

Source brightness and useful beam current of carbon nanotubes and other very small emitters

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The potential application of carbon nanotubes as electron sources in electron microscopes is analyzed. The resolution and probe current that can be obtained from a carbon nanotube emitter in a low-voltage scanning electron microscope are calculated and compared to the state of the art using Schottky electron sources. Many analytical equations for probe-size versus probe-current relations in different parameter regimes are obtained. It is shown that for most carbon nanotube emitters, the gun lens aberrations are larger than the emitters' virtual source size and thus restrict the microscope's performance. The result is that the advantages of the higher brightness of nanotube emitters are limited unless the angular emission current is increased over present day values or the gun lens aberrations are decreased. For some nanotubes with a closed cap, it is known that the emitted electron beam is coherent over the full emission cone. We argue that for such emitters the parameter "brightness" becomes meaningless. The influence of phase variations in the electron wave front emitted from such a nanotube emitter on the focusing of the electron beam is analyzed. © 2006 American Institute of Physics. [DOI: 10.1063/1.2162270]

I. INTRODUCTION

Carbon nanotubes (CNTs) are widely regarded as promising candidates to replace the presently dominating "Schottky emitter" in electron sources for microscopes and lithography machines.^{1,2} The usual arguments are "small energy spread," "high coherence," and "extremely high brightness," because generally there is a direct proportionality between useful beam current and brightness. The same arguments previously led to the investigation of nanometer-sized tungsten field emitters.³ The aim, of course, is to have a large current in a small spot for scanning electron microscopes and lithography machines or to have a high current density in a large coherent area in the transmission electron microscope. There is a problem, however: what does "brightness" mean in a coherent beam? If the concept of brightness loses meaning, the argument for CNTs as prospective electron sources must be reconsidered.

The concept of brightness is used in geometrical optics because it is a property of a beam that is constant under macroscopic changes to the beam. Because in electron optics the beam can be accelerated or decelerated, it is necessary to work with "reduced brightness:" the current per unit area into a solid angle unit divided by the electron acceleration voltage,

$$B_r = \frac{\delta I}{\delta A \delta \Omega V}. \quad (1)$$

Neither magnification, nor aperturing, nor acceleration can change B_r . The importance for probe-forming systems is clear: given the size of the source image d_I in a probe and the

aperture angle α as limited by aberrations, the current in the probe is

$$I_p = B_r \frac{\pi}{4} d_I^2 \pi \alpha^2 V \quad (2)$$

and thus directly proportional to B_r .

When the wave properties of electrons are taken into account, a complication arises because the electron wave can spread out over a larger area than geometrical optics would allow. Thus, the area A of the beam increases and the brightness would decrease. Imagine a beam with aperture angle α focused into a diffraction-dominated probe of approximate size $d_A = 0.6\lambda/\alpha$, containing current I . If the aperture is decreased to $\alpha/2$, the probe size increases by a factor of 2 and the current decreases by a factor of 4. If this beam size would be used in the brightness equation, the brightness would also have decreased by a factor of 4. In practice, we have been able to circumvent this complication because in most systems there is only one plane in which the diffraction increases the beam size significantly: the plane in which the probe is focused. Here we calculate the diffraction size and the geometrical size separately and then add them to find the resolution of the system, while only using the geometrical size to calculate the current in the probe. We think about the diffraction spot as the area in which the electron wave of one electron arrives and, obviously, the different parts of that wave are fully coherent with each other. The trajectory of this electron in the geometrical approach starts at a specific point on the emitter and ends in the center of its diffraction spot. Different electrons are never mutually coherent, nevertheless we speak of a "coherent beam" if all geometrical trajectories end in a spot much smaller than the diffraction spot ($d_I \ll d_A$) and we speak of a "partially coherent beam" if the geometrical trajectories end in a spot of the same dimen-

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sion as the diffraction spot. Clearly, every beam can be made coherent by sufficiently demagnifying the geometrical size. Equivalently, each emitter can be made to seem coherent by accepting a sufficiently small cone of electrons, such that the diffraction spot of the back-traced electrons in the virtual source plane is larger than the geometrical size of the virtual source.

For very small emitters such as the CNTs it may be that the size of the electron wave in the (virtual) source plane is already large compared to the spread in starting points of geometrical trajectories, even without aperturing the beam ($d_v \ll 0.5\lambda/\alpha_{\text{tot}}$, with α_{tot} the half emission angle). We could certainly call this an intrinsically coherent emitter. Usually, in optimizing a probe in an electron-beam instrument, we make the contribution of the geometrical spot to the probe size about equal to the contribution from other causes, such as diffraction or aberrations. However, if the geometrical size of the source is already smaller than the size of the wave function at the source, this cannot be done anymore and the analysis of probe size and probe current totally changes. We shall perform that analysis in the following sections and find that brightness becomes a meaningless characterization for these emitters.

