 50 lpm	45
4.3.1	Model Configuration and Initialisation Anomaly	45
4.3.2	$t = 0.5 \text{ s}$	46
4.3.3	$t = 4.0 \text{ s}$	49
4.4	Effect of Inlet Flow Rate: 50 lpm versus 67 lpm	53
4.4.1	Near-nozzle thermal response	53
4.4.2	Thermal penetration and spatial extent of jet influence	56
4.4.3	Comparison of hydrodynamic and thermal length scales	59
4.4.4	Mechanistic interpretation of velocity effects	62
5	CONCLUSION	63
	References	67
A	Determination of Inlet Flow Orientation	71
A.1	Calculation of Velocity Direction Vectors	71
A.2	Gunn vs Ranz-Marshall Source code	72
A.3	Python Code for Nusselt Number Plot	72
A.4	Heat Transfer UDF Source Code	73

Nomenclature

Abbreviations

Abbreviation	Definition
CFD	Computational Fluid Dynamics
EE	Euler–Euler (two-fluid model)
TFM	Two-Fluid Model
UDF	User-Defined Function
UDM	User-Defined Memory
ISA	International Standard Atmosphere
KTGF	Kinetic Theory of Granular Flow
PR-DNS	Particle-Resolved Direct Numerical Simulation

Symbols

Symbol	Definition	Unit
A	Cross-sectional area	[m ²]
a_s	Specific gas–solid interfacial area	[m ² /m ³]
Ar	Archimedes number	[-]
C_p	Specific heat capacity	[J/(kg·K)]
d_p	Particle diameter	[m]
f_r	Packet renewal frequency	[s ⁻¹]
g	Gravitational acceleration	[m/s ²]
H	Bed height	[m]
h	Heat transfer coefficient	[W/(m ² ·K)]
h_{gs}	Gas–solid heat transfer coefficient	[W/(m ² ·K)]
h_w	Wall-to-bed heat transfer coefficient	[W/(m ² ·K)]
k	Thermal conductivity	[W/(m·K)]
k_e	Effective thermal conductivity	[W/(m·K)]
Nu	Nusselt number	[-]
Pr	Prandtl number	[-]
Q	Volumetric flow rate	[m ³ /s]
$\dot{Q}_{gs}'''$	Volumetric gas–solid heat transfer rate	[W/m ³]
Re_p	Particle Reynolds number	[-]
T	Temperature	[K]
u	Velocity	[m/s]
U	Superficial gas velocity	[m/s]
U_{mf}	Minimum fluidization velocity	[m/s]
ϵ	Void fraction	[-]
ϵ_{mf}	Void fraction at minimum fluidization	[-]
γ	Cone half-angle	[rad]
λ	Thermal conductivity ratio (k_g/k_s)	[-]
μ	Dynamic viscosity	[Pa·s]
ρ	Density	[kg/m ³]
τ	Characteristic time	[s]

Introduction

Reducing the carbon footprint of industrial thermal processes is one of the more urgent engineering problems of the coming decades. As energy demand rises and emissions regulations tighten, there is real pressure to develop reactor technologies that combine thermal efficiency with operational flexibility [32]. Thermochemical conversion; gasification, fast pyrolysis, combustion, catalytic cracking, sits near the centre of this challenge. These processes underpin waste-to-energy systems, second-generation biofuels, and the production of platform chemicals from non-fossil feedstocks [11, 63]. In every one of them, the ability to deliver heat rapidly, uniformly, and controllably to a reactive solid bed is not just a matter of convenience: it directly governs reaction selectivity, product distribution, and the avoidance of localised hot-spots or thermally induced runaway [21, 38].

Gas–solid fluidised beds have been a workhorse of industrial thermal processing for more than seventy years. The vigorous mixing generated by the upward gas flow suspends and circulates solid particles continuously, creating large gas–solid interfacial areas and achieving interphase heat transfer coefficients that can exceed those of packed beds by one to two orders of magnitude [35]. The practical consequence is near-isothermal bulk temperature profiles even in strongly exo- or endothermic reactions, a property that is genuinely difficult to replicate in fixed or moving bed alternatives. As Kunii and Levenspiel [35] show, the thermal behaviour of a fluidised bed is inseparable from its hydrodynamic state: bubble size, rise velocity, interphase gas exchange, and solids circulation all modulate heat transfer simultaneously. This coupling is exactly what makes fluidised beds so thermally effective and, at the same time, so hard to model with confidence.

Despite decades of research, accurate numerical prediction of heat transfer in gas–solid fluidised beds remains an open problem. The Euler–Euler two-fluid framework, in which both phases are treated as interpenetrating continua, has matured considerably and now gives reliable predictions of hydrodynamic behaviour across a wide range of operating conditions [67, 7]. Extending these models to non-isothermal conditions, however, introduces additional closure requirements: interphase heat transfer coefficients, effective solid-phase thermal conductivities, and wall heat transfer correlations, each of which carries its own uncertainties and each of which is sensitive to local hydrodynamic conditions [19, 70]. The gap between hydrodynamic validation and reliable thermal prediction is a genuine constraint on what can be designed with confidence through simulation alone.

Non-cylindrical fluidised bed geometries, conical configurations in particular, have attracted growing attention in this context. In a conical bed the cross-sectional area increases with height, so the superficial gas velocity decreases along the axial direction even at constant volumetric flow rate. This produces a spatially varying degree of fluidisation: the region near the gas inlet operates at higher velocities with more energetic bubbling, while the upper, wider section approaches minimum fluidisation conditions. The cyclic, fountain-like solids circulation characteristic of conical spouted beds [48, 43] promotes intimate gas–solid contact throughout the bed volume, suppresses particle segregation that can afflict cylindrical systems with wide particle size distributions, and reduces defluidised agglomerate formation when processing sticky materials such as waste polymers [60, 39]. The conical geometry

also permits a wide operational range of gas velocities without transitioning to slugging, a regime that severely degrades heat transfer in narrow cylindrical beds [54].

The thermochemical processing of biomass and waste plastics illustrates why these geometric advantages matter in practice. Fast pyrolysis of lignocellulosic biomass requires heating rates of 10^3 – 10^4 K s⁻¹ to maximise liquid bio-oil yield and suppress secondary cracking [11]. Achieving this demands not only high interphase heat transfer coefficients but a well-controlled temperature environment: spatial gradients of more than a few tens of kelvin across the bed can shift product distributions appreciably [62]. Conical spouted and bubbling beds have demonstrated competitive performance in this role [1, 60], with the additional benefit that the spout's coating action on inert bed particles mitigates the defluidisation problems associated with molten plastic intermediates [Aguado2002_Polyolefins]. Nevertheless, the quantitative characterisation of heat transfer in these systems, particularly under non-standard gas injection configurations such as multi-nozzle distributors rather than perforated plates, remains insufficiently understood [60, 15].

CFD provides a complementary route to experimental characterisation. Properly validated simulations can resolve local temperature fields, gas–solid interfacial heat fluxes, and transient thermal evolution at spatial and temporal resolutions that are practically inaccessible to thermocouple arrays or optical diagnostics [19, 50]. The Euler–Euler approach, implemented in commercial solvers such as *ANSYS Fluent*, offers a computationally tractable framework for reactor-scale simulations provided appropriate kinetic theory closures for solid-phase stress and appropriate heat transfer correlations for the interphase exchange are employed [23, 28]. Developing and systematically applying such models in conical geometries therefore represents a meaningful contribution both to the fundamental understanding of fluidised bed heat transfer and to the engineering knowledge base required for reliable scale-up of next-generation thermochemical reactors.

The present thesis is motivated by this convergence of practical need and scientific gap. The aim is to deepen the physical understanding of how heating mode, gas velocity, and conical geometry jointly determine the spatially resolved thermal response of a gas–solid system, and to identify the principal modelling limitations that need to be addressed before these tools can be used with confidence for reactor design.

1.1. Problem Statement

Although the hydrodynamics of gas–solid fluidised beds have been studied extensively for decades, reliable numerical prediction of heat transfer under realistic, non-isothermal operating conditions remains a substantially open problem. This is not simply a matter of computational resolution; it reflects genuine uncertainties in physical closure models and a relative scarcity of well-characterised experimental datasets suitable for thermal validation.

The vast majority of validated Euler–Euler heat transfer studies in the literature address cylindrical geometries with uniform perforated-plate distributors, operated under either isothermal or simplified steady-state thermal boundary conditions [7, 70, 51]. These configurations provide tractable benchmarks but do not represent the full range of industrially relevant scenarios. Conical fluidised bed reactors, which introduce axial velocity gradients and qualitatively different solids circulation patterns, have received comparatively little attention from the heat transfer modelling community [15, 60]. Nozzle-based or multi-nozzle gas injection systems further distinguish these systems from conventional configurations by altering the local gas–solid contact pattern near the distributor and modifying the spatial distribution of bubble nucleation sites, effects that standard distributor correlations simply do not capture [47].

From a modelling standpoint, the Euler–Euler framework introduces several sources of uncertainty that become particularly consequential under non-isothermal conditions. The interphase heat transfer coefficient, typically expressed through the Nusselt number correlation of Gunn [28] or similar relationships, was developed primarily from fixed-bed or dilute-suspension data and may not accurately represent heat transfer in the dense, heterogeneous flow structures (bubbles, emulsion packets, particle clusters) characteristic of bubbling fluidised beds [19, 34]. The effective thermal conductivity of the solid phase and the treatment of wall heat transfer at the reactor boundary introduce further uncertainties that are rarely addressed systematically in published studies.

Against this background, the present thesis investigates heat transfer numerically in a laboratory-scale conical bubbling gas–solid fluidised bed, using porous alumina particles ($\gamma\text{-Al}_2\text{O}_3$) fluidised by nitrogen as the working system. The Euler–Euler approach is implemented in *ANSYS Fluent*, building on hydrodynamic models previously developed and validated for this geometry within the research group. Three distinct thermal operating cases are examined:

1. Gas preheating to 120 °C at the inlet, with the wall held at 16 °C, internal heating via the fluidising agent only.
2. Wall heating at a fixed temperature of 120 °C, with cold inlet gas at 16 °C, external heating through the reactor wall only.
3. Combined wall and gas heating, both at 120 °C, superposition of both modes.

In all three cases the bed is initialised at a spatially uniform temperature of 16 °C, and each case is run at two inlet flow rates (50 lpm and 67 lpm) to examine the effect of gas velocity on the thermal response.

1.2. Research Questions

The foregoing problem statement leads to the following overarching research question:

How accurately can an Euler–Euler CFD model reproduce the transient and spatially resolved heat transfer behaviour of a laboratory-scale conical bubbling gas–solid fluidised bed, and to what extent do heating mode, gas velocity, and conical geometry govern the thermal response of the system?

To address this in a structured way, five specific sub-questions are formulated:

1. How does the predicted transient thermal response differ between wall heating, gas preheating, and combined heating in a conical bubbling fluidised bed initially at uniform temperature, and what physical mechanisms account for the observed differences in heating rate and spatial temperature distribution?
2. How does the superficial gas velocity influence heat transfer under each heating configuration, and how do the corresponding changes in hydrodynamic regime, bubble frequency, size, and rise velocity, modulate interphase and wall heat transfer rates?
3. To what extent can UDFs and UDM variables within *ANSYS Fluent* be used to reliably extract local and global heat transfer metrics, including local Nusselt numbers, interphase exchange rates, and wall heat fluxes, from transient Euler–Euler simulations of a conical geometry?
4. How do the hydrodynamic features intrinsic to the conical geometry, axially varying gas velocity, particle circulation, and the spatial distribution of the bubble phase, influence the local and global distribution of heat transfer within the bed?
5. What are the principal limitations of the current Euler–Euler framework in predicting heat transfer under non-isothermal conditions in a conical geometry, and which model components, interphase heat transfer closures, solid-phase thermal conductivity, wall boundary conditions, are most in need of refinement?

1.3. Thesis Outline

The remainder of this thesis is organised as follows.

Chapter 2 presents a critical review of the relevant literature, covering the fundamentals of gas–solid fluidisation, heat transfer mechanisms in fluidised beds, the Euler–Euler modelling framework, and prior numerical and experimental studies of heat transfer in cylindrical and conical configurations.

Chapter 3 describes the reactor geometry, operating conditions, numerical model formulation, computational mesh, boundary conditions, and the UDF implementation for heat transfer metric extraction.

Chapter 4 presents and analyses the simulation results for all three heating configurations and both flow rates, with direct comparison against experimental data where available. Synthesises the findings, evaluates the model's strengths and limitations, and discusses the physical implications for reactor design.

Chapter 5 summarises the principal conclusions and offers recommendations for future work.

2

Literature Background

2.1. Gas--Solid Fluidisation: Fundamentals

2.1.1. From Packed Bed to Fluidised State

When gas flows upward through a bed of solid particles at low velocity, it percolates through the interstitial voids while the particles remain stationary [36]. The pressure drop across this fixed bed is described by the Ergun equation [20]:

$$\frac{\Delta P}{L} = 150 \underbrace{\frac{\mu_g U (1 - \varepsilon)^2}{\phi_s^2 d_p^2 \varepsilon^3}}_{\text{viscous term}} + 1.75 \underbrace{\frac{\rho_g U^2 (1 - \varepsilon)}{\phi_s d_p \varepsilon^3}}_{\text{inertial term}}, \quad (2.1)$$

where ϕ_s is the particle sphericity, μ_g the gas dynamic viscosity, ε the void fraction, and d_p the mean particle diameter. The viscous term dominates at low Reynolds numbers, while the inertial term becomes significant at higher velocities [55]. As U increases, the drag force on the bed grows until it equals the buoyancy-corrected weight per unit cross-sectional area:

$$\Delta P_{mf} = (\rho_s - \rho_g) (1 - \varepsilon_{mf}) g H_{mf}, \quad (2.2)$$

where ε_{mf} is the void fraction at incipient fluidisation and H_{mf} the corresponding bed height [36]. The velocity at which Eqs. 2.1 and 2.2 balance defines the minimum fluidisation velocity U_{mf} , the lower operational bound for any fluidised-bed process [55].

Equating the two expressions and non-dimensionalising with the Archimedes number:

$$Ar = \frac{d_p^3 \rho_g (\rho_s - \rho_g) g}{\mu_g^2}, \quad (2.3)$$

yields a quadratic in the particle Reynolds number at minimum fluidisation, $Re_{mf} = \rho_g U_{mf} d_p / \mu_g$ [36]. The correlation of Wen and Yu [68], which absorbs the dependence on ε_{mf} and ϕ_s into two empirical constants fitted to a large experimental dataset, simplifies this to:

$$Re_{mf} = \sqrt{(33.7)^2 + 0.0408 Ar} - 33.7. \quad (2.4)$$

This remains the most widely used predictive tool for U_{mf} in engineering design and is valid for Geldart Group B and D particles under near-ambient conditions [68, 55]. Its accuracy decreases at elevated temperatures, where the change in μ_g shifts Ar appreciably [36], a point directly relevant here since the simulations span 289–393 K. For the porous γ -alumina particles of this work, the internal porosity also reduces the effective particle density below the bulk alumina value, and this must be accounted for when evaluating Eq. 2.3.

Beyond incipient fluidisation, the Richardson–Zaki relation [56] connects the operating velocity to the bed void fraction:

$$\frac{U}{U_t} = \varepsilon^n, \quad (2.5)$$

where U_t is the single-particle terminal settling velocity and n is a regime-dependent exponent, ranging from ≈ 4.65 in the Stokes regime to ≈ 2.3 at high particle Reynolds numbers [56, 36]. This matters for heat transfer because ε directly governs the specific gas–solid interfacial area and thereby controls both the interphase exchange rate and bubble dynamics. In a bubbling bed ε is far from spatially uniform: the bubble phase has a void fraction approaching unity, while the surrounding emulsion phase sits close to ε_{mf} [36, 16]. In a conical reactor an additional axial gradient arises from the geometrically imposed velocity decay with height [49], so the local heat transfer rate varies continuously along the reactor axis. This spatial non-uniformity is one of the principal motivations for a CFD approach rather than a lumped or one-dimensional model.

2.1.2. Geldart Classification

The nature of gas–solid fluidisation is strongly governed by particle size and density, a dependence formalised by Geldart [22] in a classification chart that has remained the standard reference in fluidisation science for more than fifty years [13]. The Geldart diagram plots the density difference ($\rho_s - \rho_g$) against d_p to delineate four groups with qualitatively distinct behaviour:

Group C Cohesive powders ($d_p \lesssim 30 \mu\text{m}$) in which van der Waals and electrostatic forces exceed gas drag. These materials resist uniform fluidisation and channel preferentially [22].

Group A Aeratable powders ($30 \mu\text{m} \lesssim d_p \lesssim 100 \mu\text{m}$) that expand homogeneously between U_{mf} and a minimum bubbling velocity $U_{mb} > U_{mf}$. Small, rapidly splitting bubbles and vigorous mixing make Group A conditions particularly favourable for heat transfer [22, 36].

Group B Sand-like particles ($100 \mu\text{m} \lesssim d_p \lesssim 800 \mu\text{m}$) for which bubbling commences at essentially U_{mf} ($U_{mb} \approx U_{mf}$). Bubble size grows with height and superficial velocity. This is the most common group in industrial fluidised-bed reactors [22, 10].

Group D Spoutable particles ($d_p \gtrsim 600 \mu\text{m}$, high density) whose large inertia resists conventional bubbling. These are suited to spouted-bed operation, in which a stable central gas jet is established [22, 44].

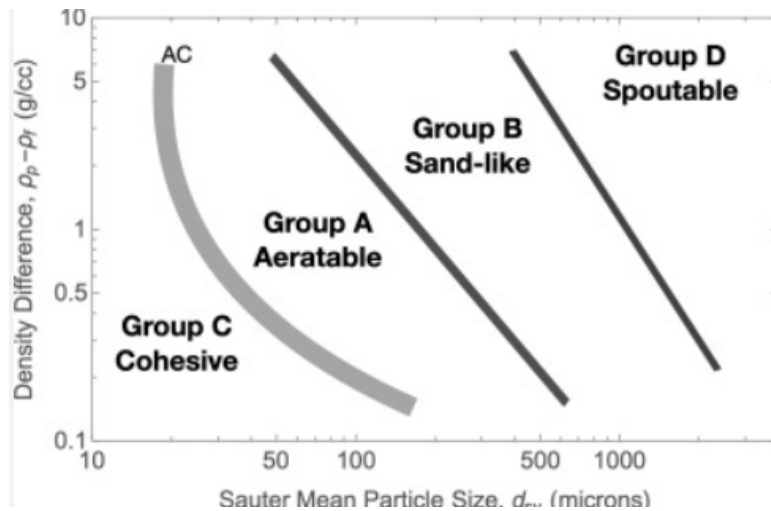


Figure 2.1: Geldart classification of gas–solid fluidisation behaviour as a function of density difference ($\rho_s - \rho_g$) and mean particle diameter d_p , redrawn from [22]. The porous $\gamma\text{-Al}_2\text{O}_3$ particles used in the present work fall within Geldart Group B.

The porous alumina particles of this work fall within Geldart Group B (Figure 2.1), meaning that bubbling commences at U_{mf} and bubble size increases with both velocity and bed height [22]. This is important for the present study because it means the Gunn and Ranz–Marshall heat transfer correlations, derived for conditions representative of Group B behaviour, are physically appropriate closures for the interphase exchange in the CFD model.

2.1.3. Flow Regimes and Their Thermal Implications

As superficial velocity increases beyond U_{mf} , a gas–solid system passes through a sequence of well-characterised flow regimes, each with qualitatively different hydrodynamic structure and heat transfer

behaviour [36]. Figure 2.2 illustrates this progression.

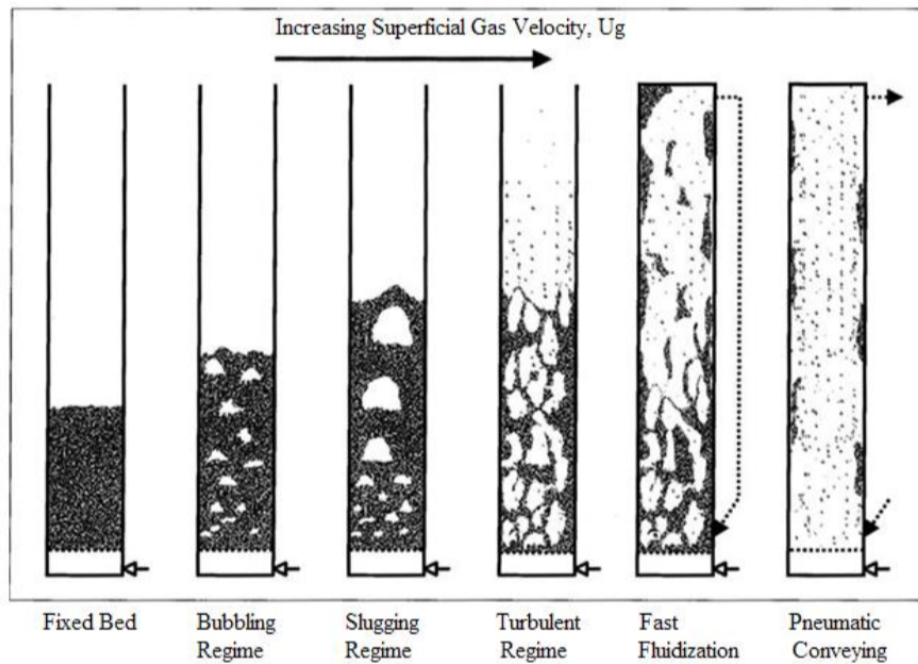


Figure 2.2: Gas–solid fluidisation flow regimes as a function of superficial gas velocity, redrawn from [36]. The superimposed dashed curve shows the qualitative variation of the wall heat transfer coefficient h_w with velocity: it rises steeply above U_{mf} as bubble-induced packet renewal begins [46], reaches a maximum near the bubbling–turbulent transition, then declines as bed dilution reduces the solids inventory near the wall [71, 10]. The shaded band marks the bubbling regime relevant to the present work.

Fixed bed ($U < U_{mf}$). Gas flows through stationary interstitial voids. Heat transfer is governed by gas-phase conduction and limited forced convection; pronounced axial and radial temperature gradients can develop [36, 10].

Bubbling fluidised bed ($U_{mf} < U \lesssim U_{ms}$). Excess gas above U_{mf} forms discrete rising bubbles through a continuous dense emulsion phase, as described by the two-phase theory of Davidson and Harrison [16]. Bubble wakes entrain and circulate solids, generating vigorous mixing that enhances both gas–solid interphase heat exchange and particle–wall heat transfer. Coefficients in this regime are typically one to two orders of magnitude higher than in a fixed bed [71, 46]. **This is the operating regime of the present thesis.**

Slugging ($U > U_{ms}$, **height-to-diameter ratio** > 2). Bubbles coalesce into gas slugs spanning the full reactor cross-section [36]. Radial mixing is severely reduced, and local heat transfer becomes oscillatory and spatially non-uniform. The conical geometry of the present reactor naturally suppresses slugging by imposing an axially decreasing superficial velocity [49], this is discussed further in Section 2.4.

Turbulent fluidisation. The bubble–emulsion boundary becomes indistinct, replaced by chaotic voids and clusters of varying size [36]. Mixing intensity is high, but the strongly unsteady, spatially heterogeneous flow makes reliable prediction of local heat transfer coefficients difficult.

Fast fluidisation and pneumatic conveying. Net upward solids transport occurs. Strong axial gradients in solids concentration produce spatially non-uniform wall heat transfer, and bed-to-wall exchange is reduced in dilute upper regions [36, 55].

The non-monotonic relationship between h_w and U in Figure 2.2 reflects a fundamental competition first systematised by Zabrodsky [71]: increasing velocity enhances particle mobility and bubble-induced wall surface renewal, raising h_w , but simultaneously dilutes the bed, reducing the thermal mass available for heat exchange near the wall, lowering h_w [71, 10]. An optimum velocity exists between these limits. This is directly relevant to the operating condition selection in the present study: both the 50 lpm and 67 lpm cases are intended to sit within the bubbling regime on the rising side of this curve, where increasing velocity improves rather than degrades heat transfer.

2.2. Heat Transfer Mechanisms in Gas--Solid Fluidised Beds

Heat transfer in gas--solid fluidised beds involves three coupled mechanisms operating simultaneously at different scales: gas--solid interphase convection, particle--wall exchange governed by transient packet renewal, and conduction through the dense emulsion phase [36, 10]. The relative importance of each depends on the local hydrodynamic state, particle properties, and reactor geometry [10].

2.2.1. Gas--Solid Interphase Heat Transfer: Ranz--Marshall and Gunn Correlations

Exchange between the fluidising gas and the suspended particles is the primary mechanism by which thermal energy, whether supplied at the wall or introduced through the gas inlet, is redistributed throughout the bed [36, 10]. In the Euler--Euler framework this exchange appears as a volumetric source term coupling the gas and solid energy equations, and its magnitude is controlled entirely by the choice of interphase heat transfer correlation [24, 14]. The exchange rate is quantified through the interphase Nusselt number [31]:

$$Nu_p = \frac{h_{gs} d_p}{k_g}, \quad (2.6)$$

where h_{gs} [$\text{W m}^{-2} \text{K}^{-1}$] is the gas--solid heat transfer coefficient, d_p [m] the particle diameter, and k_g [$\text{W m}^{-1} \text{K}^{-1}$] the gas thermal conductivity [31]. The theoretical lower bound $Nu_p = 2$ follows from the exact analytical solution for steady-state conduction from a sphere into an infinite, stationary medium [31, 9]; any physically consistent correlation must recover this limit as the relative velocity approaches zero [29, 52].

Two correlations are employed in the present work: the Ranz--Marshall correlation for dilute, single-particle conditions, and the Gunn correlation for the dense and intermediate void fraction regimes that characterise the bed interior [14, 6].

The Ranz--Marshall Correlation: Single-Particle Limit

Ranz and Marshall [52] extended the $Nu_p = 2$ diffusive limit by adding an empirical convective correction fitted to evaporation and heat transfer data for single droplets and isolated spheres:

$$Nu_p = 2 + 0.6 Re_p^{1/2} Pr^{1/3}, \quad (2.7)$$

where $Re_p = \rho_g u_{\text{slip}} d_p / \mu_g$ is based on the gas--solid slip velocity $u_{\text{slip}} = |v_g - v_s|$, and $Pr = \mu_g C_{p,g} / k_g$ [52]. The correlation is valid for $Re_p \lesssim 200$ and $Pr \lesssim 250$, and the $Re_p^{1/2}$ scaling is consistent with laminar boundary layer theory over a curved surface [31, 69].

The fundamental limitation is that Eq. 2.7 was derived for a single, isolated particle with no neighbouring solids. As a result, it contains no dependence on void fraction and produces the same Nu_p regardless of local solid concentration [29]. In the dense, heterogeneous flow structure of a fluidised bed, where particle proximity modifies the gas flow and thermal boundary layer around each particle, this makes it physically inappropriate for anything other than very dilute conditions [29, 36]. In practice, the Ranz--Marshall correlation is only justified where $\varepsilon \gtrsim 0.99$, corresponding to the effectively particle-free fountain region above the bed surface. Cruz et al. [14], in a systematic comparison of heat transfer correlations in Euler--Euler simulations of bubbling beds, found that Ranz--Marshall is a minority closure choice precisely for this reason.

The Gunn Correlation: Multiparticle Dense Beds

The void-fraction independence of Ranz--Marshall is its critical deficiency for dense bed conditions. In a real multiparticle system, neighbouring particles obstruct gas flow to each particle surface, gas channels

through locally more open regions, and adjacent boundary layers interact, all effects that make Nu_p a function of ε as well as Re_p [29, 36]. Gunn [29] developed a correlation that captures this dependence by anchoring it to four asymptotic conditions: the low- Re_p packed-bed limit; the high- Re_p power-law behaviour observed in packed beds; recovery of the single-particle result as $\varepsilon \rightarrow 1$ and $Re_p \rightarrow 0$; and the experimentally established dependence of Nu_p on void fraction across the full range from dense packed beds to dilute suspensions. Fitting these constraints to heat and mass transfer data spanning several orders of magnitude in Re_p for both gas and liquid systems, he obtained:

$$Nu_p = (7 - 10\varepsilon + 5\varepsilon^2) \left(1 + 0.7 Re_p^{0.2} Pr^{1/3}\right) + (1.33 - 2.4\varepsilon + 1.2\varepsilon^2) Re_p^{0.7} Pr^{1/3}, \quad (2.8)$$

valid for $0.35 \leq \varepsilon \leq 1.0$ and $Re_p \leq 10^5$ [29]. The essential advance is that both polynomial coefficient groups depend explicitly on ε , so Nu_p responds to the local void fraction rather than treating all operating conditions identically. As $\varepsilon \rightarrow 1$ and $Re_p \rightarrow 0$, Eq. 2.8 correctly recovers $Nu_p \rightarrow 2$, consistent with both the single-sphere analytical result and Eq. 2.7 [29].

Particle-resolved direct numerical simulations have confirmed the accuracy of Eq. 2.8 for $\varepsilon > 0.7$ [66]. For denser conditions, Sun et al. [64] reported errors up to 35% at $\varepsilon \lesssim 0.5$ using PR-DNS, attributing this to the use of bulk-averaged fluid temperature as the thermal driving force, an assumption that becomes inconsistent at high solids loading. The emulsion phase of the present bed operates at $\varepsilon = 0.59$, placing it near this threshold. Despite this, the Gunn correlation is retained as the primary closure for several reasons: it is the standard validated implementation in ANSYS Fluent [3]; its stated validity range formally encompasses $\varepsilon = 0.59$ [29]; and it is the closure used in essentially all reviewed Euler–Euler heat transfer studies [14, 6, 33]. The potential overestimation of h_{gs} in the dense emulsion phase is acknowledged as the primary interphase closure uncertainty and is assessed in Chapter ??.

Correlation Selection in the Present Simulations

Both correlations are activated through a void-fraction-based switching criterion implemented as a UDF in ANSYS Fluent. For $\varepsilon > 0.8$, conditions representative of the dilute jet core and bubble interiors, Ranz–Marshall (Eq. 2.7) is applied as the single-particle closure. For all denser regions where $\varepsilon \leq 0.8$, including the emulsion phase ($\varepsilon \approx 0.59$), the Gunn correlation (Eq. 2.8) is applied. The threshold of 0.8 was chosen because the theoretically rigorous dilute limit ($\varepsilon_g \rightarrow 1$) is realised in only negligibly small regions of a bubbling bed simulation, so using 0.99 would activate Ranz–Marshall almost nowhere; 0.8 more meaningfully separates the dilute and dense regimes within the computational domain. Since both correlations recover $Nu_p = 2$ as $Re_p \rightarrow 0$ and $\varepsilon \rightarrow 1$ [29, 52], there is no discontinuity at the switching point. This approach is consistent with the findings of Cruz et al. [14], who found Gunn to be the community standard that produces closer agreement with experimental data in heat-transfer-sensitive cases.

Volumetric Interphase Heat Transfer

The volumetric interphase heat transfer rate per unit reactor volume is:

$$\dot{Q}_{gs}''' = h_{gs} a_s (T_g - T_s), \quad (2.9)$$

where the specific gas–solid interfacial area for a monodisperse bed of spherical particles is [36, 55]:

$$a_s = \frac{6(1 - \varepsilon)}{d_p}. \quad (2.10)$$

Since $a_s \propto d_p^{-1}$, finer particles provide greater contact area per unit volume, raising the volumetric exchange rate at the same h_{gs} . However, finer particles also reduce U_{mf} through the Archimedes number (Eq. 2.3), which constrains the achievable slip velocity and therefore h_{gs} itself [68, 29]. The result is a heat-transfer optimum at an intermediate particle size, demonstrated experimentally by Zabrodsky [71] and discussed in depth by Kunii and Levenspiel [36]. For the γ -alumina particles of this work ($d_p = 2.02 \times 10^{-4}$ m), both a_s and h_{gs} are fixed by the geometry; the operating variable is ε , which varies along the reactor axis and between the bubble and emulsion phases.

Figure 2.3 illustrates the essential difference between the two correlations. Ranz–Marshall produces a single curve that is blind to void fraction, while Gunn produces a family of curves that separate with

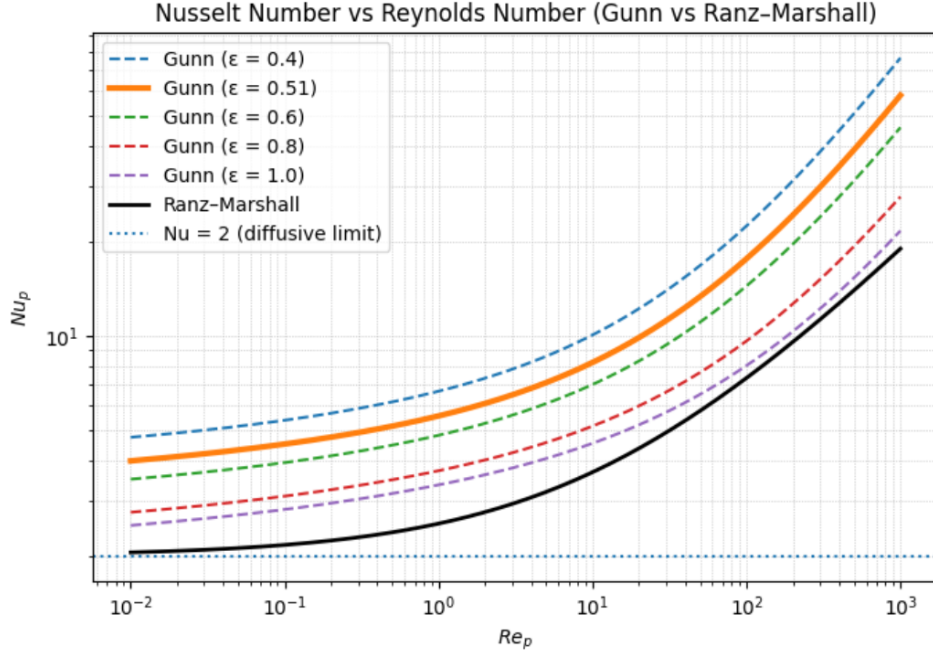


Figure 2.3: Gas–solid Nusselt number Nu_p as a function of particle Reynolds number Re_p for selected void fractions ε , computed from the Gunn correlation [29] (Eq. 2.8) at $Pr = 0.71$ (nitrogen at 20 °C). The dashed line is the Ranz–Marshall result [52] (Eq. 2.7), which contains no void-fraction dependence and is recovered by Gunn as $\varepsilon \rightarrow 1$. The highlighted curve at $\varepsilon = 0.59$ corresponds to the emulsion-phase condition of the present bed. The horizontal line at $Nu_p = 2$ marks the analytical lower bound [31, 9].

decreasing ε [29]. The emulsion-phase curve at $\varepsilon = 0.59$ lies noticeably below the Ranz–Marshall prediction at the same Re_p , reflecting the suppression of heat transfer by particle crowding. This void-fraction sensitivity is precisely what is needed to represent the spatially varying conditions along the conical reactor height, where ε decreases from the dilute upper section toward the denser base [61, 49], and it is what makes the Gunn correlation the more physically consistent choice for the present simulations.

