

Optimal design of flood defence systems in a changing climate

H.G. Voortman¹ and J.K. Vrijling²

¹ ARCADIS Infra, Amersfoort, The Netherlands

² Faculty of Civil Engineering and Geosciences, Delft University of Technology, Delft, The Netherlands,

Recent research indicates that worldwide climate change is likely, leading to changes of the mean sea level and the wind climate. The paper presents a methodology to find the most effective strategy for coastal flood protection in a changing climate. The method is based on risk-based optimisation of the flood defence design. Scenarios for climate change, including associated uncertainties, are included.

Key words: flood defence systems, risk-based design, optimisation, climate change

1 Introduction

Flood defence systems are designed and constructed to protect low-lying areas against flooding. Conceptually, the cost of the protection system should be in balance with the value of the protected area. Over the years, risk-based design and optimisation methods have proven to be useful tools to obtain a balanced level of protection.

Van Dantzig (1956) established the concept of economic optimisation of flood defences in the aftermath of a major flooding disaster in the Netherlands in 1953. Van Dantzig strongly schematised the protection system, essentially to a single dike cross section with only the crest level as a decision variable. The water level in front of the structure was considered a random variable and the only failure mode considered was overflowing.

Following the development of reliability-based methods in the 1960s and 1970s (see for instance Turkstra, 1962, 1970; Hasofer and Lind, 1974; Hohenbichler and Rackwitz, 1983) it became generally recognised that:

- The reliability level of any structure is determined by a range of random variables describing the loads on and the strength of the structure;
- A variety of different failure modes may be responsible for failure of the structure.

Furthermore, in designing structures a set of decision variables must be indicated. The geometry of a dike is determined by a number of decision variables like:

- The steepness of the slopes of the structure;
- The type and size of slope protection;
- The width and level of berms in the profile.

The large spatial scale of flood defence systems in a number of countries poses an important additional problem when performing risk-based optimisation of such systems.

Reliability-based methods were applied to flood defences by Bakker and Vrijling (1980) and TAW (2000). A method for risk-based optimisation of large-scale flood defence systems is proposed by Voortman (2002). In this paper, a method will be proposed to incorporate the effects of climate change in the risk-based design method for flood defence systems.

2 Problem outline

Recent research indicates that a change of the world's climate is likely (IPCC, 2001). The change of the climate may have important consequences for the reliability level of flood defences because of:

- A rise of the mean sea level;
- An alteration of the wind climate in the form of a changed frequency and intensity of storms and/or a change of the prevailing wind direction;
- A change of the mean amount and the variability of the rainfall.

The changes mentioned affect the loads on flood defences. The effects depend on the area in which the flood defence is located. Table 1 provides an indicative overview.

Table 1: Effects of climate change for flood defences in river areas and coastal areas

River area	Coastal area
• Change of peak water levels	• Change of peak water levels as a consequence
• Change of speed of water level changes	of sea level rise and change of wind setup
• Change of wave loading	• Change of wave loading

To date, it is not clear whether the changes mentioned will increase or decrease the loads on flood defences. Nevertheless, climate change introduces additional uncertainty that affects the reliability level of flood defences. A method needs to be established to incorporate climate scenarios and the related uncertainties in the risk-based design method for large-scale flood defences.

3 Risk-based design in a changing climate

3.1 General

In risk-based optimisation, the design of a flood defence system is determined by balancing the cost of constructing the defence system against the reduction of the flooding risk that is obtained. Cost and risk may be measured in monetary terms or in a measure of utility (see Van Gelder, 1999 and Voortman, 2002 for example applications). In this paper, cost and risk will be measured in monetary terms, which leads to the concept of risk-based cost-benefit analysis of flood defence systems. In this approach the optimal flooding probability is found by solving:

$$\min_{P_{flood}} C_{life}(P_{flood}) = I(P_{flood}) + \int_0^T P_{flood} \left(\frac{1+r_e+i}{1+r} \right)^t (c_b b_0 + c_d d_0) dt \quad (1)$$

where:

- I : Direct cost of flood defence;
- P_{flood} : Flooding probability;
- T : Reconstruction period;
- r_e : Rate of economic growth;
- i : Inflation;
- r : Interest rate;
- b_0 : Lost benefits in case of flooding;
- d_0 : Maximum possible damage to investment in protected area;
- c_b : Damage factor lost benefits;
- c_d : Damage factor investment.

Especially for flood defence systems with a large spatial scale, establishing the cost and risk as a function of the flooding probability is a complicated task. Methods for dealing with large-scale systems are described in Voortman (2002) and will be applied in this paper.

