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# Optimising rainwater harvesting systems under uncertainty: A multi-objective stochastic approach with risk considerations

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ABSTRACT

Optimising rainwater harvesting (RWH) systems' design involves sizing the storage and catchment areas to enhance cost-effectiveness, self-sufficiency, and water quality indicators. This paper considers the design of RWH systems under long-term uncertainty in precipitation and demands. In this work, we formulate and solve a multi-objective stochastic optimisation problem that allows explicit trade-offs under uncertainty, maximising system efficiency and minimising deployment cost. We use the yield after spillage (YAS) approach to incorporate the physical and operational constraints and the big-M method to reformulate the nonlinear min\max rules of this approach as a mixed-integer linear programming (MILP) problem. By posing a risk averseness measure on efficiency as a conditional value at risk (CVaR) formulation, we guarantee the designer against the highest demand and driest weather conditions. We then exploit the lexicographic method to effectively solve the multi-objective stochastic problem as a sequence of equivalent single-objective problems. A detailed case study of a botanical garden in Amsterdam demonstrates the framework's practical application; we show significant improvements in system efficiency of up to 15.5% and 28.9% in the driest scenarios under risk-neutral and risk-averse conditions, respectively, compared to deterministic approaches. The findings highlight the importance of taking into account multiple objectives and uncertainties when designing RWH systems, allowing designers to optimise efficiency and costs based on their specific requirements without extensive parameterisation.

## 1. Introduction

### *1.1. Motivation*

Some water consumers require a significant amount of water stably throughout the year. Greenhouses and chemical industries belong to this type of consumers. However, changing conditions, mainly due to climate change, force the water utilities to impose restrictions on the consumption of large consumers in some months of the year. Therefore, consumers with a high sensitivity to reliable supply must explore alternative options. Rainwater harvesting (RWH) systems can independently connect to the consumers as an alternative resource to supply during uncertain flow through the main water network. The determining factor in the RWH system is the tank size for storing rainwater. This size, in addition to the modelling paradigm, depends on the rainfall data and required demand. However, the optimal sizing of a RWH system becomes challenging when taking into account the uncertainty associated with the mentioned parameters. Furthermore, when accounting for parameter uncertainty, it is important to ensure that the system can achieve a specific output level even in the worst-case scenarios, which may include water-saving efficiency or reliability. The consumer must also consider satisfying its demand as well as the RWH system's investment cost.

### *1.2. Literature review*

This section reviews existing literature on rainwater harvesting RWH system design and optimisation, highlighting the gaps addressed by this study. The review is structured thematically, initially examining the evolution of RWH system design considerations, then focusing on the incorporation of uncertainty, and finally discussing the application of multi-objective optimisation techniques.

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*RWH system's design:* The most important aspect of designing the RWH systems is their storage size. Finding the optimal storage tank size for these systems has received special attention in recent years (Semaan et al., 2020). Furthermore, researchers have also considered optimising other aspects of RWH systems in the form of objective functions or evaluation metrics. Consideration has been given to objective functions such as reliability (Islam et al., 2021), efficiency (Zhang et al., 2018), meeting the demand Lopes et al. (2017), and roof area (Wallace et al., 2015). Efficiency can be defined differently based on the formulas used to model mass balance and the relationships between various flows in the RWH system. Two general indices used in the literature to assess efficiency are water-saving efficiency (Zhang et al., 2018), the proportion of total water usage met by the RWH system, reflecting the amount of water conserved relative to overall demand, and rainwater use efficiency (Muklada et al., 2016), the ratio of rainwater supplied for consumption to the total rainwater harvested from the catchment area, indicating the extent to which harvested rainwater is utilised. Besides these two, there exist other efficiency metrics in the studies, such as retention (Carollo et al., 2022), reduction of runoff peaks (Xu et al., 2020), and so on (Quinn et al., 2021). The evaluation metrics mainly assess the different aspects of investment costs, like payback period or return on investment, benefit-cost ratio, and net present value (Hafizi Md Lani et al., 2019). A large body of literature follows a similar direction concerning the solution approach and nature of considered parameters. From a solution approach point of view, a considerable portion of the works have used simulation-based optimisation approaches for solving the storage sizing problem. Notwithstanding the capabilities of these approaches, they possess some disadvantages, including (1) Intractability resulting from the utilisation of object-based simulation models instead of formulas (Klemmt et al., 2009); (2) Low efficiency due to the iterative procedure of repeatedly running the model, which consumes significant computational resources (Ma et al., 2014; Albanesi et al., 2018; Whitt and You, 2022); (3) Lack of guarantee for solution optimality due to the inherent stochastic nature of these approaches, often involving approximations and sampling techniques (Klemmt et al., 2009); and (4) Slow convergence caused by the computational complexity associated with running simulations repeatedly to evaluate different solutions (Nageshwaraniyer et al., 2013).

*RWH systems optimisation under uncertainty:* Despite the different objectives and design characteristics of the RWH systems that have been considered in the literature, most of them overlooked the uncertainty of the system's parameters. Two important probabilistic parameters of RWH systems are the precipitation values during the planning horizon, and the required demand of consumer(s). Hence, designing a storage tank for RWH systems without considering the uncertain nature of these two parameters may wander far from expected outcomes due to changing conditions surrounding the problem, such as climate change.

Table 1 has collected the RWH system storage sizing papers that have taken uncertainty into account. From this table, it is apparent that there is a gap in considering both rainfall and demand as uncertain parameters. The literature mainly has used other methods for modelling the mass balance equations within the system like yield before spillage (YBS) than yield after spillage (YAS) as the operational rule. Regarding output, the main difference between YAS and YBS is that the YAS operational rule is more conservative than the YBS rule (Fewkes and Butler, 2000). Additionally, Mitchell (2007) suggested employing the YAS algorithm due to its lower sensitivity to changes in storage capacity and water demand. Furthermore, it is worth noting that despite having addressed multiple objectives in their work, Soh et al. (2023), Wang et al. (2024) have not approached them in the form of a multi-objective optimisation problem. The papers that have used optimisation instead of simulation for solving the storage sizing problem are limited to Nop et al. (2021), Soh et al. (2023), which have modelled the problem as a Markovian decision process and two-player game theory, respectively, and then solved them using dynamic stochastic programming and robust optimisation. Additionally, it is crucial to emphasise that the risk considered in Soh et al. (2023) pertains to surface overflows rather than the worst-case scenarios. It is also possible to adapt the approaches to account for uncertain parameters, such as precipitation and flows from other applications (Li et al., 2024; Fei et al., 2024).

*Multi-objective optimisation of RWH systems:* Since various elements are involved in designing an RWH system, it is crucial to incorporate them into the design procedure. This can be done with multi-objective optimisation. From this perspective, only a few works have utilised this type of optimisation. Li et al. (2018) integrate life cycle assessment into a multi-objective optimisation model to design sustainable RWH systems in Beijing, considering trade-offs among RWH volume, stormwater runoff control, economic cost, and environmental impacts. Li et al. (2017) explore the multi-objective optimisation of RWH systems in China, considering factors such as water resource augmentation, waterlogging alleviation, economic investment, and pollutant reduction. The study develops a systematic model using non-dominated sorting genetic algorithm II to determine the optimal construction areas of green rooftops, porous pavements, and green lands, providing insights into urban development policymakers. By formulating a multi-objective optimisation model, Pérez-Uresti et al. (2019) addresses the challenge of sustainable water management in regions with over-exploited water resources, showcasing the potential of RWH as an alternative source to meet water demand and facilitate the recovery of depleted wells, as demonstrated in the case study conducted in Queretaro, Mexico. Regarding including multiple objectives in RWH system design, two relevant studies with a distributed approach are worth mentioning. Di Matteo et al. (2017) introduce a framework for designing distributed stormwater harvesting systems, considering objectives such as lifecycle cost, volumetric reliability, and total suspended solids reduction. On the other hand, Ali et al. (2014) focuses on analysing household-level adoption through an agent-based model and system dynamics simulation to explore the trade-offs in decentralised RWH strategies for urban water sustainability. However, both works use evolutionary algorithms for solving multi-objective optimisation problems. Furthermore, the overall cost of an RWH system in Sánchez-Zarco et al. (2021) is considered one of the objectives of a water-energy-food nexus. In contrast to the heuristic nature of evolutionary algorithms, which offer flexibility but lack optimality guarantees and can be computationally expensive, exact methods provide certifiably optimal solutions where applicable, albeit with limitations in scalability and potential challenges in handling highly nonlinear problems; this trade-off is central to the methodology employed in the field. To the best of the author's knowledge, the only paper that has utilised an exact method to solve multi-objective optimisation in this field is Zhen et al. (2023), and its superiority becomes evident when compared to evolutionary algorithms. However, it focuses on controlling a network of pre-designed rainwater storages without considering uncertainty, similar to all the above-reviewed literature about the multi-objective optimisation of RWH systems.

Where possible, many designers rely on extensive historical rainfall data for deterministic RWH system design, given the time-series encompasses extreme wet and dry scenarios sufficiently to ensure that the design would be robust to uncertainty. However, this assumption may be flawed as future conditions, like mid-century scenarios and beyond, might display non-stationary statistical properties compared to the past, due to climate change. To address this, we compare deterministic methods using historical data with stochastic models that incorporate projected future climates, highlighting the added value of considering different future scenarios explicitly.