2.2.2. Wall-to-Bed Heat Transfer: The Packet Renewal Mechanism

Heat exchange between the reactor wall and the dense bed phase is governed by the *packet renewal mechanism*, first proposed by Mickley and Fairbanks [46] and since confirmed by a range of experimental and numerical studies [33, 10]. In this model, packets of dense emulsion phase, behaving as semi-infinite porous media, periodically contact the heated wall, absorb heat conductively over a stochastic residence time τ , and are then displaced by rising bubbles that refresh the surface with cooler bulk material [46].

The instantaneous coefficient during packet contact follows from the semi-infinite conduction solution [46]:

$$h_{\text{inst}}(t) = \sqrt{\frac{k_e \rho_e C_{p,e}}{\pi t}}, \quad (2.11)$$

where k_e , ρ_e , and $C_{p,e}$ are the effective thermal conductivity, bulk density, and specific heat of the emulsion packet. Integrating over the contact time distribution gives the time-averaged coefficient [46, 10]:

$$\bar{h}_w = \sqrt{\frac{k_e \rho_e C_{p,e} f_r}{\pi}}, \quad (2.12)$$

where f_r is the bubble-driven packet renewal frequency. Equation 2.12 correctly recovers the experimentally observed proportionality $\bar{h}_w \propto \sqrt{k_e}$, demonstrated by Mickley and Fairbanks by varying the fluidising gas species at fixed particle properties [46]. It also explains why wall heat transfer responds strongly to bubble frequency: f_r scales approximately with $U - U_{mf}$, so increasing the excess velocity directly accelerates surface renewal [36, 10].

In practice a thin gas film of elevated void fraction at the wall introduces an additional series resistance, giving the composite coefficient [10]:

$$\frac{1}{h_w} = \frac{1}{\bar{h}_w} + \frac{1}{h_{\text{film}}}, \quad (2.13)$$

where $h_{\text{film}} \approx k_g/\delta_{\text{film}}$ and δ_{film} is the effective gas-film thickness, estimated as a fraction of d_p [10, 33]. In the Euler–Euler CFD model, both the packet renewal and the gas-film resistance are captured implicitly through the resolved near-wall hydrodynamics, without requiring f_r as an empirical input [33]. This is one of the more practical advantages of the CFD approach over the original analytical packet model.

2.2.3. Effective Thermal Conductivity of the Dense Phase

Within the dense emulsion phase, heat transport occurs through a combination of gas-phase conduction in the interstitial pores and solid-phase conduction through the particle contact network [72, 10]. Because heat-flow paths are strongly constricted at particle contact points, the effective stagnant conductivity $k_{e,0}$ cannot be represented as a simple volumetric average of k_s and k_g [72]. The standard engineering expression is provided by the Zehner–Schlünder model [72]:

$$\frac{k_{e,0}}{k_g} = (1 - \sqrt{1 - \varepsilon}) + \frac{2\sqrt{1 - \varepsilon}}{1 - \lambda B} \left[\frac{(1 - \lambda)B}{(1 - \lambda B)^2} \ln \frac{1}{\lambda B} - \frac{B + 1}{2} - \frac{B - 1}{1 - \lambda B} \right], \quad (2.14)$$

where $\lambda = k_g/k_s$ is the inverse conductivity ratio and $B \approx 1.25 [(1 - \varepsilon)/\varepsilon]^{10/9}$ is a geometric deformation factor for spherical particles [72]. The model has been validated against packed-bed data over a wide range of conductivity ratios and is widely used as the solid-phase thermal conductivity closure in Euler–Euler models [72, 33].

For the porous γ -alumina particles used here, the internal porosity reduces k_s well below the bulk dense alumina value [36]. The practical consequence is that the dense emulsion phase of the present bed may behave as a relatively poor conductor, placing a non-negligible fraction of the total thermal resistance on the emulsion-phase conduction pathway. This becomes particularly relevant in the upper, quasi-dense regions of the conical bed, where convective transport weakens as $U(y)$ approaches U_{mf} (see Section 2.4).

2.2.4. Combined Thermal Resistances

The three mechanisms described above can be interpreted through an approximate resistance-network framework [10, 36]:

$$\frac{1}{U_{\text{bed}}} = \frac{1}{h_{gs}} + \frac{1}{h_w}, \quad (2.15)$$

where h_{gs} represents the bulk gas–solid interphase exchange and h_w the wall-to-bed exchange (Eq. 2.13). These resistances are not strictly in series, the spatial distribution of heat deposition depends on whether wall or gas-inlet heating dominates, but the network interpretation provides a useful physical basis for identifying the rate-controlling step under each of the three heating configurations studied here.

For wall heating, $1/h_w$ dominates the early transient, before thermal diffusion has penetrated the dense bed [46, 10]. As the bed heats progressively, $1/h_{gs}$ increasingly governs the approach to equilibrium [36]. For gas preheating, $1/h_{gs}$ controls from the outset, and the transient response is therefore determined by the interphase Nusselt number and the volumetric interfacial area a_s (Eq. 2.10) [29, 36]. The combined heating case superposes both pathways and is expected to give the fastest thermal equilibration, consistent with results reported for analogous configurations in cylindrical beds [6].

2.3. Influence of Operating Parameters on Heat Transfer

2.3.1. Superficial Gas Velocity

Superficial gas velocity U is the primary operating variable governing both hydrodynamic structure and thermal transport in fluidised beds [36, 55]. At the particle scale it controls the Reynolds number $Re_p = \rho_g u_{\text{slip}} d_p / \mu_g$, which drives h_{gs} through the Gunn correlation (Eq. 2.8) [29]. Increasing U raises u_{slip} , thinning the thermal boundary layer and increasing $h_{gs} \sim u_{\text{slip}}^{0.7}$ in the inertial regime [29, 71]. At

the same time, the expanding void fraction reduces a_s (Eq. 2.10), so the net volumetric exchange rate $\dot{Q}_{gs}''' \propto h_{gs} a_s$ peaks at an intermediate velocity, producing the bell-shaped trend in Figure 2.2 [71, 10].

Beyond the optimum, strong local convection coexists with insufficient solids inventory near the wall, degrading global thermal performance [71, 36]. The 50 lpm and 67 lpm cases in this study are intended to sit on the rising side of this curve, so the model should predict increasing heat transfer with increasing velocity. Whether the two flow rates bracket the optimum or both fall below it is one of the questions addressed in Chapter ??.

There is also a time-scale argument worth making. The gas residence time scales as $\tau_{\text{res}} \sim H/U$ [36], while the particle thermal response time (treating the particle as a lumped system with $Bi = h_{gs} d_p / 2k_s \ll 1$) is approximately $\tau_p \sim \rho_s C_{p,s} d_p / (6h_{gs})$ [10]. When $\tau_{\text{res}} \ll \tau_p$, the gas exits before reaching thermal equilibrium with the solid, causing thermal bypassing even at locally high h_{gs} [36]. This ratio is worth evaluating against the present operating conditions to confirm whether the simulations are heat-exchange-limited or residence-time-limited.

Once the bed is fluidised, the pressure drop stabilises at approximately the buoyancy-corrected bed weight per unit area [36, 55]:

$$\Delta P_{\text{bed}} \approx (\rho_s - \rho_g)(1 - \varepsilon) g H, \quad (2.16)$$

and no longer rises with U . The energetic cost of increasing U beyond the optimum is therefore borne entirely by the additional gas flow, not by a higher bed pressure drop [55].

2.3.2. Particle Size and Density Effects

Particle diameter d_p controls both the convective boundary layer resistance and the available interfacial area [36, 10]. From the Gunn correlation at intermediate Re_p , where $Nu_p \propto Re_p^{0.7}$ and $Re_p \propto d_p$, the heat transfer coefficient scales as $h_{gs} \propto d_p^{-0.3}$ [29]. Combined with $a_s \propto d_p^{-1}$ (Eq. 2.10), the volumetric capacity scales as $\dot{Q}_{gs}''' \propto d_p^{-1.3}$, decreasing strongly with increasing particle size [71]. This is consistent with classical measurements by Zabrodsky [71] and Botterill [10], who both documented the marked improvement in volumetric heat transfer when moving from large Group D to intermediate Group B particles.

Particle density ρ_s enters through two competing pathways [36]. Higher ρ_s raises U_{mf} and requires higher operating velocities to achieve comparable void fractions. Simultaneously, since $\rho_s \gg \rho_g$, the solid phase dominates the volumetric heat capacity of the bed:

$$(\rho C_p)_{\text{bed}} \approx (1 - \varepsilon) \rho_s C_{p,s}, \quad (2.17)$$

so higher ρ_s increases thermal inertia and slows transient equilibration [10, 36]. This is directly relevant here: the large $(\rho C_p)_s / (\rho C_p)_g \approx 1125$ ratio for the present system means the solid heats extremely slowly relative to the gas, which shapes the interpretation of every temperature contour in Chapter ??.

2.3.3. Void Fraction and Volumetric Heat Transfer Capacity

Void fraction ε mediates all three heat transfer mechanisms simultaneously [36, 10]. For interphase convection, it appears in both a_s (Eq. 2.10) and the Gunn $Nu_p - \varepsilon$ dependence (Eq. 2.8, Figure 2.3) [29]. For wall heat transfer, ε near the wall determines the packet thermal properties entering Eq. 2.12 [46, 10]. For dense-phase conduction, it enters the Zehner–Schlünder conductivity (Eq. 2.14) through both the prefactor and the geometric factor B [72].

The net result on $\dot{Q}_{gs}'''$ is the bell-shaped trend already discussed: at low voidage (fixed bed) convection is weak; at intermediate voidage (bubbling), convection and interfacial area are optimally balanced; at high voidage (dilute transport), the collapse of a_s dominates [71, 10]. In the present conical reactor, the axial variation of ε produces a spatial gradient in all three contributions simultaneously, the high-voidage upper cone has lower interfacial area, lower packet renewal frequency, and reduced dense-phase conduction relative to the denser base near the nozzle inlets. This spatial coupling between ε , flow regime, and heat transfer is the defining thermal characteristic of the conical geometry, and it is the principal reason why a spatially resolved CFD approach is needed rather than a lumped or one-dimensional model [61].

2.4. Reactor Geometry: Cylindrical vs. Conical Beds

2.4.1. Cylindrical Beds: The Reference Configuration

In a cylindrical bed of constant cross-section A , the superficial velocity $U = Q/A$ is uniform throughout the bed height [36]. Hydrodynamic transitions, from fixed bed to bubbling to slugging, occur globally and simultaneously across the full reactor volume [36, 55]. The spatial distribution of heat transfer is determined primarily by bubble dynamics rather than by geometry, and the classical correlations, Gunn for interphase transfer [29] and the packet renewal model for wall transfer [46], were developed predominantly for this configuration. They provide the benchmarks against which conical geometry is assessed [10, 33].

The drawback of this uniformity is that slugging can disrupt heat transfer across the full reactor simultaneously, and the cylindrical geometry provides no geometric mechanism to arrest it [36, 55]. In narrow reactors with height-to-diameter ratios above approximately 2, slug formation severely reduces radial mixing and produces oscillatory, poorly controllable thermal behaviour [10].

2.4.2. Conical Beds: Axial Velocity Gradient and Spatial Regime Co-existence

In a conical bed of half-angle γ and base radius r_0 , the cross-sectional area grows with height as $A(y) = \pi(r_0 + y \tan \gamma)^2$ [49]. At constant volumetric flow rate Q , the superficial velocity decays axially as:

$$U(y) = \frac{Q}{\pi (r_0 + y \tan \gamma)^2}. \quad (2.18)$$

This inverse-square dependence was established experimentally for conical spouted beds by Olazar et al. [49] and confirmed in subsequent investigations of conical fluidised and spouted beds [61]. The result is that distinct hydrodynamic regimes coexist at different elevations within a single reactor. Near the base, where $U(y) \gg U_{mf}$, active bubbling and vigorous interphase convection dominate. Higher up the cone, where $U(y) \rightarrow U_{mf}$, the emulsion phase densifies and conduction through the particle contact network becomes increasingly significant relative to convective transport [49, 61].

The critical height y^* at which $U(y^*) = U_{mf}$ is obtained directly from Eq. 2.18:

$$y^* = \frac{1}{\tan \gamma} \left(\sqrt{\frac{Q}{\pi U_{mf}}} - r_0 \right). \quad (2.19)$$

Above y^* , the local superficial velocity falls below U_{mf} and the bed transitions from a convection-dominated to a conduction-influenced state. This geometric partitioning has no counterpart in cylindrical beds, and it is precisely why classical cylindrical correlations, developed for spatially uniform flow, require careful local application in conical systems where operating conditions vary continuously with height [61, 49].

Two further advantages of the conical geometry are relevant to the thermochemical applications motivating this thesis. First, the widening cross-section naturally suppresses bubble coalescence and slugging because inter-bubble spacing increases with height [49], enabling operation across a wider velocity range without transitioning to the slugging regime [55]. Second, the absence of a flat perforated distributor eliminates the stagnant annular dead zones that form at the wall-distributor junction in cylindrical beds, reducing the risk of agglomerate formation when processing sticky or thermoplastic particles [Aguado2002_Polyolefins].

2.4.3. Spouted Beds: Hydrodynamic Structure and Heat Transfer

When a conical bed is operated above the minimum spouting velocity U_{ms} , it enters the spouted-bed regime [44]. The hydrodynamic structure is tripartite [44, 49]: a dilute central spout where particles are transported upward pneumatically; a fountain above the bed surface where particles decelerate, disperse radially, and fall back; and a dense, slowly descending annulus where gas velocity is low and particle motion is gravity-driven. Figure 2.4 illustrates these zones.

The ordered solids circulation fundamentally changes the heat transfer distribution compared to conventional bubbling beds. In the spout core, high gas–solid slip velocities generate elevated local Nusselt numbers [29]. In the annulus, dense packing and low particle velocity reduce convective contributions

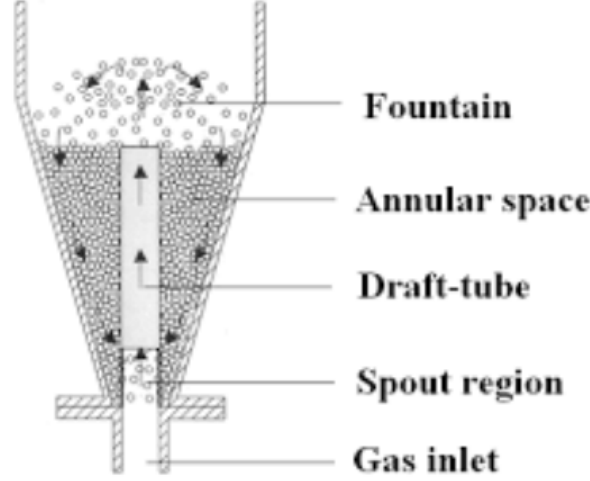


Figure 2.4: Schematic of the three hydrodynamic zones in a conical spouted bed [44, 49]. Spout core: dilute, high gas–solid slip, elevated local Nu_p governed by the Gunn correlation (Eq. 2.8) [29]. Fountain: particles decelerate and disperse radially. Annulus: dense, slowly descending phase where Zehner–Schlünder conduction (Eq. 2.14) contributes significantly to lateral heat redistribution [61]. The spout–annulus interface (dashed) sustains the highest local convective heat fluxes through continuous surface renewal. Redrawn from [44].

while Zehner–Schlünder conduction becomes the dominant lateral redistribution pathway [72, 61]. The overall heat transfer performance emerges from the superposition of these two mechanisms, mediated by the circulation loop that continuously exchanges material between the high-flux spout and the conductive annulus [44, 61].

The present reactor operates in the conical *bubbling* regime below full spouting. Nevertheless, incipient jetting near the nozzle inlet influences the local void fraction and heat transfer in the lower bed section, and the Euler–Euler model must resolve this near-distributor region adequately [49].

2.5. Numerical Modelling of Heat Transfer in Fluidised Beds

2.5.1. The Euler–Euler Two-Fluid Framework

The hydrodynamic foundation of the present simulations is the Euler–Euler (EE) two-fluid model (TFM), in which both gas and solid phases are treated as interpenetrating continua governed by volume-averaged conservation equations [2, 24]. A full derivation and justification of the hydrodynamic sub-model for the same conical reactor geometry is provided by Roy [58], whose validated model forms the basis on which the present thermal extension is built. The justification for the EE approach over Euler–Lagrange methods, the continuity and momentum equations, the drag model formulations, and the full Kinetic Theory of Granular Flow (KTGF) closures are all covered there and are not repeated here.

The present section focuses on the thermal extension of the EE–TFM: the energy conservation equations for each phase, the interphase heat transfer coupling term, and the effective solid-phase thermal conductivity closure, all of which are absent from the isothermal model of Roy [58].

2.5.2. Phase Energy Conservation Equations

When the EE framework is extended to non-isothermal conditions, a thermal energy equation is solved for each phase alongside continuity and momentum [24, 33]. The gas-phase energy equation is [24, 3]:

$$\frac{\partial}{\partial t} (\varepsilon_g \rho_g H_g) + \nabla \cdot (\varepsilon_g \rho_g \vec{u}_g H_g) = \nabla \cdot (\varepsilon_g k_g \nabla T_g) - \dot{Q}_{gs}''' + S_g, \quad (2.20)$$

and the solid-phase energy equation is [24, 3]:

$$\frac{\partial}{\partial t} (\varepsilon_s \rho_s H_s) + \nabla \cdot (\varepsilon_s \rho_s \vec{u}_s H_s) = \nabla \cdot (\varepsilon_s k_{e,0} \nabla T_s) + \dot{Q}_{gs}''' + S_s, \quad (2.21)$$

where $H_g = C_{p,g}T_g$ and $H_s = C_{p,s}T_s$ are the phase-specific enthalpies, $k_{e,0}$ is the effective solid-phase conductivity from the Zehner–Schlünder model (Eq. 2.14), and S_g, S_s are volumetric source terms, set to zero in the absence of chemical reaction. The term $\dot{Q}_{gs}'''$ (Eq. 2.9) appears with opposite signs in the two equations, enforcing conservation of thermal energy across the gas–solid interface [24, 36]. When $T_g > T_s$, energy flows from gas to solid; when $T_g < T_s$, as in the inverted transient of Case W in Chapter ??, the direction reverses.

It is worth noting what these equations do not contain relative to the momentum equations. Granular temperature Θ , which describes the kinetic energy of particle velocity fluctuations [58], does not appear explicitly in the thermal energy equations. It influences heat transfer indirectly through its effect on phase distribution and solids velocity, but at the operating conditions relevant here, viscous dissipation heating is negligible and the thermal and mechanical energy equations are effectively decoupled [24].

2.5.3. Interphase Heat Transfer Coupling

The volumetric exchange rate coupling Eqs. 2.20 and 2.21, in its expanded form, is [36]:

$$\dot{Q}_{gs}''' = h_{gs} \frac{6(1 - \varepsilon_g)}{d_p} (T_g - T_s), \quad (2.22)$$

where $h_{gs} = Nu_p k_g / d_p$. Its magnitude depends on three quantities that vary spatially and temporally: the local void fraction ε_g (through both a_s and Nu_p), the local slip velocity (through Re_p), and the local temperature difference $(T_g - T_s)$ [36, 29]. Resolving all three at every cell and every timestep is the principal advantage of the EE–CFD approach over analytical packet models: no empirical renewal frequency or bed-averaged coefficient is required as input [33].

In the present work, the Gunn correlation (Eq. 2.8) is applied where $\varepsilon_g \leq 0.8$ and the Ranz–Marshall correlation (Eq. 2.7) where $\varepsilon_g > 0.8$, with the switching logic implemented as a UDF as justified in Section 2.2. Since both correlations recover $Nu_p = 2$ as $Re_p \rightarrow 0$ and $\varepsilon_g \rightarrow 1$, there is no discontinuity at the switching point [29, 52].

2.5.4. Effective Solid-Phase Thermal Conductivity

The conductive term in Eq. 2.21 requires a closure for $k_{e,0}$, provided here by the Zehner–Schlünder model (Eq. 2.14) [72]. The model depends on the local void fraction ε_g and the conductivity ratio $\lambda = k_g/k_s$, both of which vary with height in the conical reactor due to the axial velocity gradient [72, 61].

For the porous γ -alumina particles used here, the internal porosity reduces k_s below the bulk dense alumina value, so the Zehner–Schlünder model predicts a relatively low $k_{e,0}$ for the dense emulsion phase [36, 72]. The practical consequence is increased thermal resistance on the solid-side conduction pathway relative to non-porous particles [10]. Any underestimation of lateral thermal diffusion that results from this could produce artificial localisation of temperature gradients near the nozzle inlets in the simulation results.

ChatGPT was used to generate L^AT_EX code for equations, and reference formatting. It was also used for grammar and spelling corrections

Multiphase Modelling in ANSYS Fluent

The gas–solid fluidisation dynamics were modelled using the Eulerian–Eulerian Two-Fluid Model (TFM) in *ANSYS Fluent 2024R2*. In this framework both phases are treated as interpenetrating continua, each governed by its own conservation equations for mass, momentum, and energy. The sections below describe the computational mesh, material properties, boundary conditions, constitutive models, and initialisation strategy applied throughout the study.

3.1. Description of the System

3.1.1. Mesh Description

The computational mesh used here is the same structured hexahedral grid previously established in the hydrodynamic study by Roy [59], carried over unchanged into the present heat transfer simulations. Reusing this mesh eliminates any interpolation error that would arise from mapping fields between non-conforming grids, which is a non-trivial concern when initialising a coupled thermal calculation from a hydrodynamic steady state.

A structured hexahedral topology was chosen, and this is worth a brief justification. Hexahedral elements aligned with the flow direction are known to produce lower numerical diffusion than tetrahedral cells for the same node count [27, 41], which matters here because the TFM involves multiple coupled transport equations where accumulation of discretisation error across phases is undesirable. The structured O-grid topology also allows anisotropic refinement without mesh quality degradation, which is useful for mesh sensitivity studies. These are not merely theoretical advantages: the mesh quality metrics confirm that the discretisation meets the thresholds recommended by ANSYS Fluent [4], with a minimum orthogonal quality of 0.75 and a maximum aspect ratio of 4.49.

The overall layout of the computational domain is shown in Figure 3.1. The reactor consists of a single outlet at the top and three nozzle inlets positioned symmetrically in the lower conical section (Figure 3.2). Each nozzle introduces fluid with both an azimuthal component (Figure 3.2a) and an inclination component (Figure 3.2b). The azimuthal positions of the three inlets are 30° , 150° , and 270° , measured from the positive x -axis in the horizontal xz -plane. The downward tilt of each nozzle is governed by the inclination angle θ .

For the baseline simulations $\theta = 6.7^\circ$, matching the geometry of the physical reactor. Additional runs at $\theta = 20^\circ$ were performed to replicate the nozzle orientation characteristic of larger experimental configurations. The inlet velocity components for both geometries were computed from the coordinate transformation formulations in Section A.1.

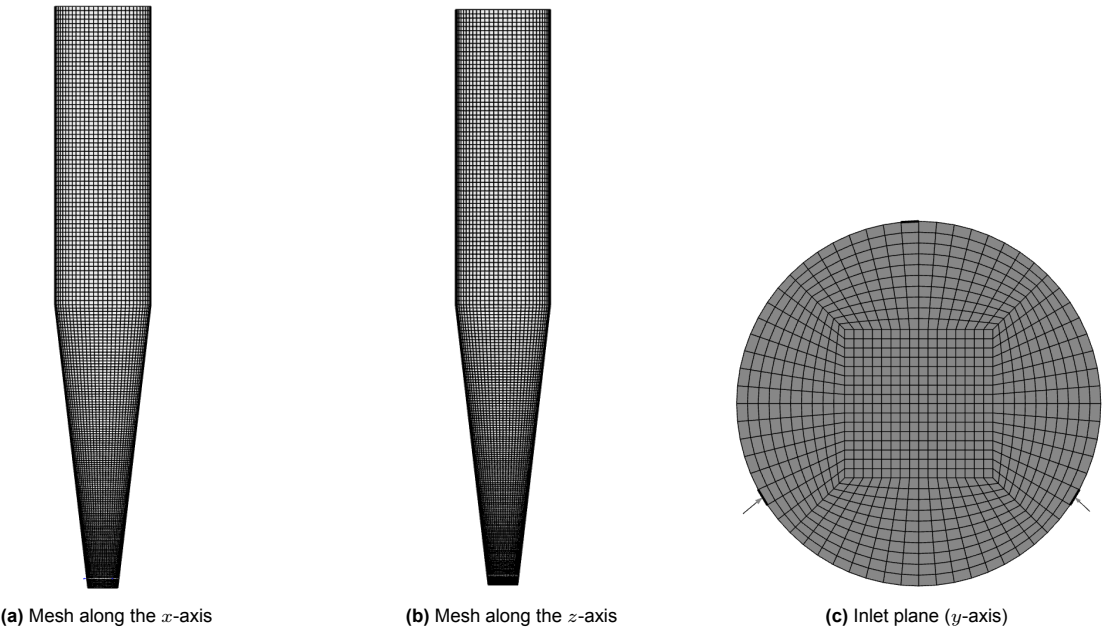


Figure 3.1: Structured hexahedral mesh shown at three orientations.

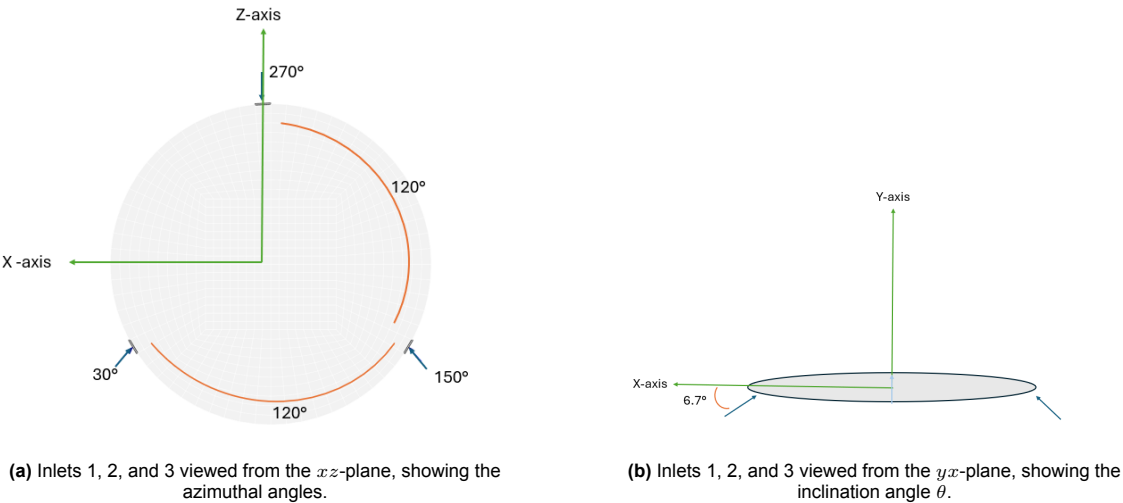


Figure 3.2: Geometry of the three gas inlets.

3.1.2. Mesh Dimensions

The key mesh parameters and quality indicators are collected in Table 3.1.

Table 3.1: Mesh parameters, geometric dimensions, and grid quality indicators for the computational domain.

Category	Parameter	Value	Unit
Grid statistics	Total cell count	188,160	–
	Conical region cell count	97,664	–
	Cell type	Hexahedral	–
	Grid topology	O-grid	–
Mesh quality	Minimum orthogonal quality	0.75	–
	Maximum aspect ratio	4.49	–
Inlet / outlet	Number of inlets	3	–
	Number of outlets	1	–
	Area of each inlet face	2.105×10^{-5}	m ²
Geometry	Axial height of inlet plane	0.0325	m
	Height of conical section	0.950	m
	Cone half-angle, $\alpha/2$	6.7	°
	Nozzle inclination, θ	6.7	°
	Radius at cone apex, r_0	0.050	m
	Radius at inlet plane	0.054	m
	Freeboard radius	0.160	m
	Wall thickness	2×10^{-3}	m

3.1.3. Material Properties

Both phases are defined as fluid materials within ANSYS Fluent R2024 [4], consistent with the Eulerian continuum treatment. The solid phase is porous alumina with constant thermophysical properties; the gas phase is nitrogen with temperature-dependent property models to account for the property variation across the operating temperature range of 289–393 K. The material properties of both phases are summarised in Tables 3.2 and 3.3; the gas-phase model equations are described in Section 3.1.3.

Table 3.2: Thermophysical properties of the solid phase (porous alumina, constant).

Property	Symbol	Value	Unit
Density	ρ_s	1372	kg m ⁻³
Specific heat	$C_{p,s}$	742.096	J (kg K) ⁻¹
Thermal conductivity	k_s	7.36	W (m K) ⁻¹
Viscosity	μ_s	0.3	kg (m s) ⁻¹
Molecular weight	M_s	28.966	kg kmol ⁻¹
Standard state enthalpy	ΔH_s	0	J kg ⁻¹
Reference temperature	$T_{\text{ref},s}$	298.15	K

Table 3.3: Thermophysical properties of the gas phase (nitrogen, N₂). Temperature-dependent properties are indicated; governing equations are in Section 3.1.3.

Property	Symbol	Value / Model	Unit	Model type
Density	ρ_g	Eq. (3.1)	kg m ⁻³	Incompressible ideal gas
Specific heat	$C_{p,g}$	Eq. (3.2)	J (kg K) ⁻¹	Piecewise polynomial
Thermal conductivity	k_g	0.0242	W (m K) ⁻¹	Constant
Viscosity	μ_g	Eq. (3.4)	kg (m s) ⁻¹	Sutherland's law
Molecular weight	M_g	28.0134	kg kmol ⁻¹	Constant
Standard state enthalpy	ΔH_g	0	J kg ⁻¹	Constant
Reference temperature	$T_{\text{ref},g}$	298.15	K	Constant

Given the 104 K temperature range of the simulations, treating the nitrogen properties as constant would introduce non-negligible errors in the energy equation, particularly for density and viscosity. The following subsections describe the temperature-dependent models selected for each property.

Density: Incompressible Ideal Gas Law

The nitrogen density is computed from the incompressible ideal gas law as implemented in ANSYS Fluent [4]:

$$\rho_g = \frac{p_{\text{op}} M_g}{R T} \quad (3.1)$$

where p_{op} is the operating pressure [Pa], M_g is the molecular weight [kg kmol⁻¹], $R = 8314 \text{ J kmol}^{-1} \text{ K}^{-1}$ is the universal gas constant, and T is the local temperature [K]. This formulation is appropriate for low-Mach-number flows where dynamic pressure variations are negligible: density varies by less than 5% when $\text{Ma} < 0.3$ [5], which is well satisfied at the nozzle velocities of 40–56 m s⁻¹ used here. The key advantage over the compressible ideal gas law is that density is decoupled from local pressure fluctuations, improving numerical stability without sacrificing accuracy at these conditions.

Specific Heat Capacity: Piecewise-Polynomial Fit

The specific heat of nitrogen varies with temperature and cannot be treated as constant without introducing errors in the enthalpy balance. $C_{p,g}$ is therefore represented by a piecewise polynomial consistent with the NASA thermodynamic database [45]:

$$C_{p,g}(T) = \sum_{n=0}^N a_n T^n \quad (3.2)$$

The fit uses two temperature ranges with a transition at 1000 K. For the range 300–1000 K relevant to the present simulations:

$$C_{p,g}(T) = 979.043 + 0.4180 T - 1.176 \times 10^{-3} T^2 + 1.674 \times 10^{-6} T^3 - 7.256 \times 10^{-10} T^4 \quad (3.3)$$

These mass-based coefficients are derived from the NASA seven-term polynomial fit of McBride, Gordon, and Reno [45] by multiplying through by R and dividing by $M_g = 28.0134 \text{ kg kmol}^{-1}$. The resulting $C_{p,g}$ increases monotonically from $\approx 1040 \text{ J (kg K)}^{-1}$ at 300 K, reflecting the progressive excitation of vibrational modes in the diatomic N₂ molecule [42].