3.2 Optimisation of flood defence strategies

Equation 1 implicitly assumes an unchanging flooding probability over time. However, due to a changing environment the flooding probability will develop over time. In case of an increasing flooding probability, climate change necessitates to:

- Adopt a lower design flooding probability for the system at the time of construction to provide reserve strength for future climate change;
- Perform reconstruction of the flood defence system after some period of time.

The combination of the reconstruction interval and flooding probability may be optimised. Vrijling and Van Beurden (1990) performed such an optimisation based on the work of Van Dantzig. In general, the following steps need to be taken to optimise the flood defence strategy in a changing environment:

1. Establish the optimal design of the flood defence for a range of flooding probabilities calculated under current conditions;
2. Quantify the effect of climate change on the flooding probability, representing the flooding probability as a function of time;
3. Establish the cost of a flood protection system as a function of time;
4. Evaluate the life-cycle cost of alternative flood defence strategies;
5. Comparison and choice of a strategy.

3.3 Step 1: optimal design of the flood defence system for current conditions

The large spatial scale of most flood defence systems complicates the optimisation process. This problem is often circumvented by strongly simplifying the system as was done by Van Dantzig (1956). On such a basis, decision-making on real-life systems is of reduced value as the simplified system loses many crucial characteristics. One has to deal with the full complexity of the system in some practical way. Such a method is proposed by Voortman (2002). The basis of this method is a decomposition of the system following the method of systems engineering (see for instance: Department of Defence, 2001). The flood defence system is decomposed in individual structures and dike sections. On the level of structures and dike sections, a further decomposition into elements of the structure is necessary. Figure 1 shows an example.

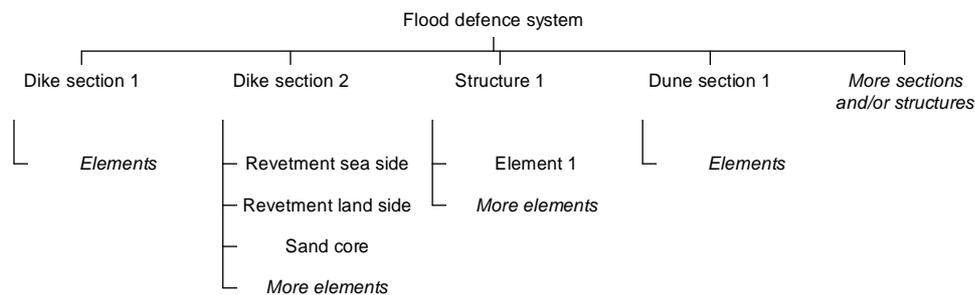


Figure 1: Decomposition of a large-scale flood defence system

At the highest level, the cost and the probability of failure of the system is determined by the costs and probabilities of failure of the underlying sections of the system. The cost and failure probability of a section are in turn determined by the costs and failure probabilities of the elements out of

which the section is composed. Voortman (2002) showed that it is possible to establish an optimal design of the system by first performing an optimisation on section level and using the results to establish the optimal design of the system. Thus, the cost of a protection system as a function of its probability of failure is established.

3.4 Step 2: flooding probability as a function of time

The actual effects of climate change are still highly uncertain. For policy studies, scenarios are defined that may be modified and updated on the basis of new observations of the changing climate. It is therefore advantageous to quantify the effects of climate change on the flooding probability independent of the climate scenarios. Following work of Casciati and Farvelli (1991) and Dawson and Hall (2001, 2002), Voortman and Vrijling (2003) proposed a method based on fragility curves. A fragility curve in this case provides the probability of flooding for a given change of the climate. The climate scenario considered should provide the mean climate change and the associated uncertainties. The current estimate of the flooding probability (hazard rate h) at some time t in the future is then easily found by:

$$h_{flood}(P_{f;0}, t) = \int_{-\infty}^{+\infty} f_{\Delta h}(\eta, \mathbf{p}(t)) P_{f|\Delta h}(\eta, P_{f;0}) d\eta \quad (2)$$

where:

- f : Probability density of effect of climate change;
- \mathbf{p} : Vector of parameters;
- Δh : Sea level rise;
- $P_{f|\Delta h}$: Fragility curve, probability of flooding for given sea level rise.

3.5 Step 3: cost of construction of a flood defence as a function of time

In this paper, the cost of maintenance of flood defences will be neglected. Therefore, the cost of construction of a flood defence system is fully determined by the construction of the flood defence at the beginning of the reference period and by the periodic reconstruction in the future. The cost in case of construction or reconstruction is determined by:

- The materials used in the flood defence system;
- The quantities of the materials used;
- The unit prices of the materials;
- The costs of starting the project (cost of design, mobilisation etc.).

It may be clear that it is hard to estimate the future cost of construction of a flood defence system. Currently unknown materials may be applied in the future. Developments in technology may

strongly influence the unit prices of materials. Therefore, the price of flood defences will probably not follow inflation.

