Free Form Structural Design

Schemes, Systems & Prototypes of Structures for Irregular Shaped Buildings

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FREE FORM STRUCTURAL DESIGN

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1 Introduction

1.1 Background

Beware! The Blob. This frightening film title from 1972 applied to a gelly mass coming from space, but did also apply to building industry in the 1990's, when they were requested to construct freely curved building shapes. By that time building structures were mostly constituted out of planar elements, often in an orthogonal arrangement at regular intervals.

Although non-orthogonal building designs exist since long – just think of prehistorical caves, nomad's tents, igloo's and renaissance vaults – the creation of curved architectural designs received a strong boost with the availability of computerized designs tools in the 1990's. These tools were introduced in the domain of architectural design through a technology transfer from film-, car and aeroplane industry. At first only for those architects that were eager to experiment, they later became commonly available. User-friendliness of modelling software and the ease of transforming conventional shapes stimulated the search for new formal expressions. The outcomes were animations, which in a frozen state depicted curved lines and irregularly (double) curved surfaces, Greg Lynn being among its pioneers (Lynn 1999).

Although obviously the virtual three-dimensional modelling tools as used in film industry were never meant to transform a real world, the whole chain of disciplines involved in design and construction nevertheless found itself confronted with curved forms to be materialised. Different from architects doing proposals, structural designers were constrained by manufacturing techniques, their analysis software, budgets and lack of experience. Buildings constructed in the last decade still prevailingly have orthogonal layouts whereas their designs do not feature any straight line in their envelope form.

This discrepancy may be covered since in parallel with the software that enabled architects to design curved shapes, technology for design and production developed as well. Some automated production methods now allow producing numerous different parts for the same cost as of identical elements. Such advancements release the constraint that manufacturing capabilities previously imposed. As a consequence construction now is challenged with new structural layouts.

1.2 Problem description

Although computer renderings showed perfectly smooth curving building envelopes seamlessly flowing into each other, reality of construction was less smooth and had to overcome numerous obstacles during engineering and construction, as well as the integration of contributions from all parties involved. One of the principal problems was related to the difficulty of gaining insight in the structural action of complex three-dimensional layouts, when precedents are few and rules of thumb are not yet established (Wagner and Bögle 2002). Transfer of geometry generated in the architect's design tool into structural engineering analysis-software was problematic and often resulted in simple geometrical data without relational or dimensional information (Koster 2003). Referring to the complications of production, exclamations of 'fluid design nightmares' are reported (Eekhout and Lockefeer 2004). Opposed to this are the results of a study group on the 'freedom of structural shape', operating within the Dutch engineering consultancy ABT responsible for the engineering of several Blob-shaped structures. It concludes with a plea for renewed application of shell-like structural typologies in shapes featuring curved surfaces – despite the numerous practical problems to be solved – as an alternative to trite structural solutions, typically consisting of large frames acting in bending (Jorissen, Wenting et al. 2002).

This value judgement hints the existence of a criterion of appropriateness, where qualifications like 'ingenious', 'clever' and 'original' would be opposites to 'trite'. Though seldom made public, the apparently unsatisfying structural quality is heard in other cases too – despite proper fulfilment of structure's indisputable requirements of providing strength, stiffness and stability. For instance in the case of the Guggenheim museum in Bilbao (Spain) completed in 1997 (architectural design by Frank Gehry, structural engineering by Skidmore, Owings and Merrill), where despite being hidden in between the architecturally defined curving interior and exterior skin, the museum's polygonal structure is often criticised for its lack of ingenuity. However, when it was publicly criticised in a paper (Yun and Schodek 2003) that its structure 'did not conform to the shape', 'resulting in both architectural incongruities and undesirable complexities in connections and needs for additional systems', this was corrected by the project's structural engineers in (Iyengar, Sinn et al. 2004), stating that the applied system actually resulted from a well-thought definition of its requirements, which included the reduction of cost and complexity through the application of straight elements and a single principle for its connections. As a solution to the problem Yun and Schodek observed in the Guggenheim Museum, in their paper they proposed a (one) novel structural system able to follow a building's curvature neatly.[1] As the proposed solution consists of curved elements, it was of the same formal vocabulary as the building itself.

Formal anticipation of the structure to the building shape has so far been achieved either by the development of custom structural systems in a one-project setting (e.g. the systems developed for the Guggenheim Museum mentioned previously or the National Stadium[2] for the 2008 Olympics in Beijing, China), or by the application of systems allowing for customisation per project. Such systems are commercialised by manufacturers (e.g. the single-layered triangulated surface structures by the German firm Mero-TSK (2006)).

Apart from such large projects or integrated facilities, technological progress was made within individual disciplines, but this did not necessarily contribute to the constructability of the building as a whole since the other disciplines were unaware of production technologies. Hence, advances in projects of moderate budgets seem to be limited: they are still being designed with conventional means encountering numerous practical deficiencies. Furthermore, architect, (structural) engineer and contractor still operate in consecutive steps, as they did before on regular projects. This single-disciplinary-approach has not been beneficial for the integration of the structural shape and the architectural shape of free form designs.

To summarize, the engineering discipline, the structural engineers being part of this, and building industry experienced a delay on the advancements made in architectural design, notably in those concerning irregularly curved building shapes. With its conventional structural principles, tools and procedures, structural designs demonstrated a lacking anticipation to the architectural shape. The need to do so is felt, but an overview of potential structural systems that would be supportive in attaining this goal is missing. Such an overview should link structural functioning to geometrical definition and way of production, including the constraints that are involved.

1.3 Mission

This thesis proposes an array of structural systems through an alternative structural design approach. The approach integrates constraints and degrees of freedom of all involved parties and the tools they use. Anticipation to production technologies in the design phase, as well as the engineering that follows, may open up the range of structural design solutions. To do so, the essential constraints from basically all design phases, must be integrated in the structural design proposal. To facilitate this, a definition of requirements on a high level of abstraction is proposed. With the set of requirements, highly abstract structural schemes are to be designed. These schemes will subsequently be elaborated into structural systems that include more specifications, and finally as structural designs for a specific application. Through this proposal, criticalities can be solved on a high level, after which implementations are expected to

[1] The system is included in the overview of precedents in chapter 3.
[2] The Chinese National Stadium is included in the overview of precedents in chapter 3.

pass on fluidly. With a systematic approach, ad-hoc solutions that involve extensive labour can be minimised.

This approach results in the following research question for the actual research:

- What are the most appropriate structural schemes, structural systems and structural designs for free form (parts of) buildings?

The denominations of 'most appropriate', 'structural schemes, structural systems and structural designs' and 'free form buildings' frame the research topic, and are defined as follows:

Most appropriate comprises optimally meeting three criteria which are: highest degrees of systematisation, formal freedom and material efficiency. The criteria for evaluation will be developed further in chapter 4, where it will also be shown that these criteria conflict.

Structural schemes, structural systems and structural designs are three levels of abstraction that are passed through when a design evolves from an abstract definition of principles (in scheme) to a materialised and specified implementation for a specific context (a design). A scheme can have many materialised applications in systems, which in turn can be implemented in an array of structures for a specific site under specific conditions.

Free form buildings are defined as building shapes that are double curved, which do not feature repetition of elements and of which the shape is not structurally optimised. The latter characteristic is included to make regular shell- and membrane structures fall outside the scope of this research, as they are a well-covered field of research already since decades. Rather than on the building shape as a whole, this research focuses on the meso- and micro level of scale.

The research will result in structural design propositions on the three levels of abstraction defined before: schemes, systems and designs, each of them being an implementation of its preceding. Together these propositions constitute an overview of structural configurations, including principles of materialisation and detailing. Only one of the structural systems is developed into a mock-up. The particular value of the research is the abstract level of the overview.

Next to proposing concrete structural systems and designs of free form buildings, the research is also adding new knowledge by the documentation of the design path how the propositions evolved. This involves constraints from the phases of design and production, which are critical and therefore relevant in free form building design. The research will therefore stretch out across several disciplines (geometrical descriptions, mechanics, manufacturing), each of them requiring a different methodology, e.g. qualitative methods for assessment of complexity, mathematical methods for geometrical description and quantitative methods for structural analysis.

1.4 Methodology

To generate the structural schemes and specify them further into structural systems and structural designs, the research project rests on the assumption that designing with three explicitly defined design variables and with knowledge on how these are interrelated, will facilitate the exploration of technically viable paths from design to realisation. The three variables are:

1) Geometrical definition (in short: geometry);
2) Structural action (in short: force);
3) Material processing (in short: material).

Throughout the research these variables will be mentioned. They will be elaborated first in the theoretical framework in the next chapter.

As the research goes across several disciplines, a combination of different methodologies, namely that of research-driven case study design in addition to theory- and precedent-based analysis, had to be applied. While a study into the design variables is covered in the theoretical background, the practical considerations are described in a number of precedent studies. These stress any of the design variables in building designs or the way in which they were interrelated. Together, the two studies provide a basis for formulating hypothetical solutions for successful structural systems.

Whereas the proposed structural schemes primarily rest on qualitative descriptions, the development from abstract schemes to concrete structural designs implies specification of materials, and finally also a quantitative specification of its geometry, dimensions and amounts. The architectural shape and intended use of an unbuilt building design, the DO Bubble, provides the context for the system's final specification onto structural design. With an increasing level of specification, the ability to generalise the proposed solutions to other applications decreases. This ultimately goes off with the system's implementations of (only) three structural systems in case studies, which allow for extensive analysis of that case, but offers little generalisable knowledge (Swanborn 2000).

1.5 Scope of the research

This thesis deals with the material implications of the load bearing structure of irregular and double curved building shapes. The style of such shapes professionally is also known as Blob-, fluid- or free form architecture. The buildings seem to lack relationships with any conventional or geometrically regular building form. As all of these denominations emphasised the building form, and not the other qualities architects attributed to them, the names were systematically rejected by architects themselves. Bögle considered 'free form' incorrect, since the building's geometry is well-defined, and consequently not free (Bögle 2006). In the actual research however, 'free' refers to its structurally unconstrained origin. Hence it does not pursue any architectural or mathematical intention, and the denominations of 'Blob' and 'free form' are used as synonyms.

This research explores various ways to design structures that follow the curvature of the building's envelope. Whether the structural shape of the proposed solutions exactly aligns with the architectural shape, is a matter of evaluation based on the criterion of formal freedom, but structures of orthogonal layouts where the formal variation of the building envelope is achieved through spacers of different lengths, are left aside. The concerned structures are primary load bearing structures of free form shaped buildings, or parts of them (notably façades and roofs). The building envelope is not just a structural surface or reticulated, but has to accommodate openings, and is anticipated to be out in the open. Evaluation thus goes beyond fixed structural requirements of strength, stiffness and stability, to include perception of building shape in its structure. In case of reticulated structures, the design, fabrication, assembly and installation of cladding on top is not considered, but the density of the underlying pattern has to be customisable to spans of two meters. Effects of structural action reaching beyond the free form part (e.g. floors resting on a curved facade) are not considered.

The research in the first place intends to exploit current techniques for design and production. However, niches where developments of either a process or a tool can have a large effect are proposed where appropriate. Despite being a major decision parameter in building practice, cost has not been taken into account since it is highly specific to local conditions. However, striving for the most systematised application, while using existing techniques, for sure is cost-effective.

Since structural designs are specific while the geometrical contours of a building and its intended use are only roughly defined, the question what the best structural solution could be, is not answered within this research. The propositions in this research include a list of improvements, and to what extent these can be implemented within the system's specifications. Thus, this research at best allows for optimising.

1.6 Organisation of the book

The book presents the research process step by step. From preliminary analysis in chapter 2 (from a theoretical point of view) and 3 (from a practical point of view), via generating alternatives on the abstract scheme-level in chapter 4, and the more applied level of systems, including implementations in a case study, in chapter 5. One of the systems, Delta Ribs, is developed further onto prototypes that are presented in chapter 6. The thesis concludes with conclusions and recommendations in chapter 7.

Although presented in a serial manner in this thesis, in the global research set-up parts were contributing in a parallel manner. For readers interested in practical solutions chapter 5, *Case study implementations of schemes onto systems and structural designs,* is most recommended.

2 Theoretical framework: design variables and their use in structural design research

The three design variables considered when analysing solutions applied in built examples (chapter 3), and when proposing new solutions (chapter 4 onwards), are:
1) Geometrical definition ('geometry');
2) Structural action ('force');
3) Material processing ('material').

Structural design is defined as assigning values or specifications to design variables. These variables focus on the elementary level of scale, but may later aggregate on structural assembly. The subdivision was chosen as it categorises precedent solutions and also serves generating new ones in an extraordinary context.

Other classifications exist, but don't fit the goal of this research. Schodek for instance classifies structures according to the geometrical nature of its elements (linear and surface-like, specified further into straight/planar and curved) and the rigidity of the material, in his book on the design and structural analysis of structures (Schodek 2004). He integrates geometrical and mechanical aspects, but does not address aspects of construction.

Rickenstorf (1972) makes a distinction based on geometry, and goes remarkably far into the curved surfaces by distinguishing between Gaussian curvatures smaller than, equal to and greater than zero, which is a logical consequence of the state of the art of shells at that time. However his examples are all highly regular (circular shells, extruded arcs and ruled surfaces), and do not feature any smaller level of scale. Polonyi (1987) combines structural and geometrical aspects, elaborating on shell structures sometimes also loaded in bending. Furthermore he identifies a geometrical category for ruled surfaces as the formwork for these can be constructed relatively simple from straight pieces of timber. He also signals that the optimal geometry for a shell structure is often modified to make it constructable, and is therefore more proper to a timber than to a concrete structure. For this class he thus anticipates manufacturing techniques.

To handle the design variables, this chapter provides a framework with sub-categories:

First, the geometrical definition is subdivided into 0-, 1-, 2- and 3-dimensional objects, commonly known as points, curves, surfaces and volumes. They are needed to define an element in geometrical sense. In addition to the n-dimensional classification, transformation techniques of extruding, scaling and rotating are included. Thus the geometrical classification contains information on how the shape was created, which is useful to couple the geometrical definition to manufacturing techniques.

Second, the structural action is subdivided into vector-, section-, surface- and form-action, of which also combinations do exist in either superposition or interaction (Engel 1999). Each class of structural action corresponds to a typical type of stress, that later will link to appropriate materials and geometrical arrangements.

Third, material processing is subdivided in additive, subtractive and formative techniques, as well in fabrication of two-dimensional elements.

The three design variables adopted in this research are highly interrelated, as is exemplified through general notions in Figure 1. Some relationships are critical: some geometrical constructs can only be materialised through specific manufacturing processes.

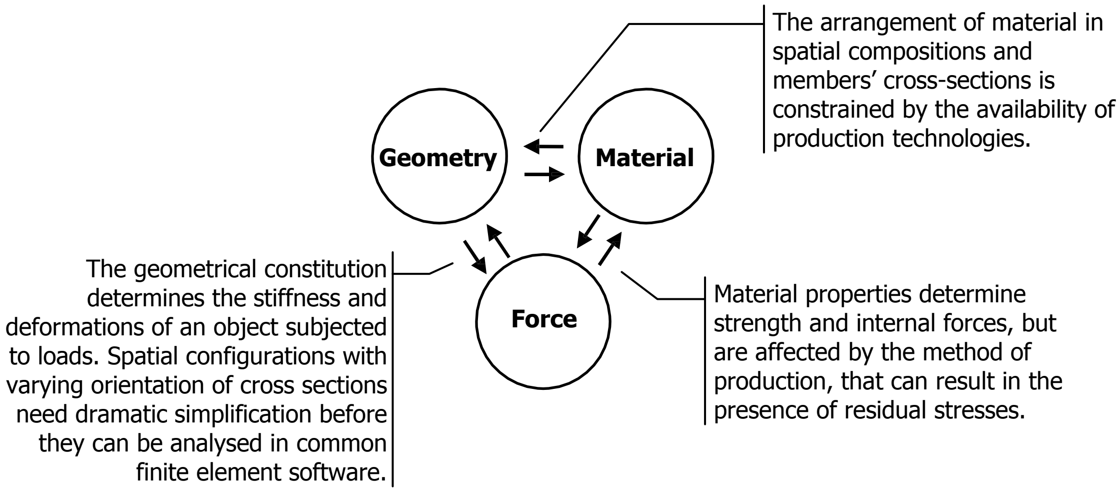


Figure 1. The design variables of geometry, material and force are interrelated. This is exemplified with general remarks.

With the theoretical framework established in this chapter, the next chapter features their practical implementations in built examples. This chapter and the next constitute a framework to which, from chapter 4 onward, alternative designs for structural schemes and systems are generated.

2.1 Geometrical definition ('geometry')

2.1.1 Constructive and descriptive classifications

In spatial constructions of any size and function, material matter is arranged in a specific way, resulting in a shape that is defined through its geometry. This geometry may be defined in a descriptive or a constructive manner. Descriptive geometries include the definition of geometrical elements with the aim of graphically representing them (Paré, Loving et al. 1965). Constructive geometries describe the way through which the geometry was constructed (Pratt 2003); for instance through Boolean operations in case of constructive solid geometry, or through operations of extrusion scaling and rotation as in this research.

Descriptive and constructive geometrical definitions can result in identical geometry. Since the latter includes intelligence on how it was constructed (in a geometrical sense), and since linking geometry to manufacturing (in a material sense) is one of the means through which this research attempts to reach its aim, a constructive classification is used for the geometrical definition of the schemes in chapter 4. While maintaining the step-wise structure of operations, and by varying only numerical values, the definitions become generic.

The absence of construction history in descriptive geometrical definitions is advantageous in the case of geometries of unknown origin, as it renders the definition universally valid. For this reason, both constructive and descriptive definitions are presented in Figure 13. The former is used for categorising precedent realisations in chapter 3, the latter to define the schemes in chapter 4.

2.1.2 Geometrical primitives

The primitives are elements of zero to three relevant dimensions. Their dimensional relevance depends on the scale and level of abstraction; a global level of scale and a high level of abstraction normally reduce the number of relevant dimensions. The classification does not deal with absolute sizes and is applicable to any scale – in the context of this research: to structural elements, structural components as well as entire buildings.

Although useful in their representation, abstract entities like lines to depict profiles should be handled carefully. In a top-down approach, from macro to micro level, with each step more geometrical dimensions become relevant. Although the transition from geometrical entity of a line to material entity of a profile is plausible, one must be careful not to give non-materialisable attributes such as implicit rotations to 1D-elements. To prevent giving unrealistic profile-attributes to curves, the appropriate representation should be used. Curved profiles featuring plane-symmetry can best be schematised as planes or surfaces with controlled curvature (e.g. extrusions of curves with constant curvature). As a contrast cross sections featuring point symmetry only are geometrically twisted and not materially twisted, and may therefore well be schematised as 1D-elements.

Points, 0D

Points have dimensions that are all relatively small: none of them is significantly bigger than another, hence codified as 0D-elements. The ultimate point has no dimensions at all, thus is no longer an element but a coordinate point which serves as geometrical reference. Nodes in a network structure, when considering the structure's macro level of scale, are an example.

Curves, 1D

Curves are geometrical elements significantly sized in one dimension and therefore coded as 1D-elements. They can be curved, also in space. Particular curves are straight or have a constant radius of curvature. A group of curves, for instance in a grid, can behave according to the same system, for instance by having a common centre point or parallelism. Curves may serve as geometrical entities with no section dimensions, or may be the abstraction of a curvilinear beam element.

Surfaces, 2D

Surfaces are geometrical elements with significant measures in two directions and are therefore codified as 2D-elements. Their thickness is non-existent or negligible. This for instance applies to a façade, which is observed as a surface entity (so not having a thickness) when observing it at the scale of an entire building. Elements of little thickness as sheet-like materials (e.g. foils or metal sheets) can keep the 2D-representation to a more detailed level than thick elements like sandwich panels. When there is no physical thickness at all, the surface acts as a geometrical reference. Zooming in onto the previously mentioned curvilinear beam element, for instance an I-section, it will disclose two surfaces of symmetry. Its representation as a 2D-element then contains information on its orientation. The combination of two of such representations provides information on the nature of the intersection.

Volumes, 3D

Volumes have 'height', 'depth' and 'width' and none of them is negligible; consequently they are 3D-elements in geometrical sense. Volumetric elements are massive and defined by their limitations, which are surfaces. By zooming in on a building part the number of relevant geometrical dimensions therefore always increases, until the volumetric level of material matter is reached. When for instance zooming in on the façade, at the meso level of scale its thickness becomes relevant, and then the façade constituted out of a structural, insulative and protective layers, should be considered as a volumetric entity.

2.1.3 Transformations of geometrical primitives

Constructive intelligence is introduced when the geometrical primitives undergo transformation, even if already all geometries could fit in either of the geometrical primitives mentioned so far. The applied transformations are extrusion, scaling and rotating, and are codified with the letters 'E', 'S' and 'R' respectively[3]. From the three

[3] The naming of 'E' for extrusion, as a distinction to 'D' for dimension was set up in collaboration with Karel Vollers.

transformations presented here, only extrusions, and extrusions combined with scaling or rotation (codified as ES and ER), create new geometrical artefacts. Elements may be subjected to several rounds of transformation, this way constructing a code as displayed in Figure 2. As mentioned earlier, the inclusion of a constructive history will prove powerful when the geometrical transformations will be coupled to manufacturing techniques, and for instance the inclusion of an extrusion-rotation will exclude the application of transformed planar elements.

The denomination of E is preferred above using fractions of D (e.g. 1½D), which is sometimes seen in practice, but on which no general agreement exists: some use 2½D for expressing that cross sections are identical at various positions (planar geometries extended with a height-parameter), others use it to denominate developable surfaces (transformed planes) (Eekhout 2004), or the approximation of nD, so 2½D meaning 'is almost 2D', equal to 'is almost planar'. Using E rather than D also avoids association with the commonly accepted term of dimensions. Also, using fractions suggest that there is a continuity of geometrical dimensions on a sliding scale from 1D to 2D and possibly onwards, whereas this is not the case.

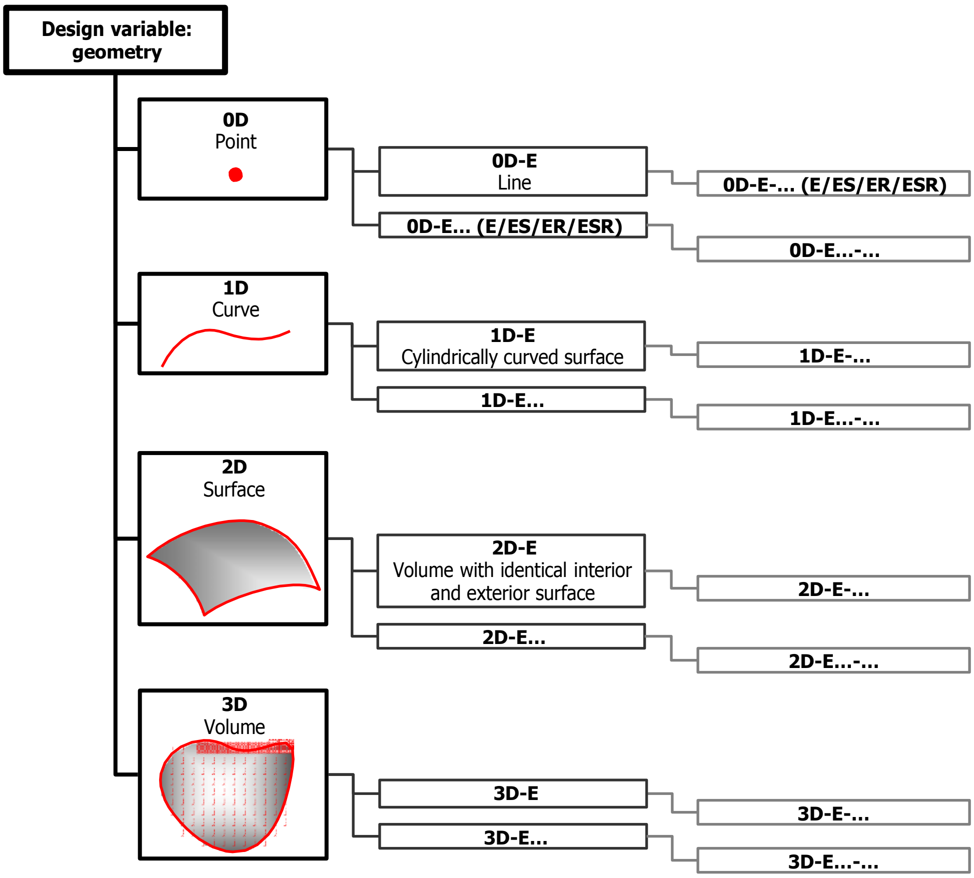


Figure 2. Constructive classification of geometrical primitives and their transformations.

Extrusion, E

In the extrusion-transformation a geometrical component is moved along a vector, this way defining a new element. The newly created geometry is described by:

- the input geometry;
- the direction of the extrusion.

The reference to a transformation by extrusion is of considerable practical value for implementation in structural design since numerous standard components are extruded geometries, originating from points, curves and surfaces. Its biggest potentials are the straightness of extruded points, and the cylindrical curvature that results from extruded curves, characterised by parallel ruling lines. Such surfaces are developable and can thus be constructed from a planar base material, which will be explored in more detail later.

Figure 3 demonstrates the extrusion operation on the base geometry of different categories. The extrusion operation applied to the 0D-, 1D- and 2D-elements results in a repetition of identical sections, namely the original element. Other features are that extruded points (0D-E) result in a rule, extruded curves (1D-E) in cylindrically curved surfaces with parallel ruling lines running in the extrusion direction. Also extruded surfaces (2D-E), resulting in a volume, feature parallel ruling lines although these are less evident and not commonly used.

The logical completion of the presented extrusion transformations of 0D-, 1D- and 2D-elements would be the extrusion of a 3D-element, but since the resulting element (a 3D-E element) has no relevant meaning in the sense of a material object (it has in the sense of the description of the volume occupied when shifting the element along a trajectory as for instance 3D-E: a lift-shaft), it has not been considered in this research.

Whereas the extrusion of a curve is a function in Rhinoceros 3.0 and Maya 6.5 (the modelling software used in this research), extrusion of points and surfaces is not. The extrusion operation was therefore executed by spanning a surface ('lofting' in 3D-modelling terminology) between the original entity and its shifted duplicate, where the shift equalled the extrusion direction.

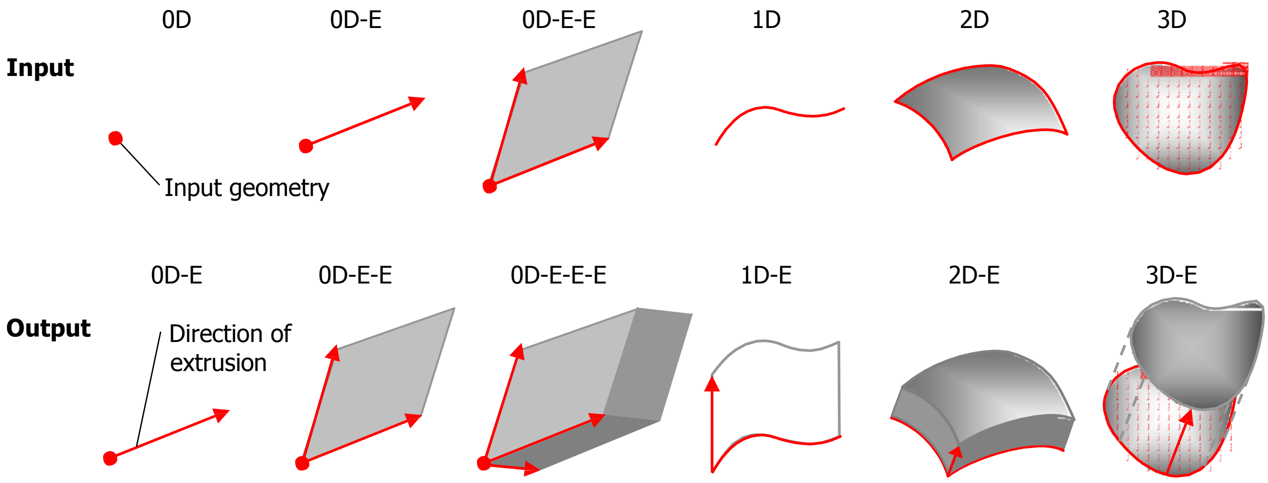


Figure 3. Examples of the constructive geometrical class obtained by extrusion.

Extrude + Scale, ES

The combined transformation of extruding and scaling shapes the element. For a limited number of examples, Figure 4 shows that it allows for geometrical adjustment of shape and type of curvature. Whereas in the case of only an extrusion, ruling lines are parallel, extrusion and scaling will undo their parallelism, and introduce a conical curvature with an intersection point in the centre of scaling. This is demonstrated in detail in Figure 5 where both an extrusion-operation and a combined extrusion-scaling-operation are applied to the same input (a 1D-element). The first operation results in parallel ruling lines, the second into converging lines. Also conical curvatures are developable and can thus be unrolled. Parameters of the combined operation extruding-scaling are:

- the input geometry;
- the direction of the extrusion;
- the position of the centre of scaling;
- the scaling factor.

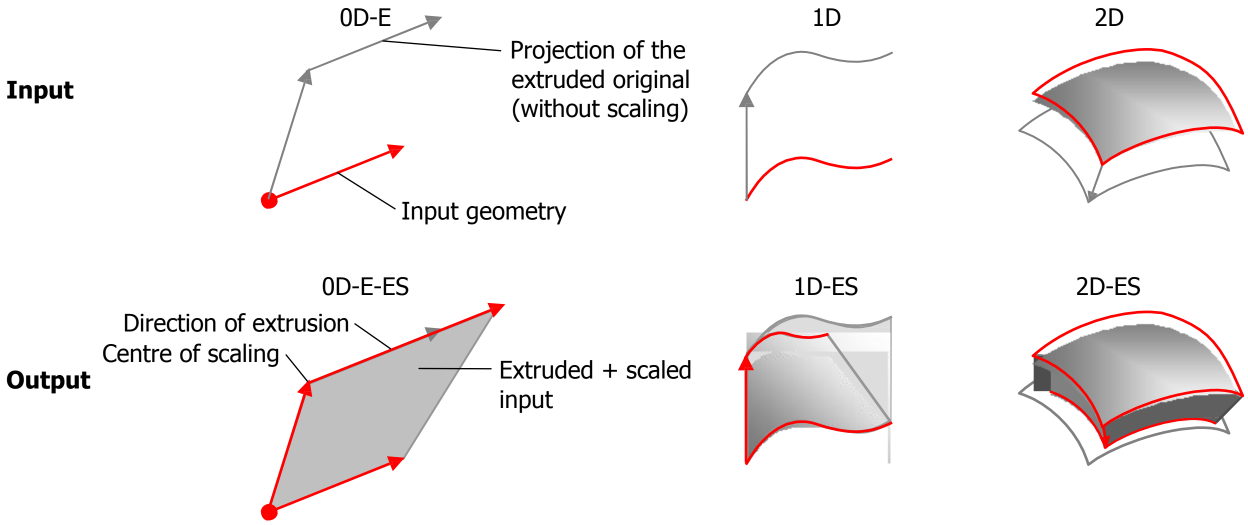


Figure 4. Constructive geometrical classes obtained by combined extrusion and scaling.

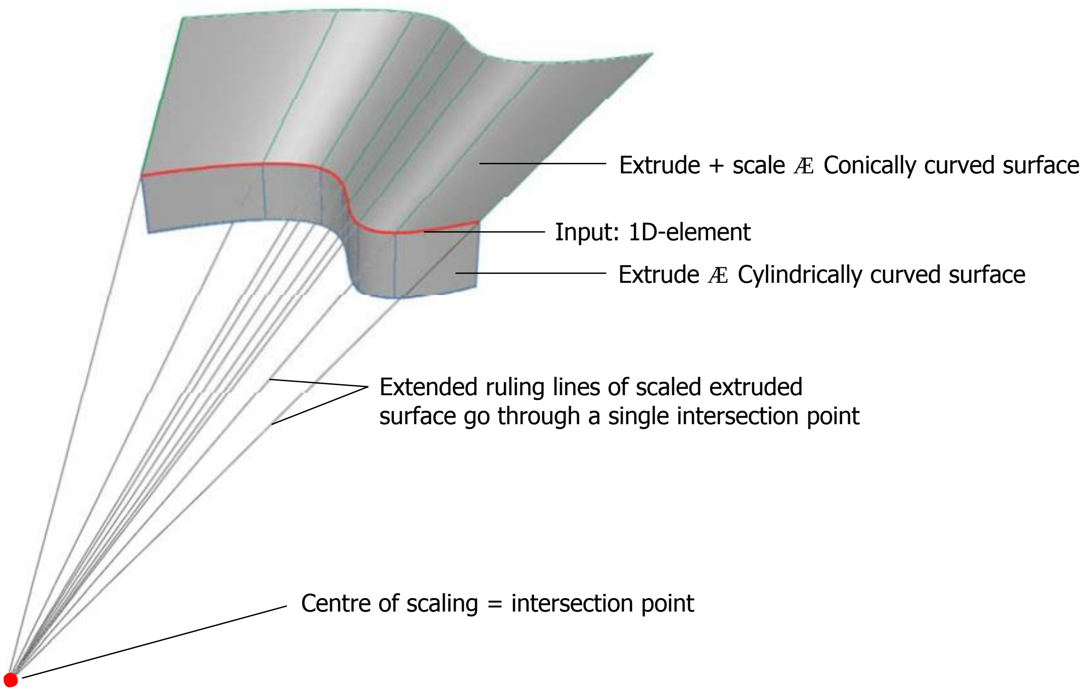


Figure 5. Extrusion and scaling operation.

Extrude + Rotate, ER

An extrusion combined with rotation results in a directional change of the input element (Figure 6). Doing so, the extrusion's characteristic of a repetitive cross section is maintained, although each of them is rotated relative to the previous. Rotations may be necessary to maintain (continuously) or restore (at intervals) the orientation of an element relative to another – e.g. the orientation of an element at a constant orientation relative to the building envelope. Parameters of the operation of combined extruding and rotating are:

- the input geometry;
- the direction of the extrusion;
- the position and orientation of the axis of rotation;
- the angle of rotation.

In a view parallel to the rotation axis, each point on the cross section describes a circle path, and its surface along that path features a double, but constant, curvature. This was explored by Vollers on the building level of scale. He combined extrusion of a single floor plan with rotation, with the aim of turning this repetition into a cost-effective feature (Vollers 2001).

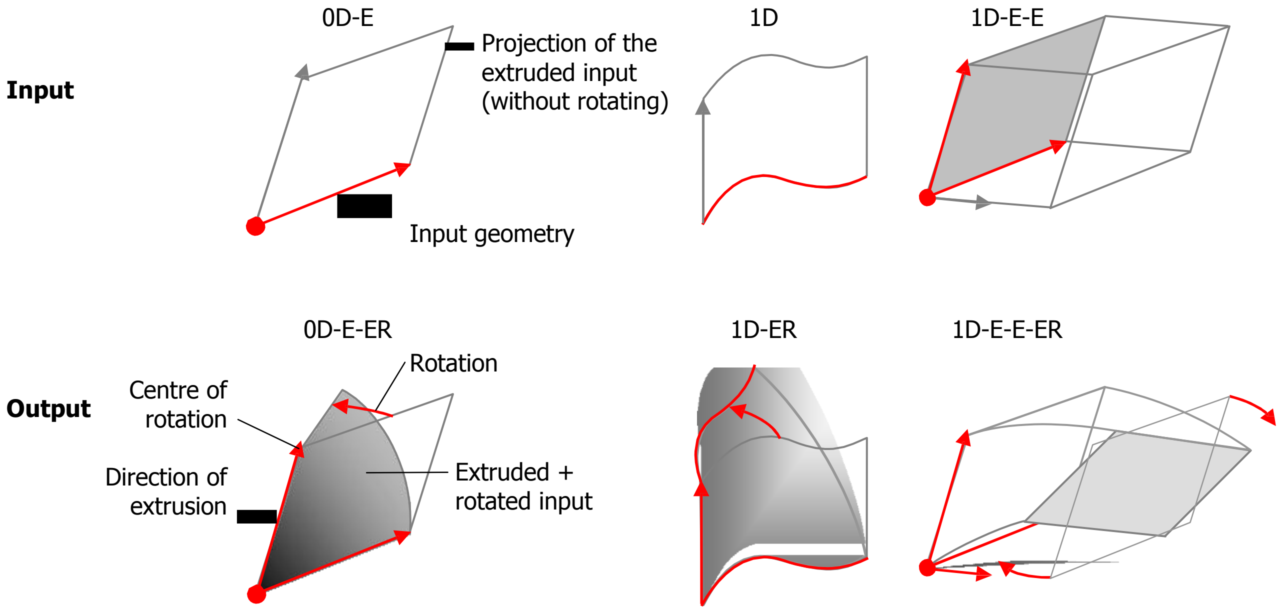
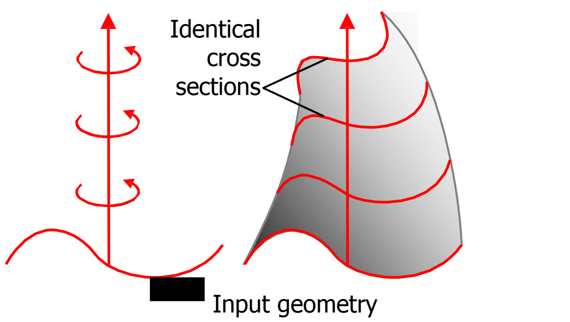
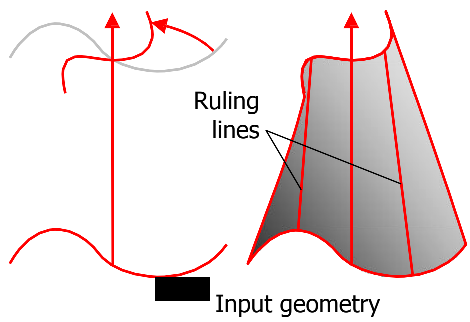


Figure 6. Constructive geometrical classes obtained by combined extrusion and rotation.

Although the elements at the extremities are identical, geometries created through extrusion and rotation cannot be described as a surface spanned between the original entity and its duplicate that underwent a shift and a rotation, as is demonstrated in Table 1.

Table 1. Comparison between extrusion and loft in rotated geometries.

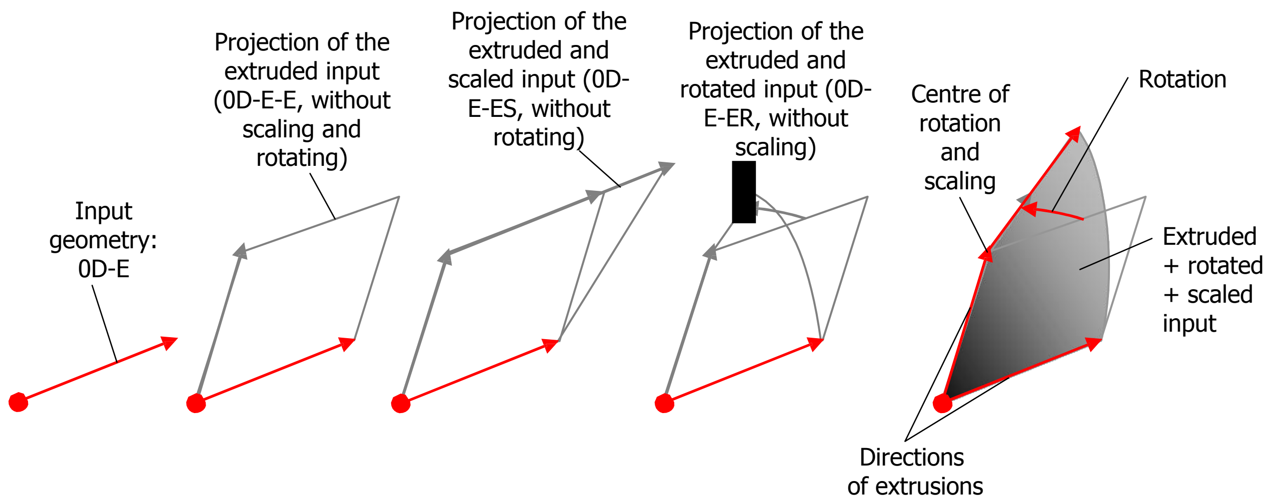
Simultaneous extrusion and rotation	**Loft between a curve and its shifted and rotated duplicate**
	
Constant cross section (rotated duplicates)	Varying cross section (no duplicates, not even scaled or rotated)
At maximum one ruling line, coincident with the axis of rotation	Ruled surface, thus filled with ruling lines
In case of curve as base geometry: the resulting surface is always double curved	The lofted surface is a ruled surface

Extrude + Scale + Rotate, ESR

The extrusion transformation combined with scaling and rotating combines the parameters of all. These parameters, exemplified on a 0D-E-ERS–element in Figure 7, are:

- the input geometry;
- the position of the centre of scaling;
- the scaling factor;
- the position and orientation of the axis of rotation;
- the angle of rotation.

Of all operations, it thus offers the largest number of opportunities for customisation. However from a rationalist point of view, the opposite is the case: whereas the scaling operation features converging ruling lines and developability of the surface, both get lost when undergoing a rotational transformation. The cross-section is scaled, but its shape is identical to the original.



A. Intermediate transformations to create the 0D-E-ESR-element: 0D-E, 0D-E-E, 0D-E-ES and 0D-E-ER.

B. Combined transformations to create the 0D-E-ESR, created from a 0D-E-element.

Figure 7. Constructive geometrical classes obtained by combined extrusion, scaling and rotation.

2.1.4 Assessing curvatures

Curvature, K

The curvature K of a curve at a point is defined as 1/R, R being the radius of the tangency circle at that point (Figure 8). A small curvature thus entails a large radius. To evaluate the curvature of a surface at a point, it is intersected by planes rotating through the surface normal at that point (Figure 9A). Each plane results in a planar intersection curve, which can be evaluated as curves (B). The smallest and biggest curvatures of all curves passing through this point are called the principal curvatures (C).

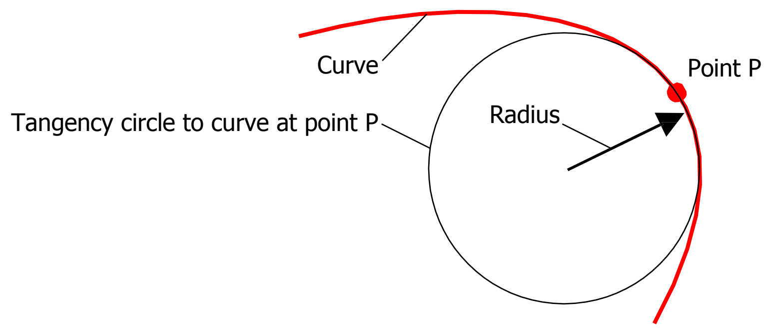
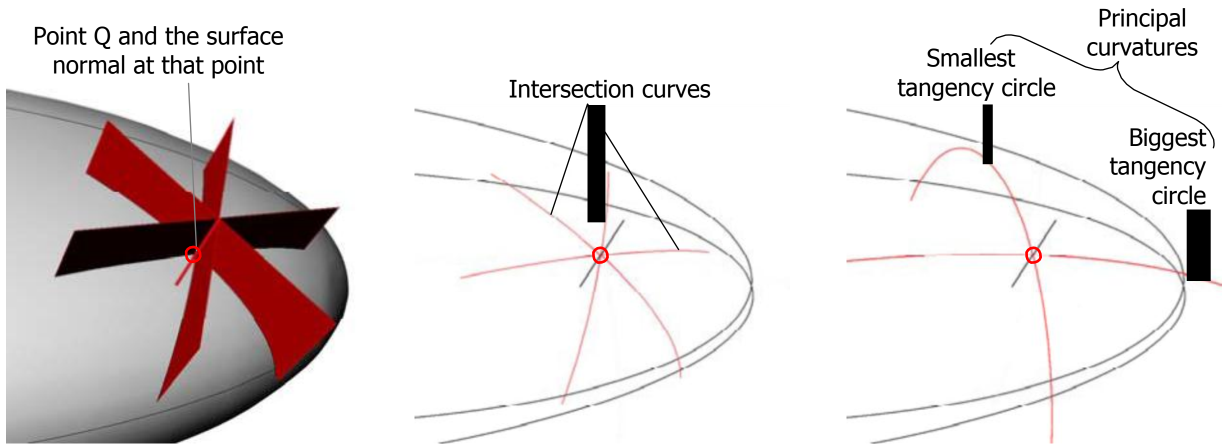


Figure 8. Determination of the curvature of a curve at point P: 1 / radius of tangency circle.



A. Planes rotating through the surface normal at point Q.

B. Intersection curves between the planes and the double curved surface.

C. Principal curvatures of point Q.

Figure 9. Determination of the principal curvatures of a point on a surface.

Gaussian curvature

Gaussian curvature is commonly used to describe surface curvature at a point. The Gaussian curvature (named after the German mathematician Karl Friedrich Gauss, living 1777-1855) is the product of the principal curvatures. It permits to classify and quantify the curvature of surfaces. As the individual curvatures can be positive, zero or negative, the product of any two curvatures can be smaller than, equal to or larger than zero. Whether curvatures are positive or negative depends on the viewing point, and therefore is only a relative denomination. What matters is whether or not the two curvatures are of the same sign (either positive or negative), thus curving towards the same side of the surface (called synclastic), or of different sign (one positive, the other negative) when directed to opposite sides of the surfaces. When the surface's Gaussian curvature is everywhere negative, it is called anticlastic and is saddle-shaped (Weisstein 1999).

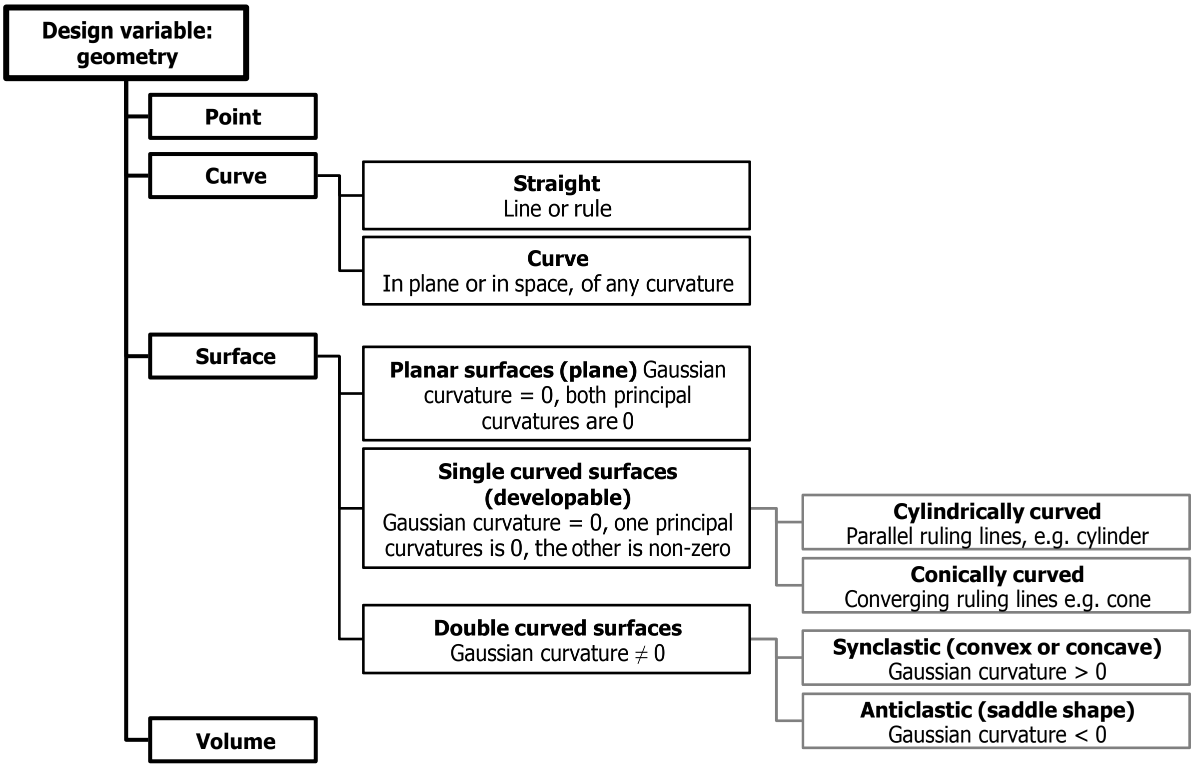


Figure 10. Descriptive classification of geometry.

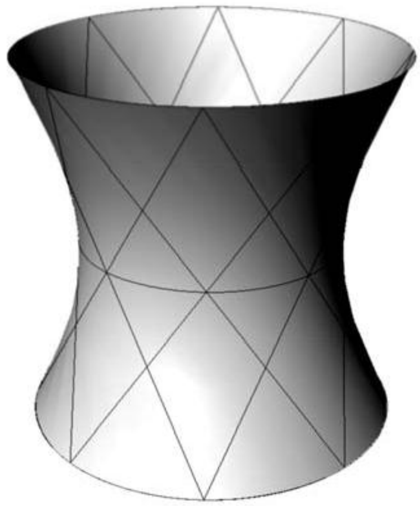
Classification of surfaces according to Gaussian curvature

For purposes of evaluation, surfaces – including planes – can be classified according to their type of curvature (see Figure 10):

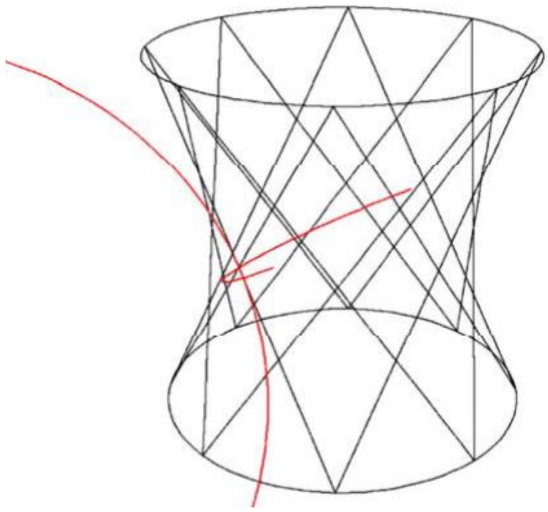
1. Planar surfaces – These surfaces have no curvature in any direction.
2. Single curved surfaces – These surfaces are curved in only one direction (one of the principal curvatures is zero) and can therefore be unrolled to a plane. These are known as developable surfaces. The merely geometrical feature of being developable is important when it comes to materialisation, for which in principle all sheet materials are candidate. Developable surfaces can be cylindrically or conically curved.
3. Double-curved surfaces – In these surfaces the two principle curvatures are non-zero. Such surfaces may be either synclastic or anticlastic. When using a formative fabrication process, only sheet materials that allow stretching are candidate to materialise this class of surfaces.

Special case: ruled surface

Ruled surfaces feature straight lines, lying side-by-side like in Figure 11A. Nevertheless the surface is double curved because the ruling lines are neither the maximum nor the minimum curvature at that point (Figure 11B). So the Gaussian curvature is not zero, and hence the surface is not developable. Zooming in on ruling lines through two points on the boundary at an infinitesimal distance from each other, one is always slightly rotated relative to the other, and are thus not in the same plane. A twisted surface (0D-E-ER) is spanning in between.



A. Ruled surface consisting of a series of straight lines.



B. The boloid from the previous figure, now highlighting its principal curvatures at its 'waist'. The rules are neither the smallest, nor the largest curvature.

Figure 11. Ruled surface, possessing straight lines but nevertheless doubly curved.

2.1.5 Mathematical definitions of geometry

Apart from the classification method and the employment of the geometry, the definition of these geometries, which is notably the field of Computer Aided Geometric Design (CAGD), has implications also the designers using them should be aware of. This notably counts for free form curves, which are all 1D-elements that are not straight ('rules', 0D-E) or circle segments ('arcs'). Depending on the modelling tool that is used, they may be nurbs-curves, splines or Bézier splines. Although from a non-expert point of view all these curves are all capable to create the same fluidly shaped curves, the mathematics behind them are different.

Before computers were being used, a spline was a flexible strip of timber placed against a series of nails, and used by carpenters to draw fluid curves. A digital spline works the same, but with the nails replaced by control points as demonstrated in Figure 12. These 'nails' are no longer on a plane, but may be anywhere in space. Also, these points can be on the curve (like a line spanned between two points, or an arc through three points), but also aside, acting as a magnet. The curve is transformed by shifting the position or weight of data points. In nurbs-definitions one control point is ruling the sections on either side of the control point.

Thus, by controlling only a few points, the designer may generate a large formal variety. This dependency on a small number of points may however also be problematic since removing a control point, for instance one at an extremity when trimming a redundant part, will result in a transformed shape. Also, making one curve out of two, generates a new set of control points, thus (often unintentionally) transforming the shape or its structure.

Like curves, also surfaces can be described through nurbs and their control points. On this surface positions are indicated through U- and V-coordinates (as they are named in the software used in this research) running from 0 to 1. Unlike conventional orthogonal coordinate systems, this system does not consist of equally spaced parallel lines. In case of a spherical surface for instance, either the U- or V-gridlines all merge into one point at the sphere's poles. Points placed in a regular UV-pattern, will therefore not result in equally spaced points. This complicates the mapping of patterns on surfaces.

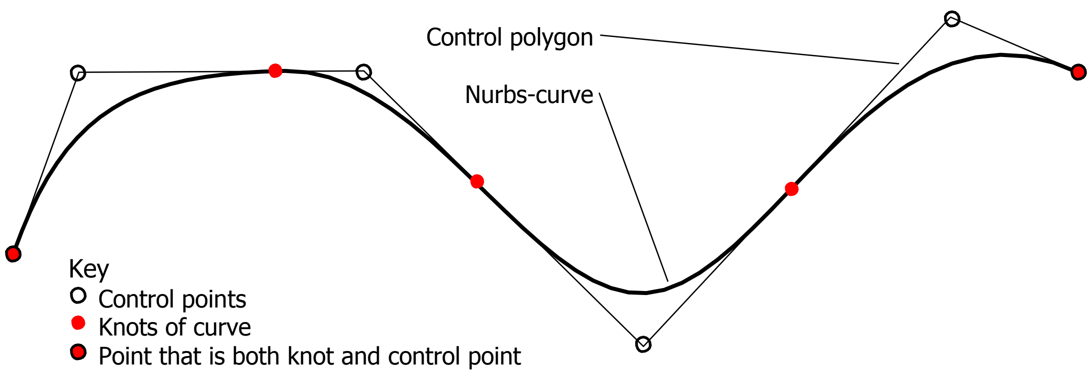


Figure 12. Spline with control points together defining a control polygon

Nurbs, acronym for Non-Uniform Rational B-splines, are the most commonly used variant of splines. 'Non-Uniform' refers to non-equal distances between control points. 'Rational' refers to the non-equal weight attributed to the control points of a b-spline, which is not the case with ordinary b-splines. Finally 'b-spline' simply stands for basis spline, which is a generalisation of the Bézier spline, named after the French engineer Pierre Bézier (1910-1999) working as head of the design department of the French car producer Renault, where he realised the need for digital representations of mechanical parts, and developed polynomial curves for this. Worth noting is that he worked in parallel with, but independent from, Paul de Faget de Casteljau who was working at Citroën, Renault's competitor, on exactly the same problem. (Piegl and Tiller 1995)

Since complex shaped curves and surfaces are generated through a limited number of points, the file size remains small. Complex geometries described as polygons usually subdivide one curve in a large number of (short) straight sections, thus requiring the coordinates of each point to be defined. This requires significantly more memory to achieve similar smoothness.

Since the curve's definitions include further specifications on the connection of control points and the weight attributed to each magnet, exchanging shapes between software is risky. A standardised file format such as IGES (Initial Graphics Exchange Standard) overcomes this, but results in a loss of information on how the geometry was constructed. The first was established in 1980 (Nagel, Braithwaite et al. 1980) after common effort by a group of CAD users and vendors, the latest version 5.3 was published in 1996.

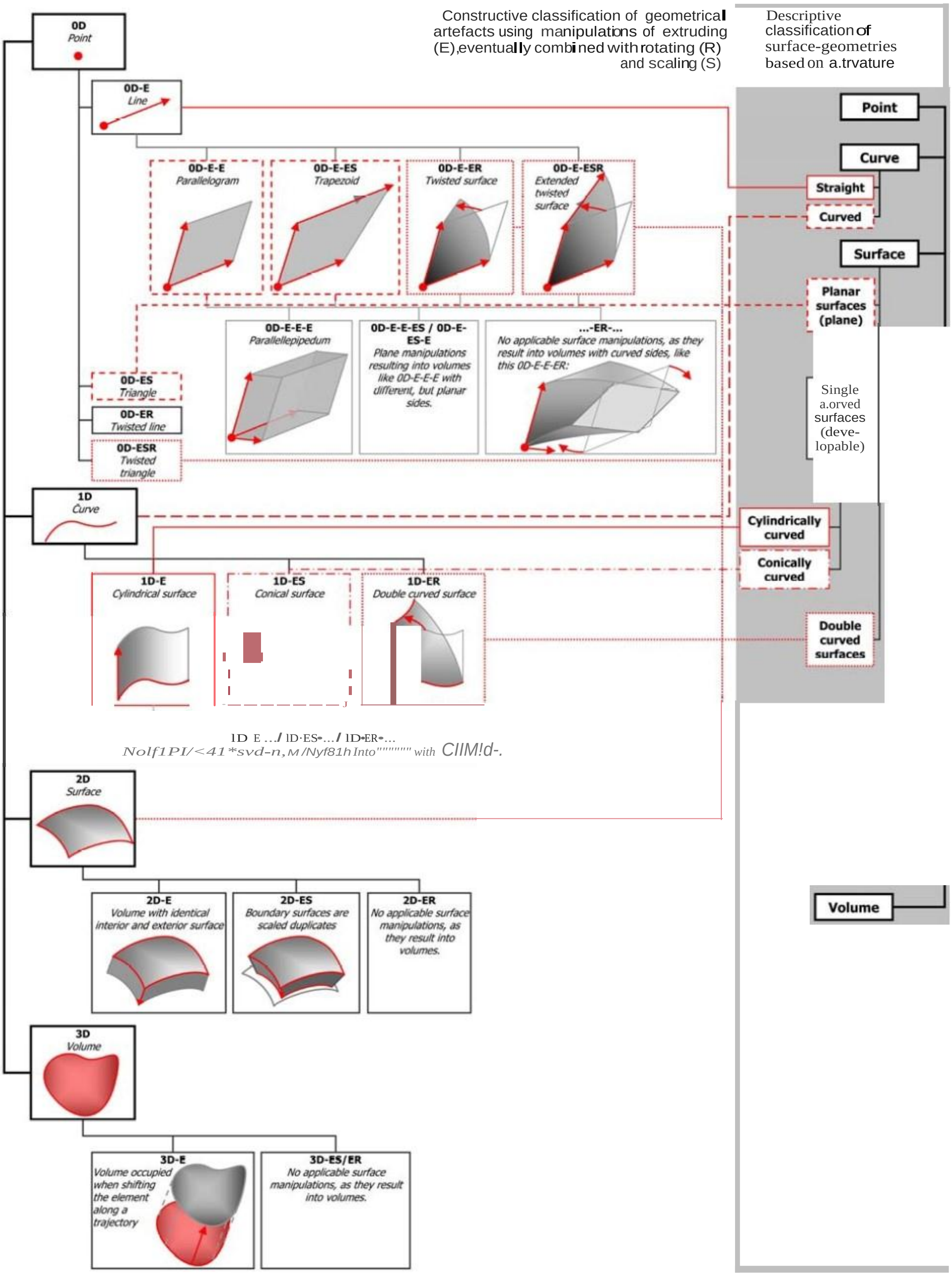


Figure 13. Classification of geometrical artefacts from a constructive (tree-diagram starting from the left) and descriptive side (diagram starting from the right).

2.1.6 Review

An overview of geometries constructed through manipulating primitives (the constructive classification) is presented in Figure 13, starting from the left. In the same figure, starting from the right side, the descriptive classification of surfaces is included. From the scheme it becomes clear that planar surfaces can only be constructed from manipulations on points, and developable surfaces only from manipulations on curves. In contrast, double curved surfaces can be constructed through manipulations on a variety of primitives, notably when a rotation is among the applied manipulations. In other words: double curved surfaces are less critical than planar and single curved surfaces – unless there are specific requirements to the curvature.

As this section on geometry proved that geometrical constructs are well-defined, the question arises whether or not one should still speak about 'free form'. We should, since Blob-designs share features (double-curved globally, irregularly shaped) but in a building design they not necessarily all are met. Also, the total building-design being a Blob-design – the striking image – does not imply that its load bearing structure shares all Blob-features. A lot of non free-form structural design is to take place before it becomes the opposite – polygonal structures in a rectangular layout. The geometrical overview presented in this chapter is based on manipulations of primitive-geometries, which are free, meaning unconstrained, forms. The formal freedom is constrained by rationalising, but this is not contradictory to the definition of free forms, stating global double curvature, irregularly shaped and a non-structurally optimised shape.

2.2 Structural action ('force')

The structural action displays how loads are received, transferred and transmitted. Primary inputs are the structure material and the shape and organisation of its elements. Of the multiple categorisations, the one adopted here is that of Engel because it is the most abstract one that nevertheless anticipates material properties and geometry. Engel identified four highly abstract 'families' of structural action (which are defined in the next section), each of them subdivided into 'types', and next into 'structure singles' (Engel 1999). Whereas the top level is highly abstract, the second and third levels implicitly refer to structures in conventional orthogonal layouts, and will therefore not be used in this research. For reasons of clarification, Table 2 lists the four structure systems and their types.

Table 2. Structure systems and structure types according to Engel (1999).

Structure systems	Types
Form-active	Cable structures Tent structures Pneumatic structures Arch structures Flat
Vector-active	trusses Curved trusses Transmitted flat trusses Space trusses
Section-active	Beam structures Rigid frame structures Beam grid structures Slab structures
Surface-active	Plate structures Folded plate structures Shell structures

As the categorisation is highly abstract, it applies to any level of scale. Per level, a structure may feature a different structural action. Scale-transitions are useful in the crude-to-fine transition that is made in the structural design process. This way the number of structural parts can be kept small on the macro-level, whereas the outcome can be used for preliminary dimensioning of elements on the meso-level. Examples are:

1. A space frame-structure transfers the applied loads through splitting them into vectors (= vector action), such that all bars in the space frame are loaded axially, in either tension or compression. Observing this structure from a distance,

disregarding the separate members, the total structure becomes a thick layer structurally acting in bending, so in section action. Zooming in on the space frame's bars, these are commonly tubes that provide strength against axial forces as well as stiffness against bending and buckling. To resist this bending, the cross section is sollicitated, resulting in the section-active mechanism. As it is the tube's curved exterior that provides the stiffness, it is also featuring surface action.

2. Another example is the discretisation of surfaces into a mesh of (curvi)linear members. Seen from the macro-level of scale, the structural action may be associated to the surface (e.g. surface-action), whereas on a lower level of scale a section-active mechanism may be present. The opposite is also possible: beams constituted out of surfaces structurally act through section-action on a macro level, but through surface-action on a meso-level.

2.2.1 Four mechanisms of structural action

Mechanisms of load transfer are – disregarding the microscopic material level – scale-less and not specific to any material. The definitions nevertheless contain implicit reference to a material class (e.g. rigid or flexible) or geometrical class (e.g. its curvilinear or surface-like nature). The four mechanisms are depicted in Figure 14 and defined next. The original definitions by (Engel 1999) contained references to implementations, which have been removed and reformulated to make them more generic.

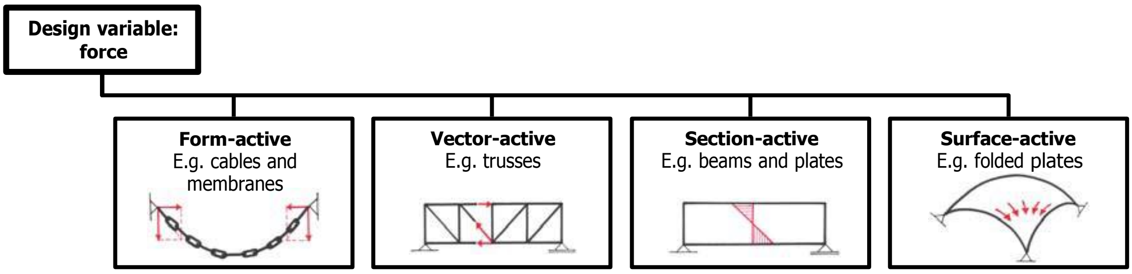


Figure 14. Four mechanisms of structural action and examples of components featuring them.

Form-action

Definition: Form-active structural systems are systems in which the redirection of forces is effected by a self-found form design and a characteristic form stabilization. They therefore have an equal distribution of axial stresses in a cross section. (Based on the definition by Engel (1999): Form-active structure systems are systems of flexible, non-rigid matter, in which the redirection of forces is effected by a self-found FORM design and characteristic FORM stabilization.)

This mechanism involves that typical form-active systems such as a wire net or an air cushion will reposition itself under loading. The stresses resulting from pure application of this mechanism are uniformly distributed axial stresses. Taking into account that there are multiple load cases, relatively large deformations and a non-uniform stress distribution will have to be accommodated through appropriate detailing and dimensioning.

Funicular lines and hanging chain-lines are among the structures that achieve their shape through form-action. Since the shape of the inverted structures, now acting in compression instead of tension, is still derived from the hanging model, arch and shell structures are counted among the form-active structural systems.

Since structures based on the form-active structural mechanism achieve their form through a loading, this shape can – by definition – not be a free form shape defined through a design. Although the quantitative deformations are controlled through the materialisation, the qualitative aspects are only controllable up to a limited degree. In pneumatic structures convex areas cannot be adjacent to concave areas, as this requires the opposite pressure. Varying curvatures within a single object can only be achieved through patterning the inflated shape or the application of external restraints. These measures will therefore be the focus when constructing a free form through a form-active mechanism.

Vector-action

Definition: Vector-active structure systems are systems of straight linear members, in which the redirection of forces is effected by multi-directional splitting of forces into vectors along compressive and tensile elements. (Based on the definition by Engel (1999): Vector-active structure systems are systems of short, solid, straight lineal members (bars), in which the redirection of forces is effected by VECTOR partition, i.e. by multi-directional splitting of single forces (compressive or tensile bars).)

The characteristic of straight members implies at best a polygonal approximation of the free form curved shapes that are envisaged to be constructed, where the size of the members determines the deviation between the intended curved shape and its approximating polygon. As no bending, torque or shear is involved, the material stresses resulting from this mechanism are equally distributed in the members' cross sections. This leads to a highly effective usage of the structural material. However, to resist instability and local bending, members should always feature some resistance to section-action.

Whereas straight members contradict formal freedom, their structural efficiency is attractive, and may nevertheless be exploited if the parts that do not allow realisation as straight members are structurally acting through another system.

Surface-action

Definition: Surface-active structure systems are systems of rigid surfaces (= resistant to compression, tension, shear), in which the redirection of forces is effected by surface resistance and particular surface form. (Based on the definition by Engel (1999): Surface-active structure systems are systems of flexible, but otherwise rigid planes (= resistant to compression, tension, shear), in which the redirection of forces is effected by SURFACE resistance and particular SURFACE form.)

Structures acting in surface-action consist of a surface, as form-active structures could be composed of too. The difference between both structural systems is defined through the nature of the material the surface is made of: form-active structures do not resist to compression, tension and shear, whereas material in surface-active structures do.

Since free form building designs are often designed as surfaces of rigid material, load transfer through surface-action seems to be an appropriate mechanism to use the enclosure in a structural way.

Section-action

Definition: Section-active structure systems are systems of rigid elements, in which the redirection of forces is effected by mobilization of sectional (inner) forces. (Based on the definition by Engel (1999): Section-active structure systems are systems of rigid, solid, linear elements – including their compacted form as slab –, in which the redirection of forces is effected by mobilization of SECTIONAL (inner) forces.)

This mechanism is by far the most versatile of all four. Contrary to Engel's definition, members do not necessarily have to be solid or linear. The mechanism applies to any rigid material composition. The variety of stresses resulting from this mechanism is characteristic for the multitude of loading types that can be transferred. The drawback of the versatility is in the inhomogeneous stress distribution within a single member, resulting into unused capacity.

As the geometry of Free Form building designs is capricious, the load bearing behaviour resulting from this may be equivalent. The versatility of the section-active mechanism seems to be appropriate – though not outstanding in efficiency – to do this.

2.2.2 Combinations of structural systems

The four principal mechanisms of load transfer are theoretical cases that seldom appear in isolated form. In reality, usually several of them are acting at the same time within the same element. This simultaneous action either takes place on one, or on different levels of scale. In the first case, the structural systems are mutually dependent and interacting. Potential benefits are reciprocal compensation of critical stresses, systems-transgressing multiple functions, increased rigidity through opposite system deflection (Engel 1999) or the increased versatility towards multiple loading conditions. In the case of structural systems acting on different levels of scale, the hierarchically lower system simply transmits its loads to (or: is given support by) the system hierarchically above.

2.3 Material processing ('material')

Three different processes for the production of elements are commonly distinguished: additive, subtractive and formative. The process of cutting planar sheets is widespread and therefore isolated from its wider class of subtractive processes and identified as two-dimensional fabrication processes (Kolarevic 2003). Also the tools needed for two-dimensional fabrication are substantially different from volumetric subtractive processes. Whereas these processes were formerly carried out through manual operations, nowadays numerically-controlled (NC) equivalents exist. Notably in the case of rapid prototyping (the creation of a material model from a digital model) the combinations of these processes extend the range of elements that can be fabricated in one go (Onuh and Yusuf 1999).

Each of the processes fits a class of similar material properties and size-range. They are often constrained by the existence and availability of machines. From the perspective of manufacturing, distinction is made between the following materials:

1. Materials that are initially behaving as a liquid aggregate, which can subsequently be poured, expanded, cast or layered, and subsequently harden or cure, in additive processes;
2. Materials of sufficient rigidity such that material can be removed in subtractive processes;
3. Materials with elastic (and eventually also plastic) properties which can be bent or stretched without breaking in formative processes;
4. Materials available in planar sheets to be cut in a two-dimensional fabrication process.

Specifications and examples of each of the processes will be discussed in the next sections. Moulding techniques are discussed separately as they may apply to both additive and formative processes.

Table 3 shows an overview which elements from the overview of geometrical classes can potentially be produced through which of the four processes. Although by definition all physical elements are volumetric, classes for (curvi)linear and surface-like elements are introduced to better match the manufacturing processes. Elements are defined as individually parts of a single material. The table applies to the material the element is made of, but in case of using a mould, the scheme also applies to the fabrication of the mould itself.

From the table it is clearly visible that additive and subtractive processes are the most versatile as they can produce elements of all geometrical classes. However, it is not necessarily needed when the processes' versatility is used for relatively simple elements. The table therefore proposes alternative processes for the applications where additive and subtractive techniques are judged inappropriate.

Each of the methods will be specified further in the following sections, however, the specifications for the required material and the manufacturing techniques applied to this are highly context-specific and in constant development. Attempting to cover all processes for all materials is therefore not appropriate. Instead, the focus is laid on the abstract principles that are more likely to remain unchanged. In the same abstract manner the possibilities for numerically controlled production is discussed.

Table 3. Candidate manufacturing processes to construct elements of identified geometrical classes.

			Manufacturing process			
			Additive Moulding constraints may apply!	**Subtractive**	**Formative** Moulding constraints may apply!	**2D-fabrication**
Geometrical class of element	**Curvilinear elements of constant cross section**	**Straight** (0D-E)	Pouring Expanding Casting Layering	Milling		Cutting
		Planar curve (1D)			Single axis bending	
		Space curve (1D)			Double axis bending	
	Surface elements of constant thickness	**Planar** (0D-E-E)				Cutting
		Single curved (1D-E)			Single axis bending	
		Double curved	〃	〃	Stretching (requires significant force)	
	Other volumetric elements		〃	〃		

Key:

Element cannot be produced through this method
Possible but little appropriate since less elaborate processes are available
Most appropriate

2.3.1 Additive processes

Additive processes shape initially shapeless material through pouring, spraying or casting of aggregate liquids, expanding of foam or deposing layers of sheets, powder or liquid, followed by solidification. Large scale applications are the casting of concrete (tenths of meters, constrained by the size of the mould and the hardening process). Medium and small scale applications are the casting of metals (up to several meters) and 3D printing (up to several decimetres, constrained by the size of the machine).