Dynamic Viscosity: Sutherland's Law

The dynamic viscosity is described using Sutherland's law [65], a semi-empirical relation derived from kinetic theory that provides accurate results over wide temperature ranges with errors of only a few percent [12]:

$$\mu(T) = \mu_0 \left(\frac{T}{T_0} \right)^{3/2} \frac{T_0 + S}{T + S} \quad (3.4)$$

where μ_0 is the reference viscosity at reference temperature T_0 , and S is the Sutherland constant. The coefficients for nitrogen are listed in Table 3.4.

Table 3.4: Sutherland's law coefficients for nitrogen as specified in ANSYS Fluent R2024 [4].

Parameter	Symbol	Value	Unit
Reference viscosity	μ_0	1.735×10^{-5}	kg (m s)^{-1}
Reference temperature	T_0	289	K
Sutherland constant	S	111	K

As expected from kinetic theory, the dynamic viscosity increases monotonically with temperature, reflecting greater molecular momentum exchange at higher thermal energies.

3.1.4. Wall Material Properties

The reactor wall is stainless steel (AISI 304) with constant thermophysical properties, defined as a separate solid zone within the solver. These properties govern the conductive heat transfer through the wall as described in Section 3.3.1. The assigned values (Table 3.5) are consistent with standard reference data for austenitic stainless steel at temperatures in the range 300–500 K [8].

Table 3.5: Thermophysical properties of the reactor wall (stainless steel AISI 304, constant).

Property	Symbol	Value	Unit
Density	ρ_w	7930	kg m^{-3}
Specific heat	$C_{p,w}$	510.79	J (kg K)^{-1}
Thermal conductivity	k_w	16.2	W (m K)^{-1}
Wall thickness	δ_w	0.002	m

3.2. Boundary Conditions

The velocity inlet conditions for the three nozzles at both flow rates are given in Table 3.6, and the three thermal configurations are defined in Table 3.7. The velocity magnitude and direction components were computed from the nozzle geometry according to Section A.1. The direction unit vectors are identical at both flow rates, they depend only on the nozzle geometry, not the flow rate, so only the velocity magnitude changes between 50 and 67 lpm.

Table 3.6: Velocity inlet boundary conditions for the three nozzle inlets at 50 lpm and 67 lpm. Gas phase: nitrogen (N_2), absolute Cartesian frame. Direction components are dimensionless unit vector components.

Flow rate	Inlet	$ \mathbf{u} \text{ [m s}^{-1}\text{]}$	$\hat{u}_x$	$\hat{u}_y$	$\hat{u}_z$
50 lpm	lowerinlet1	40.16	−0.8601	0.1167	0.4966
	lowerinlet2	39.52	0.8601	0.1167	0.4966
	lowerinlet3	41.73	0.0000	0.1167	−0.9932
67 lpm	lowerinlet1	53.43	−0.8601	0.1167	0.4966
	lowerinlet2	52.95	0.8601	0.1167	0.4966
	lowerinlet3	55.92	0.0000	0.1167	−0.9932

Table 3.7: Thermal boundary condition configurations for the three simulation cases. Each case was run at both flow rates, giving six simulations in total. All cases initialised at $T_0 = 289.15$ K.

Case	Description	Wall temperature	Inlet gas temperature
G	Gas preheating only	16 °C (289.15 K)	120 °C (393.15 K)
W	Wall heating only	120 °C (393.15 K)	16 °C (289.15 K)
C	Combined heating	120 °C (393.15 K)	120 °C (393.15 K)

3.3. Computational Model Configuration

The model settings applied in all simulations are summarised in Table 3.8. Closures marked with an asterisk (*) were implemented via User-Defined Functions (UDFs), because the default solver does not natively support the kinetic-theory-based constitutive relations required for the granular phase or the adaptive interphase heat transfer correlations described in Section 3.3.1.

Table 3.8: Computational model settings for the Eulerian–Eulerian two-fluid simulations.

Setting	Value / Description
<i>Flow model</i>	
Viscous formulation	Laminar
Multiphase approach	Eulerian–Eulerian (Two-Fluid Model)
Volume fraction formulation	Implicit
<i>Phase definition</i>	
Primary phase	Gas (nitrogen, N ₂)
Secondary phase	Solid (porous alumina, granular)
Granular temperature model	Partial differential equation
Particle diameter	2.02×10^{-4} m
<i>Granular constitutive relations (KTGF)</i>	
Granular viscosity	Gobin et al. [26]*
Granular bulk viscosity	Gidaspow [25]*
Solids pressure	Ding & Gidaspow [25]*
Granular conductivity	Gobin et al. [26]*
Radial distribution	Lun & Savage [40]*
Packing limit	0.64
<i>Phase interaction</i>	
Gas–solid drag	Gobin et al. [26]*
Restitution coefficient	0.9
<i>Heat transfer</i>	
Gas–solid exchange	Ranz–Marshall [53]* / Gunn [30]* (adaptive)
Wall heat flux	Fourier’s law [8]*

*Implemented via UDF compiled in ANSYS Fluent R2024 [4].

3.3.1. Heat Transfer User-Defined Function

The default Eulerian–Eulerian heat transfer implementation in ANSYS Fluent R2024 does not natively support: (i) adaptive switching between two interphase Nusselt correlations based on local void fraction,

or (ii) a cell-geometry-aware wall heat flux model applied to the gas phase of a two-fluid system. A UDF was therefore written and compiled within the solver to handle both. It consists of three coupled sub-models, adaptive interphase heat transfer, near-wall boundary layer thickness, and wall heat flux, described below. The complete source code is in Appendix A.4.

User-Defined Memory Allocation

Seven UDM slots are allocated to store locally computed thermal quantities at each cell centroid. These serve two purposes: passing intermediate results between the three UDF macros across successive solver calls, and providing cell-level diagnostic fields for post-processing. The slots are listed in Table 3.9.

Table 3.9: User-Defined Memory (UDM) slots allocated for inter-function communication and diagnostics.

Index	Identifier	Symbol	Description
0	UDM_RE_SLIP	Re_{slip}	Slip Reynolds number
1	UDM_PR_GAS	Pr_g	Gas-phase Prandtl number
2	UDM_NU_USED	Nu	Nusselt number from active correlation
3	UDM_H_GS	h_{gs}	Gas–solid HTC [$\text{W m}^{-2} \text{K}^{-1}$]
4	UDM_DELTA_T	δ_T	Near-wall thermal boundary layer [m]
5	UDM_Q_WALL	$\dot{q}_w$	Wall heat flux [W m^{-2}]
6	UDM_MODEL_ID	–	Correlation flag (0 = Ranz–Marshall, 1 = Gunn)

Slot 6 (UDM_MODEL_ID) is particularly useful during post-processing: visualising it as a contour plot directly shows which regions of the reactor are governed by each correlation at any given instant, providing a spatial check that the switching criterion (Section 3.3.1) is behaving as physically expected.

Adaptive Interphase Heat Transfer Coefficient

The interphase heat transfer coefficient is computed via `DEFINE_EXCHANGE_PROPERTY`, which is invoked at each iteration. At each call, the function retrieves the local gas-phase properties ($\rho_g, \mu_g, k_g, C_{p,g}$) from the cell state using native Fluent macros, ensuring full consistency with the temperature-dependent models of Section 3.1.3. The gas–solid slip velocity is then:

$$|\mathbf{u}_g - \mathbf{u}_s| = \sqrt{(u_g - u_s)^2 + (v_g - v_s)^2 + (w_g - w_s)^2} \quad (3.5)$$

from which $Re_p = \rho_g |\mathbf{u}_g - \mathbf{u}_s| d_p / \mu_g$ and $Pr_g = \mu_g C_{p,g} / k_g$ are evaluated. The local gas volume fraction ε_g is retrieved and clamped to the valid range $[0.35, 1.0]$ of the Gunn correlation before use [30]. The heat transfer coefficient follows from:

$$h_{gs} = \frac{Nu \cdot k_g}{d_p} \quad (3.6)$$

A numerical floor of $\varepsilon = 10^{-12}$ is applied to all intensive properties to prevent division-by-zero in degenerate cells. Diagnostic quantities are stored in the corresponding UDM slots.

Adaptive Correlation Switching Criterion

The switching logic follows the physical argument in Section 2.2: the Ranz–Marshall correlation is appropriate only in the dilute limit where inter-particle interactions are negligible [53], while Gunn applies in the dense and intermediate void fraction regions [30]. A void fraction threshold of $\varepsilon^* = 0.8$ is adopted. This is a practical choice: the theoretically rigorous dilute limit ($\varepsilon_g \rightarrow 1$) is only realised in negligibly small regions of a bubbling bed simulation, so using a threshold of 0.99 would in practice activate the Ranz–Marshall correlation almost nowhere. The value 0.8 more meaningfully separates the dilute and dense flow regimes, is consistent with bubble-phase void fractions ($\varepsilon_g \approx 0.85\text{--}0.95$) typical of the present operating conditions, and aligns with values reported in the fluidised bed literature for this boundary [25]. The criterion implemented is:

$$Nu = \begin{cases} Nu_{\text{RM}} & \text{if } \varepsilon_g > 0.8 \quad (\text{dilute: Ranz–Marshall}) \\ Nu_{\text{Gunn}} & \text{if } \varepsilon_g \leq 0.8 \quad (\text{dense/intermediate: Gunn}) \end{cases} \quad (3.7)$$

Since both correlations recover $Nu_p = 2$ as $Re_p \rightarrow 0$ and $\varepsilon_g \rightarrow 1$ [30, 53], there is no discontinuity at the switching point. The sensitivity of results to the choice of ε^* is acknowledged as a modelling uncertainty and is recommended for future parametric investigation.

Near-Wall Thermal Boundary Layer Thickness

A `DEFINE_ADJUST` function runs once per iteration, after the flow solution is updated, to compute the local thermal boundary layer thickness δ_T at each fluid cell adjacent to a wall. In the laminar framework employed here, no turbulent wall function is active, so near-wall heat transfer is governed by molecular conduction across the gas layer between the wall and the first interior cell [4]. The thickness of this conduction layer is approximated as the wall-normal distance from the wall face centroid to the adjacent cell centroid:

$$\delta_T = |(\mathbf{x}_c - \mathbf{x}_f) \cdot \hat{\mathbf{n}}_f| \quad (3.8)$$

where $\mathbf{x}_c$ and $\mathbf{x}_f$ are the cell and face centroid position vectors, and $\hat{\mathbf{n}}_f$ is the unit outward normal. A floor of 10^{-12} is applied to the result, which is stored in `UDM_DELTA_T`. Computing δ_T geometrically from the mesh, rather than using a fixed global length scale, means the wall heat flux model adapts naturally to local cell sizes, which is important for a mesh of non-uniform refinement.

Wall Heat Flux Boundary Condition

The wall heat flux is prescribed via a `DEFINE_PROFILE` function called at each iteration. A fixed wall temperature $T_w = 393.15$ K is imposed for the wall heating cases. The local heat flux at each wall face is computed from the discrete form of Fourier's law [8]:

$$\dot{q}_w = k_g \frac{T_w - T_{\text{cell}}}{\delta_T} \quad (3.9)$$

where T_{cell} is the gas-phase temperature at the adjacent cell centroid, k_g is the local gas thermal conductivity, and δ_T is from Section 3.3.1. A positive flux indicates heat transfer from wall to fluid. The computed flux is assigned via `F_PROFILE` and stored in `UDM_Q_WALL` for post-processing.

3.4. Solution Methods

The solution methods used for the simulations to conclude this theses are similar to the ones use in [59]: The key solver settings adopted in this work are summarised in Table 3.10. Pressure-velocity coupling is handled by the Phase-Coupled SIMPLE (PC-SIMPLE) scheme, which is well established for Eulerian multiphase flows [ref77]. Pressure interpolation uses the PRESTO! scheme, the recommended default for multiphase configurations in ANSYS Fluent [ref75]. All spatial discretization schemes, including momentum, volume fraction, and granular temperature, are set to first-order upwind, and the transient formulation is first-order implicit. These choices prioritise convergence speed over accuracy, which is appropriate for the baseline simulations presented here. The influence of higher-order schemes is assessed in Chapters 5 and 6.

Table 3.10: Solver and discretization settings.

Setting	Selection
Pressure-Velocity Coupling	Phase-Coupled SIMPLE
Gradient	Green-Gauss Cell-Based
Pressure	PRESTO!
Momentum	First-Order Upwind
Volume Fraction	First-Order Upwind
Granular Temperature	First-Order Upwind
Transient Formulation	First-Order Implicit

The simulation time step is set to $\Delta t = 1.78 \times 10^{-5}$ s, derived from the CFL condition (Equation 3.10) using a minimum cell width of 0.0011 m and a maximum expected velocity of approximately 70 m/s, corresponding to $2.5 \times u_{mf}$ at the orifice.

$$\text{CFL} = \frac{\Delta t \cdot u_x}{\Delta x} + \frac{\Delta t \cdot u_y}{\Delta y} + \frac{\Delta t \cdot u_z}{\Delta z} \quad (3.10)$$

3.5. Initialisation

All simulations were initialised using the hybrid initialisation approach in ANSYS Fluent R2024 [4]. This was preferred over standard initialisation, which requires manual specification of all field variables and is poorly suited to geometrically complex systems like fluidised beds where initial flow and pressure fields are non-trivial. The hybrid initialiser solves the Laplace equation to produce a velocity field conforming to the domain geometry, together with a pressure field that smoothly interpolates between the inlet and outlet regions. Remaining variables are set from domain-averaged values over a small number of preparatory iterations.

One practical consequence of this approach in the Eulerian multiphase framework is that all secondary (solid) phase variables, including the solid volume fraction, are automatically initialised to zero throughout the domain. The physically correct initial bed must therefore be imposed separately through the patching procedure described below.

3.6. Patching of Initial Conditions

After hybrid initialisation, the Fluent patching function was used to set physically representative initial conditions for the solid volume fraction. Patching allows specific values of any solution variable to be assigned to user-defined cell zones independently of the globally initialised field [4], and is essential for fluidised bed simulations where the physical starting point is a quiescent, gravity-settled bed occupying a defined region below a filling height.

Two cell zones were defined:

- **Full region:** the entire computational domain, including the conical bed section and the freeboard above it.
- **Bed region:** the portion of the conical section below the static filling height of $h = 644$ mm, measured from the base of the cone.

The patching was applied in two steps. First, the full region was patched with $\varepsilon_s = 0$, clearing any residual solid fraction and ensuring the freeboard is particle-free. Second, the bed region was patched with $\varepsilon_s = 0.59$, representing a moderately packed initial bed consistent with the physical filling procedure used experimentally.

The choice of 0.59 is subject to some uncertainty, as initial packing depends on filling speed, particle size distribution, and mechanical vibrations, all of which influence the degree of compaction [35]. Values reported in the literature for similar systems range from $\varepsilon_s = 0.48$ to 0.60 [26, 25], so 0.59 lies near the upper end of this range and is consistent with the approach of Roy [59] for the same reactor geometry. The solid-phase temperature is simultaneously patched to $T_s = 289.15$ K throughout the bed region, consistent with the uniform initial thermal state applied to the gas phase.

Declaration of AI: ChatGPT was used to generate L^AT_EX code for tables, equations, and reference formatting. It was also used for grammar and spelling corrections

4

Results and Discussion

4.0.1. Preliminary Note on Simulation Configuration

This section covers the gas preheating case at 50 lpm, where nitrogen is supplied at $T_{\text{in}} = 393.15 \text{ K}$ while the reactor wall is kept adiabatic. Because there is no wall heat flux, all energy input to the bed comes from gas–solid interphase exchange; the wall plays no thermal role whatsoever in these results.

One thing worth flagging before going into the data: the interphase heat transfer coefficient here is computed using the built-in Gunn correlation in ANSYS Fluent 2024 R2, applied uniformly over the full void-fraction range, rather than the adaptive UDF switching between Gunn and Ranz–Marshall described in Section 3.3.1, yet it is not possible to extract the interphase heat transfer coefficient from fluent. In practice this difference is small, both correlations recover the same limiting value as $\varepsilon_g \rightarrow 1$ [29, 52], so the near-nozzle dilute region is unlikely to be significantly affected, but it should be kept in mind when comparing these results against the UDF runs discussed later in Section ??.

Results are presented at $t = 0.5, 1.1, 1.8, 2.5,$ and 4.0 s , tracking the bed from the very first moments of jet penetration through to what looks like the onset of a quasi-periodic bubbling regime. At each snapshot, three sets of post-processing data are examined: temperature contours at the horizontal inlet plane ($y = 32.5 \text{ mm}$), solid volume fraction fields at the inlet plane, on a vertical axial slice, and on horizontal cross-sections at $y = 32.5, 100, 200, 300, 400, 500, 600 \text{ mm}$, and axial temperature profiles along the three jet trajectories.

4.1. Gas Heating Operation, Case G, 50 lpm

4.1.1. $t = 0.5 \text{ s}$

Temperature field at the inlet plane

The inlet-plane temperature fields at $t = 0.5 \text{ s}$ are shown in Figure 4.1. The solid temperature (figure 4.1a) shows a quasi-uniform background of approximately 300 K across most of the cross-section, with three small isolated hot spots at the nozzle exits reaching $T_{s,\text{max}} \approx 310 \text{ K}$, a local rise of about 10 K above the background. The footprints are slightly elliptical rather than circular, due to the nozzle inclination angle $\theta = 6.7^\circ$: the jet enters at a shallow angle and the heated zone is elongated radially inward. The three plumes are completely independent at this stage, consistent with the jets not yet having had time to interact hydrodynamically. Strong localised temperature gradients exist near the jet region; no meaningful gradient is present elsewhere at the inlet plane.

The jet penetration depth, defined by the axial position of maximum gas volume fraction, is approximately 50–60 mm (as shown in Appendix B), as indicated by the void fraction data. Temperature maxima in the solid phase coincide with this near-nozzle region, indicating that heat transfer is primarily concentrated within the jet core region where gas–solid interaction is strongest.

The gas temperature (figure 4.1b, right) tells an interesting story on its own. Although nitrogen is being injected at $T_{\text{in}} = 393.15 \text{ K}$, the gas at the inlet plane ($y = 32.5 \text{ mm}$) peaks at only $T_{g,\text{max}} \approx 330\text{--}337 \text{ K}$, a drop of roughly 56–63 K over the first 32.5 mm of bed penetration, corresponding to approximately 160

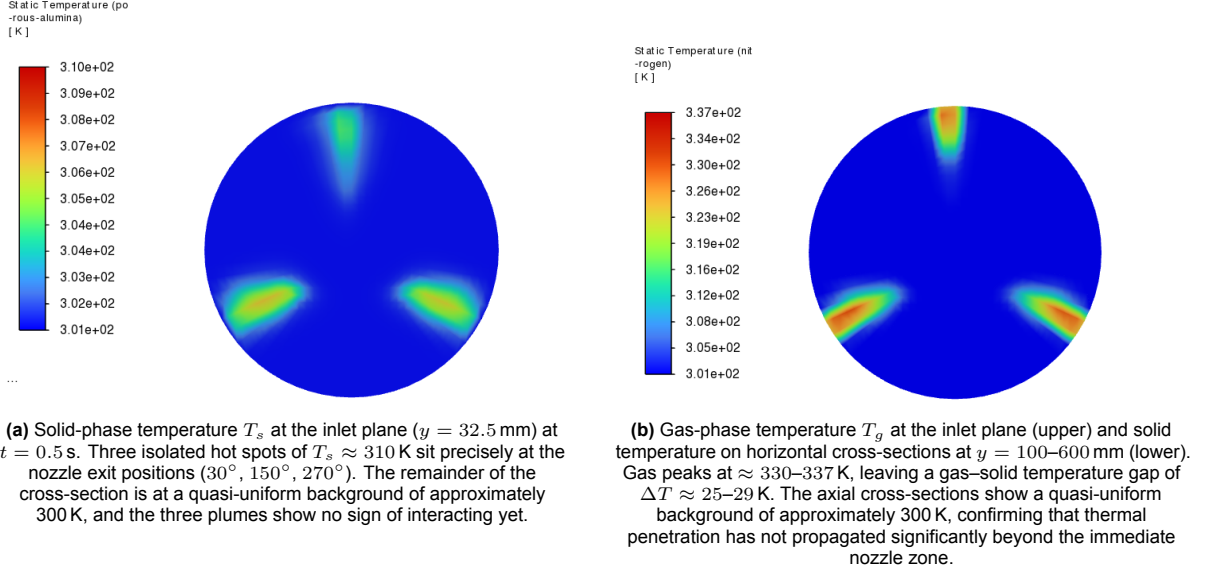


Figure 4.1: Temperature distributions at $t = 0.5$ s, Case G (gas preheating), 50 lpm, built-in Gunn correlation, adiabatic wall.

particle diameters. The reason for this rapid temperature decrease with distance from the jet core is the large mismatch in volumetric heat capacity between the two phases. At $T = 393$ K, $\rho_g \approx 0.87$ kg m $^{-3}$ from Eq. 3.1, giving:

$$\left. \frac{(\rho C_p)_s}{(\rho C_p)_g} \right|_{T=393 \text{ K}} = \frac{1372 \times 742}{0.87 \times 1040} \approx 1125. \quad (4.1)$$

Even a relatively small transfer of energy from gas to solid causes a large temperature decrease with distance from the jet core in the gas phase, while barely affecting the solid. the gas temperature decreases rapidly toward the local solid temperature within a short axial distances; the solid, with its much greater thermal mass, responds on a far longer time scale.

That said, the ≈ 25 – 29 K gap between gas and solid at the nozzle cores does not close within this first 60 mm, which at first glance might seem at odds with the high exchange rates predicted by the Gunn correlation. The apparent contradiction is resolved by looking at the interfacial area. With the particle Reynolds number $Re_p = \rho_g |\mathbf{u}_g - \mathbf{u}_s| d_p / \mu_g$ evaluated at nozzle conditions:

$$Re_p \approx \frac{0.87 \times 40 \times 2.02 \times 10^{-4}}{2.0 \times 10^{-5}} \approx 352, \quad (4.2)$$

giving $Nu_p \approx 38$ and $h_{gs} \approx 4560$ W m $^{-2}$ K $^{-1}$ at $\varepsilon_g \approx 0.8$, $Pr_g \approx 0.72$. But h_{gs} alone is not the relevant quantity: the volumetric exchange rate is $h_{gs} a_s$, and the specific interfacial area $a_s = 6(1 - \varepsilon_g)/d_p$ (Eq. 2.10) is only ≈ 5940 m $^{-1}$ inside the dilute jet core at $\varepsilon_g = 0.8$, compared with $\approx 17,300$ m $^{-1}$ in the dense emulsion where $\varepsilon_g \approx 0.41$. The jet core is therefore a rate-limited zone: high h_{gs} but poor interfacial contact. This competition is what keeps the ≈ 25 – 29 K gas–solid gap alive.

Void fraction structure

Figure 4.2b shows that the regions directly at the nozzle exits are strongly gas-dominated, with a reduced solid volume fraction ($\varepsilon_s \approx 0.12$ – 0.18) compared to the surrounding bed ($\varepsilon_s \approx 0.59$). This local depletion of porous alumina explains the elevated apparent solid temperature at the nozzle locations, which is influenced by the gas phase in these dilute regions.

At $t = 0.5$ s, the system remains in the very early stages of hydrodynamic development. The vertical slice (figure 4.2a) shows that the dense initial packing is largely preserved from the distributor up to the bed surface, with only minor local disturbances near the nozzle region caused by the incoming jets. These disturbances appear as three narrow gas channels emerging from the nozzles, with no coherent or fully developed void structures propagating upward.

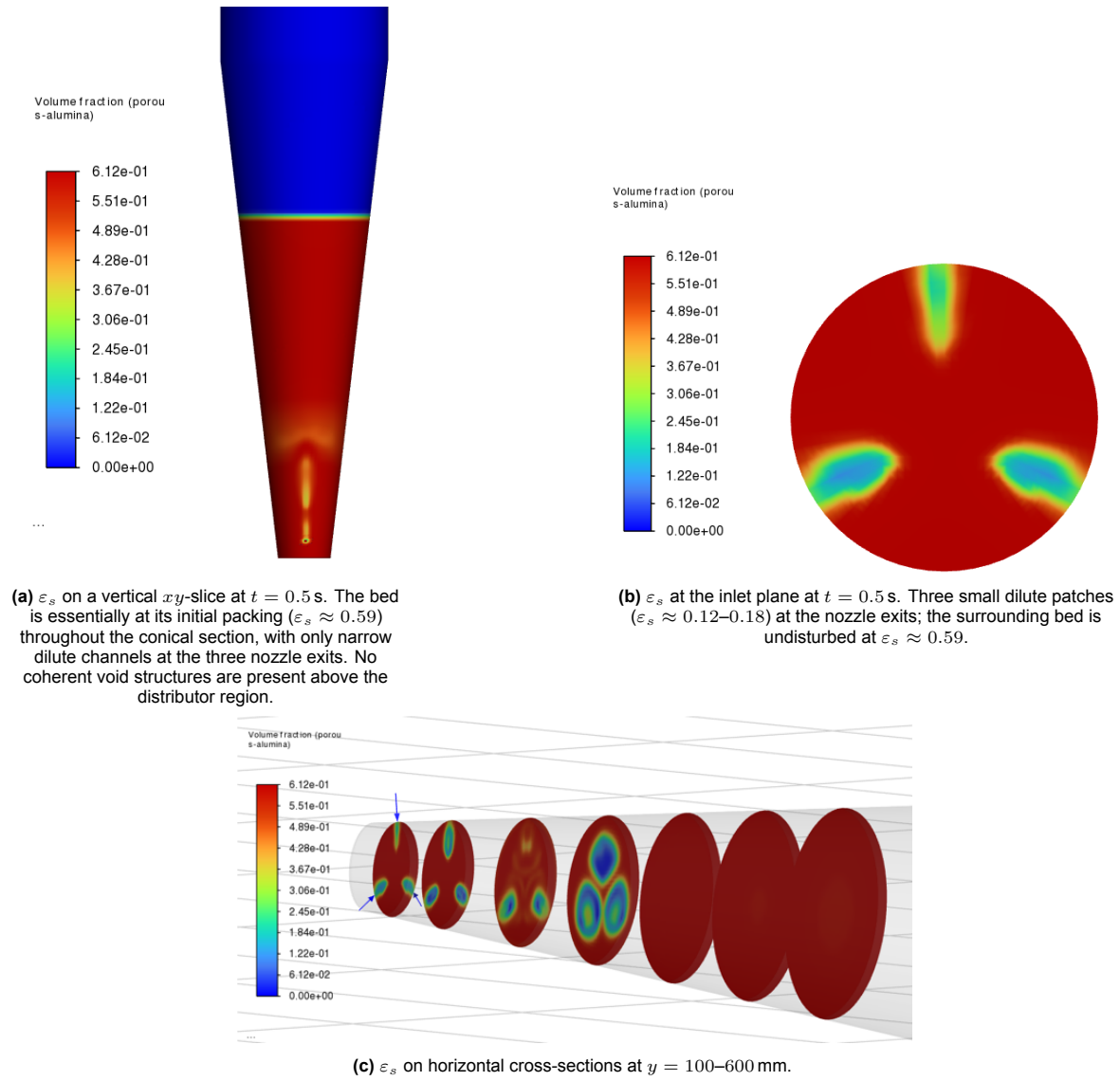
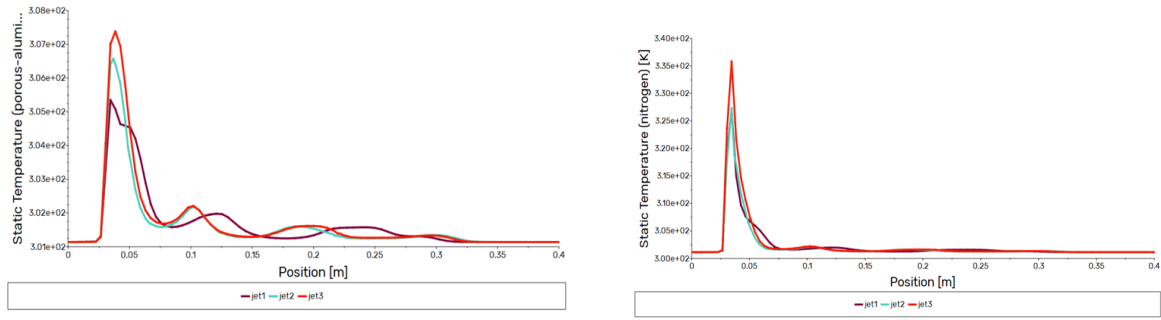


Figure 4.2: Void fraction at $t = 0.5$ s, Case G, 50 lpm.

This behaviour is further supported by the horizontal cross-sections (figure 4.2c), which show that all planes above approximately $y \approx 300$ mm remain uniformly dense. The dilute regions remain confined to the lower part of the bed, indicating that the jets have not yet penetrated significantly into the upper sections.

The heat transfer coefficient remains effectively zero throughout the bed, as expected under the imposed adiabatic and isothermal conditions.

Axial temperature profiles



(a) Axial solid-phase temperature T_s along the three jet trajectories at $t = 0.5$ s, Case G, 50 lpm. Each jet shows a primary spike at $y \approx 30$ mm; weak secondary perturbations may extend slightly beyond this, but the profiles rapidly return toward the background of approximately 300–302 K within ≈ 300 mm. Noticeable differences between the three jet lines indicate early-stage asymmetry in the thermal field.

(b) Axial gas-phase temperature T_g along the three jet trajectories at $t = 0.5$ s, Case G, 50 lpm. A sharp spike at $y \approx 30$ mm peaks at $T_g \approx 335$ K and decays rapidly. Above $y \approx 50$ mm the profiles return toward the quasi-uniform background of approximately 300–302 K.

Figure 4.3: Axial temperature profiles at $t = 0.5$ s, Case G, 50 lpm.

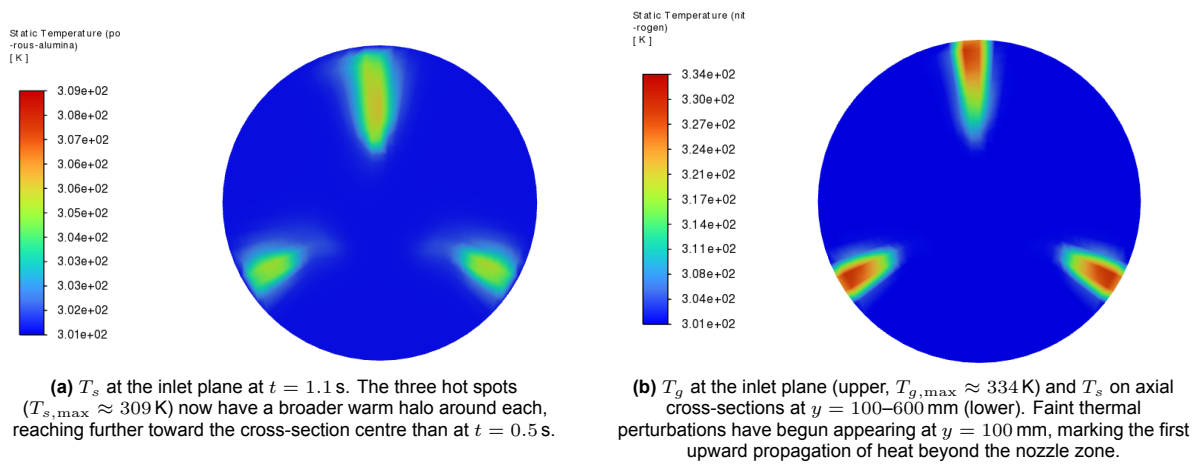
The axial temperature profiles along the jet-aligned lines (Figure 4.3) clearly reflect the jet-dominated nature of this early stage. The gas profiles (figure 4.3b) exhibit a single sharp peak near the nozzle, reaching approximately 330–337 K, followed by a rapid decrease with increasing height. Beyond $y \approx 50$ mm, the gas temperature approaches a nearly uniform background of 300–302 K, indicating fast thermal equilibration of the gas phase.

In contrast, the solid temperature profiles (figure 4.3a) show a first peak at $y \approx 30$ –50 mm, corresponding to the near-nozzle region where gas–solid interaction is strongest. As the height increases, the solid temperature gradually decreases toward the background value. This behaviour is consistent with the void fraction distribution: the most dilute regions, where gas volume fraction is highest, are concentrated near the nozzle, while further up the bed the solid phase becomes more uniformly packed.

Weak secondary oscillations observed in the solid temperature profiles can be explained with local heterogeneities in the void fraction field, reflecting small-scale variations in gas–solid contact. These perturbations are reduced with height, and the profiles return to the background temperature by approximately $y \approx 300$ –350 mm, indicating that the influence of the jets remains confined to the lower portion of the bed at this stage.

4.1.2. $t = 1.1$ s

Temperature field at the inlet plane



(a) T_s at the inlet plane at $t = 1.1$ s. The three hot spots ($T_{s,\max} \approx 309$ K) now have a broader warm halo around each, reaching further toward the cross-section centre than at $t = 0.5$ s.

(b) T_g at the inlet plane (upper, $T_{g,\max} \approx 334$ K) and T_s on axial cross-sections at $y = 100$ –600 mm (lower). Faint thermal perturbations have begun appearing at $y = 100$ mm, marking the first upward propagation of heat beyond the nozzle zone.

Figure 4.4: Temperature distributions at $t = 1.1$ s, Case G, 50 lpm.

By $t = 1.1$ s, the overall picture at the inlet plane has not changed significantly (figure 4.4a). The three hot spots are still clearly visible, with peak temperatures remaining around ≈ 305 – 307 K. However, compared to $t = 0.5$ s, each hot spot is now surrounded by a noticeably broader warm halo that extends further toward the centre of the cross-section. This indicates that heat is beginning to spread radially away from the nozzle regions, although the three jets remain clearly separated and do not interact.