An important influence on the price of future flood defences is the change of the climate. A flood defence with a given geometry will have a future yearly probability of failure that is higher than the current one. Considering the cost of such a flood defence to be the same now and in the future, the cost of protection of a flood defence system can be considered to shift horizontally over time (see Figure 2).

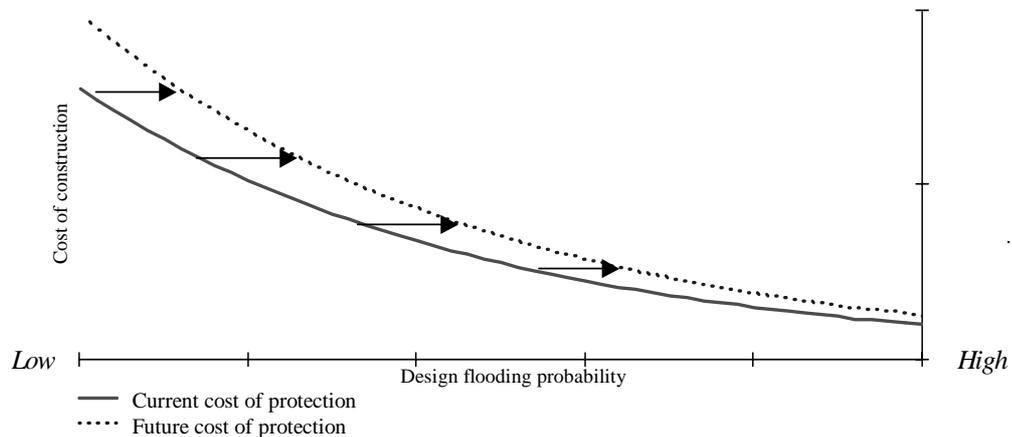


Figure 2: Due to climate change, a given value of the flooding probability now corresponds to a higher value in the future; the cost function shifts to the right

3.6 Step 4: evaluating the life-cycle cost of alternative defence strategies

A flood-prone area may be protected against flooding in many ways between the following two extreme strategies:

- Constructing a very safe flood defence system, suitable for a very long planning period (e.g. 500 years);
- Constructing a flood defence system suitable for a short interval (for instance 20 years) and reconstructing it after that period.

Furthermore, alternatives are available for the definition of the design flooding probability, like:

- Optimised, based on scenarios of climate and society for the period considered;
- A fixed maximum that should be fulfilled at any time in the future.

For every combination of reference period and design flooding probability in any strategy, the life-cycle costs, total future investment and total future risk can be established.

3.7 Step 5: comparison and choice of a strategy

Different flood defence strategies can be compared on different aspects, like:

- Total life-cycle cost;
- Total future investment in the flood defence;
- Total future flooding risk;
- Maximum occurring flooding probability;
- Actual length of the reference period.

Societal requirements may set constraints on the maximum allowable flooding probability, the maximum or minimum length of a reference period or both. The monetary effects of such constraints can easily be established by applying the methods outlined in this paper.

4 Case study: optimal defence strategy for a flood defence in the southern North Sea

In a case study, the optimal flood defence strategy will be established for a coastal flood defence system in the southern North Sea.

The joint probability distribution of the environmental conditions in the area are described in Voortman (2002). The hydraulic conditions are described by a superposition of astronomical tide and wind effects. The description is such that it can be used in reliability calculations of the defence system.

The Dutch Institute for Coastal and Marine Management established three scenarios of climate change that are used in policy studies. These scenarios are also aggregated in a probability distribution (scenario 4). Table 2 provides an overview.

Table 2: Climate scenarios used by the Dutch Institute for Coastal and Marine Management (2000)

Scenario	Distribution of climate changes	Sea level rise in 2100		Increase of wind speed in 2100	
		Mean	Std deviation	Mean year maximum	Std deviation
1. Minimal	Deterministic	0.2 m	n.a.	0 m/s	n.a.
2. Middle	Deterministic	0.6 m	n.a.	0 m/s	n.a.
3. Maximal	Deterministic	0.85 m	n.a.	1.98 m/s	n.a.
4. Probabilistic	Normal	0.6 m	0.25 m	0 m/s	1.98 m/s



Figure 3: Case study location

Data on the protected area is necessary to calculate the flooding risk. Table 3 provides an overview.

Table 3: Input for economic optimisation of the flood defence system

Parameter	Description	Value	Remark
a	Slope of investment function	0.18	Voortman (2002), see below
b	Intercept of investment function	8.81	Voortman (2002), see below
d_0	Monetary value of the area	G€34,-	Taken from PICASO study*
b_0	Taken equal to yearly gross domestic product	G€14.40	Value of 1998#
r	Interest rate	0.07 per year	Average over 1960-2001#
r_e	Rate of economic growth	0.03 per year	Average over 1960-2001#
i	Inflation	0.02 per year	Average over 1960-2001#

#: RWS, 2001

*: Data obtained from the database of the Dutch Central Bureau of Statistics.