It is very difficult to measure the brightness of (partially) coherent emitters, because it is impossible to make an image of only the geometrical source without the diffraction contribution. Mostly this will lead to an underestimation of the brightness. The method used by Fransen *et al.*⁴ and de Jonge⁵ offers the best chance of giving the real value of the geometrical source size and thus the brightness. For an emitter as analyzed by Hata *et al.*² even the definition of brightness is a problem. Here a single electron wave exits through six pentagons in the cap of the nanotube. When using the beam from this emitter, one will almost always select the part of the beam that comes from the center pentagon, so what to take for the size of the electron wave in the virtual source plane? What are the starting points for the geometrical trajectories, the distribution of which we need for the definition of brightness? Hata *et al.*² assumed a virtual source size equal to the actual size of the pentagon, without correcting for the magnification from cap surface to virtual source plane. This is rather arbitrary. We can only conclude that all electrons are emitted from approximately the same delocalized electron states inside the emitter and thus that the brightness is infinite or undetermined. We shall see that in practice this does not make a difference in the use of these emitters since brightness does not play a role anymore. There is a second question about coherent sources: Is coherence in a beam a sufficient requirement for the ability to focus the beam to a “diffraction-limited” spot? In order for a beam of aperture angle α to be focused into a spot of size $0.5\lambda/\alpha$, it is necessary that the wave front in the aperture is a perfect sphere; in other words, the phase on the sphere must be constant. In focusing laser beams this is well known: the beam must be single mode for focusing to the smallest diffraction disk. Is this the case in the beam emitted from a CNT?

II. BRIGHTNESS DEFINITION AND CURRENT IN A PROBE

The reduced brightness of an electron source is given by⁶

$$B_r = \frac{J_\Omega}{\pi/4 d_v^2 V_e}, \quad (3)$$

where V_e is the accelerating voltage at the extractor plane and d_v is the diameter of the virtual source as seen from this plane. J_Ω is the angular current density from the virtual source, measured at the extractor voltage V_e . V_e depends on the type of emitter and on the geometry of the emitter environment and extractor. Very sharp emitters work at lower V_e than larger emitters. Changing the extractor voltage has a dual effect: increasing the field at the emitter surface, which increases the brightness, and further accelerating the beam, which contracts the beam into a smaller cone.

To form a probe in an electron microscope or lithography machine, the virtual source is imaged on the target with magnification M . In the optics column, the aperture angle is limited by a variable aperture to a half-angle α at the probe, corresponding to a half-angle α_e at the source. An infinitely small point in the virtual source plane is imaged to a blurred spot on the target as a result of diffraction and aberrations. The effect of a finite source size is taken into account by convoluting the blurred spot with the intensity distribution of the source image. Since B_r is a conserved quantity through the whole system, the current in the probe is calculated from

$$I_p = B_r \frac{\pi}{4} (M d_v)^2 \pi \alpha^2 V, \quad (4)$$

where V is the accelerating voltage at the target.

Barth and Kruit⁷ have shown that for the calculation of the full width 50% (FW50) probe size, the wave optical addition of diffraction and spherical aberration contributions and subsequent convolution with the source distribution and chromatic aberration can be approximated by the following addition rule:

$$d_p = \{[d_I^{1.3} + (d_A^4 + d_s^4)^{1.3/4}]^{2/1.3} + d_c^2\}^{1/2}, \quad (5)$$

where d_I , d_A , d_s , and d_c are the contributions from the source image, the diffraction disk, the spherical aberration, and the chromatic aberration, respectively, given by Eqs. (6)–(9).

$$d_I = M d_v = \frac{2}{\pi} \sqrt{\frac{I}{B_r V}} \frac{1}{\alpha}, \quad (6)$$

$$d_A = 0.54 \frac{\Lambda}{V^{1/2}} \frac{1}{\alpha}, \quad (7)$$

with $\Lambda = 1.226 \times 10^{-9} \text{ m V}^{1/2}$.

$$d_s = 0.18 C_s \alpha^3, \quad (8)$$

with C_s the spherical aberration coefficient of the system.

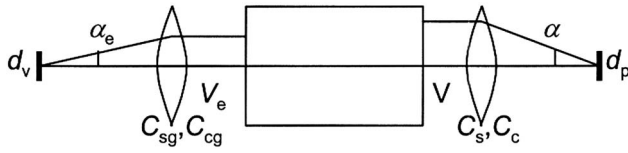


FIG. 1. Schematic diagram of the optical system that images the source onto the target of a SEM or lithography machine.

$$d_c = 0.6C_c \frac{\delta U}{V} \alpha, \quad (9)$$

with C_c the chromatic aberration coefficient of the system and δU the FW50 of the energy distribution of the source.⁸ Note that people often use the full width at half maximum (FWHM) of the energy distribution, in which case the prefactor in Eq. (9) must be 0.34.

