Quantum Synchronization of Conjugated Variables in a Superconducting Device Leads to the Fundamental Resistance Quantization

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(Received 7 August 2012; published 26 February 2013)

We propose a way to achieve quantum synchronization of two canonically conjugated variables. For this, we employ a superconducting device where the synchronization of Josephson and Bloch oscillations results in the quantization of transresistance similar to that in the (fractional) quantum Hall effect. An LC oscillator is a key component to achieve an exponentially small rate of synchronization errors.

DOI: 10.1103/PhysRevLett.110.097002 PACS numbers: 74.78.Na, 74.25.Gz, 74.81.Fa

One of the most interesting discoveries of the twentieth century was the perfect (fractional) quantization of Hall transresistance in rather imperfect 2DEG semiconducting samples [1]. The resistance as a function of electron density and magnetic field tends to be close to plateaus with values

$$R = \frac{V}{I} = \frac{2\pi\hbar}{e^2}\frac{m}{n}, \tag{1}$$

n, m being integer numbers. The accuracy is so good as to enable numerous metrological applications [2,3]. The physical explanation of the effect is the commensurability of electron density and density of the magnetic flux penetrating the sample, which takes place any time the ratio of numbers of elementary charges and flux quanta in the structure is a rational fraction n/m.

Quantum Hall samples are macroscopic, involving infinitely many degrees of freedom. Shortly after the discovery, Likharev and Zorin [4] hypothesized that similar resistance quantization may occur in a Josephson-junction superconducting device encompassing only a few quantum degrees of freedom. They foresaw it as the result of *synchronization* of two oscillations of complementary quantum variables: Bloch [5] and Josephson [6] oscillations. The Josephson frequency $\omega_J = 2eV_{\emptyset}/\hbar$ is proportional to the average voltage dropping at a part of the device while the Bloch frequency $\omega_B = \pi I_{\emptyset}/e$ is proportional to the average current in another part. A synchronization condition of the two oscillations, $n\omega_J = m\omega_B$, results in

$$R = \frac{V_{\emptyset}}{I_{\emptyset}} = \frac{\pi\hbar}{2e^2}\frac{m}{n}. \tag{2}$$

The resistance quantum is modified in comparison with Eq. (1) manifesting the double charge $2e$ of Cooper pairs in superconductors. Unfortunately, the original device suggestion [4] does not work. The reason for the failure seems fundamental. The quantities to be synchronized, the charge and flux in the device, are canonically conjugated variables. Quantum mechanics forbids them to be simultaneously certain, and the synchronization is expected to be destroyed by quantum fluctuations.

A recent burst of theoretical and experimental activities concerns quantum-coherent phase slips in thin nanowires [7]. On the theoretical side, the concept of the phase-slip junction has emerged [8,9]. Such a junction is exactly dual to a common Josephson junction with respect to charge-flux conjugation. This inspired the proposals of novel superconducting devices [10–12]. Very recently, a phase-slip qubit on InO nanowires has been realized [13]. Relevant experimental developments include observation of the predicted phenomena: phase slips in Josephson junction chains [14,15], Bloch oscillations [16], and charge sensitivity [17].

In this Letter, we demonstrate that combining a phase-slip and a Josephson junction in a single device solves the problem of quantum synchronization. A necessary element of the device appears to be an LC oscillator with high quality factor Q. With this, one can make the rate Γ of synchronization errors exponentially small, $-\ln\Gamma \simeq Q$, thereby achieving exponential accuracy of the resistance quantization. Importantly, the device suggested can also be used as both the voltage and the current standard, thereby closing the metrological triangle [18].