### 1.3. Contributions and paper structure

Considering the uncertainties surrounding an RWH system and its various design requirements, this paper proposes a multi-objective optimisation under uncertainty for storage and catchment sizing of an RWH system. For this purpose:

**Table 1**
Comparison of RWH papers w.r.t. modelling paradigms and consideration of uncertainties.

| | Uncertain parameter(s) | | Modeling paradigm | Resolution | Objective(s) | Methodology | Risk |
|---|---|---|---|---|---|---|---|
| | Rainfall | Demand | | | | | |
| Muklada et al. (2016) | ✓ | × | YAS | Daily | Efficiency | Simulation | × |
| Sim and Kim (2020) | ✓ | ✓ | Simple storage equation | Daily | | Simulation | × |
| Silva and Ghisi (2016) | × | ✓ | * | Daily | Water savings | Simulation | × |
| Nop et al. (2021) | ✓ | × | ** | Monthly | Optimal operation | Optimisation | × |
| Lopes et al. (2017) | ✓ | × | YAS | Daily | Deficit rate | Simulation | × |
| Cheng et al. (2021) | ✓ | × | *** | Hourly | Reliability | Simulation | × |
| Soh et al. (2023) | ✓ | × | Inventory modelling | Daily | Surface overflows and water availability | Optimisation | × |
| Wang et al. (2024) | ✓ | × | YBS | Hourly | Efficiency and reliability | Simulation | × |
| **This work** | ✓ | ✓ | YAS | Daily | Efficiency and cost | Optimisation | ✓ |

* The Netuno computer program was used (Ghisi et al., 2014).
** Water balance equation and using Markov chain model for rainfall.
*** StRaHaS method (simulation based stochastic rainwater harvesting system modelling).

- The non-convex formulations of the YAS operational rule, which models the flows within an RWH system, are transformed into a system of mixed-integer linear programming (MILP) equations using the big-M method (Vielma, 2015), enabling the efficient computation of optimal solutions, unlike simulation-based approaches.
- The water saving efficiency, defined as the ratio of rainwater yield to water demand, and the deployment cost, determined by storage size and required catchment area, are considered objectives in formulating a multi-objective RWH system design problem.
- Then, the uncertainty around the system is captured by defining the rainfall and demand as random variables with appropriate probability distributions, which is essential for a more robust design that accounts for variability in these key parameters.
- Conditional value-at-risk (CVaR) is employed as a risk measure to safeguard system efficiency during dry years and high-demand scenarios, ensuring the system's reliability even under the worst-case scenarios.
- A case study with realistic data from a greenhouse in a botanical garden in Amsterdam represents the applicability of the developed method.

Turning to the point that RWH systems have various applications and can be used for different purposes, it is worthwhile to mention that this work focuses on optimising the size of the storage tank to be used by a large consumer. While this study specifically addresses the technical challenges of designing them, it does not delve into the broader sustainability and circularity aspects of these systems. Future research is needed to comprehensively assess the cost-effectiveness of RWH systems, considering factors like water quality, environmental impact, and potential benefits such as flood risk mitigation. This would involve a more detailed cost–benefit analysis that incorporates these benefits in a broader context.

The following sections organise the rest of the paper. Section 2 explains the physical model of the system and subsequently formulates the deterministic and stochastic multi-objective optimisation problems. Section 2.5 makes plain the lexicographic algorithm for solving the multi-objective problems. Section 3 features a botanical garden in Amsterdam, a real-world case study illustrating how pilot data was used to generate scenarios for the developed model. Within Section 4, a comprehensive numerical analysis is conducted and the results are widely discussed. Eventually, Section 5 concludes the paper and gives direction for future works.

## 2. Methodologies

In this section, we present a model for describing an RWH system and formulate its design problem as a multi-objective mathematical optimisation in both deterministic and probabilistic forms. We then set forth an approach for solving the proposed optimisation problem. To provide a high-level overview, the key stages of the methodology are outlined in Section 2.1 and summarised in Fig. 1.

### 2.1. Overview of the methodology

To provide a clear overview of the approach used in this study, we outline the key stages of the methodology as follows:

- System Modelling: The physical and operational constraints of the RWH system are modelled using dynamic mass balance equations. The YAS operational rule is employed to govern the system's behaviour and determine the flows (inflow, outflow, overflow, and stored volume) based on rainfall and demand data. A deterministic model is first developed, followed by its stochastic extension to incorporate uncertainties.
- Optimisation Framework: The system design problem is formulated as a multi-objective optimisation problem, balancing the trade-offs between deployment cost (tank size and catchment area) and system efficiency. A risk-averse formulation is introduced using CVaR to account for extreme scenarios, and then a lexicographic approach is used to solve the bi-objective problem.
- Evaluation and Analysis: The optimised solutions are evaluated through a case study of a botanical garden in Amsterdam, highlighting the improvements in system efficiency under dry scenarios and comparing deterministic, stochastic, and risk-averse approaches.

Fig. 1 illustrates the overall framework used in this study, summarising the steps from system modelling to optimisation and evaluation.

### 2.2. Physical model of the system

Before formulating the system design, we first introduce the physical and operational constraints of the system. Subsequently, we utilise a dynamic model of tank storage and describe its input and output using water flow equations.

Collecting rainwater is possible from various surfaces, but this research specifically focuses on rainwater harvesting from available roofs. The first step in harvesting rainfall from roofs involves collecting water from the gutters. Then, water flows through pipes to a storage tank, where it undergoes treatment or remains available for direct use. Fig. 2 illustrates the simplified configuration of RWH systems, along with a mathematical conceptualisation of water use, flows, and storage.

To simulate the RWH system, we conduct a continuous simulation of the mass balance equation. Fig. 2 illustrates the various terms considered in the mass balance equation in this study. In this configuration, $r_t$ [mm] denotes the rainfall received within period $t$, while $q_t$, $v_t$, $y_t$, and $o_t$ represent the inflow into the storage, the stored volume, the outflow to meet the demand (specifically, irrigation demand in our case study), and the overflow from the storage, respectively. Additionally, $d_t$ represents the water demand at time $t$, and $S$ indicates the tank capacity, all measured in $m^3$.

According to Semaan et al. (2020), the model may utilise different time scales and operating rules, such as YBS and YAS, to estimate

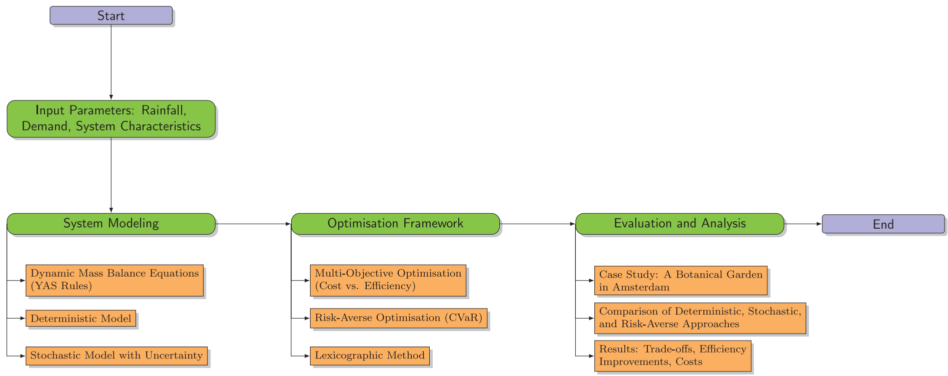



**Fig. 1.** RWH system design methodology.

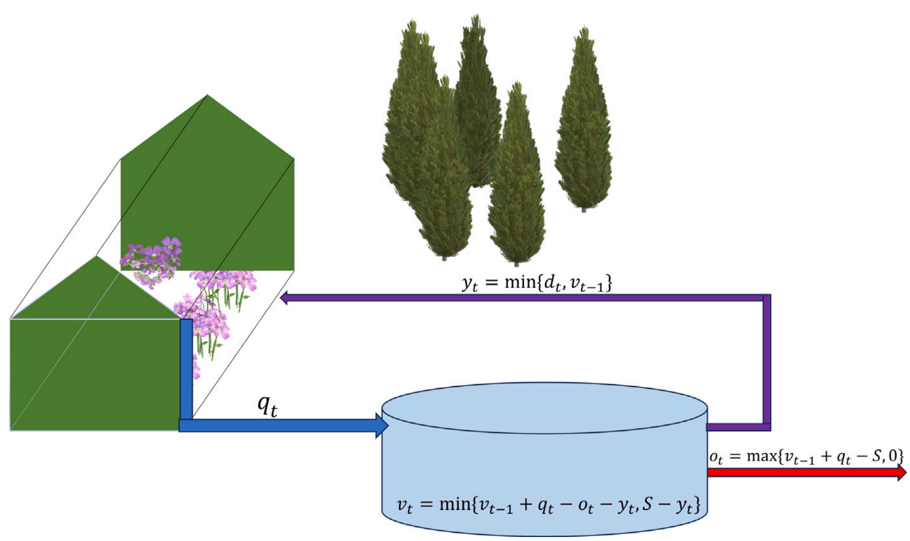



**Fig. 2.** A RWH system and different flows therein (inflow $q_t$, outflow $y_t$, overflow $o_t$ [m$^3$/time]) with their corresponding mathematical model variables (stored volume $v_t$ and storage size $S$ [m$^3$]) and parameter ($d_t$ [m$^3$/time]) based on YAS operational rule.

tank sizes (Jenkins and Pearson, 1978; Fewkes and Butler, 2000; Liaw and Tsai, 2004). Parametric techniques like the storage-reliability-yield model (SRY) are also among the rules considered in this regard. The YAS operating rule determines the yield as the lower volume of stored rainwater from the preceding time interval or the demand in the current time interval, as explained in (1). The volume of the tank at each time step is calculated by considering the minimum of the maximum capacity minus the outflow (required to meet the demand at time $t$) or the inflow minus the overflow and yield. This calculation is represented by (2).

$$y_t = \min\{d_t, v_{t-1}\} \tag{1}$$

$$v_t = \min\{v_{t-1} + q_t - o_t - y_t, S - y_t\} \tag{2}$$

The YBS operating rule calculates the yield as the smaller of two values: the volume of rainwater in storage from the previous time interval plus the runoff in the current interval, or the current demand. This is elucidated in (3) and (4).

$$y_t = \min\{d_t, v_{t-1} + q_t\} \tag{3}$$

$$v_t = \min\{v_{t-1} + q_t - y_t, S\} \tag{4}$$

The equation in both simulation modelling approaches that must be satisfied is the overflow Eq. (5).

$$o_t = \max\{v_{t-1} + q_t - S, 0\} \tag{5}$$

According to Semaan et al. (2020), 51% of simulation modelling employs the mass balance method, 29% uses the YAS operational rule, and 11% utilises the YBS operational rule. In terms of output, the YAS operational rule is more conservative than the YBS rule (Fewkes and Butler, 2000). Additionally, Mitchell (2007) suggested employing the YAS algorithm due to its lower sensitivity to changes in storage capacity and water demand. Therefore, this research adopts the YAS approach. Historical precipitation observations, primarily in daily resolution (Ward et al., 2010), serve as input for the model. However, the optimum design depends on different parameters, such as precipitation, water demand, and the system's characteristics.