In probe-size calculations, it is very often allowed to use only the aberration coefficients of the last probe-forming lens for the analysis of Eqs. (8) and (9). This is because usually the last lens demagnifies all aberration contributions of the other lenses in the system. However, for emitters with a very small virtual source size, such as the CNT emitter, the aberration coefficients of the gun lens must be taken into account explicitly. We shall assume that the gun lens directly follows the extractor, although it may be advantageous to carefully postaccelerate the beam before entering the gun lens. The equations can be kept relatively simple if we assume that the gun lens has a very large magnification and the last probe-forming lens has a very large demagnification, because in that case the aberrations of all intermediate lenses can be neglected. The contributions to the probe size, now including the aberrations of the gun lens, can be written in a number of forms. We choose to express them in terms of the accepted cone half-angle α_e at the emitter. At the same time, we want to include the possibility that the virtual source size goes to very small values or even to zero, as in the case of fully coherent emitters.

The quantity B_r becomes indeterminable in the latter case and the current can only be calculated from what is emitted into the acceptance cone at the source

$$I = J_\Omega \pi \alpha_e^2. \quad (10)$$

For an emitter with finite size and definable B_r , this is the same as in Eq. (4), since

$$M = \frac{d_I}{d_v} = \frac{\alpha_e}{\alpha} \left(\frac{V_e}{V} \right)^{1/2}. \quad (11)$$

Taking into account the aberration coefficients of the gun lens C_{sg} and C_{cg} and the aberration coefficients of the last probe-forming lens C_s and C_c (see Fig. 1),

$$d_I = M d_v, \quad (12)$$

$$d_A = 0.54M \frac{\Lambda}{V_e^{1/2}} \frac{1}{\alpha_e}, \quad (13)$$

with $\Lambda = 1.226 \times 10^{-9} \text{ m V}^{1/2}$.

$$d_s = 0.18M \alpha_e^3 \left[C_{sg} + \frac{1}{M^4} \left(\frac{V_e}{V} \right)^{3/2} C_s \right], \quad (14)$$

TABLE I. Values of parameters used in the calculation of the current vs probe-size relation for four emitters.

	Schottky emitter ^a	Monochromator Schottky	CNT1 ^b	CNT2 ^c
B_r (A/m ² sr V)	5×10^7	2.5×10^7	8×10^8	Infinite
d_v (m)	50×10^{-9}	50×10^{-9}	3.3×10^{-9}	0
J_Ω (A/sr)	0.57×10^{-3}	0.28×10^{-3}	1.6×10^{-6}	0.35×10^{-6}
δU (V)	0.4	0.2	0.2	0.2
V_e (V)	5000	5000	250	1800
C_{sg} (m)	10^{-2}	10^{-2}	10^{-3}	5×10^{-3}
C_{cg} (m)	10^{-2}	10^{-2}	10^{-3}	5×10^{-3}
V (V)	1000	1000	1000	1000
C_s (m)	0.5×10^{-3}	0.5×10^{-3}	0.5×10^{-3}	0.5×10^{-3}
C_c (m)	0.5×10^{-3}	0.5×10^{-3}	0.5×10^{-3}	0.5×10^{-3}

^aReference 10.

^bReference 1.

^cReference 2.

$$d_c = 0.6M \alpha_e \frac{\delta U}{V_e} \left[C_{cg} + \frac{1}{M^2} \left(\frac{V_e}{V} \right)^{3/2} C_c \right] \quad (15)$$

(see also Ref. 17).

