Modelling the Interaction between Structure and Soil for Shallow Foundations

-A Computational Modelling Approach-

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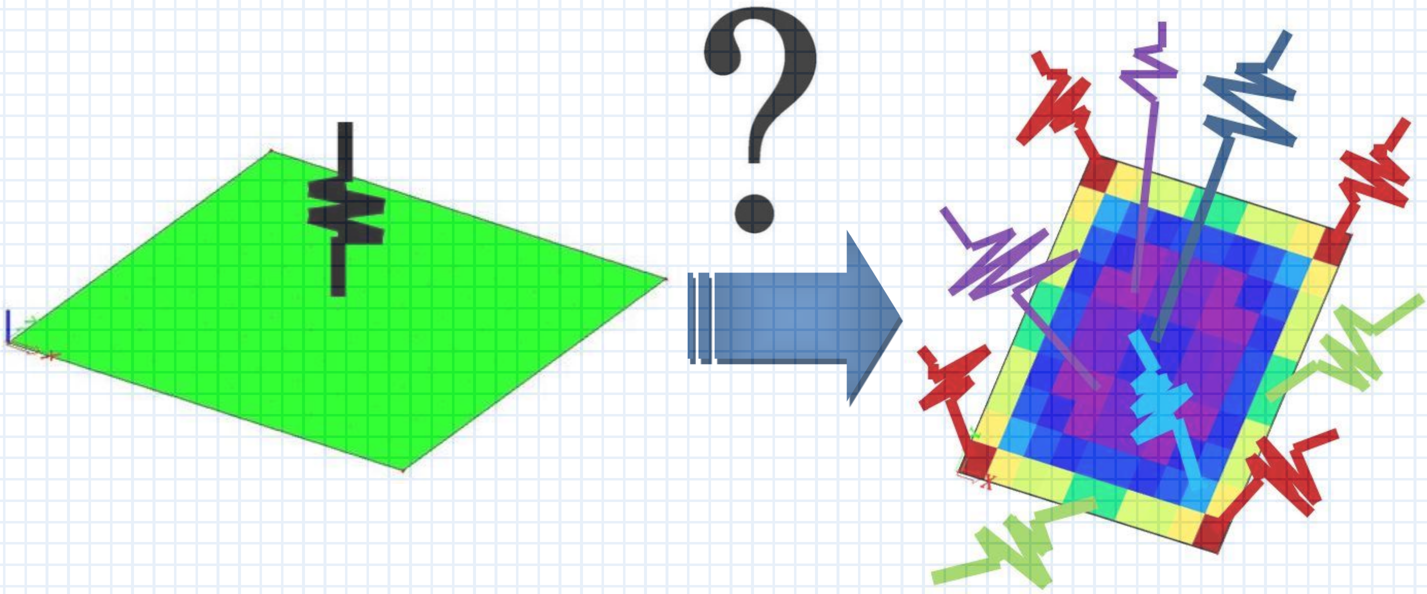

Challenge the future

Modelling the Interaction between Structure and Soil for Shallow Foundations

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By

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Master of Science Thesis

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Summary

Designing and modelling foundation structures crosses two engineering disciplines. There is the structural engineer who designs the structure and the geotechnical engineer who determines the bearing capacity of the soil. When modelling large shallow foundations, it is not always clear how to determine the stiffness of the soil and how it should be used in structural software programs. In general, the stiffness is determined by the geotechnical engineer and the determined value is used by the structural engineer. To determine the stiffness, it is important to know the intended use. For the use of the stiffness, it is also necessary to know what restrictions/assumptions are applicable. Besides uncertainties when determining the soil stiffness, other methods to model the interaction between structure and soil are also not straightforward. A realistic model of structure and soil interaction can lead to an optimal and economical structure.

The objective of this thesis is to develop applicable guidelines to analyse and model large slab foundation structures. The purpose is to have consistency in the modelling of the interaction between structures and soil. The findings of this report can, in turn, assist the structural and geotechnical engineer when modelling shallow foundation structures. It will also provide insight into the determination of structure and soil parameters. Furthermore, more knowledge on the behaviour of structure-soil interaction is gained on the basis of a parametric analysis. This study has been done within the section of Structural Engineering, specialization Structural Mechanics, at the faculty of Civil Engineering and Geosciences of the Technical University of Delft.

In Chapter 1, a brief introduction to shallow foundations is given and the related information and aspects required for modelling such structures. Finally, the research description, objectives, challenges, approach and scope of this thesis are explained. In this thesis, the analysis of shallow foundations is carried out with the spring analysis for soil response.

In Chapter 2, the fundamental theory and the literature which will be used in the report are described. The chapter begins by describing the general way of modelling. More specifically, the emphasis in this chapter is on the way of modelling the super structure and the soil. The super structure is modelled with the classical beam theory and the soil as springs.
The modelling of the soil as springs comes forth out of the Winkler foundation model. This model has some drawbacks which are improved by the Pasternak foundation model.
The chapter continues by describing the interaction between structure and soil. The stiffness of both structure and soil has an important role when modelling the interaction.
The chapter concludes with the work flow for designing a foundation slab and part of the process of which will form the focus later on in the study.

In Chapter 3, the analytical case studies are discussed. In this chapter the formulation of the Winkler and Pasternak foundation models will be studied analytically. In addition a new method will be introduced. This new method is named the Gradient foundation model. This foundation model influences the rotation of the soil surface. In the study different boundary

conditions will be used and the influence on the surrounding soil will be research for the different foundation models. Through this study, the theory and the working of the equations and the difference between the models are made clear. An important conclusion which follows in this analysis is that the Pasternak foundation model yields results which come closer to reality. This is accomplish due to the coupling of the springs in the form of the second parameter (G_p).

Chapter 4 is dedicated to the theoretical overview of the structural software program to be used in the research. The structural software program SCientific Application (SCIA) engineering has been used to model the shallow foundation. In the program the slab is modelled with 2D plate elements and the soil as springs.
The interaction of structure and soil is modelled by the interaction parameters called "*C* parameters". The theory of these *C* parameters will also be discussed in this chapter. The method to use these interaction parameters can be divided into two main groups. One group is making use of uniform coefficients and the other of non-uniform coefficients. The non-uniform coefficient models are the Eurocode 7, Pseudo-Coupled and Secant Method. These methods will be elaborately discussed in this chapter. Especially attention will be given to the process of transforming and spreading soil properties into springs. This process is called the Secant Method.
Also the settlement calculation that the program uses will be discussed. The settlement calculation which the program uses to determine the interaction parameters is different than the often used Terzaghi equation. In that context the two settlement equations will be compared and studied.

Chapter 5 is focused on the applicability of the uniform and non-uniform coefficient models, from the previous chapter. In this chapter the influence and variation of these coefficients will be analysed. The implementation of the Eurocode 7, Pseudo-Coupled and Secant Method can be seen. The difference between these models will also be discussed by comparing their results. The results which will be compared to one another are settlement, moment and contact stress.
In the end of this chapter a sensitivity analysis of the second interaction parameter (C_2) will be done. By doing this, more insight can be gained about this parameter.

Chapter 6 deals with the comparison between the Secant Method (SM), the linear finite element method (LFEM) and the nonlinear finite element method (NLFEM). The SM is implemented in the SOILin module of the program SCIA Engineer. The linear and nonlinear finite element analyses are implemented in PLAXIS. PLAXIS is a geotechnical program that is often used in everyday engineering practice. The nonlinear analysis performed by this program has been compared extensively with experimental results and are generally considered to be quite realistic [1] [2].
The SM has not been compared with experiments [J. Bucek]. It has been developed based on compliance to governing codes of practice. In this chapter the main difference between the two programs will be documented.

Chapter 7 will focus on the interaction between structural and geotechnical engineering. A practical approach which makes use of the Winkler foundation model will be worked out. Also a checklist which can assist in a better communication between the two engineers will be explained. In the end, a brief and final thought about modelling foundations will be presented.

Finally, Chapter 8 presents the conclusions of the thesis and the recommendations for further study.

Samenvatting

Het ontwerpen en modelleren van funderingen kruist twee ingenieursdisciplines. Er is de constructeur die de constructie ontwerpt en de geotechnische ingenieur die de draagkracht van de bodem bepaalt. Bij het modelleren van grote funderingen op staal, is het niet altijd duidelijk hoe de bodemstijfheid kan worden bepaald en hoe die gebruikt moet worden in constructieve programma's. In het algemeen wordt de stijfheid bepaald door de geotechnische ingenieur en de vastgestelde waarde wordt gebruikt door de constructeur. Het beoogde gebruik van de constructie is belangrijk bij het bepalen van de grondstijfheid. Voor het gebruik van de stijfheid is het ook noodzakelijk te weten welke beperkingen / aannames van toepassing zijn. Naast het bepalen van de stijfheid, zijn de methodes om de interactie tussen constructie en grond te modelleren niet eenvoudig en eenduidig. Een model dat de interactie tussen constructie en grond realistisch beschrijft kan leiden tot een optimale en economische constructie.

Het doel van dit afstudeerverslag is het ontwikkelen van geschikte richtlijnen voor het modelleren en analyseren van grote plaatfunderingen op staal. Het doel is om consistentie te hebben bij het modelleren van de interactie tussen constructie en grond. De bevindingen van dit verslag kunnen constructeurs en geotechnici assisteren bij het modelleren van funderingen op staal. Het zal ook inzicht geven in de bepaling van de constructie - en de grondparameters. Bovendien kan meer kennis over het interactie gedrag van constructie-grond worden verkregen op basis van een parametrische analyse. Deze studie is gedaan binnen de afdeling van Structural Engineering, specialisatie Structural Mechanics, aan de faculteit Civiele Techniek en Geowetenschappen van de Technische Universiteit Delft.

In hoofdstuk 1 wordt een korte introductie van funderingen op staal gegeven. Ook de bijbehorende informatie en aspecten die nodig zijn voor het modelleren van dergelijke constructies komen ter spraken. Ten slotte een beschrijving van het onderzoek, de doelstellingen, uitdagingen, opzet en reikwijdte van dit afstudeerverslag worden toegelicht. In dit afstudeerverslag wordt de analyse van funderingen op staal uitgevoerd door de grond te modelleren als veren.

In hoofdstuk 2 worden de fundamentele theorie en literatuur die gebruikt zal worden in het rapport beschreven. Het begint met een beschrijving van de algemene wijze van modelleren. De nadruk is op het modelleren van de constructie en de grond. De constructie wordt gemodelleerd met de klassieke balkvergelijking theorie en de grond als veren. Het modelleren van de grond als veren komt voort uit de Winkler fundering model. Dit model heeft een aantal nadelen, dat worden verbeterd door het Pasternak fundering model. Het hoofdstuk vervolgt met de beschrijving van de interactie tussen constructie en grond. De stijfheid van zowel constructie als grond speelt een belangrijke rol bij het modelleren van deze interactie.
Het hoofdstuk besluit met een werkschema voor het ontwerpen van een betonnen fundering vloer. In dit schema wordt aangegeven waarop er in deze studie gefocust zal worden.

In hoofdstuk 3 worden de analytische case studies besproken. In dit hoofdstuk wordt de Winkler en Pasternak fundering modellen analytisch onderzocht. Daarnaast zal een nieuwe methode geïntroduceerd worden. Deze nieuwe methode wordt de Gradient fundering model genoemd. Dit model heeft invloed op de rotatie van het bodemoppervlak. In deze studie zullen verschillende randvoorwaarden worden gebruikt. Via deze studie worden de theorie en de werking van de differentiaal vergelijkingen en de onderscheidende kenmerken duidelijk.
Een belangrijke conclusie die in deze analyse volgt is dat het Pasternak fundering model resultaten levert die dichter bij de werkelijkheid komen. Dit is te verklaren door de koppeling van de veren in de vorm van de tweede parameter (G_p).

Hoofdstuk 4 is gewijd aan het theoretische overzicht van de gebruikte constructieve software. In dit afstudeerverslag wordt het programma SCIA engineering gebruikt om de funderingen te modelleren. In dit programma wordt de plaat gemodelleerd met 2D plaat elementen en de bodem als veren.
De interactie van de constructie en de grond wordt gemodelleerd door de interactie parameters genaamd "*C* parameters". De theorie van deze *C* parameters zal ook in dit hoofdstuk worden besproken. De methode om deze interactie parameters te gebruiken kunnen worden verdeeld in twee hoofdgroepen. Een groep maakt gebruik van uniforme coëfficiënten en andere van niet-uniforme coëfficiënten. De niet-uniforme coëfficiënt modellen zijn de Eurocode 7, Pseudo-Coupled en Secant Methode. Deze methoden zullen uitgebreid worden besproken in dit hoofdstuk. Vooral aandacht zal worden gegeven aan het transformatie en het verspreiden van bodemeigenschappen in veren. Dit proces wordt het Secan Methode genoemd.
Ook de zettingsberekening dat het programma gebruikt wordt besproken. De zakking berekening die het programma gebruikt om de interactie parameters te bepalen verschilt met de vaak gebruikte Terzaghi vergelijking. In dat verband zullen de twee zakking formules worden vergeleken en bestudeerd.

Hoofdstuk 5 is gericht op de toepasbaarheid van de uniforme en niet-uniforme coëfficiënt modellen uit het vorige hoofdstuk. In dit hoofdstuk wordt de invloed en de variatie van deze coëfficiënten geanalyseerd. De implementatie van de Eurocode 7, Pseudo-Coupled en Secant Methode zal worden waargenomen. Het verschil tussen deze modellen zullen ook worden besproken door hun resultaten te vergelijken. De resultaten die zullen worden vergeleken zijn de zakking, moment en contact spanning.
Op het einde van dit hoofdstuk wordt een gevoeligheidsanalyse van de tweede interactie parameter (C_2) worden uitgevoerd. Door dit te doen, kan meer inzicht worden verkregen over deze parameter.

Hoofdstuk 6 bespreekt de vergelijking tussen de Secant Methode, de lineaire eindige elementenmethode en de niet-lineaire eindige elementenmethode. De Secant Methode is geïmplementeerd in de SOILin module van het programma SCIA Engineering. De lineaire en niet-lineaire eindige elementen analyse worden uitgevoerd in PLAXIS. PLAXIS is een geotechnisch programma dat vaak wordt gebruikt in het dagelijks vakmanschap. De niet-lineaire analyse die dit programma gebruikt is uitgebreid vergeleken met experimentele resultaten en worden algemeen beschouwd als zeer realistisch. De Secant Methode is niet vergeleken met experimenten [J. Bucek]. Het is ontwikkeld op basis van naleving van richtlijnen van praktische normen. In dit hoofdstuk worden het belangrijkste verschil tussen de twee, eerder genoemde, programma's besproken en gedocumenteerd.

Hoofdstuk 7 richt zich op de wisselwerking tussen constructeurs en geotechnici. Een praktische aanpak die gebruik maakt van de Winkler fundering model zal uitgewerkt worden. Ook een checklist die kan helpen bij een betere communicatie tussen deze twee ingenieurs zal worden samengesteld. Tot slot zal korte opmerkende punten die nodig zijn bij het modelleren worden besproken.

Tenslotte Hoofdstuk 8 bevat de conclusies van het afstudeerverslag en de aanbevelingen voor verder onderzoek.

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I would like to thank those who have contributed to the realisation of my thesis report. This work would not have been possible without the guidance and assistance of the following persons and organisations. To those I like to express my sincere gratitude.

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I also would like to thank Frank van der Woerdt, who also guided me, especially in the last phase of the thesis. During my time working on my thesis at Ballast Nedam Engineering I got a lot of support from the wonderful employees. Ben Nieman assisted me in finding the codes and literature I needed for my research. Also Dennis Grotegoed was a good sparring partner when discussing the geotechnical aspects of the research study. He helped me with getting to learn the PLAXIS 2D software and had given me a lot of material to understand the program much better and faster. I am very thankful for his assistance.
I am also very thankful for the structural engineers Sjoerd Duijst, Bart Vosslamber, Rob Grift and Jacques Geel who provided me with a lot of practical knowledge in structural engineering. During the office hours and lunch time they shared the experiences with me.

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Symbols

E	Young's modulus
E_c	Young's modulus concrete
E_s	Young's modulus soil
$Edef$	Young's modulus of soil as indicated SCIA
$Eoed$	Young's modulus Oedometer
G	Shear modulus
G_p	Shear modulus of the shear layer or Pasternak second parameter
I	Moment of inertia
EI	Bending stiffness
w	Displacement or Settlement
M	Moment (internal force)
p	Pressure
q	Distributed line load
k	Modulus of sub-grade reaction
γ_{unsat}	Unsaturated specific soil weight
γ_{sat}	Saturated specific soil weight
υ	Poisson's ratio
υ_c	Poisson's ratio concrete
υ_s	Poisson's ratio soil
k_r	Stiffness ratio
t	Thickness slab
L	Length of the structure or slab
W	Width of the structure or slab
C_1, C_2	Interaction parameters
m	Structural strength coefficient
$\mathrm{h_{soil}}$	height of the soil layer
P, F	Point load
$\mathrm{V_c}$	Shear force concrete
$\mathrm{V_s}$	Shear force shear layer (Pasternak)
R	Radius
z, d	Depth
σ_{z}	Stress in the subsoil due to an external load
σ_{zz}	Stress in the subsoil due to the Boussinesq spreading
σ_{or}	Effective stress
τ	Shear stress
κ	Curvature
γ	Shear strain
φ	Rotation or Angle of internal friction

1. Introduction

Most civil engineering structures are connected to the ground. The part of the structure where the loads are transferred to the soil is called the foundation. In civil engineering we distinguish between three types of foundations. These are the shallow, deep and piled foundations. An illustration is given in Figure 1-1 [3] were *D* is the depth and *B* the width.

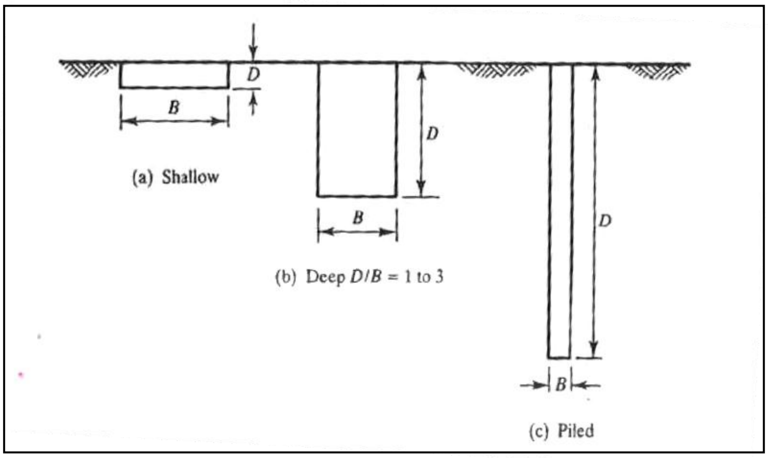

Figure 1-1 Types of foundation [3].
(a) is a shallow foundation, (b) a deep foundation and (c) piled foundation with their *D* depth and *B* the width.

A foundation structure design is realized on the basis of two civil engineering disciplines, which are: the geotechnical and the structural engineering. Often it is not clear which combination of engineering discipline will result in a final optimal foundation structure design. This can result in a conflicting point of view. The point of view of the geotechnical engineer, regarding the soil properties, can be different to that of the structural engineer who determines the structure. It is obvious that these two disciplines cannot independently realize the design.

In this thesis a systematic and practical approach for modelling, designing and analysing shallow foundations for the structural analysis will be discussed. Due to their extensive application in civil engineering projects the focus will be on large concrete slab elements on soil, also known as "slab foundations". An optimized and structured way to model this type of foundation can, finally, lead to an improved understanding between the two civil engineering disciplines.

This chapter starts in Section 1.1 with a general outline of shallow foundations. Subsequently, in Sections 1.2, 1.3 and 1.4, respectively, the research description, research objectives & challenges and research approach as well as the scope of this thesis project will be explained. Section 1.5 gives an outline of the further chapters of the Master of Science (MSc.) thesis.

1.1. Background

1.1.1. Shallow Foundation

Shallow foundations are foundations where the loads of the structure are transferred near the ground surface, unlike in the case of deep - or piled foundations where the loads are transferred into a subsurface layer or a range of depths.

There are three types of shallow foundations that can be distinguished:

- Pad foundation.
- Strip foundation.
- Raft foundation.

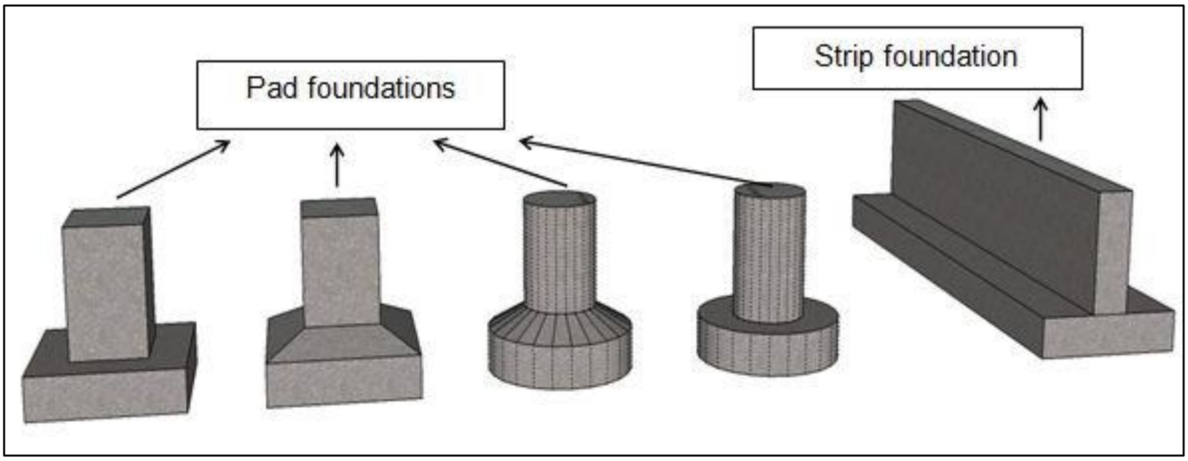

Figure 1-2 Pad and strip foundations

Pad foundations (Figure 1-2) are often seen at the foot of a column or pillar. They can be circular, square or rectangular. The column is seen as an individual point load which rests on the usual block with uniform or tapered thickness. Strip foundations are used to carry load-bearing wall types of structures. These walls are modelled as line loads. Also if columns are very close to one another a strip foundation can be used instead of a pad foundation.

A raft foundation (Figure 1-3) is essentially a slab construction on soil. Such a slab works as a medium which spreads the entire load from the structure over a large area. As with the strip foundation, the raft foundation is also used when columns - or other structural loads are situated too close together and may thus cause the individual foundations to interact. This type of foundation often provides a good solution when encountering soft or loose soils with low bearing capacity. The reason is that it can spread the load over a large area.

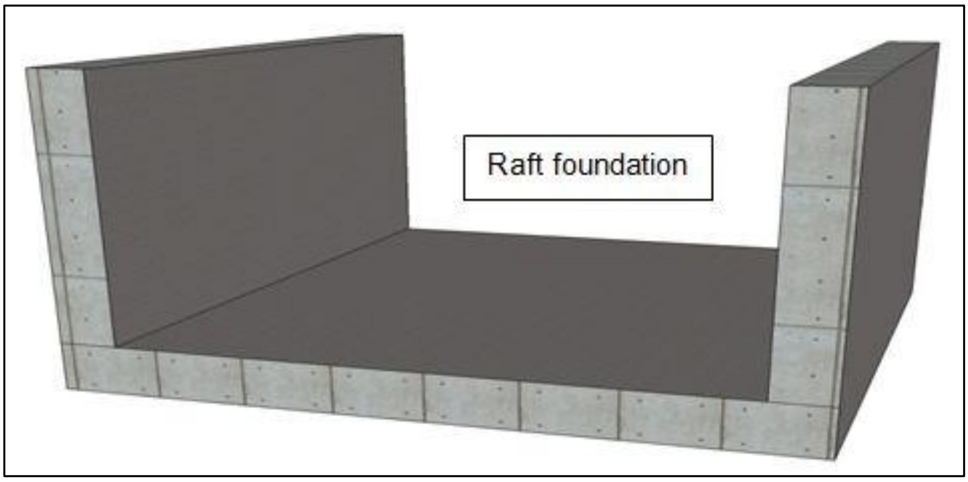

Figure 1-3 Raft foundation

The raft foundation or slab foundation is widely used in large civil engineering works. The codes of practice applied when designing and verifying shallow foundations mainly focus on building structures. This forms a dilemma for structural engineers when they have to deal with large civil engineering works, for example tunnels and piers. The foundations that have to be realized in such kinds of large civil works can be seen as superstructures, with very large dimensions. The codes do not always serve as a clear guideline to model foundations for these types of superstructures.

This thesis will focus on ways of modelling large concrete slab foundations. The models need to be generally applicable and their implementation should be possible with the help of computer software.

1.1.2. Modelling Slab Foundation

When modelling a shallow foundation, specifically "slab foundations", codes of practice are used to ensure the safety and durability of the design. In previous years the NEderlandse Norm (NEN) was used as a guideline to verify the different types of foundations in the Netherlands. In 2010 the Eurocodes became mandatory throughout the European Union. The EN 1992-1-1: Eurocode 2 (EC.2): Design of concrete structures and the EN 1997-1-1: Eurocode 7 (EC.7): Geotechnical designs are the codes that need to be used to guarantee a safe and durable foundation design.

Apart from the codes of practice, it is also important to have an overview of the actors and other factors that have to be taken into consideration when realizing such a structure. This way it is easier to formulate the constraints and functions of such a structure. The many actors and factors which are involved in modelling foundation structures are given in the concise illustration in Figure 1-4.

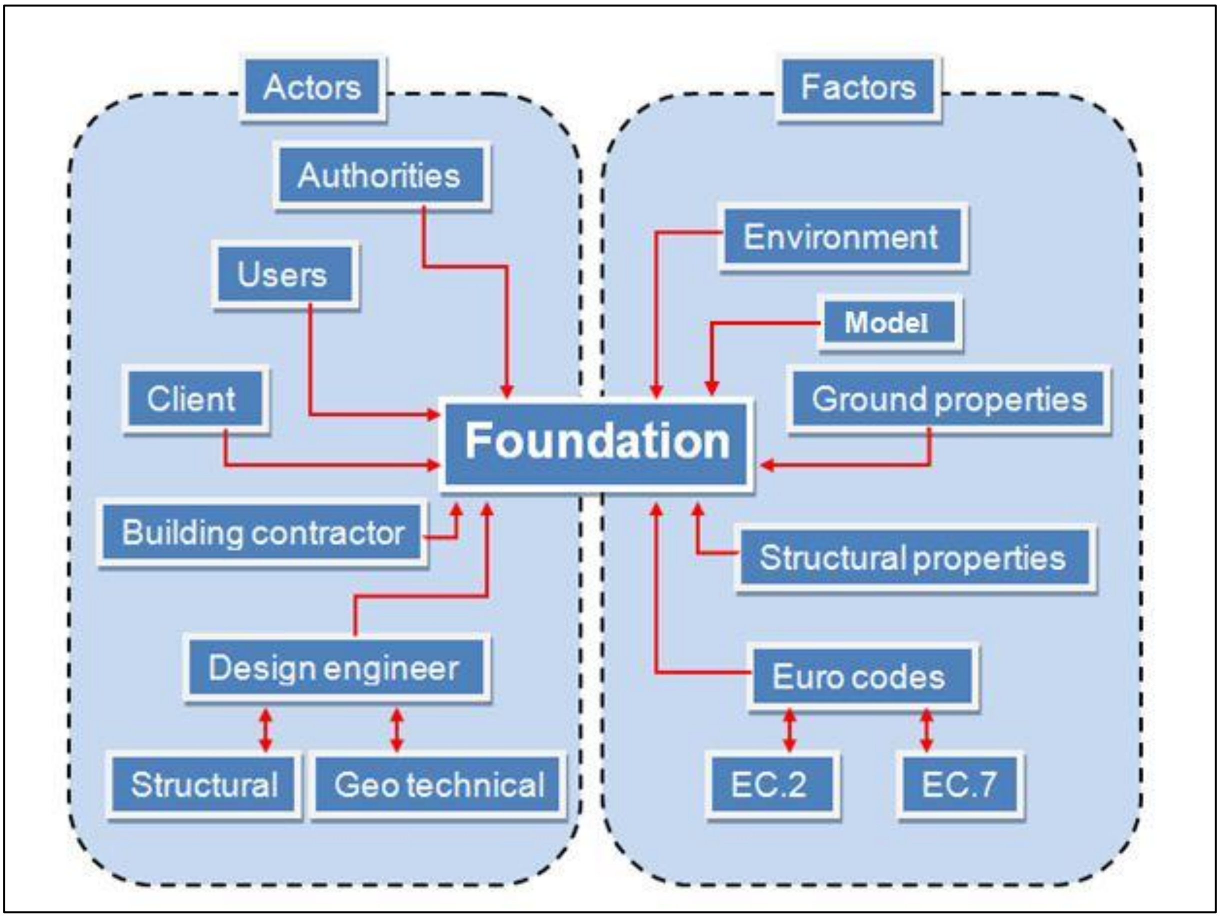

Figure 1-4 Concise representation of actors and factors involved in modelling foundations

1.2. Research Description

The communication between geotechnical and structural engineer in a foundation design process is not always optimal. The reason for this is not always clear. What can be stated is that the complexity of the interaction between structure and soil makes the process complicated and time-consuming. Also when modelling large shallow foundations it is not clear how to determine the soil stiffness and how exactly it should be used in the structural software program. In general, the stiffness is determined by the geotechnical engineer and the value used by the structural engineer. Besides uncertainties in determining the stiffness, other methods to model the interaction between structure and soil are also not straightforward.

A realistic model of structure and soil interaction can lead to an optimal and economical structure. A realistic model is a model which describes the behaviour of the structure in reality. With reality the meaning of the Stichting Bouw Research (SBR) will be used. In the SBR reality is describe as the deflection curve of the superstructure coincides with the settlement curve of the soil surface [4].

1.3. Research Objectives and Challenges

1.3.1. Research Objectives

The research objectives are formulated in two main objectives:

I. Develop a practical and consistent way of modelling large concrete slab foundations.
   - I.A. To obtain more insight into the interaction between large shallow foundation structures and soil.
   - I.B. To concretize which soil properties are needed in a structural software program and how the input works in the case of large concrete slab foundation structures.
   - I.C. Formulating the available methods to model and determine the structural and soil stiffness.
   - I.D. Describing the effect of the different parameters influencing the modulus of sub-grade reaction and how this affects the stresses and deformations of the structure.

II. Investigating an optimal way for the geotechnical and structural engineer to design a safe, reliable and economical foundation structure within a project planning scheme.

1.3.2. Scientific Challenge

The scientific challenge in this research is to study how to deal with the interaction between the structure and the soil. Soil, with its heterogeneous, anisotropic and nonlinear force-displacement characteristics makes modelling difficult [5]. The task of incorporating the soil properties, particularly the soil stiffness, into the structure model is the main scientific challenge. The soil stiffness has to be modelled accurately; an overestimation can cause unforeseen settlements and damage to the structure and an underestimation can result in an expensive design for these big structures.

1.4. Research Approach and Scope

1.4.1. Research Methodology

To overcome the scientific challenge of this study the following modelling methods will be applied. An overview of these methods can be seen in Figure 1-5. These methods were used through the software programs, SCIA and PLAXIS.

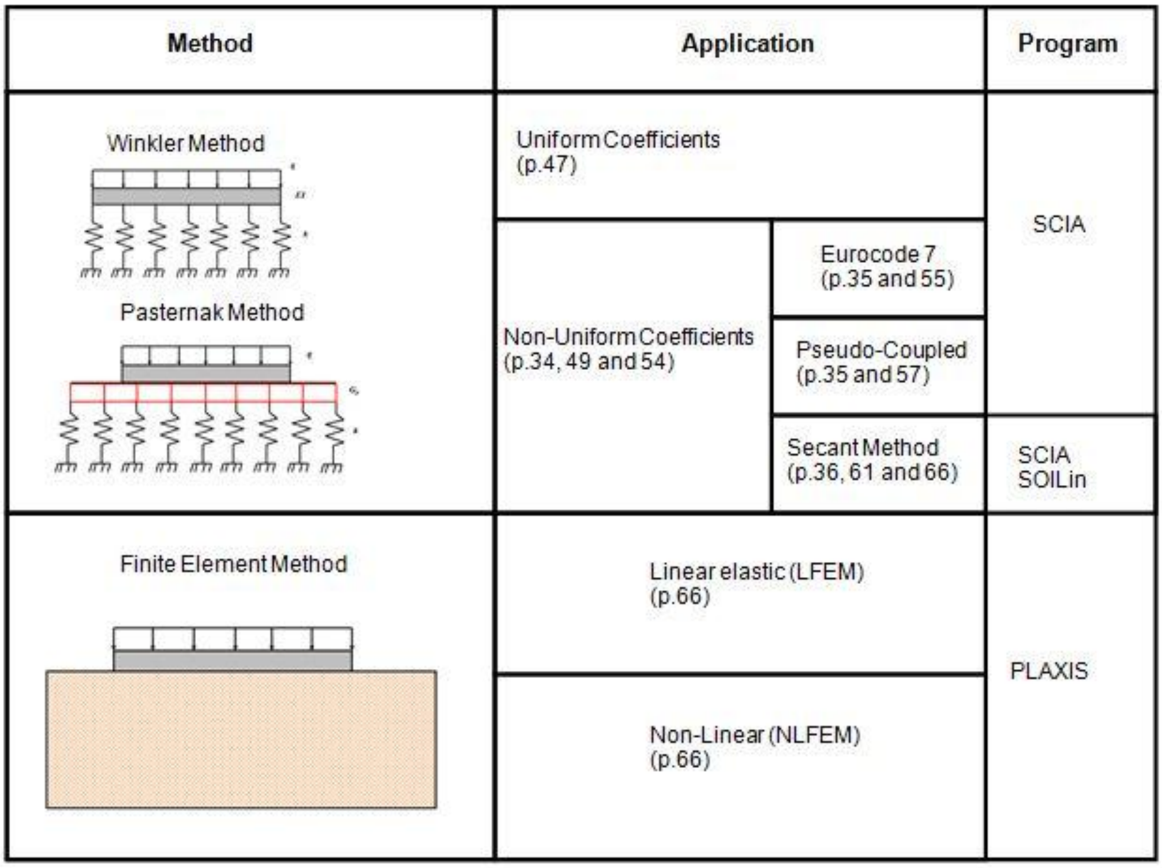

Figure 1-5 Overview of the used methods and software

1.4.2. Scope of the Project

The scope of this project is to develop a practical and consistent approach to model large concrete slab foundation. Such a model should take into account the interaction between the structure and the soil. The focus will be to find a slab foundation model which is safe and economical with regard to the final structure or the design process. The soil stiffness variation and the influence that it has on the model will also be analysed.

To make the scope more practical, case studies will be formulated, modelled and work out. The models in these cases should give more insight in the merit and demerits between the different models. In some cases a sensitivity analysis will be worked out to determine the consistency of the models. The workflow involved in realizing the entire process will also be documented and evaluated.

1.5. Outline of the Thesis

The research study contains an initial, analytical and finalizing phase (Figure 1-6). The first phase will consist of the available literature regarding the modelling of shallow foundation structures and the collection of data for the case studies. After the available literature and data has been collected, respectively, a number of large slab foundation case studies will be formulated and tested in a structural program.

In the last phase the data will be analysed and discussed. The whole process will be documented in this final report together with the important conclusions and recommendations.

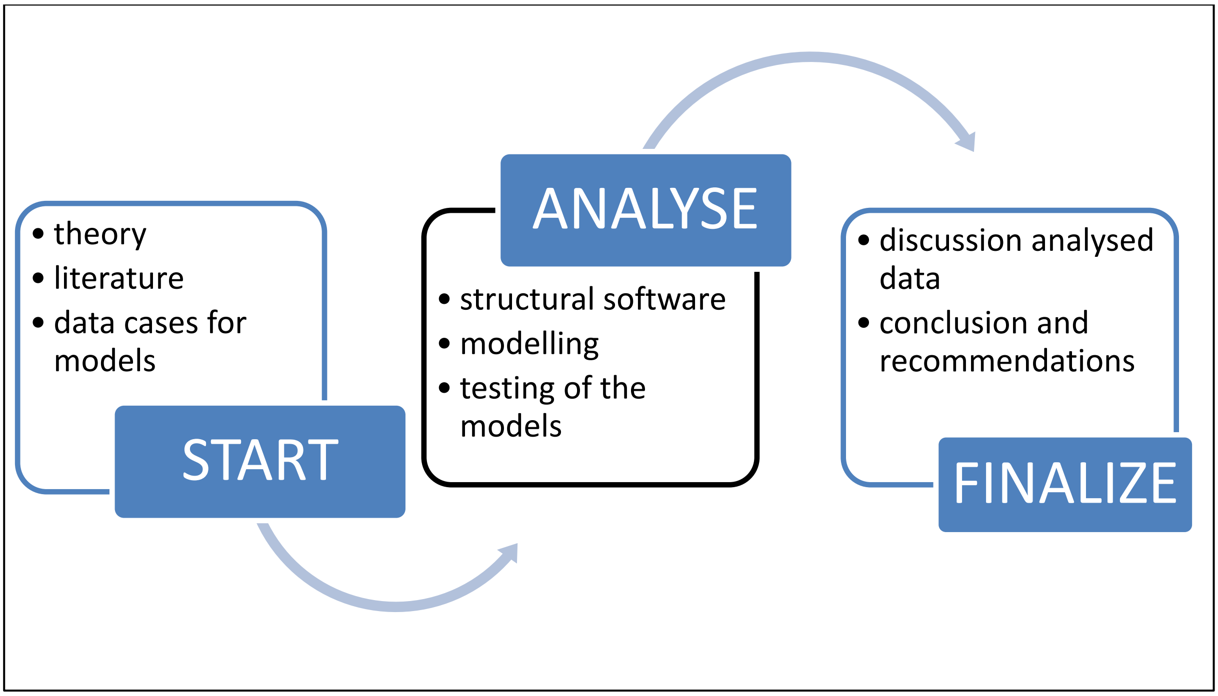

Figure 1-6 Work approach of this research study

This report comprises several chapters which follow the work approach outlined in Figure 1-6

In Chapter 2, the fundamental theory and the literature which will be used in the report are described. The chapter begins by describing the general way of modelling. More specifically, the emphasis in this chapter is on the way of modelling the super structure and the soil. It continues by describing the interaction between structure and soil. The chapter concludes with the work flow for designing a foundation slab, part of the process of which will form the focus later on in the study.