### 2.3. Deterministic storage size optimisation

Now that we have modelled the system through its dynamic equations in the previous section, we can define the optimisation problem to find the optimal storage tank size. For this purpose, we develop a deterministic problem in this section, with known system parameters. Then a stochastic model will be presented in the next section taking the uncertainty of the system parameters into account.

The deterministic cost and efficiency optimisation problem for finding both storage size and the required roof of the RWH system can be formulated as follows:

$$\begin{cases} \min\limits_{S,A,\Phi_t} & C_m S + C_c S^2 + C_A A \\ \max\limits_{S,A,\Phi_t} & \dfrac{\sum_{t=1}^{T} y_t}{\sum_{t=1}^{T} d_t} \end{cases} \tag{6a}$$

$s.t.\quad \forall t = 1, \dots, T$

$$q_t = \phi \cdot r_t \cdot R \cdot A \tag{6b}$$

$$y_t \leq d_t \tag{6ca}$$

$$y_t \leq v_{t-1} \tag{6cb}$$

$$y_t \geq d_t - M(1 - y_t^y) \tag{6cc}$$

$$y_t \geq v_{t-1} - M y_t^y \tag{6cd}$$

$$o_t \geq v_{t-1} + q_t - S \tag{6da}$$

$$o_t \geq 0 \tag{6db}$$

$$o_t \leq v_{t-1} + q_t - S + M(1 - y_t^o) \tag{6dc}$$

$$o_t \leq +M y_t^o \tag{6dd}$$

$$v_t \leq v_{t-1} + q_t - o_t - y_t \tag{6ea}$$

$$v_t \leq S - y_t \tag{6eb}$$

$$v_t \geq v_{t-1} + q_t - o_t - y_t - M(1 - y_t^v) \tag{6ec}$$

$$v_t \geq S - y_t - M y_t^v \tag{6ed}$$

The objective function (6a) consists of two terms: the first is for minimising the overall cost of the system, including the construction of a storage tank, materials used for water storage, and preparing the required roof; the second is for maximising system efficiency, a non-dimensional index used to evaluate the performance of the RWH system and defined as the ratio of rainfall yield at each time step $t$ to water demand in the corresponding time step over the system life cycle. In the first term, $C_m$, $C_c$, and $C_A$ are the cost function coefficients related to the new storage and purchase\rental and maintenance of roofs. We determined the cost functions related to material and construction of storage empirically for our case study and provide the methodological details in Appendix. The first constraint relates the inflow into the tank $q_t$ to the available catchment area $R$, the expected increase factor in that area $A$ and the rainfall $r_t$ by the runoff coefficient $\phi$. It is crucial to note that the proposed planning problem assumes the existence of a roof before its application. So, in the cases without a roof as the catchment area in the location of the demand or neighbouring areas, all terms involving the fraction of increase in the roof $A$ have to be slightly adjusted. Next, we have three sets of constraints (6c)–(6e), respectively, corresponding to the three flow equations of the systems (1), (2) and (5). Each set includes four inequalities providing a mixed-integer reformulation of logical min\max statements. In these constraints, $M$ is a sufficiently large positive number, and $y_t^y$, $y_t^o$, and $y_t^v$ are auxiliary binary variables, respectively, corresponding to rainwater supply, overflow, and stored volume in the tank storage. This problem is binary linearly constrained quadratic programming that, due to the convexity of the objective function, can be solved to optimality by off-the-shelf optimisation solvers. It gives the storage size $S$ and required roof $A$ in addition to the set of system dynamic variables $\Phi_t = \{v_t, q_t, y_t, o_t\}$. However, since there are two objective functions, the solution will be a set of optimal points. It will be discussed in greater detail in the solution approach subsection.

### 2.3.1. Single-objective optimisation versions

Although the multi-objective problem provides a trade-off between the goals of RWH system planning, the storage sizing problem can be formulated as a single-objective optimisation problem in two different modes. The first is when the RWH system planner prioritises the cost of its built infrastructure over the efficiency they can expect from it. Therefore, the objective function defines the minimisation of the cost of investment in the new RWH system as

$$\min_{S,A} \quad C_m S + C_c S^2 + C_A A. \tag{7}$$

Here, $C_m$ stands for the required materials for keeping water in itself and $C_c$ is related to the construction-related costs like digging. Since the construction cost is increasing to a higher degree than the cost required for materials, it appears in the objective function with second order. Also, $C_A$ is the cost of purchasing the necessary roof for water collection. In this mode, the planner can set a constraint for the efficiency so that its value does not fall below a certain threshold. Second, with efficiency as the objective function,

$$\max_{S,A} \quad \frac{\sum_{t=1}^{T} y_t}{\sum_{t=1}^{T} d_t}, \tag{8}$$

the cost function is imposed as a constraint, ensuring it remains within a given project budget. Solving the resulting problem may yield a continuous range of solutions for $S$, which produce the same optimal efficiency value. As will be discussed in Section 4.1.2, the problem of maximising efficiency subject to budget constraints is trivial and always has a solution on the boundary since cost is an increasing function of storage and area, and the efficiency is also an increasing function of catchment area and storage.

## 2.4. Stochastic storage size optimisation for risk quantification

The two uncertain parameters pertaining to the RWH system design, namely rainfall and demand, are driven by stochastic processes. These will be represented by a set of, potentially dependent, random stochastic variables sequentially arranged for the planning period considered. Typically, such time series of stochastic variables (Conejo et al., 2010, Sec. 2.3) can be represented by scenarios. Here we define the uncertain demand time series as $d(\omega_d)$, $\omega_d = 1, 2, \dots, N_{\omega_d}$, where $\omega_d$ is the demand scenario index and $N_{\omega_d}$ is the number of demand scenarios considered. Similarly, we can define the uncertain rainfall time series as $r(\omega_r)$, $\omega_r = 1, 2, \dots, N_{\omega_r}$, where $\omega_r$ is the rainfall scenario index and $N_{\omega_r}$ is the number of rainfall time series scenarios considered. These two variables together can form a scenario set of uncertainties of size $N_\omega$ by combining the possible set of uncertainties $\omega = \omega_d \times \omega_r$. Each such time series is considered as a realisation of the combined uncertain parameters $[d(\omega), r(\omega)]$, with associated probability $\pi(\omega)$, and with $\sum_{\omega=1:N_\omega} \pi(\omega) = 1$.

Now, the stochastic optimisation model can be formulated in multi-objective mode. For this purpose, the random parameters representing the uncertain nature of rainfall and water demand replace the corresponding values in the deterministic multi-objective model. The multi-objective stochastic model is given by (9),

$$\begin{cases} \min\limits_{S,A,\Phi_t} & C_m S + C_c S^2 + C_A A \\ \max\limits_{S,A,\Phi_t} & \mathbb{E}_{\omega}\left[\frac{\sum_{t=1}^{T} y_t}{\sum_{t=1}^{T} d_t(\omega)}\right] \end{cases} \tag{9a}$$

$s.t.\ \forall t = 1, \dots, T$ and $\forall \omega = 1, \dots, N_\omega$

(6b)–(6e) (9b)

in which the second objective of (9a) is the expected efficiency over all considered scenarios $\omega$ and constraint (9b) encompasses the satisfaction of the constraint sets (6c)–(6e) for all scenarios and time steps. Note that although all the dynamic variables are also scenario variables, for the sake of brevity, the $\boldsymbol{\omega}$ notation has been omitted. Furthermore, note that both $\boldsymbol{\omega}_r$ and $\boldsymbol{\omega}_d$ are included in the notation $\boldsymbol{\omega}$ to represent the combined uncertainty in both rainfall and water demand. In the formulation, $N_\omega$ represents the total number of scenarios considered including both rainfall and water demand. The expectations $\mathbb{E}_{\boldsymbol{\omega}}[\cdot]$ in the objective function indicate that the objective is evaluated by considering the average values over all possible scenarios.

This formulation, however, has a neutral behaviour towards different scenarios and optimises the storage tank based on the average of the scenarios. This approach overlooks the inherent risk and variability in the uncertain parameters. By not explicitly considering the probability distribution or the variability of these parameters, the resulting solution may not be robust or resilient to extreme or unexpected scenarios. Consequently, the system may be ill-prepared to handle situations where actual demand or rainfall deviates significantly from the average, leading to unacceptable performance to the RWH user. Additionally, in storage sizing, often it is required that efficiency at a certain future time is, with high reliability, at least equal to a certain value. Therefore, it is necessary to apply an appropriate risk measure to mitigate the impact of scenarios that involve high demand and dry weather conditions. One such measure is expected shortage (ES), which represents the expectation of efficiency in scenarios where the efficiency is smaller than a pre-fixed value $\eta$. ES provides insight into the average loss beyond a specified threshold and is a linear measure of risk. However, the arbitrary selection of a target value for efficiency is required, which does not align with the definition of a coherent risk measure (Artzner et al., 1999). An alternative risk measure frequently utilised for risk management is value-at-risk (VaR), which captures the $(1-\alpha)$-quantile of the efficiency distribution. Here, $\alpha$ denotes a predetermined probability level. This measure provides a threshold value that separates extreme losses from the rest of the efficiency distribution and does not require the specification of an efficiency target value. However, one fundamental shortcoming of VaR is that it only provides information up to a certain threshold.