To calculate I vs d_p curves, one proceeds as follows. For a probe current I , the half cone angle α_e follows from Eq. (10). Given a source with its emission properties and lenses with their aberration coefficients, the probe size can be minimized by optimizing the total magnification M . In effect, choosing M means determining the aperture angle α at the target and thus balancing the contributions from the probe lens aberrations, which increase with α and the contributions from diffraction, source image, and gun lens aberrations, which decrease with α . At very large currents, the aberrations will dominate all other contributions. The optimum M then balances the gun and probe lens aberrations. We shall first calculate I vs d_p curves for typical emission properties of a Schottky gun and compare these to curves for CNT emitters. After that we shall try to derive some general conclusions for the CNT emitter as a source for probe-forming systems. A typical application of high brightness sources is the low-voltage scanning electron microscope (SEM) where the acceleration voltage (beam energy) at the target is 1 kV and the aberration coefficients of the last lens are $C_s = 0.5 \text{ mm}$ and $C_c = 0.5 \text{ mm}$. The aberrations of present generation gun lenses for Schottky emitters are not always fully optimized since for probe currents less than about 300 nA, the gun lens aberrations are negligible. For CNT emitters, the gun lens aberrations are important because of the smaller virtual source size. The construction of CNT gun lenses can be different from Schottky gun lenses since the voltage of the anode for a CNT emitter may be lower than for a Schottky emitter. Thus, the aberrations of CNT emitters can be smaller. Table I gives the values used in the calculations. The values for the Schottky emitter are conservative: brightness values of 2×10^8 have also been demonstrated.⁹ Adding a monochromator to a Schottky source has been demonstrated and shown to be of value for high-resolution probes, so we add this case to the comparison, setting the energy width equal to what is obtained intrinsically from a CNT emitter. Note that we use FW50 values for the energy spread; FWHM

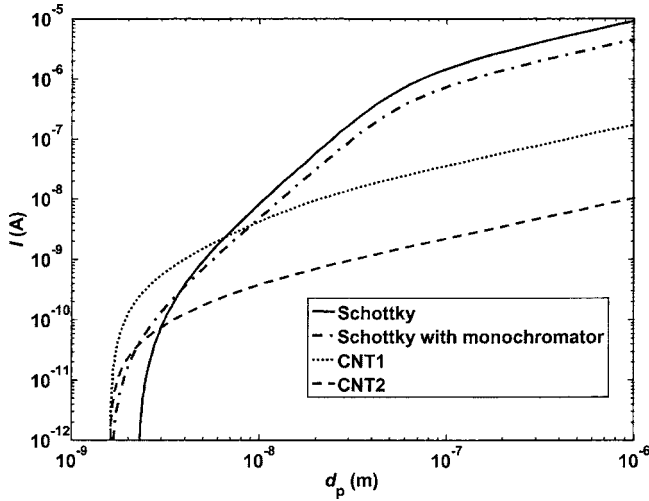


FIG. 2. Current-probe size relation for the four emitters of Table I.

values are higher but would demand a different and source-specific prefactor in Eq. (15). The values in the table for CNT1 are from de Jonge *et al.*,¹ specifically the average of CNT numbers 7 and 8 in their Table I. Apparently, 200 nA is a typical stable emission condition at an anode voltage of about 250 V. From this table we have taken the average normalized J_Ω of 3.2 nA/sr V at 100 nA emission, giving a J_Ω of 1.6×10^{-6} A/sr at 200 nA emission at 250 V. The extractor for these measurements was at a distance of about 1 mm, so we assume that aberration coefficients of the gun lens will be of that same order. The values for CNT2 are from Hata *et al.*² Here we assume the emission to be fully coherent, thus $d_v=0$. The angular current density in the reference was measured at a screen held at a potential $V_e=1800$ V at a distance of about 5 mm. Again, we take aberration coefficients of the order of the tip-extractor distance.

Figure 2 shows the current-probe size curves for the four systems of Table I. The ultimate resolution for these low-energy probe systems only depends on the balance between diffraction and chromatic aberration. At very small currents, α_e is sufficiently small to make the gun lens aberrations negligible and the magnification is sufficiently small to make the size of the source image negligible. If we assume the spherical aberration contribution to be much smaller than the chromatic aberration contribution, the probe size is

$$d_p = \left[\left(0.54 \frac{\Lambda}{V^{1/2}} \frac{1}{\alpha} \right)^2 + \left(0.6 C_c \frac{\delta U}{V} \alpha \right)^2 \right]^{1/2}. \quad (16)$$

For optimized α this is

$$(d_p)_{\min} = d_{ac} = 2.81 \times 10^{-5} \frac{C_c^{1/2} \delta U^{1/2}}{V^{3/4}} \quad \text{at} \quad \alpha = \sqrt{\frac{0.54 \Lambda V^{1/2}}{0.6 C_c \delta U}}. \quad (17)$$

Thus, the minimum probe size is not dependent on the brightness or the coherence of the emitter; the only emitter property that enters the equation is the energy spread. However, a probe without current is of no use, so we have to allow a contribution to the probe size that is related to the

current in the probe. The contribution depends on the emitter: for a Schottky emitter this is the contribution from the geometrical source image, for a fully coherent source without virtual source size it is the aberration of the gun lens, either the chromatic or the spherical aberration. We shall analyze all three cases.