5 Optimal flood defence design for present-day conditions

Voortman (2002) performed optimisation of the design of the flood defence system for present-day conditions. Optimal geometries for all components in the system were established for flooding

probabilities ranging from approximately 10^{-1} per year to approximately 10^{-6} per year. Figure 4 shows a few examples.

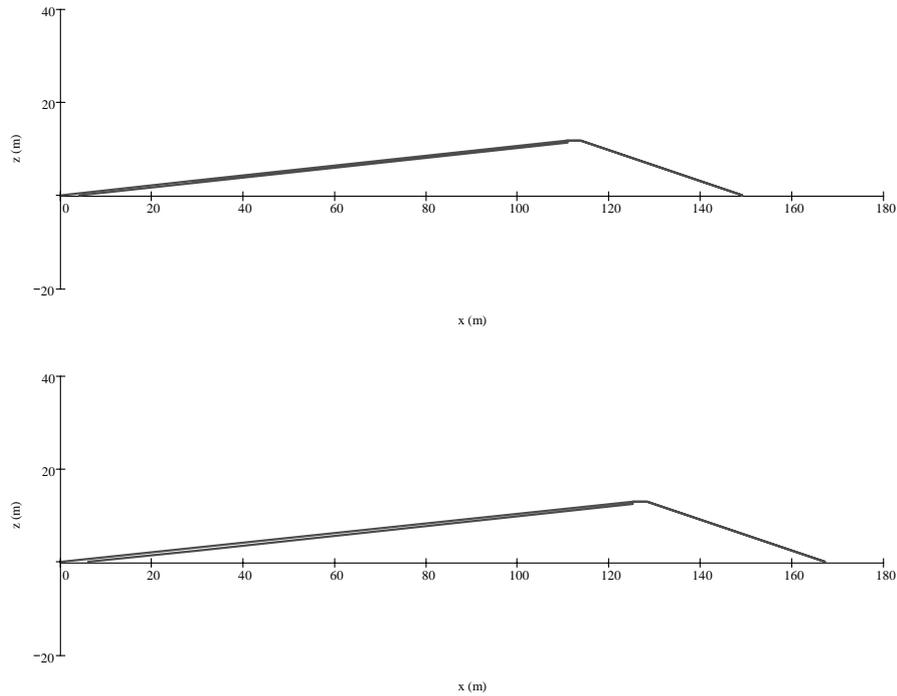


Figure 4: Optimised geometries of two dike sections for a local probability of failure of 10^{-4} per year. Upper panel: location with shallow foreshore, lower panel: location with deep foreshore

Based on the results of system optimisation, Voortman (2002) derived a parametric expression for the construction cost of the protection system as a function of the probability of flooding:

$$I(P_{f,0}) = 10^{a \log(P_{f,0}) + b} \quad (3)$$

where $P_{f,0}$ is the design flooding probability for one year and a and b are parameters of which the values are included in table 3. Expression (3) is used in a risk-based optimisation of the full system.

6 Effects of climate change

6.1 Fragility curve

The fragility curve for the full system is derived by performing reliability calculations for different combinations of:

- Initial flooding probability;

- Level of sea level rise (Δh);
- Change of mean year maximum wind speed (Δu).

The results can be summarised by (Voortman and Vrijling, 2003):

$$\log(P_{f|\Delta h, \Delta u}) = \log(P_{f,0}) + a \cdot \Delta h + b \cdot \Delta u \quad (4)$$

A comparison between the results of the numerical model and the parametric model is given in Figure 5.

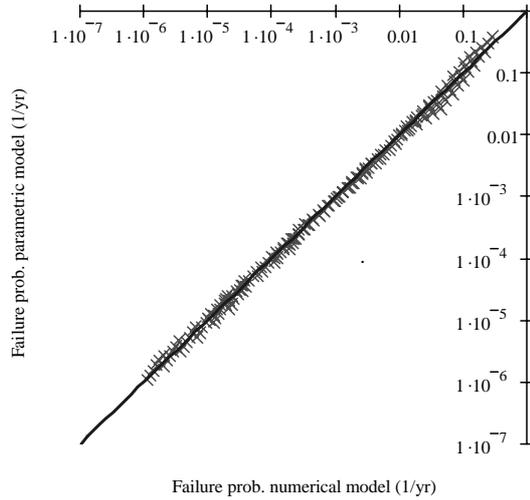


Figure 5: Comparison of parametric fragility curve with results of numerical model

6.2 Climate scenarios

The climate scenarios given in table 2 provide estimates of sea level rise and change of wind speed in the year 2100. To calculate the flooding probability as a function of time, it is necessary to model the changes of the climate as functions of time. In this paper, the assumption is made that the climate changes are linear in time. The parameters of the model are chosen such that the results in the year 2100 correspond to the scenarios of table 2.