To address these limitations, one can utilise the CVaR measure, which quantifies the expected shortfall beyond the VaR threshold and provides a more comprehensive risk assessment. It considers both the threshold and the magnitude of losses beyond that threshold, making it a more suitable risk measure. Mathematically, one can calculate CVaR by taking the sum of VaR and the expected value of losses that exceed the VaR threshold, denoted as (Conejo et al., 2010, Equation (4.73)):

$$\max \left\{ \eta - \frac{1}{1-\alpha} \mathbb{E}_{\boldsymbol{\omega}} \left[ \max \left\{ \eta - \frac{\sum_{t=1}^{T} y_t}{\sum_{t=1}^{T} d_t(\boldsymbol{\omega})}, 0 \right\} \right] \right\}, \quad \forall \alpha \in (0,1) \tag{10}$$

This measure can be embodied in the stochastic programming problem (9) as follows:

$$\begin{cases} \min\limits_{S,A,\Phi_t,\eta,s(\boldsymbol{\omega})} & C_m S + C_c S^2 + C_A A \\ \max\limits_{S,A,\Phi_t,\eta,s(\boldsymbol{\omega})} & (1-\beta)\left( \sum_{\omega=1}^{N_\omega} \pi(\boldsymbol{\omega}) \frac{\sum_{t=1}^{T} y_t}{\sum_{t=1}^{T} d_t(\boldsymbol{\omega})} \right) \\ & + \beta(\eta - \frac{\alpha}{1-\alpha} \sum_{\omega=1}^{N_\omega} \pi(\boldsymbol{\omega}) s(\boldsymbol{\omega})) \end{cases} \tag{11a}$$

$s.t.\ \forall \omega = 1, \ldots, N_\omega$

$$\eta - \frac{\sum_{t=1}^{T} y_t}{\sum_{t=1}^{T} d_t(\boldsymbol{\omega})} \le s(\boldsymbol{\omega}) \tag{11b}$$

$$s(\boldsymbol{\omega}) \ge 0 \tag{11c}$$

(6b)–(6e) $\quad \forall t = 1, \ldots, T$

The problem (11) represents a deterministic equivalent of the risk-averse version of (9). In (11), the parameter $\beta$ indicates how much risk the decision maker is willing to accept, which ranges from 0 for risk-neutral states to 1 for risk-averse states. The value of parameter $\alpha$ also explicitly indicates risk attitude, as the size of the considered set of worst cases is reflected by $1-\alpha$. The auxiliary variable $\eta$ is a threshold value and determines the risk level at which losses are considered in the CVaR calculation and $s(\boldsymbol{\omega})$ is a continuous non-negative variable equal to the maximum of $\eta - \frac{\sum_{t=1}^{T} y_t}{\sum_{t=1}^{T} d_t(\boldsymbol{\omega})}$ and 0. Also, the probability of each scenario is denoted by $\pi(\boldsymbol{\omega})$.

### 2.4.1. Single-objective optimisation versions

When considering uncertainties, one can also perceive the problem as a single-objective akin to the deterministic optimisation problem. It provides the decision-maker with a new perspective to examine the problem when they want to give precedence to one objective over another. Another reason for adopting this problem representation is to develop a solution approach for the multi-objective problem based on it in the next section.

To address the storage sizing problem, one can approach it in two modes, depending on the objective functions, as a risk-averse stochastic single-objective optimisation problem. If prioritising the cost related to storage, one can formulate the first mode of the single-objective problem as follows:

$$\min_{S,A,\Phi_t,\eta,s(\boldsymbol{\omega})} \quad C_m S + C_c S^2 + C_A A \tag{12a}$$

$s.t.\ \forall \omega = 1, \ldots, N_\omega$

$$(1-\beta)\left( \sum_{\omega=1}^{N_\omega} \pi(\boldsymbol{\omega}) \frac{\sum_{t=1}^{T} y_t}{\sum_{t=1}^{T} d_t(\boldsymbol{\omega})} \right) + \beta(\eta - \frac{\alpha}{1-\alpha} \sum_{\omega=1}^{N_\omega} \pi(\boldsymbol{\omega}) s(\boldsymbol{\omega})) \ge E \tag{12b}$$

(11b) and (11c)

(6b)–(6e) $\quad \forall t = 1, \ldots, T$ (12c)

The planner in this mode is looking to design an RWH system in which the costs associated with the construction and equipment of the water storage tank and catchment area are at their lowest. Meanwhile, it is crucial for them to consider the system's efficiency as well. In view of this, constraint (12b) imposes that the expected efficiency and risk must be greater than a predefined minimum efficiency $E$ for all scenarios. It is also necessary to include the constraints (11b) and (11c) to incorporate the risk within the problem.

In the second mode, the priority is to give emphasis to efficiency. In this regard, (13) formulates the problem in a single-objective optimisation form.

$$\max_{S,A,\Phi_t,\eta,s(\boldsymbol{\omega})} \quad (1-\beta)\left( \sum_{\omega=1}^{N_\omega} \pi(\boldsymbol{\omega}) \frac{\sum_{t=1}^{T} y_t}{\sum_{t=1}^{T} d_t(\boldsymbol{\omega})} \right) + \beta(\eta - \frac{\alpha}{1-\alpha} \sum_{\omega=1}^{N_\omega} \pi(\boldsymbol{\omega}) s(\boldsymbol{\omega})) \tag{13a}$$

$$s.t.\ C_m S + C_c S^2 + C_A A \le B \tag{13b}$$

$$\left. \begin{array}{l} \text{(11b) and (11c)} \\ \text{(6b)–(6e)} \quad \forall t = 1, \ldots, T \end{array} \right\} \quad \forall \omega = 1, \ldots, N_\omega \tag{13c}$$

In this mode, the planner wants to maximise the system's efficiency, considering the impact of the worst scenarios. For this, we have to solve a CVaR-constrained stochastic programming problem. However, ensuring compliance with the project budget requires implementing the constraint (13b) in which $B$ indicates the maximum budget available to the system planner. Nevertheless, if the planner does not care about the cost, they can disregard the budget constraint. However, in order to obtain a unique solution for the variable $S$, they must include a regularisation term in the objective function by adjusting a $\rho$ coefficient similar to (8). Further, as shown in (13b), since storage size and roof area are planning decision variables, they are independent of time $t$ and scenario.

### 2.5. Lexicographic approach for solving the bi-objective optimisation problem

As described above, the design problem that aims to simultaneously maximise the system's efficiency and minimise the system's cost results in a bi-objective quadratic program with linear constrains. Here, we present the lexicographic approach to solve this bi-objective optimisation problem, which enables the two objectives to be handled sequentially according to a predetermined order of priorities. In this case, solving the problem involves considering both scenarios: the first objective of minimising the cost is given priority over maximising the system efficiency, and vice versa.

The lexicographic method can be summarised using the following algorithm, which has been adapted from Jaimes et al. (2009, Sec. 3.1.3):

**Algorithm 1** Lexicographic Method for Solving (11)

**Initialization:**
Set all RWH design and Algorithm parameters to their values (eg. see second paragraph of Sec. 4.1.1.)
Set the priority list of objectives
Initialize best objective values set *OptObj*[1,2]
*# Objective optimization loop:*
**if** Cost is the first priority **then**
  Solve (12)
  **if** $Cost^*$ is unique **then** *# Lexicographically optimal solution for cost*
    *OptObj*[1] $\longleftarrow Cost^*$
  **else**
  *# Constraint update:*
    Solve $\{$(13) | (13b) $\longleftarrow ... \leq Cost^*\}$
    *OptObj*[2] $\longleftarrow Efficiency^*$ *# Lexicographically optimal solution for efficiency*
  **end if**
**else**
  Solve (13)
  **if** $Efficiency^*$ is unique **then** *# Lexicographically optimal solution for efficiency*
    *OptObj*[1] $\longleftarrow Efficiency^*$
  **else**
  *# Constraint update:*
    Solve $\{$(12) | (12b) $\longleftarrow ... \geq Efficiency^*\}$
    *OptObj*[2] $\longleftarrow Cost^*$ *# Lexicographically optimal solution for cost*
  **end if**
**end if**

By applying the lexicographic method as described in the Algorithm above, we obtain a set of lexicographically optimal solutions, each representing a distinct trade-off between the cost of the storage system and system efficiency. Each solution in this set prioritises the objectives in a specific order, optimising the first objective and then finding the best solution for the second objective given the optimum of the first. This set provides decision-makers with a clear understanding of the possible compromises between these two objectives, enabling them to select the solution that best aligns with their special needs and priorities.

However, it is important to note that the lexicographic method yields only the top two optimal solutions corresponding to two objective functions. As a result, it does not provide a complete set of optimal solutions in the form of the Pareto front. One can employ alternative multi-objective optimisation methods to find all Pareto-optimal solutions. These methods allow for a more extensive exploration of the objective space and offer a set of solutions that reflect the compromises between the objectives. In this case, we will utilise the widely recognised weighted sum method (Aneja and Nair, 1979). This method involves reformulating the multi-objective optimisation problem as a single-objective problem, as represented in (14), where each objective function contributes with a specific weight. Terms $Cost$ and $Efficiency$ in the objective function represent objective functions of optimisation problems (12) and (13), respectively. The sum of weights must also be one. Solving the optimisation problem (14) iteratively for different values of weights $w_C$ and $w_E$ results in the Pareto front set. This set provides decision-makers with a comprehensive understanding of the trade-offs between the objectives, allowing them to make informed choices based on their specific requirements and priorities.

$$\max_{\substack{S,A,\Phi_t,\\ \eta,s(\omega)}} \quad -w_C Cost + w_E Efficiency \tag{14a}$$

$$s.t.\ \forall \omega = 1, \ldots, N_\omega$$

$$w_C + w_E = 1 \tag{14b}$$


**Fig. 3.** The greenhouses and their roof materials in the Hortus Botanicus.

$$\begin{array}{ll}(11b) \text{ and } (11c) & \\ (6b)\text{–}(6e) & \forall t = 1, \ldots, T\end{array} \tag{14c}$$

## 3. Case study: a real application in Amsterdam

We first describe the case study area, providing details about the botanical garden, its surroundings, and relevant factors such as existing roofs. Next, we discuss the scenario generation process, which captures uncertainty in the garden's water demand and the area's rainfall patterns. By considering different climate change scenarios and utilising time-series rainwater data, we generate a range of plausible scenarios to assess the RWH system's performance under varying conditions.