For the Schottky emitter, the probe current is given by Eq. (4), which, using Eq. (7), is rewritten as

$$I_p = B_r \frac{\pi}{4} d_I^2 \pi \alpha^2 V = B_r \left(0.54 \frac{\pi}{2} \Lambda \right)^2 \left(\frac{d_I}{d_A} \right)^2 = B_r K_\Lambda \left(\frac{d_I}{d_A} \right)^2, \quad (18)$$

where $K_\Lambda = 1.08 \times 10^{-18}$ m² sr V. This is a very useful equation, because it gives a simple estimate of how much current is obtained in a probe which is close to the minimum size.

For example, for a Schottky emitter at $B_r = 5 \times 10^7$, this current for $d_I = d_A$ (we call it I_Λ , with $I_\Lambda = B_r K_\Lambda$) is 50 pA, independent of the beam energy or the lens aberrations. It is often useful to express the current in the probe as a function of probe size. For a brightness-limited probe, close to the best obtainable resolution d_{ac} , the current is

$$I_p = I_\Lambda \left[\left(2 \frac{d_p^2}{d_{ac}^2} - 1 \right)^{1/2} - 1 \right]^{2/1.3}. \quad (19)$$

For an emitter with zero virtual source size, the probe current is given by Eq. (10), so in order to obtain current in the probe, α_e must be allowed to increase, accepting a contribution to the probe size from the gun lens aberrations. Let us first assume that the gun lens chromatic aberration dominates. We then allow a contribution d_{cg} that is a fraction d_{cg}/d_A of the contribution of the diffraction disk, and using Eqs. (13) and (15) we obtain

$$\begin{aligned} I_{cg} &= J_\Omega \pi \alpha_e^2 = J_\Omega \pi \left(\frac{d_{cg}}{d_A} \right) \frac{0.54 \Lambda V_e^{1/2}}{0.6 C_{cg} \delta U} \\ &= K_{cg} \frac{J_\Omega}{V_e \delta U} \frac{V_e^{3/2}}{C_{cg}} \left(\frac{d_{cg}}{d_A} \right), \end{aligned} \quad (20)$$

where $K_{cg} = 3.47 \times 10^{-9}$ m sr V^{1/2}. The terms are separated into two groups: the first is an emitter property, since in a homogeneous accelerating field the angular current density is proportional to the acceleration; the second group is a property of the accelerator/gun lens design.

If the spherical aberration of the gun lens dominates,

$$\begin{aligned} I_{sg} &= J_\Omega \pi \alpha_e^2 = J_\Omega \pi \left[\left(\frac{d_{sg}}{d_A} \right) \frac{0.54 \Lambda V_e^{1/2}}{0.18 C_{sg}} \right]^{1/2} \\ &= K_{sg} \frac{J_\Omega}{V_e} \left(\frac{V_e^{3/2}}{C_{sg}} \right)^{1/2} \left(\frac{d_{sg}}{d_A} \right)^{1/2}, \end{aligned} \quad (21)$$

where $K_{sg} = 1.91 \times 10^{-4}$ m^{1/2} sr V^{1/4}. Again we have separated the terms in groups related to emitter properties and optical column properties.

Table II gives the numerical values of the probe currents according to Eqs. (18), (20), and (21) for the four emitter-column situations of Table I. For each source type, the lim-

TABLE II. Probe currents for four emitters calculated using Eq. (18) with $d_l=d_A$, Eq. (20) with $d_{cg}=d_A$, and Eq. (21) with $d_{sg}=d_A$.

	Schottky emitter ^a	Monochromator Schottky	CNT1 ^b	CNT2 ^c
I_A (A)	54×10^{-12}	27×10^{-12}	864×10^{-12}	NA
I_{cg} (A)	35×10^{-9}	35×10^{-9}	439×10^{-12}	36×10^{-12}
I_{sg} (A)	41×10^{-9}	21×10^{-9}	770×10^{-12}	62×10^{-12}

^aReference 10.

^bReference 1.

^cReference 2.

iting value is shown in bold. Clearly, the current at high resolution for Schottky sources is limited by the emitter brightness.

For both CNTs, the current at high resolution is already limited by the chromatic aberration of the gun lens, making the brightness of these emitters an unimportant parameter. For these CNTs, the angular current density is the parameter that represents the quality of the emitter.

We have chosen a rather arbitrary value of 1 mm for the aberration coefficients of the gun lens for the CNTs. A careful optimization may lead to a better performance. Looking again at Fig. 2, one sees that for the parameters that are used here, only CNT1 has a regime in which it outperforms the monochromated Schottky source. The zero-current probe size using either source is 1.6 nm. Initially, when increasing the probe size, the current is proportional to the brightness, following Eq. (16). Accordingly, the CNT delivers 30 times the current of the monochromated Schottky source for very small currents. However, a current of about 2 pA is considered to be the minimum for imaging. The monochromated Schottky for probe size at this current is 1.7 nm. Because of the influence of the gun lens aberrations the advantage of the CNT is already decreased to a factor of 12 at this probe size. The CNT current for this probe size equals 25 pA, which would require increasing the probe size to 2.1 nm if a monochromated Schottky source was used. The CNT current falls below that of the monochromated Schottky source for probe sizes larger than 9 nm.

