

Modelling and Optimization of Tilt-Rotor Aircraft Flight Trajectories

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Modelling and Optimization of Tilt-Rotor Aircraft Flight Trajectories

by

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Preface

After nine months of hard work, I can proudly present my thesis. It is not always easy to find the perfect research topic that keeps one's keen interest over the course of such a long period. At first I thought that my research interests lie within specific airline and airport related topics, but after multiple discussions with faculty staff, I found that my interests can better be defined by optimizing complex systems. Instead of a niche topic within the master's track, I found a topic that covers the topic of aerospace engineering in a broader sense. This resulted in the fact that I had to incorporate a wide array of topics that ranged from specific optimizations of the master's track to the flight mechanics that were already covered in the first and second year. Meanwhile, it also lead to a steep learning curve on helicopter theory: a topic that I have never touched upon before starting this thesis as it is not part of the educational curriculum.

Conducting this extensive project would not have been possible without the help of others. Therefore, I would like to thank everyone who supported me during this project, with a few in particular.

First of all I would like to thank my daily supervisor Sander Hartjes for steering me throughout this thesis project, sharing your thoughts on the research, broaden my sometimes narrowed mind by showing me the matter from a different perspective and yielding new input and methodologies. Your approachable policy of always having the door open for students to come up with questions was really helpful and also saved a lot of time. Thanks for quickly shifting gears from your own work, to an in-depth helicopter optimization question in order to come up with good feedback. In addition to this, I would like to thank Dries Visser, Ricky Curran and Mark Voskuil for reflecting on my work during the kick-off, midterm and greenlight meetings and taking place in my thesis committee.

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Thanks to the ever-changing 2.44 community for sharing the thesis life, thinking along with everyone's topic and sharing, sometimes unsolicited, thoughts on the most versatile topics varying from detailed programming and LaTeX knowledge to what paper quality and grammage I should use for this thesis.

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Delft, July 2018*

Abstract

Tilt-rotor aircraft combine the helicopter's benefit of VTOL capability and the flight performance that turboprops possess on range, speed and endurance. Tilt-rotor aircraft have the potential to decrease airport congestion and average flight delay in passenger transportation, while benefits are numerous in i.e. search and rescue, disaster relief and military application. Time, money and risk can be significantly reduced through the application of flight trajectory optimization and assessment prior to flight, as this can accurately simulate flight and its related performance limits. A numerical three-dimensional point-mass model for a tilt-rotor type aircraft has been derived to fill the gaps that currently exist in tilt-rotor modelling and the understanding of their flight mechanics. After validation, the model is applied to optimize integral flight trajectories using optimal control theory in GPOPS. It was concluded that the derived model is valid under its assumptions and limitations. From the tilt-rotor flight behaviour it was assessed that specific nacelle tilting behaviour could be observed, as the tilt-rotor can exploit its unique rotor tilting capability. Since the power required is the driving factor in most flight optimization, the nacelle angle is driven by the aircraft's velocity and altitude. This model can be used in theoretical flight trajectory optimization studies. The model can be adapted to account for specific requirements and aircraft types.

Keywords: Trajectory optimization, tilt-rotor aircraft, optimal control, flight modelling, XV-15

Acronyms

ABZ	Aberdeen Airport
AEI	All Engines Inoperative
AEO	All Engines Operative
AHE	Above Helipad Elevation
ATC	Air Traffic Control
CTO	Continued Take-Off
CTR	Civil Tilt-Rotor
DCP	Differential Collective Pitch
EAS	Equivalent Airspeed
FAA	Federal Aviation Administration
GA	Genetic Algorithm
GPOPS	General Pseudospectral Optimal Control Software
GTRS	Generic Tilt Rotor Simulation
IFR	Instrument Flight Rules
IGE	In Ground Effect
ISA	International Standard Atmosphere
L/D	Lift-to-Drag ratio
LDP	Landing Decision Point
NAS	National Airspace System
NASA	National Aeronautics and Space Administration
NCE	Nice International Airport
NLP	Non-Linear Programming
NM	Nautical Mile
MCM	Monaco Helipad
MTOW	Maximum Take-Off Weight
OEI	One Engine Inoperative
OGE	Out of Ground Effect
RTM	Rotterdam/the Hague Airport
RTO	Rejected Take-Off
SID	Standard Instrument Departure
SHP	Shaft Horsepower
SNOPT	Sparse Nonlinear Optimizer
STOL	Short Take-Off/Landing
TAS	True Airspeed
TDP	Take-off Decision Point
TPP	Tip Path Plane
VFR	Visual Flight Rules
VTOL	Vertical Take-Off/Landing

List of Symbols

Latin

a	Acceleration	m/s ²
a	Speed of sound	m/s
A	Aspect ratio	-
B	Reference frame	-
C_d	Rotor drag coefficient	-
C_D	Drag coefficient	-
$C_{D,0}$	Zero-lift drag coefficients	-
C_L	Lift coefficient	-
C_T	Thrust coefficient	-
D	Tilt-rotor Drag	N
DL	Download factor	-
e	Oswald Efficiency factor	-
f_G	Ground effect factor	-
g	Gravitational acceleration	m/s ²
h	Altitude	m
i_n	Nacelle inclination angle	deg
K_{ind}	Induced power correction factor	-
L	Tilt-rotor lift	N
m	Tilt-rotor mass	kg
m_{fuel}	Fuel weight	kg
P_a	Power available	W
P_r	Power required	W
S	Surface area	m ²
R	Rotor radius	m
sfc	Specific fuel consumption	kg/W·hr
t	Time	s
T	Rotor thrust	N
u	Horizontal velocity	m/s
U_c	Flow perpendicular to rotor TPP	m/s
U_t	Flow parallel to rotor TPP	m/s
v_i	Induced velocity	m/s
v_h	Induced velocity in hover	m/s
V	Velocity	m/s
W	Tilt-rotor weight	N
w	Vertical velocity	m/s
x	Longitudinal location of tilt-rotor	m
y	Lateral location of tilt-rotor	m
z	Vertical location of tilt-rotor	m

Greek

α	Angle of attack	deg
β_{lat}	Lateral cyclic angle	deg
β_{long}	Longitudinal cyclic angle	deg
γ	Flight path angle	deg
δ_f	Flap deflection angle	deg
ϵ	Optimization threshold	-
η_β	Cyclic angle control factor	-
η_p	Propulsive efficiency factor	-
θ_1	Cyclic blade pitch angle	deg
μ	Advance ratio	-
μ	Bank angle	deg
ρ	Air density	kg/m ³
σ	Rotor solidity ratio	-
τ_p	Pilot time delay	s
χ	Flight heading angle	deg
ω	Transformation matrix	-
Ω	Angular velocity	rad/s

Superscripts

a	in aerodynamic reference frame
I	in Inertial reference frame
-	Normalized variable
.	Time derivative
$\rightarrow$	Vector

Subscripts

$_0$	Initial variable
$_f$	Final variable

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Introduction

Around the year 400 B.C. Greek mathematician Archytas was reputed to have built a mechanically powered aircraft in the form of a pigeon, whereas the earliest design sketches for vertical flight date back to Leonardo da Vinci [20], [45]. It was not until 1930 that one started to design a concept that would combine the concept of vertical and horizontal powered flight. The benefit of being able to take-off, hover and land as helicopter and having the flight performance that turboprops possess on range, speed and endurance offers numerous applications. Implementing tilt-rotor flight in passenger transportation offers operators the opportunity to operate independently of the runway capacity of highly-congested airports, or to penetrate new markets such as long range city-center to city-center flights. Boeing projects that 20-60% of flights within the U.S. have the possibility to be conducted by tilt-rotor aircraft [7]. These tilt-rotor flights will relieve congested airports and can hence reduce the average flight delay from 21.6 to 7.2 minutes [17]. Other tilt-rotor applications lie in medical services (in remote area's), fire-fighting, search and rescue, coast guard, border patrol and the military [3].

Numerous kinds of designs have been proposed that add vertical take-off and landing capability to an ordinary airplane design, or vice-versa. Sadly, these have contemptuously been summarized in the "wheel of misfortune" [69], not only due to the many failed designs, but more importantly to the high fatality rate during development. However, thorough research and development has been conducted since the 1970s and multiple designs have proven themselves in recent history, such as NASA's XV-15 and the V-22 Osprey. Based on these two, aircraft are currently being designed to operate for civil and commercial purposes, such as the AW609 by AgustaWestland.

Introducing a new aircraft always brings along the process of certification. Initial flight tests are costly, time-consuming and potentially dangerous. A deadly flight test crash of the AW609 tilt-rotor has proven this danger again [63]. Risk, time and money can be significantly reduced through the application of flight trajectory optimization simulation and assessment prior to the flight tests, as this can accurately simulate flight and related performance limits. Because of this, engineers and test pilots can get a better understanding of optimal flight behaviour.

In order to achieve this, a numerical model for a tilt-rotor type aircraft is to be derived and coupled to an optimization technique in order to fill the gaps that currently exist in tilt-rotor modelling and the understanding of its flight mechanics. Before addressing the derivation of a tilt-rotor model, it is essential to get a better understanding on how airplanes, helicopters and tilt-rotors have been modelled so far and how these have already been applied. Countless airplane and helicopter models already exist. The current state of art of tilt-rotor modelling however, sum up to only two two-dimensional rigid-body models that optimize very short duration flights in the form of take-off and landing procedures in nominal or engine failure conditions [9], [50], [21], [15].

It can be concluded that there is a need for a tilt-rotor model that can optimize flight trajectories in all three dimensions, and furthermore optimize integral flights in their entirety. The latter implies to optimize all flight phases in succession in order to assess flight behaviour, distinct from isolated flight phases, as the imposed boundary constraints on these do not necessarily result in the optimum. In this report a tilt-rotor type aircraft model will be derived in the form of a three-dimensional point-mass model. This model will then be verified and validated in order to be able to acknowledge its outcome and trajectories. Hereafter, the model will be applied in real-life situations, in order to assess tilt-rotor flight behaviour in

these situations and conditions. It will be assessed how these tilt-rotor mechanics can best be modelled and if these assumptions and derivations result in a meaningful and valid model. Finally, conclusions will be drawn from the optimized tilt-rotor flight trajectories with respect to optimal flight behaviour. It is expected that the tilt-rotor flight trajectories will exhibit considerable differences with flight trajectories of ordinary airplanes and helicopters as the tilt-rotor can exploit its unique characteristics that make it stand out. The emphasis of this thesis research will lie however, in the derivation, verification and validation of the model. The model will be applied to a few tilt-rotor flights to investigate general behaviour.

This has lead to the following research question that will be the point of focus of this thesis research: *how can a three-dimensional tilt-rotor aircraft be modelled efficiently to optimize for flight trajectories and procedures, in order to assess and enhance safety and performance?*

The structure of this thesis is as follows. In order to gain preliminary knowledge on airplane, helicopter and tilt-rotor aircraft modelling, its operations and optimization methodologies, a literature review was performed in Chapter 2 on current literature that address the various topics of the above described challenges. This is followed by the research gap and objective that is distilled from the literature review in Chapter 3. The derivation of the tilt-rotor model is presented in Chapter 4, which is followed by the optimization methodology in Chapter 5. The derived model is validated in Chapter 6, while the results of a flight optimization are in Chapter 7. This thesis is concluded in Chapter 8 with the conclusions and recommendations.

2

Literature Review

Previous to this research a literature study was performed to gain knowledge and insights and identify research gaps, which has been used as input to this research. The following chapter will provide an overview of current and relevant literature on the topic of tilt-rotor aircraft modelling and optimization, as was summarized in “Modelling and Optimization of Tilt-Rotor Aircraft - Literature Review” [57].

The literature review was divided in four parts that addressed distinct topics. Section 2.1 presents current tilt-rotor development and its potential in usage, while Section 2.2 elaborates on how airplanes and helicopters have been modelled in current literature. Section 2.3 addresses tilt-rotor aircraft mechanics and controls and how these can be modelled. Furthermore, this section summarizes optimization studies that already have been performed so far. Finally, Section 2.4 discusses various optimization methodologies to optimize flight trajectories.

2.1. The Tilt-Rotor Aircraft

Tilt-rotor aircraft combine the benefits from helicopters and turboprop aircraft, by merging the Vertical Take-Off and Landing (VTOL) and hover capability of helicopters with the performance that turboprop aircraft have on range, speed and endurance. Although this aircraft type has already been used in the military for quite some time, there currently is interest in introducing the tilt-rotor aircraft in the civil and commercial market. This section will start by giving a basic introduction and the main characteristics of a tilt-rotor aircraft, along with the recent developments in new tilt-rotor aircraft. Hereafter, the potential benefits that tilt-rotor aircraft can have on the airspace, commercial markets and community will be elaborated. Finally, potential future missions that tilt-rotor aircraft can have in all of these markets are presented.

Characteristics

Being able to operate with the benefits of both an airplane and helicopter is the result of two tiltable propellers mounted on the wing tips and airplane fuselage. Tilted forward the propellers act as airplane propellers during cruise flight, and tilted upright they act as helicopter rotors during vertical flight. Having two contra-rotating rotors omits the need for a tail rotor to counter the torsion, as with ordinary helicopters. The tilt-rotors cruise speed and range is comparable to that of turboprop regional airliners. Moreover, the tilt-rotors flight ceiling (~25.000 feet) is more than twice as high as that of a helicopter, enabling it to circumvent bad weather [42]. All in all, aircraft characteristics and properties give the tilt-rotor a flight envelope that practically encompasses those of similar helicopters and turboprop airplanes [9]. Figure 2.1a shows the XV-15 research plane in helicopter configuration, where Figure 2.2 shows the three view of the XV-15 in cruise configuration.

Current tilt-rotor aircraft have forward swept wings in order to account for propeller blade clearance from the wing. The propeller blades of tilt-rotor aircraft are generally shorter than helicopter blades and longer than turboprop blades. This results in high disc loading in helicopter configuration and low disc loading in cruise configuration. Due to high disc loading, the hover efficiency (thrust/power) decreases. However, the tilt-rotor blades are highly twisted (~45° for the XV-15) to approach the hover efficiency of helicopters. Helicopters are not able to incorporate this high twist in their design as the blades need to produce both lift and thrust in forward flight.



(a) XV-15 during take-off in helicopter configuration.
Adapted from: [48]



(b) AgustaWestland AW609 during flight in airplane configuration [63]

Figure 2.1: XV-15 and AW609 aircraft in different configuration modes

Throughout this report a distinction will be made in the terminology for the state of the aircraft (or rotorcraft for completeness), depending on the state of the nacelle inclination, or merely the phase of flight in which the aircraft is [44]. The nacelle inclination should not be confused with the mast angle. The mast angle differs in that it has a frame of reference opposite to the nacelle angle, that starts in the vertical upright position and ends in the horizontal position.

- Helicopter and hover mode is described when the engine nacelles are tilted vertically upright in between $75^\circ \leq i_n \leq 95^\circ$.
- When the engine nacelles is locked forward ($i_n = 0^\circ$), this is referred to as the aircraft being in airplane mode or cruise flight.
- Conversion mode is defined to be (the transition) in between the latter two states, when the engine nacelles are at an angle of $0^\circ < i_n < 75^\circ$.

Recent Developments

The idea of a tilt-rotor aircraft is not entirely new. NASA, Boeing, Bell and the U.S. Army have already been on this topic since more than 60 years. This has resulted in the design and testing of multiple tilt-rotor aircraft. The XV-3 acted as a proof-of-concept in 1953, while the XV-15 research plane (1972) has been researched thoroughly in wind tunnel and flight tests, yielding huge amounts of data. 30 years of research resulted in the military V-22 Osprey which went into production in 1989 [7], [9].

In recent years, multiple tilt-rotor UAVs have been developed [17]. To penetrate the civil and commercial market, Boeing and Bell partnered up to use all gained experience from the XV-15 to develop a civil tilt-rotor aircraft. Eventually AgustaWestland ended up to be the sole developer of the AW609, a tilt-rotor aircraft with 11-person capacity, 275 kts cruise speed, 5.000 ft and 25.000 ft hover and service ceiling. With a Maximum Take-Off Weight (MTOW) of 16.800 lbs a range of 700 NM can be reached [3]. A recent crash during a flight test in 2015 proves the fact again that initial flight tests are not only costly and time consuming, but also very dangerous. This emphasizes the need for adequate simulation and optimization software in order to reduce these risks.

2.1.1. Tilt-Rotor Advantages and Potential

Nowadays, tilt-rotor aircraft are only being used for military purposes in the form of the U.S. Navy V-22 Osprey. Young et al. and Chung et al. describe the potential that Civil Tilt-Rotor aircraft (CTR) can have on the air transportation system. Provided an adequate infrastructure exists not only for ground facilities but air traffic control as well, it is expected that Civil Tilt-Rotor aircraft will successfully compete with fixed-wing aircraft. An operational concept of runway independent aircraft has the potential of increasing airport and airspace capacity, which will result in delay reduction and increased throughput throughout the entire system [73], [17]. Furthermore, it is noted that recent events, such as hurricane Katrina, demonstrated the critical need for incorporating CTR for disaster relief [73].

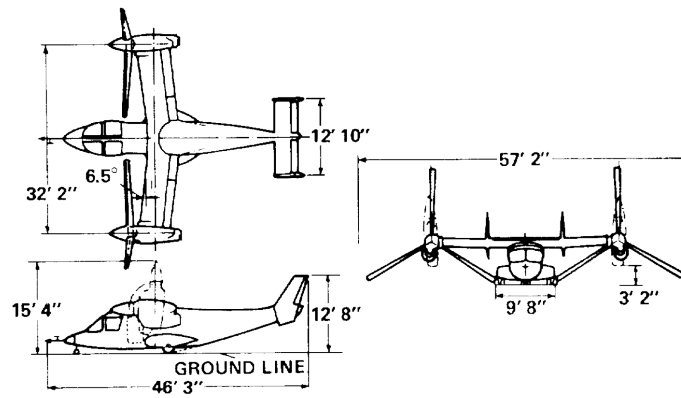


Figure 2.2: Schematic three view of XV-15 in cruise configuration [18]

Airport and Airspace Congestion

FAA projections suggest that both airport and airspace will come to their limits, which will result in more congestion. As CTR can operate in both vertical as Short Take-Off/Landing (STOL). This can add additional capacity to airports as the CTR can take-off during peak hours in STOL mode from short or stub runways, converted taxiways or in VTOL mode from vertiports, which thereby makes runway slots available to larger aircraft. This goes for on-airport vertiports, but airport capacity can also be increased by diverting passengers from crowded hubs to off-airport urban area vertiports. According to Boeing research, about 60% of the movements at the ten major hub airports in the U.S. are consumed by flights within 500 Nautical Mile (NM) and on average 41% of flights originate from 300 NM or less [60], [26]. Moreover, 20-40% of all movements are operated with aircraft with a capacity of 50 seats or less [7]. This implies the huge potential civil tilt-rotor aircraft can offer in order to increase (slot) capacity and reduce congestion. As tilt-rotor aircraft have the ability to fly terminal area trajectories that are unavailable to fixed-wing aircraft, operations will not be confined to fixed-wing aircraft trajectories.

Flight Delay

Furthermore, Young et al. studied the effect that CTR will have on system-wide delays of U.S.' entire National Airspace System (NAS). The baseline delays projected for 2025 were compared to the delays when small and medium sized aircraft were replaced with comparable CTRs into the regional networks of Atlanta, Las Vegas and the North-east corridor. The regional network consists of airports within an about 500 NM radius around the airport, whereas the North-east Corridor consists of the network around the nine major airports in Boston, Baltimore, New York, Philadelphia, Pittsburgh and Washington. The Atlanta regional network was chosen to represent a connections-based hub network, the Las Vegas regional network to represent hub supporting origin-destination traffic and the North-east Corridor network to represent a regional network, consisting of nine major airports, its inter-hub traffic and traffic to airports within 500 NM distance, as depicted in Figure 2.3a. Due to simulation resources these three networks were chosen and the results were scaled to provide NAS-wide estimates for delay metrics [72].

The average delays were substantially reduced due to the introduction of a CTR fleet in this simulation. As can be seen in Figure 2.3b, delays can be reduced from the baseline 21,6 minutes to up to 7.2 minutes [73], depending on the scale of introduction.

2.1.2. Tilt-Rotor Missions

Due to the fact that tilt-rotor aircraft have the unique benefit of operating both as a helicopter and as airplane, several missions or purposes exist for tilt-rotor aircraft, such as commercial air transport, public service and military. A complete overview of all possible applications can be found in de FAA transcript on tilt-rotor aircraft [26].

Commercial Transportation

The first possible opportunity for tilt-rotor aircraft is the commercial transportation market, including offshore and executive transportation. Three feasible potential markets were found to be urban area

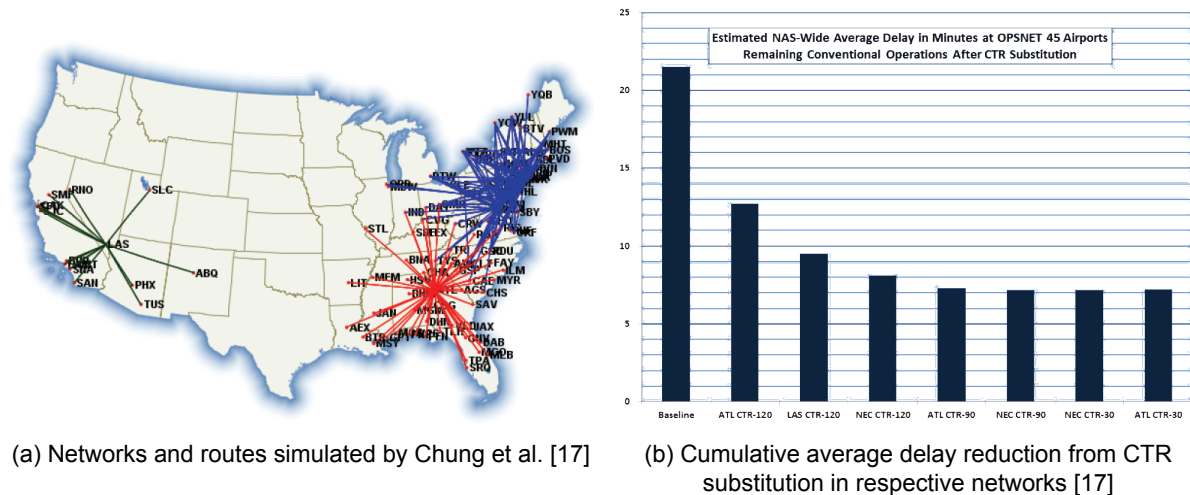


Figure 2.3: Research on average flight delay reduction due to CTR by Chung et al. [17]

to urban area, city-center to city-center and hub feeder traffic [7] with flights ranging from 500 NM to 1,500 NM.

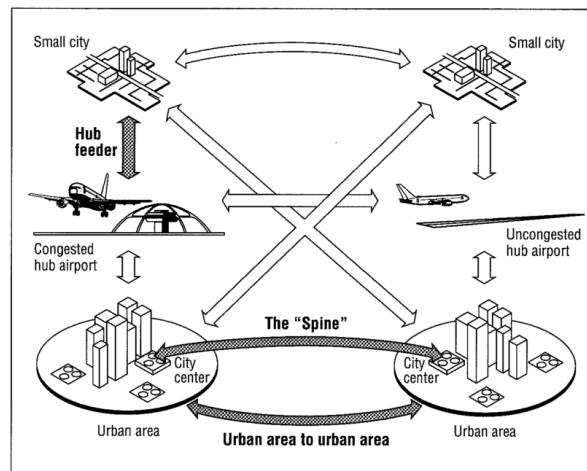


Figure 2.4: Commercial passenger transportation markets for tilt-rotor aircraft [7]

A typical passenger transportation mission profile in NextGen airspace is depicted in Figure 2.5. Whenever the tilt-rotor aircraft is designed for commercial purposes, it falls under FAA Transport Category rules. An important requirement regarding this aspect is that all rotorcraft account for One Engine Inoperative (OEI) capability during take-off and landing.

Commercial passenger transportation in Boston-New York-Philadelphia-Washington D.C. for instance could draw away up to 15% of the passengers from the airports, which accounts for around 10% of fixed-wing movements. Moreover, tilt-rotor operations within this market would yield a time-saving of 40% on an average flight [7]. Connecting two or more city centers is a market which is not quite served yet. Out-of-helicopter-range markets will yield a high origin-destination traffic flow, which demands both high frequency and short ground times [7]. For high-density hub feeders, the key advantage is that it can by-pass the slot constrained runways, gates and precision approach airspace. Boeing estimates that a combined urban area to urban area and hub feeder system could make 1,000 daily slots available in the North-east Corridor only [7].

For executive and offshore travel, the tilt-rotor can decrease the number of assets within a corporate fleet as one tilt-rotor can replace one helicopter and one turboprop. For offshore transportation, a tilt-rotor can become economically feasible for oil rigs further offshore, because their faster speed and further range. Finally, it is said that remote areas such as Alaska still possess precious resources that

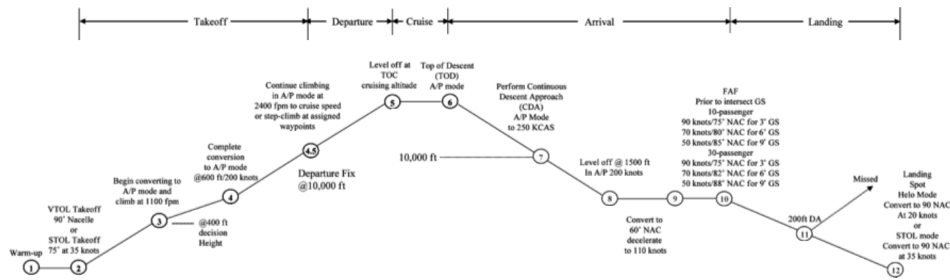


Figure 2.5: Generic mission profile for commercial tilt-rotor aircraft flights in NextGen. Adapted from:[17]

are not being exploited yet due to an non-viable transportation system [60].

Public and Military Services

Vital time can be gained in transportation of patients or organs between accident location, hospital and specialist medical center. This will especially be of benefit in remote and low-dense populated areas, with few medical centres, such as Australia. Moreover, the pressurized cabin allows for a broader range of medical interventions [3]. The tilt-rotor aircraft poses several benefits for fire-fighting as well. Not only can it drop the water quicker due to faster travelling between the fire and water sites, but it can also transport fire fighters and critical command and control equipment to the fire front quickly when the fire site is off-grid, which reduces the response time significantly. Search & Rescue, the coast guard, and border patrol will also benefit of tilt-rotor aircraft.

Finally, the tilt-rotor can be applied for military purposes and disaster relief in a similar fashion. The V-22 Osprey has already proved the advantages in disaster relief by re-supplying remote Philippine areas after typhoon Haiyan destroyed most infrastructure.

2.2. Flight and Helicopter Mechanics

Due to the fact that a tilt-rotor aircraft is a combination between an airplane and a helicopter, the aircraft will deal with the physical and mechanical phenomenon of both of these. It is therefore necessary to understand the physics and mechanics of both in order to be able to model a tilt-rotor aircraft. In the following section a brief review will be given on how aircraft and helicopters have been modelled so far and which aspect of these will also go for a tilt-rotor aircraft.

2.2.1. Flight Mechanics and Modelling

Aircraft mechanics have been widely studied. A common way of modelling a three-dimensional aircraft is with six degrees of freedom, although three degrees of freedom are sometimes used for commercial aircraft under the small angle assumption. The corresponding state variables for the kinematic and dynamic equations are x , y , h , V , γ , ψ [14], [68], [70].

Carlson and Zhao fully derived the three-dimensional equations of motion for aircraft. In general, rigid body dynamics of an aircraft are usually described by twelve state equations, consisting of six velocity and six position components. The six components of both correspond to the six degrees of freedom: rotational and transversal about all three axis. The twelve state equations accounted for x , y , h , ϕ , θ , ψ , u , v , w , p , q and r . The model is in two reference frames: the inertial reference frame and the aircraft body coordinate frame. Carlson finally notes that it is quite common for airplane flight analysis to add the time derivative of mass in order to account for fuel consumption. For rotorcraft flight, a first order equation for the rotor speed is required to account for the change of rotor speed.

2.2.2. Helicopter Mechanics and Modelling

In the following subsection first the control and mechanics of a helicopter is summarized, which is followed by how this has been modelled so far in literature.

Helicopter Mechanics

As for airplanes, helicopters have their distinct physics and therefore their distinct way of control. Helicopter controls consist of the cyclic lever, collective lever and pedals. The cyclic lever is used for

both longitudinal as lateral movement. Whereas an airplane is control through control surfaces that change the lift coefficient of the respective wing or stabilizer, a helicopter is controlled by changing the angle of attack of specific rotor blades in order to achieve a change in lift coefficient. A collective pitch input changes the angle of all blades equally by the same (absolute) amount. This implies an increase/decrease of thrust when a collective input is given to the main rotor, which results in the helicopter ascending/descending. A secondary effect is a change in torque, which will not further be elaborated. The cyclic pitch works in a similar manner. However, the change in angle of attack is not the same for all blades, but is transferred periodically to the blades, which creates a lift difference between the front-back and/or left-right side of the rotor disc. This results in a change in roll and/or pitch rate. The collective and cyclic inputs are transferred to the blades through the swashplate. Finally, a pedal input changes the collective pitch of the tail rotor in order to induce a yaw rate. As the tilt-rotor does not have a tail rotor, this will not be looked into.

Another important aspect of helicopter control is the speed governor function. Changes in collective and cyclic pitch change the drag of the rotor blades and hence the rotor rpm. The governor is a sensing device that controls the rotor rpm and keeps it constant. If switched off, the rotor speed is allowed to vary in order to have more or less thrust [9]. A correlator works in a similar fashion in order to match the collective input with the engine power. For instance, it automatically increases engine power when the collective lever is raised, in order to maintain the rotor rpm close to the desired value [27]. To conclude, helicopter flight speed is severely limited due to the influence of advancing blade shock and retreating blade stall [43].

Helicopter modelling

Several helicopter models have been derived so far by multiple researchers. In this subsection these will briefly be illustrated by stating the purpose of the model and in which state and control variables and number of degrees of freedom this resulted. Some authors used different symbol conventions for their models. Some symbol usage has been adapted accordingly for consistency in the following list. Literature yields several distinct models, which include but are certainly not limited to the following, in approximate order of increasing complexity:

- Tsuchiya conducted JAXA research in order find flight trajectories that minimize the ground noise during landing approaches. For simplicity, a two degree of freedom, point-mass model was used, limited to the vertical plane. State and Control vector equalled $[x, h, u, w]$ and $[T, \theta]$ respectively. As of 2007, it was recommended by Tsuchiya to expand the optimization with lateral motion for three-dimensionality [61].
- Chen and Zhao derived a 3 degree of freedom, two-dimensional point-mass model for an UH-60A helicopter in order to investigate OEI optimal control strategies and according trajectories in terminal-area operations. The model accounts for the eight state variables $[u, w, h, x, C_x, C_z, \Omega, P_s]$ and the two control variables $[\dot{C}_x, \dot{C}_z]$ [16].
- Lee derived a two-dimensional, longitudinal point-mass model of an UH58A helicopter in order to study optimal autorotative trajectories after power failure. The five state variables accounted for $[x, h, u, w, \Omega]$ and the control variables for $[C_x, C_z]$. The model has two degree of freedom model but it is not mentioned why the pitch angle and rate have not been used as states [40]. This model has been derived from the two-dimensional, two degree of freedom, longitudinal model by Johnson [35].
- Okuno and Kawachi have derived a similar two-dimensional, longitudinal rigid-body helicopter model. The goal of their research was to analytically predict the tilt-rotor's H-V diagram and optimize the take-off trajectories of multiple different procedures. The model has four degrees of freedom (horizontal and vertical translations, fuselage pitch angle, and rotor angular speed) as well with state variables $[h, u, w, \theta, q, \Omega]$ and control variables $[\beta_{col}, \beta_{lon}]$, the collective pitch and longitudinal cyclic pitch respectively [39], [50], [51], [49].
- Tang has derived a three-dimensional point-mass model of the Robinson R22, in order to implement a helicopter model into the NOISHHH tool to optimize noise abatement arrival trajectories. The R22 is a light, single main rotor helicopter, including tail rotor. The three-dimensional time-space model accounted for three degrees of freedom. State and control variable equalled $[u, v,$

$w, x, y, z]$ and $[C_x, C_y, C_z]$ respectively. The full derivation of the equations of motion and the 3D point-mass model can be read in [59].

- Due to the increasing availability of unmanned aerial systems, Bibic and Narkiewicz investigate new control techniques and efficient methods to control helicopter after power failure, covering both RTO as CTO. The three-dimensional model consisted of eight degrees of freedom: six of a rigid fuselage, the angular velocity of the main rotor, and the available engine power. The 14 state variables accounted for $[x, y, z, u, v, w, \phi, \theta, \psi, p, q, r, \Omega, P_s]$, while the control vector accounted for all pitch angles of the main and tail rotor: $[\beta_{col_{main}}, \beta_{lat}, \beta_{lon}, \beta_{col_{tail}}]$. One remark is made by the author on the computational time. Due to the size of the problem it was not possible to execute the calculations in real-time, what would have been a requirement for autonomous control [6].
- In order to have an efficient and accurate way to numerically optimize approach trajectories for helicopters that focusses on community noise mitigation, Hartjes developed the European Clean Helicopter Optimization software suite. The incorporated three-dimensional helicopter was modelled using an eight degree of freedom rigid-body dynamic model in which the state variables accounted for $[u, v, w, p, q, r, \theta, \psi, \phi, x, y, z, \lambda_{imr}, \lambda_{itr}]$, where the last two terms are the dynamic inflow of both the main and tail rotor. The control variables equalled $[\beta_{col_{main}}, \beta_{lat}, \beta_{lon}, \beta_{col_{tail}}]$. The author acknowledges that the use of an eight degree of freedom model leads to additional requirements on the optimization algorithm and that extra attention has to be paid on the computational efficiency in order to ensure acceptable computation time [32].

Some similarities can be noted in the above list. It can be seen that position and velocity variables are always used as state variable and to which angular position and velocity are added for a more complex model. As an extended degree of freedom, the rotor velocity and power can be added. Unsurprisingly, it is common for a helicopter model to use the rotor for the control variables. This can either be done with the rotor input (collective and cyclic pitch in respective directions) or the output of the rotor (thrust coefficient in the respective directions).

2.3. Tilt-Rotor Mechanics and Modelling

In order to optimize the operations of the tilt-rotor aircraft, it is to be modelled mathematically and numerically. Now that the basic mechanical and dynamical aspects of both airplane and helicopter are known, the model can incorporate the phenomenon of both modes to combine the helicopter with the aircraft. The following section will first elaborate on the unique features and characteristics of tilt-rotor aircraft and what the implications of these are on the aircraft performance, after which current tilt-rotor models and their optimizations will be scrutinized.

2.3.1. Tilt-Rotor Aircraft Mechanics

The following section addresses some implications of tilt-rotor aircraft. First it will be looked into how tilt-rotor aircraft are controlled, after which it will be looked into how aircraft behaviour changes due to its characteristics. Finally, the conversion is looked into.

Tilt-Rotor Control

Because a tilt-rotor aircraft quite bluntly is a mixture between an airplane and helicopter, the aircraft has to incorporate multiple control mechanisms. Figure 2.6 summarizes which control mechanisms are used in the cruise and helicopter phase of the flight for the AW609. For nacelle inclinations in between the two extremes, a combination of aerodynamic control surfaces and rotor control ensures sufficient control power in all axes. The primary controls consist of the cyclic stick, collective stick and pedals. The primary controls of the AW609 differ only slightly from the XV-15, which will be pointed out in this section.

Helicopter Mode When the aircraft is in helicopter mode, rotor controls are used: the aircraft is pitched by symmetric application of longitudinal cyclic blade pitch, yawed by differential left-right fore/aft cyclic blade pitch and rolled by differential left-right collective blade pitch. Vertical thrust is controlled by collective blade pitch. A final control that is used in the XV-15 is lateral translation. With the use of a power lever button on the control stick, the pilot can adjust the cyclic blade pitch to induce lateral

translation. This is not depicted in Figure 2.6, but comes across to the pitch control in helicopter mode, but inducing a lateral movement instead of a longitudinal one [66]. Lateral translation only is applicable in helicopter mode.

The AW609 differs with the XV-15 in that it does not possess lateral cyclic blade pitch control, which is a required control for conventional helicopters, but optional for tilt-rotors. Lateral cyclic blade pitch control can provide side force control, roll control, and lateral flapping alleviation to minimize rotor loads. The AW609 rotors, however, have a fixed 2.5° inward lateral tilt, which during hover directs rotor wash outward away from the wings, to reduce download [31].

Airplane Mode The tilt-rotor aircraft has the option of using rotor controls or conventional aerodynamic control surfaces. The AW609 pitches with elevator surface deflection and rolls by differential deflection of the flaperons. Yaw control is achieved by Differential Collective Pitch (DCP), unlike the XV-15 that uses a conventional rudder control surface. The advantage of DCP for the AW609 was substantial cost and weight saving [31].

Hybrid Mode The rotor blades of the AW609 are designed as such that rotor controls alone can produce sufficient yaw moment during conversion and in airplane mode. During the conversion process, however, the differential fore/aft cyclic blade pitch is being phased out in the AW609 as a function of the nacelle angle, while phasing out the DCP simultaneously. The rotor controls are phased out gradually to increase efficiency. It can be noted that the AW609 does not need a rudder to compensate for the asymmetric thrust during OEI, since both engines are interconnected with both rotors through a cross-shaft. This allows a single engine to generate thrust in both rotors [31].

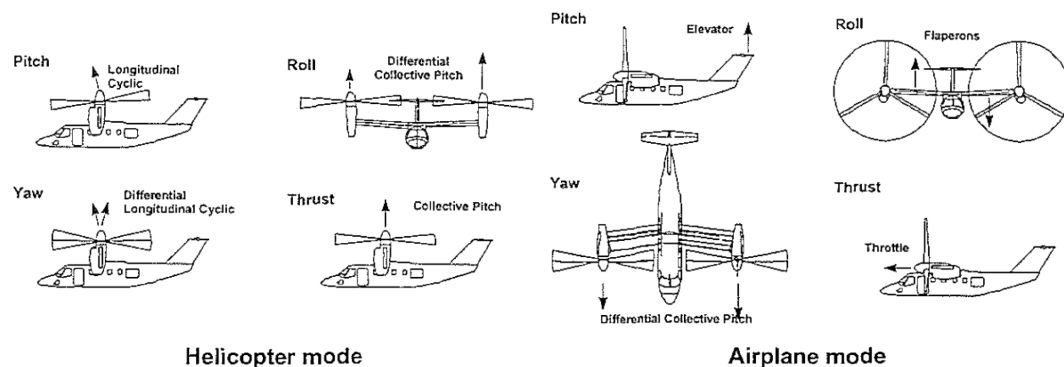


Figure 2.6: Tilt-rotor primary control mechanisms (for AW609). Note the difference between the XV-15 and AW609, in that the AW609 lacks lateral cyclic pitch for lateral translation in helicopter mode and a rudder to yaw in airplane mode. Adapted from: [31]

Conversion Corridor

The conversion corridor is a special characteristic for a tilt-rotor, in which the range of possible airspeeds for each nacelle angle are defined. The lower limit of this range is defined by wing stall or pitch attitude limit, while the higher limit is defined by power required and blade flapping. This gives an overview of the flight boundaries and it is desired to have the corridor as large as possible to provide a safe conversion.

Power and pitch attitude distributions are an important criteria that is used to define the conversion strategy. From these, Diaz has deduced the conversion corridor in Figure 2.7a, that shows the upper and lower limit as well as the iso-pitch line with 0° pitch attitude, at 10° flap deflection. The iso-pitch line at 0° pitch can become relevant in terms of passenger comfort. As mentioned before, the flap deflection is an important variable for tilt-rotor aircraft, as it can delay stall and decrease download. In Figure 2.7b, the effect of the flaps is illustrated for the lower limit of the conversion corridor. It can be seen that an increase in flap deflection leads to a new conversion corridor limit at lower speed [21].

Diaz concludes her performance code with an optimal conversion strategy as depicted in Figure 2.7a. Again, this is not an optimal solution to the XV-15.

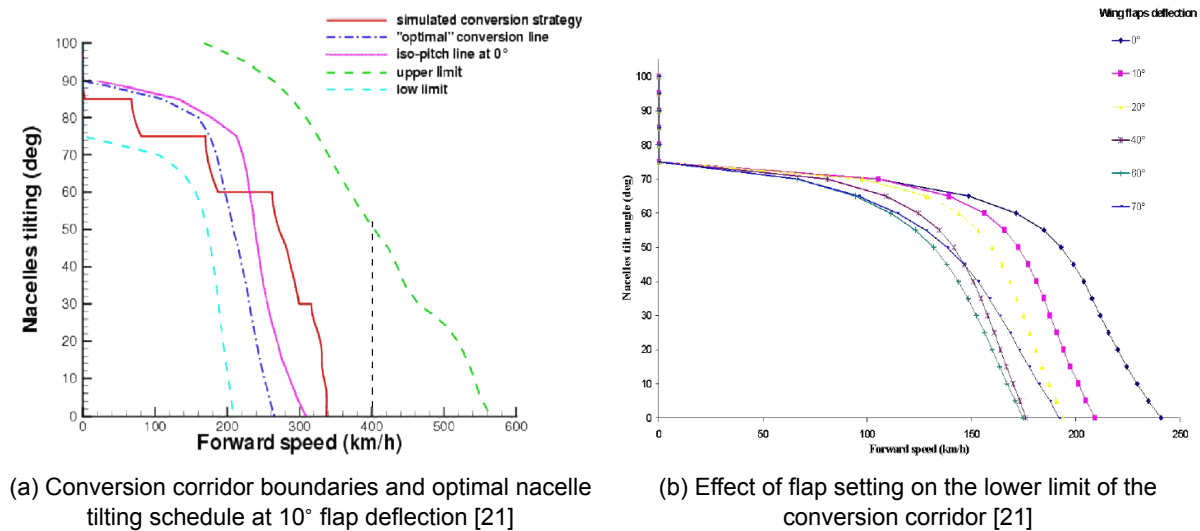


Figure 2.7: Conversion corridor and the effect of flaps on the conversion corridor as studied by Diaz [21]

Rotor Wash/Download

When the nacelle angle is at high inclination, the rotor wash during hover and low speeds impinges the fuselage and wing surface area underneath the rotor, which creates a force opposing the lift, which is download. The download can be equal to 10-15% of the total rotor thrust during hover. This is not only wasted thrust, but moreover results in a loss of lift. The air flow in hover is schematically depicted in Figure 2.8.

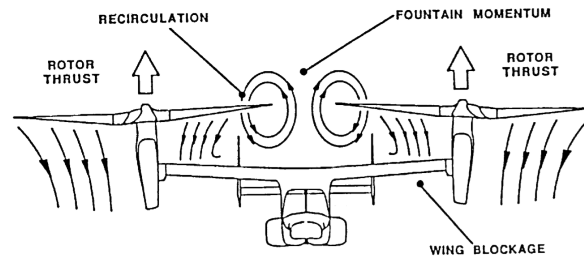


Figure 2.8: Sketch of the V-22 in hover, showing the main flow features [29]

Both Diaz and Cerbe simulate the effects of download in their studies. Diaz proposes a method to estimate the download that uses a semi-empirical model that is based on interpolation of published data curves in hover at 90° nacelle inclination. In Diaz' study, a generic tilt-rotor is used [21], while Cerbe used the Generic Tilt Rotor Simulation of the XV-15 [15].

The magnitude of the download depends on various factors such as the airspeed, flap deflection, nacelle tilt angle and rotor-ground distance, and varies as follows:

- Download decreases when the airspeed increases, since this sweeps back the rotor wake. Diaz concluded that the download disappears at an airspeed of ~ 30 m/s [21].
- The trailing edge flaps not only function as high-lift devices in airplane mode, but also reduce the wing download at low speeds by reducing the wing surface area and changing the aerodynamic flow. Diaz found this in her simulation. Cerbe simulated the wing download with respect to wing flap deflection as well. With respect to the magnitude of the download there is a noticeable difference between the findings of Diaz and Cerbe. Due to the flap deflection, less power is required during hover [15].
- Download decreases slightly when the tilt-rotor approaches the ground, due to the fact that the rotor wash impacts the ground and fountain flow effect occurs, which lifts the aircraft slightly,

reducing download. Due to the reduced rotor-ground distance, the induced velocity in the rotor disc area is reduced. This reduces the download within proximity of the ground. According to Cerbe, the power coefficient ratio $\frac{C_{p,IGE}}{C_{p,OGE}} = 0,76$ on ground. The ground effect has vanished when the tilt-rotor has reached a height of two times the rotor diameter [15].

2.3.2. Tilt-Rotor Aircraft Modelling

Now that some basic knowledge on the tilt-rotor has been acquired, the focus can be shifted towards modelling the tilt-rotor. This section addresses the models that have already been derived and what optimizations they have been used for. Moreover, it presents some useful insights and relations that can be taken along and a small subsection elaborates on the data processing of the raw XV-15 data.

Current Tilt-Rotor Aircraft Models

In current literature two tilt-rotor models are documented. the two-dimensional theoretical model by Carlson and Zhao, and the Generic Tilt Rotor Simulation (GTRS) by Ferguson. In this section these models will be set out along with some useful relations.

Two-Dimensional Rigid-Body Model Carlson and Zhao defined a mathematical model of the tilt-rotor aircraft that is based on the configuration and parameters of the Bell XV-15 research aircraft. A major benefit is that the XV-15 has been well researched and that its data is publicly available [9]. For their research Carlson and Zhao have derived a two-dimensional longitudinal tilt-rotor model. In the dissertation Carlson and Zhao research take-off and landing procedures as these are the most dangerous phases of the flight due to the low states of the kinetic and potential energy, combined with the close proximity to the ground. Since take-off and landing mainly take place in the vertical plane, they have opted to model the tilt-rotor in a two-dimensional, longitudinal model, as this model is much simpler and computationally faster than the three-dimensional model. This model had to account for forces and moments produced by rotors, wing, fuselage and horizontal stabilizer, since stick control affects both rotor inclination and elevator deflection. Moreover, since a tilt-rotor can fly as a helicopter as well, forces and moments of the aerodynamic surfaces and fuselage had to be modelled over the full range of angle of attack α , between -180° and 180° [9].