To appreciate the difficulty of quantum synchronization, we consider first a phase-slip and a Josephson junction embedded in a general linear circuit that can be represented with four (frequency-dependent) resistors (Fig. 1). The circuit parts in the dashed boxes represent the Bloch and Josephson oscillators. Let us first consider them separately by setting two coupling resistors $Z_{1,2}$ to $Z_1 = \infty$, $Z_2 = 0$. The Josephson part is then a common [19] Josephson current-biased junction shunted by the conductor G_J. If the bias current exceeds the critical one, $I_b > I_C \equiv 2eE_J/\hbar$, the circuit produces voltage oscillations with frequency $\omega_J = \frac{2eV_{\emptyset}}{\hbar} = \frac{2e}{\hbar G_J}\sqrt{I_b^2 - I_C^2}$, $V_{\emptyset}$ being the time-averaged voltage across the junction. The energy accumulated in the oscillation is of the order of Josephson energy E_J. To have a well-defined semiclassical oscillation, we shall require that the energy accumulated by far exceeds

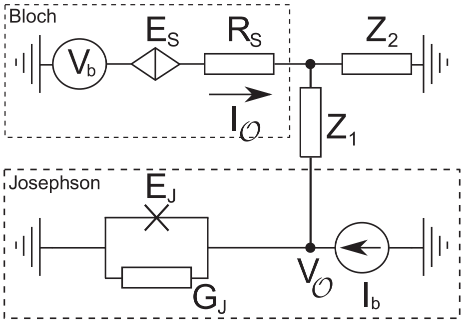

FIG. 1. A general linear circuit embedding a phase-slip (diamond) and a Josephson (cross) junction illustrates the problem of quantum synchronization of the circuit parts (dashed boxes) that generate Bloch and Josephson oscillations. The parts are coupled by the (frequency-dependent) resistors Z_1 and Z_2. The circuit is controlled with voltage and current sources V_b, I_b and provides current and voltage outputs $V_{\mathcal{O}}$, $I_{\mathcal{O}}$. The solution to the problem is presented in Fig. 2.

the quantum frequency scale $\hbar\omega_J$. The latter can be regarded as an effective noise temperature T_J^* characterizing the quantum fluctuations in the circuit (we neglect the thermal fluctuations assuming sufficiently small temperature). The condition $E_J \gg T_J^*$ amounts to $G_J \gg e^2/\hbar$: the conductance must be high at quantum scale.

The Bloch oscillator is made by connecting in series a voltage source, a phase-slip junction, and a resistor R_S. It is dual to the Josephson oscillator upon interchanging the phase and charge [9]. Upon such a transformation, the Josephson junction is replaced by a phase-slip junction, the current bias by the voltage bias, and the parallel conductor becomes a series resistor R_S. Bloch oscillations occur provided the bias voltage exceeds the critical voltage of the junction, $V_b > V_C = \pi E_S/e$. Their frequency, $\omega_B = \frac{\pi I_{\mathcal{O}}}{e} = \frac{\pi}{eR_S}\sqrt{V_b^2 - V_C^2}$, is related to $I_{\mathcal{O}}$, the average current in the junction. To have a well-defined semiclassical oscillation, we shall require that the energy accumulated $\simeq E_S$ by far exceeds the effective noise temperature $T_B^* \simeq \hbar\omega_B$. This gives $R_S \gg \hbar/e^2$: for Bloch oscillations, it is the resistance that must be high at quantum scale.

Let us now couple the circuits. The main effect of the coupling is a transfer of oscillating voltage from the Josephson to the Bloch part, or a transfer of oscillating current from the Bloch to the Josephson part, whereby the voltage or current is multiplied with the amplification coefficient $K(\omega) \equiv Z_2/(Z_1 + Z_2)$. Additionally, the effective resistance or conductance of the Bloch or Josephson part is modified, $\delta R_S = Z_2 Z_1/(Z_2 + Z_1)$, $\delta G_J = 1/(Z_2 + Z_1)$. In order to preserve well-defined oscillations, we require this modification to be small, $\delta R_S \ll R_S$, $\delta G_J \ll G_J$.

We estimate the energy scale E_{cp} associated with the coupling and synchronization of the oscillations as a product of oscillating voltage and current (denoted by a tilde) in each device times the oscillation period, assuming $\omega_B \simeq \omega_J \simeq \omega$, $E_{cp} \simeq \tilde{I}_{\mathcal{O}} K(\omega)\tilde{V}_{\mathcal{O}}/\omega$. It is important to recall that the oscillating quantities are fundamentally related to frequency, $\tilde{I}_{\mathcal{O}} \simeq e\omega$, $\tilde{V}_{\mathcal{O}} \simeq \hbar\omega/e$. With this, $E_{cp} \simeq K\hbar\omega$. A generic estimation for K is $K \lesssim 1$. Indeed, for real impedances $Z_{1,2}$ $K < 1$. In this case $E_{cp} \lesssim T_{B,J}^*$ and the envisaged synchronization in a *general* circuit is destroyed by quantum fluctuations.