With this information, a probability distribution of climate changes at any point in the future can be established (Figures 6 and 7).

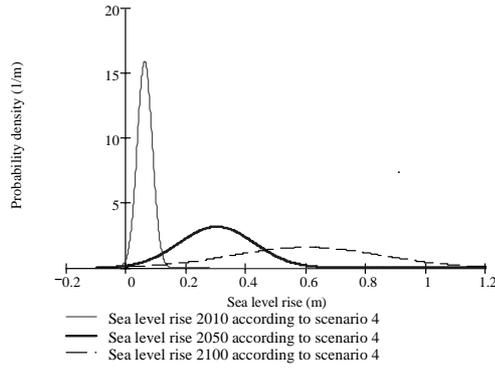


Figure 6: Distribution of future sea level rise according to climate scenario 4

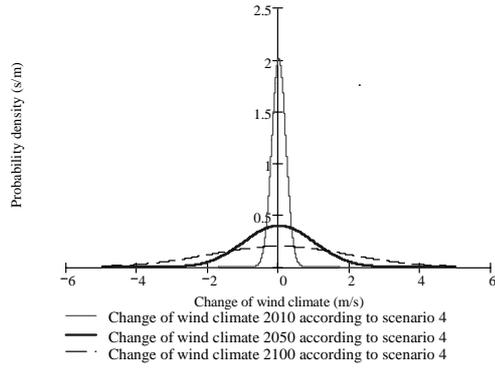


Figure 7: Distribution of future changes in mean year maximum wind speed according to climate scenario 4

6.3 Flooding probability over time

The flooding probability over time h_{flood} is found on the basis of the fragility curve and the probability distributions of future climate changes by:

$$h_{flood}(P_{f;0}, t) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f_{\Delta u}(v, \mathbf{p}(t)) f_{\Delta h}(\eta, \mathbf{p}(t)) P_{f|\Delta h, \Delta u}(\eta, v, P_{f;0}) d\eta dv \quad (5)$$

where:

- f : probability density of climate change;
- Δu : change of mean year maximum wind speed;
- Δh : change of mean sea level;
- \mathbf{p} : vector of distribution parameters;
- $P_{f;0}$: design flooding probability.

Figure 8 shows the results for the four scenarios.

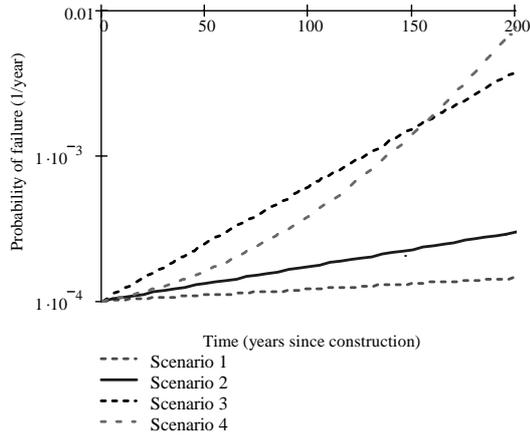


Figure 8: Hazard rate h_{flood} of the flood protection system for four different scenarios of climate change

It is convenient to express the effect of climate change on the hazard rate in a dimensionless measure by:

$$\gamma(t) = \frac{h_{flood}(t)}{P_{f;0}} \quad (6)$$

The parameter γ will be applied in the evaluation of different defence strategies.

7 Evaluating defence strategies

7.1 General

Three flood defence strategies will be evaluated. Table 4 provides an overview.

Table 4: Input for economic optimisation of the flood defence system

Strategy	Description
A	One-time construction of a flood defence system
B	Fixed maximum flooding probability
C	Optimised flooding probability in any period with constant reconstruction interval

As flood defence is intended for the far future, all strategies will be evaluated for a period of 500 years into the future. This period will be referred to as the reference period.

7.2 Life-cycle cost

In strategies B and C, investments in the flood defence system are made at several times in the future. This aspect needs to be incorporated in the life-cycle cost function. As stated earlier, the

investment consists of a fixed part and a part that depends on the design. Furthermore, a flood defence system is already present. In this paper, the assumption is made that the investment in a flood defence at a time t , equal to the sum necessary to decrease the existing flooding probability to the design value $P_{f,des}$ is given by:

$$I_{reinf}(t, P_{f,des}) = (1+i)^t \left(I_0 + I \left(\frac{P_{f,des}}{\gamma(t)} \right) - I \left(\frac{P_{f,exist}(t)}{\gamma(t)} \right) \right) \quad (7)$$

where:

- $P_{f,des}$: Design flooding probability;
- $P_{f,exist}$: Existing flooding probability at time t ;
- γ : Parameter describing effect of climate change (equation 6);
- I_0 : Mobilisation cost;
- i : Inflation.

