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Object Tracking in Images of an Airborne Wide Angle FMCW Radar

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***Abstract*—Object tracking is performed when surveillance applications have multiple observations of an object over time. An example of such a surveillance application is mounting a wide-angle Frequency Modulated Continuous Wave (FMCW) radar system on board of a General Aviation aircraft. This is done in order to observe its environment in detail, including non-cooperative objects such as birds and windmills. Data generated by such a system follows different physical laws than the images of standard visual applications. In this paper, a novel tracking algorithm is introduced which is tailor-made for FMCW applications. The algorithm is tested in a simulated crowded general aviation airspace, and the resulting tracks are qualitatively and quantitatively analysed. The proposed algorithm performs better than a traditional algorithm on all aspects, but tracking errors can still be made in rare cases. The proposed algorithm can be used in conjunction with research focusing on observation quality or assignment problems.**

***Keywords*—Object Tracking, General Aviation, Sense-and-avoid, FMCW Radar, Modelling, Kalman Filtering, Aliasing, Simulation Experiment, Independent Surveillance**

I. Introduction

Object tracking is an important step in any surveillance application [1]. In diverse fields, from weather stations to traffic cameras, linking different observations to one another through time is a crucial step in order to study and predict the medium term behaviour of any observations. Improved methods of tracking can increase the quality of the observations over time, and will improve the situation awareness of the observer with respect to the surroundings.

Much work has been done in the field of visual object tracking. Numerous studies have been performed that found high quality algorithms for handling temporary occlusions, singularities and even faulty observations [2], [3], [4]. Many of these studies are applied on visual systems such as webcams, helicopter imagery or handycams. Tracking algorithms which are tailor made for airborne FMCW applications, however, are scarce.

In this paper, an algorithm is presented which can bridge the gap between the field of visual tracking algorithms and a novel type of wide angle Frequency Modulated Continuous Wave (FMCW) radar. In section II, the application of the FMCW radar is explained, after which the theory of the algorithm improvements is explained in section III. The parameters that are used to assess the quality of the algorithms are discussed in section IV, after which the experiment is described in section V. The results are presented in section VI, followed by discussion (VII) and conclusions (VIII).

II. Application

In General Aviation (GA), many flights are performed under Visual Flight Rules (VFR), in which pilots rely on their own eyes to perform navigation and surveillance. Since a pilot's field of view is finite, possible threats may be overlooked, causing hazardous situations.

Technical applications that assist the pilot in his/her VFR tasks exist. An example of this is FLARM technology [5], in which aircraft broadcast their positions to each other using transponders. These kinds of systems are a form of dependent surveillance, and they can only work if both aircraft are equipped with the right technology. Therefore, such an application can never guarantee that no dangers are present.

A different, novel approach is to have a radar system on board, which can broadcast its own signal and use it to actively and independently scan its environment. This signal is reflected back on objects and can be observed by the system [6]. Similar to the way in which bats sense their environment, such a system empowers independent surveillance with which non-cooperative objects such as birds, towers and mountains can be observed [7].

Developments in radar technology have improved the availability, weight and pricing of FMCW radar systems, to an extent that they can be considered feasible for these tasks. If such systems are to be implemented for improving situation awareness, robust and accurate algorithms are required to perform tracking of observations, which is what this paper is about.

III. Algorithm

In this section, a new object tracking algorithm is proposed. It starts with a section on conventional object tracking, after which the FMCW radar principles are introduced. The majority of this section is found in the last part, in which the differences between visual and radar images are discussed and in which the tailor-made tracking algorithm is presented.

A. Visual Object Tracking

As discussed in section I, many publications exist in which visual tracking algorithms are discussed [2]. These are used in all kinds of applications, ranging from mobile phones to satellites. The challenge of object tracking is to link observations to each other, which are supposedly done in (short) succession to each other. A model of the object properties is used to quantify expectations about the behaviour, that are used to perform accurate assignment between observations and models [1]. The common elements of object tracking are illustrated by the image in figure 1.