The slight drop in peak temperature compared to $t = 0.5$ s, together with the widening of the halos, is physically consistent. As the flow develops, mixing becomes more effective, and the heat introduced near the nozzle exits is redistributed over a larger volume of particles. As a result, the local temperature peaks decrease slightly, even though the overall energy in the system increases.

The gas phase still shows a strongly jet-dominated pattern. The temperature peaks at the nozzle exits remain sharp, at approximately ≈ 334 – 337 K, and remain highly localised within the inlet plane. Although jet along z -direction seems more elongated than the next presenting asymmetry

Void fraction structure and incipient bubble formation

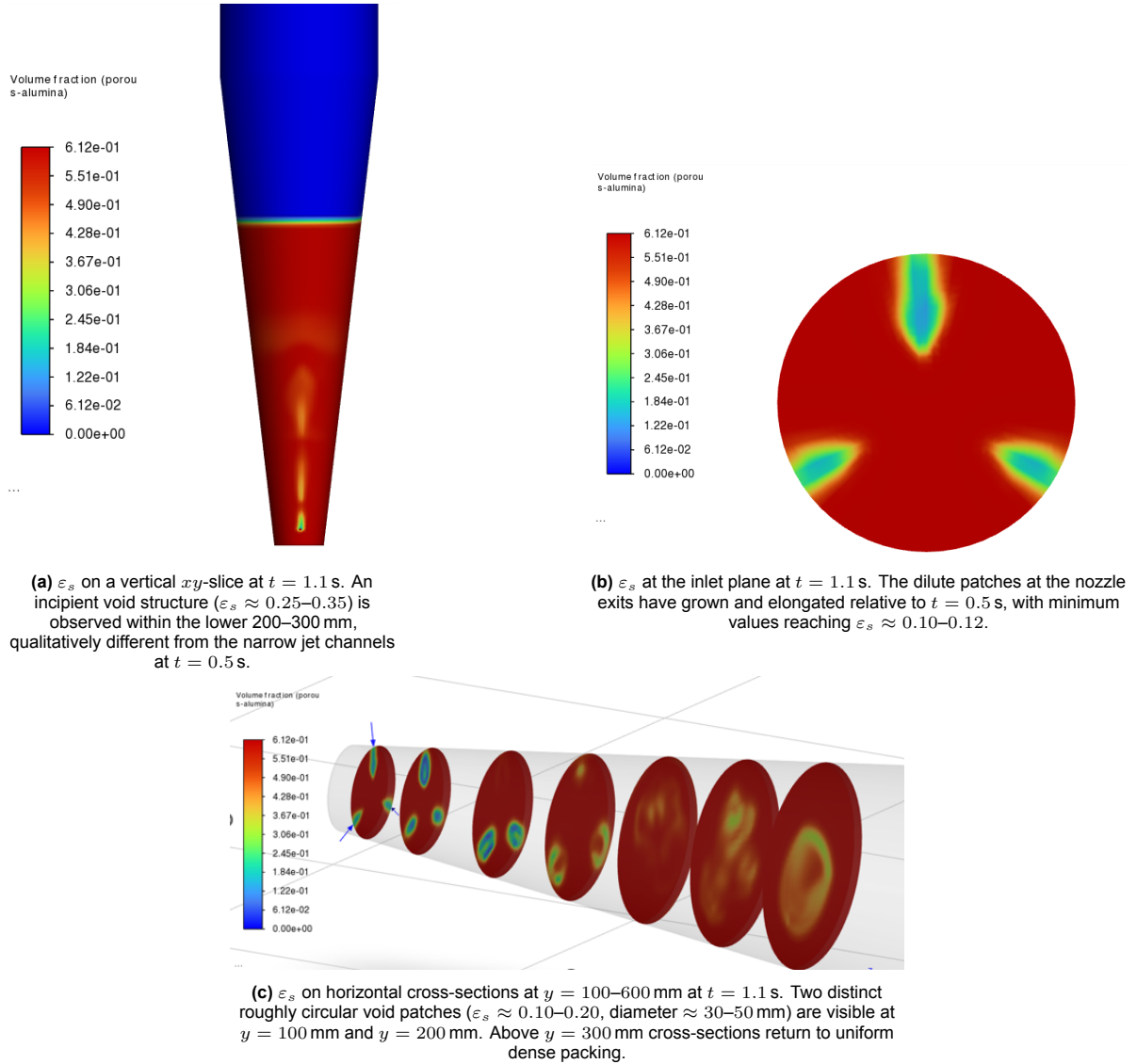


Figure 4.5: Void fraction at $t = 1.1$ s, Case G, 50 lpm.

The void fraction data at $t = 1.1$ s (figure 4.5) show the onset of a hydrodynamic transition. The vertical slice indicates the beginning of noticeable particle motion along the bed height, with the formation of a bubble-like structure in the lower region. Compared to the narrow jet channels observed at $t = 0.5$ s,

the flow is no longer strictly confined to thin, well-defined paths, although the jet influence remains clearly dominant at this stage.

The horizontal cross-sections provide further insight into this transition. At lower heights ($y \approx 100\text{--}200\text{ mm}$), three distinct, roughly circular dilute regions are visible, with minimum solid volume fractions of $\varepsilon_s \approx 0.10\text{--}0.20$. These structures are clearly different from the inlet jet footprints: they are detached from the immediate nozzle region and show a more rounded shape, consistent with incipient bubble-like behaviour. At the same time, the three jets do not evolve identically. The jet that appears strongest at the inlet plane weakens more rapidly with height, as the surrounding solid phase begins to refill the region. This highlights the asymmetric and unsteady nature of the developing flow.

Above $y \approx 300\text{ mm}$, the bed remains predominantly dense, but faint and diffuse variations in void fraction can still be observed. These weak fluctuations suggest a slight level of gas–solid interaction at these heights, indicating that the gas is beginning to reach and influence the upper sections of the bed. However, these effects remain limited, and no coherent void structures are established. Overall, the hydrodynamic activity is still largely confined to the lower portion of the bed.

This evolving void fraction field has a direct impact on the thermal behaviour. In the near-nozzle region, where the solid volume fraction is low ($\varepsilon_s \approx 0.10\text{--}0.20$), the gas phase dominates and the interfacial area available for heat transfer is reduced. As a result, although the gas carries significant thermal energy, the transfer to the solid phase remains locally limited. Further up the bed ($y \approx 100\text{--}200\text{ mm}$), the emergence of bubble-like structures introduces alternating dense and dilute regions, which leads to spatial variations in interfacial area and enhances local heat exchange. This is consistent with the first appearance of temperature perturbations observed in the thermal fields. Above $y \approx 300\text{ mm}$, the bed remains largely unaffected thermally, as the hot gas has not yet penetrated significantly into these regions, despite the presence of weak hydrodynamic fluctuations.

The fact that the more coherent void structures remain primarily below $y \approx 200\text{ mm}$ is consistent with expected bubble rise behaviour. Estimating a characteristic void size of $d_b \approx 15\text{--}20\text{ mm}$ from the spatial scale in the figure, the Davies–Taylor correlation [17] gives:

$$U_b \approx 0.711 \sqrt{g d_b} \approx 0.25\text{--}0.32\text{ m s}^{-1}. \quad (4.3)$$

Taking a representative value of $\approx 0.28\text{ m s}^{-1}$, a void nucleating near the nozzle region would be expected to rise on the order of 0.3 m over 1.1 s . This is of the same order of magnitude as, though somewhat higher than, the observed activation confined below $y \approx 200\text{ mm}$. This difference reflects the fact that the structures are not freely rising isolated bubbles, but are influenced by confinement, interaction with the surrounding dense phase, and the developing hydrodynamic field. The corresponding transit time from the distributor to the bed surface ($\approx 0.644\text{ m}$) is therefore on the order of $\sim 2\text{--}3\text{ s}$, providing a useful reference for the later timesteps.

Axial temperature profiles

The axial temperature profiles along the jet-aligned lines at $t = 1.1\text{ s}$ (figure 4.6) show a clear development compared to the $t = 0.5\text{ s}$ result. In addition to the primary peak near the nozzle, a secondary peak becomes visible in the solid temperature profiles at approximately $y \approx 100\text{--}150\text{ mm}$, depending on the jet line. This feature is not present at earlier times and indicates that thermal effects are beginning to propagate further into the bed.

The primary peak at $y \approx 30\text{ mm}$ corresponds to the direct nozzle exchange zone, as observed previously. Differences in peak magnitude between the three jets reflect slight asymmetries in the local flow and heat transfer, consistent with the jets being at different stages of their respective void development cycles.

The emergence of the secondary peak can be linked to the void fraction behaviour. As observed in the void fraction contours, the developing void structures tend to rise predominantly in the vertical direction rather than migrating toward the centre of the bed. As these voids pass through a given axial position, they locally reduce the solid volume fraction and therefore the interfacial area $a_s = 6(1 - \varepsilon_g)/d_p$ (Eq. 2.10), which lowers the local heat transfer rate within the void core. At the void boundaries, however, the interfacial area remains high and the temperature difference between phases

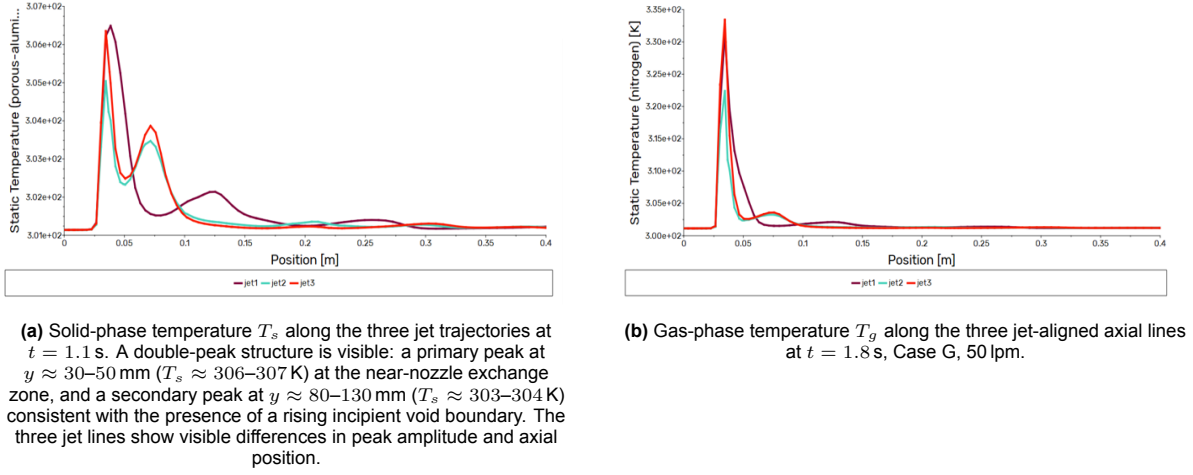


Figure 4.6: Axial temperature profiles at $t = 1.1$ s (Time = 1.068×10^0 s), Case G, 50 lpm.

is maintained, leading to more effective heat transfer. This spatial variation in heat exchange gives rise to the observed secondary temperature peak in the solid phase.

A weaker secondary feature is also visible in the gas temperature profiles $y \approx 50$ – 140 mm. Unlike the solid phase, this is not due to enhanced heat transfer, but rather to reduced heat extraction in the dilute regions. As the gas passes through a void, the reduced solid contact limits cooling, allowing the gas to retain a slightly higher temperature over a longer distance before gradually equilibrating with the surrounding bed.

4.1.3. $t = 1.8$ s

Temperature field at the inlet plane

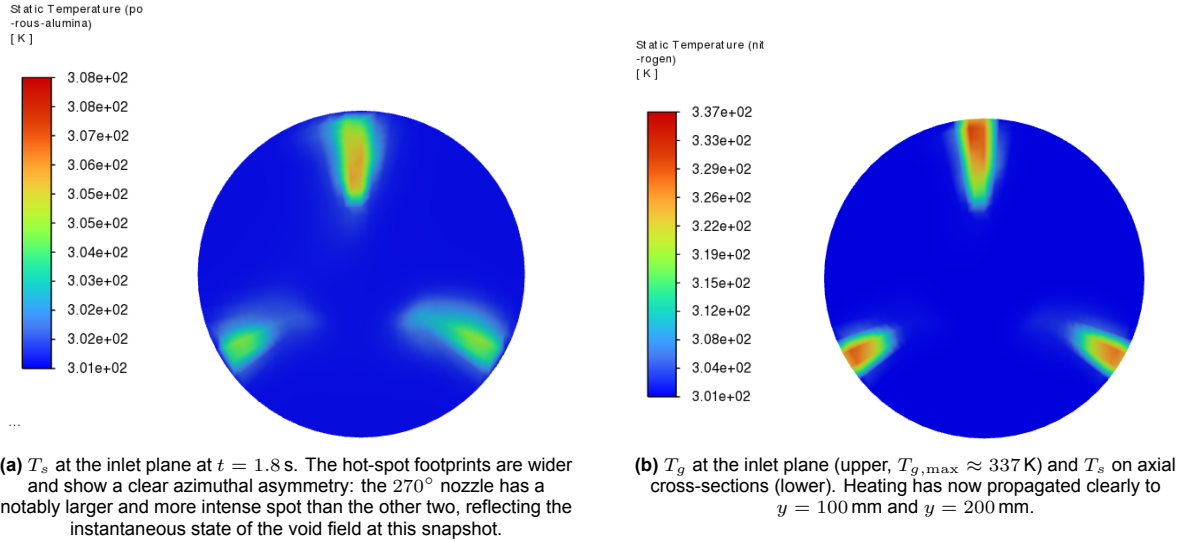


Figure 4.7: Temperature distributions at $t = 1.8$ s, Case G, 50 lpm.

At $t = 1.8$ s (figure 4.7), the inlet-plane temperature field shows a similar trend to the previous timestep, which could suggest that the mixing in the inlet region appears more developed and stable. The asymmetry associated with the nozzle located at 270° is still clearly present. While the sizes of the hot spots remain comparable, the surrounding halos are slightly more defined. This may indicate that the jet at this location is maintaining a more persistent dilute region, resulting in fewer particles locally and therefore a stronger or more sustained thermal footprint. Alternatively, it may reflect that particles in this region have experienced longer exposure to the hot gas due to their proximity to the nozzle.

The gas peak at the inlet plane remains around 333–335 K, which is very close to the values observed at earlier timesteps. This near-constancy of the gas temperature near the nozzle is an important observation: it indicates that the gas rapidly reaches a local thermal equilibrium with the surrounding solid phase shortly after entering the bed. As a result, the gas temperature at $y = 32.5$ mm is primarily controlled by the inlet conditions and the immediate gas–solid heat exchange, rather than by the slower evolution of the bulk bed temperature. The cumulative heating of the solid phase is therefore too gradual to significantly alter this local equilibrium over a timescale of 1.8 s.

Void fraction structure

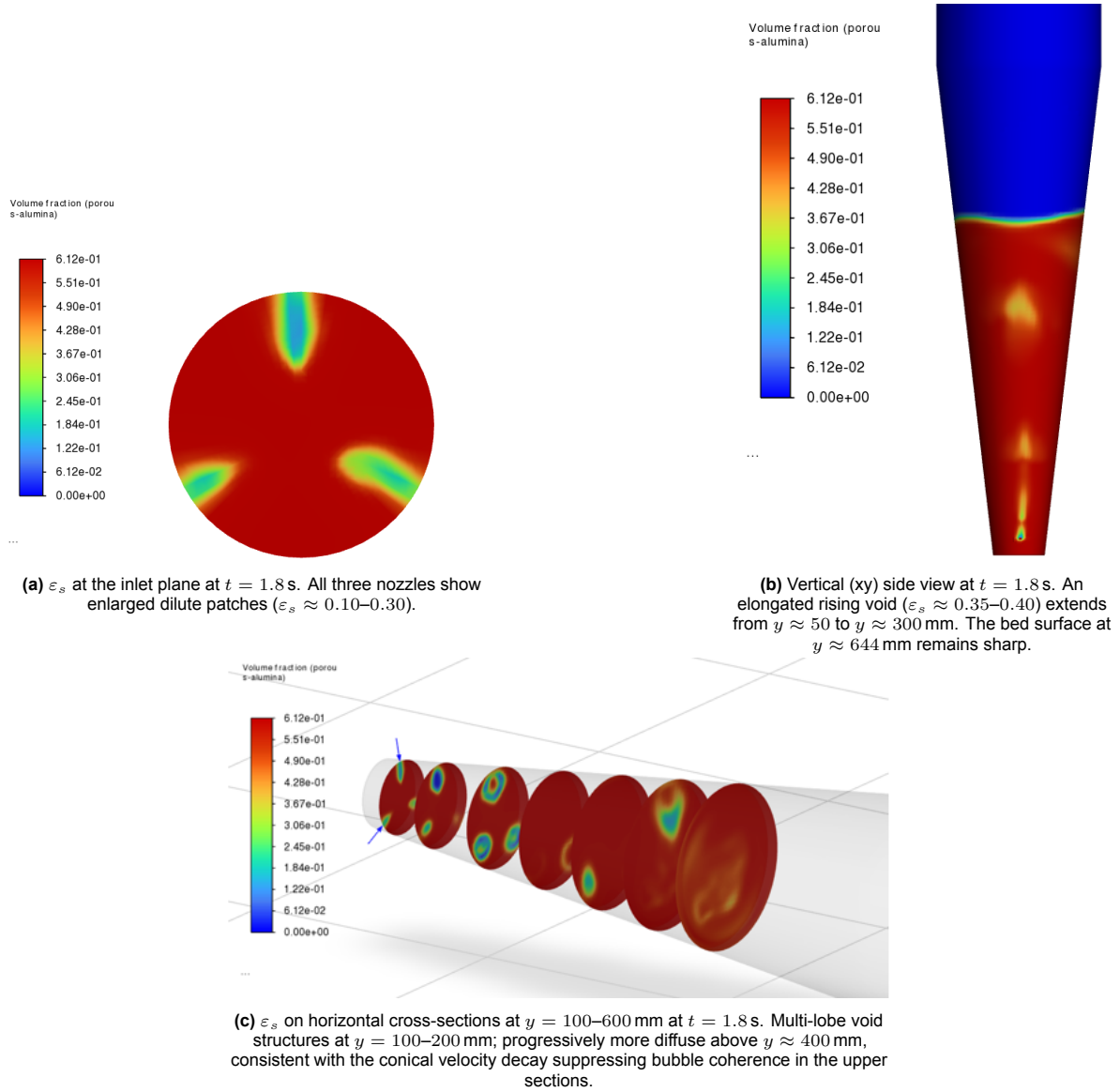


Figure 4.8: Void fraction at $t = 1.8$ s, Case G, 50 lpm.

The void fraction field at $t = 1.8$ s (figure 4.8) shows a clear progression compared to the earlier timesteps. The vertical slice (figure 4.8b) reveals an elongated rising void structure extending from the lower part of the bed up to approximately $y \approx 500$ mm. This is a significant change from the more localised and narrow features observed at $t = 1.1$ s, indicating that the hydrodynamic structures have become more developed and coherent.

The multi-height cross-sections (figure 4.8c) highlight a clear axial variation in the flow behaviour. At lower heights ($y = 100$ – 200 mm), complex multi-lobe void patterns are present, reflecting intense gas–solid interaction. Moving upward to $y = 300$ – 400 mm, these structures become more diffuse, indicating

a gradual loss of coherence. Above $y \approx 500$ mm, the bed appears nearly uniformly dense, with only very weak signs of void activity.

This behaviour is consistent with the conical geometry of the bed. As the cross-sectional area $A(y)$ increases with height, the local superficial velocity $U(y)$ decreases, reducing the ability of the gas to sustain coherent void structures. As $U(y)$ approaches the minimum fluidisation velocity U_{mf} , the flow transitions from a strongly convective, bubble-rich regime in the lower region to a weaker, more densely packed regime higher up. The cross-section data at $t = 1.8$ s begin to make this transition clearly visible.

Axial temperature profiles

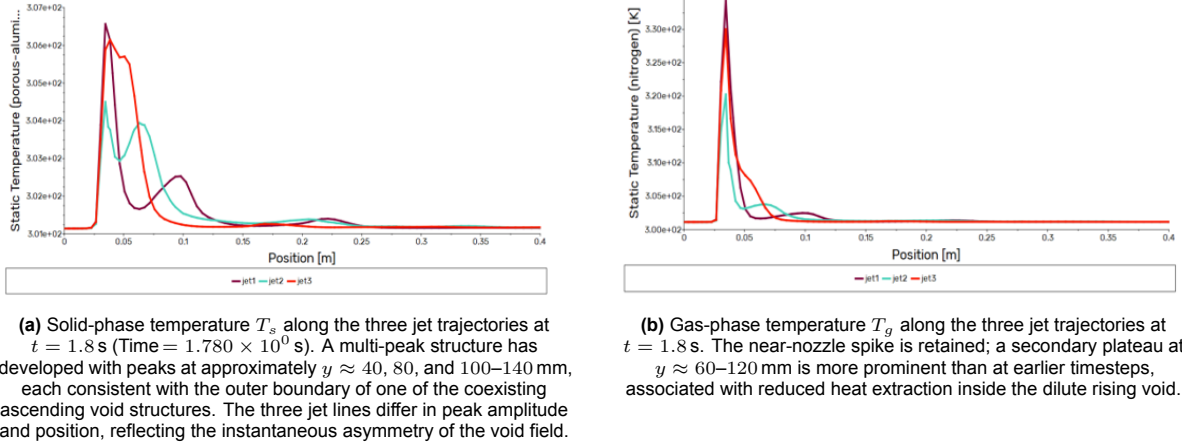


Figure 4.9: Axial temperature profiles at $t = 1.8$ s, Case G, 50 lpm.

The axial profiles along the jet-aligned lines at $t = 1.8$ s (figure 4.9) show a clear progression from the double-peak structure observed at $t = 1.1$ s. The solid temperature now exhibits at least three discernible peaks along some jet lines, located at approximately $y \approx 40, 80$, and $100\text{--}140$ mm. These peaks are consistent with the presence of multiple coexisting void structures, with each peak likely associated with regions near the void boundaries, where the interfacial area a_s is high and the temperature difference between phases remains significant.

The spacing between peaks, on the order of $\approx 40\text{--}50$ mm, is broadly consistent with the characteristic size and distribution of void structures observed in the void fraction field. However, a direct one-to-one correspondence cannot be established without simultaneous void fraction and temperature overlays. Weak additional peaks are also visible around $y \approx 200$ mm, which are consistent with the presence of more diffuse or higher-positioned void structures identified in the void fraction analysis.

It is also worth noting that, when examining temperature fields at the same axial planes, little to no variation is observed in certain regions despite the presence of void structures in the corresponding void fraction data. This can be explained by the transient nature of the flow: the voids rise rapidly through the bed, resulting in a short residence time at any given axial position. As a result, the local gas–solid interaction time is insufficient to produce a measurable temperature response in these regions. This behaviour is consistent across multiple planes and timesteps, and representative examples are provided in Appendix B for completeness.

On the gas side, the secondary plateau between $y \approx 60\text{--}120$ mm has become more pronounced compared to earlier timesteps. This behaviour is physically consistent: gas passing through a void experiences reduced interfacial area, which limits heat transfer to the surrounding solid. As a result, the gas retains a higher temperature over a longer axial distance before gradually equilibrating with the bed.

4.1.4. $t = 2.5$ s

Temperature field at the inlet plane

At $t = 2.5$ s (figure 4.10), the inlet-plane temperature field remains qualitatively similar to the previous timestep. The three nozzle hot spots are still clearly visible, with the main difference being that the surrounding thermal halos have become more defined. The underlying mechanism for this behaviour

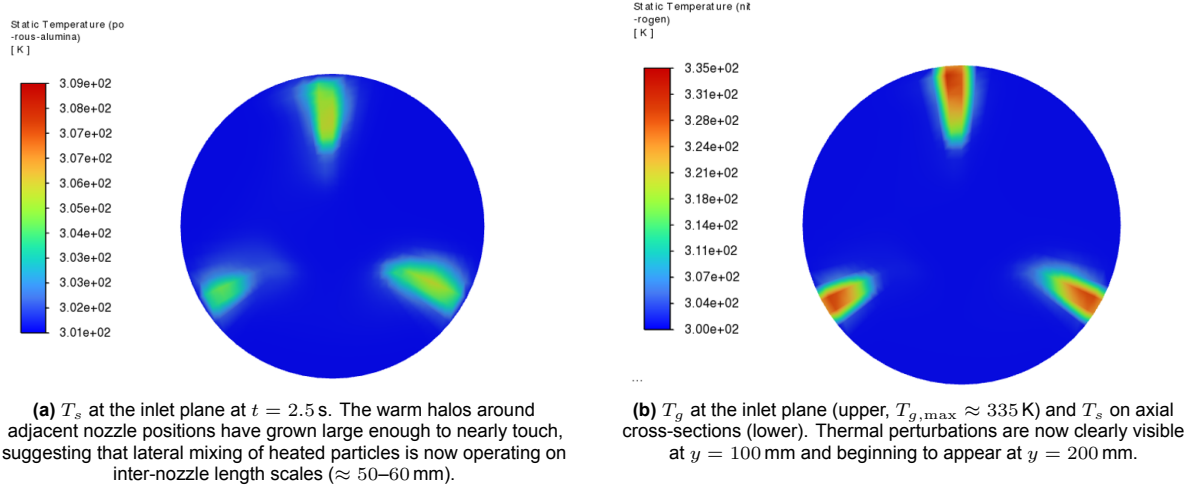


Figure 4.10: Temperature distributions at $t = 2.5$ s, Case G, 50 lpm.

has already been discussed: as mixing develops, the heat introduced by the jets is redistributed over a larger volume of particles, leading to a smoother and more spatially extended temperature field.

This progressive smoothing of the thermal field suggests that the inlet region is approaching a quasi-steady state. While local gas–solid heat exchange remains strong, the overall structure of the temperature field changes only gradually with time. As a result, further thermal evolution is no longer dominated by changes at the inlet plane, but rather by the propagation of heat into higher regions of the bed.

Void fraction structure and bubble coalescence

The void fraction field at $t = 2.5$ s (figure 4.11) shows a clear progression toward a more developed bubbling regime compared to earlier timesteps. The vertical slice (figure 4.11a) reveals a single large, elongated void structure extending approximately from $y \approx 100$ mm to $y \approx 350$ mm. This structure is significantly larger and more coherent than the smaller, isolated voids observed at $t = 1.1$ s, indicating that bubble growth and coalescence have become dominant mechanisms in the lower part of the bed.

The horizontal cross-sections (figure 4.11c) provide further insight into this behaviour. At $y = 100$ – 200 mm, the flow is dominated by a single, approximately circular dilute region, with solid volume fractions as low as $\varepsilon_s \approx 0.06$ – 0.12 . The characteristic size of this structure (on the order of 80 – 100 mm) is consistent with the expected growth of bubbles with height due to coalescence and gas accumulation. Such behaviour is well documented in classical fluidization theory, where bubble diameter increases as a function of height and excess gas velocity [37],[18]

As the height increases to $y = 300$ – 400 mm, the void structure becomes less coherent and more diffuse, indicating a gradual loss of bubble integrity. Above $y \approx 400$ – 500 mm, the bed appears predominantly dense, with only weak and diffuse void fraction variations. This axial stratification is consistent with the conical geometry of the bed: as the cross-sectional area $A(y)$ increases, the local superficial velocity $U(y) = Q/A(y)$ decreases, reducing the ability of the gas to sustain large coherent bubbles. As $U(y)$ approaches the minimum fluidisation velocity U_{mf} , bubble formation is progressively suppressed, and the flow transitions toward a dense, weakly heterogeneous regime.

The inlet-plane distribution (figure 4.11b) also reflects this evolving behaviour. The three nozzle-induced dilute regions remain visible, but are clearly asymmetric, with one nozzle exhibiting a wider and more deeply dilute core. This asymmetry is consistent with the inherently unsteady nature of multi-jet fluidization, where bubble formation and growth occur in a quasi-periodic and non-synchronous manner. Overall, the flow at $t = 2.5$ s is characterised by a dominant large-scale void structure in the lower bed and a gradual transition to a more uniform dense phase at higher elevations.

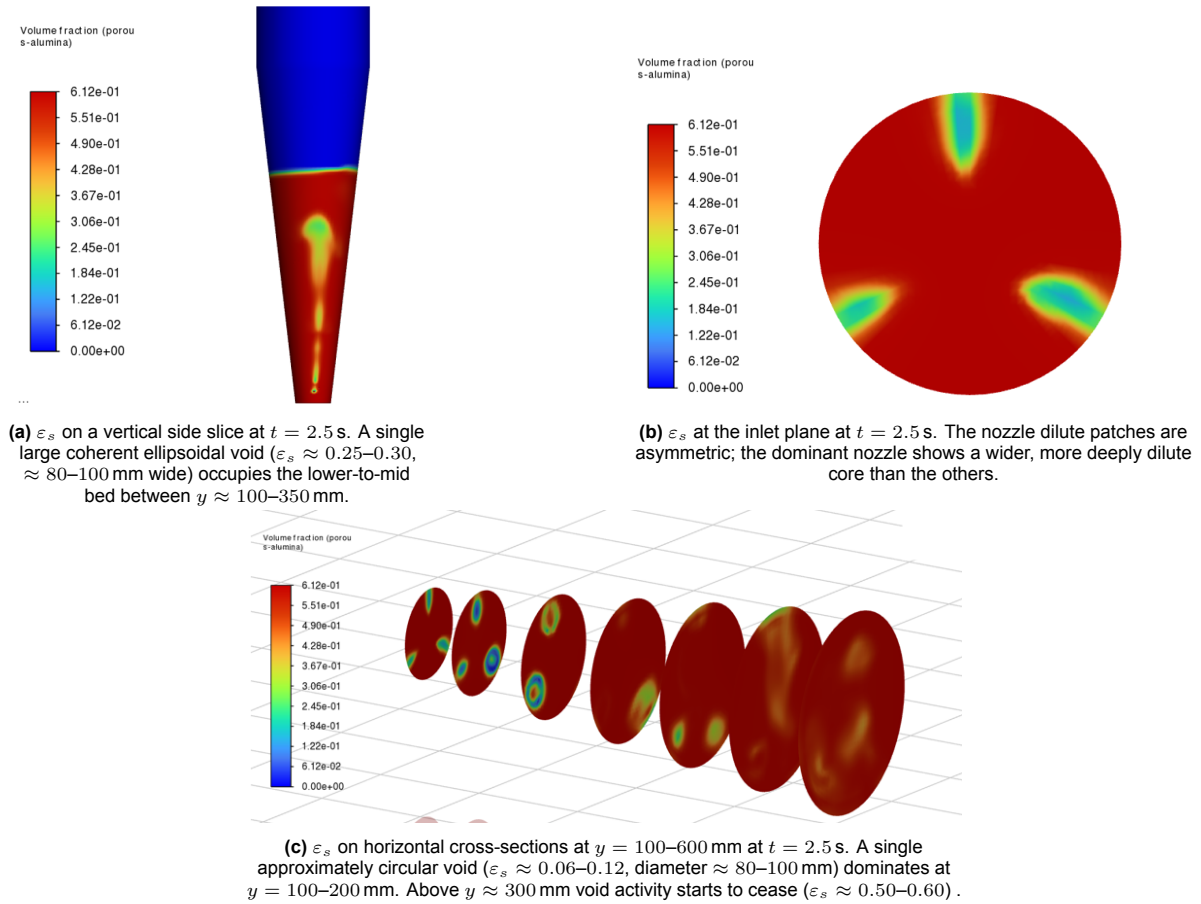


Figure 4.11: Void fraction at $t = 2.5$ s, Case G, 50 lpm.

Axial temperature profiles

The axial profiles along the jet-aligned lines at $t = 2.5$ s (figure 4.12) continue the trend of a growing thermally active zone. Jet 2's solid temperature profile is the most striking: it sustains $T_s \approx 302$ – 303 K all the way from $y \approx 60$ to $y \approx 200$ mm, a broader and longer plateau than any of the profiles at earlier timesteps. This is consistent with jet 2's trajectory running along the extended outer boundary of the coalesced void, where large a_s and sustained ΔT combine to deposit heat over a longer axial stretch.

The gas secondary hump between $y \approx 70$ – 160 mm has also grown more prominent. A larger void interior means a larger zone where a_s is suppressed, so the gas can travel further before it loses its thermal energy to the surrounding particles. The link between void size and the axial extent of the secondary gas hump is physically self-consistent.

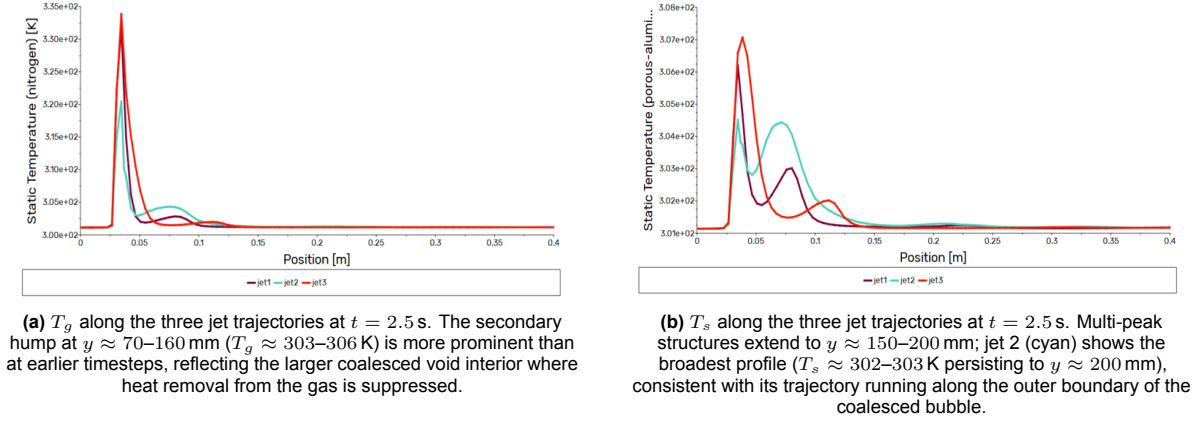


Figure 4.12: Axial temperature profiles at $t = 2.5$ s (Time = 2.492×10^0 s), Case G, 50 lpm.