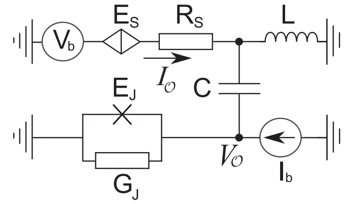

FIG. 2. The proposed quantum synchronization circuit. The resistors $Z_{1,2}$ of Fig. 1 are replaced with a capacitor and an inductor, respectively, forming an oscillator. This results in a big amplification coefficient $K \gg 1$ close to the resonant frequency Ω enabling the quantum synchronization. The dc output voltage and current $V_{\mathcal{O}}$, $I_{\mathcal{O}}$ manifest the quantized transresistance $R = V_{\mathcal{O}}/I_{\mathcal{O}}$.

To overcome this, we need large K. An active amplifying circuit could provide this but brings extra noise that increases the fluctuations. The main idea of this Letter is to use a passive amplifying circuit, an LC oscillator, replacing Z_1 with a capacitor C and Z_2 with an inductor L (Fig. 2). With this, $K(\omega) \gg 1$ near the resonant frequency $\Omega \equiv (LC)^{1/2}$. Assuming that a small real part of Z_2 gives rise to a finite quality factor Q of the oscillator, $K = [2(\omega/\Omega - 1) + iQ]^{-1}$ at $\omega \approx \Omega$. The maximum value of K is thus limited by Q, leading to $E_{cp} \simeq Q\hbar\omega \gg T_{J,B}^*$. We expect the synchronization errors to be related to the activation over this energy barrier and thus to occur at an exponentially small rate $\simeq \exp(-E_{cp}/T^*) \simeq \exp(-\alpha Q)$, α being a coefficient of the order of 1. We stress and prove further that the synchronization takes place in a rather broad interval of frequencies near Ω: the Josephson and Bloch oscillations are thus synchronized with each other rather than with the LC oscillations.

The effective quality factor in our circuit is in fact limited by dissipation in R_S, G_J. The conditions of nonobtrusive coupling $\delta G_J \ll G_J$, $\delta R_S \ll R_S$ imply that $Q \ll \min(G_J z_0, R_S/z_0)$, where $z_0 = \sqrt{L/C}$ is the effective impedance of the oscillator. In fact, the corresponding equality estimates the maximum effective quality factor $Q_m^{-1} = 1/G_J z_0 + z_0/R_S$. The choice $z_0 = \sqrt{R_S/G_J}$ optimizes Q_m to the value $Q_m = \sqrt{R_S G_J}/2$.

The synchronization persists in a finite interval of frequencies $\omega_B(V_b)$, $\omega_J(I_b)$ near the line where those satisfy a given fractional ratio $\omega_B/\omega_J = n/m$. To estimate the width of the interval, we compare E_{cp} with an energy scale characterizing the frequency deviation, which is either $(\Delta\omega_B/\omega_B)E_S$ or $(\Delta\omega_J/\omega_J)E_S$. This leads to $(\Delta\omega_B/\omega_B) \simeq K/(R_S e^2/\hbar)$, $(\Delta\omega_J/\omega_J) \simeq K/(G_J\hbar/e^2)$. We note that, for the limiting Q and at frequencies close to Ω, the width of these intervals may become comparable with the frequency itself, $\Delta\omega_J/\omega_J$, $\Delta\omega_B/\omega_B \simeq 1$.

In the remainder of the Letter, we support these qualitative estimations with quantitative illustrations.