3D printing is a numerically controlled additive method that adds multiple layers of material. When deposing material, this can for instance be starch or gypsum, or the solidification of a liquid through a laser beam. Objects that can be produced in this way are limited to the size of the machine. Printing a volume of a 0,5m-cube is about the maximum, although a machine (called 'huge' and 'unique' by the fabricator, state 2006) exists that prints a car-dashboard in one run (Materialise 2006), and the ambition of Khoshnevis' Contour Crafting method is to print entire buildings in full scale (Khoshnevis 2004), by adding layers of concrete with numerically controlled trowel. Apart from their distinct size, the latter two examples also imply that the mechanical properties of these objects are sufficiently performing to act as final product.

Additive processes other than the numerically controlled ones require at least a surface acting as mould, which in itself also has to be produced. The production of moulds thus comes down to the production of surfaces, for which production methods – specified per type of curvature – are already included in Table 3. The required surface quality (geometrical accuracy, texture) depends on the application, while requirements concerning stiffness and water-tightness depend on the material that is poured onto it: expanding foams require fully enclosed moulds, while poured liquid materials require watertight moulds, eventually with exposed topside in which the material will level itself. In case of spraying, spreading out or pouring of a liquid with high viscosity, the applied material will not level out, and only a one-sided mould is needed, which may not even need to be watertight. Precedents of additive processes are included in the precedents in chapter 3. These precedents notably concern various applications of concrete and resins used in combination with fibre reinforcements.

The tendency towards curved and non-repetitive shapes, in combination with the cost, time, effort and waste involved in custom-made moulds has increased the need for an adjustable mould, which could be re-used like conventional moulds and formwork. However, although the idea is expressed numerous times, only few attempts have been developed onto the level of a working prototype (Helvoirt 2005), and no commercially available technique is known.

Because additive and formative processes both make use of rigid master geometries (such as moulds) on, to or onto which material is applied, alternative moulding techniques for these processes are discussed together in section 2.3.5 on Moulding techniques.

2.3.2 Subtractive processes

In subtractive processes a shape is cut or carved out of a larger starting volume through the removal of material. Cutting off material is done through cutting devices that are either point- (e.g. a mill), line- (e.g. a cutting wire) or surface-shaped (e.g. a saw blade).

Line-shaped cutting devices are for instance wire cutters having a wire with sharp spots (e.g. stones or diamonds) on it, running at high speed through the element's material. As the wire is stressed, the cut is running as a straight line through the material, resulting in a ruled surface. Obviously, such cuts are always through the full depth of the object. A heated wire may melt its way through soft materials (such as foams), resulting in ruled surfaces too. With heated curved wires of sufficient rigidity also curved cutting planes can be realised. In this case the cutting wire is curved by rotating the extremities. By controlling the rotations during the cutting process, the cutting edge's shape changes from straight to curved, and smooth transitions between the layers can be produced (Broek, Horvath et al. 2002). Most formal freedom can be achieved with a thin wire as bending or rotation of this is not obstructing itself as surface-shaped cutters would. For the same reason, surface-shaped cutting tools always result in planar cutting sections.

With a mill, a rotating device of which the positioning relative to the object is controlled, far more geometries can be produced. The formal freedom that can be reached notably depends on the number of axis in or around which the milling head can move or rotate. A 1-axis mill (basically a drill) can move in one direction, a 2-axis mill in two directions and thus is able to cut out any planar shape. With a 3-axis mill also the height can be controlled. In addition to the 3 translational axes, the 4- and 5-axis mills can also rotate, such that also undercuts can be made. The number of axes includes the movements and rotations of the milled element.

Numerical controlled milling consists of determining the required path that the milling head should follow in order to achieve the required surface. The accuracy and smoothness depend on the intermediate distance between the milling paths. As less-spaced paths result in a longer path-length, first a fast and rough run with a large and coarse milling head is carried out, followed by a second run with a small tool of appropriate shape. To do so, milling machines can automatically change their mill head.

2.3.3 Formative processes

Formative processes deform geometries from one shape into another through the application of a load, be it a pressure, moisture or heat, as well as combinations of these. Formative processes are applied to transform parts prepared from starting material. These starting materials are commonly mass-produced and available as linear elements or planar sheets. A further specification of the formative process concerns the conditions at which the forming takes place. For this, cold and hot deformation processes are distinguished. Furthermore, permanent, partly-permanent and non-permanent deformations are distinguished.

In case of deforming sheets, the sole application of bending leads to a developable surface. Bending combined with compression or extension leads to a double curvature.

As the sheet's full cross section is sollicitated (in case of bending only the applied extension is a gradient across the thickness), for which relatively high-energy impact-techniques are required, such as (a combination of) heating, hammering, explosions or others means to apply pressure.

Formative processes other than through the application of force, for instance by local heating or moistening of the material, are based on the material's volumetric change due to swelling or grain repositioning (Thomson and Pridham 1995). If these phenomena occur unevenly across the element's cross section, the element will bend or twist.

The appropriateness of formative processes to produce irregular shaped elements depends on the ability of the material to undergo the required deformation without failing (e.g. breaking) or a degradation of the material properties. Notably metals are generally fit for formative processing. The material properties of metal do change during the formative process, but as the effect is known, they can be anticipated to. Numerous tools exist for rolling sheets and profiles (to create curved elements) or pressing (to create polygonal shapes). To create double curvatures, the base material is pushed into a die using the force of a press or a water pressure (e.g. caused by an explosive).

A limitation of formative processes is due to the potential overlap of formative loads and service loads, notably in cold processes where the atmospheric conditions are the same as once the element is in service in its final load-bearing constitution. In the first case the element has to be sufficiently flexible to undergo the transformation, whereas in the second case it should remain stiff (with the bottom line of not undergoing plastic transformations), resisting to the loads a load bearing structure receives and transfers.

The targeted shape is to be controlled either through controlling the (amount, positioning and direction) of the applied process, or through the control of the element's geometrical constraints. Determining the required amount of loading may be done iteratively until the final shape meets the set target shape, or through an in advance determined amount, which thus has to include anticipation to springback-effects. This anticipation requires precise knowledge of the element's material properties and the residual stresses due to the formation history of the material, and insight in the forming process. As notably the material properties vary and are hard to measure, this is a considerable obstacle for the implementation of numerically controlled formative processes (Bahloul, Ben-Elechi et al. 2006). Therefore a feedback control system is needed that measures the initially produced geometry, re-calculates the load needed and finally repeats the formative process (Thomson and Pridham 1997). This is an automation of the actually exercised manually controlled bending process.

Geometrically constraining is a far more simple method, as the constraints (= the mould) can be measured unambiguously, followed by the application of an amount of load which is large enough (rather than a specific amount) to achieve the deformation. Only in case of discrete constraints (which support the elements only locally), there may also be a maximum amount of load applied. Geometrical constraints are classified identical to those of additive processes, specifying the control of one geometrical entity in order to control another. See section 2.3.5 on Moulding techniques.

Cold and hot deformation

If the deformation is taking place under normal atmospheric conditions, one speaks of cold deformation. Consequently, hot deformation implies that conditions are tuned for the deformation to decrease the material's stiffness (such that less power is needed to achieve the deformation, and less cross sectional deformations appear[4]) or decrease the elasticity in favour of the plasticity (such that it features less springback). Such specific conditions generally imply an increased temperature, an increased atmospheric

[4] For instance in the case of bending tubes into tight curvatures, local buckling of the tube can be prevented by employing a hot deformation technique such as induction bending (Angle Ring 2006).

humidity or an increased pressure. In case of metal, hot deformations are those in which recrystallisations (the formation of new crystals in deformed metal) can occur.

If the deformed element maintains its deformed shape, the deformation is permanent, whereas the deformation is (partly) elastic if the element is featuring (some) springback. The element will have to be constrained to stay in its deformed shape. This implies that the stresses due to the deformation are still present, but – depending on the material – may disappear through relaxation. Although in stable position, also permanent deformations result in residual stresses. In the deformation process, one may also anticipate to the springback by deforming the element further than needed, such that it achieves the required shape after springback.

2.3.4 Two-dimensional fabrication processes

Two-dimensional fabrication is the process of cutting shapes from planes. Planar elements are relatively simple to make, as their only geometrical parameters are the x- and y-positioning. Such elements are usually cut from mass-produced planar sheets. Numerous cutting methods exist, featuring differences such as the materials that can be cut, the range of thicknesses it can cut, cutting tolerances, the amount of heat that is generated (and thus the degree to which the initial material properties are modified), the width of the kerf (including on which side of the line to cut) and the smoothness of the cutting edge. All of these result in consequences on speed and cost.

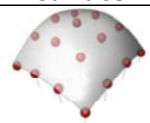
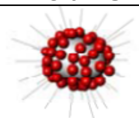
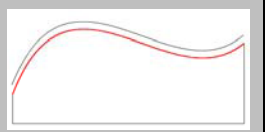
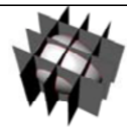
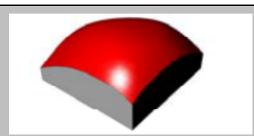
The potential for free form is considerable as – since recently – also spline-geometries can be read. Before, the input geometry was limited to lines and arcs, since the conversion of splines into the dxf-format resulted in numerous short lines which were shorter than the kerf of the process so the calculation could not be made. This is now solved by converting splines into lines and arcs (all with tangent transitions and within a user-defined accuracy), which could already be handled (Blackmon 2006). However, the success of their application depends on the assembly into spatial components.

2.3.5 Moulding techniques

Moulds are used to temporarily constrain the geometry of an element that is to be built by an already existing geometry, in additive as well as formative processes. An important distinction is to be made between directly and indirectly controlled geometries. The constraints' geometry is said to be directly controlled if the geometry of the element as it is produced, is known exactly. If not, for instance when the produced geometry is only known at points, the eventual leftover parts between the constraints may be known, but is not in control. Therefore, it can at best be an approximation of the shape that is strived for.

Possible combinations with respect to the geometrical class of the mould and the elements that can be built on them are depicted in Table 4. The options on the diagonal axis of symmetry all use a mould of the same geometrical class as the element that is to be produced on it, therefore the control is said to be direct. On the right side of this axis, the elements to be produced are of a higher geometrical class than the actual mould. The control is therefore indirect: it is constrained locally, the rest results through the stiffness properties of the element's material that is put on top. On the left side of the axis of symmetry the produced elements are of a lower geometrical class than the mould itself. Since higher geometrical classes require more definitions of boundaries, the mould is geometrically-unnecessary over-defined. The left three columns specifically apply to formative processes as the material used has initial coherence and stiffness, whereas the right two columns are appropriate for additive fabrication processes that requiring at least a surface to bear the material that is initially shapeless and non-rigid. The resulting element is classified according to typical geometrical feature that is controlled by the mould. So when spreading concrete on a curved surface, the applied layer may well vary in thickness, but this is not a result of the mould. Hence, the resulting element is classified as a surface, not as a volume. This is for instance relevant when a specific thickness or smoothness of the surface is required – it may either be left to the manufacturing procedure, or otherwise be assured through direct control by means of a volumetric mould.

Table 4. Origin-result matrix of moulding techniques according to the geometry of the mould and the geometry of the element to construct.

		Resulting geometry			
		point	**curve**	**surface**	**volume**
Controlled geometry of mould	**point**	Locator (e.g. a support of adaptable height)	 Point-controlled spline (e.g. a strip in bending)	 Point-controlled surface (e.g. a pin bed)	 Point-controlled volume (e.g. a 3D pin bed with pins in all directions)
	curve		 Curvilinear mould (e.g. a custom-cut limitation of a plane)	 A series of curvilinear moulds (e.g. a line-controlled surface)	 A series of curvilinear moulds enclosing a volume (e.g. a 3D line-controlled surface)
	surface			 Surface mould with one exposed surface (e.g. a milled surface)	 Surface mould enclosing a volume (e.g. an inflated balloon)
	volume	Example of a geometrically-unnecessary over-defined mould: milling a block to position a point in space			Negative mould: an enclosed volume (e.g. a milled object) Positive mould: investment casting

Potentially appropriate for **formative** processes

Potentially appropriate for **additive** processes

Key:

	Indirect mould-control: mould geometry is controlled locally, in order to approximate the wanted geometry
	Direct mould-control: controlled geometry = wanted geometry
	Geometrically-unnecessary over-defined since it requires more geometrical inputs than the number of geometrical dimensions of the resulting element

The scheme can be ran through multiple times within a single mould. For instance when the top of a square pattern of bars (=points) do adjust two strips (=lines) in two directions. On top of these two indirectly-controlled strips a flexible surface (=surface) is laid out. This surface is the moulding surface, which consequently produces indirectly controlled surfaces through indirectly controlled strips. The fluidity of the surface (e.g. if the strips leave a mark on the element's surface) depends on the degree to which the flexible surfaces can bend and stretch. For more fluidity, more surfaces may be stacked, each surface controlling the surface above. This leads to a high degree of indirectness of control, requiring multiple levels of corrections when adjusting the mould for the element to be produced.

To reduce an elements' geometrical complexity, a strategy may be to reduce the number of parameters to be handled to an affordable minimum. For this, one should strive to use elements of the lowest geometrical class possible – and hence lessen the number of control parameters and the associated complexity, and on the other hand strive to produce elements with the lowest number of geometrical control parameters. As controlling less geometrical parameters costs less effort, but also decreases the influence on the final geometry, a balance has to be found between manufacturing constraints and performance requirements (e.g. smoothness, accuracy).

2.4 Overview and look ahead

The design variables that will be used to associate precedent solutions to, as well as to generate alternative schemes and systems from, are displayed in Figure 15. From the subsequent overview of mechanisms of structural action and manufacturing processes, it appeared that there is notably a strong relationship between features of geometry and material. These give a primary indication of potentially viable paths to realisation, and will therefore be explored more extensively. These paths are:

Custom-shaped planar elements – Materials available as planar elements, which are constructible from input points, can be custom-cut into arbitrary shapes.

Fabricatable developable surfaces – In case of sheet material, the formative manufacturing results in developable surfaces. For this, the extrude-manipulation, eventually extended with a scaling-manipulation, is the way to generate them. Singular use of extrusion leads to cylindrical curvatures, whereas combined extrusion and scaling leads to conically curved surfaces. In case a constant curvature is required, then the curvature of its primitive curve is decisive.

Partially controlled moulding techniques – Applicable to both formative and additive manufacturing processes, the usage of a mould and its associated constraints, allows constructing geometrical artefacts of a higher dimensional class than the mould itself.

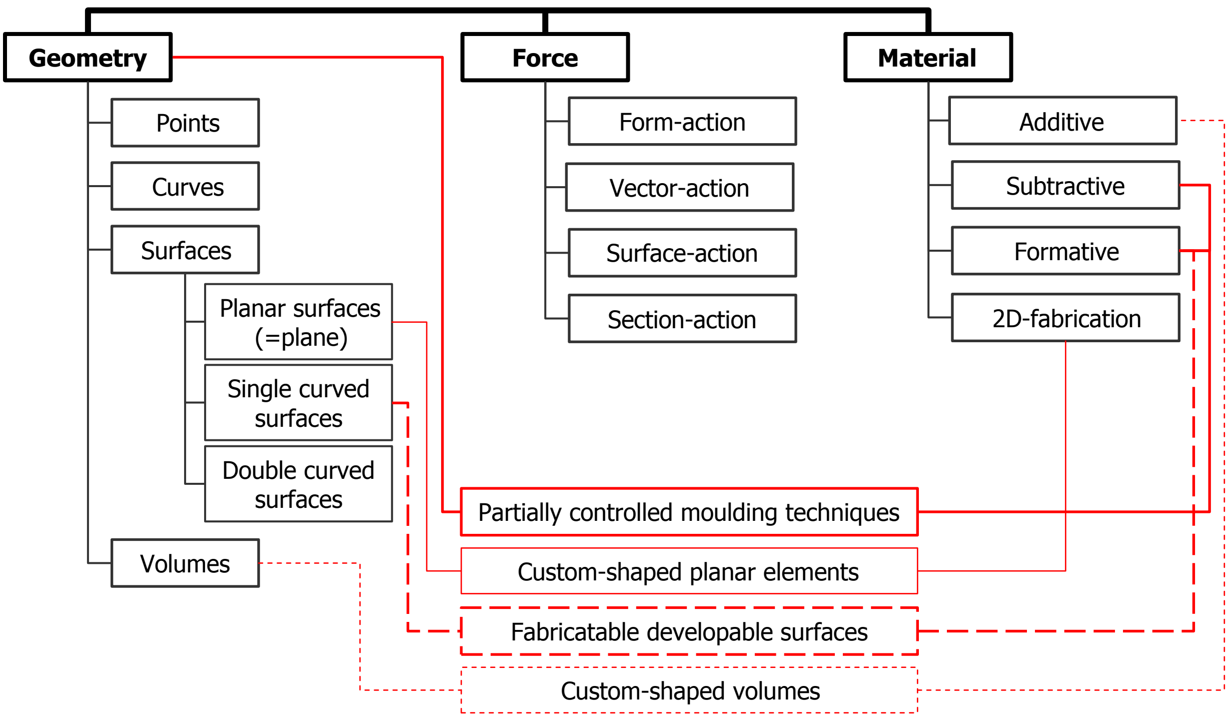


Figure 15. The design variables broken down into features, with potential combinations indicated.

3 Precedents of free form structures in the BLIP-framework

The previous chapters elaborated on the design variables geometry, force and material, as well as their subdivision into features, this chapter will specify and demonstrate their use through realised structures. To enable a proper description, the notion of the level of abstraction will first be specified. This is needed since all listed realised structures are different on the object-level, but nevertheless feature similarities on an abstract level. Among these levels of abstraction, also the class of structural systems specific to a manufacturer will be introduced. This class frequently appears, notably in parts of building structures (e.g. facades or roofing), and went parallel to an ongoing development within some leading companies of design and/or production.

The described precedents are categorised as data-documents in the framework of the Blob Inventory Project (BLIP), a database developed by the author and two fellow researchers. It records information in an overview where each document may be attached to multiple features from a tree-hierarchy, this way offering multiple entries. Once the database is filled, it facilitates analysis across different documents, needed to derive a design direction from for the next phase.

In addition to the attention given to the individual design variables, also their coherence on the overall level will be discussed, notably the development of digital tools used in either design, engineering, manufacturing or installation. These tools allow abandoning labour intensive solutions, in favour of parametric modelling followed by computer controlled customised production. This development marks the paradigm shift from mass production to custom production, in which standardisation is no longer needed to maintain the workability, but systematisation is.

The analysis of the precedents of this chapter, together with the overview of the previous chapter, results into findings. These findings form the basis for the formulation of a series of hypothetical solutions in the next chapter. Based on this, alternative structural systems will be designed.

3.1 Levels of abstraction in structures

Categorisation and comparison of structures is complicated by the fact that some have only been designed, while others have been developed as a prototype, and only some are realised. Characteristic to this range is the decreasing level of abstraction. Starting from highly abstract, the level of specification of a) the load transferring principle, b) the manufacturing process and c) the geometrical class of used elements increases. By specifying a structure, its number of degrees of freedom decreases.

Structural schemes are at the highest level of abstraction of structures. Structural systems and then structural designs are more specified, while finally structural applications, the built ones, are the most concrete (see Figure 16). This subdivision into different levels of abstraction, including the scheme that precedes it, will be exemplified in section 3.3 by means of the spaceframe structural system and its implementation in the Esplanade-project by the manufacturer Mero-TSK.

These levels used to describe different levels of abstraction are also transitory steps gone through in a design process: from conceptual principles in early design stages until the realised object. At each of the levels the structural variables material, geometry and load, as well as the relationships between them, are present, but considered differently.

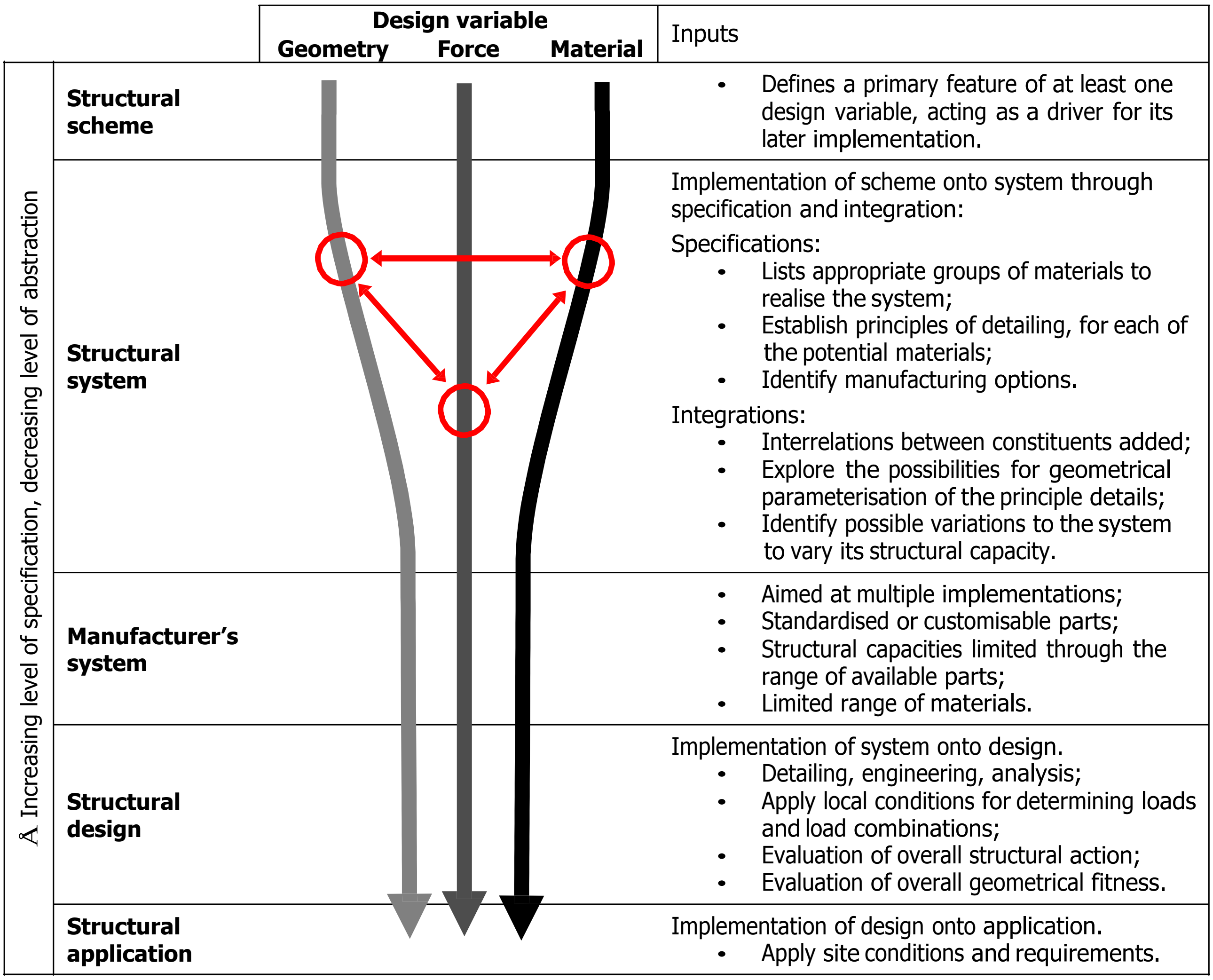


Figure 16. Structures regarded at different levels of abstraction, from abstract to specified.

Structural schemes

A structural scheme is a simplified, yet accurate representation for the driving constituents of geometrical, force or material nature, as well as combinations of these. The schemes are defined with anticipation to known or expected performance. Schemes indicate any of the constituents, but do not involve a specific choice within these.

Structural systems and manufacturers' systems

A structural system is an integral implementation of a structural scheme. It is the sum of the constituents (geometry, force and material) including their interrelations. Thus, it is of a higher degree of specification, and of a higher degree of integration. Systems as such are without size and specific material, but their application is preconceived for a certain size-range and type of material (and thus also their method of manufacturing), and they include solutions on how the capacity of members can be enlarged in case of heavy solicitation, and reduced in case of excessive ineffective capacity. Systems include principle details on how the before mentioned aspects are to be integrated and which, as a consequence, can be explored in a parametrical manner.

Manufacturers' structural systems are a sub-group of structural systems, but are specified and integrated to a more concrete level of abstraction. Manufacturers' structural systems are owned and implemented by manufacturers and are usually optimised for multiple implementations. To do this, systems consist of standardised and/or customisable parts, which put a limitation on the range of applications, as well as on the structural capacities that can be reached through them. Obviously a larger number of customisable or even custom-made parts extends the range of applications, but challenges the systematic setup of the system, as more degrees of freedom are introduced.

Structural designs

A structural design is an implementation of a structural system, comprised in a model. Through the specification of materials, dimensions, as well as the application of local conditions for determining loads and load combinations, it includes detailing, engineering and analysis. These inputs allow for the evaluation of overall structural action, and consequently also for the evaluation of the overall geometrical fitness.

Structural applications = built structures

A structural application is an implementation of a structural design. For this, all site conditions and requirements are implemented, including a sequence of construction of parts and their installation. It is a full-scale built object of the highest level of application and materialisation, and thus the lowest level of abstraction.

3.2 Storing precedents in the Blob Inventory Project (BLIP)

3.2.1 Background and aim

Much research (although not necessarily named so) on how complex structures are designed, analysed and built is done in building practice. Notably in the case of free form building designs, built structure as well as the alternatives that were rejected earlier in the design process, contain precedent information that may be helpful in a future project. Although captured in a highly specific context, abstract notions of a higher, more general validity may be distilled from them. The aim is to categorise them, and make them accessible for future use.

3.2.2 The concept

The proposed solution to retrieve precedent information is to break down precedent projects into content documents, each covering a (sub-)feature of a predefined (but extensible) taxonomy. Documents may also be related to more than one feature. This decomposition allows for presenting information on the same topic side by side, whereas they remain associated to the project they originate from. By decomposing information, and simultaneously providing relationships between them, a very short way to retrieving it has been created.

3.2.3 The application

After an initial attempt to setup a standard database using fields and cards turned out to be unsatisfying because the interface was far too limited in exploring the data, an application called the Blob Inventory Project, BLIP, was developed[5]. Although storing of specific data was its primary aim, the stored data should be presented such that it allows for later retrieval, comparison and analysis. Also it was suggested that the stored data could be made accessible to others (e.g. students and building practice) such that they could search for earlier examples with similar characteristics, when they were confronted with a new problem (Kocatürk, Veltkamp et al. 2003). However, such usage is beyond the scope of this thesis.

[5] BLIP is a project developed as a joint work by three PhD researchers (Tuba Kocatürk, Bige Tuncer, Martijn Veltkamp). Bige Tuncer's research provided a flexible and extensible framework for knowledge modelling that acts as the backbone of the information structure of BLIP. Tuba Kocatürk's and Martijn Veltkamp's research provided the main context and the related knowledge content for the application which contributed to the cross-disciplinary richness of the knowledge content due to the separate research foci and disciplinary background of the two researchers. The authors would like to acknowledge Joost Beintema for his contribution to the programming of BLIP.

The database's user interface is optimised to allow cross-referencing between multiple projects. At any time, the screen layout as shown in Figure 17 provides feedback to the user about which aspect or feature he is exploring and to which projects they are associated. This allows both browsing to a more specific feature – so called forward browsing – as well as for a more general feature – referred to as backward-browsing. Currently, the main starting point for data-retrieval is the predefined keyword network structure, in frame A, on the left hand side of the screen. The selected feature(s) will be highlighted and displayed in frame B on the top right hand side with the other features associated with it. These associated features are predefined within the keyword network structure, using a subdivision into formal aspects, structural aspects and aspects of manufacturing.

By the use of a slider, the user can choose the degree of sub-features to be displayed in this window. The actual project-related content is stored in documents that are related to one or more of the predefined features. Clicking on a feature in window A or B will display the related project documents in frame C which contain textual, graphical or numerical data containing information on the selected feature and its associated features. If one of the documents is selected in frame C, frame D will then display specific information on the selected project and will name all the features associated with this project.

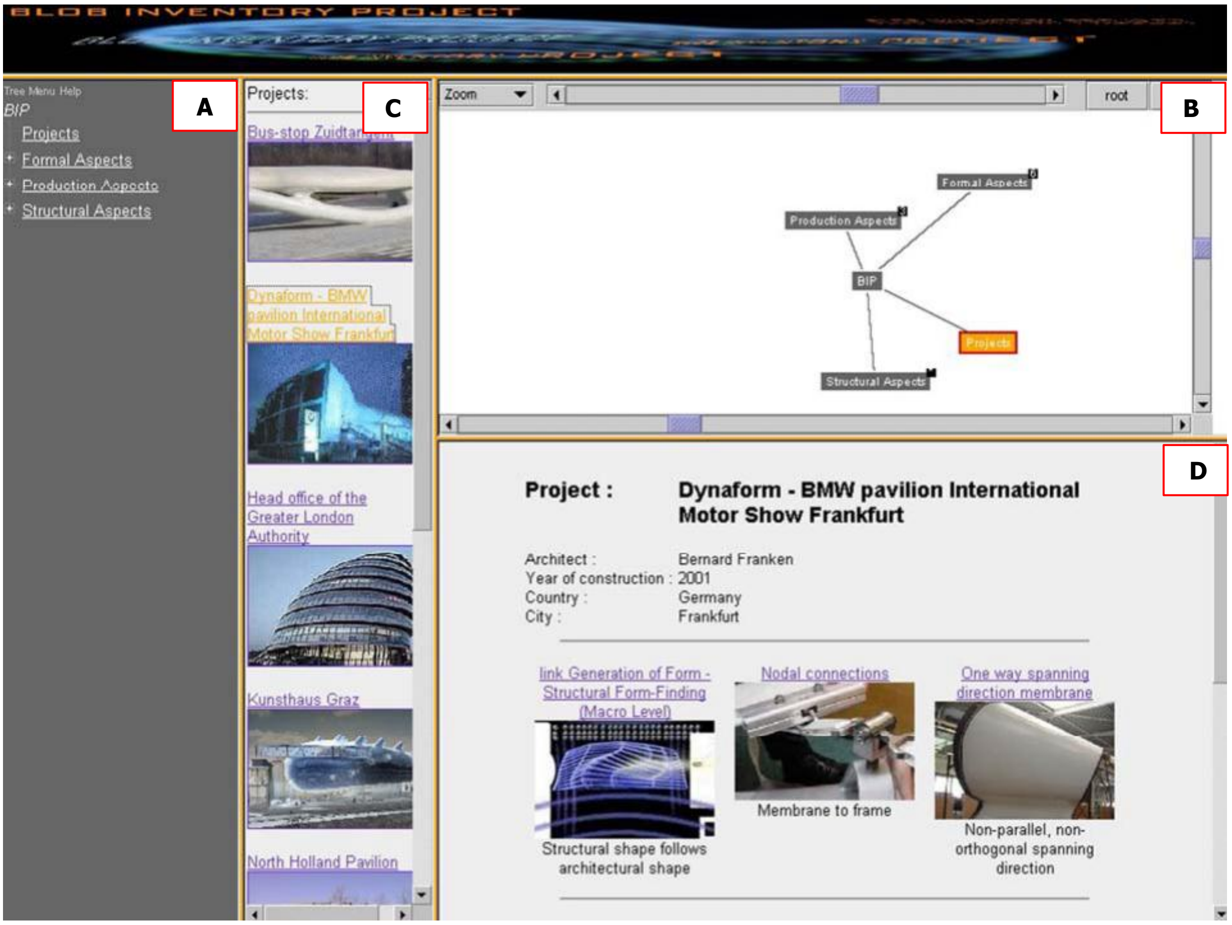


Figure 17. Interface of the BLIP-application. Frames: A) aspects and features, B) relationships between features, C) list of projects or documents, D) list of documents belonging to one project or content of document.

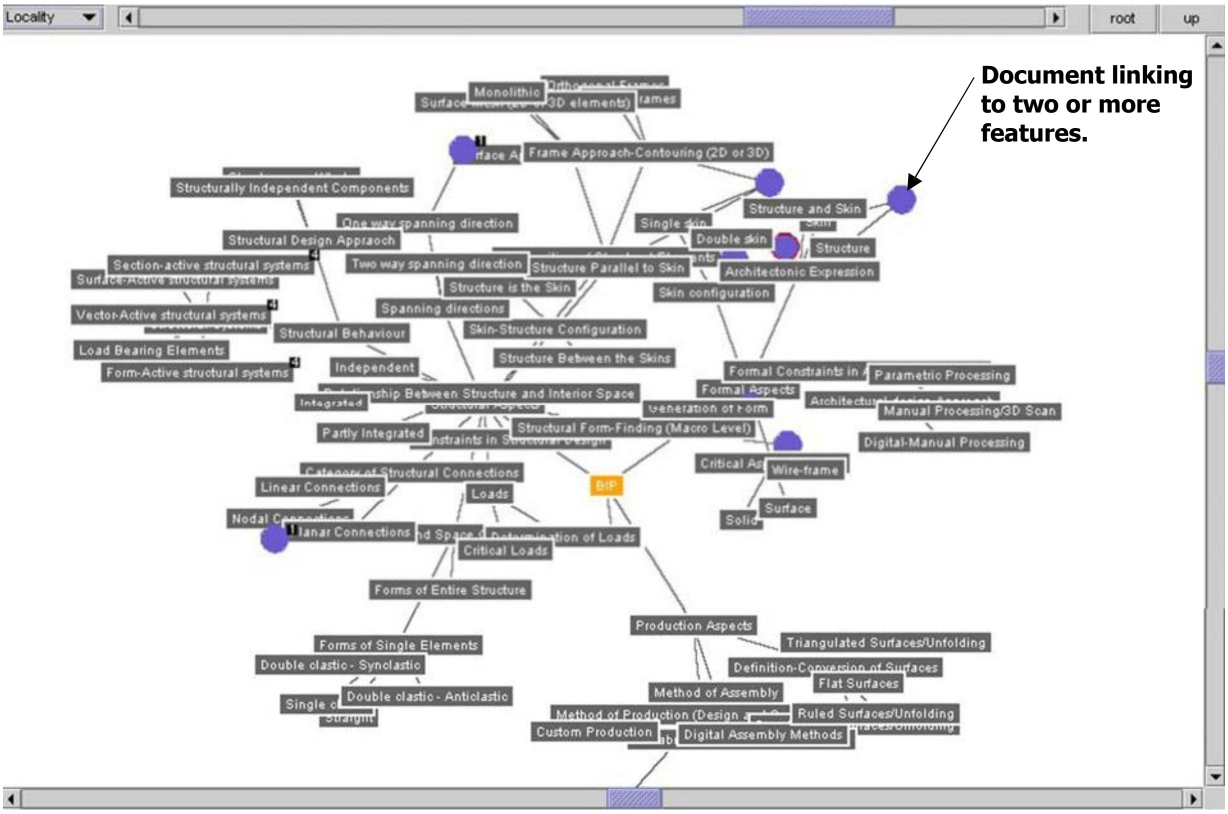


Figure 18. The relationship window of BLIP showing a link document.

3.2.4 BLIP in this thesis

In the context of the actual research project, the concept of BLIP has facilitated the structuring of information gathered from precedents, through using content documents. For this research project the original taxonomy has been replaced by the design variables of geometry, force and material as developed in chapter 2, containing information focussing on products. The sum of documents is expected to give an overview of alternative solutions and the relations between them (breadthwise), as well as the actual content (depth). Figure 19 visualises how documents are related to features of either of the three design variables. For reasons of clarity the figure only displays two documents, since the view quickly becomes crowded due to the numerous relationships, which was solved in the application of BLIP. The analysis is focussed on finding appropriate relationships between either of the features of the considered design variables, and find directions for future searches. It is expected that these are either located in those areas where no precedents are stored, as well as through combination of existing precedents.

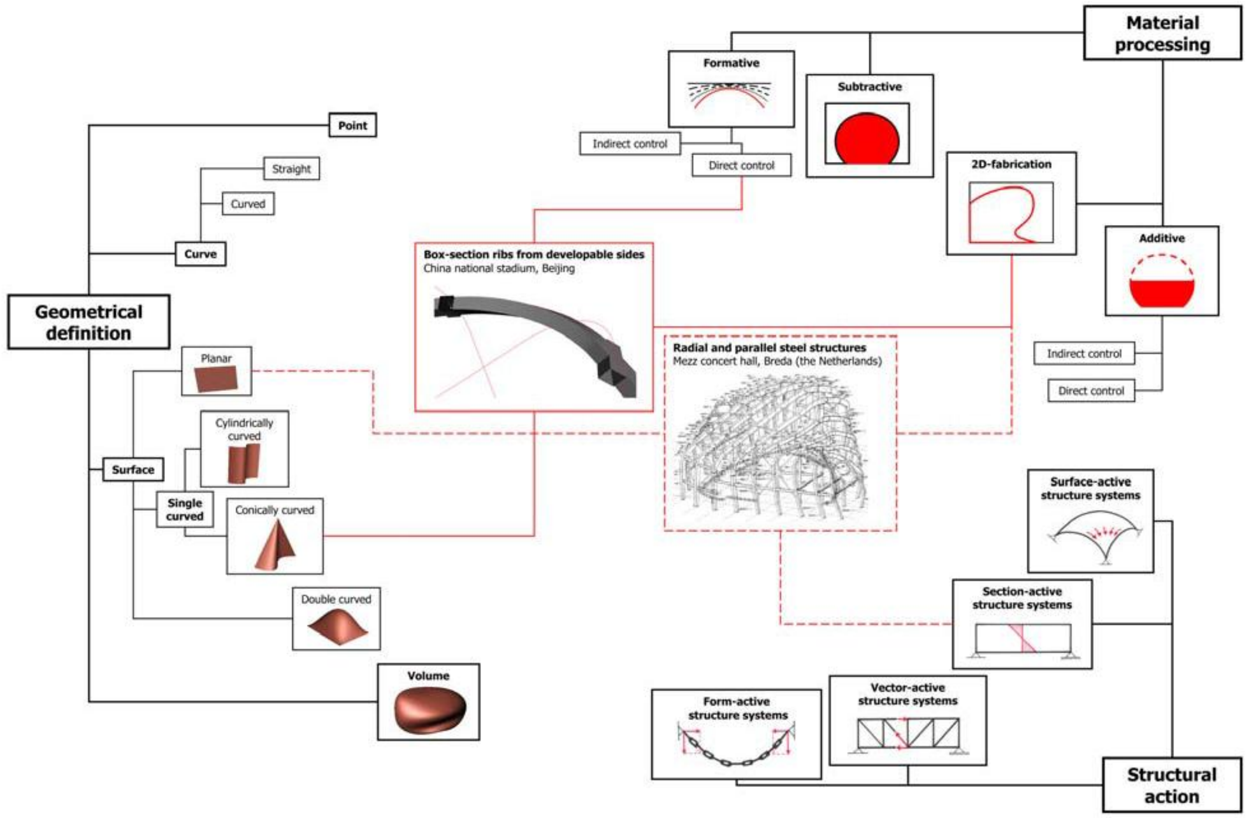


Figure 19. Structure of categorising content documents in BLIP, with two exemplary documents.

3.3 Content documents

The content documents to be presented next have been selected to represent a wide range of technologies for the design, analysis and construction of building structures. The documents have no systematic hierarchy or sequence other than through the features they are associated to, nonetheless for the sake of readability they have been clustered around topics. This chapter's final section cross-analyses the documents, hypothetical solutions for structural schemes derived from this are listed in the next chapter.

3.3.1 Four levels of abstraction in structures exemplified in the Singapore Arts Centre

Document title:	**Structural scheme of a space frame**
Parent features:	Geometry > Curve > Straight Force > Vector action
Level of abstraction:	Structural scheme
Content:	The scheme consists of geometry of at least six straight lines connected at their extremities to one point in a 3-dimensional space, which is simultaneously also its representation, and the vector-action resulting from this when the points are loaded by forces, see Figure 20. The members are laid out in two separate networks on two surfaces. Starting from the members' ends, diagonals run to the network on the other side.

Appropriate implementations as structural system are those with members of constant cross section. If the members cross section features point- or line-symmetry this will facilitate detailing of the connections. Members can either be connected directly to each other, this way including all complexity in the members, or connected through intermediate elements, nodes. With the latter approach, fixing complexity is included in the nodes.

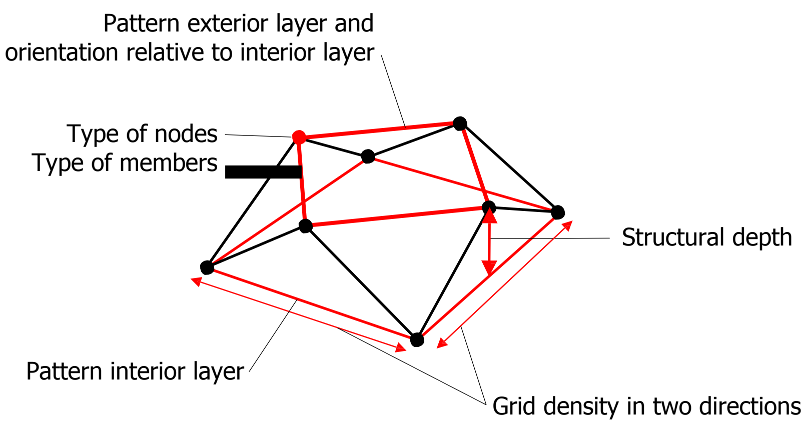


Figure 20. Degrees of freedom of the space frame structural scheme

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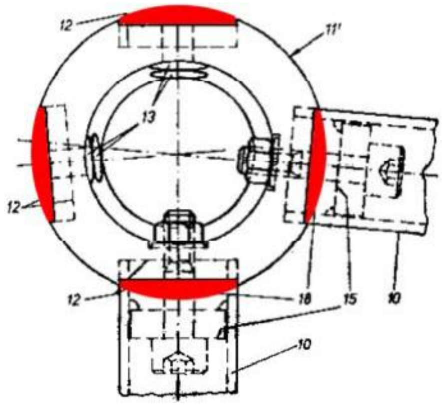
Document title:	**Mero-TSK's space frame systems**
Manufacturer	Mero-TSK
Parent features:	Geometry > Curve > Straight Force > Vector action Material > Subtractive
Level of abstraction:	Manufacturer's system

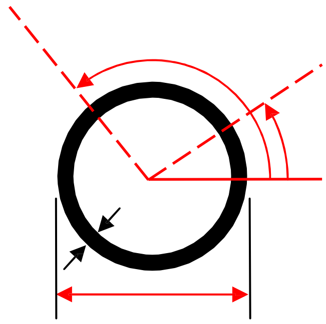
Content: A number of different systems materialising the spaceframe structural system exist, summarised and evaluated on their structural range of application in (Stephan, Alvarez et al. 2004). The German steel constructor Mero-TSK is one of the firms designing, producing and building these, and already have built a large variety of reticulated structures in steel, all based on off-site prefabrication and on-site bolted connections. Mero-TSK has a variety of systems available for this, which are customised for each application. Customisation includes the mesh-shape and its density, the size and cross section's shape of members and the type and size of nodes applied. The choice of a system depends, amongst others, on the required mesh properties (interaction with cladding sizes and support conditions), the design being single- or double layered (interaction with structural requirements) and transparency (interaction with the architects' intentions).

Focussing on double layered structures of which the exterior layer is to be cladded with planar glass panels, specifically appropriate for this domain are systems that allow rectangular members to be connected, which serve to linearly support glass panels. As the interior layer and the diagonals do not need to fulfil requirements such as supporting panes, here the system may consist of simple tubes. Some nodes will now be connected to tubular members only (spherical nodes suffice in this case), whereas more complex nodes are needed to accommodate both tubular and rectangular members. These two types of nodes have been used together multiple times in Free Form surfaces (Stephan, Alvarez et al. 2004). All connections between member and the hollow node are made by one or two bolts in the member's centre line.

For application in irregularly curved shapes, geometrical customisation of both nodes and members is needed. The customised nodes have to accommodate the members' different angles of incidence at each node, while the node itself points in the surface's normal direction. The bowl nodes used by Mero-TSK in double-layered structures consist of a spherical part and a cylindrical part, that is later milled to a cone-shape. The conical part serves to connect rectangular hollow sections to (Kraus 1994). Bowl nodes are customised by machining to create a plane on the node to which the square cross sectioned members can be connected (Figure 21A). In addition one or two bolt-holes have to be drilled normal to the connection plane. Spherical nodes require bolt holes to be custom-drilled, and surface milling there where tube ends will connect (B). The members spanning between the nodes have to be custom-cut to the length needed, and an end-plate with boltholes made in it.



A. Customisation by milling off the coloured parts of a standard bowl node, top view. Image based on (Kraus 1994)



B. Customised angle of incidence of bar connecting to a spherical node of a standard range (in cross section).

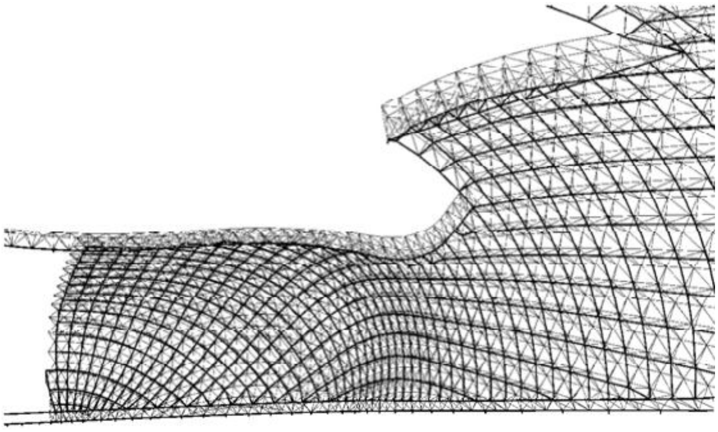
Figure 21. Geometrical customisation of the manufacturer's system for usage in irregularly curved surface structures.

Document title:	**Structural design of a space frame**
Project:	Singapore Arts Centre
Architect:	Michael Wilford and Partners (London, UK), DP Architects (Singapore)
Manufacturer	Mero-TSK
Parent features:	Geometry > Curve > Straight Force > Vector action
Level of abstraction:	Structural design
Content:	The free form façade and roof structure of the Singapore Arts Centre was to be constructed as a space frame. The choice for the grid-layout was based on the high transparency strived for by the architect. Thus, a square-on-diagonal grid (with fewer elements) was favoured over, amongst others, a square-on-square grid, after the original single-layered structure with thick beams had already been abandoned. The non-planar squares on the exterior surface are divided into planar triangles, such that planar glass panels can be laid on top. These diagonal members on the exterior surface have head plates that narrow down the member, such that the node can be kept small. In the square-on-diagonal grid the interior pattern is defined by an offset of the quadrangles' midpoints to the inner surface, through which then a diagonal grid is generated. In the applied grid all members on the exterior surface have a length of 1,5m. As a result of this the quadrangles get more and more, lozenge-shaped nearby the ground. The interior surface is at a constant 900 mm inward offset of the exterior surface. Geometrical customisation was applied as included in the system's definition. The required structural capacity was anticipated by applying multiple sizes of spherical nodes, bowl nodes, tubes and rectangular hollow sections. Although a degree of freedom of the system, the spacing between the two layers was not varied, thus no larger lever arm with lower forces in the upper and lower chord was resulting from this. Larger nodes and members were only used in exceptional cases. The largest members are applied in the most heavily loaded areas where, in order to accommodate the large dimensions, also larger nodes had to be used. (Sanchez 2002)

Document title:	**Structural application of a space frame**
Project:	Singapore Arts Centre
Architect:	Michael Wilford and Partners (London, UK), DP Architects (Singapore)
Manufacturer	Mero-TSK
Parent features:	Geometry > Curve > Straight Force > Vector action
Level of abstraction:	Structural application
Content:	In the two free form building envelopes of the Singapore Arts Centre (Figure 22), nodes and members were customised in prefabrication according to the system's degrees of freedom. To facilitate logistics during the design and engineering phases all members

were colour-coded, while for on-site logistics all nodes had a part number on them, and when needed also a mark indicating their orientation of the North-direction. The space trusses were pre-assembled in parts of 4,5 by 9 meters maximum, which were stable once erected. During assembly the positions of the nodes were permanently measured, at some places members that were intentionally fabricated with minus tolerances could be elongated. (Sanchez 2002)

A. Overview of the Singapore Arts Centre (Esplanade 2006).



B. The Lyric theatre's space frame geometry (Sanchez 2002).

Figure 22. The Lyric Theatre of the Singapore Arts Centre.

3.3.2 Twisting members

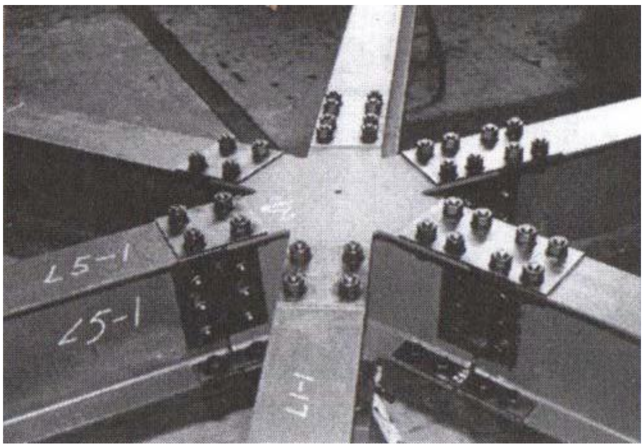
Document title: **Physically twisted I-sections**

Project: Research project Kuroiwa et al.

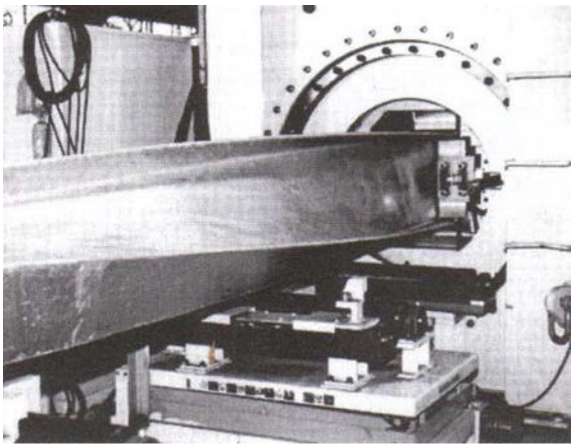
Parent features: Geometry > Surface > Double curved
Material > Formative > Indirect control

Level of abstraction: Structural system

Content: The mapping of reticulated structures on irregularly curved surfaces requires the members to twist – no matter whether the members are straight or curved. Applications where the twist is physically incorporated in the members rather than in the nodes, is advantageous for the geometry of the nodes. These nodes only need to accommodate the variation of in-plane directions of the connected beams, but their orientations are all equal (Figure 23A). The physical twist (see Figure 23B) does not affect the straightness of the member, and maintains the I-section's advantageous structural resistance against bending. Experimental research on the structural implications of twisting members has demonstrated that the ambiguous direction of the section's weak axis is compensated by the plastic deformation it underwent (Kuroiwa, Masaoka et al. 2002).



A. Standard joint with twisted members



B. The twisting operation.

Figure 23. Physical twisting of I-sections. Images from (Kuroiwa, Masaoka et al. 2002).

Document title: **Diagonally folded steel plates**
Project: Web of North Holland at its first use in 2002, reconstructed in 2006 as I-web
Architect: ONL
Steel manufacturer: Meijers Staalbouw
Year: 2002
Parent features: Geometry > Surface > Planar
Material > Formative > Direct control
Material > 2D-fabrication
Level of abstraction: Structural application
Content: The steel structure of the Web of North Holland-pavilion is a refined icosahedron – a polyhedron with 20 triangular faces sides, each of them subdivided into 6 segments – mapped onto the free form building shape (Figure 24A). The full structure follows the exterior contour of the building. The sides of the original polyhedron have been constructed as 25mm-thick continuous planar frames (Figure 24B), the members in between are fabricated as 15mm thick elements. The twist needed to span members between non-coplanar surface normals is rationalised by folding a steel sheet across its diagonal (Figure 24C). Actually the architect had foreseen two more variables than the sheet thickness only, namely the structural depth (controlled by the offset between the exterior and the interior surface of the thick-layered building envelope) and the point distribution (although constrained by the radial frames that require in-plane points, and the visual continuity of the grid). (Boer and Oosterhuis 2004)

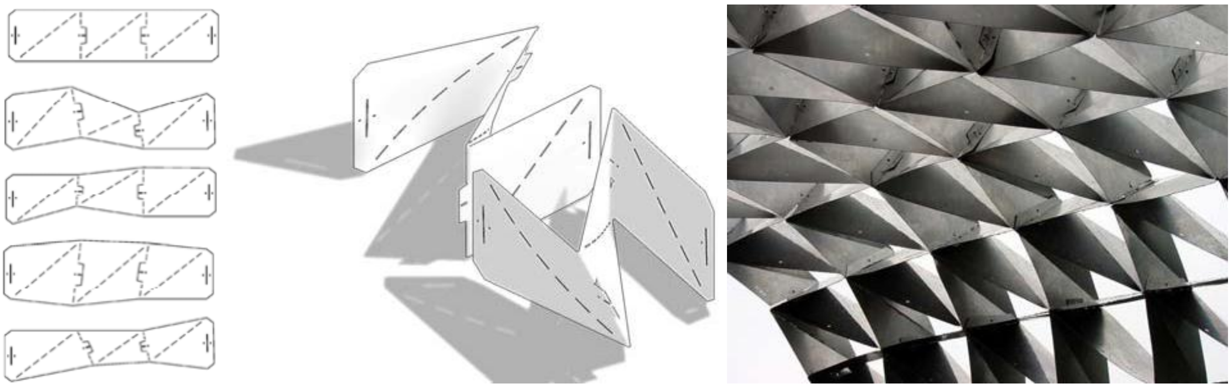
A. Digital model of the Web of North Holland: a steel sheet folded across its diagonal.

B. Primary planar frame.

C. Generation of a folded member.

Figure 24. The steel structure of the Web of North Holland. Images A and C (Boer and Oosterhuis 2004).

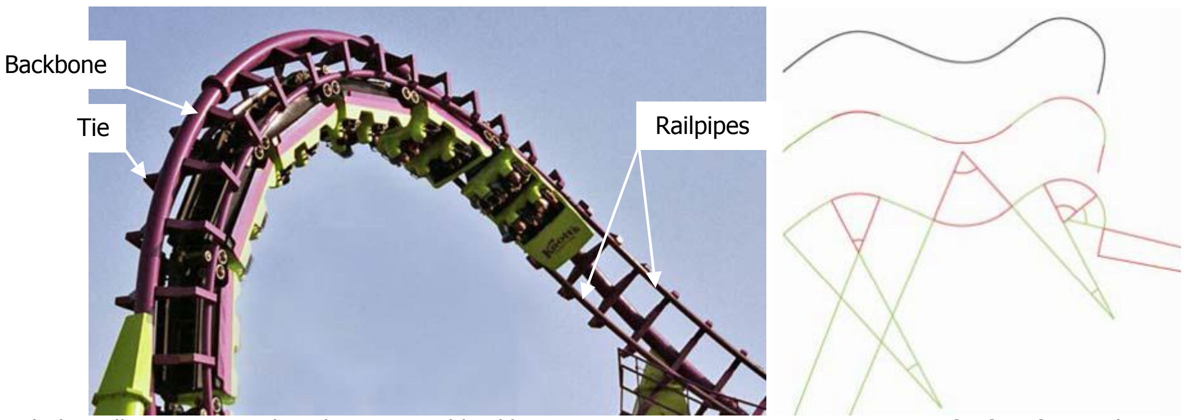
Document title: **System of interlocking diagonally folded steel sheets**
Project: Studio assignment
Design and build: ETH Zürich (Switzerland), chair for Computer Aided Architectural Design
Year: 2001
Parent features: Geometry > Surface > Planar
Material > Formative > Direct control
Material > 2D-fabrication
Level of abstraction: Structural system
Content: A structural system consisting of folded metal sheets has been developed as a studio assignment at the ETH, involving the scripting of a structure. The system consists of custom-shaped numerically cut metal sheets (Figure 25A), to be folded (B) along perforated lines. It results in a polygonal network of 4-sided boxes (C). Connections are made by a mortise-and-tenon joint, all included in the elements' cutting patterns. The tenons are fixated with a hollow rivet. In the constructed prototype, the elements suffered stability problems.



A. Metal sheets to be cut numerically. B. Folding pattern of one element. C. Part of the resulting structure.

Figure 25. System of diagonally folded steel sheets. Images courtesy ETHZ.

Document title:	**Rollercoaster track with tubular backbone**
Manufacturer:	Vekoma Rides Manufacturing
Parent features:	Geometry > Curve Material > Formative > Direct control
Level of abstraction:	Structural system
Content:	The use of a tubular backbone is one of the alternative approaches to construct tracks for roller coasters, see Figure 26A. Such tracks normally curve in the track's longitudinal direction, and rotate along the track's central axis which results in a twisted track ('banking' in coaster jargon). Except for small-scale rides, tracks normally consist of two rail pipes with the trains' wheels gripping around, and a primary load bearing backbone in between. For the latter, box sections like tubes provide the structurally required resistance against torque. Due to their point-symmetrical cross section, tubular profiles do not require to twist physically in case of a geometrical twist. This facilitates the connection between segments of the backbone, as well as between the backbone and rail pipes through ties, which maintain a constant orientation to these in stretches without excessive banking. The tube's wall thickness varies according to the structural capacity needed, but its exterior diameter is kept constant. Though apparently curving arbitrarily, practically the profile's curves consist of a series of planar arcs with constant curvature (Figure 26B), which can be bent more accurately and through less labour than space curves.[6]

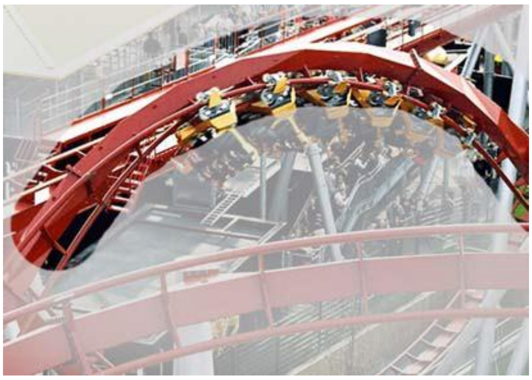


A. Tubular rollercoaster-track with 3D-curved backbone. Image courtesy (CoasterGallery 2006). B. Conversion of a free form spline into a series of arcs.

Figure 26. Usage of curved tubes in a rollercoaster track.

[6] The information was gathered during visits to Vekoma's engineering department in Vlodrop (the Netherlands) and the production facility in Sviadnov (Czech Republic). Mr. Har Kupers (manager engineering) and Mr. Leo Baaten (production director) are gratefully acknowledged for their willingness to provide information on the design and construction of roller coasters.

Document title:	**Rollercoaster track with triangulated polygonal backbone**
Manufacturer:	B+M / Intamin
Parent features:	Geometry > Curve > Straight Material > Formative > Direct control Material > 2D-fabrication
Level of abstraction:	Structural system
Content:	Folded sheet-like elements are being applied in three-dimensional applications in the square-section backbone of roller coaster tracks by the competiting Swiss fairground-design firms Intamin and Bolliger & Mabillard (B&M), see Figure 27A. The box-section provides resistance to torque, while the geometrical accuracy during manufacturing is easy to control due to the sole existence of planar triangles. The triangles run from tie to tie, this way constructing a polygonal backbone (Figure 27B). Its cross section is of constant dimensions at the ties. Although applied both by B&M and Intamin today (while the latter also produces spatially curved truss-shaped tracks), the origins of the box-shaped are at manufacturer Giovanola that was initially producing coasters for Intamin. Two of its employees founded B&M in the late 1980s, and continued applying the system (Wikipedia 2006).

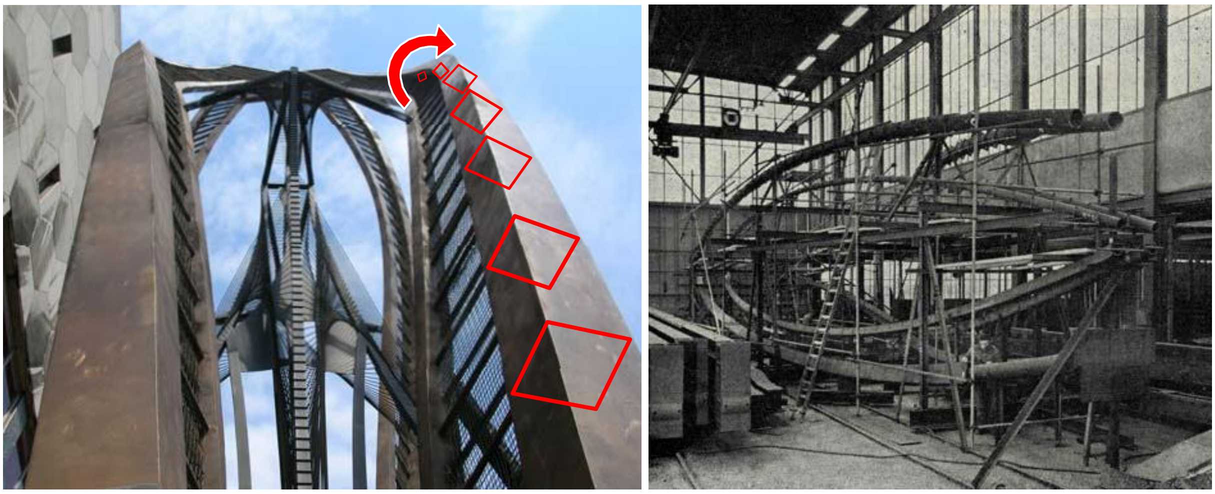


A. The Daemonen-ride in Tivoli (Copenhagen, Denmark). Image courtesy (CoasterGallery 2006)

B. Backbone with accentuated folds.

Figure 27. Polygonal beam consisting of planar triangles as the backbone of a rollercoaster track.

Document title:	**Approximation of a twisting box-section through curved sheets**
Project:	The stylised flower
Artist	Naum Gabo
Manufacturer:	Lubbers' Constructiewerkplaatsen en machinefabriek "Hollandia", Krimpen aan de IJssel (the Netherlands)
Year:	1957
Parent features:	Geometry > Curve > Curved Geometry > Surface > Single curved Material > Formative > Direct control
Level of abstraction:	Structural application
Content:	The standing box-shaped steel beams in the "stylised flower" by artist Naum Gabo twist 90° over its full height of 25m (Figure 28A). This twist has been constructed through applying curved strips on a tube with numerous plates inside the beam (Figure 28B). The construction is an approximation since constructing a twisted surface through single curved panels is mathematically impossible. Its appearance is nevertheless smooth. (Vossnack 1957)

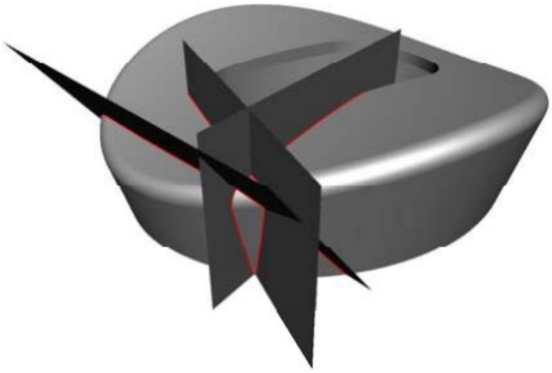


A. "The stylised flower". The rotating squares drawn on the figure are actually on the inside to facilitate fabrication.

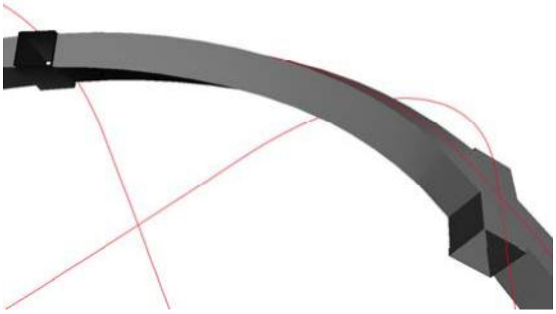
B. During construction, showing that each twisting square beam is approximated through four bent steel sheets (Vossnack 1957).

Figure 28. Twisted box sections in the "stylised flower"-artwork by Naum Gabo, Rotterdam (the Netherlands).

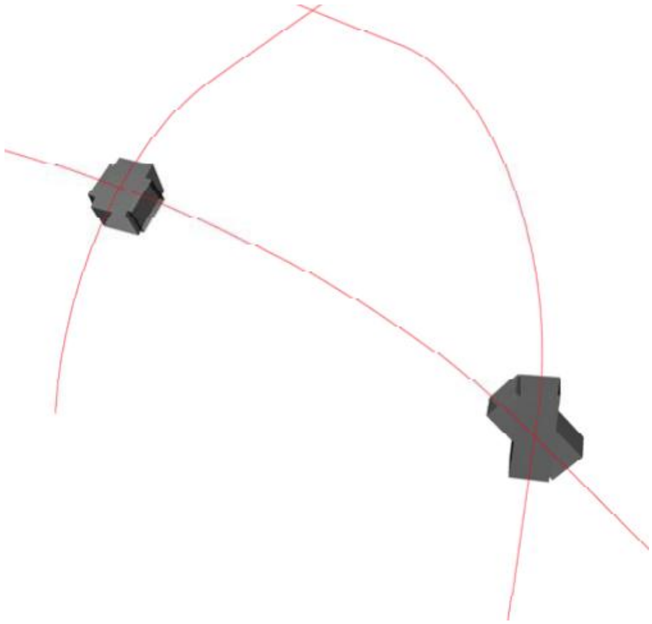
Document title:	**Box-section ribs from developable sides**
Project:	China national stadium, Beijing
Architect:	Herzog & de Meuron
Engineering	Arup and Gehry Technologies
Steel manufacturer:	BCC (China)
Year:	2007
Parent features:	Geometry > Surface > Single curved Material > 2D-fabrication Material > Formative > Direct control
Level of abstraction:	Structural application
Content:	The stadium's primary structure consists of trusses in vertical planes running from the ground level until the edge of the cantilevering roof. The chords follow the outer contour of the building. In addition, secondary beams randomly run on the exterior envelope. All beams are square box sections of 1,2m wide, constructed from steel sheets. One rib-side is tangent to the building envelope. As the global building shape is curved, the box sections – as well as their sides – have to twist in order to maintain their tangency to the building shape. The beams are constructed through four single curved surfaces of non-constant curvature. These beams are generated between the square extremities of 'nodes' (in the same construction system as the ribs, see Figure 29). Deviations of a maximum of 10mm occur at the transitions between pairs of developable surfaces, which have been eliminated during fabrication (Figure 30). (Ceccato 2006)



A. Schematised model of the stadium and three of the planes intersecting it.



B. Zoom into the twisted secondary beam.



C. 'Nodes' with square extremities. Beams have to be generated in between these.

Figure 29. Reconstructed model of an exemplary situation of a strongly twisting box section beam in the China national stadium.

A. Overview of two primary trusses and a secondary beam diagonally running through it.

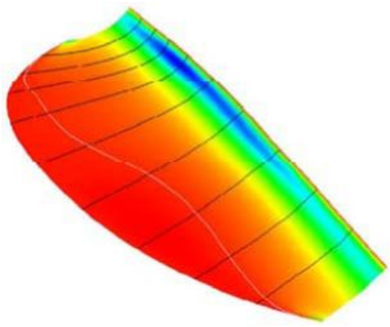
B. Zoom into the strongly curving secondary beam of the previous figure.

Figure 30. China national stadium under construction.

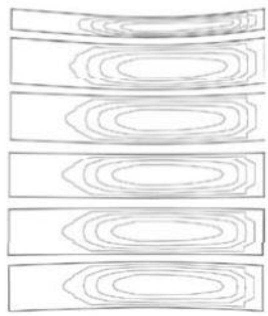
3.3.3 Curved timber structures

Document title:	**Wood-foam sandwich shells**
Project:	Research project Martin Bechthold, Harvard University (USA)
Parent features:	Geometry > Surface > Double curved Material > Subtractive Material > Formative > Direct control
Year:	2001
Level of abstraction:	Structural system
Content:	This method, a result of research by Bechthold, proposes to construct timber shells from laminating timber strips over a foam core, see Figure 31. To make the strips follow the

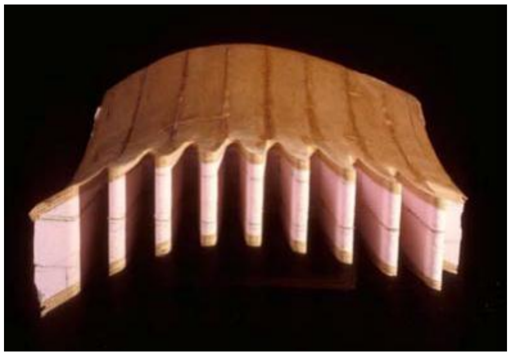
mould's curvature, the strips need to bend and twist. For an aesthetically attractive lining with wide strips, the strips need to be cut to size. With narrow strips no customised preparation is needed. The core acts as spacer between the structural layers at both extremities, and also as lost mould during the lamination of the strips onto the core. To limit the solicitation of the bonding between core and timber, and avoid special measures that can overcome this, the construction method primarily addresses shell structures acting through in-plane forces. (Bechthold 2001)



A. Digital model of curved surface showing the Gaussian curvatures.



B. Subdivision of surface into strips.



C. Mockup of the system, with fingerjoint-connection.

Figure 31. Digital model and realisation of a wood-foam sandwich shell. Images from (Bechthold 2004).

Document title:	**Cold deformed timber grid shell**
Project:	Grid shell Weald and Downland Open Air Museum (Singleton, UK)
Architect:	Edward Cullinan Architects
Structural engineer:	Buro Happold
Carpentry specialist:	Green Oak Carpentry Company
Parent features:	Geometry > Volume Geometry > Surface > Double curved Material > Formative > Indirect control (points)
Level of abstraction:	Structural system
Content:	This timber grid shell was constructed by lying out the network of laths in a rectangular mat at the height of its valleys, and then lowering it to its final position (Figure 32A, B). The network consists of four layers of 50x35mm-laths running in two directions, two laths per direction. Apart from deformation due to the shell's self-weight, also the shell's boundary and ties pulling were needed to shape the shell (Figure 32C) (Jensen 2001). During the formation, the grid-laths underwent scissoring and sliding, for which the node clamps allowed (Figure 32D). Only at the top ridge, which is in the building's vertical plane of symmetry, a single bolt goes through the layers. After the shell's geometry was formed and approved, bracings were installed that prevented further scissoring. Furthermore, timber shear blocks were introduced between parallel layers. These could transfer in-plane shear forces and thus prevent further sliding. They also activated the full depth of the two layers of laths. A structurally optimised shape was generated that approximated the initial architectural model. This form-found model was modelling the shell's resistance to shear, and was constrained to be formed from a planar mat. Later, modifications were made to this, this way sollicitating the lattice structure's resistance to bending. (Harris, Romer et al. 2003)

A. During lowering. B. Final shape. C. Props adjusting nodes. D. Node detail allowing scissoring and sliding.

Figure 32. Construction through cold deformation of laths in the Downland timber grid shell. Images courtesy (Weald & Downland Open Air Museum 2003).

3.3.4 Alternative kinds of formwork

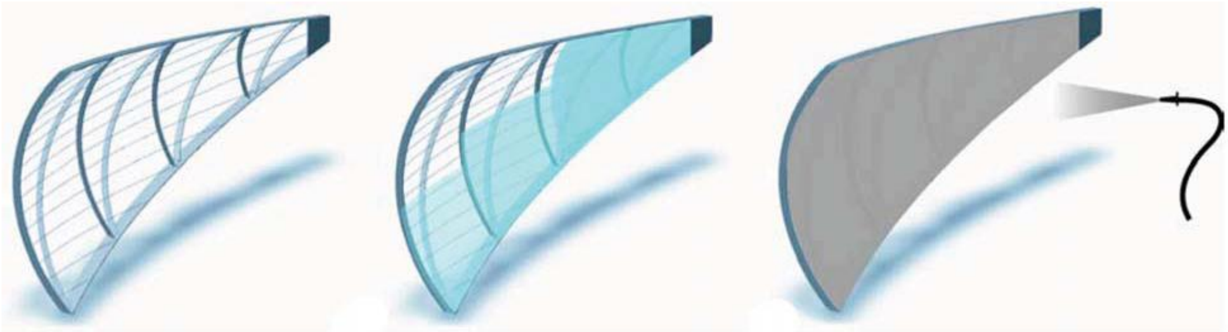
Document title: **Milled polystyrene formwork for precast concrete**
Project: Neue Zollhof (Düsseldorf, Germany)
Architect: Frank O. Gehry
Contractor: Philipp Holzmann AG (Germany)
Year: 1999
Parent features: Mould: Geometry > Surface > Double curved
Material > Subtractive > Direct control (Surface)
Artefact: Geometry > Volume
Material > Additive > Indirect control (Surface)
Level of abstraction: Structural application
Content: Formwork was milled out of polystyrene (Figure 33A). Reinforcement bars were inserted and finally concrete of low fluidity was poured, stamped and levelled manually (B), to construct curved storey-high façade panels (C). Due to the open mould, the panel's thickness could only be controlled approximately. (Cohen and Ragheb 2001)

A. Milled polystyrene moulds. B. Filling with concrete. C. Completed panels.

Figure 33. Prefabrication of concrete panels for the Neue Zollhof Project (Cohen and Ragheb 2001).