Despite the objections against it, the price development of flood protection is assumed to follow inflation. The reason for this is that innovative solutions with a major effect on the cost of protection can not be foreseen and can therefore not be incorporated. The factor γ translates future flooding probabilities to the flooding probabilities under current conditions and thus provides the shift of the investment function described earlier.

Capitalised to the decision moment, the investment over the reference period is valued by:

$$I_{cap}(T, P_{f,des}) = \sum_{n=1}^N \frac{1}{(1+r)^{(n-1)T}} (I_{reinf}((n-1)T, P_{f,des})) \quad (8)$$

where:

- T : Reconstruction period;
- r : Interest rate.

The total investment over the reference period is incorporated in the expression of the life-cycle cost (equation 1). Thus, life-cycle costs are described as a function of reference period and design flooding probability. This forms the basis for a quantitative evaluation of alternative defence strategies.

7.3 Optimisation of defence strategies

In order to optimise a defence strategy, the reinforcement period T and the design flooding probability $P_{f,des}$ need to be chosen. Together, T and $P_{f,des}$ determine the flooding probability over time. Figure 9 shows an example where strategy B is applied in combination with climate scenario 4. The maximum flooding probability is set to 10^{-4} per year.

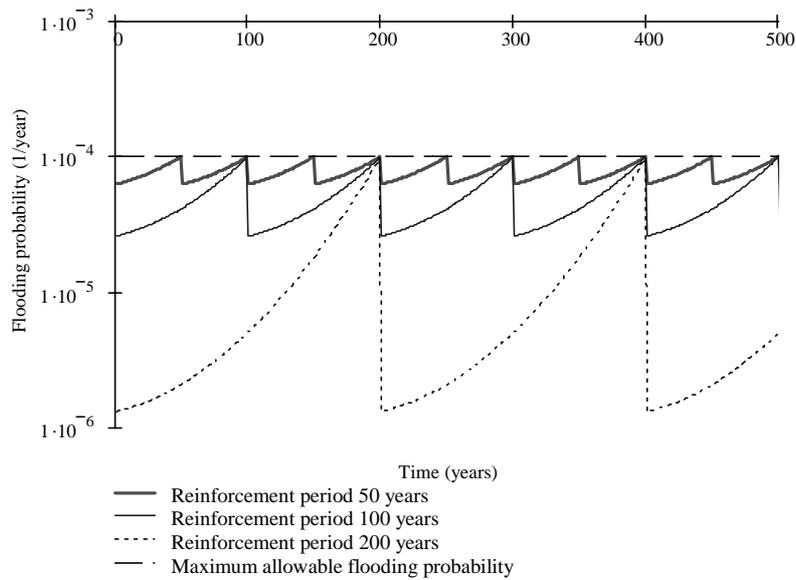


Figure 9: Flooding probability over time with a maximum flooding probability of 10^{-4} per year, applying strategy B under climate scenario 4

Climate change causes an increase of the flooding probability over time. Therefore, the design flooding probability decreases for increasing reinforcement period. The life-cycle cost as a function of the reinforcement period is shown in Figure 10.

The analysis shows that minimum life-cycle cost for this combination of protection strategy and climate scenario is found at a reinforcement period of 93 years. The corresponding probability requirement for the case study area is $1.6 \cdot 10^{-4}$ per year.

In case of strategy C, the design flooding probability is optimised for every reinforcement of the flood defence system. The flooding probability over time is shown in Figure 11.

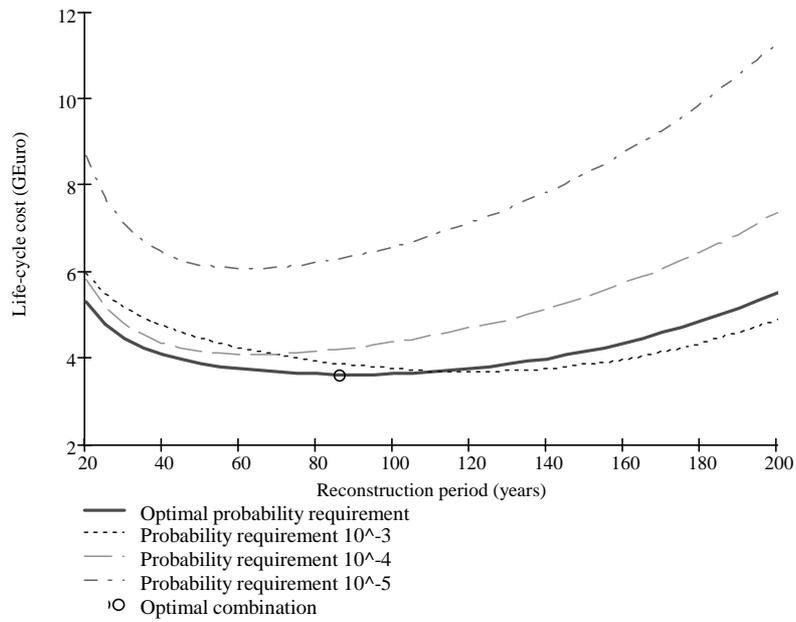


Figure 10: Life-cycle cost as a function of reinforcement period in strategy B under climate scenario 4