The two-dimensional rigid-body tilt-rotor model is depicted in Figure 2.9. Carlson's derivation of the longitudinal model ends up with seven state equations for three degrees of freedom. The six state equations and rotor speed equation account for the seven states $[w, u, \theta, q, \Omega, h, x]$ and the three control variables $[T, s, i_n]$. The three degrees of freedom account for up-down and forward-backward translation and in-plane rotation.

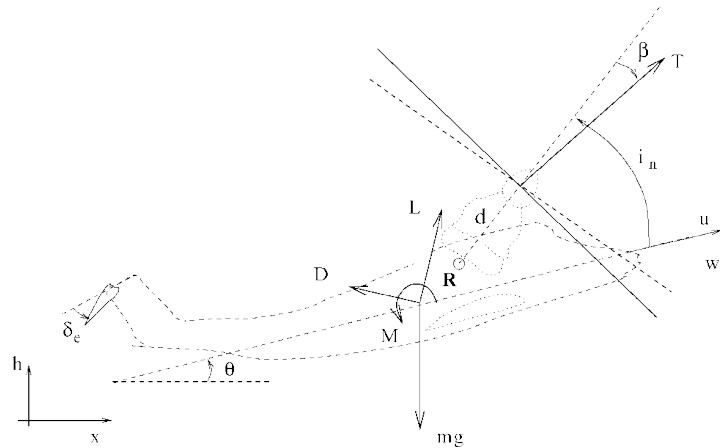


Figure 2.9: Two-dimensional tilt-rotor aircraft free body diagram [11]

Jhemi, identifies key steps for effective application of dynamic optimization for rotorcraft and tilt-rotor aircraft in his paper. Jhemi notes that a rigid-body model is required since a point-mass model would not be able to model the fact that both the rotor thrust inclination and aerodynamic forces are being

controlled by the pilot simultaneously [34]. Jhemi, however, does not deliver arguments to substantiate this point, so therefore it can be looked into to simplify the model into a point-mass model.

Okuno and Kawachi also derived a longitudinal tilt-rotor model, which they have derived from a theoretical helicopter model. Okuno and Kawachi do not state their model, but mention that their eight state variables include $[w, u, \theta, q, \Omega, i_n, h, x]$ and the three control variables $[s_{lat}, s_{col}, i_n]$. The difference of Okuno and Kawachi's model is the addition of the state variable i_n and control variable i_n and the usage of the collective lever position as control variable instead of the thrust T [51], [50].

Carlson and Zhao note that the total forces and moments are the result of summing up the forces and moments of the rotors, wing, fuselage and horizontal stabilizer. Furthermore, it should be noted that because of the fact that the nacelles can tilt the engine over a range of angles, a part of the wing is in the freestream and the other part of the wing is in the slipstream of the propeller. Hence, these two experience different flow velocities. To approximate this, both parts of the wing contribute separately to the total forces and moments. Carlson and Zhao have shown by sensitivity analysis that a simplified equation, that depends on the nacelle angle and forward speed, can be used to calculate the wing area in freestream and slipstream [9].

Whenever the aircraft finds itself in an engine failure situation, either OEI or AEI, the power available spools down to the OEI power rating or even to zero power. Hence, the power available in OEI/AEI is a function of time. Along with the power required and thrust of one rotor, it is defined to be [9]:

$$P_a = (P_{a_{AEO}} - P_{a_{OEI,AEI}}) \cdot e^{\frac{t}{\tau_p}} + P_{a_{OEI,AEI}} \quad (2.1)$$

Furthermore, the it is stated that the power required and thrust can be determined using:

$$P_r = \frac{2}{\eta_p} \rho (\pi R^2) (\Omega R)^3 C_P, \text{ in which } C_P = C_T \sqrt{C_T/2} (K_{ind} f_G \bar{v}_i + \bar{U}_c) + \frac{1}{8} \sigma c_d (1 + 4.7 \mu^2) \quad (2.2)$$

$$T = \rho (\pi R^2) (\Omega R)^2 C_T \quad (2.3)$$

Carlson and Zhao describe that the normalized induced velocity inside the Vortex-ring state, where $(2\bar{U}_c + 3)^2 + \bar{U}_t^2 \leq 1$, can be determined by an approximation. Outside the vortex-ring state, the normalized induced velocity is determined by momentum theory [9]:

$$\bar{v}_i = \begin{cases} \frac{\bar{U}_c(0.373\bar{U}_c^2 + 0.598\bar{U}_t^2 - 1.991)}{1/\sqrt{\bar{U}_t^2 + (\bar{U}_c + \bar{v}_i^2)}} & (2\bar{U}_c + 3)^2 + \bar{U}_t^2 \leq 1 \\ \text{otherwise} & \end{cases} \quad (2.4)$$

As mentioned in Section 2.3.1, the pilot's stick displacement controls both the rotor cyclic angle and elevator deflection. The rotor cyclic angle phases out slowly as a function of the nacelle angle. Due to this, it has full effect in helicopter mode and no effect in airplane mode. On the contrary, the elevator remains active in all modes, but is ineffective at low speeds. This effectiveness can be modelled as a function of the nacelle angle. Carlson and Zhao defined the stick deflection s as control variable, which leads to the cyclic angle and elevator deflection to be as follows [9]:

$$\beta = \frac{s}{s_{max}} \beta_{max} \sin(i_n) \quad \delta_e = \frac{s}{s_{max}} \delta_{e_{max}} \quad (2.5)$$

Generic Tilt-Rotor Simulation The second model was derived by Ferguson, which was used by Cerbe and as well by Diaz. The three-dimensional model was derived by Bell Helicopter Textron, under contract of the NASA for the XV-15. It is a very detailed model that consists of 20 interdependent modules that account for all defined subsystems, containing the mathematical model for the different aircraft components, such as the two rotors, the fuselage, the wing, the horizontal and vertical stabilizers, the landing gear, the two engines and the drive system, the rotor collective governor and the SCAS. All model equations and parameters are presented in the 282-page appendix, which puts the model's amount of detail in perspective [30]. The first model has been validated with XV-15 flight data, which resulted in the improved model Revision A.

The models uses the XV-15 geometric and aerodynamic data sets. Apart from the main input data, 304 input data tables are used to acquire the data of the aerodynamic coefficients from wind tunnel

tests for the different aircraft components, the interactive aerodynamics between rotor/wing/fuselage/stabilizers/ground, the gains and coefficients for the control system, etc. The model by Ferguson might be too detailed for this study as it accounts for proprotor characteristics and factors that might be too elaborate such as non-linear twist, flapping restraint, and pitch-flap coupling [15].

XV-15 Data Processing

Due to the research nature of the XV-15 aircraft, a large amount of data of the aircraft is publicly available. In the NASA paper *A Mathematical Model for Real Time Flight Simulation of a Generic Tilt-Rotor Aircraft* by Ferguson multiple aerodynamic data tables can be found for the lift and drag coefficients created by various systems [30]. This data, however, is tabular and must therefore be fitted with smooth functions in order for it to be used in the optimization. The coefficients can then be evaluated at any possible angle, instead of the discretized tabular data. A least squares fit can be used to interpolate the data. When this method is used, a adequate compromise has to be made between accuracy and simplicity, or in other words, a compromise between validity of the numerical analysis and low computational time [9].

Carlson fitted all functions using Least Squares to get the best fit. With most functions this was achieved with sinusoidal functions along with a constant term. The lift and drag coefficients of the main wing are then given in terms of the angle of attack. These coefficients then additionally have data points to all four different flap settings. Carlson differentiates between “simple” and “better” fits. The simple fits of the wing coefficients were made with three to five sinusoidal or power terms, albeit they do not capture all data points adequately. Carlson improved the accuracy of the lift and drag polars with more terms, which added up to 35 and 13 terms respectively. The lift coefficients for the horizontal stabilizer is given for seven distinct elevator deflection angles. Carlson fitted the data by fitting first for $\delta_e=0$ with two sinusoidal terms, and adding the best quadratic terms for the other elevator angles, the function of which is given to be: $C_{L_{hs}} = 1.56676\delta_e - 0.783507\delta_e^2 + 1.16175 \sin(2\alpha_{hs}) + 0.248059 \sin(4\alpha_{hs})$. The number of terms used for every function are summarized in Table 2.1 [9].

It remains unclear, however, how Carlson has chosen to specify the simple and better fits. It is not mentioned or elaborated why the fit was made up out of i.e. 5 and 35 sinusoidal terms for both accuracies, and why he did not chose for a fit with an intermediate amount of terms. He might have chosen for a standard deviation or variance threshold, but nothing has been mentioned. It has neither been mentioned if Carlson and Zhao used the simple or better fits to acquire the results.

It remains also unclear how Okuno and Kawachi have interpolated the XV-15 data since this is not clarified by the authors. Therefore, there is also the possibility that linear interpolation or tabular lookup was used. Since Diaz and Cerbe made use of the GTRS, it is unclear how the wind tunnel data of the XV-15 is used in te optimization software.

Table 2.1: Number and type of terms used for interpolation by Carlson [9]

Subsystem	Coefficient	Number of terms	Accuracy
Wing	C_L	5 Sinusoidal	Simple
		35 Sinusoidal	Better
	C_D	3 Sinusoidal	Simple
		13 Sinusoidal	Better
Horizontal stabilizer	C_L	2 Sinusoidal (+ quadratic)	-

Prior Optimization Studies

Several (two-dimensional) optimization studies have already been conducted so far by i.a. Carlson and Zhao, Okuno and Kawachi and Cerbe. Most of them confine themselves to a specific take-off or landing case, in which a distinction is made between nominal, OEI and AEI operations. For the tilt-rotor optimization several boundary and path constraints are imposed that either hold for all procedures, or for specific procedures only. Constraints that have been used in the optimization studies are the equations of motion, constraint on positive altitude and constraints on the upper and lower limit of the rotor speed, thrust coefficient, stick displacement, nacelle angle and rate of change of the latter three [13], [10]. In the following subsection the approaches and result of these studies will be presented.

Continued/Rejected Runway Take-Off Carlson and Zhao used their model to study the optimal (two-dimensional) trajectories for continued and rejected short and runway take-off. For a continued take-off in OEI, it is necessary to obtain the required conditions for Continued Take-Off (CTO) climb-out in OEI as soon as possible. Therefore, the problem is described to minimize the final forward distance x_f , at which these conditions have been met. The safe climb-out conditions are defined to have a final height of 35 ft, a final climb rate of at least 100 ft/min and steady-state climb out conditions ($\dot{w}=\dot{u}=\dot{\theta}=\dot{q}=\dot{\Omega}=0$). Moreover, a minimum forward speed of at least v_{TOSS} is required [13], which has been defined for category A certification standards by the FAA [12], [25]. The v_{TOSS} allows for a steady-state climb-out for a give take-off weight [9]. The optimization study yielded a runway take-off length of 633 ft for an aircraft take-off weight of 13,000 lbs.

For the runway rejected take-off (RTO) the goal is to minimize the used runway length to come to a full stop. Additional constraints account for a vertical speed at touchdown, touchdown speed, touchdown pitch angle, touchdown height and for a nacelle inclination that does not damage the rotor blades. Therefore, $x_f + \frac{\dot{x}_f^2}{2a}$ is to be minimized [13], [10]. The second part in the performance index accounts for the ground distance needed for a full stop. Carlson assumes the deceleration a to be $0.2g$ [9]. The rejected take-off case yielded a runway length of 802 ft.

Continued/Rejected City-Center Take-Off Carlson and Zhao also describe a similar procedure, in which the take-off takes place from city-centers or confined heliports instead of a runway. This implies a difference in the fact that the tilt-rotor cannot use the free space of an entire runway to land again after a rejected take-off. The flight has to land on the heliport again instead. Moreover, the forward speed has to be low enough for the tilt-rotor to stop within the heliport boundaries. Piloted simulations by NASA Ames Research Center have shown that aiming for the center of the heliport is the safest procedure, as the tilt-rotors altitude is susceptible to wind gusts [12].

For the CTO the performance index is equal to the runway take-off except for the fact that an additional constraint is added to account for a minimum height during transition. This yielded a take-off distance with constant rotor speed of ~ 45 m, when the TDP is at 30 m altitude.

The RTO in confined areas on the contrary has some differences with respect to the runway take-off optimization. The tilt-rotor to land back on the pad, with a vertical landing speed that will not harm the structural integrity of the landing gear, a horizontal speed low enough to decelerate on the pad and finally the pitch and nacelle angle cannot exceed their limits. To achieve this goal the performance index has been set to minimize the squared distance from the heliport center: x_f^2 . For the RTO with constant rotor speed a safe landing could be sustained with an Take-off Decision Point (TDP) at 25 m altitude. Both the runway and city-center case have been conducted with speed governor on and off. Continued and balked landing have not been investigated by Carlson and Zhao. Neither for the runway case, nor for the city-center case [9].

Take-Off Distance and Weight Okuno and Kawachi use a different approach for their tilt-rotor optimization, based on a method to analyse helicopter take-off procedures for transport category operations. A theoretical, two-dimensional helicopter model was adapted to account for tilt-rotor configuration. The goal of the optimization problem was to determine optimal take-off procedures according to the available field length in order to maximize the take-off weight, while adhering to transport regulations, such as OEI circumstances [50]. AEI is not accounted for.

Okuno and Kawachi describe three distinct take-off procedures that can be used by tilt-rotor aircraft: vertical, oblique and runway take-off. Their optimization yielded a TDP of ~ 140 ft for vertical take-off at 11,470 lbs, a runway distance of 612 ft for the oblique take-off at 13,500 lbs and a runway distance of 2,000 ft at 16,320 lbs.

Okuno and Kawachi's findings can be summarized in Figure 2.10a and 2.10b. The former shows the optimal trajectories for CTO and RTO for all three procedures, whereas the latter shows the take-off distance versus the take-off weight.

Short Take-Off Optimization Cerbe et al. state that short take-off performance, until then, was only based on flight test experience, which moreover only was done with fixed nacelle angle and fixed flap deflection. The two-dimensional model used for the optimization was an adapted form of the GTRS, the original version of which was developed by Bell and NASA for the XV-15.

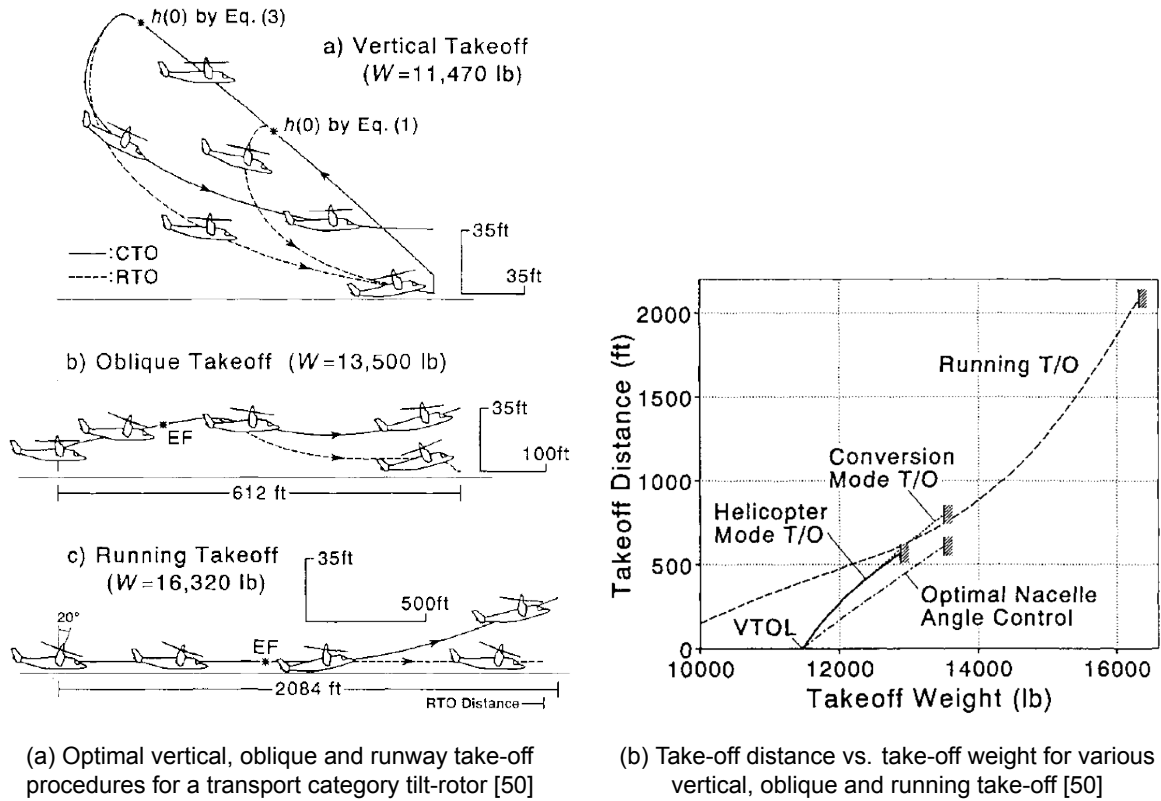


Figure 2.10: Conclusions of take-off distance and weight optimization study by Okuno and Kawachi [50]

Wing flap deflection was discretized to four the positions: 0° , 20° , 40° and 75° . Further constraints consisted of aerodynamic limitations, rotor endurance limits, wing stall and passenger comfort. The optimization task was to find the optimal longitudinal input and rotation speed in order to minimize the take-off distance. The rotor power was adapted to account for OEI in all optimization runs.

Using 80° nacelle tilt, results show slightly better short take-off performance for with 40° flap deflection with respect to 75° deflection. The differences, however, are marginal: 959 versus 1002 ft. Using 70° nacelle tilt, however, performance increases by about 25%: the 35 ft altitude is cleared already at a take-off distance of 706 and 744 ft for 40° and 75° flap deflection respectively. This performance increases can be explained due to the the fact that a larger portion of the rotor thrust is used to accelerate the aircraft. On-ground acceleration is beneficial for two reasons. Not only is the ground effect utilized, but the take-off procedures is easier as well.

Cerbe concludes that the shortest take-off is achieved with 20° and 40° flap deflection. Less nacelle tilt results in longer acceleration distances and with more nacelle tilt the aircraft has to accelerate to higher speeds so that it not violates wing stall limitations. In general the influence of the flap deflection is small [15].

2.4. Optimization Methodologies

For this research two distinct optimization methods are being considered: optimal control theory and the genetic algorithm. Jhemi describes some thoughts on the selection of optimization methodology. First of all he notes that the goal of the model is to be accurate in its objective on one hand, but on the other hand it should be simple enough to produce quick iterations of the optimization. This requires the researcher to make a good judgement regarding the model's complexity with respect to the range of validity, the balance of accuracy and complexity and required computational power.

An important aspect of aerospace modelling is the adequate handling of aerodynamic data. Experience has shown that the use of tabular data results in lower optimization efficiency, contrary to smooth data functions [34].

Another important aspect of optimizing that is to be taken into account is proper scaling of the model

parameters to end up with successful convergence in an efficient manner. According to Jhemi, the problem is properly scaled when all solution variables have their peak value close to one [34]. Carlson agrees to this. In most engineering problems, variables differ in order of magnitude by great amounts. Although this is normal, this can lead to loss of numerical precision in the computation since errors will be able to propagate to such amounts that the algorithm will fail to converge. This can be avoided by scaling the variables to be close to one. By doing this, a change in one variable has the same order of magnitude as another variable, which leads to a smaller loss of numerical precision. Moreover, it allows for the most efficient computation. Another useful insight is to combine the process of scaling with non-dimensionalizing the variables by making them unitless. "With the correct combination, these parameters should contain all possible units to allow the normalization of a variable with any possible units" [9].

The formulation of the optimization problem, or in other words the choice of the solution variables, cost functions and constraints is another important factor to bear in mind. Better choice for the state and control variables can result in simpler equations of motion, which lead to a more efficient optimization. The topic of efficient formulation is however, rarely studied in literature.

Convergence radius and speed, flexibility, complexity, computational power and accuracy are factors that are often involved in the choice for the optimization algorithm. But according to Jhemi, most often do the characteristics of the equations of motion and constraints dictate the type of suitable solution algorithm. In general, aerospace flight optimization problems contain large number of practical constraints, which suggests the use of a solution method that can handle such an amount. In the rotorcraft flight optimization study by Jhemi the equations of motion are smooth, because of which an efficient gradient algorithm can be used. When derivative information may be unavailable or expensive to calculate, researchers may need to divert to algorithms that do not require continuous derivatives [34]. Unfortunately linear interpolation is the single most widely used approach according to Betts. This interpolation method does however, yield catastrophic results as it is not differentiable at the table points and therefore an inappropriate data modelling technique [5].

Since most iterative algorithms converge to a local optimum, proper initial guesses, or repetitive initial guessing with different initial solutions are important to obtain the global solution. Sensitivity analyses are needed to gather information about the meaningfulness of the solution. The optimization may either produce solutions unconventional to pilots and engineers, or on the other hand, results consistent with flight tests. To determine the H-V diagram of a tilt-rotor aircraft, Jhemi implemented several algorithms, of which the collocation approach was found to be the most flexible for handling various kinds of constraints and was therefore chosen to generate the results [34].

2.4.1. Optimal Control

"Optimal control theory aims to find the controls that perturb a system from a fixed initial condition to a free or fixed final condition, whilst minimizing the total value of a cost functional which is itself a function of the system controls and state". These problems can be constrained by any number of path and boundary constraints [32]. Various methods exist to find the solutions of the optimization problem, including (in)direct and multiple shooting and (in)direct transcription [5]. The advantages and disadvantages of each method are summarized in Figure 2.11.

Indirect Methods

Indirect methods are basically set up by deriving the first order necessary conditions for optimality. after these have been derived, it becomes a strictly mathematical problem, and the cost function is not used directly to search for optimality [9]. In general, indirect methods are only used to solve relatively simple problems as it is quite difficult to obtain the solution since deriving the Hamiltonian boundary value problem becomes very difficult. Another drawback of this method is the fact that the problem has to be derived again after a dynamic or constraint has been added. Furthermore, indirect methods are said to be highly sensitive to the initial guess, as the solutions are often very sensitive to small changes in unspecified boundary conditions. Although indirect methods lead to the exact solution, which implies to be the global optimum, a major drawback is the inflexibility of the method and required time to derive and set-up every problem. While it remains unclear in direct methods whether a local or global optimum has been reached, the ease and robustness of setting up and solving the problem is a major advantage. Furthermore, indirect methods can have convergence problems [32], [5]. Due to the drawback of the indirect method of the difficulty of deriving and solving the Hamiltonian boundary value problem, it will

not further looked into. Other indirect methods are finite element discretization and gradient methods.

Direct Methods

Direct methods directly minimize the defined cost function and the optimal control problem is transcribed into a parameter optimization problem through finite-dimensional discretization, solved with non-linear programming [9]. The problem can therefore become very large with increasing number of constraints, states and controls. However, it still forms a very efficient way of problem solving as problem tends to be sparse, meaning that most problem derivatives are equal to zero. Contrary to indirect methods, direct methods do not require an accurate initial guess or extensive derivation of the problem. Direct methods can be divided into shooting and parametrization methods. In the former only the control variables are parametrized, while in the latter both the state as control variables are parametrized [32].

Shooting tends to be simpler because they can be described in a relatively low amount of optimization variables, but are inefficient in computational effort and tend to lead to solutions with low accuracy. Multiple shooting tries to overcome the shortcoming of both direct and indirect shooting by breaking the time domain of the trajectory into segments. This method can be implemented in both the direct and indirect approach. While the robustness increases, the number of iteration variables and constraints increases as well [5].

Parametrization methods, or collocation methods, have better accuracy and allow for complex path constraints. The collocation method can be sub-divided into local and global methods. Local methods use for instance linear interpolation of the state and control variables to locally approximate the variables. The states and controls are approximated on the entire interval in global methods [32]. Collocation methods can be extremely efficient for solving multi-point boundary value problems such as optimizing trajectory. A major advantage of direct collocation is the fact that it does not require an a priori definition of the arc sequence for path constraints [5].

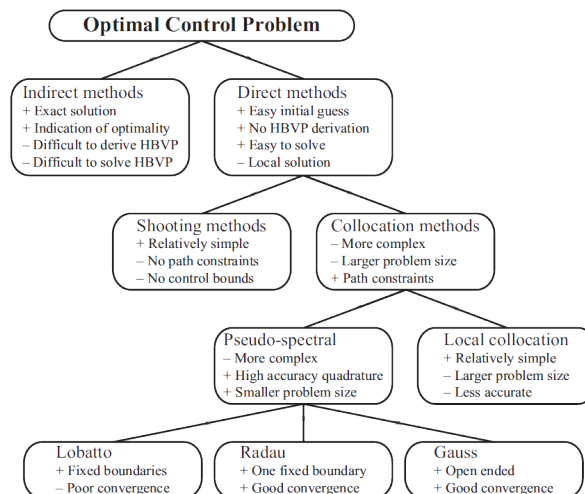


Figure 2.11: Summary of optimal control optimization methods. Adapted from: [32]

Almost all researchers in this literature review have used optimal control theory to optimize their tilt-rotor or helicopter models. The following list will set out which methods have been used:

- Carlson and Zhao use the direct collocation method in NPSOL. The advantages of this method are the fact that the user can apply inequality constraints on states, controls, parameters, linear and non-linear functions. The constraints can be applied to any point in time: at initial, interior, terminal time or throughout time. The advantages specific for his research was that initial conditions could be specified or left open, that control function continuity and rate limits could be implemented directly and that inequality constraints are directly implemented [9].
- Okuno and Kawachi have used the sequential conjugate gradient restoration algorithm. [49] [39]
- It remains however, unclear what optimization methodology is used by the GTRS, that was used by both Cerbe and Diaz.

- Chen makes use of Sequential Gradient Restoration Algorithm, developed by Miele and coded by Zhao. This direct method solves a general non-linear optimal control problem subject to terminal and path constraints on states, controls, and parameters. Inequality constraints are to be transformed into equality constraints using slack variables, as this method is only able to treat equality constraints [16].
- Tsuchiya uses the direct method as well, which in the end is solved using sequential quadratic programming, a certain gradient method [61].
- Bibic uses a direct optimal control algorithm, based on Linear Quadratic Gaussian Control approach, which allows for a change in performance index and constraints during flight and the imposition of equality or inequality constraints on the control and the state variables [6].
- Hartjes has used a (direct) pseudo-spectral collocation method for ECHO [32].
- Optimization in Tang's helicopter noise trajectory optimization is performed in EZopt, which is a toolkit that uses the collocation method [59].

2.4.2. Genetic Algorithm

Another choice for the optimization method is the genetic algorithm (GA), also known as evolutionary algorithm. As the name implies it exploits Darwinian evolution, or natural selection, to end up with a better solution in every iteration. The basic idea is that a population is assigned random values for the problem variables, which yields solutions for the performance index. 50% of the lowest solutions are dropped and the remaining 50% 'reproduces' new solutions by means of gene crossover and mutation. This process is repeated for a finite number of set generations, after which the best solution is regarded as the optimal solution.

A large difference of the genetic algorithm with optimal control is the fact that optimal control methods have well defined termination criteria for the optimal solution, whereas the genetic algorithm keeps optimizing until a predefined, finite number of iterations. This solution is then considered to be the optimal solution. The GA is the only practical alternative for applications with discrete variables. According to Betts, trajectory optimization applications are not characterized by discrete variables, and therefore there is no argument to use a method that incurs the penalty of using such an assumption. Since the GA does not exploit gradient information, it is not as computationally efficient as optimal control methods. An advantage of the GA is the fact that it is better able to escape a local optimum, in order to find the global optimum, because of the randomness introduced in the crossover and mutation of 'genes'. Another argument in favour of GA is the fact that it is incredibly simple to use, without explicitly having elaborated understanding of the to be optimized system [5]. Whereas optimal control can optimize multiple distinct variables simultaneously, GA can only optimize for one performance index. In order to optimize for the trajectory and control of the tilt-rotor, a separate black-box autopilot has to be developed to optimize for this.

Xue and Atkins have used the GA in order to investigate alternative trajectory optimization strategies to identify an efficient method for terminal area trajectory design. The GA was found to have difficulties finding the global optimum within a computational time constraint. That was for two-dimensional path searching with instantaneous transitions. Smooth transition of the trajectory worsened the computational time further [71].

Hartjes has also applied the GA to design departure trajectories that specifically take environmental impact into account. Although GA optimizations are very robust, Hartjes recognizes that the number of model evaluations required is the main issue in GA optimization. Large runtimes occur in particular when a large number of control variables are used and (in)equality constraints will further affect convergence, since more evaluations are needed. To overcome these drawbacks, Hartjes has utilized a new parametrization approach that permits a large number of optimization parameters, minimizes the required number of constraints, while still optimizing the three-dimensional trajectory. In the new approach, the vertical and lateral trajectory was parametrized [33].

Research Objective and Methodology

As was described in the introduction, the focus of this research will lie on deriving a tilt-rotor model in order to be able to optimize its flight trajectories. Before this will be conducted, an outline and project plan should be made. This chapter serves to lay out the research objective and propose the methodology to achieve this objective. First, the research gaps that followed from the literature review are summarized, after which the research objective and research question are formulated. This is followed by the expected outcome, the methodology and the experimental set-up.

3.1. Research Gap/Problem Statement

Following the review of current literature in Chapter 2, there appear to be some (sub)topics that have not yet been covered in tilt-rotor aircraft optimization, and models that can be extended in order to cover the tilt-rotor aircraft and operations in a higher degree of detail. These can be summarized to be:

- In current literature the tilt-rotor aircraft has been modelled in order to optimize its take-off procedures. Since take-off and landing primarily take place in the vertical plane, researchers have modelled the tilt-rotor by means of a two-dimensional model. Therefore, there currently is no model that accounts for three-dimensionality.
- Currently, the tilt-rotor aircraft has only been optimized for specific take-off procedures, not simulating flight longer than a few hundred feet far or high. Hence, no integral flight optimization has been conducted so far.
- An OEI situation has also only been analysed and optimized during take-off procedures. OEI has not been looked into, when it occurs during landing, conversion or cruise. Generic cruise, landing or an entire flight has not been optimized yet. AEI situations neither have been touched upon.
- Conversion corridor and strategy have been defined by a few authors, to show the outer bounds or to propose an optimum conversion strategy. Unfortunately, this has only been done for a fictitious tilt-rotor model and does not take into account the current phase of the flight or state of the aircraft.

Following the defined research gap, this research is to create a tilt-rotor aircraft model that accounts for three-dimensional flight and should be able to account for the various phases of the flight. After the tilt-rotor model has been created, it can be applied to optimize for tilt-rotor flight trajectories. Numerous optimizations can be defined ranging from isolated take-off, conversion and landing to integral flights and OEI/AEI situations. A final possibility is to include the noise and environmental emissions. It is still to be looked into whether a point-mass model, a rigid-body model or a hybrid combination of both is to be derived.

3.2. Research Objective/Question

A research objective has been formulated that aims to fill the above mentioned research gaps. The objective of this thesis research is:

To develop a numerical model for a tilt-rotor type aircraft that enables optimization studies in three-dimensional flight trajectories and procedures.

A research question is to be formulated to reach this objective. Answering this research question aids towards reaching the research objective in order to fill the research gaps that have been identified. The following research question is proposed for this thesis research:

How can a three-dimensional tilt-rotor aircraft efficiently be modelled to optimize for flight trajectories and procedures, in order to assess and enhance safety and performance?

To answer this, the main research question has been broken down into several subquestions. These subquestions contribute to the main research question and divide the research into four consecutive phases. The first phase incorporates the theoretical knowledge of aircraft and helicopter flight mechanics. The second phase account for the modelling of tilt-rotor mechanics and dynamics, while the third phase focusses on validating the model in order to acknowledge and accept its results. The last phase applies the model in a real-world environment.

Subquestion 1: *What are the principles of tilt-rotor flight mechanics, power and control, and how should they be taken into account?*

This first subquestion will not only focus on the important aspects of aircraft and helicopter flight mechanics and introduce models that have been used before in literature, but moreover should set out the tilt-rotor. It should clearly point out the distinctions and implications of a tilt-rotor aircraft, and how these implications affect dynamics and performance.

Subquestion 2: *How can these tilt-rotor principles be theoretically modelled into a point-mass or rigid-body model, in order to optimize tilt-rotor flight?*

The second subquestion aims to connect aircraft and helicopter modelling with the aspects and implications of tilt-rotor flight, and apply this to optimize the latter. The main focus is on how the tilt-rotor aircraft is modelled to represent its respective flight mechanics. Furthermore, a trade-off will be made between a point-mass model, rigid-body model or hybrid model with respect to its accuracy, efficiency and effectiveness. Finally, the model is to be verified and validated in order to assess the validity of the results.

Subquestion 3: *Does the derived model produce accurate and valid results for flight trajectory optimizations, under the imposed assumptions, simplifications and modelling?*

The third subquestion targets the phase of applying the model to isolated flight situations in order to verify and validate its outcome. In isolated flight conditions or stages of flight certain results can be expected with confidence and flight parameters and states can be determined analytically. Since the XV-15 will not be sent in the air to perform flight tests, applying the model in these situation and check it for this expectancy and analytically optimal parameters will offer the opportunity to verify the model and assess their validity. This is an important step since a very complex piece of machinery has been brought down to a simplified model under various assumptions and simplifications. Moreover, can bugs and (minor) errors bring the results of the model off course.

Subquestion 4: *Can the model be utilized to produce realistic departure and approach trajectories in a real-life utilization of the model?*

The fourth and final subquestion aims to apply the model in a case studies to assess the model and optimal tilt-rotor flight behaviour in a realistic environment.

3.3. Research Impact and Outcome

Along with the to-be determined parameters for the forces and moments acting on the aircraft, the state and control variables will be the main variables and parameters the model will work with. The data of the XV-15 will be extracted from technical reports, published by NASA [30]. The output of the model will consist of the optimal trajectories, along with the states and controls that are associated with that particular trajectory. These trajectories can then be compared to each other and to the trajectories of prior optimization studies. Moreover, it can be studied how certain mission profiles differ from classical helicopter or airplane performance, for instance in terms of time and fuel usage.

The outcome that will show the benefit or drawback of being able to tilt the rotors will be of most interest, since this information will prove the concept of tilt-rotors. This lies mostly in the entire phase before the aircraft has converted into airplane mode with its nacelles at 0° inclination. The trajectory and procedure until the conversion has ended can be very diverse as the aircraft can either convert at an early stage and climb with pitch up attitude, or it can climb with a mid-nacelle inclination and convert when the climb has ended or a certain altitude has been reached. When comparing to prior optimization studies, it is of interest if the lateral dimension plays a role in the optimization of departure procedures. When comparing the tilt-rotor to helicopters and airplanes, the entire flight will be of interest.

The relevance of the research will be to determine if the amount of complexity will have a significant impact on the optimization of departure procedures. Furthermore, it can be a relevant factor if the lateral freedom will have impact on the optimal trajectories in departures, when compared to two-dimensional optimizations. Finally, being able to optimize in three dimensions will enable studies to assess entire tilt-rotor flights (to conventional helicopter and airplane) flights. The outcome of this study can be relevant for operating procedures for the aircraft operators. Examples of this can be how to configure the aircraft with respect to nacelle inclination, flap setting, etc. in a OEI or AEI situation or what trajectory to follow to minimize the take-off distance of energy.

3.4. Methodology

Referring back to the research objective that was mentioned above, the goal is to develop a three-dimensional model that can simulate tilt-rotor aircraft flight. This allows to optimize tilt-rotor aircraft flight procedures, such as take-off, landing, conversion or emergency situations. Essentially, the problem reduces to an optimal control optimization problem that has to optimize the control input variables, while satisfying the path and boundary constraints of the states and controls in order to define certain situations and procedures. It was chosen not to optimize using the genetic algorithm since that is practical for applications with discrete variables. Since trajectory optimization problems are not described by discrete variables, there is no argument to use a method that incurs the penalty of using such an assumption. This is elaborated further on.

3.4.1. Modelling

The point-mass model will be modelled through six equations of motion for the states x , y , h , V , γ , χ . Furthermore, (pseudo-)states will be added for μ , i_n , C_L , C_T and m_{fuel} . An integral lift coefficient will be derived from the aerodynamic data of the main wing and horizontal stabilizer as these are the main contributors for the lift. From this lift coefficient the drag can be deduced. The thrust force will be modelled as a vector that finds its direction as an addition of multiple angles. First of all, the thrust vector is able to pivot in the XZ-plane as function of both the nacelle inclination and longitudinal cyclic flapping. Secondly, the thrust can be slightly vectored in the XY-plane through the lateral cyclic flapping. The model will be controlled through the variables $\dot{\mu}$, $\dot{i}_n$, $\dot{C}_L$, $\dot{C}_T$, β_{long} and β_{lat} . Although a point-mass has no directivity in the sense of a bank angle, these are used to determine the directivity of the thrust vector in order to make a turn.

Modelling as point-mass model instead of a rigid-body model adds an enormous amount of simplicity, as a rigid-body already needs 13 equations of motion for the translational and rotational states and velocities and rotor rotation x , y , z , ϕ , θ , ψ , u , v , w , p , q , r , Ω . These are then to be controlled by the aerodynamic surfaces in aircraft mode and helicopter controls in helicopter mode: δ_{ele} , δ_{ail} , δ_{rud} , β_{long} , δ_{lat} , $\dot{i}_n$ and s_{coll} . To add to the problem size, the fuselage, main wing, horizontal stabilizer, vertical stabilizer and rotors will have their own separate contribution to the total aircraft forces and moments, in order to fully account for rigid-body motion, which requires far more computational power. Flap deflection can also be taken along as control variable. It remains clear that this model will account

for far more detail, but it has to be considered what the added value of this computational effort is. The output of the model, will consist of variables of the controls (that can be translated to stick input), that are of use for pilots, but do not contribute to additional insights of determination of the flight trajectories, but rather how to fly them.

Verification and validation of the model is to be conducted, before accepting the optimization results to be sure that the numerical model represent the tilt-rotor aircraft (under the imposed assumptions). Verification is done by code verification and calculation verification. The former is done by checking the syntax, while the latter is done by having the optimization solve simplified problems of which the solution is already known or can be calculated "by hand". The model can be considered verified when the results come across to each other by a predefined degree (i.e. 10%). This can be done for the entire model, but also for smaller code blocks when complexity of the model increases, as errors might cancel each other. Validation of the model occurs by comparing the numerical solution to the actual solution. In this case it will be difficult to validate the model, as no experiments are planned and flight test data is difficult to acquire.

Modelling occurs as follows. Using engineering mechanics, the forces and moments of the aircraft will be modelled. Using engineering dynamics and differential equations, Ordinary Differential Equations (ODEs) will be extracted from the equations of motion. Both of these will be put together in the point-mass model. The forces that act on the aircraft, follow from tabular data of the XV-15 that are published by NASA. The lift data is to be interpolated in twice differentiable continuous functions before they can be used in the optimization. The model that results from this, is optimized through multi-phase optimal control theory, by employing a Legendre-Gauss-Radau quadrature orthogonal collocation method in which the problem is transcribed into a large sparse non-linear programming problem.

According to the specific optimization case the model has to optimize a specific objective function that can comprise i.e. minimum time to climb, minimize fuel flow/power required, minimize take-off distance, etc. Furthermore, in the objective function a minor penalty can be given to control inputs in order to prevent rapidly oscillating control inputs to end up with realistic controls. The output that follows from this optimization consists of the optimal flight trajectory and its according control inputs.

3.4.2. Optimization Methodology

The main objective in this research with respect to the optimization method is to be able to solve a large variety of different optimization problems with different sets of constraints and cost functions, depending on the nature of the problem. The literature review has shown that most aircraft or rotorcraft modelling research make use of optimal control theory, and only some exceptions work with the genetic algorithm, some of which only in order to assess the differences between several algorithms. This is however, not a scientific argument. Nevertheless, multiple arguments point in the direction of using optimal control theory to optimize for tilt-rotor aircraft operations.

First of all, optimal control theory is able to optimize multiple variables simultaneously, which is a very efficient way of problem solving. This amount can even go up to hundreds at the same time. This does not go for the genetic algorithm. Secondly, in trajectory optimization it is not favourable to use discrete, but continuous variables. Hence, it is unwise to make use of GA, which not only turns in on accuracy, but also on computational efficiency.

Several different optimal control theory methods can be used. In general a division within optimal control theory is made between direct and indirect methods. When comparing direct and indirect methods, it becomes apparent that indirect methods face some drawbacks that can be overcome by using a direct method. In indirect methods it is required to analytically derive expressions for the necessary conditions, which becomes more and more difficult for complicated dynamics. Secondly, the radius of convergence is quite small, which makes the initial guess very important for the final result. A third drawback is that the sequence of (un)constrained subarcs has to be guessed before iterating. Moreover, the control variables are not predefined for the optimization, which makes an indirect method hard to work with. According to Betts, direct shooting and direct collocation/parametrization are currently the most widely used methods [5].

All in all, one is inclined to use a direct collocation method. Although it yields a more complex and slightly larger problem, it offers the possibility to apply path constraints, which is not possible in shooting methods, which is necessary in tilt-rotor optimization. It has to be noted that the choice of the optimization method to be used in this research is partly dependent on the software in which the tilt-rotor model is to be implemented.

3.4.3. Experimental Set-Up

To optimize the flight trajectories of tilt-rotor aircraft no field tests or laboratory experiments will be conducted, but the aircraft will be simulated and optimized through a computer model. The simulations will be conducted to be able to answer subquestions three and four. In this research, the aircraft will be modelled in MATLAB and the optimization will be conducted using optimal control theory through the GPOPS software. This software enables the user to solve multi-phase optimal control problems using variable-order Gaussian quadrature collocation methods. “The software employs a Legendre-Gauss-Radau quadrature orthogonal collocation method where the continuous-time optimal control problem is transcribed to a large sparse non-linear programming problem (NLP)” [53].

The experimental set-up will be as follows. Referring to Figure 3.1, the tilt-rotor aircraft will be modelled as generic as possible, but will use the data of NASA's XV-15 research aircraft, since the NASA has made elaborate data of the XV-15 publicly available through Ferguson [30]. This includes the aerodynamic coefficients of the fuselage, wing-pylon, etc. and more design parameters. The equations of motion can be used in the model by inserting the ODEs and defining the limits and guesses of the states and controls. Furthermore, path and event constraints can be added to define the (physical) constraints of the aircraft. The model will be constrained and limited to the actual constraints and limitations set by aircraft manufacturer and governments.

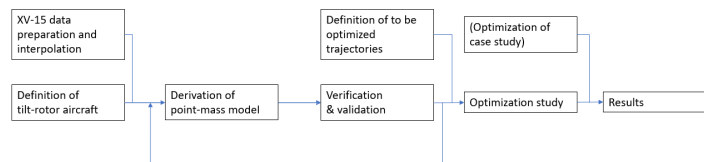
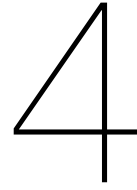


Figure 3.1: Schematic depiction of experimental set-up

It is expected however, that the software will limit the model in only a slight way. The point-mass model can be limited in the fact that a velocity of 0 m/s, cannot be used, as this results in singularity, due to the denominator in the equations of motion. A zero velocity, however, should only be the case during a hold before take-off or the full-stop after landing. This can easily be circumvented by giving the tilt-rotor a ‘headstart’ of $v = 1$ m/s. The same thing goes for a flightpath angle of $\gamma = 90^\circ$. During a vertical take-off or landing this can limit the model, and has to be assessed during the research. The same situation occurs in the equations of motion of the rigid-body model with the pitch angle θ and rotor velocity Ω . These should however, be of no concern, as it can be assumed that the engines are already started and furthermore it is not expected that a pitch angle of 90 degrees will not be anywhere near an optimal solution in any of the cases. During the optimizations, it has to be assessed whether the model is limited by the computational time it requires.

After definition and programming, the model will be used to optimize different take-off and landing procedures (vertical and runway), which is to be followed by a climb-out and cruise to a defined destination. During each phase of the optimization, boundary constraints will be given to the states, and/or path constraints will be imposed (on the controls), in order to command the aircraft to move from one to another point under limitations that account for the surroundings, landing or take-off situations, etc. Finally, the model can be implemented in a case study, to study the model in a real-life environment.



Modelling of Three Dimensional Tilt-Rotor Aircraft Point-Mass

In this chapter a description of the three-dimensional point-mass model of a tilt-rotor aircraft will be given. First, the assumptions are stated that are imposed on this model. This is followed by the derivation and elaboration of the model, with all its intermediate steps. The constraints that bound and limit the model conclude this chapter.

4.1. Model Assumptions

In this thesis, the tilt-rotor aircraft is modelled as a point-mass model with three degrees of freedom: translation in North-South direction, West-East direction and in altitude, or in x, y and h direction. Hence, this is model a simplification of the actual flight dynamics, but removes some of the complexity of higher fidelity models, in order to reduce computational effort. This has been done in a trade-off between the required computational time as input and the benefits of the output of a six degree of freedom model. The following main assumptions have been made to derive the tilt-rotor model [47]:

- **Non-oblate Earth**
Although the Earth in fact is an ellipsoid, the Earth is assumed to be non-oblate, which simplifies the transformation between reference frames and hence equations of motion. Furthermore, it yields a constant gravity field.
- **Flat Earth**
The fact that the range of the XV-15 equals 825 km [45] implies a short duration of motion, in which the influence of the Earth's curvature is negligible. Due to this, the Earth can be assumed to be flat, which will enable the vehicle carried normal Earth reference frame to coincide with the normal Earth-fixed reference frame.
- **Non-rotating Earth**
Again, due to the short duration of motion, it is allowed to assume a non-rotating Earth. This allows the angular velocity of the Earth (i.e. Coriolis and centripetal acceleration) to be neglected.
- **Constant mass**
The tilt-rotor model is assumed to have constant mass which implies no fuel consumption. If the aircraft was to be modelled as a rigid-body, this assumption would also imply no deformations. Although fuel consumption is modelled, it is not subtracted from the total aircraft weight.
- **(Aircraft has plane of symmetry)**
Although a point-mass model will be used, for completeness it is assumed that the aircraft has a plane of symmetry in the XZ-plane.