Document title: **Mesh-formwork for curved concrete surfaces**
Project: Research project Martin Bechthold, Harvard University (USA)
Parent features: Geometry > Surface > Double curved
Material > Formative > Indirectly control (curve)
Material > Additive > Indirect control (surface)
Level of abstraction: Structural design
Content: This method, a result of research by Bechthold, proposes to construct a concrete shell from lost formwork. To do so, a curved surface sufficiently small to be transported is constructed from its contours and a limited number of intersections, all curved and derived from a 3D CAD-model. These contours are realised as a system of steel ribs, onto which first rods and secondly a mesh is applied which thus approximates the curved surface. Concrete is then sprayed onto the mesh, for which the mesh has to be sufficiently dense to capture it, and the mesh and wireframe sufficiently rigid to

withstand the load involved with this. Once a total thickness of 15mm has been reached, they constitute the prefabricated formwork for a larger concrete shell to be built with conventional reinforcement, which is – contrary to ferrocement – covered by the standards. (Bechthold 2004)



A. Network of ribs. B. Covered with a mesh. C. Concrete sprayed on the mesh.

Figure 34. Using a mesh to construct lost formwork for concrete shells. (Bechthold 2004)

Document title: **Developable surface of timber as formwork for concrete**

Project: Mercedes Benz Museum (Stuttgart, Germany)

Architect: UN studio (Amsterdam, the Netherlands)

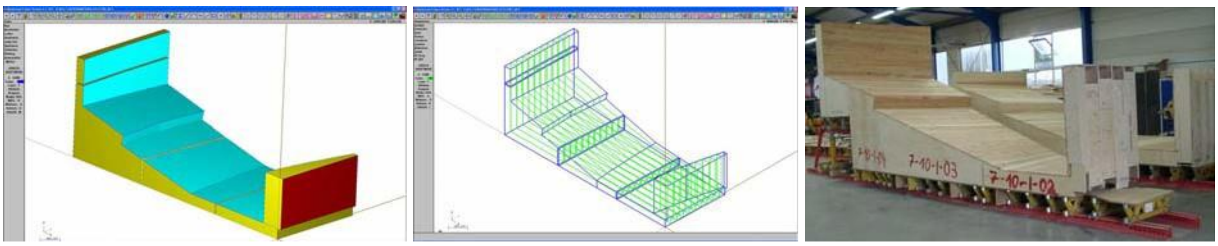
Formwork: Firmengruppe Thaleck (Raubling, Germany) commissioned by PERI GmbH (Weißenhorn, Germany)

Year: 2006

Parent features: Mould: Geometry > Surface > Single curved
Material > Formative > Indirect control (curve)
Artefact: Material > Additive > Direct control

Level of abstraction: Structural application

Content: Formwork for large curved concrete surfaces is subdivided into segments that are fabricated off-site from sheets of plywood (Figure 35A-C). Since the formwork's curvature is small, timber panels are curved through application of force until they touch the rig. Once positioned on the building site, either formwork panels with a watertight coating or plastic foil is laid out over the prefabricated formwork (Figure 36). The lay out of the panels covers the gaps between formwork segments. Through force these panels are put in place, this way deviating from the theoretical developable surface, in order to approximate a double curved surface. (Bau & Technologie 2006)



A. Digital model. B. Model showing subdivisions. C. Completed Formwork-segment.

Figure 35. Digital model and a completed Formwork-segment based on curved timber surfaces. (Bau & Technologie 2006)

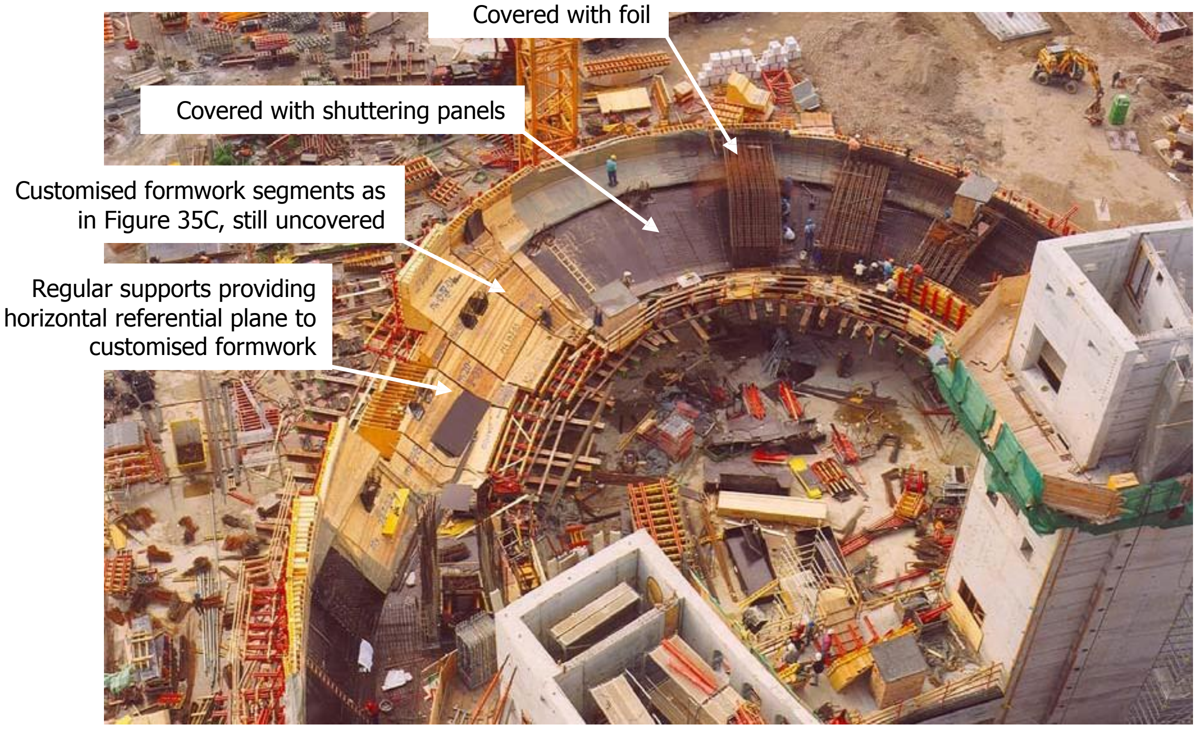


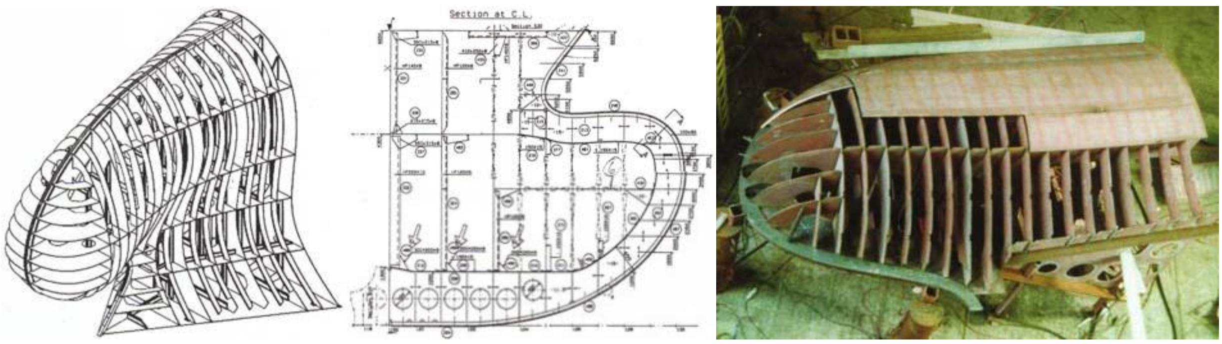
Figure 36. Building site of the Mercedes Benz Museum showing on-site installation of sheets on top of customised formwork. (Bau & Technologie 2006)

Document title:	**Ruled surface of timber as formwork for concrete**
Project:	Mercedes Benz Museum (Stuttgart, Germany)
Architect:	UN studio (Amsterdam, the Netherlands)
Formwork:	Firmengruppe Thaleck (Raubling, Germany) commissioned by PERI GmbH (Weißenhorn, Germany)
Year:	2006
Parent features:	Mould: Geometry > Surface > Double curved (ruled surface) Material > Formative > Indirect control (curve) Artefact: Material > Additive > Direct control
Level of abstraction:	Structural application
Content:	A curved surface is approximated by positioning a large number of timber strips side by side, and slightly deform each of them until they touch to a rig (Figure 37). If the timber strips are only twisted (not bent), the surface is double curved but ruled, sole bending approximates a single curved surface, and combined bending and twisting allows for the approximation of a double curved surface. To assure tight fitting, the width of the strips requires non-constant rasping along their sides. The rig's geometry thus evokes the surface geometry. It consists of a series of planes, placed parallel, their spacing being dependent on the curvature strength, the load of the freshly poured concrete and the required accuracy. The rig's geometry is normally orthogonal, allowing for simple positioning on standard supports. (Bau & Technologie 2006)

Figure 37. Formwork for a ruled surface, consisting of planks placed on top of a rig. (Bau & Technologie 2006)

3.3.5 Planar sections

Document title: **Orthogonal frames in ship construction**
Project: General ship construction in steel and timber
Parent features: Geometry > Surface > Planar
Material > 2D-fabrication
Force > Section-action
Level of abstraction: Structural system
Content: Ship construction is typically dealing with the materialisation of structures within double curved envelopes – the hull of a ship. In the large majority of ships the hull plates (steel) or planks (timber) are applied on an orthogonal grid of frames. The frames are primarily running in the ship's transverse direction, with more widely spaced horizontal frames in between them. The frames act in bending; the ship's hull acting as flange on the outside. When bulb-profiles are used to construct the frames, the bulb (a thickened edge) is placed on the frame's inner contour for increased strength and stiffness. The frames' positioning is as perpendicular to the hull as possible, and may be rotated 90° in regions of strongly varying curvature, such as the forepeak in Figure 38A-C. (Dokkum 2003)

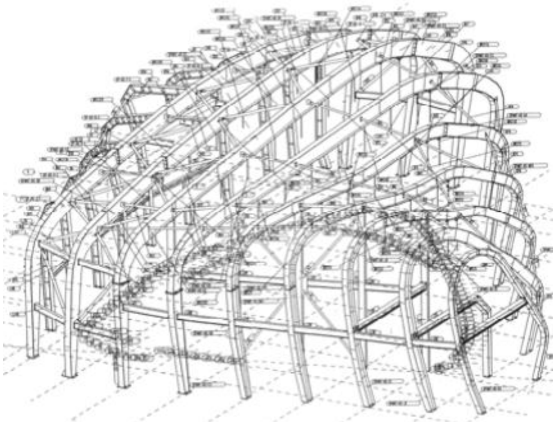


A. Assembly. B. Longitudinal cross section. C. During construction.

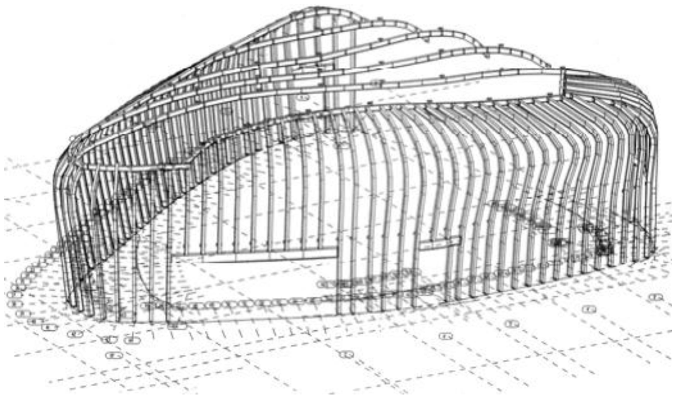
Figure 38. Typical construction method of a forepeak of a ship. Images from (Dokkum 2003). Images A and B courtesy Vuyk Engineering Center, C courtesy K. van Dokkum.

Document title: **Radial and parallel steel structures Mezz concert hall**
Project: Mezz concert hall, Breda (the Netherlands)
Architect: Erick van Egeraat Associated Architects
Steel manufacturer: Smulders Staalwerken
Structural engineer: Pieters Bouwtechniek, preliminary design with Arup
Year: 1992
Parent features: Geometry > Surface > Planar

	Material > 2D-fabrication
	Force > Section-action
Level of abstraction:	Structural system
Content:	The Mezz concert hall has a free form shape without any window openings. It is constructed from primary steel frames in a parallel layout spanning across the entire building. Each frame consists of I-sections in a polygonal layout. The structure exclusively bears through section action. Timber planks are placed between the ribs, oriented basically perpendicular to the exterior, to be covered with two more layers of timber, waterproofing and finally the copper cladding. For reasons of sound insulation, the actual concerts take place inside a second free form building envelope. This internal free form is obviously not loaded by external loads, justifying another construction method. Its ceiling is hung from the external building envelope, its walls are constructed from polygonal steel frames in a radial layout. (Veltkamp 2002)



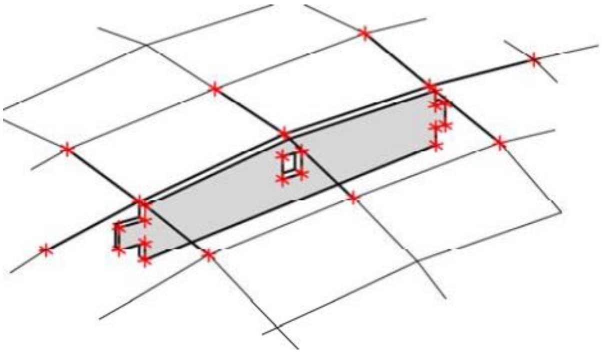
A. Steel structure exterior building envelope.



B. Steel structure interior building envelope.

Figure 39. Parallel and radial arrangement of the exterior and interior steel frames in the Mezz concert hall. Images courtesy Pieters Bouwtechniek and Smulders Staalwerken.

Document title:	**Custom-shaped timber panels in Lamella structure**
Project:	Serpentine Gallery, London (UK)
Architect:	Alvaro Siza, Eduardo Soto de Moura
Timber construction:	Finnforest Merk
Structural engineer:	Arup
Year:	2005
Parent features:	Geometry > Surfaces > Planar
	Material > 2D-fabrication
	Force > Section-action
	Force > Form-action
Level of abstraction:	Structural application
Content:	The 2005-edition of the Serpentine Gallery was a lamella structure, consisting of planar 69mm-thick panels of Kerto-Q, a laminated veneer lumber (Figure 40A). Though only featuring significant curvature in the roof, the building shape nonetheless departs from a regular box. The elements are simply supported at their extremities on other elements, and connected by mortice-and-tenon joints, subsequently fixed on one side. On a macroscopic scale, the structure acts through arching as well as bending. The geometry of each element was automatically generated from the master model, and defined by 36 points (Figure 40B). This definition includes the twisting upper edge of an element, and the mortice oblique to the panels' side, this way allowing a directional change of the grid (Figure 40C). The elements were cut using a circle blade on a numerically controlled robot arm. (Self and Stone 2005)



A. 36 points defining an element, based on (Self and Stone 2005).

B. Change of direction in the roof structure (Olll-Architecture-Gallery 2005).

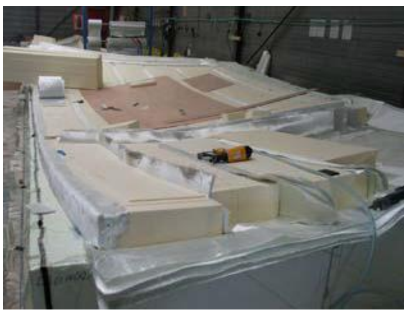
Figure 40. Lamella structure in timber of the 2005-Serpentine Gallery.

3.3.6 Miscellaneous

Document title:	**Sandwich panels on milled single-sided moulds**
Project:	Yitzhak Rabin Center, Tel Aviv (Israel)
Architect:	Moshe Safdie
GRP-manufacturer:	Holland Composites Industrials
Engineering:	Octatube International
Year:	2003-2005
Parent features:	Geometry > Surface > Double curved Material > Subtractive Material > Formative > Direct control
Level of abstraction:	Structural application
Content:	In total 5 shells, shaped as abstract dove tails and –wings cover two spaces in the Rabin Center, see Figure 41A. The shells are constructed as sandwich panels consisting of an inner and outer surface of glass fibre reinforced polyester, and a core of polyurethane. The shells were prefabricated in segments in sizes dictated by transportation constraints, connected and finished on the building site and finally hoisted into their final positions. The segments were fabricated on polystyrene moulds with a milled topside. As the mould's geometry is controlled directly, the mould's side would later be the outside of shells. Its surface was covered with a foil, a thick layer of coating and glass fibre mats. Using vacuum-injection, the glass fibre is impregnated with polyester resin (B). Once cured, blocks of polyurethane are applied (C), and the layering-sequence is repeated in reversed order. These blocks are ultimately controlling the thickness of the sandwich. Glass fibre mats and resin is also applied in between the blocks to create stringers, oriented as parallel slices, connecting the inner and outer surface to transfer shear forces and to prevent delamination of skin and core material. (Eekhout and Visser 2006)

A. Overview.

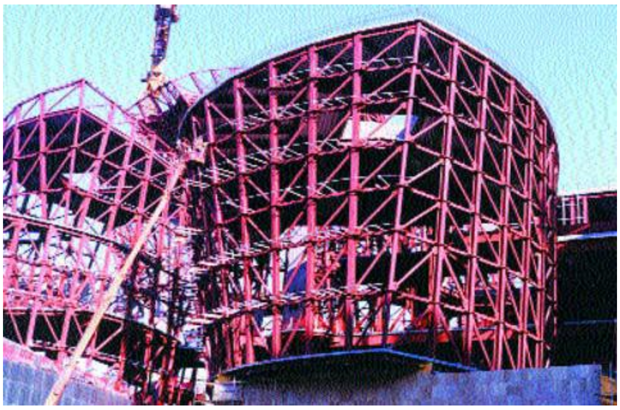
B. Vacuum injection of the exterior surface of a shell segment.



C. Laying out of polyurethane blocks on the cured exterior surface.

Figure 41. The sandwich shells of the Yitzhak Rabin Center. (Eekhout and Visser 2006)

Document title:	**Polygonal irregular shaped steel structure**
Project:	Guggenheim Museum, Bilbao (Spain)
Architect:	Frank O. Gehry and Associates
Structural engineer:	Skidmore, Owings & Merrill
Steel manufacturer:	URSSA
Year:	1997
Parent features:	Geometry > Curve > Straight Material > 2D fabrication
Level of abstraction:	Structural application
Content:	The Guggenheim Museum in Bilbao features irregularly curved facades, like most of Gehry's building designs clad with metal or otherwise opaque sheets. Hence its load bearing structure is hidden in between an architecturally defined interior and exterior skin. For a feasible, reliable and economic solution, the structural system was (amongst others) required to be applicable to a variety of architectural forms, be as thin as possible while following the forms as closely as possible without impacting them – all this while being constructed from simple and conventional members and details. This resulted in a system consisting of straight segments running between nodal points (allowing simple exchange between different pieces of software), spaced approximately 3 meters, at a constant 600mm distance from the exterior clad surface (Figure 42A). I-section columns would have a constant orientation along their full height, the web averagely oriented perpendicular to the exterior surface. All details were parametric variations to a handful of different joint types: connections between columns segments are made through horizontal bearing plates, to which also the tubular bracings were connected (B). (Iyengar, Novak et al. 1998)



A. Overview of the structure.

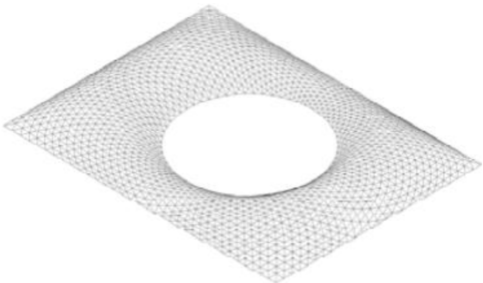
B. Connection between two column segments and bracings.

Figure 42. The polygonal steel structure of the Bilbao Guggenheim museum. (Iyengar, Novak et al. 1998)

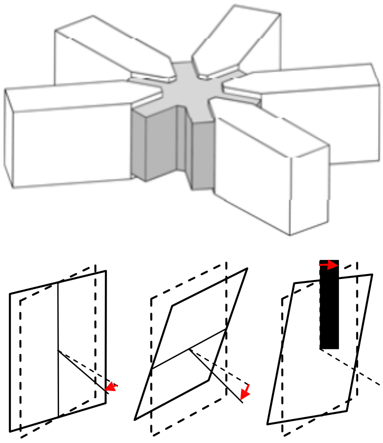
Document title:	**Reticulated network structure**
Project:	Great court yard British Museum, London (UK)
Architect:	Foster and Partners
Steel manufacturer:	Waagner Biro
Year:	2000
Parent features:	Geometry > Curve > Straight Material > Subtractive Material > 2D fabrication
Level of abstraction:	Structural system
Content:	The covering of the court yard is a triangulated double curved reticulated steel structure (Figure 43A) consisting of straight beams between nodes. Glass panels are laid on top of the beams. The global shape is irregular and double curved, hence all beams and nodes are unique. The beams are 80mm wide with depths varying from 80 to 200mm. They are constituted from steel plates to achieve the architecturally desired sharp edges, but also to reduce dead weight. The finger-shaped nodes (B) are perpendicularly cut from

thick steel plates. Beams are placed between the 'fingers', and welded to the node. Only in-plane rotations are accommodated by the node's cutting contour. The beam's out-of-plane angle of incidence, as well as its rotation, requires custom-cutting of the beam ends. For reasons of cost as well as elimination of errors, engineering and production of elements were automated. Beams and nodes were pre-assembled in the workshop up to components of transportable dimensions. On the building site these components were positioned and welded. (Sischka, St. Brown et al. 2001)

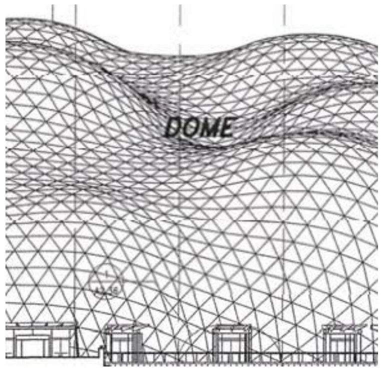
In 2007 he same system was applied again in the covering of the Zlote Tarasy shopping centre in Warsaw, Poland (Figure 43C).



A. Gridlines roof structure Great Court Yard. (Williams 2001)



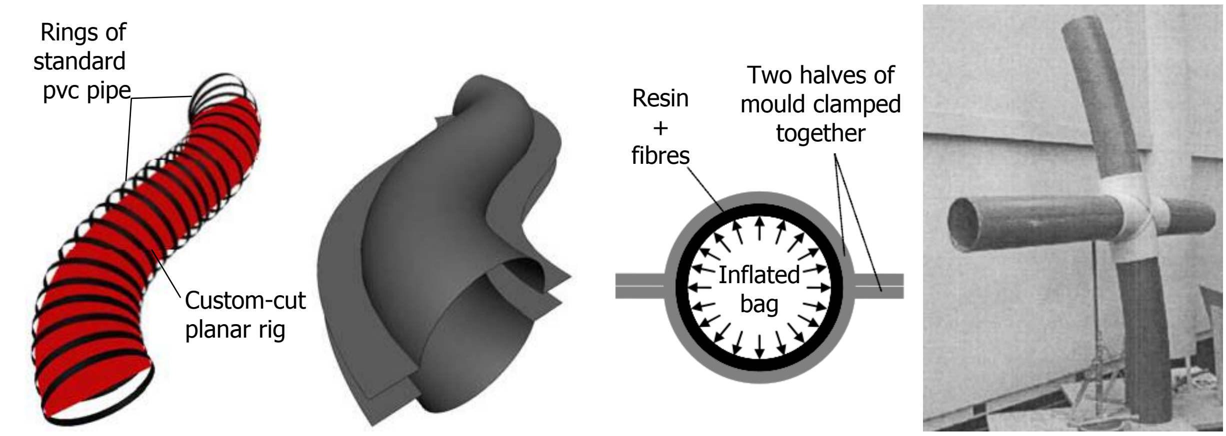
B. Node cut from steel plate and the degrees of freedom of members connected to it. Based on (Stephan, Alvarez et al. 2004)



C. Front view of the central part of the Zlote Tarasy roofing.

Figure 43. Roofing structures using a solid node.

Document title: **Curved tubular structures in FRP**
Project: Research project Yong Gib Yun, Harvard University (USA)
Year: 2001
Parent features: Geometry > Curve > Curved
Material > Additive > Indirect control (curve)
Level of abstraction: Structural system
Content: Aimed at a low number of structural components, Yun proposes boundary structures which wrap the entire building, and consequently have to follow their shape. To do so, he developed a method to fabricate tubes in custom curvatures of glassfibre reinforced polymers through an additive process. The method employs an internal mould which is a simple inflated cylindrical bag, and a custom-made external mould, which is the most elaborate part of the proposed method. The external mould is made in two halves by placing rings around a custom-cut planar rig (see Figure 44A, thus producing custom-shaped planar curved components), followed by layering glassfibre and resin around it (B). The tubular structural components are finally produced by placing the pressure bag inside the mould, and adding a mixture of resin and chopped fibres in between (C). Connections between members are shown in (D), made by layering mats with resin, similar to how the mould was produced. (Yun and Schodek 2003)



A. Construction of external mould.

B. External mould made by layering mats around the rings.

C. Cross section of during curing.

D. Mock up of two connecting tubes. (Yun and Schodek 2003)

Figure 44. Curved tubular structures in FRP.

3.4 Analysis

Analysis of the content documents and the features they are parented to, reveals a series of phenomena. As the number of cases is limited, the analysis is entirely qualitative, and does not allow any quantitative statement to be made about them. First, three relationships of general validity (marked by red dots) are described through the content documents, followed by phenomena specific to two or more content documents.

3.4.1 Link 1, geometry-material: Planar surfaces – 2D-fabrication

The relationship that planar geometries can be constructed using a 2D-fabrication process is obvious, and demonstrated in:

- **Diagonally folded steel plates** Web of North Holland
- **Orthogonal frames in ship construction**

Less evident is the usage of geometrical planes as a reference for the positioning of material planes, such as webs of I-sections (**Radial and parallel steel structures** Mezz concert hall) and the stringers inside the **sandwich panels** of the Rabin Center. In the case of the Serpentine Gallery 2005 the planes refer to the sides of structural elements, rather than to their centre planes.

3.4.2 Link 2, geometry-material: Planar surfaces – formative process

The following three cases demonstrate how a custom-shaped planar pattern is formed into a spatial shape:

- **Diagonally folded steel plates** Web of North Holland
- **System of interlocking diagonally folded steel sheets** ETHZ studio assignment
- **Rollercoaster track with triangulated polygonal backbone** Intamin, B&M

All three cases are the result of the need to rationalise a twisting plane through planar elements. In the first two cases, the increased structural stability is a necessary by-effect, although not sufficient for the cases in which they were applied and therefore requiring additional measures. In the third case, structural stability is not solely obtained from the panel, but also from its assembly with other elements.

3.4.3 Link 3, geometry-material: Single curved surfaces – formative process

In addition to the materialisation of planar surfaces from sheet-like materials, these surfaces may subsequently be formed, as has been demonstrated in:

- **Box-section ribs from developable sides** China national stadium
- **Wood-foam sandwich shells** Research Bechthold

- **Approximation of a twisting box-section through curved sheets** Artwork Naum Gabo

The first case is prepared such that it will exactly fit in its desired curved shape. In the second case both prepared and non-prepared unrolling patterns are proposed. The latter case only concerns an approximation of a double curved shape through a series of single curvatures.

3.4.4 Automated systematic setup

Although more documents probably have a systematic setup, these documents explicitly demonstrate an automated implementation of a parametrically defined detail:

- **Custom-shaped timber panels in lamella structure** Serpentine Gallery 2005
- **System of interlocking diagonally folded steel sheets** ETHZ studio assignment
- **Rollercoaster track with triangulated polygonal backbone** Intamin, B&M
- **Diagonally folded steel plates** Web of North Holland
- **Polygonal irregular shaped steel structure** Guggenheim Museum, Bilbao (Spain)

The automation of all examples concentrates on geometry, notably the extraction of single elements (micro-level of scale) from a global model (meso/global-level of scale). Only the latter example also allowed parameters affecting the structural performance to be fed into the system, although in the end it was not implemented. In the other cases anticipation to the required structural capacity takes place per element (e.g. varying the wall thickness of a tube, or the thickness of a sheet). The advantage of including structural parameters in an automated setup is that it fully integrates structural, geometrical and material characteristics – ultimately leading to programmed (as opposed to drawn) structures.

3.4.5 Cold deformation

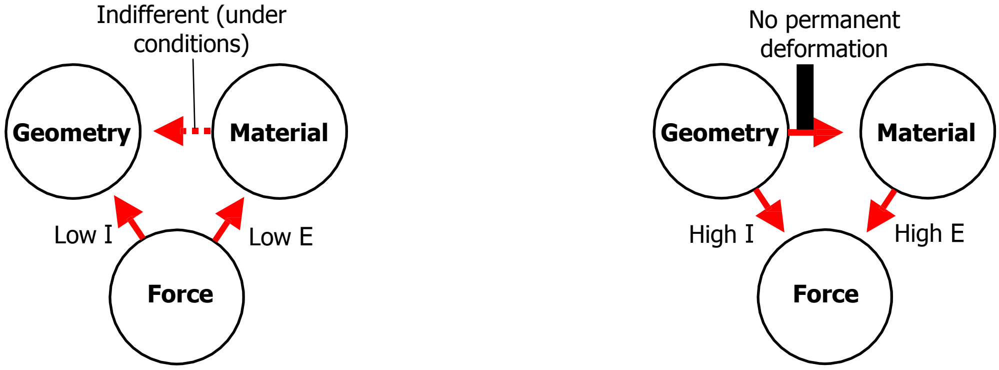
Cold deformation of structural elements is described in the following content documents:

- **Wood-foam sandwich shells** Research Bechthold
- **Sandwich panels on milled single-sided moulds** Rabin Center
- **Cold deformed timber grid shell** Weald and Downland

Unless measures are taken, cold deformation is problematic. This arises from contradictory performances required during formation, compared to that required in its final stage of a completed structure. In the formation phase, deforming a material requires elongation of a material, which is easier with a low modulus of elasticity (E). Furthermore the material properties should allow the deformation while maintaining sufficient redundant capacity to be sollicitated during the structural action it should perform later. This is notably critical in the case of the non-plastic deformation of timber, which thus accumulates stresses due to the formation process and functional loading. The materials from the examples, steel (as well as metals in general) and timber feature this property.

Apart from the material properties, the magnitude of the load (which is most commonly a rotational moment) applied to physically deform a member also depends on the geometry to be deformed (bent/twisted). Bending a thick sheet or torqueing a closed section obviously encounters higher resistance (expressed by the area and polar moment of inertia I respectively) requires a larger load than a thin sheet or open section. The contradiction now is that elements to be deformed should feature low resistance to keep the formation operation in control (Figure 45A), whereas to resist functional loading once fulfilling its structural role in a structure, they should feature a high resistance (Figure 45B).

Each of the listed exemplary precedents overcomes this contradiction through rigidising the element after formation, in the listed examples carried out through coupling multiple formed elements. Through this coupling, the elements' structural action shifts to a higher level of scale, and may be of different nature, e.g. from solely section-action on the micro level of scale to combined section and form action on the meso level of scale in the case of the Weald and Downland grid shell.



A. During formation. To keep the force to be applied low, the material's modulus of elasticity and the geometry's area/polar moment of inertia should be low. The geometry may deform permanently under the applied loads.

B. During final functioning. As permanent deformations under loading are not acceptable, the material's modulus of elasticity and the geometry's polar moment of inertia should be high.

Figure 45. Contradictory requirements during formation and in functional service.

3.4.6 Alternative approaches to twisting

Several examples of twisted (curvi)linear members have been recorded, approaching the problematic phenomenon of twisting differently. The examples are categorised in Table 5. Three approaches to materialise twisting curvilinear members have been observed:

1. through a stepwise-approach in which the twist is incorporated in the node rather than in the member between them;
2. through a geometrical rationalisation onto a geometrical construct which is rationally constructable;
3. through physical twisting, or any other way resulting in a gradual twist along the element's length. Within this class, tubes are a particular case as they do by nature feature a gradual rotation of the cross section, thus eliminating the need to twist them.

A second categorisation is made by the eventual sollicitation by torque. If it would be sollicitated this way, the problematic of cold deformation, as described before, occurs.

Table 5. Alternative approaches to constructing twisting curvilinear elements.

	Stepwise twist	**Geometrically rationalised twist**	**Physical or gradual twist**
Not sollicitated by torque	No example included in precedents	z **Diagonally folded steel plates** Web of North Holland	z **Cold deformed timber grid shell** Weald and Downland z **Physically twisted I-sections** Research Kuroiwa et al.
Sollicitated by torque*	z **Reticulated network structure** Great court yard	z **Rollercoaster track with triangulated polygonal backbone** Intamin, B&M	z **Rollercoaster track with tubular backbone** Vekoma z **Approximation of a twisting box-section through curved sheets** Artwork Naum Gabo**

* To be classified as 'sollicitated by torque', the load transfer through torque should be among the primary structural actions, which consequently should be accommodated by the structural elements.
** Both the tube inside and the strips around it feature a gradual twist, despite the strips are an approximation of it.

4 The design of alternative structural schemes and systems

The framework of structural design in the context of free form buildings has been described in the previous two chapters, together with the variables that play a role in this. In this chapter five hypothetical solutions are discussed that relate to the geometrical class of structural elements, a structural principle and a method for forming material. The further research consists of testing the proposed solutions.

The data to be evaluated on specified evaluation criteria is generated partially through combinatorics, but mostly through explorative design, driven by the hypothetical solutions. In a first stage attention is given to principles, in structural schemes with the constraints and degrees of freedom explicitly stated. Implications in case of developing the schemes further onto systems are given, but only three of them will be developed to this stage. This is reported in the next chapter.

4.1 Methodology of using design in research

So far the design variables of geometry, force and material have been used in a descriptive manner. The next step is to employ them to generate alternative structures (on any level of abstraction) for free form building designs. Then, the variables respectively address how such structures…

- … are designed – which in this case means how and through which aids their geometry is defined;
- … act structurally – meaning through which mechanisms they fulfil their load bearing function;
- … are manufactured - which includes the choice of a material or a class of materials and appropriate manufacturing techniques.

The three variables thus address design technology, product technology and manufacturing technology respectively (Poelman 2005). To ensure the alternative structural systems are realistic, each of the variables must be addressed. This is achieved by, when exploring the potentials of a feature of one variable, ensuring that the remaining two are anticipated too. This is also required for the reason that a high degree of interrelation between the design variables has been observed in the previous two chapters, and the criticality of some relationships between them. Following an integrative approach from the earliest stage onward, as a by-effect, positively constrains the search directions.

To generate the alternative structures, an analytic-systematic method will be pursued. Such a method is characterised by initially structuring the problem through 'distinguishing aspects of subproblems' and 'by seeking common characteristics of solutions to analogous problems' on one hand (these have been reported in the previous chapter), and the generations of solutions taking place 'by combining or varying subsolutions to a of the problem' on the other hand (Roozenburg and Eekels 1995). The generation of alternatives is driven by hypothetical solutions formulated on the basis of the previous two chapters. The alternatives aim to provide possible solutions of esteemed performance that are not yet covered by already existing structural systems, typified as the exploration of possible but not probable future scenarios (Jong T.M. de and Voordt D.J.M. van der 2002).

A mixture of methods has been used for the generation, their efficacy being dependent on the nature of the explored path. Hypothetical solutions allowing a well-defined and finite subdivision into a limited number of variables related to this solution may be explored through combinatorics, or a morphological chart. However, the multitude of variables, their interrelatedness and applicability on different levels of abstraction are all troubling a straightforward application. Design studies, as they are more malleable and open to various inputs, proved more flexible and successful instead.

4.2 Formulating hypothetical solutions on the scheme-level

Based on the theoretical framework and the analysis of precedents, hypothetical solutions are formulated. These will act as design drivers in the generation of alternatives. The hypothetical solutions are as follows:

1. Planar elements: Since cutting of forms out of planar elements is not geometrically constrained, while its 2D-definition is unambiguous on any level of scale, the usage of this geometrical type is potentially helpful to construct elements.

2. Developable surfaces: In the application of curved surfaces, surfaces of single curvature are the first to be looked at as they can be obtained from planar raw materials, while their transition from curved to planar is digitally mastered.

3. Rib-shell collaboration: The closed envelope of buildings may be used structurally when combined with an additional curvilinear structure, needed to compensate for the sub-optimal curved geometry typical to free form building designs.

4. Twistless tube: Due to the feature of tubes' circular cross sections being rotationally symmetric, twisting geometries do not require the material to twist.

5. Partial geometrical control: By using a cold formative process on a support of which the control concerns a limited number of variables, an object of a higher geometrical class than that of the mould can be controlled, be it indirectly.

Altogether these hypothetical solutions cover each of the three design variables, see Figure 46. Geometry is addressed by hypothetical solutions 1, 2, 4 and 5. The middle two of these anticipate the material transformation in two ways; solution 2 by departing from a formative technique, solution 4 by deliberately avoiding material twisting and proposing an alternative of geometrical nature instead. Force is addressed by the third hypothetical solution. Finally hypothetical solution 5, in addition to geometrical consideration only, it also addresses aspects of material and force in an integrated manner: the shape of elements produced depends on the supporting geometry, the material properties and the amount of force that is applied.

Hypothetical solutions 1 and 2 are related since through a formative operation (more particular: rolling/unrolling), a planar surface becomes a developable surface. The two hypothetical solutions together thus set out an expectation for unrolled patterns with no constraints of geometrical nature.

Design variable Æ È Hypothetical solutions	**Geometry**	**Material**	**Force**
1. Planar elements			
2. Developable surfaces			
3. Rib-shell collaboration			
4. Twistless tube			
5. Partial geometrical control			

Figure 46. Coverage of the design variables by the sub-hypotheses.

Hypothetical solutions 1 to 4 are suitable for scheme-generation, whereas hypothetical solution 5 introduces a consideration that could further increase the performance (notably on the criterion of formal freedom) of any of the alternatives developed before. In the following sections each of the drivers has been explored to derive alternative structural systems from it, starting on the scheme-level. This way, one hypothetical solution yields to several schemes, while each of these schemes potentially yields to several systems that can be applied in structural designs. Each of the levels of abstraction has its own degrees of freedom, meaning an explicit possibility for the designer to modify a design, without affecting the scheme's or system's integrity.

4.3 Evaluation criteria for schemes, systems and structural designs

Three performance-indicators are used to assess the structural systems' appropriateness for their application in curved building structures. Any of the alternative solutions, either as scheme, system or structural design, is supposed to be physically possible to be built. Buildability is therefore a requirement, and not considered for evaluation. All evaluation criteria relate to any of the three design variables, or their integration:

1. Material efficiency: The degree to which all available material performs a structural function. Apart from obviously advantageous mechanisms of force transfer, such as vector action that transfers loads through axial stresses equally distributed over the element's cross section, the material efficiency can also be enlarged by the system's allowance to adapt its cross section to the local structural solicitation. This adaptation may consist of quantitative characteristics (e.g. structural depth) as well as qualitative characteristics (e.g. members' topology and orientation).

2. Formal freedom: The degree to which the intended double curved geometry of the building envelope can be followed, rather than through approximations and pre- and post-rationalisations.

3. Degree of systematisation: The degree to which the structural elements that compose the structure can be described by rational (logical and quantifiable) relationships (rather than custom definition through intervention of the designer). Such a rational setup, with explicit input parameters, is a precondition for automated application through a parametric design system. In such methods the values of parameters may vary per element, while the systematic setup is maintained. The systematic setup should best go beyond geometrical relationships only and be associative to other inputs (for which the geometrical implications have to be quantified), notably the required structural performance.

The nature and diversity of these indicators make clear that the structural design activity is perceived as an operation of multi-criteria optimisation, rather than that of maximising a sole performance indicator. However, the targets, as well as their relative weights, are ill-defined. The so-called "optimal design solution" is therefore more of an abstract idea rather than a specified design.

4.4 Schemes based on hypothetical solution 1: planar elements

4.4.1 Introduction and overview

When proposing schemes based on using planar elements, the variables that were identified, followed by their possible values, are listed in Table 6. The vertical inclination being a separate instance, rather than only one value out of a range from 0° to 180°, is justified by its structural implication, as it distinguishes between in-plane and out-of-plane structural action. In case the number of sets is larger than one, these sets do interact by default. If they wouldn't, these would be two interwoven singular sets, which cannot be considered as a new scheme.

Table 6. Variables in the generation and classification of schemes and systems based on hypothetical solution 1, planar elements.

Variable	Possible values
Orientation in plan	Parallel / non-parallel
Orientation in elevation	Vertical / inclined
Number of sets	Single set / multiple sets

All possible combinations of these variables are displayed in Figure 47 by using combinatorics. It starts with typically regular schemes with few degrees of freedom. As all exceptions to the regular layout (which typically consists of a parallel, vertical, single set) cumulate in the lower part of the scheme, the ability of the proposed schemes increases to make the schemes and systems derived from these, locally anticipate irregular curvature. This local anticipation implies formal freedom and thus potential for appropriate systems, especially if further variables address structural properties, which would also increase performance on material efficiency.

Figure 47 covers a single level of scale only – without specifying which, since the high level of abstraction does not require so. Furthermore it only presents singular systems, and not with system transitions as these require specific boundary conditions that cannot be dealt with in general terms. Finally, it only lists the schemes, and not the systems that are derived from it. When the scheme is judged to possess potential for further development onto a system that meets the performance criteria, the resulting systems are described together with the scheme. All systems, except for the latter two, derived from the schemes envisage an application on the global level of scale. The latter two schemes instead anticipate the local curvature of the building envelope. For this reason they offer the largest formal freedom, and are therefore both developed further as structural system in the next chapter.

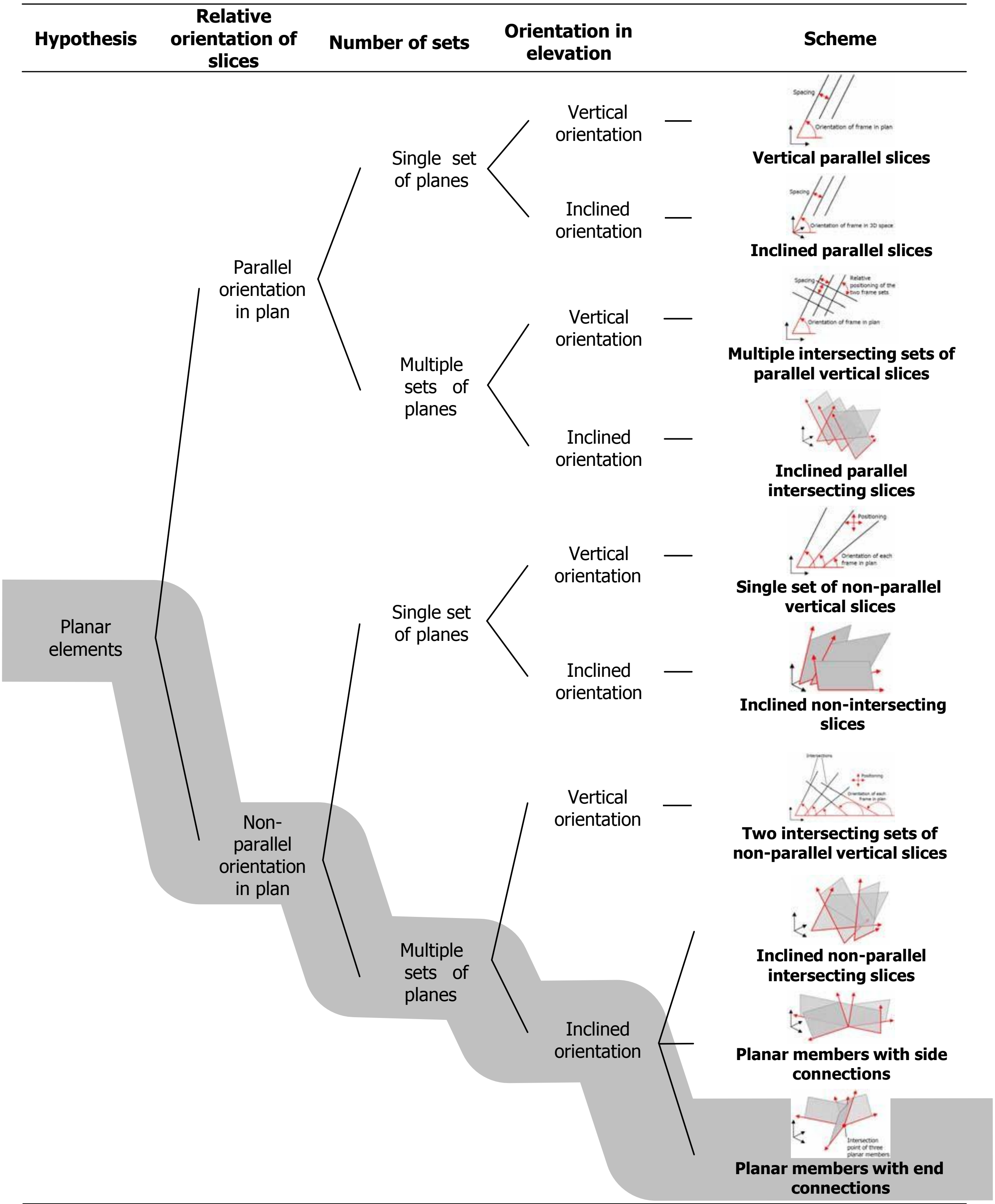


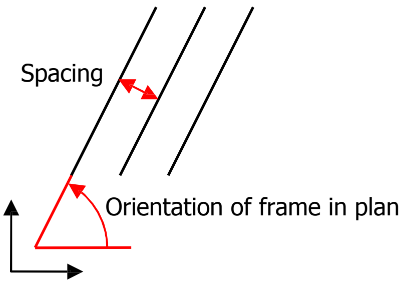
Figure 47. Schemes and systems based on hypothetical solution 1: planar elements. The trajectory marked in grey leads to the scheme that has been developed further onto structural system.[7]

[7] In this and all further graphics, all geometry in red belongs to the degrees of freedom (input). All other tones of grey show the resulting geometry (output). Note that pictures that are graphically identical, but with different elements coloured red, consequently demonstrate different effects. Arrows indicate extrusions along straight lines. The number of axes in the left hand bottom-corner indicates if the scheme is depicted as a single view (2 axes), or in 3 dimensions (3 axes).

4.4.2 Schemes imposed on the global-level

Vertical parallel slices

The most basic scheme is that of the interior and/or exterior envelope of a free form building design sliced vertically at (commonly regular) intervals. This egg-cutter approach results in planar slices with free form contours that are unique for each slice. Given the planarity of the slices, their vertical orientation and their parallel, non-intersecting positioning all being included in the scheme's definition (Figure 48A), the only formal degrees of freedom left are the slices' direction in the horizontal plane as well as their spacing. The structural action is still open, but constrained to any system spanning in one direction.



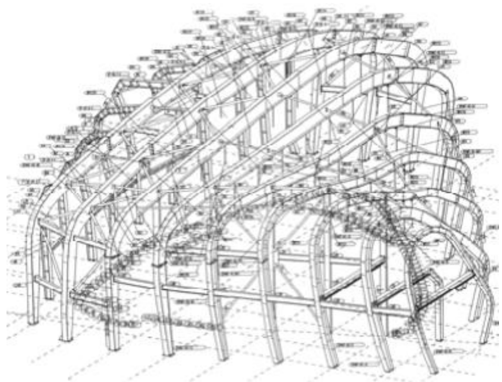
A. Scheme in top view.

Formal
z Planar intersection
z Vertical position
O Spacing
O Orientation of frame in plan
O Contour

Structural
z One-way load transfer
O Structural action

Material
z Planar base materials

B. Scheme's variables: z = fixed, O = degree of freedom.



C. Example of the scheme implemented in a realised structure: the Mezz concert hall. Image courtesy Pieters Bouwtechniek and Smulders staalwerken.

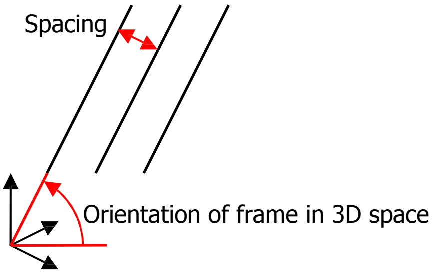
Figure 48. The structural scheme of parallel vertical slices, its degrees of freedom and a typical realised structure.

Further development onto the level of a structural system results in the schematic slices being realised as structural frames. The frame's vertical orientation allows them to stand independently, however since primarily in-plane loads can be transferred, transverse bracing may be needed for out-of-plane load transfer. These additional elements fall beyond the scheme's logic. In addition to those of the scheme, new degrees of freedom are the structural action the frames display, and the material in which they can be realised. In principle any material available in planar elements suits, while the material properties will affect the required dimensions. In case of additional elements at the contours (such as flanges), these need to be connected to the slices as well as formed to shape, this way introducing new constraints.

The system has known multiple realisations, one of them being the Mezz concert hall (Breda, The Netherlands), see Figure 48C and the BLIP-document in section 3.3.5. Here the scheme's slices were realised as steel frames, the free form contour being approximated through an assembly of standard steel sections, thus acting in section-action. The scheme's slices served as geometrical centre-plane through the sections' webs.

Inclined parallel slices

Compared to the previous scheme, now the constraint of placing the slices vertically is released, such that it now is the more generic class of inclined parallel slices (Figure 49A). While it keeps most of its rationality, the degree of freedom to freely place the parallel slices in space gives a first potential to respond to the global geometry. As Figure 49B shows, implementations onto systems are mostly similar to the previous scheme, although any structural system based on it, most likely to be realised as structural frames, has to anticipate the out-of-plane structural action that will occur if the slices in Figure 49C are realised as structural frames. This anticipation is nevertheless not part of the scheme's systematic.



Formal
z Planar intersection
z Inclined position
O Spacing
O Orientation of frame in space
O Contour

Structural
z One-way load transfer
O Out-of-plane structural action

Material
z Planar base materials

A. Scheme in view perpendicular to slices.

B. Scheme' variables:
z = fixed, O = degree of freedom.

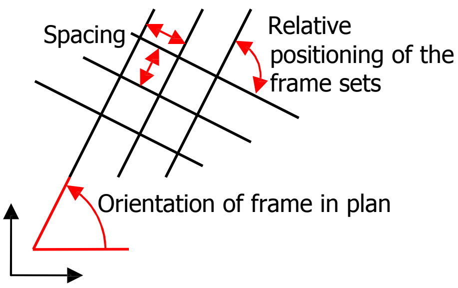
C. Implementation of the scheme to the DO Bubble.

Figure 49. The structural scheme of parallel inclined slices, its degrees of freedom and an implementation of the scheme in a case study design.

Multiple intersecting sets of parallel vertical slices

In addition to the characteristics described for the earlier scheme consisting of a single set of vertical parallel slices, one or more additional sets of parallel slices are introduced, all intersecting with the others (Figure 50A). With an additional frame set, the number of degrees of freedom slightly expands: each additional set of slices allows for positioning, thus able to optimise the members' orientation at a second point. Also for structural performance this is beneficial, as it now allows for a two-way load transfer (Figure 50B).

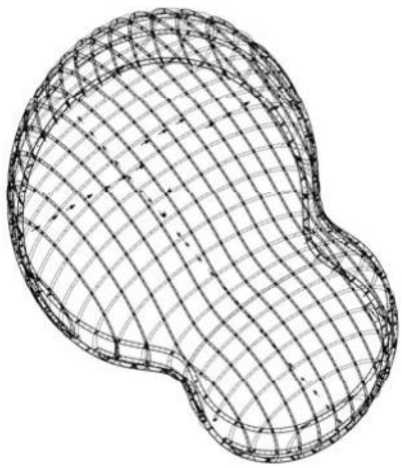
When the scheme is implemented as a system of intersecting structural frames (their structural composition to be determined), it features the same angle of incidence at all intersections, which simplifies detailing. Numerous precedents exist where this angle of incidence equals 90°, the BMW H2O-Bubble (Frankfurt, Germany) depicted in Figure 50C being one of them.



Formal
z Planar intersection
z Vertical position
O Spacing
O Orientation of each frame set in plan
O Contour

Structural
z Two-way load transfer
O Structural action

Material
z Planar base materials



A. Scheme in top view.

B. Scheme's variables:
z = fixed
O = degree of freedom.

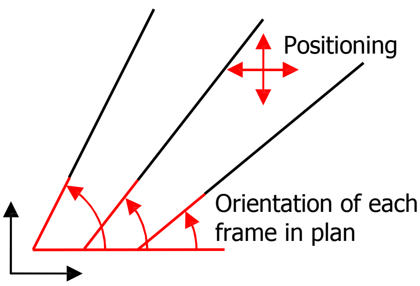
C. Example: the BMW H2O Bubble. Image courtesy Bollinger + Grohmann Ingenieure.

Figure 50. The structural scheme of multiple intersecting sets of parallel vertical slices, its degrees of freedom and a realised structure based on this scheme.

Single set of non-parallel vertical slices

By placing the slices in a non-parallel manner (Figure 51A), the degrees of freedom that first applied to a set of parallel slices presented before, now apply to each individual frame. This variation could be exploited for local anticipation to the building form. No intersections are foreseen in the actual scheme. As listed in Figure 51B, the scheme’s variables for structural and material aspects are the same as for the parallel layouts presented before.

Fan-orientations of slices are among the examples of implementations of this scheme, this way attempting to achieve an approximately constant orientation of the slices relative to the building envelope, without introducing the complexity of intersecting frames. Architectural rather than geometrical or technical considerations were decisive in the orientation of the frames of the BMW Dynaform-pavilion (Frankfurt, Germany) shown in Figure 51C (Franken 2002). Reducing the frame’s thickness to a curve on the referential plane resolves the varying angle of incidence – typical for globally imposed layouts – between the building envelope (in this case a membrane) and the frame’s plane.



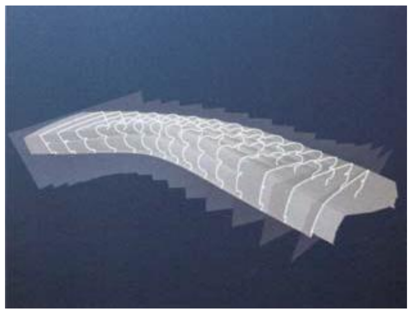
A. Scheme in top view.

Formal
z Planar intersection
z Vertical position
O Individual positioning of frames
O Individual orientation of frames
O Contour

Structural
z One-way load transfer
O Structural action

Material
z Planar base materials

B. Scheme’s variables:
z = fixed, O = degree of freedom.



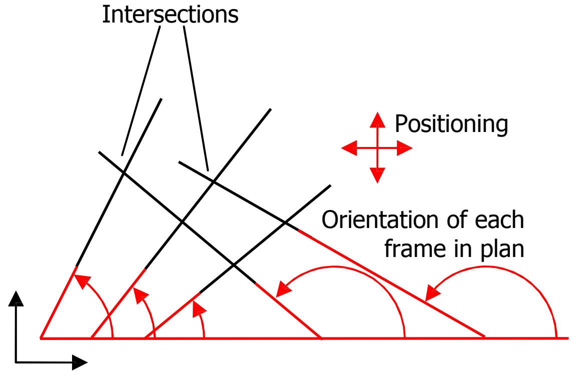
C. Example of the implemented scheme: frames of the BMW Dynaform-pavilion. Image courtesy ABB Architekten/Bernhard Franken.

Figure 51. The structural scheme of two intersecting sets of parallel vertical slices, its degrees of freedom and a typical realised structure.

Two intersecting sets of non-parallel vertical slices

When the slices of the previous scheme intersect to create a new scheme as in Figure 52A, the already present geometrical degrees of freedom are maintained, while structurally two-way load transfer is enabled (Figure 52B).

The scheme’s implementation in a system yields to connections at different angles, although well along straight lines. The China National Stadium (Beijing, China) is an example of the scheme implemented in a realised structure (also see the content document in section 3.3.2). Here, the slices through a global shape (Figure 53A) are realised as planar trusses with square box-sections. Particular detail is that after application of the scheme to the global level of scale (resulting in a series of planar intersecting slices), the orientation of the box-section is aligned with the contour of the building, this way requiring the exterior chord member to twist, and the diagonals to attach to it with varying angles in 3D space (Figure 53B).



A. Scheme in top view.

Formal

z Planar intersection
z Vertical position
z Non-repetitive angle of incidence
O Individual positioning of frames
O Individual orientation of frames
O Contour

Structural

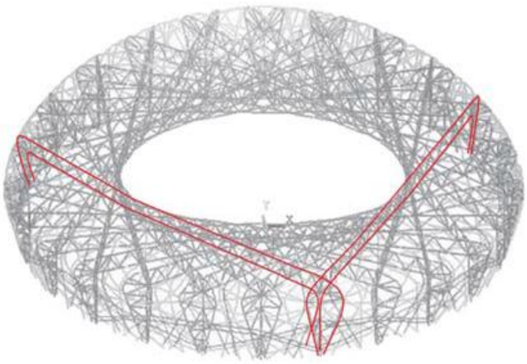
z Two-way load transfer
O Structural action

Material

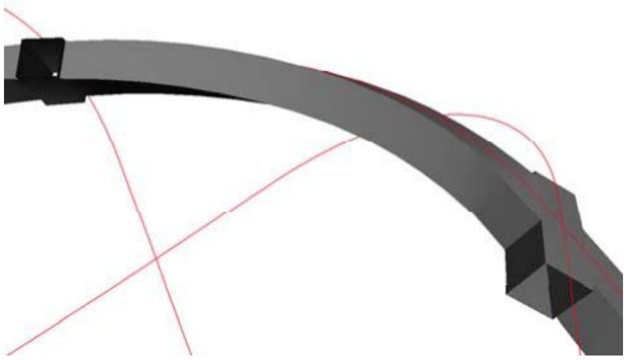
z Planar base materials

B. Scheme's variables:
z = fixed, O = degree of freedom.

Figure 52. The structural scheme of two intersecting sets of non-parallel vertical slices and its degrees of freedom.



A. Digital model of the structure, showing the primary frames in a fanning-layout.

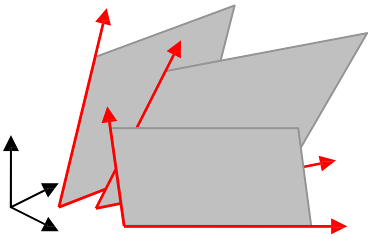


B. Zoom into the realised structure, showing a twisting exterior chord member, the centre line of which is nevertheless well in the planar slice.

Figure 53. Example of the implemented scheme: China National Stadium (after rationalisation).

Inclined non-intersecting slices

All schemes of non-intersecting slices can also be realised with inclined slices, instead of vertically oriented ones, as in Figure 54A. However, as the schemes with inclined slices described earlier, the inclined orientation has repercussions on the frames' structural action, since the inclination causes out-of-plane structural action. When implementing this scheme as a system, the out-of-plane capacity has to be provided by the materialisation, or otherwise assured through a system not included in the scheme's definition.



Formal
z Planar intersections
z Inclined positioning
z No intersections between frames
O Individual positioning of frames
O Individual orientation of frames
O Contour

Structural
z One-way load transfer
O Structural action

Material
z Planar base materials

A. Scheme in 3D view.

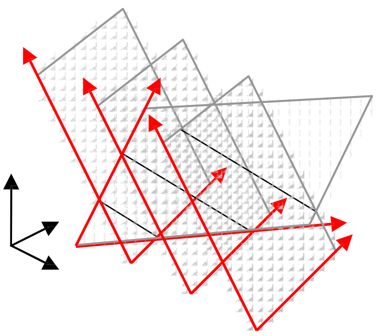
B. Scheme's variables: z = fixed, O = degree of freedom.

C. The scheme implemented on a free form building shape.

Figure 54. The structural scheme of inclined non-intersecting slices, its degrees of freedom and a demonstration of the implemented scheme.

Inclined parallel intersecting slices

The scheme of inclined parallel intersecting slices, see Figure 55A, is the more generic version of vertical parallel intersecting slices. With systems derived from this scheme, their variables being listed in Figure 55B, more customisation is possible since the planes' orientation has become a degree of freedom.



Formal
z Planar intersection
z Inclined positioning
z Repetitive angle of incidence
O Positioning per set
O Orientation per set
O Contour

Structural
z Two-way load transfer
O Structural action

Material
z Planar base materials

A. Scheme in 3D view.

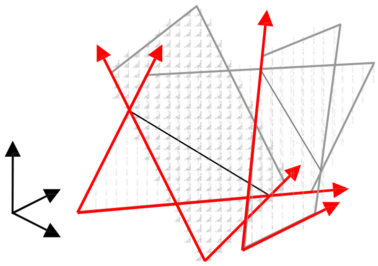
B. Scheme's variables: z = fixed, O = degree of freedom.

C. Application in a case study structural design.

Figure 55. The structural scheme of inclined parallel intersecting slices, its degrees of freedom and a demonstration of the implemented scheme.

Inclined non-parallel intersecting slices

When the inclined slices from the previous scheme are allowed to intersect, this results in a new geometrical scheme depicted in Figure 56A. Apart from the non-vertical positioning which has now become a degree of freedom (Figure 56B), it is geometrically identical to the scheme of vertical non-parallel intersecting slices, but allows for more local anticipation through the variable inclination that can be varied per slice, rather than per set of slices. The scheme has been implemented in the case study model depicted in Figure 56C, using the slices to construct the diagonals of a global truss.



Formal
z Planar intersection
z Inclined positioning
z Non-repetitive angle of incidence
z Intersections between frames
O Individual positioning of frames
O Individual orientation of frames
O Contour

Structural
z Two-way load transfer
O Structural action

Material
z Planar base materials

A. Scheme in 3D view.

B. Scheme's variables: z = fixed, O = degree of freedom.

C. Model of the case study structural design.

Figure 56. The structural scheme of inclined intersecting slices, its degrees of freedom and the case study structural design described extensively in the next chapter.

4.4.3 Schemes able to anticipate to the local geometry of the building envelope

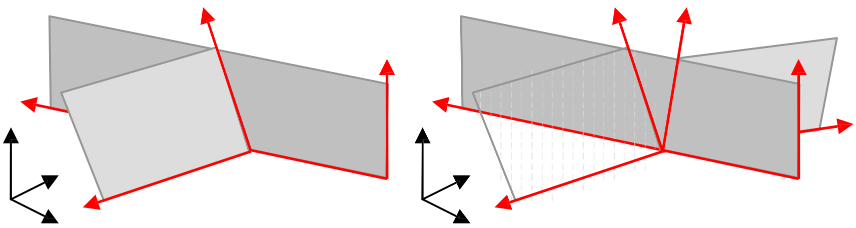
The denomination of 'local anticipation' that will be explored in this section simply refers to the ability of exploiting the grid's degrees of freedom to make it anticipate to the local geometrical circumstances. This concerns for instance curvature and clear span. In the rare cases that the building design has been set up with a referential grid system in mind, the system's degrees of freedom may be exploited to a higher degree than compared with an application of the same system but in a design not anticipated to this system.

Planar members with side connections

In this scheme, the extremity of one planar member connects to the side of another, see Figure 57A. This way, relatively simple connections can be made for the reason that per intersection no more than two members intersect. Each of these members can be individually oriented without affecting the scheme's integrity. If members are attached to both front and backside of a member as in Figure 57B, forces may be transferred directly between them. The member's pattern is to be decided upon by the designer. The scheme resembles the class of Mutually Supported Elements (or (multi) reciprocal frame), consisting of each linear element simply supported on another. Its primary driver has been to construct spans larger than one single element (Brown 1989).

The scheme may be realised as a system in any material available in sheets, the material deciding upon the required detailing and dimensioning. In doing so, sufficient resistance due to eccentric connections has to be taken care of. In fact, since the members are oriented individually, from one to the next in line they may feature a fold or twist (if it wouldn't, there would be no need to realise them as two separate members), or both. A twist may result from the desire to orientate each member at a constant orientation (e.g. perpendicular) to the building envelope. Structurally, this results in an eccentric connection at either the top or bottom chord, resulting in bending out of the element's plane. Apart from in-plane bending capacity, the elements should therefore also feature sufficient out-of-plane bending capacity, notably at those spots where members are connected. To limit this effect, it is proposed that, in case of connections on the front and backside of a continuous member, their projections should intersect on the continuous members as in Figure 58A or Figure 58B, but not beyond its contours as in Figure 58C.

In defining the members' grid, variation of structural performance can be realised through the grid's pattern (quadrangles, hexagons, …) and its density (by decreasing the length of members and thus increasing the number of members per unity of area). It can be made self-stabilising by introducing triangles, whereas with a quadrangular pattern additional bracing may need to be introduced. When realised in a foldable material, connections can be made by folding the members' extremities. Three implementations of the scheme using folding are demonstrated. Figure 59A and Figure 59B use two members (as in Figure 57A), while Figure 60A employs three members (as in Figure 57B). For realisation however, it was proposed to be constructed from two sheets, partially lying side by side.



A. Scheme of two members in 3D view.

B. Scheme of three members in 3D view.

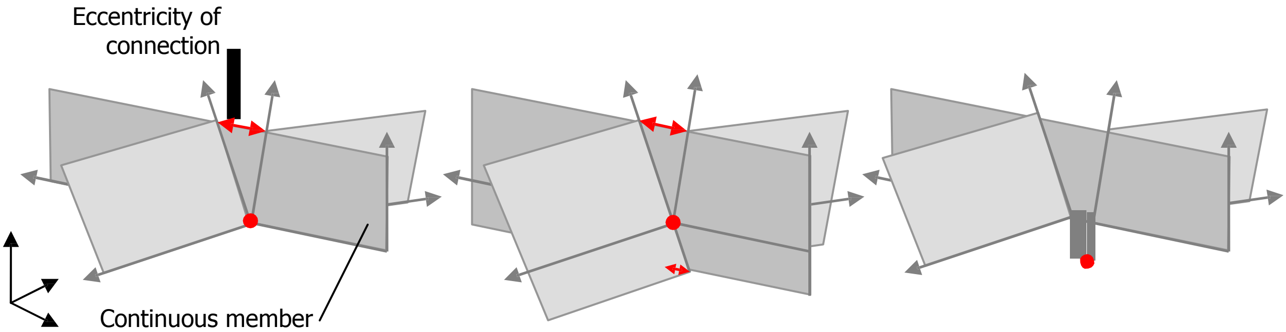
Formal
- z Planar intersection
- z Inclined positioning
- z Non-repetitive angle of incidence
- z Pattern of continuous polygons
- O Individual positioning of members
- O Individual orientation of members
- O Upper and lower contour

Structural
- z Two-way load transfer
- z Non-centered connections
- z Section action
- O Structural depth

Material
- z Planar base materials

C. Scheme's variables:
z = fixed, O = degree of freedom.

Figure 57. Two variants of the structural scheme of inclined intersecting slices and its degrees of freedom.

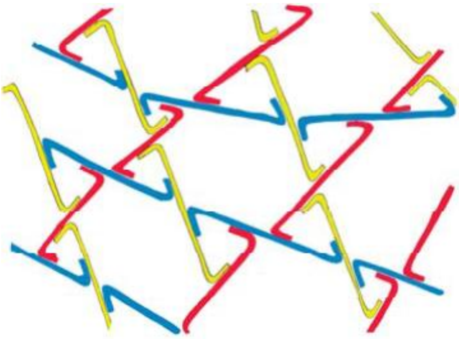


A. Joint point at boundary of the continuous member.

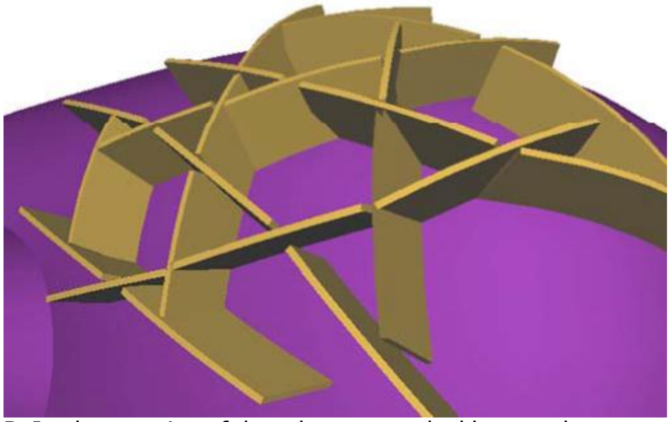
B. With extended planes, joint point within planes.

C. Joint point beyond the contours of the continuous member, to be avoided to limit eccentricity.

Figure 58. Connection of planar members to the front and backside of a continuous member.

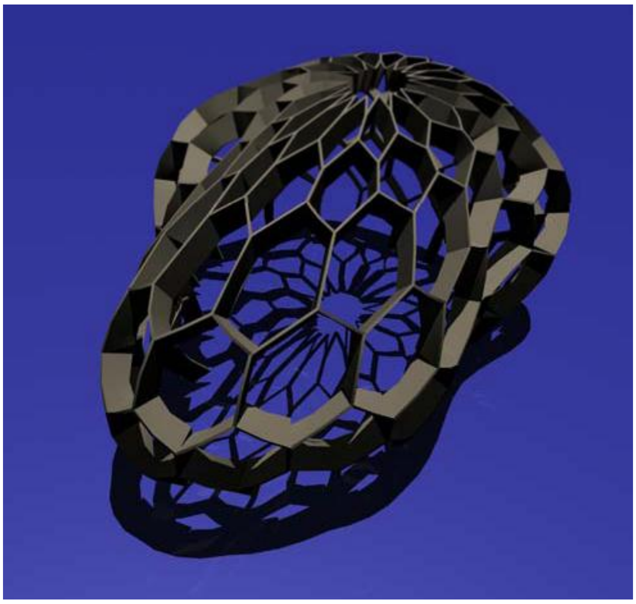


A. Irregular pattern of hexagons and triangles based on three polygonal lines.

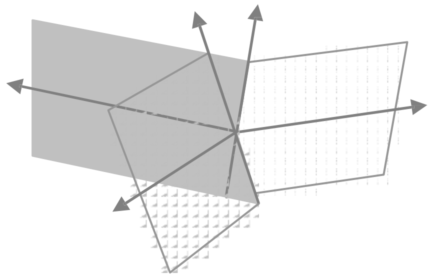


B. Implementation of the scheme on a double curved surface, using timber panels.

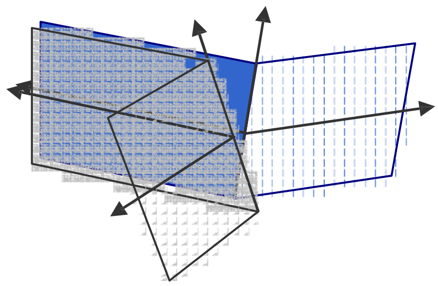
Figure 59. Demonstration of the scheme of planar members with side connections with members realised in a foldable material.



A. Continuous members folded into a hexagon pattern.



B. Detail of a connection according to the principle of B.



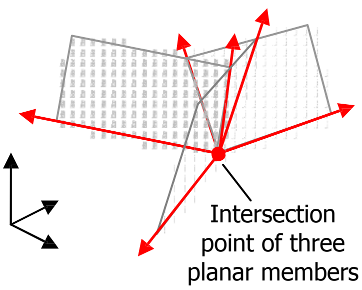
C. Proposed realisation of two folded members partially lying side by side.

Figure 60. Cardboard pavilion as competition entry for the new Stylos Bookshop, an implementation of the scheme of planar members with side connections. Design team: Jop van Buchem, Pim Marsman, Taco van Iersel and the author.

Planar members with end connections

This scheme involves the connection of the extremities of at least three planar members. The members intersect in one point, as shown in Figure 61A. The extremity of each member is in the plane of another member, this way creating a pyramid-shaped node in the middle.

As the previous scheme, also this scheme can be realised in any planar material which also features resistance to out-of-plane strength due to eccentricity of connections. Depending on the material and detailing chosen in its implementation, the location of the intersection point may be at an upper or lower extremity as in Figure 62A, such that the node consists of one pyramid. When extending the planes at the intersection point into two directions, pyramid-nodes appear at both sides of the node. This way the connection's eccentricity becomes located centrally as in Figure 62C, rather than on an extremity. Structurally this is beneficial, but it requires a particular, more elaborated detailing from both top- and bottom side. Both options observe the scheme. The planar elements may have arbitrary contours (Figure 62B). Different from the scheme of Planar members with side connections described before, neither the scheme nor the system is constrained by the pattern, as the pattern is unstructured and thus requires no logic in continuity of members or repetition.