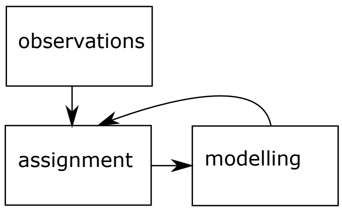


Fig. 1. Generic elements of tracking software

1) Elements of Tracking: The three elements of figure 1 form a tracking system together. Firstly, the objects that should be tracked must be observed by a sensor. Generally, this is a camera system but in this project, the sensor is an FMCW radar.

The second block is the assignment of observations to the internal models of the objects, usually named 'tracks'. Assignment theory is a research field on its own, but one of the most frequently used algorithms is the Hungarian Algorithm, a well-known optimization algorithm that can solve assignment problems [8]. This algorithm will also be used in this study. Many object tracking research focuses on this aspect of object tracking [9], [4], [3], [10], [11].

The third block, the modelling of the tracks, is the focus of this paper. An observation is usually described as a state vector, containing all important information. Amongst the information is, at least, the location of the observation in the image. The colour of the object, its shape and its structure can be possible extra information in the state.

2) Object Modelling: The progression of the state parameters of an object is usually computed by using a Kalman Filter [12], [10], [9], [13], [2], [6]. An internal linear model of the object parameters is used to predict the next expected state vector of the track. An internal model, frequently denoted F, is usually simply defined in the following way:

$$\vec{s}_{t+1} = F \cdot \vec{s}_t = \begin{bmatrix} 1 & & dt & \\ & 1 & & dt \\ & & 1 & \\ & & & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ v_x \\ v_y \end{bmatrix} \quad (1)$$

In equation 1, the parameters x and y in state vector $\vec{s}$ are used to describe the position of the observation in the frame, and v is used for the time derivative of the position, in both directions. The time of the measurement is called t, and the time since the previous measurement is called dt. It can be seen that even though v_x and v_y may not be directly observed, they can indeed be part of the internal Kalman model.

3) Standard visual algorithm: Combining all the descriptions based on the literature in section III-A, it is possible to describe a 'standard approach' for object tracking in visual systems. Such an algorithm typically looks like the following:

1) Load observations
2) For all existing tracks: prediction current state
3) Evaluate all combinations of observation + track
4) Assign the best matches of observations to the tracks
5) Update internal models of assignments
6) Initialize new tracks of unassigned observations
7) Close tracks without observations
8) Start again at 1.

B. *FMCW Radar Imaging*

A Frequency Modulated Continuous Wave (FMCW) radar system operates by broadcasting a radio wave using a transmitting antenna. The signal is reflected on the environment, and observed by a receiving antenna. The basic signal, the 'carrier wave' can be modulated by another frequency, with can be varied over time [14], [15].

Comparing the observed frequencies with the broadcast ones, information about the environment can be deduced. Phase and frequency shifts between the signals are caused by the travel times of the radio waves and by the Doppler effect. Fourier transforms can be used to infer those properties from the incoming signals, once they are converted to a digital signal.

The result of this is that the axes system of an FMCW radar image are different than those of traditional visual imagery, as indicated in figure 2 [16]. Where the traditional axes of a visual image contain information about the location of an object, or its elevation and azimuth with respect to the camera, this information is absent in this type of radar images.

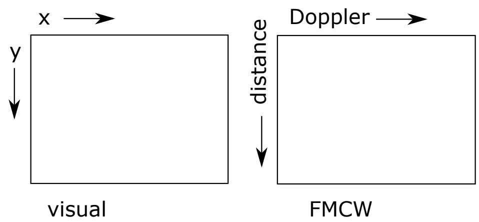


Fig. 2. Differences in image axes

Algorithms for Direction of Arrival (DoA) estimation of incoming radar signals may be applied to such an FMCW radar system, in order to increase the knowledge about the observed objects [6], [15]. They are dependent on the quality and number of receiving antennas on the aircraft, and the stiffness of the wing[1] and presence of background noise [17]. These factors influence the performance in determining the DoA. It is therefore beneficial to be able to perform tracking directly in the radar image, so as not to be dependent on the quality of DoA estimation.