The adequate quantum description of the circuit involves two variables: superconducting phase drop at the Josephson junction $\hat{\phi}$ and dimensionless charge $\hat{q} = \frac{\pi}{e}\hat{Q}$ flown in the phase-slip junction. The action is obtained in the framework of Keldysh action formalism [20] where variables are doubled $\hat{\phi} \to \phi^{\pm}(t)$, $\hat{q} \to q^{\pm}(t)$ corresponding to two parts of the Keldysh contour. It is convenient to use "classical" and "quantum" variables defined as $2\phi, \phi_d = (\phi^+ \pm \phi^-)$, $2q, q_d = (q^+ \pm q^-)$. The total Keldysh action

$$\mathcal{S} = \mathcal{S}_B + \mathcal{S}_J + \mathcal{S}_{cp} + \mathcal{S}_N$$

is contributed by the Bloch and Josephson parts,

$$\mathcal{S}_J = \int dt\left(2E_J \sin\phi \sin\frac{\phi_d}{2} - \frac{I_b}{2e}\phi_d + \dot{\phi}\phi_d\frac{G_J}{4e^2}\right), \quad (3)$$

$$\mathcal{S}_B = \int dt\left(2E_S \sin q \sin\frac{q_d}{2} - \frac{eV_b}{\pi}q_d + \dot{q}q_d\frac{e^2R_S}{\pi^2}\right), \quad (4)$$

the coupling part

$$\mathcal{S}_{cp} = \int\frac{d\omega}{2\pi}\left\{\phi^d_{-\omega}\frac{\delta G}{4e^2}(\dot{\phi})_\omega + q^d_{-\omega}\frac{e^2\delta R}{\pi^2}(\dot{q})_\omega + \frac{K(\omega)}{2\pi}[q^d_{-\omega}(\dot{\phi})_\omega - \phi^d_{-\omega}(\dot{q})_\omega]\right\}, \quad (5)$$

and the noise part $\mathcal{S}_N$ that is quadratic in q_d, ϕ_d and satisfies the fluctuation-dissipation theorem (see Ref. [21] for concrete expressions). The resulting action is nonlocal in time and therefore cannot be treated exactly.

The saddle point equations of the Keldysh action [21] neglect the noise and are the classical circuit-theory equations. To start with, we study these nonlinear equations. This approximation gives a good estimation of the positions and widths of the synchronization domains while disregarding rounding of large and vanishing of small domains. We solve the equations numerically at given V_b, I_b and assess if the solution is periodic. If it is the case, we note the corresponding n, m. We repeat the procedure to scan the V_b, I_b plane and to find the synchronization domains. Typical results are presented in Fig. 3. For this plot, we made (mostly for esthetic reasons) a symmetric choice of parameters $E_S = E_J$, $G_J\hbar\pi/4e^2 = e^2R_S/\pi\hbar$, so that output current and voltage, and correspondingly the oscillation frequencies, are symmetric in the plane of V_b and I_b, $\omega_B(I_b/I_C, V_b/V_C) = \omega_J(V_b/V_C, I_b/I_C)$. On average, these frequencies are close to those of uncoupled oscillators, $\bar{\omega}_B(V_b)$, $\bar{\omega}_J(I_b)$; the deviations are mostly due to synchronization. We observe the domains corresponding to the fractions n/m. They are centered at the curves where $m\bar{\omega}_B(V_b) = n\bar{\omega}_J(I_b)$.

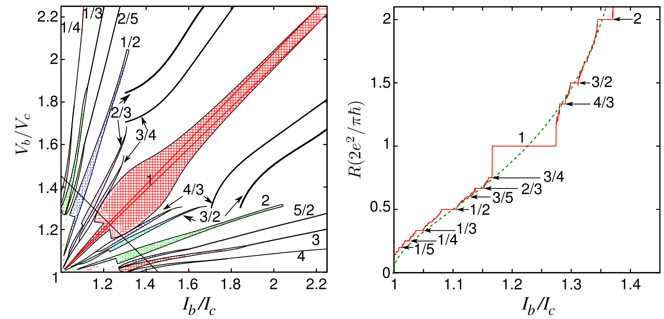

FIG. 3 (color online). Left: Synchronization domains (n/m) in the plane of normalized bias voltage and bias current. Right: Quantized plateaus of transresistance $R = V_{\mathcal{O}}/I_{\mathcal{O}}$ along the cut given by the line in the left-hand figure. Dashed curve: Continuous transresistance as set by uncoupled Bloch and Josephson parts, $R = (\pi\hbar/2e^2)\bar{\omega}_J(I_b)/\bar{\omega}_B(V_b)$.