Formal
- z Planar intersection
- z Inclined positioning
- z Non-repetitive angle of incidence
- z Three planes intersecting in one point
- O Polygonal pattern
- O Individual positioning of members
- O Individual orientation of members
- O Upper and lower contour

Structural
- z Multi-way load transfer
- z Section action
- O Structural depth

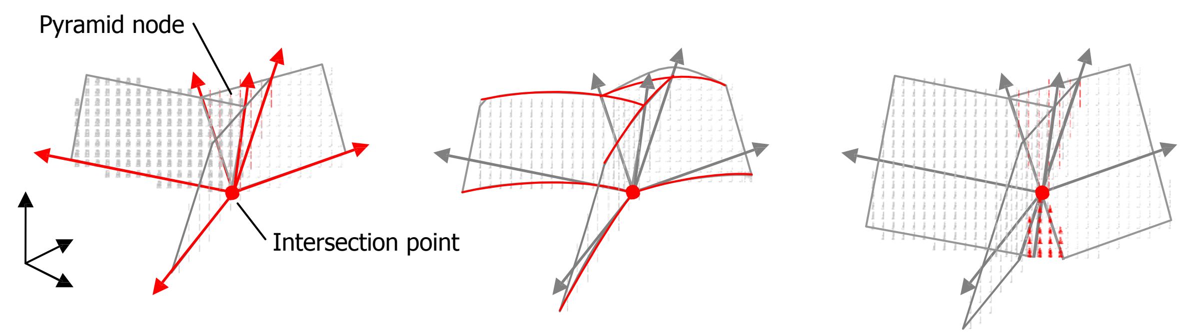
Material
- z Planar base materials

A. Scheme of two members in 3D view.

B. Scheme's variables:
z = fixed, O = degree of freedom.

C. Typical application of planar members in a case study structural design.

Figure 61. The structural scheme of Planar members with end connections, its degrees of freedom and a typical realised structure.



A. The schematic basis: 3 planes without any extension and a single pyramid node.

B. Free form contours of the upper and lower edge.

C. Intersection point inside joint.

Figure 62. Geometrical consequences of extended planes and free form contours.

4.5 Schemes based on hypothetical solution 2: developable surfaces

Through using developable surfaces, the first hypothetical solution of exploiting the free form contour of planar elements is extended with the introduction of two more degrees of freedom. These additional degrees of freedom are the curvature class and its eventual variance, and are included in Table 7, containing the list of variables used to categorise the upcoming schemes. While the notion of parallelism was among the categorising variables in the first series, it was not so evident in the case of the planar elements-hypothesis. It was replaced by the notion of set-wise manipulations, meaning that the curved parts have a common centre point. New variables are the curved parts that may either form a closed loop or a part of it, and finally the number of intersecting parts. Though numerous, the variables being so explicit will be helpful in the categorisation of the schemes.

Table 7. Variables in the generation and classification of schemes and systems based on hypothetical solution 2, developable surfaces.

Variable	**Possible values** Level
of scale	Globally / locally
Curvature class	Cylindrical / conical
Curvature variance	Constant / non-constant
Full or partial loop	Full / partial
Set wise	Individual / set
Number of intersecting sets/parts	Single (=no intersections) / double / triple / …

By using combinatorics as was done with the previous hypothesis, theoretically a number of (2 x 2 x 2 x 2 x 2 x 3 =) 96 alternative schemes can be generated with the variables listed. This number is too large to be assessed individually on their potential. Instead, a creative design process is used to generate the alternative schemes. The eight schemes generated this way are depicted in Figure 63, which includes the values of the variables according to which they are categorised. The case study object envisaged for the implementation and evaluation of systems (see next chapter) is used to direct the searches by direct evaluation of appropriateness in double curved building shapes. Although creative implementations are strongly directed by the specific context of the design, the assessment finally employs predefined criteria, this way allowing an evaluation in a scientific manner.

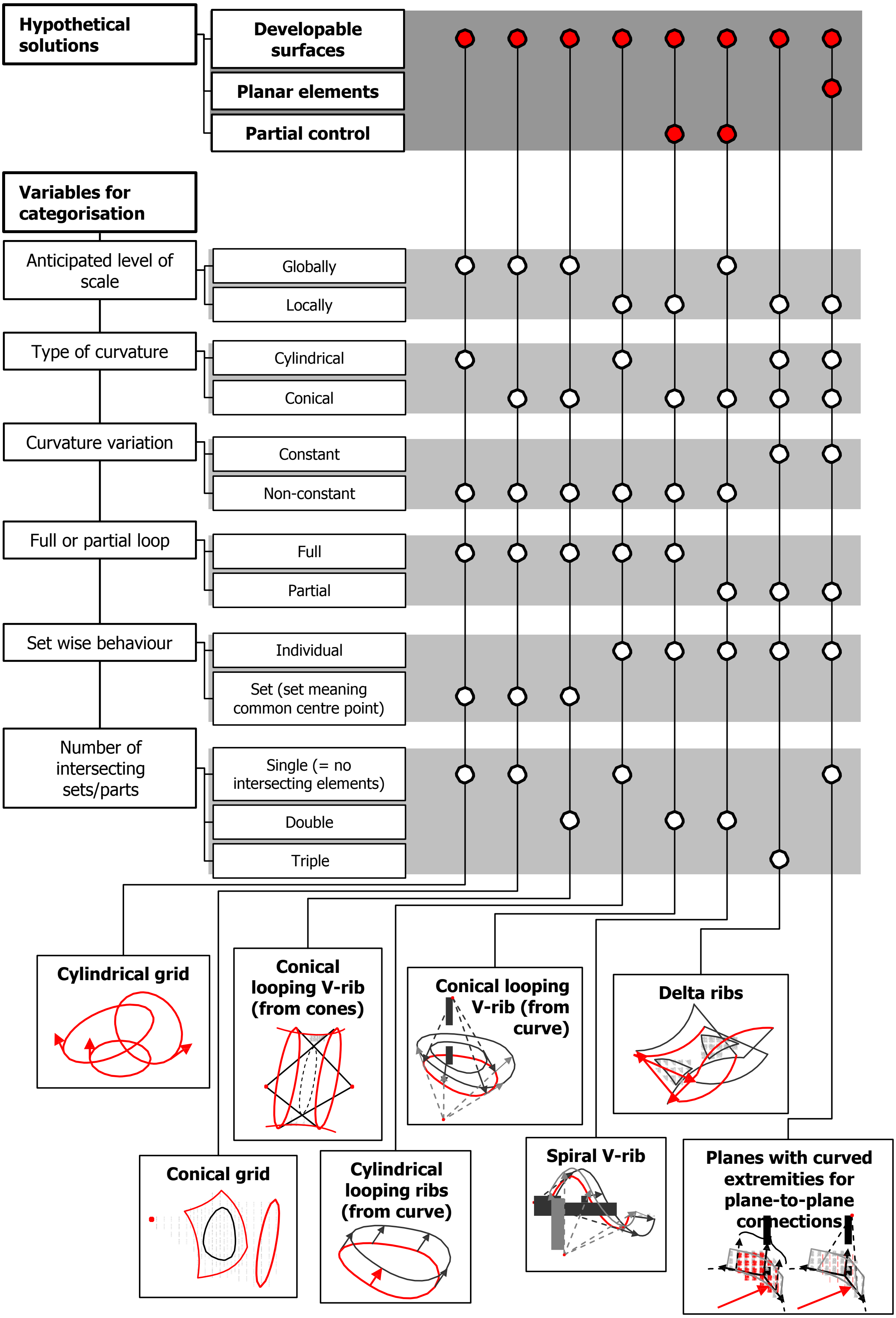


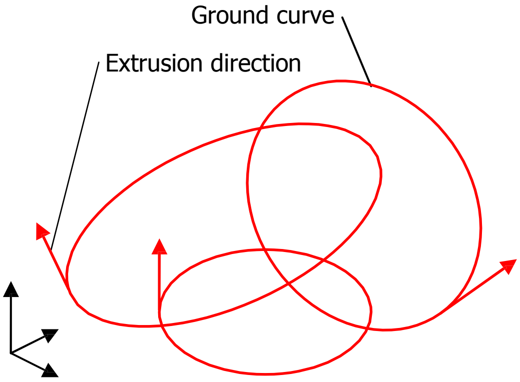
Figure 63. Schemes based on hypothetical solution 2: developable surfaces.

4.5.1 Schemes imposed on the global-level

Cylindrical grid

Curved grids are proposed as an extension to the grids consisting of planar slices in either orientation as included in the first series of schemes, and as seen in numerous precedent projects. The scheme results in free form contours when superimposed to building shapes. Curved grids' primary difference with planar grids is their ability to choose the ground curves in accordance with local geometrical circumstances. When extruded, the ground curves result in cylindrically curved surfaces. Its schematic principle is depicted in Figure 64A. The variance in curvature depends on the ground curve and the extrusion direction, circles extruded perpendicularly to the circle's plane result in a constant cylindrical curvature.

While the curvatures evidently result in additional degrees of freedom as listed in Figure 64B, their structural implementation is little evident because the scheme only consists of a curved referential surface with no inherent structural properties. Figure 65 demonstrates stepwise how the grid could be literally translated into a structural layout. To make these curved sheets resistant to out-of-plane loadings, it needs additional elements not included in the scheme.



A. Scheme of intersecting curves to be extruded to cylindrically curved members, in 3D view.

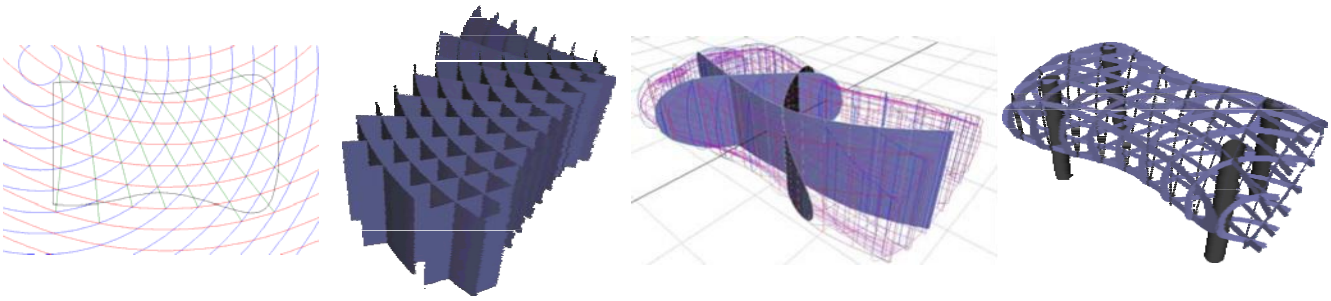
Formal
z Cylindrical curvature
O Number and shape of ground curves
O Extrusion direction
O Individual positioning and orientation

Structural
z Sheet-like cross section
O Structural depth

Material
z Planar base materials

B. Scheme's variables:
z = fixed, O = degree of freedom.

Figure 64. The structural scheme of cones in a grid setup, its degrees of freedom and a structural design in which it is implemented.



A. Grid of circles.

B. Extrusion of the grid to cylindrically curved surfaces.

C. Trimming the extruded curves to the building envelope.

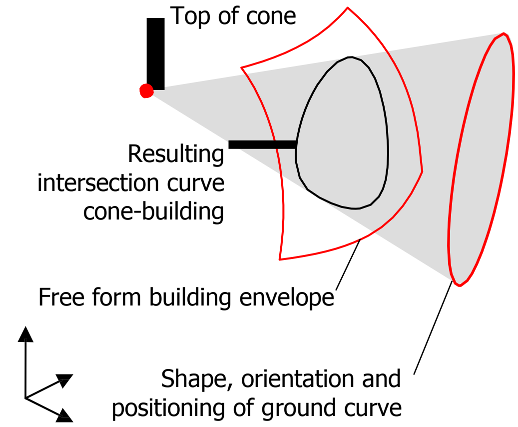
D. Cylindrically curved surfaces proposed as ribs.

Figure 65. Implementation of cylindrically curved surfaces in a referential grid system. Although the structure is spatial, the geometrical variety is fully located in its ground plan.

Conical grid

Cones feature offer more formal variety than cylinders because cones have a top. This is employed in the scheme of a conical grid depicted in Figure 66A, which exploits the conicality as an additional degree of freedom compared to the previous scheme. The scheme's formal variables, listed in Figure 66B, thus become even more potential. As the previous scheme, the scheme of a conical grid is to be superimposed to any shape, which determines the grid's contour.

Implementations of the scheme onto systems may exploit the conical grid's variables by making the grid systems anticipate the building envelope's local curvature. An implementation of a conical grid is shown in Figure 67, where the cones are oriented to avoid tangency with the building envelope.



A. Scheme of cones in 3D view.

Formal
z (Parts of) cones
O Top of cone
O Shape of cone's ground curve
O Individual positioning and orientation

Structural
z Sheet-like cross section
O Structural depth

Material
z Planar base materials
O Eventual multilayer realisation of the surface representation

B. Scheme's variables: z = fixed, O = degree of freedom.

Figure 66. The structural scheme of cones in a grid setup and its degrees of freedom.

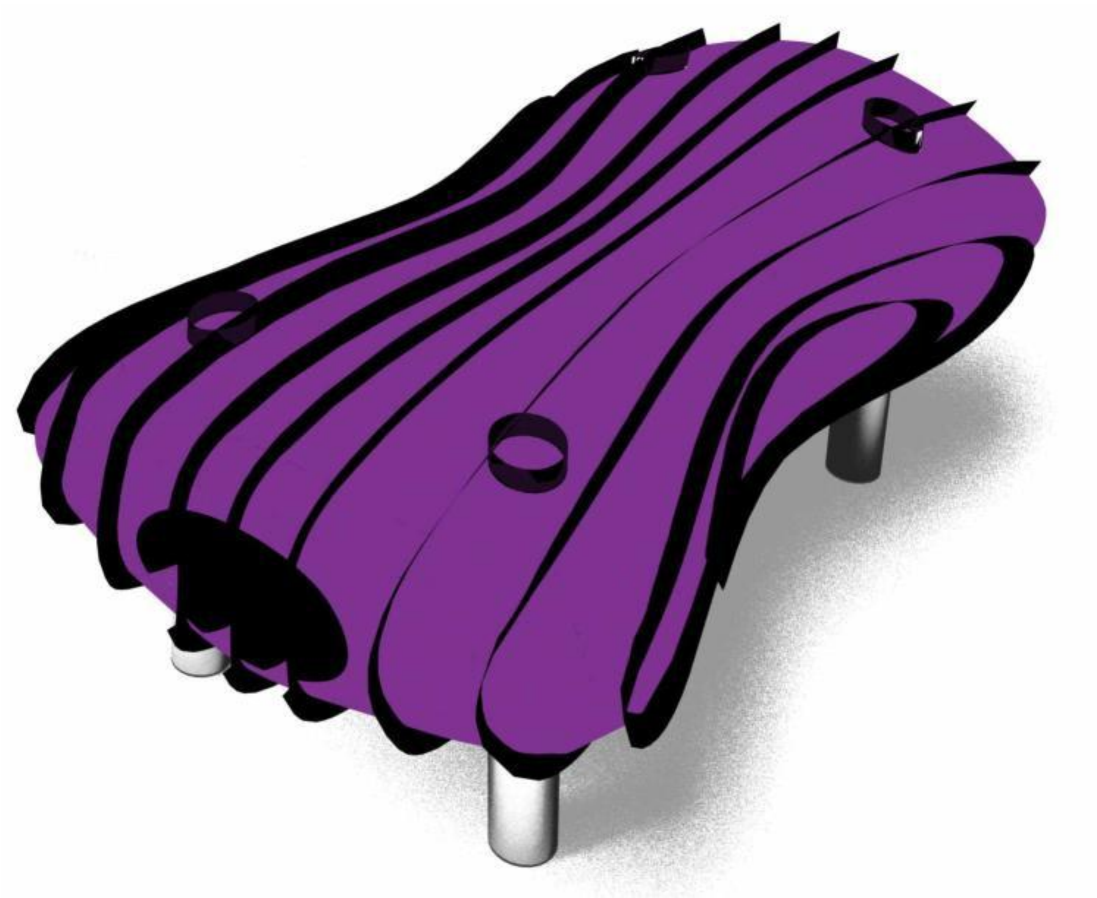
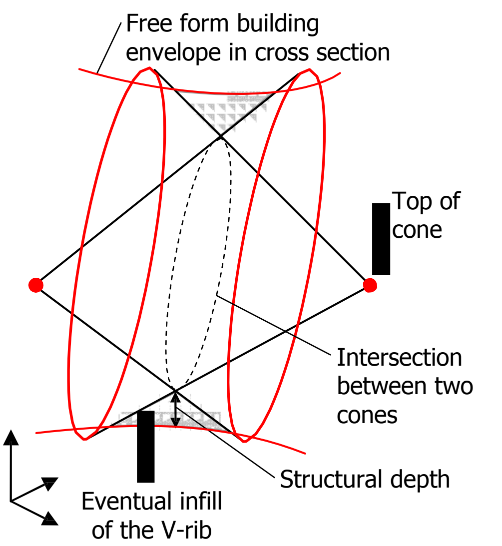


Figure 67. Implementations of a conical grid in a case study design, attempting to position the conical grid perpendicularly to the building envelope.

Conical looping V-rib (from cones)

This scheme intersects two cones, this way anticipating structural performance of what was primarily a geometrical setup in the earlier schemes. When the cones' tops point in opposite directions they constitute a V-section in a closed ring, as demonstrated in the scheme's schematic cross section in Figure 68A. The V-shaped cross section is delimited at the intersection with the building envelope, while the V-angle can be manipulated through each of the cones. These geometrical variations have structural implications, as listed in the scheme's degrees of freedom in Figure 68B.

Applied as a system, the V-shaped cross section could for instance be filled to give strength and/or stiffness to the rib. The V-shape is thus used as a moulding. This way also the open side of the V is addressed, for which the scheme itself does not provide any systematic solution. An implication of changing the cones is that the intersection curve will change, thus affecting the available options for materialising its eventual articulation: the intersection curve may for instance no longer be a planar curve. This makes this scheme suitable for situations when the conical surfaces are constrained, for instance to achieve a constant curvature of the surfaces. Figure 68C shows an implementation of the scheme using repetition: all cones are identically shaped, but trimmed differently.



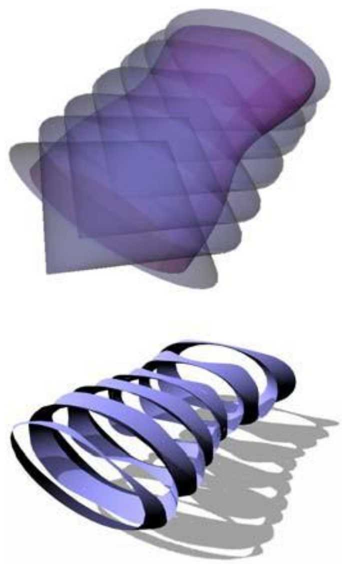
A. Scheme of conical looping V-ribs in 3D view.

Formal
z Intersection curve
z Closed loop
O Top of each cone
O Shape of each cone's ground curve
O Individual positioning and orientation

Structural
z V-shaped cross section
O Structural depth

Material
z Planar base materials
O Eventual infill

B. Scheme's variables: z = fixed, O = degree of freedom.



C. Application in a case study structural design.

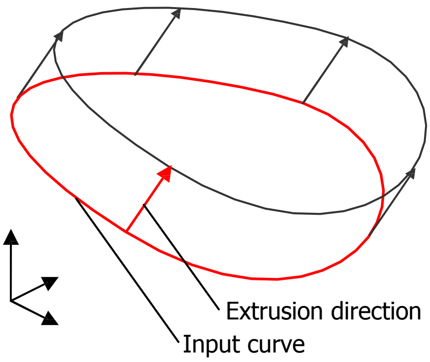
Figure 68. The structural scheme of a conical V-rib, its degrees of freedom and a structural design in which it is implemented.

4.5.2 Schemes able to anticipate to the local geometry of the building envelope

Cylindrical looping ribs (from curve)

Starting from a curve, extruding it into a direction, the scheme depicted in Figure 69A generates cylindrically curved ribs. The scheme is geometrically powerful due to its input of any curve (including space curves). However, its structural characteristics are fully determined by the cross section of the single surface, as listed in Figure 69B.

Applied on a sufficiently small level of scale in a building design, the extrusion direction could respond to a local curvature condition, for instance to be nearby a perpendicular orientation. Figure 69C exemplifies the scheme's implementation, showing that perpendicular orientation is hard to achieve with looping ribs. The scheme of looping ribs with highly structurally potential V-shape cross section cannot be obtained with two cylindrical surfaces. This is due to the fact that the section created this way will vary between a V and a straight line, the latter obviously lacking all structural potential the V-shaped cross section had. For non-looping ribs this V-shaped cross section is well possible, which is demonstrated in the Delta rib-scheme presented later.



A. Scheme in 3D view.

Formal
z Single cylindrical surface
z Closed loop
O Extrusion direction
O Shape of curve

Structural
z Sheet-like cross section
O Structural depth

Material
z Planar base materials
O Eventual multilayer realisation of the surface representation

B. Scheme's variables:
z = fixed, O = degree of freedom.

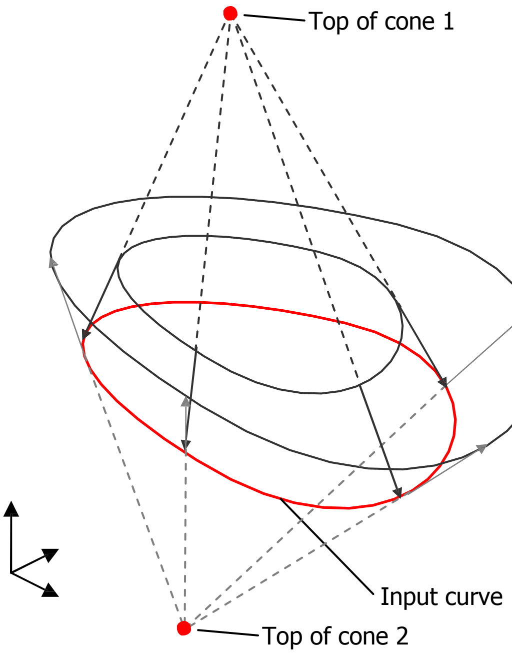
C. Application in a case study structural design.

Figure 69. The structural scheme of a cylindrical looping ribs, its degrees of freedom and a structural design in which it is implemented.

Conical looping V-rib (from curve)

By changing the cylindrical curvature of the previous scheme into a conical curvature, a second looping surface can be created. This scheme is depicted in Figure 70A. Together the two cones form a V-shaped cross section with structural potential. The scheme of the Conical looping V-rib (from cones) described before, differs from the actual scheme regarding the required input. Whereas the former is based on cones imposed on the global-level and the intersection curve is a result of this, the actual scheme departs from a curve and creates the attached surfaces. As the actual scheme departs from the edge curve, for materialisations in which the edge curve's geometry is critical best this scheme is followed. Contrarily, when constraints on the curved surfaces apply, e.g. regularity, best the former scheme should be followed.

Although the scheme suggests the generation of a looping cone towards one side of the curve, the outcome is not restricted to this. Through the same scheme, V-shaped sections can be created in any direction relative to the input curve. As remarked earlier at the conical looping V-rib, neither that scheme nor the actual scheme includes a systematic solution to finalizing the open side of the 'V', and integrating it to the double curved building envelope by which it is delimited. A possible way to overcome this is to fill the V-shape with a structural material. This allows the sides to be relatively flexible as the filling will render them stiff. Such relatively flexible sides could be formed through using a locally controlled formation method. The local controls could be a few triangular cross sections custom-cut to shape, together with the edge where the cones intersect, as shown in Figure 71.



A. Scheme in 3D view.

Formal
z Two intersecting cones
z Cone’s curvature depends on curve
z Open loop
O Input curve
O Top of cone

Structural
z V-shaped cross section
O Structural depth

Material
z Planar base materials
O Eventual infill

B. Scheme’s variables:
z = fixed, O = degree of freedom.

Figure 70. The structural scheme of a conical looping V-rib and its degrees of freedom.

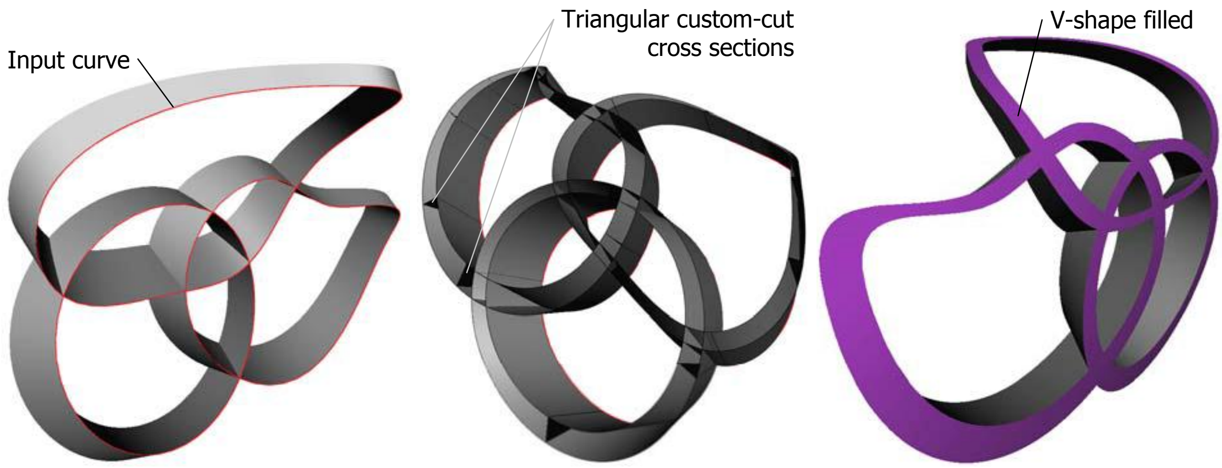
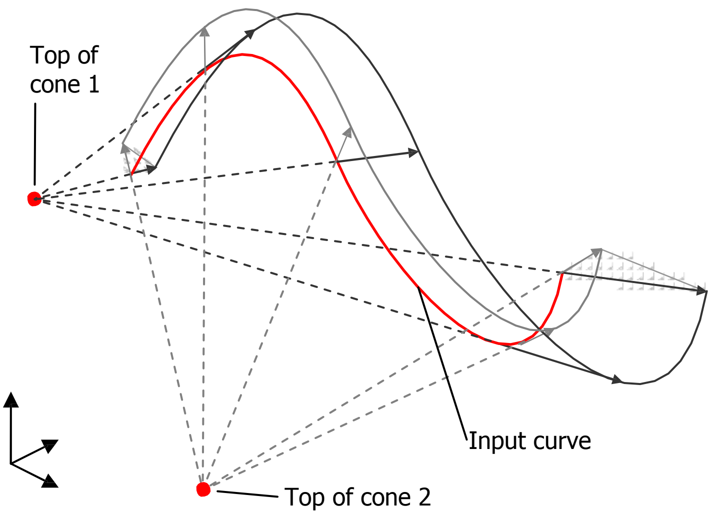


Figure 71. Structural design showing an implementation of the conical looping V-rib–scheme, using a locally controlled formation technique.

Spiral V-rib

As the previous scheme, also the actual scheme of a spiralling V-rib is composed by intersecting two conically curved surfaces, both departing from the same input curve. Customisation is possible through defining the two points where to scale from (Figure 72A). The scaling points each become the top of a cone. Among the degrees of freedom as listed in Figure 72B, the primary difference with the closed loop V-rib is the additional formal freedom to implement such ribs in non-looping applications, whereas the structural potential of the rib’s V-shaped cross section is maintained.

Implementing the scheme in a structural layout, the single scaling point per cone causes the rib’s V-shaped cross section to change shape at larger distances from the scaling point. This necessitates a system-restart to redress the V-rib’s cross section, resulting in an abrupt transition as shown in Figure 73. Remarks made at the previous scheme on an eventual infill of the V-shape apply to this scheme too.



A. Scheme of two members in 3D view.

Formal
z Two intersecting cones
z Non-constant curvature
z Open loop
O Input curve
O Shape of cone's ground curve

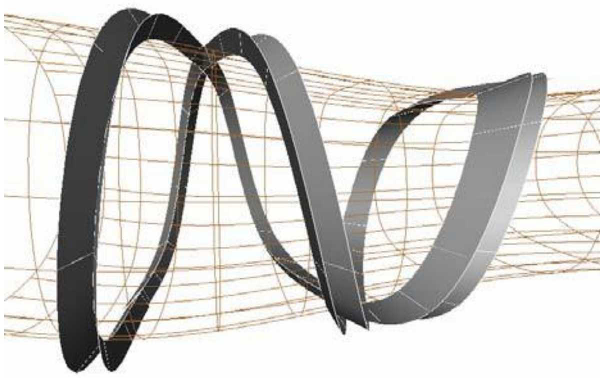
Structural
z V-shaped cross section
O Structural depth

Material
z Planar base materials
O Eventual infill

B. Scheme's variables:
z = fixed, O = degree of freedom.

Figure 72. The structural scheme of a spiralling conical V-rib and its degrees of freedom.

A. System-restart to redress the V-rib's cross.



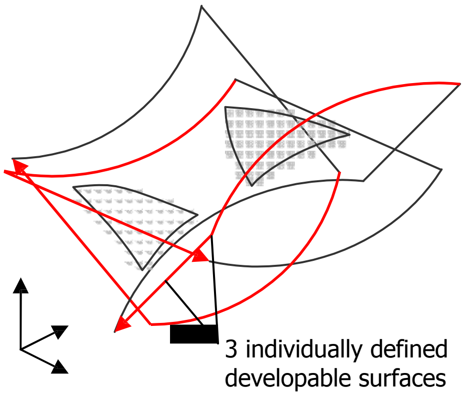
B. Without system-restart the cross section's shape of the V-rib tends to distort at larger distances from the scaling point.

Figure 73. Spiral V-ribs with and without system-restart.

Delta ribs

The scheme for ribs with triangular cross section (hence delta) is based on three intersecting developable surfaces, thus either conically or cylindrically curved, as depicted in the schematic view of Figure 74A. The surfaces are not looping, and are therefore not constrained by a looping arrangement needing appropriate orientation to the building envelope along its full length. This enhances the formal freedom. By varying the curvature, position and orientation of each of the three surfaces (or the cylinders or cones these surfaces are part of), curving members with different height-width-ratios along their length are generated. These degrees of freedom are listed in Figure 74B.

Implementation of the scheme results in segments that are structurally self-stable and, unlike the schemes with V-sections listed before, require no additional structures for stabilisation. As the members are not looping, they allow for application as singular elements (various rib shapes are shown in Figure 75), as well as in a network-arrangement as demonstrated in Figure 74C.



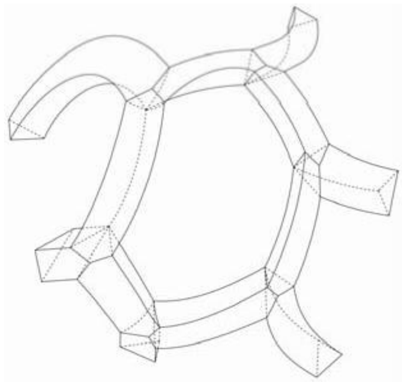
A. Scheme of Delta Ribs consisting of three cylindrically curved surfaces in 3D view.

Formal
z Three developable surfaces
z Open loop
O Individual positioning and orientation

Structural
z Triangular cross section
O Structural depth
O Height/width-ratio

Material
z Planar base materials

B. Scheme's variables:
z = fixed, O = degree of freedom.



C. Application of the Delta Rib-scheme in a network.

Figure 74. The structural scheme of a Delta Rib, its degrees of freedom and their implementation in a network.

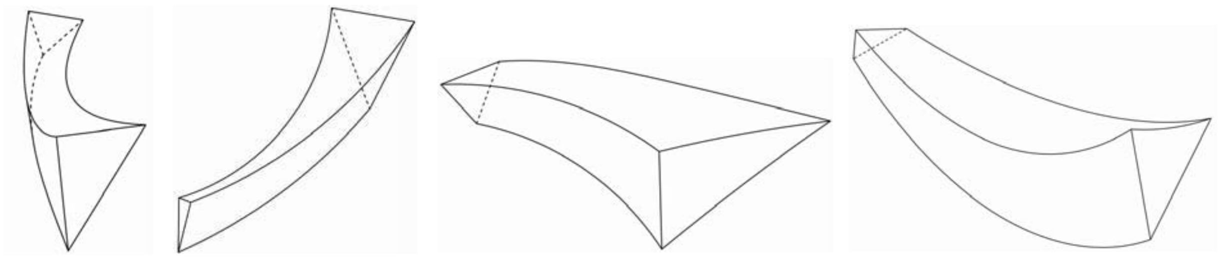
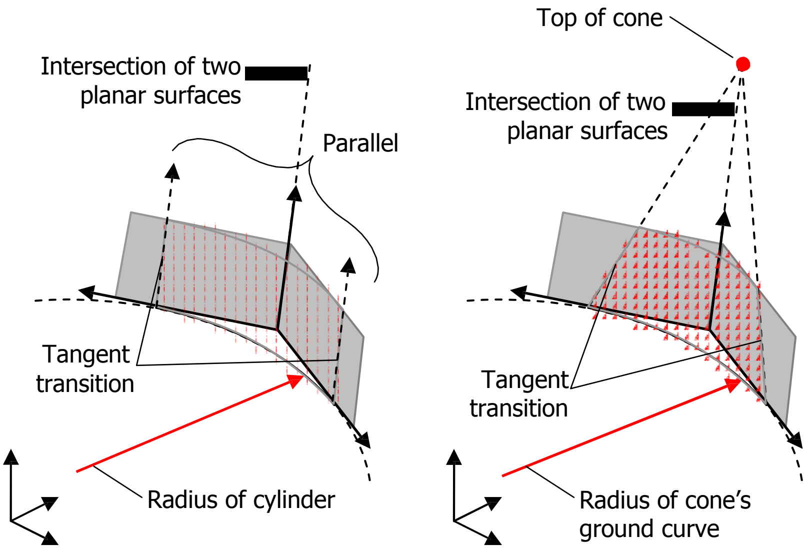


Figure 75. Examples of rib-shapes constructed with the Delta Rib-scheme.

Planes with curved extremities for plane-to-plane connections

Rather than surfaces being entirely curved as generated so far from the hypothetical solution of potentialities of developable surfaces, the actual scheme proposes a curved transition between two planar elements, as displayed in Figure 76A. When used at an extremity of a planar member, it is added to elements like those presented in the earlier scheme 'planar members with end connections'. With this curved end, the extremity of one element gets positioned side-by-side with another element, enabling a plane-to-plane connection.

Implementations of the scheme in systems notably address materials where such plane-to-plane connections (rather than connections for instance along one line) are preferential. To limit structurally undesirable effects of non-centric connections, the cylinder's or cone's radius should be kept small. Also for structural reasons, the connection is expected to be most performing in case of three interlocking planes, as an alternative for the scheme of Planar members with end connections that uses pyramid-shaped nodes. These curved extremities work with either position of the intersection point as shown in Figure 62. For application in a system, timber is an appropriate material as it allows for both curving and plane-to-plane connections, with the connector loaded in shear, see Figure 77.



Formal
z Cylindrically or conically curved transition
O Radius

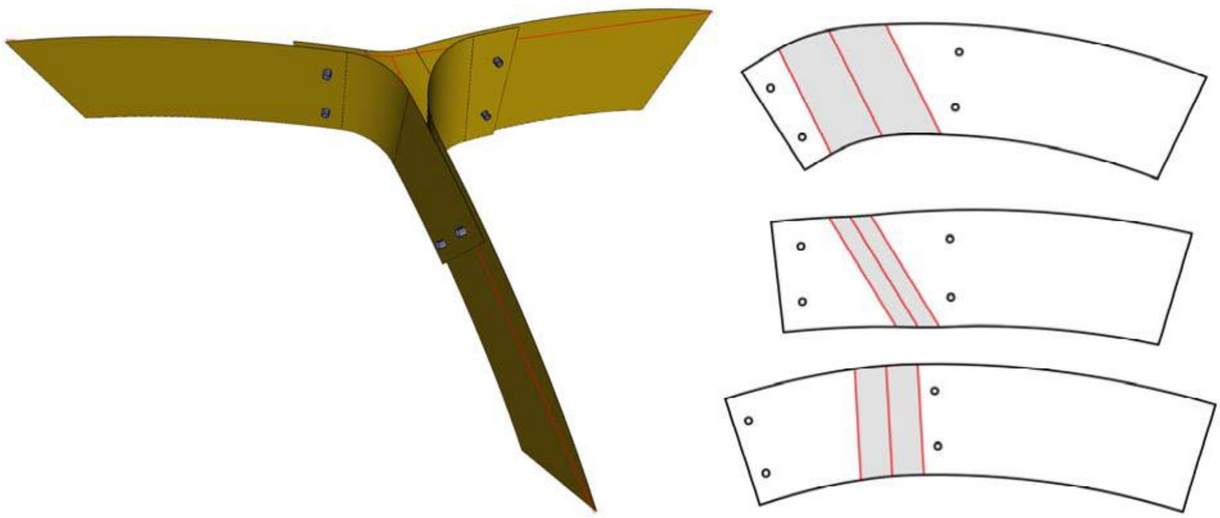
Structural
z Non-centered connections
z Section action
O Structural depth

Material
z Planar base materials

A. Scheme of two planar members with a cylindrically curved and a conically curved transition, seen in 3D view.

B. Scheme's variables:
z = fixed, O = degree of freedom.

Figure 76. The structural scheme of planes with curved extremities for plane-to-plane connections and its degrees of freedom.



A. 3D-view.

B. Unrolled ribs (curved parts in grey).

Figure 77. Implementation into a timber structural system of the structural scheme of planes with curved extremities for plane-to-plane connections.

4.6 Schemes based on hypothetical solution 3: rib-shell collaboration

In the series of schemes based on hypothetical solution 3, rib-shell collaboration, the building enclosure is integrated in the load bearing structure. As stated in the introduction, this research concerns double curved building enclosures of which the geometry is not optimised to the fulfilment of its structural function (unlike the thin concrete shells from the 1950's and 1960's). The primary way to which the proposed alternatives have been categorised vary in the eventual addition of structural ribs. This way, the number of ways through which the structural needs can be anticipated substantially grows. Alternatives are ranging from monocoque shells (the building envelope is the structure), via semi-monocoques (stiffening ribs are added to the building envelope) to rib structures in which the building envelope only spans at a lower level of scale and thus acts structurally in a secondary way. Further categorisation involves the constitution of the shell, as this raises differentiation towards how the shell is to be constructed (e.g. by using formwork) or functions in a structural way (e.g. differentiation of the size and materials in the shell's cross section to the degree to which they are loaded).

The considered categories are listed in Table 8. The overview of the alternative schemes generated with them is depicted in Figure 78, in which conventional monocoque shells, working in form-action are included too, to pinpoint the additional parameters for constructing surface structures that are proposed in this scheme. As the proposition for shells and ribs involves a large number of parts and functions, also the other hypothetical solutions have been called upon to direct the explorative search for alternatives, as is depicted in the mentioned figure.

Monocoque shells, despite geometrically simple as they are one single surface, have not been researched in particular in this thesis since the number of variables available for such shells only would be worth a research on its own.[8] The variables involved in such a research should concern the material properties, how these materials can be put in place and how the nature of the structural action and its capacity can be varied within the system, which could be of composite nature. Furthermore, when applied in a free-form multilayer sandwich, a particularly critical point is the transfer of structural actions other than in-plane loadings. Out-of-plane loadings will always result in shear and bending in the shell's cross section, and thus sollicitate the bonding between the core and the skin. So far, the problem is solved by making stringers, which implies the usage of planar elements.

In building construction, composite structural action is best known between steel and concrete: the former in areas of tension, the latter in zones where compression occurs. Not all fields of application have nevertheless been explored yet. As such structures mostly act in section-action, the transfer of shear forces is a common task, for which dowels are a common solution. However, while using various kinds of dowels has been the topic of many research projects, these primarily focussed on straight beam-like applications, where shear forces primarily act in one direction. The three dimensional application as proposed here will result in a spatial structural action, not covered by simple linear models. So even these well-known materials still have their indefinite areas.

Table 8. Variables in the generation and classification of schemes and systems based on hypothetical solution 3, rib-shell collaboration.

Variable	Possible values
Hull-rib-collaboration	Monocoque / semi-monocoque (rib-shell collaboration) / enclosure not or only secondarily structurally active
Edges and/or out of plane ribs	Yes / no
Shell constitution	Monolithic (1-zone) / tensile-compression (2-zone) / sandwich (3-zone)

[8] To indicate the vast-ness of this field of structures: a doctoral research dedicated entirely to structural applications of sandwich configurations realised in fibre-reinforced polymers has recently started at Delft University of Technology. This particular combination of a material, a structural class and a geometry, is only one combination in this research.

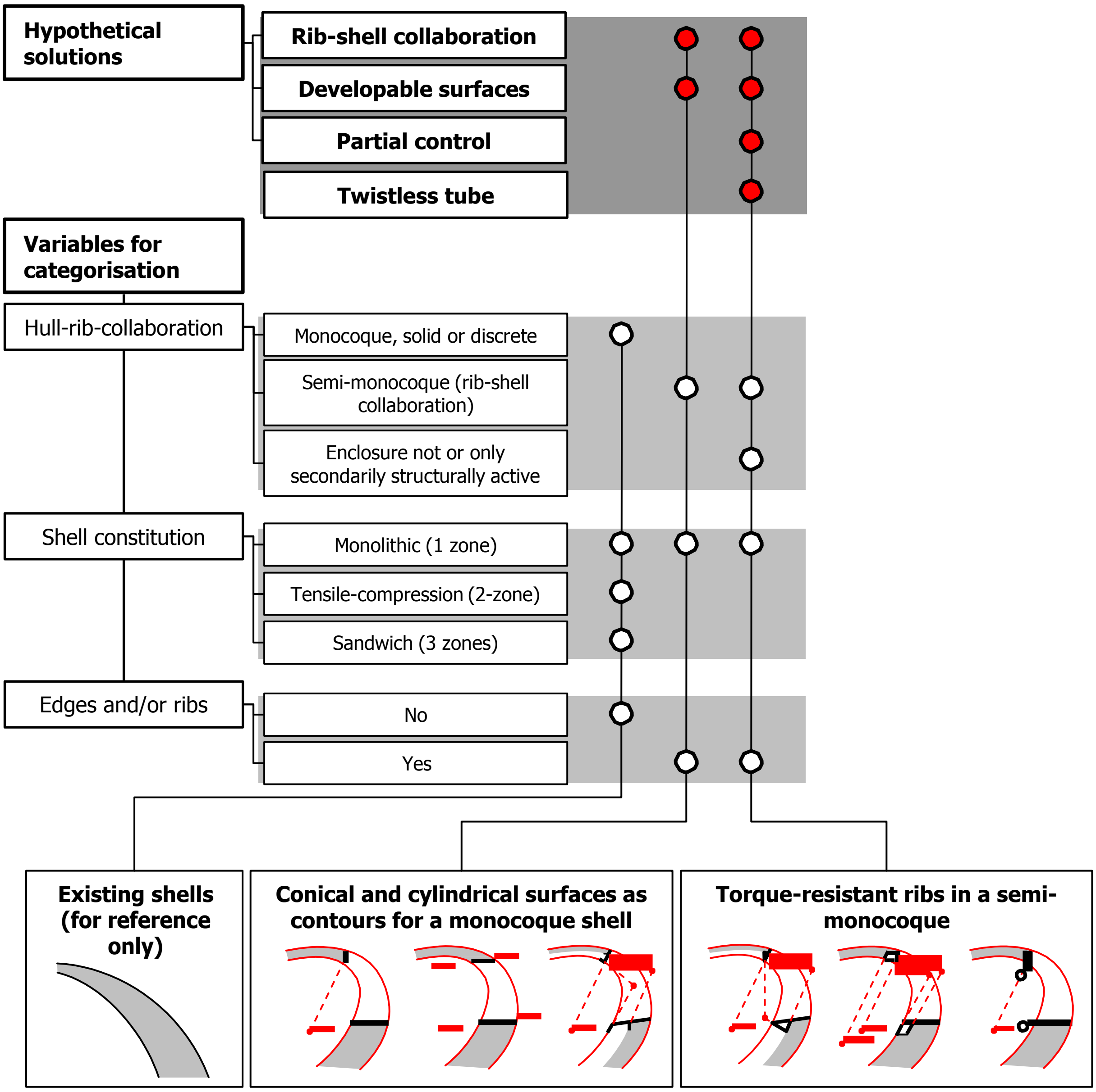
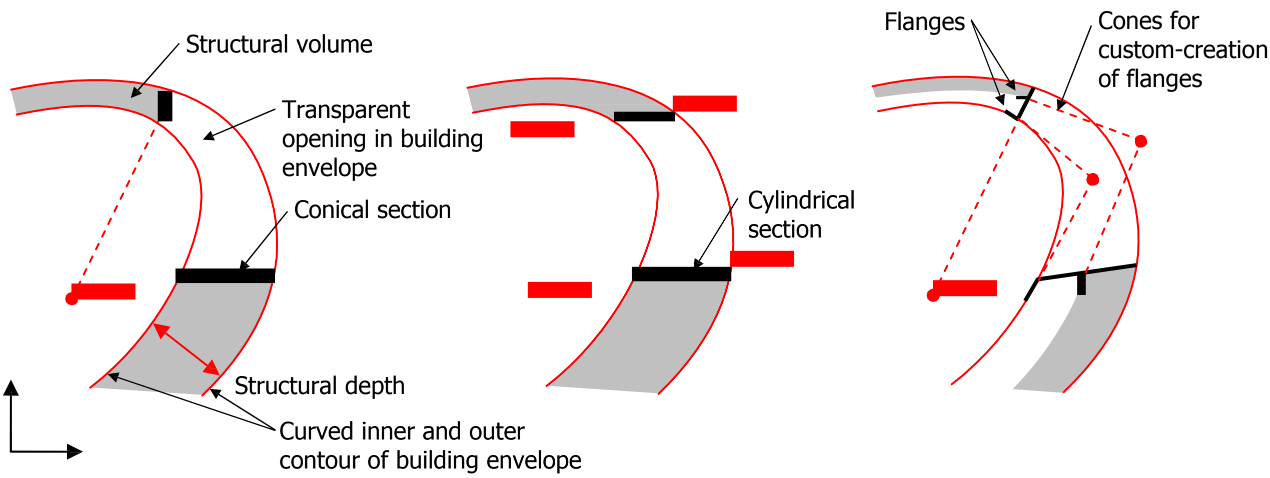


Figure 78. Schemes based on hypothetical solution 2: developable surfaces.

Conical and cylindrical surfaces as contours for a monocoque shell

Rather than solely acting as structural elements, developable surfaces in this scheme are acting as delimitation between a volumetric enclosure and an opening, see Figure 79A and B. Such single sheet-like elements need additional means to act structurally, as was already remarked several times in earlier schemes. The additional structural capacity of this scheme is provided by making the curved building envelope structurally collaborative with the boundary elements. The advantage of introducing these boundary elements is that the geometry is directly controlled at the edges, which facilitates geometrical accuracy where it is needed in the first place, namely at interfaces between components. Furthermore, in a structural sense the edges now act as stiffeners to the enclosure, to compensate for the loss of stiffness due to the openings. Flanges as displayed in Figure 79C, which should be conically curved to accommodate the curved contour, may further stiffen the edges.

The scheme's degrees of freedom are listed in Figure 80A, but despite potentials on formal freedom and the structural considerations being included in the scheme's definition, an implementation of the scheme onto a structural system still requires many specifications on detailing and material choice. How to bring in place the volumetric structural enclosure is a key one among them. The design shown in Figure 80B proposes to use the flanges as in Figure 79C, to put a custom-milled volume on top acting as lost formwork for a composite-material. Instead of a full mould, also a mesh of sufficient density to withstand freshly poured concrete is imaginable. The mesh could be shaped using the hypothetical solution of local-control, by being supported at regular intervals through a rig. The latter solution is explored in the further development onto a system in the next chapter.



A. Conically curved boundary.

B. Cylindrically curved boundary.

C. Conically curved boundary extended beyond structural shell-thickness, with flanges.

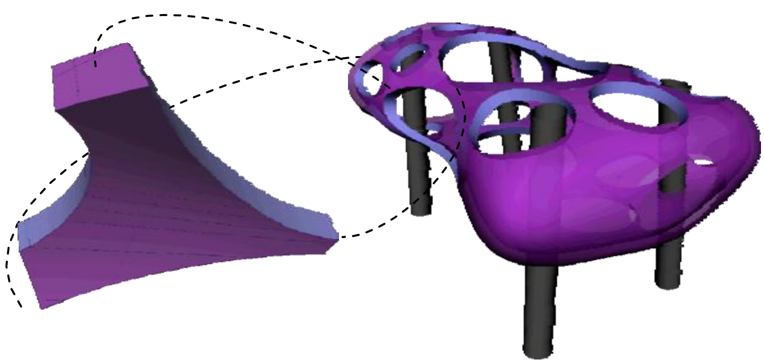
Figure 79. Scheme of developable surfaces as boundary edges for subdividing a monocoque shell, displayed in cross section.

Formal
z Developable surfaces
z Closed loop
O Individual positioning and orientation

Structural
z Semi-monocoque
O Structural depth of shell
O Structural depth of rib

Material
z Planar base materials for ribs
O Additive technique for shell

A. Scheme's variables:
z = fixed, O = degree of freedom.



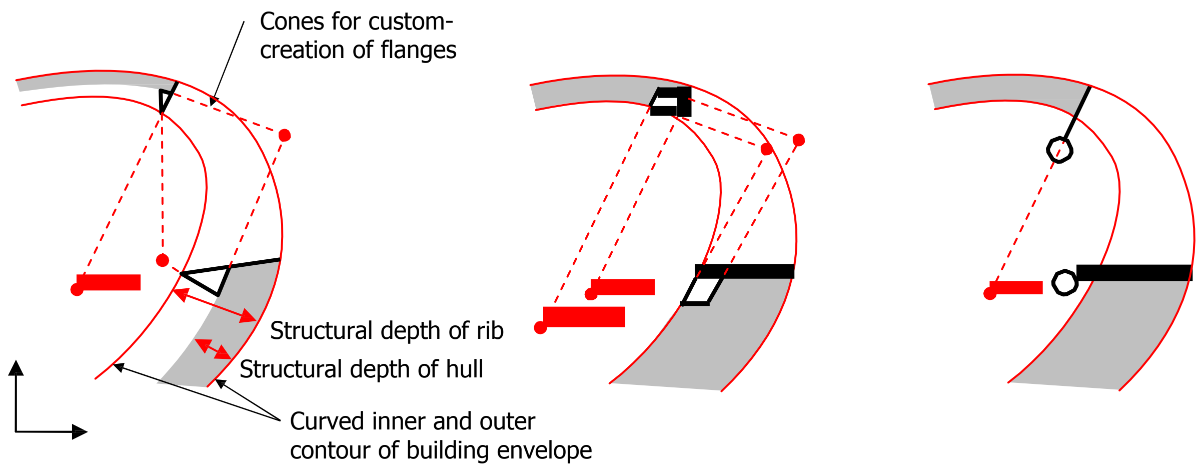
B. Typical application in a case study structural design.

Figure 80. The scheme's variables, and an implementation of it in a structural design.

Torque-resistant ribs in a semi-monocoque

Whereas the previous scheme required the building envelope to be structural on the global level, in this case the edges are articulated such that they become more flexible in their use as load bearing structure. The shell may still be structurally activated, but is no longer the only means. Hence, the structure may be positioned more freely. To allow for this, the edge articulations should be resistant to torque, and allow for a variable bending capacity. Figure 81A to C is depicting schemes of three alternative ways to increase the structural capacity, stiffness and stability of the curved boundary. The first comes back to one of the schemes generating conical looping V-ribs, this time with one side extended further than the other. As discussed earlier in the case of ways to generate V-shaped ribs, the designer may control either the cone's curvature (scheme: Conical looping V-rib (from cones)) or the intersection (scheme: Conical looping V-rib (from curve)), but not both since one is resulting from the other.

When implementing the scheme onto a materialised structural system, all considerations made earlier on components of the actual scheme apply: the need to control the edge-geometry, the control on either the variation of curvature or geometry of intersecting parts, and alternative ways to construct the enclosure, for instance by using partial geometrical control.



A. With triangular beams.

B. With quadrangular box sections.

C. With tubes. Without the structural shell, this scheme is also included in section 4.7 Schemes based on hypothetical solution 4: twistless tube.

Figure 81. Alternative setups of the scheme of torque-resistant ribs in a semi-monocoque, all comprising stiffened ribs alongside an opening in a semi-monocoque building envelope. Openings as well as all other sheets are based on cones.

Formal

z Three developable surfaces
z Closed loop
O Individual positioning and orientation

Structural

z Triangular cross section
O Structural depth

Material

z Planar base materials

A. Scheme's variables:
z = fixed, O = degree of freedom.

B. Typical application in a case study structural design.

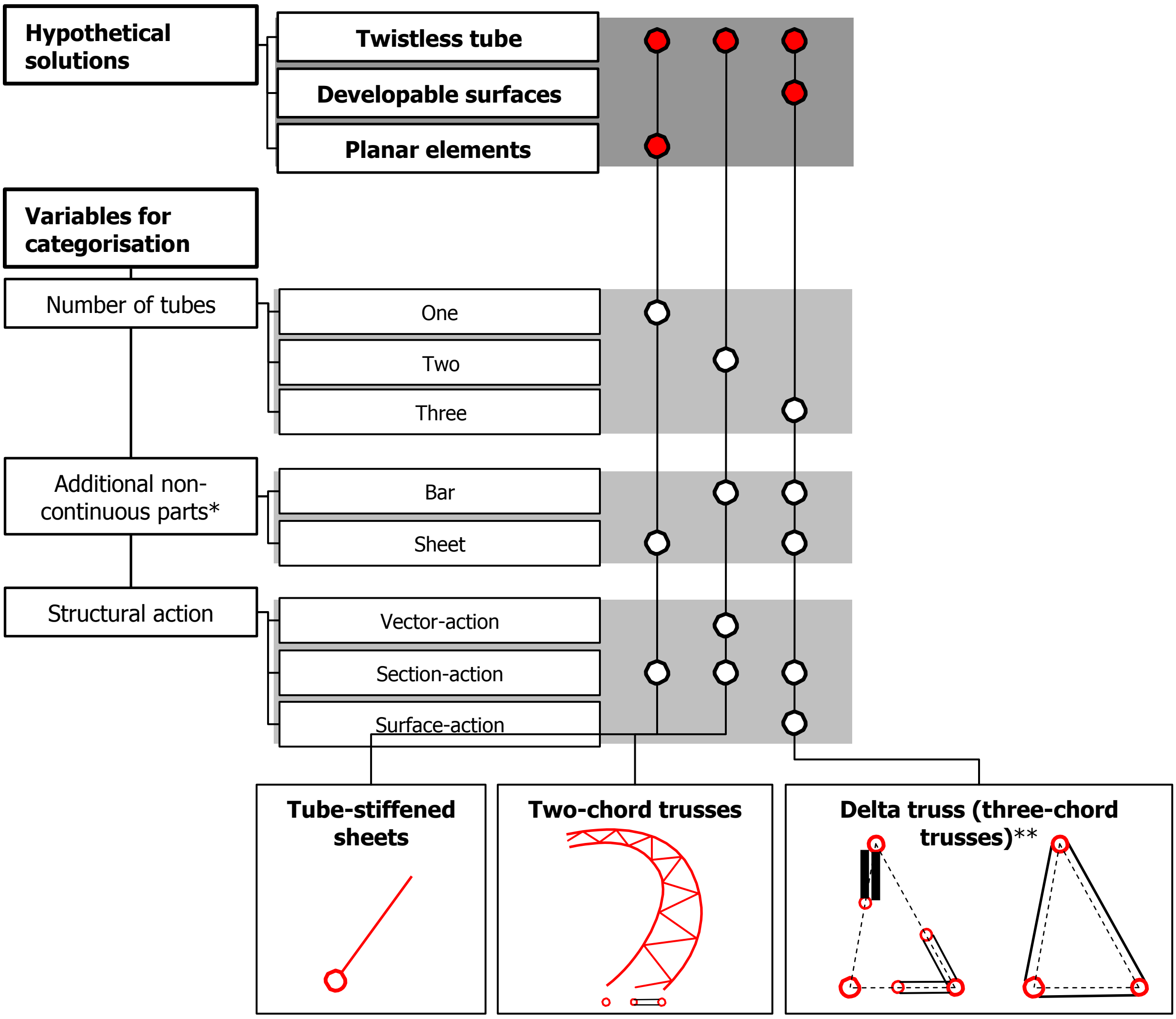
Figure 82. Degrees of freedom of the scheme of torque-resistant ribs in a semi-monocoque, and a structural design in which it is implemented.

4.7 Schemes based on hypothetical solution 4: twistless tube

Tubes allow for easy connections to be made to and between them (in the building context be it structural or cladding) since for connections without eccentricity, all detailing follows the same systematic. Once the intersection line can be determined automatically (based on input parameters of diameters and angle of incidence) and sent to a cutting machine, the connections are no longer critical. This is a routine operation in steel construction, reason for which these schemes can be easily implemented as structural systems in steel.

Table 9. Variables in the generation and classification of schemes and systems based on hypothetical solution 4, twistless tube.

Variable	Possible values
Number of primary tubes	One / two / three
Additional non-continuous parts	Bar / sheet
Structural action	Vector action / section action / surface action



* A scheme without any additional parts would have had the layout of the continuous members as the only parameter, and was considered of insufficient potential for systematisation anticipating formal variety and varying structural capacity.
** The scheme of Delta ribs is displayed here to demonstrate how the schemes derived from different hypothetical solutions come back to similar solutions. The Delta rib-scheme has been included in section 4.5 Schemes based on hypothetical solution 2: developable surfaces.

Figure 83. Schemes based on hypothetical solution 4: twistless tube.

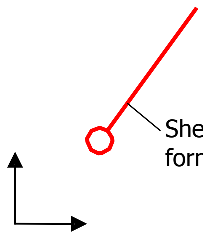
In the proposed schemes, sheets are connected to a tube, or applied as trusses with tubular chords. Apart from the already mentioned advantageous connecting, trusses feature the important potentials for structures in free form buildings that their material efficiency is generally advantageous, and secondly because all geometrical variation is condensed in the chords.

Only three variables, listed in Table 9, have been used to categorise schemes. Opening up the range of alternative structures using trusses has primarily been achieved in the implementations that are included with each scheme, of which an overview is given in Figure 83.

Tube-stiffened sheets

Connecting tubes to a sheet stiffens the sheet in sideward direction. Simultaneously the sheet serves as web and thus enlarges the structural depth and structural capacity of the composed member. These advantages are maintained in curved applications, where the tubes follow the sheet's contour. This scheme is depicted in Figure 84A. The scheme's variables are listed in Figure 84B.

Precondition for a systematic application is to handle the geometry consistently, e.g. by assuring that the (virtually) extended sheet always intersects the tube's central axis. By using offsets tangency connections between a surface and a tube may be created (see for instance the Delta truss-scheme further on in this section), but impose a constraint on the offset direction. An implementation of the scheme is depicted in Figure 85, and demonstrates how the scheme of Cylindrical looping ribs (from curve) can be made more structurally viable.



A. Scheme seen in cross section.

Formal
z Tubular edge to sheet
O Free form chords

Structural
z Structural action: section action
O Structural depth
O Diameter, sheet- and wall-thickness of individual members

Material
z Linear base materials formed to (space-) curves

B. Scheme's variables:
z = fixed, O = degree of freedom.

Figure 84. Scheme of tube-stiffened sheets and its degrees of freedom.

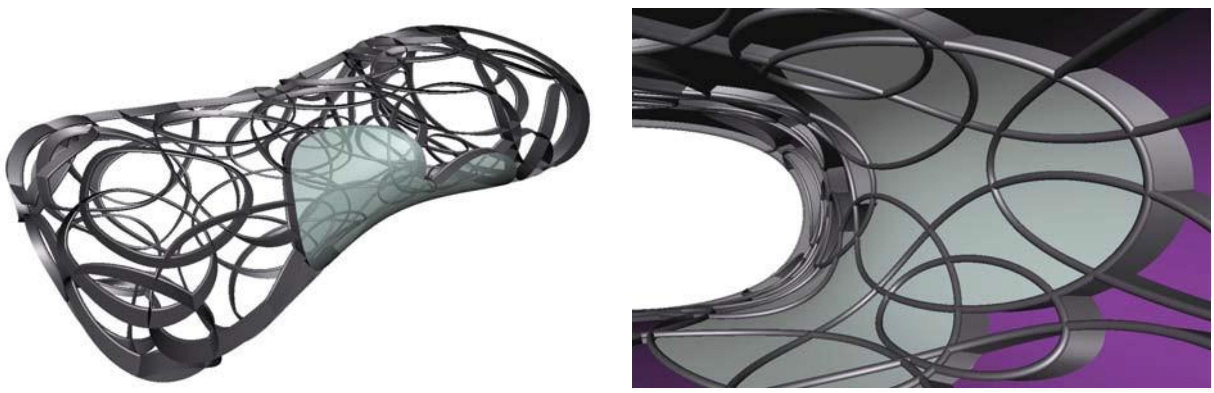


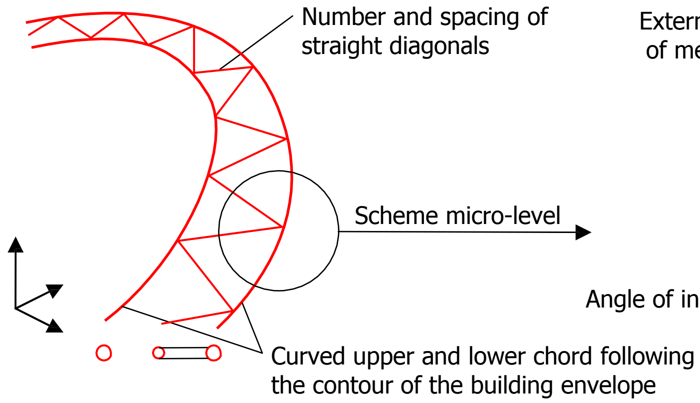
Figure 85. Exemplary implementation of the scheme of tube-stiffened sheets in a structural design. In the zones with a transparent cladding the tubular structure continuous, whereas the sheets are interrupted.

Two-chord trusses

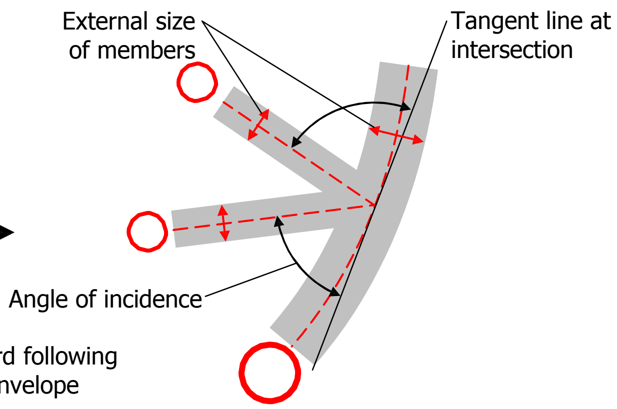
Well known in orthogonal applications, trusses consisting of one upper and one lower chord can be customised by curving the upper and lower chord, and connect them via straight members diagonally running between the chords (see Figure 86A displaying the scheme on a meso-level). The free form is thus fully concentrated in the chords. The scheme proposed for this presupposes members of circular cross section, as this allows for a connection to be generalised easily. Namely, when the members' cross sections are small relative to the members' curvature, the angle of incidence can be schematized to two straight members with intersecting centre lines. The angle of incidence and the diameters of the attached tubes are the only geometrical variables in the connections, as is displayed closely in Figure 86B.

Despite the formal freedom resulting from the curved chords, this well has structural implications. Namely by introducing a freely curving chord, it deviates from the force-line running between nodes. This introduces bending moments compromising the advantageous vector action, requiring anticipation in the dimensioning of the chord member. These and the scheme's further degrees of freedom are listed in Figure 87A.

An implementation of the scheme is shown in Figure 87B. It shows a non-planar truss in a spiralling arrangement, demonstrating the formal freedom of this scheme. Nevertheless, the truss' structural capacity does not anticipate the twisting geometry, which will result in torque. The next scheme introduces torque resistance, needed to solve this.



A. Meso-level.



B. Micro level: connection of multiple members.

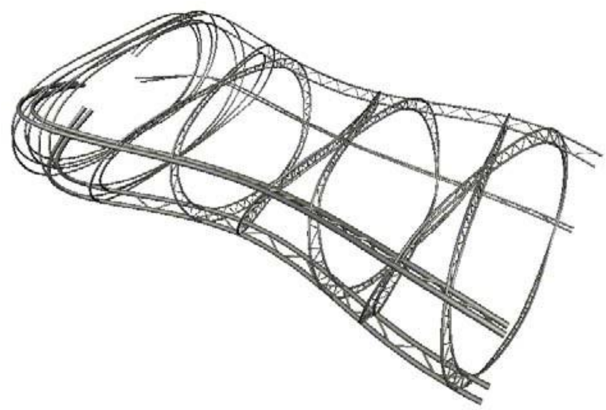
Figure 86. Scheme of two-chord trusses seen in cross section to the building envelope, on meso- and micro-level of scale.

Formal
z Two tubular chords
O Free form chords

Structural
z In-plane global structural action: vector action
z Local structural action: section action
O Diameter and wall-thickness of individual members

Material
z Linear base materials formed to (space-)curves

A. Scheme's variables:
z = fixed, O = degree of freedom.



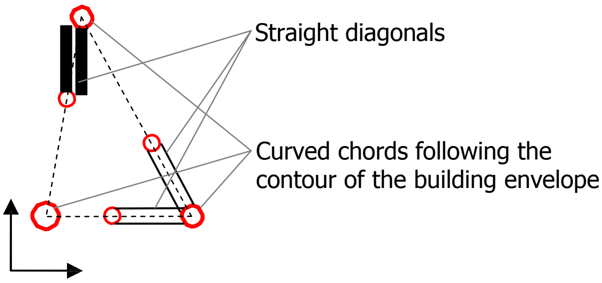
B. Typical application in a case study structural design.

Figure 87. Degrees of freedom of the scheme of two-chord trusses, and a structural design in which it is implemented.

Delta truss (three-chord trusses)

Introducing a third chord to the scheme of two-chord trusses presented before increases the versatile structural ability of the truss: Its stiffness in all directions (including torque) can now be controlled through the system rather only than through the dimensions of the elements in it, furthermore it has also become resistant to torque. The newly proposed scheme is depicted in Figure 88A, its degrees of freedom in Figure 88B. Like the scheme of Two-chord trusses, also the Delta trusses could be applied in a spiralling as well as in a looping arrangement. Two implementations of looping arrangements are shown in Figure 89.

When the chords are not connected through straight bars but through curved surfaces as demonstrated in Figure 90, we again achieve the earlier described scheme of Delta Ribs (in section 4.5.2 Schemes able to anticipate to the local geometry of the building envelope). This demonstrates the close relationship between the proposed alternatives, although the considerations of each scheme are different. Also the seemingly – from a formal perspective – minor difference between defining curved sides through or around tubes, has a vast impact on the systematics generating these geometries, and more important: in the degrees of freedom available to the designer.



A. Scheme seen in cross section. On the level of the connections, this scheme is equal to the previous one.

Formal
z Three tubular chords
O Free form chords

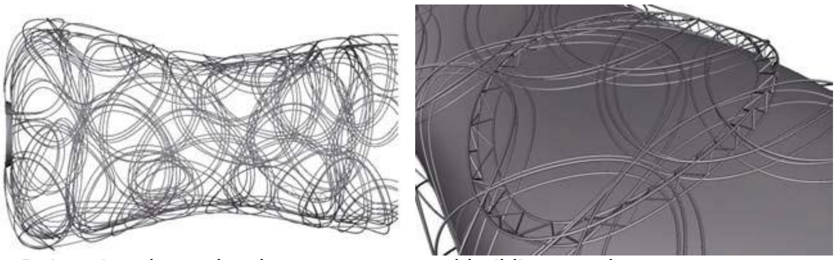
Structural
z Global structural action: vector action
z Local structural action: section action
O Number and distribution of diagonals
O Diameter and wall-thickness of individual members

Material
z Linear base materials formed to (space-)curves

B. Scheme's variables:
z = fixed, O = degree of freedom.

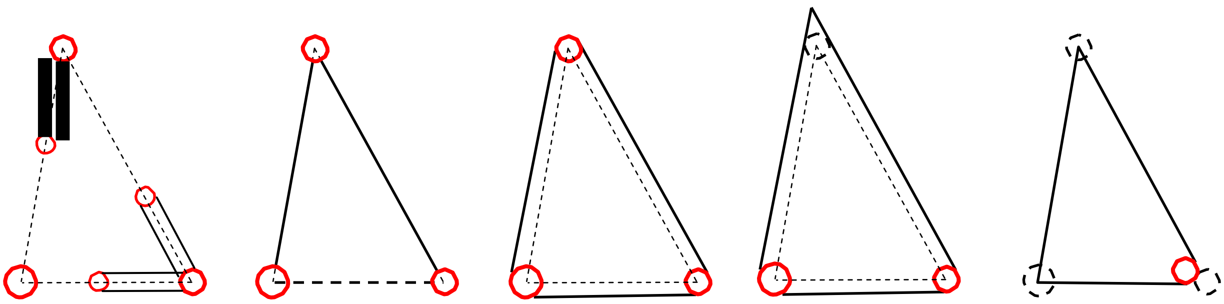
Figure 88. Scheme and degrees of freedom of the scheme of three-chord trusses.

A. Looping three-chord truss to contour a building shape.



B. Looping three-chord trusses on curved building envelope.

Figure 89. Applications of the scheme in a model and a structural design.



A. Scheme of three-chord trusses.

B. As previous, with (curved) sheets instead of linear members between the chords.

C. As previous, with the sheets at the exterior boundary of the chords.

D. As previous, with one chord removed, and the sheets extended to a new (sharp) edge.

E. As previous, with only one tubular chord and the sheets defined through the chords centre lines.

Figure 90. Gradual transition from the scheme of three-chord trusses to the scheme of Delta Ribs, all seen in cross section.

4.8 Schemes based on hypothetical solution 5: partial geometrical control

No schemes were developed based on the driver of using partial geometrical control only. Instead, it was applied in conjunction with other hypothetical solutions, namely 2, 3 and 4 as only in these hypothetical solutions curved elements are proposed. Application of partial geometrical control appears to be notably fruitful in case of interconnected non-constantly curved surfaces, as these cannot be formed through automated fabrication. Furthermore, the interconnection provides stiffness, and thus in itself a framework to control the geometry.

4.9 Evaluation

Using the hypothetical solutions as drivers, in total 23 alternative structural schemes have been generated. All of these schemes are at least addressing one potential quality for appropriate application in free form building design. Based on the expected performance on the three evaluation criteria defined in section 4.3 Evaluation criteria for schemes, systems and structural designs, three of these schemes will be developed further onto structural system in the next chapter. In addition to, also the followed method will be reviewed.

Method-wise, it was concluded that generating schemes through a driver, despite being canalised through the solution space by denominating variables with a limited number of alternative values (in a qualitative sense), was nevertheless a creative process. Only the first hypothetical solution did lend itself to the use of combinatorics, as the set characteristics had a high level of general validity. Furthermore, as most conventional structural layouts are derived from this scheme, it was judged helpful to include these as a reference to known schemes, despite the fact that they were not new. Using combinatorics to explore the other hypothetical solutions was problematic due to the involvement of more than one driver, the numerous invalid combinations resulting from it, and the characteristics that only applied to specific (rather than general) circumstances.

Although satisfying from the designer’s point of view, a major disadvantage of being dependent on creativity is that it does not assure that all possible solutions have truly been found, whereas a fully systematic search would (with the variables given by the operator beforehand). In addition to this, it has been experienced that despite starting from different hypothetical solutions, nevertheless similar alternative solutions were found.[9] Is this the designer’s bias directing him to an existing solution, or a logical outcome of the given variables? On the other hand, a designerly process allows for custom interventions whenever judged appropriate, which a systematic search would not. Given the fact that the design action involves many inputs of diverse nature, and that a fully systematic (mathematically rigid) approach has limitations as well, it appears that the use of creative approach is justified.