[1]The distance away from the aircraft makes the wingtips a very suitable place to mount a radar system, but this has consequences for the observation quality

C. Algorithm Improvements

Working in the radar image frame, important differences exist that distinguish this project from a standard camera application. Three changes in standard visual tracking models are made in order to accommodate those differences. They are discussed in the paragraphs below.

1) Aliasing: When inferring a frequency from a Fourier transform of sampled data, the maximum observable frequency, called the Nyquist frequency, is dependent on the sampling rate of the data. Frequencies that lie outside the range will be observed as their aliases: frequencies with a difference of $n \cdot f_N$ (an integer times the Nyquist frequency) [14], [18].

This is illustrated in figure 3, where three rotating discs are depicted, with different rotational velocities. If the sampling frequency is such that each disc rotates half a circle between each sample, the discs will be observed the same, and an observer cannot distinguish which one has which rotational velocity.

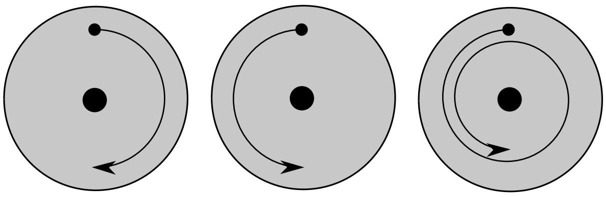


Fig. 3. With the right sampling rate, these three discs with different rotational frequencies will be observed identically

For this project, this is applicable for the Fourier shift in Doppler direction. This means that it is impossible to estimate the Doppler frequency exactly, since a signal with a Doppler frequency outside of the observable spectrum will be seen as its alias.

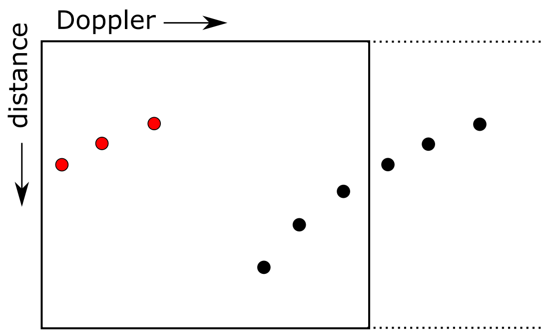


Fig. 4. Observations of an object in the range-Doppler image over time, with aliases indicated in red

When tracking of an object is performed over multiple time instances, it is possible that the Doppler shift of the object changes such that the object moves out of the image frame, and that an alias becomes visible. This is indicated in figure 4, where an example track under influence of aliasing is shown.

The problem of the aliasing can be solved using a simple but precise alteration in the algorithm, to change the definition of **innovation** in Doppler direction. Innovation is a term used in signal processing, which describes the difference between the predicted state of the observation and the observed one [12], [1], [13]. Traditionally, innovation ($\vec{y}$) is computed in the following way, using the notation $:=$ to indicate a computational assignment:

$$\vec{y} := \vec{o} - \vec{p} \tag{2}$$

Innovation is used in the Kalman filter, to update the track model, and in the assignment algorithm, as mentioned in section III-A1: the Hungarian Algorithm is fed with the l^2 norms of the innovation vectors.

Realising that aliasing may make an object appear at the other side of the image, the component of the innovation in Doppler direction is computed in the following way:

$$y_D := mod\Big(\Big(o_D - p_D + \frac{D_{width}}{2}\Big), D_{width}\Big) - \frac{D_{width}}{2} \tag{3}$$

In equation 3, $mod(a, b)$ indicates the modulo of a divided by b and D_{width} is the size of the image in Doppler direction. This definition ensures that the innovation is computed either with the direct distance, or with the distance around the outside of the image, whichever is shorter.

2) Relation between image axes: When looking at the standard x, y linear model shown in equation 1, it can be observed that x and y are independent of each other - a common element of visual systems, in which the different axes of the generated images are uncoupled. Some research includes perspectivity in the model [19], but in general, movements in x and y direction are independent.