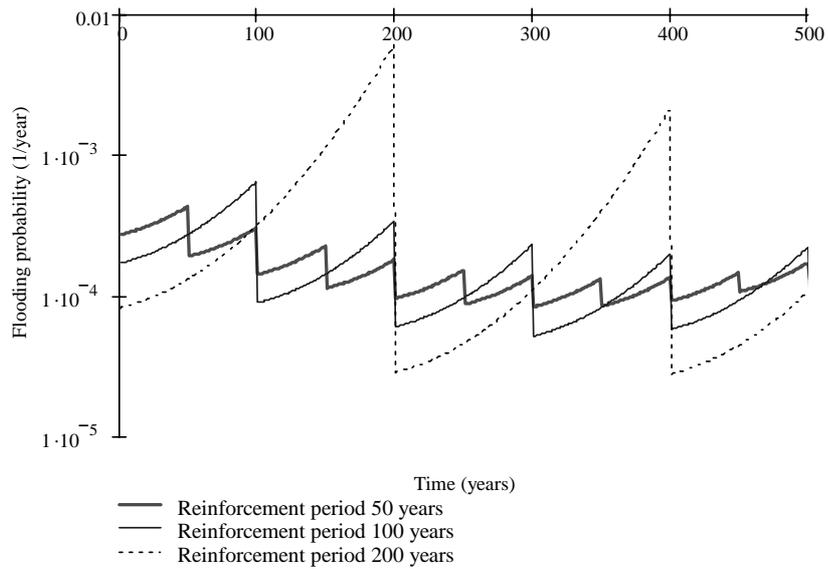


Figure 11: Flooding probability over time, applying strategy C under climate scenario 4

Initially, the damage in case of flooding increases more rapidly than the cost of reinforcement of the defence system, resulting in a decreasing design flooding probability over time. After approximately 350 years however, climate change causes the cost of reinforcement to increase more rapidly than the damage in case of flooding, resulting in an increasing design flooding probability over time.

Figure 12 shows the life-cycle cost as a function of the reinforcement period. The minimum cost level is found at a reinforcement period of 77 years. In that case, the design flooding probability for the first period is $2 \cdot 10^{-4}$ per year.

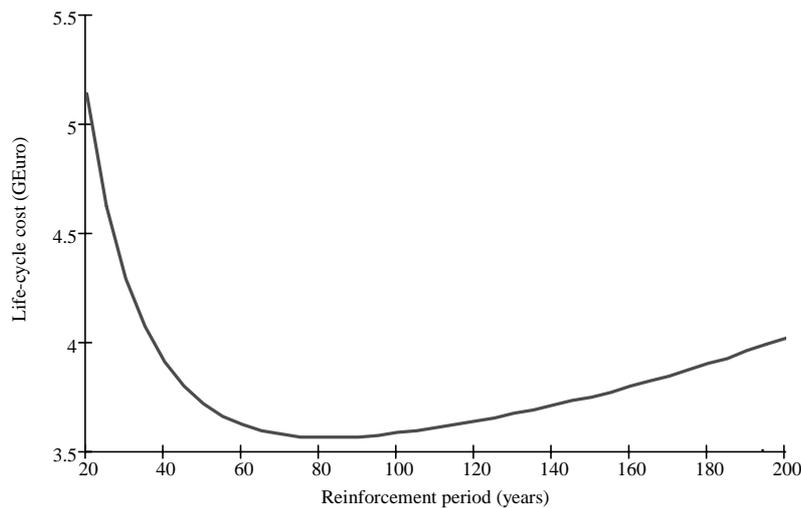


Figure 12: Life-cycle cost as a function of reinforcement period in strategy C under climate scenario 4

8 Choice of defence strategy for the case study area

The grounds for choosing a defence strategy depend on the decision-maker and can in principle not be left to the analyst. However, the analysis provides a great deal of insight in the consequences of the choice of the strategy. In table 5, a number of characteristics of every strategy are summarised. The optimised characteristics of every strategy are shown. In every row, the value that can be considered most favourable to a decision-maker is shaded.