Concerning the search for a product and the choice for schemes for further development, the following remarks should be made. Schemes derived from the same hypothetical solution form classes with similar characteristics, like minor variations on a theme. However the most performing ones combine multiple hypothetical solutions, and thus propose not a solution to only form, or structure, or material, but to all of these at the same time. In particular dominant characteristics should be anticipated by an associated degree of freedom of a scheme. In several of the schemes it was evaluated that one of the aspects was well-addressed, e.g. formal freedom through sufficient degrees of freedom on the formal-aspects, but also that the generated forms could not be given structural meaning, and consequently not increase the performance on structural efficiency. To improve the performance on this aspect, additional

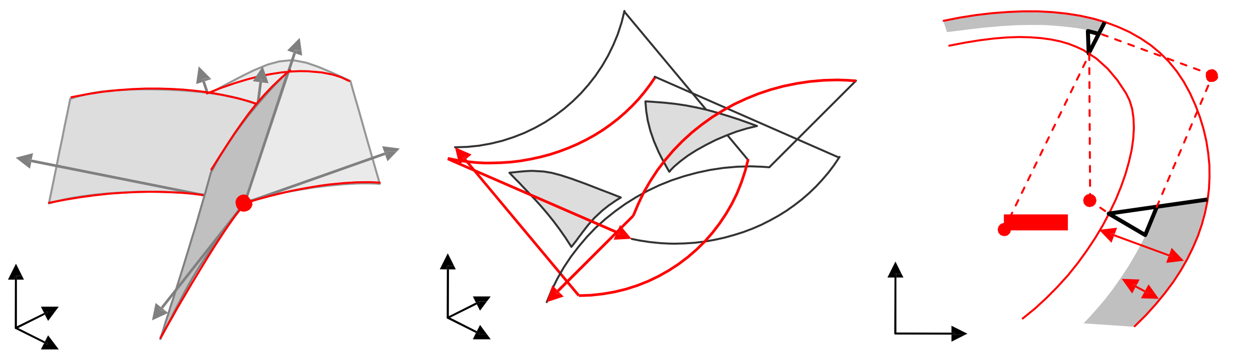
[9] For instance the scheme of Delta ribs that was initially generated from three intersecting developable surfaces, while it was also reached by the scheme of the Delta truss (three-chord trusses), with the particular case of sheet-like webs rather than linear chord members.

elements had to be introduced, this way compromising the systematic setup. This was specifically the case in several schemes involving planar or curved slices in inclined orientations. The implementation of these would be susceptible to out-of-plane structural action. If this is not a degree of freedom of the scheme, the scheme's implementation needs custom interventions, and consequently will perform poorly on the degree of systematisation.

Another general remark concerns the application of schemes on the global level of scale. Although planar sections (hypothetical solution 1) are favourable for their geometrical and structural clarity, they have only their contour which can continuously anticipate local geometry and curvature. The member's orientation can only be optimised at one spot, thus generally resulting in a less-favourable orientation elsewhere. Introducing curvature, notably one that is not constant, brings along degrees of freedom to anticipate orientation. Locally anticipated schemes however overcome the problem of constrained orientation in a more fundamental way; only the curvature-variation within the length of a single element is to be incorporated.

Other than through offering geometrical degrees of freedom, performance on the criterion of formal freedom is achieved when the scheme allows for non- (or little-) constrained input geometry. Notably free form curves, including space curves, feature potential here. The task thus evolves into the question which schemes allow to construct rational load bearing elements starting from free form space curves.

Based on these findings, the schemes to be developed further are the schemes of Planar members with end connections (generated from hypothetical solution 1), Delta ribs (from hypothetical solution 2) and Torque-resistant ribs in a semi-monocoque (from hypothetical solution 3), listed in Figure 91. All of these include sheet-like elements; the first in planar configurations, the latter two in curved applications. In the curved applications, strength and stiffness are provided by a connection with the shell, and a self-stable assembly through multiple sheets, both included in the scheme-definition and thus including measures to assure structural efficiency.



A. Scheme of Planar members with end connections.

B. Scheme of Delta ribs.

C. Scheme of Torque-resistant ribs in a semi-monocoque.

Figure 91. The three structural schemes to be developed further onto structural systems in the next chapter.

5 Case study implementations of schemes onto systems and structural designs

In this chapter three selected structural systems, based on the hypothetical solutions defined in the previous chapter, are developed further onto the more concrete level of that of a structural design. As in the phase described in the previous chapter, the systems are applied to a structural design for the DO Bubble, a free form building design. As part of this, the system design was further developed including setups for detailing and structural analysis.

The first structural system is a network of planar elements, made in timber. Since the system has no nodes but only spanning members, the conventional complexity of nodes has been suppressed. The second system is a network of steel ribs with triangular cross section. The sides of these ribs are complex, but nevertheless single curved, and made out of rolled steel sheets. The latter structural design has been realised as a prototype for a structure, reported in the next chapter. The third system features a setup in which the building envelope, in concrete, and the steel ribs underneath structurally collaborate.

Once the systems are realised, each system is individually evaluated with respect to the sub-hypothesis it is based on. The conclusions and recommendations on the overall level, including the answering of the initial research question, are included in the last chapter, chapter 7.

5.1 Presentation of the case study object: the building design of the DO Bubble

The three selected structural schemes to be developed further onto structural systems are applied to the DO Bubble. The DO Bubble is a design made by Aukett Architects[10], resembling a 50 metre long mannequin lying horizontally, supported on four columns: one under each 'shoulder' and two under the 'hips' (Figure 92). The volume is to be filled with three floors of leisure or sport functions, and for this to be equipped with daylight openings. When the architects came to the Blob Technology Group at TU Delft in early 2002, they had no idea on how to technically realise this project.

The proposed building design featured all characteristics of a blob design:

- on a global level of scale its shape double curved;
- it is irregularly shaped, possessing a large variation of single and double curvatures (both synclastic and anticlastic), see Figure 93B;
- the shape lacks reference to a structurally optimised shape.

These made this building design an ideal case to develop alternative structural systems for, and map them to. Throughout the course of this research, numerous variants of the building shape have emerged, stressing different formal features through exaggeration (see for instance Figure 93C). Common to all building shapes was a thick layered building envelope of variable thickness to place a structure in. No supportive structures outside this structural hull are considered. Neither the building bridging between the supports, nor the floors and its load bearing structure were considered as generic characteristics of free form building designs, and have subsequently not been studied.

In this research, the DO Bubble acts as a case needed to provide a context to the implementation of structural systems, as it is unnatural to judge their merits in isolation. This context consists of the intended free form building shape and a technologically challenging structure inside. While this building design cannot be considered as a natural context as is typical to conventional case study research (Swanborn 2000), it is more natural than a conditioned laboratory experiment.

[10] Since 2006: Group A - Aukett, Rotterdam (the Netherlands).

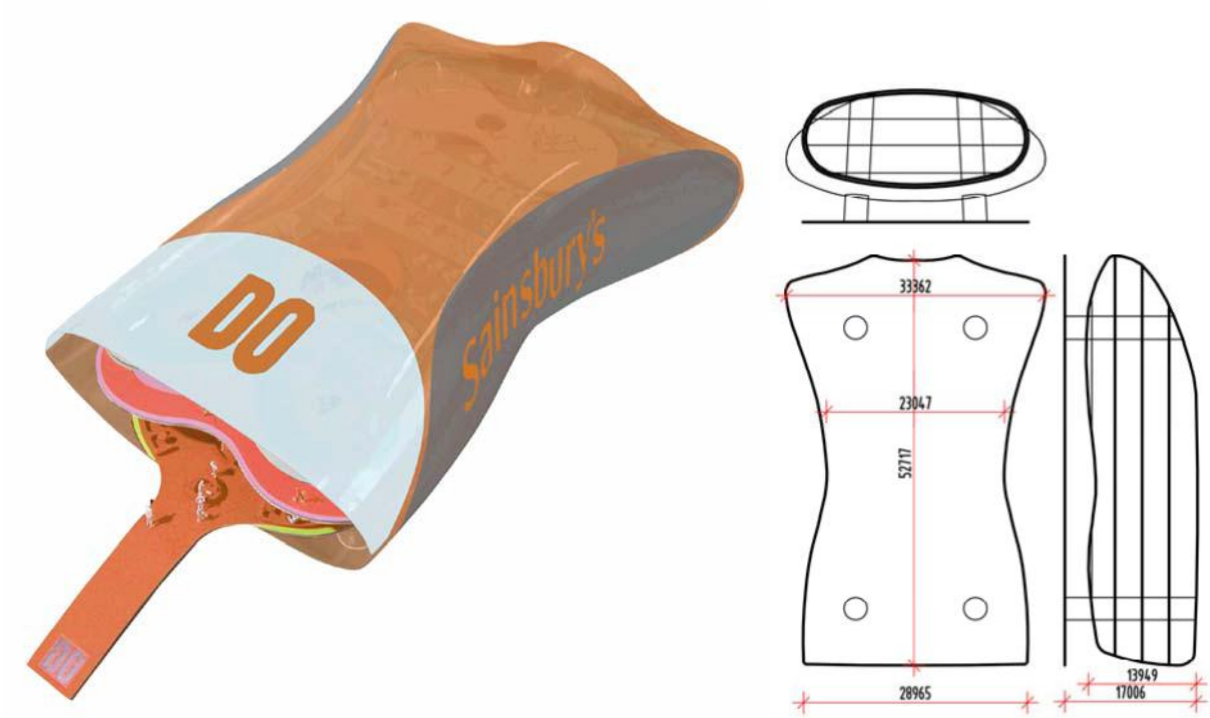
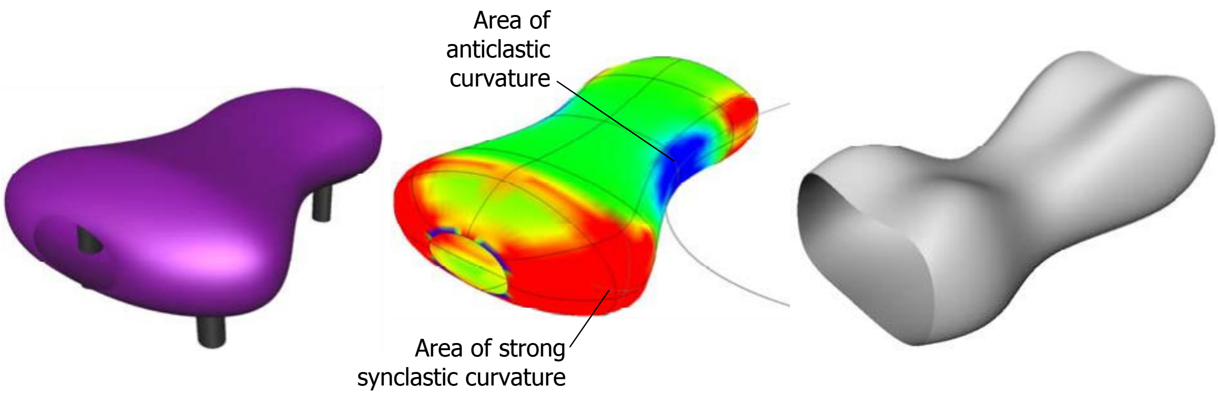


Figure 92. The DO Bubble, a 50 metre long, three storeys high Blob-design. Images courtesy Aukett Architects.



A. Digital model used as template for test-implementations.

B. Gaussian curvature analysis of the previous model.

C. Alternative model with stronger curvatures.

Figure 93. Digital model of the DO Bubble.

5.2 Case study 1: implementation of the Planar members with end connections-scheme onto a structural system

The scheme of Planar members with end connections has been implemented in panels of cross-laminated timber. The additionally required inputs and all associated specifications and inputs are depicted in Figure 94.

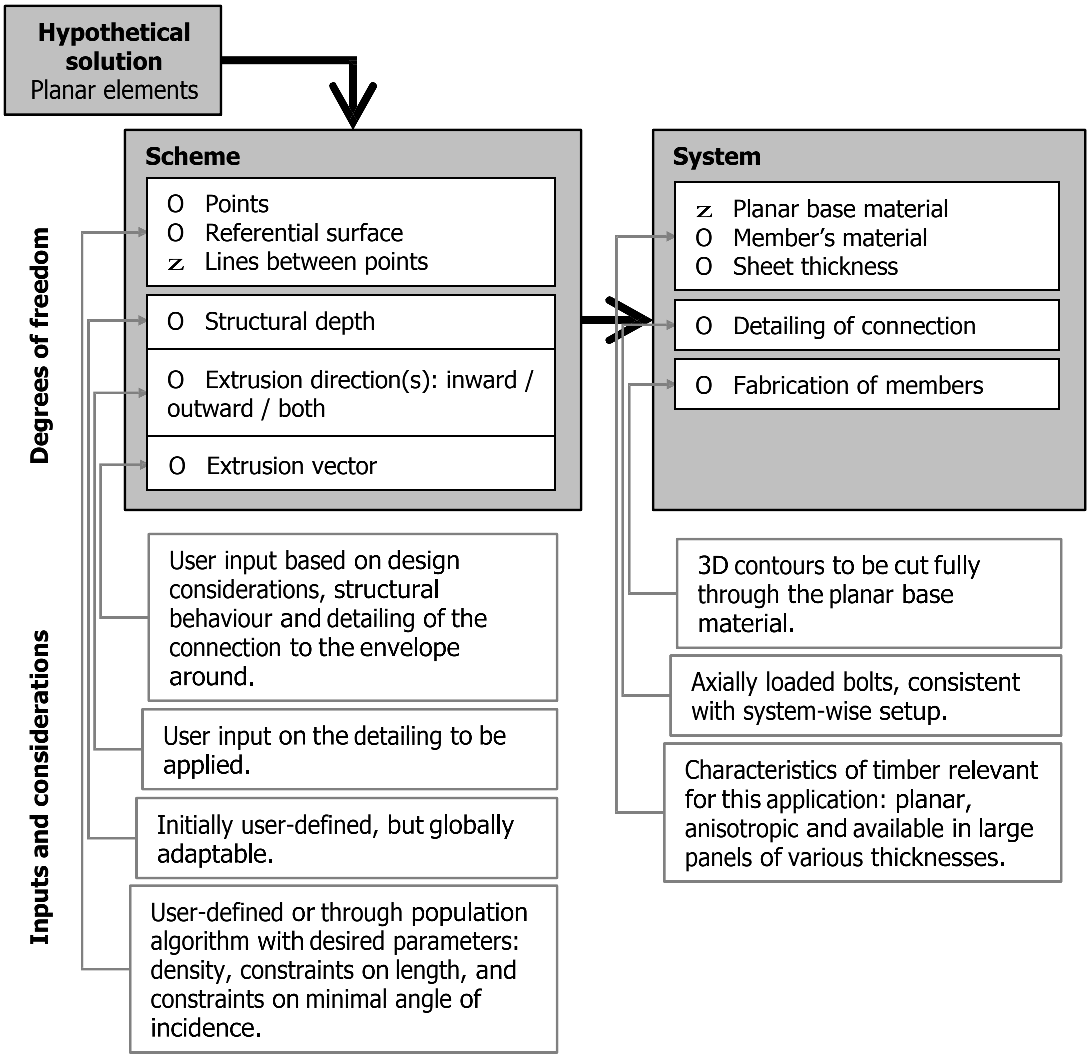


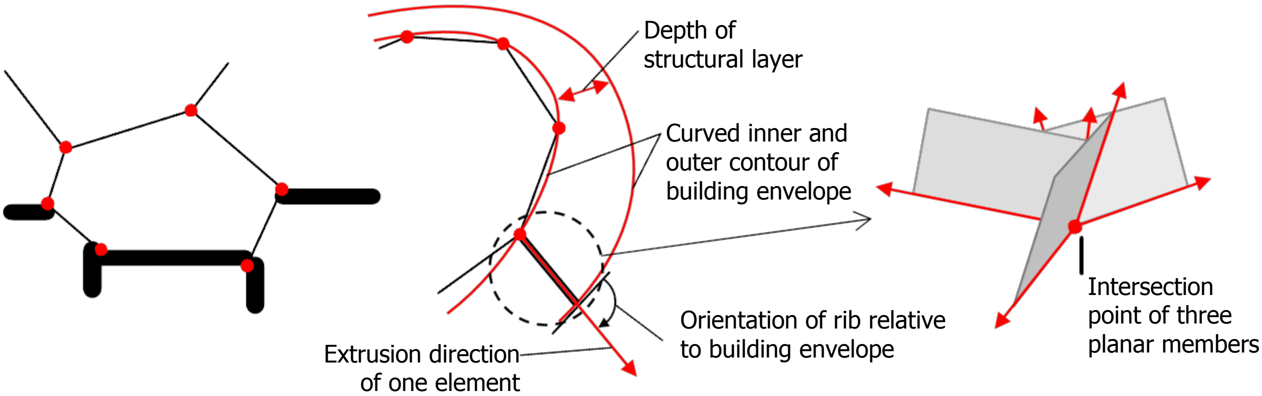
Figure 94. Inputs and definitions of the scheme of Planar members with end connections implemented as structural system.

5.2.1 Geometrical design rationale

In the scheme of Planar members with end connections three planar elements intersect at their extremities in one point under arbitrary angles of incidence. The scheme is derived from hypothetical solution 1 that promotes the usage of planar elements, and was geometrically defined in Figure 61 in section 4.4.3. To obtain this layout, on one higher level of scale than that of the scheme, straight lines connect points that are arbitrarily positioned in 3D-space (Figure 95A). The number and position of the points are the scheme's input, and may thus be decided upon by external considerations such as design intentions and structural necessity for the pattern's density. Specialised algorithms may be used to map a pattern on the surface, or populate it with points. In the implementation of the scheme presented here, the input-points are chosen to lie on the interior envelope of the structural layer around the free form building shape (Figure 95B). This interior boundary is positioned at a non-constant distance from the exterior envelope, using the in-between space for the rib's structural depth. As a consequence of this choice, the nodes' tops will be pointing inward once the planar members have been generated.

Once the linear grid is defined, each of the grid lines is extruded to generate planes. This is the primary characteristic of the scheme, and was described before (Figure 95C). In this case study implementation all extrusions are chosen to be directed towards the exterior envelope, although extruding some gridlines outward and others inward would still respect the scheme's systematic. The extrusion direction (within the constraint of the in- or outward extrusion decided upon before) of each individual line is a degree of freedom, which could be exploited to anticipate the orientation to the exterior envelope. This orientation could serve structural reasons or facilitate the connection to façade panels to be put on top. In the latter case however, since per planar element only one extrusion direction can be followed, the element's orientation relative to the building envelope will not be constant.

Finally the planar elements are trimmed at the intersection with the interior and exterior surface, as well as at the intersections between the elements. In case that any of the planes spanned between a pattern line and an extrusion direction does not intersect a contour or another element, the plane can be extended unambiguously until it does intersect.



A. Macro-level: points on a surface connected by lines.

B. Meso-level: specifications (building envelope in cross section).

C. The original scheme, which defines the element-level.

Figure 95. Degrees of freedom of the 'planar ribs'-system on three levels of scale.

5.2.2 Materialisation and fabrication

Material

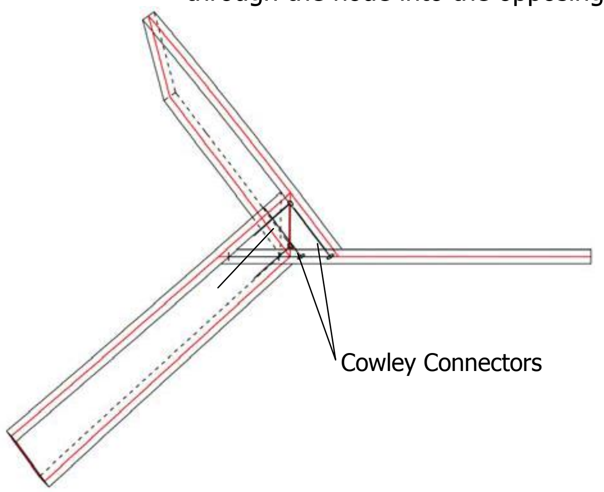
The scheme's geometrical definition employs a surface representation, and consequently does not include any information on the thickness of the applied elements. To maintain the system's consistency, best elements with a constant thickness without any geometrical irregularities (such as flanges) should be used. Furthermore, a solid material seems most appropriate such that connections can be made at various positions. From the conventional structural materials, it was decided to explore a materialisation of the scheme in wood, more particular the usage of relatively thick sheets (anticipating out-of-plane structural action that will be described in the next section) of anisotropic nature. Due to the isotropic nature, orientation of the element during manufacturing is irrelevant, this way allowing the parts to be nested optimally on the fabricator's standard panel sizes. One of the materials meeting these requirements is LenoTec. It is a crosswise laminate of soft spruce produced by Finnforest, available in sheets of maximum 4,80 by 20m, and a thickness of up to 300mm (Finnforest 2006). LenoTec is similar to Kerto (from the same producer) from which the timber frame of the 2005 Serpentine Pavilion (see content document Custom-shaped timber panels in Lamella structure in section 3.3.5) were constructed, but has a different layering system such that it is more isotropic than LenoTec. Though advantageous for an optimal use of the base material, the sheet's anisotropic material properties are an average between the material strength's parallel and perpendicular to the grain. For highest capacity, best the more isotropic Kerto should be used.

Connections

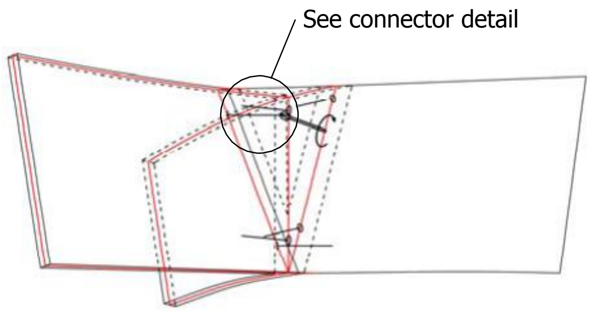
Two alternatives appear for the detailing of the connections: the first involving a direct connection from one element onto another, the second introducing an intermediate node. The two alternative ways of connecting could also be used simultaneously to combine the advantages of both.

When implementing a direct connection, the basic idea is to screw a pair of connectors into the extremity of an element, through the connecting element. As the elements are loaded by shear as well as axial forces, and connect to each other in a non-perpendicular layout, the connecting element may tend to slip away. This is to be prevented by placing a toothed plate connector at each element's extremity, or otherwise directly by the connector. A commercialised connecting system capable doing so is the Cowley Connector (developed by Cowley Timberwork, UK) (Cowley Timberwork 2006). The detail, depicted in Figure 96, consists of a captive bolt in a tube that is epoxy bonded into an element's extremity. In assembly, the bolt is fixed into an internally threaded socket captured in the opposing element. The socket is sunk into the opposing element, but as it is aligned with the element, the embedment will be inclined depending on the angle of incidence of the two members. The connection is entirely prefabricated, and has demonstrated to possess sufficient tolerance in network structures and sufficient rigidity to transfer bending moments when used in pairs. (cowley 2006)

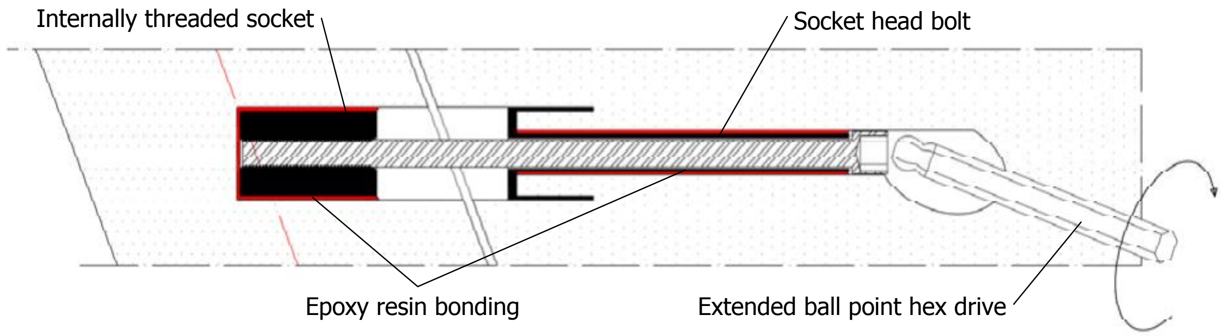
Another way of connecting introduces a volumetric four-cornered node that is placed between the extremities of three members. Screws within toothed plate connectors connect the two faces (Figure 102C). As the top of the node will normally be sharp and dependent on the orientation of each of the connecting elements, it has little space to accommodate a bolt, and it may be required to create the connection through the node into the opposing element.



A. Top view of three interconnected planar elements.



B. Front view of three interconnected planar elements, showing the use of two Cowley Connectors per edge.



C. Detail of one Connector in cross section.

Figure 96. Connection detail using the Cowley Connector.

Fabrication of notches in areas of clashing elements

If the centre planes of the material members are aligned with the system's referential planes, then the materialised elements intersect and require notching in the area where the elements' referential planes intersect in a point. The resulting clashing volumes are marked red in Figure 97.

In automated fabrication, notching may be executed in a CNC-process of inclined cutting, provided that the notches can be described parametrically and their values for each individual element be derived from the system. Apart from the socket sunk into an element which require milling (which is nevertheless always in the same plane, be it in different directions), all cuts go entirely through the element.

The clashing material could either be removed from the facing member (Figure 98A) or from the connecting member (Figure 98B). The most straightforward way to do this is to notch all elements over their full height, although this implies the removal of more material than strictly necessary, leaving gaps as can be seen in the bottom-views in the two mentioned figures.

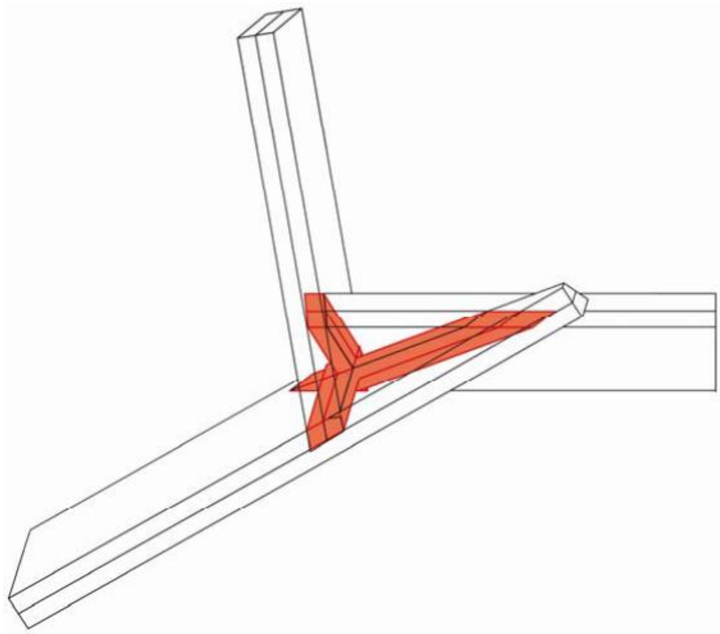
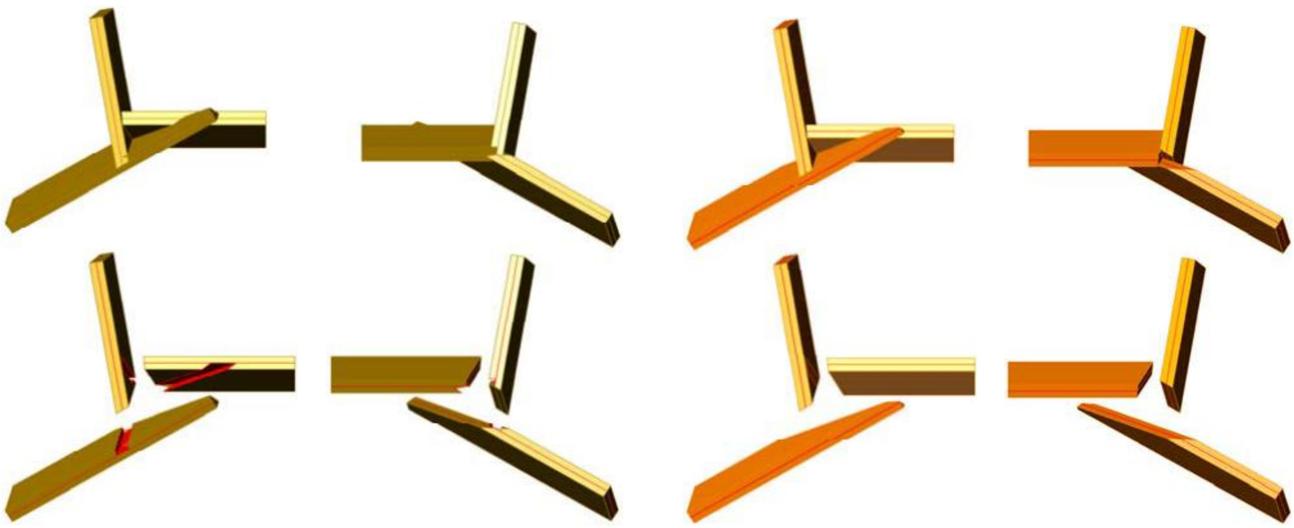


Figure 97. Required notching (marked red) in areas of clashing elements.



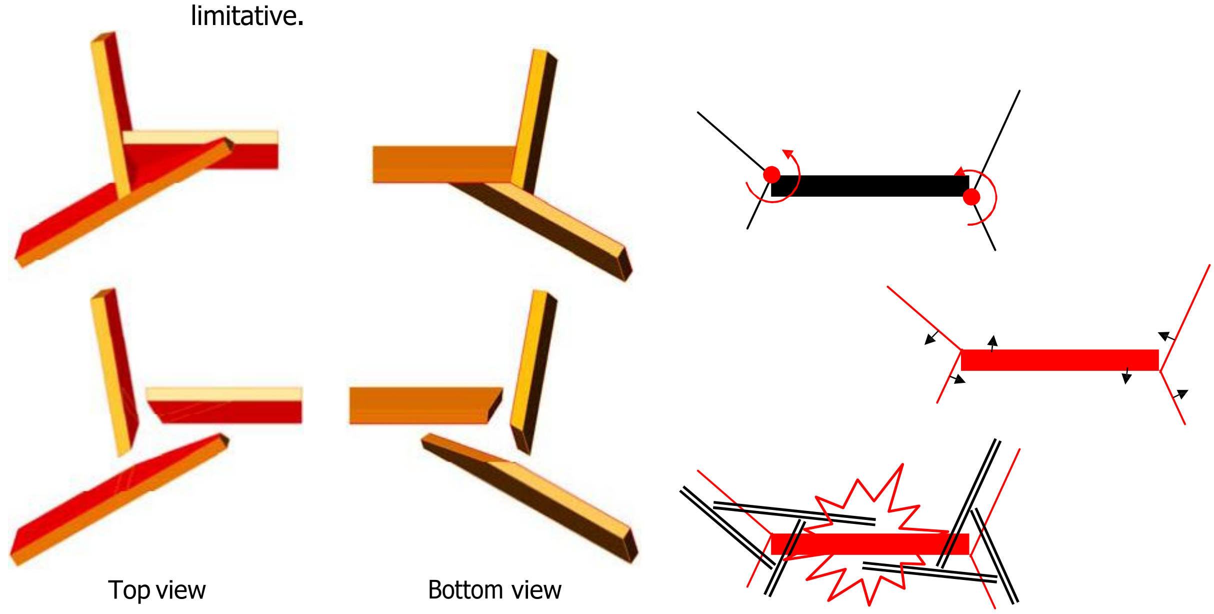
Top view Bottom view Top view Bottom view

A. The facing member is notched until the referential plane. In return, the connecting member is unnotched.

B. The facing member is unnotched. In return, the connecting member is notched until the surface of the facing member.

Figure 98. Two alternatives for notching: the facing member or the connecting member.

An alternative way is to place the members on the referential planes with an eccentricity of half the thickness of a member, Figure 99A. Now, the outsides of a member just touch in the top and notching is only needed to overcome the inclined intersection of two planes. This makes the notched geometry more simple, but has unfavourable implications on the structural action due to the required enlarging of already-existing eccentricities. Furthermore this notching principle compromises the locally anticipative setup since the shifting of the member to one side of the referential plane also affects the member's other extremity, and may lead to a conflict as demonstrated in Figure 99B. This could be overcome by defining the referential grid system such that it already anticipates the placing of the members. The existing polygonal grid could well be used if the side on which the panels are placed are pre-defined, such that is defined beforehand which pairs should intersect. This implies a constraint on the member's orientation. Moreover, to allow looping only loops of a pair number of elements can be used – for which loops of 6 are proposed. To implement the system on curved surfaces, then jumps from 6 to 4 elements in a loop have to be made, rather than from 6 to 5 (see Figure 100). For the usage in free form building designs, the constraints yielding from eccentrically positioned elements may be too limitative.



A. Elements placed eccentrically on the referential plane (marked in red). Notching only to accommodate inclined intersection of two planes, not the clashing at the top. This results in a clear top-detailing seen in the bottom view.

B. Discontinuity in case of conflicting eccentric positioning of members.

Figure 99. Simplified notching in case of eccentric positioning on the referential grid.

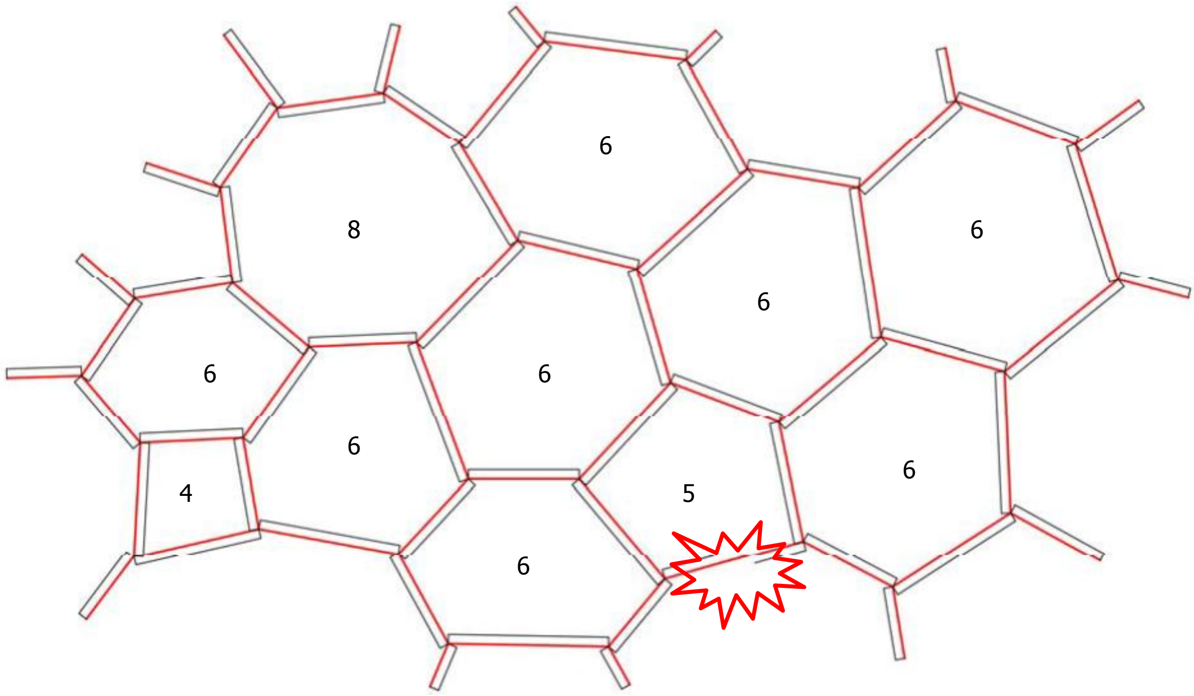
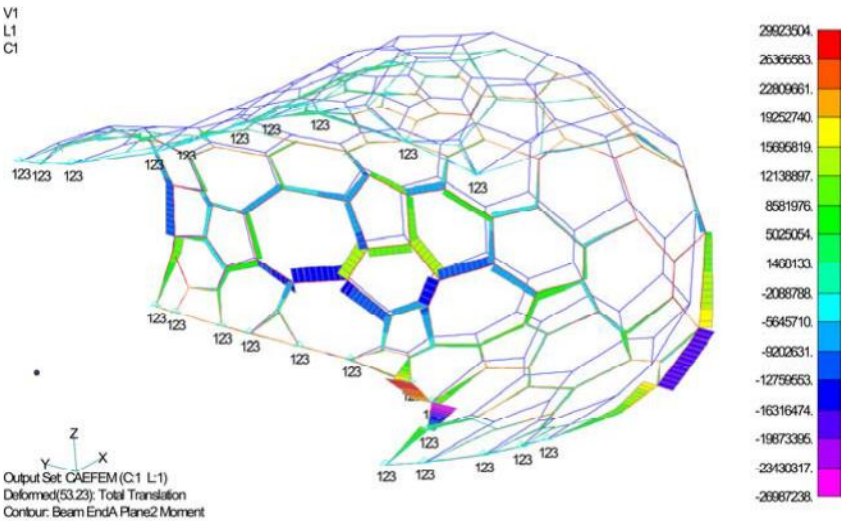


Figure 100. Schematic view of a polygonal pattern with predefined eccentric positioning of members. The number of elements per loop is indicated. Clashing occurs in case of an odd number.

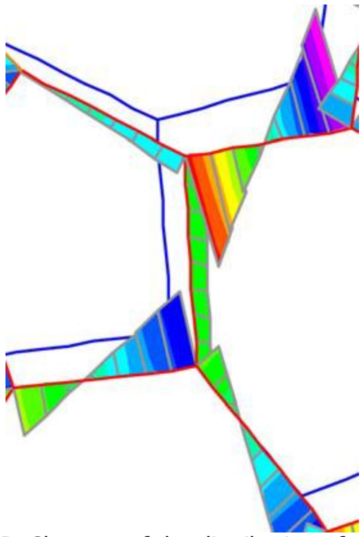
5.2.3 Structural action

The system's primary load bearing mechanism is through section action, on the building's as well as on the element's level of scale. The systems structural capacity can be customised by in- and decreasing the structural depth, through changing the spacing of the interior and exterior envelope on the building-level of scale. On the element-level of scale the member's thickness can be changed, or another composition of the laminate material used – which constrains the field of application. Furthermore also the pattern layout could be re-organised, and its density in- or decreased. Also the detailing of the scheme – bolts in the members' centre planes as described in the previous section – has introduced an additional constraint on the minimally required inclination between two connecting members.

To analyse the types and magnitudes of structural action, and assess the possibilities to anticipate it through the variables included in the system, a structural analysis was carried out on a finite element model of the system implemented to a part of the DO Bubble (see next section). As notably the observation of qualitative aspects was targeted, the planar elements were modelled as linear elements with the stiffness properties of planes of different heights, attributed to the geometry according to its expected structural behaviour and oriented such that the elements' intersections create inverted triangular pyramids. The material properties were that of in-plane loaded LenoTec and anisotropic, while in reality its out-of-plane modulus of elasticity is smaller. The elements were rigidly connected in a single node, which is only a rough model of the actual connection. In reality the upper side of the connection (at the pyramids' base) is less rigid than at the lower side, as at the upper side the distance between connection points is larger and the element loaded and bending sideward provides little resistance. The model was loaded by dead load only, which resulted in elements being loaded in a wide variety of directions.



A. Distribution of bending moments in the elements' planes, depicted on the deformed model.



B. Close-up of the distribution of bending moments out of the elements' planes.

Figure 101. Results of the finite element analysis of the planar members-system implemented on a quarter of the DO Bubble.

As expected, the analysis results displayed in Figure 101A show an outward and downward deformation, with the largest out-of-plane bending moments in the elements that are leaning outwards most. The in-plane distribution of bending moments shows that the rotational fixation of the element's extremities causes a gradient in the moment line between a positive and a negative bending moment (B). As the model is loaded vertically and also deforming in this direction, notably the non-vertical members display this behaviour.

In the analysed model, the out-of-plane bending moments were approximately four times larger than the in-plane bending moments. Whereas the variation of bending moments in-plane of the elements (which is out of plane with respect to the building envelope) can be anticipated by the elements' height, the elements' capacity to transfer the out of plane bending moments can only be anticipated directly through the orientation of the timber panel and its thickness.

5.2.4 Implementation on the DO Bubble

The system of planar members has been implemented on a quarter of a strongly curved variant of the DO Bubble. To do this, manually an irregular pattern of quadrangles, pentagons and hexagons was generated with their nodal points lying on the referential surface (see Figure 102A). The actual building envelope is outside this referential surface, their spacing deciding upon the ribs' structural depth. Next, the rules were extruded outward to create planes. Extruding often had to be combined with extending to assure that the constructed panels would well intersect. Once constructed the planes' extremities and the upper and lower edge were trimmed off with respect to the neighbouring elements and the surfaces on the interior (the referential surface) and exterior respectively. The resulting geometry of the rib's referential planes is depicted in Figure 102 B and C.

Whereas most of this generation was carried out in a straightforward way, the iterative adjustment of the extrusion direction was most problematic. On the one hand the inclinations affected the triangular ground that had to be plane sufficiently large to practically construct a node – which was most easily achieved with strongly inclined elements producing the least risk of conflicting orientations. On the other hand, from a structural point of view inclinations had to be minimised as this would deviate least from the surface normal, and thus induce the smallest eccentricities.

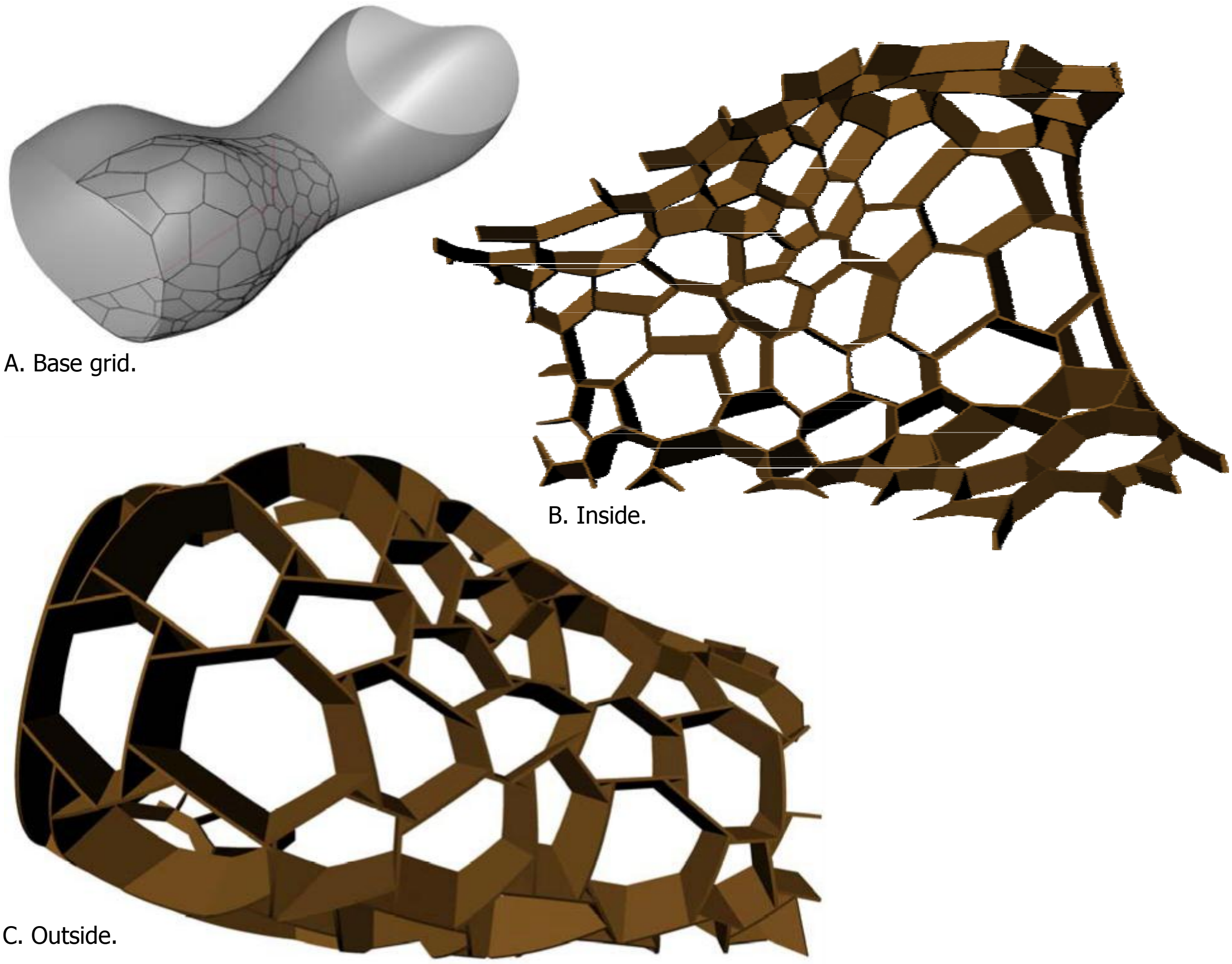

A. Base grid.
B. Inside.
C. Outside.

Figure 102. The planar members-system applied to a quarter of the DO Bubble.

5.2.5 Evaluation

The scheme's implementation onto a structural system involved its materialisation and detailing. It resulted in a structural system in which the features anticipated in the underlying scheme are maintained, although within a constrained domain. The scheme's primary features are the individual positioning and orientation of elements such that they are able to respond to the local geometrical conditions of the free form building envelope. In addition, the interconnection between three elements of varying inclination allows for systematic detailing.

In the systematisation additional criticalities came to light that were not foreseen on the scheme's higher level of abstraction, thus constraining the system's application. These criticalities involved the necessity of introducing volumetric material transformations in addition cutting operations of planes of zero-thickness to which the scheme was anticipated. New insight was also gained from the system's implementation in the case study, where orientation of the members favourable to both manufacturing (preferring large nodal eccentricities to reduce force in bolts) and structural action (preferring small nodal eccentricities to reduce sideward loading of members) appeared contradictory.

The opposing requirements on the nodal eccentricities are depicted in Figure 103, which also gives an overview of the other constraints, departing from the design variables of geometry, form and material. From this it becomes clear that also material aspects of locally required structural capacity and the fibre orientation needed to provide this capacity, are to be included in the system's setup.

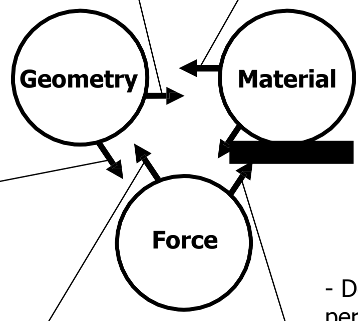


Figure 103. Requirements and constraints of the system of planar elements.

5.3 Case study 2: implementation of the Delta Ribs-structural system

The scheme of Delta ribs has been implemented onto a system with the same name, in curved sheets of steel. The additionally required inputs and all associated specifications and inputs are depicted in Figure 104.

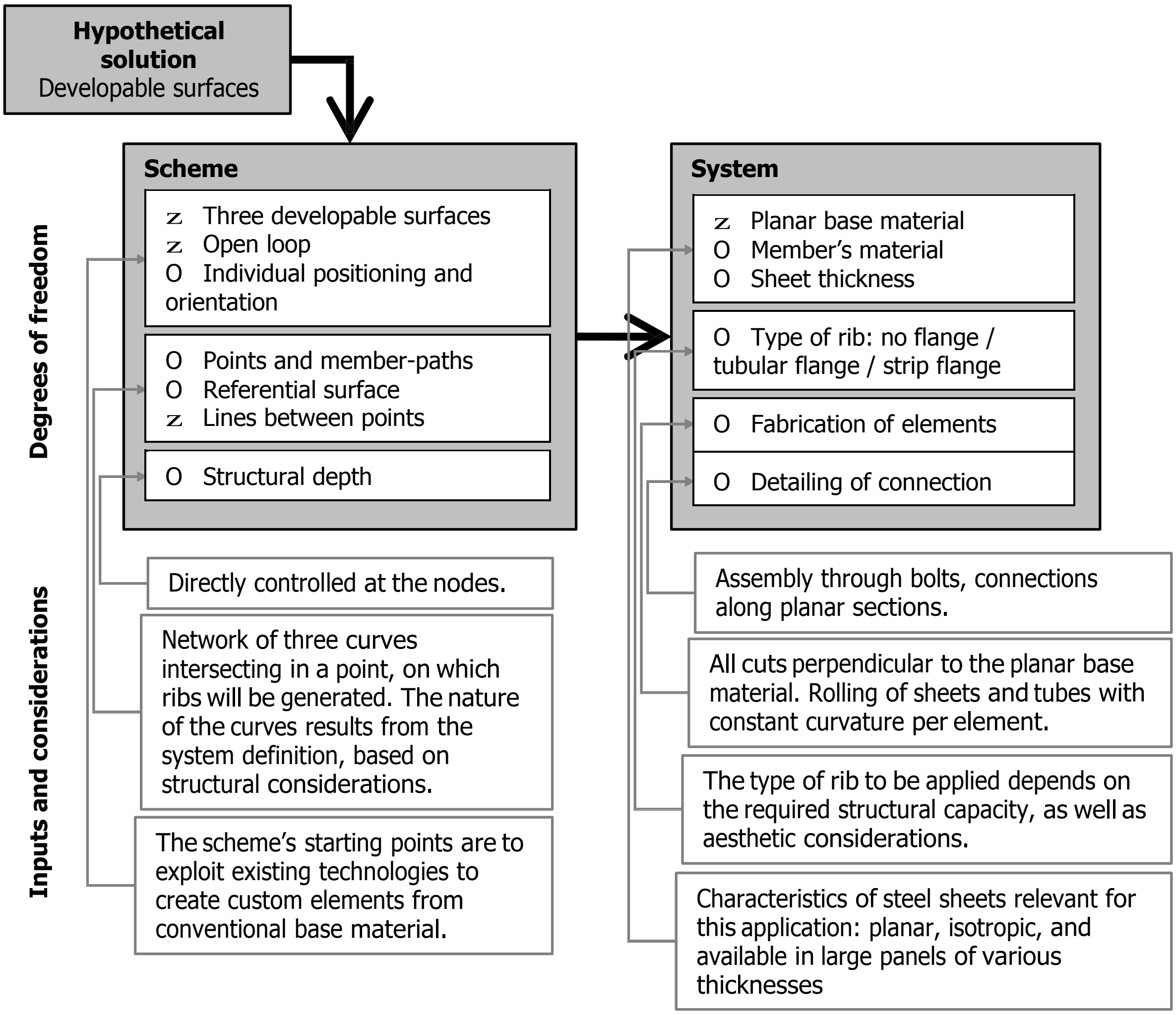


Figure 104. Inputs and definitions of the scheme of Delta ribs implemented as structural system.

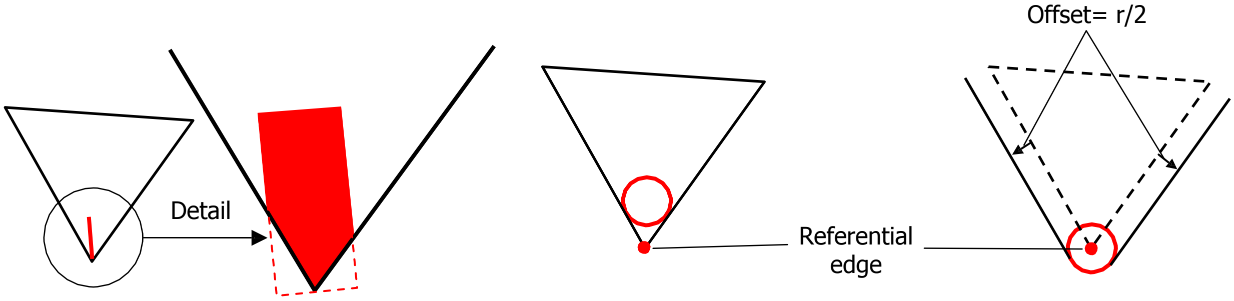
5.3.1 Geometrical design rationale

The primary characteristic of the scheme of Delta ribs (see previous chapter) is a beam element consisting of three developable sides, creating a triangular cross section. In the further development onto a system this characteristic remains unchanged, but is specified further. This specification concerns the system's construction from a curvilinear grid onto the ribs corner points, and finally to individual sheets. While doing so, the domains of material and structural considerations affect the geometrical setup through constraints they impose. They are derived from a rational manufacturing process, while additional constraints apply when flange-like elements are added to the rib's cross section. Each case results in different degrees of freedom that are at the designer's disposition.

Additional flange-like elements

A strip or tube may be added to the edge of a rib to increase its structural capacity. Anticipating construction, it is imposed that the strips are planar, and the tubes of constant curvature. To introduce the strip in the rib's edge, the strip will be chamfered until it touches the rib's curved sides, see Figure 105A. Hence introducing a strip does not influence the rib's geometry, on any level of scale. Two options exist for adding a tube inside a rib: by placing the tube inside a rib (Figure 105B) or by using the original edge curve as the tube's axis (Figure 105C). To achieve a tangent transition from side to tube, the original sides have to be offset at a distance of the tube's radius. This operation results in a different curvature, but maintains the curvature being constant. Obviously the modification of the sides also affects the geometry of the rib's third side, as well as its connection to adjacent ribs at both extremities.

The rib's curved sides have to connect tangent to the tube, this way assuring a smooth transition. Using an arc as lower edge, and the sides as scaled extrusions of this, appears to have a favourable effect on the detailing: including the tube inside the rib (as depicted in Figure 106A) results in a tangent line at a constant height relative to the plane of the ground curve. This is demonstrated in the unrolled pattern in Figure 106B. This distance depends on the angle of the rib and the tube's exterior diameter, and can be derived from a simple section orthogonal to the tube as depicted in Figure 106C.

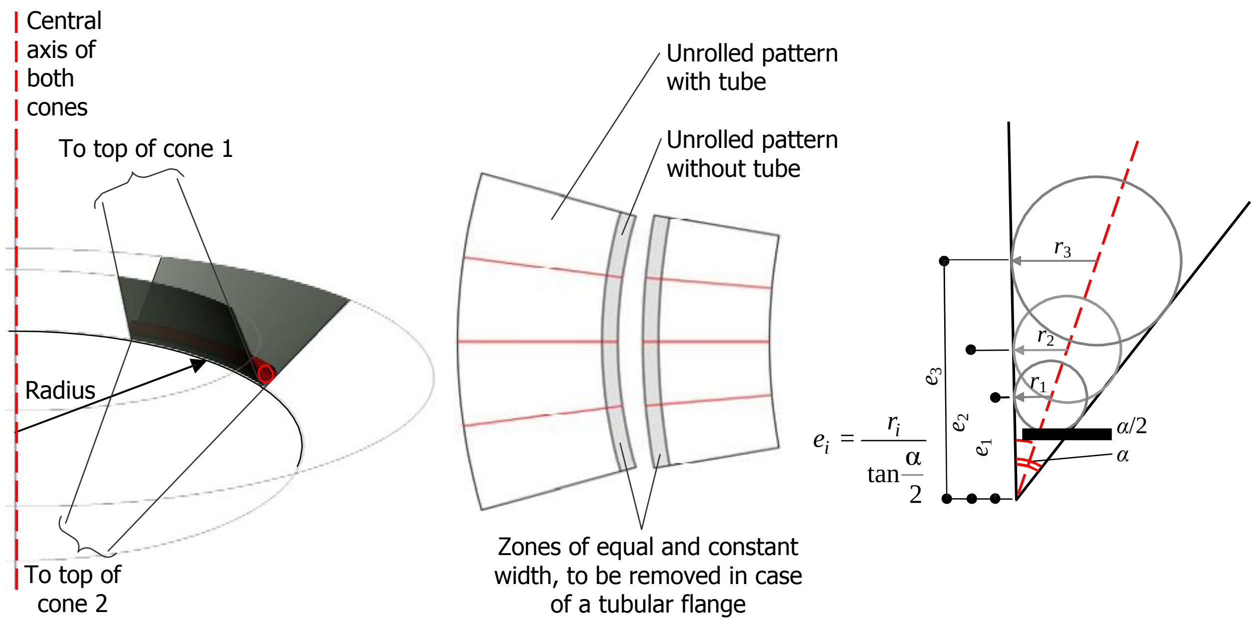


A. Strip included in the rib's edge, and detail showing the chamfered strip.

B. Tube placed inside a rib.

C. Using the original edge curve as the tube's axis. New sides are constructed at an offset.

Figure 105. Inclusion of a flange and a tube at an edge of a Delta Rib.



A. Constructing a rib with a tubular flange, using and arc as base-curve and conically curved sides.

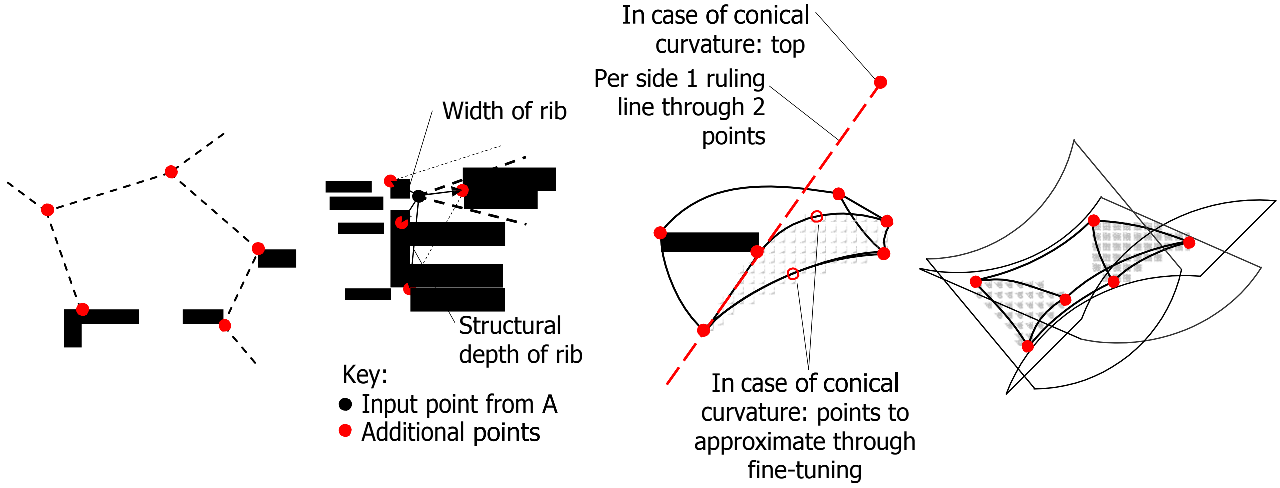
B. Unrolled pattern of the conically curved sides in previous figure, showing the effect of the tube.

C. Cross section showing how e depends on the rib's cross section and the tube's diameter.

Figure 106. Repercussions of the inclusion of a tube inside a Delta Rib.

Repercussions of flange on rib-geometry

The eventual addition of flange-like elements is translated into the geometrical constraint of using input curves (Figure 107-2A). In cases without additional element the edge is not constrained, and input points as in Figure 107-1A suffice. Both approaches go through the same phases of definition. Departing from points or curves affects the rib-generation already in the first phase: the definition of a network on the macro level. Next, from each input point additional points are generated based on for instance the designer's considerations or the required structural performance (Figure 107-1B and 2B). Between these points, the actual Delta Rib will be generated. Then, on the element-level, the primary difference between the point- and the edge-based approach appears: in the former, each of the three sides of a rib is generated in the same manner by using four input inputs (Figure 107-1C). In the latter case the two sides adjacent to the edge curve use this curve as input (Figure 107-2C). The third side of the rib is created individually, most logically sharing the two points needed to define the sides, but still require one more input point to be provided by the designer. The sides may be either cylindrically or conically curved, the latter providing more degrees of freedom to the user.

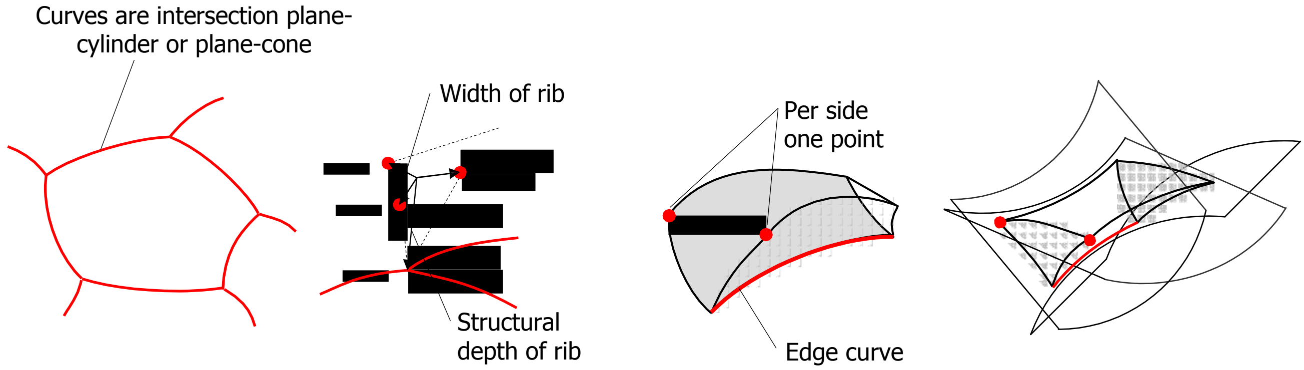


1A. Macro-level: points on a surface.

1B. Meso-level: rib's corner points derived from input point by user-controlled structural depth and width.

1C. Element-level: each side generated individually from four (corner) points.

1D. The original scheme, using input points.



2A. Macro-level: curves (with constrained properties) connected at extremities.

2B. Meso-level: curves become edge with additional element.

2C. Element-level: two sides adjacent to edge generated from edge curve.

2D. The original scheme, using input curves.

Figure 107. Degrees of freedom of the Delta Rib-system on three levels of scale in case of using either input points (series 1) or input curves (series 2).

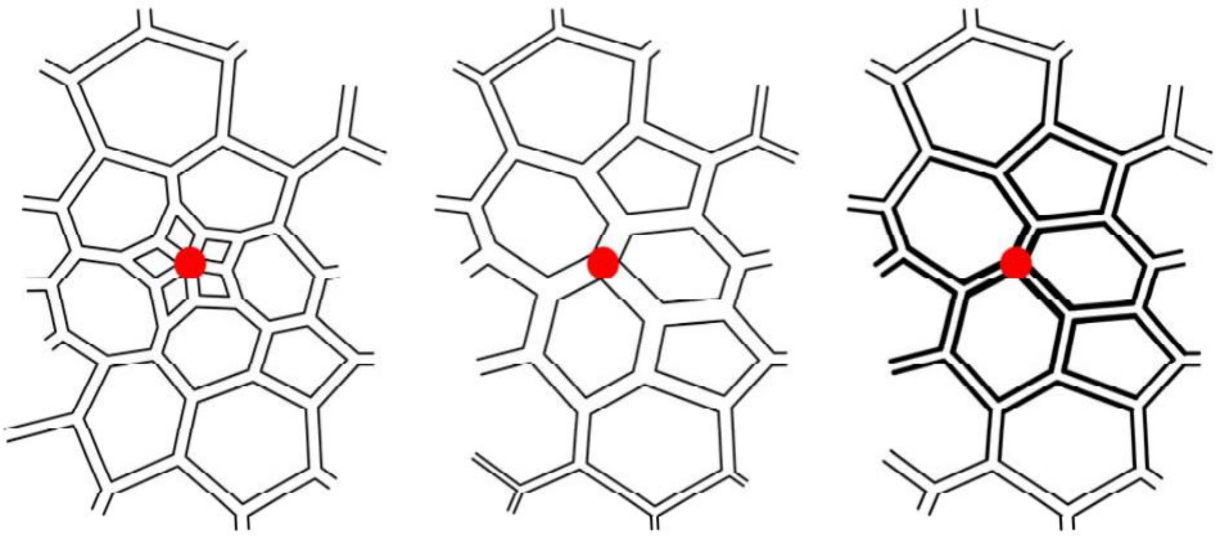
Although the input curve presented here is exploited to rationally construct the rib's lower edge, the designer may obviously use the presented procedures to his own intention. When for instance he wishes to assure that the lower edges of a network of Delta Rib are all aligned on one virtual surface, he will give priority to directly control these edges, and hence require it as input. For this he will apply a procedure which involves an input curve rather than only input points, without using the strip-flange for which the procedure as described here was intended.

Cylindrical and conical curvature

The constant curvature required to comply with rational manufacturing methods are achieved by referencing to circles. To achieve a cylindrically or conically curved surface, these circles are extruded to cylinders or used as ground curve with a top defined perpendicularly above its centre. Figure 109 demonstrates how to construct a cylinder and a cone of constant curvature from input points, while Figure 111 gives a full overview which inputs are needed to construct conical and cylindrical curved sides of a Delta Rib, also in case of additional elements on the rib's edge. The step-by step-approach of Figure 109 is helpful in an implementation in a parametric system.

Application of ribs in network

The layout of ribs in a network structure is unconstrained. Hence, the network could be optimised with respect to material efficiency, by scheming for a denser pattern in areas where the structural action is the most demanding. This is exemplified in Figure 108A where the ribs' pattern is densified nearby a support. Alternatively, structural capacity could also be increased by higher ribs (B) or ribs constructed from thicker sheets (C).



A. Higher density nearby support. B. Uniform density, variation in height of ribs. C. Uniform density, variation in sheet thickness.

Figure 108. Alternatives for increasing structural capacity of a network of ribs (schematic representations).

Repercussions of flange on network-layout

Except for the accidental case where ruling lines of two adjacent sides coincide, two adjacent sides of ribs in a network intersect along a space curve. In the unrolled pattern, this will be a free form curve, which is a degree of freedom. Hence, for the basic case of ribs without additional elements, no constraints apply to their application in a network configuration. Also no constraints apply in case of strip-like flanges, since the strips are placed inside the ribs, without affecting the rib's geometry. Inside the connection, the three strips are connected similar to the system presented previously, where three planar elements enclose a pyramid-shape. Its detailing will be discussed in the next chapter, on prototyping of the Delta Rib system.

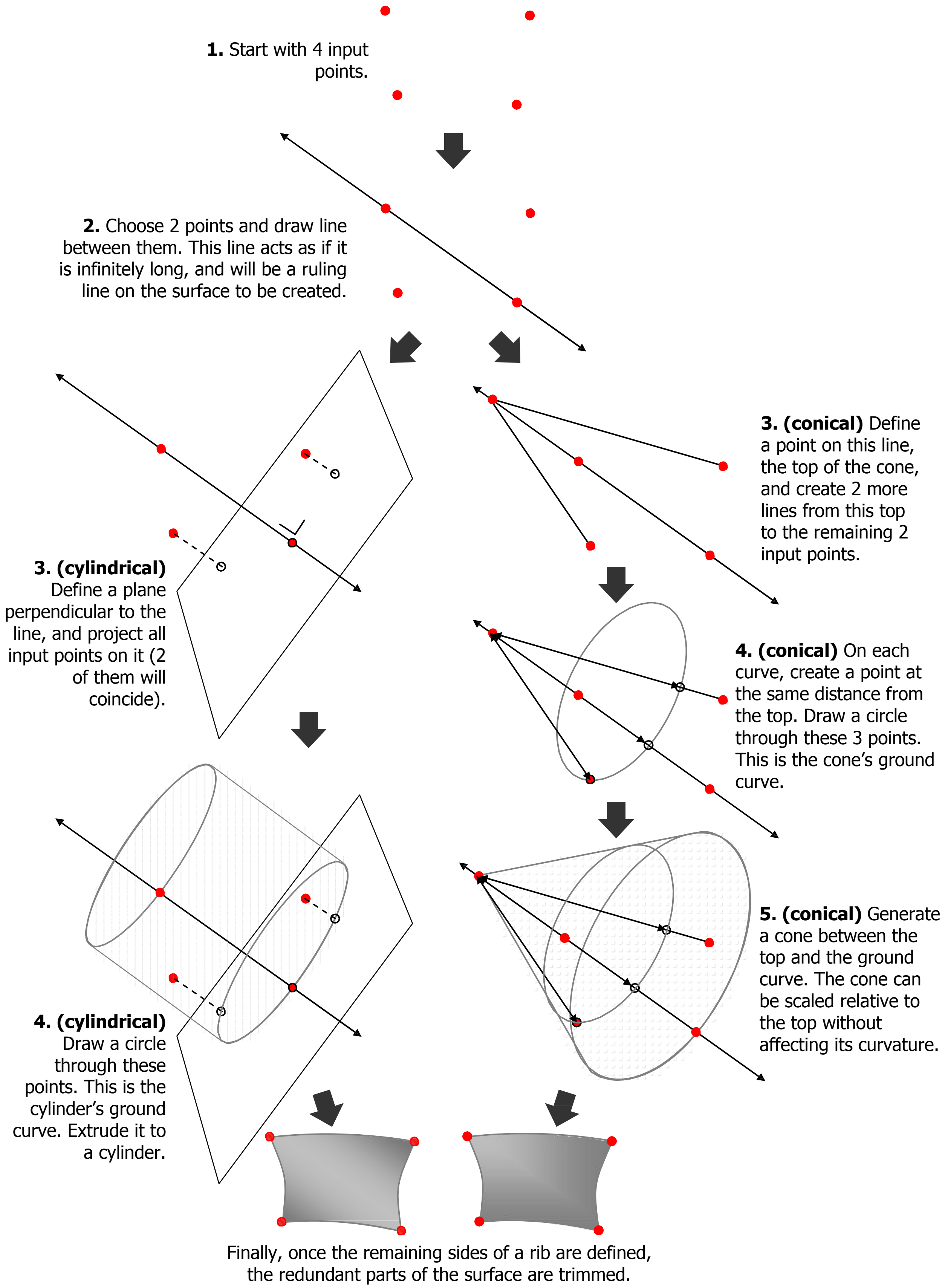
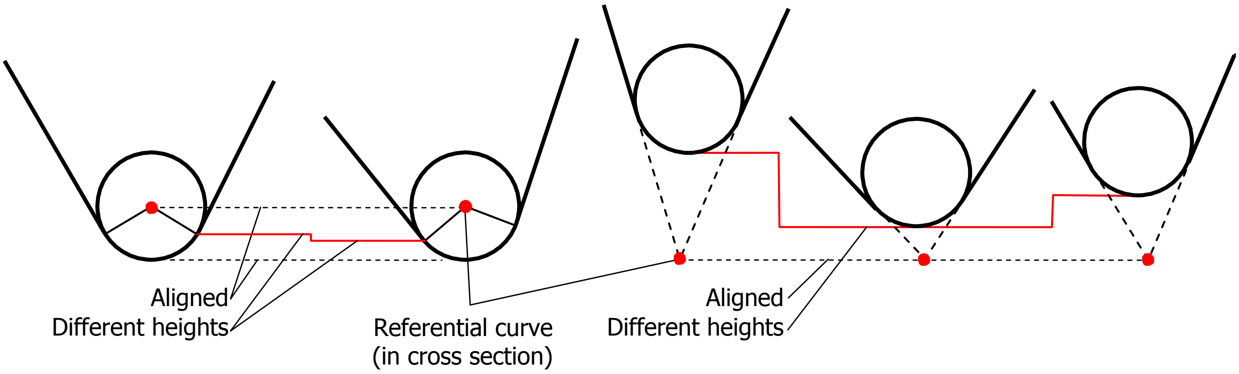


Figure 109. Generation of curved sides based on different inputs.

The connection of ribs with tube-flanges however is critical, since apart from a tangent connection between a curved side and the tube, also an orderly connection between ribs is desirable. Tubes of identical external diameter always fully intersect, thus do not leave open ends, regardless their orientation. Although it constrains the tube, this decision does not constrain the formal freedom of the network configuration. Regarding the positioning of the tube, the tube is placed on the centre line. The tubes will then connect exactly. A discontinuity occurs at the lower edge of two adjacent sheets, which do not align due to different angles of the ribs V-shapes (Figure 110A). In construction, this is to be resolved by filling the gap. Placing the tube inside a rib will cause the tube to be placed at different heights (relative to the referential curve) due to the cross section that is unique per rib (Figure 110B). Since this could only be avoided by imposing a constant angle to the rib's V-shape, this alternative is rejected.