The widest domain the one with $n = 1$, $m = 1$ and is centered at the diagonal. The domains with higher n, m are increasingly narrower, as is also the case in quantum Hall effect. The parameters are chosen such that the resonant frequency Ω is achieved at $I_b/I_C = V_b/V_C = \sqrt{2}$, where the domains are widest. $R_S = 10\pi\hbar/e^2$ and the oscillator impedance is optimized, $z_0 = \sqrt{R_S/G_J}$, so that $Q_m = \sqrt{R_S/G_J}/2 = 10$. In accordance with the above estimations, the widest synchronization domain spreads at the scale of Ω itself. The widths of the domains decrease at much higher and much lower frequencies $\bar{\omega}_B$, $\bar{\omega}_J$ owing to a decrease of $K(\omega)$. More details and finer steps can be seen in the right-hand panel where the transresistance is plotted along the cut in the $V_b - I_b$ plane showing a typical devil's staircase curve. As a side note, the domains are not precisely single connected; there is a fine structure of small "islands" of the same n, m near each domain. This structure is, however, too fine to be resolved at the scale of the plots.

To address the quantum effects, we restrict ourselves to narrow synchronization domains where a new long time scale $\simeq (\Delta\omega_{B,J})^{-1} \gg (\omega_{B,J})^{-1}$ emerges. Our purpose is to find the rate of synchronization errors Γ with exponential accuracy (Fig. 4). At this time scale, one can disregard the dispersion of quantum noise and amplification coefficient and end up with a local-in-time action which is formally equivalent to that of a classical system subject to a white noise. A similar approach has been applied to narrow Shapiro steps [19]. The slow variables in our case are the *phases* $\theta(t)$, $\Psi(t)$ of Bloch and Josephson oscillations, respectively. With those, the time-dependent current [voltage] is represented as $I_{\mathcal{O}}(t) = I_{\mathcal{O}} + \tilde{I}_{\mathcal{O}}[\bar{\omega}_B t + \theta(t)]$ $[V_{\mathcal{O}}(t) = V_{\mathcal{O}} + \tilde{I}_{\mathcal{O}}[\bar{\omega}_J t + \Psi(t)]]$. We derive the effective action in the vicinity of the point in the $I_b - V_b$ plane where $n\bar{\omega}_J = m\bar{\omega}_B = \omega$ aiming to describe the (n, m) domain (in the formulas for the action, $\hbar = 1$ for compactness).

$$\mathcal{S} = \mathcal{S}_B + \mathcal{S}_J + \mathcal{S}_{cp}; \quad (6)$$

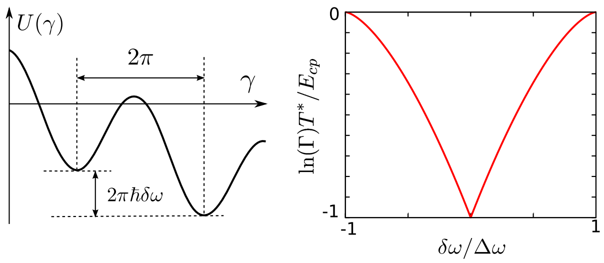

FIG. 4 (color online). Left: Washboard potential for the phase difference γ. The hops over the barriers are synchronization errors. Right: The logarithm of the error rate across the synchronization domain.

$$\mathcal{S}_B = r \int dt[\dot{\theta}\theta_d - iT_B^*\theta_d^2 - (\delta\omega_B)\theta_d], \tag{7}$$

$$\mathcal{S}_J = g \int dt[\dot{\Psi}\Psi_d - iT_J^*\Psi_d^2 - (\delta\omega_J)\Psi_d], \tag{8}$$

$$\mathcal{S}_{cp} = \omega\frac{|K|}{2\pi}\int dt[-A_B\cos(m\theta - n\Psi + \kappa)\theta_d + A_J\cos(m\theta - n\Psi - \kappa)\Psi_d]. \tag{9}$$