A decision-maker can choose a strategy for a number of reasons. It may be expected that a decision-maker aims for a low cost level but is at the same time sensitive for the value of the flooding probability at different times in the reference period.

Strategy A is preferred over the other two strategies with respect to the long reinforcement period and the low risk over the full life-cycle. However, this strategy must be considered unattractive

since the costs of protection are excessive and fall completely in the beginning of the reference period. Due to capitalisation, the risk for 200 years and further into the future hardly influence the decision, leading to a flooding probability as high as 1 per year occurring after 450 years. Strategy A is therefore not sustainable in the far future and will not be considered any further.

Strategy B shows the lowest values of the design flooding probability and of the maximum flooding probability over the reference period. Strategy C is fully optimised and therefore shows the lowest life-cycle cost, life-cycle investment and investment in the first period. Comparison of strategies B and C shows that in C cost reductions are obtained by accepting slightly higher flooding probabilities and slightly higher risk. A comparison of the two strategies is shown in table 6 where the results of strategy B relative to the results of strategy C are presented.

Table 5: Characteristics of alternative defence strategies after optimisation of every strategy

	Strategy		
	A	B	C
Design flooding probability first period (1/year)	$5.8 \cdot 10^{-9}$	$1.7 \cdot 10^{-4}$	$1.9 \cdot 10^{-4}$
Reinforcement period (years)	500	86	83
Maximum flooding probability (1/year)	1	$5.0 \cdot 10^{-4}$	$5.3 \cdot 10^{-4}$
Total life-cycle cost (GEuro)	20.5	3.61	3.56
Life-cycle investment (GEuro)	20	2.99	2.94
Life-cycle risk (GEuro)	0.49	0.62	0.63
Investment first period (GEuro)	20	2.93	2.86
Risk first period (GEuro)	0.49	0.50	0.55

Table 6: Characteristics of strategies B and C, relative to strategy C

	Strategy	
	B	C
Design flooding probability first period (1/year)	89%	100%
Reinforcement period (years)	104%	100%
Maximum flooding probability (1/year)	94%	100%
Total life-cycle cost (GEuro)	101%	100%
Life-cycle investment (GEuro)	102%	100%
Life-cycle risk (GEuro)	98%	100%
Investment first period (GEuro)	102%	100%
Risk first period (GEuro)	91%	100%

Strategy B provides 11 % reduction of risk (19 % over the first period only) by allowing an increase of the investment of 2 % (2 % over the first period) and an increase of 1 % of the life-cycle costs. Strategy B is sub-optimal from a mathematical point of view. However, from a practical point of view, the difference in flooding probability for the first period for strategies B and C is negligible. Also the differences in investment and risk, both for the first period and over the life-cycle are small. It appears that in this case study, there is no significant distinction between strategies B and C.

9 Conclusions and discussion

Risk-based design methods for large-scale flood defence systems are extended with a method to deal with scenarios for climate change. This opens the possibility to perform quantitative analysis of different flood defence strategies over time. The method shows a modular structure. If the optimal design of the flood defence system is available for a range of predefined conditions, different climate scenarios as well as different defence strategies may be evaluated rapidly without the necessity of repeating the full optimisation of the flood defence system itself.

The method is applied to support the societal decision-making process with respect to an appropriate flood defence strategy for a case study area in the southern North Sea. The following conclusions are valid for the case study only.

Three strategies are evaluated with respect to:

- Minimum and maximum values of the flooding probability;
- Cost of the flood defence;
- Risk over the reference period.

In strategy A, a very safe flood defence system is constructed at the beginning of the reference period of 500 years. Reinforcement of the flood defence at some time in the future is not considered in this strategy. Quantifying the characteristics of this strategy shows that it leads to very high initial investments and therefore to a very high level of life-cycle cost. Despite the very low value of the flooding probability at the beginning of the reference period, this strategy will lead to highly unsafe situations in the far future.

The starting point of both strategy B and strategy C is that at some point in the future reinforcement of the flood defence system will be necessary. In strategy B, fixed values for the design flooding probability and the reinforcement period are adopted. In strategy C, the design flooding probability is optimised for every reconstruction. The life-cycle costs and life-cycle investment are the lowest under strategy C. The price for this is a minor increase of the risk. The reconstruction periods and

the probability of flooding in the first period are comparable for both strategies B and C. In fact, for the first period there is hardly a distinction between the two strategies.

The quantitative results of the analysis depend on a set of input parameters, describing both environmental conditions, economic and societal characteristics of the protected area at the beginning of the reference period and changes of these parameters over the reference period. If necessary, the sensitivities for such input parameters can be investigated. In a qualitative sense, the analysis already leads to the important conclusion that in a changing climate, an adaptive strategy like B or C is more cost-effective than a static strategy like strategy A.

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