### 3.1. Description of the case study

There are many projects throughout the world for collecting and reusing rainwater. In Europe, increasing numbers of individuals desire to lessen their environmental impact by utilising collected rainwater because of adopting RWH can protect against the expected damage caused by an increase in the frequency and severity of precipitation in Western Europe (Hofman-Caris et al., 2019). For instance, Amsterdam is exploring all public spaces where rainwater could be collected during big surges, to increase the co-benefits of adaptation by complimenting decentralised water supply as well as reducing urban flooding (Amsterdam Rainproof, 2024).

The Hortus Botanicus, a renowned botanical garden in central Amsterdam (Fig. 3), for example aims to transition from using drinking water to rainwater for its irrigation needs.

The Hortus Botanicus heavily relies on potable water for irrigating its gardens. In recent years, the Hortus has felt the effects of more summer days without rainfall. During these dry periods, they had to use drinking water for irrigation. They could not efficiently utilise rainfall as an additional source for plant irrigation because they did not have an efficient system to harvest and treat rainwater for irrigation purposes. This realisation has heightened their awareness of the need for change to continue providing their services in the future. The Hortus, in consequence, is pursuing sustainability and climate adaptation goals by renovating the largest greenhouse in the garden and the three-climate greenhouse. In addition, they are exploring the construction of rainwater harvesting storage in other greenhouses. Rainwater collected from roofs and other possible catchment areas may serve as an alternative to drinking water.

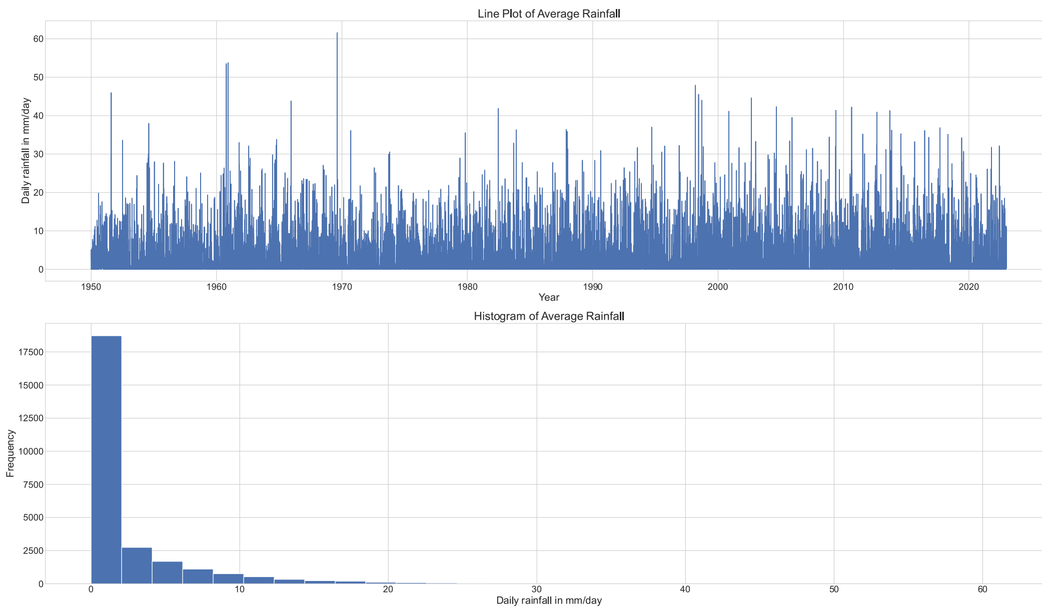



**Fig. 4.** Rainfall distribution in the final rainfall time series.

**Table 2**
The distribution of the rain gauge stations surrounding the Hortus Botanicus.

| Name of station | Distance from the Hortus in km |
|---|---|
| Schellingwoude | 6.4 |
| Zaandam-Hembrug | 10.5 |
| Lijnden NH | 10.5 |
| Weesp | 11.2 |

### 3.2. *Scenarios data preparation*

The backbone of stochastic programming is the input scenarios that model the uncertain parameters of the problem. Moreover, the accuracy of the problem's solutions is directly related to the quality of these scenarios. This makes scenario generation a crucial part of stochastic programming. Hence, in this work, scenarios are utilised in a way that effectively reflects the uncertain parameters of the problem.

As it will be described in Section 3.2.2, the rainfall scenarios come from The Royal Netherlands Meteorological Institute (KNMI) rainfall scenarios generated to support climate adaptation. As such, these scenarios aim to capture a broad set of possible futures and so are assumed to reflect climate uncertainty in our problem effectively. On the other hand, in consultation with the stakeholder based on their upgrade planning, it is assumed that the average demand may change significantly (e.g., 10%–40%). However, the stochastic variation of demand is assumed to have similar seasonal variation as was historically the case. As such, we characterise the historical demand and sample future demand variations (both average and stochastic) from this distribution. This process will be described in more details in Section 3.2.3.

#### 3.2.1. *Historical rainfall and demand*

To find the right size for rainwater storage at the Hortus, the study began by choosing nearby meteorological stations that measure daily rainfall. The average daily precipitation we used can be downloaded from the KNMI website (KNMI, 2024). As shown in Table 2, there are four different rain gauges, each at a different distance from the Hortus Botanicus, with Schellingwoude being the closest station. This effort resulted in a detailed rainfall dataset that covers 72 years, from 1950 to 2022, offering insights into long-term rainfall trends.

The model used in this study relies on historical rainfall data, recorded daily (Ward et al., 2010), as illustrated in Fig. 4.

The Hortus monitors its water demand in different parts of the garden biweekly, categorising it into two main areas: greenhouses and outdoor gardening. Typically, outdoor irrigation ceases from October to March, while the greenhouses require daily watering. The peak demand months at the Hortus are from May to August, during which both the greenhouses and the garden require irrigation, significantly straining the water supply. Fig. 5 illustrates the measured demand in 2021 and 2022.

In this study, outdoor gardening was not included in the analysis because it did not accurately reflect the true demand for water. Also, during data collection, some cleaning activities were incorrectly labelled as gardening. Furthermore, the daily water demand was estimated by dividing the total monthly usage by the number of days in each month.

#### 3.2.2. *Rainfall scenarios*

Typically, we use historical rainfall data to determine the appropriate size of a rainwater harvesting tank. However, for planning for the future, relying solely on historical data cannot account for the impact of climate change on future rainfall patterns. Therefore, it is necessary to also take into consideration future rainfall scenarios when sizing RWH systems. KNMI is a Dutch national weather service in charge of forecasting and monitoring weather, climate, air quality, and seismic activity in the Netherlands. The report of KNMI'14 climate change scenarios (Attema et al., 2014) provided four different future climate scenarios, which were determined by the interaction of two crucial factors: global temperature rise and alterations in air circulation patterns.

These scenarios were designed to outline potential changes in weather patterns, temperature, and sea levels. The First one is assumes moderate temperature rise, and low change in air circulation (GL). This scenario projects a small rise in global temperatures together with little variations in air circulation patterns. The effects are rather little in comparison to other situations. The second one is moderate temperature rise, high air circulation change (GH). In this scenario, there is a mild temperature rise together with notable air circulation changes. The change of noticeable seasonal and weather extremes may be greater as a result than in the GL scenario. The third one was known as warm temperature increase and low change in air circulation (WL) which predicts a faster rate of warming with little changes in air circulation. The last one is about significant warming and significant changes in

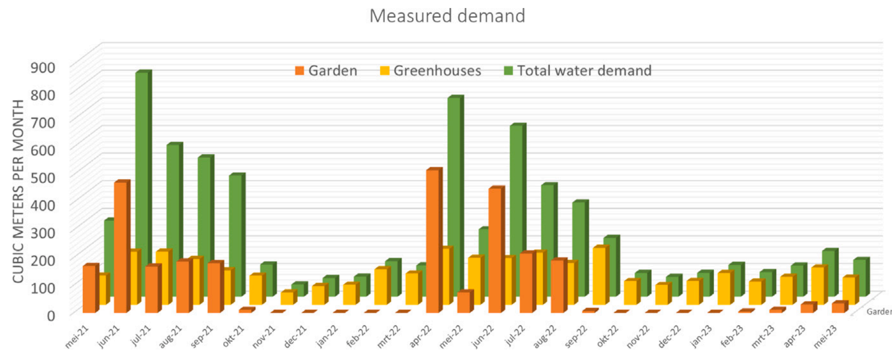



**Fig. 5.** Demand in the Hortus Botanicus for both gardens and greenhouses.

air circulation patterns characterise the most extreme changes, known as warm temperature increase and high change in air circulation (WH). This situation might have a major impact on precipitation and sea levels as well as severe weather events by significantly changing the climate and weather patterns. Each of these four scenarios are parameterised and have three different levels of uncertainty in their severity, ranging from low to high leading to a total of 12 different climate change scenarios. There were two horizons for the mentioned climate change scenarios, 2050 and 2085. Since the lifetime of rainwater harvesting is about 30 years the projections to 2050 from these scenarios were employed in this work.