Here, $\mathcal{S}_{B,J}$ describe the Brownian motion of the phases in the absence of the coupling, with g, $r \gg 1$ being $g \equiv (\hbar/4e^2)(dI_b/dV_{\mathcal{O}})$, $r \equiv (e^2/\pi^2\hbar)dV_b/dI_{\mathcal{O}}$ the dimensionless differential conductance and resistance, respectively, and $T_{J,B}^* \simeq \hbar\omega$ the effective noise temperatures that depend on the bias current and voltage. $\mathcal{S}_{cp}$ gives the energy ($\simeq \hbar|K|$) gained by synchronization and $\kappa \equiv \arg(K)$. The coefficients $A_{B,J}$ depend on I_b, V_b as well as on n, m. We concentrate on the relevant variable $\gamma = m\theta - n\Psi$ to reduce the action to the form

$$\mathcal{S} = \int dt[a(\dot{\gamma}\gamma_d - iT^*\gamma_d^2 - \delta\omega) - E_{cp}\gamma_d\sin\gamma]. \tag{10}$$

Here, the susceptibility $a = gr/(gm^2 + rn^2)$, noise temperature $T^* = (T_B^* m^2 g + T_J^* n^2 r)/(gm^2 + rn^2)$, the energy barrier $E_{cp} = \hbar\omega|A_B nrK + A_J mgK^*|/(gm^2 + rn^2)$, and $\delta\omega = m\delta\bar{\omega}_B - n\delta\bar{\omega}_J$. This action is formally equivalent to that of an overdamped particle moving in a trapping washboard potential $U(\gamma) = -E_{cp}\cos\gamma - \gamma\hbar a\delta\omega$ (Fig. 4) and being subject to thermal noise. If we neglect the noise, the motion obeys $a\dot{\gamma} + \partial U(\gamma)/\partial\gamma = 0$. The stationary solutions of this equation where γ is trapped in one of the minima correspond to the synchronization of the oscillations. They occur within a strip $|\delta\omega| \equiv E_{cp}/\hbar a$, in accordance with the estimations made. Beyond the strip, γ increases with time corresponding to two unsynchronized frequencies.

The synchronization errors are thermally activated hops between the neighboring minima and their rate governs the accuracy of the resistance quantization. To estimate this rate one needs to compute the energy barrier separating the minima and the effective temperature T^*. Clearly, this rate is exponentially small, $\Gamma = \exp(-E_{cp}/T^*)$, in the center of the synchronization domain. This guarantees the high quality of the resistance quantization. The rate increases towards the strip edge owing to the lowering of the barrier in the washboard potential, $\ln\Gamma = -(E_{cp}/T^*)[(1 - y^2) + y\arccos(y)]$ (see Fig. 4), $y \equiv |\delta\omega|/\Delta\omega$. The coefficient $E_{cp}/T^* \simeq K$ depends on the bias current and voltage, as well as on n, m. We provide extensive illustrations of this dependence in Ref. [21].

In fractional quantum Hall effect, the excitations at the background of a certain plateau bear fractional charge or flux. The synchronization errors may also be considered as excitations at the background of a synchronization domain. One might conjecture that *extra* charge or flux induced by a hop over the barrier is fractional: this would be the case if the 2π change in γ is equally split between the two phases θ, Ψ. In fact, the situation is more complex since the hop takes a relatively long time $\simeq a\hbar/E_{cp}$ during which the charge and flux (related to the superconducting phase difference ϕ) may fluctuate. Owing to this, the average extra charge and flux transferred in the course of a hop do not exhibit a strict quantization,

$$\frac{\delta q}{2e} = \frac{mg}{gm^2 + rn^2}, \qquad \frac{\delta\phi}{2\pi} = \frac{-nr}{gm^2 + rn^2}. \tag{11}$$

However, in the limit $g \gg r$ the extra charge approaches fractional values $2e/m$, while the extra flux approaches $-1/n$ in the opposite limit.

In conclusion, we have proven the feasibility of synchronization of quantum conjugated variables. The superconducting device suggested shall manifest a quantum-Hall-like (fractionally) quantized transresistance owing to the synchronization of Bloch and Josephson oscillations. The high amplification coefficient required for stable synchronization is achieved by using an LC resonator with high quality factor Q. The minimum synchronization error rate is shown to be exponential in Q.

We acknowledge fruitful discussions with J. E. Mooij, K. K. Likharev, and A. V. Zorin. Our research was supported by the Dutch Science Foundation NWO/FOM.

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