- (No rotating masses)
Even though both prop-rotors rotate in opposite direction and a point-mass is assumed, for completeness it is assumed that the model does not own any rotating masses so that gyroscopic effects can be ignored.
- Constant zero wind velocity
By assuming zero wind velocity, the aircraft travels through undisturbed air that is at rest with respect to the Earth's surface. The result of this is that the aircraft's kinematic velocity equals the aerodynamic velocity. By assuming constant zero wind, gusts and turbulence can be ignored. The calculated helicopter airspeeds are the true airspeed.
- International Standard Atmosphere (ISA)
The International Standard Atmosphere is used to calculate all atmospheric parameters. All optimizations will be done at standard day atmospheric conditions. These imply a sea-level air temperature of $T_0 = 288.15$ °K, air pressure of $p_0 = 101.325$ Pa and air density of $\rho_0 = 1,225$ kg/m³.

The following assumptions and simplifications have been made in the derivation that deviate from the actual data of the XV-15 test aircraft. Some assumptions depend on the to be followed derivation, but are compiled here for the sake of organisation.

- Although the nacelle angle of the XV-15 can increase up to 95°, the flightpath angle cannot equal 90° due to $\cos \gamma$ being in the denominator of the equations of motion in order to avoid singularity. Therefore, it is assumed that the tilt-rotor is not able to fly backwards. Hence, $|\gamma| < 90^\circ$.
- No data was available on the limits of the (acceleration of the) bank angle μ . Therefore, using an engineering sense these have been assumed to be $\pm 60^\circ$ with a limit on the rate of $\pm 5^\circ/\text{sec}$ in order to end up with realistic banking.
- A minimum thrust coefficient value is set in order to avoid singularity in the calculation of the normalized induced velocities. This leads to the minimum value of the thrust coefficient to be $C_{T_{min}} = 0.0001\sigma = 8.9 \cdot 10^{-6}$. The maximum value of the thrust coefficient remains unchanged at $C_{T_{max}} = 0.17\sigma = 0.01513$ [9]. This assumption leads to a minimum thrust at sea-level of about 50 N, which is negligibly small compared to an aircraft weight of 13.000 lbs (5.900 kg).
- Although the XV-15 is able to hover in place, a minimum velocity of $V_{min} = 1$ m/s is imposed to the model to avoid singularity. A minimum velocity of 0.01 m/s could also have been imposed, but to avoid steep gradients, the former has been chosen. This will lead to some impracticalities in hover optimizations, but these will be elaborated.

4.2. Derivation of Three-Dimensional Point-Mass Model

The following section will set out the steps that are taken to derive the tilt-rotor model. First, the appropriate reference frames and the corresponding transformations are elaborated. Section 4.2.2-4.2.5 describe the derivation of the state equations in the form of their kinematics, dynamics and pseudo-states. The equations of motion that follow from this are summarized in Section 4.2.6. The forces and other parameters that appear in these equations of motion and how these have been modelled are elaborated upon after this. These forces have been divided into the aerodynamic forces (Sec. 4.2.7), propotor power and thrust (Sec. 4.2.8) and other parameters (Sec. 4.2.9).

4.2.1. Reference Frames

To describe the forces and motion of the point-mass model, two distinct coordinate systems, or reference frames, are used: the inertial reference frame B^I and aerodynamic reference frame B^a . These reference frames will be introduced in this section, to be followed by the transformation between the two.

First, the inertial reference frame B^I , or Earth axis system, is used to describe the position of the aircraft's states in space relative to Earth. Displacements are positive in the positive sense of the axes and angles are positive in clockwise direction when looking along the respective axis in positive direction. The respective velocities, accelerations and angular velocities are positive in the same direction.

The origin of the inertial reference frame is taken at a fixed point on the Earth, with $\vec{X}^I$ pointing in a forward direction, $\vec{Y}^I$ pointing to the right and $\vec{Z}^I$ pointing downward. For convenience, the altitude h is added to the reference system as $-\vec{Z}^I$. This completes the right-handed reference frame. This reference frame is moreover needed as the time derivatives of Newton's second law are only valid in a non-accelerating and non-rotating reference frame [9].

The second reference frame to be used is the aerodynamic reference frame B^a . This reference frame is coupled to the aerodynamic velocity, being the velocity of the centre of mass relative to the undisturbed air. The origin of the aerodynamic reference frame is fixed at the centre of gravity of the point-mass, with X^a pointing in positive direction of the velocity vector relative to the atmosphere, Z^a being perpendicular to the velocity vector and the XZ-plane of symmetry of the aircraft, being positive below the aircraft. Y^a completes the right-hand reference frame by being perpendicular to the latter two vectors, positive in right direction [47], [68] en [22].

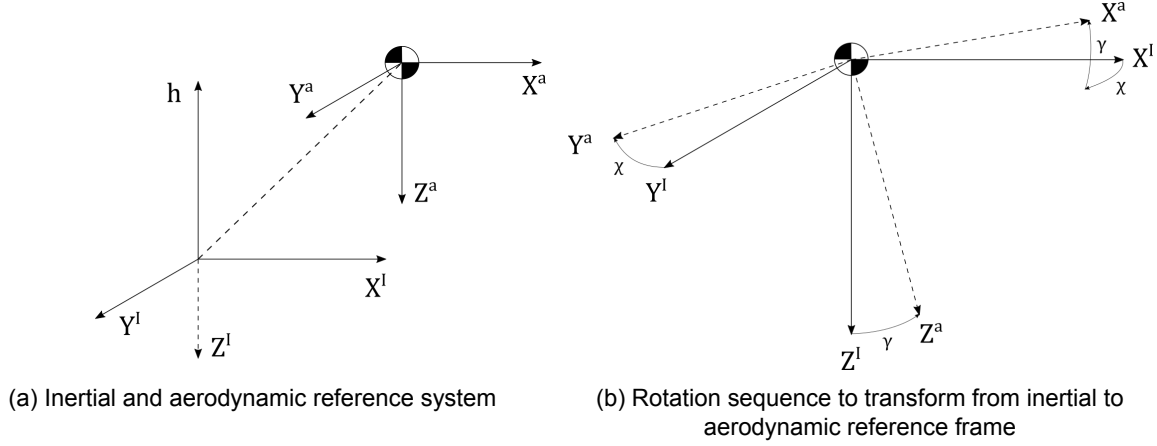


Figure 4.1: Reference coordinate systems

A sequence of the following two consecutive rotations about the heading angle χ and flight path angle γ transforms the inertial into the aerodynamic reference frame. Normally, this would be followed by a rotation of μ degrees aerodynamic bank angle about the x-axis, but since a point-mass does not possess a bank angle this is omitted. The bank angle μ will only be used to direct the thrust in order to make a heading angle change. This transformation sequence is the most commonly used sequence in the aerospace industry [47].

1. χ degrees aerodynamic heading angle about the z-axis
2. γ degrees aerodynamic flight path angle pitch about the y-axis

Using this, forces, velocities, accelerations and rotations can easily be transformed from one into the other reference frame. This transformation from inertial to aerodynamic reference frame is depicted in Figure 4.1b. A vector in the inertial reference frame can hence also be coordinatized in the aerodynamic reference frame through multiplication with the rotation matrices show in Equation 4.1 [22], [47], [68].

$$\begin{pmatrix} \vec{X}^a \\ \vec{Y}^a \\ \vec{Z}^a \end{pmatrix} = \begin{pmatrix} \cos \gamma & 0 & -\sin \gamma \\ 0 & 1 & 0 \\ \sin \gamma & 0 & \cos \gamma \end{pmatrix} \begin{pmatrix} \cos \chi & 0 & \sin \chi \\ -\sin \chi & \cos \chi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \vec{X}^I \\ \vec{Y}^I \\ \vec{Z}^I \end{pmatrix} \quad (4.1)$$

This can be reduced to the following integral transformation matrix, $\omega_{I/a}$:

$$\begin{pmatrix} \vec{X}^a \\ \vec{Y}^a \\ \vec{Z}^a \end{pmatrix} = \begin{pmatrix} \cos \gamma \cos \chi & \cos \gamma \sin \chi & -\sin \gamma \\ -\sin \chi & \cos \chi & 0 \\ \sin \gamma \cos \chi & \sin \gamma \sin \chi & \cos \gamma \end{pmatrix} \begin{pmatrix} \vec{X}^I \\ \vec{Y}^I \\ \vec{Z}^I \end{pmatrix} \quad (4.2)$$

By inverting the transformation, the transformation from aerodynamic to inertial reference frame can be established with the transformation matrix $\omega_{a/I}$:

$$\begin{pmatrix} \vec{X}^I \\ \vec{Y}^I \\ \vec{Z}^I \end{pmatrix} = \begin{pmatrix} \cos \gamma \cos \chi & -\sin \chi & \sin \gamma \cos \chi \\ \cos \gamma \sin \chi & \cos \chi & \sin \gamma \sin \chi \\ -\sin \gamma & 0 & \cos \gamma \end{pmatrix} \begin{pmatrix} \vec{X}^a \\ \vec{Y}^a \\ \vec{Z}^a \end{pmatrix} \quad (4.3)$$

4.2.2. Point-Mass Kinematics

The aircraft's position P and the aerodynamic velocity of the aircraft V can be written in its position and velocity components in the inertial reference frame as follows:

$$\vec{P} = x^I + y^I + z^I \quad (4.4)$$

$$\vec{V} = \dot{x}^I + \dot{y}^I + \dot{z}^I \quad (4.5)$$

This velocity is the true airspeed (TAS), which is equal to the ground speed since there is no wind velocity components. Using the transformation matrices from Section 4.2.1, it follows that the velocity vector is given by:

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix} = \begin{pmatrix} \cos \gamma \cos \chi & -\sin \chi & \sin \gamma \cos \chi \\ \cos \gamma \sin \chi & \cos \chi & \sin \gamma \sin \chi \\ -\sin \gamma & 0 & \cos \gamma \end{pmatrix} \begin{pmatrix} V \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \cos \gamma \cos \chi \\ \cos \gamma \sin \chi \\ -\sin \gamma \end{pmatrix} V \quad (4.6)$$

or re-introducing $h = -w$:

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{h} \end{pmatrix} = \begin{pmatrix} \cos \gamma \cos \chi \\ \cos \gamma \sin \chi \\ \sin \gamma \end{pmatrix} V \quad (4.7)$$

The inertial accelerations $\ddot{V}$, $\dot{\gamma}$ and $\dot{\chi}$ that are needed to derive the equations of motion using Newton's second law, can be determined using Coriolis' theorem:

$$\frac{d}{dt} \vec{V}^I = \frac{d}{dt} \vec{V}^a + \vec{\omega}_{I/a} \times \vec{V} \quad (4.8)$$

where $\omega_{I/a}$ is the vector relating the angular velocity of the aerodynamic frame to the inertial frame. This vector is constructed by the following equation, in which C is the appropriate transformation matrix, as in Equation 4.1:

$$\omega_{I/a} = [\dot{\phi}, 0, 0]^T + C_\phi [0, \dot{\gamma}, 0]^T + C_\phi C_\gamma [0, 0, \dot{\chi}]^T \quad (4.9)$$

Since $\dot{\phi}$ is not part of the point-mass, as it cannot have an attitude, this is set to zero. This leads to the angular vector to be:

$$\omega_{I/a} = \begin{pmatrix} 0 & 0 & -\dot{\chi} \sin \gamma \\ 0 & \dot{\gamma} \cos \phi & \dot{\chi} \cos \gamma \sin \phi \\ 0 & -\dot{\gamma} \sin \phi & \dot{\chi} \cos \gamma \cos \phi \end{pmatrix} \quad (4.10)$$

Substituting this vector back into Equation 4.8 yields the relation for the inertial acceleration:

$$\ddot{V}^I = \begin{pmatrix} \ddot{V} \\ V(-\dot{\gamma} \cos \phi + \dot{\chi} \cos \gamma \sin \phi) \\ -V(\dot{\gamma} \sin \phi + \dot{\chi} \cos \gamma \cos \phi) \end{pmatrix} \quad (4.11)$$

4.2.3. Point-Mass Dynamics

A three dimensional Cartesian reference frame has been established to formulate the force equations for F_x , F_y and F_z . The forces that are acting on the tilt-rotor point-mass are shown in Figure 4.2a and 4.2b. The aircraft figure acts only for illustrative purposes. The drag force D is aligned with the aerodynamic velocity vector $\vec{X}^a$, while the lift force L is aligned with negative $\vec{Z}^a$ direction. The aircraft's weight is in $\vec{Z}^l$ direction. The thrust force T is variable in the XZ-plane and acts at an angle of $i_n - \beta_{long}$ with respect to the airspeed vector, consisting of the nacelle inclination i_n and longitudinal cyclic (flapping) angle β_{lat} . In Figure 4.2b it can be seen that L and T also have a component in the YZ-plane due to a bank angle μ . In addition to this the T can also be tilted laterally by the lateral cyclic (flapping) angle β_{lat} .

In cyclic control the blade pitch of each blade is changed in a periodic fashion as a function of the blades position in the cycle. This creates a difference in lift at different positions of the rotor plane. This effectively tilts the thrust vector at some cyclic angle β_{lat} in the lateral or β_{long} longitudinal plane[4]. This should not be confused with the lateral and longitudinal cyclic blade pitch angle, commonly denoted as θ_{1c} and θ_{1s} .

A second point of notion is the assumption that the angle of attack is approximately equal to the flightpath angle. The tilt-rotor flight model has to accommodate for both airplane and helicopter flight and hence has to be able to have velocity in both modes. The nacelle inclination is measured with respect to the aircraft's X^a axis. Furthermore, since a point-mass does not possess any attitude (and zero wind velocity is assumed), it is safe to assume that $\theta \approx 0$. This yields the fact that $\alpha \approx \gamma$. To accommodate for both of these the angle of attack is subtracted from the nacelle inclination in order to decompose the thrust vector along the velocity direction.

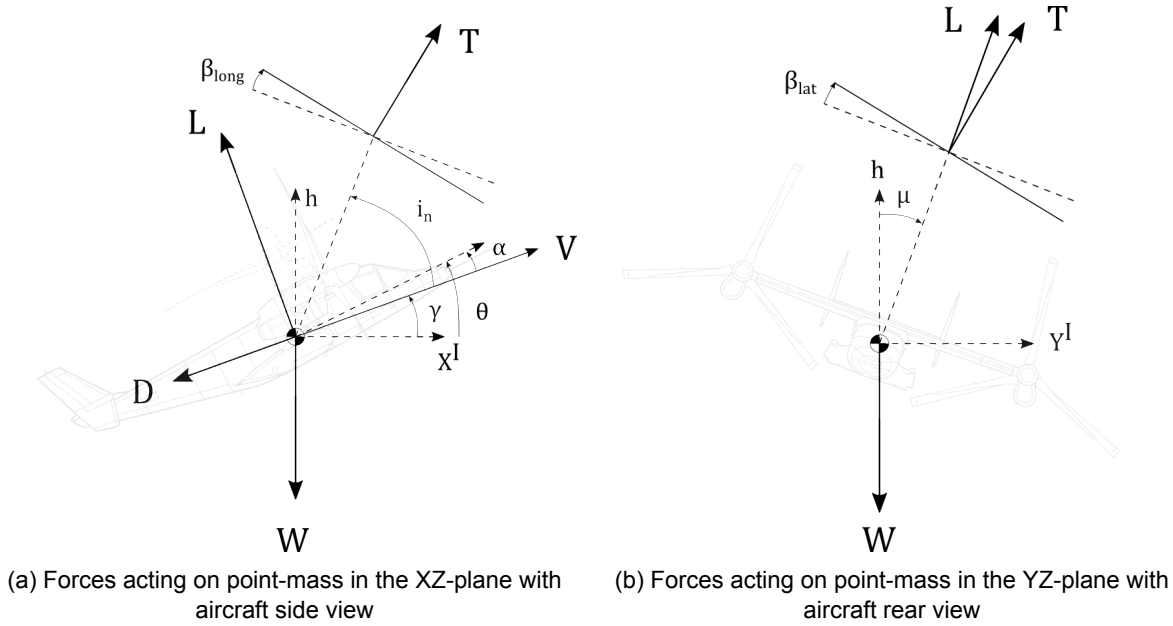


Figure 4.2: Free body diagram of the point-mass model. (Forces not to scale)

Newton's second law states that the sum of forces acting on a body is equal to the change in momentum of that body, or more commonly known as the mass times acceleration:

$$\sum \vec{F} = \begin{pmatrix} T \cos(i_n - \beta_{long} - \alpha) - (D + mg \sin \gamma) \\ (L + T \sin(i_n - \beta_{long} - \alpha)) \cos(\mu - \beta_{lat}) - mg \cos \gamma \\ (L + T \sin(i_n - \beta_{long} - \alpha)) \sin \mu - \beta_{lat} \end{pmatrix} = m \vec{V} \quad (4.12)$$

Here F consists of all aerodynamic forces, propulsive forces and aircraft weight. External forces such as wind could also be included but will be neglected within this thesis. Moreover, the aircraft weight is assumed to be constant throughout the flight, as mentioned in Section 4.1. Note that the small angle assumption does not hold, as the tilt-rotor will manoeuvre over the entire range of flight path angle γ .

Newton's second law yields the following dynamic equations using the inertial accelerations from Equation 4.11 together with the above mentioned forces:

$$T \cos(i_n - \beta_{long} - \alpha) - (D + mg \sin \gamma) = m\dot{V} \quad (4.13)$$

$$(L + T \sin(i_n - \beta_{long} - \alpha)) \cos(\mu - \beta_{lat}) - mg \cos \gamma = mV(-\dot{\gamma} \cos \phi + \dot{\chi} \cos \gamma \sin \phi) \quad (4.14)$$

$$(L + T \sin(i_n - \beta_{long} - \alpha)) \sin \mu - \beta_{lat} = mV(-\dot{\gamma} \sin \phi - \dot{\chi} \cos \gamma \cos \phi) \quad (4.15)$$

To decouple Equations 4.14 and 4.15, Equation 4.14 was multiplied by $\sin \phi$ and Equation 4.15 by $\cos \phi$. The resulting equations were added to end up with a dynamic equation for $\dot{\gamma}$. Similar to this, Equation 4.14 was multiplied by $\cos \phi$ and Equation 4.15 by $\sin \phi$. The resulting equations were subtracted to end up with a dynamic equation for $\dot{\chi}$:

$$mV\dot{\gamma} = (L + T \sin(i_n - \alpha - \beta_{long})) \cos(\mu - \beta_{lat}) - mg \cos \gamma \quad (4.16)$$

$$mV \cos \gamma \dot{\chi} = (L + T \sin(i_n - \alpha - \beta_{long})) \sin \mu - \beta_{lat} \quad (4.17)$$

4.2.4. Point-Mass Pseudo-States

For the purpose of flight path design the latter six state equation suffice to optimise translational motion [70]. However, in order to prevent some control variables to make instantaneous changes in their value, a pseudo-state is introduced with the rate of change of the respective state to act as control variable. This will for example keep the model from making instant corners, instead of gradual ones. Hence, the bank angle μ , nacelle inclination i_n and lift and thrust coefficients C_L and C_t are treated as pseudo-states, which leads to more realistic flight paths.

$$\dot{\mu} = \mu_{rate} \quad (4.18)$$

$$\dot{i}_n = i_{n_{rate}} \quad (4.19)$$

$$\dot{C}_L = C_{L_{rate}} \quad (4.20)$$

$$\dot{C}_T = C_{T_{rate}} \quad (4.21)$$

4.2.5. Point-Mass Fuel Flow

Although constant mass is assumed for model simplicity, the last state of the point-mass model is its fuel consumption. The state equation for the fuel flow is given by the following formula, that is based on the specific fuel consumption stated by the manufacturer.

$$\dot{w}_{fuel} = sfc \cdot P_r \quad (4.22)$$

4.2.6. Equations of Motion

The previous derivations have lead to the equations of motion of the point-mass model of the tilt-rotor aircraft in the form of the eleven first-order differential equations as depicted below.

$$\dot{x} = V \cos \gamma \cos \chi \quad (4.23)$$

$$\dot{y} = V \cos \gamma \sin \chi \quad (4.24)$$

$$\dot{h} = V \sin \gamma \quad (4.25)$$

$$\dot{V} = \frac{T \cos(i_n - \beta_{long} - \alpha) - (D + mg \sin \gamma)}{m} \quad (4.26)$$

$$\dot{\gamma} = \frac{(L + T \sin(i_n - \beta_{long} - \alpha)) \cos(\mu - \beta_{lat}) - mg \cos \gamma}{mV} \quad (4.27)$$

$$\dot{\chi} = \frac{(L + T \sin(i_n - \beta_{long} - \alpha)) \sin \mu - \beta_{lat}}{mV \cos \gamma} \quad (4.28)$$

$$\dot{\mu} = \mu_{rate} \quad (4.29)$$

$$\dot{i}_n = i_{n_{rate}} \quad (4.30)$$

$$\dot{C}_L = C_{L_{rate}} \quad (4.31)$$

$$\dot{C}_T = C_{T_{rate}} \quad (4.32)$$

$$\dot{w}_{fuel} = sfc \cdot P_r \quad (4.33)$$

where T is the total thrust, L is the total lift, D is the aircraft drag, P_r is the power required, m is the total aircraft mass and g the gravitational constant.

The model is controlled by six control variables: while μ_{rate} , $i_{n_{rate}}$, $C_{L_{rate}}$ and $C_{T_{rate}}$ control the rate of change of the bank angle, nacelle inclination and lift and thrust coefficient, β_{long} and β_{lat} control the cyclic angle in longitudinal and lateral direction.

The cyclic angle of the rotor phases out slowly as a function of the nacelle angle, since these become less effective at higher speeds than control surfaces. Due to this, it has full effect in helicopter mode and no effect in airplane mode. To model this, the rotor cyclic phases out as follows, in which η_β can be chosen within the blade flapping limits. This goes for both the lateral as for the longitudinal flapping.

$$\beta = \eta_\beta \sin i_n \quad (4.34)$$

Therefore, the model consists of eleven state and pseudo-control variables and six control variables. The state and control vector are:

$$s = [x, y, h, V, \gamma, \chi, \mu, i_n, C_L, C_T, m_{fuel}] \quad (4.35)$$

$$u = [\mu_{rate}, i_{n_{rate}}, C_{L_{rate}}, C_{T_{rate}}, \beta_{long}, \beta_{lat}] \quad (4.36)$$

4.2.7. Aerodynamic Forces

From the tabular aerodynamic data of the XV-15 the lift and drag coefficient can be extracted [30]. Figure 4.3a shows the lift coefficient for the wing-pylon assembly for the full range of angle of attack for both helicopter and airplane mode with various flap settings.

Since a point-mass does not have an attitude, there is no angle of attack. Therefore, the lift coefficient was chosen to be a control variable (the lift coefficient being a state and the rate of change of the lift coefficient being a control variable). In order to have one equation for the lift coefficient, these have to be merged into one equation for the lift and drag coefficient.

The lift coefficient has been composed by the two lifting wing areas: the wing-pylon combination and the horizontal stabilizer. Lift of the fuselage and of the two vertical fins have been neglected, unlike the drag of the fuselage which has been incorporated. Clearly, the lift is calculated only with the airspeed of the air flowing over the wings (Equation 4.46). Although the lift coefficient is higher in airplane mode than it is in helicopter mode, helicopter flight occurs at a low forward speed u , which results in a low amount of lift. Which is not a problem since most of the vertical force is produced by the rotors. Transitioning to airplane flight the lift starts to increase and less power is required by the engines as the wings start to carry the weight. Moreover, the lift coefficient in helicopter mode at $\pm 90^\circ$ equals zero. Therefore, it is chosen to model the lift coefficient from airplane mode in between $-40^\circ < \alpha < 40^\circ$. The lift is calculated by:

$$L = \frac{1}{2} \rho u^2 (S_{wp} + S_{hs}) C_L \quad (4.37)$$

The data yields the fact that the minimum and maximum lift coefficient yield values of -1.15 and 1.99, which result in the bounds for the state. The bounds for the control variable result from thin airfoil theory. Since the lift coefficient can be chosen in the optimization, only the resulting drag is to be modelled, which will be done in two separate ways. By means of the induced drag equation and by interpolating XV-15 data. For the induced drag equation (Equation 4.38), a parasite drag value of 0.017 was found from XV-15 data and a wing efficiency factor of $e = 0.85$ was assumed. The polynomial interpolant results with an R^2 -value of 0.900 and a RMSE of 0.06531. Not all data points were used to interpolate this polynomial, but only the lift and drag coefficient data points that describe the outer contour, as in Fig. 4.3b.

$$C_D = C_{D,0} + \frac{C_L^2}{\pi A e} \quad (4.38)$$

$$C_D = 0.05795C_L^4 - 0.0513C_L^3 - 0.01675C_L^2 + 0.02555C_L + 0.03022 \quad (4.39)$$

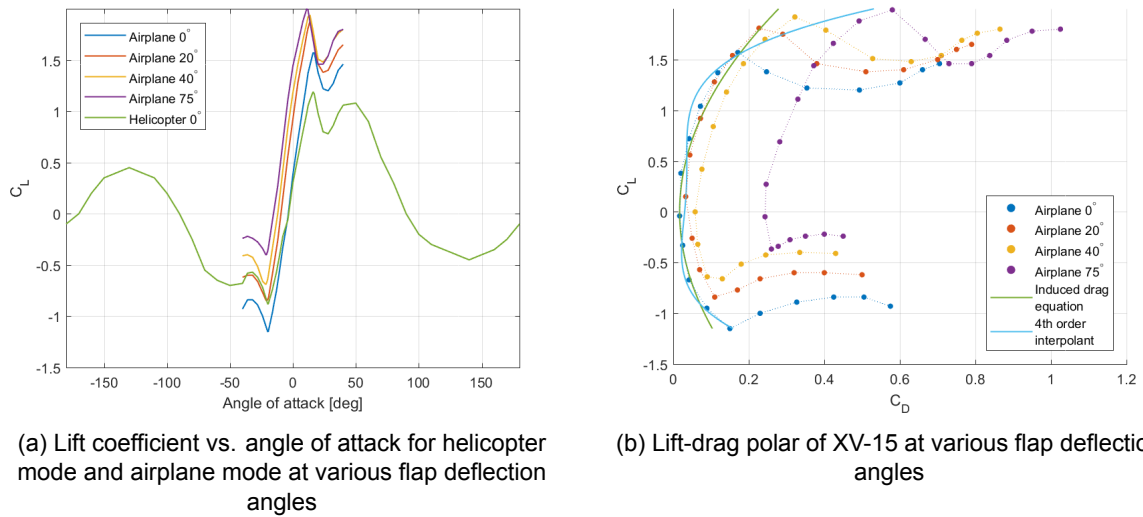


Figure 4.3: Aerodynamic coefficients of the XV-15

The drag of the fuselage is as well given by Ferguson in tabular data as a function of the dynamic pressure. Since the fuselage drag is only a small portion of the total drag (in the order of 10%), it must definitely be incorporated but will not be calculated over the entire range of angle of attacks. The following formula will yield the total drag of the wings and fuselage:

$$D = D_{wing-pylon} + D_{horz.stab} + D_{fuselage} = \frac{1}{2} \rho u^2 ((S_{wp} + S_{hs}) C_D + 0.1449) \quad (4.40)$$

4.2.8. Proprotor Power

This subsection will set out the derivation of the power the engines have available and require in varying flight conditions and thrust they can produce.

Power Available

The proprotors are powered by two Lycoming LTC1K-41K turboshaft engines, which are a modified version of the Lycoming T53-L-13B [45]. NASA describes the engine specifications of the XV-15 in the Tilt Rotor Familiarization Document [44]. The maximum operating tip speed is provided in Table 4.1, while.

Table 4.1: Maximum tip speed specification of the Lycoming LTC1K-41K per flight mode [44]

Flight mode	Max. tip speed [rpm]
Helicopter and STOL take-off	565
Helicopter and conversion, cruise and climb	565
Airplane cruise	458

Since the engines are turboshaft gas turbines, the power available reduces with altitude as the air becomes less dense. Corresponding to the engine performance graphs in the aircraft documentation [44], the engine power available can be modelled with the following polynomials, which are depicted in Figure 4.4. Notice: the altitude used in Equations 4.41-4.44 is in feet and the power in shp.

$$P_{a_{norm}} = -4.46 \cdot 10^{-7} h_{ft}^2 - 0.01329 h_{ft} + 1125 \text{ [shp]} \quad (4.41)$$

$$P_{a_{mil}} = -3.655 \cdot 10^{-7} h_{ft}^2 - 0.01812 h_{ft} + 1260 \text{ [shp]} \quad (4.42)$$

$$P_{a_{t/o}} = -1.422 \cdot 10^{-7} h_{ft}^2 - 0.0271 h_{ft} + 1393 \text{ [shp]} \quad (4.43)$$

$$P_{a_{cont}} = 3.903 \cdot 10^{-8} h_{ft}^2 - 0.03886 h_{ft} + 1619 \text{ [shp]} \quad (4.44)$$

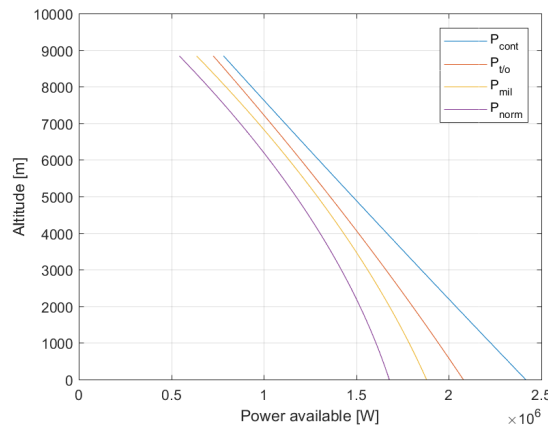


Figure 4.4: Power available of two engines due to a change in altitude

In the event of an engine failure, the power available spools down from nominal operations rating to its One Engine Inoperative power rating of even zero power in the event of a double engine failure, or All Engine Inoperative. Hence, the power available is a function of time and can be modelled by [9]: where τ_p is the pilot's time delay, which has conservatively been set to 0.6 second.

$$P_a = \left(P_{a_{AEO}} - P_{a_{OEI,AEI}} e^{\frac{t}{\tau_p}} \right) + P_{a_{OEI,AEI}} \quad (4.45)$$

Induced Velocity

Before determining the power required, the rotor induced velocity v_i is to be calculated. The induced velocity describes the wind velocity that is developed by the rotors turning. In order to derive the induced velocity, the velocity components relative to the rotor Tip Path Plane (TPP) are needed. Figure 4.5a and 4.5b show the horizontal and vertical components of the aerodynamic velocity and the relation of the aerodynamic velocity to the TPP. It can be seen that the parallel component consists of a velocity component in longitudinal and lateral direction. Due to the fact that a point-mass model is used and no wind field exists, the kinetic and aerodynamic velocity are aligned and no lateral component exists. This would exist in lateral translation, but again, lateral translation is omitted since the aircraft has no

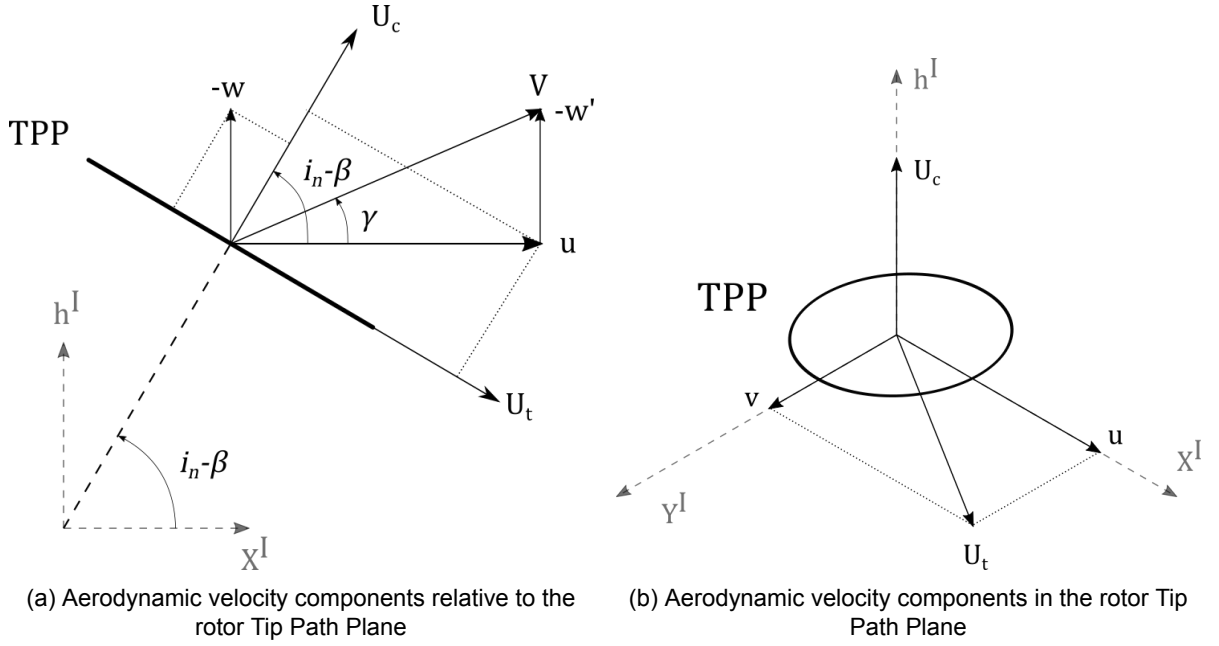


Figure 4.5: Schematic depiction of velocity components relative to the rotor

attitude except for its flightpath and heading angle. Therefore, $U_{ty} \approx 0$. From the figures it can be derived that the velocity normal to the TPP U_c and the velocity parallel to the TPP U_t can be derived to be:

$$u = V \cos \gamma, \quad \dot{h} = -w = V \sin \gamma \quad (4.46)$$

$$U_c = u \cos(i_n - \beta_{long}) + \dot{h} \sin(i_n - \beta_{long}) \quad (4.47)$$

$$U_t = u \sin(i_n - \beta_{long}) - \dot{h} \cos(i_n - \beta_{long}) \quad (4.48)$$

From moment theory follows that the induced velocity at the rotor disc during hover can be related to the thrust by the following equation [41], which is defined to be the normalization factor to normalize the relative and induced velocities:

$$v_h = \sqrt{\frac{T}{2\rho\pi R^2}} \quad (4.49)$$

Consequently, the induced velocity, and velocity parallel and perpendicular to the rotor tip plane are normalized with the induced velocity in hover. The determination of the normalized induced velocity follows hereafter.

$$\bar{v}_i = \frac{v_i}{v_h}, \quad \bar{U}_c = \frac{U_c}{v_h}, \quad \bar{U}_t = \frac{U_t}{v_h} \quad (4.50)$$

Momentum theory dictates that the normalized induced velocity can be determined by solving the following fourth order polynomial or empirical formula depending on the fact if the rotor plane is in its own wake, or in other words, if the rotor plane is situated in the vortex-ring state [75], [74].

If $(2\bar{U}_c + 3)^2 + \bar{U}_t^2 > 1$, the rotor is not located in its own wake and $\bar{v}_i$ is described by:

$$\bar{v}_i^4 + 2\bar{U}_c \bar{v}_i^3 + (\bar{U}_c^2 + \bar{U}_t^2) \bar{v}_i^2 - 1 = 0 \quad (4.51)$$

Otherwise if $(2\bar{U}_c + 3)^2 + \bar{U}_t^2 \leq 1$, the rotor is in vortex-ring state and can be calculated by:

$$\bar{v}_i = \bar{U}_c (0.373\bar{U}_c^2 + 0.598\bar{U}_t^2 - 1.991) \quad (4.52)$$

The relation between the normalized velocity parallel and perpendicular rotor plane and the normalized induced velocity is depicted in Figure 4.6, for $-5 < \bar{U}_c < 5$ and $-5 < \bar{U}_t < 5$ since $\bar{v}_i$ slowly converges to zero outside of this range. To illustrate the vortex-ring state, the region inside the vortex-ring state comprises of roughly the region $-2.5 < \bar{U}_c < -0.5$ and $-0.5 < \bar{U}_t < 0.5$.

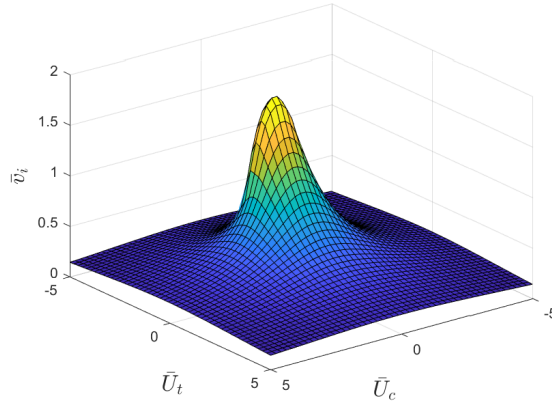


Figure 4.6: Normalized induced velocity $\bar{v}_i$ due to $\bar{U}_c$ and $\bar{U}_t$

Power Required

The power available only changes with altitude, but the power required varies in a different way. The power required for the engine to keep the aircraft in the air and maintain its current flight condition, can be said to come from four distinct sources [41]:

1. The rotor induced power P_i , which is the power required to produce the rotor thrust.
2. The rotor profile power P_{pr} , which is the power required to overcome viscous losses at the rotor.
3. The rotor parasite power P_p , which is the power required to compensate for the aircraft drag.
4. The rotor climb power P_c , which is the power required to increase the gravitational potential of the aircraft.

Hence, the total power required can be defined as [41]:

$$P_r = \frac{2}{\eta_p} (P_{pr} + P_i + P_p + P_c) \quad (4.53)$$

where, η_p accounts for the transmission losses, while the factor two accounts for the fact that two rotors are used. Common values for losses in helicopter transmission comprise of $0.91 \leq \eta_p \leq 0.96$, according to Johnson [36]. Without further explanation, Carlson and Zhao have used a propulsive efficiency value of 0.95 [12].

Apart from this, Johnson has also derived an analytical expression for P_i , P_p and P_c , using momentum theory and conservation of energy [36], [58], [4]:

$$P_i + P_p + P_c = T (U_c + K_{ind} f_G v_i) \quad (4.54)$$

The term TU_c equals the power required to overcome aircraft drag and climb, while Tv_i accounts for the induced power required. The latter term is adjusted with a correction factor K_{ind} to align the model with flight test data. These flight tests have shown that the actual induced power loss was 5-20% higher than anticipated with momentum theory. According to literature, the correction factor K_{ind} holds a value of 1.15 [41], [16], [59], [64]. Although the ground effect is neglected within this thesis, the ground effect

factor f_G has been added to the correction for completeness. To keep the tilt-rotor out-of-ground effect (OGE), f_G is kept equal to a value of one.

Now that relations have been determined for the rotor induced power, rotor parasite power and the rotor climb power, the profile power remains to be determined, which can be done using blade element theory [41], [58]:

$$P_{pr} = P_{pr_0} (1 + \mu^2) \quad (4.55)$$

where P_{pr_0} and the advance ratio μ equal:

$$P_{pr_0} = \frac{1}{8} \sigma \pi R^2 \rho (\Omega R)^3 c_d \quad (4.56)$$

$$\mu = \frac{U_t}{\Omega R} \quad (4.57)$$

where σ is the rotor solidity ratio, which equals 0.089 for the XV-15 [44], [30]. c_d is the rotor drag coefficient, which is determined to be 0.015 [30], [12]. Through experimental data it was found however, that the profile power had a stronger dependency on the advance ratio. Due to this a correction factor of 1.7 was applied [9]:

$$P_{pr} = P_{pr_0} (1 + 1.7\mu^2) \quad (4.58)$$

Finally, the H-force is included in the profile power required. The H-force is the backward force that is the result of the imbalance of the advancing and retreating blade due to their speed relative to the air. Because the retreating blade has a lower airspeed, it has to operate at larger angle of attack. Therefore, the profile drag will decrease, but the induced drag will increase. On the contrary the advancing blade will have larger profile drag due to the larger airspeed, but will have lower induced drag due to lower angle of attack [67]. This can be calculated to be $3P_{pr_0}\mu^2$. Therefore, the profile power required results to be:

$$P_{pr} = \frac{1}{8} \sigma \pi R^2 \rho (\Omega R)^3 c_d (1 + 4.7\mu^2) \quad (4.59)$$

Finally, the previous relations that were found for the TPP velocities, induced velocity and power required can be rewritten to end up with one relation for the power coefficient:

$$C_P = C_T \sqrt{C_T/2} (K_{ind} f_G \bar{v}_i + \bar{U}_c) + \frac{1}{8} \sigma c_d (1 + 4.7\mu^2) \quad (4.60)$$

Using the previous relation for the power coefficient, the total power required by the rotors can be determined by the following formula according to Johnson. Needless to say, the power required cannot exceed the power available.

$$P_r = \frac{2}{\eta_p} \rho (\pi R^2) (\Omega R)^3 C_P \quad (4.61)$$

Thrust

The thrust produced by one rotor is calculated with the following equation [41], [58], [4]. The thrust coefficient is a control variable in this optimization. This means that the thrust coefficient C_T is to be chosen as such that the power required that follow from this, does not exceed the power available, while satisfying all other constraints and optimization goals.

$$T = \rho (\pi R^2) (\Omega R)^2 C_T \quad (4.62)$$

4.2.9. Other Model Factors

The last subsection will provide details about the other factors contributing to or acting on the point-mass model such as the fuel flow, rotor wash/wing download and the ground effect.

Fuel Flow

The fuel consumption from the XV-15 can directly be taken from the aircraft documentation [44], where it is stated for its different engine settings as given in Table 4.2. The specific fuel consumptions are converted from the Imperial $\frac{lb}{shp \cdot h}$ to the metric $\frac{kg}{kW \cdot h}$. The fuel flow that follows from this is described by the following equation, with P_r in Watt and the resulting fuel flow in kg/s. All in all, the two engines of the XV-15 will consume 0.17633 kg/s when in full power. When the engines require no power and run idle, it is assumed that the engines will still have a fuel consumption at 10% of maximum power.

Table 4.2: Specific Fuel Consumption of the Lycoming LTC1K-41K [44]

Engine setting	sfc [kg/kW·h]
Contingency rated	0.343
Take-off rated	0.355
Military rated	0.366
Normal rated	0.378

$$\dot{w}_{fuel} = \frac{P_r \cdot sfc}{1000 \cdot 3600} \quad (4.63)$$

Download

When the nacelle angle is at high inclination, the rotor wash during hover and low speeds impinges the fuselage and wing surface area underneath the rotor, which creates a force opposing the lift, which is download. The download can be equal to 10-15% of the total rotor thrust during hover. This is not only wasted thrust, but moreover results in a loss of lift. The air flow in hover is schematically depicted in Figure 4.7.

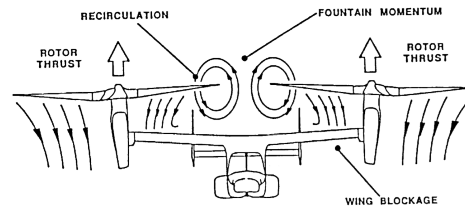


Figure 4.7: Sketch of the V-22 in hover, showing the main flow features [29]

According to Jordan, the download of the XV-15 research aircraft has been determined experimentally with the use of flight tests in conjunction with wind tunnel tests of a representative two-dimensional airfoil. hover flight tests have been conducted at an altitude of 2.5 times the rotor diameter to assure out-of-ground conditions. Furthermore, ambient test conditions were monitored. This has lead to a download over thrust ration, out of ground effect for a 0° flap deflection of $\frac{DL}{T} = 0.132$ [38]

Diaz states that the magnitude of the download can depend on various factors such as airspeed, flap deflection, nacelle tilt angle and rotor-ground distance. The download will be determined by the airspeed and nacelle inclination. However, since the use of flaps is not modelled to full extend, this will not be included in the calculation of the download. Furthermore, the tilt-rotor will spend very little time within the rotor-ground region that is of influence for the download, that it is omitted from this study.

Diaz follows by investigating the effect of forward velocity on the rotor download. Diaz has used a semi-empirical method that interpolates published data curves of tilt-rotor hover at 90° nacelle inclination. From these download values, her model determines the evolution of the download with nacelle inclination and aircraft forward speed. It was determined that the interaction between download and velocity disappears at a forward velocity of $V_{lim} \approx 30m/s$. Hence, for $0 \leq V \leq 30$ the download can be determined by [21]:

$$DL = DL_{hover} \left(1 - \sin^2 \left(\frac{\pi V}{2V_{lim}} \right) \right) \sin i_n \quad (4.64)$$

Combining the determined download value by Jordan with the relation between download and forward speed by Diaz leads to the XV-15 download as depicted in Figure 4.8.

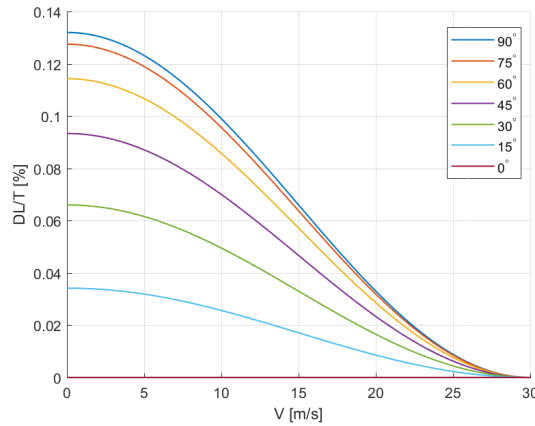


Figure 4.8: Download versus forward speed for various rotor nacelle inclinations

Ground Effect

As the helicopter is close to the ground, the flow of air interacts with the ground and due to this the flow field changes. This leads to the induced velocity and the rotor tip vortices, and thus rotor drag, being reduced in force. The change in air flow resulting from the ground is visualized in Figure 4.9. This leads to a reduced power required and increase of available thrust when the rotors are in-ground-effect (IGE). Literature has shown that this positive effect is present within an elevation from the ground equal to two times the rotor diameter [15], [41], [36]. A rotor radius of 12.5 ft, yields to the fact that the ground effect for the XV-15 is present within 50 ft altitude.