4.1.5. $t = 4.0$ s

Temperature field at the inlet plane

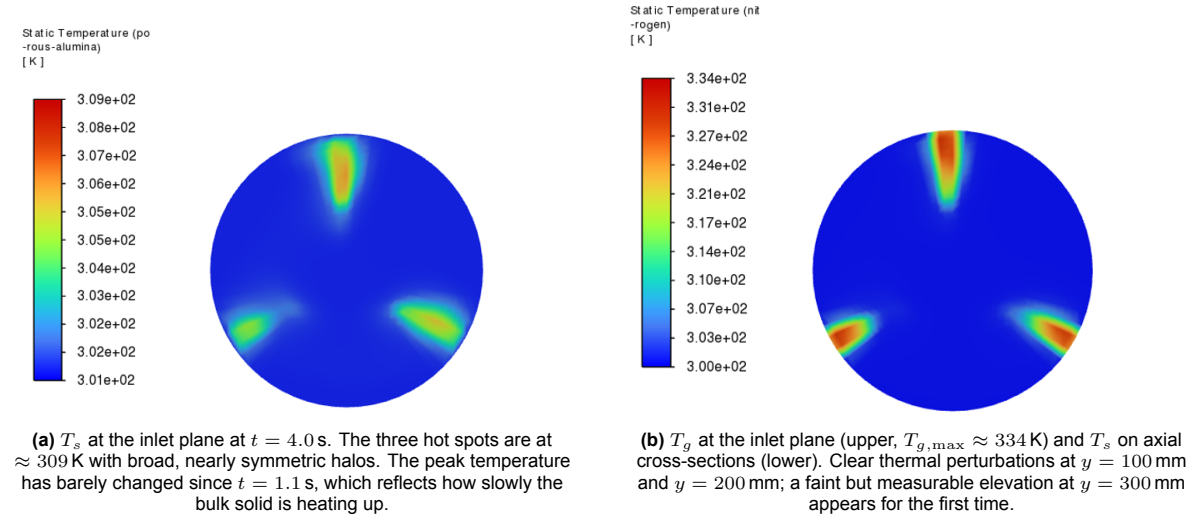


Figure 4.13: Temperature distributions at $t = 4.0$ s, Case G, 50 lpm.

At $t = 4.0$ s (figure 4.13), the inlet-plane temperature field shows only modest changes compared to earlier timesteps. The three nozzle-induced hot spots remain clearly visible, with peak solid temperatures of approximately $T_s \approx 309$ K. The most noticeable evolution is the further broadening of the surrounding thermal halos, which have expanded radially while maintaining a similar overall structure. In addition, the solid phase now shows a slight increase in background temperature, visible as a faint orange tint suggesting the beginning of shift to temperatures of 307 K. This indicates that the bulk solid has begun to heat up, although the temperature rise remains relatively small compared to the bulk.

The limited increase in peak temperature between $t = 1.1$ s and $t = 4.0$ s suggests that the inlet region has largely reached a quasi-steady thermal state. While local gas–solid heat transfer remains active, the temperature field is now governed by a balance between continuous heat input from the jets and its redistribution into the surrounding bed. As a result, further thermal evolution is no longer dominated by changes at the inlet plane, but by the gradual propagation of heat to higher regions.

The gas-phase temperature at the inlet plane remains nearly unchanged, with peak values of $T_g \approx 334$ K. This reinforces the earlier observation that the gas temperature rapidly reaches a local equilibrium with the surrounding solid phase shortly after entering the bed. Since the solid temperature increases only gradually over time, this local equilibrium remains largely unchanged, leading to a nearly constant gas temperature at this height.

Void fraction structure

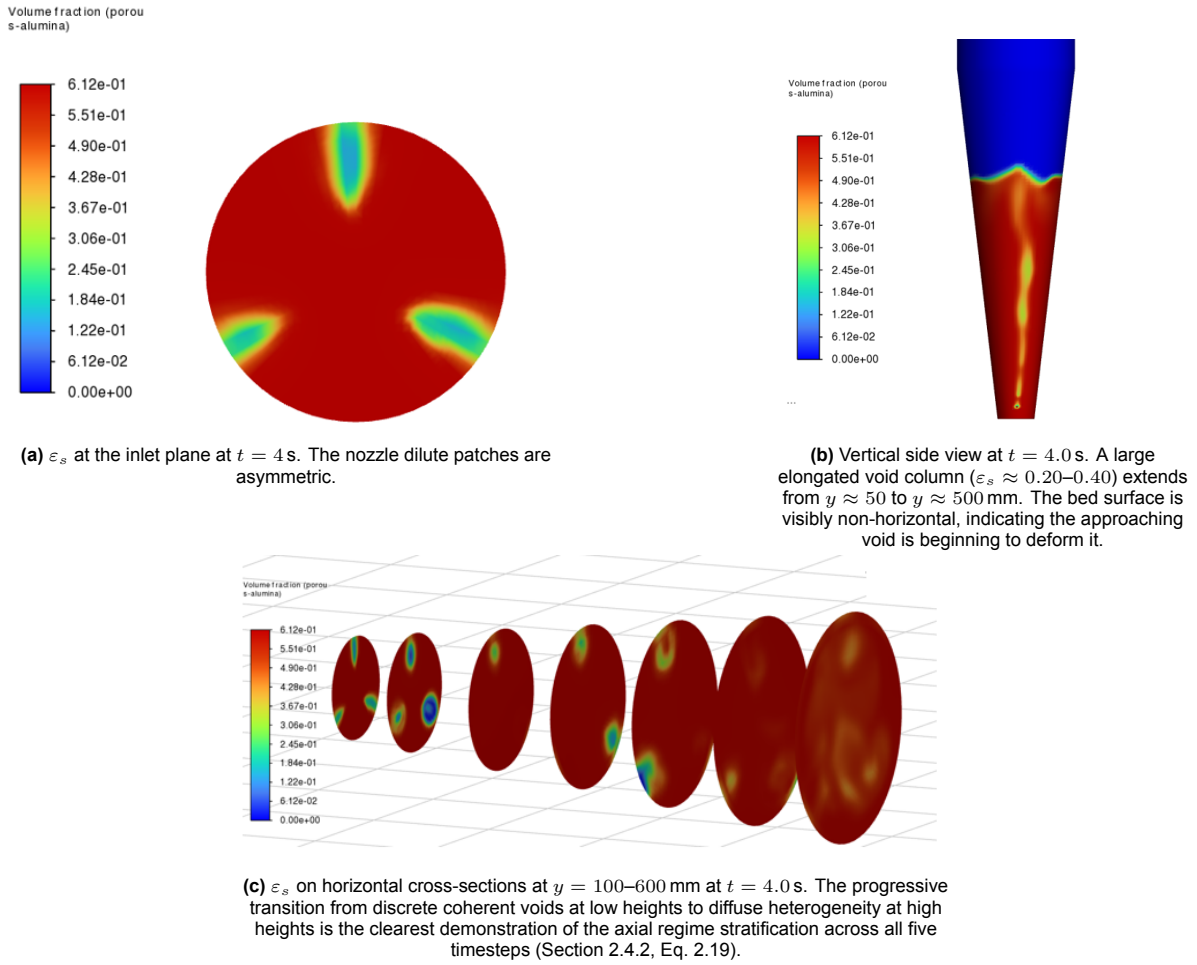


Figure 4.14: Void fraction at $t = 4.0$ s, Case G, 50 lpm. The most hydrodynamically active state in the simulation window.

The void fraction field at $t = 4.0$ s (figure 4.14) represents the most developed hydrodynamic state within the simulation window. The inlet-plane distribution (figure 4.14a) shows that the three nozzle-induced dilute regions are still present, although they remain asymmetric, with one nozzle exhibiting a slightly stronger and more elongated structure. This asymmetry reflects the inherently unsteady nature of the multi-jet interaction, where each jet evolves at a different stage of its void development cycle.

The vertical slice (figure 4.14b) reveals a large, elongated void column extending from approximately $y \approx 50$ mm up to $y \approx 500$ mm. Compared to earlier timesteps, this structure is now more continuous and coherent, indicating that bubble coalescence and vertical growth have become dominant. The bed surface is no longer perfectly horizontal and shows visible deformation, which is consistent with the upward motion of the large void and its interaction with the free surface.

The horizontal cross-sections (figure 4.14c) highlight a clear axial stratification of the flow. At lower heights ($y = 100$ – 200 mm), coherent void structures are still present, although they are now part of a larger, connected system rather than isolated bubbles. Moving upward to $y = 300$ – 400 mm, the void structures become increasingly diffuse, indicating a loss of coherence. Above $y \approx 500$ mm, the bed is dominated by dense packing with only weak heterogeneity.

This behaviour is consistent with the conical geometry of the bed. As the cross-sectional area increases with height, the local superficial velocity decreases, reducing the ability of the gas to sustain discrete bubbles. As a result, the flow transitions from a strongly bubbling regime in the lower region to a more diffuse, weakly heterogeneous state higher up. At this stage, the system exhibits a fully developed hydrodynamic structure, where large-scale void formation, coalescence, and axial stratification are all

clearly established.

Axial temperature profiles

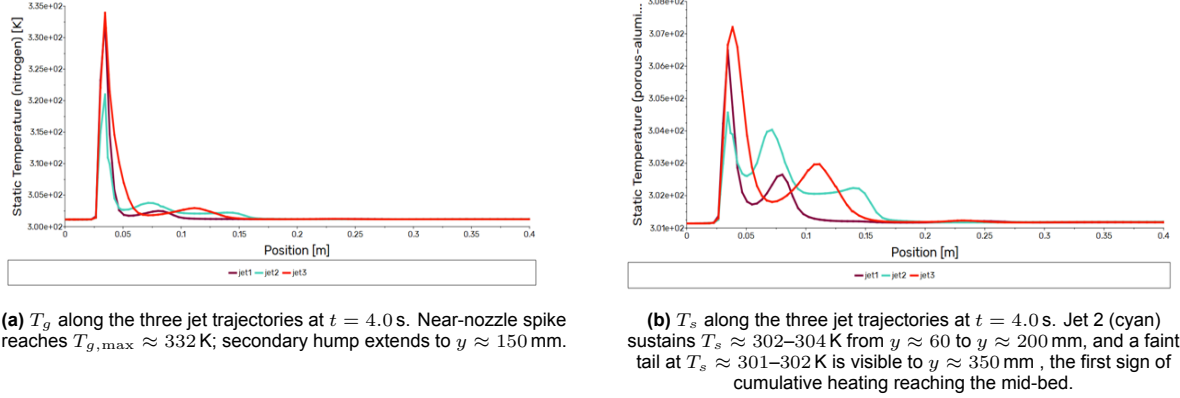


Figure 4.15: Axial temperature profiles at $t = 4.0$ s, Case G, 50 lpm.

The axial temperature profiles along the jet-aligned lines at $t = 4.0$ s (figure 4.15) show the most developed thermal field within the simulation window. The primary near-nozzle peak remains at $y \approx 30$ – 40 mm for all three jets, with gas temperatures reaching $T_g \approx 332$ – 334 K and corresponding solid peaks slightly above $T_s \approx 307$ – 309 K.

Beyond the nozzle region, the solid temperature profiles now extend significantly further into the bed compared to earlier timesteps. In particular, Jet 2 exhibits the most pronounced thermal penetration, maintaining elevated temperatures of $T_s \approx 302$ – 304 K from approximately $y \approx 60$ mm up to $y \approx 200$ mm. This broader profile reflects local asymmetry in the flow, consistent with the interaction between the jet and the elongated void structure observed in the void fraction field. In such regions, the reduced solid fraction lowers the interfacial area a_s , leading to a slower rate of heat transfer and allowing the gas to retain its energy over a longer axial distance.

A secondary feature is also visible in the gas phase, where a weak plateau extends to $y \approx 120$ – 150 mm. This is consistent with reduced heat extraction in the dilute regions, where limited gas–solid contact delays thermal equilibration.

More notably, a faint but measurable temperature tail now appears in the solid phase above $y \approx 250$ mm, reaching approximately $T_s \approx 301$ – 302 K as far as $y \approx 300$ – 350 mm. This feature is absent at earlier timesteps and represents the first clear indication of cumulative heating reaching the mid-bed region. Although the temperature increase is small, it confirms that thermal energy is gradually propagating upward, even if the dominant heat transfer processes remain confined to the lower, jet-driven region of the bed.

4.1.6. Local and global heat transfer behaviour.

Taken together, the results highlight a clear difference between the local and global thermal response of the bed.

Close to the nozzles, heat transfer is strongly dominated by the gas jets. The highest solid temperatures are found in the near-nozzle region ($y \lesssim 100$ mm), with the thermal influence extending along the jet trajectories up to around $y \approx 300$ mm. These regions coincide with strong gas–solid interaction and the presence of coherent void structures. The resulting variations in interfacial area lead to the multi-peak temperature profiles observed in the axial data, although the amplitude of these peaks remains small (on the order of ± 2 – 3 K).

Further up the bed, however, the behaviour changes noticeably. Even though the void fraction field shows that gas structures and flow heterogeneity persist up to $y \approx 400$ – 500 mm, the temperature field remains almost uniform. Only very weak temperature elevations are visible around $y \approx 100$ mm, appearing as faint thermal tails ($T_s \approx 301$ – 302 K). Above this level, the bed remains close to its initial temperature, even at $t = 4.0$ s.

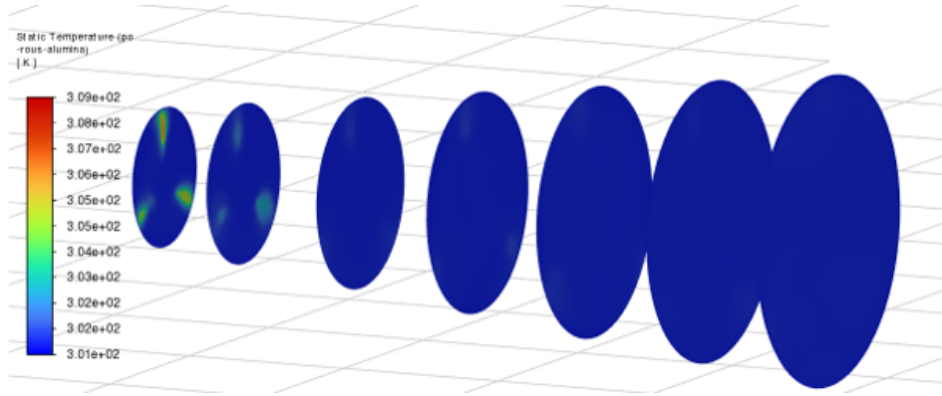


Figure 4.16: xz planes from $y=100, 200, 300, 400, 500, 600$ at $t=4$ s showing the solids temperature along the axial direction of the bed

This difference can be understood from the governing gas–solid heat transfer relation:

$$\dot{Q}_{gs}''' = h_{gs} a_s (T_g - T_s), \quad (4.4)$$

which shows that the local heat transfer rate depends on the temperature difference, the interfacial area, and the heat transfer coefficient.

Near the nozzle region, all three terms contribute to strong heat exchange: the temperature difference is large, the slip velocity is high, and the interfacial area is significant. Further up the bed, however, the temperature difference decreases rapidly as the gas cools, reducing the driving force for heat transfer.

At the same time, the interfacial area

$$a_s = \frac{6(1 - \varepsilon_g)}{d_p} \quad (4.5)$$

is reduced in the dilute regions associated with void structures. As a result, even though gas continues to flow through the bed, the effective heat transfer rate becomes very small.

A timescale argument helps explain why these void structures do not leave a thermal signature at higher elevations. Gas parcels are transported rapidly through the bed, with a characteristic residence time on the order of ~ 0.1 s, whereas the thermal response of the solid phase occurs over longer timescales. As a result, the gas does not remain in contact with the solid long enough to produce measurable local temperature changes.

Overall, this leads to a strongly localised thermal response, where most of the heating remains confined to the lower, jet-dominated region, while the upper bed remains largely unaffected.

Another feature that remains consistent across all timesteps is the temperature difference between the gas and solid phases near the nozzle region, which stays at approximately 25–30 K. This gap does not close with time, indicating that it is not a transient effect, but rather an inherent feature of the flow.

The reason lies in the local structure of the jets. Inside the jet cores, the solid fraction is reduced, which lowers the available interfacial area a_s for heat transfer. Although the local heat transfer coefficient is high due to strong slip velocities, the overall heat exchange rate remains limited by this reduced contact area.

As a result, the gas leaves the nozzle region at a relatively high temperature, while the solid heats up more gradually.

In practical terms, this means that temperature measurements near the nozzles will consistently show the gas phase to be significantly hotter than the surrounding solid. This should be interpreted as a characteristic feature of the flow, rather than a sign of incomplete thermal equilibration.

4.2. Combined Heating Configuration: Case C, 50 lpm

4.2.1. Initialisation Anomaly and Its Consequences for Interpretation

Before going into the results, there is a model configuration issue that needs to be addressed upfront, because it fundamentally changes what these results actually show.

The intended initial condition for Case C is a uniform bed temperature of $T_0 = 289.15$ K, with both the reactor wall and gas inlet then held at $T_{bc} = 393.15$ K (Table 3.7). But inspection of the earliest available output ($t = 0.5$ s) shows both gas and solid phases already sitting at $T \approx 393$ K throughout the entire bed interior. This cannot be physical. Heating the full solid inventory by 104 K in 0.5 s would require a volumetric power input of

$$\dot{Q}''' \sim \frac{\rho_s C_{p,s} \Delta T}{\Delta t} = \frac{1372 \times 742 \times 104}{0.5} \approx 2.1 \times 10^8 \text{ W m}^{-3}, \quad (4.6)$$

which is completely unachievable through interphase convection at these conditions.

The most likely explanation, is that the fixed-temperature wall Dirichlet boundary condition ($T_w = 393.15$ K) interfered with Fluent's hybrid initialiser. In the Eulerian multiphase framework, the hybrid initialiser solves the Laplace equation to generate a smooth initial velocity field [4]. When a fixed-temperature wall BC is active during this process, the initialiser appears to propagate the wall temperature as a domain-wide constraint on the energy equation, overwriting the cold-bed patch ($T_s = 289.15$ K, applied in Section 3.5) before the first physical timestep runs. The result is a bed that starts pre-heated at 393.15 K rather than cold.

This has three direct consequences for everything that follows.

First, **there is no heating transient**. The progressive rise of the bulk solid temperature from 289 K toward 393 K is simply absent. With the bed and all boundaries already at 393.15 K, the thermal driving force $\Delta T = T_{source} - T_{bed}$ is essentially zero everywhere.

Second, **what the simulation does capture is the near-isothermal hydrodynamic response** of a pre-heated bed at 393.15 K. The void fraction fields, bubble timelines, and recirculation patterns are physically real. The elevated temperature shifts the gas thermophysical properties through Eqs. 3.1 and 3.4, dropping the nitrogen density from 1.18 kg m^{-3} at 289.15 K to 0.87 kg m^{-3} at 393.15 K, and this modifies the Archimedes number:

$$Ar = \frac{d_p^3 \rho_g (\rho_s - \rho_g) g}{\mu_g^2}, \quad (4.7)$$

lowering U_{mf} relative to the cold conditions of Case G. All hydrodynamic observations in Case C should therefore be read as properties of a near-isothermal bed running at a higher effective excess velocity $(U - U_{mf})/U_{mf}$ than Case G.

Third, **temperature variations in the figures reflect void-fraction-mediated phase-averaging effects**, not real heat transfer. In the Eulerian framework the phase-averaged solid temperature in a cell is weighted by the local ε_s . Where $\varepsilon_s \rightarrow 0$, jet cores, bubble interiors, the reported T_s can deviate from 393.15 K without any actual cooling having occurred. All temperature depressions discussed below are interpreted in that light.

A corrected simulation, activating the fixed wall BC only after confirming the cold-bed patch, is recommended as future work and discussed in Chapter 5.

4.2.2. $t = 0.5$ s

Temperature field at the inlet plane

At $t = 0.5$ s (figure 4.20), the temperature field confirms the anomalous initial condition described earlier. Both the gas and solid phases are nearly uniform at $T \approx 393$ K across the entire inlet plane, with no evidence of a physical heating transient.

Small localised depressions are visible at the nozzle positions (30° , 150° , 270°) in 4.17b, with temperature differences of approximately $\Delta T \approx -3$ to -5 K. These features coincide with the dilute jet cores observed in the void fraction field. In these regions, the solid volume fraction ε_s is reduced, and the

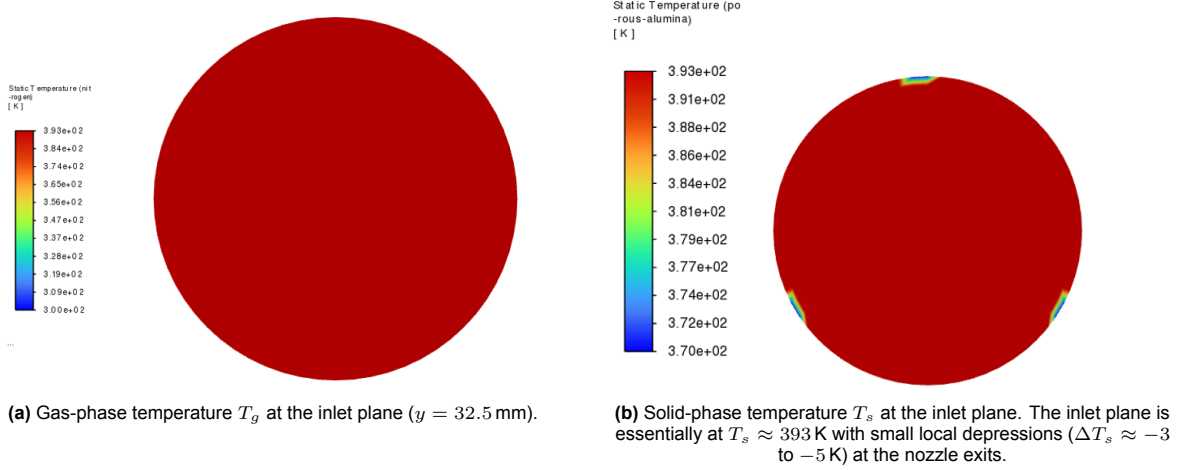


Figure 4.17: Temperature distributions at $t = 0.5$ s, Case C (combined heating), 50 lpm, built-in Gunn correlation. The near-uniformity at 393 K across both phases and all heights is a consequence of the initialisation anomaly described in Section 4.2.1.

Eulerian phase-averaged temperature becomes sensitive to the local phase distribution. As a result, the apparent decrease in temperature does not represent actual cooling, but rather a consequence of phase-averaging in low- ε_s regions.

The gas phase remains essentially uniform, with only very weak structure visible at the nozzle positions. This further supports the conclusion that no meaningful thermal gradients are present in the system at this stage.

Overall, the temperature field at $t = 0.5$ s should be interpreted as near-isothermal. The observed variations are not the result of heat transfer, but instead reflect the underlying void fraction distribution. The temperature field therefore acts as a diagnostic of the hydrodynamic structure rather than an indicator of thermal evolution.

Void fraction structure

At $t = 0.5$ s (figure 4.18), the void fraction field shows the early jet-dominated behaviour, consistent with the initial stages of fluidisation. The vertical slice (figure 4.2a) indicates that the bed remains essentially at its initial packing condition, with a nearly uniform solid volume fraction of $\varepsilon_s \approx 0.59$ throughout most of the domain. Only narrow dilute channels are present at the nozzle exits, extending a short distance into the bed. No coherent void structures are observed above the distributor region.

The inlet-plane distribution (figure 4.18b) confirms the presence of three small dilute patches at the nozzle positions (30° , 150° , 270°), with local values of $\varepsilon_s \approx 0.12$ – 0.18 . Outside these regions, the bed remains essentially undisturbed and densely packed. In contrast to Case G, however, these void structures do not provide direct insight into heat transfer behaviour. Due to the near-isothermal condition of the bed in Case C, there is no meaningful thermal gradient driving gas–solid heat exchange. As a result, any apparent temperature variations associated with these regions arise from phase-averaging effects linked to the local void fraction, rather than from physical heating or cooling.

The horizontal cross-sections (figure 4.18c) further illustrate the limited extent of the jet penetration. At lower heights ($y \approx 100$ – 300 mm), small, localised dilute regions are visible, corresponding to the upward extension of the jet cores. However, these structures do not persist with height, and by $y \gtrsim 400$ mm the bed returns to a uniform dense state with no discernible void structures.

Overall, the flow at this stage is characterised by narrow jet channels confined to the lower portion of the bed, with no evidence of bubble formation or large-scale hydrodynamic structures. This behaviour is consistent with the early jet inception phase and confirms that, despite the elevated temperature in Case C, the initial hydrodynamic development follows the same pattern as observed in Case G. In this case, the temperature field should therefore be interpreted as a diagnostic of the hydrodynamic structure rather than as a measure of thermal evolution.

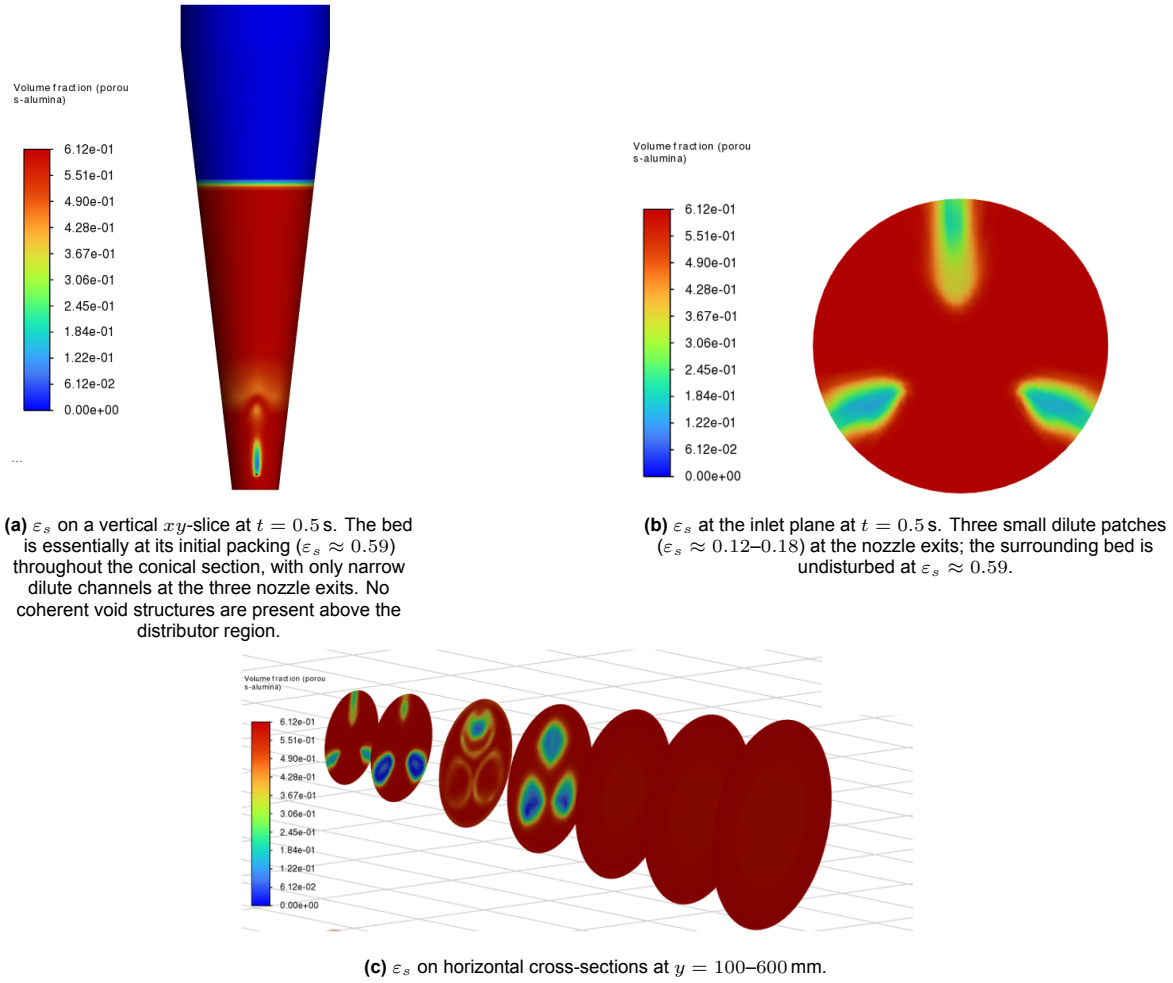


Figure 4.18: Void fraction at $t = 0.5$ s, Case G, 50 lpm.

Axial temperature profiles

The axial temperature profiles along the jet-aligned lines at $t = 0.5$ s (figure 4.19) reflect the near-isothermal condition of the bed. Both gas and solid phases remain close to $T \approx 393$ K along the entire height, with only small local deviations near the nozzle region.

A sharp dip is observed in both gas and solid temperature profiles at $y \approx 30$ – 50 mm, corresponding to the location of the jet cores. This feature coincides with the regions of reduced solid volume fraction identified in the void fraction field. In these dilute regions, the phase-averaged temperature becomes sensitive to the local value of ε_s , leading to an apparent decrease in temperature. This effect is particularly pronounced in the gas phase, where the dip is sharper and more localised.

Beyond the nozzle region, the profiles quickly return to the uniform background temperature, with only very weak oscillations visible along some jet trajectories. These minor variations correspond to small-scale fluctuations in the local void fraction, but do not indicate any meaningful thermal gradient or heat transfer process.

It is important to emphasise that these features do not represent physical cooling or heating of the phases. Instead, they arise from the phase-averaging formulation of the Eulerian model in regions where the solid fraction is low. As such, the axial temperature profiles should be interpreted as indirect indicators of the hydrodynamic structure, rather than as evidence of thermal evolution.

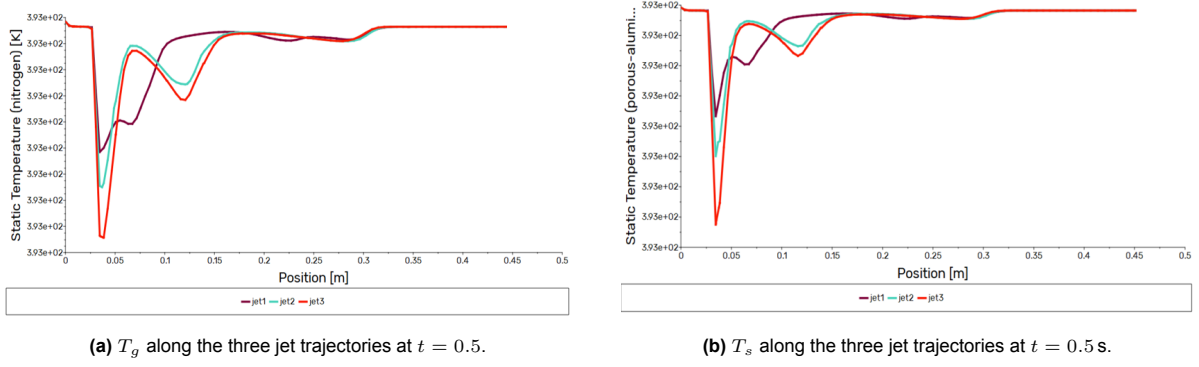


Figure 4.19: Axial temperature profiles at $t = 0.5$ s, Case G, 50 lpm.

4.2.3. $t = 4.0$ s

Temperature field at the inlet plane

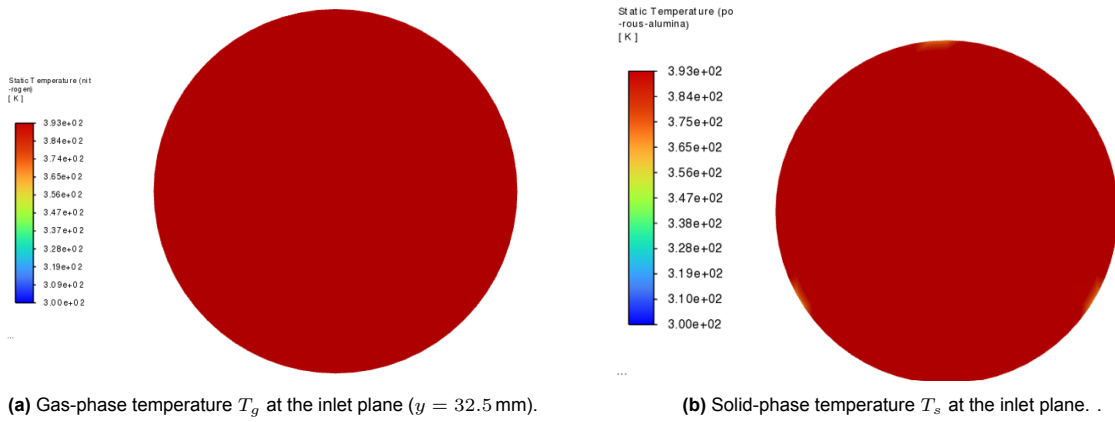


Figure 4.20: Temperature distributions at $t = 4$ s, Case C (combined heating), 50 lpm, built-in Gunn correlation. The near-uniformity at 393 K across both phases and all heights is a consequence of the initialisation anomaly described in Section 4.1.1

At $t = 4.0$ s (figure 4.20), the temperature field is completely uniform across the inlet plane, with both gas and solid phases at $T \approx 393$ K. No localised variations or structures are visible at the nozzle positions or elsewhere in the domain.