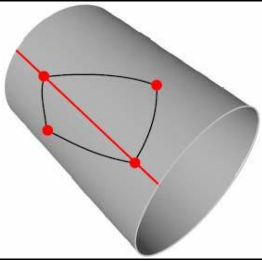
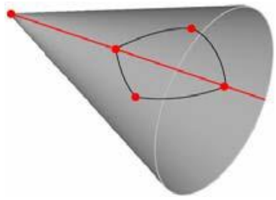
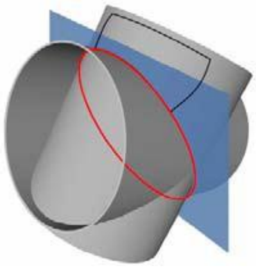
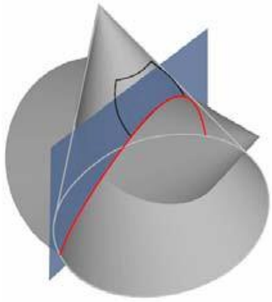
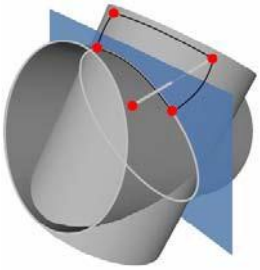
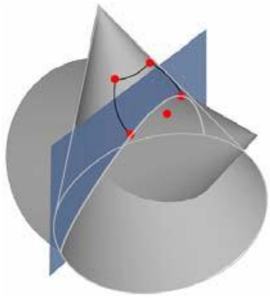
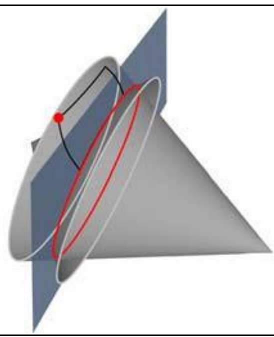
A. Tube placed on the referential curve. Cross sections showing non-aligning lower edges of two adjacent sheets.

B. Tube placed inside rib, causing the tube to be at different heights relative to the referential curve.

Figure 110. Implications of two alternative ways to position tubular edges in ribs with V-shapes of different angles.

Overview of alternative systems

Figure 111 gives an overview of the possible variations that can be constructed with Delta Ribs. They are based on the variable of curvature (cylindrical/conical) and eventual additional elements (none/planar strip/tube). It becomes clear that each combination has its own constraints and degrees of freedom. Adding additional elements imposes constraints, or 'consumes' degrees of freedom. Alternatively, a conical curvature brings in additional degrees of freedom. Hence, a tubular edge cannot be constructed with constant cylindrically curved sides, whereas it can with constant conically curved sides.

	Constant cylindrical curvature		Constant conical curvature	
No additional elements at rib edges		O 4 points O Definition of ruling line between points This scheme has been evaluated through prototyping.		O 4 points O Definition of ruling line between points O Top of cone
Planar strip based on input curve		O Planar curve, derived from the intersection between a cylinder and a plane, but no arc z Two extrusions onto cylinders, the two being mirrored relative to the edge-plane		O Planar edge curve, which is derived from the intersection between a cone and a plane O 1 point per side z Per side: scaled extrusion relative to a point on the cone's axis, such that the input point lies on the cone. This results in two cones, which intersect at the input curve.
Planar strip based on input points		For one side: O 4 points O Definition of ruling line between points For adjacent side: O 1 point z First side is a regular constant cylindrically curved surface. The additional point for the other side, through construction, defines the plane of symmetry between the two sides. The edge is on this plane.		For one side: O 4 points O Definition of ruling line O Height of cone For adjacent side: O 1 point z First side is a regular constant conically curved surface. The additional point for the other side, through construction, defines the plane of symmetry between the two sides. The edge is on this plane.
Tube		N/A, extrusion only perpendicular to arc's plane, thus no possibility to construct V-shaped cross section.		O Arc O 1 point O Tube radius This scheme has been evaluated through prototyping.

System's variables: O = degree of freedom, z = fixed.

Figure 111. Alternative systems based on the Delta Rib-scheme.

5.3.2 Materialisation and fabrication

Material

The material has to be available in sheets in a range of thicknesses that can be cut through a numerically controlled cutter, has to allow for plastic deformation and should possess a known behaviour when curving the sheets. For construction, it has to allow for a continuous connection along the sheet's free form contours. Sheet metal features these characteristics, where welding is the associated connection for the curved contours.

Rolling of curved sides

The common manufacturing method to curve metal sheets is the usage of rolling mills. By adjusting the position of the mills relative to each other, the sheet led between them undergoes a plastic deformation, after initial elastic deformation. The amount of deformation is determined by the mill's settings, the sheet's geometry and its material properties - notably the stress level where yielding occurs. As the latter property varies per sheet the rolling process requires constant supervision by a worker. He measures the chord between reference points and interprets this to modify the mill's settings. To comply with this fabrication method, the curvatures of the Delta Rib's sheet's should be constant and in one direction only. This way, metal sheets can be rolled with the equipment's settings kept constant for the whole sheet. Fabrication of conical curvatures requires more attention than cylindrical curvatures since the sheets' radii of curvature at both extremities requires verification, however both conical and cylindrical curvatures are judged acceptable. Although specialised rolling companies are able to deliver custom-shaped sheets with non-constant irregular curvature, they require intensive labour investment, at the associated cost.

Flange-like elements

In case of section action, the structural material in the member's extreme fibres is the most active. As triangular shaped ribs have little material there, increasing the amount of material at that point is favourable for the ribs', and therefore the system's structural efficiency. Two alternative solutions are proposed: to include a strip or a tube in the edge. Both additional elements may be given the dimensions that are structurally needed. For the strip these are its width and height, for the tube its diameter and wall thickness. In its extreme case, also a solid bar may be used.

To enable rational production, the strip is constrained to be a planar element (with contours that can be cut numerically controlled), while the tube is constrained to a constant radius for the same reason as the sheets' constant curvature. Both the strip and the sheet are welded to the sides.

Connections

Although rationally generated, the intersection of three ribs constitutes a complex joint. In the joints the sides intersect along a space curve; in the case of cylindrical curvature by default, in the case of conically curved sides as a consequence of the designer's degree of freedom to choose the ruling line. To construct these joints, a permanent connection through welding seems to be the most appropriate. Furthermore, to facilitate prefabrication the structure has to be subdivided, and assembled on the building site, preferably through bolted connections. It is anticipated to make these connections in the ribs, along a planar intersection. Their detailing will be discussed in the next chapter, on prototyping of the Delta Rib system.

5.3.3 Structural action

Due to its nature of closed box sections, the system is naturally fit for transferring torque, in addition to its bending resistance. As thin-walled box sections have a little favourable distribution of material to resist bending, the proposed flanges are most welcome. It is proposed to position the ribs with their topsides approximately aligned with the building envelope, and place the flanges opposite to the rib's topside, thus in the lower edge. This way, the interconnected curved topsides transfer in-plane forces through their full width, balanced with a force of opposite sign in the lower edge – the rib's height acting as lever-arm. This orientation anticipates bending action beyond the building envelope, which is likely in the vast majority of cases.

Assumptions on the system's structural behaviour were confirmed through finite element analysis, of which Figure 112 displays the stress distribution and exaggerated deformations. The model[11] primarily aims at highlighting qualitative characteristics, rather than a quantitative assessment. It consists of elements of a single thickness of sheet metal, and is loaded by its self weight only. The following characteristics of the systems' structural action are observed:

1. Peak stresses occur in the three upper corners of the node, as well as in the lower corner. These are due to the unfavourable connection of two plates, not providing a third direction for equilibrium of in-plane forces. Directional splitting of internal forces is only possible under acute angles at the corner of the model, resulting in stress concentrations. The discretisation into elements further heightens this. In reality, stresses will cause local plastic deformation, followed by redistribution of stress, but it is nevertheless recognised that this is a critical spot;
2. Little effective connections of curved sides in the nodes due to absence of directional splitting of internal forces, along the sides as well as in the top. As a result, internal forces are concentrated along the relatively stiff rib-edges;
3. The edges connected through welding provide mutual stiffness, this way stabilising the curved sides. This expresses the need for a connection that is continuous and rigid, rather than hinged and interrupted;
4. Connection plates provide stiffness against the before mentioned-effects of straightening sides and rotating edges, leading to a more effective use of the width of sides and increased stability. Adding more plates inside the ribs would further improve the stiffness and stability of the rib sides;
5. The curved topsides tend to straighten, pointing at a relatively low out-of-plane stiffness;
6. As it is only restrained at its lower edge, the strip-flange's upper edge tends to buckle. A flange of larger thickness will overcome this.

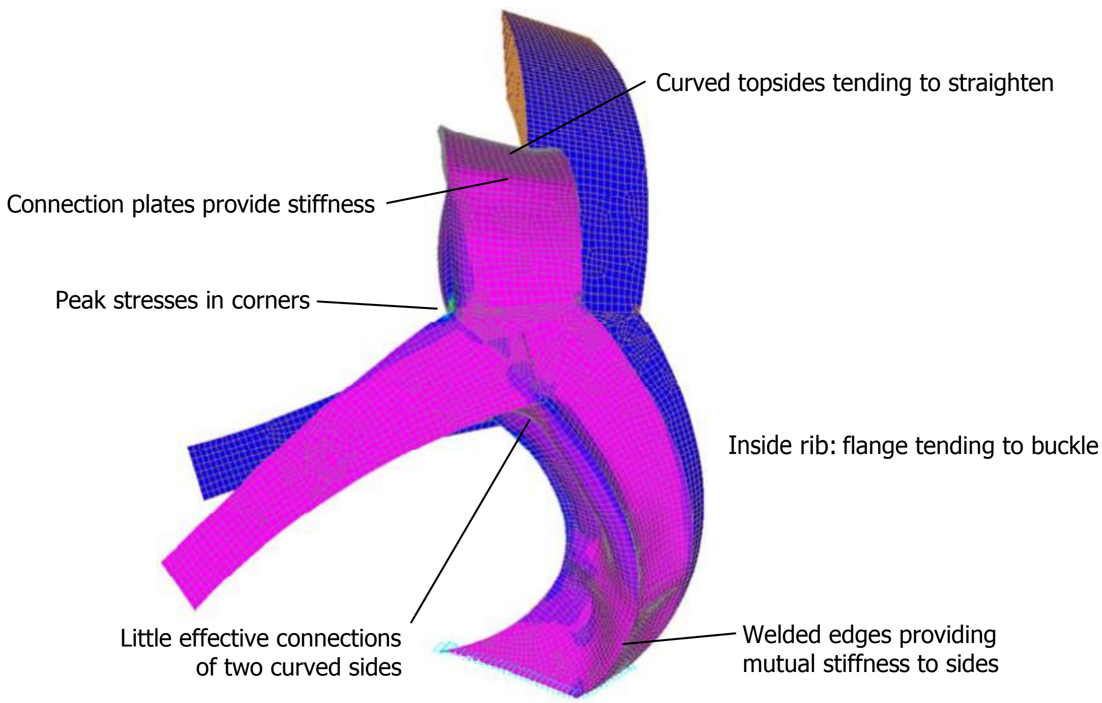


Figure 112. Characteristics and structural analysis of three connected ribs under dead load showing the VonMises-stress distribution and exaggerated deformations.

[11] The model made for the prototype-implementation was used for this. It will be discussed further in the next chapter.

5.3.4 Implementation on the DO Bubble

The Delta Rib system has been implemented in two variations on the DO Bubble, the first in a dense pattern without using any flanges, the second in a more open spaced network that uses ribs with a tubular lower flange. The resulting structures are displayed in Figure 113 to Figure 115.

As no replicatable algorithm that generates Delta Ribs from input curves has been developed yet, all ribs have been generated based on arcs, of which extremities and midpoints are lying on the interior building envelope. Straight profile curves were extruded along these arcs using a Maya-script. By setting the extrusion perpendicular to the arcs, the surfaces generated are cones of constant curvature. As the orientation of the arcs relative to the building envelope varies, rib sides were created at 10° intervals (Figure 116A). Once created all, the appropriate ones among them were selected manually (Figure 116B). For the ribs with tubular flanges, the newly created surfaces were offset. All rib-sides had to be trimmed to its adjacent ones, and for this extended where needed. Finally the ribs' upper sides were generated through spanning a surface between the rib sides' upper edges. Although the surface thus created does not have a constant curvature, the visual effect of this simplified method is comparable.

In both implementations, the structural depth was manipulated by varying the angle between two profile curves, as well as by varying the distance between the interior and exterior referential surface. General criticalities that appeared during the system's implementations are listed in Figure 117. These are the capricious intersection between two adjacent conical surfaces, conical surfaces that become of no use when they include the cone's top and the width of ribs that is hard to foresee in case both the angle between the two rib sides and the depth is varied.

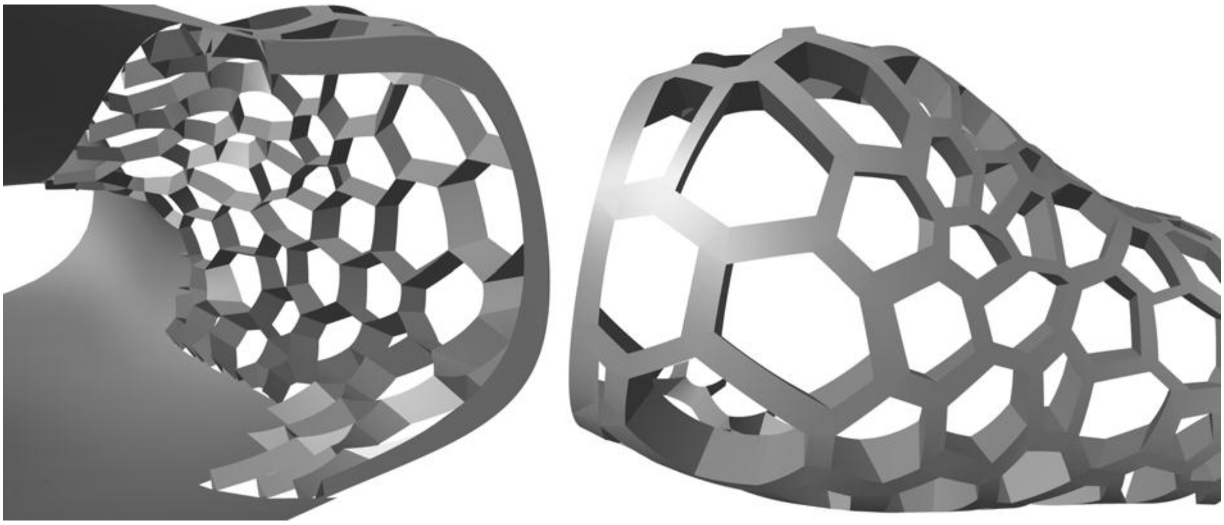


Figure 113. Implementation of the Delta Ribs structural system on the DO Bubble, using ribs with no additional flanges.

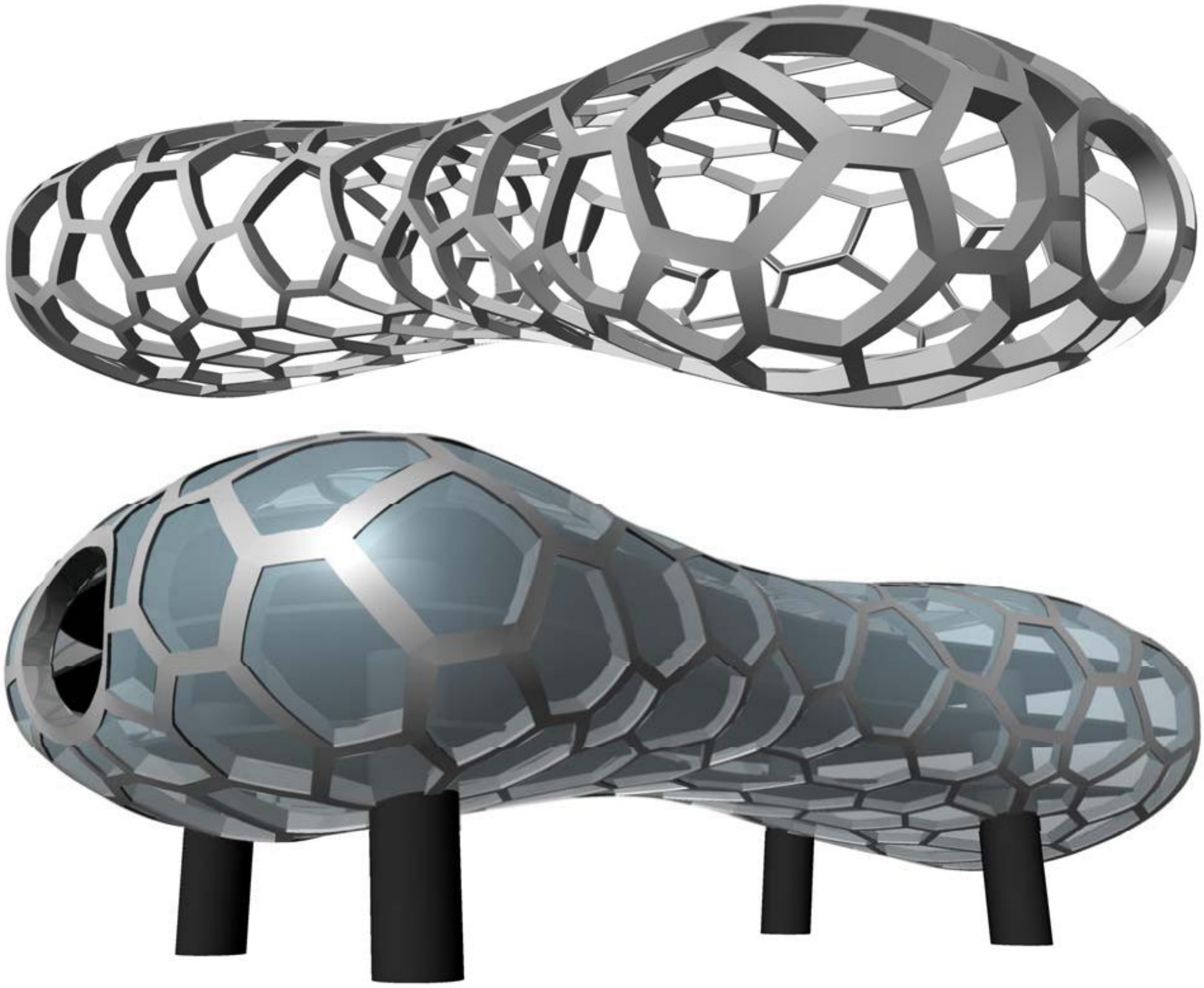


Figure 114. Delta Ribs with tubular flanges implemented on the DO Bubble, the top figure depicting the Delta Ribs only, the lower including floors, supports and a transparent envelope.

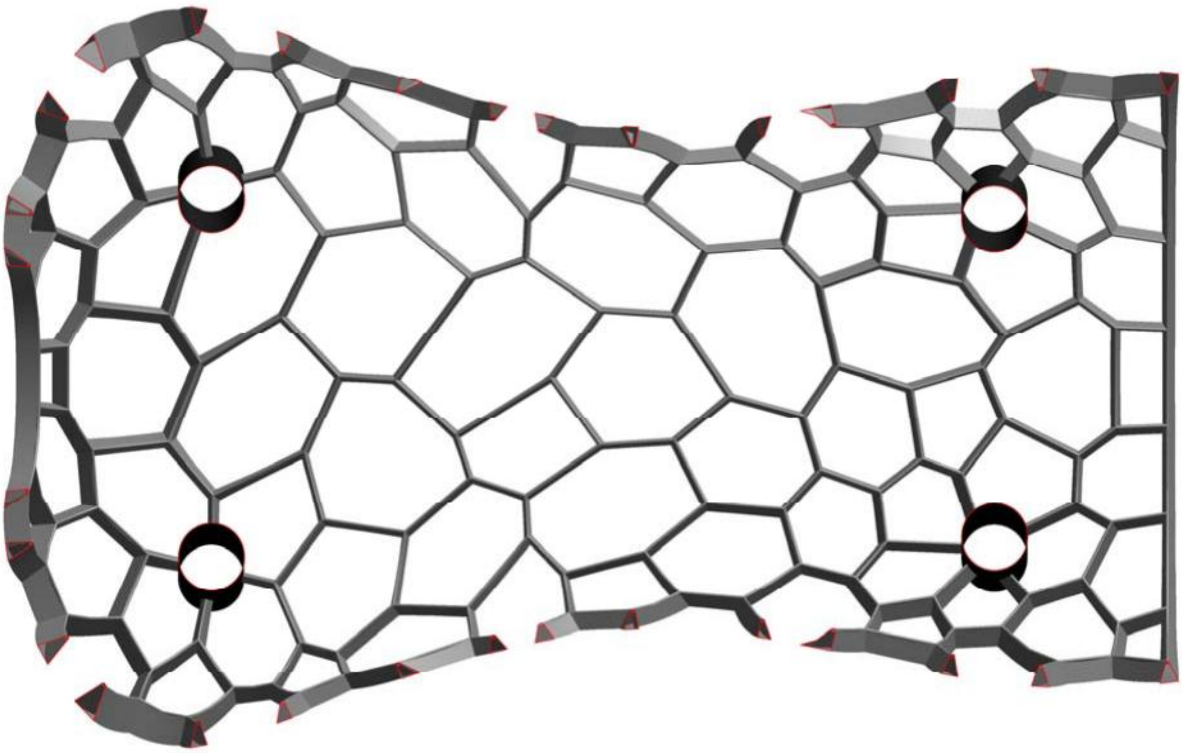
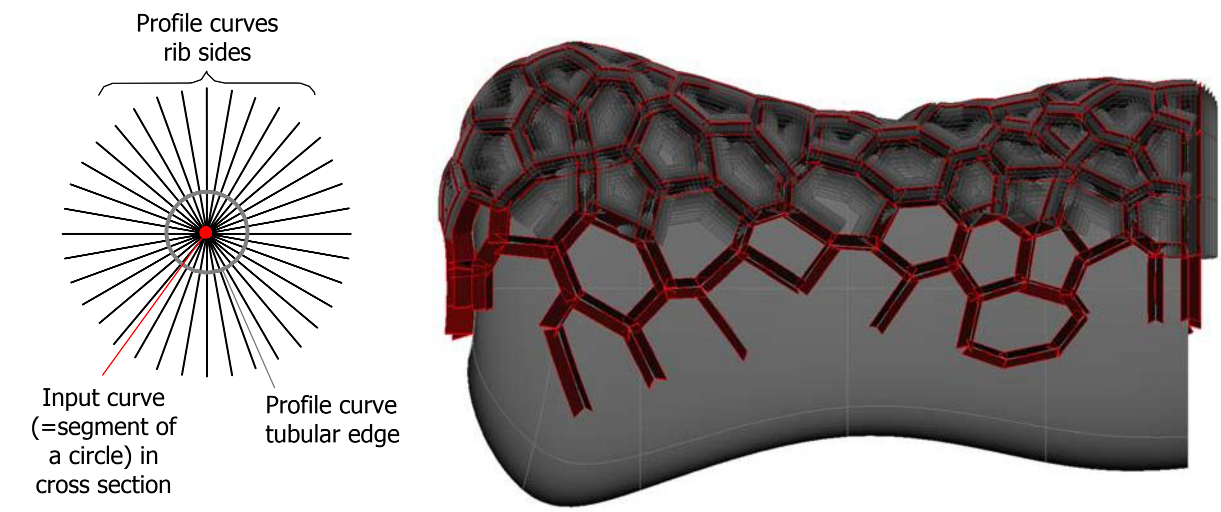


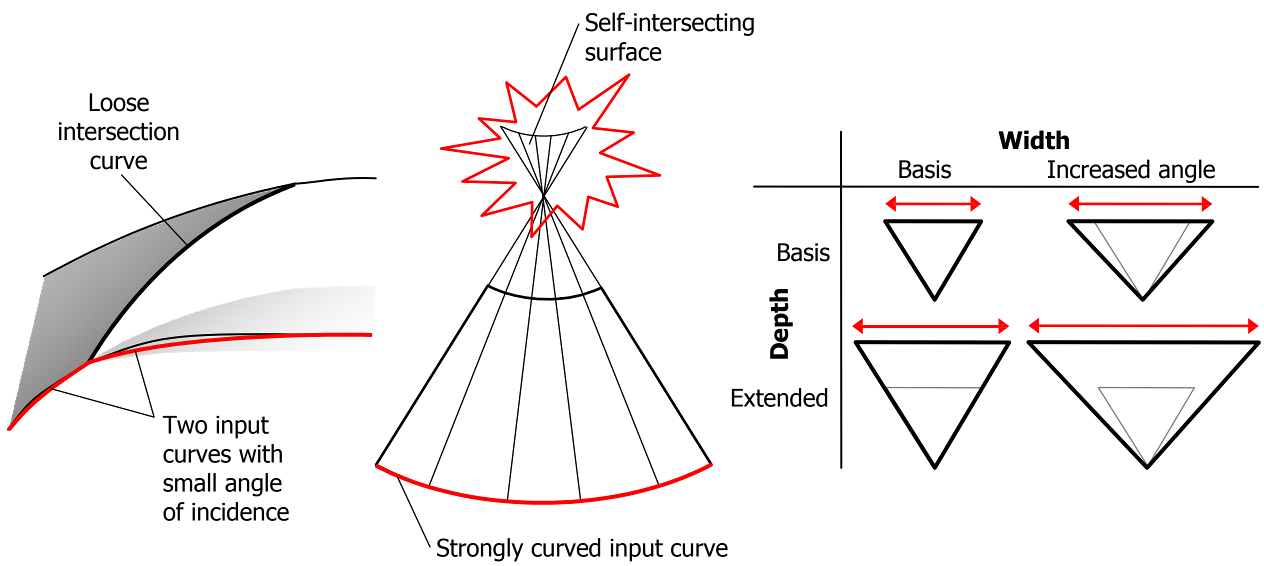
Figure 115. Horizontal cross section of the DO Bubble with Delta Ribs.



A. Profile curves to generate rib sides and tubes through batch processing.

B. Created rib sides, of which appropriate ones have been coloured red.

Figure 116. Extrusion of profile curves along the arcs.



A. Loose intersection curve resulting from two input curves with small angle of incidence.

B. Self-intersecting surface (conical surface on two sides of the cone's top) caused by the combination of a great depth and an input curve of relatively small radius.

C. Generation of unintentionally wide cross sections due to the accumulation of an increased angle between rib sides and an extended depth.

Figure 117. Problematic issues when generating Delta Ribs.

5.3.5 Evaluation

The Delta Rib system offers considerable formal freedom while using conventional methods of fabrication. To maintain this rationality, constraints and requirements between the design variables as depicted in Figure 118 apply. The formal freedom allows generating ribs between several kinds of input geometry – depending on the type of rib. The generation of ribs is based on a sequence of systematic operations, and thus potentially fit for automated application. Although automatable, the rational operations leave degrees of freedom to be controlled by the designer. As the operations go across levels of scale, this implies a controlled gradually increasing specification.

Since the geometry of the ribs is influential on their structural performance, the formal freedom may be employed to make the geometry anticipate the required structural capacity by varying the depth and width of ribs, as well as through adding a flange. This anticipation positively affects the structural efficiency that can be achieved through the system.

As geometrical and structural characteristics of the system are promising, prototypes of the system will be built to assess the system's buildability. Through the prototyping the system's underlying assumptions will be validated.

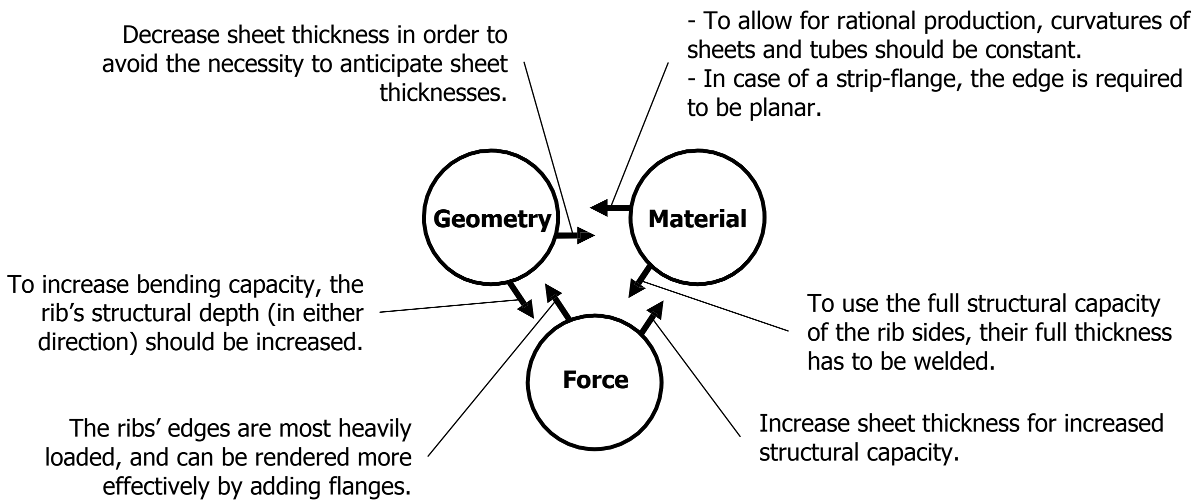


Figure 118. Requirements and constraints of the system of Delta Ribs.

5.4 Case study 3: implementation of the 3D components-structural system

The scheme of Torque-resistant ribs in a semi-monocoque has been developed onto the 3D-components-system. It employs thin metal sheets to create the irregular curvature of the sides for triangular ribs. Once filled with concrete, the parts are stiff and can be assembled into a load bearing building envelope. The inputs that are required in addition to those already defined in the scheme-stage are depicted in Figure 119.

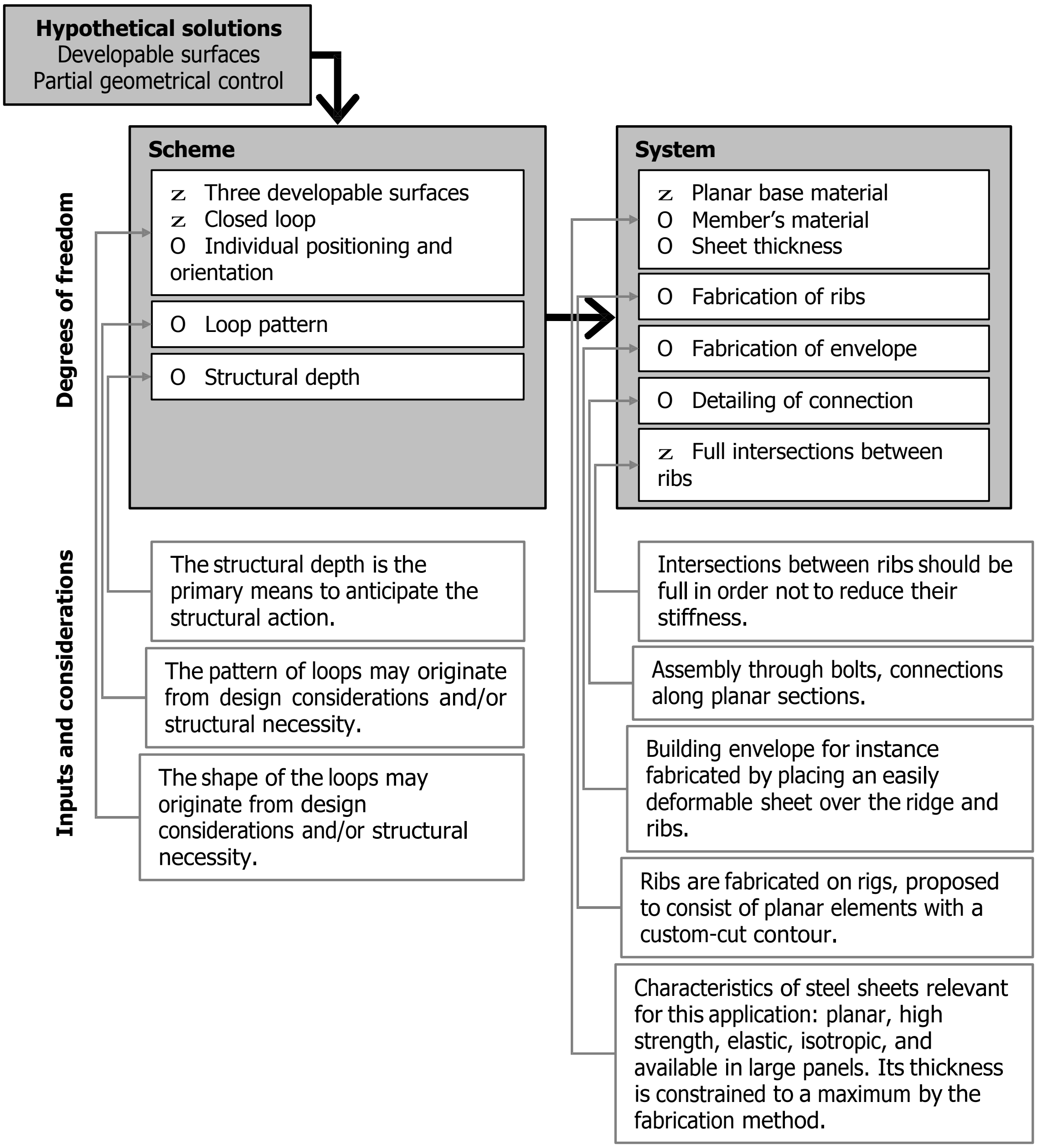
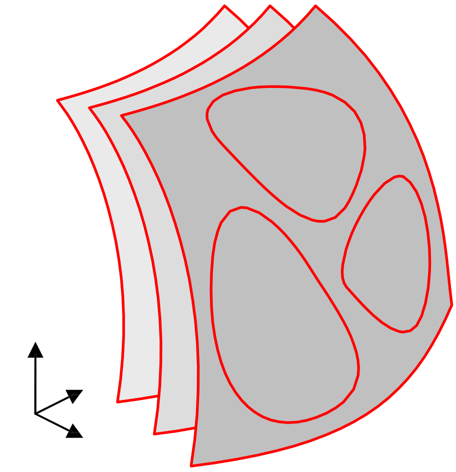


Figure 119. Inputs and definitions of the scheme of Torque-resistant ribs in a semi-monocoque implemented as the 3D component-structural system.

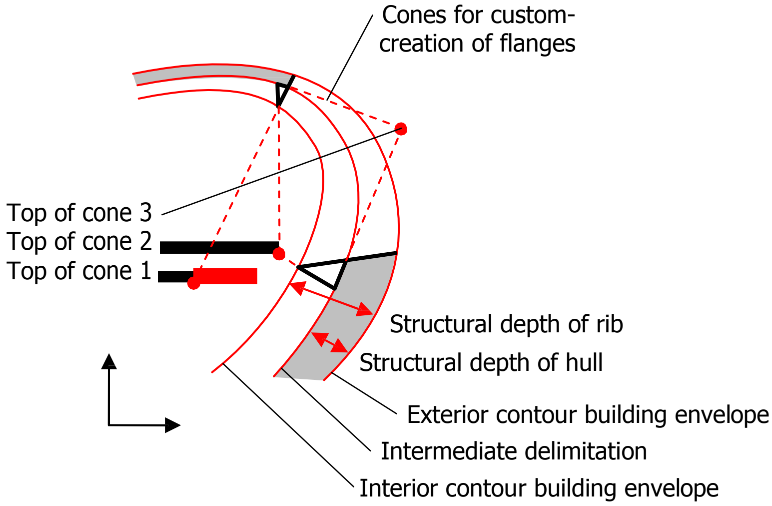
5.4.1 Geometrical design rationale

The system of 3D components entails the partitioning of a structural layer through conically curved bordering surfaces. The partition results in meso-sized elements of which the lower and upper extremities are exactly aligned with the respective free form envelope. All geometry in between anticipates the hull's local curvature. Additional flanges, each being a conically curved surface too, are created to reinforce the partitioning borders (Figure 81). These borders are constructed as box-sections resistant to torque.

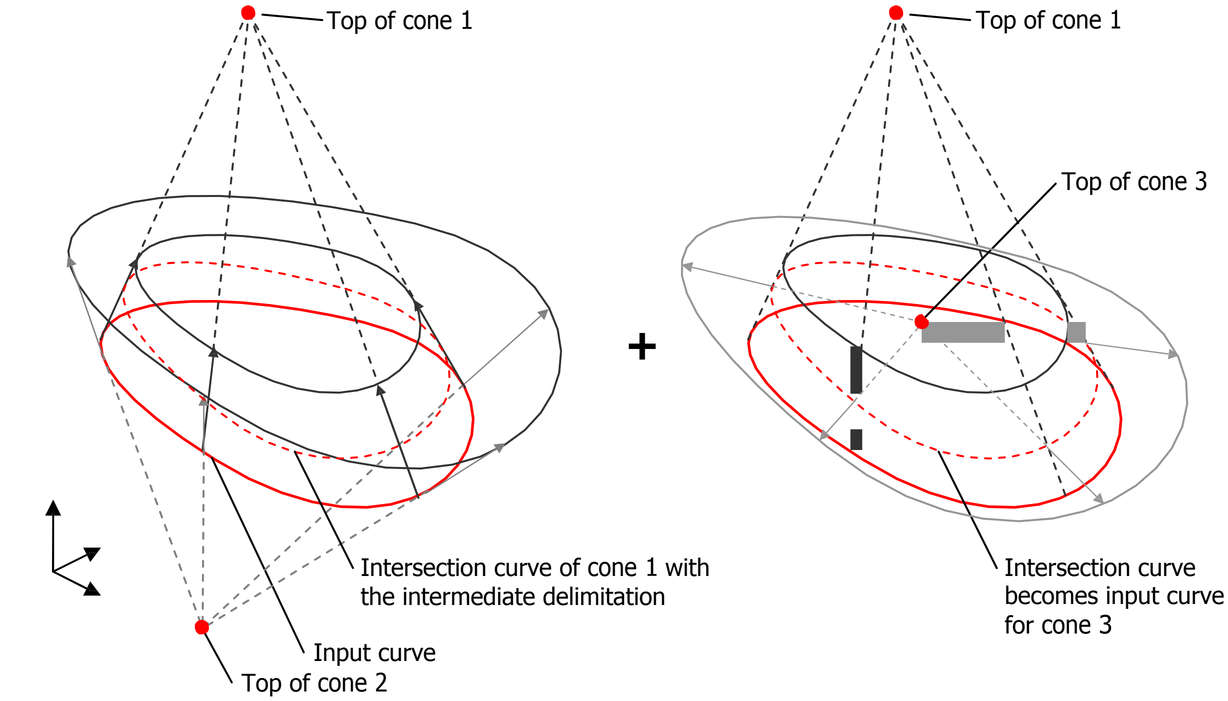
The definition of the scheme as defined in the previous stage is depicted in Figure 120B. To this scheme, an additional higher level-definition is added. This comprises the definition of two non-intersecting surfaces in addition to the architecturally defined building envelope, and a curve on any of these surfaces, see Figure 120A. Together they serve the generation of three cones that together constitute the triangular rib: from the initial input curve a cone is generated, which, when intersecting it with any of the layers, creates a new intersection curve. This curve then serves as input for generating the next cone. The system demonstrated in Figure 120C generates two cones from one input curve. The third cone is generated from the intersection of cone 1 with the intermediate layer. Since the system allows for irregular curvatures, usage of another input curve for each new cone is not constrained.



A. Macro-level: three non-intersecting surfaces, with closed curves on one of them.



B. Meso-level, showing the building envelope and the ribs' delimitation the 3D-component in cross section.



C. Meso-level, showing the generation of cone 1 and cone 2 from the input curve in the left figure. The right figure (which fits on top of the left figure, but for reasons of clarity is depicted separately) demonstrates how the curve that results from the intersection between cone 1 and the intermediate delimitation of the building envelope, serves as input curve for cone 3.

Figure 120. Degrees of freedom of the 3D-components-system on two levels of scale.

Despite presuming non-intersecting ribs, intersections can well be made using this system. The intersections do not all have to constitute a boundary around an opening, but may also simply be introduced as a support below the building envelope – the only difference being the trimming boundary. Figure 123 demonstrates an exemplary implementation to the DO Bubble. As connections between ribs will provide mutual support, they are beneficial for the system's stiffness in the construction phase.

5.4.2 Materialisation and fabrication

Material and fabrication of curved sides

From the scheme-phase onward, the system's implementation presupposes a formative fabrication method for the curved surfaces. The surfaces' material should allow for elastic out-of-plane deformation, should enable a tight connection at the ridge of two adjacent surfaces and should possess strength to act as tensile zone once the ribs are filled. Furthermore it should be able to cut them in a digitally controlled manner, such that the initial hypothetical solution of unconstrained geometry in 2D is realised. When using thin sheets the material thickness causes only negligible deviations from the referential geometry. Hence, the geometrical referential surfaces can be used directly as cutting pattern for the sides.

Metal sheets typically feature these characteristics, reason for which this implementation intends to apply steel sheets. A typical thickness of around 1 mm balances between sufficient stiffness to temporarily position the curved sheets, and sufficient flexibility to allow cold deformation of the sheets into irregular curvatures. Notably in case of strongly curved sheets, best steel with an elevated yield stress should be used, such that plastic (permanent) deformations are prevented while the other properties are maintained. Fabricating the curved sides from steel facilitates their connection to the nodes between components, which is discussed in a later section.

An additive fabrication is anticipated for the structural enclosure to be laid inside, between and over the V-shaped ribs. Since initially formless, the materials to be poured is initially shapeless, and inherently appropriate for application in free form shapes. The material is required to have resistance to shear and compression, and should be sufficiently viscous (thus not level out) such that the surface can be sculpted to approximate the free form building envelope. As a result, the structural layer will most likely require a finishing before obtaining the intended free form envelope.

Material and fabrication of structural enclosure

Concrete is chosen to be the material for the structural enclosure. In consequence, the system becomes a curved application of existing steel sheet-concrete structures, and may benefit from knowledge available about their regular applications. Furthermore, concrete exists in various kinds and can be applied in different ways: through pouring as well as spraying. Finally, since concrete is a well known material in building industry, the proposed system only requires the industry to get accustomed with the system, and saves time and effort the familiarize with a new material. The composition of the concrete has not been studied as part of this research since it was expected that this would not implicate the choice for a system, but only affect the specification of the system itself. Among these specifications, notably the option to use fibres as a partial or complete substitute for reinforcement bars is worth researching. This would remove the constraints imposed by the reinforcement layout (such as the varying curvature that has to be given to the rebars), and alleviate the system's dependency on the bonding between the steel sheet and the concrete poured in it.

Although custom–milled polystyrene moulds have proven to enable fabrication of custom-shaped concrete surfaces, it is proposed to consider using an intermediate layer that is shaped on site. This way the hypothetical solution of using a partially-controlled formation method is applied another time. When using the intermediate layer's bending stiffness, it has to meet the contradictory requirement to be sufficiently flexible to approximate a curved surface while simultaneously being sufficiently stiff to bear the load of the freshly poured concrete on top.

The use of an intermediate layer was assessed in a scale model by using an eggcrate-profiled panel[12]. The panel was laid over a rig, after which polyurethane foam was applied for waterproofing and additional stiffness, finally covered with gypsum. Just to mention some alternatives, also inflated air cushions or change-phase

[12] An eggcrate-like profile was proposed as double curved envelope for the Bloboard-bookshop competition entry (see Figure 60). Experiments showed that the shape-changes occurred due to local bending in between the stiff tops and valleys.

assemblies could be applied to provide a custom-shaped temporary double curved surface: an easily deformable airtight compartmented bag filled with sand that gets rigid when the air is sucked out. As there were several ideas on how the intermediate layer could be realised, and the problem of creating a continuously curving surface between point-like supports is not unique to this system[13], this has not been topic of further research.

Connections between components

After off-site fabrication of the components, they are transported to their final location for on-site assembly. Connections are made between along planar sections, realised as rigid endplates in steel. While their upper and lower contours are defined by the intersection with respectively the building's interior-, intermediate- and exterior envelope and therefore curved, the sides are preferably constructed as straight lines to facilitate positioning. Accuracy of the connection then depends on the accurate positioning and orientation of the endplates during the component's construction. A straight side is achieved by aligning the endplate with a ruling line of the cone determining the contour at that place (see Figure 121). Only when the cones intersect, and the endplate is constructed between intersecting ruling lines, then the endplate can have two straight sides.

Creating a rigid connection between components requires connectors nearby the section's contours in addition to axial and shear forces also bending moments and torque can be transferred. Difficulties similar to those when connecting Delta Ribs arise; notably the lower edge of the V-shaped rib is hard to connect due to limited space.

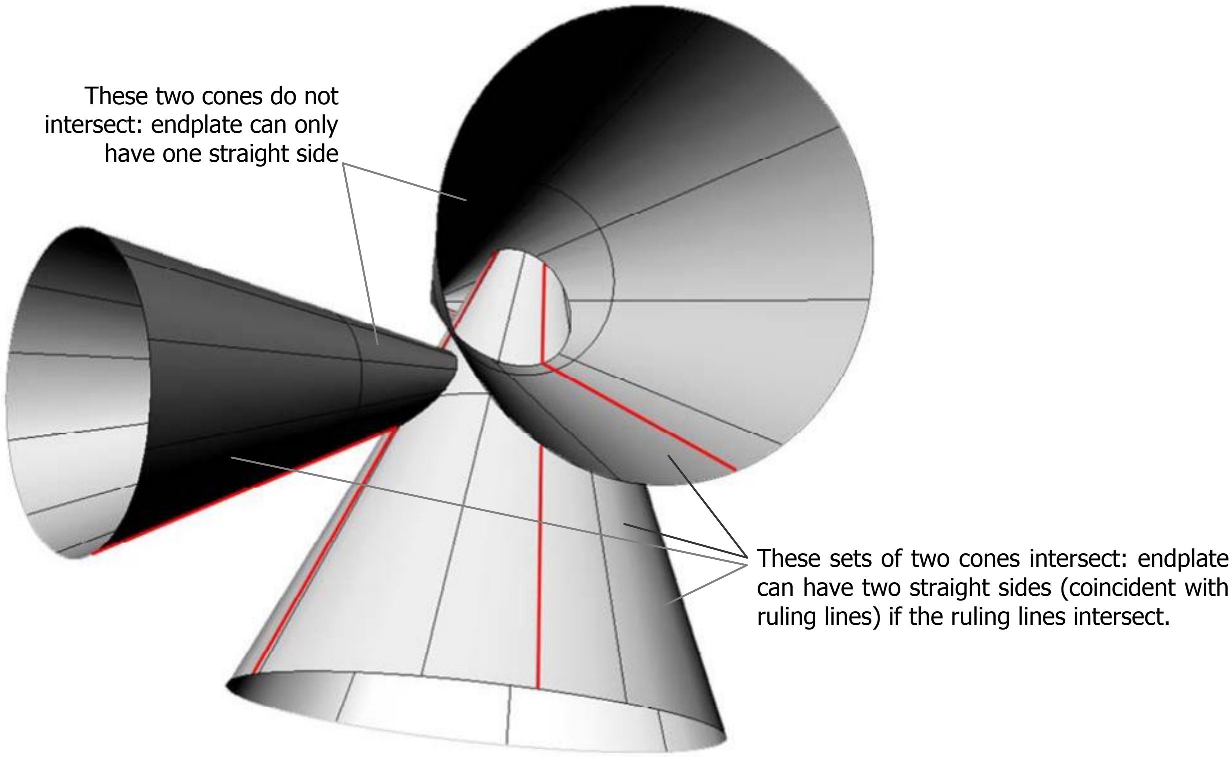


Figure 121. Creating endplates with one or two straight sides.

Positioning and construction

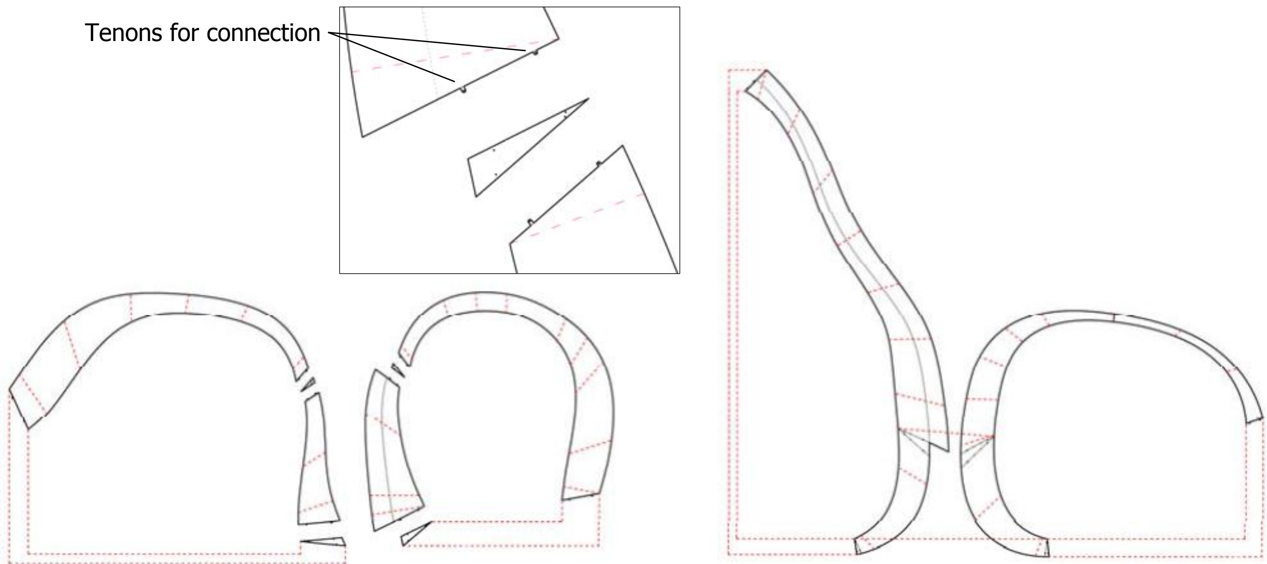
Until assembly onto triangular ribs, the curved sides are highly flexible and require intermediate support. This support is proposed to be provided by a rig for each element, constructed as a series of planar intersections in an orthogonal arrangement, each frame having a unique contour. As the elements are produced off-site and

[13] It is, for instance, also a prerequisite in an adjustable mould using point-like actuators.

individually, they will be orientated to facilitate construction, such that open topside is indeed directed upward. To facilitate positioning and avoid deformations under the loading, the frames should be placed vertically on the ground. The elements' geometry determines if their spacing can be regular.

Connections between intersecting ribs

In a layout with intersecting ribs, where the secondary (interrupted) rib connects to a primary (uninterrupted) rib, tenons from the first rib could slot into the latter, this way preliminary fixing the two, while simultaneously a correct relative positioning is achieved. Two ribs will intersect along a (space-)curve, where tenons at the interrupted rib's extremity punch through the side of the uninterrupted rib. There the tenons could be folded, or used to connect them internally to the next sheet to create an ongoing curved sheet. When folding them, they will do so along a straight line, hence the tenons have to be sufficiently narrow not to compromise the curvature of the connecting rib's extremity. The tenons' positions only requires a start and endpoint (and is independent of further local geometry such as orientation and angle of incidence), and may thus reference to the intersection curve only. Further engineering may simply take place in 2D through additions to the unrolled pattern as in Figure 122.



A. Secondary interrupted ribs with tenons at their extremities. B. Primary uninterrupted rib.

Figure 122. Unrolled patterns of ribs in sheet metal.

5.4.3 Structural action

Variation in magnitude and nature of the structural action occurring in the system is anticipated in several ways, in the construction phase as well as upon completion.

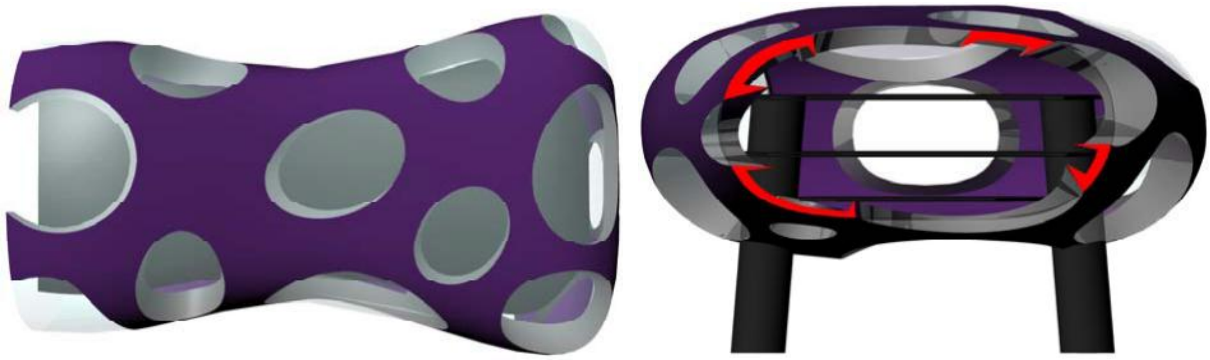
During construction, only the curved sheets provide stiffness to the system. Since the sheets should be flexible to allow for local controlled formation, the required stiffness cannot be derived from the sheets proper stiffness, but instead has to be obtained from either their constitution, or external means. Both ways are proposed, to be used in combination: first, the sides contouring the holes are assembled from three surfaces together creating a triangular cross section.[14] This section resists torque, and may thus be applied in structural members loaded out of plane. Second, the rig used to position the coils until their assembly onto ribs also serve as temporary support during the construction phase.

[14] The Delta Rib-system uniquely rests on this assembly. In the actual case of the 3D-element however, the system also derives load bearing capacity from the building envelope. Furthermore the actual system employs looping surfaces – whereas the Delta Rib-system uses only segments of cones and cylinders, and therefore allow for the more restrained usage of constant curvatures. The two systems consequently share one principle, but are implemented differently.

Structural action of the completed structure is anticipated on several levels of scale: on the building level of scale, the spacing of the three layers determines the structural depth of the ribs and of the structural envelope (Figure 120B). With alternative configurations of the triangular cross section, the middle layer may be left out resulting in a single structural depth for both the rib and the envelope. Also the curves' pattern is defined on the building level. A denser pattern and curves running in favourable directions increase the structural capability. Obviously such modifications intervene with the building's architectural and functional design.

5.4.4 Implementation on the DO Bubble

An exemplary implementation of the 3D-components system on the DO Bubble is depicted in Figure 123A. Figure 123B and Figure 124 highlight the component's structural infill, respectively the edges that are characteristic to the system. Figure 125 shows a single 3D-component. Such 3D-components could be constructed on an orthogonal rig as shown in Figure 126.

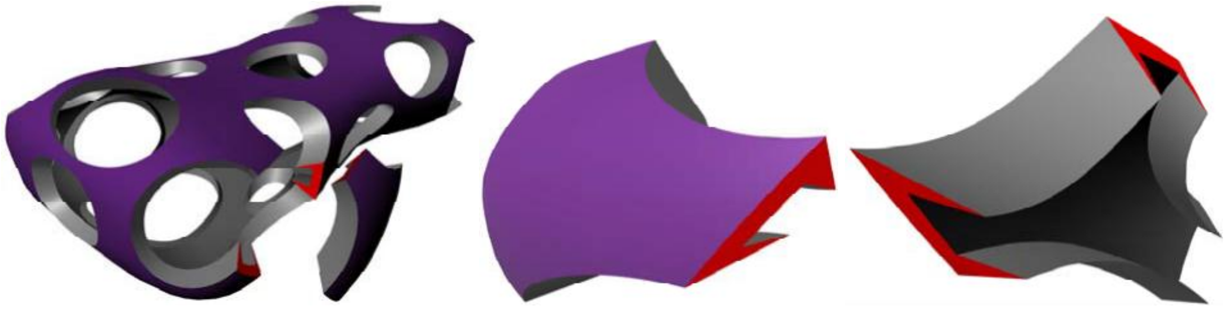


A. Top view.

B. Cross section at the DO Bubble's 'waist', the 3D components are marked red.

Figure 123. Implementation of the 3D-components structural system on the DO Bubble.

Figure 124. Implementation of the 3D-components showing only the components' edges.



A. Overview. B. Outside. C. Inside.

Figure 125. 3D-component.

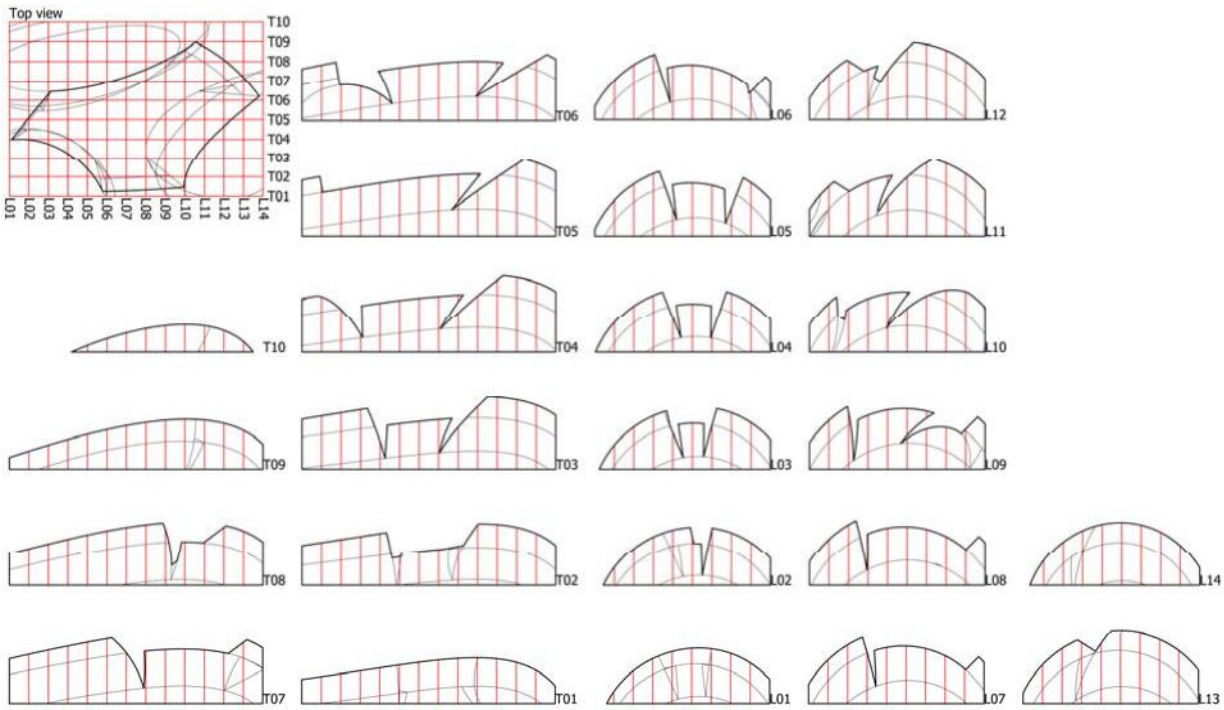


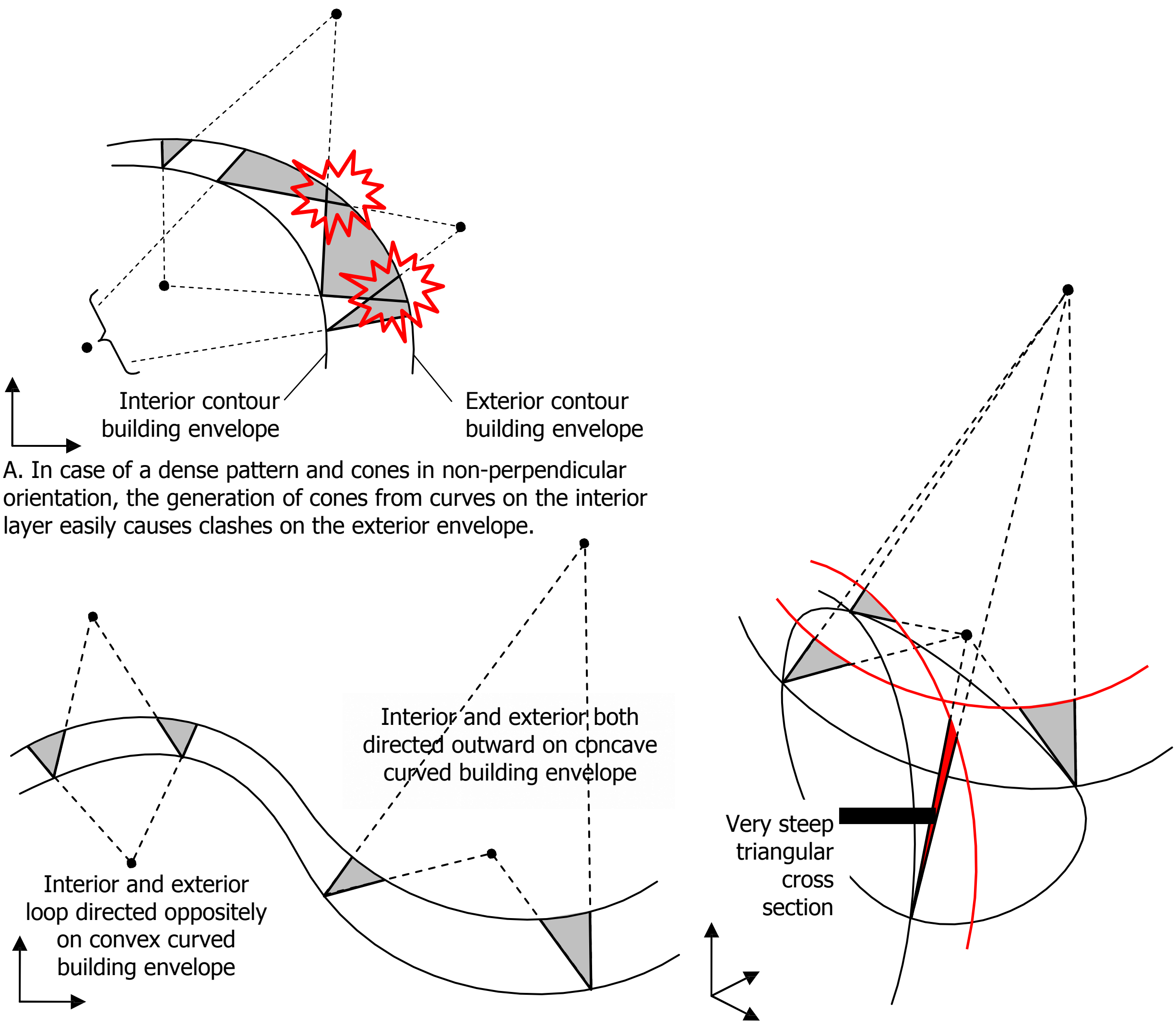
Figure 126. Rig for constructing a 3D-component.

In the implementation of the system to the building envelope of the DO Bubble, several criticalities of the system came to light:

When orienting the cones irregularly to the building envelope in a dense pattern (rib's structural depth of the same order of magnitude as spacing between ribs), it has appeared difficult to meet the system's constraint that two ribs should either not or fully intersect. Figure 127A shows a schematised example of this situation. Though intersections between cones occurring on one referential surface can still be overseen, doing so on each of the three referential surfaces requires a sequential iterative approach, going from one surface to the next. This problematic positioning occurs least when orientating the cones in an orthogonal orientation to an externally convex building envelope, since in that case the cones open up outward, and secondly the intersections with each of the surfaces remain close together. Any deviation from this ideal setting potentially yields into complications.

The non-constant curvature of the building envelope is anticipated in the system by allowing for cones of different heights, the interior and exterior loop directed oppositely in areas of slight and convex curvature. Contrarily, in case of a concave curvature, the cone's direction may need inversion to maintain its orientation relative to the building envelope, see Figure 127B. Though manually the same routine, in an automated implementation this requires input from local geometrical data.

The complication mentioned previously appears it its ultimate extent when the system is applied on a double curved surface with curvatures directed oppositely. Here, the optimisation of the cone's section with respect to the building envelope's concave direction may yield in an unfavourably awkward rib section for the other, convex, direction. This is demonstrated in Figure 127C.



B. Swapping from inversely directed cones (left) to two cones orientated in the same direction.

C. Wide variation in aspect ratios triangular cross sections.

Figure 127. Domains of problematic system implementation.

5.4.5 Evaluation

The system of 3D components offers a great potential in the integrated construction of load bearing structure and building envelope. However, still numerous contradictions as depicted in Figure 128 were found, and need to be resolved before a test application could be constructed. The primary contradiction is the usage of a manually deformable material to construct the irregularly curved sides of the ribs. For its structural functioning in the completed stage, the structure should instead be stiff. This stiffness is to be obtained fully through interconnecting the three rib sides onto triangular box-sections, hence making the curving and assembly of rib-sides a highly critical phase. Furthermore, also filling the space between the ribs requires a moulding technique of unprecedented flexibility. As this moulding is highly specific to the material to be moulded, only a suggestion is made that employs the ridge that is already in place for the construction of the rib sides.

The usage of a formative fabrication technique also constrains the system's structural functioning, as elements of higher flexibility will also be more vulnerable to local instabilities within the component. Finally, the composite action between the ribs and the load bearing envelope is to be assured.

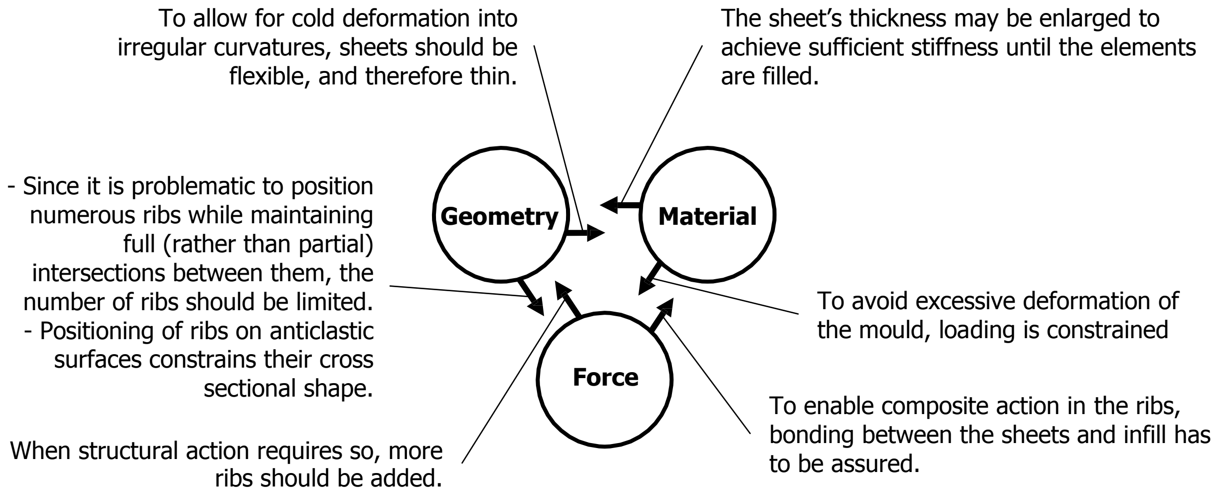


Figure 128. Requirements and constraints of the system of 3D components.

6 Prototyping and further development of Delta Ribs

In this chapter, the prototyping[15] of the Delta Rib structural system is presented and evaluated. The system is based on the scheme of three intersecting curved surfaces as presented in chapter 4, and implemented onto a structural system in chapter 5. Based on the results, modifications to the system are proposed. Furthermore, explorations on a parametrical implementation are presented. The prototyping is evaluated both on the product-level as well as on the process-level, and serves the final conclusions and recommendations in the next, last, chapter.

6.1 Aim

Constructing the prototypes served testing the validity of the assumptions made during the development of the Delta Rib system. So far, despite anticipating manufacturing constraints, the system was entirely a geometrical construct. The prototypes will be evaluated on the following aspects:

Buildability – Although buildability has been set as a primary requirement of any system proposed within this research, whether or not it indeed is buildable is assessed unambiguously by its full scale construction. This requires each variant of the Delta system: without flange, with strip flange and with tubular flange. In this context, buildable comprises both the preparatory phases and the actual construction from the base material onto elements and components, as well as all data transfers between these.
Feasibility of assembly – To allow for industrialised application coherent with the tools used for design and engineering, on-site building activities should best consist of assembly connections (in steel construction conventionally through bolts), despite welding involving less formal criticalities. Therefore, the feasibility of the assembly using either a strip- or tubular flange and an endplate needs to be assessed.
Neglecting sheet thickness – The surface representation (of zero thickness) of sheet material (of non-zero thickness) that has been adopted so far stems from the linkage between the design variables of developable geometries and material processing using a formative process. As the sheet thickness to be applied was small compared to the size of the prototype and it was unknown how the sheet thickness could practically be anticipated, it was decided to use the reference surfaces as cutting patterns without any anticipation to the material thickness. Actual construction should explicate to what degree neglecting the sheet thickness is allowed. Where needed, directions for the inclusion of sheet thickness need to be formulated.
Dimensional accuracy – Assuring dimensional accuracy involves the comparison of the built geometry with the digital model used to construct it. Notably since bolted connections are envisaged, sufficient dimensional accuracy is critical for the buildability of the system, but since the dimensional accuracy can be assessed numerically, it is listed as a separate criterion. Acceptable tolerances are to be defined per element of operation.

Construction of the prototypes and its evaluation according to these criteria forms part of the overall assessment of the Delta Rib-structural system, according to the criteria of material efficiency, formal freedom and degree of systematisation.

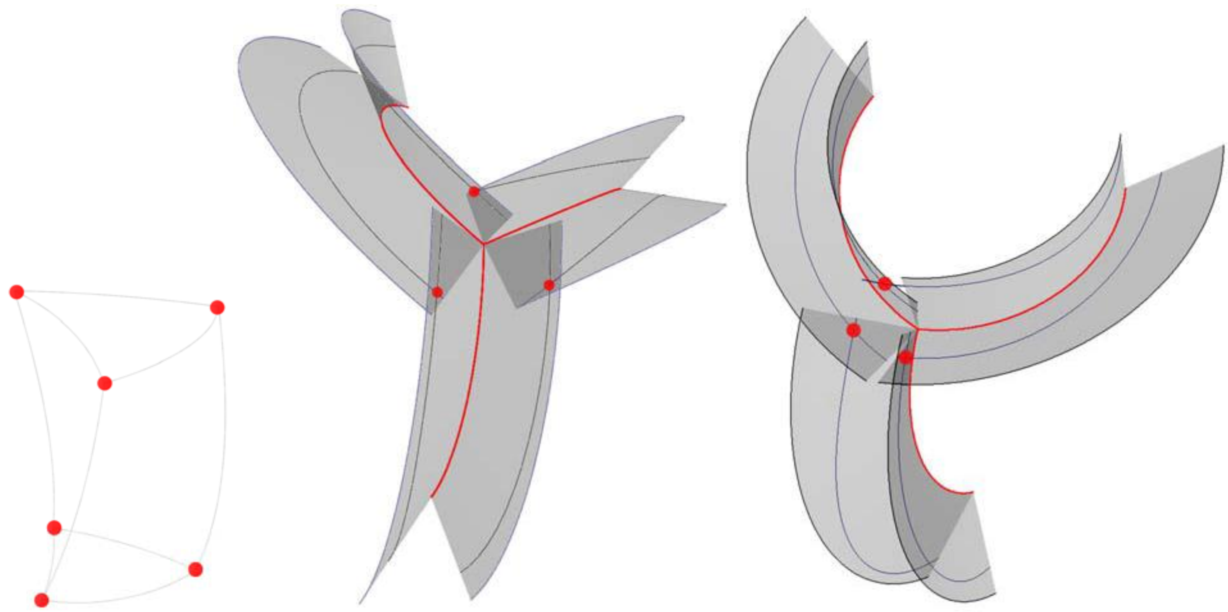
[15] The author acknowledges Corus, Kersten Europe and Delft University of Technology, all from the Netherlands, for their contribution (steel supply and laser cutting, steel rolling, financial support, respectively) to the realisation of the prototypes.

6.2 Realisation of the prototypes

6.2.1 Design

Three prototypes were designed, each representing a variant of the Delta Rib system: prototype 1 not involving any additional element, prototype 2 including a strip flange, and prototype 3 including a tubular flange. As the first one was obviously the most simple, it was only constructed as a rib-segment, whereas the remaining two prototypes consisted of three ribs joining in a node – this way including all complexity resulting from this node. It was decided not to test the constructability of a network configuration, but instead concentrate on a single node.

Prototype 1 was designed by rotating and transforming a triangular cross section, this way simulating a twisted rib (Figure 129A). In the engineering phase, surfaces were to be constructed between these points. The contours for prototype 2 and 3 were prepared in Maya (AliasSystemsCorp. 2005), where the parametric linkage of the enclosing surface intersecting with the conical surfaces provided direct feedback on the resulting rib shapes (see Figure 130). The lower edges of the ribs in this prototype were constructed from arcs, although this was only required for the prototype with the tubular edge (Figure 129B and C). All arcs were identical, which in the case of prototype 3 would lead to bending a single tube, and then cutting it into three parts, this way limiting waste. The rib's extremities were defined by the conically curved surface spanning between the input arc and its scaled duplicate, and were therefore straight. Whereas the constantly curved sides could be employed directly in the engineering phase, the ribs' upper sides were constructed later, in the engineering phase. For this the corner points obtained from intersecting the conically curved sides with a double curved enclosing surface were provided. This enclosing surface was positioned such that the ribs would display a variety of structural depths.



A. Prototype 1. B. Prototype 2. C. Prototype 3.

Figure 129. Input points and curves for the three prototypes, as preparation for the surfaces to be generated in the engineering phase.

6.2.2 3D-engineering

All 3D-engineering was carried out in Rhino (McNeel 2002), see Figure 131. The position of elements as well as their naming is depicted in Figure 132.