As the reader might already have read in the previous section, the ground effect will be neglected in this thesis due to the following reason. First of all does the tilt-rotor only spend a limited amount of time within this altitude compared to the rest of the flight. Moreover, as will be elaborated further on in Section 4.3.4, will the take-off and landing be modelled with the TDP and LDP as start and endpoint of the optimization. These points fall outside of the ground effect region and therefore ground effect can be neglected. Hence, all flight throughout this thesis will be out of ground effect.

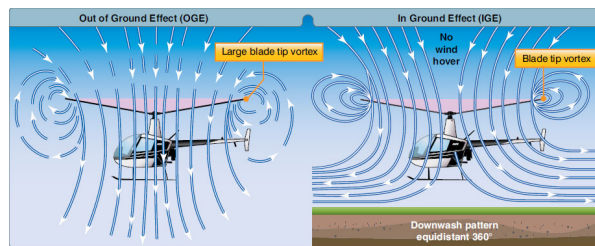
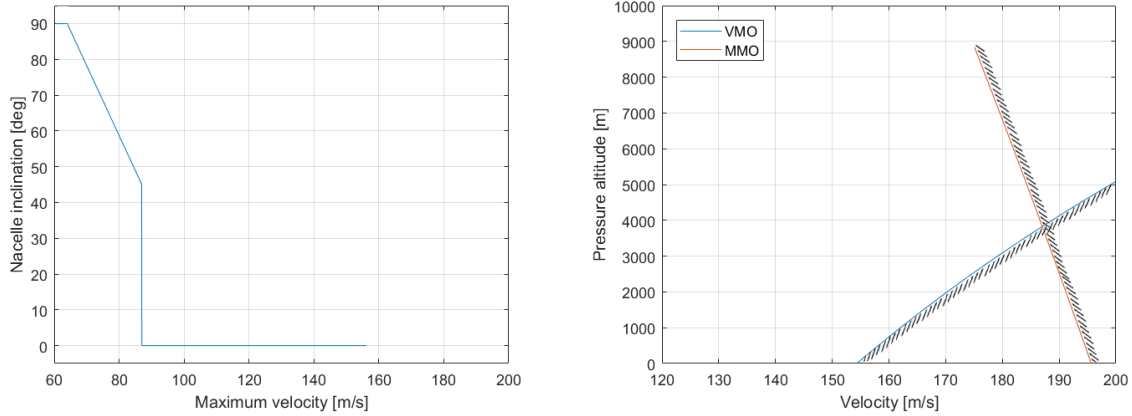


Figure 4.9: Visualization of airflow fields out of and in ground effect [28]

4.3. Constraints

Now that the model's dynamics and forces have been established, some constraints have to be imposed on the model so that it will be able to simulate realistic flight behaviour. Constraints will be imposed in two distinct manners. First of all, the states and control will be bounded by their respective minimum and maximum value. Secondly, path constraints will be imposed on the model to assure the tilt-rotor does not exceed its velocity and power limits. Finally, boundary constraints will be proposed for take-off and landing flight, that have been deduced from an operational standpoint.



(a) XV-15 limit design speed with respect to rotor inclination on a standard day at sea-level

(b) Maximum velocity due to design diving speed and maximum Mach operating number

Figure 4.10: Constraints on the maximum velocity

4.3.1. Maximum Velocity

The maximum allowable velocity of the XV-15 is limited in a multitude of manners. First of all as function of the nacelle inclination, since the rotor tip Mach number cannot exceed the speed of sound (including a margin). Because of this a speed limit is impinged on the XV-15, consistent with XV-15 documentation [44]. This has been modelled by the following velocity values or Mach numbers as function of nacelle angle. A linear interpolation has been used to model the maximum velocity in between 45° and 90°. The maximum velocity is depicted in Figure 4.10a.

$$V_{max_{i_n}} = \begin{cases} 0.575 \cdot a \\ 87 \\ \left(\frac{64-87}{45}\right)(i_n - 90) + 64 \\ 64 \end{cases} \quad \text{for} \quad \begin{cases} i_n = 0^\circ \\ 0^\circ < i_n \leq 45^\circ \\ 45^\circ < i_n < 90^\circ \\ i_n \geq 90^\circ \end{cases} \quad (4.65)$$

Apart from the maximum velocity, the tilt-rotor also has a minimum velocity bound with respect to the nacelle inclination that is imposed due to the wing stall. Although this lower limit of the velocity has not been imposed as hard constraint, the tilt-rotor automatically obeys to this limit since some combinations of inclined thrust and lift simply cannot keep the aircraft in the air.

The XV-15's speed is not only limited to the inclination of the rotor blades, but also as a function of its structural and vibrational limits. These are translated in the design diving speed and the maximum Mach operating number. This implies that the equivalent airspeed (EAS) cannot exceed 300 kts, or 154.33 m/s. Adding to this, the Mach number of the aircraft cannot exceed 0.575 times the speed of sound [44]. The limits of the latter two can be seen in Figure 4.10b. As can be seen in the graph it is expected when optimizing cruise for minimum time, that the tilt-rotor will cruise at approximately 4.000 m as the aircraft will achieve its highest velocity at that respective altitude. These three velocity constraints are implemented in the model by applying the following path constraint at all times:

$$V \leq V_{max_{i_n}} \quad (4.66)$$

$$M \leq M_{MO} \quad (4.67)$$

$$V \leq V_{MO} \quad (4.68)$$

4.3.2. Power Available

The power available and power required as elaborated upon in Section 4.2.8 are variable with respect to the atmospheric conditions, thrust setting and flight condition. It is clear that the power required to maintain or change the tilt-rotors flight condition can never exceed the power available that the engine

can deliver at that time. Therefore, at all time during the flights the following path constraint applies to the model:

$$P_r \leq P_a \quad (4.69)$$

4.3.3. Generic Limits

The XV-15 aircraft is bound to its own limitations. Various optimizations will be conducted in the following chapters and for a specific optimization study, specific constraints have to be imposed to account for the specific conditions. Most constraints, however, will apply for most problem formulations. The constraints for states and controls listed below apply for generic flight, and will therefore be called *generic constraints* from here onward. These generic constraints apply unless stated otherwise.

State Limits

The lateral and longitudinal position of the tilt-rotor is free, unlike the altitude that is obviously bounded by the ground, which is assumed to be at sea-level, and the service ceiling of 29,000 ft (8,840 m). A minimum velocity of 1 m/s has been addressed to avoid singularity in the equations of motion (Eq. 4.23-4.33). The same thing goes for the flightpath angle γ , that is not allowed to take on a value of 90° to avoid singularity. The heading angle of the tilt-rotor is allowed to vary over the entire compass rose, but instead of limiting it to $0^\circ \leq \chi \leq 360^\circ$, the heading angle is set to vary between $\pm 720^\circ$ in order to allow the model to cross over from 0° to 360° and vice versa. The data for the lift coefficient has been taken from the XV-15 tabular data [30]. The upper and lower bound for C_T has been determined by Carlson and Zhao [9]. The lower bound also preventing from singularity while calculating the power required. The limit for μ have been estimated with common sense. All other values for the upper and lower limits of the states have been determined from XV-15 specifications [44].

$$\begin{array}{ll}
 x_{min} \leq x \leq x_{max} & x = \text{free} \\
 y_{min} \leq y \leq y_{max} & y = \text{free} \\
 h_{min} \leq h \leq h_{max} & 0 \leq h \leq 8.840 \text{ m} \\
 V_{min} \leq V \leq V_{max} & 1 \leq V \leq V_{max}^{\rightarrow} \\
 \gamma_{min} \leq \gamma \leq \gamma_{max} & -89.9^\circ \leq \gamma \leq 89.9^\circ \\
 \chi_{min} \leq \chi \leq \chi_{max} & -720^\circ \leq \chi \leq 720^\circ \\
 \mu_{min} \leq \mu \leq \mu_{max} & -60^\circ \leq \mu \leq 60^\circ \\
 i_{n,min} \leq i_n \leq i_{n,max} & 0^\circ \leq i_n \leq 95^\circ \\
 C_{L,min} \leq C_L \leq C_{L,max} & -1.15 \leq C_L \leq 1.99 \\
 C_{T,min} \leq C_T \leq C_{T,max} & 0.0001\sigma \leq C_T \leq 0.17\sigma \\
 m_{fuel,min} \leq m_{fuel} \leq m_{fuel,max} & 0 \leq m_{fuel} \leq 675 \text{ kg}
 \end{array} \quad (4.70)$$

Control Limits

The maximum rate of change of the nacelle inclination and blade flapping are from XV-15 specification [44]. The minimum and maximum rate of change for the lift coefficient has been assumed using thin airfoil theory, where it is assumed that $C_{L\alpha}$ has a value of $2\pi\alpha$ [2]. Furthermore, it has been assumed that a fictitious angle of attack change α of 5° per second can be achieved. A guesstimate has been made for the rate of change of the thrust coefficient through the time it takes for the engine to spool up completely to full thrust. Again, the values for the limit for μ_{rate} have been guesstimated with an engineering sense. The generic constraints on the control variables are as follows:

$$\begin{array}{ll}
 \mu_{rate,min} \leq \mu_{rate} \leq \mu_{rate,max} & -10^\circ \leq \mu_{rate} \leq 10^\circ/\text{sec} \\
 i_{n,rate,min} \leq i_{n,rate} \leq i_{n,rate,max} & -7.5^\circ \leq i_{n,rate} \leq 7.5^\circ/\text{sec} \\
 C_{L,rate,min} \leq C_{L,rate} \leq C_{L,rate,max} & -2\pi \cdot (5^\circ) \leq C_{L,rate} \leq 2\pi \cdot (5^\circ) \\
 C_{T,rate,min} \leq C_{T,rate} \leq C_{T,rate,max} & -0.001 \leq C_{T,rate} \leq 0.001 \\
 \beta_{long,min} \leq \beta_{long} \leq \beta_{long,max} & -12^\circ \leq \beta_{long} \leq 12^\circ \\
 \beta_{lat,min} \leq \beta_{lat} \leq \beta_{lat,max} & -12^\circ \leq \beta_{lat} \leq 12^\circ
 \end{array} \quad (4.71)$$

Time and Linkage Constraints

Finally, a rather obvious constraint is imposed on the time in the optimization that it should be positive, starting at zero. No upper limit for the time has been set, unless stated otherwise.

$$t \geq 0 \quad (4.72)$$

In case the optimization is multi-phased, a constraint is applied to link the phases. Since there can be no instantaneous changes in any of the states, the states at t_0 of the succeeding phase have to have equal value to the states at t_f of the preceding phase to have a seamless transition from phase to phase. The controls of the phase do not have to equal the controls of the either preceding or succeeding phase.

$$\vec{x}(t_f)^p = \vec{x}(t_0)^{p+1} \quad (4.73)$$

4.3.4. Take-Off and Landing Boundary Constraints

In Appendix A, current take-off and landing procedures have been elaborated. It will now be discussed how these procedures will be modelled. The constraints of the take-off and landing are summarized in Table 4.3, where Imperial units have been converted to SI units and rounded.

Helicopter Take-Off

As can be read in Section A.1.1, a vertical take-off is performed by a linear backup until the TDP, which is then followed by the climb-out. In rigid-body modelling this would have been possible to model, but due to the nature of the equations of motion (Equations 4.23-4.33, it is not possible to have or cross the flightpath angle γ of 90° . Therefore, it was decided to have $|\gamma| \leq 89.9^\circ$ to avoid singularity (Equation 4.70). Therefore, the helicopter take-off procedure will be omitted and the optimization will start at the TDP with an initial altitude of 120 ft and airspeed of 300 fpm, in arbitrary direction of γ and χ . The initial value for i_n is 90° .

Helicopter Landing

The LDP in Figure A.4 will be taken as final point in optimizations that end in helicopter landing. From the approach guide points of the Eurocopter, it can be deduced that the LDP is approached with and approach angle of roughly 6° . Literature showed that suitable approach angles for the XV-15 lie within 3° and 12° [26]. Considering the other factors contributing to the landing, the aircraft has to be decelerated to 20 kts with a rate of descent smaller than 300 fpm at an altitude of 100 ft at the final time of the phase. Again, with a nacelle inclination of i_n is 90° .

Airplane Take-Off

The point at which the aircraft rotates will be taken as initial point for the runway take-off. The aircraft rotates at an altitude of 0 m and 40 kts velocity. It is rotated to an angle of $\gamma = 8^\circ$, which comes across to most airliners [65]. To provide ground clearance for the rotor tips, the nacelle inclination cannot be lower than 60° [10], [13].

Airplane Landing

The runway landing is chosen to be modelled as follows. Referring back to Section A.1.4, the endpoint of the optimization is set to be at an altitude of 100 ft with an airspeed of 40 kts. The rate of descent should not exceed 500 fpm [16], while the flight path angle should remain between -3° and -9° . The flight path constraint has been set to remain consist with other literature, but it can be noted that the combination between the airspeed and maximum rate of descent yields a maximum glideslope of $\sim 7.3^\circ$. Finally, as for the runway take-off, the nacelle inclination should be higher than 60° to provide rotor tip clearance.

Table 4.3: Initial or final parameters for take-off and landing modelling

Flight mode	Constraints
Helicopter take-off	$h_0 = 36 \text{ m}$ $V_0 = 2 \text{ m/s}$ $i_{n,0} = 90^\circ$
Helicopter landing	$h_f = 30 \text{ m}$ $V_f = 10 \text{ m/s}$ $h_f \geq -2 \text{ m/s}$ $-12^\circ \leq \gamma_f \leq -3^\circ$ $i_{n,f} = 90^\circ$
Airplane take-off	$h_0 = 0 \text{ m}$ $V_0 = 20 \text{ m/s}$ $\gamma_0 = 8^\circ$ $i_{n,0} \geq 60^\circ$
Airplane landing	$h_f = 30 \text{ m}$ $V_f = 20 \text{ m/s}$ $h_f \geq -2 \text{ m/s}$ $-9^\circ \leq \gamma_f \leq -3^\circ$ $i_{n,f} \geq 60^\circ$

Non-Linear Optimal Control

Numerous optimization approaches and techniques exist that can solve the trajectory optimization problems that will be imposed on the model in validation and application. The problems in this study will be optimized using a gradient-based optimization method based on optimal control theory. The following chapter will give an brief introduction to optimal control and GPOPS and will elaborate the general optimal control problem formulation in GPOPS.

5.1. Optimal Control

The use of optimal control theory optimization methods is favourable, as trajectory optimization problems are generally not described by discrete variables. The main benefit of optimal control is clear termination criteria and the use of gradient information to end up with the search direction. Generally, optimal control theory has low computational time. Optimal control theory furthermore allows for the implementation of bound, event and path constraints on the problem. “Optimal control theory aims to find the controls that perturb a system from a fixed initial condition to a free or fixed final condition, whilst minimizing the total value of a cost functional which is itself a function of the system controls and states” [32]. More specifically, the tilt-rotor flight trajectories will be optimized using a direct method with pseudospectral collocation in GPOPS (General Pseudospectral Optimal Control Software). The optimal control problem is defined by the cost functional, the differential algebraic equations, the boundaries and the initial guess. Other input data follows from the model. This process is depicted in Figure 5.1. GPOPS will iterate until the solution to the problem has crossed both its feasibility and optimality threshold.

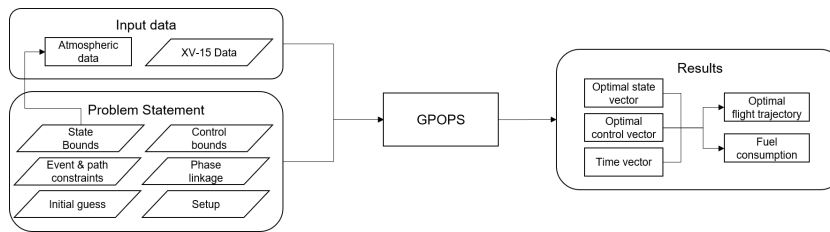


Figure 5.1: Schematic depiction of Optimization in GPOPS

The state and control vectors in pseudospectral methods “are approximated using global polynomials and collocation of the differential-algebraic equations is performed at orthogonal collocation points” [53]. These pseudospectral methods converge spectrally, which in essence means that the solution converges faster than any power of N^{-m} , where N is the number of collocation points and m can have any value.

The Legendre-Gauss (LG), Legendre-Gauss-Lobatto (LGL), and Legendre-Gauss-Radau (LGR) points are the most commonly used set of orthogonal collocation points in pseudospectral methods, which are obtained from the roots of a Legendre polynomial or linear combination of Legendre polynomials and its derivatives. These sets of points are defined on the domain $[-1, 1]$ but are dissimilar

in the fact that the LG points do not possess either of the endpoints, LGR points include one of the endpoints and LGL include both of the endpoints. Furthermore, the LGR points are asymmetric. These three collocation methods have lead to the following mathematical methods: the Legendre pseudospectral method (LPM), the Radau pseudospectral method (RPM), and the Gauss pseudospectral method (GPM) [53].

5.1.1. GPOPS

The optimization software used in this research is GPOPS. This is a MATLAB-based hp-adaptive pseudospectral optimization software. The GPOPS software employs the Radau pseudospectral method to solve the non-linear equations of motion with boundary conditions, path constraints, event constraints, linkages and cost functions. GPOPS is used in together with the IntLab automatic differentiation and the Non-Linear Programming (NLP) solver Sparse Nonlinear Optimizer (SNOPT). The software discretizes optimal control problems using spectral collocation method. These collocation points are defined at the roots of Legendre-GaussRadau functions. Furthermore, the software automatically refines the mesh by dividing the time segments of the states and controls to arrive at a denser solution without using higher-order polynomials. This discrete problem is then solved by an NLP problem solver, in this case SNOPT. This method is ideal for generating rapid solution due to simultaneously solving the entire trajectory, based on an initial guess and small number of nodes. For most problems the amount of nodes was set to amount to 80 with a feasibility and optimality threshold of $\epsilon = 1 \cdot 10^{-6}$.

The optimizations were conducted on a personal computer with a 2.40 GHz dual core CPU. The code was written in MATLAB environment. Solving times of the optimizations varied. Straightforward constrained, simple flights were solved in 30-90 seconds, while the more complex, multi-phased, lesser constrained problems ran up to 30 minutes.

5.1.2. General Problem Formulation in GPOPS

GPOPS is set up as follows: a P -phase optimal control problem can be stated in its general form. The states, controls and time are to be determined for an optimum of the cost functional J , which is a function of the state and control functions $\vec{x}^{(p)} \in \mathbb{R}^{n_p}$ and $\vec{u}^{(p)} \in \mathbb{R}^{m_p}$. The cost functional, or performance index, holds both the Mayer term $\vec{\Phi}^{(p)}$ as the Lagrange term $\vec{L}^{(p)}$ [54], [52]. This dynamic system is subjected to its dynamic constraints, boundary constraints and path constraints.

$$J = \sum_{p=1}^P J^{(p)} = \sum_{p=1}^P \left[\vec{\Phi}^{(p)}(\vec{x}^{(p)}(t_0), t_0, \vec{x}^{(p)}(t_f), t_f; \vec{q}^{(p)}) + \int_{t_0}^{t_f} \vec{L}^{(p)}(\vec{x}^{(p)}(t), \vec{u}^{(p)}(t), t; \vec{q}^{(p)}) dt \right] \quad (5.1)$$

This cost function is subjected to the dynamic constraints $\vec{f}^{(p)}$

$$\dot{\vec{x}}^{(p)} = \vec{f}^{(p)}(\vec{x}^{(p)}, \vec{u}^{(p)}, t; \vec{q}^{(p)}), \quad (p = 1, \dots, P) \quad (5.2)$$

and the boundary conditions, or event constraints $\phi^{(p)}$

$$\phi_{min} \leq \phi^{(p)}(\vec{x}^{(p)}(t_0), t_0^{(p)}, \vec{x}^{(p)}(t_f), t_f^{(p)}; \vec{q}^{(p)}) \leq \phi_{max}, \quad (p = 1, \dots, P) \quad (5.3)$$

the inequality path constraints $\vec{C}^{(p)}$

$$\vec{C}^{(p)}(\vec{x}^{(p)}(t), \vec{u}^{(p)}(t), t; \vec{q}^{(p)}) \leq \vec{0}, \quad (p = 1, \dots, P) \quad (5.4)$$

and the phase continuity constraints, or linkages, $\vec{P}^{(s)}$

$$\vec{P}^{(s)}(\vec{x}^{(p_l^s)}(t_f), t_f^{(p_l^s)}; \vec{q}^{(p_l^s)}, \vec{x}^{(p_u^s)}(t_0), t_0^{(p_u^s)}; \vec{q}^{(p_u^s)}) = 0, \quad (p_l, p_u \in [1, \dots, P], s = 1, \dots, L) \quad (5.5)$$

where $\vec{q}^{(p)} \in \mathbb{R}^{q_p}$ and $t \in \mathbb{R}$ are the static parameters and time in phase $p \in [1, \dots, P]$. L is the number of to be linked phases, $p_l^s \in [1, \dots, P]$, ($s = 1, \dots, L$) are lower or left phase numbers, while $p_u^s \in [1, \dots, P]$, ($s = 1, \dots, L$) are the upper or right phase numbers [54], [52]. Although most optimizations will require sequential phases, GPOPS allows for not-sequential phases, on the requirement that the independent variable does not change direction.

GPOPS applies an *hp*-adaptive version of the *Legendre-Gauss-Radau (LGR) orthogonal collocation method*. This method is a Gaussian quadrature implicit integrator method where collocation is performed at the LGR points [52].

All states and controls are evaluated at a discrete number of points in time, defining the number of nodes N . At each of these nodes $\tau_1, \tau_2, \dots, \tau_N$ the states and controls are parametrized. The nodes are unequally distributed at the LGR points at the roots of the Legendre polynomial. The number of nodes is user-defined by the number of intervals and number of nodes per interval. A combination of intervals and nodes per interval that produces a number of nodes in the order of 80 to 100 is said to yield the best trade-off between accuracy and computational efficiency.

Model Verification and Validation

In order to acknowledge and accept the output of the model with confidence, the model's separate modules and general output of flight behaviour is to be validated. First, the parameters for optimal aerodynamic performance are calculated, to be followed by analytically and dynamically validating the proprotor with respect to its hover performance, ceiling, endurance and fuel consumption. After this, the model is validated if the analytical, optimal aerodynamic parameters are being achieved during airplane climb, cruise and descent. In the next step distinct flight procedures are isolated to assess these for their flight behaviour and results, such as take-off and landing in both helicopter and airplane mode, conversion and turning performance. Furthermore, the cruise and climb-out are further assessed by varying conditions to evaluate different flying behaviour. Finally, the model is validated against an runway length optimization problem.

6.1. Optimal Aerodynamic Performance

A first step in the validation of the model is to analytically determine parameters that are expected in the optimization study. It will be useful to compare the analytical and optimized results to explain model behaviour. An effective metric that surfaces in many optimizations, especially when optimized for minimum fuel consumption, is the ratio between lift and drag as this reflects the aerodynamic efficiency of the aircraft. The higher the ratio the less thrust is needed, to put it quite bluntly, as less thrust is needed to provide the same amount of lift. It will be analytically be proven below that the following metrics are desired optimal behaviour [56], [2]:

- When flying at minimum airspeed it is necessary to fly at $C_{L,max}$.
- When the flying goal is to maximize the range or to have a maximum glide distance, it is desirable to have the aircraft fly at $(L/D)_{max}$.
- When it is desired to maximize the endurance of the aircraft or to maximize the rate of climb, the metric $(L^3/D^2)_{max}$ is to be flown at.

The lift-to-drag of the tilt-rotor point-mass is calculated by the Equation 6.1. Through analytical derivation, the values of optimal lift-to-drag can be coupled to their respective optimal airspeeds through Equation 6.2. Since the air density ends up to be the only variable in this equation, this results in the expected optimal airspeeds, as in Figure 6.1b.

$$\frac{L}{D} = \frac{1/2\rho V^2 SC_L}{1/2\rho V^2 (SC_D + 0.1449)} \quad (6.1)$$

$$\left(\frac{L}{D}\right)_{max} \quad \text{for} \quad \left(\frac{SC_L}{SC_D + 0.1449}\right)_{max}$$

$$V_{opt} = \sqrt{\frac{W}{S} \frac{2}{\rho} \frac{1}{C_{L,opt}}} \quad (6.2)$$

As can be seen in Figure 4.3b in Section 4.2.7, the induced drag equation accurately captures the drag polar for 0° and 20° flap deflection between $-20^\circ < \alpha < 20^\circ$, but does not capture the curve beyond those points, which makes sense as fixed wing aircraft only rarely operate outside of these limits. For the tilt-rotor however, this is not the case and therefore the drag needs to be modelled differently. It is decided to do this by interpolating the outer contour of the drag polar as the optimizer will always choose the lowest drag coefficient for a given lift coefficient value and thus with optimal L/D for a given C_L . Only when speed has to be decreased deliberately for instance, this is not the case.

6.1.1. Minimum Airspeed

The maximum lift coefficient value equals 1.99. Using this in Eq. 6.2, the stall speed for the flight envelope can be determined. It is shown in Figure 6.1b that the XV-15 has a sea-level stall speed of 47 m/s, which increases with altitude as the air density decreases. The sea-level stall speed agrees with the stall speed of ~ 50 m/s [44].

6.1.2. Optimal Cruise

In order to avoid mixing up the optimality parameters for jet and propeller aircraft, a short derivation is given [2], [56]. In cruise flight that is optimized for fuel efficiency, it is desired to fly a given distance with the minimum amount of fuel. Therefore, the specific range $\frac{V}{\dot{m}_{fuel}}$ is to be maximised, or in other words, to maximise the distance per Newton of fuel. The fuel consumption is given by:

$$\dot{m}_{fuel} = sfc \cdot \frac{P_a}{\eta_p}$$

In steady horizontal flight $P_a = P_r$, and assuming that $P_r \approx DV$

$$\dot{m}_{fuel} \approx sfc \cdot \frac{DV}{\eta_p}$$

from this follows that the specific range is given by

$$\frac{V}{\dot{m}_{fuel}} \approx \frac{\eta_p}{sfc \cdot D}$$

since the propulsive efficiency η_p and the specific fuel consumption sfc can be assumed to be constant it can be concluded that to maximize the specific range, the drag is to be minimized. In steady horizontal flight this comes across to maximizing L/D .

When considering the optimal $(L/D)_{max}$, it is therefore expected that the tilt-rotor will fly at a lift coefficient value of 0.92 during cruise with a respective drag coefficient value of 0.0411, as this results in $(L/D)_{max}$. The resulting lift-to-drag-ratio equals 22.375. This value is comparable to the lift-over-drag of high-performance, intercontinental aircraft such as the Boeing 777 and Airbus A340 [46]. It is expected that the L/D of the XV-15 should be lower than this, since the aircraft is not as optimized for aerodynamics as the compared aircraft and has a large amount of additional drag due to the large rotors that have not been modelled. But considering the fact that the aerodynamic forces of the point-mass have only been modelled by the lift and drag of the wing-pylon, horizontal stabilizer and fuselage the obtained L/D is quite accurate. The optimal lift-to-drag ratio results in an optimal airspeed for this parameter of 70 m/s at sea-level, which increases with altitude up to 120 m/s at 10 km altitude, as shown in Figure 6.1b.

6.1.3. Optimal Climbing Flight

When optimizing climbing flight for maximum rate of climb, it is desired to maximise the excess power, that can be used to climb. In other words the rate of climb is maximised for a maximum $\frac{P_a - P_r}{W}$.

$$\frac{P_a - P_r}{W} = V \sin \gamma$$

Under the assumption that the power available is constant with airspeed, the power required is to be minimised for a maximum rate of climb. Again under the simplifying assumption that $P_r \approx DV$, this corresponds to the following for propeller aircraft:

$$P_r \approx DV = D \sqrt{\frac{W}{S} \frac{2}{\rho} \frac{1}{C_L}} = \frac{C_D}{C_L} W \sqrt{\frac{W}{S} \frac{2}{\rho} \frac{1}{C_L}} = \sqrt{\frac{W^3}{S} \frac{2}{\rho} \frac{C_D^2}{C_L^3}}$$

Therefore, P_r is minimized by maximizing the ratio L^3/D^2 . Considering Figure 6.1a, the optimal value for L^3/D^2 occurs at a lift coefficient value of 0.996, slightly higher than optimal L/D . The optimal airspeed that results from this is slightly lower than the airspeed for optimal L/D with an offset of approximately -3 m/s.

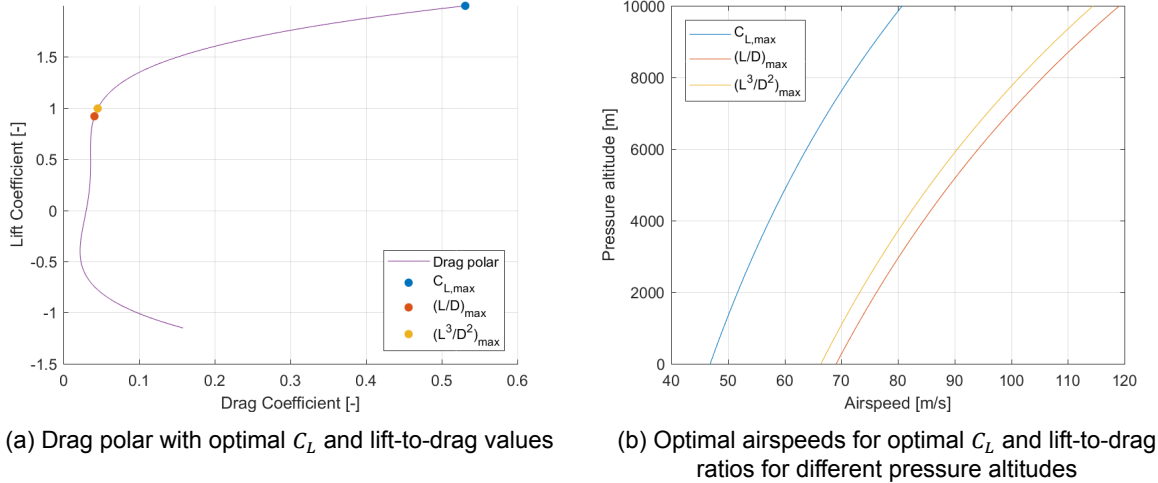


Figure 6.1: Analytical determination of optimal lift-to-drag ratios and corresponding airspeeds

6.1.4. Optimal Glide Angle

During descent it is desired to use as little power as possible to fly as far as possible. In other words it is wanted to get the most horizontal distance per kg of fuel. For an optimal descent, it can be shown that an optimal angle exists, that is to be adhered to in order to achieve this. The same thing holds in case of a total loss of thrust: it can be desired to either glide as far as possible in order to reach the airport, or to glide as long as possible to be able to try to fix the failure and restart the engine. The latter will not further be discussed here.

Hence, to glide as far as possible, it is desired to minimize the glidepath angle γ . It can be derived analytically that the optimal glide angle is given by the following relation. From simplified equations of motion it can be deduced that [56]:

$$\frac{-DV}{WV} = \sin \gamma$$

Therefore it can be concluded that to minimize the flightpath angle γ , the drag is to be minimized, which corresponds to $(L/D)_{max}$. The corresponding speed can be read from the curve in Figure 6.1b. In the case of the point-mass the optimal glidepath angle yields to be:

$$\gamma_{glide} = \sin^{-1} \left(\frac{-C_D}{C_L} \right) = -2.56^\circ \quad (6.3)$$

6.2. Proprotor Power and Fuel

The validation of the engine is divided in four parts. First, the model's hover performance can be validated analytically with another model that was validated against test data. Secondly, the hover ceiling of the point-mass can be determined both analytically and dynamically and validated with XV-15 documentation. The third point is to ballpark the fuel consumption with comparable engine data. Finally, the fuel and engine are validated collectively by validating the model's hover performance.

6.2.1. Hover Performance

Johnson developed a model to test conceptual designs to satisfy specific design conditions of rotorcraft: NASA Design and Analysis of Rotorcraft tool (NDARC). The comprehensive analysis used for the proprotor is CAMRADII. This model has been based upon the models developed using wind tunnel measurements of the JVX rotor performance, a rotor comparable to the XV-15 proprotor [37].

Figure 6.2 shows the isolated rotor performance in hover comparison of XV-15 flight tests, the CAMRADII hover performance analysis and the point-mass model. It can be seen that the model comes across to the test data and that only marginal differences exist in the mid-power section. As this is hover performance test data, the velocity components u and w are set to zero.

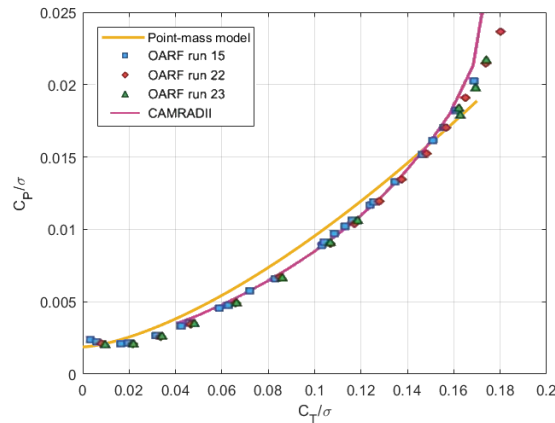


Figure 6.2: Comparison of point-mass model with XV-15 rotor hover performance (OARFs) and CAMRADII calculations

6.2.2. Hover Ceiling

Due to the fact that the air becomes less dense with altitude, the thrust the proprotors deliver decreases as well. Therefore, the power required increases as well up to the point that power required equals power available. At that particular point the tilt-rotor cannot climb any higher, which forms the hover ceiling. Key factors in the hover ceiling are the altitude, thrust and power available and required.

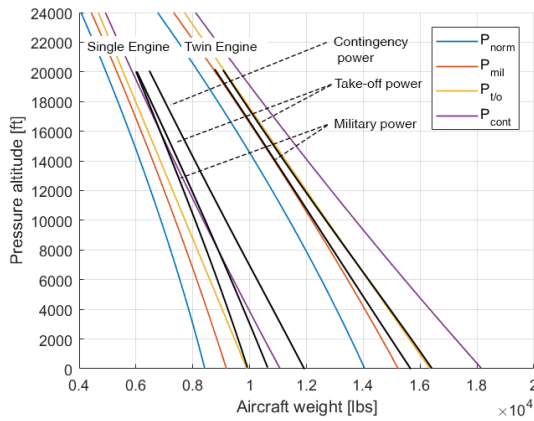
The graph in Figure 6.3a depicts a comparison between the point-mass model and XV-15 data of the amount of thrust the rotors can produce over the given range of pressure altitudes in terms of aircraft gross weight ($W = 1.0 \cdot T$) for the four engine settings. These four settings being: normal rated, military rated, take-off rated and contingency power, for single and for twin engine operation. As can be seen, the power curves of the point-mass model come across to the data specified by NASA for twin-engine operations. The point-mass model however, consistently underestimates the actual power of the XV-15 when the aircraft is in single-engine operations by 10-20%. It was expected that the thrust in single-engine operations equals exactly the half of twin-engine operation. This is due to the fact that the proprotors can be operated at a higher C_T when only one engine is used, while keeping the power required smaller than the power available. This can be seen in the derivation of the thrust and power coefficient (Eq. 4.62 and 4.60), which results in the maximum thrust coefficient to alter by a factor of $2^{2/3}$. This analytical factor however, does not match the actual difference. It could be that the maximum thrust coefficient value in OEI is higher than the previously anticipated maximum thrust coefficient value of 0.17σ .

Although the latter graph depicts the force the rotors can produce at the specified altitudes, this does not necessarily mean that the tilt-rotor is capable of hovering at that altitude. Especially, given the fact that the minimum operating weight of the XV-15 comes across to 10,000 lbs. As before, the power required to do so, cannot exceed the power available. In order to validate the available thrust, the model is used to determine the maximum altitude the tilt-rotor can reach under the constraints in Table 6.1, which entail the following: the aircraft has initial position and an initial velocity of 1 m/s. The initial and final flightpath angle and nacelle inclination are set vertically upward and furthermore the nacelle inclination is not allowed to vary. In order to avoid singularity the final velocity is again 1 m/s and in order to avoid the model from increasing the velocity and use this excess velocity to follow a

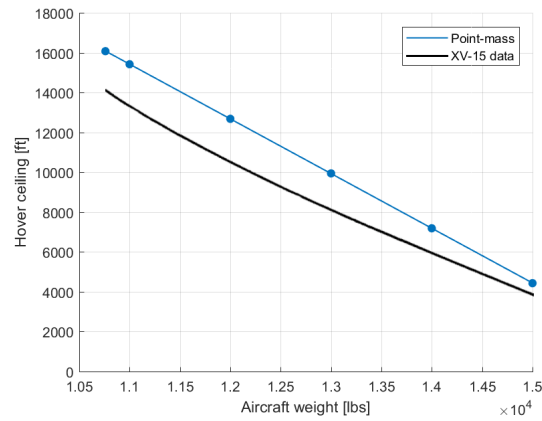
ballistic trajectory and acquire a higher speed, a maximum velocity of 2 m/s has been set to minimize the profit from this to a bare minimum.

Table 6.1: Boundary constraints for hover ceiling

Initial	Final
$x_0 = 0$ m	$x_f = \text{free}$
$y_0 = 0$ m	$y_f = \text{free}$
$h_0 = 0$ m	$h_f = \text{free}$
$V_0 = 1$ m/s	$V_f = 1$ m/s
$\gamma_{0,\dots,f} = 89.9^\circ$	
$i_{n,(0,\dots,f)} = 90^\circ$	
$m_{fuel,0} = 0$ kg	$m_{fuel,f} = \text{free}$



(a) Comparison of power available of XV-15 [44] and point-mass model



(b) Comparison of point-mass hover ceiling with XV-15 [44] specification

Figure 6.3: Rotor power with respect to pressure altitude (left) and dynamically determined hover ceiling (right)

Due to the simplicity of the procedure and the small freedom of this optimization the control penalty has been omitted, which results in the performance index to be to only maximize altitude. 4 intervals and 20 nodes per interval resulted in 80 nodes for this problem.

$$\min -h_f \quad (6.4)$$

The results of the optimization is depicted in Figure 6.3b. The values for the point-mass model have been acquired using test sampling at maximum gross weight, from which steps of 1.000 lbs have been taken up to the minimum flight weight. These points have been interpolated linearly. Although a similar trend is followed, the point-mass' hover ceiling differs from the XV-15 documentation's hover ceiling, with the difference of the point-mass model at some points being up to 20% higher, resulting in an error up to 600 m. The error for all samples varies between 600 and 300 m.

To check the dynamic model, the hover ceiling is calculated analytically by first calculating the required thrust coefficient to have the total thrust equal the weight. Hereafter, the power required can be determined using this previously acquired thrust coefficient. Finally, a check has to be conducted that the power required does not exceed the power available. In this process some parameters have been set: $\gamma = 89.9^\circ$, $i_n = 90^\circ$ and $DL = 0.07$. The latter due to this being stated in NASA documentation, that assumes out-of-ground effect hover with 7% download [44]. All steps in this process are shown in the derivation in Equations 4.46 to 4.61. The hover ceiling occurs when the power required intersects the power available. From this point on, the tilt-rotor requires more power to be available to climb higher, which subsequently forms the ceiling. Figure 6.4 shows the analytically determined power required, that results in the hover ceiling, with the XV-15 specification data shown in black. A sensitivity analysis is performed on the download in Figure 6.4a and on the propulsive efficiency in Figure 6.4b, which will be elaborated upon below.

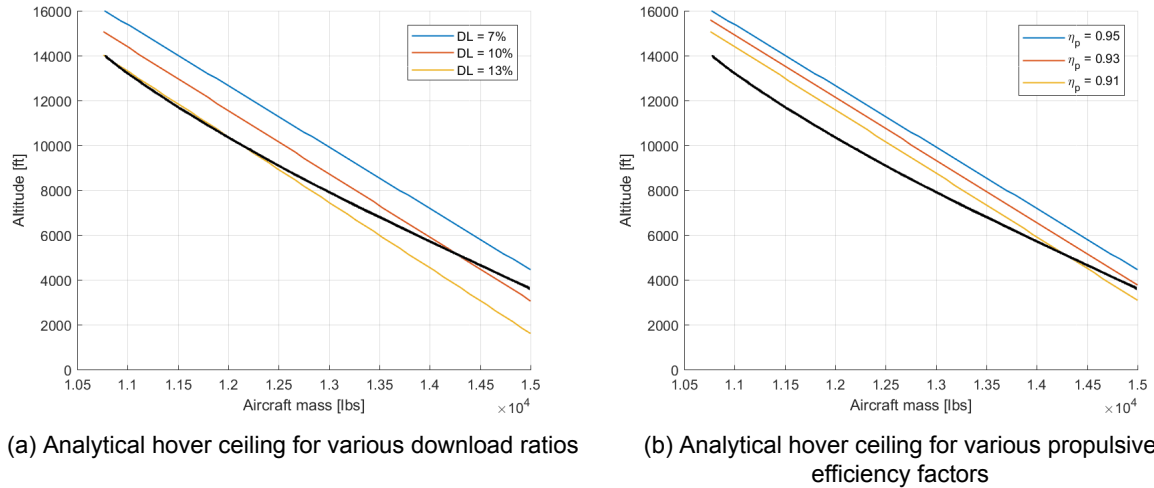


Figure 6.4: Analytically determined hover ceiling for point-mass model at take-off rated power for various aircraft masses [lbs], compared to XV-15 specification data (black). The baseline model is in blue.

As shown previously in Figure 6.3a, the maximum thrust that can be produced at each altitude matches the engine data from the documentation. However, when the power required and power available are taken into account the hover ceiling starts to deviate from the specifications. No anomalies have been recorded in the optimization for the hover ceiling, with all parameters remaining constant except for speed and altitude. There can be multiple reasons for or a combination of these for the difference:

- First of all, the XV-15 documentation assumes a constant download of 7%. Consulting Figure 4.8, Diaz and Jordan estimate a download value of 7% to already be produced at 30° nacelle angle [21]. In hover, at 90° nacelle, they estimate the download value to be almost twice as high. Figure 6.4a shows the sensitivity analysis of the assumed download ratio, where the baseline of the model is in blue. A higher download results in more power required to hover at the same altitude. This can clearly be seen in the graph. It can be deduced that an increase of download of 1% results in a hover ceiling loss of 100-140 m. This effect is larger at lower altitude. It can be concluded that an increase in download ratio in the order of 10-13% yields a much better fit to the hover ceiling as proposed in the XV-15 documentation. This download values agree to the expected download values by Diaz.
- The same thing goes for the propulsive efficiency. In literature it was stated that to determine the power required a factor of 0.95 is appropriate (Section 4.2.8). This could as well be a parameter that impacts the error in the hover ceiling. Obviously, the hover ceiling decreases when the propulsive efficiency decreases. Figure 6.4b shows the hover ceilings for distinct propulsive efficiencies, where the baseline of the model is in blue. Just as for the download, a lower propulsive efficiency approaches the XV-15 specification data. It can be deduced that a decrease of 0.01 in η_p results in a hover ceiling loss of 80-100 m. The hover ceiling however, is less sensitive to the propulsive efficiency of the prop rotor than it is to the rotor wash.
- The tilt-rotor being in hover results in the normalized tip plane velocities to have very small values: $\bar{U}_c$ in the order of 0.1 and $\bar{U}_t$ in the order of 0.001. After consulting Figure 4.6 for the determination of the normalized induced velocity, it becomes evident that these normalized TPP velocities lie in a region where a small error will yield a large offset. As can be seen in the determination of the power coefficient (Eq. 4.60), the impact of $\bar{v}_i$ relatively high since $\bar{U}_c$ has a small value.
- Other factors that have not been taken into account comprise more advanced aerodynamics such as aerodynamic interaction between prop rotor and fuselage.
- Finally and most importantly, the documentation does not state how the hover ceiling has been determined (analytical, flight test, etc.). Moreover, is it feasible to say that the graph functions as

manual for pilots, instead of analysis of the aircraft, as for instance can be deduced from the x-axis being “Useful load”. Combining these two facts makes it realistic to say that the XV-15 hover ceiling in the documentation [44] makes up a more conservative ceiling than the actual operating extreme of the aircraft.

6.2.3. Fuel Consumption

As can be read in Section 4.2.9, the fuel flow of the XV-15 is given by the specific fuel consumption. The values from the specific fuel consumption from the XV-15 documentation can be checked against the emission model by the Swiss Federal Office of Civil Aviation (FOCA), which is based upon their own engine test data and confidential engine manufacturer data. The following polynomial describes the approximate fuel consumption with respect to the available horsepower of turboshaft engines above 1000 shp [55].

$$\begin{aligned} \dot{m}_{fuel} = & 4.0539 \cdot 10^{-18} \cdot shp^5 - 3.16298 \cdot 10^{-14} \cdot shp^4 + 9.2087 \cdot 10^{-11} \cdot shp^3 \\ & - 1.2156 \cdot 10^{-7} \cdot shp^2 + 1.1476 \cdot 10^{-4} \cdot shp + 0.01256 \quad \left[\frac{kg}{s} \right] \quad (6.5) \end{aligned}$$

Figure 6.5 compares the fuel consumption stated by the XV-15 documentation versus the fuel consumption modelled by FOCA. As can be seen the FOCA and point-mass model agree fairly well on average, and especially within the range of 500 and 1200 SHP. There is, however, a mismatch when the power required either undershoots or exceeds the latter section. It is however difficult to compare a the fuel flow data in the XV-15 documentation with a fuel flow model that incorporates all sorts of engines. It can be known in advance that the two models will not coincide exactly. This comparison however, acts to ballpark and verify the modelled fuel flow module of the point-mass to see that it globally agrees with engine data of comparable engines.