Note that if the CNT emitter is placed in a SEM at the position of the Schottky emitter without reoptimizing the gun lens almost all advantages compared to a monochromated Schottky source will be lost. With gun lens aberration coefficients of 1 cm, the CNT current falls below that of the monochromated Schottky at 2.1 nm. A disadvantage of the CNT emitters is that even at large probe sizes, the probe current is relatively small. In the probe-current versus probe-size curves, it is possible to distinguish different regimes where different contributions to the probe size dominate. It is sometimes useful to have explicit analytical relations for the probe current as a function of probe size, so we shall derive these relations here. These relations are obtained by optimizing α or α_e in Eq. (5). If the chromatic aberration of the probe lens dominates a system with emitter brightness B_r and emitter FW50 energy spread δU , the current in a probe of FW50 size d_p is

$$I_p = 1.71 \frac{d_p^4 B_r V^3}{C_c^2 \delta U^2}. \quad (22)$$

Note that if the FWHM of the energy distribution is used, the prefactor in Eq. (22) is 5.4.¹¹

If the spherical aberration of the probe lens dominates a system with gun brightness B_r , the current in a probe of FW50 size d_p is

$$I_p = 2.44 \frac{d_p^{8/3} B_r V}{C_s^{2/3} \delta U^2}. \quad (23)$$

If the brightness does not play a role, but the chromatic aberration of the gun lens needs to be balanced with the chromatic aberration of the probe lens, we find that the optimized magnification of the system is independent of I and the contribution of the gun lens is equal to the contribution of the probe lens, or $C_{c,tot}=2C_c$:

$$I_p = 2.18 \frac{d_p^2 J_\Omega V_e^{1/2} V^{3/2}}{C_c C_{cg} \delta U^2}. \quad (24)$$

If the spherical aberration of the gun lens needs to be balanced with the spherical aberration of the probe lens, we find again that the optimized magnification of the system is independent of I . The contribution of the gun lens is three times the contribution of the probe lens, or $C_{s,tot}=4C_s$, and

$$I_p = 6.77 \frac{d_p^{2/3} J_\Omega V_e^{1/4}}{C_s^{1/6} C_{sg}^{1/2} V_e^{1/4}}. \quad (25)$$

Studying Fig. 2, we see that most of these regimes are represented in the plotted curves. A danger of high brightness electron beams is that stochastic Coulomb interactions disturb the trajectories, causing an additional blur of the probe. The effects can be approximated by analytical equations.¹¹ We shall only estimate whether Coulomb interactions play a role or not but shall not attempt to precisely calculate the contribution to the probe sizes or to reoptimize the magnification. Only the beam segments directly next to the emitter and next to the probe will be taken into account, assuming that the contributions from other beam segments can be minimized by sufficient acceleration in these segments. An additional approximation is that the electrons are assumed to be accelerated to V_e in the whole gun segment. There are analytical equations for different parameter regimes of the beam. For the gun section, the appropriate regime is the Holzmark regime and the contribution to the apparent source size is

$$(FW_{50H})_{tg} = 1.87 \frac{I^{2/3} L^{2/3}}{V_e^{4/3} \alpha^{4/3}} = 4.01 \frac{J_\Omega^{2/3} L^{2/3}}{V_e^{4/3}}, \quad (26)$$

where L is the length of the segment up to where we assume that the beam is apertured and accelerated. As a numerical value for L , we shall take the same as for the gun lens aberrations. For the probe section, the regime may be either Holzmark or pencil beam, so that the contribution to the probe size is

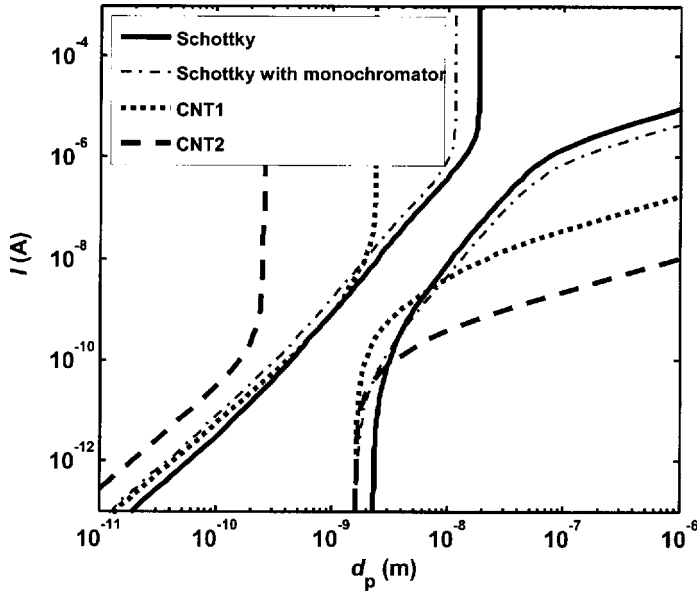


FIG. 3. Current-probe size relation for the four emitters of Table I. The four curves on the left give the estimated contributions from stochastic Coulomb interactions.