Although these scenarios are for the whole of the Netherlands, they do not necessarily capture spatial variations within the country. So, in 2019, KNMI released a new regional classification system to provide a more detailed picture of climatic trends, to distinguish both year-round and particularly winter climatology (KNMI, 2019). The report gives three classifications for the year-round climatology compared to DeBilt station: Low (L), Regular (R), and High (H). Four classes are given to differentiate winter season precipitation compared to DeBilt: Very Low (LL), Low (L), Regular (R), and High (H).

The Foundation for Applied Research in Water Management (STOWA, the acronym in Dutch) serves as a knowledge hub for regional water managers, and has worked to disaggregate these scenarios spatially for operational water management. STOWA therefore applied the statistics of the mentioned climate change scenarios from KNMI on the De Bilt station (KNMI station De Bilt has the longest homogeneous dataset in the Netherlands) to generate rainfall time series for each scenario and regional classification; these rainfall time series can be accessed online (KNMI, 2019).

Amsterdam is the location of the case study under consideration in this research, and has the regional classification of 'RR', indicating 'Regular' classification for both seasonal and winter climates. This classification can be used to download the time series data of each 12 scenarios from Meteobase (2024).

The STOWA-generated rainfall time series for the case study location were directly utilised as input scenarios in our stochastic model. These time series represent daily rainfall projections under each of the 12 KNMI scenarios (4 main scenarios × 3 uncertainty levels) for the 2050 horizon. No further manipulation of historical data was performed; instead, the scenarios serve as standalone climate-conditioned rainfall sequences.

### 3.2.3. Demand scenarios

We employed a dataset of the Hortus Botanicus consisting of monthly historical demand data spanning one year to generate demand scenarios. Considering a 10-year planning horizon, we consider two types and four scales of demand increase in this period. We consider two categories of demand increase: gradual and uniform increase over the desired years and a one-time surge in demand at the beginning of the design. Regarding the scale, we have included an increase of 1, 2, 3, and 4 tenths of a percent to the current amount of demand.

In this study, the demand scenarios were generated by assuming increases in base demand by certain percentages and sampling from the empirical distribution specific to the case study garden. This approach allowed us to capture realistic variations in both average and stochastic aspects of demand. If we want to describe in the context of notations of Section 2.4, for generating each one of eight scenarios $d^t(\omega_d^t)$ in day $t$, we added random numbers $\omega_d^t$ sampled from the garden demand distribution $\omega_d^t \sim \mathcal{N}(0, \Sigma_d^t)$ to the demand of the corresponding day $t$ in the time series data. We have found the standard deviation $\sqrt{\Sigma_d}$ of the distribution using the maximum likelihood estimation function `fit_mle` of Distributions.jl package (Besançon et al., 2021). Fig. 6(b) represents the range of generated demand scenarios and the used historical demand data of the Hortus. As can be seen, the historical data is relatively far from the maximum and minimum scenarios and demonstrates a characteristic comparable to that of the average. In addition, if we compare the annual intervals together as well as Fig. 5, we note the seasonality in the generated scenarios.

## 4. Results and discussions

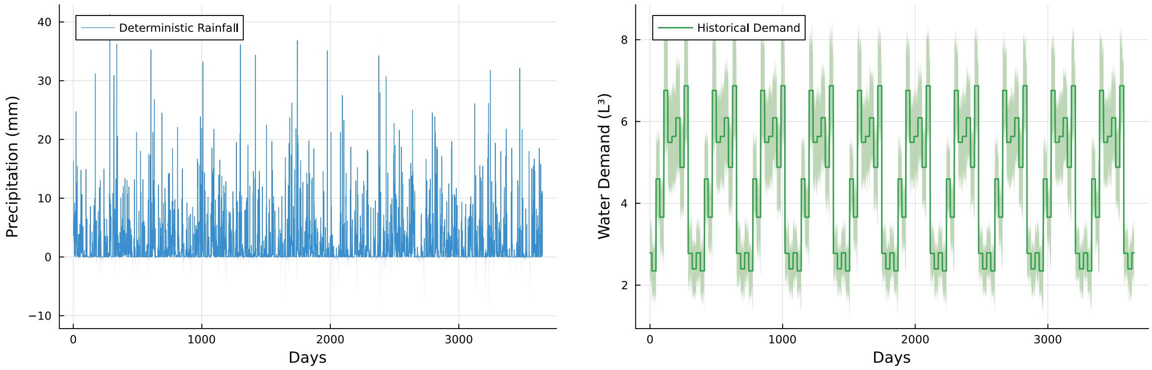
This section presents the numerical results obtained from analysing the RWH system. Building on the data described in the previous section, we explore three cases studies: minimising cost, maximising efficiency, and conducting a multi-objective analysis. For each case, we examine deterministic, stochastic, and risk-averse versions of the optimisation problem to evaluate the system's performance under different decision-making paradigms. These numerical results provide valuable insights into the optimal storage sizing of the RWH system and its potential to address water scarcity and sustainability concerns.

### 4.1. Design experiments

In this subsection, we analyse the optimal storage sizing of the RWH system using three optimisation cases of minimising cost, minimising efficiency and a multi-objective analysis. All three cases will consider and compare deterministic conditions, and conditions with uncertainty, where we also explore both risk-neutral and risk-averse approaches.

#### 4.1.1. Case 1: Minimising cost

In this case, the storage sizing problem will be exhaustively examined, with the highest priority given to the cost of the RWH project. The planner solves an optimisation problem with the objective function (7) and constraints (6b)–(6e), when they do not consider uncertainty. However, focusing only on cost reduction without considering the system's efficiency reduces its effectiveness significantly. The decision maker can compensate for this by imposing a minimum efficiency constraint, $\frac{\sum_{t=1}^{T} y_t}{\sum_{t=1}^{T} d_t} \geq E$, on the defined optimisation problem. In addition, the planner must formulate and solve the problem (12) to address the uncertainty condition. It provides risk-neutral and risk-averse stochastic solutions when $\beta$ equals zero and one, respectively. By




(a) (b)

**Fig. 6.** The range of scenarios changes with (a) the mean value of rainfall, and (b) the historical value of water demand.

**Table 3**
Results for minimising cost with a minimum efficiency value of 0.8.

| | | Deterministic | Stochastic | |
|---|---|---|---|---|
| | | | Risk-neutral | Risk-averse |
| S ($m^3$) | | 69 | 139 | 221 |
| A | | 1.3 | 1.3 | 1.3 |
| Cost (€) | | 27 895 | 57 340 | 93 174 |
| | ◊ Simulation | 0.81 | 0.89 | 0.95 |
| Efficiency | ◊ Worst year-scenario | 0.473 | 0.628 | 0.762 |
| | ◊ 8 worst scenarios | 0.695 | 0.77 | 0.827 |
| running time (s) | | 258 | 15 393 | 54 928 |

setting a minimum efficiency constraint of $E = 0.8$, the design results for all three problems are shown in Table 3. The cost coefficient for roof area is negligible compared to storage cost and is set to 1 arbitrarily; in all cases, the optimisation results in roof area use at the maximum available $A$.

These are obtained by $M = 10^6$, $\phi = 0.8$, only four demand scenarios corresponding to an initial one-time increase at the beginning of 10-year design period, and 12 rainfall scenarios based on KNMI climate change projections applied to historical rainfall data with equiprobability, and $\alpha = 0.8$ (meaning 20 percent (10 of 48) of scenarios are the worst scenarios). In addition, in all the simulations of this work, the value 2617 as the available catchment area is multiplied by $A$ as the expected increase factor of the development plan. The cost function coefficients are $C_m = 400$, $C_c = 0.1$, and $C_A = 1$. All optimisation problems have been modelled in Julia\JuMP (Dunning et al., 2017) and solved by Gurobi on a laptop with an Intel Core i5 2.6 GHz processor and 8 GB RAM.

Table 3 indicates that the deterministic optimisation formulation results in a smaller storage size than the stochastic ones. This difference arises because the stochastic problem must satisfy constraints under all 48 scenarios, whereas the deterministic problem only needs to account for historical demand and rainfall time series. Also, under conditions of uncertainty, incorporating risk measures leads to a more expensive plan for a guarantee of given efficiency level $E$. However, the higher cost of storage of the RWH system in the risk-averse formulation is justified when considering the system under worst-case scenarios. For this purpose, consider the rows corresponding to the efficiency in those scenarios. The row labelled *worst year scenario* shows simulated performance of the optimal design specifically for the driest year from the 72 years of historical rainfall data set. Similarly, the row labelled *eight worst scenarios* simulates performance under the worst future climate rainfall time series from the twelve rainfall scenarios (i.e. time-series representing the four dry rainfall scenarios and the two highest increases in demand). Comparing the results of these rows of Table 3 illuminates a justification for using the stochastic formulation and

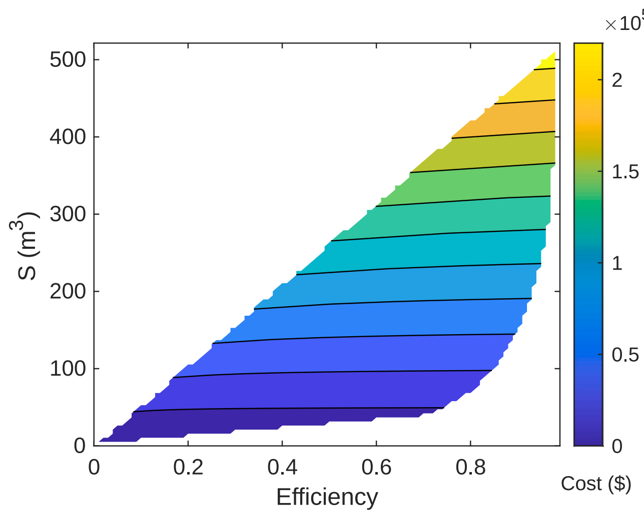



**Fig. 7.** Sensitivity of the planning cost and storage size to the minimum efficiency $E$ in the deterministic mode.

a measure of risk. As can be seen from these results, the stochastic problem performs better than the deterministic one in the most adverse circumstances. For instance, we have a 13% increase in the efficiency for the eight worst scenarios and a 29% increase for the worst year-scenario compared to the deterministic case when we utilise the CVaR measure in the stochastic case.