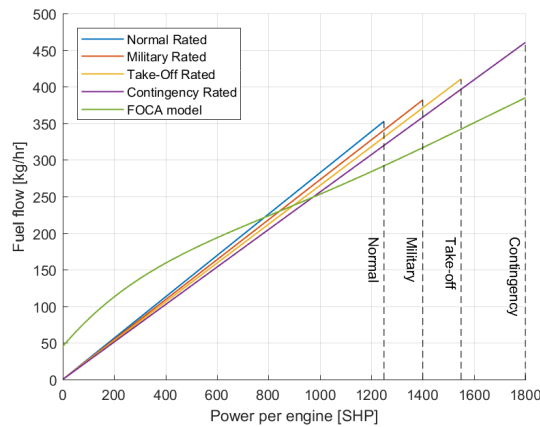


Figure 6.5: Comparison of point-mass model and FOCA fuel flow models

When the aircraft has converted from helicopter/conversion mode to airplane mode, the aircraft requires less power. To improve aircraft efficiency, the engine rpm is reduced which will reduce the power required which then results in a lower fuel consumption. Engine rpm is reduced from 565 rpm to 458 rpm. Looking at Equations 4.60 and 4.61 it is approximated that the power required is proportional to the engine rotational velocity squared. Since the fuel consumption is linear to the power required it can be stated that: $\dot{m}_{fuel} \propto \Omega^2$. Hence the power required and fuel consumption reduction by converting to airplane mode can be approximated by Equation 6.6, which implies that the aircraft is 35% more fuel efficient in airplane mode than in helicopter mode (flying in an equal situation). This has been determined under the assumption that the thrust coefficient remains equal and that $\bar{v}_i \ll \bar{U}_c$.

$$\left(\frac{\Omega_{aircraft}}{\Omega_{helicopter}} \right)^2 \approx 0.65 \quad (6.6)$$

6.2.4. Hover Endurance

To add both the hover and the fuel consumption to each other and validate this, the hover endurance will be validated. As already mentioned before it is impossible for the model to have zero velocity, due to the nature of the equations of motion. Hence a minimum velocity of $V_{min} = 1$ m/s has been set. In order to assess the fuel consumption in hover, only a small alteration will be made by setting the minimum velocity back to 0.1 m/s.

Since the hover endurance data of the XV-15 is given at sea-level, the tilt-rotor is set to fly from 0 to 10 m altitude, while maximising the final time, or in other words, do this as slow as possible to approach hover conditions. The tilt-rotor is constrained to fly in helicopter mode and adhere to a vertical flight trajectory. These constraints are summarized in Table 6.2. According to Cunha, it is sufficiently accurate to determine the endurance with the available fuel and the average fuel flow [19]. Hence, the results of the optimization's fuel consumption can be extrapolated according to its respective maximum fuel mass and flow.

Table 6.2: Boundary constraints of hover endurance helicopter flight

Initial	Final
$x_0 = 0$ m	$x_f = \text{free}$
$y_0 = 0$ m	$y_f = \text{free}$
$h_0 = 0$ m	$h_f = 10$ m
$V_0 = 0.1$ m/s	$V_f = 0.1$ m/s
	$88^\circ \leq \gamma_{0,\dots,f} < 90^\circ$
	$i_{n,(0,\dots,f)} = 90^\circ$
$m_{fuel,0} = 0$ kg	$m_{fuel,f} = \text{variable}$
	$m_{aircraft} = \text{variable}$

$$\min -t_f + \int_{t_0}^{t_f} \vec{u} dt \quad (6.7)$$

In the payload-endurance diagram (Fig. 6.6) two distinct gradients can be identified. The steep, rightmost gradient comes across to the endurance the tilt-rotor has when flying with maximum fuel. This leads to a reduction in endurance when more payload is on-board. Until the point of maximum weight has been reached, a trade-off can be made between payload and endurance. After the point of maximum weight, the conditions change to the moderate gradient in the graph. From this point on, the tilt-rotor always operates at maximum weight, and a trade-off between fuel and payload results in the hover endurance. This continues up to the point that maximum payload weight and zero fuel weight results in zero endurance.

The maximum allowable fuel or payload can be determined using either of the relations below between the fuel, payload and weights, in which the empty weight equals 10.073 lbs, the crew 400 lbs and the trapped fuel and oils 138 lbs. The maximum fuel weight is 1.490 lbs, or 675 kg. Imperial units are used for the weights to be able to compare the results directly to the specification data.

$$\begin{aligned} m_{fuel} &= m_{gross} - (m_{empty} + m_{crew} + m_{payload} + m_{trapped\ fluids}) \\ m_{payload} &= m_{gross} - (m_{empty} + m_{crew} + m_{fuel} + m_{trapped\ fluids}) \end{aligned} \quad (6.8)$$

The optimized flight trajectories are as they were expected to be and no further adjustments to the constraints are necessary. The trajectories consist of a vertical flightpath at γ_{max} and V_{min} . As can be seen in Figure 6.6, the results of the point-mass model come across to the hover endurance given in the aircraft specifications. It can be noticed that the point-mass model slightly undershoots the data of $T/W = 1.0$, that was approached. This however, was expected as the optimization is not able to hover in the air, but as this could only be approached by a 'climbing' flight to 10 m altitude at 0.1 m/s.

A second thing that can be noticed is that the model's gradient differs from the specification's gradient in the part of maximum fuel. It can be deduced that the ratio of payload over endurance is slightly lower for the point-mass. During analysis it was seen that the power required increases minimally as the aircraft 'climbs' from 0 to 10 meters altitude. This difference in power required becomes larger with smaller aircraft mass. This tiny difference however, is extrapolated over the course of more than

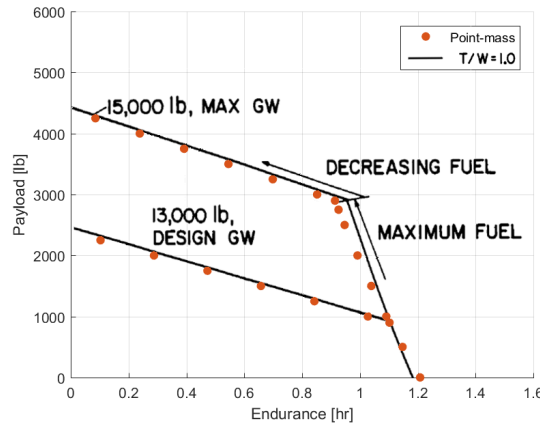


Figure 6.6: Comparison of point-mass and XV-15 hover endurance variation with payload

an hour, and thus cumulates. All in all however, it can be concluded that the combination of hover performance and fuel consumption in the hover endurance can be assumed to be validated.

6.3. Isolated Flight Procedures

Now that the model has been validated for its aerodynamic and propulsive properties, these can be applied in a broader scope. In the following section flight phases and procedures have been isolated in order to assess the results the point-mass model yields and to verify if expected results appear and if not, the unexpected flight behaviour can be explained. First, helicopter take-off and landing are isolated, to be followed by airplane take-off and landing. The process of conversion, cruise and turning are further assessed to be followed by an analysis of the effect that the cruise distance and initial velocity have on these respectively. Finally, the model is applied to optimize a continued runway take-off after an engine-failure in order to equate the point-mass model with Zhao's rigid-body model.

6.3.1. Minimum Airspeed

In Section 6.1 the minimum airspeed was calculated analytically, and to validate the model, the model is set to dynamically minimize the airspeed, while the aircraft is flying level and holding its altitude in airplane mode. Sample testing have been conducted in the range of 0 - 10 km altitude in steps of 1000 m. It is expected that the model will optimize to fly at $C_{L,max}$ and the corresponding minimum airspeed as determined before. The boundary constraints are set to be as in Table 6.3.

Table 6.3: Boundary constraints of minimum airspeed airplane flight

Initial	Final
$x_0 = 0$ m	$x_f = \text{free}$
$y_0 = 0$ m	$y_f = \text{free}$
$h_0 = \text{variable}$	$h_f = h_0$
$\gamma_{0,...,f} = 0^\circ$	
$i_{n,(0,...,f)} = 0^\circ$	
$m_{fuel,0} = 0$ kg	$m_{fuel,f} = \text{free}$

The problem is to optimize for minimum airspeed with 4 intervals and 20 nodes per interval resulting in 80 nodes for this problem.

$$\min v_f + \int_{t_0}^{t_f} \vec{u} dt \quad (6.9)$$

As can be seen in Figure 6.7, the dynamic model is able to match capture the analytically determined stall speed with almost exact overlap. Needless to say, the aircraft flies at $C_L = C_{L,max} = 1.99$. The aircraft's specification yields a stall speed at sea-level of slightly less than 100 knots (51 m/s) [44]. It

can be seen that the this is slightly higher than the model's stall speed by about 3 m/s. It was however, expected that the model's stall speeds would not match the aircraft's stall speed exactly as the lift and drag of the point-mass have only been modelled by the main wing, horizontal stabilizer and the fuselage. Other aircraft elements such as the vertical fins have not been taken into account but add to the global aerodynamic of the aircraft. Moreover, do the large rotors change the aerodynamic conditions of the wing behind the rotor wake. These factors add up and lead to a higher stall speed. Hence, it can be concluded that the model can be validated in this module.

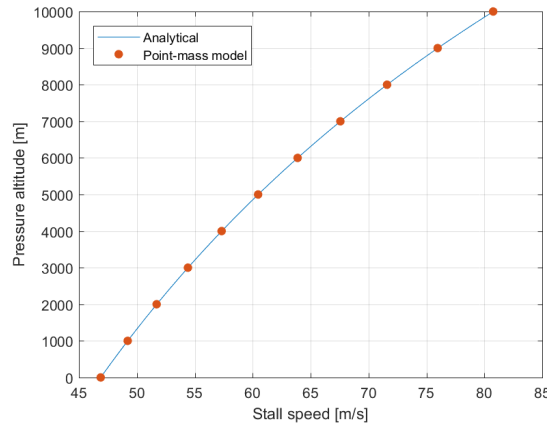


Figure 6.7: Comparison of point-mass model and analytically determined stall speed

6.3.2. Airplane Climb

In order to validate the model's performance, it can be validated against the previously analytically determined climb performance in Section 6.1. It was determined that the maximum rate of climb is achieved by flying at $(L^3/D^2)_{max}$. Maximum rate of climb is optimized for by minimizing the final time to reach the final altitude. The model is optimized for during an isolated climb to 8.000 m, with the nacelle inclination being restricted to airplane mode. The point-mass is free to choose optimal conditions for the remainder of the parameters. The problem is optimized with 8 intervals of 20 nodes per interval resulting in 160 nodes for this problem.

Table 6.4: Boundary constraints of airplane climbing flight

Initial	Final
$x_0 = 0 \text{ m}$	$x_f = \text{free}$
$y_0 = 0 \text{ m}$	$y_f = \text{free}$
$h_0 = 0 \text{ m}$	$h_f = 8 \text{ km}$
	$i_{n,(0,...,f)} = 0^\circ$
$m_{fuel,0} = 0 \text{ kg}$	$m_{fuel,f} = \text{free}$

$$\min t_f + \int_{t_0}^{t_f} \vec{u} \, dt \quad (6.10)$$

As can be seen in Figure 6.8a the model chooses maximum initial velocity, which is unsurprisingly as this will give the model a 'free' kickstart of approximately 2.000 m altitude. The endphase also yields unconventional behaviour, which however is explicable as the aircraft exchanges its velocity for altitude in order to gain the slightest time advantage.

The part of most interest however is the middle part, where the point-mass conducts its steady-state climb. It can be seen that tilt-rotor's velocity during the climb approaches the velocity of optimal $(L^3/D^2)_{max}$, but offsets it by about 5 m/s. This is reflected in the lift coefficient graph on the right, where the aircraft does not fly at its expected lift coefficient of 0.996, but slightly lower at 0.96. The entire climb is conducted at a climb angle between $2.5^\circ \leq \gamma \leq 15^\circ$.

The offset of the expected and actual value can be explained though. As mentioned during the determination of the analytical parameters, a few simplifications have been imposed. The power required for instance is dependent on a multitude of parameters that are interconnected with each other (Eq. 4.61). To come up with the concise lift-to-drag ratio, a simplification for the power required has been imposed that assumes $P_r = DV$. Analysis has shown that this assumption is valid when the tilt-rotor is in airplane mode, but is invalid for helicopter mode, due to the low velocity. In airplane mode, the modelled P_r can be up to 10% larger than DV , such as the assumption that $P_r = DV$.

In addition to this, the same problem was also optimized for minimum fuel consumption. Interestingly, the flight trajectories for both optimization targets agreed with each other. In the trade-off between climbing faster with high power and slower with less power, it turns out that these coincide. Optimizing for minimum time yielded a final fuel consumption and time of 55.00 kg and 403.10 seconds respectively, while flying for minimum fuel consumption results in a trajectory of 55.40 kg and 398.83 seconds.

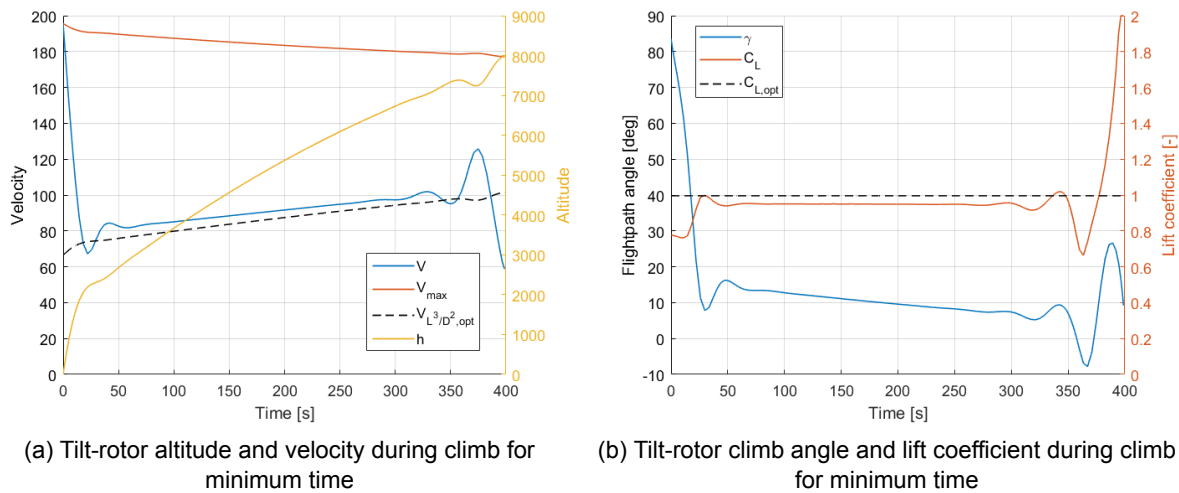


Figure 6.8: Simulation of airplane climbing flight for minimum time

In the previous optimization the tilt-rotor was set to climb in airplane mode, in order to assess if the optimal lift-to-drag ratio was matched. Since the tilt-rotor could benefit from the tilting the rotors during climb, it is inspected how the fuel consumption and time behaves with varying nacelle inclination. This is done by performing the same optimization at nacelle angles other than airplane mode. The results at a ten degrees interval and the optimum nacelle angle are summarized in Table 6.5. From 70° nacelle angle on, the tilt-rotor does not reach the pre-set 8 km altitude anymore.

Table 6.5: Development of fuel consumption and time at varying nacelle inclination for free climb to 8 km altitude

Nacelle angle	Fuel consumption	Time
60°	70.33 kg	491.95 s
50°	69.89 kg	488.80 s
45°	69.65 kg	486.62 s
40°	69.83 kg	487.96 s
30°	70.38 kg	492.26 s
20°	71.31 kg	499.59 s
10°	72.73 kg	510.86 s
0°	55.40 kg	398.83 s

It can be concluded that the tilt-rotor has a global optimum nacelle angle for climb at 0° nacelle inclination and a local optimum at 45° nacelle angle. When looking at $i_n > 45^\circ$, it is seen that fuel consumption is lower for lower i_n . Referring back to Eq. 4.60, it can be determined that a lower nacelle inclination reduces the advance ratio μ . Since this term is squared, it is beneficial to lower this term. At high speeds, the normalized induced velocity $\bar{v}_i$ is generally low as well. Only the normalized velocity

component normal to the TPP increases, but as the optimization has shown not proportionally to the decrease of the two former. When looking at $i_n < 45^\circ$, the fuel consumption increases again. Referring back to the maximum velocity the tilt-rotor is bounded by in Figure 4.10a, it can be seen that the tilt-rotor has the same velocity constraint between 45° and 0° nacelle angle. This means that when the nacelle is decreased beyond 45° , the maximum velocity is the same, but reached at an earlier stage. Since the aircraft still climbs further and air density decreases, more lift is to be generated by increasing the lift coefficient since velocity cannot be increased further. This results in an increase of drag and therefore more fuel. Although the difference might be small, this can explain model behaviour.

6.3.3. Airplane Cruise

In a similar manner the cruise phase can be validated. Cruise flight is often characterized by an only very slightly climbing flight path since the aircraft becomes lighter, as this allows the aircraft to fly in thinner air which is faster and more economical. This however, will not be the case for the point-mass, since aircraft mass is assumed to be constant. Fuel consumption is only a calculated metric. Hence it can be safely assumed that the cruise phase is defined by level flight with $\gamma_{0,\dots,f} = 0^\circ$. The only other constraints are listed below and consist of initial position, flying in airplane mode and the final position at 50 km.

Table 6.6: Boundary constraints of airplane cruise flight

Initial	Final
$x_0 = 0 \text{ m}$	$x_f = 50 \text{ km}$
$y_0 = 0 \text{ m}$	$y_f = \text{free}$
	$\gamma_{0,\dots,f} = 0^\circ$
	$i_{n,(0,\dots,f)} = 0^\circ$
$m_{fuel,0} = 0 \text{ kg}$	$m_{fuel,f} = \text{free}$

The flight will be optimized for both minimum fuel consumption and for minimum time to validate. 5 intervals and 20 nodes per interval resulted in 100 nodes for this problem.

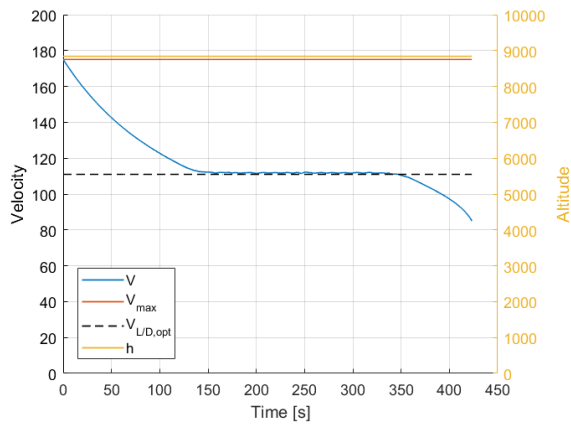
$$\min m_f + \int_{t_0}^{t_f} \vec{u} \, dt \quad (6.11)$$

$$\min t_f + \int_{t_0}^{t_f} \vec{u} \, dt \quad (6.12)$$

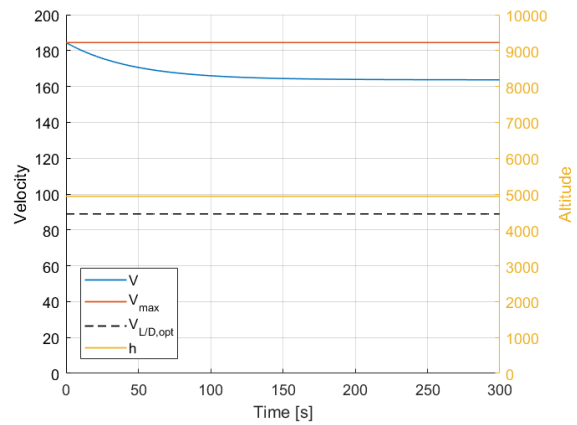
The results for both performance indices are shown in Figure 6.9, minimum fuel consumption on the left, minimum time on the right. Just as for the climb optimization, three different phases can be distinguished, of which the middle one is of most interest for the optimal result of cruise flight.

To start with minimum fuel consumption, it can be seen that the point-mass has opted for the maximum altitude of 8.840 m, which equals the service ceiling of the XV-15. This is logical behaviour as the air density is lower at higher altitude, which results in a lower power required. After an initial maximum velocity, the aircraft converges to its optimal cruise velocity of 111-112 m/s, which comes across to the speed for optimal L/D. The lift coefficient only slightly varies from 0.90 to 0.91, which comes across to the optimal corresponding L/D value. It can be seen in Figure 6.9e that the aircraft still has excess power available at that particular altitude, but that it does not climb further due to its service ceiling. It can be noted that there was slight noise in the control history, as the accuracy was not met to its full extend.

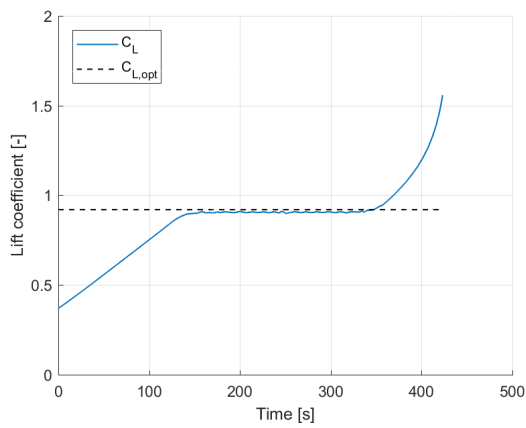
When comparing the latter results to the results when optimizing for time, some differences become apparent. First of all, it is clear that when flying as fast as possible, a significantly lower altitude is flown at: 5.000 m. Although the lower altitude yields higher drag, but it also results in the ability to generate higher thrust. Referring back to Section 4.3, it was expected that the optimal altitude when minimizing for time was approximately 4.000 m instead of 5.000 as a larger velocity can be achieved. This however can be explained because the power available and power required equal each other (Figure 6.9f) and therefore it is impossible to increase the velocity even further, which removes the reason to fly at 4.000 m. Although having maximum initial velocity, the velocity converges to a value of approximately 165 m/s, which is driven by the available power. Analysis has yielded that enforcing an



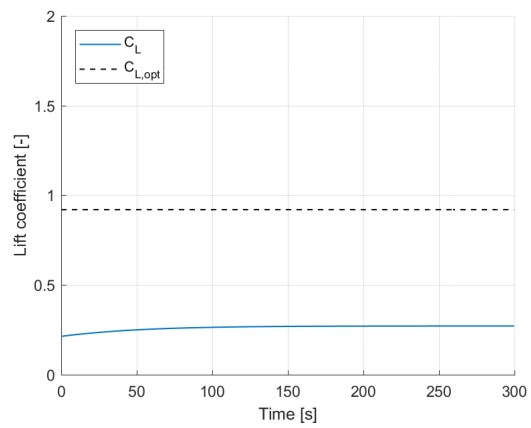
(a) Tilt-rotor altitude and velocity during cruise optimized for fuel consumption



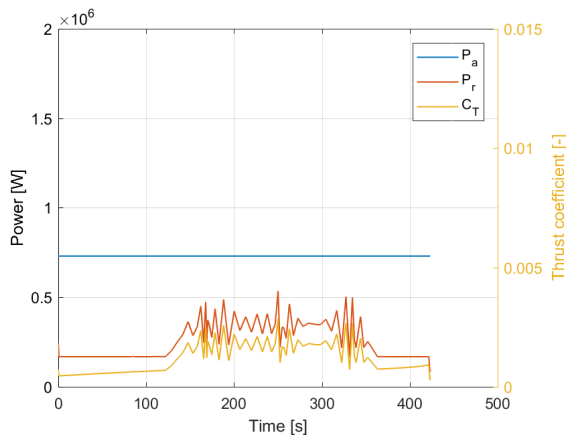
(b) Tilt-rotor altitude and velocity during cruise optimized for minimum time



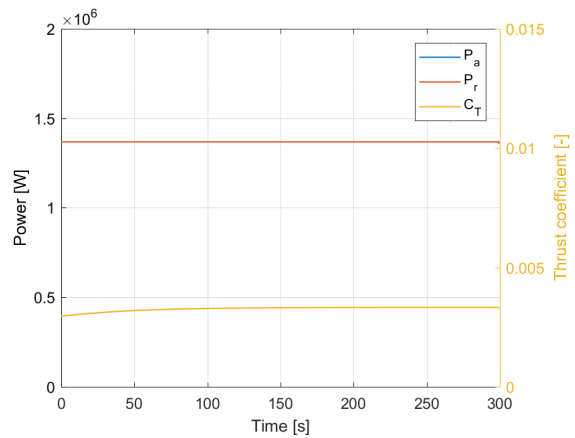
(c) Tilt-rotor lift coefficient during cruise optimized for fuel consumption



(d) Tilt-rotor lift coefficient during cruise optimized for minimum time



(e) Tilt-rotor power during cruise optimized for fuel consumption



(f) Tilt-rotor power during cruise optimized for minimum time

Figure 6.9: Simulation of airplane cruise flight

altitude of 4.000 m results in an only slightly larger final time (299.75 vs. 300.45 seconds). Lastly, the tilt-rotor flies at a lift coefficient value of just about 0.25. It can be shown analytically that the values of air density, lift coefficient and velocity equal a lift force equal to the aircraft weight. This implies that the tilt-rotor flies at a lift coefficient to remain in the air, and not inducing additional drag.

Unsurprisingly, these two distinct procedures yield a different outcome. When optimizing for minimum fuel the 50 km cruise lasts 423 seconds and consumes 11.53 kg of fuel, while the minimum time cruise consumes 43.11 kg of fuel to conduct the cruise in 299 seconds. Other performance indices can be set up to balance the time and fuel consumption. Preliminary optimizations have resulted in intermediate cruise altitudes, consumed fuel and times. These optimizations have not been taken up in this section.

6.3.4. Airplane Descent

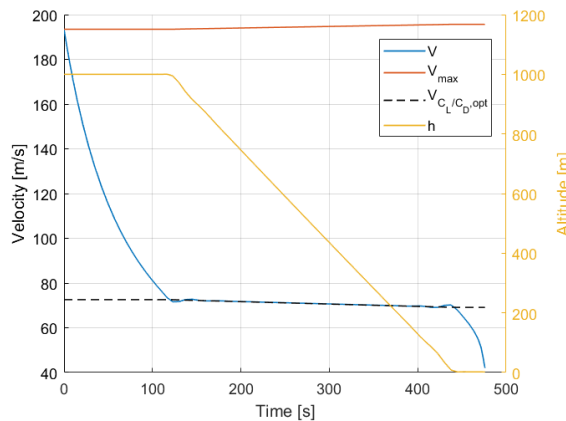
Just as with the isolated climb, the model's performance can be validated for the analytically determined optimal glidepath angle. It was determined that an optimal glideangle in Section 6.1 equals -2.56° at a lift coefficient value of $C_L = 0.92$. Throughout the entire flight no power, nor thrust is available $P_a = T = 0$. The following set of constraints is imposed to simulate the gliding flight:

Table 6.7: Boundary constraints of airplane powerless glide flight

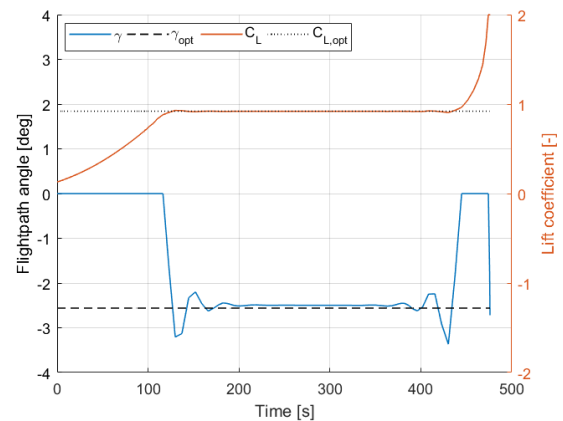
Initial	Final
$x_0 = 0 \text{ m}$	$x_f = \text{free}$
$y_0 = 0 \text{ m}$	$y_f = \text{free}$
$h_0 = 1000 \text{ m}$	$h_f = 0 \text{ m}$
$\gamma_{0,\dots,f} \leq 0^\circ$	
$i_{n,(0,\dots,f)} = 0^\circ$	
$m_{fuel,0} = 0 \text{ kg}$	$m_{fuel,f} = \text{free}$

The model is enforced to behave as airplane at 1.000 m altitude. The optimizer is kept free to choose initial speed but is however constrained to have negative flightpath angle to prevent the model from choosing high initial speed and climb to higher altitude. The problem is optimized to achieve maximum distance with 5 intervals and 20 nodes per interval resulting in 100 nodes for this problem.

$$\min -x_f + \int_{t_0}^{t_f} \vec{u} dt \quad (6.13)$$



(a) Tilt-rotor altitude and velocity during powerless glide



(b) Tilt-rotor glide angle and lift coefficient during powerless glide

Figure 6.10: Simulation of airplane powerless glide

The results of the powerless glide optimization are shown in figure 6.10. Since the model was to maximize the horizontal distance, the point-mass chooses the maximum allowable speed as initial

velocity, as this will give extra distance. The airspeed then gradually drops as the point-mass tries to keep the aircraft at 1.000 m. When the tilt-rotor reaches the velocity corresponding to optimal L/D, it initiates the descent. As can be seen in Figure 6.10b the descent occurs at the exact parameters as calculated analytically: $\gamma = 2.56^\circ$ and $C_L = 0.92$. This results in constant airspeed, and constant vertical velocity. The speed only changes slightly to adapt to the optimal airspeed corresponding to the current altitude. When it reaches the ground the tilt-rotor increases the lift coefficient and ‘flares’ until it reaches zero velocity, just as it would do in real-life. It can be concluded that this part of the model can be validated.

6.3.5. Helicopter Take-Off

To verify the model further, it will be used to optimize isolated operational procedures to check whether the model will reach expected tilt-rotor behaviour, with only minimal constraints. This will be done by simulated isolated airplane and helicopter take-off and landing. For helicopter take-off, the model is subjected to the generic constraints as in Equation 4.70 and 4.71 in addition to the extra imposed constraints. The performance index for the following optimizations is to minimize the fuel consumed and furthermore a control penalty is imposed to slightly limit control inputs. The control penalty should be about 3-5% of the Mayer term. 4 intervals and 20 nodes per interval resulted in 80 nodes for this problem.

$$\min m_{fuel} + \int_{t_0}^{t_f} \vec{u} dt \quad (6.14)$$

To start with a vertical take-off, that is constrained only by the following initial and final constraints as in Table 6.8, that describe its initial position on ground with an initial velocity of 1 m/s. The tilt-rotor is to start and end the procedure with a nacelle inclination of 90° . The simulation ends at an arbitrary set altitude of 100 m and obviously the initial fuel consumed is 0 kg.

Table 6.8: Boundary constraints of vertical take-off

Initial	Final
$x_0 = 0$ m	$x_f = \text{free}$
$y_0 = 0$ m	$y_f = \text{free}$
$h_0 = 0$ m	$h_f = 100$ m
$V_0 = 1$ m/s	$V_f = \text{free}$
	$i_{n,0,\dots,f} = 90^\circ$
$m_{fuel,0} = 0$ kg	$m_{fuel,f} = \text{free}$

The results of this simulation are shown in Figure 6.11. The flightpath of the tilt-rotor is shown in Figure 6.11a, where it can be seen that the tilt-rotor model is capable of performing a dead-straight vertical ascent to 100 m altitude in roughly 18 seconds. It is obviously not the safest procedure to climb straight upwards, but proves the model in a vertical take-off. Zooming in on Figure 6.11b shows that the velocity increases from 1 m/s to a maximum velocity of 8 m/s, which is a rapid vertical take-off, but is expected because no further operational constraints are being imposed. A final fuel consumption of roughly 3.5 kg is required for this procedure. As can be seen in Figure 6.11c, there is no use of cyclic angle, which is not desired as no forward or lateral movement is desired. The nacelle inclination is kept constant at 90° , while the flightpath angle slightly fluctuates in between 89.5° and 89.9° . Concluding with Figure 6.11d, it can be seen that the model keeps the power required constant at the level of power available by slightly decreasing the thrust coefficient as the tilt-rotor gains altitude. In the last seconds of the flight the model decreases the thrust coefficient as sufficient velocity has already been gained to reach the final altitude, which allows for a slight reduction in fuel consumed.

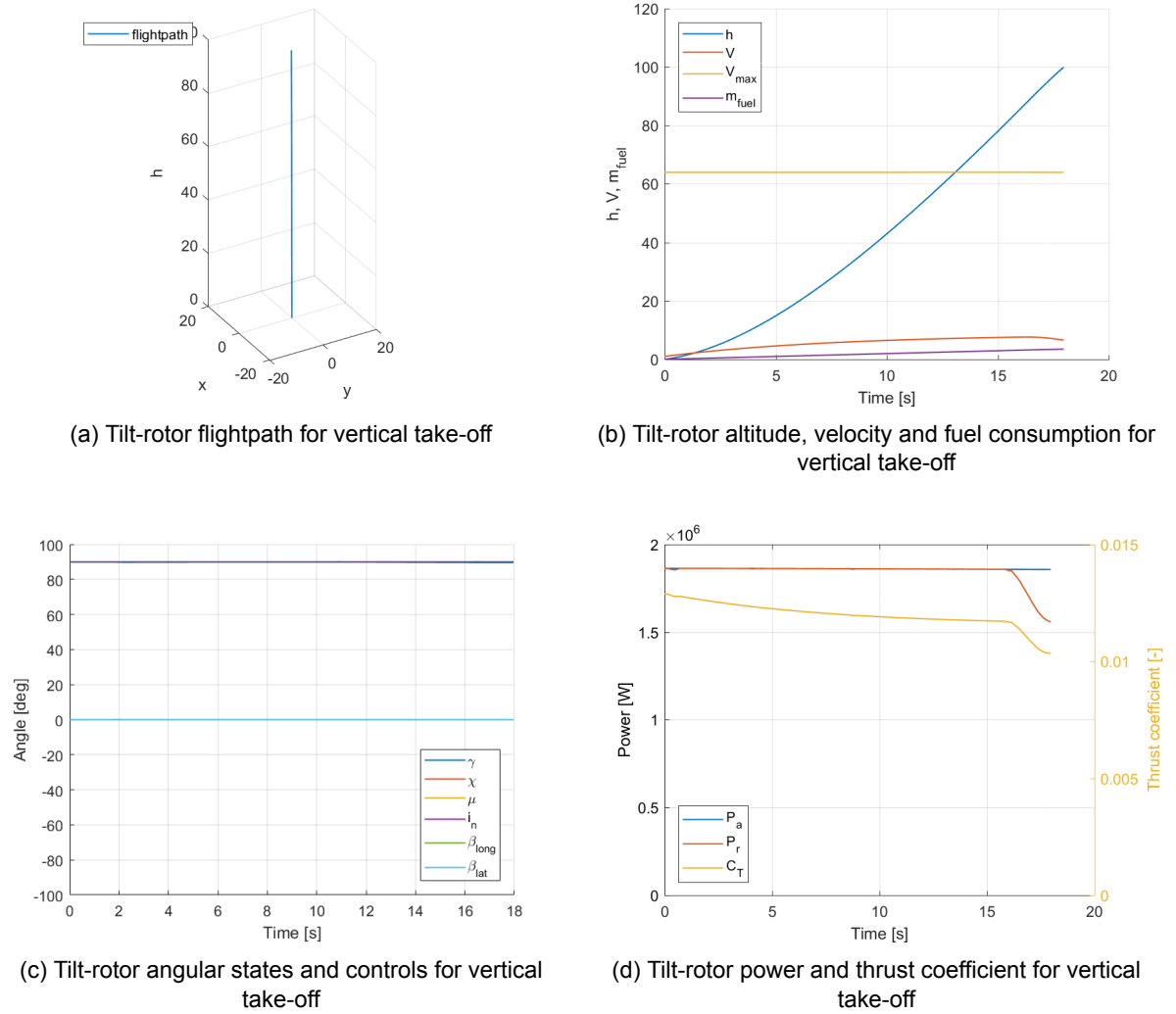


Figure 6.11: Simulation of isolated vertical take-off

6.3.6. Helicopter Landing

The vertical landing is constrained similar to the vertical take-off, but in an opposite way, where the only differences are the initial and final altitude. A reference initial speed of 10 m/s has been adopted from Figure A.4.

Table 6.9: Boundary constraints of vertical landing

Initial	Final
$x_0 = 0$ m	$x_f = \text{free}$
$y_0 = 0$ m	$y_f = \text{free}$
$h_0 = 100$ m	$h_f = 0$ m
$V_0 = 10$ m/s	$V_f = 1$ m/s
	$i_{n,0,\dots,f} = 90^\circ$
$m_{fuel,0} = 0$ kg	$m_{fuel,f} = \text{free}$

The results of the simulation of the vertical landing are shown in Figure 6.12. The model shows expected behaviour for this flight procedure. The flightpath of the tilt-rotor that descends 100 m in slightly more than 10 seconds is depicted in Figure 6.12a. As can be seen in Figure 6.12b, the model allows the helicopter to gain some extra velocity before slowing the tilt-rotor down to its touch-down velocity of 1 m/s. This is the result of the thrust coefficient being slowly increased up to the point that the power required equals the power available, as can be seen in Figure 6.12d. Unsurprisingly the tilt-rotor uses

two times less fuel in vertical landing than vertical take-off operation, which is expected as well. As for the vertical take-off there is no bank or cyclic input and, while the flightpath angle approaches -90° , the nacelle inclination remains constant at 90° (Figure 6.12c). 4 intervals and 20 nodes per interval resulted in 80 nodes for this problem.

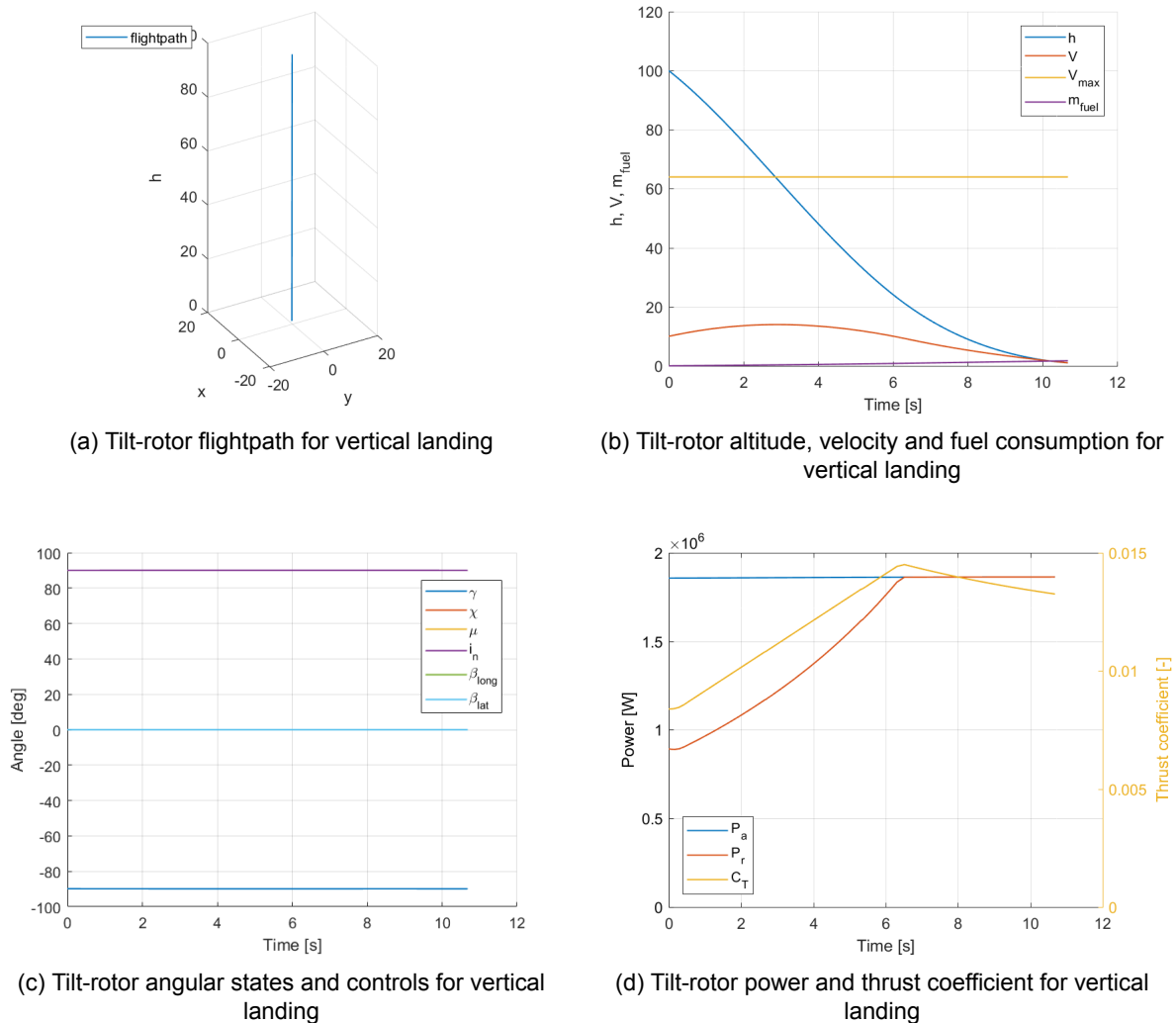


Figure 6.12: Simulation of isolated vertical landing

6.3.7. Runway Take-Off

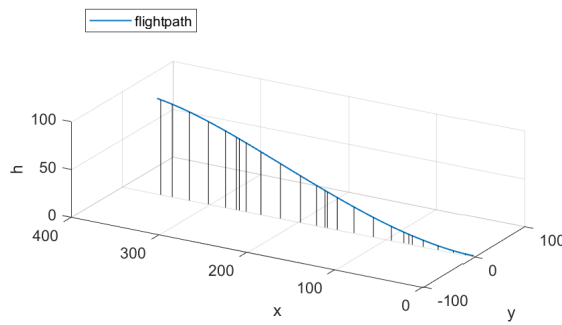
The simulated runway take-off starts with the moment of rotation with a lift-off speed of 20 m/s and ends at again, an arbitrary altitude of 100 m and a climb-out flightpath angle of 8° . The initial nacelle angle is fixed at the minimum angle to provide ground clearance. Hereafter, the nacelle inclination is free. 4 intervals and 20 nodes per interval resulted in 80 nodes for this problem.

Figure 6.13 depicts the results of the simulation of the runway take-off. The flightpath of the tilt-rotor is shown in Figure 6.13a, which comes across to ordinary aircraft take-off profiles. In a time domain of roughly 12 seconds the tilt-rotor climbs 100 m in a horizontal distance of 360 m, which results in an average flightpath angle of 15° , before settling on the climb-out angle of 8° . This comes across to what most airliners do for safety reasons, by gaining altitude quick through a steep first climb. Although this is not programmed, this has been the result because the model chooses a maximum power required in the first part to gain speed and altitude quick, which is used to follow the trajectory to end up with the final altitude, speed and flightpath angle as in Figure 6.13b. The tilt-rotor reaches a maximum velocity of 37 m/s before throttling down to save some fuel, which is reflected in the plot for the power required

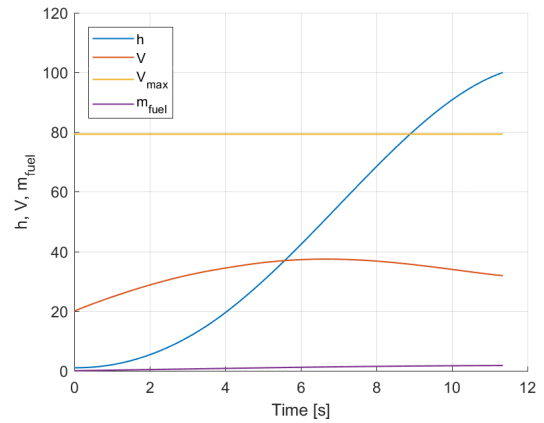
Table 6.10: Boundary constraints of runway take-off

Initial	Final
$x_0 = 0$ m	$x_f = \text{free}$
$y_0 = 0$ m	$y_f = \text{free}$
$h_0 = 1$ m	$h_f = 100$ m
$V_0 = 20$ m/s	$V_f = \text{free}$
$\gamma_0 = 0^\circ$	$\gamma_f = 8^\circ$
$i_{n,0} \geq 60^\circ$	$i_{n,f} = \text{free}$
$m_{fuel,0} = 0$ kg	$m_{fuel,f} = \text{free}$

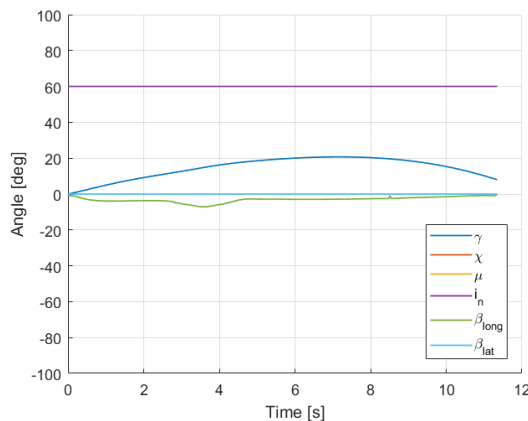
(Figure 6.13d). In the end the tilt-rotor consumes roughly 2 kg of fuel, which is half the amount the vertical take-off consumes. This emphasizes the efficiency that the lift of the wings have during runway take-off over the sole thrust of the rotors during vertical take-off. As can be seen in Figure 6.13c the model does not change the nacelle to a higher inclination which could have been done in order to climb faster. However, the angle remains constant at 60° , which means that the model recognizes that it is more efficient to use the lift of the wings instead of the vertical thrust of the rotors. Finally, it should be mentioned that the tilt-rotor starts flying at $C_{L,max}$ to remain airborne as it flies at low speed, which virtually implies the use of full flaps.