This is not the case for an FMCW radar system. One of the two parameters measured, the Doppler shift, is caused by relative movements of the object with respect to the observer. The Doppler shift is a direct measure for the radial velocity of the object, its speed in the direction along the distance vector. This means that there is a relation between the distance R and the Doppler speed V_R:

$$V_R = \frac{dR}{dt} \tag{4}$$

The relation between R and V_R from equation 4 means that the internal linear model must be adapted, as seen in equation 5:

$$\begin{bmatrix} 1 & & dt & \\ & 1 & & dt \\ & & 1 & \\ & & & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ v_x \\ v_y \end{bmatrix} \rightarrow \begin{bmatrix} 1 & dt & & \\ & 1 & & dt \\ & & 1 & \\ & & & 1 \end{bmatrix} \begin{bmatrix} R \\ V_R \\ dR/dt \\ dV_R/dt \end{bmatrix} \tag{5}$$

3) Aliasing and Axes Relations Combined: When the two improvements from the previous sections are implemented, a new problem rises. If the relationship between R and V_R is used, but due to aliasing V_R can have multiple values, how to predict R?

When an observation is assigned to an existing track, the values of R between the current and most recent observations can be compared. The change in R per time unit, $\Delta R/\Delta t$ can be used to find the proper value of V_R. This means that an extra step should be taken after the assignments are computed by the Hungarian Algorithm, just before observations are appended to the internal models:

$$V_R := V_R + round\Big(\frac{\Delta R/\Delta t - V_R}{D_{width}}\Big) * D_{width} \qquad (6)$$

In equation 6, $round()$ indicates rounding off to the nearest integer. This computation step ensures that all tracks that consists of two or more observations have found the right value of V_R to use in their internal model. When a track still consists of only one observation (it was just formed in the previous step), it is not possible yet to have an accurate estimate of V_R.

This means that this should be incorporated by the tracking algorithm. When computing the innovation, as the difference vector between expected and observed states, one extra step should be added that is only executed if the track only consists of one observation yet. In this step, the innovation component in Range direction should be recomputed:

$$y_R := mod\bigg(\Big(y_R + \frac{D_{width}}{2}dt\Big), D_{width} \cdot dt\bigg) - \frac{D_{width}}{2}dt \quad (7)$$

IV. Quality Assesment

To assess the performance of visual tracking algorithms, two different dependent variables are used. They are discussed in the paragraphs below. Next to these parameters, it is beneficial to plot the course of the tracks in the radar image frame. This will provide an insight in the tracking results, and it allows for human verification of the achieved tracks.

A. Innovation

Innovation, as described in section III-C1, is the variable which describes the vector difference between the predicted and observed state vectors. The l^2-norm of the innovation describes the pixel distance between the states in the radar image, and it describes how accurate the internal linear model is: lower innovation is better. Therefore, the average value of the innovation will be used as a quality parameter in this research.

Next to that, the distribution of the innovation is also relevant. If the innovation is consistently low, this means that the gate size of the tracking algorithm can be reduced: the border distance at which an observation and a track may still be linked. A low gate size means that new tracks can be initiated when new observations occur close to existing tracks. In other words: a low gate size means that the algorithms can follow more tracks at the same time. In order to monitor this, the 95% border of the innovation distribution will be used as a quality parameter.

B. Number of Tracks

Nothing says more 'Tracking Failure' than losing track of an object. Therefore, it is important to monitor the number of lost tracks in an experiment. Any time an existing track is ended too soon, the remaining observations will form a new track together. The number of observed tracks can therefore be used to describe the amount of lost tracks, of which fewer is better.

V. Experiment

An experiment is conducted to assess the performance of the proposed tracking algorithm. In order to test only the 'modelling' element from figure 1, 'assignment' will be done by the standard practice of the Hungarian Algorithm and in order to guarantee proper 'observations', a computer will be used to simulate the radar response in different flight conditions.

The radar simulator is a high-precision wave generator, in which all important factors are incorporated: the positions and angular rates of the aircraft, terrain structure and hardware properties: from the specific antenna configuration to wavelength and position on the aircraft frame. The simulator is able to compute the effects of millimetre-scale design and compute the Doppler and range properties of all objects in the vicinity of the aircraft, with a maximum range defined at 5km, and generated radar images with a resolution of 250x250 pixels.