The sensitivity analysis on the deterministic formulation assessed the impact of the minimum efficiency value $E$ on the storage size and system cost. This is done by changing the value of $E$ and solving the problem for each new value and obtaining the objective function value (cost) and variable $S$. As seen in Fig. 7, as $E$ increases, the system's need to store water to meet the demand increases, leading to more storage volume and, thus, planning cost.

### 4.1.2. Case 2: maximising efficiency

In this section, we investigate the condition where maximising efficiency is the primary goal of the RWH system designer. Although it is possible to solve this problem without restricting the designer's budget if the objective function includes the designing variables with weighting coefficient, this approach may not be so efficient and realistic. It is inefficient since adjusting the weighting coefficient for $S$ and $A$ is based on the designer's experience and is case-sensitive, and finding their best value requires solving another optimisation problem. In addition, overlooking the budget constraint can lead to impractical designs or inadequate capacity, hindering the system's effectiveness.

**Table 4**
Results for maximising efficiency with budget constraint.

| | | Deterministic | | | | | | Stochastic (Risk-neutral) | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Budget ($\times 10^5$) | 0.5 | | 1 | | 4 | | 0.5 | | 1 | | 4 | |
| | $C_c$ | 0.1 | 0.5 | 0.1 | 0.5 | 0.1 | 0.5 | 0.1 | 0.5 | 0.1 | 0.5 | 0.1 | 0.5 |
| $S$ ($m^3$) | | 121 | 110 | 236 | 200 | 707 | 580 | 121 | 110 | 236 | 200 | 828 | 580 |
| $A$ | | 1.3 | 1.3 | 1.3 | 1.3 | 1.3 | 1.3 | 1.3 | 1.3 | 1.3 | 1.3 | 1.3 | 1.3 |
| Efficiency | ◊ Optimisation | 0.877 | 0.865 | 0.953 | 0.937 | 1 | 0.994 | 0.763 | 0.755 | 0.853 | 0.832 | 0.95 | 0.938 |
| | ◊ Simulation | 0.82 | 0.81 | 0.886 | 0.869 | 0.98 | 0.972 | 0.82 | 0.81 | 0.886 | 0.869 | 0.985 | 0.972 |
| Objective function | | 0.877 | 0.865 | 0.953 | 0.937 | 1 | 0.994 | 0.763 | 0.755 | 0.853 | 0.832 | 0.95 | 0.938 |
| Running time (s) | | 1.68 | 1.66 | 1.72 | 1.8 | 2.2 | 3.61 | 525 | 421 | 423 | 576 | 862 | 1114 |

With this in mind, to perform the numerical analyses of this case in the deterministic mode, we need to solve an optimisation problem that incorporates all the constraints of (6), with the objective function defined as (8) while taking $\rho_S$ and $\rho_A$ equals to zero and (13b) as the budget constraint. Solving stochastic programming under similar conditions is necessary to facilitate result comparison. So, we solve problem (13) in both risk-neutral ($\beta = 0$) and risk-averse ($\beta = 1$) modes. Considering the same parameter values as Case 1, the results of simulations can be presented as the Table 4. The results have been obtained based on different project budgets and the $C_c$ coefficient. Two coefficients 0.1 and 0.5 for $C_c$, respectively, correspond to the construction of the storage tank on the ground or below it.

We only show the risk-neutral results in this table because they are the same as the risk-averse case. Comparing the results of deterministic and stochastic problems shows similar results except for one case. The different result, related to the project budget $4 \times 10^5$ and $C_c = 0.1$, exposes that when we spend a large amount of money and have a low construction cost, we can expect a larger storage size for the condition with scenarios. But even though the deterministic and stochastic results for these parameter values are slightly different, they reach the same efficiencies for the worst year-scenario (0.717) and eight worst scenarios (0.722). By examining the following figure, we can gain further clarity on the reason behind the similar outcomes observed in deterministic and stochastic problems when maximising efficiency.

By examining the curves depicted in Fig. 8, which correspond to two different values of $C_c$ in the quadratic cost function, one can discern the similarity between the results of the deterministic and stochastic cases. Both lower budgets lead to similar storage sizes for two different values of $C_c$, given the overlap of the two curves. However, with the increase in the project budget, we can expect different values according to the distance taken between the two graphs. The enlargement of the solution space makes it possible for the deterministic and stochastic modes to reach different results according to the corresponding columns of the Table 4.

Also, similar to case 1, a sensitivity analysis for the objective functions and planning variable is carried out here. For this purpose, the problem of case 2 is solved iteratively by changing the project budget parameter $B$ and the values of the objective function (efficiency) and variable $S$ are recorded. In this regard, Fig. 9 shows the change of the efficiency and resulting storage size with respect to the project budget for the deterministic formulation. Comparing the $x$ and $z$ axes of this figure with Fig. 7 specifies which objective is the intention of the optimisation problem. As can be seen in Fig. 9, we have high values of efficiency with high planning cost in contrast to lower values of planning cost with lower efficiency in Fig. 7.

#### 4.1.3. Case 3: Multi-objective approach

We assess the trade-off between cost and efficiency in this case by solving the multi-objective deterministic optimisation (6) for the case without uncertainty and the stochastic optimisation (11) for the case with uncertainty. In both cases, we use Algorithm 1 to solve this problem. To implement the algorithm, we utilise Julia's MultiObjectiveAlgorithms package and keep the `MOA.Algorithm()` attribute in its default. However, since the lexicographic method only gives two solutions with respect to the best value of each objective, as described in Section 2.5, we need to employ another approach to obtain Pareto front solutions. To achieve this, we modify the algorithm setting to `MOA.Dichotomy()`, which utilises a weighted sum algorithm suitable for the proposed problem due to its convexity in continuous variables.

In the weighted sum approach (`MOA.Dichotomy()`), the weights $w_C$ and $w_E$ are iteratively adjusted to explore the trade-offs between cost and efficiency, satisfying the condition $w_C + w_E = 1$, as shown in (14b). Points on the Pareto front correspond to specific weight combinations, where solutions with higher $w_C$ prioritise cost minimisation and those with higher $w_E$ emphasise efficiency. The density and spread of points along the Pareto fronts in Fig. 10 illustrate the achievable range of compromises between these two objectives. For example, points closer to the lower-left corner emphasise cost minimisation, while points closer to the upper-right corner represent higher efficiency values.

Fig. 10 shows the Pareto-front solutions of the multi-objective approach, the left, middle, and right plots represent the deterministic, risk-neutral, and risk-averse cases, respectively. While the optimisation results of the deterministic case exhibit higher efficiency values compared to the stochastic cases, notable differences emerge in scenarios characterised by dry weather and high demand. Like the two previous cases, we used eight worst scenarios and the worst year scenario for this purpose. For the eight worst scenarios, the Pareto-front solutions of risk-averse have higher values of efficiency than risk-neutral, and they have higher values than deterministic for the same amount of cost. For the worst-year scenario, although the efficiency values are relatively similar for all three cases, the risk-averse case has significantly higher values for points related to costs less than $10^5$. This is also true for the risk-neutral case when compared to the deterministic case. The results further highlight the importance of adjusting weights $w_C$ and $w_E$ to explore different trade-offs, enabling decision-makers to select solutions that align with their priorities. From another perspective, one can make a notable observation when comparing the efficiency of different scenarios for lower cost values. While there is a distinct disparity between the cost optimisation results and those obtained for worst-case scenarios in the deterministic case, the efficiency values across different scenarios in both stochastic cases are relatively similar. This is the reason for the lower efficiency values in the two stochastic cases.

When we compare the outcomes of case 3 with those of the preceding two cases, noticeable similarities emerge. For example, we expected a higher cost for the stochastic case compared to the deterministic case and for the risk-averse case compared to the risk-neutral case at a similar efficiency. In return, and as a result of paying this higher cost, we would expect a higher efficiency from the RWH system designed for the stochastic case compared to the case without considering the uncertainty. One can make these observations by carefully examining Fig. 10, which follows the results of Table 3 but with a multi-objective approach.

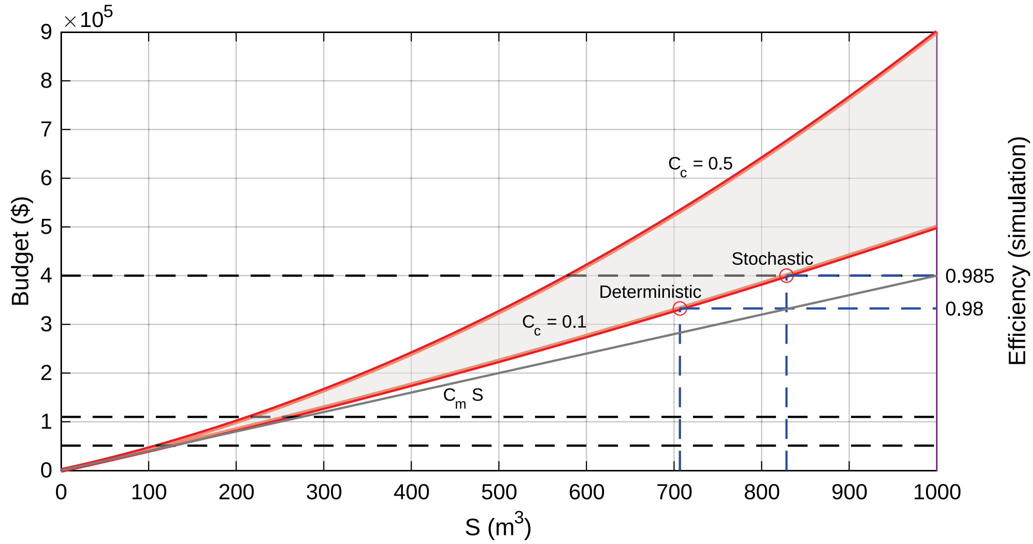



**Fig. 8.** The effect of budget and construction cost on the results of maximising efficiency; we also depict the linear, in storage $S$, component of the objective $C_m S$ for comparison.