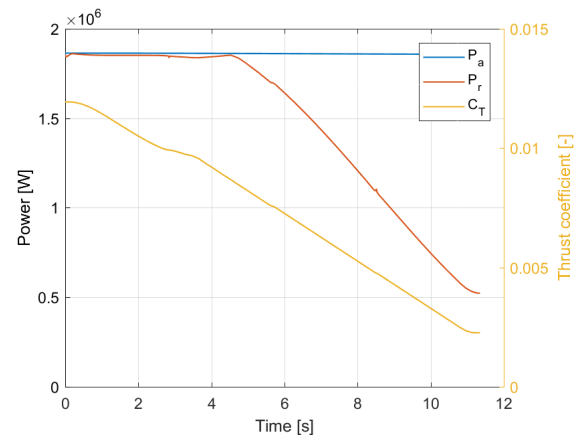
(a) Tilt-rotor flightpath for runway take-off



(b) Tilt-rotor positional and translational states for runway take-off



(c) Tilt-rotor angular states and controls for runway take-off



(d) Tilt-rotor power and thrust coefficient for runway take-off

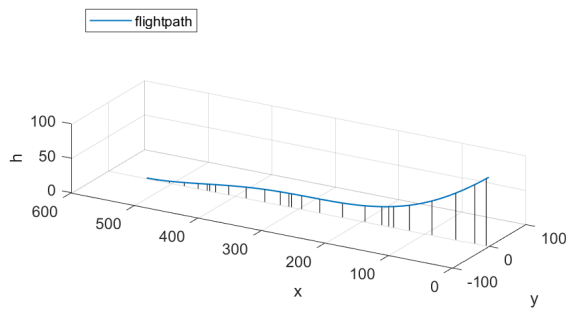
Figure 6.13: Simulation of isolated runway take-off

6.3.8. Runway Landing

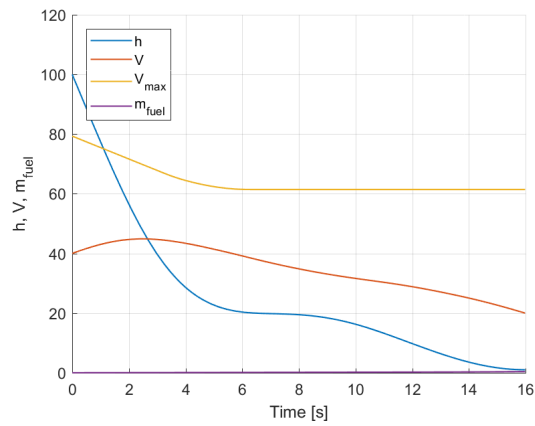
The runway landing is simulated as follows. Starting at an initial position at 100 m altitude with a reference speed of 40 m/s, the tilt-rotor is to reach sea-level while flying parallel to the ground for a smooth touchdown at the same speed as lift-off speed. Furthermore, rotor ground clearance is to be maintained. This is described in Table 6.11. 4 intervals and 20 nodes per interval resulted in 80 nodes for this problem.

Table 6.11: Boundary constraints of runway landing

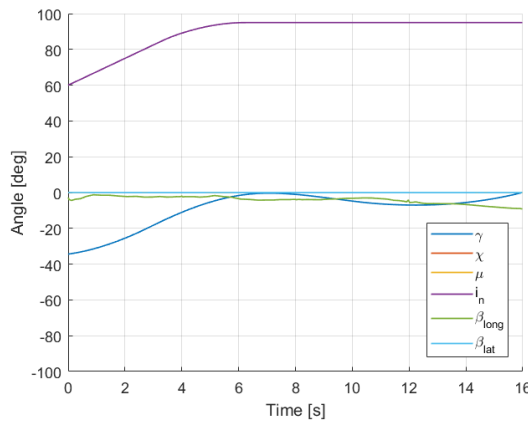
Initial	Final
$x_0 = 0$ m	$x_f = \text{free}$
$y_0 = 0$ m	$y_f = \text{free}$
$h_0 = 100$ m	$h_f = 1$ m
$V_0 = 40$ m/s	$V_f = 20$ m/s
$\gamma_0 = \text{free}$	$\gamma_f = 0^\circ$
$i_{n,0} = \text{free}$	$i_{n,f} \geq 60^\circ$
$m_{fuel,0} = 0$ kg	$m_{fuel,f} = \text{free}$



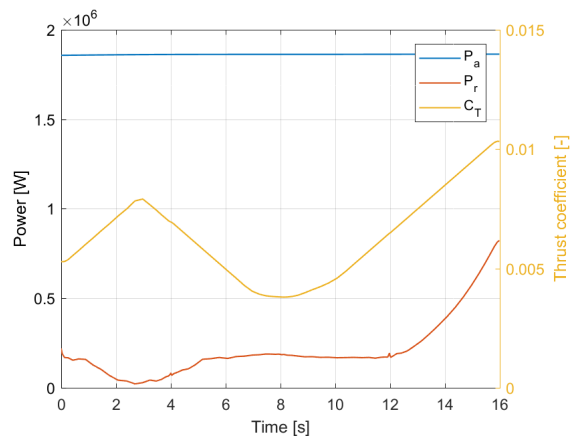
(a) Tilt-rotor flightpath for runway landing



(b) Tilt-rotor positional and translational states for runway landing



(c) Tilt-rotor angular states and controls for runway landing



(d) Tilt-rotor power and thrust coefficient for runway landing

Figure 6.14: Simulation of isolated runway landing

The results of this simulation are shown in Figure 6.14, which shows the aircraft descending to sea-level in approximately 17 seconds, in a horizontal distance of 550 m. It can be seen in Figure 6.14b-

6.14d that the nacelle inclination is immediately increased to 95° in order to decrease velocity. Because of this the thrust coefficient is increased. It is briefly lowered though, as the drop in altitude results in an increase in speed. Hereafter, the thrust increases again to decrease velocity. Again $C_{L,max}$ is flown at in order to be able to remain airborne. Interestingly, in the last second the nacelle returns to 90° , whereas the longitudinal cyclic input changes to -7° . It is concluded that this yields a slight numerical advantage opposed to a nacelle angle of 95° and longitudinal cyclic of -2° . The same thing goes for the first four seconds where a positive longitudinal cyclic cancels the increase in nacelle angle. From Figure 6.14d it can be deduced that the tilt-rotor does not have a lot of power required, which is in contrast to the vertical landing, but is logical since the lifting surfaces carry a lot of the weight.

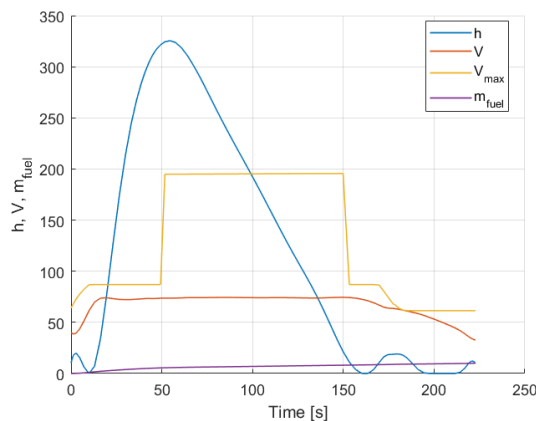
6.3.9. Conversion

To verify aircraft behaviour in cruise, the aircraft is set to start at an initial position with initial velocity and is set to end at 15 km further on. The aircraft starts and ends in helicopter mode. It is expected that the model ascends to higher altitude and converses to airplane mode because the thinner atmosphere and lower rotor rpm will have a positive effect on the fuel consumption, while lift is provided by the wings and hence efficient flight. 5 intervals and 20 nodes per interval resulted in 100 nodes for this problem.

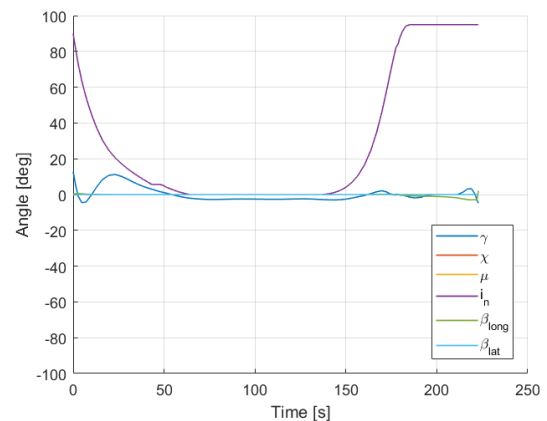
Table 6.12: Boundary constraints of conversion flight

Initial	Final
$x_0 = 0$ m	$x_f = 20$ km
$y_0 = 0$ m	$y_f = \text{free}$
$h_0 = 10$ m	$h_f = 10$ m
$V_0 = 40$ m/s	$V_f = \text{free}$
$\iota_{n,0} = 90^\circ$	$\mu_f = 90^\circ$
$m_{fuel,0} = 0$ kg	$m_{fuel,f} = \text{free}$

As can be seen in Figure 6.15a and 6.15b, the model performs as expected by immediately converting the nacelle angle to airplane mode to profit from the lower rotor rpm and climbs to roughly 350 m, which is not a lot, but considering the distance of only 15 km which is covered in about 225 seconds, it is a feasible altitude. Interestingly, the aircraft is able to speed up more than it currently does, which can be seen in Figure 6.15a. At that particular point, the tilt-rotor flies with a constant airspeed of 74 m/s and at a flightpath angle of -2.52° , which is extremely close to the optimal glidepath angle of -2.56° . The tilt-rotor does however, not fly at optimal lift-to-drag ratio for glide. This is down to two reasons. First of all, ensures a minimum thrust coefficient value a minimum amount of power required. Secondly, does the lower limit on the fuel consumption restrain the tilt-rotor from cutting off the engine, which models a minimum fuel consumption of an idle engine.



(a) Tilt-rotor positional and translational states during conversion



(b) Tilt-rotor angular states and controls during conversion

Figure 6.15: Simulation of isolated conversion

Conversion for Minimal Time

The results for the conversion as stated above have been optimized for minimum fuel consumption. It can be seen in Figure 6.15a that, although the state of the aircraft allows to do so, the aircraft does not accelerate beyond 74 m/s. When optimizing for minimal time (Eq. 6.15), the results differ entirely. As can be seen in Figure 6.16, the aircraft does not climb anymore. Contrarily, the aircraft remains flying at sea level, since there is no benefit of climbing with respect to time. It can be argued that the lower air density will have a profit in long distance flights. The aircraft flies constantly at its maximum speed at that particular moment. However, the aircraft does not convert up to 0° nacelle angle, but converts to 20° and slowly converts back to the final inclination. This flight procedure yields a time advantage of 45 seconds, or 2 seconds per kilometer. This time profit is paid by a 27% increase in fuel consumption: from 9.95 to 12.48 kg. 30 intervals and 4 nodes per interval resulted in 120 nodes for this problem.

$$\min t_f + \int_{t_0}^{t_f} \vec{u} dt \quad (6.15)$$

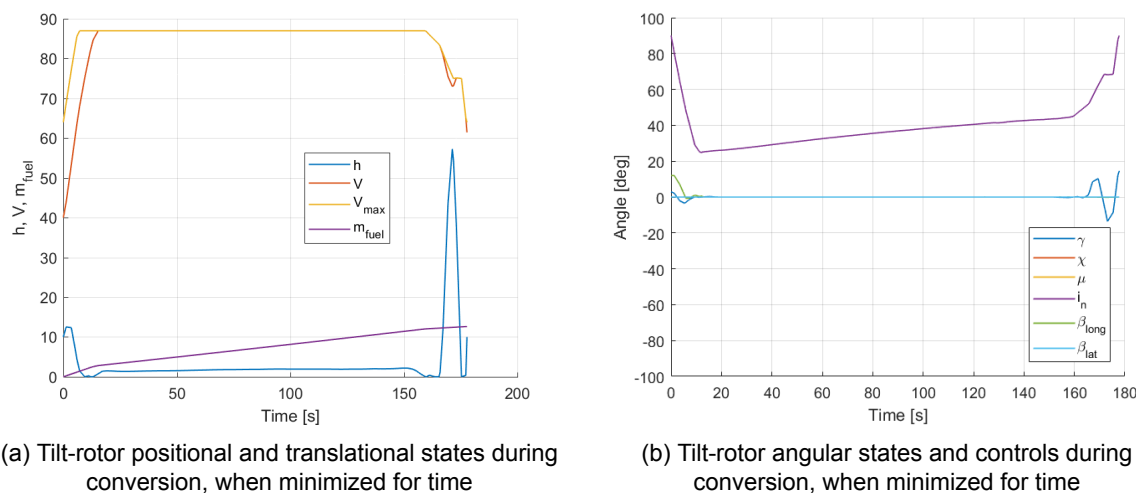


Figure 6.16: Simulation of isolated conversion, when minimized for time

6.3.10. Turning Performance

To validate the models turning behaviour in flight, a flying turn can be isolated in order to assess the flying behaviour. Botasso has conducted research on numerical procedures for trajectory optimization problems in rotorcraft flight mechanics. In his research he has applied the developed numerical procedures in some simple rotorcraft flight manoeuvres such as a 90° turn, 180°, 360° pirouette and a slalom through a parcours. Moreover, the 180° turn has been performed for both a helicopter and a tilt-rotor aircraft. It is known that the model consisted of a rigid-body model. Therefore, the point-mass can be validated by means of performing a 180°.

It is merely difficult to exactly reproduce the 180° results as, minimal data on the rotorcraft and constraints are given. It is stated that a minimum time 180° turn is considered. Initial and final conditions correspond to straight and level flight at 5 m/s. The latter is doubtful, since in the paragraph before a 90° turn is made at 50 m/s, after which it is stated that “the procedure can be applied unchanged”. Botasso follows by stating that the problem is complemented by “suitable bounds on the vehicle states, controls and control rates”. The problem is optimized for minimum time, including a penalty on the control rates [8]. Finally, only the flightpath is depicted. The optimized time to perform the turn is neither given.

As can be seen in Figure 6.17, the helicopter initiates the turn, climbs slightly while banking, after which it descends again to end up at the same altitude in the opposite direction. The tilt-rotor trajectory is slightly different in that it flares to slow down, starts turning, while losing altitude, and ascends again at an incredibly high sideslip angle. It is interesting to see that although the tilt-rotor has a maximal lateral displacement of 20 m, the final displacement is only 5 m. While the helicopter only has a slight

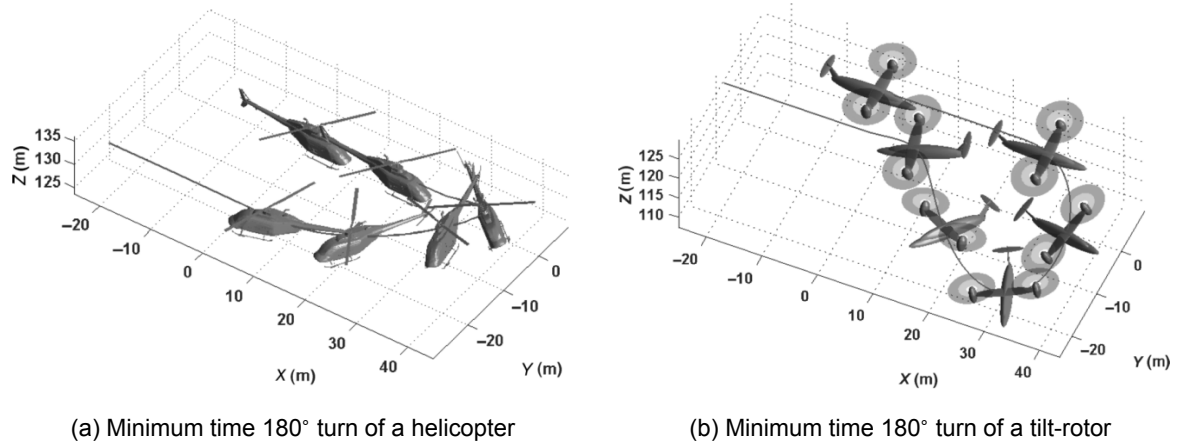


Figure 6.17: Minimum time 180° turn optimization of rotorcraft by Botasso [8]

altitude differential, the tilt-rotor loses almost 15 m only to regain it afterwards. Botasso acknowledges that these trajectories are not desirable in real life and can hence be refined by imposing desirable constraints [8].

To approach Botasso's optimization study the following constraints are imposed in order to start and end in straight and level flight. 4 intervals and 20 nodes per interval resulted in 80 nodes for this problem.

Table 6.13: Boundary constraints of 180° turn flight

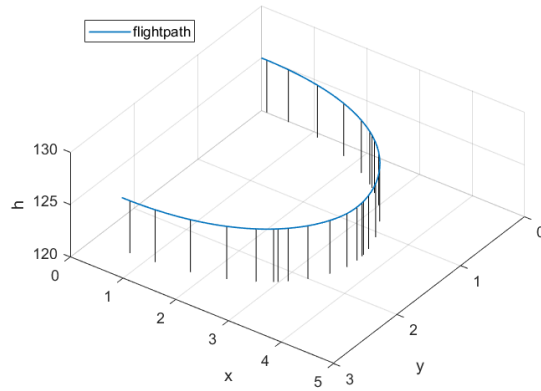
Initial	Final
$x_0 = 0$ m	$x_f = \text{free}$
$y_0 = 0$ m	$y_f = \text{free}$
$h_0 = 125$ m	$h_f = 125$ m
$V_0 = 5$ m/s	$V_f = 5$ m/s
$\gamma_0 = 0^\circ$	$\gamma_f = 0^\circ$
$\chi_0 = 0^\circ$	$\chi_f = 180^\circ$
$\mu_0 = 0^\circ$	$\mu_f = 0^\circ$
$m_{fuel,0} = 0$ kg	$m_{fuel,f} = \text{free}$

$$\min t_f + \int_{t_0}^{t_f} \vec{u} \, dt \quad (6.16)$$

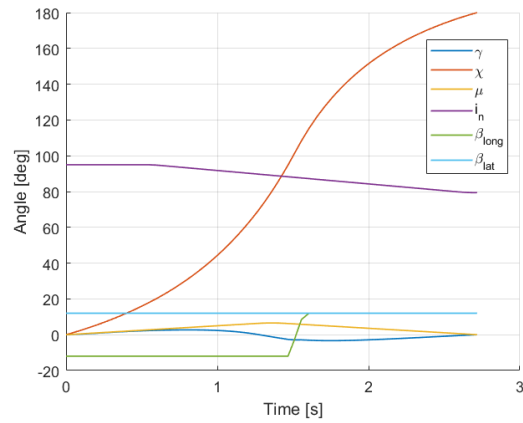
Figure 6.18 shows the result for the optimization. As mentioned above, it remained unclear what parameters were used by Botasso. To try to reproduce Botasso's results, a distinction has been made between flight with and without the use of β_{lat} . The upper two graphs show flight with the use of lateral cyclic flapping.

As can be seen in Figure 6.18a and 6.18b, a steady turn is made. The tilt-rotor chooses to enter the turn with 95° nacelle inclination, in order to slow the aircraft down, which accelerates $\dot{\chi}$. While doing so, the aircraft immediately increases the bank angle, and uses maximum lateral cyclic pitch. In the middle of the turn, the aircraft reverses the process to end up in its final state. However, the nacelle inclination does not increase again to helicopter mode, but decreases in order to pick up speed, as expected. The same thing holds for the longitudinal cyclic angle, which in the first half of the turn is used to slow down, while it accelerates the aircraft in the second part of the turn. The timestamp of $V = 1$ m/s, $\chi = 90^\circ$, $i_n = 90^\circ$, μ_{max} and the switching point of the longitudinal cyclic coincide at the centre of the turn. Although small flightpath angles are present, the aircraft's height differential is within 1 m.

Apart from the distances, the flight trajectory shows strong similarity with the Botasso's helicopter trajectory, which is not expected at first, since a tilt-rotor was modelled. However, this makes sense for the following two reasons. The tilt-rotor is modelled as a point-mass model and has one thrust vector. Therefore, the two rotors do not create an additional moment during the turn from which Botasso's



(a) Flightpath of 180° turn



(b) Angular states and controls for 180° turn

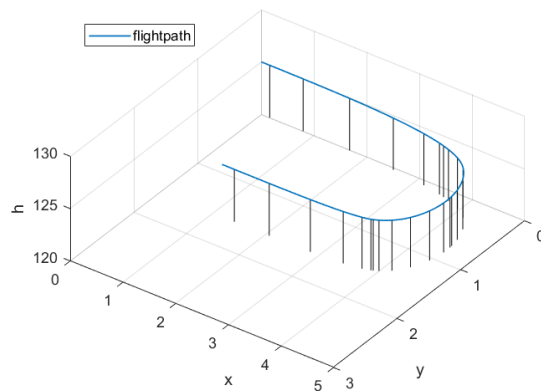
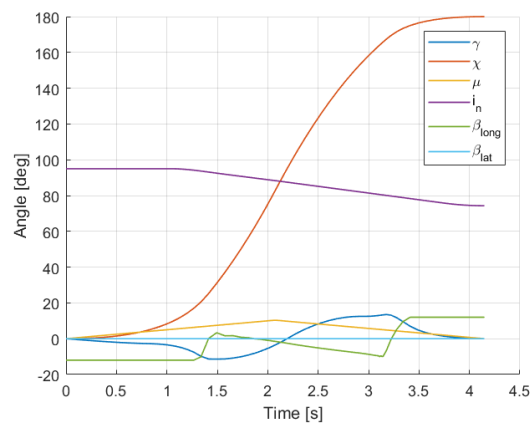
(c) Flightpath of 180° turn with restricted lateral cyclic β_{lat} (d) Angular states and controls for 180° turn with restricted lateral cyclic β_{lat}

Figure 6.18: Simulation of 180° turn for 5 m/s initial velocity

rigid-body tilt-rotor seems to benefit. Secondly, from XV-15 data, the point-mass a maximum cyclic angle of $\pm 12^\circ$ and the banking rate was guesstimated at $5^\circ/\text{second}$. This implies that during the turn it only reaches a maximum banking angle of about 7.5° , as can be seen in the graph. This results in the fact that the turning contribution of the cyclic is greater than the banking angle, which comes closer to helicopter flight, than tilt-rotor flight.

To verify the difference when no lateral cyclic is used, the use of this has been restricted in the second optimization. It is expected that the optimized flight trajectory of the point-mass will make shift from Botasso's helicopter flightpath and slightly approach the result from the tilt-rotor. The results of this have been shown in Figure 6.18c and 6.18d. It can be seen that slight distinctions occur already. Instead of immediately turning, the focus lies on decreasing the airspeed, as this increases the rate of change of the heading angle (Eq. 4.28). This can also be seen as the aircraft decelerates longer with the nacelle angle and longitudinal cyclic. It is not shown in the graph, but $1 \leq t \leq 3$ s the aircraft flies at the minimum airspeed of 1 m/s. In the previous optimization, the change in the heading angle χ is nearly linear. Now however, the rate of change is much steeper. The aircraft hence almost makes a turn in hoversea level, since there is no benefit of climbing with respect to time. It can be argued that the lower air density will have a small profit in long distance flights. The aircraft flies constantly approaches its maximum speed at that particular moment. Similar to Botasso's tilt-rotor, the point-mass flies at negative flightpath angle γ in the first part of the turn, while the turn is completed with pitch up attitude. This is unlike the first optimization and may seem a bit odd, as the tilt-rotor wants to slow down in order to rapidly make the turn. However, just as for the previous optimization, the altitude differences

are negligible, which reduces the gravity of the negative flightpath angle.

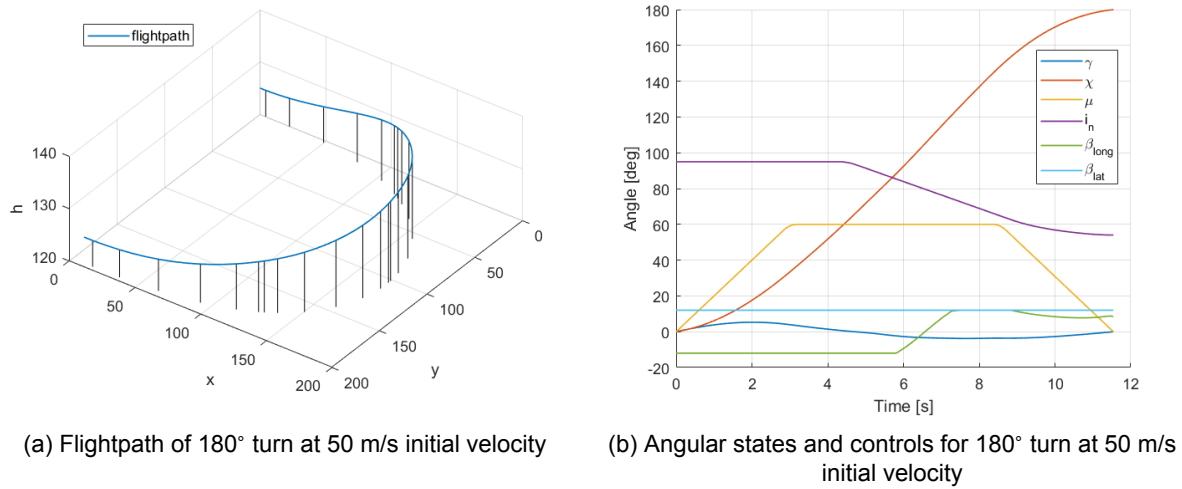


Figure 6.19: Simulation of 180° turn for 50 m/s initial velocity

To investigate the flight behaviour with higher velocity, Figure 6.19 shows another optimization that is performed with 50 m/s initial velocity. This shows the same behaviour at however, another scale. One point that stands out is that the aircraft reaches its banking limit. This is however as expected, since the rate of change of the heading angle is smaller because of the higher velocity (Eq. 4.28).

6.3.11. Effect of Cruise Distance

To further validate the point-mass model, the global flight behaviour of long distance flight will be looked into and it will be assessed if the point-mass will have similar behaviour to aircraft, or how it differs from them. While keeping all (boundary) conditions the same and by adjusting the horizontal distance to be flown, dissimilar flying behaviour is expected. This can be scrutinized to assess model validity.

At the start of the flight, it is expected that the tilt-rotor gains speed while converting to airplane mode to profit with respect to fuel flow from both the lift of the wings and the lower engine rpm. While doing so, it is expected that the tilt-rotor gains altitude in order to profit from lower air density. Depending on the horizontal distance to be flown, the altitude changes. When flying longer distance, the benefit of flying higher outweighs the drawback of spending more power to climb and therefore every horizontal distance will have its own optimal altitude. After this it is expected that the aircraft will keep its altitude and until it approaches its goal. Then when the speed allows for it, the aircraft reconverts to 95° to slow the aircraft down to its final speed and altitude. During the flight phases with both low airspeed and high nacelle inclination, it is expected that β_{long} is used to give an extra 'kick' to the conversion process to expedite it.

The constraints of the simulations have been given in Table 6.14. The aircraft's initial and final position, velocity and nacelle inclination are constrained in order to have helicopter to helicopter flight. The distance of the final position has been varied in the order of 10, 20, 30, 35, 40, 50, 60, 70 and 80 km to explore cruise behaviour.

Table 6.14: Boundary constraints of cruise flight

Initial	Final
$x_0 = 0$ m	$x_f = \text{variable}$
$y_0 = 0$ m	$y_f = 0$ m
$h_0 = 10$ m	$h_f = 10$ m
$V_0 = 20$ m/s	$V_f = 20$ m/s
$i_{n,0} = 90^\circ$	$90^\circ \leq i_{n,f} \leq 95^\circ$
$m_{fuel,0} = 0$ kg	$m_{fuel,f} = \text{free}$

The flight is optimized for minimum fuel consumption. The optimizations for smaller final distances

were optimized with 4 intervals and 20 nodes per interval. However, as the final distance increased, up to 240 nodes were used (12 intervals with 20 nodes per interval) in order to acquire convergence.

$$\min m_f + \int_{t_0}^{t_f} \vec{u} dt \quad (6.17)$$

The results of the optimization are shown in Figure 6.21. The flight paths of the cruise are depicted in Figure 6.21a. Although the to be flown distances have been equally spaced, except 35 km, there is a clear distinction between distances higher than 35 km. (The exact distance for the tipping point of the flight procedure was found to be in between 37.4 and 37.5 km.) This distance is the tipping point at which the benefit of the higher altitude outweighs the power required of the extra climb and deceleration at the end, due to the use of another climb and conversion strategy.

This can clearly be seen in Figure 6.21b, which depicts the different changes in nacelle inclination. Distances lower than 35 km convert straight to airplane mode and reconvert back to maximum nacelle angle to land at the designated spot. The single variance between the lower distances is the length at which the aircraft flies at 0° nacelle angle, as the descent procedure is the same. When the to be flown distance exceeds 37.4 km, the optimal flight path lies different. The aircraft starts to convert towards airplane mode, but at approximately 40° nacelle angle it converts at a slower pace to make use of the vertical force of the thrust for about a minute. Hereafter, conversion recommences just as the others to airplane mode. The re-conversion is the same as the others, except for the fact that the aircraft flies longer at maximum nacelle angle to cushion the landing. It can be noticed that this time grows with the altitude of the cruise, while the 'braking' for the smaller distances is the same length since the aircraft glides towards the final destination.

To conclude: the benefit of the lower air density at higher altitude is worth the combination of stopping the conversion for a brief moment to climb longer and using more power to make the safe descent from higher altitude. This is underlined when plotting the trend of the fuel consumption of the two different procedure with respect to the cruise distances. Figure 6.20 clearly shows that the fuel gradient of the second procedure is lower than the gradient of the first procedure. The values for the fuel consumption are given below in Table 6.15. The fuel data for the optimizations just above and below the tipping point are given, but are omitted from the other graphs to have the graphs remain clear and concise.

Table 6.15: Fuel consumption for cruise flight optimization

Cruise distance	Fuel consumption
10 km	5.51 kg
20 km	10.10 kg
30 km	14.62 kg
35 km	16.83 kg
37.4 km	18.03 kg
37.5 km	18.09 kg
40 km	19.08 kg
50 km	22.99 kg
60 km	26.85 kg
70 km	30.66 kg
80 km	34.64 kg

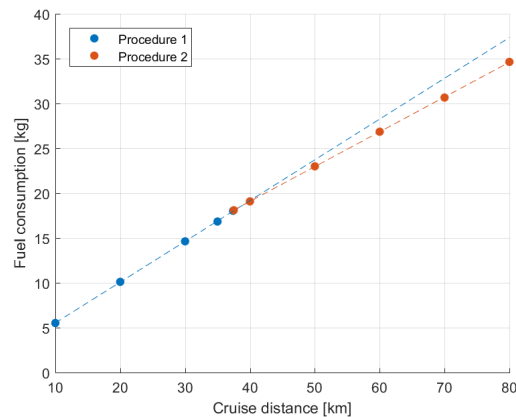


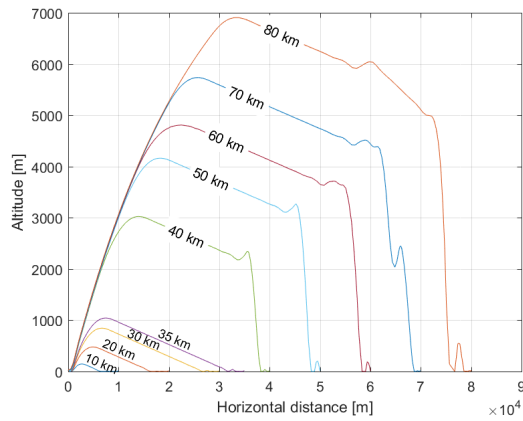
Figure 6.20: Fuel consumption with respect to cruise distance for cruise flight optimization

Referring back to Section 6.1 where the optimal lift-to-drag ratios had been determined, it was determined that in terms of a fuel efficient climb maximum L^3/D^2 is desired. The velocity profiles of the various climbs of the cruise optimizations have been plotted in Figure 6.21c, while the corresponding optimal height-velocity profile that had been determined analytically (as in Figure 6.1b) has been added too. The height-velocity combinations of the descent have been omitted from the figure. It can be said that the various optimizations match the optimal climb speed, considering the fact that the offset is minor (~ 3 m/s) and the simplifications that were made to come up with the lift-to-drag ratios. This validates the model to fly at optimal speed to climb. It can be noticed in the graph that the tilt-rotor

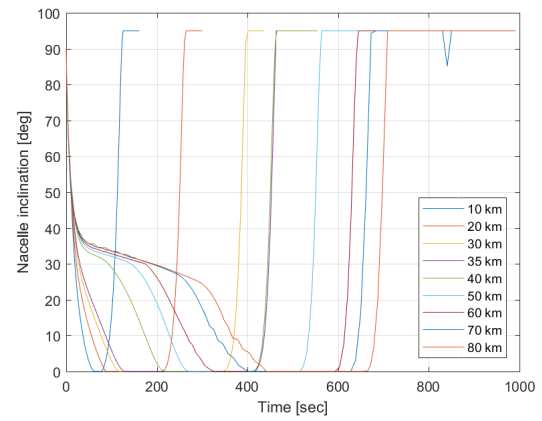
speeds up at low altitude (up to 200 m) before climbing, until it reaches the optimal speed to climb at which it begins the climb during which it further accelerates depending on its altitude.

To conclude the analysis, the used nacelle inclination has been plotted against the velocity in the climb in Figure 6.21d, to assess the conversion used by the tilt-rotor. As can be seen in the figure, the different flights use the same conversion strategy until the point that the top of the climb has been reached, at which point the nacelle is converted to 0° . Higher velocity is flown at at altitude, which explains the only difference. The second question is why this particular conversion is used. Sample testing of the force balance of the forces in the vertical direction yielded what was expected: namely, that the amount of force of lift, drag, thrust and weight in these conditions result in a force surplus that accelerates the aircraft in vertical direction. Hence, it can be concluded that the conversion highly depends on the velocity, which has been adapted to show the horizontal velocity.

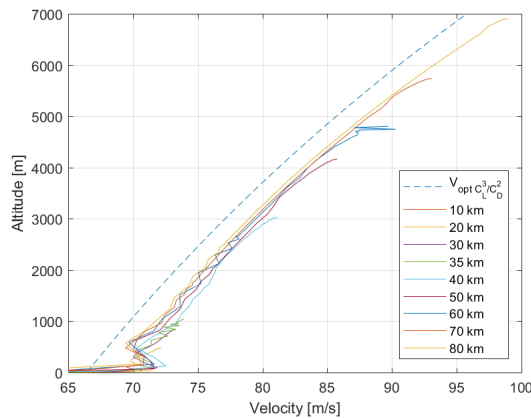
All in all, this confirms the expected outcome as described in the fact that the tilt-rotor converts to airplane mode, when it has sufficient lift, to profit from lower engine rpm. Furthermore, the tilt-rotor climbs to higher altitude when it is summoned to fly a further distance, as the benefits increase with higher altitude. The conversion strategy used for further flight was not expected though, but after analysis it makes sense that the tilt-rotor slightly balances the vertical force of the thrust and lift to climb further with minimal power required. Finally, the expectation of optimal speeds that was made before was validated with this analysis.



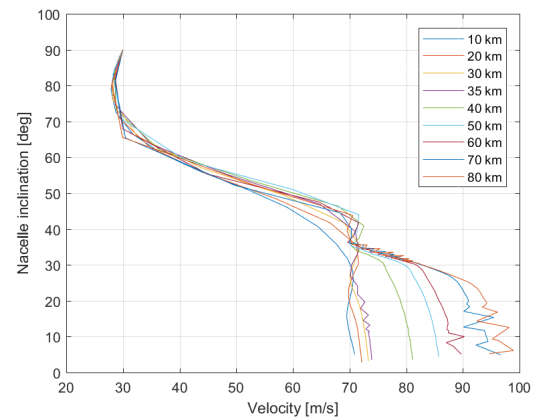
(a) Flight profiles of cruise flights with varying final distance



(b) Conversion strategy of cruise flight of various distances



(c) Airspeed versus altitude during climb of the cruise phase of various distances compared to optimal climb speed at maximum L/D



(d) Rotor nacelle inclination versus airspeed during climb

Figure 6.21: Results of cruise flight optimization

It was desired to increase the cruise distance further in order to validate the fact that the tilt-rotor

continues to increase its altitude, until it reaches its service ceiling at which it cruises in level flight. It was expected that the service ceiling would have been reached when the cruising distance exceeds approximately 100 km. Increasing the cruise distance however, resulted in the optimizer having more and more difficulties to converge to the optimal solution due to numerical difficulties. When optimizing the same problem for minimal time the optimizer successfully converges to the optimal solution in 62 major iterations in a computational time of about 200 seconds

The numerical difficulties with convergence arise when optimizing for minimum fuel usage, which implies minimizing the power required. Calculating the power required calls for a complex procedure. Two bottlenecks are identified in the calculation of the power required. It is either expected that the numerical difficulties arise from the determination of the induced velocity when the rotor is outside its vortex-ring state (Eq.4.51). The model solves for the fourth order polynomial by solving $v_i = \text{roots}[a \ b \ c \ d \ e]$; for initial guess, normalizing and final solution and $v_i = \text{roots}[a.x \ b.x \ c.x \ d.x \ e.x]$; for the gradients during the optimization's iterations. a , b , c , d and e represent the coefficients for the polynomial. The solution yields four solutions, one positive real number, one negative real number and 2 complex numbers. From these the positive real number is to be used. The other bottleneck is that one or more linearities arises in the calculation. It is said that GPOPS has difficulties dealing with linearities. This topic will further be elaborated in the recommendations.

To illustrate the amount of computation, the GPOPS output of the 20 km optimization is shown in Appendix C.

6.3.12. Effect of Initial Velocity on Climb-Out

Similar behaviour can be found when investigating the tilt-rotor behaviour when the initial velocity is varied during climb-out. The optimization is constrained to its initial and final position, as in Table 6.16. This time, however, the initial velocity is varied over [1, 2, 5, 10, 20, 30 m/s] in order to analyse the effect it has on the start of the climb of the tilt-rotor. The results of the optimization have been shown in Figure 6.22. The graphs have been cropped to emphasize the effect of the initial velocity. All in all, the tilt-rotor flight takes between 50 and 60 seconds, achieving a top speed between 70 and 80 m/s. The top speed of this flight is reached with an acceleration of ± 0.3 g. The numerical optimality was close to, but did not reach the $1e - 6$ threshold value, resulting in a not perfectly smooth curve.

Table 6.16: Boundary constraints of climb-out flight

Initial	Final
$x_0 = 0$ m	$x_f = 3$ km
$y_0 = 0$ m	$y_f = \text{free}$
$h_0 = 0$ m	$h_f = 1$ km
$V_0 = \text{variable}$	$V_f = \text{free}$
$m_{fuel,0} = 0$ kg	$m_{fuel,f} = \text{free}$

The optimization goal is again to minimize the consumed fuel. 8 intervals with 20 nodes per interval were used.

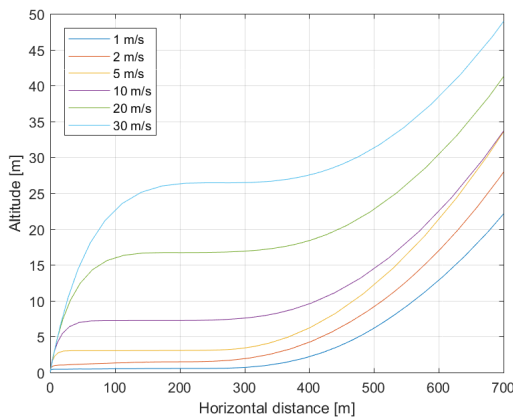
$$\min m_f + \int_{t_0}^{t_f} \vec{u} \, dt \quad (6.18)$$

As it can be seen in Figure 6.22a, the tilt-rotor does not instantly starts to climb, but levels off for about 300 m horizontal distance. Interestingly, this comes across to the STOL take-off as described in Figure A.1. The primary concern of the tilt-rotor is to gain velocity instead of altitude. All trajectories begin their climb in between the 300 and 400 m mark. As can be seen in Figure 6.22b, this is the moment that the tilt-rotor reaches a velocity of 60 m/s, which is the optimal speed to climb fuel friendly. The tilt-rotor then follows the optimal height-velocity profile to climb optimally. It can be seen that flights with smaller initial velocity have a steeper velocity gradient as they put in more power to acquire the optimal speed faster.

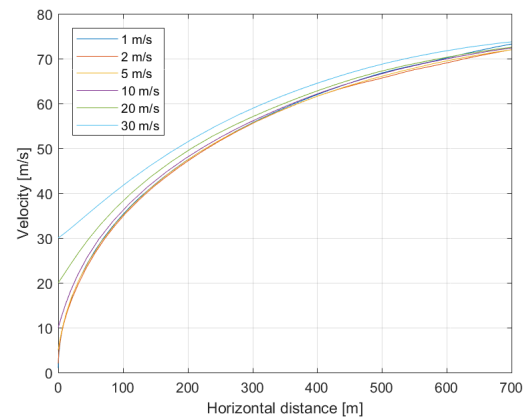
Figure 6.22c shows the optimal nacelle angle the model chooses depending on its initial velocity. It can be concluded that a 90° nacelle inclination is not required for take-off with minimal initial velocity. Using 60° might not be expected for taking off with minimal airspeed, but is a logical choice. With this nacelle setting, the rotor is able to use the horizontal thrust to pick up speed, while still producing

sufficient vertical force to remain airborne. Doing this enables the tilt-rotor to use the lift that will be generated by the wings to alleviate the use of the proprotor and hence the fuel. Furthermore, download on the wing is reduced by using this nacelle setting over 90° . It should be mentioned that a minimum nacelle inclination of 60° is required for ground clearance, but the optimized initial nacelle inclinations are feasible if the tilt-rotor has already acquired sufficient ground clearance. No significant difference in the use of the longitudinal cyclic angle was found. It is difficult to justify the fact that the nacelle angle increases at first, and after that decreases again. It can be noticed however, that a clear trend can be identified, across the tops of the curves.

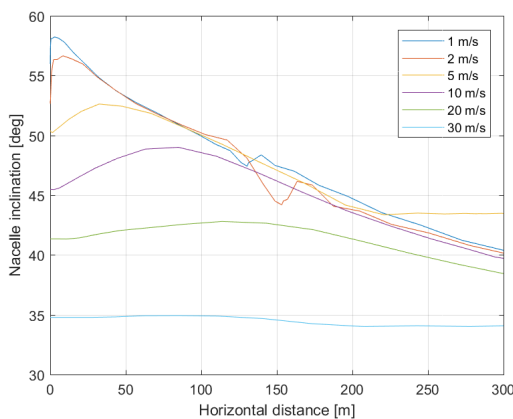
As for the nacelle angle, the model also chooses dissimilar initial flight path angles. The flight path of the point-mass in the first 100 m is depicted in Figure 6.22d, which comes across to the first 5 seconds of flight. It can be seen that the higher the initial velocity is, the lower the initial flight path angle is. The velocity v is in the denominator of the equation of motion for γ (and χ). This enables the tilt-rotor to make rapid changes in the flightpath and heading angle, while at higher speeds it takes longer to induce a change in the respective angles. This explains why at slower speeds the changes in angle are so rapid. Therefore, the higher initial speeds, choose a lower initial flightpath angle in order to converge faster to 0° and hereafter, the flight path angles follow the same climb trajectory.



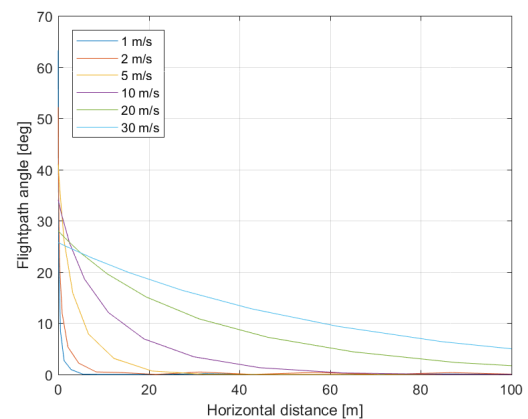
(a) Flight paths of climb-out flight behaviour with various initial velocities



(b) Velocity profiles of climb-out flight behaviour with various initial velocities



(c) Optimal nacelle conversion of climb-out flight behaviour with various initial velocities



(d) Optimal flight path angle of climb-out flight behaviour with various initial velocities

Figure 6.22: Flight behaviour of climb-out flight with varying initial velocity

6.3.13. OEI Continued Runway Take-Off

The entire dynamics of the model can be verified by means of another numerical model by Carlson and Zhao, who have made an two-dimensional rigid-body model of the XV-15. Although the point-mass model is three-dimensional, the aircraft can be restricted to two-dimensional flight to be able to validate the model.

The following optimization optimizes runway take-off in the form of a continued take-off after one engine failure. This is defined by Carlson and Zhao as to minimize the runway length required for the tilt-rotor to complete a safe take-off, as stated in Section A.1. Carlson and Zhao conducted optimizations with a variable rotor speed as the allowed for the option to switch the rotor governor on and off. Since this was not the scope of the point mass model, the results will only be compared to the optimizations with rotor speed governor on. Therefore, the optimization was to minimize the final distance, which was conducted using 20 intervals with 4 nodes per interval were used.

Subjected to the generic constraints, an initial nacelle angle of 70° and an initial lift coefficient corresponding to an angle of attack of zero degrees at 40 degrees flaps. The results of this optimization and the results of the rigid-body model have been depicted in Figure 6.23. In order to accommodate for comparison with the results by Carlson and Zhao, the graphs have been depicted in Imperial units.