The difficulty here is to develop a testing environment which is both challenging and realistic. It is important to test the performance of the tracking algorithm under difficult circumstances, where many aircraft are in each others vicinity. Many simultaneous observations may lead to mixing of tracks, which should be prevented.

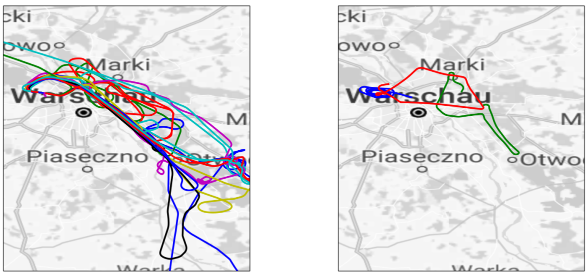
On the other hand: the testing environment should be realistic and not appear set up, so as not to raise questions about the independence of the test.

A solution was found to meet both criteria simultaneously. Using flightradar24.com, a website that displays live aircraft locations, a Cirrus SR22T aircraft was found which performs mostly local VFR flights. A picture of such an aircraft model is seen in figure 5.

Fig. 5. Image of a Cirrus SR22T Aircraft

The flight history of this aircraft can be downloaded, and 10 flights were selected that have been performed under VFR around the same airport, Warsaw-Babice Airport in Poland (ICAO code EPBC). These flights form the basis for the experiment. Therefore, the experiment is conducted with aircraft that are accelerating, making turns, changing altitude and performing other manoeuvres which were not discussed in the model in section III-C. The routes are displayed in figure 6.

Next to those flights, three flights have been simulated using the off-the-self computer simulator Xplane, a high-fidelity flight simulator, in the same airspace. In the radar simulations, the radar system will be simulated to be on board of these aircraft, since the simulated data has a higher quality than the ADS-B data. These three flights will form three distinct test scenarios, in which all Cirrus flights are implemented. The simulated flights are also indicated in figure 6.



(a) Cirrus Flights

(b) Simulated Observer Flights

Fig. 6. Flights around Warsaw-Babice plotted on a map from Google, the first letter 'a' in Warschau is on the location of the airfield

The first simulated flight is called the Circuit flight, as it performs a simple circuit above the airfield (without landing). All Cirrus flights are simulated to be at the end of their route and nearing the airfield again, and they will land with intervals of only 60 seconds. This simulates a busy situation near an airfield.

The second flight (the red line in figure 6) revolves around point Zulu, the entrance/exit point of the airfield, located just after crossing the river Wisla. The flights will be simulated to depart one minute after each other, and they will all fly towards point Zulu. This simulation creates a dense airspace.

The last simulated flight (the green line in figure 6), is performed in the free airspace north-west of the river Wisla. The flight is still in alongside direction of the river, as the majority of the Cirrus traffic flies in that direction. Flying at more diverse altitudes and with the possibility of sudden turns, this is the approximation of a crowded Free airspace.

VI. Results

The resulting tracks in the $R - V_R$ frame of the flights are seen in figures 7, 8 and 9, with the results of the traditional and proposed tracking algorithms. In these figures, each track is randomly assigned a colour. This means that all line segments with the same colour belong to the same track.

From figures 7, 8 and 9, two direct observations are made: the first is that the proposed algorithm connects track data