In Chapter 3, the analytical case studies are discussed. In these chapter different beams resting on an elastic foundation cases will be presented and verified. The goal is to explain the different analytical foundation models and the differences between them. The idea will also be to make clear the parameters needed to model the interaction between structure and soil when modelling analytically. In the end, the important conclusions of the different models will be presented.

Chapter 4 is dedicated to the theoretical overview of the structural software program to be used in the study. The working mechanisms of the programs interaction parameters are described.
The settlement equation of the program has an important role in determining the interaction parameters. This equation will be compared to the settlement equation from Terzaghi, which is often used in the Dutch codes.

Chapter 5 is focused on the implementation of the different computational spring analysis. The spring can be applied as uniform or non-uniform coefficients. In this chapter the difference between the two will be made clear. The non-uniform methods: Eurocode 7, Pseudo-Coupled method and Secant Method will be compared to each other.
In the end, a sensitivity analysis of the second interaction parameter will be done. This will give more insight in this unknown parameter.

Chapter 6 deals with the comparison between the Secant Method (SM), the linear finite element method (LFEM) and the non-Linear finite element (NLFEM). The SM is implemented in SCIA and the LFEM and NLFEM in PLAXIS. This chapter will be dedicated to the differences between the two computational software programs. The working approach, results and conclusions will be presented and discussed.

Chapter 7 will focus on the interaction between structural and geotechnical engineering. A checklist will be presented. This checklist will assist the two engineers in the communication process. Also the practical work process involved in designing and modelling a shallow foundation will also form a brief part of the discussion in this chapter.

Finally, Chapter 8 presents the conclusions of the thesis and the recommendations for further study.

2. Theory and Literature Overview

2.1. Introduction to Foundation Models

In this chapter an introduction to the literature regarding modelling foundations is given. Subsequently, the modelling of the super structure, soil media, the interaction between the two mediums and steps to design shallow foundation are the point of discussion. Foundations are designed to spread the load of the structure in the soil. The general approach to designing an adequate foundation structure is to make a model which describes reality [4].

The structure part of a foundation can be modelled as a flexible or a rigid plate. The flexible theory of plates can be categorized as the thin and thick plate theory as described in the book "*Plate analysis vol.1*" [6]. In a number of consulted literatures [5] [7] about foundation models there are two main approaches to model the soil beneath a foundation, these models are known as the Winkler (Section 2.4) and the continuum model.

The continuum model is computationally difficult to exercise and often fails to very closely represent the physical behaviour of soil [5]. Also the time factor, both in modelling and computation, can be exhausting.
The Winkler model, however, is not that difficult to exercise. The physical behaviour still cannot be presented clearly, but the time factor is not as exhausting as with the continuum model. To achieve the goal set in this thesis the focus will be on the Winkler model.

2.2. Modelling Structures in General

2.2.1. Model/Design Structures

In practice, two methods are used to model structure-soil interaction. One method is the beam/plate resting on an elastic foundation and the other is the continuum method which makes use of the finite element analysis (FEA). These methods take into account the deformation of soil and structure.

The beam/plate resting on an elastic foundation is related to the Winkler foundation model (Section 2.4.). The FEA is an advanced way of calculating mechanical problems. The outcome of the finite element (FE) calculation depends on the constitutive equation (relation stress - deformation). Although, the FEA is an advanced way of modelling the interaction, it is still a complex approach and, depending on the model, may require many input parameters. On the one hand, in some cases these input parameters are not known beforehand and they are also difficult to determine. On the other hand the Winkler model only needs one parameter to model the interaction between structure and soil.

A foundation model can be modelled in one, two or three dimensions. Each one of these models has its own boundary conditions, calculation approach and ultimately its merits and demerits.

2.2.2. One-Dimensional Model

A one-dimensional (1D) model (Figure 2-1) can be calculated on the basis of analytical or numerical approaches [8]. An ordinary differential equation can be set up to describe the behaviour of the system. With the help of boundary conditions the differential equations can be solved. One example might be that the classical beam theory (section 2.3.) is used to model the plate and the Winkler model (assuming that the soil behaviour is purely linear-elastic) is used to model the soil. This approach is often used when analysing slender structures that rest on an elastic foundation.

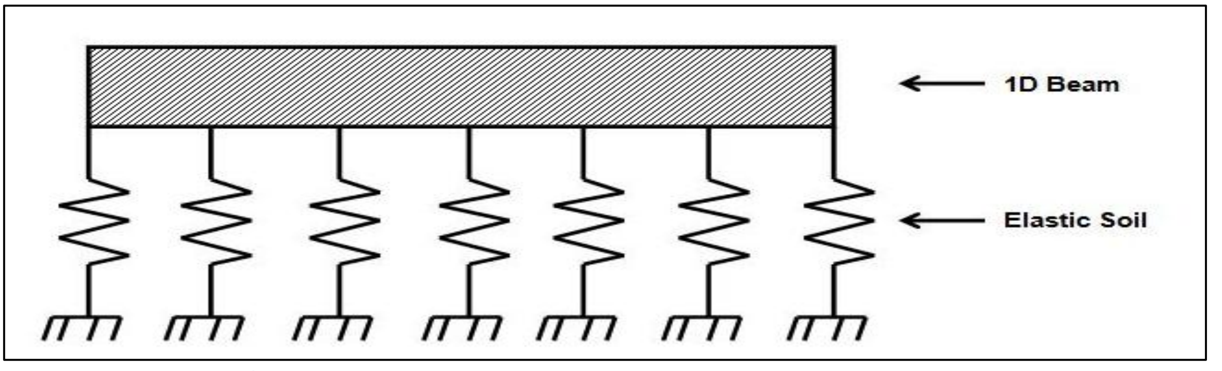

Figure 2-1 Example of a 1D model

A merit of this model is that it requires less time, but the demerit is that for superstructures, especially slabs, it often is a too simplistic approach. The input of the stiffness parameters for the structure and soil is also very important, because they are the only parameters that represent the structure and the soil medium in the model.

2.2.3. Two Dimensional Models

This 2D model adds another dimension to the system (Figure 2-2) and is useful to model plate structures. When using 2D elements in structural software special attention needs to be paid to the soil-structure interaction.

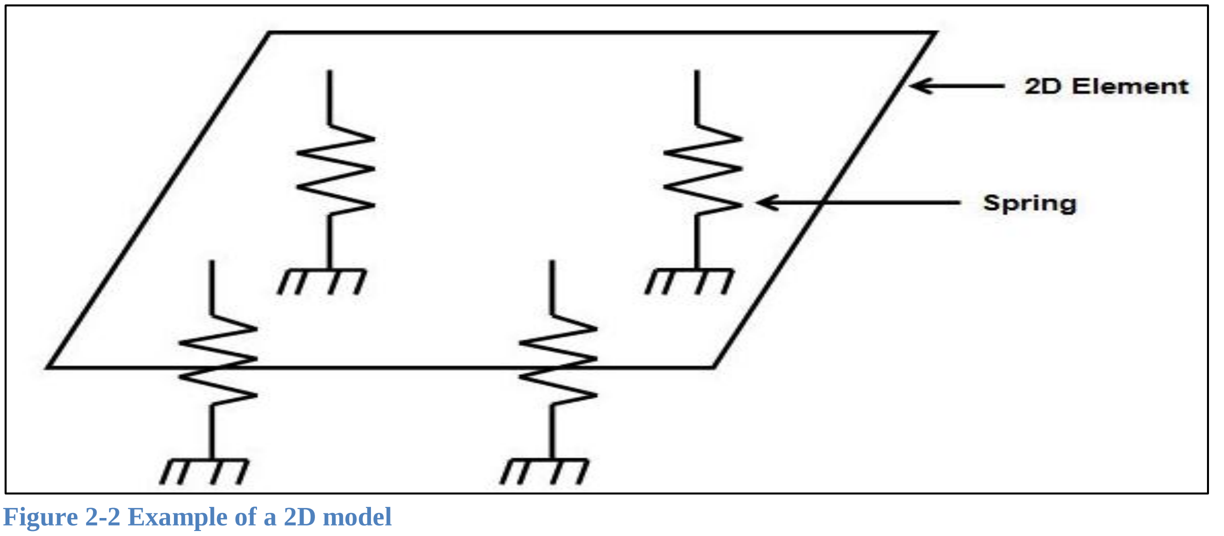

Figure 2-2 Example of a 2D model

A merit of this model is that it comes close in approximating reality due to the added dimension. The output results are convenient to interpret. The demerits are that the interaction between the soil and the structure are still difficult to model in 2D spaces.

2.2.4. Three Dimensional Model

These types of models are closest to representing the reality. All the dimensions are taken into consideration. In Figure 2-3 the structure and soil are both model with 3D elements [2].

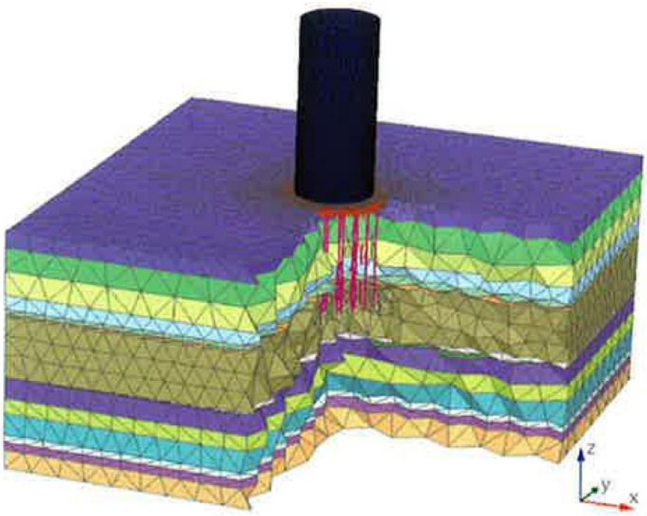

Figure 2-3 Example of a 3D model [2]

The merits of these 3D models are [2] [5] [1]:

- More spreading of the loads which can lead to the saving of material.
- Load interactions in multiple directions are taken into account.
- The model is more realistic.

The demerits are:

- Time intensive (modelling and calculating).
- Not all parameters might be known in the design phase.
- The results are more difficult to control than in a 2D model.
- There can be apparent accuracy.

2.3. Approach to Model the Super Structure

2.3.1. Classical beam theory

The structure part of the foundation has been modelled as a plate structure. The plate theory is closely related to classical beam theory [6].

$$EI\left(\frac{d^4w}{dx^4}\right) = q \qquad (2.1)$$

E = young's modulus
I = moment of inertia
w = displacement
q = distributed line load

Kinematic equation	Constitutive equation	Equilibrium equation
$\kappa = -\frac{d^2w}{dx^2}$ (2.2)	$M = EI\kappa$ (2.3)	$-\frac{d^2M}{dx^2} = q$ (2.4)

w = displacement
EI = bending stiffness
κ = curvature
M = moment
q = distributed line load

A plate is loaded in the direction of its plane (for example a wall). A slab is a plate structure loaded perpendicular to its plane (for example a floor). The plate theory categorizes the thin and the thick plate theory. The thin plate, or Kirchoff theory, assumes that a plane cross-section normal to the undeformed mid-surface would remain normal to the deformed mid-surface. This assumption ignores the shear effect. For a thick plate, or Mindlin-Reissner theory the shear effect is taken into account [6]. In appendix A, the kinematic, constitutive and equilibrium equation for the thin and thick plate theory are given.

2.3.2. Bending Stiffness (*EI*)

The stiffness of the structure is represented by the bending stiffness *EI*. The *E* is Young's modulus and it is a material property which indicates the stiffness of the material. The *I* is the moment of inertia it is a geometric property and indicates the stiffness created by the profile shape and dimension.

In a state of bending the concrete will crack when its tension limit is reached. At that point the reinforcement will carry the tension. Due to the crack phenomenon the bending stiffness of the concrete will decrease. This loss has to be taken into account. A practical rule in design theory is to lower the *E* value by multiplying *E* by ½ or by 1/3. This rule-of-thumb also takes the creep factor into consideration. This is an engineering approach for determining the bending stiffness in a preliminary design stage. In the final design stage the Eurocode 2 guidelines must be used to determine the Young's modulus. In Figure 2-4 a sketch of the cracking process is shown.

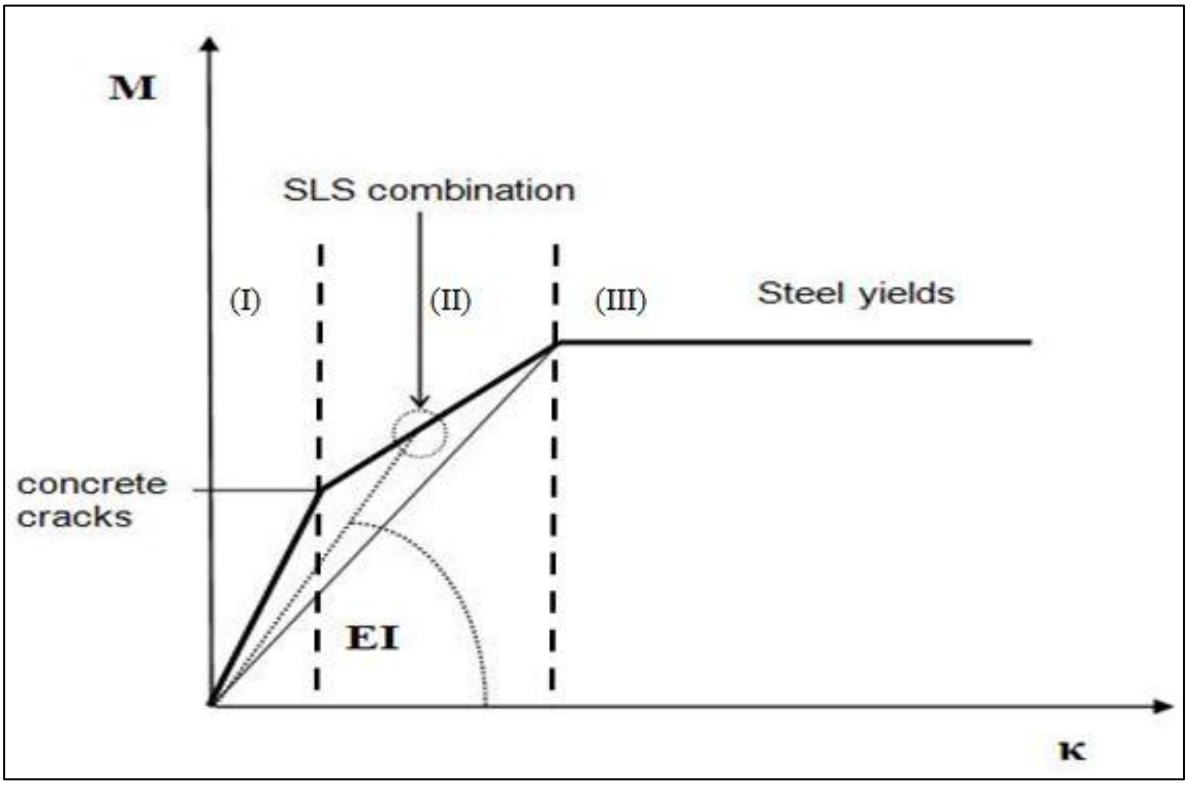

Figure 2-4 Bending stiffness for reinforce concrete

The concrete will carry the load in phase (I). After the tension strength is reached the concrete will crack as can be seen in phase (II). The reinforcement will be activated in phase (III) due to the cracking of the concrete.

2.4. Approach to Model the Soil

2.4.1. Winkler Model

The idea of the Winkler foundation model is to idealize the soil as a series of springs which displace due to the load acting upon it. A demerit of the model is that it does not take into account the interaction between the springs. The soil is also described according to the linear stress-strain behaviour. This linear relation makes calculation easier, but in practice soil does not behave linear elastically.

This model does not give a very realistic representation of the settlement, but it still gives an indication of what will happen in reality. The merit of this model is that it uses only one parameter (the modulus of sub-grade reaction, better known as the”*k*” parameter) to represent the soil (Figure 2-5). This is why it is also called the one parameter model.

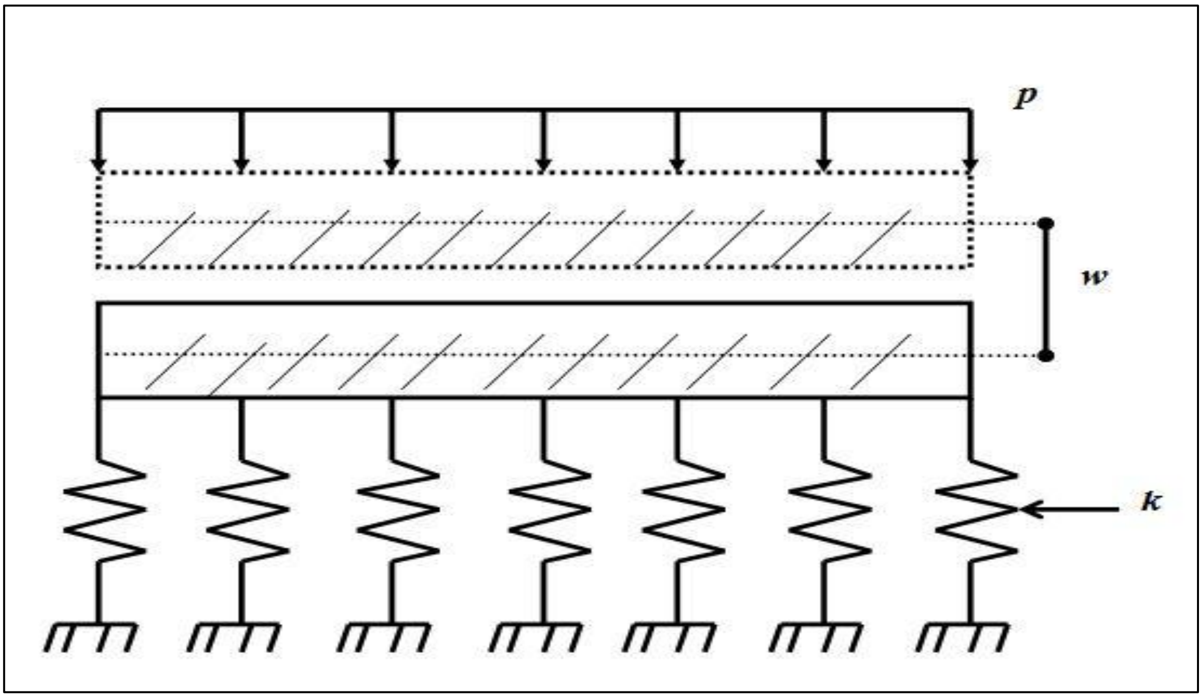

Figure 2-5 Winkler model (1 parameter)

$$p = wk \tag{2.5}$$

p = pressure
w = settlement
k = modulus of sub-grade reaction

2.4.2. Modulus of Sub-Grade Reaction (*k*)

The modulus of sub-grade reaction is a model parameter (*k*) that describes the stiffness of the soil. This parameter is not a soil property. This value is determined by dividing the pressure by the settlement (*k*=p/w). The settlement can be determined by different methods. A few of these methods are the formulas of Koppejan and Terzaghi. It can also be determined with software programs (for example PLAXIS, DSettlement (predecessor of MSettle)).
Determining the *k* value to replace the soil below the foundation structure is not a simple task. This parameter not only depends on the nature of the soil, but also on the dimensions of the load area and the type of loading. Also a time aspect plays a role as all soil settlements do not always occur immediately. It is important to know that the modulus of sub-grade reaction is not a constant value and that it varies under the same slab.

2.4.3. Pasternak Model

To overcome the Winkler model shortcomings improved versions [5] [7] have been developed. One of these versions is the Pasternak foundation model (Figure 2-6). In this model the springs are coupled with one another. This model is also known as a two parameter model because it takes another term into account. This term is “the G_p parameter”. Physically, this parameter represents the interaction due to shear action among the spring elements [7]. With this extra parameter the displacement of the model can be more realistic compared to the one parameter model.

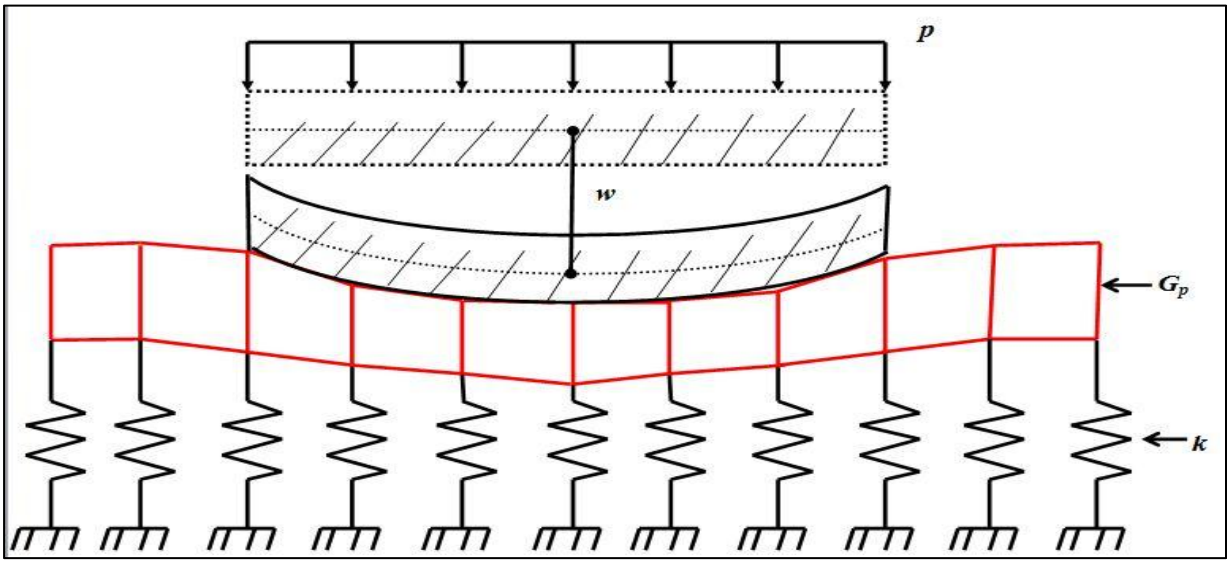

Figure 2-6 Pasternak model (2 parameter)

The differential equation is:

$$p = wk - G_p \frac{d^2w}{dx^2} \tag{2.6}$$

p = pressure
k = modulus of sub-grade reaction
G_p = shear modulus of the shear layer [5]

2.4.4. Shear Modulus of the Shear Layer (G_p)

The material response to shear strain is given by the shear modulus (G). Figure 2-7 shows the illustration of the shear deformation.

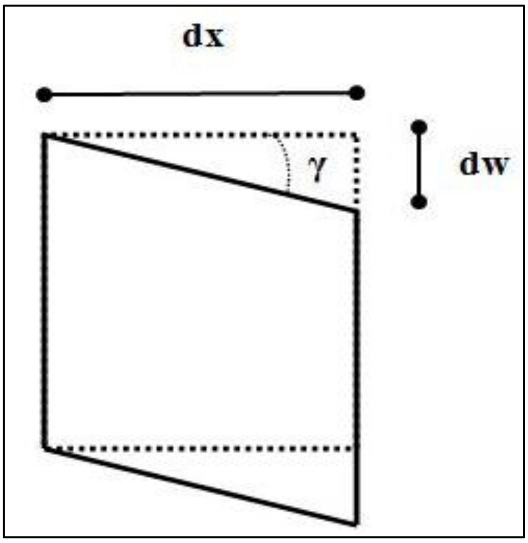

Figure 2-7 Shear deformation

$$\gamma \approx \frac{dw}{dx} \tag{2.7}$$

w = displacement
γ = shear strain

$$\tau = G\gamma \tag{2.8}$$

τ = shear stress
G = shear modulus

$$G = \frac{E}{2 * (1 + \upsilon)} \tag{2.9}$$

E = young's modulus
υ = Poisson's ratio

The G_p value is related to the shear modulus (G) but they are not the same. That they are not the same can be seen in their dimensions, G is kN/m^2 and G_p is kN/m, in a three dimensional space. G_p is G times an effective depth over which the soil is shearing.
Not a lot of literature and theory on this G_p parameter is available. The available and consulted articles regarding the Pasternak foundation model states that G_p is an interaction parameter. This parameter takes into account the interaction of the springs. The shortcoming of the Winkler model is thus improved with this extra parameter.
Physically, this parameter represents the interaction due to shear action among the spring elements as stated in the article of [7]. In [5], G_p is named the shear modulus of the shear layer.

2.4.5. Stress Distribution

The stress distribution in the soil plays a role when determining the settlement of a foundation. The stress in the soil closely beneath the foundation slab will almost be the same as the stress acting on the foundation. This stress will however decrease in larger depths of soil. In 1885, Boussinesq developed a method to determine the stress distribution in the deeper soil layer. This method is based on an ideal homogenous half space model. Also Newmark and Flamant found a solution to determine the stress distribution in a half space model. In the book "Grondmechanica" [9] more information can be found about these methods.

In Figure 2-8 [10] a Boussinesq stress distribution is shown. The σ_z is the vertical axial component of stress in an elastic homogenous infinite half space. It is observed that the deeper the σ_z is in the soil, the smaller stress (σ_z) gets.

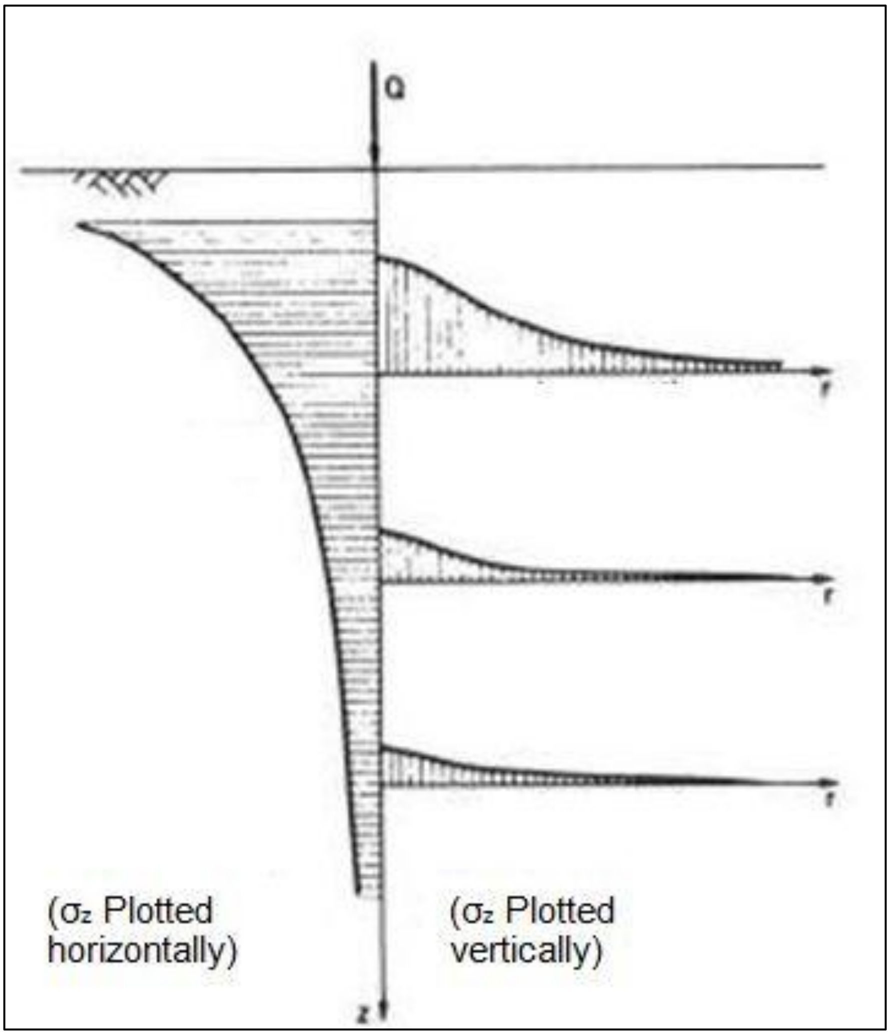

Figure 2-8 Boussinesq stress distribution [10]

2.5. Structure and Soil Interaction

The structure and soil interaction can be described by the relationship of their stiffness. Figure 2-9 shows the comparisons of settlements, contact stresses and bending moments for a uniform load on a flexible and stiff foundation slab. A flexible slab foundation has the largest settlement in the middle, together with a uniformly distributed contact stresses and low moments. A stiff slab foundation settles equally across its length. The contact stresses at the edge are larger because the soil at the edge behaves more stiffly, due to the fact that the load can spread there. Therefore, the contact stress of the slab has a parabolic form, with the maximum stresses at the edge and the minimum values in the middle of the slab. The bending moment in the stiff slab is much larger than that in a flexible foundation slab [11].

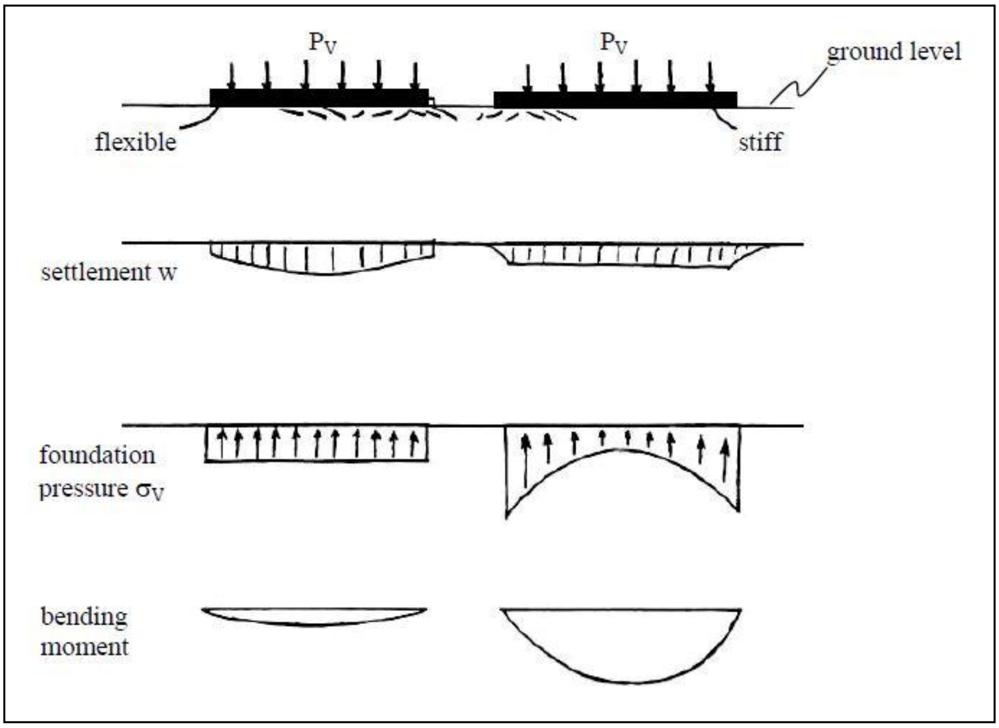

Figure 2-9 Stiffness interactions [11]

In [12] it stated that in general it can be said that the linear elastic behaviour of the foundation slab and the non-linear elastic behaviour of the soil are the cause of the interaction between soil and foundation.

Figure 2-9 and the following literature [11], [4] and [13] however indicate that the stiffness is the cause of the interaction between structure and soil. In this report the stiffness will be used to describe interaction between structure and soil. In the same literature also a method is given to determine the stiffness category of a system. This method makes use of the stiffness ratio (k_r) and the formula is:

$$k_r = \frac{Et^3}{12E_sL^3} \tag{2.10}$$

k_r = stiffness ratio
E = young's modulus of the slab
E_s = young's modulus of the soil
t = thickness of the slab
L = length of the slab

The t and L can be straightforwardly determined by the designer. To determine the E and E_s can however be complex. Still the E value can be determined with the methods discussed in section 2.3.. The E_s can be determined with the help of correlations or experimental soil samples. In case of different soil layers there are also practical rules available, however these also depend on the load acting on soil surface. With a FE more research can be done to determine this E_s value.

For a $k_r \leq 0,01$ the structure may be defined as flexible and for $k_r > 0,1$ the structure may be defined as stiff. In literature [11] the following conclusion of the k_r is given through the following Figures. Figure 2-10 shows the contact stresses and the moment line along the slab foundation. Figures 2-11 and 2-12 illustrates the difference in settlement with respect to the average settlement and the contact stresses with respect to the average pressure as a function of the relative stiffness. Out of the Figures it can be concluded that [11]:

- A stiff structure will give small differential settlements, but will show larger redistribution of loads, causing large moments and stresses in the structure.
- A flexible structure will show large differential settlements and low moments and stresses.

Interaction can thus, be considered when a structure is relatively stiff compared to the average stiffness of the soil and the foundation. For flexible structures this interaction will not lead to an optimized foundation structure.

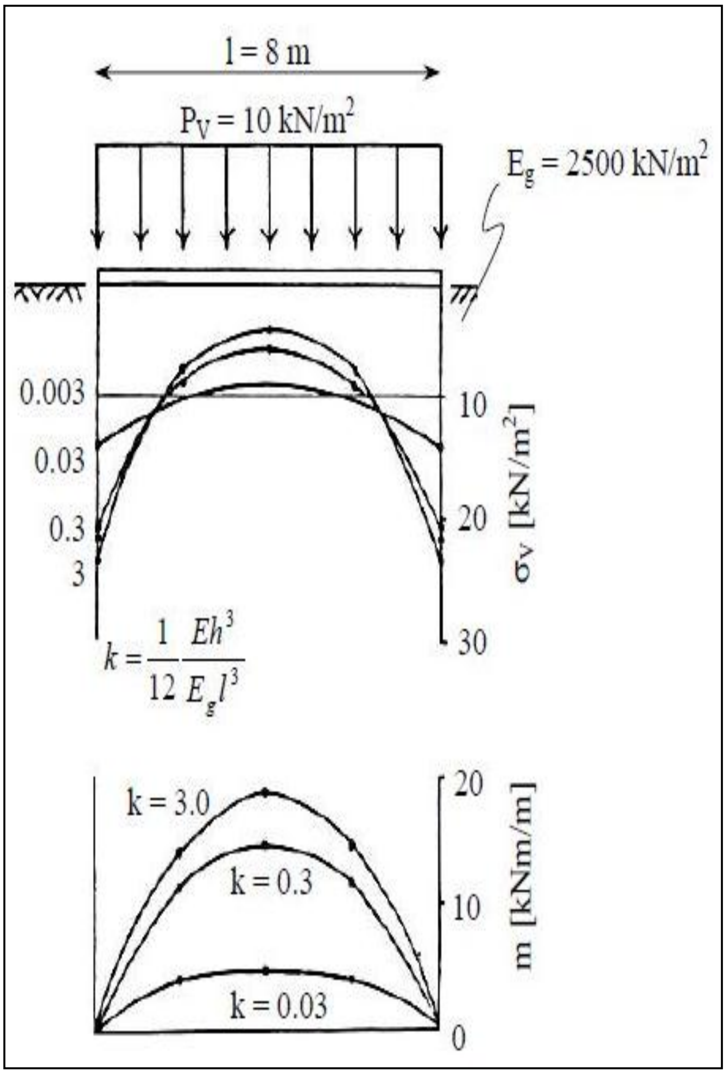

Figure 2-10 Contact stress and moment line against relative stiffness [11]

The k in the figure is the stiffness ratio as given in [11]. The σ_v is the contact stress and P_v the average pressure. The E_g is the young's modulus of the soil.

Figure 2-11 illustrates the difference in settlement with respect to the average settlement with respect to the average pressure as a function of the relative stiffness.

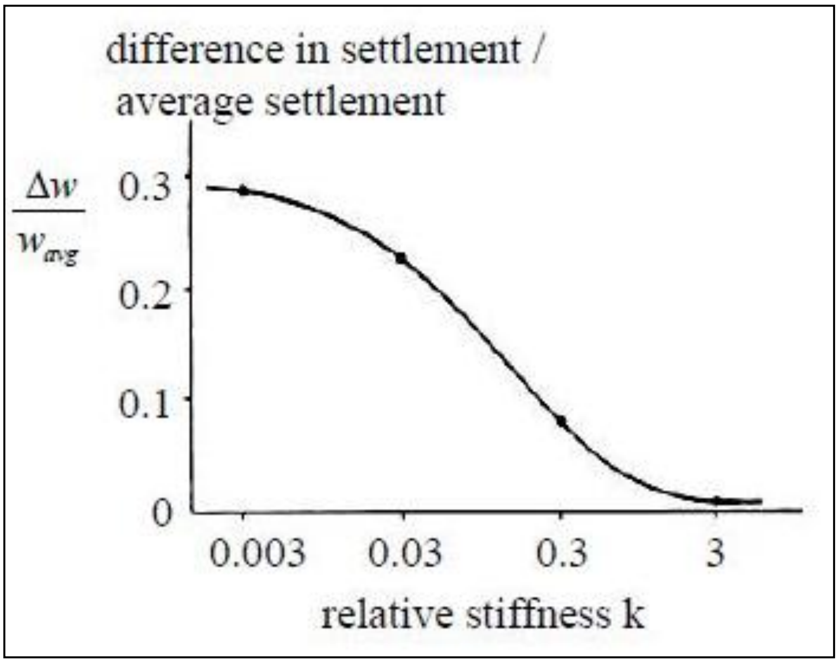

Figure 2-11 Difference in settlement/average settlement against relative stiffness [11]

Figure 2-12 shows the contact stresses with respect to the average pressure as a function of the relative stiffness.

