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# Closed-Form Solutions for the Analysis of Artificial Dielectric Layers under Generic Field Incidence

Daniele Cavallo, Waqas H. Syed, and Andrea Neto

Microelectronics dept.  
Delft University of Technology  
Delft, The Netherlands

d.cavallo@tudelft.nl, w.h.syed@tudelft.nl

**Abstract**—We present an analytical method to model artificial dielectric layers (ADLs) of finite height. Starting from the closed-form solution for the scattering from a single layer under plane wave illumination, the formulation is extended to the multi-layer case, by including the higher-order interaction between parallel layers in analytical form. The method can be used to describe the radiation of a source located in the close proximity of the ADL. Experimental data obtained from a prototype demonstrator are presented and show a good agreement with the results of the theoretical analysis.

## I. INTRODUCTION

Recently the growing demand for silicon integrated antennas is driving a renewed interest for planar artificial dielectrics, to improve the performance of such antennas. In integrated technology, artificial dielectric layers (ADLs) can be realized by multiple layers of patches with small electrical dimensions, as depicted in Fig. 1. These structures can be used to enhance the front-to-back ratio and the gain of integrated antennas, without supporting surface waves [1], [2].

The scattering from a single layer of periodic electrically small square patches has been extensively treated in the literature, for example in [3]–[6]. Some of these methods provide analytical formulas for the equivalent reactance of the layer under generic plane-wave incidence. Since ADLs consist of multiple layers that are closely spaced, the inter-layer interaction is dominant and significantly changes the reactance that the layer would have in isolation. Therefore the reactive coupling between parallel adjacent layers must be rigorously taken into account to correctly characterize the structure. This coupling can be described by adopting methods such as multi-mode equivalent networks or generalized admittance matrix [7], [8]. However, such methods lead to rather complex equivalent networks to represent the ensemble of metallic layers that constitute the ADL, which renders the analysis difficult for arbitrary excitation. Recently, an approach based on multi-modal equivalent network was presented in [9]. This method accounts for the higher-order Floquet modes represented as lumped elements. However, it is assumed that the aperture field on the unit cell is known a priori.

In this work we derive an analytical method to describe artificial dielectric layers (ADL) of finite thickness under generic field incidence. The method is based on the generalization of the single-layer solution to the multi-layer

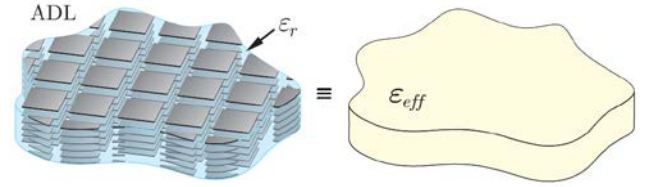


Fig. 1. Artificial dielectric layer structure and equivalent anisotropic medium.

case, accounting for the reactive coupling between parallel layers in analytical form. Equivalent circuit model for finite ADL slabs are given and used to evaluate the dispersion characteristic of finite slabs, and the radiation pattern of a generic antenna in the presence of the ADL. The calculated patterns were validated both with simulations performed with commercial electromagnetic solvers and with measured results from a 300 GHz antenna prototype.

## II. EQUIVALENT CIRCUIT OF THE ADL

With reference to Fig. 2(a), let us consider a plane wave with magnetic field  $\mathbf{h}_{\text{dir}}$  incident on an array of electrically small patches from the direction  $(\theta, \phi)$ . By decomposing the incident magnetic field into transverse electric (TE) and transverse magnetic (TM) components

$$\mathbf{h}_{\text{dir}} = I_{\text{TE}}^+ \mathbf{h}_{\text{TE}} + I_{\text{TM}}^+ \mathbf{h}_{\text{TM}} \quad (1)$$

the equivalent circuit representation in Fig. 2(b) can be used [10]. Two uncoupled transmission lines describe the propagation of TE and TM waves, while the patch array is represented by an equivalent layer susceptance given by:

$$B_s = \frac{2k_0}{\zeta_0} \sum_{m_y \neq 0} \frac{\left| \text{sinc}\left(\frac{k_{ym} w_x}{2}\right) \right|^2}{|k_{ym}|} \quad (2)$$

where  $k_{ym} \approx 2\pi m_y / d_y$  and  $k_0, \zeta_0$  are the characteristic wavenumber and impedance of the medium, respectively.