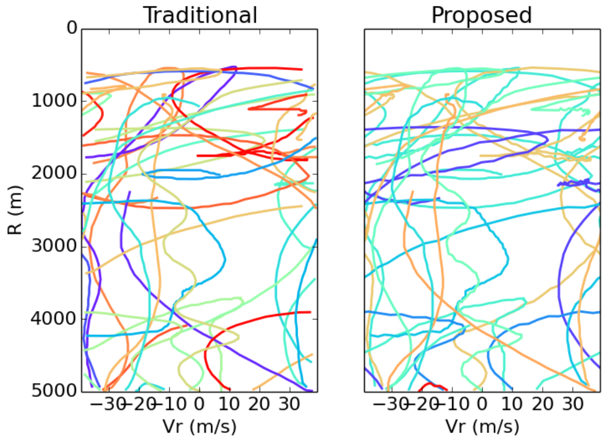


Fig. 7. FMCW Radar image tracks of the Circuit flight

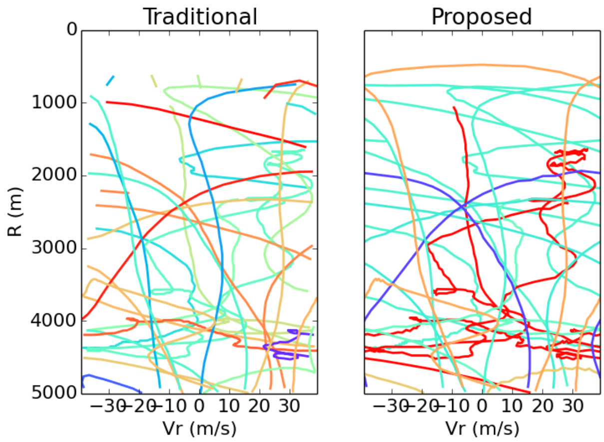


Fig. 8. FMCW Radar image tracks of the Zulu flight

together which may be seen as separate tracks by the traditional algorithm (an example of this are the tracks which are closer than 2000m in figure 9). The second observation can be done when looking at the side edges of the figures, at $V_R = + - 40m/s$. Here it can be seen that many of the tracks end when observed with the traditional algorithm, but with the improvements they are connected to their aliases on the other side of the image.

In figures 10, 11 and 12, the same tracks from figures 7, 8 and 9 are plotted, but now in the horizontal aircraft body fixed frame of reference. These tracks are plotted assuming that perfect Direction of Arrival estimation is performed. In other aspects they are identical to the tracks in the previous figures; only the axes are changed. These images are made because they are slightly easier to interpret than the $R - V_R$ plots. In the images, the X-axis points in the direction of the nose and the Y-axis points towards the right wing tip of the aircraft.

In figure 13, a histogram is given of the distribution of the definitive innovations used by all tracks' Kalman filters. It can be seen that the majority of all innovations, both in the

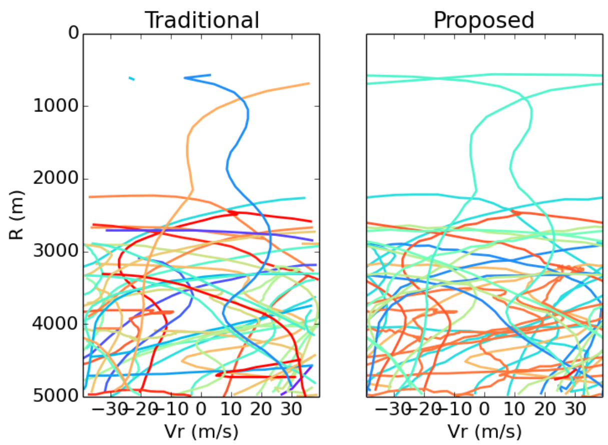


Fig. 9. FMCW Radar image tracks of the Free airspace flight

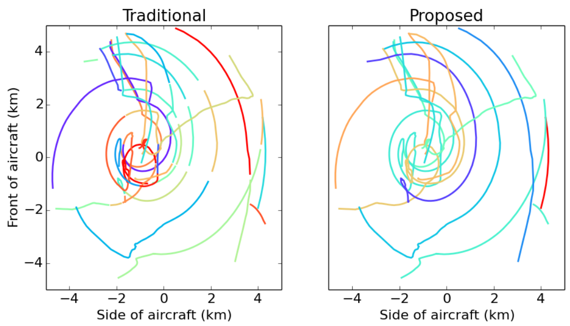


Fig. 10. Tracks in horizontal aircraft frame of Circuit flights

traditional and proposed algorithms, are less than 5 pixels. The histograms of the other two flights look very similar to the one in figure 13, and are therefore omitted. The exact data on the average innovation and 95% percentile innovation are given in table I.