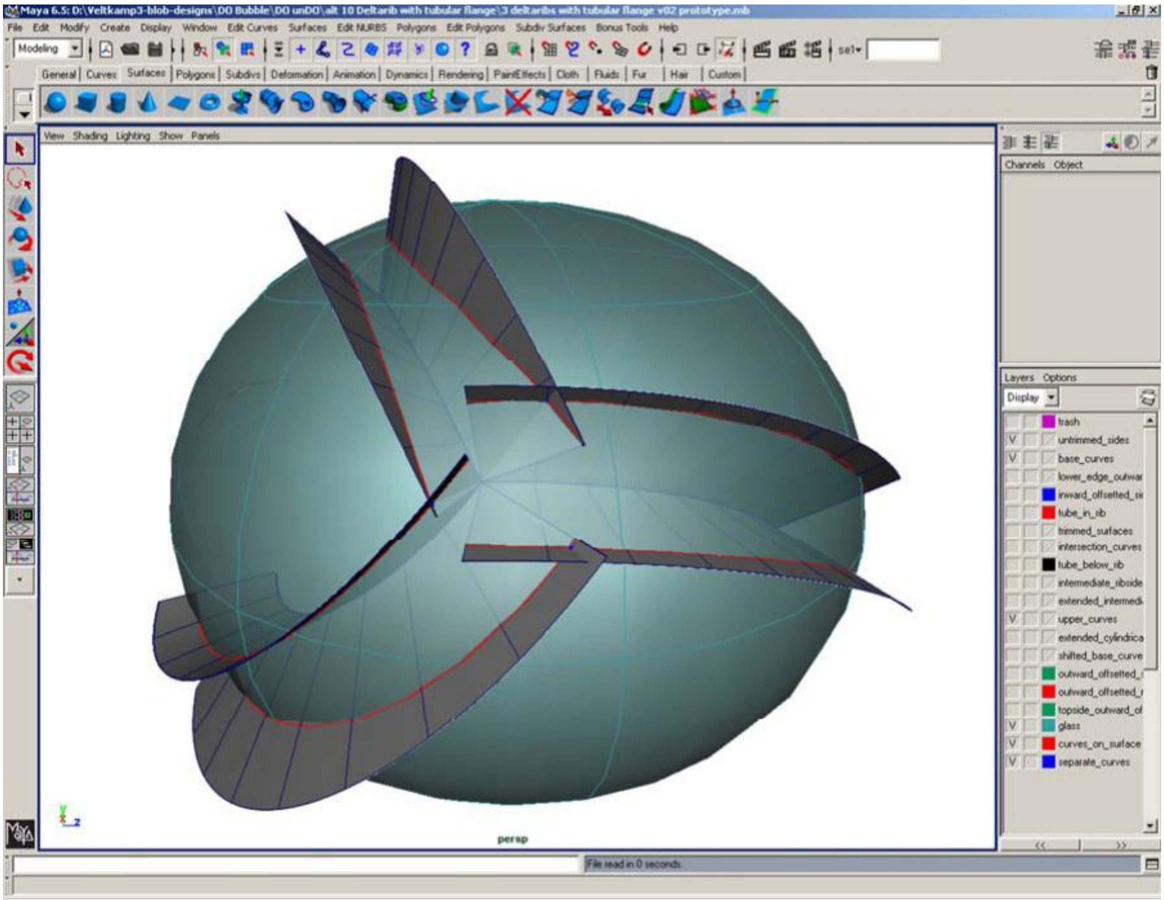


Figure 130. Shaping of the contours for prototype 2 in Maya, to be approximated through constantly curved conical surfaces.

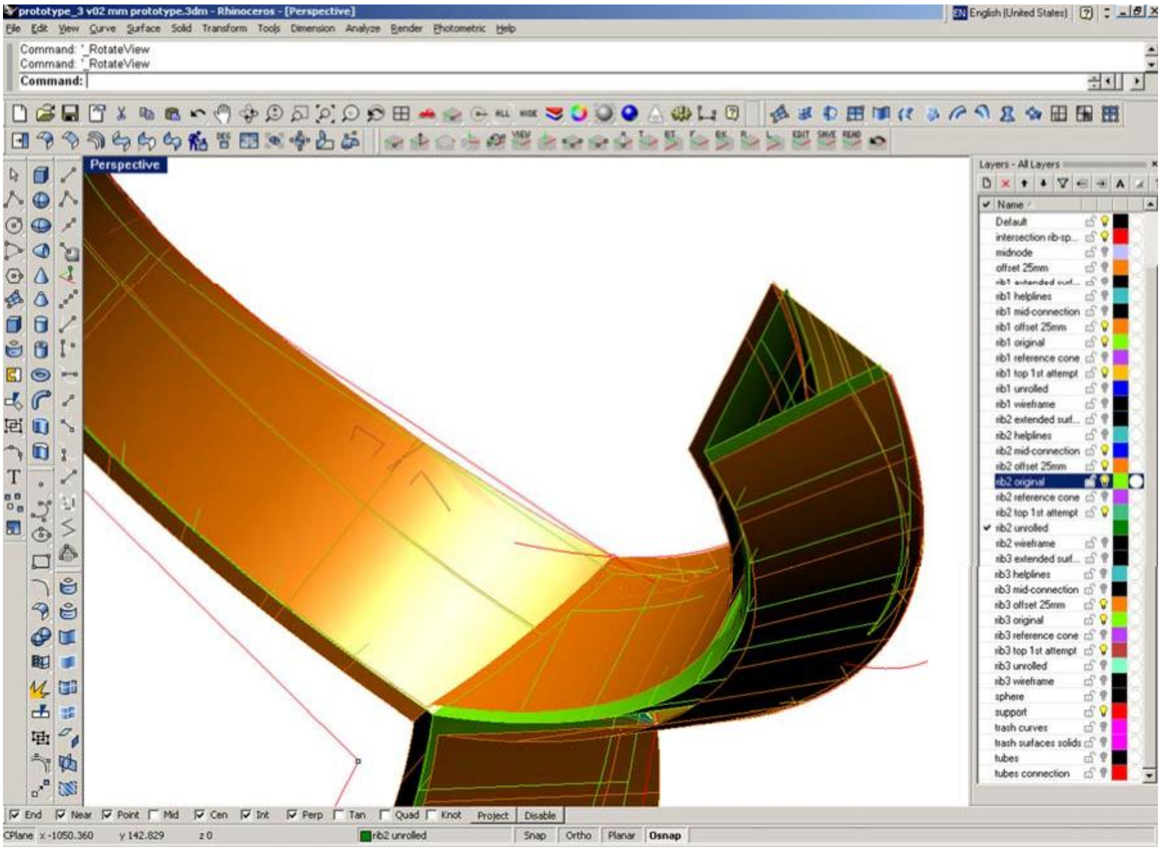


Figure 131. 3D-engineering of prototype 3 in Rhino.

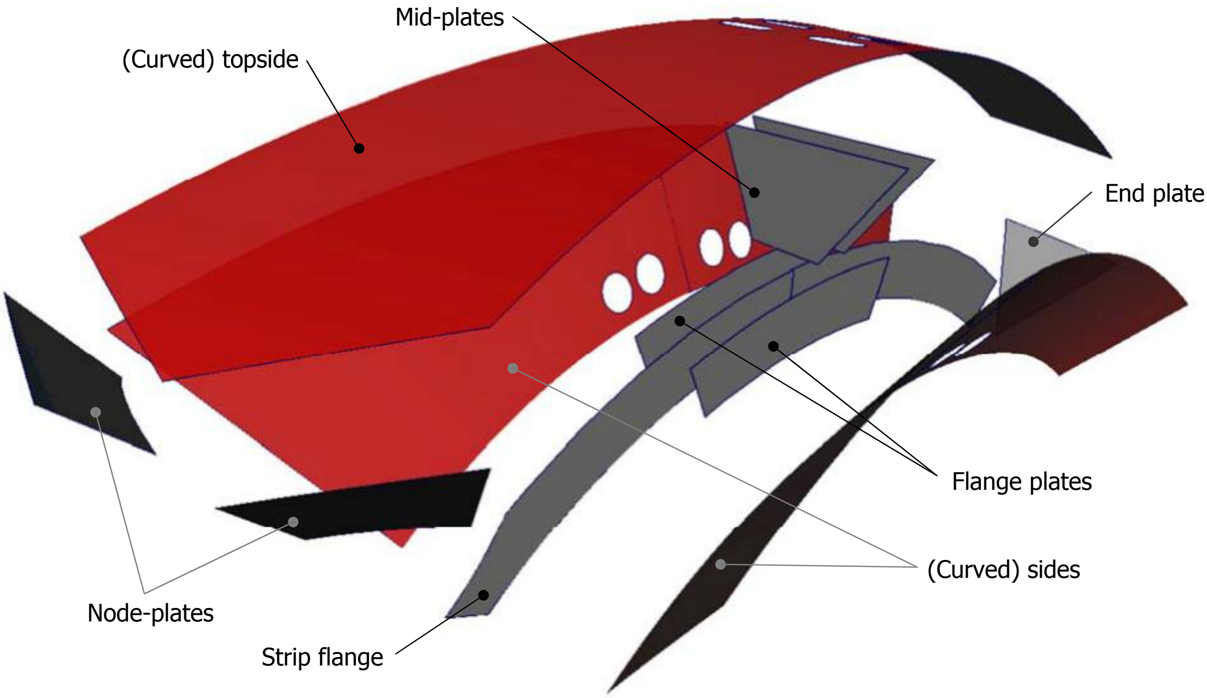


Figure 132. Naming of the elements in an exploded view of a rib from prototype 2.

Generating developable surfaces of constant conical curvature in between input points

Transferred as IGES-file, the points and input curves were imported into Rhino to generate constantly curved surfaces between them. In prototype 1 these were constantly curved cylindrical surfaces, while all curved sides of prototype 2 and 3 were conically curved. Only the top surfaces of the latter two prototypes still had to be constructed. Whereas the corner points were constraints, the cone's top was employed to approximate the intersection curve with the double curved enclosing surface as defined in the engineering phase.

Detailing anticipating the tubular edge

The curved surfaces of prototype 3 tangent to the tube were offset 25mm (= the tube's radius) in outward direction. Next, intersections with adjacent sides were reconstructed and redundant parts trimmed off.

Detailing sectioning plane including strip flange and tubular flange

The sectioning plane was modelled at which the rib would be split. From this section the splitting lines on the three rib sides and the strip were retrieved, and marked with 'split'. The centres of boltholes were indicated on the strip flange, and projected on the curved sides, perpendicularly to the strip to indicate the centres of the access holes. Centres of boltholes and access holes were also modelled for the sectioning plane and the curved topside. As no markings could be made on the tube, no 3D-engineering was to be made for this connection.

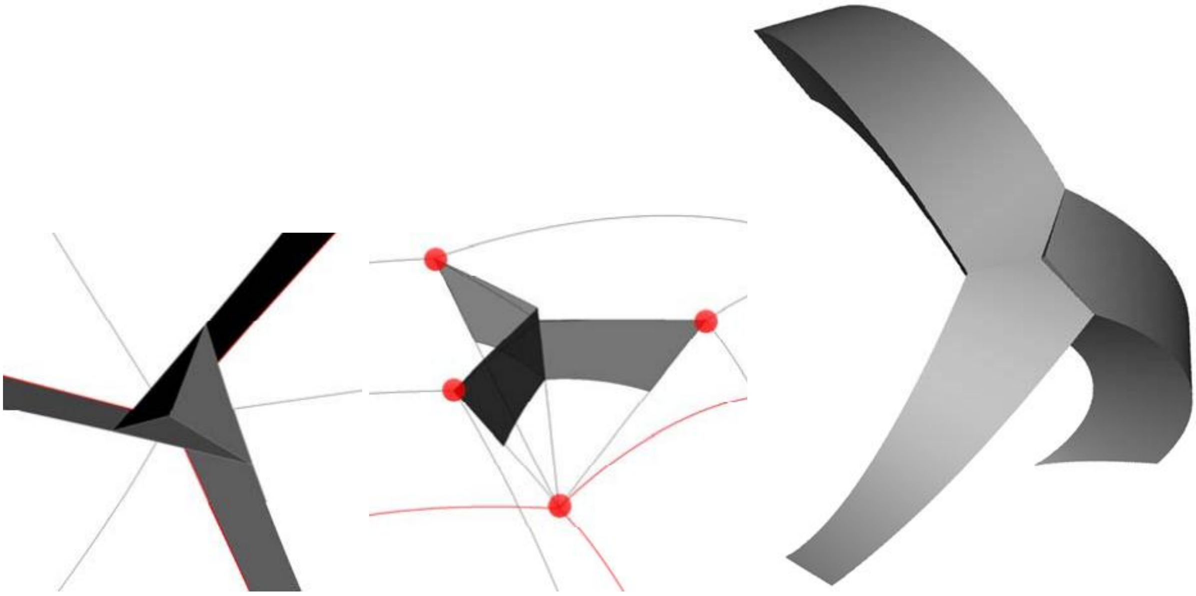
Detailing intersection three ribs

The detailing of the intersection of three ribs was the most geometrically complex part due to the large number of elements coming together along curved lines, and meeting at different angles.

In the lower edges, the strip flanges were modelled to intersect in one point, while their inclination creates a pyramid with triangular ground plane. This connection type is universally applicable, independent of the angle of intersection (it is also proposed as connection detail for planar timber elements, see section 5.2 Case study 1:

implementation of the Planar members with end connections-scheme onto a structural system). No engineering preparation was made to connect the tubes.

As the curved topsides of the ribs are all constantly curved, they can only be connected seamlessly if each pair of adjacent sides shares a ruling line. In these ribs, this is not the case. Although transient approximations could have been made, it was decided to explicitly model the interruptions as the logical outcome of the rationalisation process. This detailing involved the introduction of three node-plates, intersecting in a straight line pointing at the ribs' lower intersection, and going through the upper corner of two neighbouring rib sides, see Figure 133. The upper extremities were aligned with the topsides of the adjacent ribs. The side boundaries were trimmed, but would only accidentally align with the intersection between two rib sides.



A. Intersection of three planar strips.

B. Reference points of the connection detail through midplates.

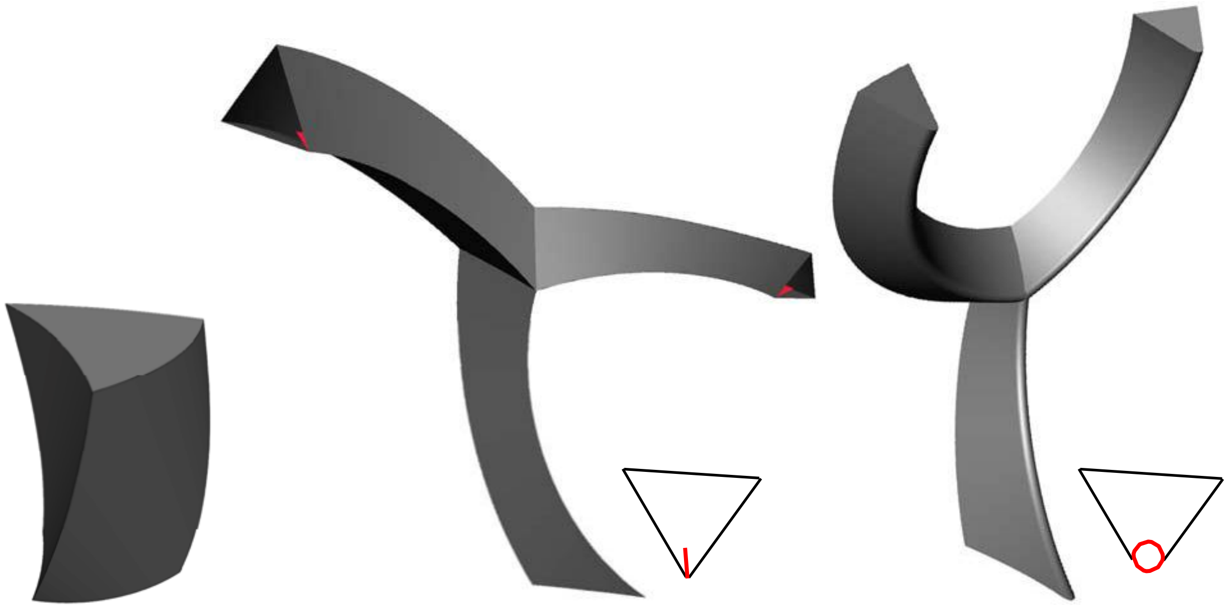
C. Digital model of the resulting external geometry in prototype 2.

Figure 133. Connection detail ribs' upper sides through three planes.

Miscellaneous

Positions of boltholes and holes providing access to these are made in the 2D-engineering, but their positions were already indicated in the 3D-model using polygonal cut-outs (after unrolling these were still traceable, whereas points and curves on surfaces weren't). Furthermore the height and radius of a cone, at a location traceable once unrolled, have to be measured, serving determination of the rolling radii.

To make the prototypes standing stable, the digital model was imported into a finite element analysis-software, where meshing attributes (material and thickness) were given to the surfaces such that the centre of gravity could be determined. The model was then oriented such that its centre of gravity was above the cross section of a rib, where a footplate was generated. The resulting models are depicted in Figure 134. For a realistic impression, also the tubular edge is modelled, although its engineering was entirely based on its centre line (and not its surface).



A. Prototype 1. B. Prototype 2. C. Prototype 3.

Figure 134. Digital models of the three prototypes.

Unrolling

All curved surfaces of the ribs are unrolled to a plane using the function unroll developable surface. When all surface normals of the 3D-model are pointing inward, they will be positioned in the XY-plane with their outside in positive Z-direction.

6.2.3 2D-engineering

All 2D-engineering was carried out in AutoCAD 2004 (Autodesk 2003).

Importing

The unrolled surfaces are transferred as AutoCAD2004-file using Rhino's export-function. By this transfer, the surfaces are converted to curves, and become a graphical representation of the surface to be rolled.

Rolling- engineering

Rolling engineering was carried out by the author, instructed by Marc Wijnhoven, head of the engineering department of Kersten Europe (Wanssum, the Netherlands). It involves:

- marking of ruling lines (straight lines that should remain parallel to the rolling mills, and thus facilitate the operator of the rolling mill);
- checking that the ruling lines well intersect in a single point;
- the generation of additional area to facilitate rolling: two of the sheet's extremities should coincide with ruling lines, and a third edge should describe a planar curve (an arc) once rolled;
- markings to be used to asses the curvature of the curved sheet;
- making codifications of the sheet needed for assembly;
- drawing boltholes and access holes;
- providing bridges to small holes and cuts that are positioned with an angle of incidence less than 45° with the rolling mills whereas large cut-outs parallel to the mill should be prevented at all to prevent uneven curving, the experience of the rolling-firm prescribed;
- providing at least 20mm space between the sheet edge and cuts.

All markings are to be made on the concave side as this will be the visual side during rolling, convex panels thus need to be mirrored, see for an exemplary engineering drawing Figure 135. From the cones' height and radius measured in 3D and the length of ruling lines measured in 2D (Figure 136A), the radius of circles can be calculated, as well as the chords between them (Figure 136B). Furthermore, by comparing the angle the cone's segment fits in at the height of the greater and of the smaller radius, the consistency is checked.

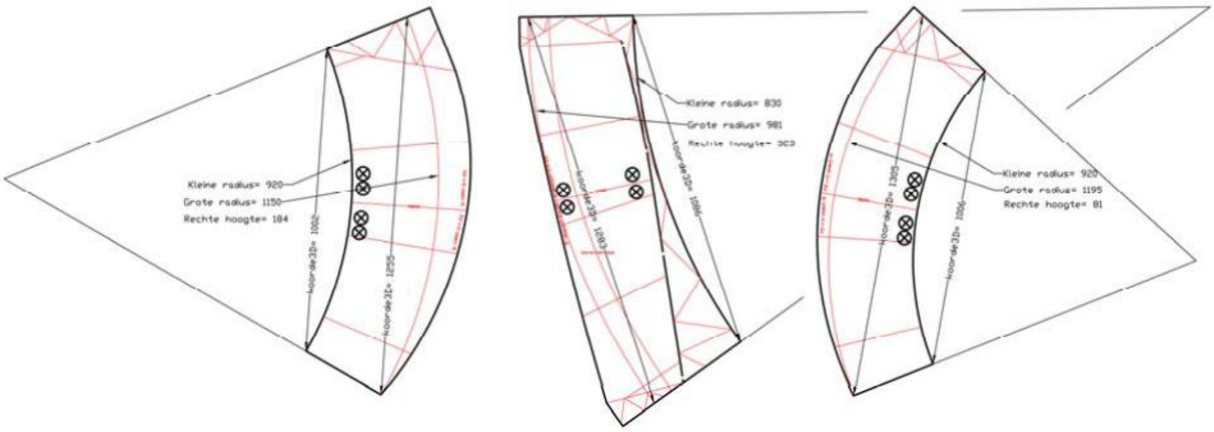
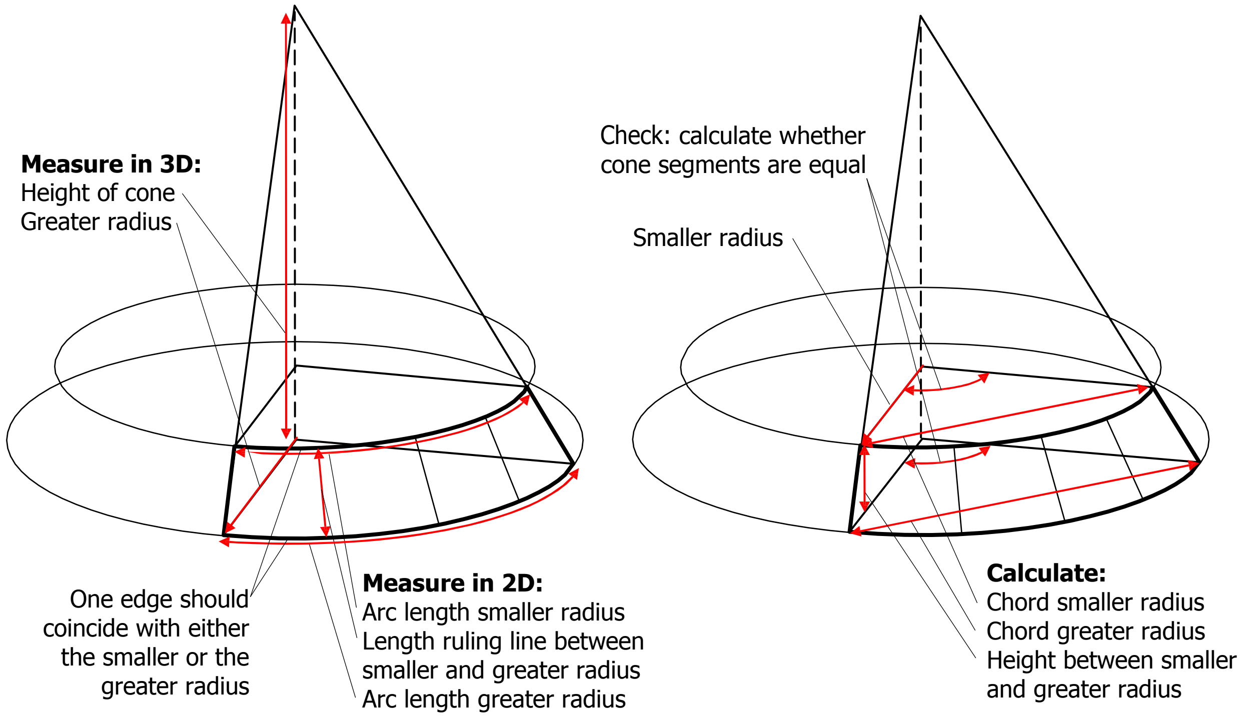


Figure 135. Engineering drawing of sheet 'P2-r1-topside AB convex'.



A. Dimensions to be measured for calculating the curvature of a conically curved sheet.

B. Calculated dimensions for communicating the curvature of a conically curved sheet.

Figure 136. Dimensions for communicating the curvature of a conically curved sheet.

Cutting-engineering

Cutting engineering was carried out at Corus Feijen Fabrication (Maastricht, the Netherlands) and involves the distinction between cutting lines and marking lines and the creation of closed contours. In this case, it also involved the conversion of splines and ellipses, which the cutting machine could not read, into lines and arcs. Later versions of the software are able to do this conversion by itself. The programmer operating the cutting machine then imports this geometrical data, indicates which lines to cut and which to mark, as well as the path the tool should follow when making these cuts and marks. This results in a file as displayed in Figure 137.

Although being treated sequentially here, obviously already in the engineering phase the line type and colours are anticipated to the cutting engineering taking place next.

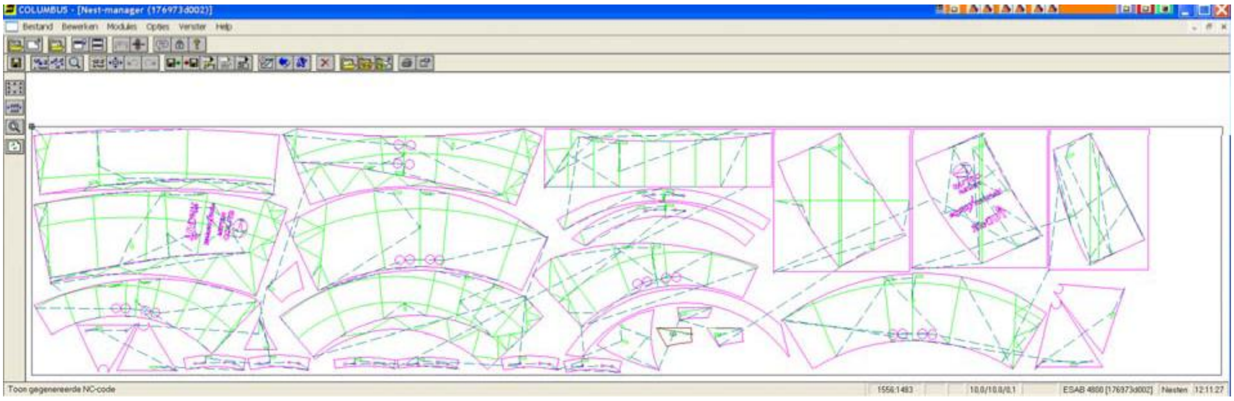


Figure 137. Shapes to be laser cut nested on a steel sheet. Image courtesy Corus Feijen Fabrication.

6.2.4 Production of elements

The elements were laser cut from 3mm thick sheets of conventional S235 JRG2-steel at Corus Feijen Fabrication (Maastricht, the Netherlands). The planar elements were then transported to Kersten Europe (Wanssum, the Netherlands) to be rolled to the required shapes using the information provided by the rolling engineering, see Figure 138.

A. Rolling operation.

B. Verification of dimensions.

Figure 138. Rolling of the sheets.

6.2.5 Construction of the prototypes

Construction of the prototypes took place in the Laboratory of Building Technology in the Faculty of Architecture (TU Delft, the Netherlands) during four days, by a professional MIG/MAG welder specialised in sheet metal, assisted by the author. The elements were first constructed onto ribs and then assembled onto node-segments of three intersecting ribs.

Removing redundant areas

Redundant areas that were needed for rolling were removed with a hand grinder. In some cases only narrow 'bridges' had to be cut through, whereas in case of cuts (nearby) parallelism, the full cut had to be made.

Positioning of elements

For positioning, two curved sides were held in their approximately position by hand by the assistant and then tack welded by the welder. The tack welds underwent plastic deformation when deforming them to positioning the third curved side. Once the latter was also correctly positioned, it was connected through tack welds too.

Using the referential surfaces to create the sheet patterns not only resulted in dimensional inaccuracies (see sections 6.3.3 Allowability to neglect the sheet thickness and 6.3.4 Dimensional accuracy), they also caused impracticalities as the corner points of sheets only matched in their (referential) middles but not on their surfaces. This occurred strongest when rib sides were inversely curved (e.g. a concave side with a convex topside), and the shortened concave edge needed to be attached to the extended convex edge. As the mid-plates (used for the connection) were

Apart from the before-mentioned ambiguous interpretations, positioning of the three curved sides of a rib went straightforward in prototype 1, where the three sides included an inherent dimensional check. Considerably more complex was the positioning of prototype 2 and 3 due to the additional element at the edge (Figure 139). These four elements together created an instable mechanism, of which the positioning could not be verified internally, but for which external references had to be employed: by means of mid- and end plates (conceived for the connection to be made later), as well as two cardboard moulds. Whereas the positioning of sharp-edged parts could be verified explicitly, the tangent positioning of a curved sheet to a tube was problematic and definitely needs to be rethought. In- or external moulds (e.g. the mid-plates with the flanges' contours indicated on them), as well as parallel marking lines on the tube's surface, would facilitate the positioning.

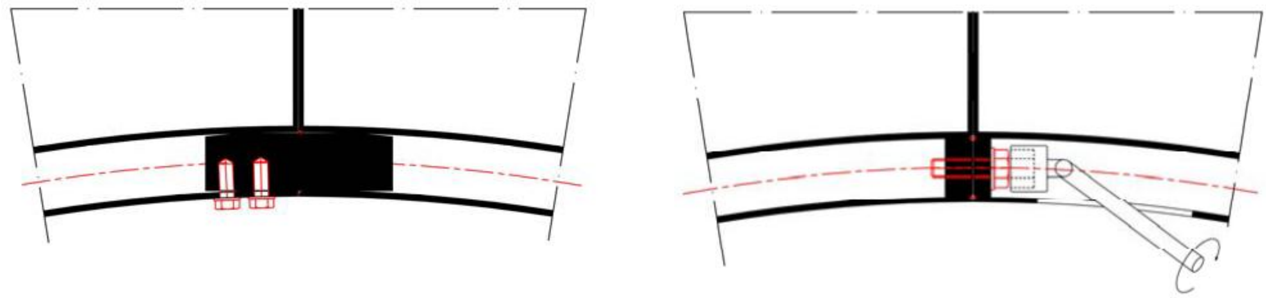
A. Positioning of curved sides and strip. B. Three rib sides prior to positioning the tube inside it.

Figure 139. Prototyping: positioning of sides.

Construction of assembly connections

The connections were made by cutting the rib in two using a hand grinder. On convex sheets, the marking line was transferred beforehand from the concave side. Next, mirrored mid-plates were inserted into both rib-ends. Despite the obvious continuity of the rib before splitting and the usage of identical mid-plates on both sides, the boltholes did not align and had to be enlarged. This was due to thermal stresses causing deformation once the rib was divided in two.

The connection at the lower edge was made through two flange plates (copies of the flange itself, though shorter and of lesser height to accommodate the strip inside the rib), while the connection in the tubular flange was made by placing a solid bar in one extremity, fixated by two bolts running through the tube into screw-thread in the bar (Figure 140A). The detail originally foreseen, capable to transmit higher forces, consisted of an axially loaded bolt inside the tube, which is accessible through an opening in the tube (B).



A. Connection as realised: solid bar welded to one tube, slotting into the other and fixated by two bolts.

B. Connection of higher capacity: axially loaded bolt inside the tube.

Figure 140. Assembly connection in tubular flange.

A. View inside the node of prototype 2, showing the intersection of the three flanges.

B. The completed prototype 2.

C. Ribs of prototype 3 prior to connection.

D. Positioning of two ribs of prototype 3.

Figure 141. Prototyping: connections of flanges and ribs.

Construction of connection between three ribs

Once the ribs were constructed, they were test-positioned to determine fitting. It turned out that narrow strips had to be grinded off from the extremities of the strip flanges (Figure 141A). The tubular flanges were cut off mitred (D), through trial and error. Once fit, all connections were tack welded. To make welds between the flanges and insert plates at the rib's upper sides, both in prototype 2 and 3 one of the three rib-topsides had to be temporarily removed.

Conservation

After construction, the prototypes were protected by a transparent powder coating, preceded by lightly sandblasting the surfaces.

6.2.6 Finite element analysis of a typical node

Structural analysis was carried out on prototype 2, using the commercial finite element software packages Femap for pre- and Caefem for post-processing (Concurrent Analysis Corporation 2005; UGS Corporation 2006). The finite element model was constructed by converting the 3D-engineering model into the STEP-file format, and importing this into Femap. No geometrical modifications were needed after this data transfer. Available meshing tools were sufficient, but it was found that inaccuracies inside the engineering-model were directly propagated to the analysis model, where they caused non-matching nodes on two neighbouring surfaces. The problem was overcome by automatically merging proximate nodes, resulting in modelling errors. As the system is largely based on section-action, it is not very sensitive to eccentricities. Hence, it is expected that the modelling errors do not worsen the results to a significant extent.

6.3 Evaluation of the buildability of the prototypes

6.3.1 Buildable

The ability to build the proposed system is not a performance criterion but a primary requirement. The successful construction of the prototypes suggests that the proposed system is buildable. Nevertheless improvements are to be made on the system's elements and components, as well as on the processing of the intermediate stages. These improvements should comprise:

1. The development of a tool for replicated operations of engineering procedures (generation of surfaces between points, upgrading of 'raw' unrolled patterns to rolling drawings);
2. The marking of parallel lines on the bent tubes as a reference for the sheets to be tangently attached to it;
3. For applications including closed loops the material thickness needs to be taken into consideration (see section 6.3.3 Allowability to neglect the sheet thickness);
4. A simplified detailing of the connection between ribs' topsides. The detailing of the prototyping requires extensive engineering, while its result is highly vulnerable to dimensional inaccuracies. Furthermore, structurally it is little efficient since the in-plane forces in the rib's upper sides can, in the connections, only be transferred through the rib edges;
5. The before-mentioned simplified detailing should eliminate the temporary removal of already assembled elements;
6. The development of an adjustable mould that facilitates positioning of parts;
7. A setup (possibly in conjunction with the adjustable mould described previously) which allows assessing the dimensional accuracy of the constructed components already during intermediate phases. This is most needed in the case of ribs with either kind of flange.

6.3.2 Feasibility of assembly

The engineering and construction using planar strip-flanges went smoothly. This also applies to the actual assembly operation itself. To eliminate play in the bolt hole, these may simply be drilled at a tight diameter. The connection in the tubular flange as applied now should be replaced by one that is more structurally performing, by placing an axially loaded bolt inside the tube. Such connections are commonly applied thus no criticalities are expected.

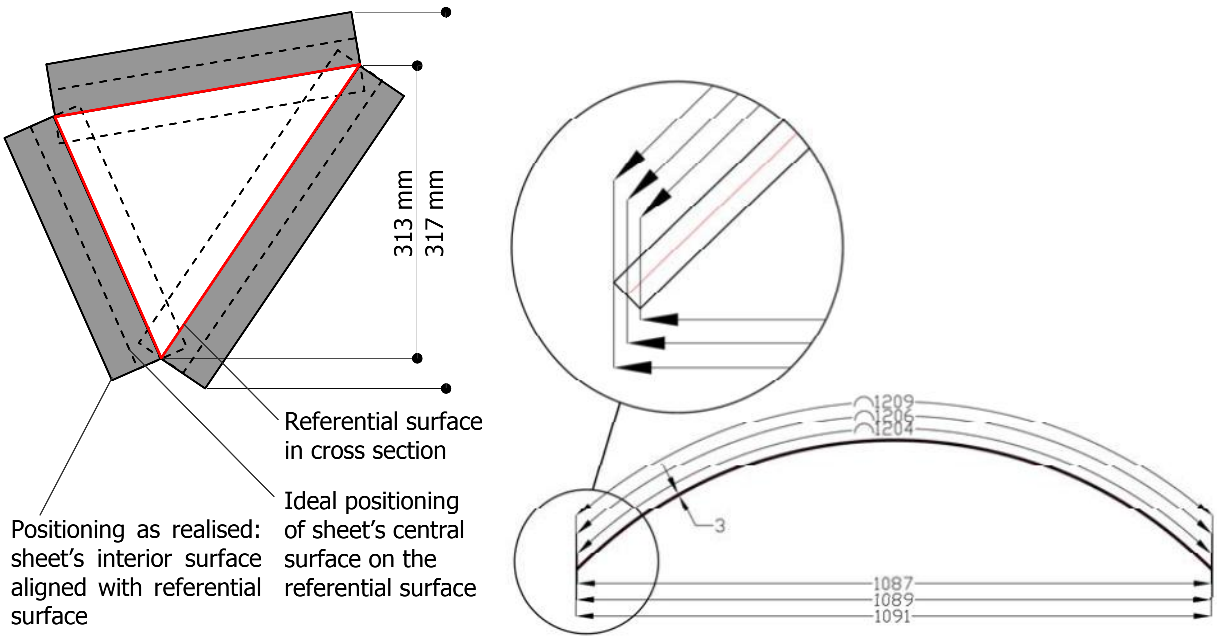
The connections in the mid plates are badly accessible for fingers as well as tools. As they are also little structurally efficient (as they load the mid plates in bending in the first place), they should best be replaced by another type. Propositions for the improved connection details are described in section 6.5 Improvements to the system.

6.3.3 Allowability to neglect the sheet thickness

Not considering the sheet thickness causes dimensional divergences in planar as well as in curved applications: two or more surfaces touch or intersect elsewhere than where they were envisaged in a single curve. As metal (unlike timber) has equal stiffness in compression and tension, the non-deformed reference plane is in the middle of the curved sheet. This allows rational anticipation.

Regardless of eventual curvature, the exterior dimensions of triangular sections (like those of the Delta Ribs) are too large as elements are positioned on the interior edge. Positioning elements on their convex edges causes that the component will be too long in longitudinal direction (convex edge is longer than the neutral edge), and contrariwise too short in case positioned on the concave edge. Figure 142 shows for an exemplary rib that divergences are approximately 4mm in cross section and not dependent on the size of the cross section. Dimensional divergence of a single sheet equals 2mm in longitudinal direction in this specific case, but will increase in case of smaller radii or thicker sheets. Dimensional divergence will double in case of connecting a convex and a concave side.

Once elements are assembled onto components (ribs), no adjustments can be made to these. The assembly of three ribs in nodes only allows minor adjustments by grinding the edges, or filling through welding material. Accumulation of dimensional errors is therefore a major risk. Processing of elements prior to positioning is therefore the most accurate solution. A solution for this is proposed in section 6.5 Improvements to the system.



A. Cross section of a typical rib of prototype 1, not to scale. The dimensional divergence equals (317mm – 313mm =) 4mm.

B. Difference in arc lengths and chords on inside, neutral axis and outside in case of a representative sheet. Dimensional divergence in case of a single sheet = (1091mm – 1089mm =) 2mm. In case of connecting a convex and a concave edge the dimensional difference equals (1091mm – 1087mm =) 4mm.

Figure 142. Causes of dimensional divergences through positioning on offset and curved edges.

6.3.4 Dimensional accuracy

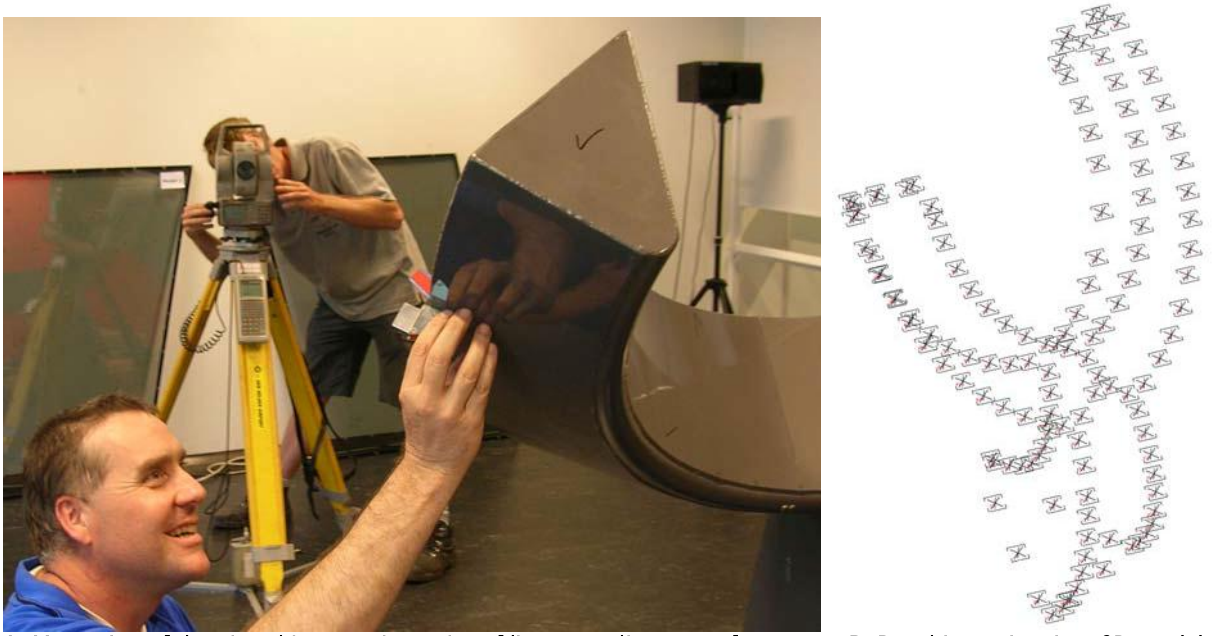
Although dimensional divergences were observed during construction, these were all sufficiently small to be resolved aesthetically. To quantify the divergences, prototype 2 and 3 have been dimensionally surveyed. The prototypes originally were not conceived to verify their dimensions in a quantitative manner. Hence no precise reference points were included on neither the physical model, nor the digital model. This required some manual manipulations: the digital model has been reconstructed such that the original model represents the referential surfaces. To make the reconstructed model represent the exterior surface, all curved sides were offset outward at a distance of half the sheet thickness. Next, these surfaces were extended along the sides' upper and lower edges to make them intersect again. A large extension was made at the lower edges of prototype 3, which were originally adjacent to the tube but now entirely disregarding it. As the surfaces were all single curved and the full cones were stored for possible future referencing, these extensions could be made accurately.

Surveying was carried out by three-dimensionally measuring the position of the virtual intersection point of two lines on two adjacent surfaces using a total station (see Figure 143A). This resulted in a series of points (B) with an accuracy of +/-0,5 mm[16], which was considered sufficiently for the dimensional deviations which were expected to be at magnitudes of scale of 10 mm. Three-dimensional laser scanning of the full objects has been considered as it would result directly in a complete digital model without the need for any re-engineering. However its accuracy in capturing sharp edges was less and the method therefore abandoned.

The newly constructed intersection curves along the rib's edges are to be compared with the edges reconstructed from the surveyed points. For this, nurbs-curves have been drawn through the surveyed points, followed by three-dimensionally positioning the surveyed model inside the engineering model based on the corners of the triangular endplates that were surveyed too. The two models were aligned with respect to the extremities of the ribs, as positioning the surveyed model on the ground plane would have resulted in a systematical error onward. Next, on the start, middle and end of each rib edge as surveyed, the distance to the digitally constructed edge was measured. The results are depicted in Figure 144A and B for prototype 2 and 3 respectively.

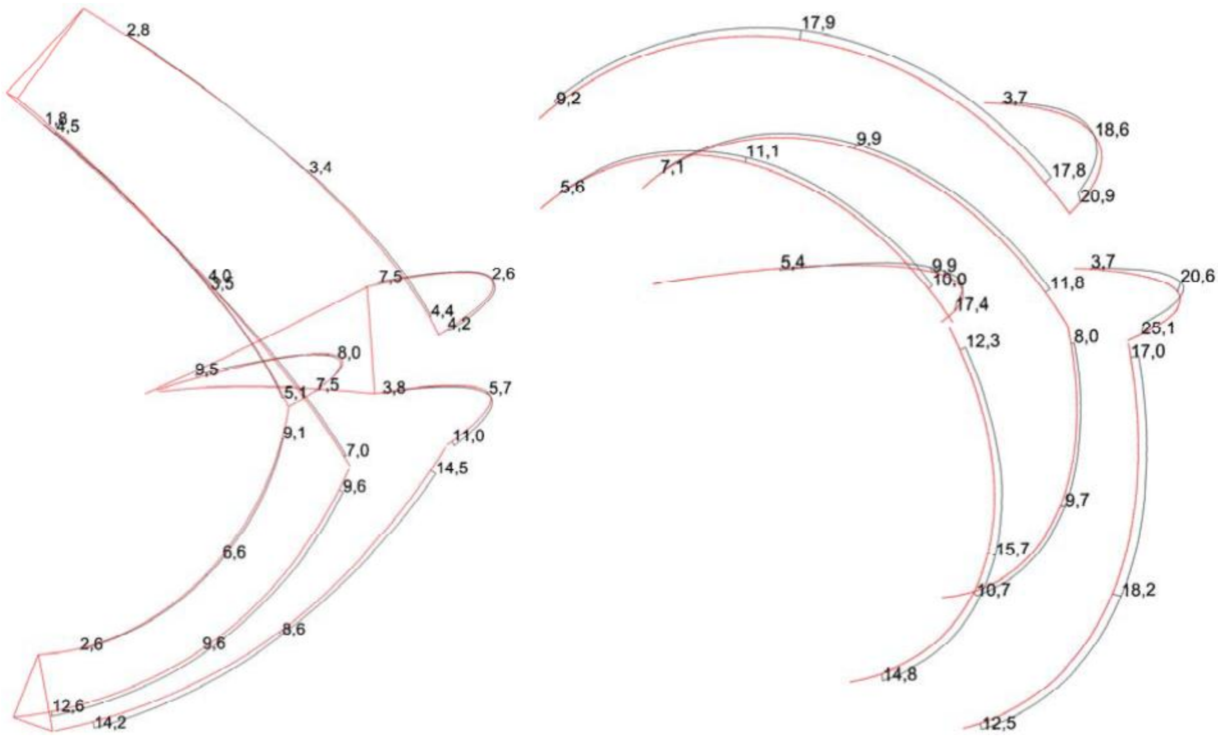
The measured dimensional deviations reached at maximum 14,5 mm in prototype 2, and 25,1 mm in prototype 3. However, visual comparison of the sets of edge curves teaches that in both prototypes the deviations gradually increased in the direction of the node in the majority of the cases. This points at the nodes (rather than the ribs) being the primary cause of dimensional deviations. This goes with the experience of the construction, where it was found that positioning the three curved sides had its checking mechanism included in the cutting patterns itself, whereas geometrical references were lacking when combining three ribs onto nodes. Therefore, while the ribs have their checking mechanism inside the ribs itself, dimensional accuracy of the system as a whole has to be achieved through a primary focus on the correct positioning of segments during construction – including intervention where needed. To assess the correctness of positioning, reference points have to be defined from the engineering model onward.

[16] Personal communication between the surveying company Passe-Partout and the author, prior to the dimensional surveying.



A. Measuring of the virtual intersection point of lines on adjacent surfaces. B. Resulting points in a 3D-model.

Figure 143. 3D-surveying of the prototypes.



A. Prototype 2. B. Prototype 3.

Figure 144. Dimensional deviations (in mm) between the surveyed points and the reconstructed engineering model.

6.4 Performance of the Delta Rib system per criterion

6.4.1 Highest degree of material efficiency

The system is predominantly acting in section action, but its variables enable the system's layout and its components to anticipate the required structural capacity:

1. The layout of the network structure, anticipating areas of demanding structural action by a denser pattern;

2. The size of the cross section, anticipating the overall increase or reduction of capacity;
3. The height-width ratio, anticipating the dominant bending direction;
4. The optional addition of a strip-like or tubular flange, the first of variable height and thickness, the second with a single external diameter, but with different wall thicknesses;
5. The sheet thickness, to be customised per side. In case a flange is applied, notably the side opposite to it may be constructed thicker to balance its cross-sectional area with that of the flange.

These variables may also be combined, and may vary along the rib. While the stiff cross sections provide resistance against buckling in case of axial compressive loading, bending capacity is available through the flanges. This is demonstrated in Figure 145 and Figure 146, where strip- and tubular flanges are added to a plain Delta Rib, and sheet thicknesses are varied. Furthermore, due to its closed box-sections, the system's ribs are naturally capable of transferring torque, to which adding flanges has no significant effect, as shown by approximately equally high bars for the polar moment of inertia (providing resistance to torque) in Figure 146.

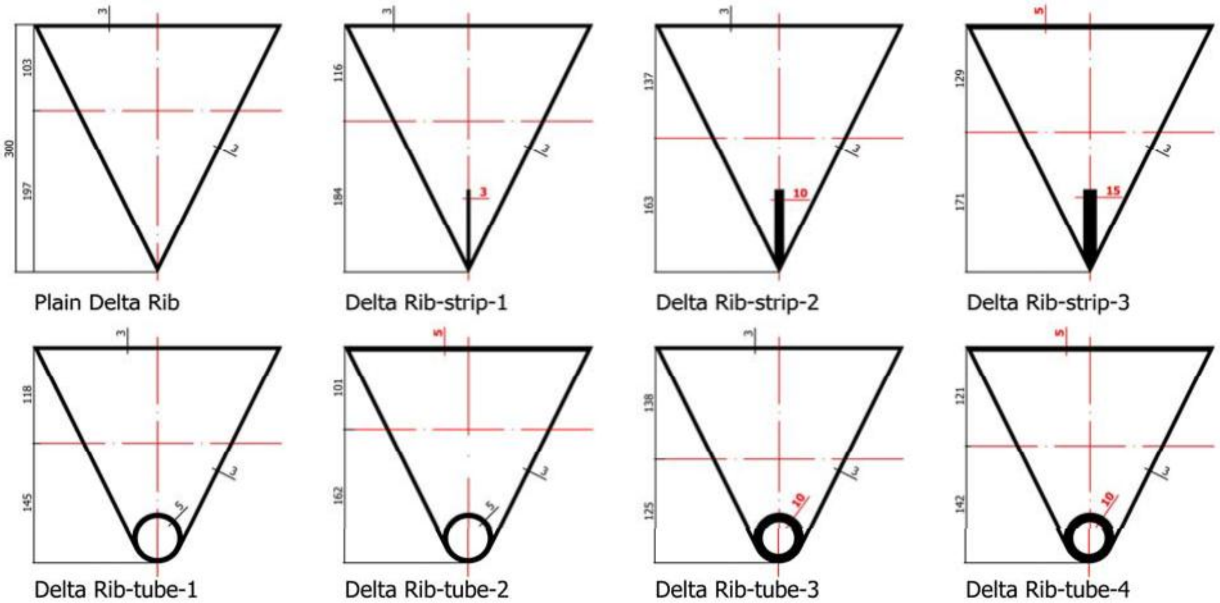


Figure 145. Cross sections of a plain Delta Rib, with a strip flange and a tubular flange. Their moments of inertia are listed in Figure 146 and Figure 147. The axes indicate the centre of gravity per rib.

Delta Ribs possess a versatile structural capacity, and may be applied in situation with an extended range of structural demands. This is demonstrated in Figure 146 which compares the cross sectional properties of conventional profiles with Delta Ribs of equal height. Whereas I sections typically feature a very strong dominant resistance to bending in one direction, resistance to the perpendicular direction as well as torque is limited, or even negligible. Hence, the resistance to bending in a vertical plane (I_z) of Delta Ribs cannot compete with that of I-sections, but instead Delta Ribs provide a considerable resistance to bending in horizontal direction (I_y), as well as to torque (I_p).

When the moments of inertia are divided by the sections' weights per length (see Figure 147), it appears that Delta Ribs provide significant stiffness for relatively little weight. Now the gap to I-beams is less acute, and it appears that the bending stiffness of Delta Ribs is comparable to that of tubes.

The critical phenomenon determining the system's capacity is local instability, as finite element analysis has demonstrated. On the system-level this is anticipated by the sheet thickness. Strong curvatures do also increase the resistance against local instability, but this would be conflicting with the design's formal freedom.

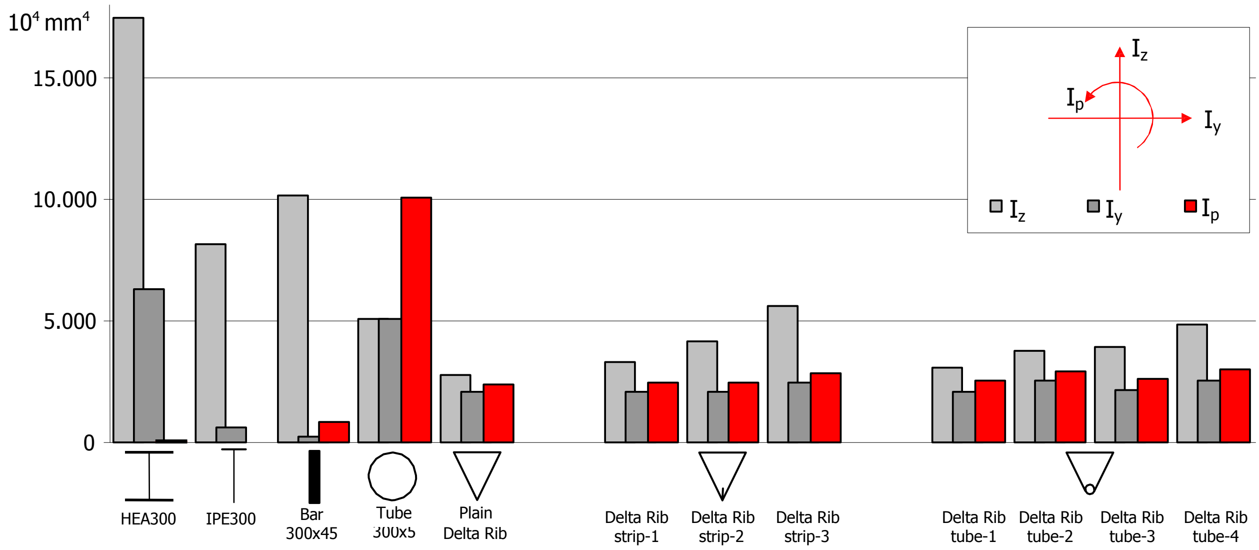


Figure 146. Area and polar moments of inertia of Delta Ribs and conventional steel sections, all approximately 300mm high.

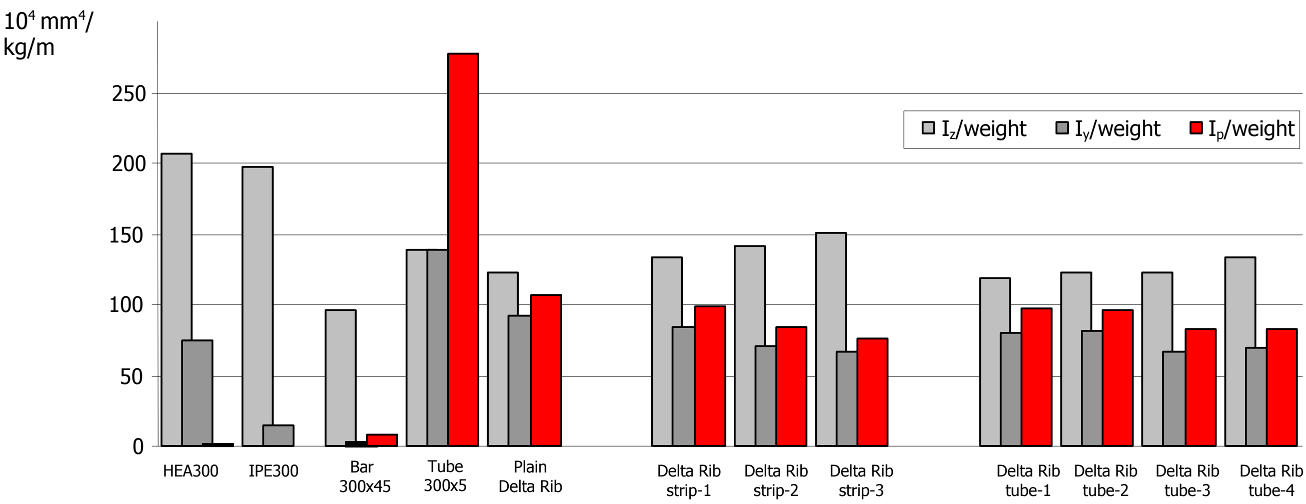


Figure 147. Area and polar moments of inertia of Delta Ribs and conventional steel sections, relative to the section's weight.

6.4.2 Highest degree of formal freedom

Depending on the variant of the system that is used, the ribs that compose the system may be constructed from input points or input curves, or a combination of the two. Both feature their own advantages. Four arbitrary input points per side obviously provide most flexibility in determining the corner points of a reticulated structure, enabling to construct curvilinear structures with twisting members of non-constant cross section. The corner points may in turn have been generated from a curvilinear base. The designer may intervene in the fine-tuning of the generated surfaces, by shifting the cones' tops.

In contrast, the usage of input curves, that will eventually become the edge of a rib, gives more direct control to the designer in shaping the most visible parts, which are above all the ribs and their edges. The corner points then become a result (and not an input) of this.

These alternative ways of generation may be exploited, thus demonstrating the versatility of the system in terms of design procedure, over and above the geometrical versatility of the construct.

6.4.3 Highest degree of systematisation

The Delta Rib system results from increased specification on both the level of scale and the level of abstraction, and the coverage of all transitions between them. Each transitory step involves new parameters. From the global to the meso-level of scale the points of a curvilinear network are upgraded to corner points or edges of ribs, using primary rib-dimensions as input. Then, from the meso- to the micro-level of scale, the structure's geometry is fine-tuned by adjusting the amount (and in some cases also the class of) curvature and assigning thicknesses. Specifications that decrease the level of abstraction during the definition of the structural design are the assignment of cross sections along the reference lines, the principle of connections and the (possible) application of a flange.

Though deciding upon the input parameters takes place outside the system, once inside the system all relationships are logically grounded and are in principle ready to be implemented in an automated system. Furthermore they are all established in coherence with rational fabrication processes, this way assuring a continuous path from design to realisation. Whereas in most preparatory steps the degree of systematisation allows for an automated implementation, the positioning of parts and its subsequent welding is manual labour, thus depending on craftsmanship.

6.4.4 Review and comparison with other systems

Numerous examples of three sheets together creating a triangular box section used as beam have been found. However none of them were connected to create a network structure, not with irregular curvature but also not with constant curvature. Therefore, a patent has been applied on the Delta Rib system.

Table 10 compares the Delta Rib system to other systems in steel able to construct structures for irregular building shapes. The systems are compared using the targeted performances as evaluation criteria. All systems are single-layered and primarily act in bending. The highest degree of material efficiency therefore has to be reached through a variable structural capacity. The highest degree of formal freedom involves local rather than global geometrical approximation and the exact following the contour of the building envelope. Approximations through polygons or arcs all decrease the performance on this criterion. The highest degree of systematisation concerns the anticipation of the system to manufacturing techniques. Moreover, it evaluates the aptness of the system for a replicated implementation, for which explicitly defined parameters are required.

From the evaluation it becomes clear that – given the targeted criteria – the Delta Rib structural system is the only system that performs well on all criteria.

Table 10. Comparison of the Delta Rib system with other systems.

Performance criterion Æ **È System**	**Degree of material efficiency**	**Degree of formal freedom**	**Degree of systematisation**
Global frames (e.g. the radial and parallel steel frame structures of the Mezz concert hall*)	**+** The structural action is entirely in-plane. Its capacity is varied through varying the structural depth, based on standard profiles as well profiles constituted out of plates.	**-** The global geometry can only be adjusted to a configuration that is locally optimal.	**-** Elaborate assembly of straight and curved elements onto segments.
Diagonally folded steel plates North Holland Pavilion*	**-** As the system lacks flanges, its structural capacity can only be anticipated within a limited range through the thickness and depth of plates.	**0** The building shape is locally anticipated in a polygonal fashion, but only its contour is exactly followed.	**+** Customisable parts and detailing, allowing for a parametric setup.
Reticulated network structure with straight elements (e.g. roof Great Court Yard*)	**+** The structural capacity is varied through varying the structural depth of box sections constituted out of plates.	**0** The building shape is locally anticipated in a polygonal fashion, but only its contour is exactly followed.	**+** Customisable parts and detailing, allowing for a parametric setup.
Rollercoaster track with tubular backbone*	**0** The tubular elements' shape is appropriate for the torque and bending forces that occur, but its capacity can only be varied per element through its wall thickness.	**+** Curved shapes are locally anticipated through curved elements.	**0** The system's implementation depends on local curvature conditions, and thus requires analysis beforehand.
Rollercoaster track with triangulated polygonal backbone*	**0** The elements' box-shape is appropriate for the torque and bending forces that occur, but its capacity can only be varied per element through its sheet thickness.	**0** The building shape is locally anticipated in a polygonal fashion.	**+** Customisable parts and detailing, allowing for a parametric setup.
Box-section ribs from developable sides in the China national stadium*	**0** The elements' box-shape is appropriate for the torque and bending forces that occur, but its capacity can only be varied per element through its sheet thickness.	**+** The building shape is locally anticipated in a polygonal fashion.	**0** Customisable parts and detailing, allowing for a parametric setup. However its curvatures are not constant and their production is thus labour intensive.
Delta Ribs	**+** The structural action is 3-dimensionally, its capacity in all directions can be varied through the rib's structural depth, its height/width-ratio, the addition of flanges and the thickness of steel plates.	**+** Curved shapes are locally anticipated through curved elements.	**+** Customisable parts and detailing, allowing for a parametric setup.

Key: + performing well 0 average - poor
* Exemplary implementations of these systems are included in section 3.3 Content documents.

6.5 Improvements to the system

Based on the completion of a full cycle from design via engineering to construction, and the evaluation of this in the previous two sections, solutions to troublesome tasks are proposed. These improvements involve both topological and procedural developments. Further development is also to take place on cladding systems to be installed between or on top of Delta Ribs.

6.5.1 Facilitated positioning due to straight connections between curved sides

To simplify the positioning of curved sides during construction, and this way finally serve the geometrical accuracy, it is proposed to impose connections between curved sides to be straight. With straight sides, the dimensional accuracy can be verified by three points only. Furthermore, it also allows insertion of planar end-plates with a tight fit along its full perimeter. Finally, imposing straight extremities will also eliminate the geometrical interdependency between two adjacent curved sides, which so far always had to be defined together.

Adding these end plates affects the generation of a curved rib side, since it 'consumes' a degree of freedom. This can only be realised if a degree of freedom is generated at the same time. Two alternative ways to achieve this according to exact geometry are proposed: the first by subdividing the constant curvature of one rib-side into two planes, the second by making the curved rib-side to be created dependent on an earlier constructed side. In addition to these exact methods, also approximate methods may be applied, e.g. by accepting deviations from the straight connection line up to a certain value, for instance the sheet thickness. This way, the smoothness of the connection, and moreover its structural continuity, depends on the application of the welding. As this dependency on craftsmanship conflicts with the aim of a systematised application, it will not be explored further.

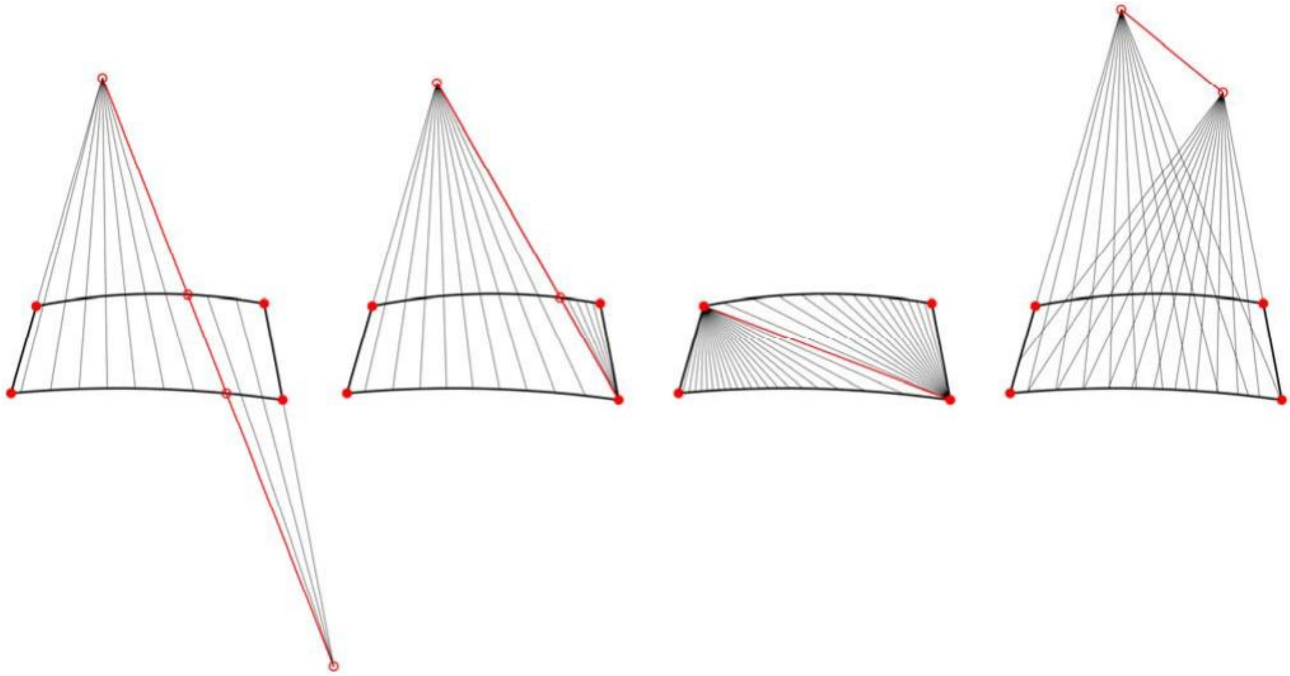
Subdividing a single constantly curved rib-side into two tangent surfaces

Alternative splitting lines for the operation of subdividing the constant curvature of a rib-side into two planes of constant curvature are depicted in Figure 148. Although all four alternatives meet the geometrical constraint of a tangent transition from one sheet to the next, only the first (alternative A) can be constructed through rolling. Alternative B includes a cone's top, which cannot be rolled with conventional rolling mills. Alternative C does not allow for a realistic subdivision into two segments as it would splice half of the rib. Alternative D finally results in a tangent, but not continuous connection. As this is an outcome likely to result from an insufficiently constrained automated implementation, it is nevertheless included.

The tangent transition is achieved when the two curved sides share a ruling line, and when the cones' central axes are coplanar. Degrees of freedom are four input points, the position of the shared ruling line and the curvature of one cone, which are to be provided by the designer. The tangent second cone can then be created from it, as is demonstrated in Figure 149. This procedure is to be repeated for each of the three rib-sides, after which redundant parts are trimmed off. The degrees of freedom for each side may be employed to control the resulting shape, as the procedure tends to result in theoretically valid, though formally unexpected constructs featuring irregularly curved edges. This is demonstrated in Figure 150, where one of ribs from the constructed prototype featuring curved extremities is reconstructed within the same corner points, but with straight extremities. By operating the degrees of freedom it was attempted to approximate the mid-sides of the original rib's edges, but the result nevertheless strongly deviates from the original.

From the reconstruction of a rib, the method's primary disadvantage that was experienced was that it is difficult, if possible at all, to position each pair of shared ruling lines from two adjacent sides in such a way that they intersect (rather than cross), and do so at a desirable position on the rib. If this is accomplished, the rib can be subdivided along this planar section and both parts will only consist of curved sheets of constant curvature. This way, the rational rolling of one constant curvature per sheet is maintained.

Rather than through constructional operations between given points, intersecting shared ruling lines could also be achieved by applying a parametric model of a universal rib as depicted in Figure 151A. All relationships are embedded in this model, and are maintained when control points are shifted or any of the other degrees of freedom as listed in Figure 151B are operated. When constructing such a model, apart from geometrical relationships also their hierarchy (defining which of the associated geometrical rules changes first, and what happens to the others) needs to be embedded.



A. Two inverted cones. Can be rolled and practical connection is possible.

B. Two inverted cones, one top coincident with a corner point. Cannot be rolled.

C. Ruling line between diagonally opposite points. Cannot be rolled and no practical connection possible.

D. Theoretically valid construct, though not of practical use.

Figure 148. Alternative splitting lines for generating a surface between four non-planar points with two opposite straight extremities, through assembling the surface from two tangent conically curved surfaces.

A. Four input points are defined, as well as the first cone (the large circle is its base curve), a shared ruling line, the plane through the ruling line and the circle's midpoint and the bisection plane of two points at equal distance from the second cone to be generated.

B. The intersection of the two planes results in the central axis of the second cone. By shifting the extremities of the shared ruling line, the initial cone needs to be redefined, the intersection with its bisection plane will shift and also the second cone will be redefined.

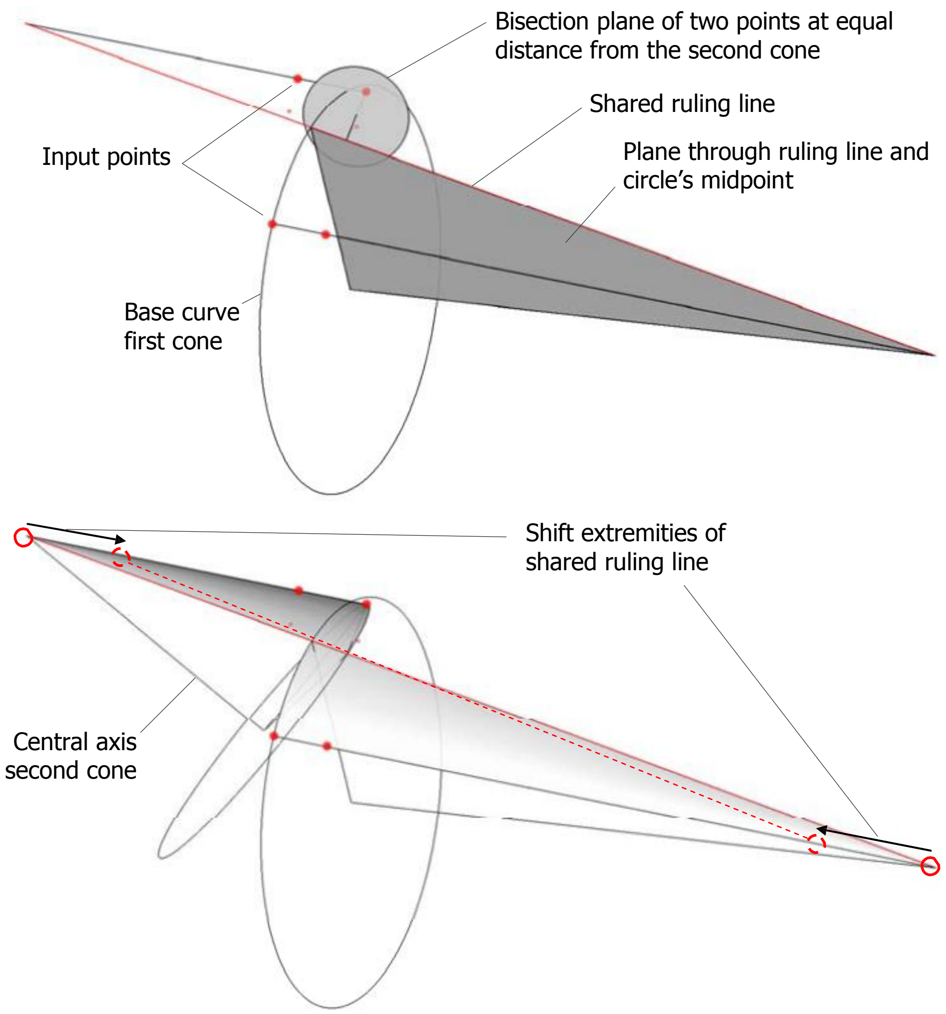
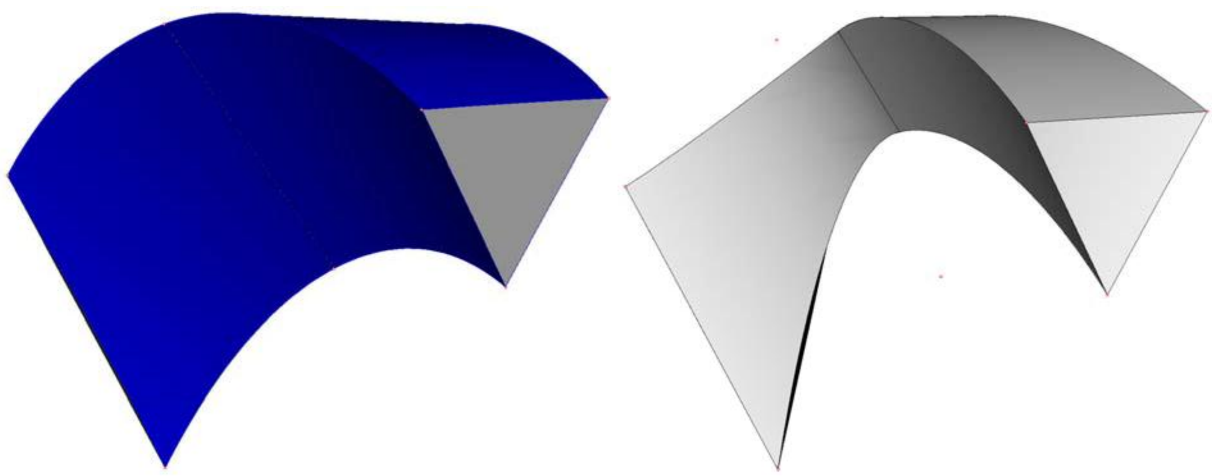


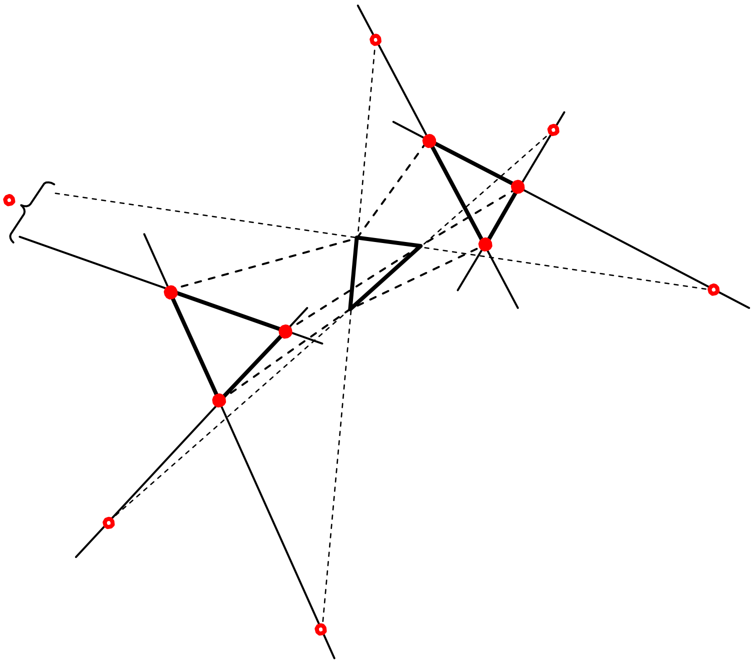
Figure 149. Construction of two tangent cones, each going through the ruling line between two input points.