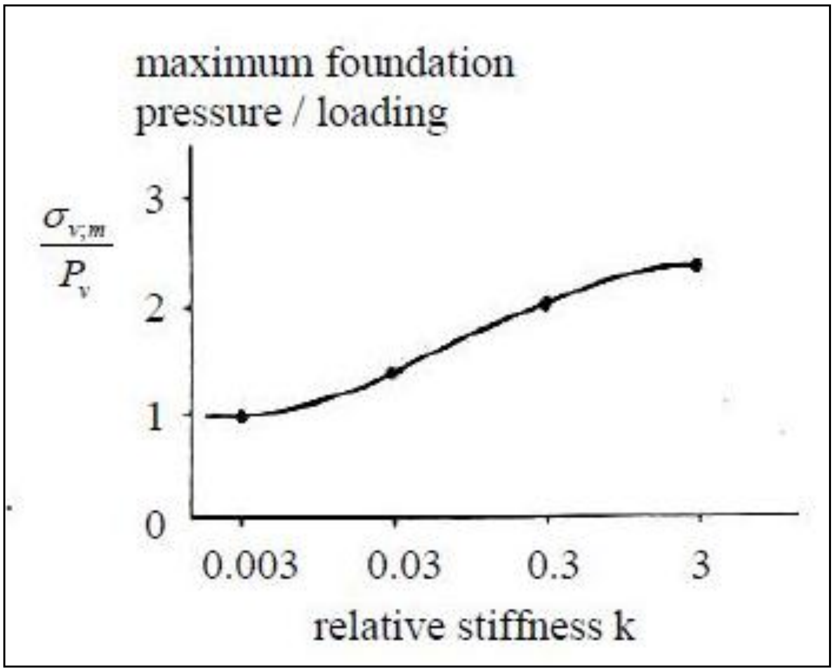

Figure 2-12 Max. contact stresses/loading against relative stiffness [11]

The Δw is the difference in settlement and the w_{avg} is the average settlement.
The σ_v is the contact stress and P_v the average pressure.

2.6. Design Code

The foundation structure considered in this work is built out of reinforce concrete. The guide lines for designing this type of structure are stated in the Eurocode 2. Table 2-1 shows the designing steps for slabs which are according to literature [14].

Table 2-1 Designing steps for slabs (according to Eurocode 2) [14].

Step	Task	Standard
1	Determine design life	NEN-EN 1990 Table NA.2.1
2	Assess actions on the slab	NEN-EN 1991 (10 parts) and National Annexes
3	Determine which combinations of actions apply	NEN-EN 1990 Tables NA.A1.1 and NA.A1.2 (B)
4	Determine loading arrangements	NEN-EN 1992-1
5	Check cover requirements	NEN-EN 1992-1: Section 5
6	Calculate min. cover for durability, fire and bond requirements	NEN-EN 1992-1
7	Analyse structure to obtain critical moments and shear forces	NEN-EN 1992-1-1 section 5
8	Design flexural reinforcement	NEN-EN 1992-1-1 section 6.1
9	Check defection	NEN-EN 1992-1-1 section 7.4
10	Check shear capacity	NEN-EN 1992-1-1 section 6.2
11	Check spacing of bars	NEN-EN 1992-1-1 section 7.3

NA= National Annex

This study will focus on step 7 (Analyse structure to obtain critical moments and shear forces). In the following chapter the theory of the analytical analysis which is explained in this chapter will be worked out.

3. Analytical Analysis

3.1. Introduction

In this chapter the formulation of the Winkler and Pasternak foundation models will be studied analytically. In addition the rotation of the soil surface will be included in the models. The method which influences the rotation will be named the Gradient foundation model. In the study a free-fee and very stiff boundary condition will be used. Through this study the theory and the working of the equations and the difference between the models can be investigated.

3.2. Winkler Foundation Model

3.2.1. Boundary Conditions

The differential equation of a beam resting on an elastic foundation is a combination of the classical beam theory (2.1) and the Winkler foundation model (2.5) (Figure 3-1).

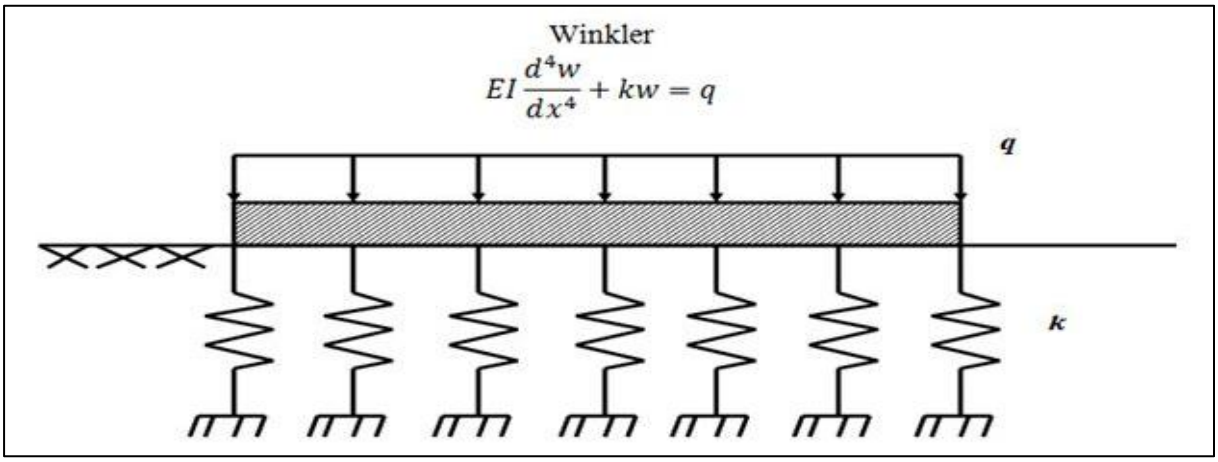

Figure 3-1 Winkler foundation model

The Winkler foundation model is a one parametric model. The boundary condition at the edge of the slab is assumed to be free-free and very stiff. Free-free boundary conditions are one where the rotation and shear is zero in the mid of the beam. At the edges the moments are equal to zero. A model of the free-free boundary condition can be seen in Figure 3-2. The *r* is the surrounding soil in the different models. This surrounding is equal of length at both sides of the foundation.

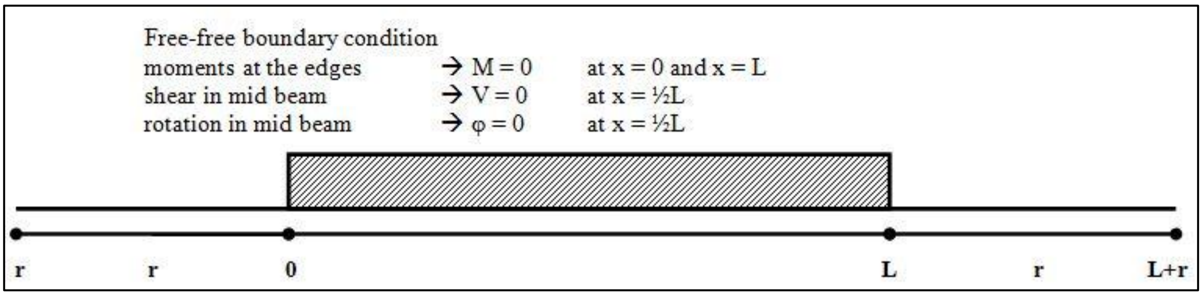

Figure 3-2 Model of a free-free boundary condition

A more detailed illustration of the free boundary condition for a Winkler model can be seen in Figure 3-3. The Figure shows the internal forces, of a small element of the beam, at the right side edge of a Winkler foundation with a free boundary. The V_c is the shear in the concrete and the M the moment. Both are equal to zero.

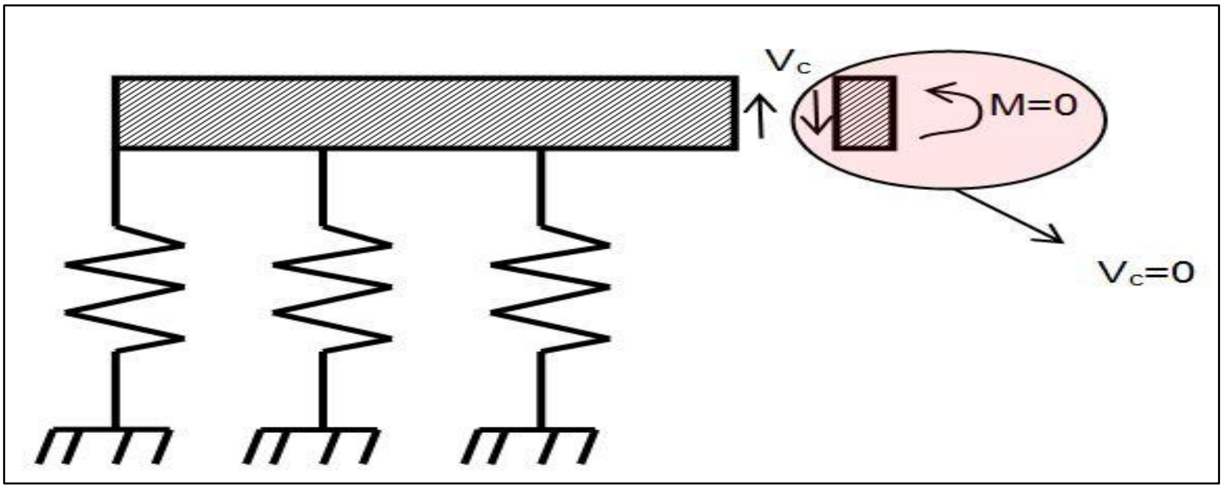

Figure 3-3 Free Winkler boundary detailed internal forces on a small element

To relate the free boundary condition to practice one can think about the building stage when the foundation is in place without any top structures or top loads working on the structure except its own weight. This can be seen in Figure 3-4

Figure 3-4 Free boundary in reality [link 1]

A very stiff boundary condition is for example the connection between a wall of a tunnel and its foundation. A boundary condition for these type of systems can be modelled as having no rotation $\left(\frac{dw}{dx} = 0\right)$ and the shear force to be equal to the force $\left(EI\frac{d^3w}{dx^3} = P\right)$ which the wall carries as can be seen in Figure 3-5 and Figure 3-6.


Figure 3-5 Very stiff boundary in reality [link 2]

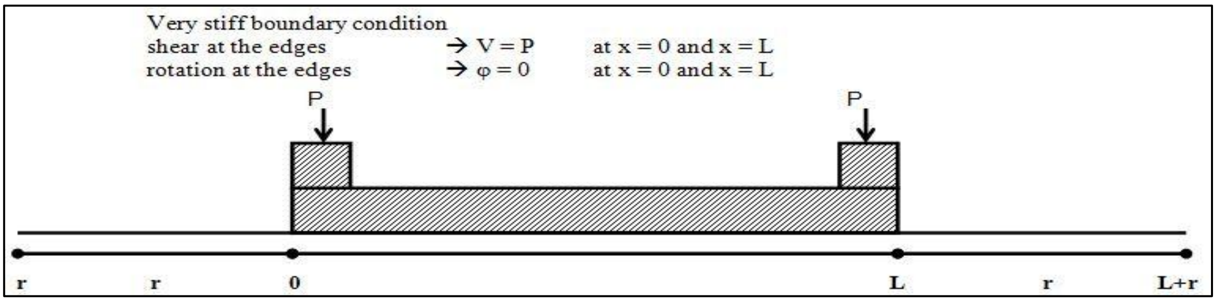

Figure 3-6 Model of a very stiff boundary condition

A more detailed illustration of the very stiff boundary condition for a Winkler model can be seen in Figure 3-7.

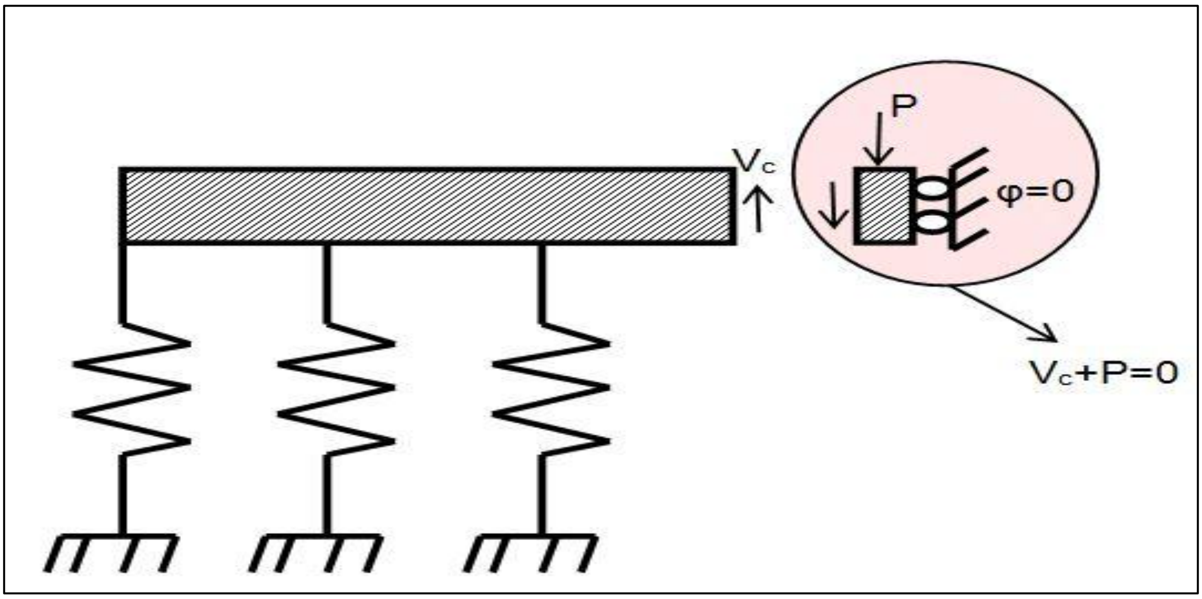

Figure 3-7 Very stiff Winkler boundary detailed internal forces on a small element

The shear force will be equal to the loads which the wall carries plus its own weight. The load (*P*) is the area of half of the tunnel (Figure 3-8) multiplied with the specific weight of the concrete. The area of half of the top structure is (10*0,8)+(5*0,8) = 12m^2. The load is thus
P = 12*25= 300kN/m.

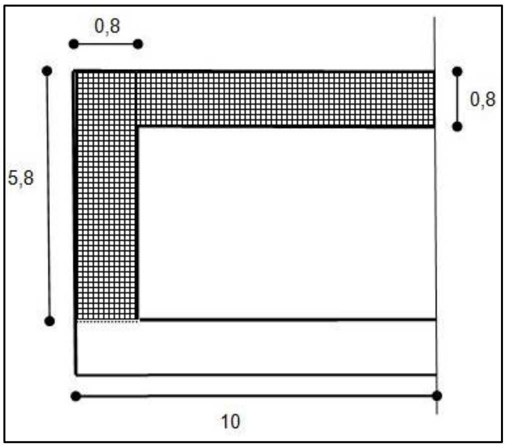

Figure 3-8 Dimensions of half of the tunnel structure

The horizontal translation will not be taken into account in these case studies. The focus will be on the vertical translation on the foundation.

3.2.2. Input Parameters

- The length of the beam is 20m the width and the height are 1m
- E_c = 10000N/mm^2 (cracked concrete)
  I = 1/12bh^3
  $EI = E_c*I$
- E_s = 45N/mm^2
- $k = E_s$*b/d
- b = 1m width beam
- d = 6m soil depth
- r = 10m surrounding soil
- q is a distributed line load which acts on the whole beam. q = 10 kN/m

The governing differential equation for a beam resting on a Winkler foundation is:

$$EI\frac{d^4w}{dx^4} + kw = q \qquad (3.1)$$

3.2.3. Results Winkler Model

With the boundary conditions the differential equation can be solved. The differential equations where analysed with the help of Maple software. The source codes of these maple files can be seen in Appendix B. The figures which can be observe in the Winkler Appendix are the displacement and contact stress for the beam with free boundaries.

For the free-free boundary conditions the displacement was uniform and the moment zero which is as expected because the beam does not have any curvature due to the fact that the springs do not interact with each other.

When the boundaries are fixed the displacement at the edges is large due to the loads of the wall. Also the moments are not zero for very stiff boundary conditions. The largest moments are at the edges near the boundary.

3.3. Pasternak Foundation Model

3.3.1. Boundary conditions

The Pasternak foundation model on a free-free boundary condition can be seen in Figure 3-9. What can directly be observed in this figure is that the shear layer of the Pasternak foundation model works outside the slab foundation. This is necessary to make the G_p active in the model. This is not necessary for the Winkler model, because the spring do not interact which each other.

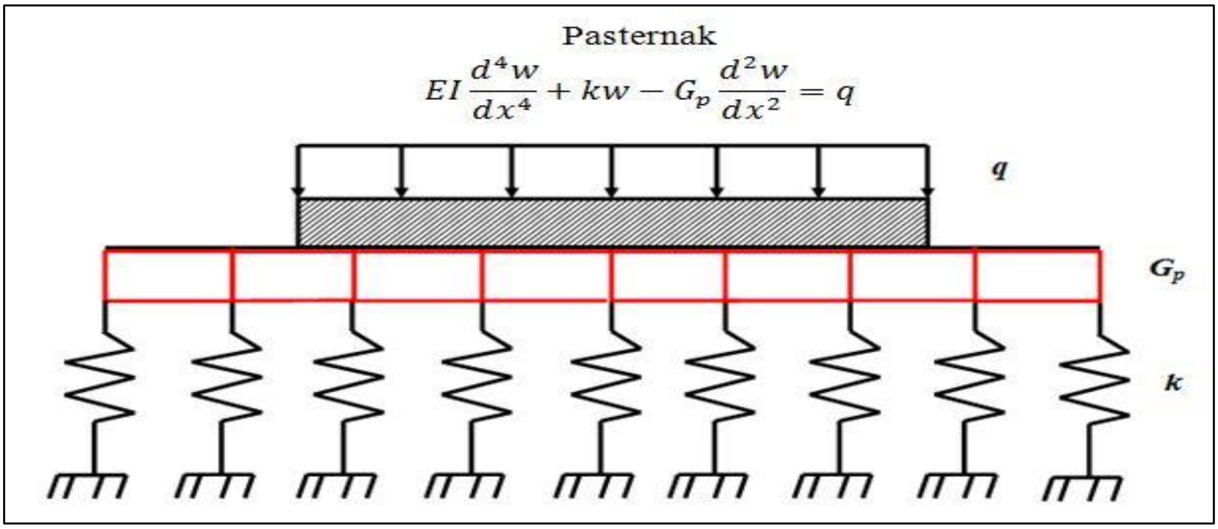

Figure 3-9 Pasternak foundation model

The same boundary of the previous section will be used. In the details of the boundary the shear layer has to be taken into account. A detail illustration at the right side edge for the free-free boundary can be seen in Figure 3-10.

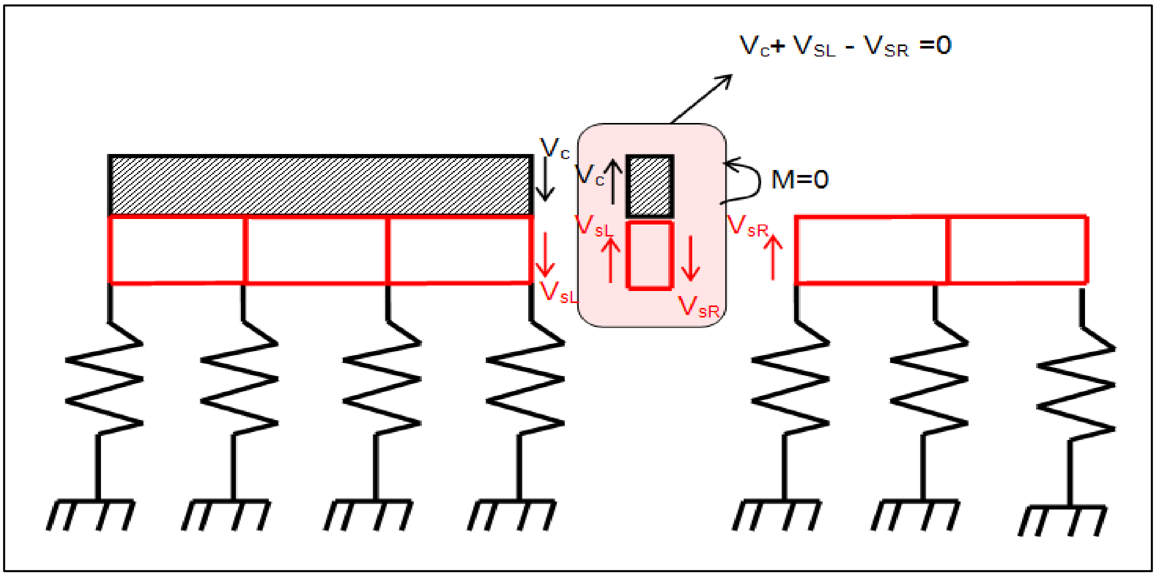

Figure 3-10 Free Pasternak boundary detailed internal forces on a small element

A more detailed illustration of the very stiff boundary condition for a Pasternak model can be seen in Figure 3-11.

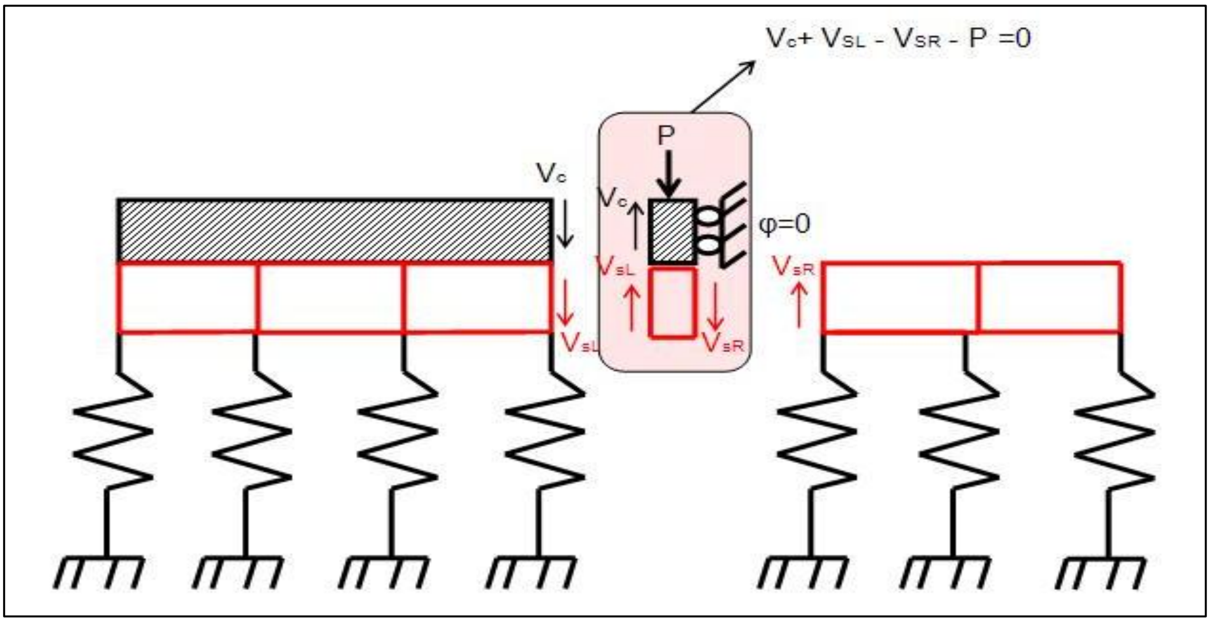

Figure 3-11 Very stiff Pasternak boundary detailed internal forces on a small element

3.3.2. Input Parameters

The governing differential equation for a beam resting on a Pasternak foundation is:

$$EI\frac{d^4w}{dx^4} + kw - G_p\frac{d^2w}{dx^2} = q \qquad (3.2)$$

The same material and stiffness input parameters of section 3.2.2. will be used.

- The G_p value will be related to the E_s. This value can be estimated through the following relation $G_p = (E_s*b*d)/2$

The boundary conditions are the same as the Winkler case.

3.3.3. Results Pasternak Model

The Pasternak foundation model is a two parametric model. The objective of this case is to see the first effect of the G_p parameter in the model. This is done by coupling both modulus of sub-grade reaction and G_p value to the Young's modulus of the soil and by observing the results of the displacement and moments. It is expected from literature that the Pasternak foundation model will give results which come closer to reality than the Winkler foundation model [5] [15] [16].

In appendix B the maple calculations can be seen. For the Pasternak foundation model the effect of the G_p can be observed in Figure 3-12. The results are different then when compared to the Winkler model which has a uniform displacement. The Pasternak foundation model also presents moments as can be seen in Figure 3-13.

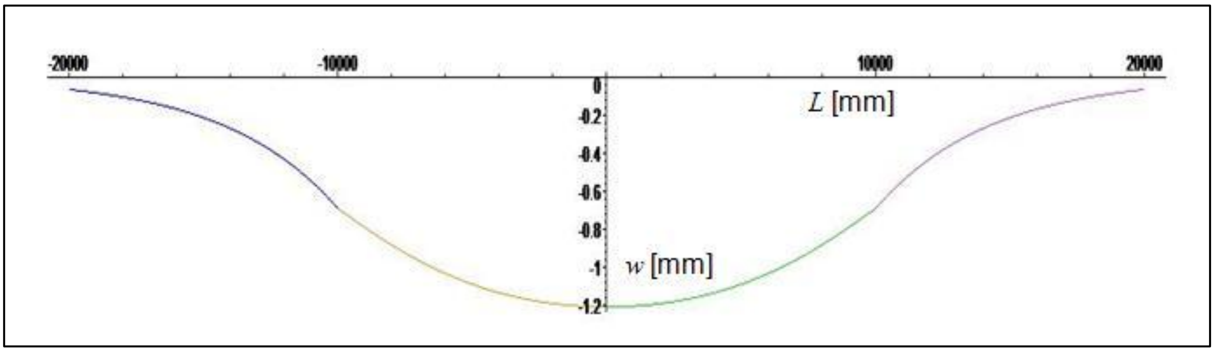

Figure 3-12 Displacement of Pasternak model

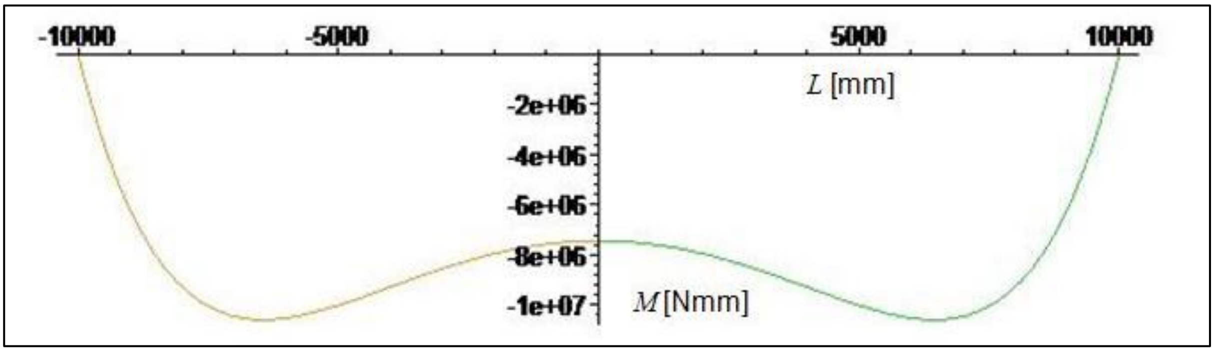

Figure 3-13 Moment Pasternak model

Out of the model it can be concluded that the Pasternak foundation model comes close in modelling the effects of the edges. The soil surrounding the foundation can be approximated with this analytical formula of Pasternak. The challenge what still remains is determining the right G_p value.

3.4. Gradient Foundation Model

The Pasternak foundation model improves the Winkler foundation model by influencing the curvature. In this study also a new parameter which influences the rotation is researched. This new model will be called the Gradient foundation model due to the fact that it influences the rotation. This new parameter is "C_φ". The C_φ effect on the foundation model will be studied. Through the following formula an attempt is made to formulate the differential equation of the Gradient foundation model.

$$EI\frac{d^4w}{dx^4} + kw - C_\varphi\left|\frac{dw}{dx}\right| = q \tag{3.3}$$

During the study it was concluded that this new parameter did not have any interesting physical effects on the model. Variation in the C_φ value gave results which were physically difficult to interpret. In appendix B the steps of the analysis can be observed.

3.5. Conclusions Analytical Analysis

The Winkler foundation model with free- free boundaries gives a uniform displacement and moments equal to zero. This is because the springs are not coupled to each other. The moment is zero because there is no curvature in the model.

In this chapter a new foundation model called the Gradient foundation model is introduced. This model effects the soil surface rotation. The analyses however showed that this model gives results which are physically difficult to interpret.

The Pasternak foundation model did however produce results which models the behaviour of the beam realistically. The soil surrounding the foundation can be approximated with this analytical formula of Pasternak. The challenge what still remains is determining the right G_p value. An important conclusion in this analysis is the coupling of the springs in the form of the second parameter (G_p).

In the next chapter the software to be used to model the different foundation methods, which uses the spring analysis, will be discussed. Special attention will be given to modelling the interaction between structure and soil with uniform and non-uniform coefficients. The software program which will be used also makes use of the Winkler and Pasternak foundation model.

4. Spring Analysis for Soil Response

4.1. Introduction

In the previous chapter the models were analytically studied. In this chapter the spring analysis will be used with the help of computer-aided engineering (CAE) tools. The FEM is a CAE tool which assists the engineer in analysing and modelling structures. In this graduation work the FEM tool SCientific Application (SCIA) engineering structural software has been used to model the shallow foundation.

In the program the slab can be modelled with 2D plate elements and the soil as a modulus of sub-grade reaction or by the help of boreholes with a geological profile. The interaction of structure and soil is modelled by the interaction parameters called "*C* parameters". In this part of the report the theory of these *C* parameters will be discussed.

The method to use these interaction parameters can be divided into two main groups. The first are the uniform coefficients and the second of non-uniform coefficients (Eurocode 7, Pseudo-Coupled and Secant Method).

The settlement equation in SCIA plays an important part in the transformation form soil parameters to interaction parameters. This formula is however different then the often used Terzaghi equation, which is used in the Dutch practice. In that context the two settlement equations will be compared to each other. This will bring to light the effect it can have on the calculations of the interaction parameters.

4.2. Interaction Parameters

In the software program there are two approaches to model a structure - soil interaction for slabs foundations [17]. One approach is to use uniform coefficients and the other one is to make use of non-uniform coefficients. The working method of both interaction approaches makes use of springs to simulate the soil stiffness.

The soil stiffness parameters are [17]:

C_{1z}, C_{1x}, C_{1y}	- resistance of environment against the displacement	[in MN/m^3]
C_{2x}, C_{2y}	- resistance of environment against the rotation	[in MN/m]

Also usually $C_{2x} = C_{2y}$ and $C_{1x} = C_{1y}$ [18].

Further on in this report the C_{1x} and C_{1y} will be kept to zero, because horizontal forces will not be taken into account in this study. Only vertical loads working on the foundation will be researched. To make the report also a bit more understandable the term C_1 and C_2 will be used in the other chapters instead of C_{1z}, C_{2x} and C_{2y}.

In SCIA, the ground under the foundation is called subsoil. This is the medium which is directly under the structure. When running an analysis the program makes use of the *C*-parameters (C_1, C_2) which represent the subsoil properties in the form of springs. The C_1 can be seen as the one parametric parameter of Winkler "the *k* value" and C_2 as the second parameter of the Pasternak "the G_p value" theory this is according to SCIA manual [18] and SCIA helpdesk. For both parameters this is true. But in the way SCIA explains it can cause some confusion. Because according to the theory of Pasternak the G_p is related to the curvature (d^2w/dx^2). In the SCIA manual the C_2 is related to the rotation (dw/dx) [17] [18]. To avoid confusions in the report the term interaction parameters will be used for C_1 and C_2.

The C's are the interaction parameters assigned to structural elements that are in contact with the subsoil. These parameters influence the stiffness matrix of the soil. The C parameter depends on the dimension and stiffness of the structure and stiffness of the soil, load and subsoil properties [18]. A change in any of these parts causes different C parameters.
Vertically the whole slab is supported by the soil stiffness-parameter C_1 and also in the shear direction-parameter C_2. The edges are more supported by the C_2 parameter. This takes care of the edge effect. The edge effect is the phenomenon that occurs near the edges of the foundation. These effects are often difficult to model.

The designer has the possibility to fill in the input parameters (C_1 and C_2) manually. The Winkler foundation model can be simulated when the C_2 is equal to zero (C_2=0) and C_1 is not equal to zero ($C_1 \neq 0$). According to the SCIA it is not recommended for users to fill in their own C_2 value because experimental data is not available. If the designer still wants to make use of the second interaction parameter he should use the module SOILin which will be explained in section 4.4.

4.3. Eurocode 7 and Pseudo-Coupled Method

4.3.1. Introduction Non-Uniform Coefficients

The Winkler model does not take into account the edge effect and the interaction between the springs. Not taking the edge effects and spring interaction into account does not give realistic results as discussed in chapter 2. The shortcoming of Winkler is overcome by using non-uniform coefficients.
The following non-uniform coefficients approaches are described in Eurocode 7 and developed by SCIA (Pseudo-Couple). The idea both approaches use is to use at the corners and edges stiffer modulus of sub-grade reaction and less stiffer "*k* values" in the middle of the foundation. By using this method the edge effects and interaction between springs can be realized.
In this section the steps to model foundations using Winkler foundation model with the help of Eurocode 7 and SCIA (Pseudo-Couple) methods are explained. Although the Winkler foundation model does not realistically model the reality, it is still often used in practice because it a quick method which yields results that is usable for the structural engineer.

4.3.2. Eurocode.7

In Eurocode 7 the following method to go about modelling shallow foundations is explained. For modelling rectangular slab foundation the contact stress can be distributed as seen in Figure 4-1:

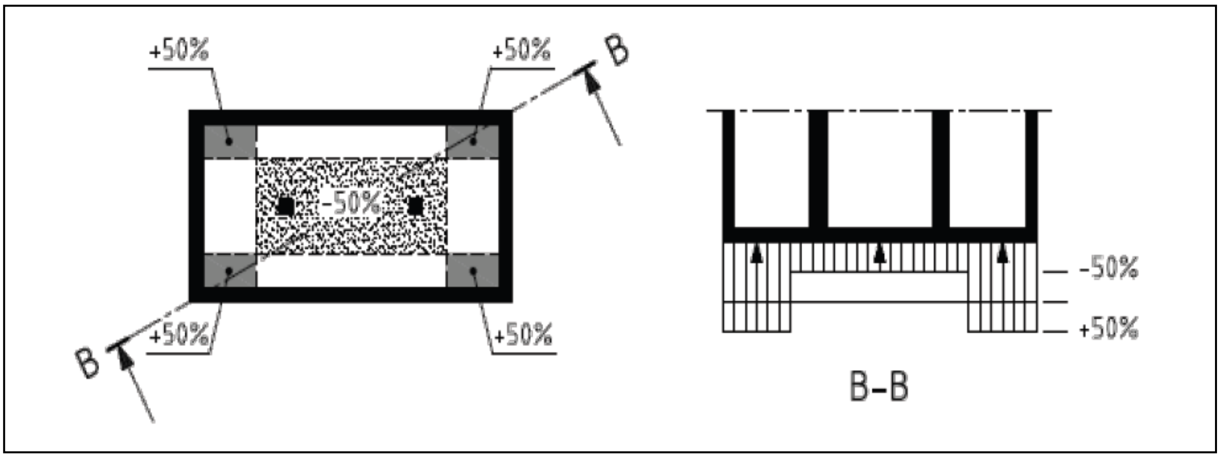

Figure 4-1 Soil pressure distribution at a deformation analysis of a foundation plate [19]

The slab should be divided into nine parts with distances of 0,25**b*' and 0,25**l*' from the slab edge. The *b*' and *l*' are the effective width and length. On the four corner surfaces the calculated average foundation pressure $\sigma_{gem;d}$, which is determined with the fundamental load combination, according to Eurocode 7 section 6.6.2(b), should be increased with 50%. In the middle of the nine surfaces the $\sigma_{gem;d}$ should be decreased with 50%. The other four surfaces should have the average foundation pressure $\sigma_{gem;d}$.

One should take care that the code does not give information about the spreading of the modulus of sub-grade reaction but about the contact stress. This method can also be used to distribute the stiffness of the subsoil. To do this first the average modulus of sub-grade reaction has to be determined. In Chapter 7 more in depth information will be given how to determine the modulus of sub-grade reaction.

4.3.3. Pseudo-Coupled

The method that is proposed by SCIA has the same philosophy as the Eurocode 7 method. The idea is to use the Winkler model and trying to couple the springs by taking different stiffness under the foundation. The average modulus of sub-grade reaction (k_s) should first be determined. This can be determined in the same manner as described in Chapter 7. After the k_s is determined the k_A, k_B and k_C can be found by substituting the known values in the following equation [SCIA helpdesk]:

$$A_A k_A + A_B k_B + A_C k_C = (A_A + A_B + A_C) k_s \quad (4.1)$$

With: $k_B = 1{,}5 * k_A$

$k_C = 2 * k_A$

This will give the distribution of the modulus of sub-grade reaction for the different zones as can be seen in Figure 4-2. By doing this the Winkler model is used and the springs are coupled through a Pseudo-Coupled method.