This behaviour is consistent with the anomalous initialisation discussed previously, which results in a fully pre-heated bed with no thermal driving force ($\Delta T \approx 0$). As a consequence, the temperature field does not evolve in time and remains spatially uniform.

At this stage, the temperature field provides no information about the underlying flow behaviour and cannot be used to infer hydrodynamic features. All observable dynamics in Case C must therefore be interpreted purely from the void fraction field.

Void fraction structure

At $t = 4.0$ s (figure 4.21), the void fraction field shows a fully developed hydrodynamic state, markedly different from the early jet-dominated regime observed at $t = 0.5$ s.

The vertical slice (figure 4.21a) reveals a large coherent void structure extending from the nozzle region up to approximately $y \approx 300$ – 400 mm. This structure is significantly larger than the initial jet channels and indicates that bubble growth and coalescence have become the dominant mechanisms in the lower part of the bed. The bed surface is visibly deformed, further confirming the presence of strong upward gas motion.

At the inlet plane (figure 4.21b), the three nozzle-induced dilute regions are clearly visible and have expanded compared to earlier timesteps. The structures are asymmetric, indicating that the jets are

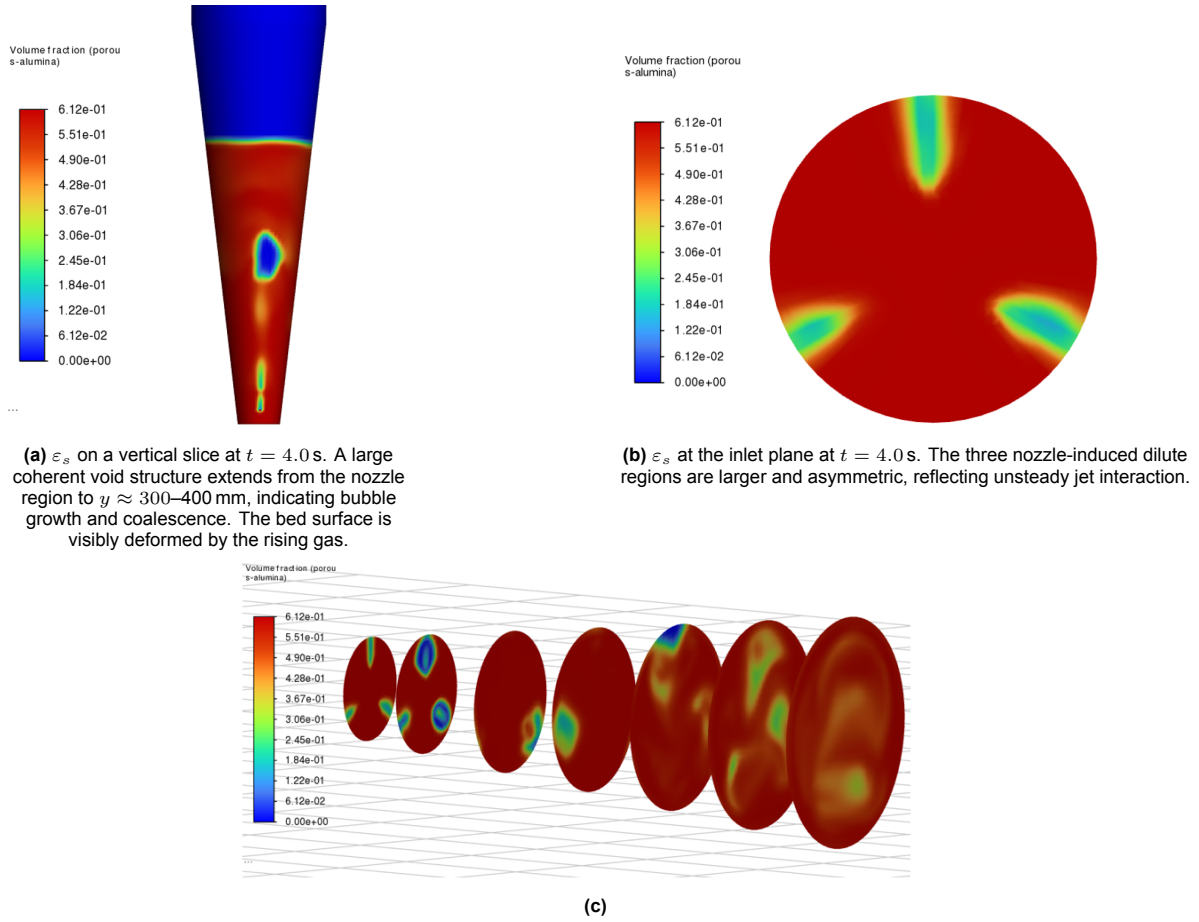


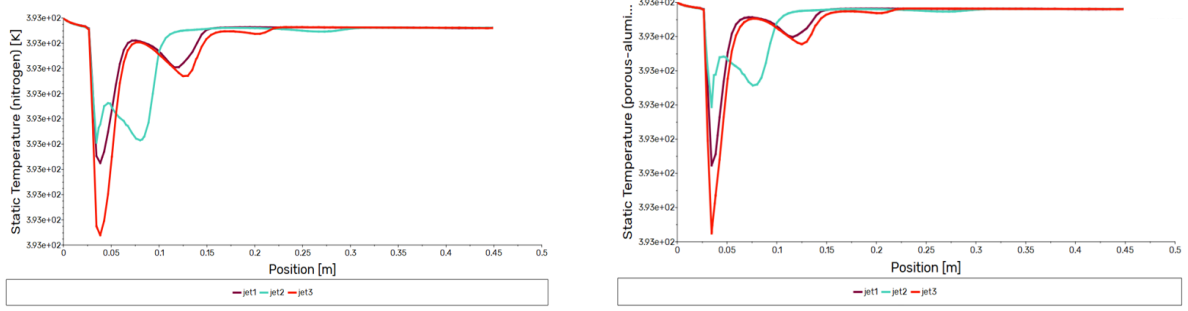
Figure 4.21: Void fraction at $t = 0.5$ s, Case G, 50 lpm.

no longer evolving identically and that the flow has become unsteady.

The horizontal cross-sections (figure 4.21c) show a clear axial evolution of the flow structure. At lower heights ($y = 100$ – 200 mm), large and coherent void regions dominate, characteristic of a bubbling regime. At intermediate heights ($y = 200$ – 400 mm), these structures become more diffuse and irregular, suggesting ongoing coalescence and breakup. Above $y \approx 500$ mm, the bed transitions to a more uniform dense phase with only weak heterogeneity, as the gas loses its ability to sustain discrete void structures.

Overall, the flow at this stage is characterised by large coherent voids in the lower bed and a progressive transition to diffuse heterogeneity with height. This behaviour reflects a fully developed bubbling regime governed by jet interaction, coalescence, and the expanding cross-sectional area of the conical geometry.

Axial temperature profiles



(a) T_g along the three jet-aligned axial lines at $t = 4.0$ s. The profiles remain nearly uniform at $T \approx 393$ K, with a sharp localised dip at $y \approx 30-50$ mm corresponding to the jet cores.

(b) T_s along the three jet-aligned axial lines at $t = 4.0$ s. A primary dip near the nozzle region and a weaker secondary deviation at $y \approx 80-150$ mm are visible, both associated with low- ε_s regions.

Figure 4.22: Axial temperature profiles at $t = 4.0$ s, Case C (combined heating), 50 lpm. The profiles remain essentially isothermal, and all apparent variations arise from phase-averaging effects linked to the local void fraction.

The axial temperature profiles along the jet-aligned lines at $t = 4.0$ s (figure 4.22) remain nearly uniform at $T \approx 393$ K across the entire bed height, consistent with the near-isothermal condition imposed by the initialisation.

Despite the absence of any thermal gradients, distinct features are visible near the nozzle region. A sharp temperature dip is observed at $y \approx 30-50$ mm for all three jets, corresponding to the jet cores. In these regions, the solid volume fraction ε_s is strongly reduced, and the Eulerian phase-averaged temperature becomes sensitive to the local phase distribution. The apparent temperature decrease therefore does not represent physical cooling, but is a direct consequence of the phase-averaging formulation in dilute regions.

A secondary, broader deviation is also visible at $y \approx 80-150$ mm, particularly for jet 2. This feature is associated with larger void structures above the nozzle, where the solid fraction remains lower than in the bulk. As with the primary dip, this behaviour reflects the local void fraction distribution rather than any heat transfer process.

Beyond $y \approx 200$ mm, the profiles return to the uniform background temperature, indicating that the bed is predominantly dense in these regions. The absence of any persistent temperature variation confirms that the thermal field does not evolve with height or time.

Overall, the axial temperature profiles act solely as an indirect indicator of the void fraction distribution. The observed features correspond to regions of reduced solid fraction, and should not be interpreted as evidence of thermal gradients or heat transfer.

4.2.4. Thermal observations throughout the cycle.

The bulk solid and gas temperatures remain close to 393 K at all heights above $y \approx 200$ mm throughout all four phases. Temperature perturbations are primarily confined to the near-nozzle zone and are strongly influenced by void-fraction-mediated Eulerian phase-averaging effects rather than genuine interphase heat transfer. The gas-to-solid thermal driving force $\Delta T_{gs} \approx 0$ throughout, rendering the Gunn exchange rate (Eq. 2.22) effectively negligible from a thermal standpoint. Collectively, these observations indicate that the combined-case simulation does not capture a significant heating transient under the present initialisation, and that its physical content is limited to the near-isothermal hydrodynamics of a pre-heated bed at 393 K.

4.3. Wall Heating Configuration: Case W, 50 lpm

4.3.1. Model Configuration and Initialisation Anomaly

Case W uses the same built-in Eulerian multiphase framework as Case C, ANSYS Fluent 2024 R2, no UDFs, with the Gunn correlation [29] applied uniformly across the full void-fraction range. The reactor wall was set to a standard fixed-temperature Dirichlet boundary condition of $T_w = 393.15$ K. The one

critical difference from Case C is the gas inlet, which was kept at the intended cold-gas condition of $T_{in} = 289.15 \text{ K}$ (16°C).

As discussed in Section 4.2.1, the fixed-temperature wall Dirichlet BC interacts with Fluent's hybrid initialiser, causing the wall temperature to be propagated domain-wide and overwriting the cold-bed patch before the first physical timestep. Case W therefore shares the same initialisation artefact as Case C: both phases begin at $T \approx 393.15 \text{ K}$ throughout the entire bed interior.

What makes Case W more interesting than Case C is precisely this combination of a pre-heated bed and a cold gas inlet. In Case C, the gas inlet was also at 393.15 K , so the anomalously hot bed faced essentially zero thermal driving force on every boundary. In Case W, the gas inlet remains cold. The pre-heated solid at 393.15 K therefore faces nitrogen entering at 289.15 K , a temperature 104 K below the bed. The Gunn exchange term

$$\dot{Q}_{gs}''' = h_{gs} a_s (T_g - T_s) \quad (4.8)$$

is negative throughout the near-nozzle zone ($T_g < T_s$), meaning energy flows from solid to gas rather than the reverse. Case W therefore produces a genuine, physically consistent heat transfer transient, it just runs in the opposite direction from the intended experiment. Rather than the wall heating a cold bed, the cold inlet gas is extracting thermal energy from the pre-heated solid, driven by the initial temperature difference $\Delta T = T_{in} - T_{bed} = 289.15 - 393.15 = -104 \text{ K}$.

All results in this section are interpreted within this inverted-transient framework. The wall BC at $T_w = 393.15 \text{ K}$ maintains the anomalously high bed temperature but contributes no additional heat flux; it plays no dynamic role in what follows. The fix recommended in Section 4.2.1, setting the wall BC to adiabatic during initialisation and activating it only after confirming the cold-bed patch, applies equally here.

Results are presented at $t = 0.5, 1.1, 1.8, 2.5$, and 4.0 s , spanning jet inception through established periodic bubbling to a quasi-steady inverted thermal state. At each instant the analysis covers the gas and solid temperature fields at the inlet plane ($y = 32.5 \text{ mm}$), the void fraction structure, and the axial temperature profiles along the three jet trajectories.

4.3.2. $t = 0.5 \text{ s}$

Temperature field at the inlet plane.

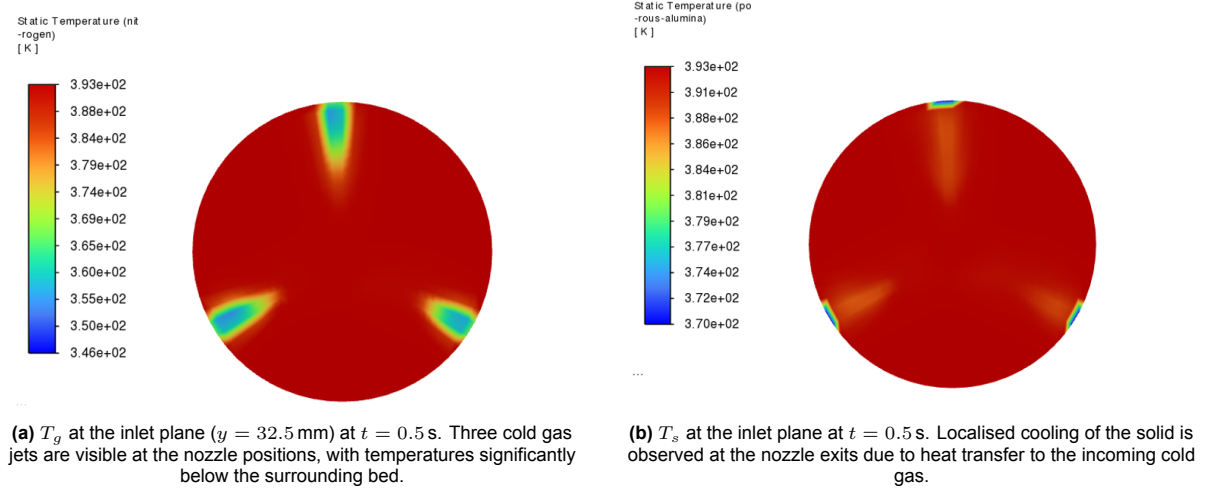


Figure 4.23: Temperature distributions at $t = 0.5 \text{ s}$, Case W (cold gas inlet, pre-heated bed), 50 lpm. The inverted transient drives heat transfer from the solid to the gas.

Figure 4.23 provides the first direct evidence that the inverted transient in Case W is both real and physically meaningful. Unlike Case C, where temperature variations were purely numerical artefacts, the temperature field here reflects a genuine heat transfer process driven by the large temperature difference between the pre-heated bed ($T \approx 393 \text{ K}$) and the incoming cold gas ($T = 289.15 \text{ K}$).

The solid temperature field (figure 4.23b) shows three compact cooling regions at the nozzle positions, with local temperature reductions of approximately $\Delta T_s \approx -15$ to -25 K relative to the bulk. These regions correspond to zones of intense gas–solid interaction, where the cold nitrogen extracts thermal energy from the surrounding particles. While phase-averaging effects may still contribute to the exact magnitude of the observed depressions, the dominant mechanism is physical heat transfer from the solid to the gas.

The gas temperature field (figure 4.23a) makes this process explicit. The gas enters at 289.15 K and is rapidly heated as it passes through the near-nozzle region, reaching temperatures on the order of $T_g \approx 340$ – 350 K within the first 60 mm of the bed. This represents a temperature increase of approximately 50–60 K over a very short distance, indicating a high volumetric heat transfer rate.

This behaviour is the direct inverse of that observed in Case G, where hot gas was cooled by a colder bed over the same spatial scale. In both cases, the underlying mechanism is governed by the same gas–solid heat transfer correlation (Gunn model), with large slip velocities and moderate void fractions producing high values of h_{gs} and therefore rapid thermal exchange as explained in Section 4.0.2.

Beyond the inlet plane, the temperature field quickly returns to the bulk value of approximately 393 K. The cooling of the solid remains confined to the immediate nozzle region at this early stage, which is consistent with the large volumetric heat capacity of the solid phase ($\rho_s C_{p,s} \approx 1.0 \times 10^6 \text{ J m}^{-3} \text{ K}^{-1}$), limiting the spatial extent of temperature change over short timescales.

Overall, the temperature field at $t = 0.5$ s clearly demonstrates a physically consistent cooling process, in which thermal energy is extracted from the solid phase by the incoming cold gas. The strongest thermal gradients are localised near the nozzle region, while the bulk of the bed remains largely unaffected at this early time.

Void fraction structure.

At $t = 0.5$ s (figure 4.24), the void fraction field shows that the flow is still in an early jet-dominated regime. The vertical slice (figure 4.24a) reveals three narrow jet channels emerging from the nozzle exits, with no coherent rising void structures above the distributor region. The bed remains largely at its initial dense packing ($\varepsilon_s \approx 0.59$).

The inlet-plane distribution (figure 4.24b) confirms the presence of three localised dilute regions at the nozzle positions, with $\varepsilon_s \approx 0.10$ – 0.20 . Outside these regions, the bed is essentially undisturbed, and the jets remain well-defined and symmetric.

The multi-height cross-sections (figure 4.24c) show that void structures are already visible at $y \approx 100$ – 300 mm, although they remain localised and do not yet form coherent bubbles. Above $y \approx 300$ mm, the bed returns to a nearly uniform dense state.

Compared to Case G at the same timestep, the void structures in Case W appear slightly more developed. This behaviour is consistent with the elevated bed temperature, which reduces the gas density and therefore the Archimedes number (Eq. 4.7). As a result, the minimum fluidisation velocity U_{mf} is lowered, increasing the effective excess velocity $(U - U_{mf})/U_{mf}$ for the same inlet flow rate. This promotes earlier void nucleation and slightly deeper jet penetration.

The void fraction field explains the localisation of the cooling observed in the temperature field. The cold gas is confined to the jet channels within the lower 100–200 mm of the bed, limiting the spatial extent of heat extraction from the solid phase. The strongest heat transfer occurs near the nozzle region, where the temperature difference is largest and the gas–solid interaction is most intense.

Within the jet cores, the reduced solid volume fraction limits the available interfacial area a_s , while the jet boundaries provide more favourable conditions for heat exchange. As a result, the cooling of the solid remains highly localised at this stage, and the bulk of the bed retains its initial temperature.

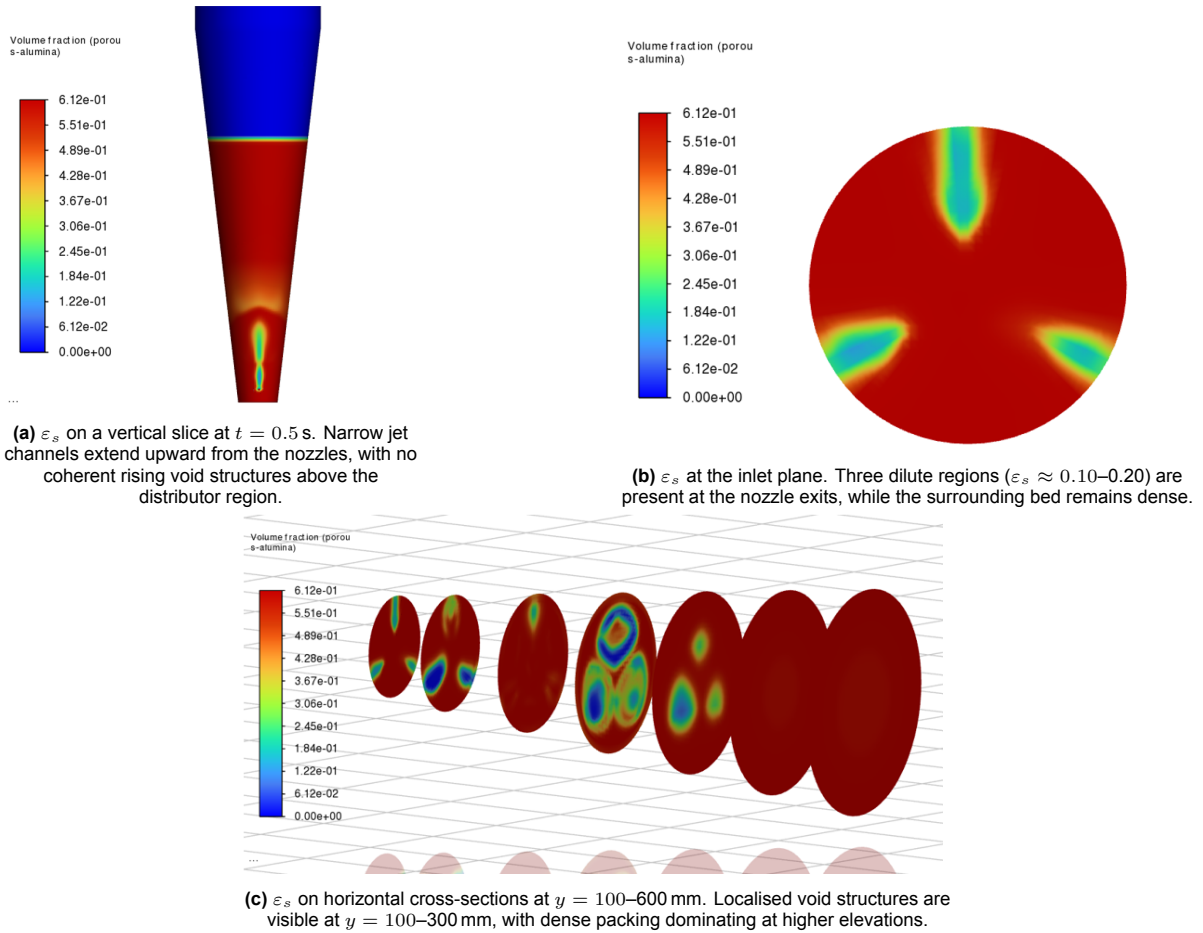


Figure 4.24: Void fraction distributions at $t = 0.5$ s, Case W (cold gas inlet, pre-heated bed), 50 lpm. The flow remains jet-dominated, with slightly enhanced void development compared to Case G.

Axial temperature profile

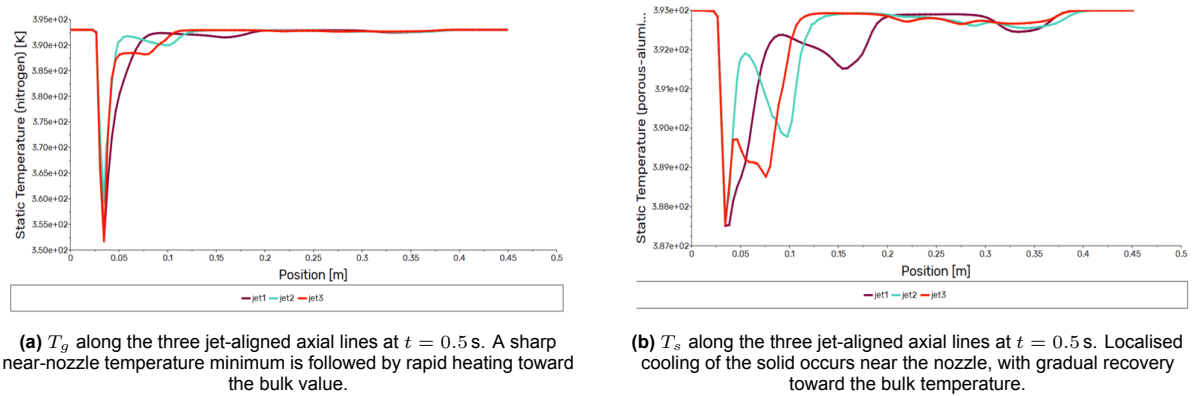


Figure 4.25: Axial temperature profiles at $t = 0.5$ s, Case W (cold gas inlet, pre-heated bed), 50 lpm. The profiles show a strong near-nozzle cooling zone and rapid thermal equilibration along the jet trajectories.

The axial temperature profiles at $t = 0.5$ s (figure 4.25) provide a detailed view of the inverted heat transfer process along the jet trajectories. Both gas and solid phases exhibit a pronounced temperature minimum near the nozzle region, followed by a rapid recovery toward the bulk bed temperature.

In the gas phase (figure 4.25a), a sharp temperature minimum is observed at $y \approx 30$ – 50 mm, where the incoming cold gas first contacts the hot particles. The gas temperature at this point reaches its

lowest value in the domain, after which it increases rapidly as heat is transferred from the solid. Within approximately 100–150 mm, the gas temperature approaches the bulk value of ~ 393 K, indicating very efficient heat uptake.

The solid phase (figure 4.25b) shows a corresponding but less pronounced temperature dip in the same region. The magnitude of the cooling is smaller due to the large volumetric heat capacity of the solid, which resists rapid temperature change. As the gas heats up and the temperature difference decreases, the solid temperature recovers toward the bulk value.

Differences between the three jet trajectories are visible in both phases, particularly in the depth and width of the temperature minima. These variations reflect local differences in hydrodynamics, including jet strength, void fraction distribution, and gas–solid contact patterns.

The location of the primary temperature minimum coincides with the near-nozzle jet region identified in the void fraction field. This confirms that the strongest heat transfer occurs where the temperature difference is largest and the gas first penetrates the bed. Further along the jet trajectory, the diminishing temperature gradient leads to a reduction in the local heat transfer rate.

Overall, the axial profiles demonstrate that the cooling effect is highly localised near the nozzle region and that thermal equilibration between the phases occurs rapidly within a short axial distance. This behaviour is consistent with the high values of the gas–solid heat transfer coefficient predicted by the Gunn correlation under these conditions.

4.3.3. $t = 4.0$ s

Inlet temperature field

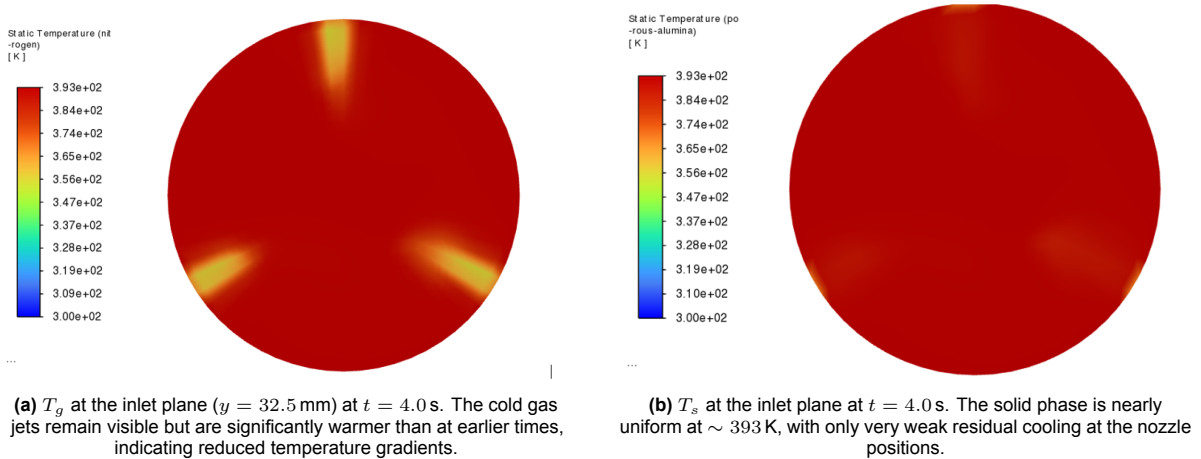


Figure 4.26: Temperature distributions at $t = 4.0$ s, Case W (cold gas inlet, pre-heated bed), 50 lpm. The system approaches a quasi-steady thermal state with reduced temperature gradients.

At $t = 4.0$ s (figure 4.26), the inverted thermal transient has reached a quasi-steady state. The temperature field shows significantly reduced gradients compared to earlier timesteps, indicating that the gas and solid phases have approached local thermal equilibrium in the near-nozzle region.

The gas phase (figure 4.26a) still exhibits three distinct jet structures at the nozzle positions, but the temperature difference relative to the surrounding bed is much smaller than at $t = 0.5$ s. The incoming cold gas is rapidly heated upon entering the bed, and by this stage, the near-nozzle gas temperature has increased substantially, reducing the local driving force for heat transfer.

The solid phase (figure 4.26b) is nearly uniform at $T \approx 393$ K, with only very weak residual cooling visible at the nozzle exits. This indicates that the rate of heat extraction by the gas has decreased, as the temperature difference between the phases has diminished.

Overall, the temperature field reflects a balance between the continuous cooling effect of the incoming gas and the rapid thermal equilibration within the bed. The system has reached a quasi-steady regime

in which temperature variations are small and confined to the immediate vicinity of the nozzle region.

At $t = 4.0$ s (figure 4.26), the inverted thermal transient has reached a quasi-steady state. The temperature field shows only weak spatial variations, indicating that the gas and solid phases have approached local thermal equilibrium in the near-nozzle region.

The gas phase still exhibits three jet structures at the nozzle positions, but the temperature difference relative to the surrounding bed is much smaller than at $t = 0.5$ s. The incoming cold gas is rapidly heated upon entering the bed, reducing the local driving force for heat transfer.

The solid phase is nearly uniform at $T \approx 393$ K, with only very weak residual cooling at the nozzle exits. This reflects the reduced rate of heat extraction as the temperature difference between phases decreases.

Void fraction field.

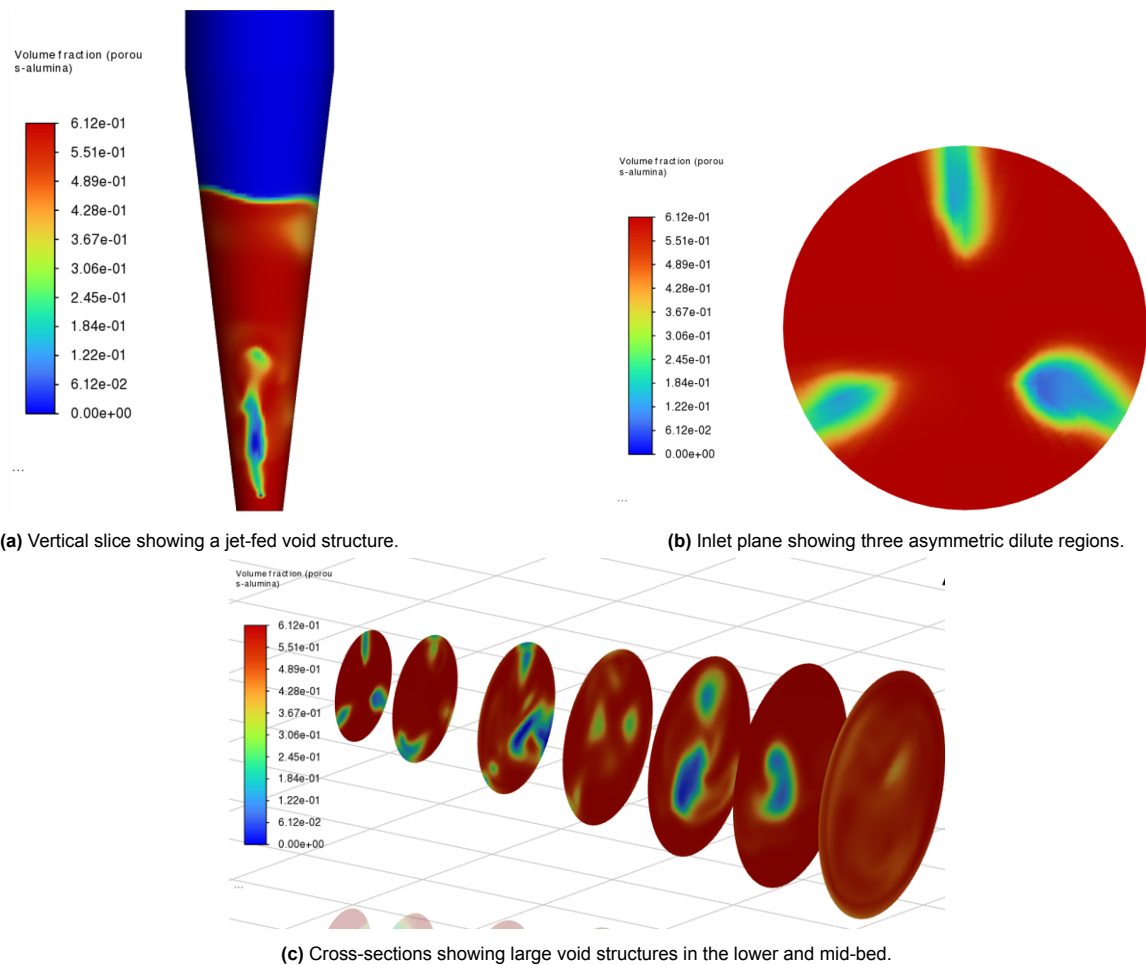


Figure 4.27: Void fraction distributions at $t = 4.0$ s, Case W.

The void fraction field (figure 4.27) shows that the flow has evolved into a developed bubbling regime. The vertical slice reveals a clear two-stage structure: a narrow feeder channel near the distributor feeding into a larger void structure higher in the bed. Between these regions, a denser emulsion zone is visible.

This behaviour is consistent with jet-driven bubble formation, where gas initially penetrates as a jet before expanding into larger void regions. However, the voids remain irregular and asymmetric, indicating that the flow is still unsteady.

The cross-sections confirm that large void structures dominate the lower and mid-bed, while the upper region becomes more diffuse due to the expanding conical geometry and decreasing gas velocity.

The heat transfer between phases is governed by

$$\dot{Q}_{gs}''' = h_{gs} a_s (T_g - T_s), \quad (4.9)$$

where h_{gs} is the gas–solid heat transfer coefficient and a_s is the interfacial area.