A. Original rib.

B. Reconstructed rib.

Figure 150. Reconstruction of an original rib featuring curved extremities with a rib featuring straight-edged extremities, using the same corner points.



Ruling lines on rib
Extended ruling lines
Ruling lines shared between two cones
Edges to be defined by surfaces
Input points
Sliding points (by designer)

A. Scheme

z Ruling lines through rib's extremities
z Direction of shared ruling lines
O Shift rib's corner points
O Slide extremities of shared ruling lines
O One surface-curvature per rib-side

B. Model's variables:
z = fixed, O = degree of freedom.

Figure 151. Design for a parametric model of a universal rib, based on generating two tangent cones between four non-coplanar points per side.

Generating a curved rib-side based on an extremity of an earlier constructed side

By constraining an upper input point on the ruling line departing from the intersection between three lower edges of ribs, this extremity of the newly created rib-side will by definition be straight (see Figure 152AB). This way, a sequence of straight extremities is defined. Only when closing the loop, no control is possible, and the extremity will consequently not be straight (Figure 152C). The conical curvature is still a degree of freedom to the designer (a cone is undetermined when only two intersecting ruling lines are given). To define it, an additional point is needed. This point may for instance be derived from the envisaged lower edge. This is particularly advantageous when this input point is employed for constructing both sides of a rib: the rib's lower edge will go through this point, and hence firstly gives direct control to the designer, and secondly this way creates a planar triangular cross section with two straight sides (see Figure 153).

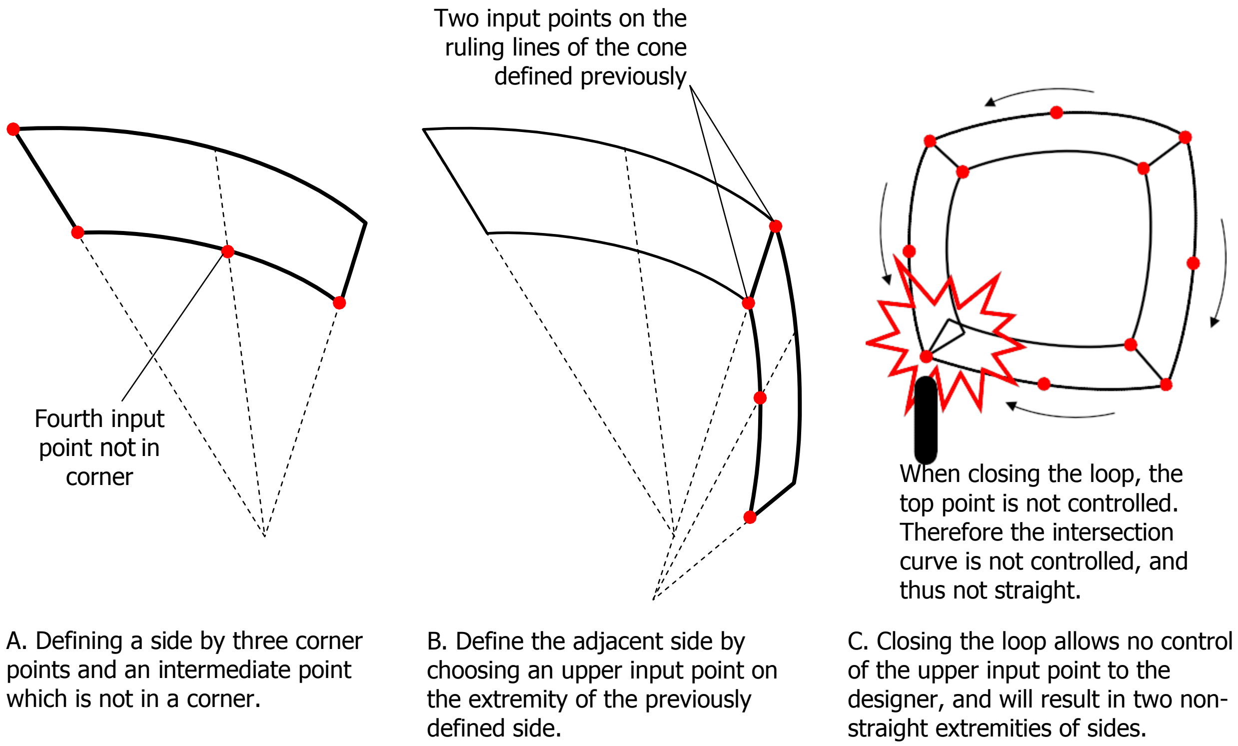


Figure 152. Creating a straight side by constraining an upper input point on the ruling line of an adjacent, previously defined side, and its limitation when closing a loop.

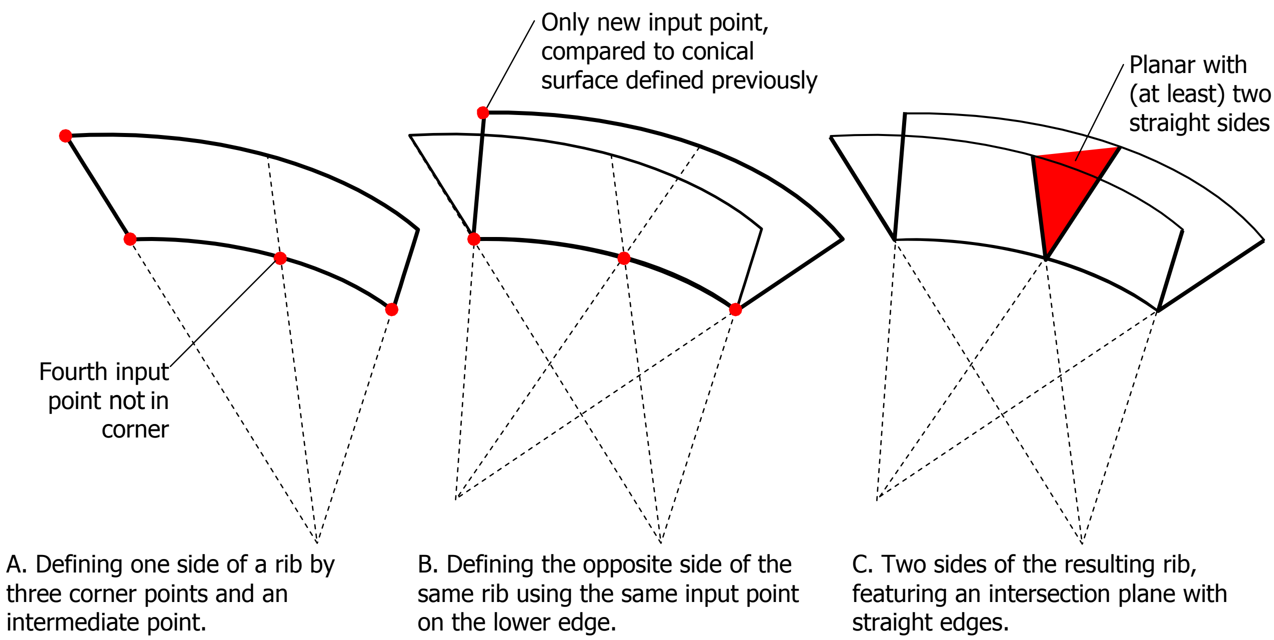


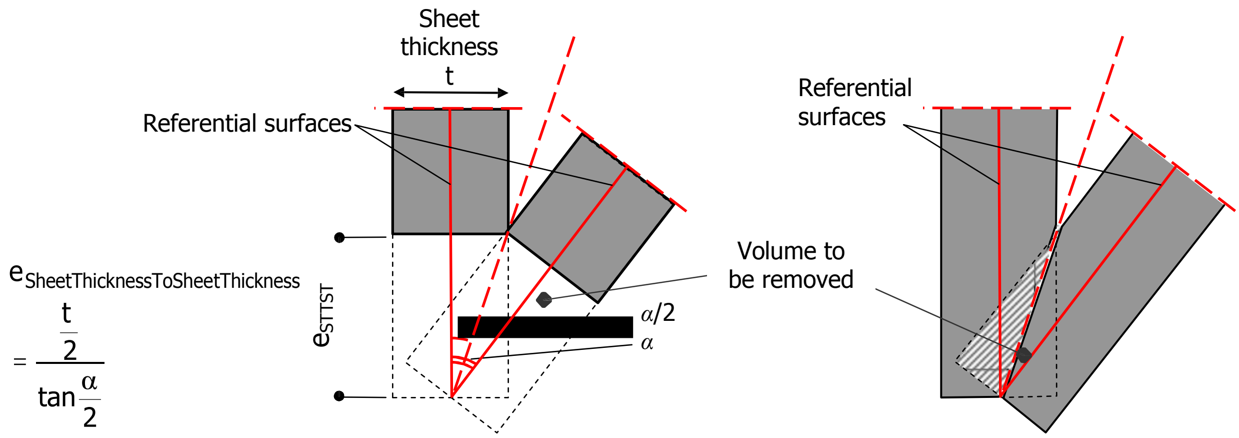
Figure 153. Creating an intersection plane through an input point on the rib's lower edge.

6.5.2 Anticipate sheet thickness in unrolled pattern in case of two adjacent curved sides

Anticipating material thickness in the 3-dimensional engineering model (by making offsets) based on surface-representations is not possible since non-intersecting surfaces cannot be trimmed (there is no edge to trim to). The anticipation should therefore either be made during the 2D-engineering, or during the construction phase.

Anticipating the sheet thickness in 2D-engineering involves the removal of a coil along the edge. The width to be removed ($e_{SheetThicknessToSheetThickness}$) is the same for both sheets, and is a working of the sheet thickness and the angle between the two sheets, see Figure 154A. If the rib's edge is an arc, the angle α will be constant along that edge. In all other cases the angle varies, and so will the coil-width that is to be removed.

Anticipating the sheet thickness during the construction phase involves chamfering all inside edges up to the middle of the sheets. The sheets can then be positioned on the referential edge without clashing, see Figure 154B. Apart from introducing a dependency on craftsmanship, this approach has the drawback that the area to be welded on is limited to only half of the sheet thickness, and is unlikely to fully fuse the material of the two rib sides.



A. In 2D-engineering: removal of a coil along the edge.

B. During construction: chamfering all inside edges up to the middle of the sheets.

Figure 154. Anticipating the sheet thickness either in the phase of 2D-engineering or in the phase of construction. Details of the connection between two sheets.

6.5.3 Anticipate sheet thickness in unrolled pattern of curved sides in case of tubular edge

To allow for an accurate positioning of the curved side to the circular perimeter of a tube or bar…

1. the offset from the referential surface without tube has to be the tube radius minus half of the sheet thickness. This is to be applied in the phase of 3D-engineering;
2. a coil has to be removed from the edges of the neighbouring sides, such that the inner edge of the side poses on the perimeter. The remaining triangular space is to be filled with welding material.

Since the application of a tube or bar is subjected to the constraint of a planar and constant curvature, the coil will have an equal width ($e_{SheetThicknessToTube}$) which can be determined from the sheet thickness and the radius of the tube, as demonstrated in Figure 155. This anticipation comes on top of the zones to be removed as in Figure 106, which marked the difference between a rib without and a rib with a tubular flange. In construction accuracy of positioning is to be achieved with an external mould, as tangency is hard to verify. In the case of a tubular edge (with a constant radius of curvature) the angle α is constant per rib, and thus a single mould suffices.

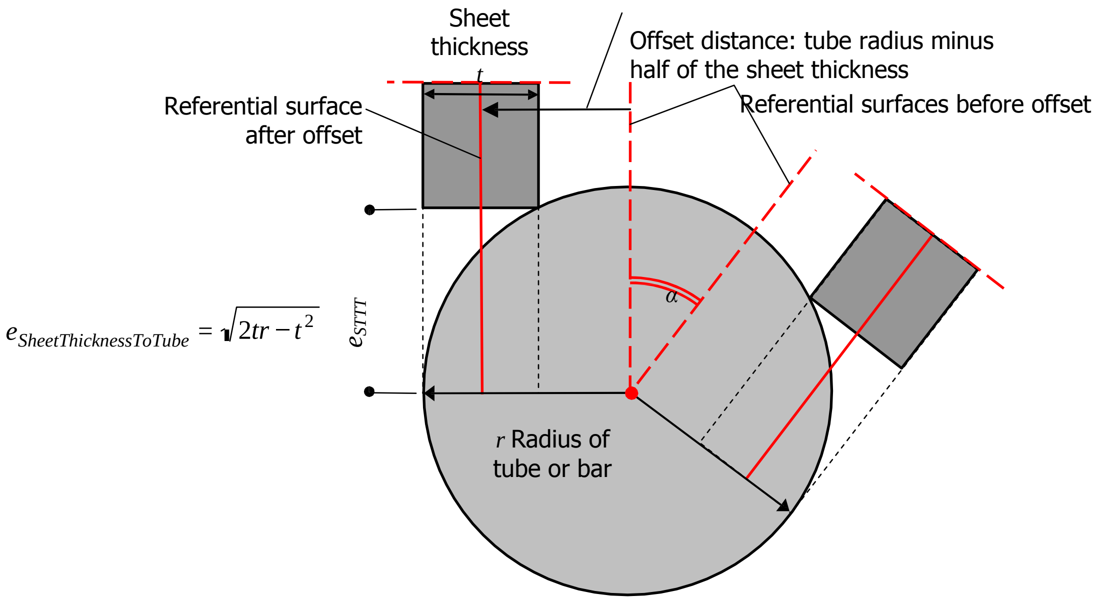


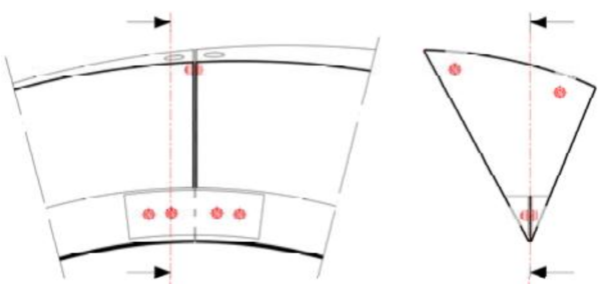
Figure 155. Removal of a coil from the sides neighbouring a tubular edge. Detail of the connection between two sheets and a tube or bar.

6.5.4 Increased rigidity of bolted assembly connection through local flange

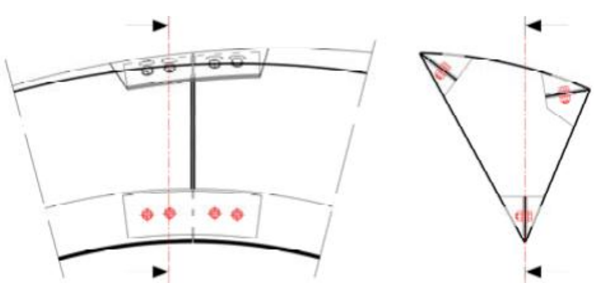
It was observed that bolted connection using bolts in the mid plates primarily caused bending of these plates, resulting in large deformations when transferring the in-plane forces from the curved topside to the next. Apart from the obvious solution to employ thicker mid plates or adding stiffeners perpendicular to the plates, the connection's capacity and stiffness may also be increased by:

1. Positioning bolts at the level of the rib's upper surfaces, rather than below. This will cause the bolts and the mid plate around it to jut out, which may not be desirable;
2. Introducing local strip-flanges in the remaining two edges, and connect these through a side plate. These strip flanges cannot be applied to the full length of the edge since the edge in which they will be placed is not planar.

Figure 156 demonstrates the application of such local flanges in a Delta Rib. After a first attempt to construct the strip in a plane defined by a midside and two extremities of the strip, it turned out that it would almost align with the rib's upper side, and hence would obstruct the three plates. Therefore a more inclined orientation to the edges with a larger eccentricity (thus less favourable) had to be employed. Connection takes place via boltholes aligned with the bolts in the ribs' sides.



A. Connection as realised: one flange and two bolts through mid plates.



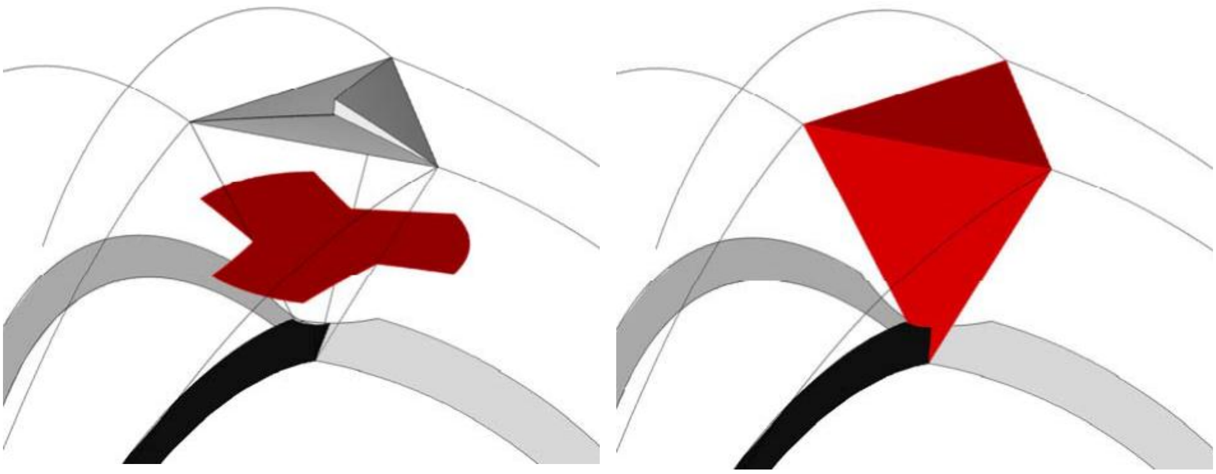
B. Improved connection: flanges in all three rib-edges.

Figure 156. Flanges in rib edge.

6.5.5 Increase nodal stability of rib-rib connection

Increasing the nodal stability of the rib-rib connection will activate the full structural depth, and is therefore beneficial for the system's material efficiency. This could either be achieved by:

1. Introducing a planar cross-plate partially penetrating each rib (Figure 157A), this way creating a path of load transfer in addition to the already existing one along the ribs' edges. As this solution does not attempt to align with the intersection curve between two sides, its application does not put any constraints on the connection;
2. Using end plates to enable redirection of in-plane forces, rather than through out-of-plane bending. For the most structurally effective solution there should not be any eccentricity in the connection. Disregarding custom-curved endplates results in the requirement for a planar endplate that intersects with the curved side along a straight line (Figure 157B). The implications and solutions to this requirement have already been addressed in section 6.5.1 Facilitated positioning due to straight connections between curved sides.



A. Planar cross-plate, the node's topside simply consists of the three extremities of the ribs' topsides.

B. Planar top and end-plates.

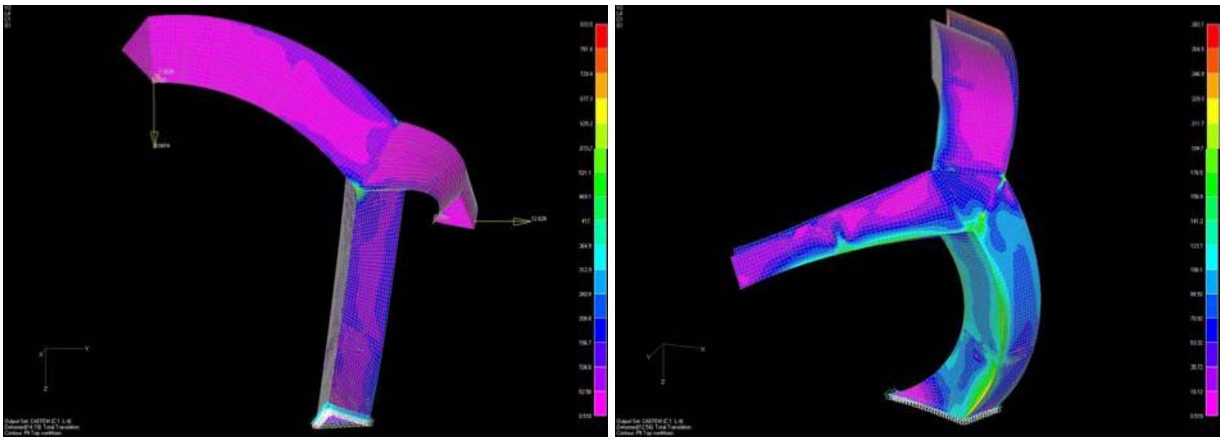
Figure 157. Increased nodal stability by additional plates. The attached ribs are depicted through their (strip-)flange and edges only.

To assess the effectiveness of the sheets, finite element models of both alternatives have been compared to the situation without additional plates.[17] As the structural analysis is carried out to investigate solutions to the limitation of structural capacity through local instability, the quantitative focus of the analysis has been on the stiffness. Stress distribution and eventual peaks have only been addressed in a qualitative sense. It was found that insertion of a cross plate results in lower local deformations in the region of the node, but that this barely results in additional stiffness on a larger level of scale: compared to the model without cross plate, the average displacement of the rib's edges only decreased 1%. Furthermore it was observed that the model with cross plate featured high peak stresses in the connection's lower corner, like the model without cross plates (Figure 158A). This indicates that the cross plate is not very effective in the system's global structural action.

In the model with endplates and a planar topside, deformations decreased significantly (12%) compared to the original model. Furthermore it displayed a fairly equal stress distribution along the ribs without featuring any extraordinary high peak

[17] The analysis was performed using finite element models based on the digital model of prototype 2. The model already featured straight lines between the upper corners of the node, which were needed to model a planar top plate between them. The curved intersection between two adjacent side plates is not in accordance with the proposed alteration, and has necessitated minor local changes to place the planar end plates inside. Due to these changes, the stiffness of the connection of rib sides is slightly underrated.

stresses (Figure 158B). This is probably due to the existence of two additional plates at the edge between two adjacent rib sides allowing directional splitting of internal forces. This suggests that this measure results in a more efficient use of structural material than the system without end or cross plates.

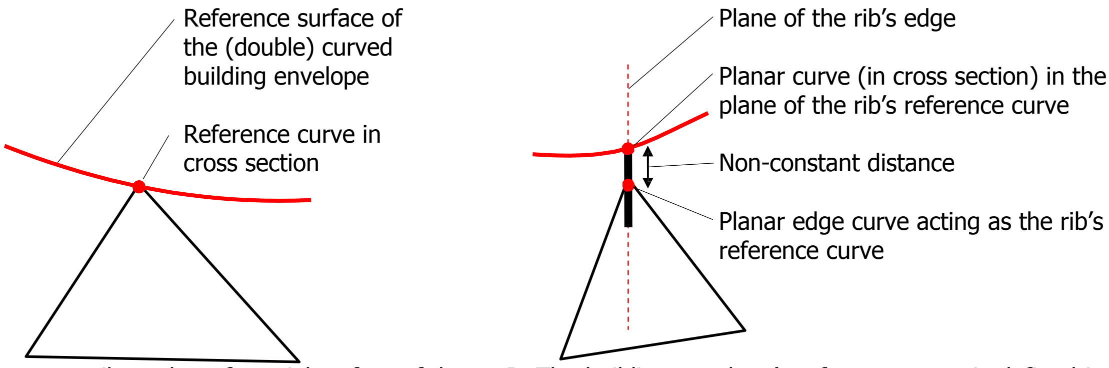


A. Model with planar cross-plate. B. Model with planar top and end-plates.

Figure 158. Results of the finite element analyses comparing the effects of additional plates to increase the system's overall stiffness.

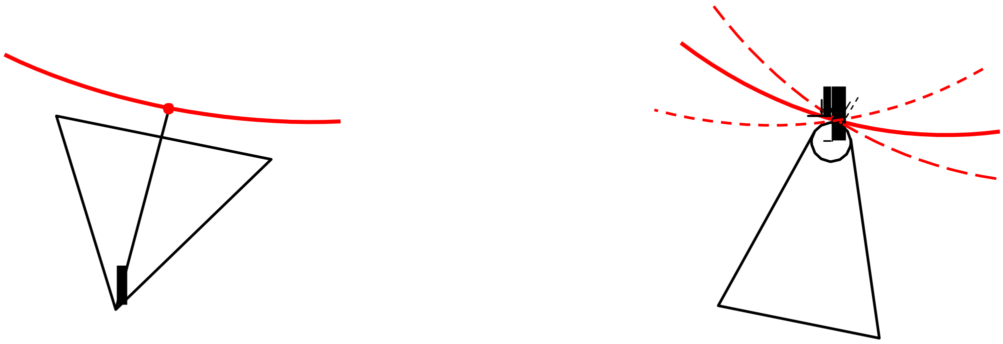
6.5.6 Cladding on top of Delta Ribs

Cladding used to cover the open areas between the Delta Ribs may employ the same geometrical basis as used for the ribs, either for the contours of the panels themselves or their connection to the Delta Ribs. The former is constrained since – depending on the type of rib that is used – the edge curve may be an arc (thus not free form), or an outcome of two intersecting rib sides. Several options are displayed and assessed in Figure 159, for ribs with and without flanges.



A. Reference curve rib on the referential surface of the building envelope. As Delta Ribs with constant curved sides cannot be generated from arbitrary input curves, the building envelope cannot be of any shape.

B. The building envelope's reference curve is defined in the same plane as a planar edge curve (e.g. in the case of a strip-flange whose contour is a segment of a circle). Different from the previous scheme, this new curve is unconstrained, and thus can follow any curvature of the building envelope.

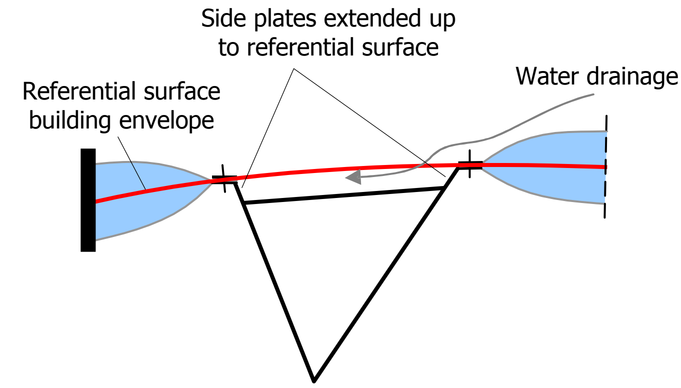


C. Different from option B, the planar free form curve may also be described at the other side of the rib. The portion inside the rib does not necessarily have to be a full plate, but may be only locally connected.

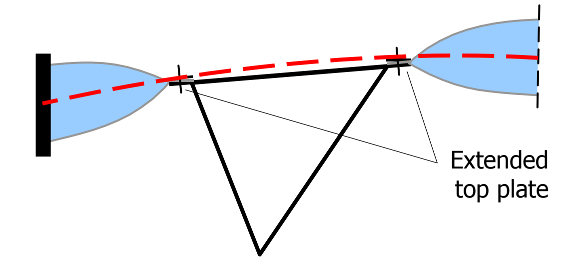
D. As it is used for the connection from rib sides to the tubular flange, the tube's circular cross section may be employed for a uniform connection detail, despite. As in option A, the tubular edge is always an arc, and thus a constraint on the shape of the building envelope.

Figure 159. Principles (in cross section) for connecting cladding to Delta Ribs.

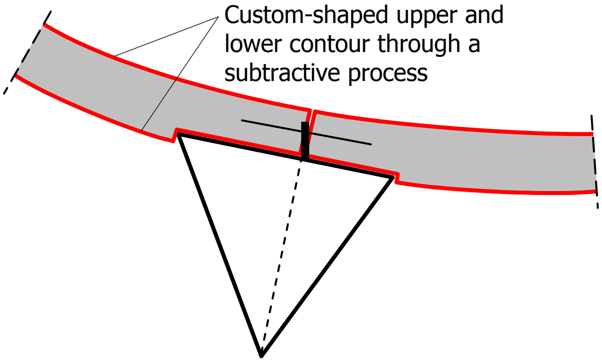
Many alternatives can be imagined for double curved cladding, of different size, material and structural capacity. Thin-layered panels of a single material (that allow modelling as a surface as in Figure 159) will typically require the smallest distance between ribs, whereas multi-layered infill-elements as in Figure 160 allow for wider spacing. When using infill with a significant structural capacity, these may also be structurally employed on the global level of scale. The alternative shown in Figure 160D proposes to use Delta Ribs as stiffeners below a load bearing concrete hull, acting in composite action through dowels. When doing so, the construction and positioning of formwork becomes critical.



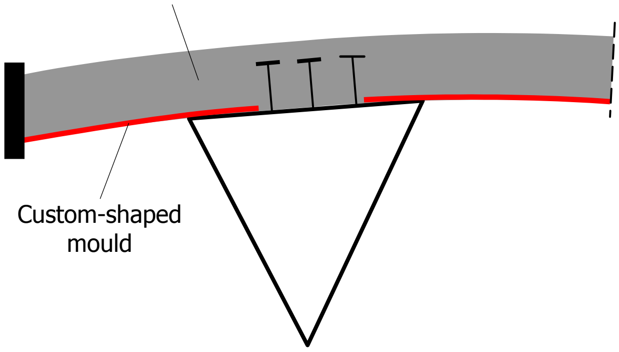
A. Infill with inflated cushions connected to extended rib sides. The sides may have free form contours, obtained through extending them up till the referential surface.



B. Rather than the sides, also the rib's top-sheet may be extended, however this does not allow for any anticipation to a free form shape.



C. Volumetric cladding elements, custom-shaped through (for instance) a subtractive manufacturing process. The rib's geometrical setup may be employed for defining the connection between the elements.



D. Structural acting in composite action with the Delta Ribs below through dowels.

Figure 160. Alternative cladding systems (in cross section) on top of Delta Ribs.

6.6 Setup for an implementation in a parametric design system

From the earliest scheme-phase onward, the alternatives proposed in this research have been set up anticipating a later automated custom-replication of system components. First, this implementation has to handle the relationships between components within as well as across levels of scale and abstraction. Second, it should relate the system's geometry to external inputs such as the structural capacity needed.

The primary variables of this research are geometry, material and force. The relationships between them so far have been handled manually. Despite all modelling resulting in geometry, the considerations behind them are of non-geometrical nature. In the case of the Delta Ribs for instance, the generating of surfaces of constant curvature, and the subsequent generation of unrolled patterns with characteristics of one circular edge and two straight extremities aligned with a ruling line, originate from constraints imposed by the rolling process.

Inclusion of generic properties (eventually leading to geometry) in digital product models has been proposed extensively under the common denominator of features in the 1980's and 1990's, in parallel with the increasing availability of computers for daily use. These features comprise 'generic shapes or characteristics of a product with which engineers can associate certain attributes and knowledge useful for reasoning about that product' (Shah and Mäntylä 1995). Although named components instead of features, Bentley's pre-beta version of the parametric and associative design system GenerativeComponents (BentleySystems 2006) follows the same logic. Parametric implies that its designs are based on explicit parameters of which the values may change. Furthermore it says that components made with it are associative, meaning that it propagates changes carried out in one through all the associated components automatically. Using GenerativeComponents, the activity of modelling consists of establishing relationships (being displayed in a relationship-graph) rather than defining geometry. As these relationships are based on considerations of the designer, the system claims to capture design intent (Aish 2005).

Different categories of parametric models exist: when based on quantitative variations (e.g. dimensions or coordinates), they are named parametric variations, whereas when based on qualitative variations (e.g. replacing a component of type a by one of type b) they are named parametric combinations. Finally, hybrid models combine both variations (Barrios Hernandez 2006). Despite being limited to quantitative variations, the first category strongly opens up the design space by separating the quantitative value of parameters from the topological layout (Vermeij 2006). To allow topological changes too – like the application of a tubular flange instead of a strip-like flange in the case of Delta Ribs, a model with hybrid characteristics is needed.

Table 11. System model and geometric model of the Delta Rib system. (Based on the distinction between a feature model and a geometrical model as described in (Shah and Mäntylä 1995), applied to the Delta Rib system).

System model	Geometric model
Manufacturing logic translated into geometrical procedures	Usage of two straight extremities per rib-side
Rolling logic translated into geometrical procedures	Usage of cones with circular ground curve
Generating ribs' corner points out of network	Offset distances in-plane (upper corner points) and perpendicular-to-plane (lower corner point)
Designating the applicable set of rules based on the type of rib (no flange, strip flange, tubular flange) and inputs (4 points, arc + 1 point, etcetera)	N/A
Generating rib-sides out of ribs' corner points	Geometry, height of cone, radius of ground circle
Generating cutting patterns out of rib-sides	Contours, distinction between marking and cutting
Generating rolling instructions out of rib-sides	Geometry, dimensions of chords, radii and heights
Generating structural centre line out of rib	Geometry

For the proposed model of the Delta Rib-system, Table 11 distinguishes between the characteristic behaviours of the system on one hand, and the resulting geometrical representation on the other hand. The former consist of knowledge related to structural action, rolling, cutting and fabrication that are translated into geometrical rules needed to construct a model. Whereas some of options result in the selection of rules for further automated specification, other degrees of freedom remain controllable by the designer. A small-scale implementation was made to evaluate this, using GenerativeComponents as modelling tool. The parametric model constructs a cone through four arbitrary points in space. The cone's height and the direction of the ruling line are available as degrees of freedom (Figure 161).

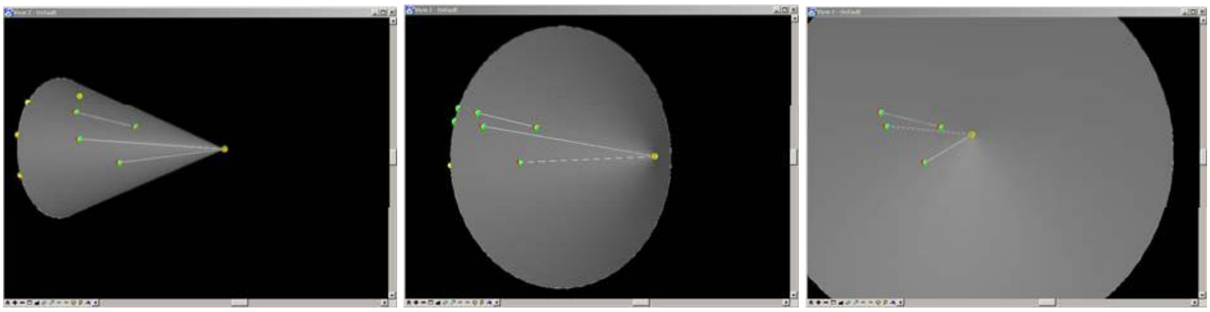


Figure 161. Implementation (in GenerativeComponents) of a parametric system generating a cone through four points in space. The top of the cone is shifted according to a numerical parameter.

A single level of parameterised components does not suffice to model Delta Ribs since the ribs cannot be generated from the network points on the global level of scale only. For such transitions of scale-levels first operations need to be performed from which the ribs result, and in a later stage also the unrolled patterns. This is done by calling components within components (named composite features in (Shah and Mäntylä 1995)). As the system model allows making transitory steps across multiple levels of scale, there are multiple levels from which replicatable components are constructed. Hence, the designer is not forced to think on a very detailed level at a very early stage. Instead the input of network points on the global level of scale is gradually specified onto the micro level of scale.

Figure 162 depicts an overview of the proposed model for the Delta Rib-system. Starting from the left hand upper corner, the input-network is further specified by the application of replicatable components. User interference is required at moments when topological choices are to be made or dimensions are to be provided. Clearly visible is the input from either of the involved manufacturing processes in the stage of generating the ribs' sides.

In the model, and also in the working procedure pursued to construct the Delta Ribs, the transition from a 3-dimensional model to the 2-dimensional working space of the unrolled patterns is unidirectional, meaning that the reverse step cannot be made unambiguously (a planar sheet can be rolled into numerous curved shapes). A bidirectional relationship would increase the system's versatility, and would for instance allow mapping a location indicated on the 2D-pattern back to the 3D-shape.

Also depicted in Figure 162 is the feedback loop starting from the generated ribs back to structural capacity. This loop is envisaged to facilitate structural design by making the realised structural capacity available to analysis. Since the ribs' shapes are fundamentally determined by geometrical rules, their geometry can only locally be dictated, and their structural properties hence only locally be known beforehand. The capacity that is really available is therefore only known when the ribs' sides are generated. It is proposed that the system calculates the neutral point and cross sectional properties at a discrete number of sections, and associates these to the neutral axis of the ribs. Structural analysis of the structure as a whole may then be carried out with linear finite element with equivalent stiffness properties. Also global structural analysis of rollercoaster tracks is carried out this way (Kupers 2005). Iteration of structural design and structural analysis will therefore always be needed, stressing the need for a parametrical associative setup once again. As far as stresses are concerned, these can drive a mechanism to modify any of the parameters determining the structural capacity. Concerning stiffness, the location where the designer's intervention is needed, has to be determined through analysis leading to an understanding of the design's structural behaviour.

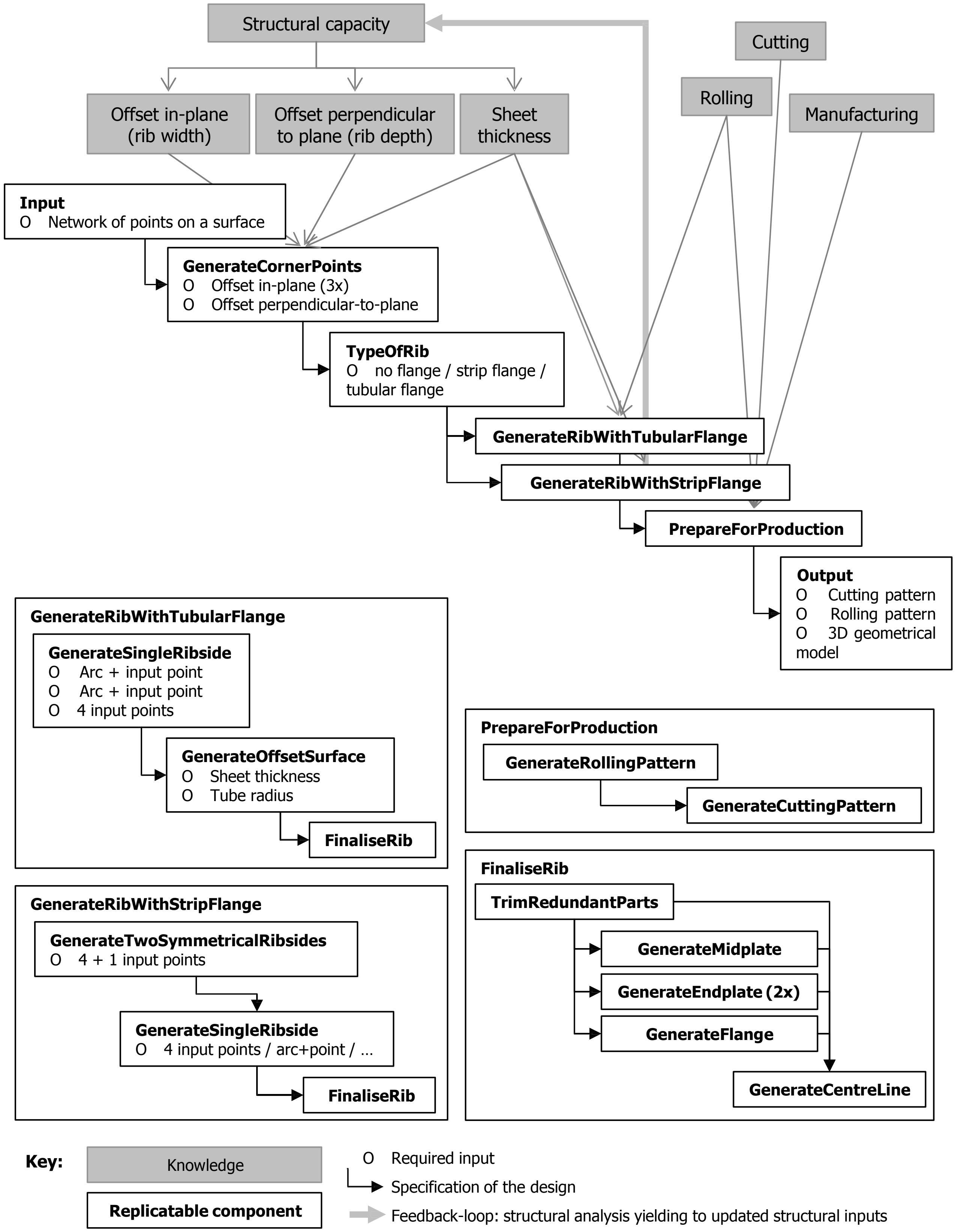


Figure 162. Proposed setup for the implementation of a product model for the Delta Rib-system.

7 Conclusions and recommendations

This chapter reports the final evaluation of the research, by looping back to the research question formulated in the introduction: What are the best-suited structural schemes, structural systems and structures for free form (parts of) buildings? In addition to proposing these schemes, systems and structures, the research has also resulted in a generic framework through which they were generated. Its validity is discussed, as well as the degree to which the outcomes can be generalised.

The research has not only resulted in propositions for schemes and systems, but has also gained experience with the methodology of using design in research, notably in transitions of abstraction and levels of scale. These findings related to the procedure are of interest to any field of design based on a limited number of design variables.

7.1 Conclusions

7.1.1 On the followed method

The method followed to generate alternative schemes and systems consisted of a design-based exploration. It was directed by highly abstract hypothetical solutions formulated on the basis of a theoretical study of the design variables, and precedent constructed structures that provided the relationships between these. The exploration has additionally been framed by the designation of variables valid within the considered hypothetical solution, and subsequent variation on these. These variables in themselves were only defined after some schemes had been generated, and thus risking a bias of the designer's self-fulfilment. Despite using this analytic-systematic approach to the generation of alternative schemes, it was found that the alternative structural schemes could not be logically derived from the variables designated per hypothetical solution alone. Adding designerly creativity, in design studies that were directed (by the hypothetical solutions) and rooted (in performance criteria) proved more flexible and successful. This does not affect the validity of the found alternatives, but prevents drawing conclusions on any schemes beyond those that are generated; a search with the same hypothetical solutions by a different designer, might well result in different schemes of equal validity.

Apart from during the phase of generating alternative schemes, the activity of design has served multiple goals in this research:

1. First, it served the generation of alternative schemes, using hypothetical solutions as a driver towards possible but not probable future scenarios (Jong T.M. de and Voordt D.J.M. van der 2002);
2. Second, a selection of the proposed schemes has been developed further onto systems through the application in a case study design. The case provided the context needed for an evaluation of the underlying schemes on the designated performance criteria;
3. Thirdly, the preparations for the construction of prototypes advocated the integration of design technology, product technology and manufacturing technology (Poelman 2005);
4. Fourth and last, the construction of prototypes implied a validation of the developed system for its conceived goal.

The sequence of levels of abstraction is depicted in Figure 163. The upstream transitions between these levels of abstraction consist of the implementation of the lower level, through specification and materialisation. To emphasise that the levels of abstraction anticipate each other, they are depicted overlapping. The upstream specification occurs simultaneously to a downstream search for degrees of freedom. This search is characterised as a quest for creativity. For the validity of the proposed solutions, it is required that both the upstream and downstream connections are present.

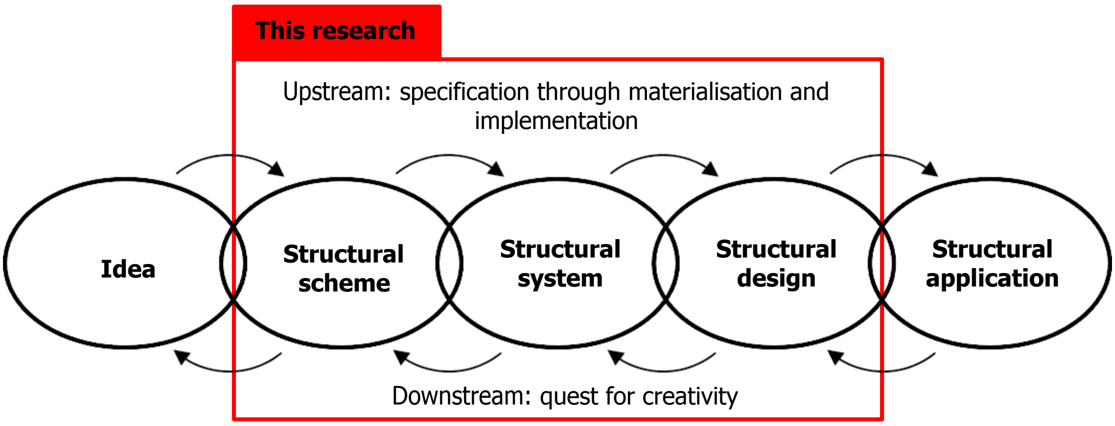


Figure 163. Upstream and downstream effects of this research's approach.

Applicability of the developed systems as structural systems, and thus validity for a larger number of cases than the explored one only, has been achieved through a number of ways:

1. In the application of the systems the macro level of scale has been decoupled from lower levels, meaning that it requires context-dependant gradual specification: first the grid (macro level of scale), then the components (meso) and finally the elements (micro). Relationships between these are embedded in the system's definition. In a future automated implementation, these relationships are parametrically defined and act as 'carrier' of non-geometrical information;
2. The transitions of levels of scale also entail transitions of level of abstraction: whereas the most abstract level, structural schemes only consisted of the definition of a pattern, further development onto systems and implementation onto structural designs also comprised the definition of materials, fabrication, detailing and dimensions. Until the level of systems they posses are applicable to a wide variety of cases;
3. The design space is enlarged by decoupling the quantitative and qualitative aspects from design, and replacing this by a parametrically defined system. In this system, both structural and material (including their processing in manufacturing) aspects are translated into context-independent parametrically defined relationships of geometrical nature. Designation of values to the parameters is required as soon as structural analysis is to be executed;
4. Variation of structural capacity is assured by including a customisable structural performance in the qualitative structural design, later to be implemented and quantitatively linked through parametrics.

7.1.2 On the proposed structural schemes and structural systems

The research question asked for the best-suited structural schemes, structural systems and structures for free form (parts of) buildings. A series of each has been proposed. All proposals are suitable for the context since they can follow the building geometry, they can be optimised to the required load carrying capacity, and their setup is systematic. The proposals that meet these criteria best have been developed to the furthest extent.

The proposed schemes, systems and structures have been generated based on the distinction of the design variables of geometry, material and force. These have been subdivided further into (sub)features, which has enabled establishing links between these on a high level of abstraction. Doing so, it was found that – in principle – numerically controlled cutting of sheet materials has released the constraints of straight cuts and of mass production. Free form contours and mass customisation came instead, although equipped with new criticalities of unambiguously defining curved contours and handling large amounts of information. For structural design, this stimulated the exploration of structural geometries based on planar geometries, which is the first proposed system.

When combining the cutting of free form contours with a formative technique, the geometrical design space of rationally producible elements is extended. Thus, the next search aimed at exploring beneficial structural properties of these geometrical constructs. The use of generic geometrical descriptions proved successful to anticipate a variety of geometries through a single geometrical principle. The connection of curved parts has been identified as the next criticality, which in turn could be handled through an additive technique. This has been implemented in the Delta Rib structural system which consists of ribs constituted out of three single curved sides, their curvilinear edges being connected through welding.

The formal freedom of the Delta Rib system is constrained by the application of constant curvatures. This constraint could be released if a locally controlled formation method is used instead. This was the underlying principle of the 3D-components, the third proposed system. In this system the required stiffness after the formation is achieved by filling the elements with a hardening, initially shapeless material, by which at a time also a load bearing building envelope is constructed.

Whereas the first two systems anticipate the irregular building shape through customised shaping of elements on a micro level of scale, the components of the third system are meso-sized, and therefore had to be released from a curvature constraint to achieve the same formal freedom.

Different from many of the precedents that were analysed, the proposals of this research are based on the exploration of a solution space, rather than a straightforward search to a single answer as generally found in building practice. The solution space is specifically defined from scratch, which is a strong distinction with the commonly practices method of adapting conventional systems to an unconventional context.

7.1.3 On the Delta Rib system

The Delta Rib system offers unprecedented freedom to design reticulated network structures with members that curve and twist, with the necessary variety in structural capacities and while being designable and fabricatable with commonly available methods. Furthermore, the Delta Rib's systematic setup is fit for a replicated implementation involving customisations of both dimensional nature (through adapting sizes and thicknesses) as well as qualitative nature (through adapting pattern configuration and additional elements).

Due to its customisable setup, the system allows for an automated implementation of the generation procedure. Feeding information on the realised structural performance back into the system, allows for an adaptation to the capacity locally needed. Furthermore, the automated implementation saves considerable engineering efforts, and therefore is a precondition for commercial application.

Despite the advantages of an automated replication of a parametrically defined element, its results cannot be taken for granted. Instead, critical assessment is required as results may be mathematically correct but formally undesirable or even structurally adverse. To prevent this, the designer should be able to control the geometry he is generating through at least one degree of freedom. The latter notably risks to occur in case only the ribs' corner points are controlled, and the rib in between logically results from imposed conditions. User intervention does not affect the systematic application, but challenges its automation. If such a degree of freedom is not available, it could be obtained by permitting slight geometrical deviations, notably by accepting slightly curved intersections as an alternative for straight intersection lines.

Prototyping of the system has demonstrated the constructability of the system. Constructing the system on the basis of referential surfaces without anticipating the material thickness leads to systematic dimensional deviations between the material object and the digital model, in the rib's longitudinal as well as transverse direction.

Whereas the latter could be logically related to the construction manner, this was not the case for the deviations in longitudinal direction.

7.2 Recommendations

7.2.1 On the approach from schemes to systems

Although different designers prefer different styles of working, based on the experience of this research it is recommended that creative searches are executed with an exemplary application in mind. This way, it is assured that the highly abstract level of schemes stays in contact with its final goal.

7.2.2 On proposed schemes and systems

The exploration of the design space starting from hypothetical solutions has led to a wide variety of potential structural systems for application in free form building designs. When only a selection of them is developed further, obviously numerous tracks are left open ended.

A potential aptness of additive fabrication processes for usage in load bearing surfaces has been recognized, but as constraints highly specific to the material and geometry emerged, it requires a study entirely devoted to it. Once criticalities and potentials of moulding techniques (or their substitutes) are inventoried, hypothetical solutions may be generated in a similar fashion to the approach followed in the actual research.

Concerning the planar members-system, further research is notably needed on the integration of the material properties into the element. The proposed timber laminate has anisotropic stiffness properties which may be influenced by the choice for a laminate, as well as by its orientation on the base-plate it is cut from. A parameter study has to reveal the macroscopic effects of these material variations on the micro level of scale. Further development of the system of 3D components in first instance concerns the accurate forming and fixation of the rib sides, as well as the choice for a material of the building envelope and the development of a mould for this within the rib layout.

7.2.3 On the Delta Rib system

Depending on the position, constructing the Delta Rib system from a representation of referential surfaces is acceptable. As the deviations in transverse direction are well-predictable, it is suggested not to anticipate the material thickness on the sides' upper and lower edges. Instead, as deviations in longitudinal direction would lead to a propagation of errors from the component level onto the macro level, the material thickness is recommended for anticipation here. In case of adding a tubular flange, the sheet thickness always has to be taken into account along the edge joining to the tube. The lower edges of strip flanges are recommended to be custom shaped too.

The Delta Rib system has been prepared for a customised replicated implementation of parametrically defined elements and components. It is recommended to implement the system in a digital parametrically controlled model, such that the design space can be explored by the designer (rather than by the system developer). However, as it was concluded that results of the automated implementation still require critical assessment, mechanisms for designer-intervention have to be installed.

Material efficiency of the system can be further increased if the parametric model is linked to a structural analysis model. Whereas critical stresses may initiate a simple modification that increases the structural dimensions, the anticipation to the global structural action involves more influential factors and thus is more complex to automate. Instead, it is suggested to make forces as well as deformations be interpreted by a structural engineer, who is to be enabled to manipulate the layout of the system. Thus, the digital parametric model of the system does not lead to more efficient structures per se, but fulfils the requirement of adaptation and subsequent assessment of its results.

Now that the system's potential is demonstrated, further development of the Delta Rib system should include the closing of the outer surface. This is likely to require changes to the system.

7.3 Perspectives for the structural engineering practice

The structural design alternatives presented in this thesis are the result of academic reasoning, guided by the research question. It is therefore not immediately suitable to be assessed through building industry's conventional criteria. Instead, on a more abstract level, this final section highlights some of this research's major impacts on the building engineering practice.

The most significant effects result from the proposed method of structural design, which entails splitting a conventional structural design phase into a phase of system design and a phase of system application. To do this, performance requirements for the application as well as manufacturing constraints have to be translated into a generic setup. This way, the total complexity – which is too extensive to be overseen entirely – is split in parts (two in this case), down to a level that is more likely to be manageable. This splitting has several effects on the use, order and flexibility of the scheme-to-system approach:

1. Rather than a steady growth of knowledge as in conventional design processes, using schemes and systems requires considerable effort-investment within limited time, subsequently followed by extensive project-specific explorations of each as depicted in Figure 164. These developments are not linked or equivalent to conventional decision moments of schematic design, design development and construction documents.
2. Due to its generic setup, schemes and systems may be re-used in future projects.
3. Concerning the order of tasks, it enforces reversing a traditional design process by investigating a fabrication process prior to making the detailed design.
4. As a result of the changed order, systematic choices (with a wide-reaching impact) have to be taken in an early stage, while decisions on implementation of the system may be postponed.
5. Concerning flexibility, the system is flexible in the sense that its application can easily be re-generated. However its use is rigid in the sense that it can only accommodate changes that match the degrees of freedom foreseen in the setup of both scheme and system. Ad-hoc solutions compromise the advantages of the systematic setup and should therefore be avoided.

Apart from technological developments, the application of for example the Delta Rib system, also involves organisational changes. Therefore, the primary contribution of this thesis lies in the ability to explore the technical solution space. By using the proposed method, previously unknown alternatives, and therefore un-evaluated solutions, have come to light.

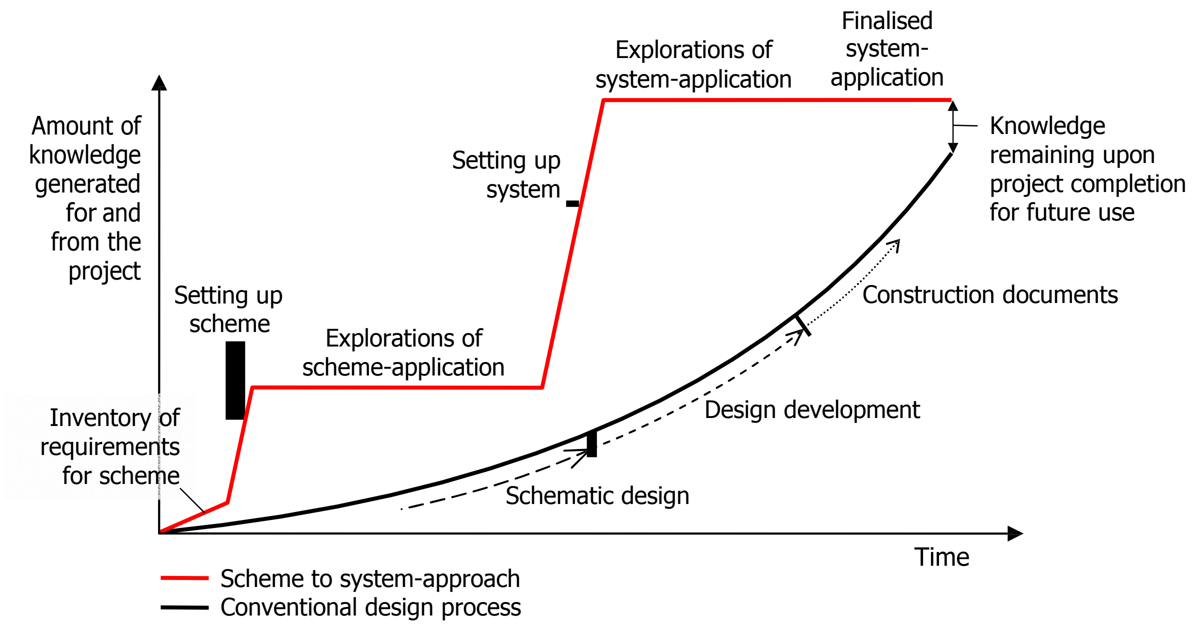


Figure 164. Comparison of knowledge generated over time in the scheme-to-system approach, compared to that of a conventional design process.

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Summary

Free Form Structural Design
Schemes, systems and prototypes of structures for irregular shaped buildings
Martijn Veltkamp

Irregularly shaped building designs with surfaces curving in two directions ('double curved'), and also known as Free Form, Blob or liquid architecture, have gained renewed interest in the last decade due to the then emerging availability and user-friendliness of computerized design tools in the 1990's. These tools were introduced in the domain of architectural design through a technology transfer from film-, car and aeroplane industry. Whereas architects explored the tools' formal consequences and proposed building designs with them, the disciplines involved in the technical realisation of such building designs underwent little change, as they focussed on digitising their existing working methods. This has resulted in structural designs which, despite fulfilling the functional and structural requirements, are not perceived as fully satisfying by the engineering disciplines. Notable shortcomings are the non-conforming of structural shape and architectural shape (e.g. straight beams in a curved building), the structure's inability to anticipate the locally required load bearing capacity, and finally the low level of systemisation, which for instance impedes re-using details in a customised manner.

This observed lacking defined the primary research question of this thesis: What are the most appropriate structural schemes, structural systems and structural designs for free form (parts of) buildings? 'Appropriate' is defined here as the highest degrees of systematisation, formal freedom and material efficiency, while 'schemes', 'systems' and 'structural designs' are three levels of abstraction, from merely abstract to highly specified. Through these transitory steps, solutions to be developed may initially be material-independent, thus keeping the range of applications wide.

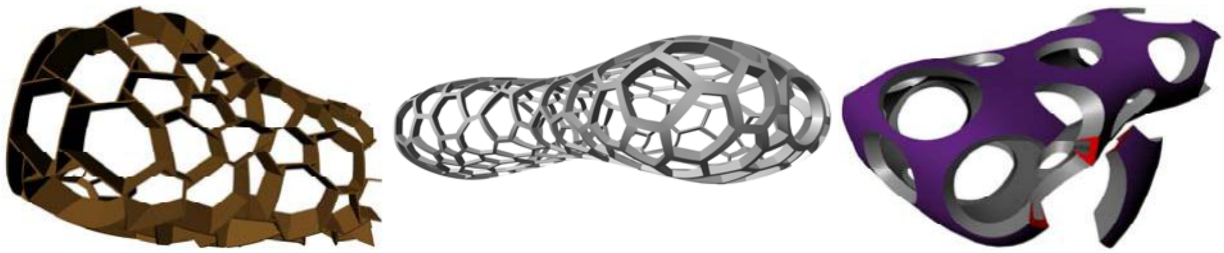
To answer the research question, three domains out of numerous influential parameters have been targeted specifically:
1. The definition of the structure's geometry;
2. The applied material and how it is brought into shape;
3. The structural action through which loads are transferred.

A study of precedent projects, structures and systems addressing sub-solutions on these three domains has resulted in the definition of search directions for structures that meet the requirements. The three most important potential solutions that have contributed to the final three systems were:
1. The use of planar elements, whose contours in plan are not constrained by manufacturing techniques;
2. The usage of developable surfaces. These surfaces can be unrolled to a plane without tearing or wrinkling, thus allowing use of commonly available sheet-like materials;
3. The use of partially controlled techniques to form materials. This means that to for instance curve a flat panel, only the position of a few points should be secured, while the rest of the panel settles itself in between. This greatly simplifies the fabrication of such panels.

Several variables per potential solution, as well as multiple values that this variable can take, were defined. By varying on them, the potential solutions, and their combinations, were systematically explored. As the potential solutions are inherently related to one or more aspects of geometry, structural action or the processing of a material class, the 23 highly abstract structural schemes that resulted, were technically viable.

The next step comprised the development of three structural schemes into structural systems, namely the systems:

1. Planar members with end-connections, comprising a polygonal network of custom-cut timber members of variable height, while connections are kept simple by joining only two members at a time;
2. Delta Ribs, comprising a network of curving and twisting members. Its members are triangular in cross section, and are composed of custom-cut steel sheets that are rolled into a single curved shape;
3. 3D components, comprising prefabricated structural elements that are both structure and building envelope. They are based on a curved frame following the same principle as the Delta Ribs system, while the space in between is filled in with a structurally-active layer, which fixates the frame underneath.



Planar members with end-connections. Delta Ribs. 3D components.

The three systems applied to a free form building.

All systems consist of medium-sized elements that allow the system to locally anticipate the building shape. By doing so it has been demonstrated how the geometrical setup of the schematic setup are propagated to a system design which comprises a specific material and a method to process it, continually maintaining the system's technical viability. To assess the ability to apply the system in a systematic manner, and validate its versatility in an irregular shaped building design, each system has been implemented on an imaginary building design. The application in a specific context resulted in findings on the systems' limitations.

The Delta Rib-system has been developed further into three full-scale prototypes, made of sheet metal. Building the prototypes was a test of the underlying system. Although the anticipated primary principles of the system were confirmed, notably the implications of the steel sheets' thickness on the dimensional accuracy necessitated a further specification of the system.

This study demonstrates how structures consisting of systematically generated components fulfil the needs for structures appropriate for free form building designs. The systems resulting from this research give unprecedented freedom of shaping, while maintaining rational fabrication standards. This can be implemented through parametric modelling, in which systems adapt themselves to the local geometry, as well as to local structural needs. Technically speaking, structures of irregularly shaped buildings no longer need to be constructed as 2-dimensional frames that slice through a building with total disregard of the building's geometry.

Samenvatting

Het constructief ontwerpen van vrije vormen
Schema's, systemen en prototypes van constructies in onregelmatig gevormde gebouwen
Martijn Veltkamp

Onregelmatig gevormde gebouwen met vlakken die in twee richtingen krommen ('dubbel gekromd'), ook wel bekend als vrije vormen en Blobs, zijn in de laatste tien jaar opnieuw in de belangstelling komen te staan dankzij nieuwe en gebruiksvriendelijke modelleringssoftware. Deze software komt uit de film-, auto- en vliegtuigindustrie, en is nu dus ook bij architecten in gebruik. Terwijl architecten vooral de vormmogelijkheden ervan benutten om er gebouwontwerpen mee te maken, ging de trend voorbij aan de technische disciplines betrokken bij de realisatie van de gebouwen. Voor wat betreft de constructies in dergelijke gebouwen, leidde dit ertoe dat zij weliswaar aan alle functionele en constructieve eisen voldeden, maar toch geen voldoening gaven. Als tekortkomingen werden onder andere gezien dat de constructieve vorm en de architectonische vorm niet overeen kwamen (bijvoorbeeld een spantenconstructie die het gebouw meedogenloos in platte vlakken snijdt, zelfs al zit er in het hele gebouw geen enkel plat vlak), de onmogelijkheid om de sterkte van de constructie lokaal aan te passen (of: grotendeels onnodig zwaar te dimensioneren), en als laatste de beperkte systematiek waardoor eenmaal ontwikkelde details niet op meerdere plaatsen toegepast konden worden.

Het verhelpen van deze tekortkomingen komt terug in de onderzoeksvraag: wat zijn de meest geëigende constructieve schema's, constructieve systemen en constructieve ontwerpen voor vrijgevormde (delen van) gebouwen? Met "geëigend" worden de hoogst mogelijke graden van systematisatie, vormvrijheid en materiaalbenutting bedoeld, terwijl "schema's", "systemen" en "constructieve ontwerpen" drie verschillende abstractieniveaus zijn, van hoog-abstract tot specifiek-toegepast. Door deze abstractieniveaus kunnen oplossingsrichtingen zo lang mogelijk materiaal-onafhankelijk worden gehouden, en daardoor breed toepasbaar.

Alhoewel er op het gebied van constructief ontwerpen vele variabelen te benoemen zijn, zijn er in dit onderzoek specifiek drie onderzocht:
1. Het vastleggen van de geometrie;
2. Het bouwmateriaal en hoe dit bewerkt kan worden;
3. De wijze waarop krachten worden ontvangen, overgedragen en afgedragen.

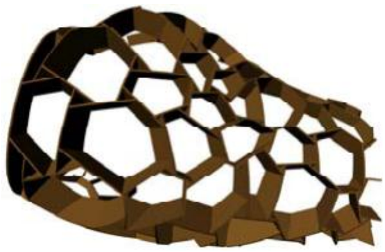
Op basis van studie naar eerdere oplossingen in gebouwen, constructies of systemen, op de drie voorgenoemde gebieden, heeft geresulteerd in zoekrichtingen voor constructies die aan de gestelde eisen voldoen. De drie oplossingsrichtingen die zijn terug te vinden in de drie systemen die uiteindelijk zijn ontwikkeld, zijn:
1. Het gebruik van vlakke elementen. Productietechnieken leggen geen beperkingen op de contouren hiervan;
2. Het gebruik van afrolbare vlakken. Dergelijke vlakken kunnen (in tegenstelling tot dubbelgekromde vlakken) zonder scheuren en plooien afgerold worden, en zijn dus te maken uit standaard plaatmateriaal;
3. Het gebruik van gedeeltelijk beheerste omvormingstechnieken. Hiermee kan bijvoorbeeld een vlak worden gekromd door het instellen van slechts een beperkt aantal punten, terwijl de rest van het vlak zichzelf ertussen vormt. Dit vereenvoudigt de productie van dergelijke elementen.

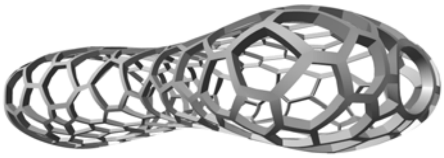
Per oplossingsrichting zijn vervolgens meerdere variabelen bepaald, alsmede de waardes die elk van deze variabelen aan kan nemen. Door hiermee te variëren zijn er 23 abstracte constructieve schema's in kaart gebracht. Omdat alle variabelen al vanuit de oplossingsrichting intrinsiek gerelateerd waren aan geometrie en/of materiaal en/of

constructieve werking, waren alle schema's die hiermee gegenereerd werden in principe technisch mogelijk.

De volgende stap omvatte de verdere ontwikkeling van drie geselecteerde constructieve schema's tot constructieve systemen. Dit betrof:

1. Vlakke ribben die op de uiteinden op elkaar aansluiten. De ribben worden in een netwerk toegepast, waarbij de verbindingen simpel worden gehouden doordat telkens maar twee ribben op elkaar aansluiten;
2. Deltaribben, een netwerk van krommende en torderende ribben. De ribben hebben een driehoekige doorsnede, en zijn samengesteld uit drie op maat gesneden gekromde staalplaten;
3. 3D-componenten, die tegelijkertijd constructie en geveldichting zijn. Ze zijn gebaseerd op verstijvingen volgens het principe van de Deltaribben, waarbij ook de ruimte ertussen constructief actief is, en tegelijkertijd de verstijvingsribben stabiliseert.


Vlakke ribben.


Deltaribben.


3D componenten.

De drie systemen toegepast op een vrijgevormd gebouwontwerp.

Alle drie de systemen bestaan uit componenten die op maat gemaakt worden voor een specifieke plek in de constructie. Op deze manier is aangetoond hoe de geometrische opzet van het constructieve schema doorwerkt in een materiaal-specifiek systeem. Om te kunnen beoordelen of het echt werkt, en zo de systemen te valideren voor toepassing in een onregelmatige gebouwvorm, is elk systeem ook toegepast op een gebouwontwerp. Hiermee werd ook inzicht verkregen in het toepassingsgebied, bijvoorbeeld of het systeem ook werkt bij heel sterk gekromde gebouwdelen.

Het Deltaribben-systeem is verder uitgewerkt tot drie prototypen op ware grootte, gemaakt van plaatstaal. Het bouwen van de prototypen was de proef van het onderliggende systeem. Alhoewel de belangrijkste principes werden bevestigd, bleek het omwille van de maatvastheid noodzakelijk om de materiaaldikte al in de ontwerpfase mee te nemen.

Dit onderzoek heeft aangetoond hoe constructies die bestaan uit systematisch gegenereerde componenten voldoen aan de eisen die aan constructies voor vrije vorm gebouwen worden gesteld. De systemen die in dit onderzoek zijn gesteld, bieden ongeëvenaarde vormvrijheid, waarbij nog altijd een rationele productiewijze mogelijk is. De systemen kunnen parametrisch worden toegepast, waarbij ze zich vormen naar de locale geometrie en de aldaar vereiste constructieve capaciteit. Het zijn dus efficiënte systemen. Technisch gezien kan de draagconstructie nu volledig in de vrijgevormde gebouwvorm worden geïntegreerd.

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With the help of Corus, the building of the prototypes of the system eventually took place. Therefore I am grateful to Corus, which was represented by Bauke Hoekstra Bonnema, as well as to the other contributors to the prototypes: Kersten Europe which was represented by Bart Simonse, and to the department of Building Technology. All contributed materials or financial support, as well as advice. The prototypes were built in the laboratory of Building Technology by Léon Kolfers and Kees Baardolf and I was glad to assist them in doing so.

During my years at Bouwkunde, I enjoyed giving workshops and organising research-events together with Walter Lockefeer, Tuba Kocaturk, Bige Tuncer and Andrew Borgart. Everyday motivation was greatly enabled by the social interaction and the Friday morning cookie time with colleagues in room 7.17. Thanks to all of you, doing a PhD was never a lonely endeavour.

Though only one day a week, my work at Octatube during the years of my research enabled me to see my research in a non-academic context. The focus on the ability to physically build things which is proclaimed throughout this thesis is to a large extent due to the 'can do' attitude at Octatube's. Since such a small appointment is not common in industry, I am grateful that this opportunity was given to me.

Finally, I am thankful to my parents, sister and further family, Robert and friends, for the connection to real life and the support they have given me.

Martijn Veltkamp
London, May 2007

About the author

Martijn Veltkamp was born in 1977 in Paramaribo, Surinam. He grew up in the Netherlands after spending periods in Surinam and Columbia in his early childhood. From 1995 to 2001 he studied Civil Engineering at Delft University of Technology. As part of his degree he spent an extended period at the Ecole Nationale des Ponts et Chaussées in Paris. His MSc degree was awarded with the qualification cum laude.

The research for this thesis was conducted from 2002 to 2007 within the chair of Product Development at the faculty of Architecture, Delft University of Technology, on a four-days-per-week basis. The other weekday was spent working as structural engineer at the design & build company Octatube Space Structures.

At the faculty of Architecture, Martijn Veltkamp has been a consultant for graduate students, taught in master courses, and co-initiated the sub-Working Group Free Form Design, part of the International Association for Shell and Spatial Structures (IASS). He has also been actively involved in numerous voluntary activities, including memberships of the boards of the student association Delftsche Zwervers, the representative body of PhD candidates in Delft PROMOOD, and the association of students in building technology BouT.

After the completion of the manuscript, Martijn Veltkamp has been working as a structural analyst at Adams Kara Taylor, London.