Table 6.17: Boundary constraints of OEI continued runway take-off

Initial	Final
$x_0 = 0 \text{ m}$	$x_f = \text{free}$
$y_0 = 0 \text{ m}$	$y_f = \text{free}$
$h_0 = 0 \text{ m}$	$h_f = 10.7 \text{ m (35 ft)}$
$V_0 = 20 \text{ m/s}$	$V_f = \text{free}$
$\gamma_0 = 3^\circ$	$\gamma_f = \text{free}$
$i_{n,0} = 70^\circ$	$i_{n,f} = \text{free}$
$C_{L,0} = 1.18$	$C_{L,f} = \text{free}$
$m_{fuel,0} = 0 \text{ kg}$	$m_{fuel,f} = \text{free}$

$$\min x_f + \int_{t_0}^{t_f} \vec{u} \, dt \quad (6.19)$$

As can be seen the point-mass fairly represents the results of the two-dimensional rigid-body optimization, although some differences exist. To start off with, it can be seen in Figure 6.23f that the authors did not take the (transmission) losses of the engines into account, which results in the fact that the rigid body model ends up with more power. Furthermore, it can be seen that the power required almost entirely equals the power available and hence makes use of the available power in an optimal sense. Since no large changes in altitude or velocity occur, and the rotor velocity is kept constant, the power required is mainly governed by the thrust (coefficient) as in Figure 6.23e. It can be seen that it follows the same trend, but that the rigid-body model has a lower thrust coefficient in the first part of the flight. Referring to Figures 6.23a to 6.23d it can be noticed that the rigid-body model follows a different approach than the point-mass, in that the point-mass prioritizes increasing the nacelle angle and climb with relatively higher vertical and lower horizontal speed and a higher initial flight path angle. Where the point-mass model uses it higher energy state to gain altitude, the rigid-body, which has a slight power advantage as mentioned above, uses its energy to gain forward speed to use the advantage of the lifting wings. Regarding Figure 6.23a, the flight path was only partially depicted by Carlson and Zhao. It has been adapted and added to the graph to best knowledge by deducing the time and place of the flightpath angle.

Finally, the optimization of the flight trajectory of the point-mass model yields a final distance of 271 ft, obtained in 4.36 seconds, while the rigid-body model yields 340 ft by minimal time difference in 4.5 seconds.

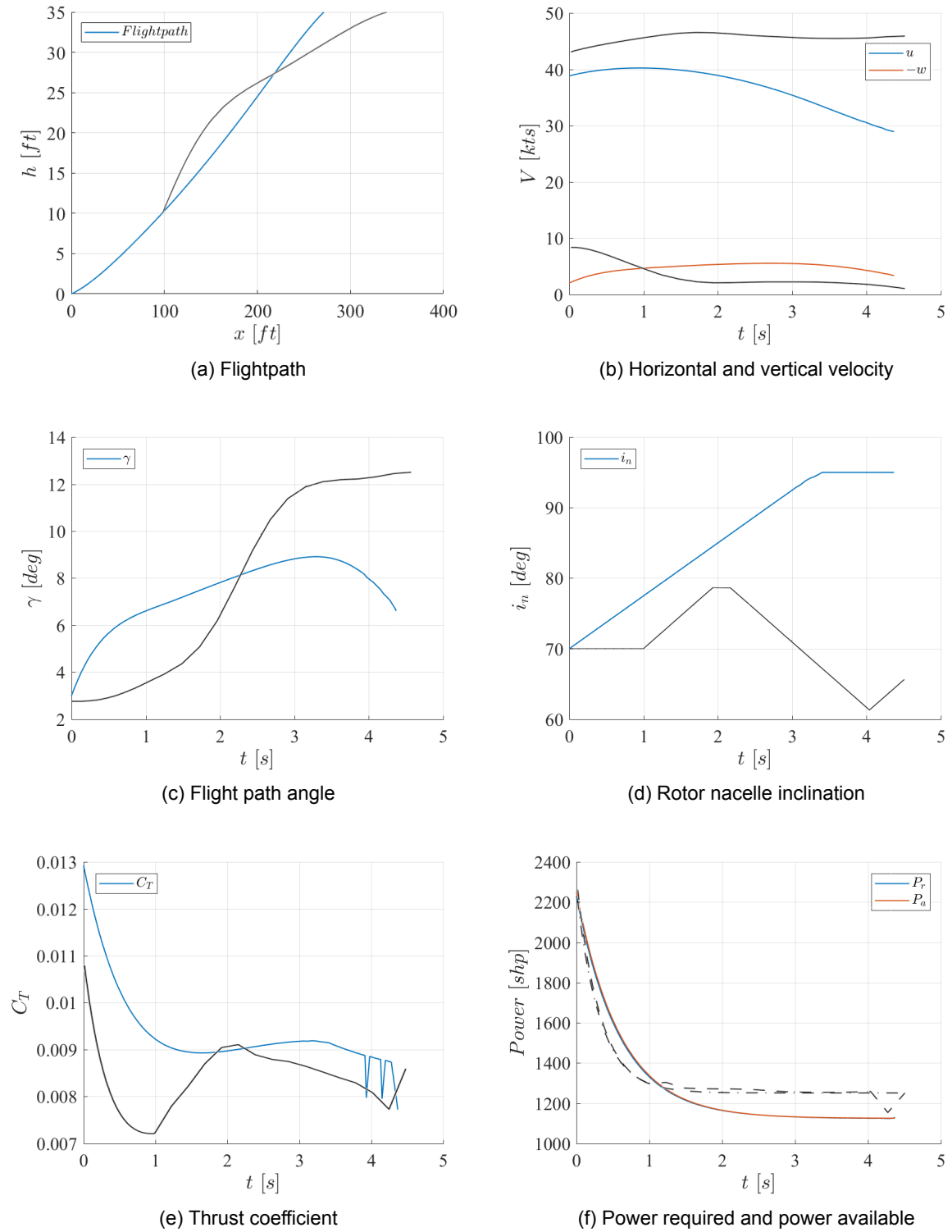


Figure 6.23: Comparison of point-mass model (blue/red) with two-dimensional rigid body model (black) by Carlson and Zhao [10], [9] for continued OEI runway take-off

Tilt-Rotor Model Optimization Applications

Now that the model has been verified and validated, it can be applied to optimize flight trajectories for possible missions. This will be conducted through two distinct mission analyses. The first application is VIP transport from the Monaco heliport to Nice International Airport, as this has been identified to be a suitable market for the tilt-rotor due to the high-demand and upmarket passengers, willing to pay the extra buck. The second application is a medical emergency on an offshore oil rig in the North Sea. Numerous helicopters fly daily from the Netherlands, Norway or the United Kingdom to all the oil rigs and vessels in the North to transport the employees. In the case of an emergency when every second counts, the tilt-rotor can perform the flight much faster and is therefore an extremely beneficial alternative for the helicopter.

7.1. Monaco - Nice VIP Flight

Nice Côte d'Azur Airport (NCE/LFMN) is the primary airport in the region to serve the Côte d'Azur. Due to its proximity to Monaco it also serves as a stopover for helicopter travellers to Monaco. Nice operates two runways and two helipads, and therefore being ideal for tilt-rotor operations, not only because of the high-volume demand, but also due to its facilities. The parallel runways point in North-East and South-West direction, being 040° and 220° respectively. The airport's elevation is 4 m at $43^\circ 39.55'N$ $007^\circ 12.54'E$ ¹.

Monaco Heliport (MCM/LNMC) is the only aerodrome in the Principality of Monaco and operates eight helipads due to the high helicopter volume to and from Monaco. A scheduled service is being operated between Monaco and Nice. It is located on the waterfront at $43^\circ 43.35'N$ $007^\circ 25.14'E$, with an airport elevation of just 6 m. Arrival and departure of helicopter flights take place over sea to minimize the noise footprint in the small state².

From the latitudes, longitudes and map data the distances between both airports could have been determined and converted to the x, y, h coordinate system of the model. The addressed problem is to determine the optimal flight paths and strategies for nominal tilt-rotor flight operations for runway and vertical take-off and landing. The route between Nice and Monaco offers a perfect and realistic case to determine this.

7.1.1. Problem Formulation

Apart from the generic constraints in Equations 4.70 and 4.71, the constraints in Table 7.1 have been imposed to in order to optimize the flight from Monaco to Nice. The flight has been split into two phases, being the departure and approach. Normally, a cruise phase could be added to enforce cruise, but due to the short duration of the flight this is omitted and the model can choose to cruise without being enforced to. In the first phase, the aircraft is only allowed to either climb or fly level, while in the second

¹From: <https://airportguide.com/airport/info/NCE>. Acquired April 04, 2018

²From: <https://airportguide.com/airport/info/MCM>. Acquired April 04, 2018

phase it is only allowed to descend or fly level. The flight is initiated with vertical take-off and ended with a runway landing, with the corresponding boundary constraints as discussed in Table 4.3. As mentioned before, the latitudes and longitudes have been converted to cartesian coordinates due to the small distances.

Table 7.1: Boundary constraints for Monaco - Nice flight optimization

Phase 1: departure		Phase 2: arrival	
$x_0 = 7 \text{ km}$	$x_f = \text{free}$	$x_0 = \text{free}$	$h_f = 0 \text{ km}$
$y_0 = 16 \text{ km}$	$y_f = \text{free}$	$y_0 = \text{free}$	$h_f = 0 \text{ km}$
$h_0 = 36 \text{ m}$	$h_f = \text{free}$	$h_0 = \text{free}$	$h_f = 30 \text{ m}$
$V_0 = 2 \text{ m/s}$	$V_f = \text{free}$	$V_0 = \text{free}$	$V_f = 20 \text{ m/s}$
$\gamma_{0,\dots,f} \geq 0^\circ$		$\gamma_{0,\dots,f} \leq 0^\circ$	
$i_{n,0} = 90^\circ$	$i_{n,f} = 0^\circ$	$i_{n,0} = 0^\circ$	$i_{n,f} \geq 60^\circ$
$\chi_0 = \text{free}$	$\chi_f = \text{free}$	$\chi_0 = \text{free}$	$\chi_f = 220^\circ$
$m_{fuel,0} = 0 \text{ kg}$	$m_{fuel,f} = \text{free}$	$m_{fuel,0} = \text{free}$	$m_{fuel,f} = \text{free}$

The flight is optimized for minimum fuel usage, including a control penalty, using 4 intervals with 20 nodes per interval.

$$\min m_{fuel} + \int_{t_0}^{t_f} \vec{u} dt \quad (7.1)$$

7.1.2. Results

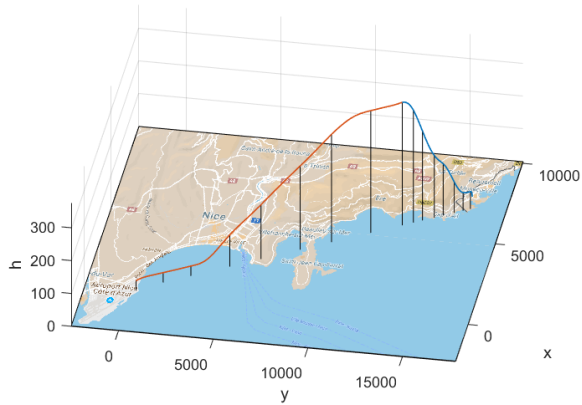
The results of the optimization are shown in the graphs in Figure 7.1. The different colours in the flightpath plot, and the dotted line in the state and control plots indicate the change in phases. The flightpath of the flight can be seen in Figure 7.1a. As can be seen four distinct and expected stages can be observed: climb-out, cruise, descent and braking before landing.

This expected behaviour is reflected in more detail in the states and controls. Figure 7.1b shows that the tilt-rotor immediately climbs and levels off to 50 m, in order for the tilt-rotor to accelerate. After this, the tilt-rotor initiates the climb towards 375 m, while continuing to accelerate until it settles at 74 m/s. This concludes the first phase of a little more than 50 seconds. In the second stage a small cruise is conducted after which the aircraft is constantly descended towards the final altitude. Here, the tilt rotor reduces its speed before landing. During climb, cruise and descent, the aircraft flies at 74 m/s, which is the optimal speed for $(L/D)_{max}$, as was shown in Chapter 6. However, the lift coefficient is not at its optimal value for this. The total fuel consumption of the flight equals 11.7 kg, 6.2 kg of which was during the climb.

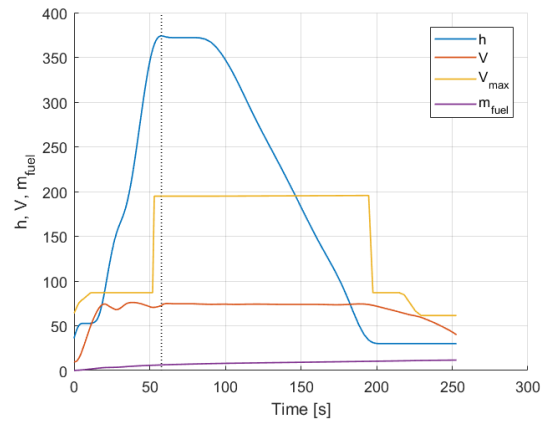
It can be noticed that the climb is not smooth, but is slightly sinusoidal. This is reflected when looking at the curves for h , γ , C_L and V . After analysing the curves it is expected that this combination of altitude, flightpath angle, airspeeds and lift coefficient yields a slight advantage in terms of performance index with respect to a smooth and consistent climb.

While looking at the parameters for longitudinal translation and rotation in Figure 7.1d, it can be seen that the tilt-rotor uses its (pseudo-) controls in an expected fashion to achieve the above mentioned flight. After a helicopter take-off the nacelle converts immediately to 20° and profits from inclined thrust during the climb-out, after which it converts slower towards airplane mode. After this it remains in airplane mode until it re-converts to its maximum inclination to reduce airspeed. It can be seen that the longitudinal cyclic angle is used in the expected fashion, but only in a minor extend: the maximum recorded deflections are $\pm 4^\circ$. A positive pitch during take-off to expedite the acceleration and a negative pitch during landing to reduce airspeed further. In line with the validation of the model, the tilt-rotor descends at its optimal glide angle of -2.56° . It should be noted that passenger comfort was not part of the constraints on tilt-rotor flight. The changes in the flightpath angle γ in the first 50 seconds are not expected to be extremely comfortable. This could however be incorporated in the optimization by imposing upper and lower bounds on the change in flightpath angle $\dot{\gamma}$ using a path constraint.

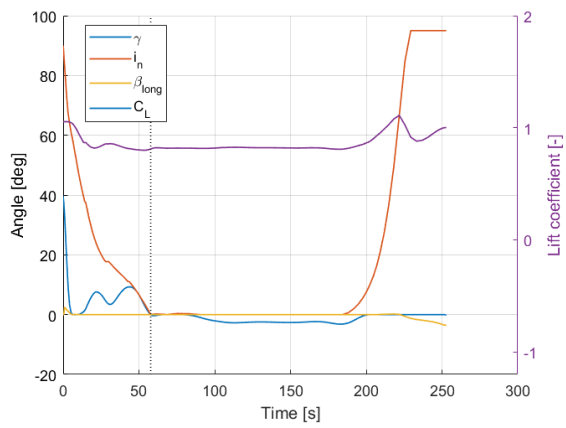
Another method is to implement an additional phase with the sole purpose of conversion. This phase could start and end with 0° and 90° respectively at the pre-set $7.5^\circ/\text{sec}$ conversion rate, while keeping the flightpath angle constant. This iso-pitch conversion is comfortable for passengers and



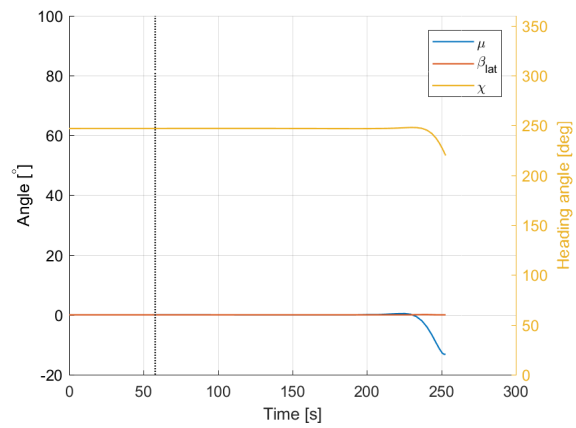
(a) Tilt-rotor flightpath of Monaco - Nice flight



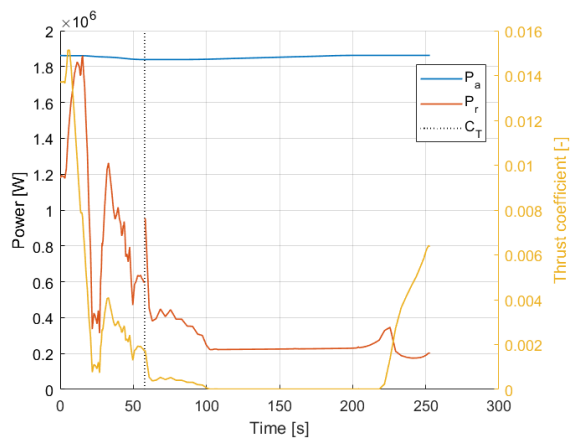
(b) Tilt-rotor altitude, velocity and fuel consumption during Monaco - Nice flight



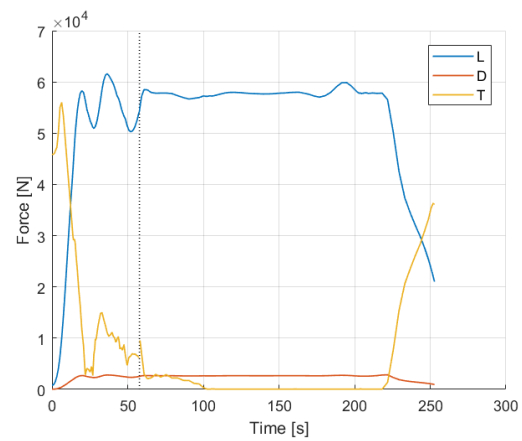
(c) Tilt-rotor longitudinal states and controls and lift coefficient during Monaco - Nice flight



(d) Tilt-rotor heading angle and lateral states and controls during Monaco - Nice flight



(e) Tilt-rotor powers and power coefficient during Monaco - Nice flight



(f) Tilt-rotor lift, drag and thrust during Monaco - Nice flight

Figure 7.1: Results of Monaco - Nice flight optimization

crew and hence forces the aircraft to increase or decrease speed while converting. Another point is the absolute value of the vertical velocity, which is now unconstrained. Similar to the latter, this can be resolved by implementing maximum bounds for $\dot{h}$ that are commonly used for vertical speed in aviation such as 1.800 feet per minute for regular operation and around 5.000 feet per minute in emergency.

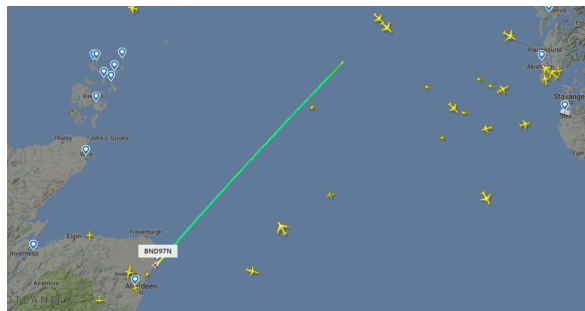
Figure 7.1c depicts the states and controls for lateral movement of the tilt-rotor. It can be seen that the model opts to take-off in the direction of 250° and at the last moment makes the turn to 220° to land at runway 22. The turn is made by banking the aircraft at the last stage of the flight for two reasons. Not only because of its ability to make the turn faster than mid-flight due to its lower speed, but more importantly because this trajectory in the XY-plane yields the shortest path. The lateral cyclic is not used.

Figure 7.1e depicts the power available and the power required with its corresponding thrust coefficient. It can be seen that maximum power is required during the helicopter take-off, which reduces during the climb, as the velocity increases, more lift is created and less thrust is needed. Only a small amount of power is required during the cruise, and the engines run in idle during descent. Before arriving at the runway the thrust coefficient is increased to reduce airspeed. This is reflected in the curves for lift, drag and thrust in Figure 7.1f, where the lift and thrust balance each other to carry the weight of the tilt-rotor, while using the surplus to accelerate and climb and the deficit to decelerate and descend.

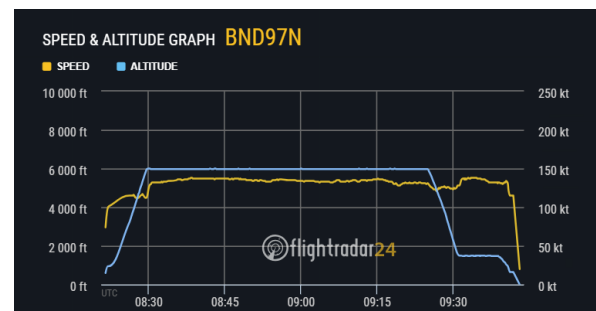
7.2. Offshore Medical Emergency Flight

The ground underneath the North Sea holds a lot of natural resources in the form of oil and gas. Because of this reason, numerous oil platforms have been erected in order to extract this from the Earth. A huge drawback of these platforms however, is their desolateness as they are located hundreds of kilometres offshore. In case of a (medical) emergency medical equipment and staff is available, but in the worst case scenario a medical evacuation is necessary.

To optimize for such an event a typical helicopter flight to or from an oil platform has been identified from flightradar24.com³. In Figure 7.2a a flight from an oil platform to Aberdeen is depicted. This flight is an ordinary flight, that most likely transports crew to and from the platform. Its altitude and velocity is depicted in Figure 7.2b. The helicopter that was used on this rotation was a Sikorsky S-92A, a typical helicopter in the offshore industry. The Sikorsky S-92A is slightly larger than the XV-15, with a maximum take-off weight of 27.700 lbs (12.568 kg) and a maximum speed of 85 m/s. It is propelled by two 2.520 shp engines. Using offshore map data, the oil platform was identified to be the Beryl-A, exploited by ExxonMobile⁴.



(a) Groundpath of typical Beryl-A - Aberdeen helicopter flight



(b) Altitude and velocity profile of typical Beryl-A - Aberdeen helicopter flight

Figure 7.2: Flight data of typical Beryl-A - Aberdeen helicopter flight

Aberdeen Airport (ABZ/EGPD) is a medium large airport that is however, more renowned as world's busiest heliport. This is due to the fact that numerous helicopter operators connect North Sea oil platforms with mainland through Aberdeen. Aberdeen operates four runways, three of which specifically for helicopters. Two helicopter runways H05/H23 and H14/H32 hold 476-581 meters, while the third runway H36 amounts 260 meters. The regular runway 16/34 is 1.829 meters. All runways are suitable for the tilt-rotor, with the smallest runway H36 being tight in the margins. The airports elevation is at 66

³From: flightradar24.com. Acquired May 01, 2018

⁴From: Offshore Magazine, North Sea Offshore Oil & Gas Map, 2013

and is located at 57°12'09"N 002°11'53"W [62]. Beryl-A is an oil platform operating for ExxonMobile, which location is approximated at 59°31'00"N 001°28.5'E, about 335 km North-East of Aberdeen. The rig features a helideck, that is 50 meters above sea-level.⁵

7.2.1. Problem Formulation

Similar to the previous optimization, specific constraints are imposed on the model to optimize the flight from Aberdeen to the oil platform, which are documented in Table 7.2.

Table 7.2: Boundary constraints for oil platform medical evacuation

Phase 1: departure		Phase 2: arrival	
$x_0 = 0$ km	$x_f = \text{free}$	$x_0 = \text{free}$	$x_f = 257$ km
$y_0 = 0$ km	$y_f = \text{free}$	$y_0 = \text{free}$	$y_f = 205$ km
$h_0 = 65$ m	$h_f = \text{free}$	$h_0 = \text{free}$	$h_f = 50$ m
$V_0 = 20$ m/s	$V_f = \text{free}$	$V_0 = \text{free}$	$V_f = 10$ m/s
$\gamma_{0,\dots,f} \geq 0^\circ$		$\gamma_{0,\dots,f} \leq 0^\circ$	
$i_{n,0} \geq 60^\circ$	$i_{n,f} = \text{free}$	$i_{n,0} = \text{free}$	$90^\circ \leq i_{n,f} \leq 95^\circ$
$m_{fuel,0} = 0$ kg	$m_{fuel,f} = \text{free}$	$m_{fuel,0} = \text{free}$	$m_{fuel,f} \leq 675$ kg

The flight is set to minimize time, including a control penalty:

$$\min t_f + \int_{t_0}^{t_f} \vec{u} dt \quad (7.2)$$

7.2.2. Results

The results of the optimization of a medical emergency at ExxonMobile's Beryl-A platform are depicted in Figure 7.3. Comparing the tilt-rotor's flightpath in Figure 7.3a to the flightpath of the Sikorsky in Figure 7.2a, they show comparable flightpaths in that they both choose the direct route. The tilt-rotor's flightpath shows that the tilt-rotor gradually climbs towards its cruising altitude and after cruising descends in a two-staged descent from cruising altitude to the platform.

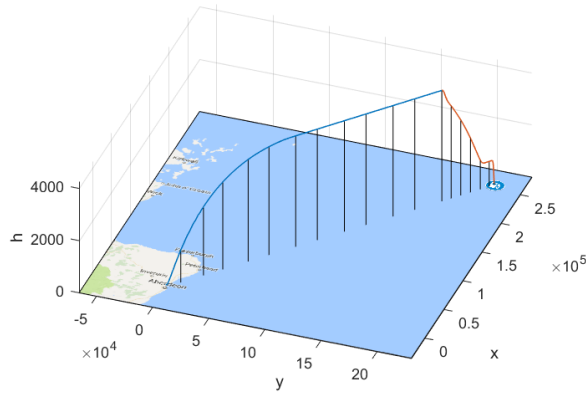
As Figure 7.3b shows, the tilt-rotor smoothly climbs to 4.250 meters, which is the same ceiling as was previously determined in the cruise validation for minimum time (Sec. 6.3.3). The combination of the altitude and velocity is limited due to the tilt-rotor flying at $P_a = P_r$. After the first phase, the tilt-rotor immediately starts its first stage of the descent. In this stage the velocity increases. The tilt-rotor levels off at 1.000 meter to use drag to slow down. In the last minute the last velocity and altitude is dumped with 95° nacelle inclination, as helicopter.

This can better be seen in the velocity-altitude profile in Figure 7.3c. This shows clearly that the tilt-rotor does not start to climb until it has increased its velocity. The velocity increases as the tilt-rotor gains altitude. The two-staged descent is clearly visible: the first part that only decreases the altitude, and the second part that focusses on decreasing the velocity. From this it could be concluded that 1.000 meters is the optimal altitude for the point-mass model to start its final stage of landing and to get rid of a velocity of 180 m/s, due to the combination of air density and remaining altitude. Descent takes about six minutes implying a descent rate of roughly 2.200 feet per minute, which is on the high side but in ballpark range, especially due to a medical emergency.

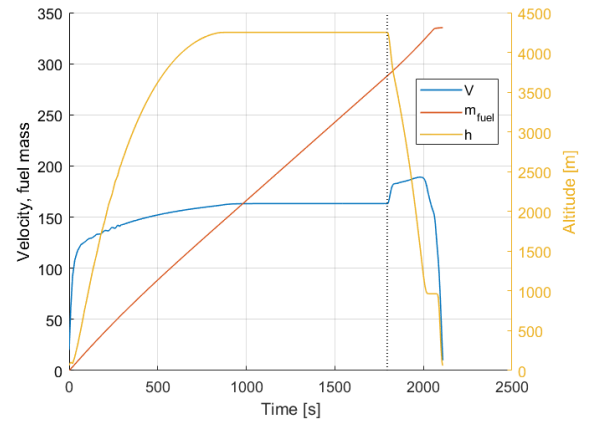
The flight trajectory and states can be compared to the ones that currently are being operated by helicopters. Comparing to Figure 7.2b, the tilt-rotor flies approximately twice as high because its pressurized cabin allows to do so and a velocity approximately twice as high because of its airplane capability. The helicopter flight's duration was 1 hour and 23 minutes, while the tilt-rotor flew the same distance in 35 minutes. It should be mentioned that the reference flight did not fly as fast as it could, but with its maximum speed of 165 kts (85 m/s), the difference still would be significant. Finally, 335 kg of fuel is used for this flight to arrive at the oil platform in 2110 seconds. 335 kg is half of the total fuel weight, which means that the tilt-rotor needs to refuel to be able to land in Aberdeen with a fuel reserve.

The states corresponding to the longitudinal plane in Figure 7.3d show the behaviour that was already seen in the previous graphs. The tilt-rotor starts at 60° inclination and converts to 10° and the

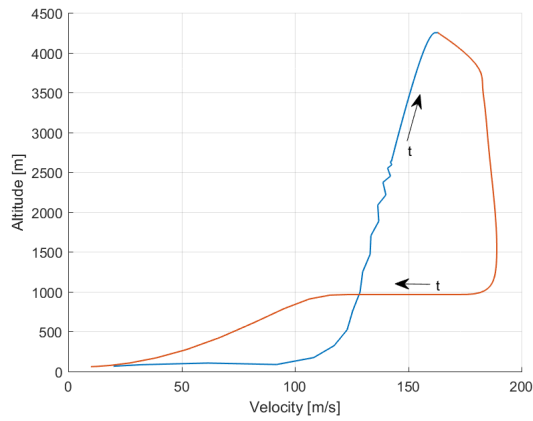
⁵From: helidecks.org. Acquired June 07, 2018



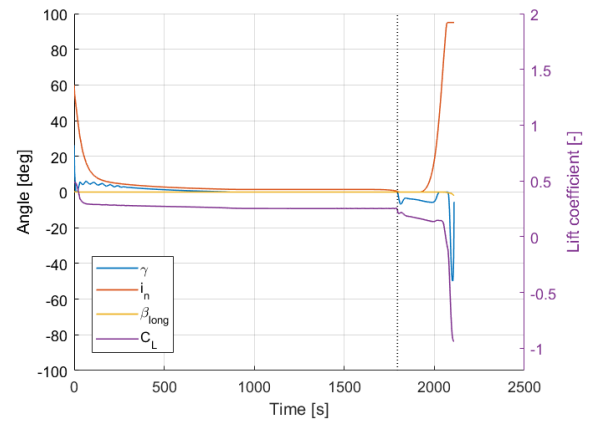
(a) Tilt-rotor flightpath of Aberdeen - Beryl A flight



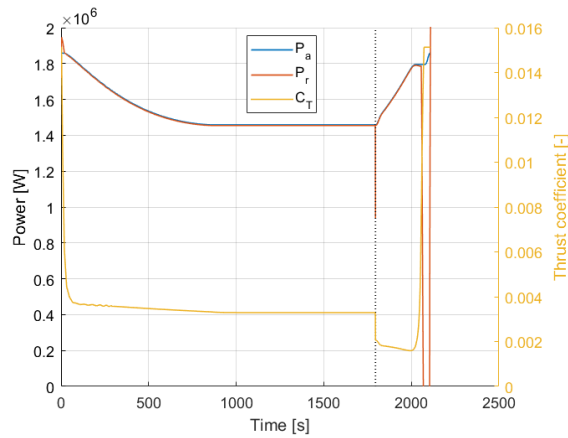
(b) Tilt-rotor altitude, velocity and fuel consumption during Aberdeen - Beryl-A flight



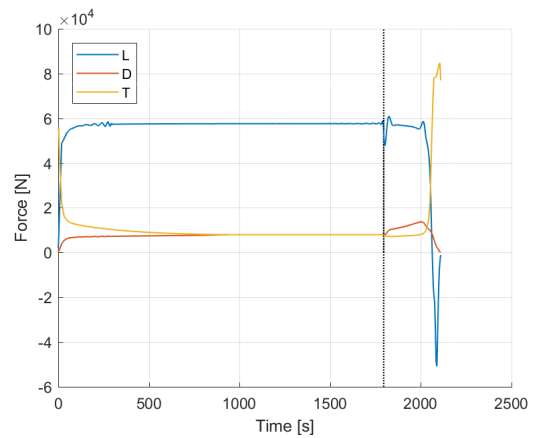
(c) Tilt-rotor altitude-velocity profile during Aberdeen - Beryl-A flight



(d) Tilt-rotor longitudinal states and controls and lift coefficient during Aberdeen - Beryl-A flight



(e) Tilt-rotor powers and power coefficient during Aberdeen - Beryl-A flight



(f) Tilt-rotor lift, drag and thrust during Aberdeen - Beryl-A flight

Figure 7.3: Results of Aberdeen Airport- Beryl-A medical evacuation flight optimization

slowly converges to 0° . During cruise the nacelle's inclination remains at 1.5° . The benefit of airplane mode was modelled to start at a nacelle inclination of 5° . The inclination of 1.5° allows the aircraft to have a slightly lower lift coefficient and corresponding drag, which results in a tiny time advantage. During the landing it converges straight from 0° to 95° to be able to dump speed as rapidly as possible.

The aircraft starts to climb at 5° , which decreases slowly to 0° . In the last seconds it descends with a flight path angle of up to 50° , which is reduced in the very last seconds which acts as a flare. The same lift coefficient is used as was proven in the validation. In the second phase the lift coefficient is drastically reduced to descend. The longitudinal cyclic is minimally used in the last seconds to decrease velocity further.

Figure 7.3e tells us that the aircraft flies at maximum power the entire time, except for a few seconds during the last seconds of the descent. From the thrust coefficient it can be concluded that full thrust is given in the first part, after which the thrust coefficient settles at the value that was determined in the cruise validation. This reduces during the descent, only to spike up during the last seconds of the flight. This is reflected in the lift, drag and thrust in Figure 7.3f. The first phase features ordinary behaviour, but the second phase shows different behaviour. Contrary to the expected does the increase in velocity and decrease in lift coefficient result in an increase of drag. The negative lift allows the tilt-rotor to descent more rapidly.

7.3. RTM Standard Instrument Departure

As discussed in the literature review, tilt-rotor aircraft have a large potential, especially when it comes to passenger transportation, either commercial, offshore or VIP transportation. Regardless of its application, it has to adhere to international air traffic control. When the operator files a flight plan as helicopter under Visual Flight Rules (VFR), the tilt-rotor can fly with more freedom, but flying with Instrument Flight Rules (IFR), flight is more regulated and more rules need to be adhered to. Most airports therefore have standardized approach and departure routes.

Table 7.3: To be followed waypoints for REFSO and TULIP SID

Waypoint	Downrange [NM]
THR 24 (Runway threshold)	0
EH159	2.8
EH158	5.9
EH156	11.0
SOMEL	14.7

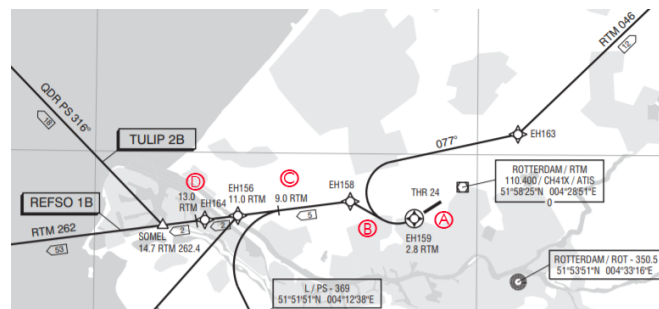


Figure 7.4: Cut-out of REFSO and TULIP SID of Rotterdam/the Hague airport runway 24. Adapted from: [1].

There is high business traffic from Rotterdam/the Hague airport (RTM/EHRD) to a few European cities, especially London. Due to the congested airspace in London there could be a market for business flights from Rotterdam/the Hague airport to the city-centre of London or one of the London airports.

In order to assess how the tilt-rotor fits in the current air traffic system, a Standard Instrument Departure (SID) from Rotterdam/the Hague's runway 24 will be optimized for. Both the westbound REFSO-1B and north-westbound TULIP-2B SID, follow the same departure trajectory up to the SOMEL waypoint. Concise information regarding the REFSO and TULIP SID is shown in Figure 7.4, while the to be followed waypoints are summarized in Table 7.3. The complete SID chart can be consulted in Appendix D.

7.3.1. Problem Formulation

The coordinates of the waypoints have been translated to the models coordinate system, and the remaining boundary constraints for the SID and take-off as in Table 4.3 have been added. Constraints for the optimization are given below in Table 7.4. From flightradar24 data it was determined that an aircraft passes the waypoint EH163 at approximately 3000 ft, which resulted in the final altitude for this optimization, moreover is this the transition altitude.

Normally, an airline operator would perform such a take-off with the goal to spend as little fuel

Table 7.4: Boundary constraints for REFSO and TULIP SID up to SOMEL waypoint

State	A: THR 24 - EH159		B: EH159 - EH158		C: EH158 - EH156		D: EH156 - SOMEL	
	Initial	Final	Initial	Final	Initial	Final	Initial	Final
x	0 km	-1.6 km	-1.6 km	-0.2 km	-0.2 km	-1.5 km	-1.5 km	-2.4 km
y	0 km	-3.2 km	-3.2 km	-10.6 km	-10.6 km	-23.5 km	-23.5 km	-32.1 km
h	0 m							≥ 3000 ft
V	20 m/s							
γ	8°	$\geq 0^\circ$	$\geq 0^\circ$	$\geq 0^\circ$	$\geq 0^\circ$	$\geq 0^\circ$	$\geq 0^\circ$	$\geq 0^\circ$
χ	237°							
i_n	$\geq 60^\circ$							
m_{fuel}	0 kg							

as needed. However, the combination of the complexity in determining the power required and the complexity of the four-phased problem, leads to high difficulty to converge the problem to an optimal solution. This will be elaborated in further detail in the recommendations. Therefore, this problem will be optimized for minimum time, including a control penalty. Nevertheless, this still will form a relevant outcome as there are sufficient situations where it is plausible that an aircraft would like to take-off as fast as possible. To emphasize this, examples of plausible situations of this are a private courier to deliver organs, stem cells or legal documents as soon as possible or a delayed aircraft that flies at maximum cost index in order to avoid paying compensation after arriving too late.

$$\min t_f + \int_{t_0}^{t_f} \vec{u} dt \quad (7.3)$$

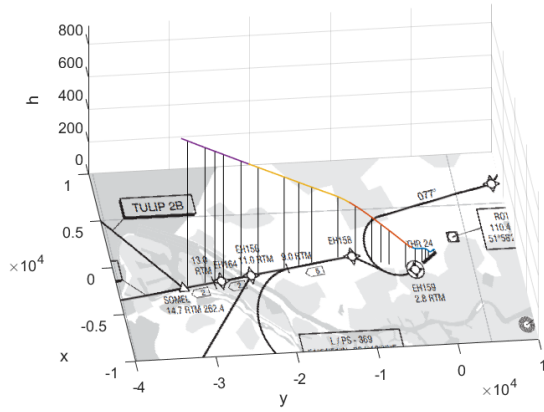
7.3.2. Results

The results of the optimization have been shown in Figure 7.5. The 3D flightpath shows that point-mass model is able to follow the SID quite correctly and that there is only a slight offset can be observed in the second segment, shown in red in Figure 7.5a.

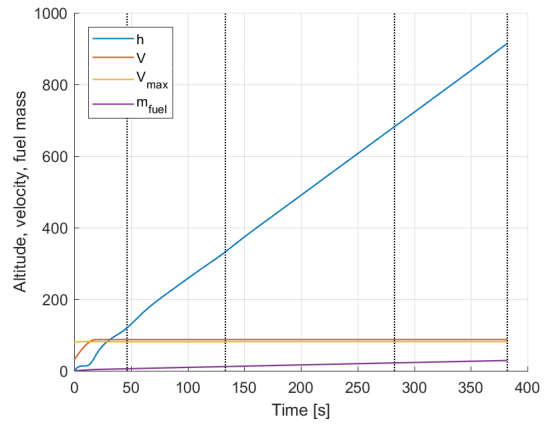
Interestingly though, the tilt-rotor levels off at 12 m altitude in the first segment, where it accelerates up to its maximum speed, as it can obtain high speed faster while in level flight than during climb. This can be seen in Figure 7.5b. After having obtained this, the tilt-rotor initiates the climb and climbs steadily to its final altitude. Obviously, a direct continuous climb is desired, and it is expected that optimizing for minimum fuel consumption or a combination of minimum fuel consumption and minimum time will improve this behaviour. The tilt-rotor ends the SID at 3.000 ft in 380 seconds, after having used 29 kg of fuel, which corresponds to $\sim 4\%$ of its maximum fuel weight; a feasible value for the first part of the climb-out, especially since it is optimised for minimum time.

Figure 7.5c depicts the parameters in longitudinal plane. It can be seen that the tilt-rotor starts at the minimum inclination for runway take-off of 60° and decreases slightly to 55° , while the tilt-rotor steadily climbs through segments B, C and D at a flightpath angle of 1.5° , and a lift coefficient of 0.55. Both of these are lower than the optimal flightpath angle and lift coefficient for the climb in minimum time in the validation. This however, is due to the fact that the tilt-rotor is more constrained with respect to its position and altitude in the SID, while in the validation it was only bounded by its altitude. The tilt-rotor does not directly convert to 0° nacelle inclination, which implies that, in this case, the optimal climb strategy is at 55° nacelle inclination which balances the lift and vertical and horizontal thrust. No longitudinal cyclic input was given.

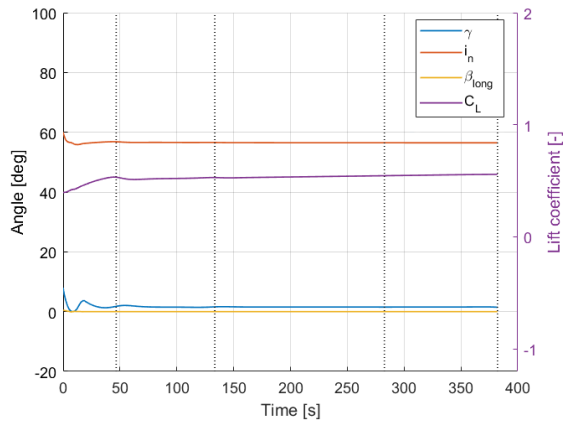
The lateral parameters in Figure 7.5d show the lateral movement of the tilt-rotor. It can be determined that the tilt-rotor achieves a total heading change of about 75 degrees ($237^\circ - 287^\circ - 263^\circ$) in approximately 140 seconds. In the Monaco optimization for minimum fuel this was about half the value (30° in about 30 seconds) This emphasises the incredibly smooth turning of the model contrary to the tight and fast turns when optimized for minimum fuel consumption. When the aircraft banks, the lift and thrust vector are tilted at the bank angle in order to coordinate the turn. This results in a loss of vertical force as part of the lift and thrust are in the longitudinal plane and part of them in the lateral plane. Now that the flight is optimized for minimum time, it does not matter that the engines need to provide more power to equal the less of vertical force, whereas the for minimum fuel consumption the point-mass



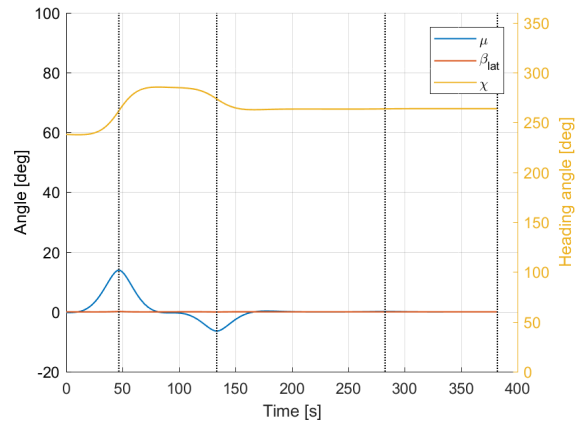
(a) Tilt-rotor flightpath of REF/SO/TULIP SID



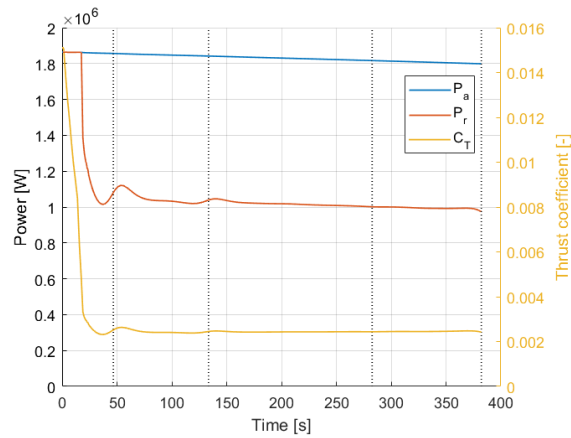
(b) Tilt-rotor altitude, velocity and fuel consumption during REF/SO/TULIP SID



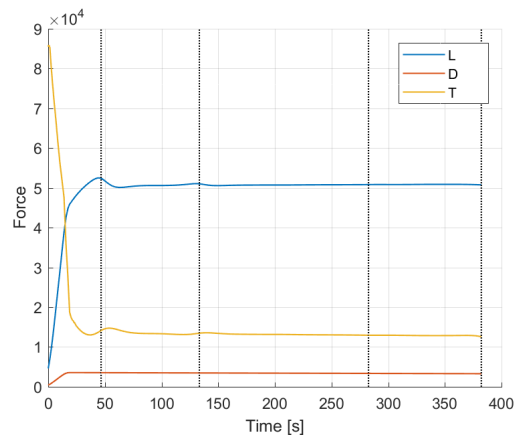
(c) Tilt-rotor longitudinal states and controls and lift coefficient during REF/SO/TULIP SID



(d) Tilt-rotor heading angle and lateral states and controls during REF/SO/TULIP SID



(e) Tilt-rotor powers and power coefficient during REF/SO/TULIP SID



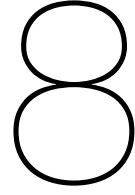
(f) Tilt-rotor lift, drag and thrust during REF/SO/TULIP SID

Figure 7.5: Results of runway 24 REF/SO/TULID SID optimization

wants to minimize the time the aircraft spends in the turn. As for the longitudinal cyclic, no lateral cyclic input was given.

Finally, the earlier findings on the velocity are confirmed in Figure 7.5e. It can be seen that the aircraft flies at maximum power in the first segment in order to increase velocity as rapidly as possible. Until it has reached its maximum velocity, less power is required, and the thrust coefficient is reduced until the power required converges to its steady-state value for the climb-out. This is reflected by the lift, thrust and drag in Figure 7.5f. It can clearly be seen that a large thrust force in the first segment is used to increase velocity until the maximum velocity is reached. This is reflected in the drag that increases similarly to the velocity, as the drag coefficient does not alter significantly throughout the flight. Lift and vertical thrust balance each other to equal a slight surplus over the weight, to increase velocity and climb, until the steady-state is reached. Although the lift still varies slightly, the drag has already settled at a constant values, which comes across odd at first, as these are correlated to each other. However, when consulting the the drag polar (Fig. 6.1a), it can be seen that the drag coefficient does not change at a lift coefficient between 0.25 and 0.75. Finally, it can be noticed that the lift-to-drag ratio is in the range of 6.0 to 7.0, which is quite to the contrary of the L/D of up to 19.0 that is achieved when optimizing for minimum fuel consumption. This was expected however.