At this stage, two key mechanisms limit further cooling:

- The temperature difference ($T_g - T_s$) has decreased significantly as the gas heats up.
- The interfacial area

$$a_s = \frac{6(1 - \varepsilon_g)}{d_p} \quad (4.10)$$

is reduced in the void regions, limiting heat exchange.

The system has now reached a state where hydrodynamics and thermal behaviour are effectively decoupled.

The flow field is highly active, with large void structures and strong gas circulation, but the temperature field remains nearly uniform. This occurs because the cold gas rapidly heats up near the nozzle region, reducing its ability to extract further energy from the solid phase.

In other words, the system becomes self-limiting: the same mechanisms that enhance mixing and transport also reduce the temperature difference and interfacial area available for heat transfer.

As a result, the bed exhibits strong fluidisation behaviour while remaining close to isothermal conditions.

Axial temperature profiles.

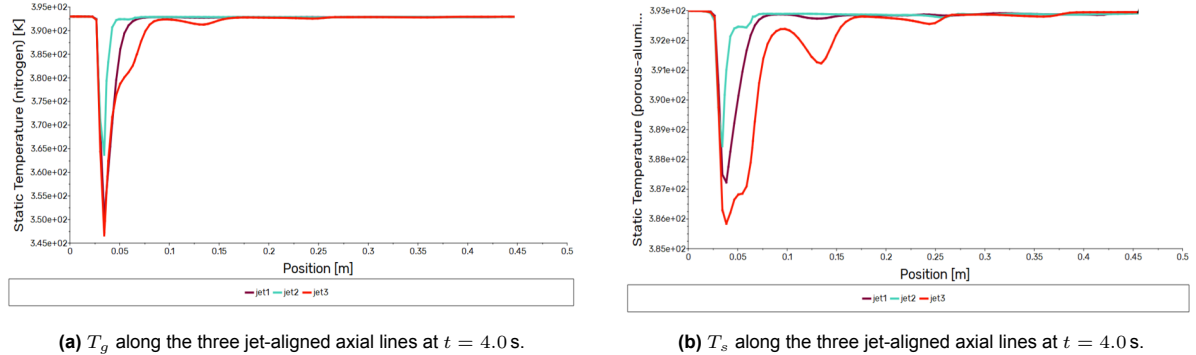


Figure 4.28: Axial temperature profiles at $t = 4.0$ s, Case W (cold gas inlet, pre-heated bed), 50 lpm.

The axial temperature profiles at $t = 4.0$ s (figure 4.28) confirm that the system has reached a quasi-steady thermal regime. Both gas and solid temperatures show strong spatial variations only within a short distance from the nozzle, followed by rapid convergence to the bulk temperature.

In the gas phase (figure 4.28a), a pronounced temperature minimum is observed at $y \approx 30\text{--}50$ mm, corresponding to the entry of cold nitrogen into the bed. The gas temperature rises steeply downstream, approaching the bulk value of ~ 393 K within approximately $y \approx 80\text{--}120$ mm. Beyond this point, the profiles are nearly flat, indicating that the gas has reached thermal equilibrium with the surrounding solid phase.

The solid phase (figure 4.28b) exhibits a corresponding but much weaker temperature depression in the same region. The magnitude of the solid cooling remains limited, reflecting the large volumetric heat capacity of the solid phase, which damps rapid temperature changes. As in the gas phase, the solid temperature recovers to the bulk value over a short axial distance.

Differences between the three jet trajectories are visible in the depth and width of the temperature minima, particularly in the near-nozzle region. These variations are consistent with local hydrodynamic asymmetries, such as differences in void fraction and jet structure, as observed in the void fraction field.

The rapid recovery of both T_g and T_s demonstrates that the characteristic thermal penetration length remains small, even at this later stage. This behaviour can be interpreted using the volumetric heat transfer relation

$$\dot{Q}_{gs}''' = h_{gs} a_s (T_g - T_s), \quad (4.11)$$

where the strong initial temperature difference near the nozzle drives intense local heat transfer. As the gas is heated and $(T_g - T_s)$ decreases, the heat transfer rate drops rapidly, leading to the observed flattening of the temperature profiles.

Overall, the axial profiles confirm that, despite the presence of developed hydrodynamic structures, the thermal response remains highly localised. The system therefore exhibits a clear separation between hydrodynamic and thermal length scales: gas and void structures extend into the bed, but effective heat transfer is confined to the near-nozzle region.

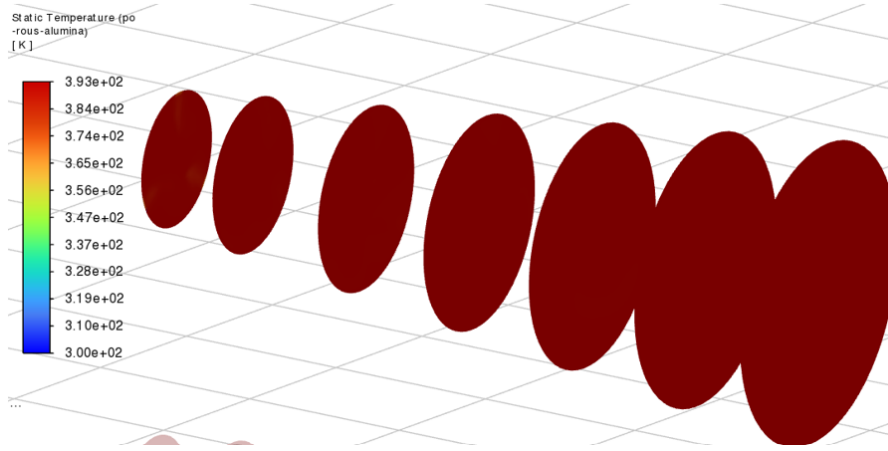


Figure 4.29: Solid-phase temperature T_s on horizontal cross-sections at $y = 100\text{--}600$ mm at $t = 4.0$ s, Case W. The temperature field remains nearly uniform despite the presence of void structures.

Global vs local thermal behaviour. The horizontal cross-sections (figure 4.29) show that the temperature field is essentially uniform across all heights, even in regions where the void fraction field indicates strong hydrodynamic activity. This demonstrates a clear decoupling between flow structure and thermal response.

The absence of measurable temperature variation can be understood from the governing volumetric heat transfer:

$$\dot{Q}_{gs}''' = h_{gs} a_s (T_g - T_s). \quad (4.12)$$

At $t = 4.0$ s, the temperature difference has already been reduced to

$$|T_g - T_s| \lesssim 5\text{--}10 \text{ K},$$

compared to the initial ~ 100 K at $t = 0.5$ s. Even if h_{gs} and a_s remain of the same order of magnitude, this reduction alone decreases the heat transfer rate by at least an order of magnitude.

A simple timescale argument further explains the absence of thermal signatures in the cross-sections. The characteristic thermal response time of the solid phase can be estimated as

$$\tau_s \sim \frac{\rho_s C_{p,s}}{h_{gs} a_s}. \quad (4.13)$$

Using typical values for the present system, $\rho_s C_{p,s} \approx 10^6 \text{ J m}^{-3} \text{ K}^{-1}$ and $h_{gs} a_s \sim 10^6\text{--}10^7 \text{ W m}^{-3} \text{ K}^{-1}$, gives

$$\tau_s \sim 0.1\text{--}1 \text{ s.}$$

In contrast, the residence time of gas within individual void structures is much shorter. For a characteristic void size of $d_b \sim 20\text{--}50 \text{ mm}$, the rise velocity can be estimated by the Davies–Taylor [57]:

$$U_b \approx 0.71 \sqrt{g d_b} \sim 0.3\text{--}0.6 \text{ m s}^{-1}. \quad (4.14)$$

This gives a residence time on the order of

$$\tau_{\text{void}} \sim \frac{d_b}{U_b} \sim 0.05\text{--}0.1 \text{ s.}$$

Since $\tau_{\text{void}} \ll \tau_s$, gas passes through the voids too quickly to induce any measurable change in the solid temperature. As a result, even though void structures are clearly present, they leave no thermal imprint in the cross-sectional temperature field.

From a global perspective, the bed behaves as nearly isothermal. From a local hydrodynamic perspective, however, the system remains highly active. This confirms that, at this stage, thermal behaviour is governed by the exhaustion of the temperature driving force rather than by the instantaneous distribution of the gas phase.

4.4. Effect of Inlet Flow Rate: 50 lpm versus 67 lpm

4.4.1. Near-nozzle thermal response

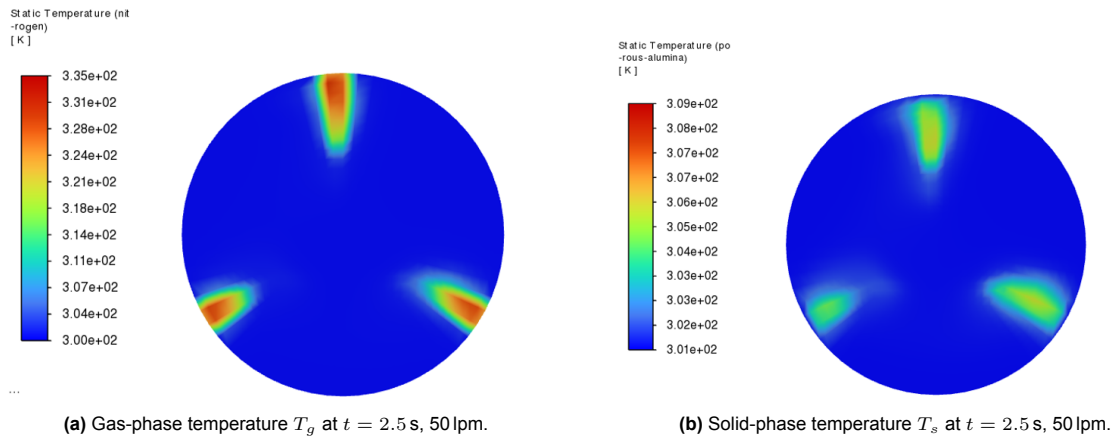
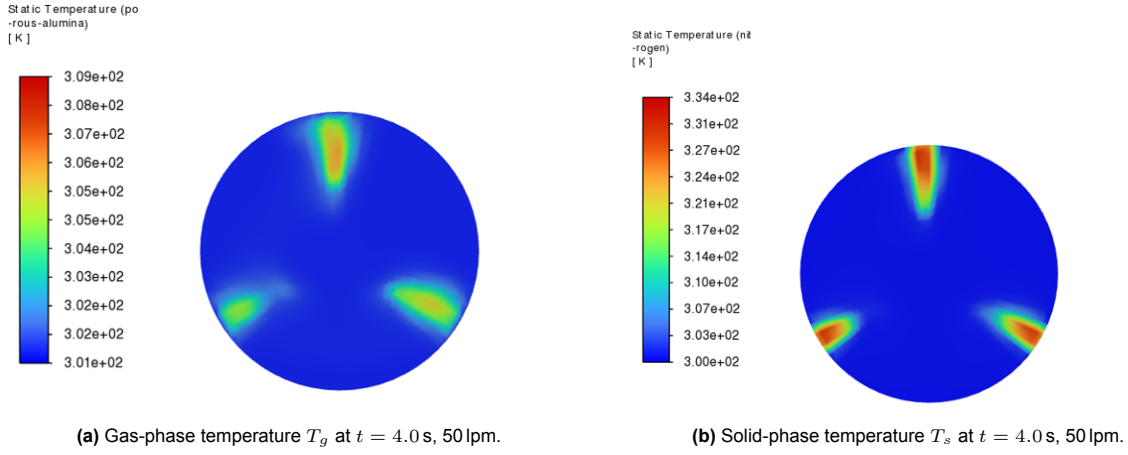
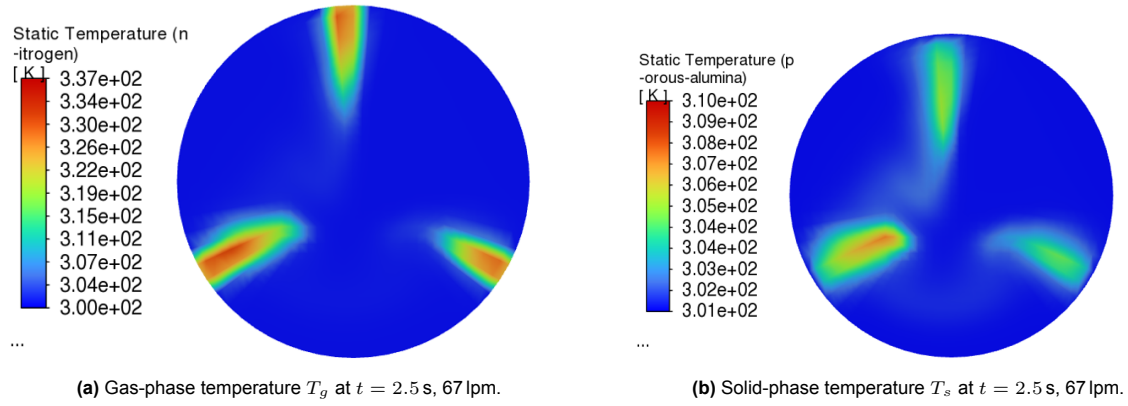
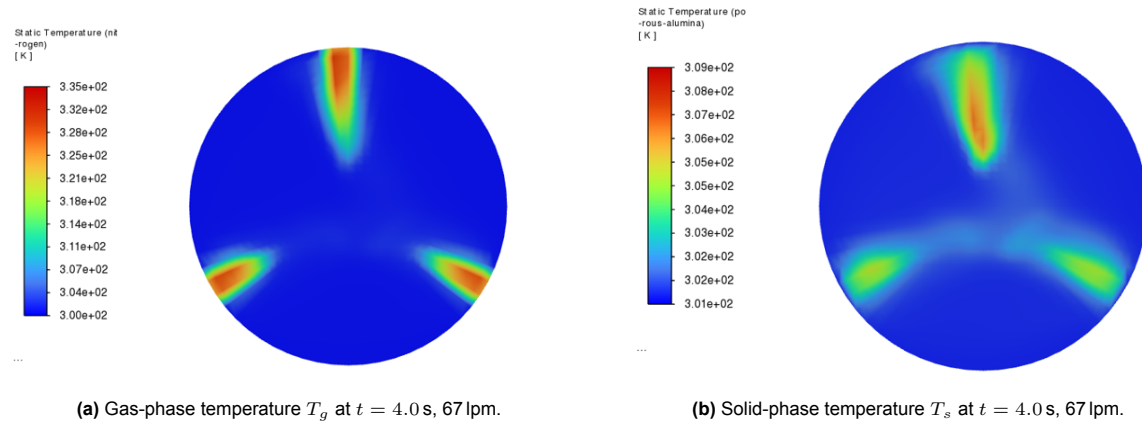


Figure 4.30: Temperature distribution at the inlet plane for 50 lpm at $t = 2.5 \text{ s}$.

Figure 4.31: Temperature distribution at the inlet plane for 50 lpm at $t = 4.0$ s.Figure 4.32: Temperature distribution at the inlet plane for 67 lpm at $t = 2.5$ s.Figure 4.33: Temperature distribution at the inlet plane for 67 lpm at $t = 4.0$ s.

$$\frac{Q_{67}}{Q_{50}} = \frac{67}{50} = 1.34. \quad (4.15)$$

Assuming a constant inlet cross-sectional area, the superficial gas velocity scales linearly with flow rate, giving

$$\frac{U_{67}}{U_{50}} = 1.34, \quad (4.16)$$

which corresponds to a 34% increase in velocity.

The jet momentum flux scales with $\rho_g U^2$, such that

$$\frac{\rho_g U_{67}^2}{\rho_g U_{50}^2} = \left(\frac{U_{67}}{U_{50}} \right)^2 = (1.34)^2 \approx 1.80. \quad (4.17)$$

This corresponds to an approximate 80% increase in momentum flux, assuming similar gas density at the inlet. The quadratic dependence on velocity explains why relatively modest increases in flow rate lead to significantly stronger jet penetration and enhanced hydrodynamic activity. The temperature fields at the inlet plane provide a direct measure of how the inlet flow rate influences local heat transfer in the immediate vicinity of the gas injection points. Figures 4.30 to 4.33 compare the gas- and solid-phase temperature distributions for 50 and 67 lpm at $t = 2.5$ and 4.0 s.

At both flow rates, three distinct high-temperature regions are observed at the nozzle locations, corresponding to the jet cores. The gas-phase temperature reaches peak values of approximately $T_g \approx 333$ – 337 K, while the bulk bed remains close to 300 – 302 K. Importantly, this peak temperature range is essentially unchanged between 50 and 67 lpm, indicating that the maximum thermal intensity of the jet cores is largely independent of inlet velocity.

A similar trend is observed in the solid phase. The solid temperature near the nozzle increases only modestly, typically by a few Kelvin above the background level. This limited response reflects the large volumetric heat capacity of the solid phase, which resists rapid temperature changes despite the strong local heat flux.

In contrast to the temperature magnitude, the spatial distribution of the thermal field shows a clear dependence on flow rate. At 67 lpm, the heated regions exhibit greater axial elongation and a wider radial spread compared to the 50 lpm case. The thermal halos surrounding the jet cores are more diffuse and extend further into the surrounding bed, indicating that thermal energy is transported over a larger volume.

The temporal evolution further shows that, for both flow rates, the near-nozzle temperature field reaches a quasi-steady state by $t = 4.0$ s. The peak temperature and overall structure remain largely unchanged between $t = 2.5$ and 4.0 s, despite continued evolution of the flow field. This indicates that the near-nozzle thermal behaviour is governed by a local balance between convective transport and interphase heat exchange, rather than by cumulative heating effects.

This behaviour can be interpreted using the volumetric gas–solid heat transfer relation

$$\dot{Q}_{gs}''' = h_{gs} a_s (T_g - T_s), \quad (4.18)$$

where h_{gs} is the interphase heat transfer coefficient and a_s is the interfacial area per unit volume. The latter is given by

$$a_s = \frac{6(1 - \varepsilon_g)}{d_p}, \quad (4.19)$$

showing that it decreases as the gas volume fraction increases.

An increase in inlet velocity enhances the gas mass flux and the slip velocity between phases, increasing the particle Reynolds number

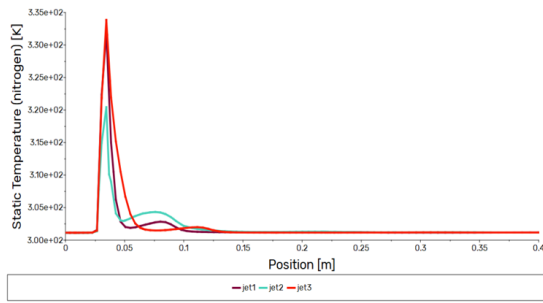
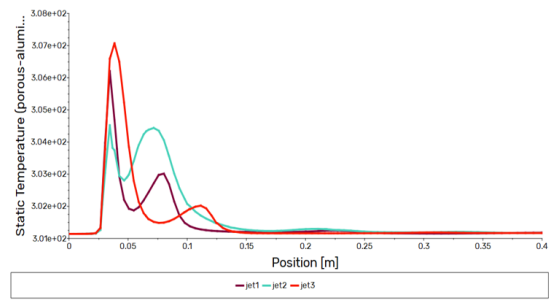
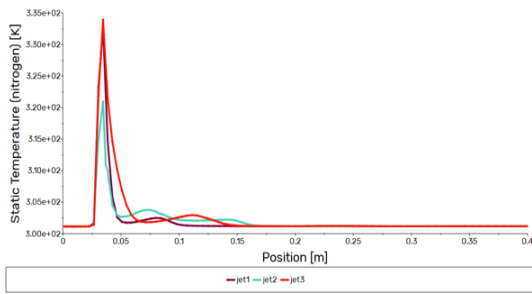
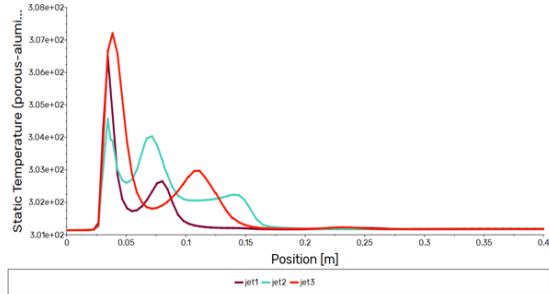
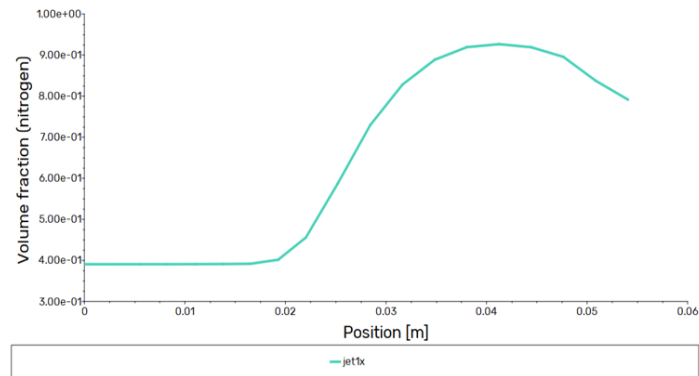
$$Re_p = \frac{\rho_g |\mathbf{u}_g - \mathbf{u}_s| d_p}{\mu_g}, \quad (4.20)$$

and therefore the Nusselt number through correlations such as the Gunn relation. This leads to an increase in the local heat transfer coefficient h_{gs} . However, the same increase in velocity also produces more dilute jet cores, reducing the interfacial area a_s .

As a result, the volumetric heat transfer rate does not scale linearly with velocity. While convective transport is enhanced, the reduction in interfacial area and the short residence time of the gas within the interaction region limit the amount of heat that can be transferred to the solid.

Overall, increasing the flow rate primarily affects the *distribution* of thermal energy rather than its *intensity*. The jets penetrate further and spread heat over a larger region of the bed, but the peak temperature levels remain largely unchanged.

4.4.2. Thermal penetration and spatial extent of jet influence

(a) T_g at $t = 2.5$ s (50 lpm).(b) T_s at $t = 2.5$ s (50 lpm).**Figure 4.34:** Axial temperature profiles at $t = 2.5$ s for 50 lpm.(a) T_g at $t = 4.0$ s (50 lpm).(b) T_s at $t = 4.0$ s (50 lpm).**Figure 4.35:** Axial temperature profiles at $t = 4.0$ s for 50 lpm.**Figure 4.36:** Void fraction along a jet-aligned line at $t = 2.5$ s (50 lpm).

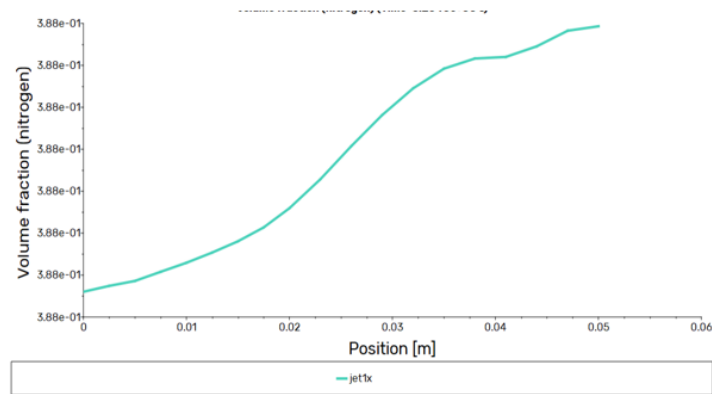
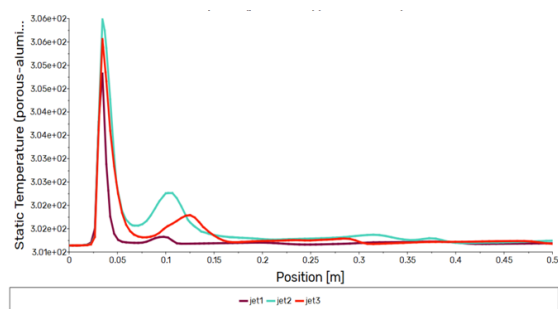
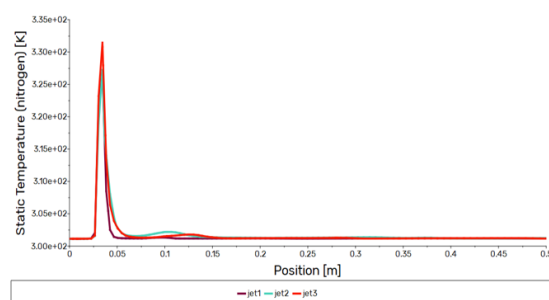


Figure 4.37: Void fraction along a jet-aligned line at $t = 4.0$ s (50 lpm), showing a moderately dilute jet core ($\varepsilon_g \approx 0.85\text{--}0.9$).

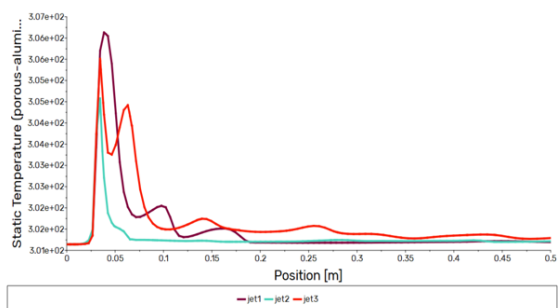


(a) T_s at $t = 2.5$ s (67 lpm).

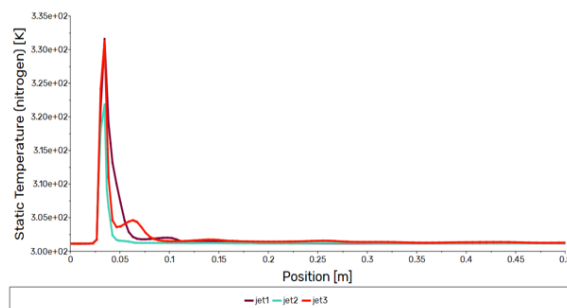


(b) T_g at $t = 2.5$ s (67 lpm).

Figure 4.38: Axial temperature profiles at $t = 2.5$ s for 67 lpm.



(a) T_s at $t = 4.0$ s (67 lpm).



(b) T_g at $t = 4.0$ s (67 lpm).

Figure 4.39: Axial temperature profiles at $t = 4.0$ s for 67 lpm.

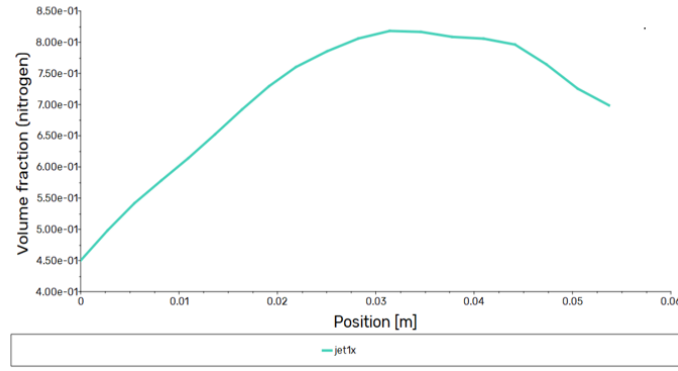


Figure 4.40: Void fraction along a jet-aligned line at $t = 2.5$ s (67 lpm).

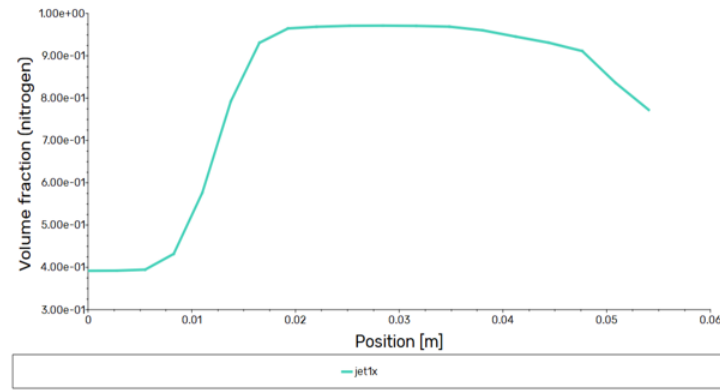


Figure 4.41: Void fraction along a jet-aligned line at $t = 4.0$ s (67 lpm), showing a highly dilute jet core ($\varepsilon_g \approx 0.95$ – 1.0).

The axial temperature profiles provide a quantitative measure of thermal penetration into the bed and its dependence on inlet velocity.

Across all cases, the near-nozzle peak is located at $y \approx 0.04$ – 0.045 m. The gas temperature reaches $T_{g,\max} \approx 332$ – 335 K, while the solid phase reaches $T_{s,\max} \approx 305$ – 307 K. These values remain essentially unchanged between 50 and 67 lpm.

Secondary temperature peaks are observed at $y \approx 0.08$ – 0.15 m, corresponding to regions of reduced heat transfer inside dilute void structures. At 67 lpm, these peaks are broader but do not increase in magnitude.

The thermal penetration depth is defined as

$$y_T = \{y \mid |T_s(y) - T_{\text{bulk}}| < 1\text{--}2 \text{ K}\}. \quad (4.21)$$

For 50 lpm, $y_T \approx 0.15$ – 0.20 m. For 67 lpm, $y_T \approx 0.18$ – 0.22 m, corresponding to only a 10–20% increase despite the higher flow rate.

At $t = 4.0$ s, a weak thermal tail extends to $y \approx 0.30$ – 0.35 m in both cases, with a magnitude of only $\Delta T_s \approx 1$ – 2 K. This indicates cumulative heat transport rather than strong local heat transfer.

The void fraction profiles reveal a key difference between the two flow rates. At 50 lpm, the jet core reaches $\varepsilon_g \approx 0.85$ – 0.9 , whereas at 67 lpm it reaches $\varepsilon_g \approx 0.95$ – 1.0 , indicating a more dilute and gas-dominated structure.

The interfacial area is given by

$$a_s = \frac{6(1 - \varepsilon_g)}{d_p}, \quad (4.22)$$

and therefore decreases significantly as ε_g approaches unity.

The volumetric heat transfer rate,

$$\dot{Q}_{gs}''' = h_{gs}a_s(T_g - T_s), \quad (4.23)$$

is controlled by competing effects. Increasing velocity enhances h_{gs} through higher Reynolds number, but simultaneously reduces a_s and decreases the residence time of the gas within the interaction region.

Although the inlet velocity increases by 34% and the momentum flux by approximately 80%, the thermal penetration depth increases only marginally. This demonstrates that heat transfer is limited by interfacial area and contact time rather than convective transport capacity.

Overall, increasing the inlet flow rate enhances hydrodynamic penetration and gas transport, but does not significantly improve heat transfer to the solid phase. The thermal field remains confined to the lower region of the bed, confirming a clear decoupling between hydrodynamic and thermal behaviour.

4.4.3. Comparison of hydrodynamic and thermal length scales

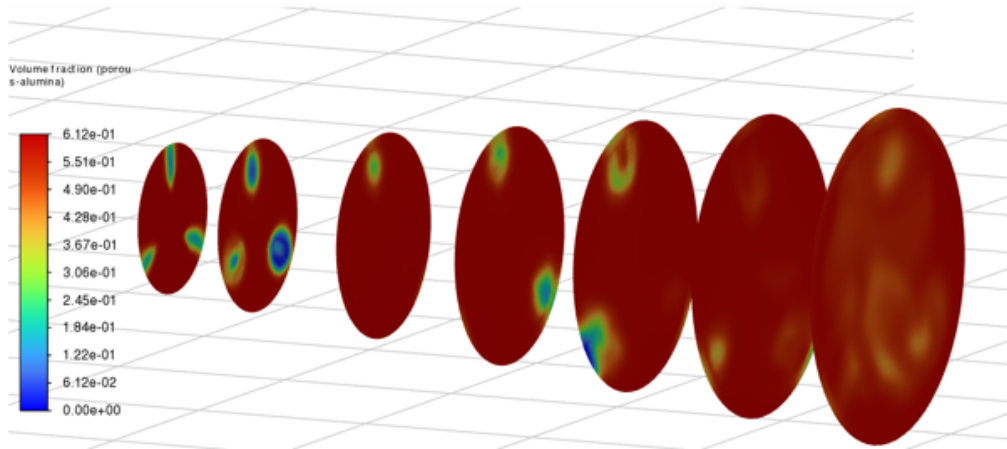


Figure 4.42: Void fraction on horizontal cross-sections at $y = 100\text{--}600$ mm at $t = 4.0$ s (50 lpm). Distinct but relatively compact void structures are visible, with moderate spatial extent and limited coalescence.

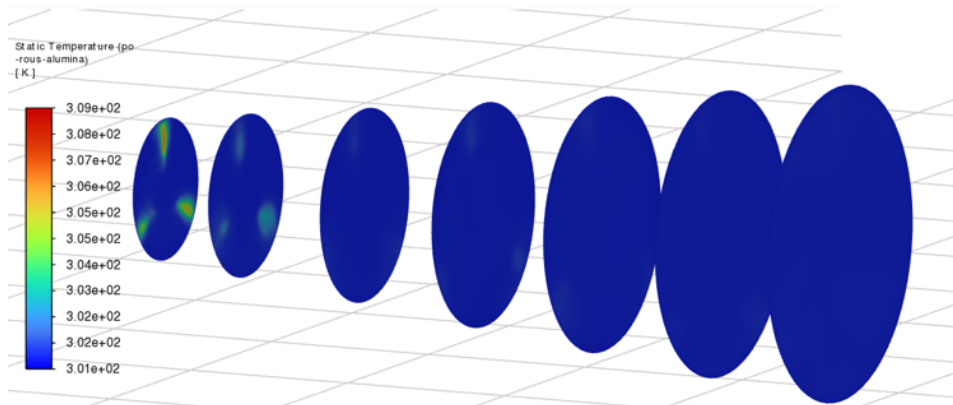


Figure 4.43: Static Temperature on horizontal cross-sections at $y = 100\text{--}600$ mm at $t = 4.0$ s (50 lpm). Mostly homogeneous temperature distribution.