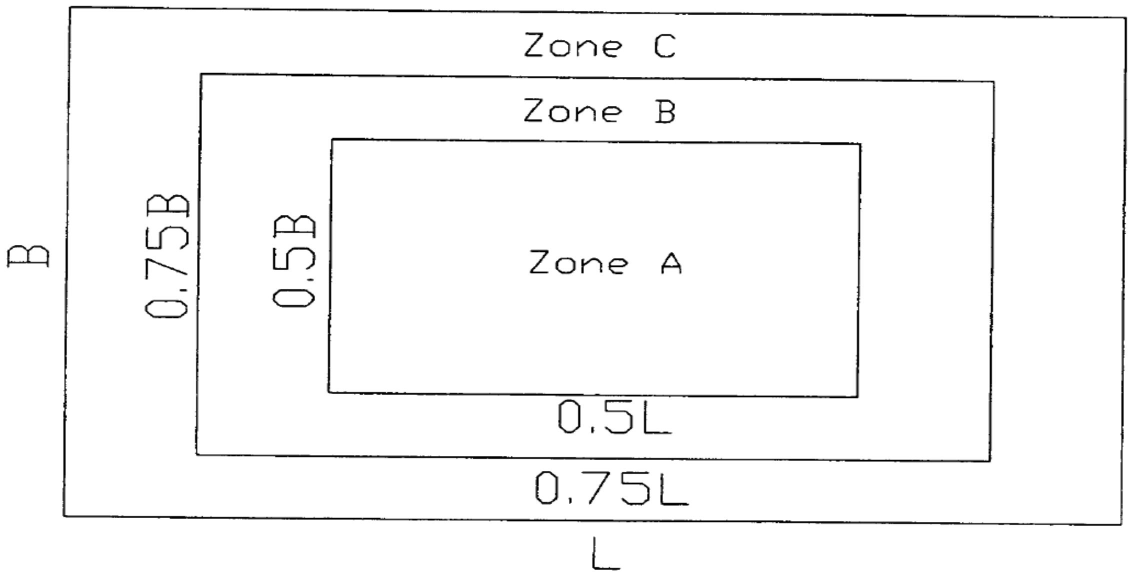

Figure 4-2 Pseudo-Coupled distribution of the *k* [SCIA helpdesk]

4.4. Secant Method

The other method to model the structure and soil interaction in SCIA is by using the SOILin module. SOILin stands for "Analysis of structure-**SOIL IN**teraction". It is a tool which uses a borehole and geological profile to calculate and spread the C_1 and C_2 parameters of the subsoil under the surface of the support. In other words it transforms soil properties into non-uniform coefficients. The process to transfer the soil properties into non-uniform coefficients will be called the Secant method (SM). This method will be explained in this section.

The SM can be used to give a prognosis about the settlement and deformation of the load on the soil surface. It can also be used as a pre-processor to calculate the soil stiffness. In the pre-processing stage the C_1 and C_2 parameters will be calculated for the given load combination. After this is done the same stiffness (C_1 and C_2) will be used for the other load combinations. The output of the module can give a visualization of the deformation, internal force, contact stress, settlement and C parameters.

The C parameters are calculated by a predictor-corrector algorithm [SCIA helpdesk]. The steps which are taken to calculate the interaction parameters are given in Figure 4-3:

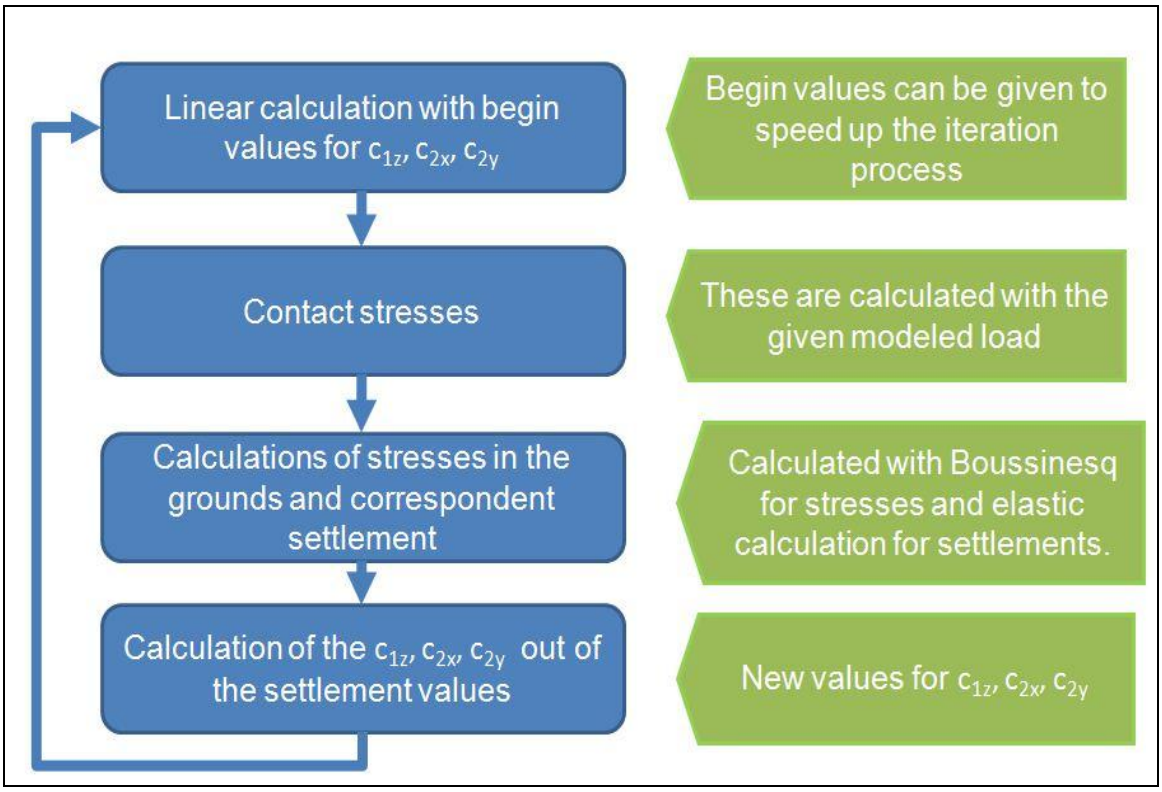

Figure 4-3 SM process [SCIA helpdesk]

The C parameters are determined by the help of an iterative process. The iterative steps are for example for the C_1 parameter as follow:

1. First a C_1 value is assumed
2. Then the contact stresses and stress distribution in the subsoil is calculated
3. Afterwards the settlement in the subsoil with the soil property is calculated
4. With the info of step 2 and 3 the new C_1 is calculated and used again.

This iteration continuous until the difference between the C_1 values is very small. The program stops when $\boldsymbol{\varepsilon}_\sigma$ < 0,001 or $\boldsymbol{\varepsilon}_u$ < 0,001. Where $\boldsymbol{\varepsilon}_\sigma$ and $\boldsymbol{\varepsilon}_u$ are:

$$\varepsilon_\sigma = \frac{\sum_{i=1}^{n}\left(\sigma_{z,i,j} - \sigma_{z,i,j-1}\right)^2 A_i}{\sum_{i=1}^{n}\left|\sigma_{z,i,j} * \sigma_{z,i,j-1}\right| A_i} \tag{4.2}$$

$$\varepsilon_u = \frac{\sum_{i=1}^{n}\left(u_{z,i,j} - u_{z,i,j-1}\right)^2 A_i}{\sum_{i=1}^{n}\left|u_{z,i,j} * u_{z,i,j-1}\right| A_i} \tag{4.3}$$

n = number of nodes
$\sigma_{z,i}$ = contact stress in node i
A_i = area corresponding to node i
$u_{z,i}$ = global displacement of node i in the z-direction

The iterative process to calculate the C_1 value in the z-direction is called Secant method (SM). The SM process can be better explained by the illustration in Figure 4-4.

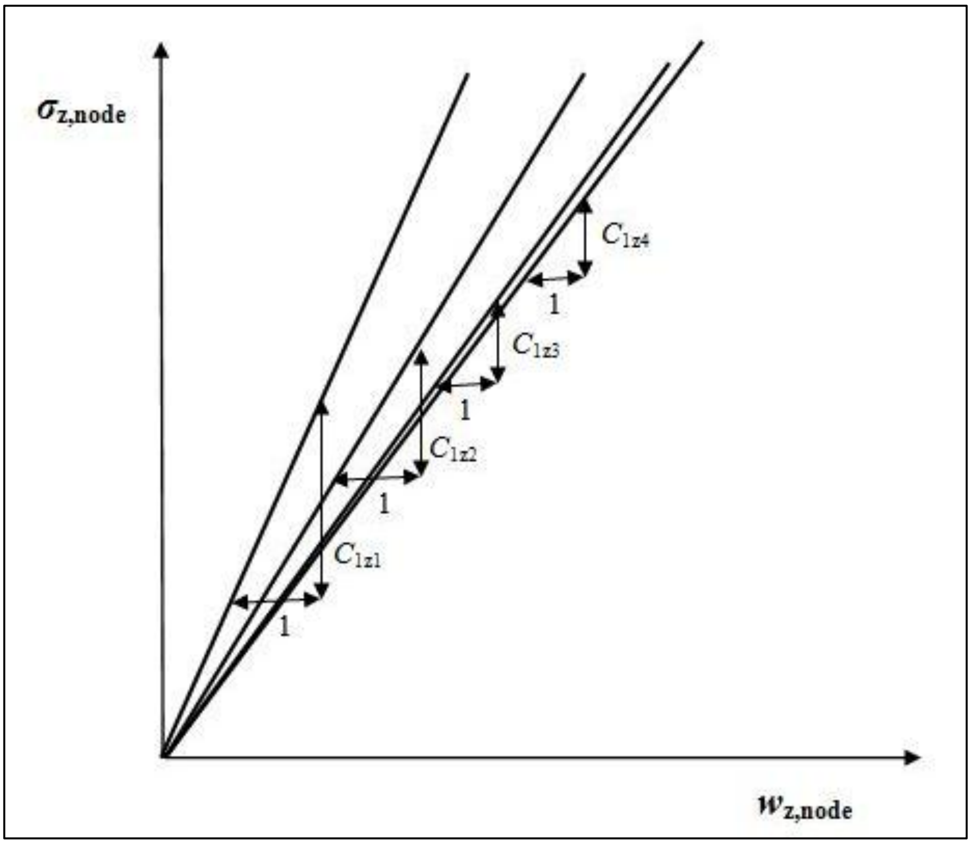

Figure 4-4 Secant analysis response

In general the *C* parameters are depending on the position of the interface point, the variable value of the limit depth and the settlement. The settlement and the strain in the subsoil are related to each other. This relation is also coupled to the load acting on the 2D interface surface which is also coupled to the structural properties and the contact stresses. So this concludes that the *C* parameters are not purely soil parameters, but they are dependent on the total properties of structure, soil and the external loads. This is the reason that the term "***interaction parameter***" is better suited for these parameters [20].

The input parameters needed when using the SM are:

- Height soil layer
- Young's modulus (E_{def})
- Poisson's ratio
- Dry and wet specific soil weight
- Structural strength coefficient (*m*)
- Ground water level

The value for E_{def} should be determined by a geotechnical engineer. The program uses this E_{def} together with the Poisson's ratio to calculate the E_{oed} [SCIA helpdesk] which will later be used by the settlement formula to determine the interaction parameters.

The transformation from E_{def} to E_{oed} is:

$$E_{oed} = E_{def} \frac{(1 - \upsilon)}{(1 + \upsilon)(1 - 2\upsilon)} \tag{4.4}$$

The Poisson ratio has a range of $0 < \upsilon < 0{,}5$. This value cannot be 0,5 because it will lead to an infinite E_{oed} value.

The structural strength coefficient value is 0,2 according to the EN 1997-1 §6.6.2 (Eurocode 7). In the norm it states:

> (5) The depth of the compressible soil layer to be considered when calculating settlement should depend on the size and shape of the foundation, the variation in soil stiffness with depth and the spacing of foundation elements.
>
> (6) This depth may normally be taken as the depth at which the effective vertical stress due to the foundation load is 20 % of the effective overburden stress.

[SCIA helpdesk] [19]

In the CSN731001 a table is given for different *m* values. The range in the CSN norms is $0{,}1 \leq m \leq 0{,}5$. The *m* value is set by default to 0,2 in "*SCIA engineering 2012*" and cannot be changed while using the Eurocode norms. Only by changing to the Czech norms can the *m* value of 0,2 be changed. In appendix C the additional information is given how to determine the input parameters for the SM.

The SM transfers the soil data into C_1 and C_2. This process also takes into account the spreading of the subsoil stiffness (non-uniform coefficients under the foundation) and the different soil layers under the foundation. This process makes use of an additional plate around the foundation. This additional plate is 10m long in all directions outside the foundation perimeter as can be seen in Figure 4-5

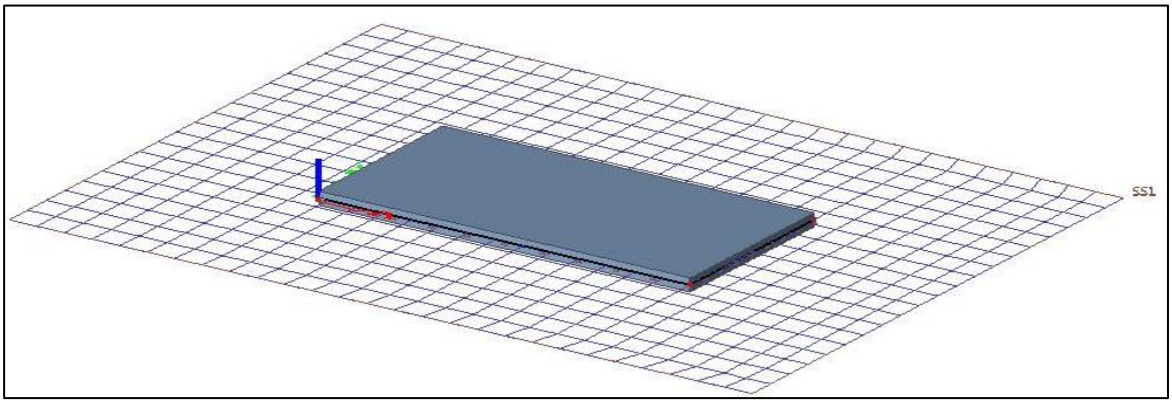

Figure 4-5 Additional plate around the foundation slab

This procedure allows taking into account the interaction of soil and structure at the edge of the foundation. The interaction causes the soil to settle less at the edges due to the redistribution of loads as a result of the C_2 interaction parameter.

4.5. Theory of the Secant Method

These C interaction parameters were formulated by V.Kolar and I.Nemec. [16]. They proposed a model which can be practically used by the structural engineer. According to them the two-dimensional model is the simplest one, considering the subsoil properties into the surface where it is connected with the structure. The model must however be based on proven mechanics theorems for example the principle of virtual work. With this idea in mind they developed the subsoil model [16]. Their subsoil model is away to take the structure and soil interaction into account. After the interaction parameters are calculated the subsoil itself is a "black box" for the structure designer. To get a quick design this method is very useful. Afterwards the bearing capacity can be checked by a geotechnical engineer. In chapter 1 of the book "*Modelling of Soil-Structure interaction*" [16] the reduction of the three-dimensional model to the two-dimensional mode is given. More information can also be found in the scientific publication of [15].

A reduction of the three-dimensional soil model to the two dimensional soil model the following functions are introduced: [16]

$$\boldsymbol{u}(x,y,z) = \sum_{i=1}^{n} \boldsymbol{u}_i(x,y) g_i(x,y,z), \tag{4.5}$$

$$\boldsymbol{v}(x,y,z) = \sum_{i=1}^{n} \boldsymbol{v}_i(x,y) h_i(x,y,z), \tag{4.6}$$

$$\boldsymbol{w}(x,y,z) = \sum_{i=1}^{n} \boldsymbol{w}_i(x,y) f_i(x,y,z), \tag{4.7}$$

Where g_i, h_i, f_i, i = 1, 2… n are selected functions and u_i(x,y), v_i(x,y), w_i(x,y) are unknown functions of two variables x and y. The functions g_i, h_i, f_i determine the course of displacement components along the variable z, i.e. under the subsoil surface [16].
The special case arises when the horizontal displacement components u, v of the soil mass have practically no influence on the amount of energy of internal forces in the subsoil. This special case gives the following function which takes into account the settlements:

$$f(z) = \frac{w(x,y,z)}{w_0(x,y)} \tag{4.8}$$

For this chosen function only two conditions need to be fulfilled $f(0) = 1$ and $f(H) = 0$, which $H \rightarrow \infty$, modelling a half space. Mostly the depth H will be finite. This function is called ***the damping function*** [20]. w_0(x,y) is the settlement of the ground surface and w(x,y,z) is the settlement in a certain soil depth.

The settlement formula which is used is:

$$w(z) = \sum_i \frac{\sigma_{z,i} - m_i \sigma_{or,i}}{E_{oed,i}} h_{soil,i} \qquad (4.9)$$

$\sigma_{z,i}$ = stress in the subsoil due to an external load
$\sigma_{or,i}$ = effective stress
E_{oed} = young modulus Oedometer
m_i = structural strength coefficient
$h_{soil,i}$ = height of the subsoil layer
w = settlement

The settlement in SCIA is not calculated by Hooke's law, but with a non-linear physical stress-strain relationship [21] [15] [12] as indicated in Figure 4-6. The stress variation in the soil, for elastic and homogeneous subsoil, is described by the influence function of Boussinesq.

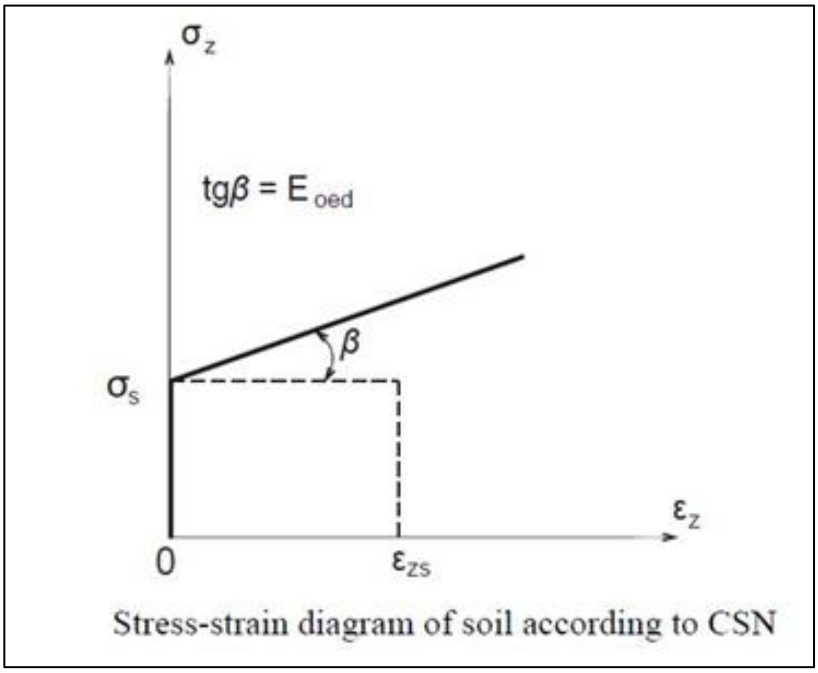

Figure 4-6 Stress-strain diagram of soil which is used in the SCIA [20]

The damping function is a very important assumption for the transformation from three to two dimensional spaces. By making use of the potential energy of the internal forces of the 3D model [15] [16]:

$$\pi_{3D}^{i} = \frac{1}{2}\int_V \underline{\sigma}^T \underline{\varepsilon} dV = \frac{1}{2}\int_V \underline{\varepsilon}^T \underline{D} \underline{\varepsilon} dV \qquad (4.10)$$

By neglecting the horizontal components as indicated in the special case:

$$\sigma = [\sigma_z, \tau_{zx,} \tau_{yz}]^T = \boldsymbol{D\varepsilon} \quad (4.11)$$

$$\boldsymbol{\varepsilon} = [\varepsilon_{z,} \gamma_{zx}, \gamma_{yz}]^T = \left[\frac{\partial w}{\partial z}; \frac{\partial w}{\partial x}; \frac{\partial w}{\partial y}\right]^T \quad (4.12)$$

The matrix of physical constants ***D***

$$\boldsymbol{D} = \begin{bmatrix} E & 0 & 0 \\ 0 & G & 0 \\ 0 & 0 & G \end{bmatrix} \quad (4.13)$$

With the help of the damping function the problem can now be transformed into a 2D model. The formula for the potential energy of body V=ΩH. Ω is the extent of the 2D model. ***H*** is the depth of the deformed zone of the 3D model.

$$\pi_{2D}^i = \pi_{3D}^i = \frac{1}{2} \int_V [\sigma_z \varepsilon_z + \tau_{zx} \gamma_{zx} + \tau_{yz} \gamma_{yz}] dV \quad (4.14)$$

$$\pi_{2D}^i = \pi_{3D}^i = \frac{1}{2} \int_\Omega \left[w_0^2 \int_0^H E_z \left(\frac{\partial f}{\partial z}\right)^2 dz + \left(\frac{\partial w_0}{\partial x}\right)^2 \int_0^H f^2 G dz + \left(\frac{\partial w_0}{\partial y}\right)^2 \int_0^H f^2 G dz \right] d\Omega \quad (4.15)$$

Integrating over the z-axes we get the formula for the potential energy of internal forces of the 2D model with two parameters C_1 and C_2:

$$\pi_{2D}^i = \frac{1}{2} \int \int_\Omega [C_1 w_0^2(x,y) + C_{2x} \left(\frac{\partial w_0(x,y)}{\partial x}\right)^2 + C_{2y} \left(\frac{\partial w_0(x,y)}{\partial y}\right)^2] d\Omega \quad (4.16)$$

This is the derivation process for getting from the general 3D to the surface 2D model. In this process we get the interaction parameters:

$$C_1 = \int_0^H E_z \left(\frac{\partial f(z)}{\partial z}\right)^2 dz \quad (4.17) \qquad C_{2x} = C_{2y} = \int_0^H G f^2(z) dz \quad (4.18)$$

A visualization of the C_1 parameter is given below in Figure 4-7. C_1 parameter is the resistance of the spring working in the vertical direction. The Δz1 is the displacement of the spring due to the point load in the z direction (F_z).

$$F_z = C_1 * \Delta z1 \quad (4.19)$$

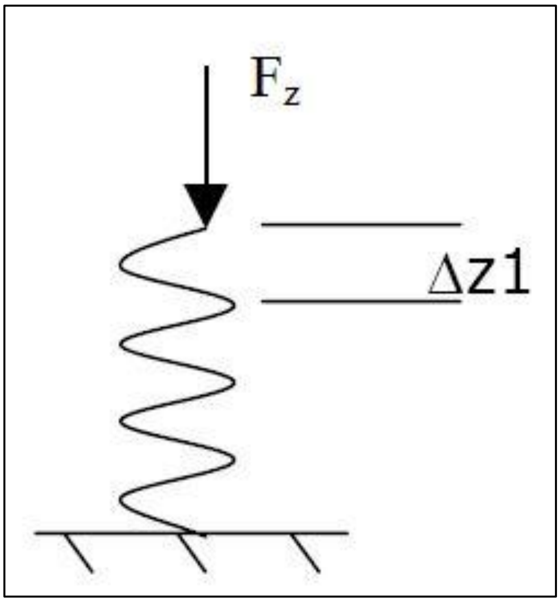

Figure 4-7 C_1 parameter visualization [SCIA helpdesk]

A visualization of the C_2 parameter is given in Figure 4-8. These springs represent the coupling between the vertical springs and take the shear interaction of the neighbouring spring into account. In this figure you can observe that on the right side the displacement (Δz2) is less than near the point load.

$$F_z = C_2 * \Delta z2 \quad (4.20)$$

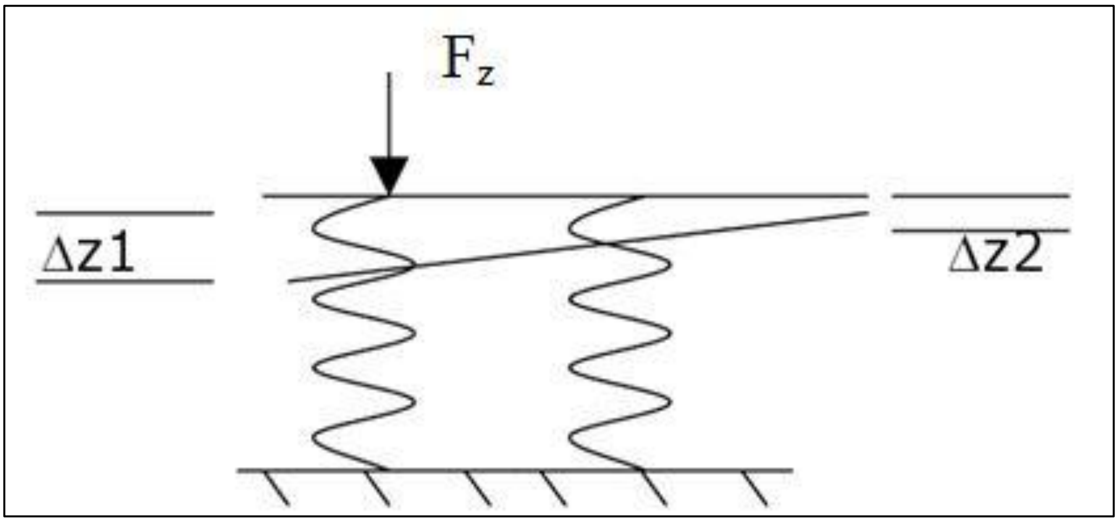

Figure 4-8 C_2 parameter visualization [SCIA helpdesk]

4.6. Settlement Comparison

The settlement calculation is important when determining the interaction parameters (C_1 and C_2). The settlement equation in SCIA is [21]:

$$w(z) = \sum_i \frac{\sigma_{z,i} - m_i \sigma_{or,i}}{E_{oed,i}} h_i \tag{4.21}$$

It will be compared to the settlement equation from the Dutch code. The Dutch code calculates the settlement with the Terzaghi formula which is [19]:

$$w(z) = \sum_{i=0}^{i=n} \frac{1}{C'_{p;i}} h_i ln \left(\frac{\sigma'_{v;z;0;d} + \Delta\sigma'_{v;z;d}}{\sigma'_{v;z;0;d}} \right) \tag{4.22}$$

Table 4-1 Input parameters:

Radius [m]	**σ_z [kN/m]**	**E_{def} [N/mm^2]**	**E_{oed} [N/mm^2]**	**C'$_p$ [-]**	**γ_{unsat} [kN/m^3]**	**γ_{sat} [kN/m^3]**	**υ_s [-]**	**m [-]**
10	100	45	60,58	600	18	20	0,3	0,2

The foundation structure is a round slab type structure with a radius of 10m. The dead load is neglected. It is loaded externally with a load of 100kN/m^2. The other input data are given in Table 4-1. The E_{def}, C'_p, γ_{unsat} and γ_{sat} are determined from EC.7 [19]. With the equation of chapter 4.4 E_{def} is transformed to E_{oed}. The Poisson ratio value is chosen to be that of a sand type soil. The *m*-factor is by default 0,2 when using the Eurocode codes in "*SCIA engineering 2012*". The h_i is the height of the soil layer. For this case layer heights of 2 meters will be used.
The stress distribution of the external load in the soil was taken into account with the assistance of the Boussinesq formula [9].

$$\sigma_{zz} = p\left(1 - \frac{z^3}{b^3}\right) \tag{4.23}$$

The *p* is the external load ($\boldsymbol{\sigma_z}$). The stress in the soil, due to the Boussinesq spreading, is indicated as $\boldsymbol{\sigma_{zz}}$. The *z* is the depth of the soil and $b = \sqrt{z^2 + R^2}$, where *R* is the radius of the circular slab.

Table 4-2 gives the settlement with a soil layer thickness of 2m which is indicated in column four. In the first column the depth of the soil is given with a height difference of 2m. In the second column the effective stresses of the soil is calculated. The third column is the external stress in the soil according to the Boussinesq theory. The last column is the settlement for each layer. The total settlement is the sum of all the layers which is **12,33mm**. This value is close to the SCIA settlement **10,8mm** (Figure 4-9). By making the layers step sizes smaller (for example 1 or 0,5 meters) the settlement comes closer to the SCIA value.

Table 4-2 SCIA settlement calculation scheme (h = 2)

z [m]	σ_{or} [kN/m^2]	σ_{zz} [kN/m^2]	*h* [m]	*w* [m]
1	18	99,90	2	0,00318
3	54	97,62	2	0,00287
5	90	91,06	2	0,00241
7	126	81,14	2	0,00185
9	162	70,06	2	0,00124
11	198	59,49	2	0,00066
13	234	50,20	2	0,00011

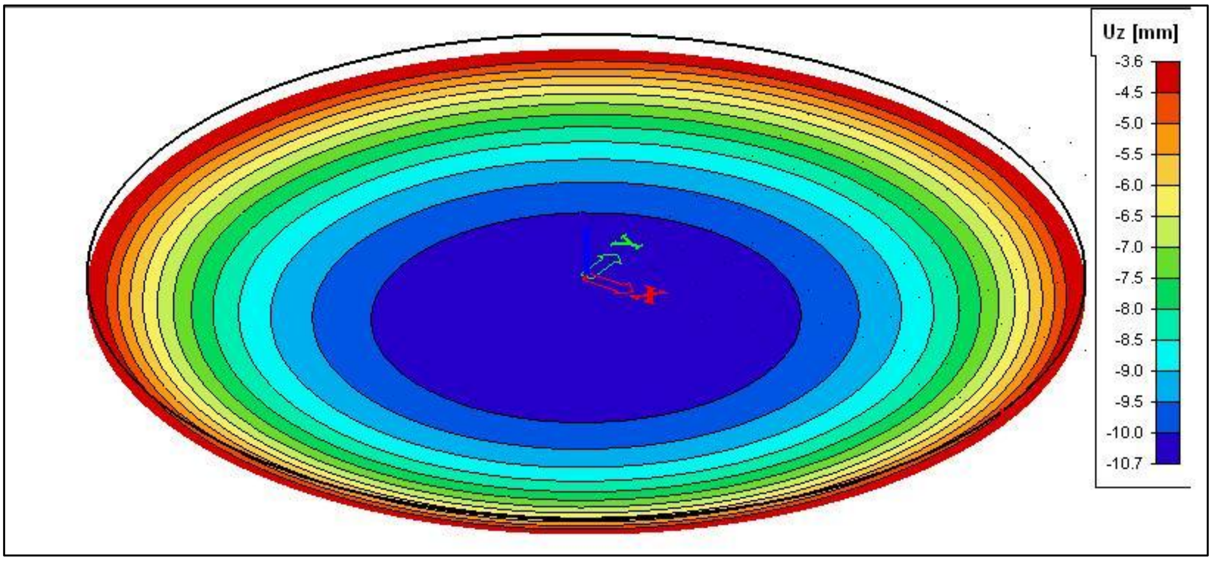

Figure 4-9 SCIA settlement

The sand layer is taken until *z* is 13m. This is done because afterwards the settlement becomes negative in the SCIA settlement calculation. The settlement calculation is based on the assumption that under the limit depth no settlement will occur.

> (5) The depth of the compressible soil layer to be considered when calculating settlement should depend on the size and shape of the foundation, the variation in soil stiffness with depth and the spacing of foundation elements.
>
> (6) This depth may normally be taken as the depth at which the effective vertical stress due to the foundation load is 20 % of the effective overburden stress.

[SCIA helpdesk] [19]

The above assumption is also implemented in the Terzaghi settlement. Besides the limit depth assumption also the stress distribution of the external load in the settlement calculation is taken into account with the Boussinesq formula.

In Table 4-3 the first four columns are as that of the SCIA settlement calculations and the last column contains the results of the settlement from the Terzaghi formula. The sum of this settlement on the same depth is **16,42mm**.

Table 4-3 Terzaghi settlement calculation scheme (h = 2)

z [m]	$\sigma'_{v,z,d}$ [kN/m^2]	$\Delta\sigma'_{v,z,d}$ [kN/m^2]	*h* [m]	*w* [m]
1	18	99,90	2	0,00626
3	54	97,62	2	0,00344
5	90	91,06	2	0,00233
7	126	81,14	2	0,00166
9	162	70,06	2	0,00120
11	198	59,49	2	0,00088
13	234	50,20	2	0,00065

The difference between these two settlement formulations, on the same depth, is 25%. What can be concluded is that the SCIA formula gives an underestimation of the settlement compared to the Terzaghi formula. The stress distribution of both models is taken into account through the Boussinesq theory. If we relate the settlements to the modulus of sub-grade reaction the SCIA settlement calculations will give higher *k* values ($k=p/w$). The settlement calculation of SCIA does not take into account the time depending factor. The Terzaghi can take this factor into account. This can be observed in art. 6.6.2 of the Eurocode 7 norm.

In the settlement comparison it is revealed that the *m*-factor has a significant influence on the settlement. The *m* has different values for different soil types. The influence the *m* has on the displacement of a slab foundation with the same input of Table 4-1 can be seen in Table 4-4.

Table 4-4 Displacement for different m factors

Type	m [-]	Uz [mm]
-	0	24,6
Fine Grained Soils	0,1	14,6
Sands	0,2	10,7
Sands, Gravel	0,3	8,3
-	0,4	6,8
Leem	0,5	5,7

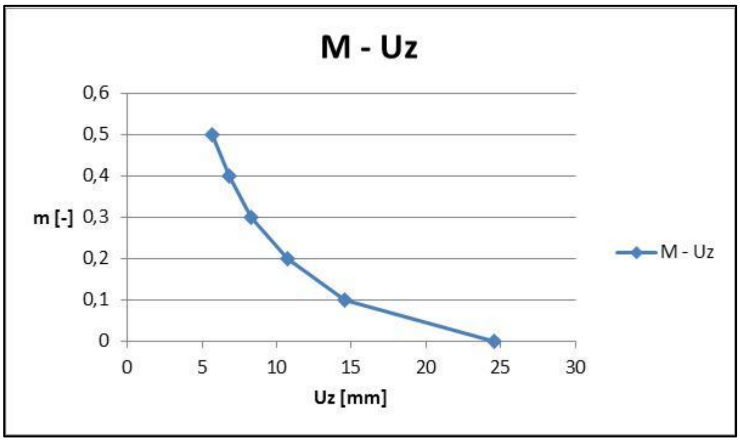

Figure 4-10 Displacement against different *m* values

It can be observed in Figure 4-10 that if the *m* increases the displacement (Uz) decreases. This should be taken into account when using the Secant Method.

In the next chapter the uniform and non-uniform coefficients will be implemented. The methods discussed in this chapter will be worked out with examples. This will give more practical insight in the working mechanisms of the different spring models.

5. Comparison of Spring Analyses

5.1. Introduction

In the previous chapter different foundation models were discussed. The two main groups were the uniform and non-uniform coefficients (Eurocode 7, Pseudo-Coupled and Secant method). In this chapter the influence and variation of these coefficients will be analysed. The non-uniform coefficients can be put in manually or they can be determined by making use of the SM. The non-uniform coefficients study will be done by comparing the Eurocode 7 and Pseudo-Coupled model to the SM. In this case the conditions will be kept similar to be able to compare the result. The results which will be compared to one another are settlement, moments and contact stress.
In the end of this chapter a sensitivity analysis of the second parameter C_2 will be done. By doing this, more insight can be gained about this parameter.

5.2. Influence of the Interaction Parameters

5.2.1. Input Data:

In this case the differences between uniform and non-uniform coefficients are researched. The approach is to observe the results for the displacement and moments for square slabs. This is done by, in one case, first varying the thickness and in another case the load.

The input parameters are:

- The slab foundation which is modelled with a 2D plate element.
  The variations of the slabs are 5 by 5 meters, 10 by 10 meters, 20 by 20 meters and 40 by 40 meters. The thickness varies from 250, 500, 750 and 1000 millimetres.
- Concrete class C35/45; Young's modulus concrete (E_c) = 34100N/mm^2
- A uniform surface load with a variation of 25, 50, 75, 100kN/m^2 is taken.
  The dead load is fixed at zero.
- The modulus of sub-grade reaction for subsoil is 10MN/m^3. This value is an often-used value in practice when making a preliminary foundation design. In appendix D a few tables from the literature are represented to determine the modulus of sub-grade reaction. Depending on the dimension of the structure, type of load and the soil properties a designer can choose to use an applicable value from one of the tables. It is advice to read through the reference to which these tables belong to. There the applicability of the *k* values can be stated.

5.2.2. Uniform Interaction Parameters

In following case a uniform Winkler foundation model will be simulated. This will be done by making $C_1 \neq 0$ and the $C_2 = 0$. For different slab dimensions and loads the stiffness of the slab is researched. This is done by varying the thickness which has influence on the moment of inertia so also on the bending stiffness (*EI*) of the slab. The C_1 value is uniform under the whole slab foundation.

An example of the displacement and moment result of one of the cases can be seen in Figure 5-1 and Figure 5-2 for a 10by10by0,75 meter slab with a load of 25kN/m^2. In Figure 5-1 the displacement is uniform for a uniform coefficient approach. This was also the case for the other models with a different thickness with a uniform load.