In table II the total number of observed tracks for the traditional and proposed tracking algorithms are shown. Next to that, a manual tracking count is also performed. This is done by simulating the radar output step by step for the complete flight, and counting the number of tracks within the looking distance. Although 10 flights were simulated, the number may be lower because some aircraft happen to never come close enough, or the number of tracks may be higher because aircraft enter the observable area multiple times.

VII. Discussion

The track counts from table II are very clear. With the proposed tracking algorithm, the number of observed tracks is reduced. Since the experiment was set up in such a way that the observations are identical for both algorithms, this means that the proposed algorithm is better at linking observations together to form a consistent track.

This is also reflected in the figures 9, 10, 11 and 12, where the observed tracks are plotted in the radar image and aircraft body frame. It is seen that often observations that form distinct tracks in the traditional algorithms are seen as one single track by the proposed algorithm.

The images in figure 10 need further eludication: in these plots many concentric circle segments are observed. Contrary to what may be intuitive, this is a correct display of the observations. In the circuit flight, around 50% of the time the aircraft is making turns. In these manoeuvres, the range to other aircraft is not significantly affected but the relative direction of the aircraft is. And since the plots are made in an aircraft body fixed frame of reference, this results in concentric tracks.

In figure 11, another phenomenon can be observed: two simultaneous tracks of which the observations are mixed up. This can be seen in the top-left corner of the image, and in the centre, where a link is drawn between two distinct tracks.

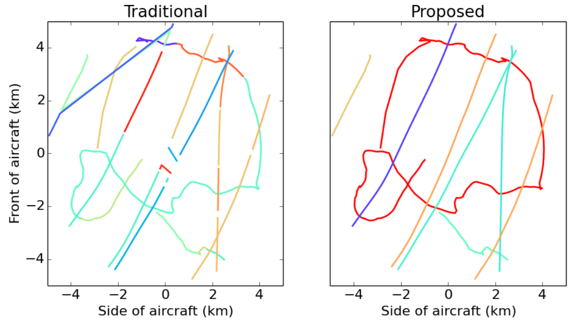


Fig. 11. Tracks in horizontal aircraft frame of Zulu flights

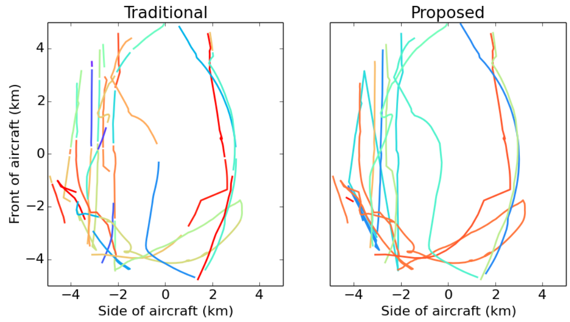


Fig. 12. Tracks in horizontal aircraft frame of Free flights

		Circuit	Zulu	Free
Average (pix)	Traditional	2.1	2.7	4.3
	Proposed	2.0	2.5	3.6
95% percentile (pix)	Traditional	7.1	10.5	14.4
	Proposed	4.9	7.1	10.7

TABLE I
Results of the l^2 norm of the innovation in the experiments

	Circuit	Zulu	Free
Traditional	42	42	54
Proposed	14	9	13
Manual	14	9	12

TABLE II
Number of observed Tracks

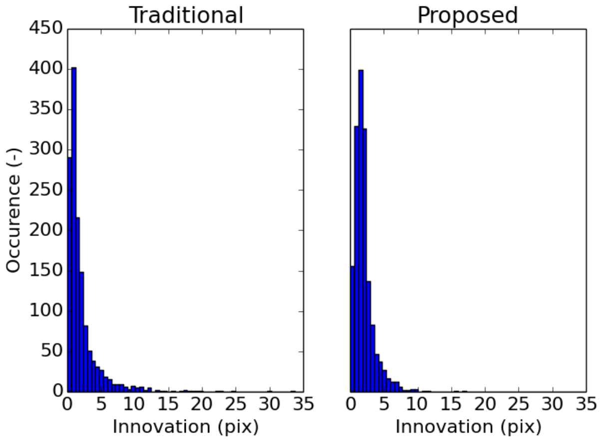


Fig. 13. Histogram of innovation of the Circuit flight

These tracks are easily distinguishable in the aircraft body frame, but in the $R - V_R$ image this distinction cannot be made and they are interchanged by the traditional algorithm. The proposed tracking algorithm improves on this and prevents mixing of tracks.