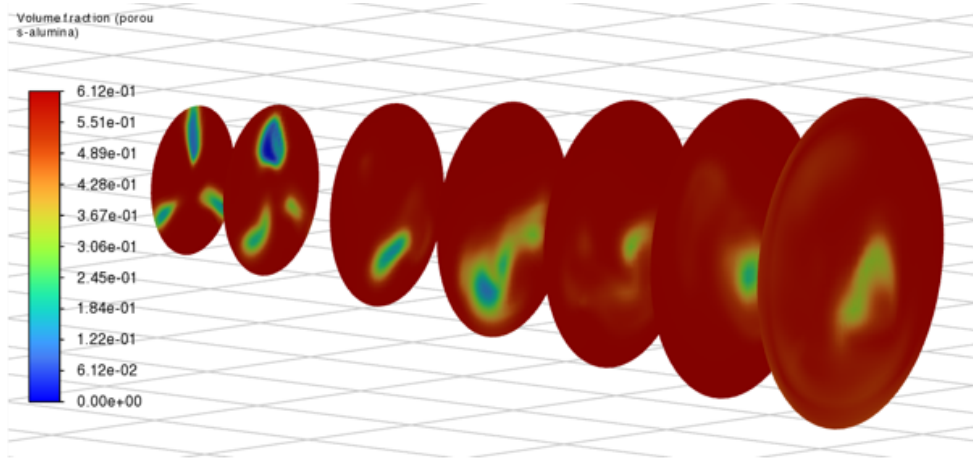


Figure 4.44: Void fraction on horizontal cross-sections at $y = 100\text{--}600$ mm at $t = 4.0$ s (67 lpm). Larger, more diffuse and more numerous void structures are observed, extending further into the upper bed region.

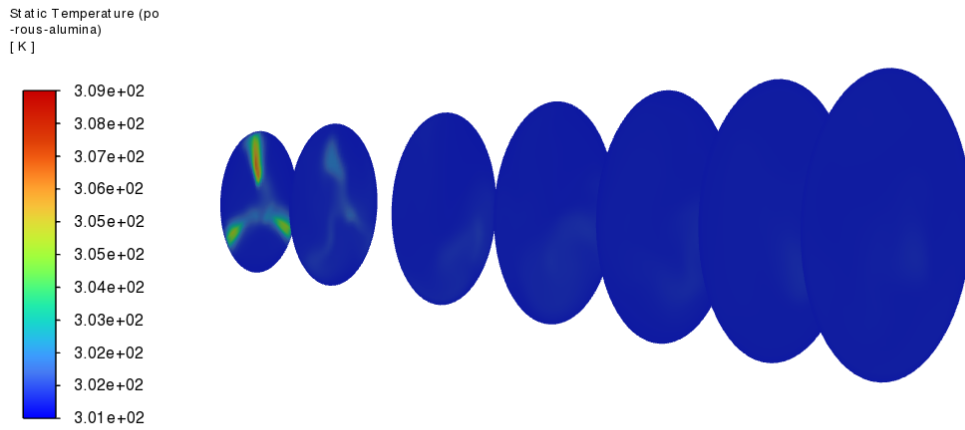


Figure 4.45: VTemperature on horizontal cross-sections at $y = 100\text{--}600$ mm at $t = 4.0$ s (67 lpm)

The effect of inlet velocity on heat transfer can be understood by combining the axial temperature data with the void fraction structure observed in figures 4.42 and 4.44.

From the axial temperature profiles, the near-nozzle peak occurs consistently at $y \approx 0.04\text{--}0.045$ m in all cases. The gas temperature reaches $T_{g,\max} \approx 332\text{--}335$ K, while the solid phase reaches $T_{s,\max} \approx 305\text{--}307$ K. These values are essentially identical for both 50 and 67 lpm, indicating that increasing velocity does not affect the initial heat transfer intensity at the nozzle.

Further along the bed, secondary temperature peaks are observed at $y \approx 0.08\text{--}0.15$ m, with magnitudes of $T_g \approx 302\text{--}304$ K and $T_s \approx 302\text{--}303$ K. These peaks correspond to regions of reduced heat transfer inside dilute void structures. At 67 lpm, these peaks are visibly broader, extending over a larger spatial range, but their amplitude remains within the same 1–2 K band.

The thermal penetration depth can be defined as the location beyond which the solid temperature deviation falls below $\sim 1\text{--}2$ K:

$$y_T = \{y \mid |T_s(y) - T_{\text{bulk}}| < 2 \text{ K}\}. \quad (4.24)$$

From the profiles, the values are:

- 50 lpm: $y_T \approx 0.15\text{--}0.20$ m

- 67 lpm: $y_T \approx 0.18\text{--}0.22\text{ m}$

This corresponds to an increase of only 10–20%, despite the increase in flow rate.

At $t = 4.0\text{ s}$, both cases exhibit a weak thermal tail extending to $y \approx 0.30\text{--}0.35\text{ m}$, with a magnitude of $\Delta T_s \approx 1\text{--}2\text{ K}$. This indicates that heat is transported upward over time, but not transferred efficiently to the solid phase.

The void fraction data explains this behaviour. At 50 lpm, the jet core reaches approximately $\varepsilon_g \approx 0.85\text{--}0.9$. At 67 lpm, this increases to $\varepsilon_g \approx 0.95\text{--}1.0$, indicating a highly dilute, gas-dominated region.

The interfacial area available for heat transfer is given by

$$a_s = \frac{6(1 - \varepsilon_g)}{d_p}. \quad (4.25)$$

Substituting typical values:

- At 50 lpm ($\varepsilon_g \approx 0.9$): $a_s \propto 0.1$
- At 67 lpm ($\varepsilon_g \approx 0.98$): $a_s \propto 0.02$

This represents a reduction in interfacial area by a factor of approximately 5 within the jet core.

The volumetric heat transfer rate is

$$\dot{Q}_{gs}''' = h_{gs} a_s (T_g - T_s). \quad (4.26)$$

While the heat transfer coefficient h_{gs} increases with velocity through the Reynolds number,

$$Re_p = \frac{\rho_g |\mathbf{u}_g - \mathbf{u}_s| d_p}{\mu_g}, \quad (4.27)$$

this increase is not sufficient to compensate for the reduction in a_s and the decrease in gas residence time.

The residence time can be estimated as

$$\tau_{\text{res}} \sim \frac{L}{U}. \quad (4.28)$$

For a 34% increase in velocity, τ_{res} decreases by approximately 25%, further limiting the time available for heat transfer.

The void fraction cross-sections provide direct visual evidence of this mechanism. In figure 4.42, void structures are present but remain relatively compact. In contrast, figure 4.44 shows larger and more diffuse structures that extend to higher bed regions, indicating stronger hydrodynamic penetration.

However, these upper regions do not exhibit corresponding temperature variations. The solid temperature remains nearly uniform beyond $y \approx 200\text{--}250\text{ mm}$ in both cases, despite the clear presence of voids.

This demonstrates that the presence of gas structures does not guarantee effective heat transfer. Instead, heat transfer is limited by the combination of reduced interfacial area and short residence time in highly dilute regions.

Although the inlet velocity increases by 34% and the momentum flux by approximately 80%, the thermal penetration depth increases only marginally. The system therefore operates in a regime where heat transport is efficient, but heat transfer to the solid phase is constrained.

In summary, increasing velocity enhances jet penetration and void formation, but does not significantly improve heat transfer. The gas phase penetrates deeply into the bed hydrodynamically, but its thermal influence remains confined to the lower jet-dominated region.

4.4.4. Mechanistic interpretation of velocity effects

The effect of inlet velocity on heat transfer can be understood through the volumetric interphase heat transfer rate,

$$\dot{Q}_{gs}''' = h_{gs} a_s (T_g - T_s), \quad (4.29)$$

which depends on the heat transfer coefficient h_{gs} , the interfacial area a_s , and the temperature difference between phases.

Increasing the inlet velocity enhances gas transport and mixing, leading to higher slip velocities and therefore an increase in the particle Reynolds number. Through standard correlations (e.g. Gunn), this results in an increase in the heat transfer coefficient h_{gs} . From a purely convective perspective, this would suggest improved heat transfer.

However, the simulations show that this increase in h_{gs} is offset by two competing effects. First, higher velocities produce more dilute jet cores, with gas volume fractions approaching $\varepsilon_g \approx 0.95$ –1.0. Since the interfacial area scales as

$$a_s = \frac{6(1 - \varepsilon_g)}{d_p}, \quad (4.30)$$

this leads to a strong reduction in the available contact area for heat exchange.

Second, the increased gas velocity reduces the residence time within the interaction region,

$$\tau_{\text{res}} \sim \frac{L}{U}, \quad (4.31)$$

thereby limiting the duration of gas–solid contact. As a result, even though the gas is transported more rapidly through the bed, it spends less time exchanging heat with the solid phase.

Finally, the temperature difference $(T_g - T_s)$ is reduced more rapidly at higher velocities due to faster local equilibration in the near-nozzle region. This further decreases the driving force for heat transfer downstream.

These combined effects explain the observed behaviour: although increasing velocity enhances mixing and hydrodynamic penetration, it reduces both the interfacial area and the effective contact time, while also weakening the temperature driving force. Consequently, the overall heat transfer does not scale with velocity, and the thermal penetration remains limited despite significantly stronger flow conditions.

5

CONCLUSION

This thesis investigated heat transfer numerically in a laboratory-scale conical bubbling gas–solid fluidised bed, using an Euler–Euler two-fluid model implemented in ANSYS Fluent 2024 R2. Three heating configurations, gas preheating, wall heating, and combined heating, were examined at 50 lpm and 67 lpm, with the bed initialised at a uniform temperature of 289.15 K. What follows addresses each of the five research sub-questions in turn, drawing directly on the simulation results.

SQ1: How does the predicted transient thermal response differ between heating configurations? The three cases behave quite differently, and in two of them not in the way that was originally intended.

Case G is the only case that ran as designed. With nitrogen injected at $T_{\text{in}} = 393.15$ K and an adiabatic wall, all energy input is carried by the gas. The bed heats from the base upward, and the rate of heating is controlled entirely by interphase convection near the nozzles. The gas drops roughly 56–63 K within the first 32.5 mm of penetration, a direct consequence of the volumetric heat capacity mismatch between phases, $(\rho C_p)_s/(\rho C_p)_g \approx 1125$ at inlet conditions. Near the nozzle, $h_{gs} \approx 4560$ W m⁻² K⁻¹ at $Re_p \approx 352$, which is high, but the volumetric exchange rate is constrained by the reduced interfacial area inside the dilute jet cores: $a_s \approx 5940$ m⁻¹ at $\varepsilon_g = 0.8$, compared with $\approx 17,300$ m⁻¹ in the surrounding dense emulsion. The result is that the gas–solid temperature gap of ≈ 25 –29 K near the nozzle never closes within the simulation window, the jet core is a rate-limited zone not because the heat transfer coefficient is low, but because there simply are not enough particles in contact with the gas to absorb its energy quickly. By $t = 4.0$ s, the bulk solid has risen to only ≈ 307 K, and measurable heating does not extend beyond $y \approx 350$ mm. The upper half of the bed is essentially untouched.

Cases C and W both suffered from the same initialisation anomaly. The fixed-temperature Dirichlet wall condition interacted with Fluent’s hybrid initialiser, causing the wall temperature to propagate domain-wide and overwrite the cold-bed patch before the first physical timestep ran. Both cases started at $T \approx 393.15$ K throughout. In Case C, with both the wall and gas inlet at 393.15 K, the thermal driving force was essentially zero everywhere and the temperature field never evolved. The apparent depressions of -3 to -5 K at the nozzle exits are phase-averaging artefacts, not physical cooling, they track the local void fraction distribution rather than any real heat exchange. The hydrodynamics of Case C are physically real, but its thermal content is absent.

Case W is more interesting, despite the same error. Because the gas inlet remained cold at 289.15 K, the pre-heated solid was immediately exposed to a 104 K temperature difference in the wrong direction: the cold nitrogen extracts heat from the bed rather than delivering it. This inverted transient is physically consistent and clearly visible in the data, the solid cools by 15–25 K at the nozzle positions at $t = 0.5$ s, and the gas heats by ≈ 50 –60 K within 60 mm of bed penetration. By $t = 4.0$ s the driving force has largely exhausted itself: $|T_g - T_s| \lesssim 5$ –10 K and the temperature field is nearly uniform. The system becomes self-limiting in the sense that the same mixing and transport that sustain high h_{gs} values also dilute the interfacial area and reduce the temperature difference. What Case W does not tell us

is anything about heating a cold bed through the wall, that transient still needs to be run with the initialisation fixed.

Across all three cases, the solid phase responds slowly. The characteristic thermal response time $\tau_s \sim \rho_s C_{p,s} / (h_{gs} a_s)$ is on the order of 0.1–1 s, which is comparable to or longer than both the gas residence time and the bubble transit time. This scale separation is the dominant reason why the upper bed remains cold in Case G and why the near-nozzle driving force depletes so quickly in Case W.

SQ2: How does superficial gas velocity influence heat transfer? Going from 50 to 67 lpm raises the inlet velocity by 34% and the jet momentum flux by roughly 80%, the quadratic scaling with velocity makes even modest flow rate increases hydrodynamically significant. Yet the effect on heat transfer is surprisingly weak.

The near-nozzle gas temperature peaks at $T_{g,\max} \approx 332\text{--}337\text{ K}$ and the solid at $T_{s,\max} \approx 305\text{--}307\text{ K}$ in both cases. These values are essentially the same, which at first seems odd given the large difference in momentum input. The explanation lies in the structure of the jet core. At 50 lpm, $\varepsilon_g \approx 0.85\text{--}0.9$ in the core; at 67 lpm, it reaches $\approx 0.95\text{--}1.0$. Since $a_s = 6(1 - \varepsilon_g)/d_p$, this dilution reduces the available interfacial area by a factor of roughly five within the core. The higher h_{gs} from the increased Re_p is not sufficient to compensate. The gas residence time in the interaction zone also drops by $\approx 25\%$, which further limits contact time with the solid.

The thermal penetration depth, defined here as the axial height where $|T_s - T_{\text{bulk}}| < 2\text{ K}$, increases from $y_T \approx 0.15\text{--}0.20\text{ m}$ at 50 lpm to $\approx 0.18\text{--}0.22\text{ m}$ at 67 lpm. That is a 10–20% improvement for an 80% increase in momentum flux, which is a poor return. At $t = 4.0\text{ s}$ both cases show a faint thermal tail extending to $y \approx 0.30\text{--}0.35\text{ m}$ with magnitudes of only 1–2 K, this looks more like heat diffusing upward cumulatively over time than any active convective mechanism. The hydrodynamic penetration depth, by contrast, extends to $y \approx 500\text{ mm}$ in both cases. The gap between where the gas structures go and where the thermal effect reaches is striking and probably the clearest demonstration in this dataset that void fraction and heat transfer are not simply correlated. Gas can penetrate deep into the bed hydrodynamically while contributing almost nothing thermally, if its interfacial area and contact time are too small.

Whether the two flow rates bracket an optimal operating point or both sit on the same inefficient side of the $\dot{Q}_{gs}''' - U$ curve cannot be determined from these results alone. That question warrants a more systematic parametric study.

SQ3: To what extent can UDFs and UDM variables reliably extract local heat transfer metrics? In short: they could not, at least not in ANSYS Fluent 2024 R2 using the approaches tried here.

A UDF written to compute and store $\dot{Q}_{gs}'''$ and h_{gs} via `DEFINE_PROPERTY` and `DEFINE_SOURCE` compiled without problems and then crashed the solver at the first timestep, every time, across multiple fresh attempts. The most likely cause is that the Eulerian multiphase framework in this version does not expose phase-specific cell data for energy source terms to user code, but it was not possible to confirm whether this is a deliberate restriction or a bug, since the error message gave little to work with. A second attempt with `DEFINE_ADJUST` and User-Defined Memory registers also compiled cleanly, but the stored values reset to zero at every new timestep. This suggests the hook executes at the wrong point in the Eulerian phase-coupling loop, or that the solver overwrites UDM data during its internal coupling iterations. Either way, no useful field data came out of it.

The practical consequence is that h_{gs} , $\dot{Q}_{gs}'''$, and local Nusselt numbers were reconstructed analytically from the Gunn correlation using the reported ε_g and velocity fields, rather than extracted directly. This is a reasonable fallback, but it assumes quasi-steady local conditions at the snapshot instants and misses the transient fluctuations associated with bubble passage. It also means the thermal interpretation relies on post-processing the temperature and void fraction fields together, which is more indirect than one would want.

The most straightforward fix within the existing framework is an offline post-processing step: export cell-level ε_g , $|\mathbf{u}_g - \mathbf{u}_s|$, and T_g at each timestep of interest, then reconstruct h_{gs} and $\dot{Q}_{gs}'''$ externally using the Gunn expression. This requires no solver modification. A migration to OpenFOAM, where

all computed field variables are accessible by design, would be the more complete solution but is a substantially larger undertaking.

SQ4: How do the hydrodynamic features of the conical geometry influence the distribution of heat transfer? The conical geometry has a clear and measurable impact, and it is probably the most physically interesting finding of this study.

Because the cross-sectional area $A(y)$ grows with height, the local superficial velocity decays as $U(y) = Q/\pi(r_0 + y \tan \gamma)^2$. The bed therefore never operates at a single fluidisation regime: the lower section, near the nozzles, runs well above U_{mf} with vigorous bubbling and active interphase convection; the upper section approaches U_{mf} and transitions toward a denser, conduction-influenced state. This stratification is directly visible in the void fraction data. Coherent bubble structures with $\varepsilon_g \approx 0.95$ –1.0 dominate the lower 200 mm, become progressively diffuse above 400 mm, and effectively disappear above 500 mm, all within the same reactor, at the same instant. The temperature fields reflect exactly this pattern: heating is strong and rapid near the nozzles, and essentially absent in the upper cone at $t = 4.0$ s.

The geometry also naturally suppresses slugging. None of the simulations produced slug-like structures, and the void phase remained coherent over at most ≈ 500 mm of a 644 mm bed height, consistent with the expanding cross-section dispersing inter-bubble spacing as the gas rises. This agrees with the behaviour reported by Olazar et al. [49] for conical spouted systems and suggests that the stabilising property extends into the non-isothermal bubbling regime.

The thermal implication of all this is that conventional correlations, most of which were derived for cylindrical beds with uniform distributor injection, are not straightforwardly applicable here. The local heat transfer coefficient varies continuously along the axis, and the transition height between convection-dominated and conduction-influenced behaviour depends on both the flow rate and the cone angle. A spatially resolved model is not just convenient for this geometry; it is probably necessary.

SQ5: What are the principal limitations of the current framework, and which model components most need refinement? Several issues emerged, at different levels of severity.

The most fundamental uncertainty is the interphase heat transfer correlation. The Gunn correlation is applied for $\varepsilon_g \leq 0.8$, which formally covers the emulsion phase at $\varepsilon_s \approx 0.59$, but particle-resolved DNS results suggest errors of up to 35% at solids fractions this high [Sun2015]. Given that the analytical reconstruction of h_{gs} throughout this work uses the Gunn expression, any systematic bias in that correlation feeds directly into every estimate of volumetric exchange rate presented here. The void-fraction switching threshold $\varepsilon^* = 0.8$ between Gunn and Ranz–Marshall is physically motivated but untested by a sensitivity study; it would be worth checking whether the results change meaningfully at, say, $\varepsilon^* = 0.75$ or 0.85.

The initialisation anomaly is a practical rather than a fundamental problem, but it invalidated two of the three heating cases and is worth documenting clearly as a warning for future users of this workflow. Activating the fixed-wall Dirichlet condition only after confirming the cold-bed patch, ideally by writing the initial T_s field to a file before any boundary conditions are set, would prevent a recurrence. The corrected Cases C and W both need to be re-run before the three heating modes can be meaningfully compared.

The short simulation windows ($t \leq 4.0$ s) are a third constraint. The large-bubble cycle period is estimated at $\tau_{\text{cycle}} \approx 1.3$ –1.8 s, meaning only two to three cycles were captured. This is not enough for statistically converged time-averaged temperature fields, which is why instantaneous snapshots were used throughout. Extending to $t \approx 15$ –20 s would be needed before cycle-averaged heat transfer coefficients could be computed with any confidence.

Finally, the effective solid-phase thermal conductivity from the Zehner–Schlünder model is uncertain for porous γ -alumina because the internal porosity reduces k_s substantially below the bulk dense-alumina value. This is likely to matter most in the upper cone where conductive transport becomes the dominant redistribution mechanism, but it was not possible to quantify the effect from the present data.

In terms of priority, the refinement hierarchy that emerges from this work is: (i) the interphase heat transfer correlation at high solids loading; (ii) the initialisation procedure for fixed-wall cases; (iii) the solid- phase thermal conductivity for porous particles; and (iv) the post- processing access to h_{gs} and $\dot{Q}_{gs}'''$, ideally resolved by migrating to an open-source solver or implementing the offline reconstruction described above.

Overall Assessment

The Euler–Euler model captures the qualitative thermal behaviour of this system correctly. Interphase convection near the nozzles drives early heating in Case G, the conical geometry produces a clear and physically consistent axial stratification of both flow and temperature, and the competing effects of increased velocity on h_{gs} and a_s explain the weak dependence of thermal penetration depth on flow rate. These observations are mutually consistent and grounded in the data.

What the model cannot yet provide is reliable quantitative output. The post-processing constraints of ANSYS Fluent 2024 R2, the initialisation anomaly affecting Cases C and W, and the short simulation windows all limit how much can be extracted. The thermal interpretation presented here is therefore qualitative in character, and results should not be used for reactor design purposes without first addressing the corrections outlined above. As a diagnostic tool for understanding the coupling between hydrodynamics and heat transfer in conical fluidised beds, though, the framework is useful, and the gap it reveals between hydrodynamic and thermal penetration depth is, on its own, a result worth reporting.

Declaration of AI: ChatGPT was used to generate $\LaTeX$ code for equations formatting. It was also used for grammar and spelling corrections

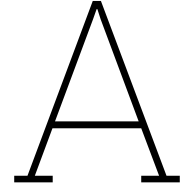
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Determination of Inlet Flow Orientation

A.1. Calculation of Velocity Direction Vectors

The orientation of the inlet velocity vectors was defined to correctly impose the inflow conditions in the CFD simulations. The reactor model features three equally spaced inlet nozzles distributed around the perimeter of the vessel. Each nozzle is inclined at an angle θ relative to the horizontal plane, introducing flow with both vertical and horizontal components.

To express these directions in Cartesian form, the velocity for each inlet was decomposed into its horizontal and vertical components. The vertical contribution corresponds to $\sin \theta$, while the projection within the horizontal plane has a magnitude of $\cos \theta$:

$$\begin{aligned} v_y &= |\vec{v}| \sin \theta, \\ v_{xz} &= |\vec{v}| \cos \theta \end{aligned} \quad (\text{A.1})$$

The azimuthal orientation of each nozzle (ϕ_i) determines the horizontal direction components in the xz -plane, leading to the following generalized vector expression:

$$\hat{\mathbf{v}}_i = \begin{pmatrix} \cos \theta \cos \phi_i \\ \sin \theta \\ \cos \theta \sin \phi_i \end{pmatrix} \quad (\text{A.2})$$

where ϕ_i represents the angular position of the i^{th} inlet, measured from the x -axis. For the present configuration, the three inlets are located at 150° , 30° , and 270° , respectively:

Inlet 1 ($\phi_1 = 150^\circ$)

$$\hat{\mathbf{v}}_1 = \begin{pmatrix} \cos \theta \cos 150^\circ \\ \sin \theta \\ \cos \theta \sin 150^\circ \end{pmatrix} \quad (\text{A.3})$$

Inlet 2 ($\phi_2 = 30^\circ$)

$$\hat{\mathbf{v}}_2 = \begin{pmatrix} \cos \theta \cos 30^\circ \\ \sin \theta \\ \cos \theta \sin 30^\circ \end{pmatrix} \quad (\text{A.4})$$

Inlet 3 ($\phi_3 = 270^\circ$)

$$\hat{\mathbf{v}}_3 = \begin{pmatrix} \cos \theta \cos 270^\circ \\ \sin \theta \\ \cos \theta \sin 270^\circ \end{pmatrix} \quad (\text{A.5})$$

These direction vectors are normalized and subsequently multiplied by the specified inlet velocity magnitude to represent the actual boundary condition in the model. This formulation preserves the geometric symmetry of the system and can easily be adapted for alternative nozzle configurations or inclination angles.

A.2. Gunn vs Ranz-Marshall Source code**A.3. Python Code for Nusselt Number Plot**

The following script was used to generate Figure 2.3, showing the variation of the particle Nusselt number with Reynolds number for different void fractions using the Gunn correlation, together with the Ranz–Marshall correlation for an isolated particle.

Listing A.1: Python script used to generate the Nusselt number comparison plot.

```

1 # -----
2 # Generation of Nu_p vs Re_p plot (Gunn vs -RanzMarshall)
3 # -----
4
5 import numpy as np
6 import matplotlib.pyplot as plt
7
8 # Prandtl number (air/nitrogen at ~20°C)
9 Pr = 0.71
10
11 # Reynolds number range
12 Re_p = np.logspace(-2, 3, 200)
13
14 # Void fractions
15 epsilon_values = [0.4, 0.51, 0.6, 0.8, 1.0]
16
17 # -RanzMarshall correlation
18 def ranz_marshall(Re, Pr):
19     return 2 + 0.6 * Re**0.5 * Pr**(1/3)
20
21 # Gunn correlation
22 def gunn(Re, Pr, eps):
23     term1 = (7 - 10*eps + 5*eps**2) * (1 + 0.7 * Re**0.2 * Pr**(1/3))
24     term2 = (1.33 - 2.4*eps + 1.2*eps**2) * Re**0.7 * Pr**(1/3)
25     return term1 + term2
26
27 # Plot
28 plt.figure(figsize=(7,5))
29
30 for eps in epsilon_values:
31     Nu = gunn(Re_p, Pr, eps)
32
33     if eps == 0.51:
34         plt.plot(Re_p, Nu, linewidth=3, label=f'Gunn ( = {eps} )')
35     else:
36         plt.plot(Re_p, Nu, linestyle='--', label=f'Gunn ( = {eps} )')
37
38 # -RanzMarshall
39 Nu_RM = ranz_marshall(Re_p, Pr)
40 plt.plot(Re_p, Nu_RM, 'k-', linewidth=2, label='-RanzMarshall')
41
42 # Lower bound
43 plt.axhline(2, linestyle=':', label='Nu = 2')
44
45 # Formatting
46 plt.xscale('log')
47 plt.yscale('log')
48

```

```

49 plt.xlabel(r'$Re_p$')
50 plt.ylabel(r'$Nu_p$')
51
52 plt.legend()
53 plt.grid(True, which="both", linestyle=':')
54
55 plt.tight_layout()
56 plt.savefig('fig_gunn_nusselt.pdf', dpi=300)
57 plt.show()

```

A.4. Heat Transfer UDF Source Code

The complete source code of the User-Defined Function described in Section 3.3.1 is listed below. The code was compiled and linked within ANSYS Fluent R2024 [4] using the built-in UDF compiler. All physical constants and model parameters are defined at the top of the file for clarity and ease of modification.

```

1  #include "udf.h"
2  #include <math.h>
3
4  #define EPS_SMALL      1e-12
5  #define D_P            2.02e-4
6  #define T_WALL_FIX    393.0
7  #define EPS_G_SWITCH  0.8
8
9  enum
10 {
11     UDM_RE_SLIP   = 0,
12     UDM_PR_GAS    = 1,
13     UDM_NU_USED   = 2,
14     UDM_H_GS      = 3,
15     UDM_DELTA_T   = 4,
16     UDM_Q_WALL    = 5,
17     UDM_MODEL_ID  = 6
18 };
19
20 static real nusselt_ranz_marshall(real Re, real Pr)
21 {
22     return 2.0 + 0.6 * sqrt(MAX(Re, 0.0)) * pow(MAX(Pr, EPS_SMALL), 1.0/3.0);
23 }
24
25 static real nusselt_gunn(real Re, real Pr, real eps_g)
26 {
27     eps_g = MIN(MAX(eps_g, 0.35), 1.0);
28     Re    = MAX(Re, 0.0);
29     Pr    = MAX(Pr, EPS_SMALL);
30
31     real Pr13 = pow(Pr, 1.0/3.0);
32     real term1 = (7.0 - 10.0*eps_g + 5.0*eps_g*eps_g)
33                 * (1.0 + 0.7 * pow(Re, 0.2) * Pr13);
34     real term2 = (1.33 - 2.4*eps_g + 1.2*eps_g*eps_g)
35                 * pow(Re, 0.7) * Pr13;
36     return term1 + term2;
37 }
38
39 DEFINE_EXCHANGE_PROPERTY(htc_adaptive, c, mixture_thread, i, j)
40 {
41     Thread *t_g = THREAD_SUB_THREAD(mixture_thread, i);
42     Thread *t_s = THREAD_SUB_THREAD(mixture_thread, j);
43
44     real rho_g = C_R(c, t_g);
45     real mu_g  = MAX(C_MU_L(c, t_g), EPS_SMALL);
46     real k_g   = MAX(C_K_L(c, t_g), EPS_SMALL);
47     real cp_g  = MAX(C_CP(c, t_g), EPS_SMALL);
48
49     real du = C_U(c, t_g) - C_U(c, t_s);
50     real dv = C_V(c, t_g) - C_V(c, t_s);
51     real dw = C_W(c, t_g) - C_W(c, t_s);
52     real u_slip = sqrt(du*du + dv*dv + dw*dw);

```

```

53
54     real Re_slip = rho_g * u_slip * D_P / mu_g;
55     Re_slip = MAX(Re_slip, 0.0);
56
57     real Pr_g = cp_g * mu_g / k_g;
58     Pr_g = MAX(Pr_g, EPS_SMALL);
59
60     real eps_g = C_VOF(c, t_g);
61     eps_g = MIN(MAX(eps_g, EPS_SMALL), 1.0);
62
63     real Nu;
64     real model_id;
65
66     if (eps_g > EPS_G_SWITCH)
67     {
68         Nu          = nusselt_ranz_marshall(Re_slip, Pr_g);
69         model_id = 0.0;
70     }
71     else
72     {
73         Nu          = nusselt_gunn(Re_slip, Pr_g, eps_g);
74         model_id = 1.0;
75     }
76
77     real h_gs = Nu * k_g / D_P;
78
79     C_UDMI(c, mixture_thread, UDM_RE_SLIP) = Re_slip;
80     C_UDMI(c, mixture_thread, UDM_PR_GAS) = Pr_g;
81     C_UDMI(c, mixture_thread, UDM_NU_USED) = Nu;
82     C_UDMI(c, mixture_thread, UDM_H_GS) = h_gs;
83     C_UDMI(c, mixture_thread, UDM_MODEL_ID) = model_id;
84
85     return h_gs;
86 }
87
88 DEFINE_ADJUST(wall_delta_t, domain)
89 {
90     Thread *t;
91     cell_t c;
92
93     thread_loop_c(t, domain)
94     {
95         if (!FLUID_THREAD_P(t)) continue;
96
97         begin_c_loop(c, t)
98         {
99             real xc[ND_ND];
100             C_CENTROID(xc, c, t);
101             real L = 0.0;
102             int n;
103
104             c_face_loop(c, t, n)
105             {
106                 face_t f = C_FACE(c, t, n);
107                 Thread *tf = C_FACE_THREAD(c, t, n);
108
109                 if (THREAD_TYPE(tf) == THREAD_F_WALL)
110                 {
111                     real xf[ND_ND], A[ND_ND];
112                     F_CENTROID(xf, f, tf);
113                     F_AREA(A, f, tf);
114
115                     real A_mag = NV_MAG(A);
116                     real nvec[ND_ND];
117                     NV_V(nvec, =, A);
118                     NV_S(nvec, /, MAX(A_mag, EPS_SMALL));
119
120                     real dx[ND_ND];
121                     NV_VV(dx, =, xc, -, xf);
122                     L = fabs(NV_DOT(dx, nvec));
123                     break;

```

```

124     }
125 }
126
127     C_UDMI(c, t, UDM_DELTA_T) = MAX(L, EPS_SMALL);
128 }
129     end_c_loop(c, t)
130 }
131 }
132
133 DEFINE_PROFILE(wall_heat_flux_cpd, t, i)
134 {
135     face_t f;
136
137     begin_f_loop(f, t)
138     {
139         cell_t    c0 = F_C0(f, t);
140         Thread    *tc = F_C0_THREAD(f, t);
141         Thread    *tg = THREAD_SUB_THREAD(tc, 0);
142
143         real k_g    = MAX(C_K_L(c0, tg), EPS_SMALL);
144         real T_cell  = C_T(c0, tg);
145         real delta_t = MAX(C_UDMI(c0, tc, UDM_DELTA_T), EPS_SMALL);
146
147         real q_wall  = k_g * (T_WALL_FIX - T_cell) / delta_t;
148
149         F_PROFILE(f, t, i) = q_wall;
150         C_UDMI(c0, tc, UDM_Q_WALL) = q_wall;
151     }
152     end_f_loop(f, t)
153 }

```

Listing A.2: Heat transfer UDF: Ranz–Marshall interphase coefficient, near-wall thermal boundary layer thickness, and wall heat flux profile.

Declaration of AI: ChatGPT was used to generate $\LaTeX$ code for equations formatting. It was also used for grammar and spelling corrections. Codex was used to debug the code