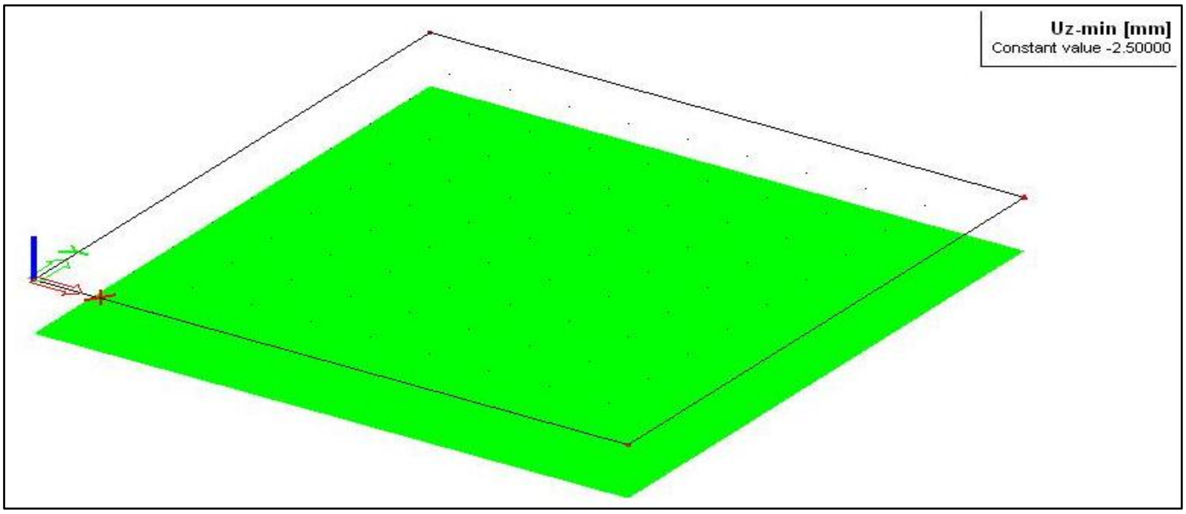

Figure 5-1 Uniform displacement using a uniform coefficient

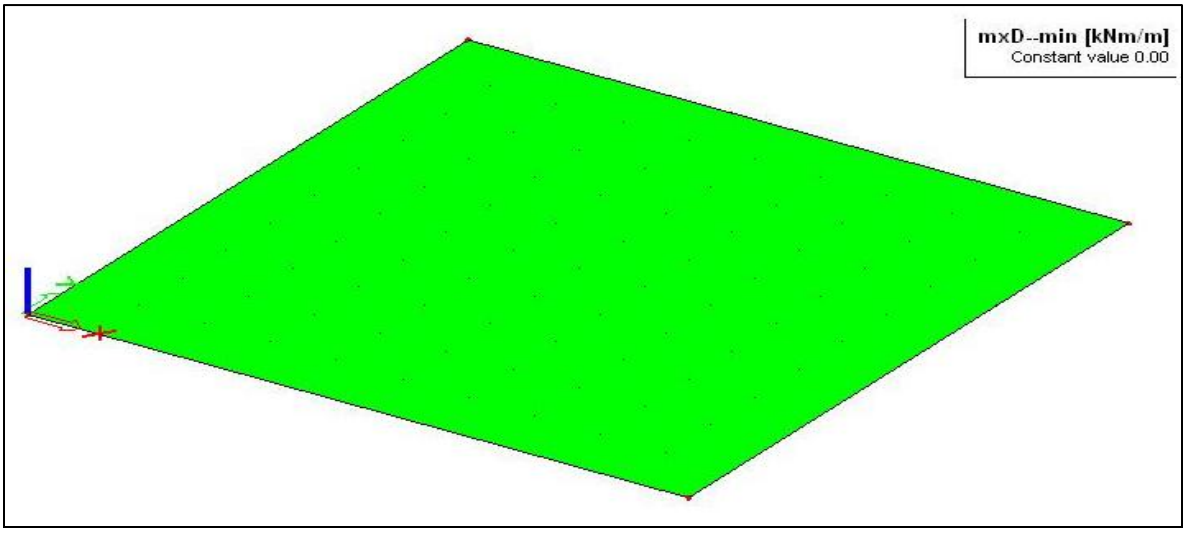

Figure 5-2 Moment using a uniform coefficient

This could be expected by the fact that the springs do not interact with each other. For this same slab the moment is also zero as indicated in Figure 5-2. The result in this case proved that the variation in thickness did not influence the displacement and moments.

5.2.3. Non-Uniform Interaction Parameters

For the non-uniform coefficients the SM procedure will be used, so both interaction parameters will be used. By using both interaction parameters a non-uniform Pasternak foundation model can be simulated. This model also includes the surrounding soil as programed in the module.

The input parameters are:

- For the slab and load the same input is used as the uniform Winkler foundation case.
- The following borehole values are:
  height soil layer = 3*width of the structure [22]
  The soil type that was chosen is of the type sand (strong and claylike)
  E_{def} = 15 N/mm^2 [19]
  υ = 0,3
  γ_{unsat} = 18 kN/m^3 [19]
  γ_{sat} = 19 kN/m^3 [19]
  The ground water level is not present in the model
  Structural strength coefficient (m) = 0,2

What could be observed in the results of this model is that the displacement is not uniform and moment not equal zero. This can be explained by the fact that the coupling of the springs are taken into account as well as the surrounding soil. The results can be observed in Figure 5-3 and Figure 5-4 for a 10by10by0,75 meter slab with a load of 25kN/m^2. It can be seen that the edges of the slab displace less than the middle of the slab. This model comes closer to reality.

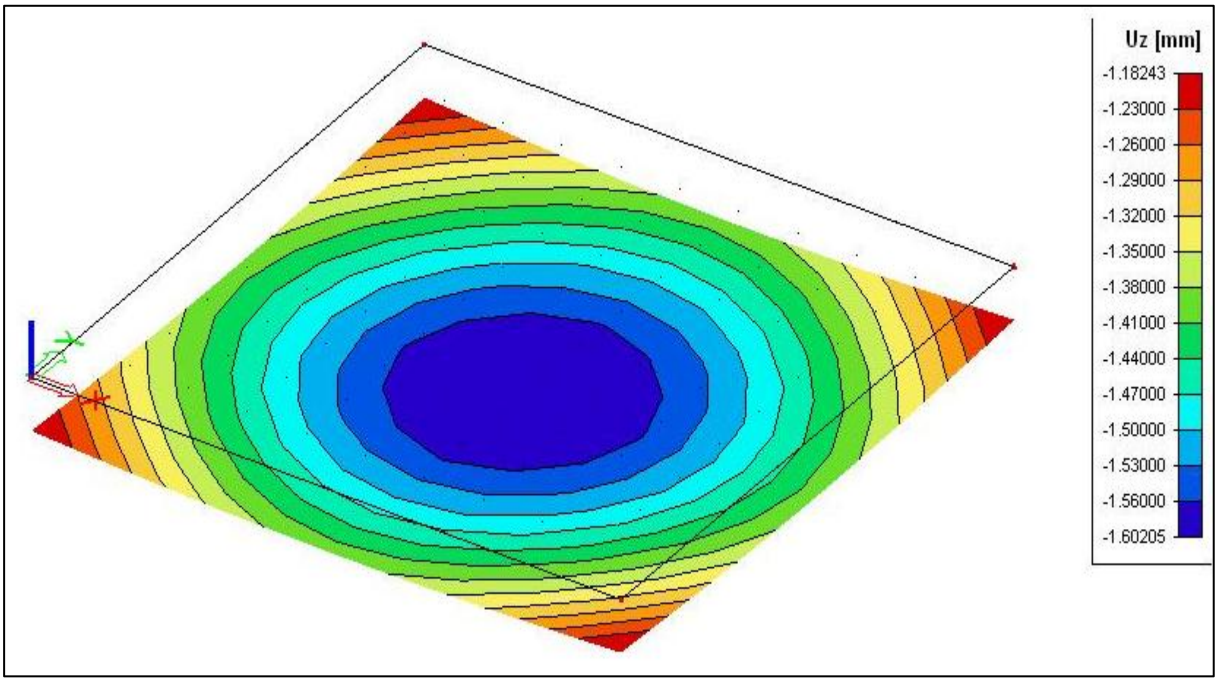

Figure 5-3 Non uniform displacement for non-uniform coefficients

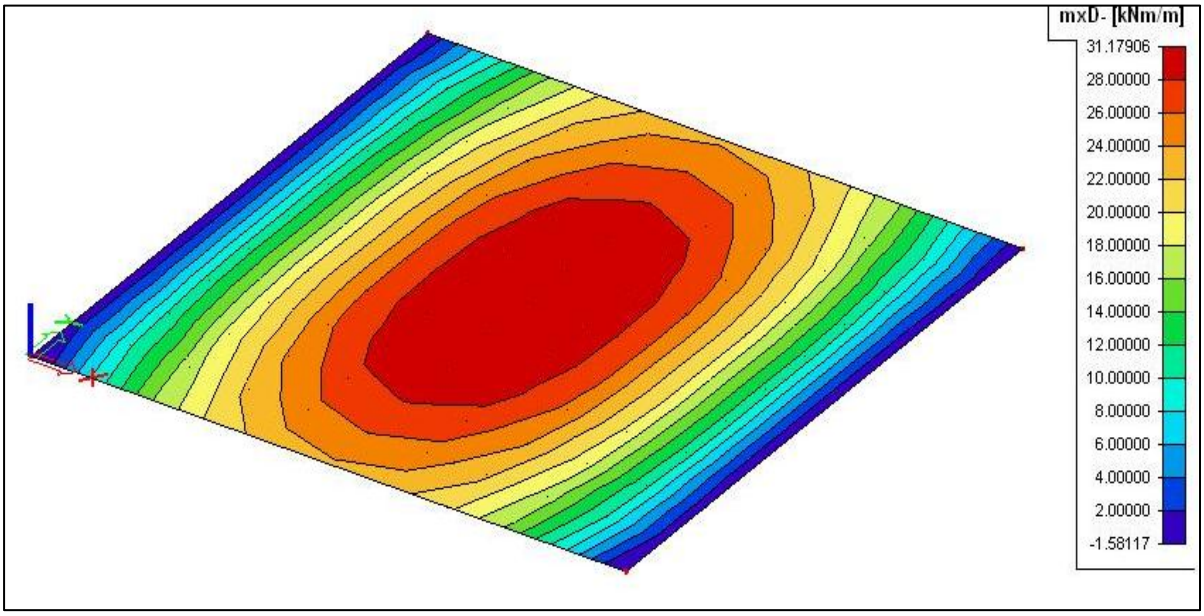

Figure 5-4 The moment using of non-uniform coefficients

The displacement of the slab is influence by the thickness. This can be seen in the Figure 5-5. The Figure gives the results of the maximum displacement – thickness relation for a load of 100kN/m^2. For the other models the same trend could be observed. The thicker the slab got the less it displaced due to the same load.

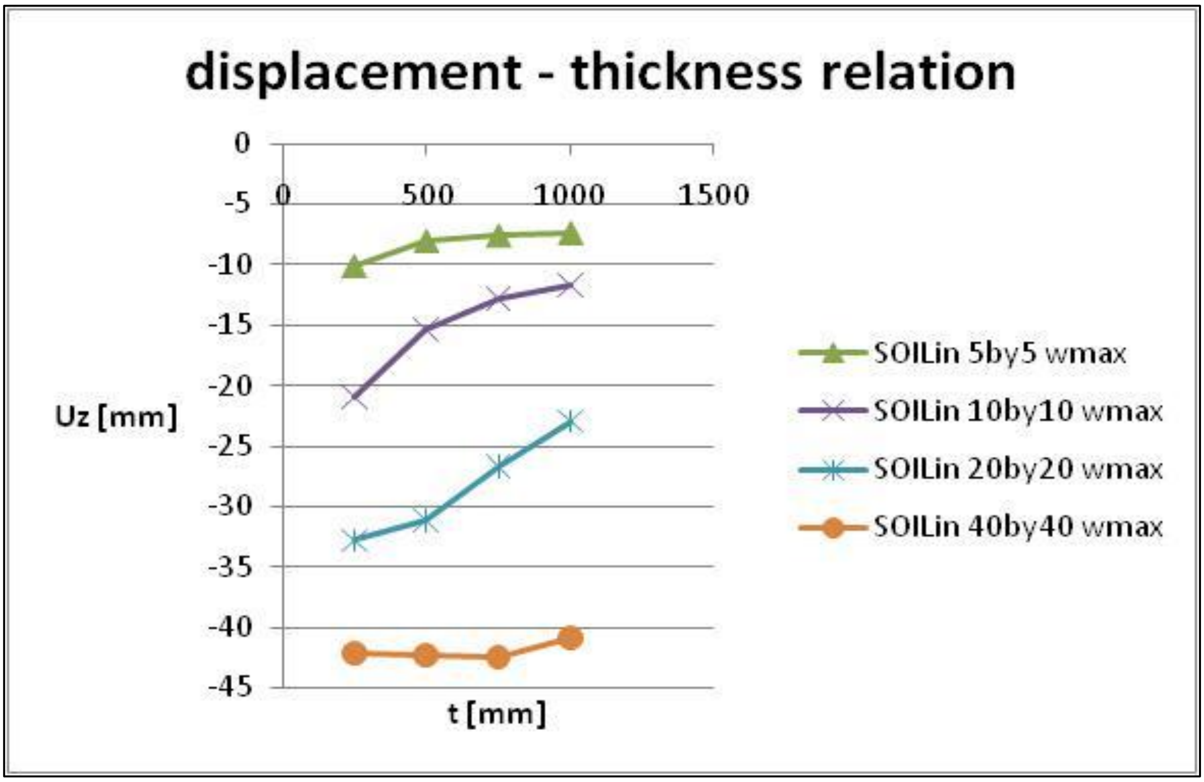

Figure 5-5 Maximum displacement – thickness relation

What is also interesting to look at in SM is the spreading of the interaction parameters under a foundation slab. This spreading of the stiffness makes it possible for the model to be more realistic. The following two Figure 5-6 and Figure 5-7 give the distribution of the C_1 parameter under a 10 by 10 slab with a thickness of 0,25 and 1 meter due to a uniform load of 100kN/m^2. In Figure 5-8 and Figure 5-9 the distribution of the C_2 parameter can be seen for the same slabs.

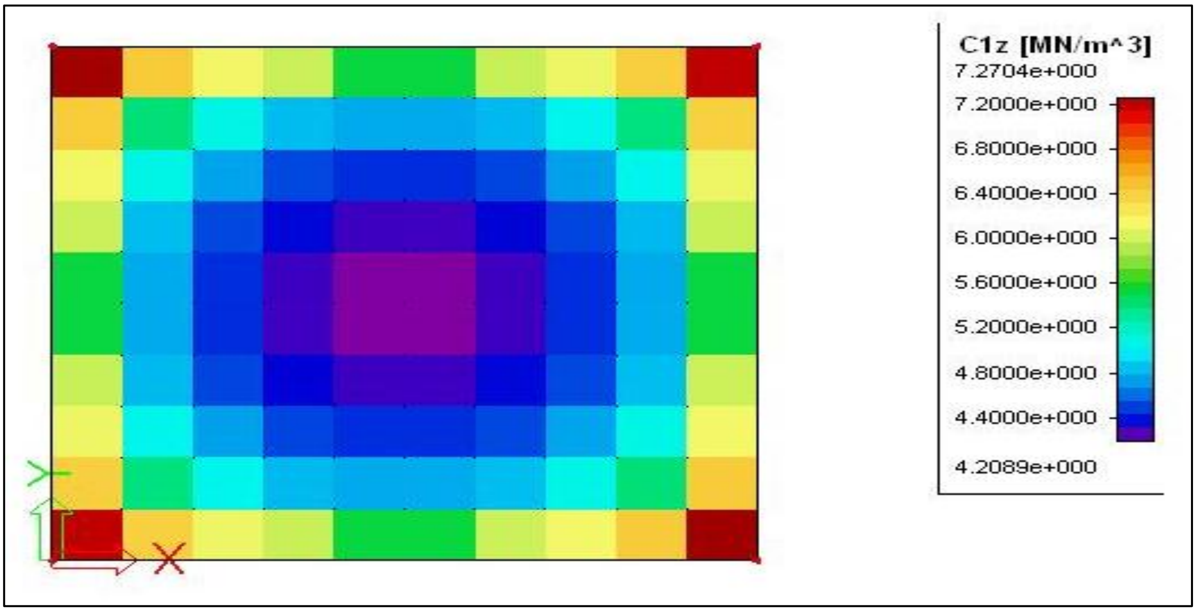

Figure 5-6 Distribution C_1 thickness 0,25 meter

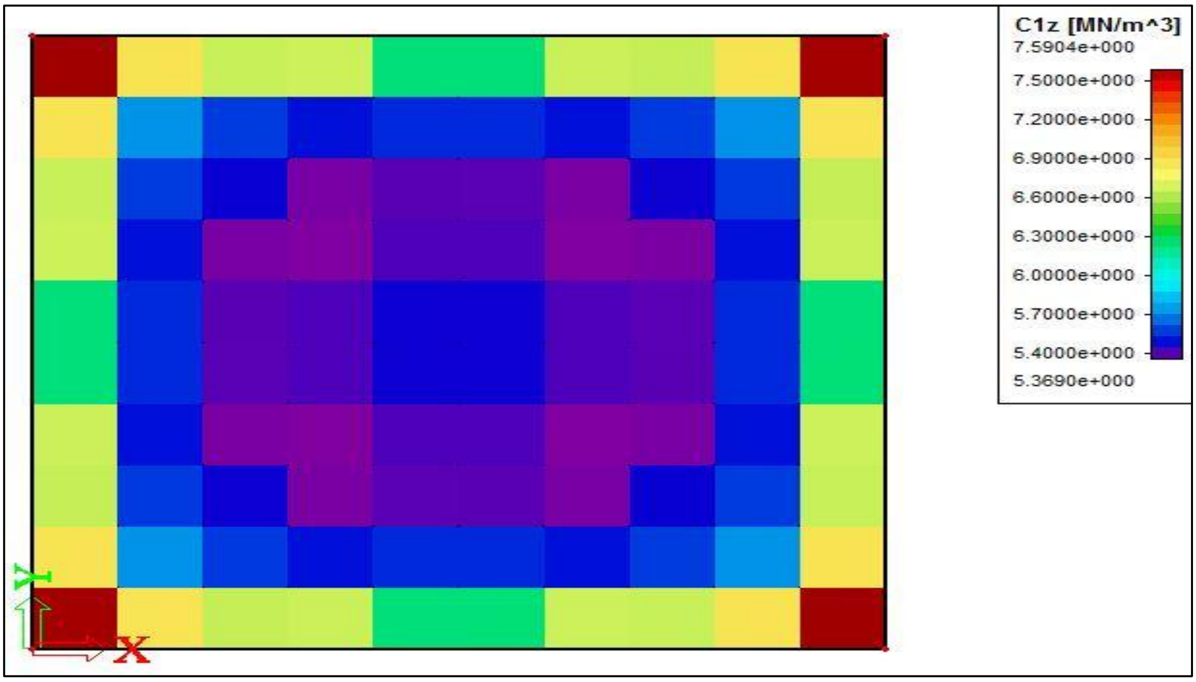

Figure 5-7 Distribution C_1 thickness 1 meter

In the C_1 distribution figures we can see that the corners of the slabs have the maximum values and in the middle of the slab the minimum. This also occurs regularly for the different slabs with different dimensions. It can also be observed is that for a thicker slabs the area of the C_1 minimum becomes bigger in diameter.

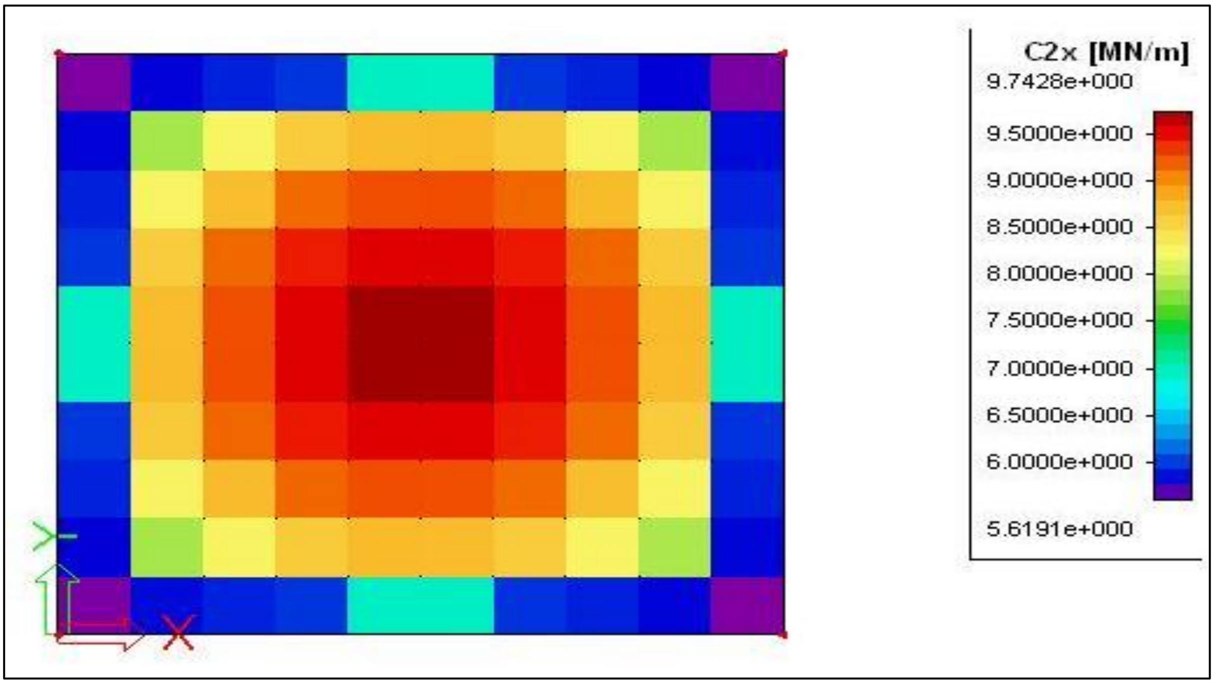

Figure 5-8 Distribution C_2 thickness 0,25 meter

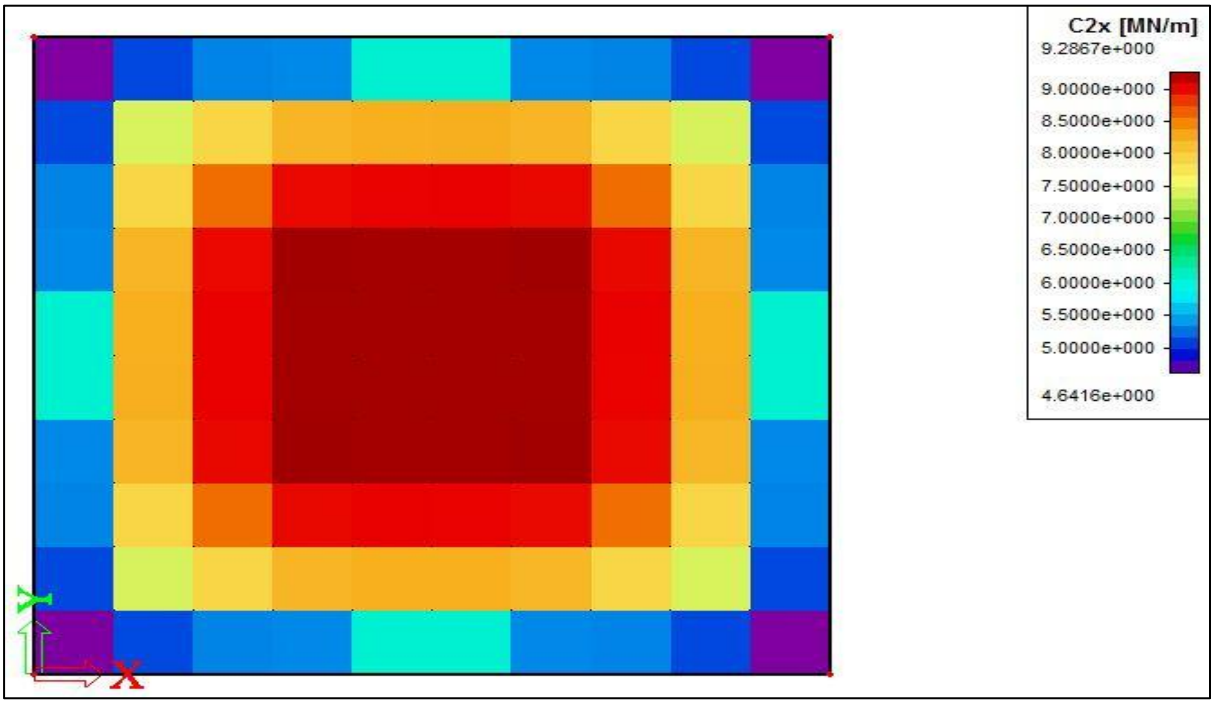

Figure 5-9 Distribution C_2 thickness 1 meter

For the C_2 values the maximum is in the middle of the slab and the minimum values in the corner. This can also be observed in almost all the other models. The spreading for a thin and thick slab also influences the C_2 parameter. That can be clearly seen in the middle were the red area is much larger and darker for the thick slab. In Table 5-1 the maximum values of all the different models with different loads can be observed. The most interesting models are the 10 by 10 and 20 by 20 models because these are the often used dimensions for tunnels. What can be observed in the table is that in those two dimensions the C_1 varies between 6 and 20MN/m^3 and the C_2 between 5 and 15MN/m.

Table 5-1 The maximum values of C_1 and C_2 for (load 25, 50, 75 and 100kN/m^2)

SOILin	Load 25kN/m^2		Load 50kN/m^2		Load 75kN/m^2		Load 100kN/m^2	
(L/W/t) [m]	C_1 [MN/m^3]	C_2 [MN/m]	C_1 [MN/m^3]	C_2 [MN/m]	C_1 [MN/m^3]	C_2 [MN/m]	C_1 [MN/m^3]	C_2 [MN/m]
5/5/0,25	16,89	4,244	11,12	5,758	8,90	6,730	7,74	7,356
5/5/0,50	17,56	4,177	11,68	5,953	9,47	7,038	8,30	7,781
5/5/0,75	18,84	4,062	12,27	5,877	9,89	6,982	8,64	7,743
5/5/1	20,23	3,941	12,89	5,764	10,31	6,865	8,98	7,628
10/10/0,25	18,43	5,684	11,28	7,727	8,47	8,955	7,27	9,743
10/10/0,50	16,38	5,150	10,37	7,310	8,27	8,571	7,21	9,537
10/10/0,75	17,04	5,069	10,28	6,950	8,44	8,270	7,39	9,399
10/10/1	18,20	4,950	11	6,956	8,70	8,155	7,59	9,287
20/20/0,25	16,08	7,896	10,69	11,740	7,56	14,290	6,23	15,901
20/20/0,50	15,89	7,700	10,38	11,330	7,41	13,686	6,29	15,275
20/20/0,75	16,56	7,320	9,55	10,830	7,05	13,050	6,16	14,832
20/20/1	16,73	6,850	8,87	10,267	6,86	12,390	6,01	12,434
40/40/0,25	66,01	10,749	9,06	15,709	6,27	19,988	8,73	23,630
40/40/0,50	33,28	10,508	9,61	15,596	6,82	19,896	5,37	23,261
40/40/0,75	35,08	10,268	9,97	15,454	7,02	19,639	5,42	22,865
40/40/1	23,20	10,027	10,09	15,212	7,09	19,223	5,36	22,416

5.2.4. Conclusions on the Influence of the Interaction Parameters

The following conclusion can be made on the basis of the studies done above. The variation in thickness does not influence the displacement of a uniform Winkler foundation model for a uniform distributed load. The displacement is directly coupled to the load and the modulus of sub-grade reaction ($w=p/k$). The displacement stays uniform and has the same maximum value. The results of the moments were also zero. This can be concluded because there is no bending, thus zero curvature, in the structure.

When both interaction parameters are activated, and distributed non-uniformly, different maximum displacements for different slab thickness at the same load occur. Also if the thickness increases the displacements decreases and the moments in slab increases. This can be explained through the bending stiffness. The bending stiffness gets larger if the thickness gets larger. This is due to the moment of inertia ($1/12bt^3$). Also the moments will get larger due to the bending stiffness ($m=EI\kappa$).

SM makes it possible to distribute the interaction parameters non-uniformly under the foundation slab. What can be observed in almost all the models is that in the corners the C_1 value is maximum and in the middle minimum. For the C_2 value the maximum and minimum are opposite to C_1. In all the different cases no relationship is found for the interaction parameters.

In the next section the non-uniform coefficient foundation models will be worked out further. Here the comparison will be made between manually and computational determining the non-uniform coefficients.

5.3. Non-Uniform Coefficient Foundation Models

5.3.1. Introduction

By spreading the stiffness of the modules of sub-grade reaction we can improve the Winkler foundation model. This can be done by making use of non-uniform coefficients. The idea of these non-uniform coefficients is to take stiffer k or C_1 value at the edge and corner of the foundation and a les stiffer value in the middle. The Eurocode.7 and the SCIA give guidelines how to apply these approaches (section 4.3.).

In this case the average C_1 value for the different models is determined with the help of the SM. A one slab element with a borehole and homogeneous geological profile is used to determine the non-uniform coefficients. Through the maximum and minimum from C_1 value the average value has been calculated. This determined value is used for the different non-uniform models. In the end of the case the different models will be compared to the SM model. The maximum and minimum displacement, moment and contact stress will be compared.

Input parameters for the slab, borehole and geological profile:

- Dimensions for all models are L= 20m; W = 10m; t = 1m.
- Concrete class C35/45; Young's modulus concrete (E_c) = 34100N/mm^2
- The distributed load = 100kN/m^2.
  The dead load is zero.
- The soil input parameters are:
  Height soil layer = 40m
  The soil type that was chosen is of the type sand (strong and claylike)
  E_{def} = 15 N/mm^2
  υ = 0,3
  γ_{unsat} = 18 kN/m^3
  γ_{sat} = 19 kN/m^3
  The ground water level is not present in the model
  Structural strength coefficient (m) = 0,2

The average C_1 value which is calculated from the SM is 6,085MN/m^3. In Figure 5-10 the maximum (7,23MN/m^3) and minimum (4,94MN/m^3) value of the C_1 can be observed for a SM model. This average value is used for the ***Eurocode 7*** and ***Pseudo-coupled*** models.

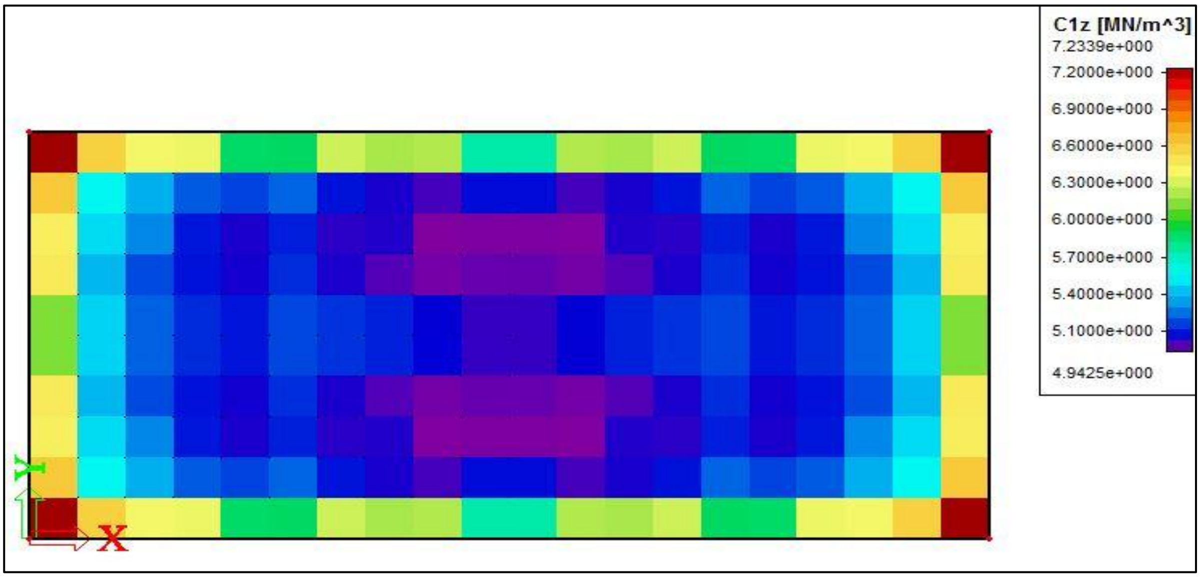

Figure 5-10 Maximum and minimum value for C_1 for a SM model

5.3.2. Eurocode 7 Model

The Eurocode 7 describes a way to model the interaction of structure and soil with the help of stresses acting on the structure. This method can be used without a modulus of sub-grade reaction, but in practice it is difficult to indicate in the design how the load will distribute over the foundation. This is the reason that in this case study the same method is used but for different modulus of sub-grade reactions under the slab instead of stresses on the slab.
Figure 5-11 gives the way the way the *k* value is distributed under the slab.

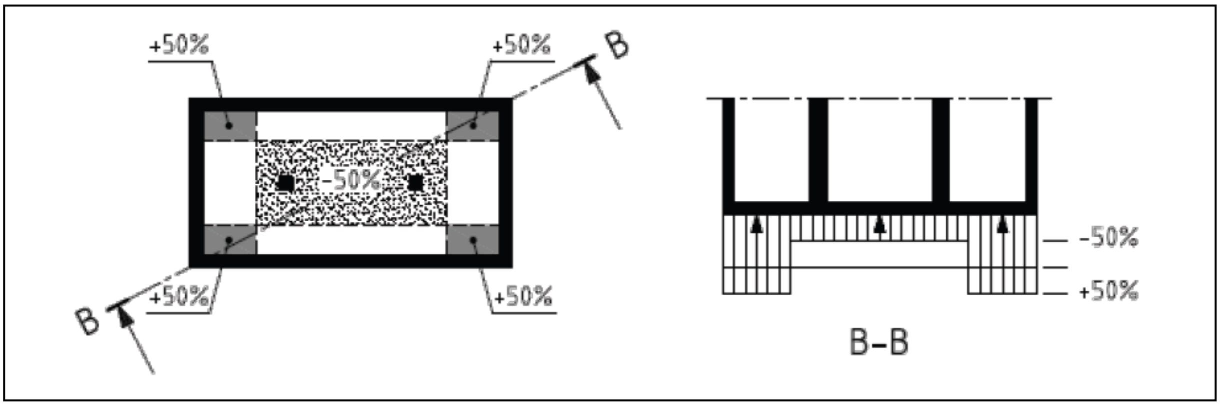

Figure 5-11 Distribution *k* value [19]

The average stiffness is 6,085 MN/m^3. In the corners the stiffness is 9,1275MN/m^3 (150%*average value) and in the middle 3,0425 MN/m^3 (50%*average). The result of the displacement, moment and contact stresses can be seen in Figure 5-12, Figure 5-13 and Figure 5-14. The lateral view is made with a section line from the middle of the y-direction.

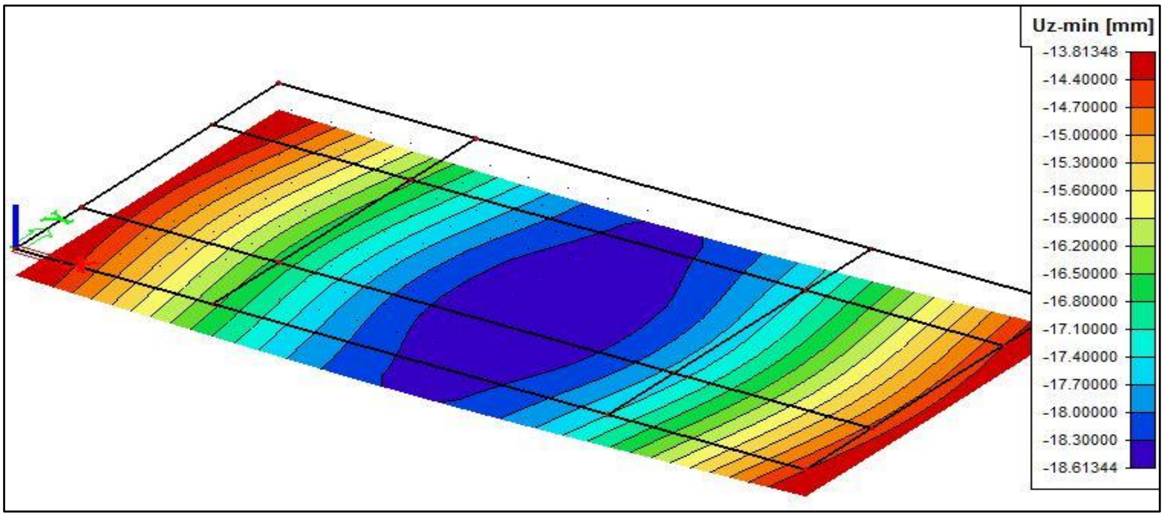

Figure 5-12 Displacement of the Eurocode 7 model

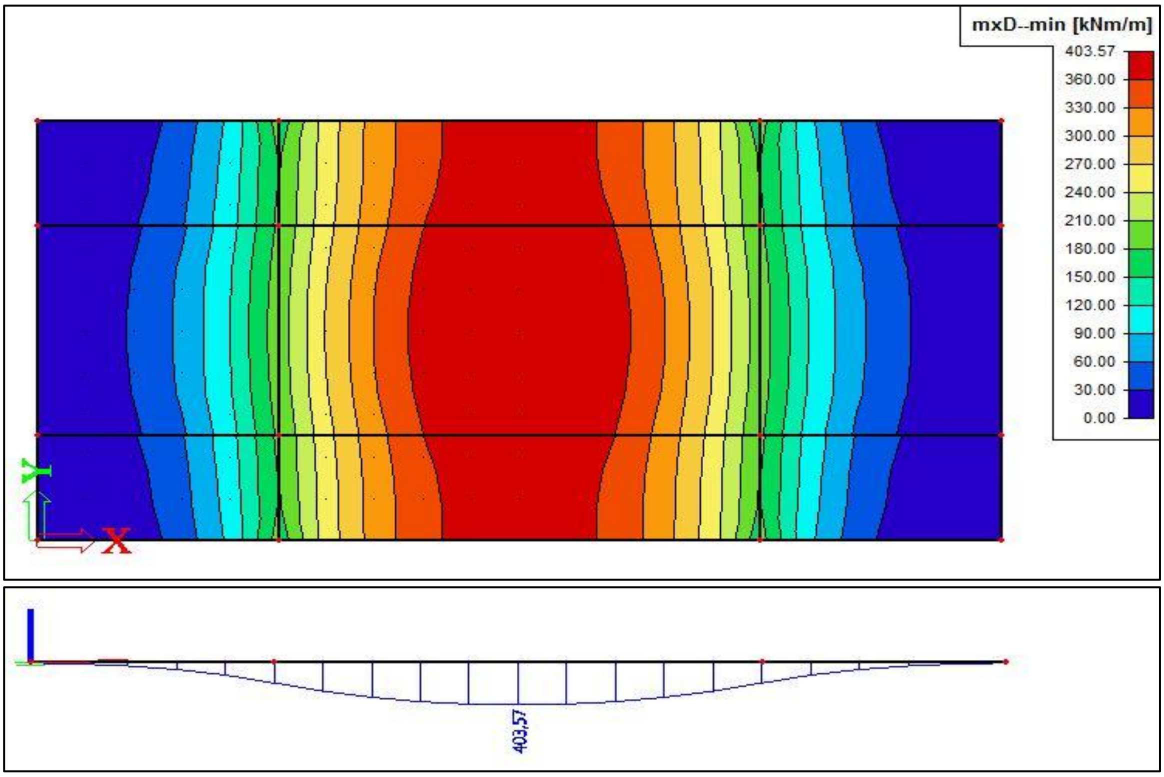

Figure 5-13 Moments top and lateral view of the Eurocode 7 model

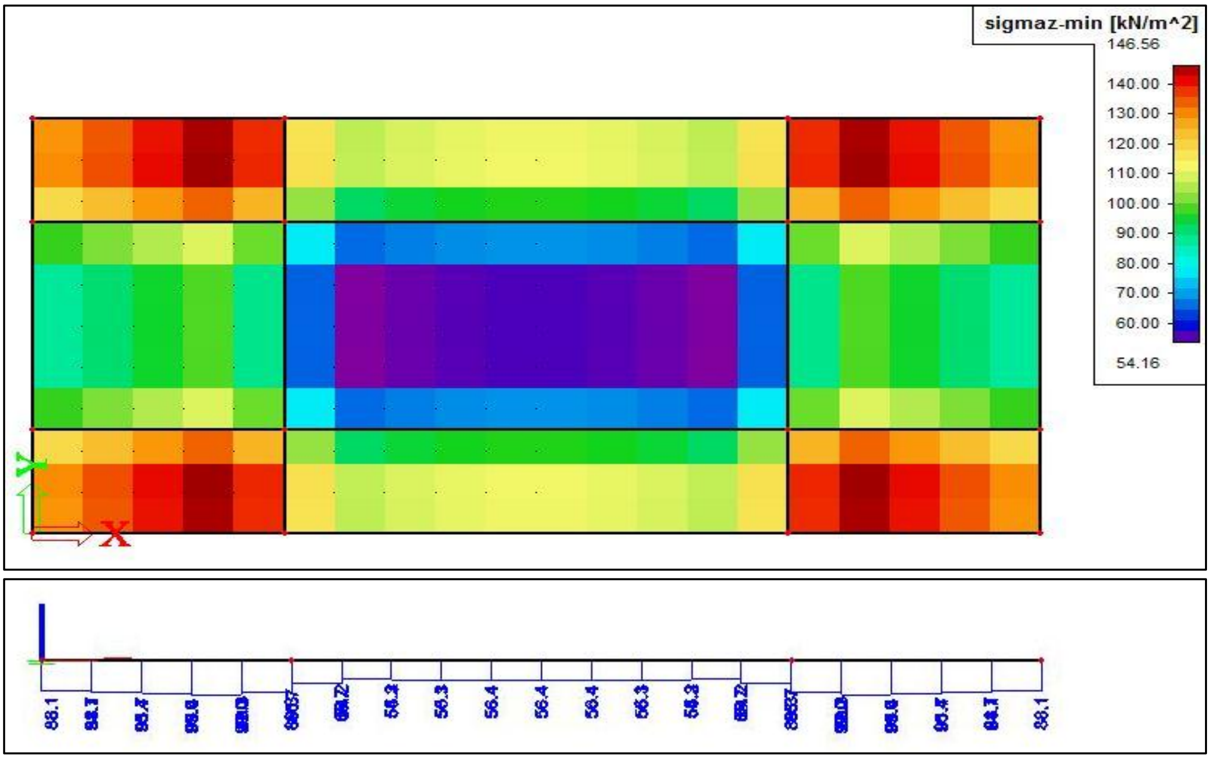

Figure 5-14 Contact stresses top and lateral view of the Eurocode 7 model

The contact stresses in Figure 5-14 are also accordance to the Eurocode. The average stress is around 100kN/m^2 and the corners are 150% larger and the centre of the slab is 50% smaller.