All in all, it can be concluded that flight behaviour differs significantly when the tilt-rotor is bounded by predefined waypoints and altitudes compared to the situation where the tilt-rotor has more freedom. The main differences comprise the conversion strategy and banking. Comparing the SID departure with the Aberdeen departure, it can be seen that where the Aberdeen departure immediately converts to airplane mode, the SID departure converts to 55° nacelle inclination, although both the aircraft fly at comparable speed and flightpath angle. Furthermore, turning results in far more smooth curves.



Conclusions and Recommendations

Now that the research is concluded, a concise overview of the conclusions is elaborated in Section 8.1. This is followed by the limitations and recommendations of this research that can result in future work in Section 8.2.

8.1. Conclusions

In recent years, the tilt-rotor concept has been further developed by several companies to exploit their potential. In order to do so, modelling of the aircraft and the trajectory optimization that follows from this give powerful insights for flight testing and further development. Currently, tilt-rotor aircraft have only been modelled in two dimensions as rigid-body model in order to optimize short duration flights such as take-off and landing. These flights were in the order of just a few hundred meters and less than half a minute.

From this follows the need for a three-dimensional model that goes beyond the vicinity of the runway and thus can perform integral flight optimizations. Referring back to the research objective in Chapter 3, it was the objective to develop a numerical model for a tilt-rotor type aircraft that enables optimization studies in three-dimensional flight trajectories and procedures. To achieve this goal the following research question was asked: *How can a three-dimensional tilt-rotor aircraft efficiently be modelled to optimize for flight trajectories and procedures, in order to assess and enhance safety and performance?* To be able to answer this question efficiently, the project was split in four distinct subquestions.

First, it was determined what the main principles are of tilt-rotor flight and its power and control, how these differ from ordinary airplane or helicopter flight and how they affect tilt-rotor flight behaviour. This was followed by looking into how their mechanics and dynamics can be modelled. It was concluded that in order to optimize its flight trajectories, a tilt-rotor type aircraft, in the form of the XV-15, can best be modelled as a three-dimensional point-mass model using three degrees of freedom. The model consists of eleven states that account for its position, airspeed, flightpath and heading angle and fuel consumption. Four pseudo-controls have been added to the states to account for the bank angle, nacelle inclination, lift coefficient and thrust coefficient. The model is controlled through the four controls belonging to these pseudo-controls and two controls for the longitudinal and lateral cyclic flapping and is concluded with the tabular data of the XV-15 on the lift, drag and power and integrating other factors such as download. The state and control vector being:

$$s = [x, y, h, V, \gamma, \chi, \mu, i_n, C_L, C_T, m_{fuel}]$$
$$u = [\mu_{rate}, i_{n_{rate}}, C_{L_{rate}}, C_{T_{rate}}, \beta_{long}, \beta_{lat}]$$

Before acknowledging and accepting the derived model however, it was to be validated in order to be able to use the results with confidence. A multitude of both qualitative and quantitative validations have been performed in order to accept the model. Qualitative in that the results were assessed for expected and realistic flight behaviour and if not, if the unexpected behaviour can be explained due to tilt-rotor mechanics or the modelling of the latter. Validation was quantitatively in that flight parameters that had been determined analytically, resurfaced in the dynamic optimizations. It was concluded that

the results of the model can be acknowledged and accepted with confidence for its purpose to optimize flight trajectories for tilt-rotor type aircraft under the assumptions and simplifications that were imposed in the definition of the model. This could be concluded due to the verification and validation of the model's mechanics and dynamics. It was concluded that it correctly represents the aerodynamic and propulsive properties of the XV-15, as the model is able to correctly match expected and analytically determined flight behaviour and flight parameters.

It was however also concluded that the model is not able to match the XV-15 documentation's hover ceiling as it overshoots this by 300-600 m. This could however be expounded by assessing the relevant parameters. A second point that could not be validated in its entirety is the fact that the model underestimates the single engine power available by 10-20%. It is foreseen that single engine operation allows for a higher $C_{T,max}$.

A few conclusions can be drawn from the flight behaviour and strategy that the model shows under the imposed flight goals. The nacelle inclination and conversion strategy is the most interesting part of the results of the optimizations. It could be seen that the point-mass model always wants to convert its nacelle to airplane mode. This goes primarily for the cases when distance is to be covered and flight is being optimized for either minimum time or minimum fuel consumption. In both cases it is desired to have a lower power required. For an as low as possible fuel consumption, a low power required is required. For minimum time, a lower power required results in more excess power that can be used to speed up until the velocity constraints are met. Low power required is the result of two things: not only is the rotor speed reduced in airplane mode but the velocity component parallel to the rotor plane is close to zero, resulting in an equally low advance ratio.

The point-mass model always ends an optimization with low final velocity at 95° nacelle angle and the use of thrust vectoring through blade flapping, as this is its only way of reducing velocity, apart from the aerodynamic drag.

During the climb in various optimizations it was seen that the nacelle does not convert to airplane mode immediately, however. Rotor thrust was initially tilted up to 45° nacelle angle, from which point the nacelle further converted at a slower rate. Analysis of the trajectories showed that as soon as the optimal climb velocity is reached, the lift of the wings carry almost the entire weight of the aircraft. Instead of increasing velocity or increasing the lift coefficient, which would increase drag, the thrust is used to keep excess vertical force. In this first part of the climb, the nacelle angle is kept at an intermediate value. At the point that the positive flightpath angle is reduced, no excess vertical force is needed anymore. At this point the nacelle is slowly reduced to 0° to balance the vertical forces. This decrease in thrust happens at the same rate as the lift force increases due to the higher velocity.

Earlier it was concluded that the tilt-rotor climbs more optimally the lower the nacelle angle is as this raises the maximum velocity and also reduces the power required, which is beneficial for both flights optimized for both fuel and time. Since maximum velocity is bounded between 45° and 0° at the same value, the global optimal climb angle is at 0°, while a local optimum occurs at 45°. This was reflected in the results where the model quickly converts to 45°, and hereafter converts to 0 degrees at a slower pace to balance forces as it has to incorporate other factors and conditions. At the point where i.e. the climbing rate reduces again, and no excess vertical force is needed, the nacelle is slowly reduced to keep the balance of vertical forces together with lift and weight.

Furthermore, conclusions could be drawn on the combination of flight profile and conversion strategy used. In cruise flight assessment it was seen that the point-mass model starts to fly a different flight trajectory when the flying distance increases. It could be identified that under the imposed boundary constraints there exists a tipping point in the cruise distance that yields lower fuel consumption from that point onward. When flying less than 37.5 km, a small climb is combined with a slow descent towards the final distance. When the final distance is further than the aforementioned 37.5 km, the tilt-rotor climbs higher to be able to profit from the less dense air, resulting in a lower fuel consumption. Contrary to the flight trajectories belonging to smaller flight distance, the tilt-rotor approaches the landing site as helicopter with slightly higher speed which is dissipated with a flare.

A further conclusion that can be drawn from the various flight optimizations is the fact that the tilt-rotor does not start to climb until the optimal climb velocity, either for minimum fuel or time, has been reached. This proves the hypothesis that, if the surroundings allow to do so, it is more beneficial to take-off by means of a STOL take-off or runway take-off and that a vertical take-off should only be performed if that is the only possible procedure. STOL and runway take-off also forms a safer method in the event of an engine failure, as the aircraft has additional vertical force through the generated lift.

Finally, it was looked into whether the model can be utilized for realistic applications such as commercial flights. Three scenarios have been optimized: a flight from the Monaco Helipad to Nice International Airport, a medical emergency flight to an oil platform in the North Sea and a Standard Instrument Departure at Rotterdam/the Hague airport.

- For the Monaco - Nice flight it was concluded that by bisecting the problem into a climbing and descending phase and by imposing only the initial and final constraints on the position, velocity and nacelle inclination an expected and realistic flight trajectory with a climb, cruise and descend was yielded.
- The same thing goes for the medical offshore emergency, which was a comparable problem that however differs in a longer distance, different take-off and landing and that it is optimized for minimum time instead of fuel consumption. Similar conclusions could be drawn from this problem. It was concluded that the trajectory forms a similar flightpath to the nowadays used flightpaths, this however under tilt-rotor assumptions.
- Thirdly, tilt-rotor flight was optimized for a Standard Instrument Departure to assess tilt-rotor flight behaviour in these predetermined conditions. It was concluded that when the tilt-rotor is more bounded by ATC flight rules, the flight behaviour differs significantly to flight when the tilt-rotor has more, or even complete, freedom to choose its flightpath. This difference lies mainly in the conversion strategy and turning behaviour. Contrary to complete freedom, the tilt-rotor does not convert directly to airplane mode and banks more smoothly.

After applying the optimization model to these three situations, it was concluded that the model is capable of optimizing flights in their entirety and that the model produces meaningful results when being applied to real-life conditions. The most interesting outcome of the optimizations is the strategy the model employs for the rotor nacelle inclination, the unique aspect of the tilt-rotor. In general, the tilt-rotor wants to convert from helicopter mode to airplane mode, and vice versa, as quick as possible. There is however, an exemption to this: the rate of change of the nacelle angle during climbing flight is smaller than during descending flight. To overcome gravity in climbing flight, the tilt-rotor wants to profit longer from the benefit of being able to tilt the thrust vector, as it needs to put force in both the vertical as horizontal direction to increase the vertical velocity and increase lift. In descent the tilt-rotor re-converts as quick as possible to decrease velocity. From various optimizations it was determined that climbing flight, the tilt-rotor slows down the conversion process in the region between 40° and 10° . This is due to the fact that the optimal lift-to-drag ratio and corresponding velocity are leading in the fuel consumption. A certain amount of generated lift relates to this optimal velocity and therefore, the tilt-rotor wants to fly at this optimal climbing velocity. In order to minimize the performance index while adhering to the constraints, the tilt-rotor supplements its vertical force with rotor thrust. This is an interesting finding as the actual XV-15 only has one pre-set angular conversion speed of 7.5° .

A second conclusion that can be drawn from all three optimizations is that the tilt-rotor shows the same flight behaviour during initial climb, regardless if the aircraft is optimized for minimum fuel or minimum time, in that it tends to level off directly after take-off in order to increase velocity faster. Velocity is increased either to the optimal climb velocity for minimum fuel consumption, or to maximum velocity for minimum final time. It is difficult to counter this behaviour when only a departure and arrival phase are imposed, or when flight is conducted in a single phase as the model will always find its way to the optimal trajectory. It is moreover undesired to overconstrain the problem with predefined sections of trajectories since the optimal flight behaviour is wanted.

A third conclusion that was drawn from these optimizations was that increasing problem complexity, through more phases or smaller node density because of large distances, increases the importance of the initial guesses and factor for the Lagrange term in the cost function. It turned out that this could result in the problem successfully converging to the optimal solution and diverging. Oddly, a more accurate initial guess did not always lead to a quicker or a more successful optimization. Furthermore, in a minimum time optimization a difference was found between overestimating or underestimating the final time.

8.2. Recommendations

Throughout the development and application of the point-mass model several assumptions and subjects have limited the model's thoroughness and some topics have emerged that should be altered

or looked into in order to increase understanding of tilt-rotor flight and its respective modelling. These limitations are recommended to investigate and add in addition to this thesis research to fully cover the subject, while the recommendations for further research are recommended to conduct further research in, in order to extend our knowledge of modelling and optimizing tilt-rotor aircraft and their flight trajectories.

As discussed in the derivation of the model in Chapter 4, the model is limited in a few ways. In deriving the model, some assumptions for generic aircraft modelling and some tilt-rotor specific assumptions were made in Section 4.1. A few of which, can be altered or omitted to enhance the model and its corresponding results:

- Although the fuel consumption is calculated throughout this thesis, the mass of the aircraft was assumed to remain constant. This was done in order to have a state variable less and prevent the model from an increment of computational time. For completeness of the model, this assumption could be dropped. It is expected that this will enhance results especially for long range optimizations. It is for example expected that cruise behaviour will differ in that the aircraft will climb slightly during the cruise as the aircraft will become lighter and lighter.
- Secondly, no wind field was assumed. This resulted in the fact that the aerodynamic velocity and kinetic velocity of the aircraft are the same and that no wind created a resultant aerodynamic velocity unequal to the aircraft's heading, which simplifies the problem. However, a wind field will not only demand the three-dimensional capability of the model to a larger extend, but can also change tilt-rotor behaviour slightly, as for instance more power is needed during a helicopter take-off or landing, a situation that already asks most of the tilt-rotor's power. It is not expected that omitting the assumption of non-oblate Earth, flat Earth or non-rotating Earth will have a significant impact on a model to optimize tilt-rotor flight trajectories due to their validity for short duration flights.
- Furthermore, the model was limited to manoeuvre inside the flightpath angle envelope of $|\gamma| < 90^\circ$, since a flightpath angle of $\gamma = 90^\circ$ results in singularity of the equations of motion. Although it is thought that flying backwards is only rarely used, especially in optimized flight, it prevents the point-mass model from flying and optimizing nominal, vertical take-off procedures, as these include a linear back-up. The same thing holds for the velocity and thrust coefficient which ought to have a minimal value to prevent singularity with $V \geq 1$ m/s and $C_T \geq 0.0001\sigma$ respectively. The issue of the velocity and flightpath angle can be solved by deriving a rigid-body model since these respective parameters are not divided by γ . Alternatively, the quaternion coordinate system can be used to solve the singularity due to γ .
- The aerodynamic forces for the model have been taken from tabular data of the XV-15 and were modelled to consist of the lift of the main wing and horizontal stabilizer, while the drag was produced by the main wing, horizontal stabilizer and fuselage. This results in higher lift-to-drag ratio than anticipated. Due to the rather large rotors and the two vertical fins, the drag of the aircraft will be higher than anticipated and should be adjusted for.
- Furthermore, the model assumes constant rotor speed. Since the rotor speed governor can be turned on and off in a helicopter or tilt-rotor, the rotor speed can vary as well. In order to fully model the tilt-rotor this should be taken along as extra state variable.
- The XV-15 has several power settings. In this model the power setting is set constant per optimization problem. It is however normal for the XV-15 to take-off in take-off power setting, and lower the power setting to normal after take-off. This should therefore be taken along for a thorough model.
- The XV-15 has a pre-set nacelle conversion speed, while the model can use any conversion rate as desired. Although the conversion process can be paused at any time, in nominal conversion the nacelle is converted directly from helicopter to airplane or vice versa. To accurately model the XV-15, both of these conditions should be implemented. Obviously, the results are expected to differ when only an entire conversion can be applied.

- By solely using lateral cyclic flapping in hover, the XV-15 is capable of performing lateral translation in hover. Due to the nature of point-mass modelling, the model only has an attitude for flightpath and heading. Hence, it is not capable of having a velocity in any other direction than the combined flightpath and heading vector. Therefore, the model is limited in that it is not able to perform a lateral sideways translation in hover. Although this manoeuvre is only used seldom, it is important to safely land the aircraft in helicopter mode (in the presence of wind). This can be implemented though in a rigid-body model.
- In the current model, the lateral velocity component relative to the TPP has been derived, but neglect due to its small magnitude as it only has a magnitude in lateral translation. Due to the fact that a point-mass model has been used, the kinetic velocity is always in the same direction as the aerodynamic velocity. Due to this the lateral component to U_t has been neglected but should be borne in mind.
- Finally, the limits for the bank angle μ , the rate of change of the bank angle μ_{rate} , lift coefficient $C_{L_{rate}}$ and thrust coefficient $C_{T_{rate}}$ and the propulsive efficiency η_p have either been taken from literature, or been estimated with an engineering sense since no data was available on that. In order to derive the model with full confidence, these parameters should be verified, or a sensitivity analysis should be performed over the range of the parameter in order to accept the estimated value.

Apart from the limitations of the model that were known upfront and can be altered in order to increase the detail and completeness of complete the model, some additional recommendations can be done to perform further research within this topic:

- The point-mass model has huge advantages in its simplicity and therefore in its computational time. However, as already briefly discussed above, this imposes quite some drawbacks as well. When further research will be conducted a three-dimensional rigid-body model can be derived to give a more thorough understanding of tilt-rotor aircraft flight behaviour.

Before doing so, it does need to be evaluated what the purpose of the model is. It is of little use to apply such an extensive model under the penalty of large computational time, when the output of the rigid-body model (aerodynamic surface deflections, input to rotor speed, etc.) is of little use for the flight trajectory itself, but does however give huge insights on how to fly them. It can be considered to derive a hybrid version of the model that optimizes the flight trajectory using a point-mass model and optimizes the aircraft motion using a rigid-body model. Or in other words, a simplified point-mass is used to determine the optimal flight trajectory, while a rigid-body model is subsequently used to follow that particular trajectory and minimizes the error with the respective trajectory.

- An issue that was encountered often throughout this thesis research was the functioning and convergence of the optimization software GPOPS. There seemed to be a fine line between the model successfully converging to its optimal solution and diverging due to numerical difficulties, which will be elaborated below. GPOPS seems to be able to work correctly with the model, but as soon as the problems complexity grows, odds of converging successfully decrease. Other optimization software or even methods can be looked into, although it seems that the problem should be able to be fixed by some alterations of the model and code.
 - Plenty optimizations have been jumping back and forth in the proximity of the threshold of feasibility and optimality, only to end in divergence due to numerical difficulties. This behaviour is not only highly unwanted, but also difficult to counter. It appears, depending on the problem, that gradients were either extremely shallow or steep. More research can be put into the surface of the performance indices, but it is plausible that in this surface steep dimples exist and that the optimizer jumps back and forth on the opposing surfaces, unable to reach the bottom. It could also be that the surface is formed as cascading terraces, as if they were partly discretized. This could be fixed by using i.e. surrogate loss functions. (Disproportionally) increasing nodes and intervals did not seem to overcome this behaviour. Another option is to switch the optimization setting to 'numerical' as this will exploit gradient information differently. Drawback of this is that computational time takes significantly longer.

- During the cruise distance optimization, it has been identified that the numerical problems arise when the tilt-rotor is optimized for minimum fuel use, and hence minimum power required. The calculation of the power required is quite complex and involves a few interdependent variables, that trigger a singular basis in the end. Another reason for this behaviour are linearities in the optimization. It is said that GPOPS is not performing well with linear behaviour. At this point it is not known where the linearities arise, but a few measures could be implemented and tested for, such as transforming the controls into sinusoidal controls or in this case to square the equation for the fuel consumption or power required, in order to get rid of the linearity. Another solution can be to derive a simpler method to determine the power required.
 - In the model a few conditional statements have been made in order to determine i.a. the maximum velocity, fuel consumption and induced velocity. These conditional statements do not only degrade computational performance but also worsen convergence. It was noticed that the use of logical operators, or approximating the maximum velocity with an exponential approximating function yield better performance. Therefore, it is recommended to revise the model to be able to discard the if-loops and conditional statements and furthermore polish it to improve the computational efficiency.
 - The sensitivity of the initial guess seemed to have a larger effect to the final solution than anticipated in the literature study. Minor changes in the initial guess sometimes were the difference of no solution due to numerical difficulties on one hand and the optimal solution on the other. It is desired to use a different optimizer, different setting or adapt the model accordingly to do away with this sensitivity of the initial guess. Behaviour of two identical optimizations, one converging and the other diverging, was also not uncommon. It is therefore advised to perform a simpler optimization and feed the results of this optimization as initial guess for the more complex optimization.
 - The same thing holds for the constraints. It was noticed that behaviour of the optimization and convergence differed when constraints were applied as either initial and final limits of the phase or as event constraints. Furthermore, it is said that the model performs better when a small margin is given for a certain constraint instead of a single value.
- Although the proprotor was thoroughly validated for its performance, hover ceiling and hover endurance, there are two imperfections that need to be altered in order to finish the model. First of all, does the single-engine power not match the actual power available. It is suspected that the maximum thrust coefficient value is higher for single-engine operations than it is in twin-engine. The second point regarding the engines is the fact that the model not only overshoots the hover ceiling, but also follows a nearly linear trend, whereas the documentation's hover ceiling is described by a polynomial. Multiple factors can be appointed to further look into to solve for this. This includes the download, propulsive efficiency, induced velocity and other factors that have not been taken into account that could influence the hover ceiling. Finally, as already was mentioned above, the complexity of determining the power required is to be looked into in order to improve numerical convergence.
 - The longitudinal and especially the lateral cyclic flapping have only rarely been used. The longitudinal cyclic to acquire and decrease velocity faster during take-off and landing and the lateral cyclic has only been used in the isolated minimum time turn. Although these two are actual controls that are of utmost importance in helicopter mode, the controls are very important and effective in the optimization of flight trajectories. They do however, add two control variables to the problem, which increases the problem complexity and its computational time. This could lead to the problem having more difficulty to converge. It should be investigated if the lateral cyclic can be neglected, and how the longitudinal cyclic can be incorporated into i.e. i_n or C_T , which will reduce the problem size by one or two control variables.
 - During the optimizations that were performed in this study, it was tried to run the simulations accurately using as few constraints as possible. This was done in order to not dictate parameters to the model that the model should be solving for. Flying at the service ceiling is for instance expected but not guaranteed for the aircraft when cruising far. On the contrary however, do the

problems have better convergence when the phases are well defined. Preliminary optimizations could be performed to optimize a global trajectory, which after that can be split into a few well defined phases to solve for the detail. Small sensitivity analysis could be performed on the defined parameters.

- A specific point that follows from this is passenger comfort during flight which has not been taken into account. Especially due to the close relation of the flightpath angle and the nacelle inclination during conversion this could result in uncomfortable optimal flight trajectories. Rapid changes in flightpath angle and rapid climb and descend can be rather uncomfortable for passengers and crew. This can be incorporated by imposing upper and lower bounds on the change of flightpath angle $\dot{\gamma}$.
- Fuel consumption is one of the most common metrics for an aircraft to optimize for, as this is the largest cost factor for aviation operators. The next step in tilt-rotor modelling, is to add the noise that the aircraft produces to the model, to be able to assess this and optimize for noise abatement. This plays an important role for optimizations of take-off and landing and departure and arrival trajectories. Moreover, noise will become an important factor in the approval of tilt-rotor flights to and from city-centres, not only due to residential buildings being in the very close vicinity, but especially since the higher disc loading of tilt-rotors creates more noise with respect to ordinary helicopters. Factors such as the power required, rotor speed, nacelle inclination, velocity, TPP velocities and altitude, are a few factors that influence the amount of noise the rotors produce. Addition of noise calculation with these parameters in the model does add to the complexity however.
- Finally, it is recommended to apply the model in more situations to conduct further research in tilt-rotor flight behaviour. Since the focus of this research was to derive a numerical model for tilt-rotor type aircraft, lesser emphasis lied on the application of the model to gain in-depth knowledge of tilt-rotor flight. A few conclusions could be drawn from these applications, but these give some leads to conduct further research. This research could be conducted in assessing the importance and behaviour of the nacelle angle, what the driving factor of changing the nacelle angle is and when it makes sense to fly at an intermediate nacelle angle.

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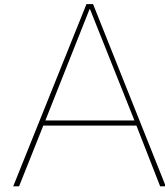
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Tilt-Rotor Aircraft Operations

The following chapter will set out current procedures for landing and take-off and the certification requirements as set by the FAA. From these current procedures the boundary or path constraints for the to be followed optimizations can be deduced. Although currently four distinct procedures can be used, the take-off and landing procedures in this thesis will be merged to either vertical or runway take-off and landing. This goes for the normal procedure as for the procedure with engine failure. The chapter concludes on conversion from helicopter to airplane mode and vice versa.

A.1. (Emergency) Take-off and Landing Procedures

A tilt-rotor aircraft has four distinct ways of taking-off, which are schematically depicted in Figure A.1. First of all, the tilt-rotor can accelerate on a runway until lift equals weight, similar to ordinary fixed-wing aircraft. If a runway is not available the tilt-rotor can accelerate close to the ground with the advantages of ground effect. Both of these are short take-off procedures, but for convenience the first is referred to as runway take-off and the latter as STOL take-off. A third method is the vertical take-off, which is a benefit in confined areas, vessels or oil rigs, where the aircraft can only take-off vertically. Finally, take-off can be performed by oblique take-off, which is a procedure in between a vertical and short take-off. As the runway take-off is most efficient, the highest maximum take-off weight is achieved with runway take-off, followed by STOL take-off and oblique take-off. Due to the high power required during hover, the lowest maximum take-off weight is associated with vertical take-off [9]. Depending on the facilities and surrounding area, a choice for take-off procedure is made based on a trade-off between take-off weight and take-off distance.

The FAA will require civil tilt-rotor aircraft to be certified in similar or even identical fashion as Category A rotorcraft, because of One Engine Inoperative (OEI) capability, which ensures safe operations during take-off and landing in the case of an engine failure. The Take-off Decision Point (TDP) is the last point in the take-off procedure at which a safe rejected take-off (RTO) is assured and furthermore, the first point at which a safe continued take-off (CTO) is assured [23]. Hence, take-off must be rejected if an engine failure occurs before the TDP and continued if that happens after the TDP. During CTO, the aircraft must acquire the take-off safety speed at a minimum height of 35 feet, along with a positive rate of climb of 100 feet/minute.

Similarly during the landing approach, the landing must be continued if the engine failure occurs after the Landing Decision Point (LDP) and it can be either continued or balked with a go-around after LDP [25]. The LDP is the point at which the combination of height and altitude permits the decision to either proceed the initiated landing or accomplish a safe climb-out [23]. No flight manual data of the XV-15 is known. Therefore, take-off and landing procedures will be deduced from literature regarding the XV-15

A.1.1. VTOL and Oblique Take-Off Operations

In general, the vertical take-off is only used when the surroundings do not allow for a runway take-off, such as in very confined areas or when the other options are not possible. This is because it allows only the smallest maximum take-off weight since the power required is very large. In the case of a confined

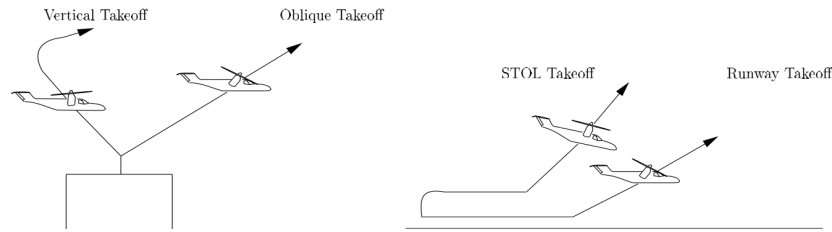


Figure A.1: Schematic depiction of four tilt-rotor take-off procedures [9]

heliport, the vertical take-off is often commenced with a linear backup so that the pilot can always have the platform in sight and bring the tilt-rotor back to the heliport in the case of a rejected take-off, as can be seen in Figure A.1.

Oblique take-off is commonly used when the take-off area is not very confined and the tilt-rotor has sufficient space to land the aircraft in case of RTO, apart from the heliport. Oblique take-off is dissimilar to VTOL in that it omits the linear backup, but climbs out directly after attaining a few feet altitude, as can be seen in Figure A.1.

The following procedure for a helicopter vertical take-off has been deduced from the flight manual of the Eurocopter BO 105. The helicopter hovers 3 ft in ground effect, after which the helicopter is backed up approximately 3 m. Now, a slow vertical climb of 10 ft above helipad elevation (AHE) is induced, after which a rearward climb with $\gamma = 150^\circ$ flight path angle is initiated, maintaining the helipad in sight, until the TDP at 120 ft AHE is reached. From here on, the helicopter is accelerated to 60 kts airspeed and climb-out is performed as desired [23], [16]. This take-off is depicted in Figure A.2.

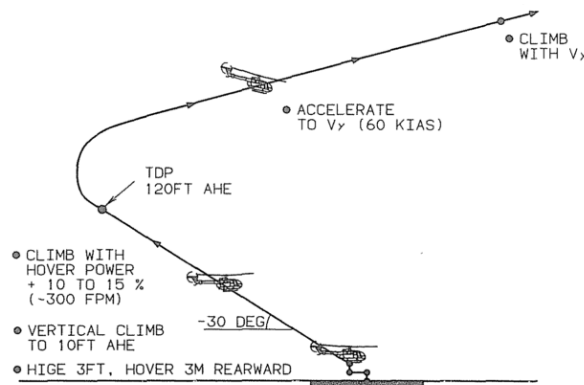


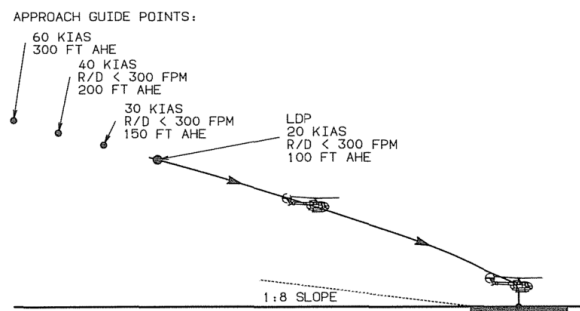
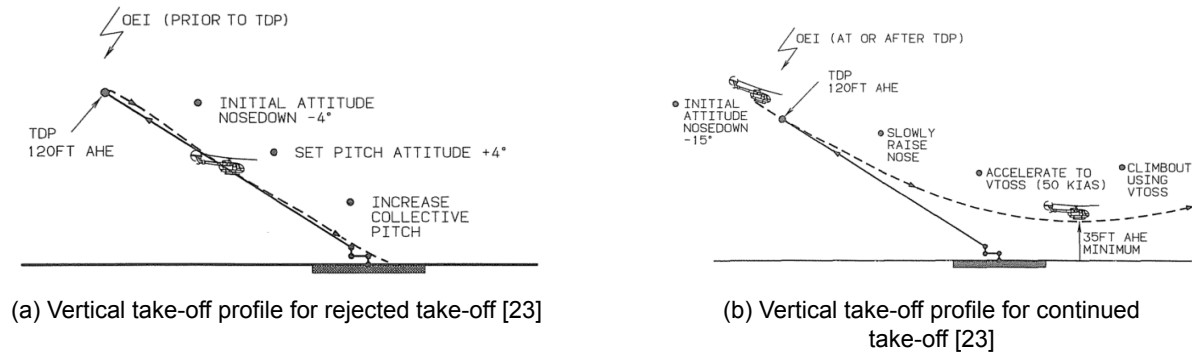
Figure A.2: Schematic helicopter vertical take-off profile [23]

In the event of an engine failure during take-off, special procedures apply to conduct safe flight. If the engine failure occurs before reaching the TDP, an initial nosedown attitude is attained, after which the nose attitude is increased closer to ground to minimize ground speed and cushion the landing [23], as depicted in Figure A.3a.

Figure A.3b, shows the emergency procedure after the TDP. After engine failure a 15° nosedown attitude is induced to pick up speed. While accelerating to V_{TOSS} , the attitude should be slowly levelled out. A minimum altitude of 35 ft is to be adhered to. During climb-out the aircraft should be accelerated to the best rate of climb speed and should climb to 1.000 ft AHE and the aircraft should be landed as soon as possible [23].

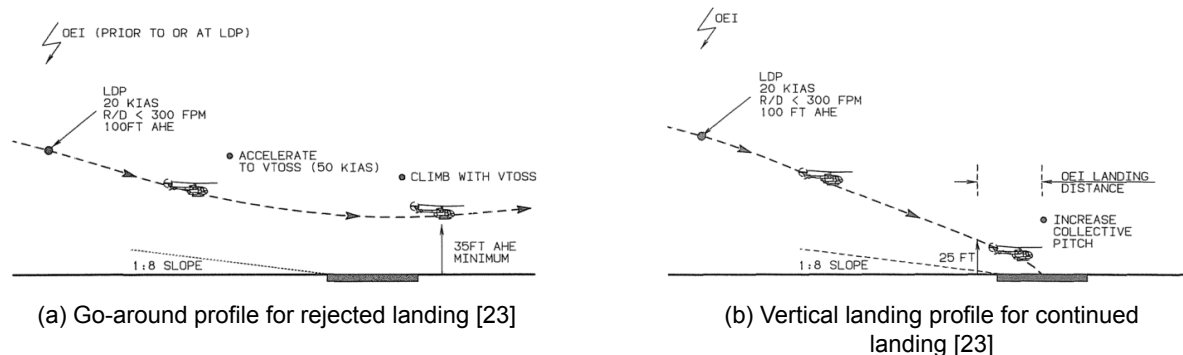
A.1.2. VTOL and Oblique Landing Operations

For vertical helicopter landing, the following procedure is given. The LDP approached is initiated with 60 kts and 300 ft AHE and, passing through first 40 kts airspeed at 200 ft followed by 30 kts at 150 ft, the helicopter arrives at the LDP with 20 kts at 100 ft AHE. All of these at a rate of descent slower than 300 fpm. After the LDP, the speed is slowly decreased to end up in a 3 ft hover, after which the touchdown is initiated [23], as depicted in Figure A.4.



For engine failure during landing, Eurocopter's flight manual prescribes roughly the same as for take-off procedures. Prior to the LDP, the pilot can elect to either go-around or continue landing, but in case of a go-around, the aircraft is to be accelerated and airspeed for best rate of climb is to be achieved to gain 1,000 ft AHE, after which the craft is to be landed as soon as possible (Figure A.5a).

When the engine failure occurs after the LDP, a landing attitude should be established for minimum ground speed and a flare should be conducted to cushion the landing (Figure A.5b). Both the OEI procedures for take-off and landing are in accordance with FAA regulations [25].



A.1.3. STOL and Runway Take-Off Operations

Regarding short and runway take-off, the aircraft accelerates with a ground run or in ground effect respectively, which reduces the required power and increases the maximum take-off weight with respect to VTOL or oblique take-off. For the runway take-off, the tilt-rotor accelerates by tilting the nacelles up to 60° nacelle inclination, to accommodate for rotor tip clearance. For STOL, the aircraft can hover in

ground effect and accelerate by tilting the nacelle or by pitching the aircraft forward. An obstacle-free field is required though. Rotation and climb-out occurs at the lift-off speed, which is assumed at 40 kts ≈ 20.58 m/s. Once airborne, the tilt-rotor maintains its acceleration and flightpath angle of $\gamma = 8^\circ$ until it reaches the unspecified TDP. According to Carlson and Zhao, these take-off assumptions are consistent with the short take-offs described by NASA [13]. After the climb-out, an altitude of 35 ft has to be reached with positive climb rate. The take-off has formally ended when a 1,000 ft altitude has been reached. Carlson and Zhao argue that the TDP should be as close to the lift off altitude $h_0 = 0$ in order to minimize the runway length for both CTO and RTO [10]. However, extrapolating their results still yields feasible RTO distance with a TDP at 50 m altitude.

Figure A.6 shows the tilt-rotor take-off procedure for short and runway take-off, where the first segment is either an in ground effect hover for short take-off or a ground run for the runway take-off. The dashed line shows normal operation, while the solid line shows the continued take-off and the dotted trajectory depicts the rejected take-off.

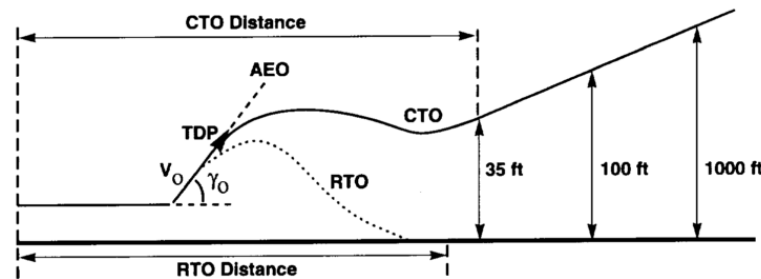


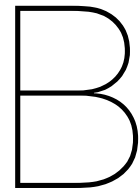
Figure A.6: Schematic tilt-rotor short and runway take-off profile in normal operations and CTO/RTO in OEI [10], [16], [34], [64]

A.1.4. STOL and Runway Landing Operations

The landing procedure is slightly dissimilar. Again, there is no official data of the procedures, but multiple sources have overlap that leads to a decent procedure. The LDP is usually set as such, that the combination of flight condition and decreasing power allows for a balked landing not exceeding a minimum altitude of 35 ft. In general, this leads to an LDP at about 100 ft altitude, with a moderate airspeed of approximately 40 kts and a rate of descent not exceeding 500 fpm [16]. This is followed by continuing to descent in between 3° and 9° to reach 15 ft with 30 kts airspeed and 500 fpm descent rate, before slowly decreasing airspeed and descent rate to end up with zero airspeed on the ground [21], [24].

A.2. Conversion Operations

Once in the air with low speed, the tilt-rotor's forward can be controlled by rotation of the nacelles or by a change in pitch attitude. Attitude is maintained by primary helicopter controls. With sufficient altitude and speed, the tilt-rotor can start the conversion process in order to change from helicopter into aircraft mode. XV-15 test pilots developed a conversion technique because of its ease, safety and efficiency: the aircraft is accelerated forward up to 60-80 knots by tilting the nacelles 10-20 degrees forward. From this point, a continuous conversion with 7.5 degrees per second is possible. While this transition takes place, the rotor controls are mechanically phased out and the airplane control surfaces are used. Once the rotors are completely tilted forward, they are locked into place and rotor speed is reduced to increase efficiency and reduce vibrations. The conversion can also be stopped any time and the aircraft can remain flying with nacelles anywhere within its tilting range. Re-conversion is done by the same process in reverse order [9].



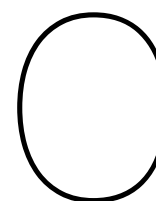
Conversions and Aircraft Parameters

XV-15 parameters

Design gross weight	5896.7	kg
Empty weight	4569	kg
Crew weight	181	kg
Trapped fluids weight	63	kg
Maximum fuel weight	675	kg
Wing-pylon wingspan	9.80	m
Wing-pylon surface area	16.816	m ²
Horizontal stabilizer wingspan	3.91	m
Horizontal stabilizer surface area	4.668	m ²
Rotor radius	3.81	m
Rotor tip speed (airplane mode)	47.96	rad/s
Rotor tip speed (helicopter mode)	59.17	rad/s
Maximum rotor cyclic flapping	12	deg
Rotor solidity	0.089	
Rotor blade drag coefficient	0.015	
Induced power correction factor	1.15	
Ground effect factor	1	
Propulsive efficiency	0.95	
Rotor time delay	0.6	s
Normal rated horsepower	1250	shp
Military rated horsepower	1401	shp
Take-off rated horsepower	1550	shp
Contingency rated horsepower	1802	shp

Conversion factors

ft to m	0.3048
m to ft	3.2808
NM to m	1852
kts to m/s	0.5144
m/s to kts	1.9440
fpm to m/s	0.00508
shp to W	745.699871
W to shp	0.00134102



Optimization Output

Summary of Problem Written to File: 3DOF_PointMass.txt

Using Default Sparsity

Automatic Scaling Turned On

Objective Gradient Being Estimated via INTLAB Automatic Differentiation

Constraint Jacobian Being Estimated via INTLAB Automatic Differentiation

| Grid Refinement Iteration: 1 |

Nonlinear constraints	2091	Linear constraints	4
Nonlinear variables	2746	Linear variables	0
Jacobian variables	2735	Objective variables	2746
Total constraints	2095	Total variables	2746

The user has defined 42552 out of 42552 first derivatives

Major	Minors	Step	nCon	Feasible	Optimal	MeritFunction	nS	Penalty	
0	2429		1	3.8E+02	2.9E-02	5.0000000E+00	112		r T
1	764	4.5E-01	2	2.0E+02	2.7E-03	1.1925481E+03	142	2.4E-04	n r l T
2	1306	5.8E-01	3	8.7E+01	6.1E-03	7.4497795E+02	118	2.4E-04	s l
3	2080	1.0E+00	4	1.6E-01	9.9E-02	3.5617468E+02	133	1.4E-02	T
4	275	1.0E+00	5	7.9E-02	1.6E-01	-1.2593066E+02	213	1.3E+03	T
5	27	1.0E+00	6	2.1E-02	1.7E-01	2.3982891E+01	227	6.3E+02	
6	45	1.0E+00	7	1.8E-03	2.6E-03	1.6347414E+01	258	1.1E+02	
7	15	1.0E+00	8	4.1E-05	1.2E-03	1.6258188E+01	258	7.9E+01	
[...]	[...]	[...]	[...]	[...]	[...]	[...]	[...]	[...]	[...]
Major	Minors	Step	nCon	Feasible	Optimal	MeritFunction	nS	Penalty	
130	5	1.0E+00	168	2.2E-06	1.1E-05	1.1923266E+01	340	7.9E+01	
131	2	1.0E+00	169	3.0E-05	1.1E-05	1.1921260E+01	339	7.9E+01	
132	2	1.0E+00	170	5.5E-05	2.0E-06	1.1918583E+01	338	7.9E+01	
133	1	1.0E+00	171	5.2E-06	5.0E-06	1.1917623E+01	338	7.9E+01	R
134	1	1.0E+00	172	(2.5E-07)	3.9E-06	1.1917573E+01	338	7.9E+01	s
135	1	2.1E-01	174	(2.2E-07)	2.7E-06	1.1917569E+01	338	7.9E+01	
136	1	3.9E-08	184	(2.2E-07)	(1.5E-06)	1.1917569E+01	338	7.9E+01	

SNOPTA EXIT 0 – finished successfully

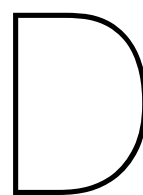
SNOPTA INFO 1 – optimality conditions satisfied

Problem name

No. of iterations	8660	Objective value	1.1917476083E+01
No. of major iterations	136	Linear objective	0.0000000000E+00
Penalty parameter	7.894E+01	Nonlinear objective	1.1917476083E+01
No. of calls to funobj	184	No. of calls to funcon	184
No. of superbasics	338	No. of basic nonlinears	1772
No. of degenerate steps	593	Percentage	6.85
Max x	1 1.0E+00	Max pi	831 1.6E+04
Max Primal infeas	1960 1.3E-07	Max Dual infeas	1373 2.4E-02
Nonlinear constraint violn	1.5E-06		

Solution printed on file 9

Elapsed time is 197.223559 seconds.



Rotterdam RWY 24 Departure Chart

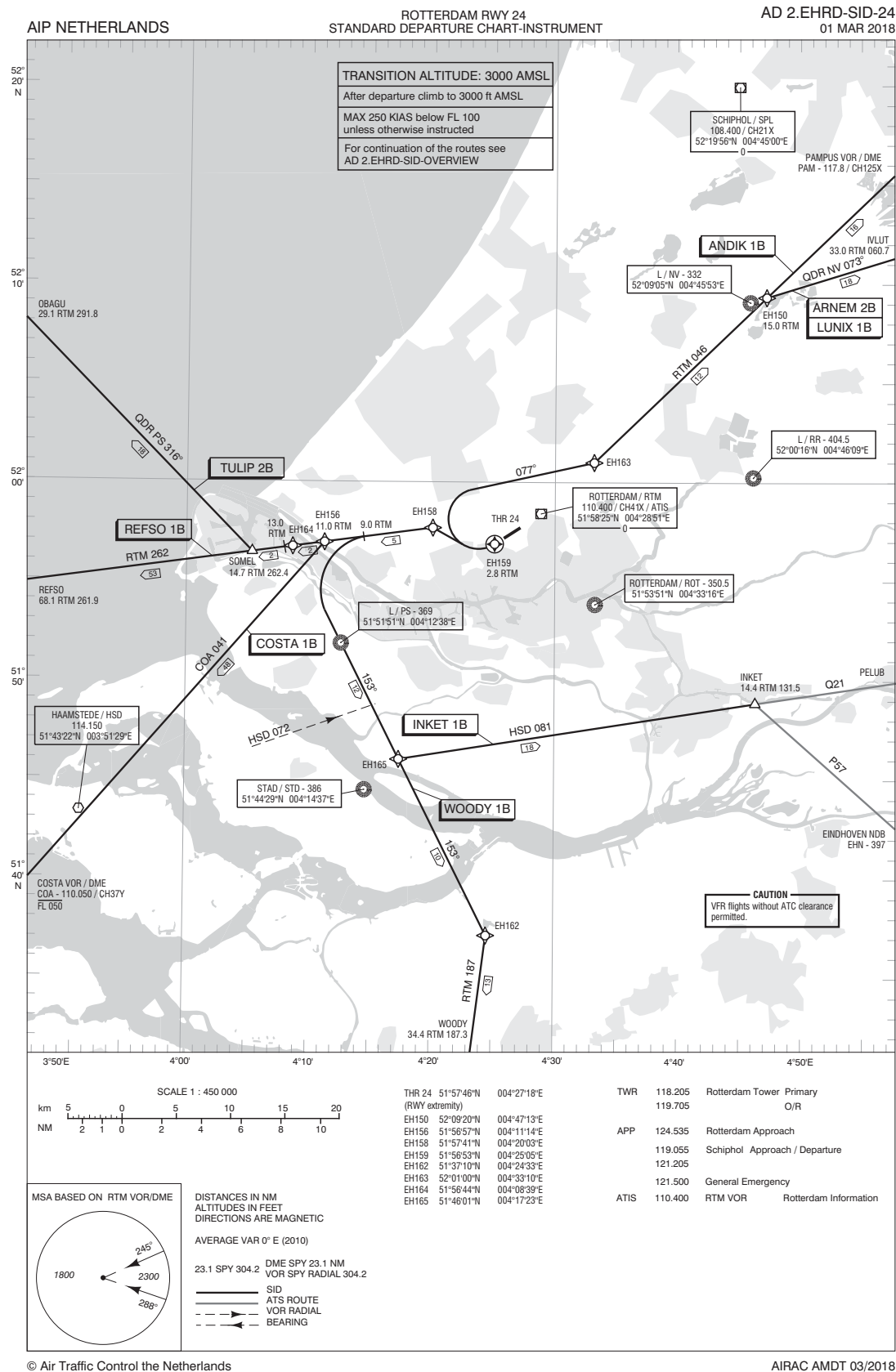


Figure D.1: Standard Instrument Departure chart for Rotterdam/the Hague airport runway 24. From: [1].