$$(\text{FW}_{50T})_{\text{tp}} = \left[\left(\frac{1}{\text{FW}_{50H}} \right)^{6/7} + \left(\frac{1}{\text{FW}_{50PB}} \right)^{6/7} \right]^{-7/6}, \quad (27)$$

with

$$\text{FW}_{50H} = 1.87 \frac{I^{2/3} L^{2/3}}{V^{4/3} \alpha^{4/3}}, \quad (28)$$

$$\text{FW}_{50PB} = 8.56 \times 10^{30} \frac{I^3 \alpha L^3}{V^{5/2}}. \quad (29)$$

In order to find the total contribution of the Coulomb interactions d_{ci} to the probe size, the contributions from the two segments are added:

$$d_{ci} = [M^2(\text{FW}_{50H})_{\text{ig}}^2 + (\text{FW}_{50T})_{\text{tp}}^2]^{1/2}. \quad (30)$$

This contribution to the probe size is plotted versus current I_p in Fig. 3. Note that d_{ci} is on the horizontal axis. Since d_{ci} is always smaller than the probe size, it may be concluded that for the emitter parameters and column design that were analyzed here, Coulomb interactions do not play a role. However, it is known that in situations where a Schottky emitter is operated at higher brightness, Coulomb interactions can limit the effective brightness.⁹ In addition, the energy spread may increase because of Boersch effects.¹¹ Optimizing a system with a Schottky source, taking all Coulomb effects into

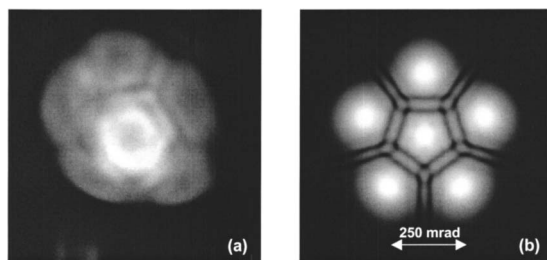


FIG. 4. Emission patterns of CNT; (a) experimental and (b) simulated.

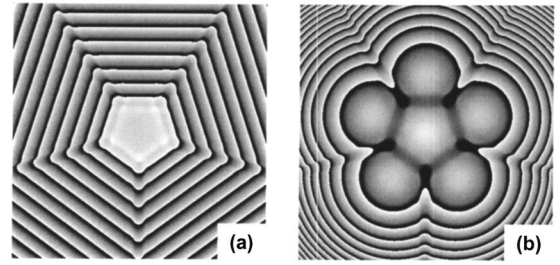


FIG. 5. Phase distributions in simulated emission pattern on a sphere (a) around the center of the six sources and (b) through points that are at an equal distance from all six sources.

account with a correctly varying acceleration voltage in the gun segment, requires a separate study.

III. PHASE AND AMPLITUDE OF EMITTED WAVE FROM A CNT

Various experiments from different groups around the world have shown that electrons from carefully prepared CNTs may emit in a typical pattern consisting of a central lobe and five other lobes surrounding the central lobe.^{3,12-15} An example of this emission pattern in the far field, measured in our laboratory,¹⁶ is shown in Fig. 4(a).

Not only are the six lobes clearly visible but the pattern also shows what looks to be interference fringes on the borders between the lobes. When the virtual source is modeled as six mutually coherent point sources at a distance of 5×10^{-10} m, emitting electrons at 2000 V into cones with a Gaussian amplitude distribution of $\sigma=75$ mrad, we find the emission pattern of Fig. 4(b). It closely matches the pattern shown in Fig. 4(a), except for the intensity distribution in each separate cone.

The simulation also yields the phase distribution on any plane of choice. Figure 5(a) shows the phase distribution on a sphere around the center of the six sources. Figure 5(b) shows the distribution on a sphere through points that are at an equal distance from all six sources.