5.3.3. Pseudo-Coupled Model

The Pseudo-Coupled model uses the terms k_s, k_A, k_B and k_C to spread the soil stiffness underneath the foundation. The k_s in this case is the C_1 parameter which is 6,085 which is used for the earlier method (section 5.3.2.). By using the spreading distribution (section 4.3.3.) the following values are found: k_A is 3,818 MN/m^3, k_B is 5,727 MN/m^3, k_C is 7,636MN/m^3. The result of the displacement, moment and contact stresses can be seen in Figure 5-15, Figure 5-16 and Figure 5-17.

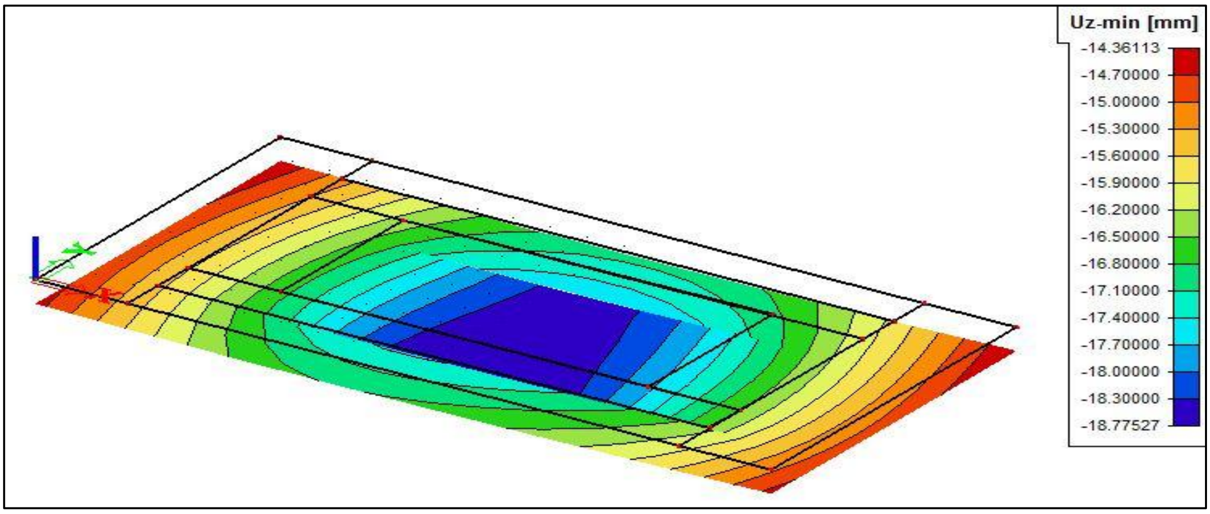

Figure 5-15 Displacement of the Pseudo-Coupled model

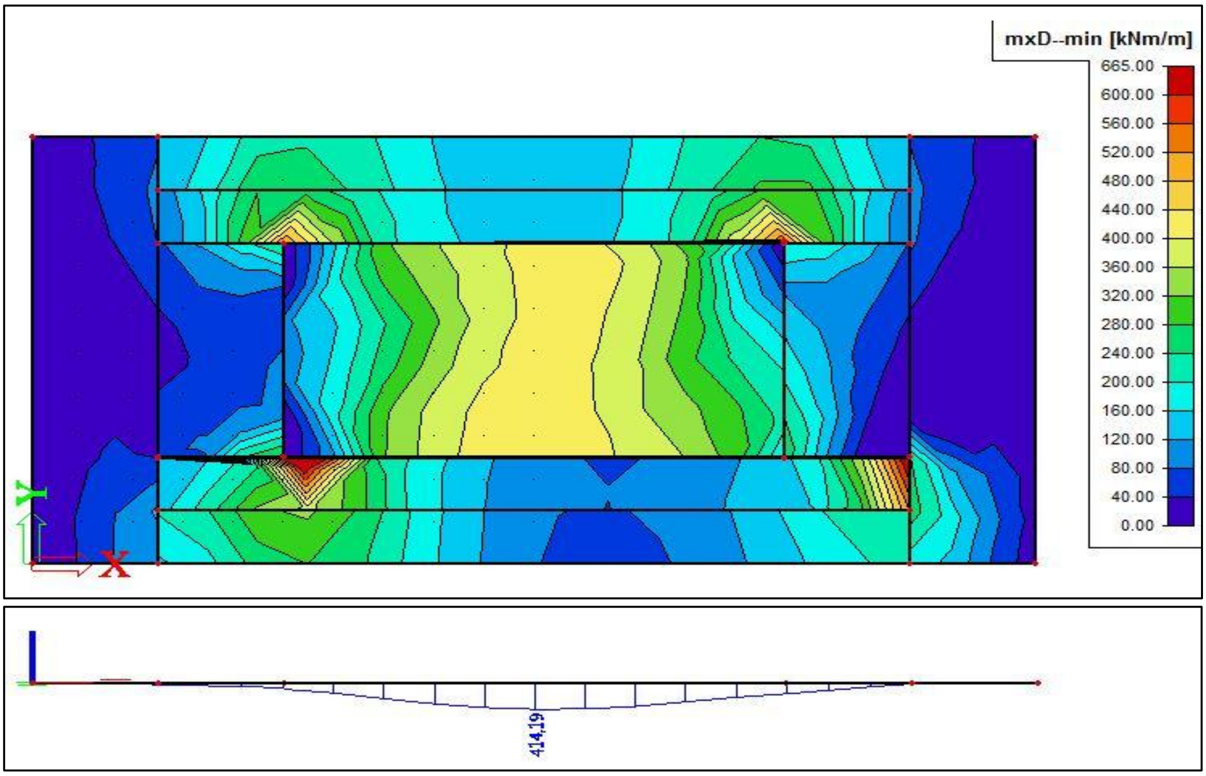

Figure 5-16 Moments top and lateral view of the Pseudo-Coupled model

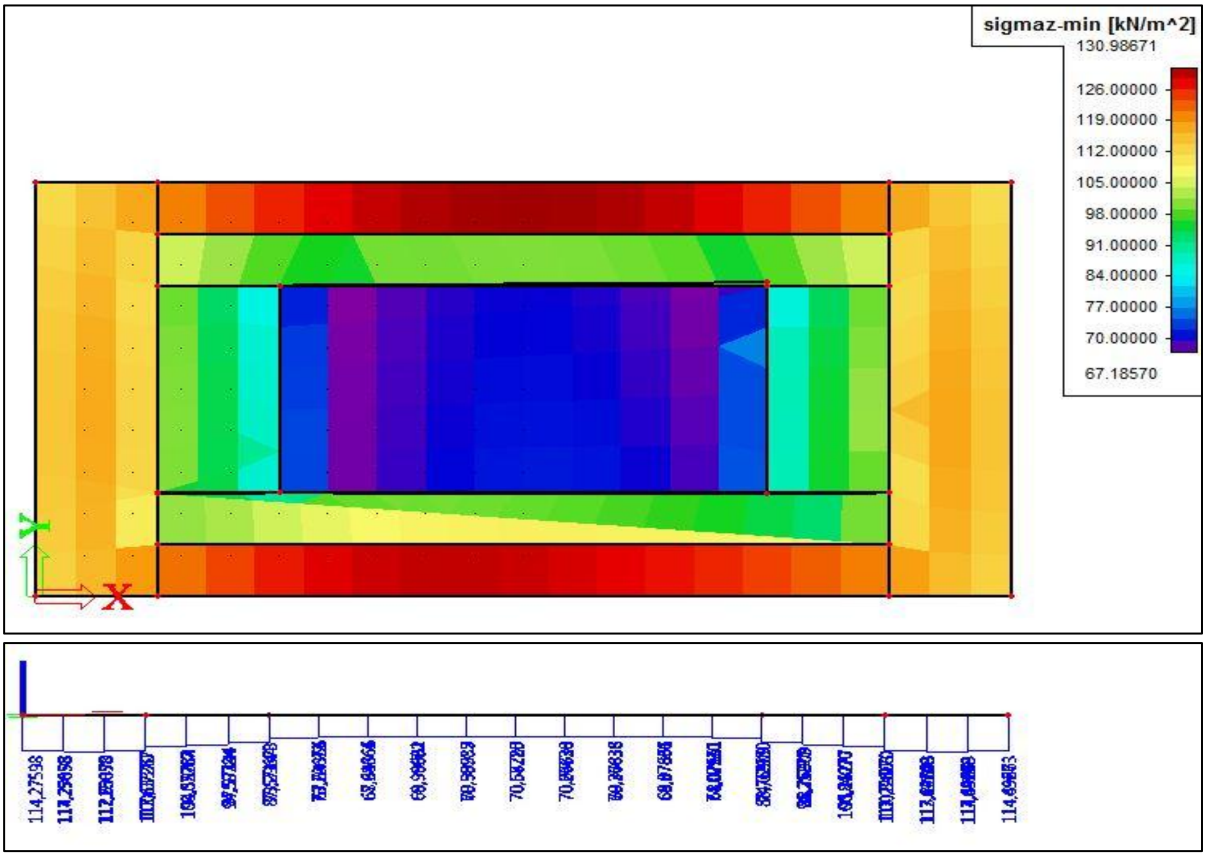

Figure 5-17 Contact stresses top and lateral view of the Pseudo-Coupled model

The above two models use non-uniform coefficients of the modulus of sub-grade reaction and are so improving the Winkler foundation model. The improvement helps with the drawbacks of the uncoupled springs and edge effect. Both models try to couple the Winkler springs by using different stiffness under the slab.

What can also be looked at is an additional model which uses a C_2 parameter. To be able to use such a model in SCIA an additional plate has to be activated along the foundation perimeter. This plate should be really thin (1mm) and can have the same structural values as the foundation slab. It is really important to use a very flexible plate, otherwise the plate will act too stiff and the model will give bad results.

The same C_1 parameter value as of the ***Eurocode 7*** and ***Pseudo-Coupled*** model is used. The C_2 value which will be used is determined the same way as the C_1 value. Through the maximum and minimum value, see Figure 5-18, of the SM model the average C_2 value is determined. The C_2 value which is used in the additional plate model is 7,3711MN/m.

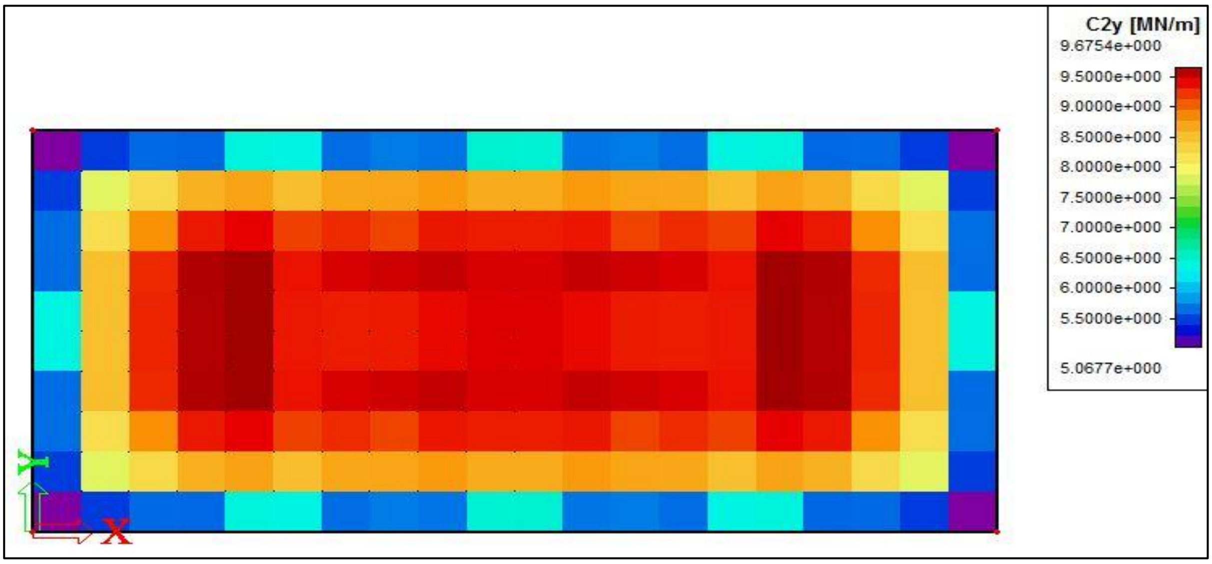

Figure 5-18 Maximum and minimum value for C_2 for a SM model

5.3.4. Additional Plate Model

The distance of these additional plate are 10m in each direction. The C_1 (6,085 MN/m^3) and C_2 (7,3711 MN/m) value are spread out under the whole foundation and the additional plate. The result of the displacement, moment and contact stresses can be seen in Figure 5-19, Figure 5-20 and Figure 5-21.

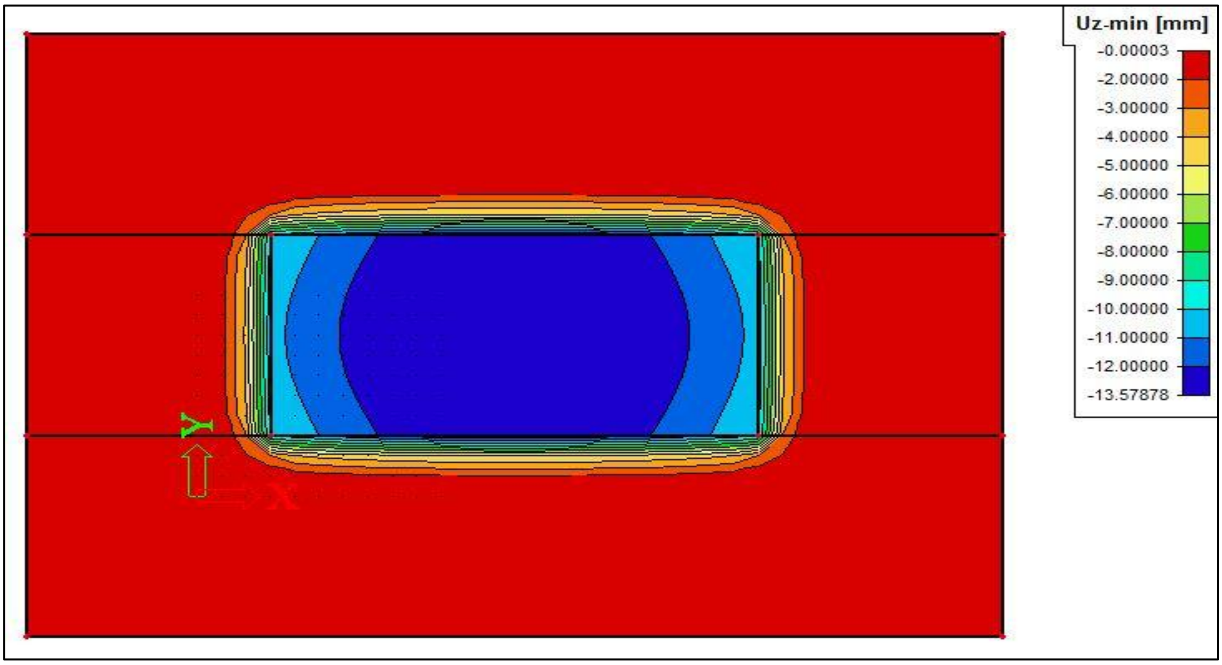

Figure 5-19 Displacement of the additional plate model (C_2≠0)

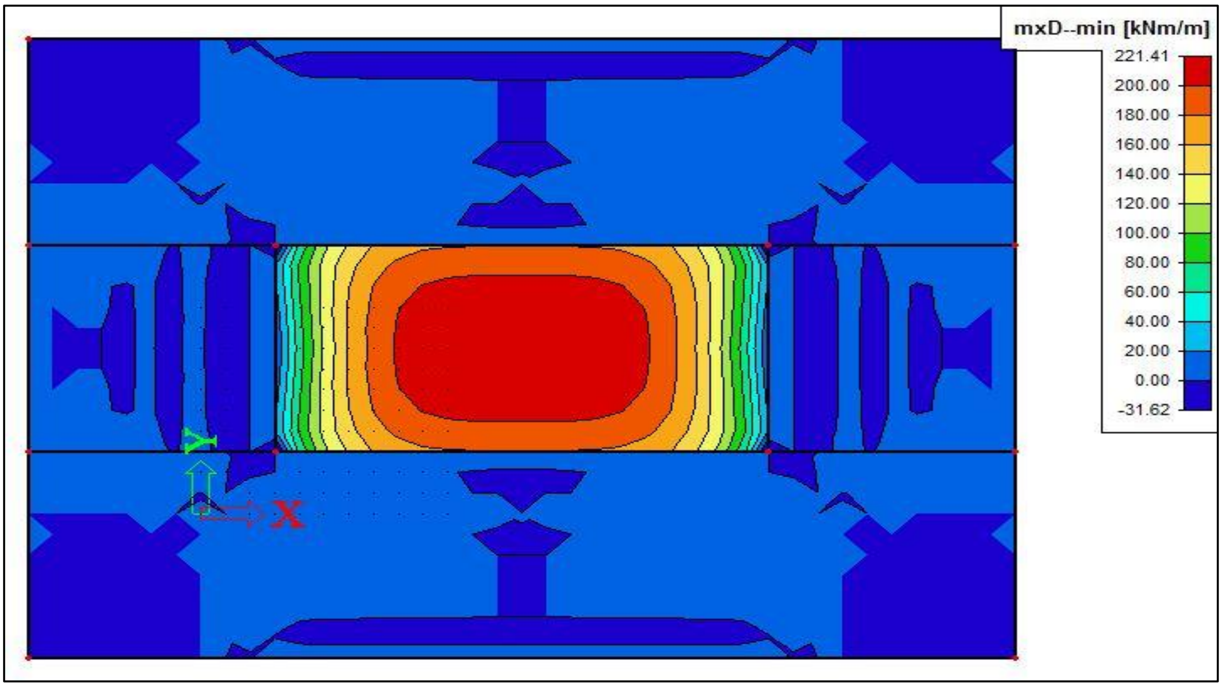

Figure 5-20 Moments of the additional plate model (C_2≠0)

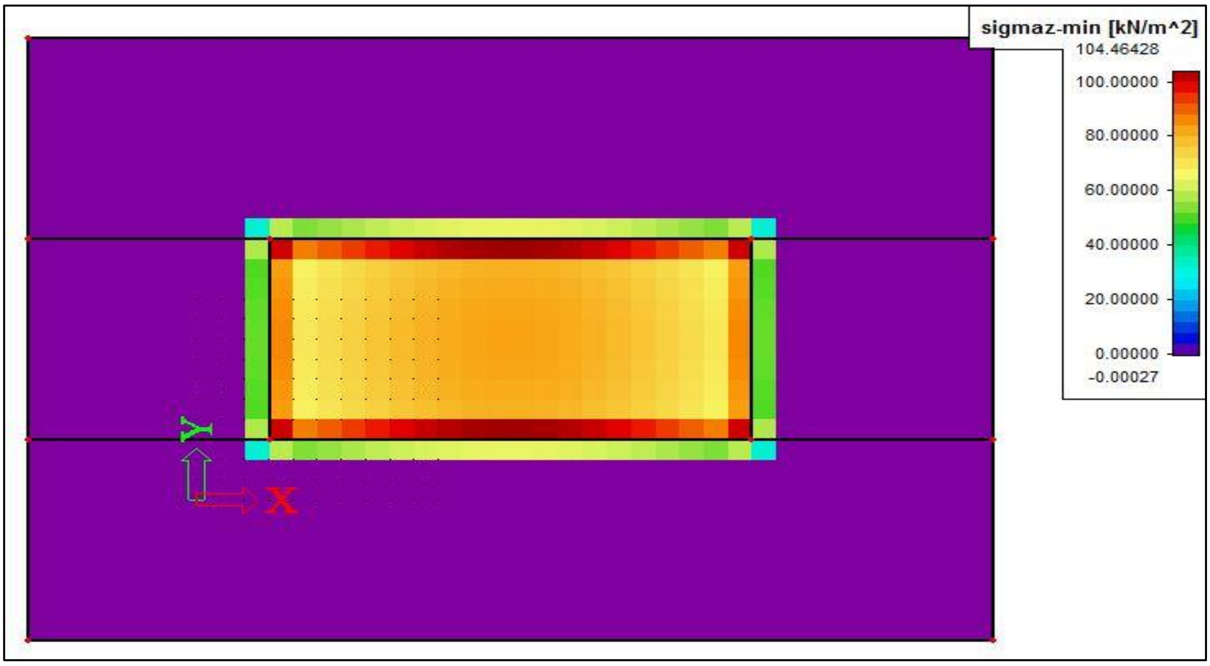

Figure 5-21 Contact stresses of the additional plate model ($C_2 \neq 0$)

The conclusion that can be drawn out of the following model is that the edge effects are taken into account. The foundation deforms around the edge. Also the moments are not zero as in the Winkler model. What still is an issue in this model is determining the C_2 parameter.

5.3.5. SM Model

In this model the non-uniform coefficients are computational determined. The displacement, moment and the contact stresses of the SM model can be seen in the following Figure 5-22, Figure 5-23 and Figure 5-24.

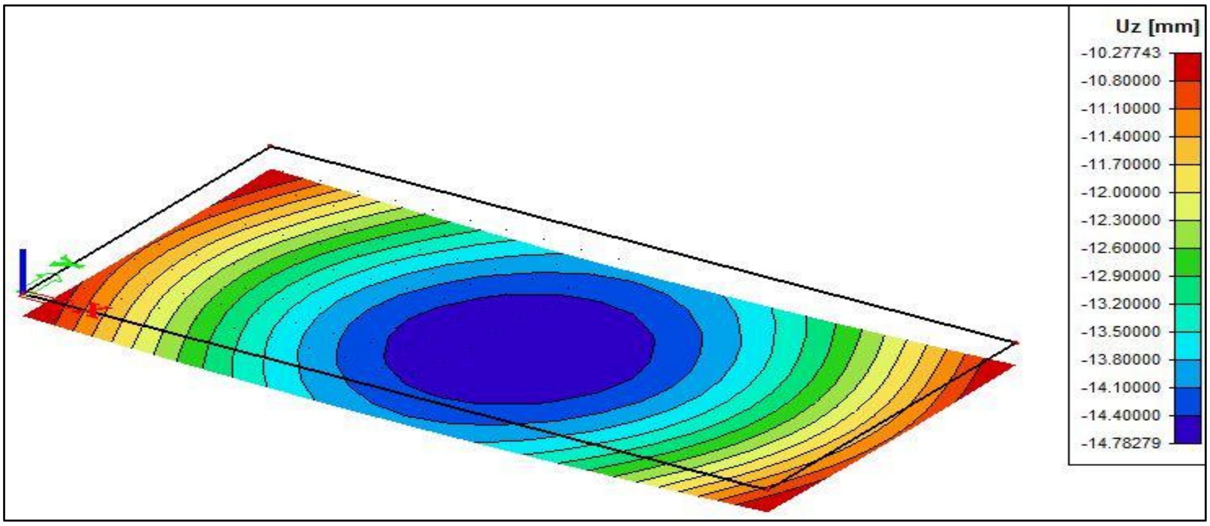

Figure 5-22 Displacement of the SM model

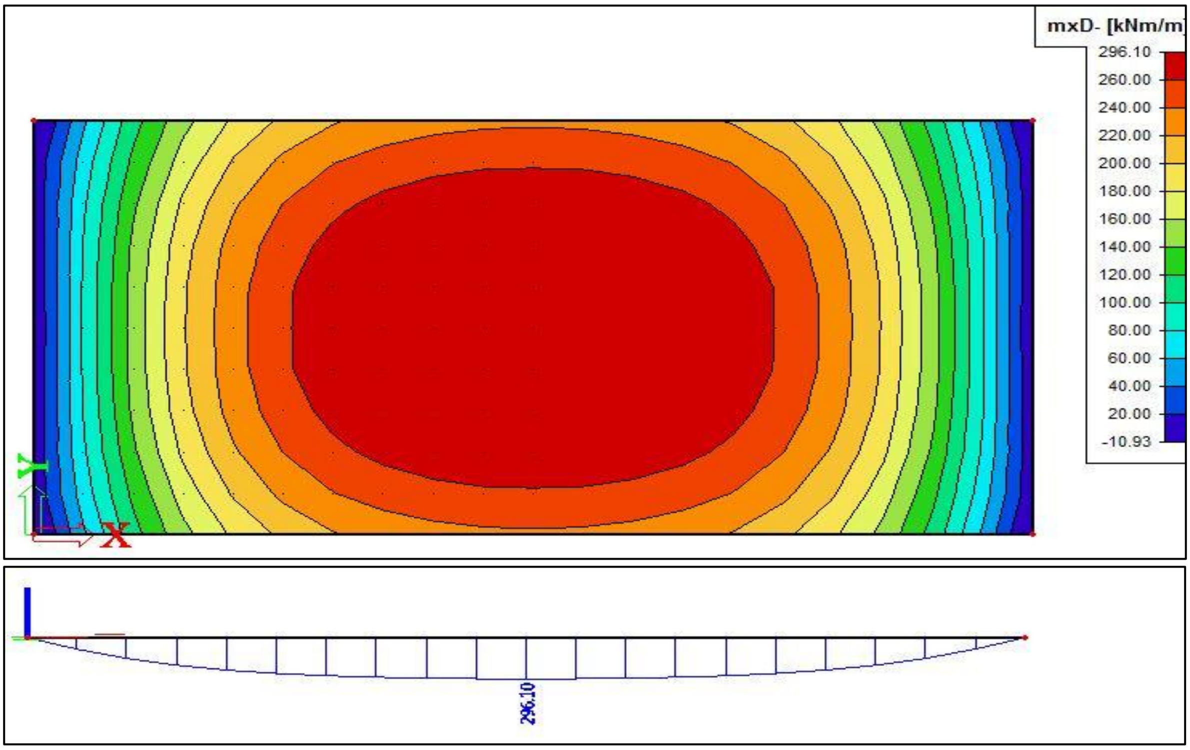

Figure 5-23 Moments top and lateral view of the SM model

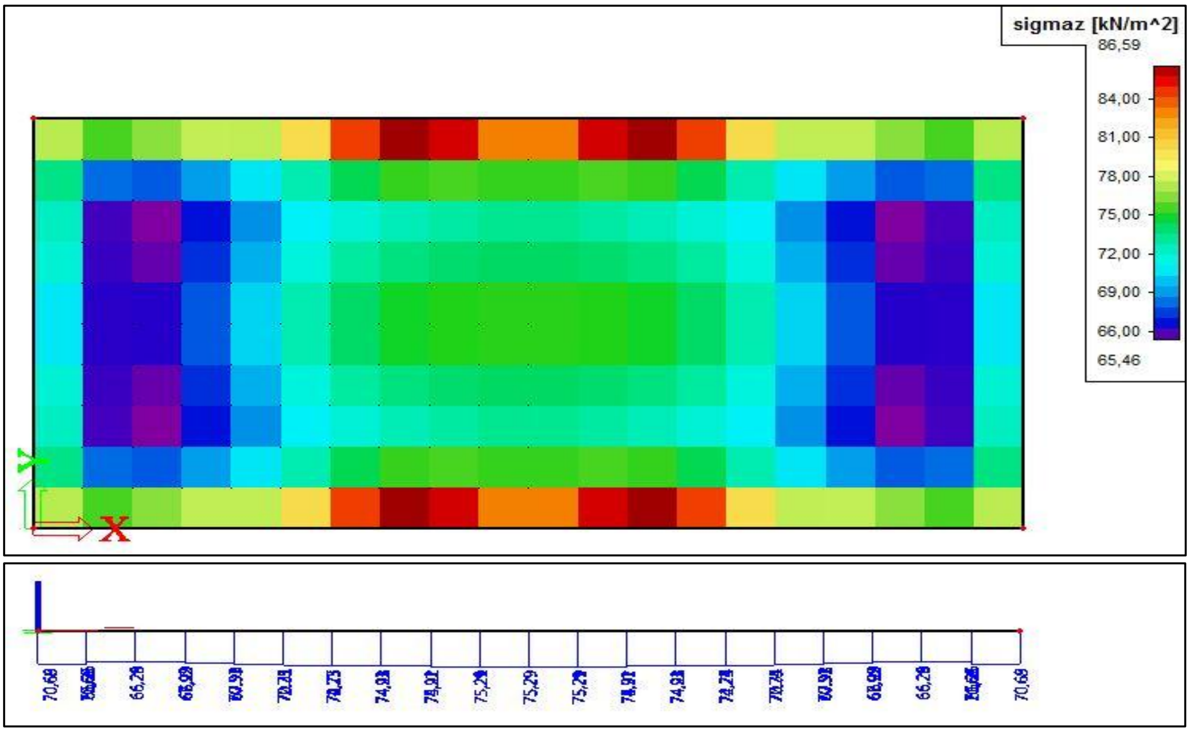

Figure 5-24 Contact stresses top and lateral view of the SM model

Figure 5-24 indicates that the contact stress is less than the external force working on the foundation surface. The external load on the foundation is 100kN/m^2 and the maximum contact stress is 86,59kN/m^2. The SCIA help desk explained that the reason for this disappearance of contact stress is due to the very thick slab and the presence of the C_2 parameters. They also indicated that it is not possible to see were the contact stress disappears to.

In Table 5-2 the results of 4 models is compared with one another. This table gives an overview of the maximum and minimum displacement, contact stress, moments between the different models.

Table 5-2 Results for the 4 interaction approaches

	Uzmax [mm]	Uzmin [mm]	σmax [kN/m^2]	σmin [kN/m^2]	Mmax [kNm]	Mmin [kNm]
1. Eurocode 7	-18,61	-13,81	146,56	54,16	403,57	0
2. Pseudo-Coupled	-18,78	-14,36	130,99	67,19	400,00	0
3. Add. plate	-13,58	-10,00	100,00	80,00	221,41	0
4. SM	-14,78	-10,28	86,59	66,00	296,10	-10,93

5.3.6. Conclusions Non-Uniform Coefficient Foundation Models

Looking at the four models the following results can be concluded:

- The results of the displacement, contact stress and moments of the ***Eurocode 7*** and ***Pseudo-Coupled*** models were C_2=0 (models 1 and 2) are close to each other. Also the results of the other two models were $C_2 \neq 0$ (models 3 and 4) are close to each other.
- In the settlement and moment figures it can be observed that the foundation has a different distribution in one direction compared to the square slabs distribution (section 5.2.).
- The load acting on the foundation is uniform and has a value of 100kN/m^2. In the additional plate and SM model the average of the contact stress is not equal to the external force acting on the foundation surface. According to the SCIA helpdesk this is due to the presence of the C_2 parameter and the thickness of the foundation. Part of the stress will be taken by the soil around the foundation slab, because the slab is very thick (1m). The amount of stress that disappears cannot be seen in the final calculation results. This can cause complications for designers because they cannot approximate the amount of stress that is transferred to neighbouring structures.
- The moments are favourable for the Additional plate and SM model. This is due to the redistribution of loads in the structure. This redistribution is possible because of the interaction of the springs. What can be concluded out of the results for the bending moments is that the C_2 parameter has a favourable effect on the internal force distribution.

5.4. Sensitivity Analysis for the C_2 Parameter

In the previous case it is proven that a C_2 parameter gives a favourable internal force. There is however no experimental data for manually determining a C_2 value. In this case the sensitivity of the C_2 parameter will be studied. Through manually filling in a C_2 value and keeping the C_1 value constant the behaviour and interaction of the two parameters can be observed.

The foundation slab has a dimension of 10 by 10. The thickness 0,25 and 1 meters (The reason was to research a thin and a thick slab). To be able to activate the C_2 parameters a very thin additional plate has to be activated around the foundation. The distance of this plate is 10m from the perimeter of the foundation. If no additional plate is put in the C_2 parameters has no effect that can be observe in Table 5-3 and Table 5-4.

The input parameters are:

- The slab foundation is modelled with a 2D plate element.
- The slab dimensions are 10 by 10 meters. The thickness varies from 250 and 1000 millimetre.
- The modulus of sub-grade reaction is 10000kN/m^3 and is kept constant.
- The C_2 value varies from 0; 2,5; 5; 10; 20; 40; 100; 1000; 10000; 100000; 1000000MN/m.
- The loads are 25kN/m^2 and 100 kN/m^2 and are uniform over the whole foundation slab.
  The dead load is zero.
- Concrete class C35/45; Young's modulus concrete (E_c) = 34100N/mm^2

Table 5-3 Displacement without an additional plate (load 25kN/m^2)

Dimensions [m]	σ [kN/m^2]	C_1 [MN/m^3]	C_2 [MN/m]	Uz [mm]
10/10/0,25	25	10	0	-2,5
10/10/0,25	25	10	2,5	-2,5
10/10/0,25	25	10	5	-2,5

Table 5-4 Displacement without an additional plate (load 100kN/m^2)

Dimensions [m]	σ [kN/m^2]	C_1 [MN/m^3]	C_2 [MN/m]	Uz [mm]
10/10/0,25	100	10	0	-10
10/10/0,25	100	10	2,5	-10
10/10/0,25	100	10	5	-10

When the C_2 parameters are activated by the additional plate the following displacements can be observed in Figure 5-25 (10 by 10 slabs with a load of 25kN/m^2 and 100kN/m^2).