The tracks from figure 12 are also suited for further discussion. It can be seen that in the flight in the free airspace, one other aircraft happened to come very close to the radar-equipped one. But exactly at the closest point, when awareness of the position of the other aircraft is most critical, the track was lost. This does not happen with the improved algorithm. The reason for this is the link between R and V_R, as described in paragraph III-C2. When an aircraft is close, the change in Doppler can be very quick, because the time between a (near) heads-on situation and flying away from each other can be only a few seconds. The improved model is capable of computing that change during the flight, and is therefore capable of tracking the other aircraft.

Lastly, the changes in innovation need to be discussed, which were found in table I. It is seen that the l^2 norm of the innovation is reduced by the proposed algorithm for all experiments. This is an indication that the system is better at predicting the short-term object behaviour, and therefore it can more accurately couple observations to tracks. This may help in order to prevent switching of tracks such as seen in figure 11.

Additionally, it can be seen that the 95% percentile innovation is lower than in the traditional algorithm. As discussed in section IV-A, a consistently low innovation means that the gate size of the tracking algorithm can be reduced, enabling a higher capacity in terms of number of simultaneous tracks in the simulation. It is seen that the proposed algorithm improves on this aspect, so this is beneficial for application in dense airspaces.

As discussed in section III-A, it is found that the problem of object tracking usually consists of three elements in collaboration: observations, assignment and modelling (as illustrated in figure 1). In this paper, the focus was on the modelling aspect of object tracking. The quality of observations was assumed to be perfect, generated by a high-detail radar simulator. More research to the real generation of FMCW radar data, including the development of proper signal filters, should be performed.

As for the 'assignment' element: so far the Hungarian Algorithm was used in the research, which demonstrated to provide results of good quality. Many research has been performed in the field of visual tracking assignment, and this knowledge may be used in the assignment algorithms, independent from the algorithm developed in this paper. Therefore, different types of research may supplement and amplify each other.

VIII. Conclusion

In this paper, a novel object tracking algorithm is proposed which is suitable for tracking objects in a radar image, such as can be generated by an FMCW (Frequency Modulated Continuous Wave) radar. The algorithm makes use of specific radar properties such as the Doppler effect, which are taken into account in the predictions of the tracks.

A novel application for this is to use a wide-angle on-board FMCW radar in General Aviation, in order to track the positions of other aircraft in the vicinity of the observer, even if those aircraft are not equipped with any special equipment. This empowers the possibility of independent surveillance, with which objects such as birds, towers and windmills can also be observed. A simulation experiment is set up to assess the performance of the algorithm in dense General Aviation circumstances, and the results are investigated.

It is found that the proposed algorithm outperforms a traditional object tracking algorithm. Often, a traditional algorithm is not able to connect segments of observations to each other when the observations are aliased to the other side of the radar image or at close distance. The new algorithm is almost always able to make these connections, but one case was found where the new algorithm misconcluded that a series of observations consisted of two tracks instead of one.

Additionally, the accuracy of the proposed algorithm was tested. It was found that the new algorithm is better in predicting the progression of a track than standard optical imagery, and that systems using the new algorithm may have a higher tracking capacity than traditional algorithms applied to FMCW radar images.

In conclusion, the proposed tracking algorithm improved the internal Kalman model of the observed tracks to include aliasing and the relation between distance and radial velocity. It is found that this is an important improvement for the object tracking in FMCW radar images. This new model may be combined with existing and upcoming research on the quality and the assignment of observations.

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