When both the amplitude and the phase of a beam are known, the wave can be propagated to other planes at arbitrary position. This enables us to redefine the virtual source. In Fig. 6, we plot the intensity and phase of the wave in several planes close to the original source plane in the model. Figure 6(a) shows the original virtual source plane, where we now include the size of the diffraction disks corresponding to the emission cones as seen in Fig. 4. The phase in this same plane shows that the center cone is emitted on the axis, while the five outside cones are emitted under an angle with the axis. In Figs. 6(a)–6(c), there is also a phase pattern in areas where there is no appreciable amplitude. That phase pattern has no practical meaning.

Figure 6(c) is interesting, because here all six spots interfere constructively in the center spot, concentrating all energy of the wave in a tiny spot, even smaller than the original spots in Fig. 6(a), because the effective emission cone angle from this plane is larger than that of the separate cones, so that the related diffraction disk is smaller. The minor distur-

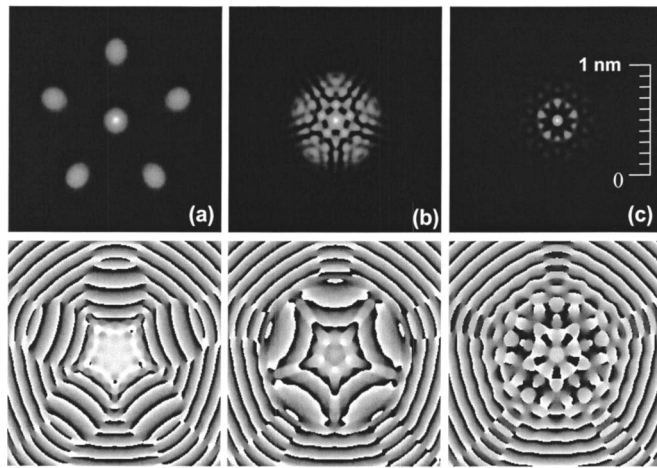


FIG. 6. Intensity (top) and phase (bottom) distributions of the emitted electron wave at different virtual source planes.

bances in a small circle around the spot are a consequence of the fact that the full emission cone has a structure in the angular intensity.

In the previous section, the probe current was calculated for a model electron optics system. It was shown that the aberrations of the gun lens limit the usable current from this type of emitters to a small fraction of the total emission current. The accepted emission angle is typically only 10–30 mrad. The diffraction spot at the virtual source plane related to this angle has a diameter of $0.5\lambda/\alpha$, which is about 15–5 nm using the 2000 V acceleration of our simulation. This is much larger than the size of the patterns in Fig. 6. This implies that the exact form of the amplitude distribution in the source is irrelevant.

From Fig. 5 it is clear that the phase of the wave is fairly constant in the part of the beam that is accepted into the system, which means that the diffraction disk in the probe is not increased by phase variations in the aperture.

IV. CONCLUSIONS

We have calculated the probe size in a typical high-resolution low-voltage SEM from all constituting contributions. The gun lens aberrations always become important at high currents. In that regime it is not the brightness but the angular current density at the source that is the determining parameter for the performance of the source. For very small electron emitters the gun lens aberrations may also be dominant at low currents, that is at the highest resolution. Obviously, when gun lens aberrations are important, it is worthwhile to make designs that minimize these aberrations. For probe lenses, we know reasonably well how small these aberrations can be made since it is one of the basic parameters on which SEM manufacturers compete. For gun lens aberrations, there is space for improvement and it is not directly clear how small these can be made.

For certain electron emitters such as the carbon nanotube with a perfect cap of hexagons and six pentagons the electrons seem to be emitted from a delocalized state and all electrons seem to come from roughly the same state. For

these emitters the concept of brightness loses its meaning. The brightness concept is useful when the size of the beam is determined by the position of the geometrical trajectories, but it is not useful when the wave properties of the electrons determine the size of the beam. For these emitters the angular current density at the source is the performance characterization parameter at all currents.

The analysis of probe-size versus probe-current relations for different emitters throws doubt on the assumption that carbon nanotubes will necessarily be better emitters for electron microscopes and lithography machines than Schottky emitters. Of course, they offer the advantage of a lower energy spread, just like metal cold field emitters, but their extremely small virtual source size is not useful. It is unclear at this moment if the angular current density of CNTs can still be improved compared to the values that we have used in our analysis. There are reports of CNTs emitting up to 20 μA of current, but it is not known how stable this current can be made, what the energy spread is, and what the lifetime of such an emitter is.

For tubes that emit into six coherent cones, our model seems to give results in reasonable agreement with the experimental data on intensity distributions, so we feel confident that the phase images that are simulated also represent the reality. These make it possible to show that the beam has a nearly constant phase over the typical apertures used in microscopy. The simulations also confirm that the size of the electron wave at the emitter surface is much larger than the spread function of the distribution of geometrical trajectories.

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