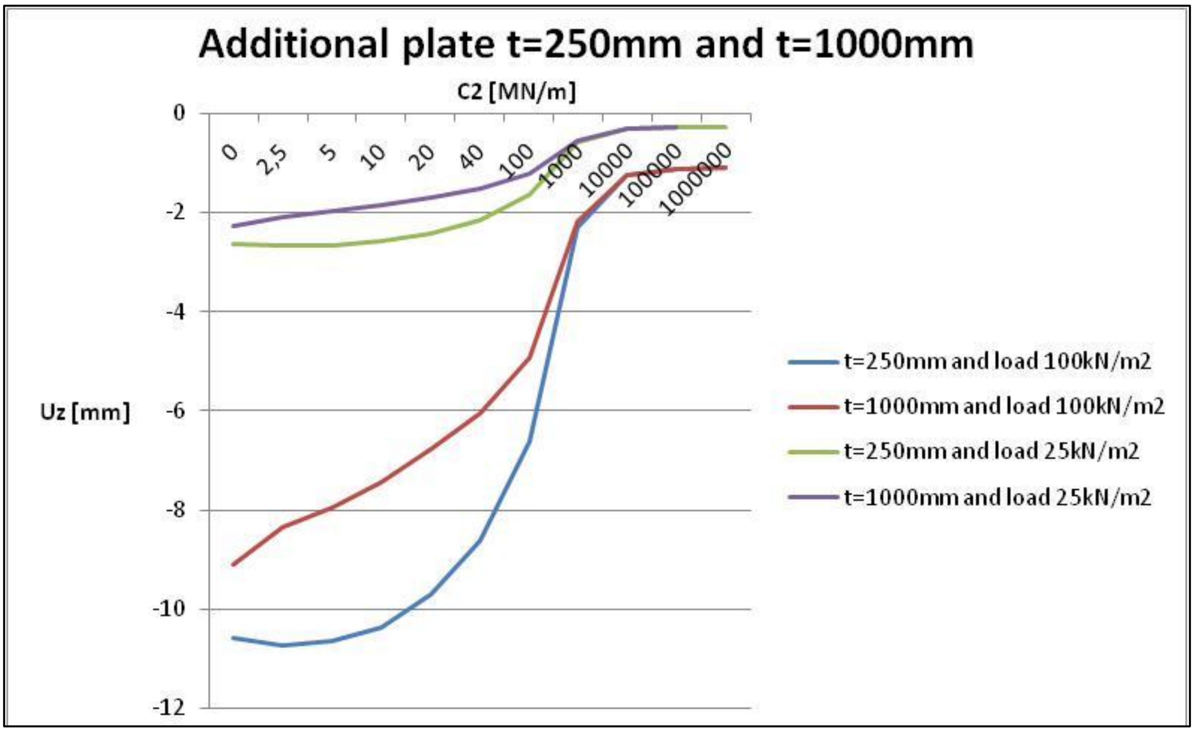

Figure 5-25 Displacement due to the C_2 variation

The influence of C_2 on the displacement for both slabs is between 0 and 10000MN/m. This is the range were a change can be observed in the test.

6. Comparison of Secant Method, LFEM and NLFEM

6.1. Introduction

In this chapter a comparison will be made between the Secant Method (SM), the linear finite element method (LFEM) and the nonlinear finite element method (NLFEM). The SM is implemented in the SOILin module of the program SCIA Engineer. The linear and nonlinear finite element analyses are implemented in PLAXIS. PLAXIS is a geotechnical program that is often used in everyday engineering practice. The nonlinear analysis performed by this program has been compared extensively with experimental results and are generally considered to be quite accurate. The SM has not been compared with experiments [J. Bucek]. It has been developed based on compliance to governing codes of practice.

6.2. Work Approach and Overview SM, LFEM and NLFEM

6.2.1. Introduction

For the SM and the LFEM the following soil parameters are needed: E, υ, $\gamma_{unsaturated}$ and $\gamma_{saturated}$. For the NLFEM Mohr-Coulomb plasticity has been applied with the following soil parameters: E, υ, $\gamma_{unsaturated}$, $\gamma_{saturated}$, φ and c.

Each of the methods has been applied to a circular slab on a soft soil. In the SM the models are two dimensional plates on sub-grades. In the LFEM and NLFEM the models are two dimensional plates on a three dimensional subgrade. However, due to axi-symmetry these latter models could be simplified to two dimensions in PLAXIS 2D (Figure 6-1).

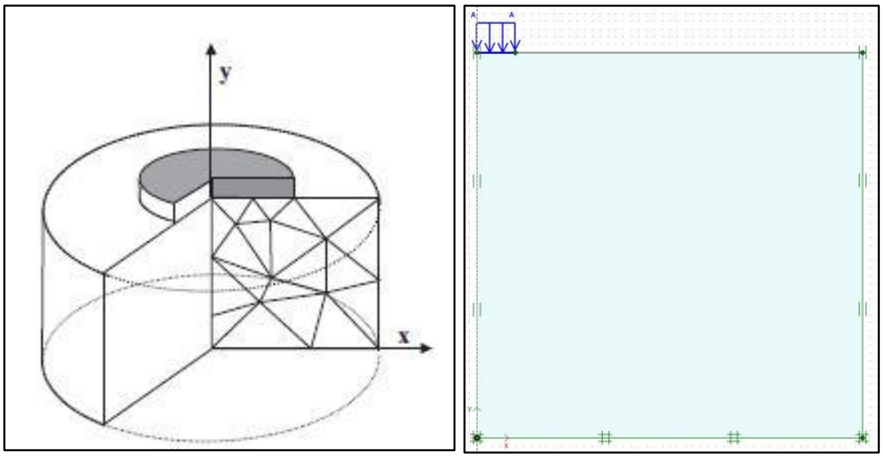

Figure 6-1 PLAXIS 2D axi-symmetric model [1]

The left Figure 6-1 gives a general view of an axi-symmetric model and the right figure the actual model that is used to compare it to the SM model. In Figure 6-2 the circular slab can be seen with a borehole and the additional plate.

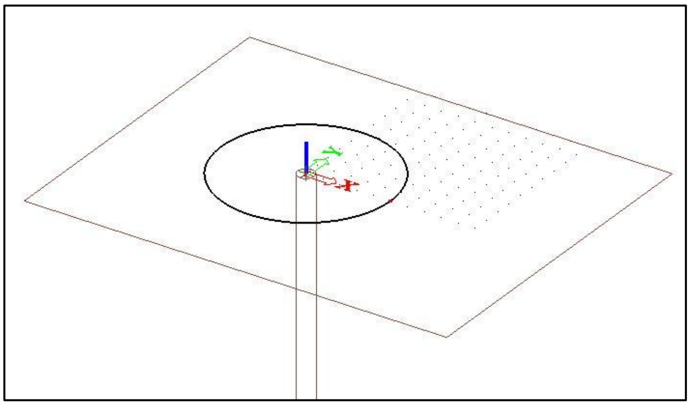

Figure 6-2 SCIA Circular slab model

A 15-nodes triangular element (Figure 6-3) is used to model the soil in PLAXIS. This element is very accurate and produces high quality stress results for difficult problems. The demerits of this element are that it uses more memory, is slower in calculations and operation performance than elements with less nodes.

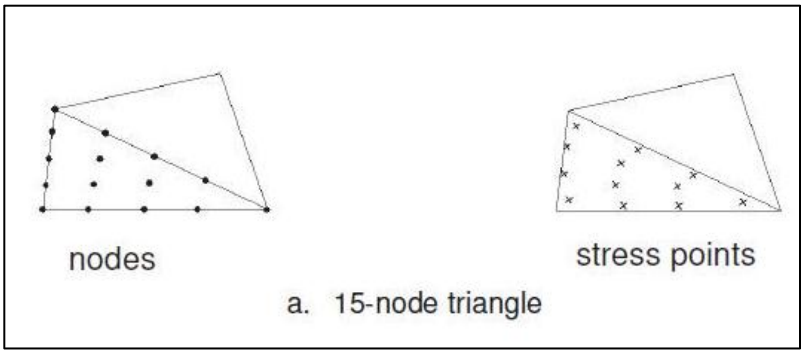

Figure 6-3 15-node triangle element [1]

The slab is modelled with 5 node plate elements (Figure 6-4), which matches with the 15-nodes soil elements. The slab is also based on the Mindlin's plate theory (Chapter 2).

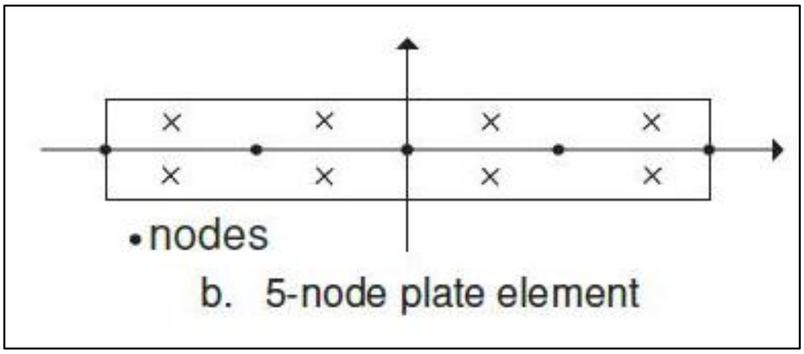

Figure 6-4 5-nodel plate element [1]

6.2.2. The PLAXIS Input Parameters

Model:	axisymmetric
Geometry dimensions:	Xmax = 50m and Ymax = 50m.
Grid spacing:	1m
Boundary:	Standard fixities

Soil properties:		
Name:	Sand	
Material model:	1. Linear elastic	(LE)
	2. Mohr-Coulomb	(MC)
E_s:	45000kN/m^2	
υ_s:	0,3	
$\gamma_{unsaturated}$:	18kN/m^3	
$\gamma_{saturated}$:	20kN/m^3	
φ:	30°	
	80°	
c:	1 kN/m^2	
	1000 kN/m^2	

The unrealistic high values for φ (80°) and c (1000 kN/m^2) are chosen for a comparison between LFEM to a NLFEM made linear. By selecting these high values the plasticity part of the NLFEM model almost does not become active. The results should be very close to those of the LFEM material model. This happens to be the case which gives extra confidence in the correctness of the analyses.

Table 6-1 Mesh output data PLAXIS

Model	Nr. of Soil elements	Nr. of nodes	Average elem. Size [m]
(LE and MC) thick and thin	786	6453	1,783

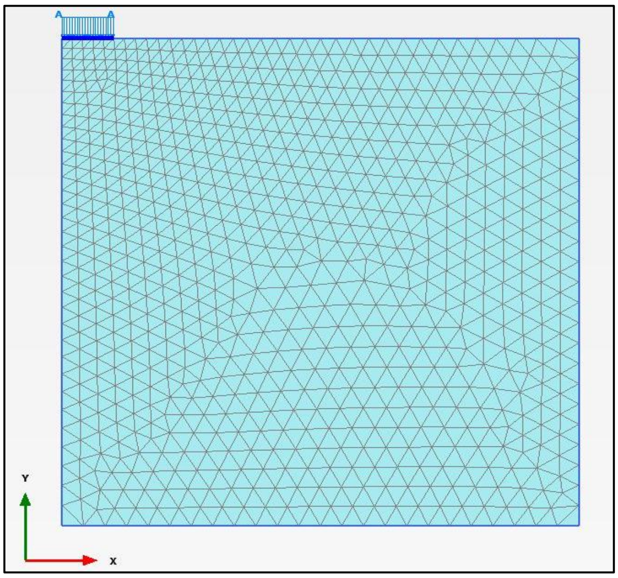

Figure 6-5 Mesh generator output

Structure properties:

Name:	Slab
Radius:	5m
Model:	Elastic
υ_c:	0,2
EA thick:	34100000kN/m → Thickness: 1m
EI thick:	2842000kNm^2/m → Thickness: 1m
EA thin:	6820000kN/m → Thickness: 0,2m
EI thin:	22730kNm^2/m → Thickness: 0,2m

6.2.3. The SCIA Input Parameters:

Structure properties:

E_c:	34100000kN/m^2
υ_c:	0,2
Radius:	5m
Thickness:	1m and 0,2m

Table 6-2 Mesh output data SCIA

Model	Nr. of Plate elements	Nr. of nodes	Average elem. Size [m]
Thick and thin slab	1332	1341	0,25

Soil properties:

Borehole with geological profile

Support:	Elastic support
E_{def}:	45000kN/m^2
υ_s:	0,3
$\gamma_{unsaturated}$:	18kN/m^3
$\gamma_{saturated}$:	20kN/m^3
m:	0,2

6.2.4. Results of the SM, LFEM and NLFEM Analysis

The load is 100kN/m^2 in all the cases and the ground water level is not taken into account. The load – displacement diagram of the LE PLAXIS model can be seen in Figure 6-6. The point (A) is in the middle of the slab and point (C) at the edge.

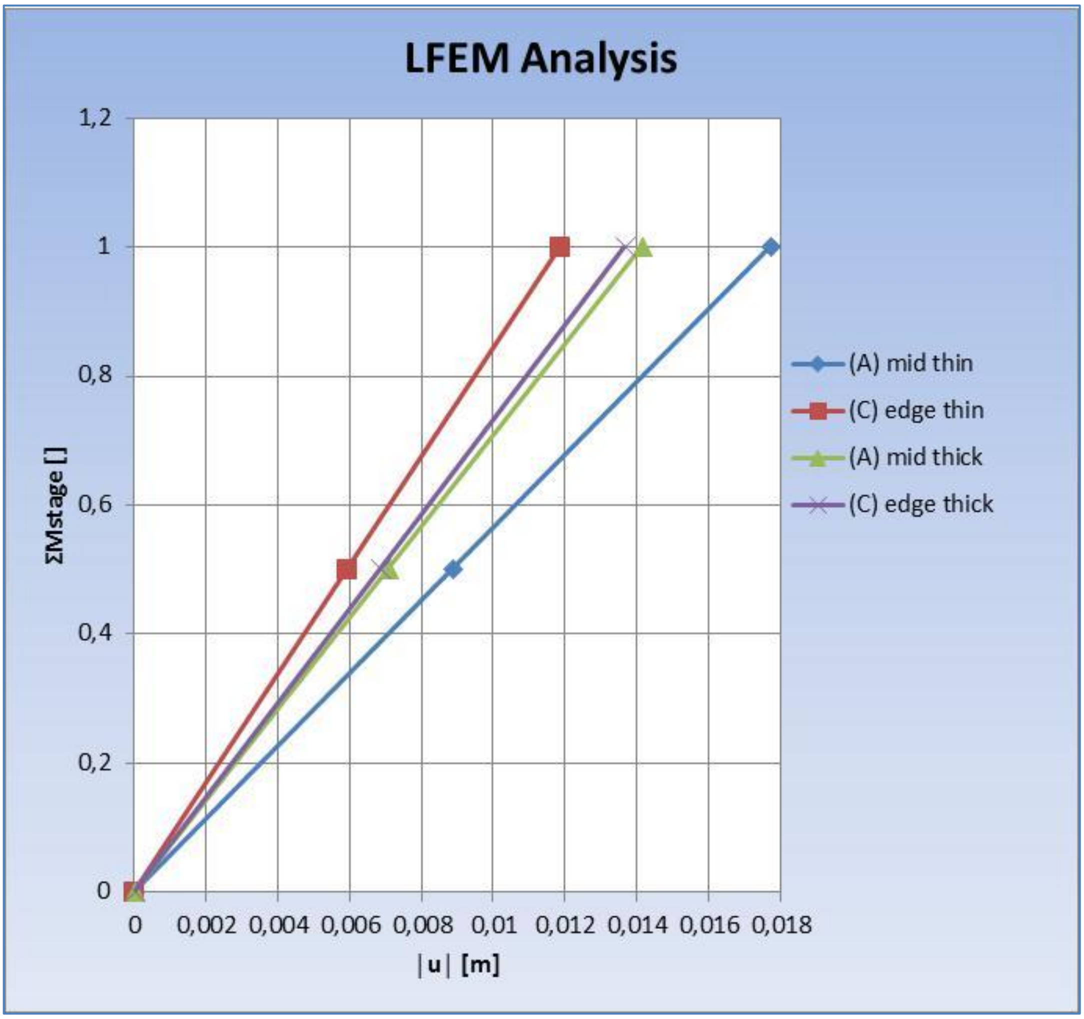

Figure 6-6 Load – displacement of the different LFEM Analysis

In Figure 6-6 the settlement is given on the horizontal axis. On the vertical axis the stage due to the loading is given. The LFEM thick slab analysis shows almost the same displacement at the mid and edge of the slab. The thin slab case however shows a smaller displacement at the edge and a larger displacement in the middle.

Table 6-3 and Table 6-4 gives an overview of the maximum displacement, moment and contact stress from the SM, LFEM and NLFEM calculations for a thick and thin slab.

Table 6-3 Overview results THICK slab model

			σcontact stresses [kN/m^2]	
Model	|u| [mm]	Mmax [kN/m]	max	min
(LFEM) thick	14,15	172,9	193,5	55,96
(NLFEM) thick φ=30° and c=1kN/m^2	16,76	69,79	108,5	73,16
(NLFEM) thick φ=80° and c=1000kN/m2	14,21	169,3	231,8	56,28
SM thick	3,98	171,71	81,55	66,48

Table 6-4 Overview results THIN slab model

			σcontact stresses [kN/m^2]	
Model	|u| [mm]	Mmax [kN/m]	max	min
(LFEM) thin	17,76	13,02	103,3	94,24
(NLFEM) thin φ=30° and c=1kN/m^2	18,20	9,10	99,61	71,16
(NLFEM) thin φ=80° and c=1000kN/m^2	17,73	15,02	116,8	86,26
SM thin	7,19	15,72	104,08	62,25

In a NLFEM material model plastic points can occur (Figure 6-7). The points arise because of the redistribution of forces in the soil model. Due to this redistribution the soil will fail. This also explains the not expected contact stresses for the NLFEM model with the c=1kN/m^2 and φ=30° (Figure 6-8). It would be expected that the contact stress of a very thick slab would be as explained in Figure 2-9 [11]

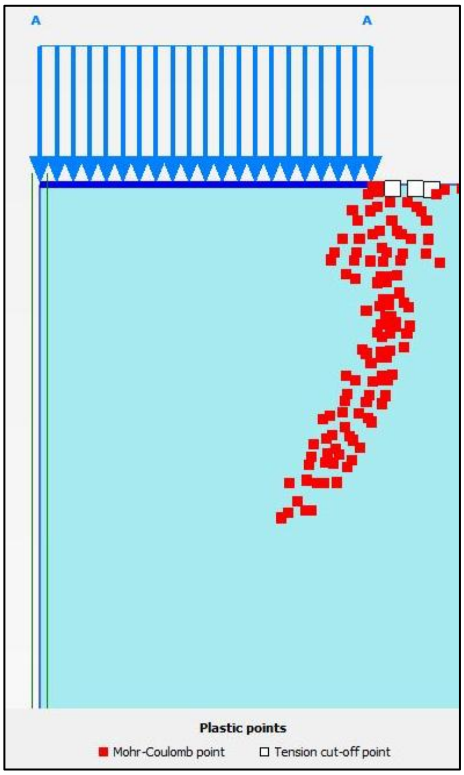

Figure 6-7 Plastic points NLFEM model

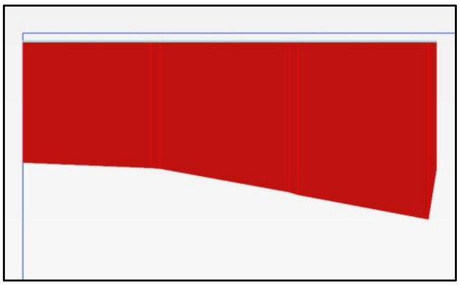

Figure 6-8 Contact stress NLFEM model vs. expected thick plate model

In the PLAXIS models there are no interface element. In reality the slab and the soil will have a sliding surface. The interface element models this sliding surface. The interface element is left out to make the PLAXIS and SCIA models more comparable.

The following figures give a more visual insight in the comparisons between the two programs. First the thin structures results are presented and afterwards the thick structure results. The most visual comparable data from PLAXIS to the SCIA results was the LFEM analysis. So the choice was made to present only the results of the LFEM analysis. This is the better choice because the results are close to each other and the difference between the results can then better be understood when visualise. The left figures are the result as presented in PLAXIS and the right figure the results as presented in SCIA. The values can be found in Table 6-3 and Table 6-4.

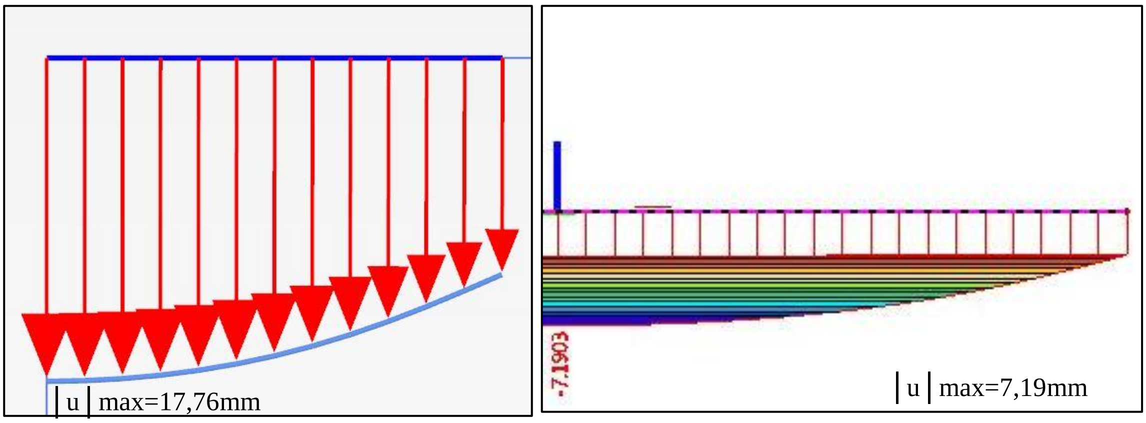

Figure 6-9 PLAXIS and SCIA displacement (t = 0,2m)

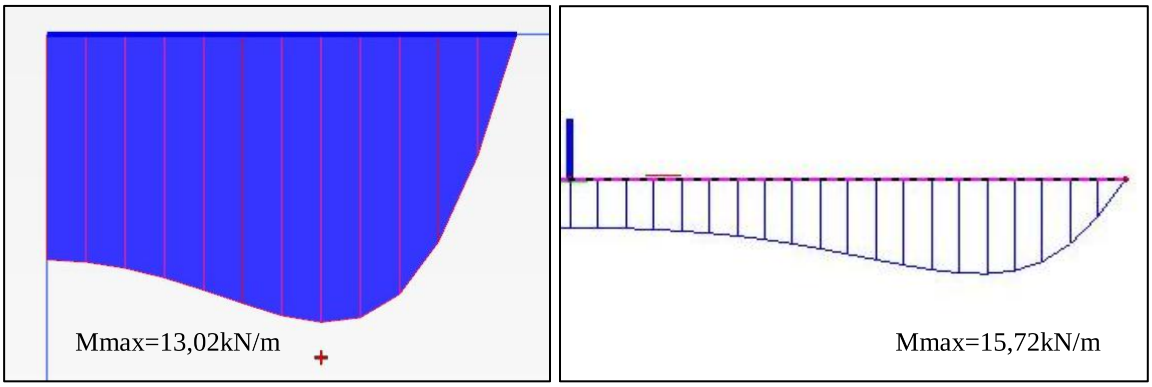

Figure 6-10 PLAXIS and SCIA moments (t = 0,2m)

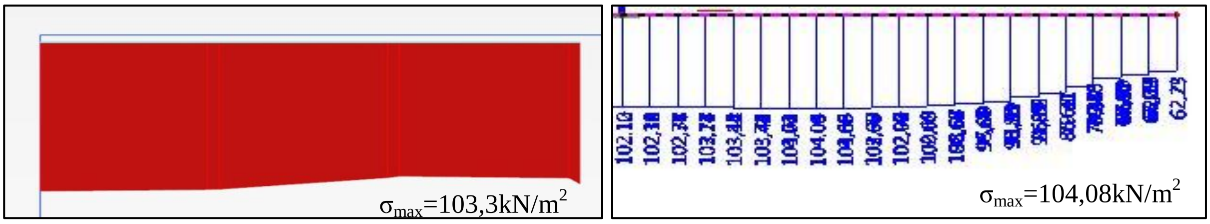

Figure 6-11 PLAXIS and SCIA contact stress (t = 0,2m)

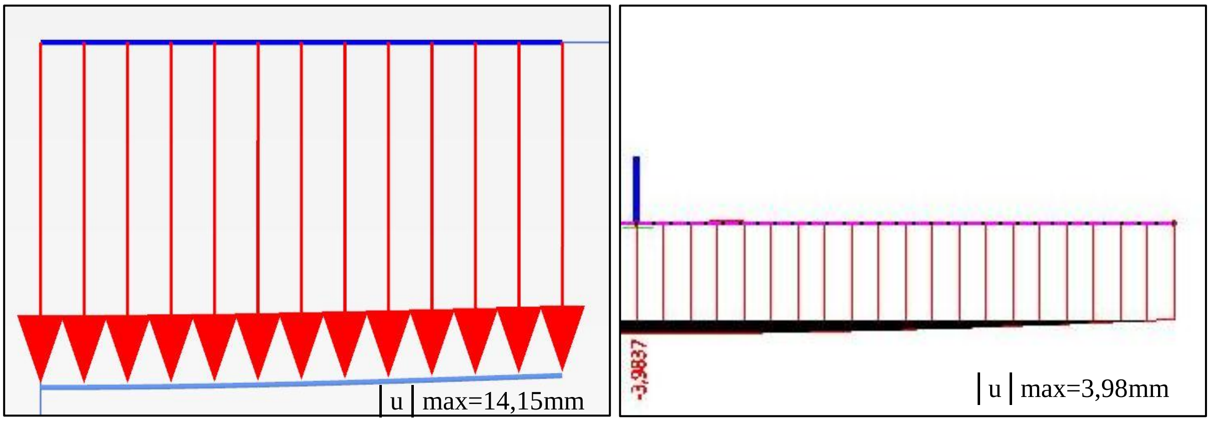

Figure 6-12 PLAXIS and SCIA displacement (t = 1m)

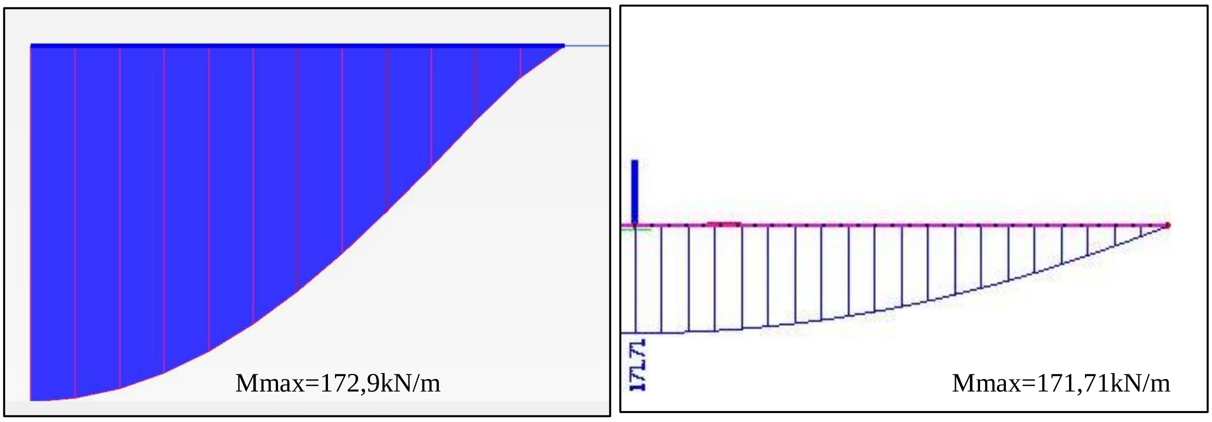

Figure 6-13 PLAXIS and SCIA moments (t = 1m)

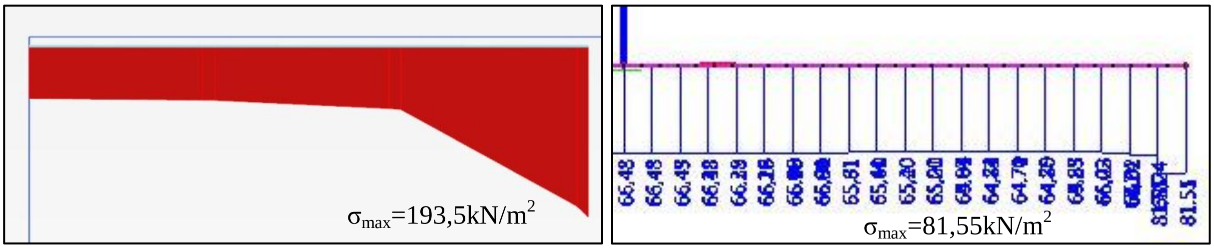

Figure 6-14 PLAXIS and SCIA contact stress (t = 1m)

6.3. Conclusion Comparison SM, LFEM and NLFEM

Comparing the maximum result of the displacement, moments and contact stress between the different models the following can be concluded:

- The maximum displacements for the all methods occur in the middle of the slab. The SM however gives significantly smaller displacements (2 to 4 times smaller) then the LFEM and the NLFEM. The explanation is that both software's use different settlement calculations.
  The small displacement in SM is also coupled through the structural strength coefficient (*m*-factor). SM takes an estimation when determining the height were settlement may take place. This height is called the limit depth. The displacement can increase if a smaller *m*-factor is chosen. To change this *m*-factor in SM the norms have to be set to the Czech Republic norms. In this study the Eurocode norms were used.
- The LFEM moment results come close to the SM values. This could be expected because SM also calculates linearly. The moments of the NLFEM model did differ a lot with SM. This difference was proven to be caused by the plastic branch. The NLFEM model became elastic by increasing the plastic values. This step made it possible to verify the LFEM model and caused the NLFEM model results to come closer to the SM moments.
- In the SBR 270 (interaction of structure and soil) literature the moment and displacement relationship was given for foundation structures. The relation between the two was the stiffness. It stated that the stiffer the foundation the bigger the moment in the slab and also the smaller the displacement in the model. This is also something that can be observed in the result [13].

General conclusion of using SCIA and PLAXIS for analysis:

- The big difference between the models can raise concern. Both models are an approximation of the reality. So it is difficult to say which one is correct. Also the SCIA module is not a software program to describe the behaviour of the soil medium accurately. It only uses the SM tool to give the structure a support to rest on which behaves approximately as the ground would. This support is the non-uniform coefficients.

6.4. Overview all foundation models

Different foundation models are presented and studied in chapter 5 and 6. These models can be categories by their applicability:

- Uniform coefficient
- Non-uniform coefficient:
    - Eurocode 7
    - Pseudo-Coupled
    - SM
- LFEM
- NLFEM

The uniform coefficient model gave settlements, but no moments. This spring model does not give results which are close to what is expected in reality. The non-uniform coefficients methods are a good way to improve the spring model. The different soil stiffness distribution beneath the foundation make it possible to optimise the spring models.

The non-uniform coefficients can be applied manually. This is done by determining the modulus of sub-grade reaction and distributing it beneath the foundation. The Eurocode 7 and Pseudo-Coupled models, which are explained in section 4.3., are possible manual approaches. The SM determines and spreads the soil stiffness automatically. This methods transforms soil properties into spring stiffness.

During the research and evaluation of the case study a few merits, demerits and doubts were observed from the SM.

Merit/advantages using the SM:

- Geological data can be input instead of a modulus of sub-grade reaction. The program calculates from soil data the stiffness in the subsoil.
- The coupled analyses (coupling of springs) have a favourable influence on the moment results. The coupling of the springs makes it possible that the loads are redistributed in the structure, thus having a positive influence on the design results.
- It is a quick method for the structural engineer to get insight in the influence of the subsoil on the foundation structure.

Demerit/disadvantages using the SM:

- The stiffness in the subsoil is calculated on the basis of the first given load combination. Afterwards the same soil stiffness is used for the other load combinations.
- Compared to the 2D PLAXIS analysis the settlements are much smaller.
- The model does not fail (no failure mechanism).

In Table 6-5 all six foundation models studied in this thesis are compared (see also Figure 1-5). The categories are safety, accuracy and usability. A model receives an extra + for safety if it predicts larger settlement values or larger moment values then another model. For example on one hand the NLFEM gave the largest settlements so if one would design using this model it would be on the safe side. On the other hand, the Eurocode 7 and Pseudo-coupled method gave larger moments. These are thus on the safe side when the moments are evaluated.

The accuracy score is based on using the NLFEM as a reference. The NLFEM in PLAXIS is a method which has been compared extensively with experimental results and are generally considered to be quite accurate. The SM has not been compared with experiments [J. Bucek]. It has been developed based on compliance to governing codes of practice.

The usability score gives an indication on how convenient a method is to work with. It is based on the amount of input data needed, the modelling time and the computation time.

Table 6-5 Evaluation all foundation models

		Safety		Accuracy		Usability
	Type of Model Application	Settlement	Moment	Settlement	Moment	
1	Uniform	++	-	-	-	++++
2	Eurocode 7	++	+++	+	+	+++
3	Pseudo-Coupled	++	+++	+	+	+++
4	SM	+	++	+	+	++
5	LFEM	+	++	++	+	+
6	NLFEM	+++	+	+++	+++	-

From Table 6-5 it can be concluded that method 2, 3 and 4 obtain a high overall score. These are the non-uniform spring models. Especially the Eurocode 7 method and Pseudo-Coupled method are recommendable. Not surprisingly, the latter methods are often used in current practice. The secant method (SM) can be interesting to predict smaller moments and smaller settlements at the expense of entering extra soil data.

7. Interaction Structural and Geotechnical Engineer

7.1. Introduction

In this chapter the interaction of structural and geotechnical engineers will be discussed. Focused is on a practical approach which makes use of the Winkler foundation model. In the end of the chapter a checklist will be formulated to assist designers in their communication on modelling. Finally, a brief observation on foundation modelling will be presented.

7.2. Interaction Engineering Modelling Process

The first step when designing a foundation is determining the centre lines or structural dimensions of the complete structure. The structural engineer will take responsibility for this phase. He also has to use his engineering judgement to determine the types of load (static, dynamic, permanent, short term, cycling) that will work on the structure and the locations of these loads. Afterwards the load distribution of the top structure on the foundation can be determined. With the structural dimensions and the loads the structural engineer can determine the contact stresses. To determine them he also needs to take the interaction of the soil into account. This will be done through the modulus of sub-grade reaction (*k*)

In section 2.4.1 the modulus of sub-grade reaction is already briefly highlighted. In this part of the report the important aspects of this parameter is summarize as follow:

- It is not a soil property but a model value.
- It is not a constant value and it varies under the foundation slab.
- It is determined by pressure divided by the settlement ($k=p/w$).

The aspects which are important to determine the modulus of sub grade reaction are [23]:

- Dimensions of the foundation slab.
- Dead load of the foundation slab.
- Loads working on the foundation slab (static, dynamic, permanent, short term, cycling).
- The location of the loads on the foundation slab.
- The stiffness and strength properties of the structure and subsoil.
- History (excavation and embankment)
- Building execution sequence.
- Permeability of the sub soil influences the settlements (time dependence)

The modulus of sub-grade reaction is a value that has to be determined iteratively. A practical way to determine this value is as follow:

- **i.** Structural engineer estimates the modulus of sub-grade reaction (usually 10000kN/m^3) and calculates the contact stresses.
- **ii.** Structural engineer gives the geotechnical engineer the dimensions of the structure, estimated contact stress values including soil information (sounding graph, soil investigation report, history of the area).
- **iii.** Geotechnical engineer determines a new value or values for the modulus of sub-grade reaction and the soil properties. This can be done by using the soil information and making a settlement calculation. Afterwards the pressure is divided by the settlement and that value is the new sub-grade of reaction or *k* value.
- **iv.** Structural engineer compares the new *k* and uses it to the earlier assumed *k* value. If it comes closely to the used value then the procedure is finished. If there is a difference then step **i**, **ii** and **iii** should be done again (iterative procedure)

The iterative procedure can be seen in Figure 7-1. The structural engineer needs to keep in mind that if the loads or the dimensions of the structure change the *k* value needs to be determined again. These two factors have a very big influence on the *k* value [23].

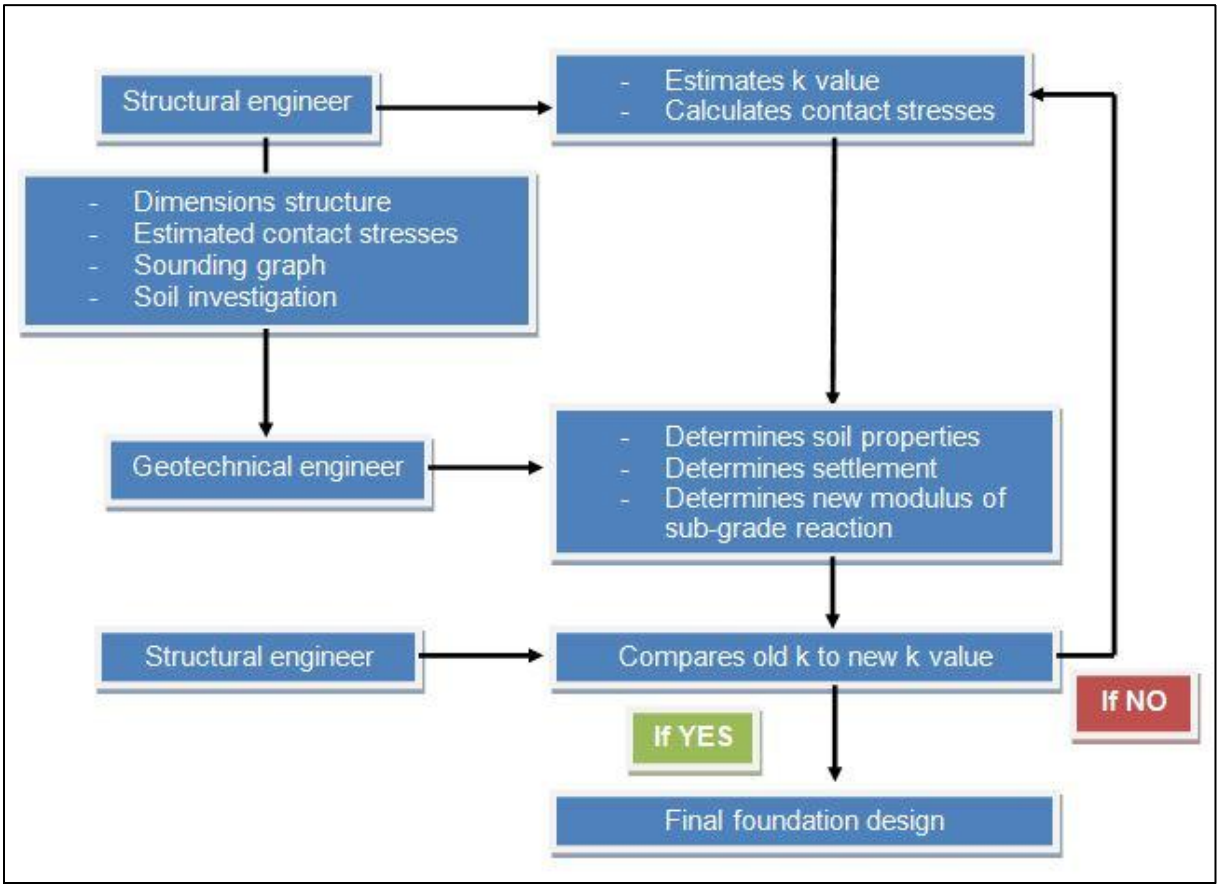

Figure 7-1 Iteration procedure to determine the *k* value

The designers should also take into account the execution process. Procedure as for example excavation and drainage can influence the soil structure thus influencing the modulus of sub-grade reaction. Temporary sheeted excavation (damwandkuip) can also influence the modulus of sub-grade reaction when removed. This information is relevant for the geotechnical engineer to determine the right *k* value with which the structural engineer will use to calculate the final foundation design.