When an infinite number of layers is considered, as shown in Fig. 3, the expression in (2) can be generalized to account for the inter-layer reactive coupling [11]. The only variation with respect to the single layer involves modifying the higher order

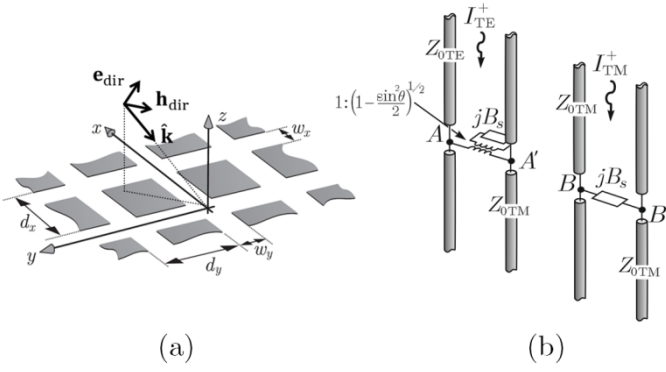


Fig. 2. (a) Periodic array of electrical small patches under plane-wave incidence; (b) equivalent transmission lines for TE and TM incidence.

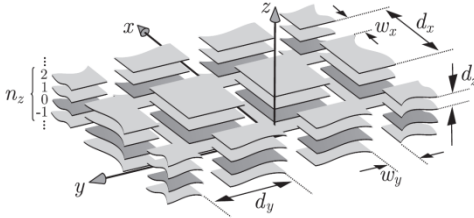


Fig. 3. Multi-layer artificial dielectric with geometrical parameters.

Floquet modes of the Green's function to account for the infinite sum on the indexes  $n_z$ . The layer susceptance becomes

$$B_{s\infty} = \frac{j2k_0}{\zeta_0} \sum_{m_y \neq 0} \frac{\left| \text{sinc}\left(\frac{k_{ym}w_x}{2}\right) \right|^2}{|k_{ym}|} \tan\left(\frac{k_{zm}d_z}{2}\right) \quad (3)$$

where  $k_{zm} \approx -j|k_{ym}|$ .

Whenever dealing with realistic problems, the number of layers is finite. The accuracy of the infinite approximation in (3) can be improved by describing the first and last layers of the finite stack with the semi-infinite solution:

$$B_{s,\text{semi}\infty} = \frac{k_0}{\zeta_0} \sum_{m_y \neq 0} \frac{\left| \text{sinc}\left(\frac{k_{ym}w_x}{2}\right) \right|^2}{|k_{ym}|} \left(1 + j \tan\left(\frac{k_{zm}d_z}{2}\right)\right). \quad (4)$$

The equivalent circuit becomes then the one in Fig. 4, for example for a cascade of four layers. A generic field radiated by a near source can be expanded in a spectrum of plane waves. For each plane wave we can use the equivalent network in Fig. 4 to calculate the asymptotic (far-field) radiated field. As a validation, we compare our method with the measured radiation patterns from the demonstrator presented in [2]. This consists of a double slot antenna loaded with an ADL (Fig. 5(a)). The results of the analytical method are compared with measurements in Fig. 5(b), showing a good agreement.

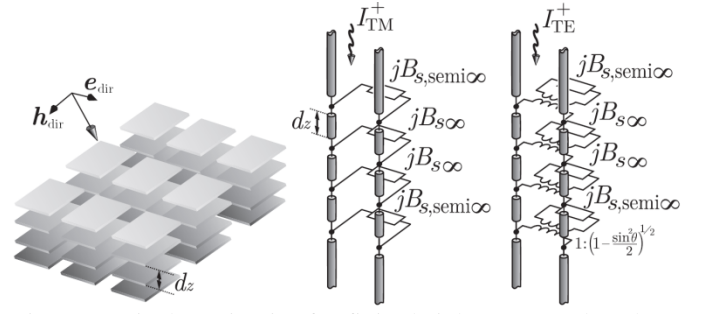


Fig. 4. Equivalent circuit of a finite-height ADL under plane-wave illumination.

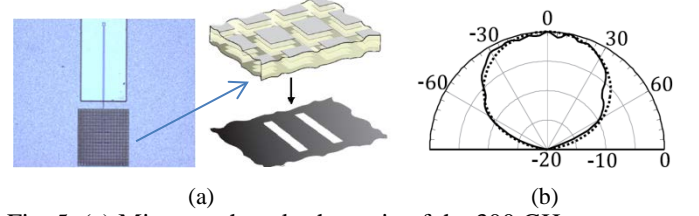


Fig. 5. (a) Micrograph and schematic of the 300 GHz prototype in [2]; (b) measured (--) and the simulated (--) radiation pattern.

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