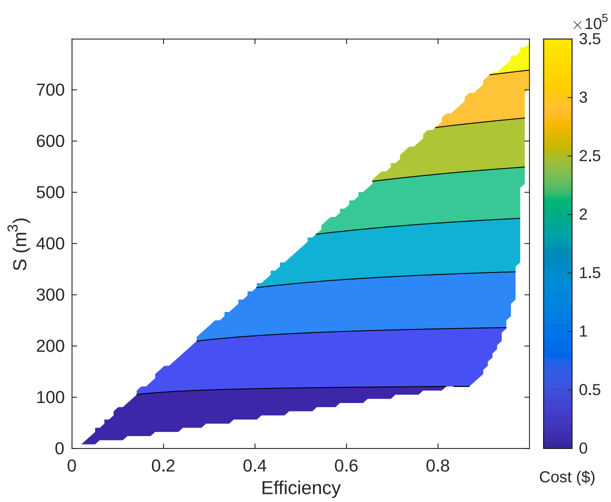



**Fig. 9.** Change in the efficiency and storage size considering the project budget in the deterministic mode.

## 5. Conclusions

In this study, we formulated a multi-objective optimisation for sizing the storage and catchment required for an RWH system for supplying a botanical garden as a large water consumer. The objectives are to minimise the total cost of storage deployment and catchment area purchase, as well as to maximise system efficiency. As part of the solution approach for this problem, we also formulated the single-objective mode of the optimisation problems. The results of this mode show a significant increase in efficiency, with a 15.5% and 28.9% improvement in the driest scenario and year when employing risk-neutral and risk-averse stochastic programming, respectively, compared to deterministic programming. Additionally, while it may be possible to obtain a range of results by conducting a considerable number of simulations on the data used for cases under uncertainty, including the results obtained in this study, the determination of optimal solutions considering different parameters of the RWH system remains challenging. It highlights the superiority of our optimisation approach in providing a prioritised and optimal solution compared to existing simulation-based approaches. Moreover, leveraging the multi-objective mode gives insight to the system designer for choosing the optimal solution based on their preference without having to get involved with setting up various parameters. Future directions also may encompass exploring step tariffs for purchased water from the network, assessing water quality using indicators such as water age, and comparing the results of CVaR with alternative risk consideration methods like chance-constrained programming.

### 5.1. Limitations and future research directions

While this study offers a framework for multi-objective RWH system optimisation under uncertainty, some of the limitations can guide future research. First, the assumed stationary distributions for demand may not fully capture real-world variability or non-stationary trends due to changes in water use or climate. Future work could explore non-stationary models to inform the scenario-based approaches. A particularly promising avenue would be to explore distributionally robust optimisation, which explicitly accounts for uncertainty in the underlying probability distributions. Second, the current focus on cost and efficiency neglects other important factors, such as water quality (e.g., water age), environmental benefits, and maintenance costs. Extending the model to incorporate these elements, including the assessment of water quality using indicators such as water age, would provide a more holistic assessment. Third, the YAS operational rule simplifies certain physical processes; although this may not have impact on the results and conclusions made, future research could investigate more detailed operational models or hybrid approaches combining optimisation with simulation. Fourth, the case study is focused on a botanical garden in Amsterdam; the generalisability of the findings can be explored through other case studies in diverse settings (residential areas, arid regions) to validate the framework's broader applicability. Investigating the sensitivity of the results to the assumed probability distributions and exploring alternative risk measures (e.g., chance-constrained programming) and comparing the results of CVaR with these alternative methods would also enrich the analysis. Furthermore, incorporating the economic impact of step tariffs for purchased water from the network could provide a more realistic cost assessment.

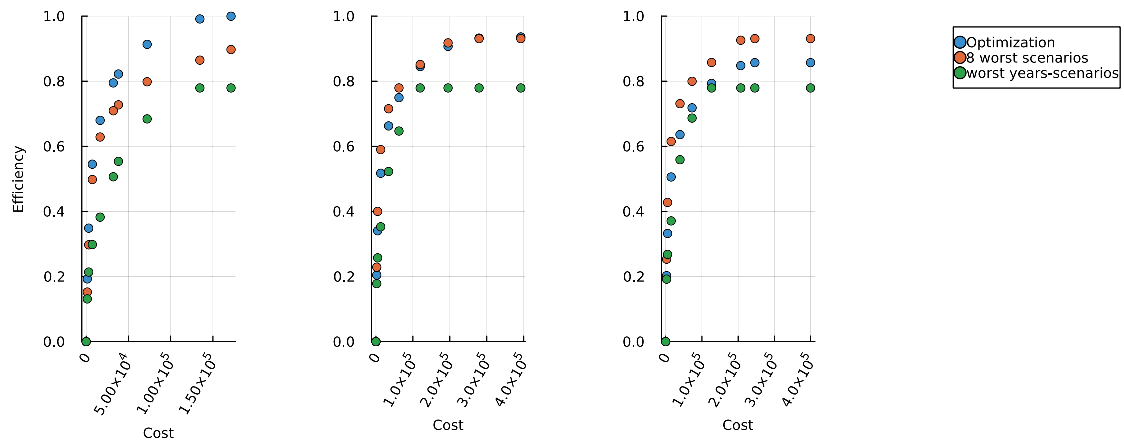



**Fig. 10.** Pareto-front of multi-objective problems for the deterministic (left), stochastic risk-neutral (middle), and risk-averse (right) modes. The coloured dots show the objective values from the optimisation, and simulations with bad (i.e. dry) years.

## CRediT authorship contribution statement

**Alireza Shefaei:** Writing – original draft, Visualization, Software, Methodology, Conceptualization. **Arash Maleki:** Writing – original draft, Visualization, Software, Resources, Conceptualization. **Jan Peter van der Hoek:** Writing – review & editing, Supervision, Funding acquisition, Formal analysis. **Nick van de Giesen:** Writing – review & editing, Formal analysis. **Edo Abraham:** Writing – original draft, Validation, Investigation, Funding acquisition, Conceptualization.

## Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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## Appendix. Supplementary material

### A.1. Linear approximation for material cost

The material cost coefficient discussed in Section 2.3.1 is derived from a market analysis in the Netherlands, supplemented by consultations with the Hortus, which has previously purchased storage tanks. In this research, when constructing large storage capacities (e.g., 250 $m^3$), smaller storage units (ranging from 1 to 50 $m^3$) can be utilised to achieve the desired total storage volume. Fig. A.11 illustrates the relationship between storage size and corresponding cost, showing varying prices for different storage capacities.

After consulting with the Hortus and comparing the purchased storage sizes by them from another companies, assuming a coefficient of $C_m = 400$ appears to be reasonable for the material costs.

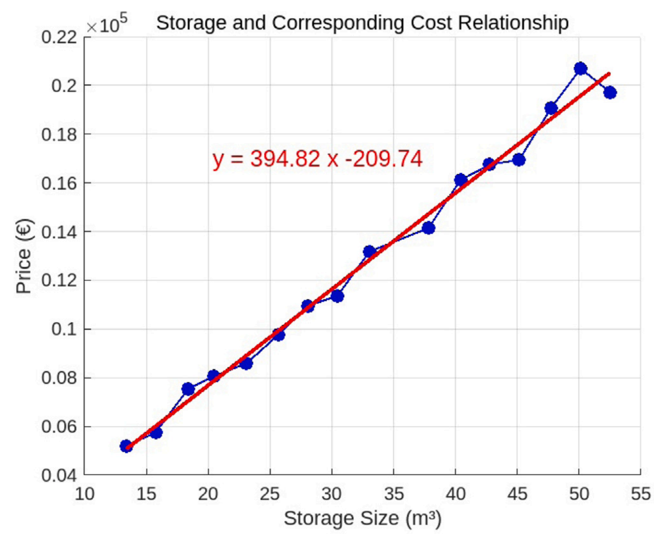


**Fig. A.11.** Storage versus cost relationship Postma kunststof tank (Anon, 2024).

### A.2. Approximation of construction cost of storage

The construction cost is typically determined by many factors, and will depend on location specific properties and labour costs, as well as the type of RWH system considered for installation. If it is a simple storage tank above ground, only the installation costs may apply. However, for underground installations, additional expenses such as digging and underground wall preparation need to be included, making it more costly. At the Hortus, the space available for a rainwater harvesting tank is limited, and the presence of existing plants and trees means that removing them would also be expensive. Consequently, in this research, the available space for placing the storage tank is restricted. Furthermore, there is no empirical data to identify the costs associated with construction for different storage sizes.

In general, we can assume that the cost of the construction is a function of the storage size (as represented in the optimisation problems we consider in (7) of the main body of the manuscript). If we then assume that the available area for installing or building storage tanks is limited, we can represent the depth dug and constructed as $d_m = (S/A_{available})$. The cost of digging and construction was here assumed as increasing super-linearly with depth; without any assumption of generality, we assume here a quadratic cost:

$Cost = c * (S/A_{available})^2$,

where $c$ is a constant. So $C_m = c * (1/A_{available})^2$ is a constant number that needs to be determined. After the interview with the Hortus, it was determined that the mentioned coefficient would be of the order 0.1 for above the ground and 0.5 for below the ground. The methodology of the optimisation in principle would work with any quadratic approximations of cost. This term and its coefficients must be evaluated prior to using the optimisation framework. If the same function is used by other researchers, the characteristic of the case study and costing needs to fit the case; for example, if material costs dominate the overall cost, then a linear cost function would suffice.

## Data availability

Data will be made available on request.

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