The following practical procedure is not the definite or only way to model the modulus of sub-grade reaction. Depending on the project, geological category, information that is available and the requirements the procedure can differ. But the described procedure is a general way of how the structural and geotechnical engineer can work together in determining the soil stiffness.

In appendix D, a list of available values for the modulus of sub grade reaction values are given. These tables, collected from available literature [24] [4] [25], are based on experimental values and can be used by a designer for determining a *k* value in the preliminary phase. He should however always discuss the chosen value with the geotechnical engineer, because these values were derived with a certain approach and constraints. The geotechnical engineer together with the structural engineer can on the basis of the available project specification and the literature choose the best approximated value.

7.3. Checklist

7.3.1. Checklist Sheet

Not all foundations will be designed in the same manner. Each project will be different and have its own obstacles which the engineers will have to overcome. Besides those differences there will also be similarities. In this part of the section the focus will be on those similarities. The focus will be on summarising the similarities of the modelling process and thus making a checklist to assist the structural and geotechnical engineer in the modelling phase. This checklist is based on: Theory of foundation modelling [26] [4] [8] and on discussion with structural and geotechnical engineers of Ballast Nedam Engineering (BNE).

Checklist	Mark
General	
Information about the location of the project	
Project specifications	
Sequence of the building process	
History of the area	
Specific	
Global dimensions foundation structure	
Loads	
▪ **Magnitude of the load**	
▪ **Location of the load**	
▪ **Type of load (stat. or dyn.)**	
▪ **Type of load combination**	
Soil data	
▪ **Soil properties**	
▪ **Sounding graph**	
▪ **Soil investigation**	
Model that will be used	
▪ **Dimensional model (1D,2D or 3D)**	
▪ **Software** ▪ **Structural engineer: SCIA, Technosoft or Excel** ▪ **Geotechnical engineer: PLAXIS, DSettlement or Excel**	
▪ **Determining input parameters for structural software**	

7.3.2. Point of Interest

The checklist above is a list which can stimulate the communication between designers when modelling a large shallow foundation design. This check list is straightforward and should serve as a guideline. Important in the checklist is the model that will be used. This can be based on engineering judgement and it will differ for each design.

The key is to keep it simple and not to choose a too complex model. Especially in the preliminary stage this is an important factor, because a lot can still change during the brainstorming period. A design benefits most from a fast, correct and simple analysis which covers the important aspects for designing a save and reliable structure. After the preliminary phase and when more information is known about the project an optimisation can result in an economical design.

8. Conclusions and Recommendations

8.1. Conclusions

The goal of this thesis is to develop a consistent way of modelling large concrete slab foundations and optimize the interaction between structural and geotechnical engineers. The model should take into account the interaction between structure and soil and should be able to yield correct, fast (computational and modelling time) and useable results for the designer. The conclusions of this thesis can be split up into three parts. The first is related to the analytical analyses of different foundation models that were investigated, the second is related to the computational analyses and the third is related to the communication between structural and geotechnical engineer.

8.1.1. Analytical Analyses

- Three simple foundation models with constant parameters have been analysed: A Winkler foundation, a Pasternak foundation and a Gradient foundation. The models were represented by one dimensional differential equations and have been analysed analytically.

- It is found that each of these models have their own important drawbacks:
  - The Winkler model does not take spreading of load in the soil layers into account.
  - The Pasternak model does take spreading into account, however it is difficult to determine the second parameters, which is the shear modulus of the shear layer (G_p).
  - The Gradient model results were physically difficult to interpret, thus making the results unusable.

- In this study it is found that the results of Pasternak foundation model represents reality more consistently. Therefore, the interaction between structure and soil and the surrounding soil can be analysed with this model. This model only needs three input parameters (section 3.3.) for which the Gp value is difficult to determine.

8.1.2. Computational Analyses

- The program SCIA Engineer has a module called SOILin which can automatically determine the interaction parameters in a Pasternak foundation model. These interaction parameters are not constant over the slab area. The method includes a number of iterations in which linear elastic structural analysis are performed. In this report this method is called the Secant Method (SM). It can be concluded from the cases discussed in chapter 5 and 6 that this method is computational fast and it yields usable result.

- Large circular slabs on soft soil have been analysed using three methods; the SM, the linear finite element method (LFEM) and the nonlinear finite element method (NLFEM). The plate thickness and the nonlinear soil properties have been varied. The displacements predicted by the SM are much smaller (up to 0,25) than the NLFEM. The SM also predicts a larger moment (up to 2,5) than the moment by the NLFEM.

- What can be concluded from the computational study is that, if the displacement is important, the SM will calculate/provide values which are significantly smaller, compared to the Terzaghi and PLAXIS results, and another method must be used to verify the displacement. If the moment is important, the SM values can be used.

8.1.3. Communication between Structural and Geotechnical Engineer

- The interaction between the structural and geotechnical engineer can be guided by a checklist (page 80). This list summarises the important aspects which need to be discussed to come to a quick and an appropriate model for a final design. This is an initial step towards improving and optimizing the work approach. What both engineers should keep in mind is that a Winkler model with non- uniform coefficients appears most suitable for most slab foundations (section 6.4.).

8.2. Recommendations

- A remarkable error was found in the Pasternak model as implemented in "*SCIA Engineer 2012*". The contact stress between a slab and the subgrade is not in equilibrium with the load. This alone would be a reason to doubt the result of the program. However, the unbalance seems to be caused by a programming bug. The contact stress displays the Winkler part of the subgrade only (C_1 springs). It does not include the stresses in the Pasternak part (C_2 springs). The resultant of the contact stress is only in equilibrium with the load when the gradient spring has almost zero stiffness ($C_2 \approx 0$). It is recommended that this bug is fixed.

- The manual of the program SCIA Engineer has led to much confusion. The manual clearly states that the C_2 parameters act on the first derivative of the slab deflection [Manual Foundations and Subsoil 2012, p. 26, 27]. However, the name used for these parameters is Pasternak, which is inconsistent because G_p in a Pasternak foundation model acts on the second derivative of the slab deflection. The unit of the C_2 parameters also shows that they must act on de second derivative of the slab deflection. Moreover, this study shows that a foundation model including the first derivative does not make much sense. It is concluded that the manual must be wrong. Probably, it has been correctly programed in SCIA Engineer that the C_2 parameters act on the second derivative of the deflection. It is recommended that this apparent error in the manual is removed.

- In the studied cases the secant method (SM) strongly underestimates the settlements compared to the nonlinear finite element method (NLFEM). It is recommended that more cases are analysed. This can provide valuable information for geotechnical engineers to interpret secant method results, which is important because the secant method is applied by structural engineers and its results are communicated to geotechnical engineers without details on the working of the secant method.

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Link:
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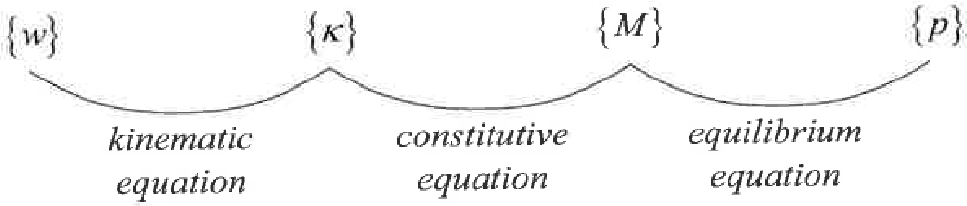
Glossary

BNE	Ballast Nedam Engineering
CAE	Computer-aided engineering
EC.2	Eurocode 2
EC.7	Eurocode 7
FE	Finite element
FEA	Finite element analysis
FEM	Finite element method
MC	Mohr-Coulomb
NEN	Nederlandse Norm
NLFEM	Nonlinear finite element method
LE	Linear Elastic
LFEM	Linear finite element method
SCIA	Scientific application
SBR	Stichting Bouw Research
SM	Secant Method
SOILin	Structure soil interaction

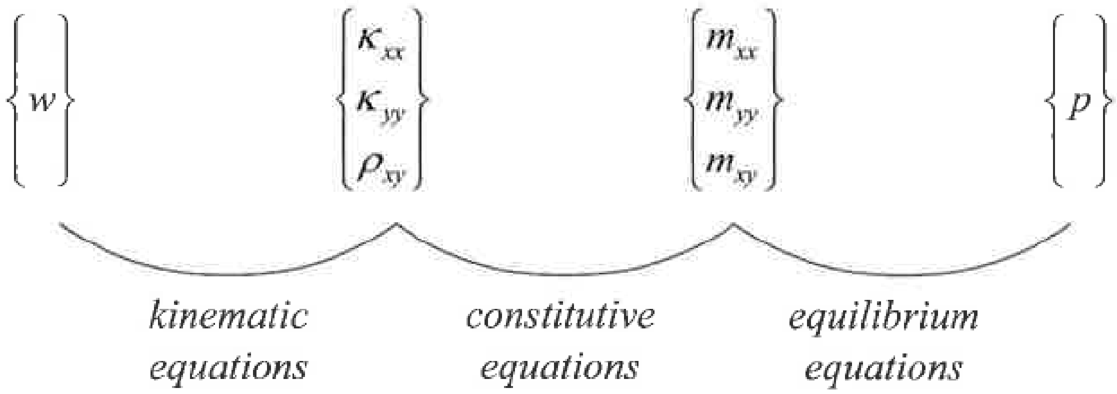
List of Figures and Tables

A. Constitutive, Kinematic and Equilibrium Equations for Thin and Thick plates

Relation scheme for slender beams (bending deformation only)



For **thin** plates there is only the bending deformation so the relationship is as follow:



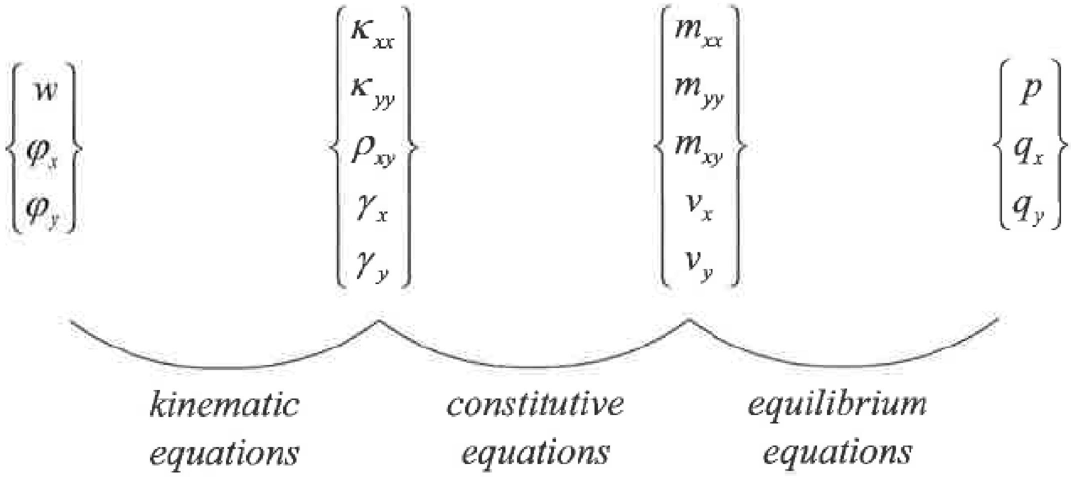
Kinematic equation Constitutive equation Equilibrium equation

$$\kappa_{xx} = -\frac{\partial^2 w}{\partial x^2}$$
$$\kappa_{yy} = -\frac{\partial^2 w}{\partial y^2}$$
$$\rho_{xy} = -2\frac{\partial^2 w}{\partial x \partial y}$$

$$m_{xx} = D\left(\kappa_{xx} + \upsilon\kappa_{yy}\right)$$
$$m_{yy} = D\left(\upsilon\kappa_{xx} + \kappa_{yy}\right)$$
$$m_{xy} = \frac{1}{2}(1-\upsilon)D\rho_{xy}$$

$$-\left(\frac{\partial^2 m_{xx}}{\partial x^2} + 2\frac{\partial^2 m_{xy}}{\partial x \partial y} + \frac{\partial^2 m_{yy}}{\partial y^2}\right) = p$$

Relation scheme for **thick** plates with both bending and shear deformation



The kinematic -, constitutive – and equilibrium equation for **thick** plates are given below.

Kinematic equation
equation

$$\kappa_{xx} = \frac{\partial \varphi_x}{\partial x}$$
$$\kappa_{yy} = \frac{\partial \varphi_y}{\partial y}$$
$$\rho_{xy} = \frac{\partial \varphi_x}{\partial y} + \frac{\partial \varphi_y}{\partial x}$$
$$\gamma_{xz} = \varphi_x + \frac{\partial w}{\partial x}$$
$$\gamma_{yz} = \varphi_y + \frac{\partial w}{\partial y}$$

Constitutive equation

$$m_{xx} = D\left(\kappa_{xx} + \upsilon\, \kappa_{yy}\right)$$
$$m_{yy} = D\left(\kappa_{yy} + \upsilon\, \kappa_{xx}\right)$$
$$m_{xy} = \frac{1}{2} D\left(1 - \upsilon\right) \rho_{xy}$$
$$v_x = D_\gamma\, \gamma_x$$
$$v_y = D_\gamma\, \gamma_y$$

Equilibrium

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + p = 0$$
$$\frac{\partial m_{xx}}{\partial x} + \frac{\partial m_{yx}}{\partial y} - v_x + q_x = 0$$
$$\frac{\partial m_{yy}}{\partial y} + \frac{\partial m_{xy}}{\partial x} - v_y + q_y = 0$$

w = displacement
φ = rotation
ρ = curvature in the $_{xy}$, $_{xz}$, $_{yz}$ direction
γ = strain in the $_x$ and $_y$ direction
m = moment
v = shear
p, *q* = loads
D = plate stiffness
υ = Poisson ratio (lateral contraction coefficient)
$_{x,\, y,\, z}$ = directions

B. Maple Calculations

B.1. Winkler Foundation Model

B.1.1. Free-free boundary

```
> restart:
> # Differental equation of the surrounding soil
> DE:= k*w(x) - G[p]*diff(w(x),x,x) - C[phi]*diff(w(x),x) = 0;
```

$$DE := k\,\mathrm{w}(x) - G_p\left(\frac{d^2}{dx^2}\mathrm{w}(x)\right) - C_\phi\left(\frac{d}{dx}\mathrm{w}(x)\right) = 0$$

```
> # Solution of the differential equation with two constants C1 and C2
> dsolve(DE,w(x));
```

$$\mathrm{w}(x) = \_C1\,\mathrm{e}^{\left(\frac{1}{2}\frac{\left(-C_\phi + \sqrt{C_\phi 2 + 4\,k\,G_p}\right)x}{G_p}\right)} + \_C2\,\mathrm{e}^{\left(-\frac{1}{2}\frac{\left(C_\phi + \sqrt{C_\phi 2 + 4\,k\,G_p}\right)x}{G_p}\right)}$$

```
> # The constant C1 must be zero because otherwise the solution explodes for x going to infinity
> ws:=C2*exp(-1/2*(C[phi]+(C[phi]^2+4*k*G[p])^(1/2))/G[p]*x);
```

$$ws := C2\,\mathrm{e}^{\left(-\frac{1}{2}\frac{\left(C_\phi + \sqrt{C_\phi 2 + 4\,k\,G_p}\right)x}{G_p}\right)}$$

```
> # The shear force V in the shear layer is Gp times slope.
> V:=-G[p]*diff(ws,x);
```

$$V := \frac{1}{2}\,C2\left(C_\phi + \sqrt{C_\phi 2 + 4\,k\,G_p}\right)\mathrm{e}^{\left(-\frac{1}{2}\frac{\left(C_\phi + \sqrt{C_\phi 2 + 4\,k\,G_p}\right)x}{G_p}\right)}$$

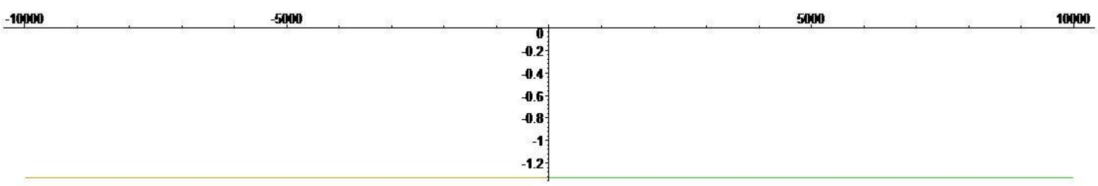
```
> # The surrounding soil can be replaced by a spring with stiffness K
> x:=0: K:=V/ws;
```

$$K := \frac{1}{2}\,C_\phi + \frac{1}{2}\sqrt{C_\phi 2 + 4\,k\,G_p}$$

```
> restart:
> l:=20000:        # mm     plate length
> h:=1000:         # mm     plate thickness
> b:=1000:         # mm     strip width (has no influence on the results)
> Es:=45:          # N/mm2  Young's modulus soil
> Ec:=0.1e5:       # N/mm2  Young's modulus concrete
> d:=6000:         # mm     depth of the soil layer
> p:=0.01:         # N/mm2  load on the plate (10 kN/m2 = 0.01 N/mm2)
> r:=10000:        # mm     width of the surrounding soil (has no influence)
>
> Ix:=1/12*b*h^3:  # mm4    moment of inertia
> EI:=Ec*Ix:       # Nmm2   bending stiffness
> k:=Es*b/d:       # N/mm2  modulus of subgrade reaction
> G[p]:=0:         # N      stiffness of the soil shear layer
> C[phi]:=0:       # N/mm   gradient stiffness
> q:=p*b:          # N/mm   strip load
>
> K:=1/2*C[phi]+1/2*sqrt(C[phi]^2+4*k*G[p]):
> # Differential equation of the plate with soil below without surrounding soil.
> DE:= EI*diff(w(x),x,x,x,x) + k*w(x) - G[p]*diff(w(x),x,x) - C[phi]*diff(w(x),x) = q:
> RV:= D(w)(0)=0,                     # in the middle no slope
      (D@@3)(w)(0)=0,                 # in the middle no shear force
      (D@@2)(w)(l/2)=0,               # at the edge no moment in the plate
      -EI*(D@@3)(w)(l/2)+G[p]*D(w)(l/2)+K*w(l/2)=0:
                                      # shear force in plate plus shear layer continues into the surrounding soil
> OP:=dsolve({DE,RV},w(x)):
> assign(OP):
> C2:=1.4; # This number needs to be manually adjusted such that the deflection figure is continues.
```

$$C2 := 1.4$$

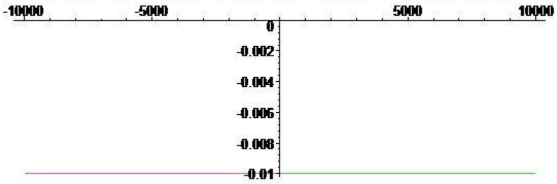
```
> ws:=C2*exp(-1/2*(C[phi]+(C[phi]^2+4*k*G[p])^(1/2))/G[p]*x):
> # deflection [mm]
> plot({[x,0,x=0..l/2],[x,-w(x),x=0..l/2],[-x,-w(x),x=0..l/2],[x+l/2,-ws,x=0..r],[-x-l/2,-ws,x=0..r]});
```



```
> # contact stress N/mm2
> s1:=-(k*w(x)-G[p]*diff(w(x),x,x)-C[phi]*diff(w(x),x))/b:
> s2:=-0/(1/2-0.9*1/2)/b; # This number needs to be manually adjusted. It is the shear force at the edge of the plate
```

$$s2 := -0.$$

```
> plot({[x,0,x=0..1/2],[x,s1,x=0..1/2],[-x,s1,x=0..1/2],[x,s2,x=0.9*1/2..1/2],[x,s2,x=-1/2..-0.9*1/2]});
```




```
> # The stress due to the concentrated force entering the plate edge is displayed as horizontal lines.
```

B.1.2. Very stiff boundary

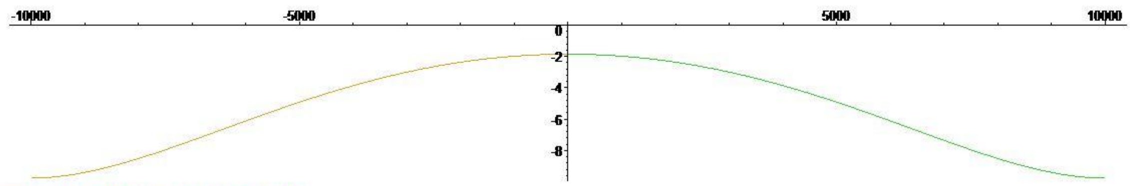
```
> restart:
> # Differental equation of the surrounding soil
> DE:= k*w(x) - G[p]*diff(w(x),x,x) - C[phi]*diff(w(x),x) = 0:
> # Solution of the differential equation with two constants C1 and C2
> dsolve(DE,w(x)):
> # The constant C1 must be zero because otherwise the solution explodes for x going to infinity
> ws:=C2*exp(-1/2*(C[phi]+(C[phi]^2+4*k*G[p])^(1/2))/G[p]*x):
> # The shear force V in the shear layer is Gp times slope.
> V:=-G[p]*diff(ws,x):
> # The surrounding soil can be replaced by a spring with stiffness K
> x:=0: K:=V/ws;
```

$$K := \frac{1}{2}C_{\phi} + \frac{1}{2}\sqrt{C_{\phi}^2 + 4\,k\,G_p}$$

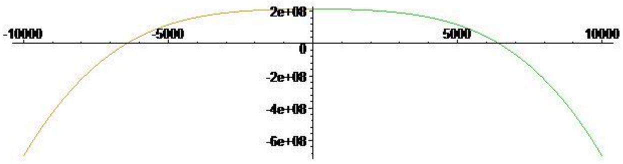
```
> restart:
> l:=20000:        # mm     plate length
> h:=1000:         # mm     plate thickness
> b:=1000:         # mm     strip width (has no influence on the results)
> Es:=45:          # N/mm2  Young's modulus soil
> Ec:=0.1e5:       # N/mm2  Young's modulus concrete
> d:=6000:         # mm     depth of the soil layer
> p:=0.01:         # N/mm2  load on the plate (10 kN/m2 = 0.01 N/mm2)
> r:=10000:        # mm     width of the surrounding soil (has no influence)
> f:=300:          # kN/m   line load = N/mm
>
> Ix:=1/12*b*h^3:  # mm4    moment of inertia
> EI:=Ec*Ix:       # Nmm2   bending stiffness
> k:=Es*b/d:       # N/mm2  modulus of subgrade reaction
> G[p]:=0:         # N      stiffness of the soil shear layer
> C[phi]:=0:       # N/mm   gradient stiffness
> q:=p*b:          # N/mm   strip load
> P:=f*b:          # N      end load
>
> K:=1/2*C[phi]+1/2*sqrt(C[phi]^2+4*k*G[p]):
> # Differential equation of the plate with soil below without surrounding soil.
> DE:= EI*diff(w(x),x,x,x,x) + k*w(x) - G[p]*diff(w(x),x,x) - C[phi]*diff(w(x),x) = q:
> RV:= D(w)(0)=0,                # in the middle no slope
      (D@@3)(w)(0)=0,            # in the middle no shear force
      (D)(w)(l/2)=0,             # at the edge no rotation of the plate
      -EI*(D@@3)(w)(l/2) + G[p]*D(w)(l/2) + K*w(l/2) - P =0:
                                 # shear force in plate plus shear layer continues into the surrounding soil
> OP:=dsolve({DE,RV},w(x)):
> assign(OP):
> C2:=8.3; # This number needs to be manually adjusted such that the deflection figure is continues.
```

$$C2 := 8.3$$

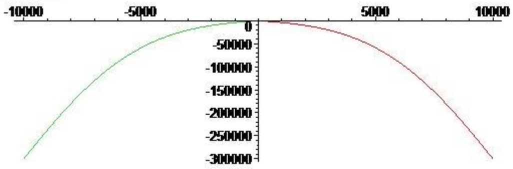
```
> ws:=C2*exp(-1/2*(C[phi]+(C[phi]^2+4*k*G[p])^(1/2))/G[p]*x):
> # deflection [mm]
> plot({[x,0,x=0..l/2],[x,-w(x),x=0..l/2],[-x,-w(x),x=0..l/2],[x+l/2,-ws,x=0..r],[-x-l/2,-ws,x=0..r]});
```



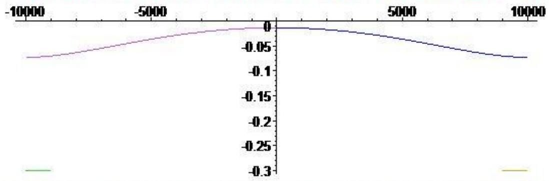

```
> # moment distribution in the plate
> M:=-EI*diff(w(x),x,x):
> plot({[x,0,x=0..l/2],[x,-M,x=0..l/2],[-x,-M,x=0..l/2]});
```




```
> # shear force in the plate
> V:=diff(M,x):
> plot({[x,-V,x=0..l/2],[-x,-V,x=0..l/2]});
```




```
> # contact stress N/mm2
> s1:=-(k*w(x)-G[p]*diff(w(x),x,x)-C[phi]*diff(w(x),x))/b:
> s2:=-300000/(l/2-0.9*l/2)/b; # This number needs to be manually adjusted. It is the shear force at the edge of the plate
                    s2 := -0.3000000000
> plot({[x,0,x=0..l/2],[x,s1,x=0..l/2],[-x,s1,x=0..l/2],[x,s2,x=0.9*l/2..l/2],[x,s2,x=-l/2..-0.9*l/2]});
```



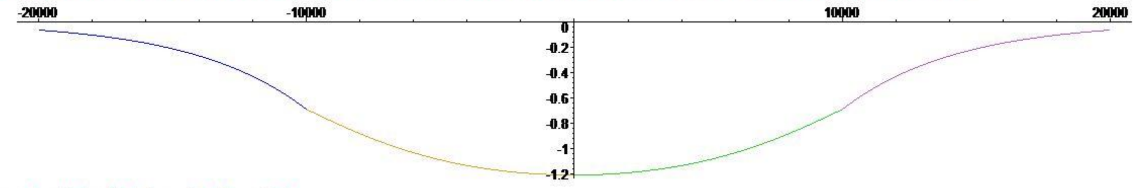

```
> # The stress due to the concentrated force entering the plate edge is displayed as horizontal lines.
```

B.2. Pasternak Foundation Model

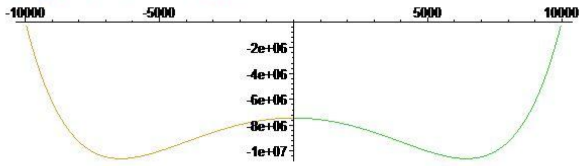
B.2.1. Free-free boundary

```
[> restart:
[> # Differental equation of the surrounding soil
[> DE:= k*w(x) - G[p]*diff(w(x),x,x) - C[phi]*diff(w(x),x) = 0:
[> # Solution of the differential equation with two constants C1 and C2
[> dsolve(DE,w(x)):
[> # The constant C1 must be zero because otherwise the solution explodes for x going to infinity
[> ws:=C2*exp(-1/2*(C[phi]+(C[phi]^2+4*k*G[p])^(1/2))/G[p]*x):
[> # The shear force V in the shear layer is Gp times slope.
[> V:=-G[p]*diff(ws,x):
[> # The surrounding soil can be replaced by a spring with stiffness K
[> x:=0: K:=V/ws:
[> restart:
[> l:=20000:        # mm    plate length
[> h:=1000:         # mm    plate thickness
[> b:=1000:         # mm    strip width (has no influence on the results)
[> Es:=45:          # N/mm2 Young's modulus soil
[> Ec:=0.1e5:       # N/mm2 Young's modulus concrete
[> d:=6000:         # mm    depth of the soil layer
[> p:=0.01:         # N/mm2 load on the plate (10 kN/m2 = 0.01 N/mm2)
[> r:=10000:        # mm    width of the surrounding soil (has no influence)
[>
[> Ix:=1/12*b*h^3: # mm4   moment of inertia
[> EI:=Ec*Ix:       # Nmm2  bending stiffness
[> k:=Es*b/d:       # N/mm2 modulus of subgrade reaction
[> G[p]:=Es/2*d*b: # N     stiffness of the soil shear layer
[> C[phi]:=0.00001:# N/mm  gradient stiffness
[> q:=p*b:          # N/mm  strip load
[>
[> K:=1/2*C[phi]+1/2*sqrt(C[phi]^2+4*k*G[p]):
[> # Differential equation of the plate with soil below without surrounding soil.
[> DE:= EI*diff(w(x),x,x,x,x) + k*w(x) - G[p]*diff(w(x),x,x) - C[phi]*diff(w(x),x) = q:
[> RV:= D(w)(0)=0,                  # in the middle no slope
       (D@@3)(w)(0)=0,              # in the middle no shear force
       (D@@2)(w)(l/2)=0,            # at the edge no moment in the plate
       -EI*(D@@3)(w)(l/2)+G[p]*D(w)(l/2)+K*w(l/2)=0:
                                    # shear force in plate plus shear layer continues into the surrounding soil
[> OP:=dsolve({DE,RV},w(x)):
[> assign(OP):
[> C2:=0.69; # This number needs to be manually adjusted such that the deflection figure is continues.
                                                   C2 := 0.69
```

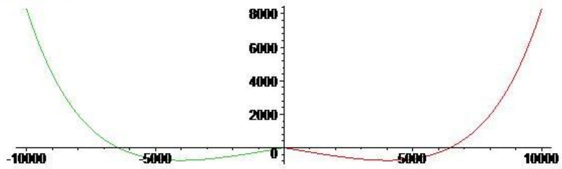
```
> ws:=C2*exp(-1/2*(C[phi]+(C[phi]^2+4*k*G[p])^(1/2))/G[p]*x):
> # deflection [mm]
> plot({[x,0,x=0..l/2],[x,-w(x),x=0..l/2],[-x,-w(x),x=0..l/2],[x+l/2,-ws,x=0..r],[-x-l/2,-ws,x=0..r]});
```



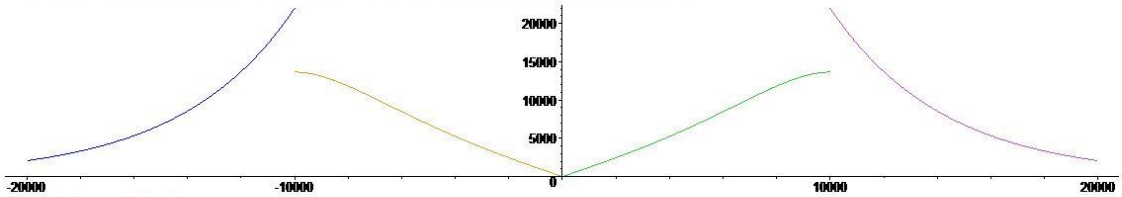

```
> # moment distribution in the plate
> M:=-EI*diff(w(x),x,x):
> plot({[x,0,x=0..l/2],[x,-M,x=0..l/2],[-x,-M,x=0..l/2]});
```




```
> # shear force in the plate
> V:=diff(M,x):
> plot({[x,-V,x=0..l/2],[-x,-V,x=0..l/2]});
```



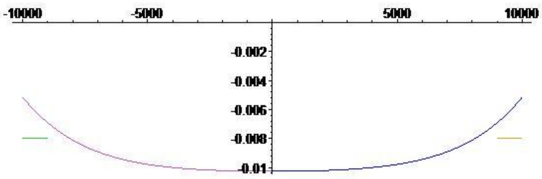

```
> # shear force in the soil shear layer
> plot({[x,0,x=0..l/2],[x,-G[p]*diff(w(x),x),x=0..l/2],[-x,-G[p]*diff(w(x),x),x=0..l/2],
       [x+l/2,-G[p]*diff(ws,x),x=0..r],[-x-l/2,-G[p]*diff(ws,x),x=0..r]});
```




```
> # contact stress N/mm2
> s1:=-(k*w(x)-G[p]*diff(w(x),x,x)-C[phi]*diff(w(x),x))/b:
> s2:=-8000/(l/2-0.9*l/2)/b; # This number needs to be manually adjusted. It is the shear force at the edge of the plate
```

$s2 := -0.008000000000$

```
> plot({[x,0,x=0..l/2],[x,s1,x=0..l/2],[-x,s1,x=0..l/2],[x,s2,x=0.9*l/2..l/2],[x,s2,x=-l/2..-0.9*l/2]});
```



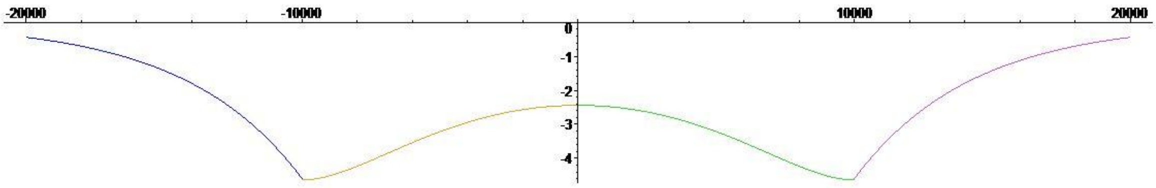

```
> # The stress due to the concentrated force entering the plate edge is displayed as vertical lines.
```

B.2.2. Very stiff boundary

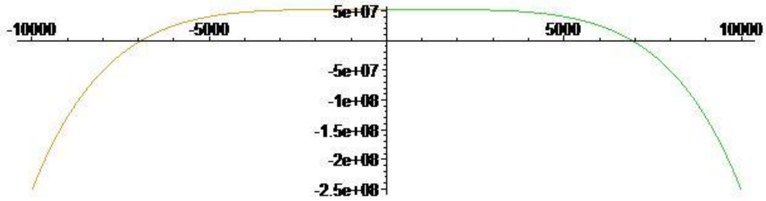
```
> restart:
> # Differental equation of the surrounding soil
> DE:= k*w(x) - G[p]*diff(w(x),x,x) - C[phi]*diff(w(x),x) = 0:
> # Solution of the differential equation with two constants C1 and C2
> dsolve(DE,w(x)):
> # The constant C1 must be zero because otherwise the solution explodes for x going to infinity
> ws:=C2*exp(-1/2*(C[phi]+(C[phi]^2+4*k*G[p])^(1/2))/G[p]*x):
> # The shear force V in the shear layer is Gp times slope.
> V:=-G[p]*diff(ws,x):
> # The surrounding soil can be replaced by a spring with stiffness K
> x:=0: K:=V/ws:
> restart:
> l:=20000:        # mm     plate length
> h:=1000:         # mm     plate thickness
> b:=1000:         # mm     strip width (has no influence on the results)
> Es:=45:          # N/mm2  Young's modulus soil
> Ec:=0.1e5:       # N/mm2  Young's modulus concrete
> d:=6000:         # mm     depth of the soil layer
> p:=0.01:         # N/mm2  load on the plate (10 kN/m2 = 0.01 N/mm2)
> r:=10000:        # mm     width of the surrounding soil (has no influence)
> f:=300:          # kN/m   line load = N/mm
>
> Ix:=1/12*b*h^3:  # mm4    moment of inertia
> EI:=Ec*Ix:       # Nmm2   bending stiffness
> k:=Es*b/d:       # N/mm2  modulus of subgrade reaction
> G[p]:=Es/2*d*b:  # N      stiffness of the soil shear layer
> C[phi]:=0.00001:# N/mm   gradient stiffness
> q:=p*b:          # N/mm   strip load
> P:=f*b:          # N      end load
> K:=1/2*C[phi]+1/2*sqrt(C[phi]^2+4*k*G[p]):
> # Differential equation of the plate with soil below without surrounding soil.
> DE:= EI*diff(w(x),x,x,x,x) + k*w(x) - G[p]*diff(w(x),x,x) - C[phi]*diff(w(x),x) = q:
> RV:= D(w)(0)=0,                # in the middle no slope
       (D@@3)(w)(0)=0,           # in the middle no shear force
       (D)(w)(l/2)=0,            # at the edge no rotation of the plate
      -EI*(D@@3)(w)(l/2) + G[p]*D(w)(l/2) + K*w(l/2) - P =0:
                                 # shear force in plate plus shear layer continues into the surrounding soil
> OP:=dsolve({DE,RV},w(x)):
> assign(OP):
> C2:=4.6; # This number needs to be manually adjusted such that the deflection figure is continues.
```

$$C2 := 4.6$$

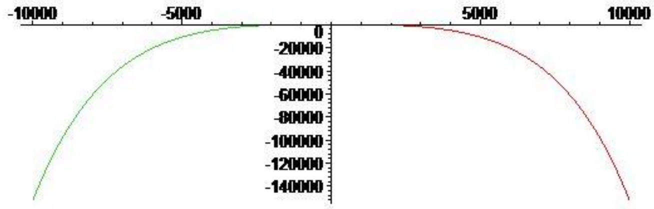
```
> ws:=C2*exp(-1/2*(C[phi]+(C[phi]^2+4*k*G[p])^(1/2))/G[p]*x):
> # deflection [mm]
> plot({[x,0,x=0..l/2],[x,-w(x),x=0..l/2],[-x,-w(x),x=0..l/2],[x+l/2,-ws,x=0..r],[-x-l/2,-ws,x=0..r]});
```



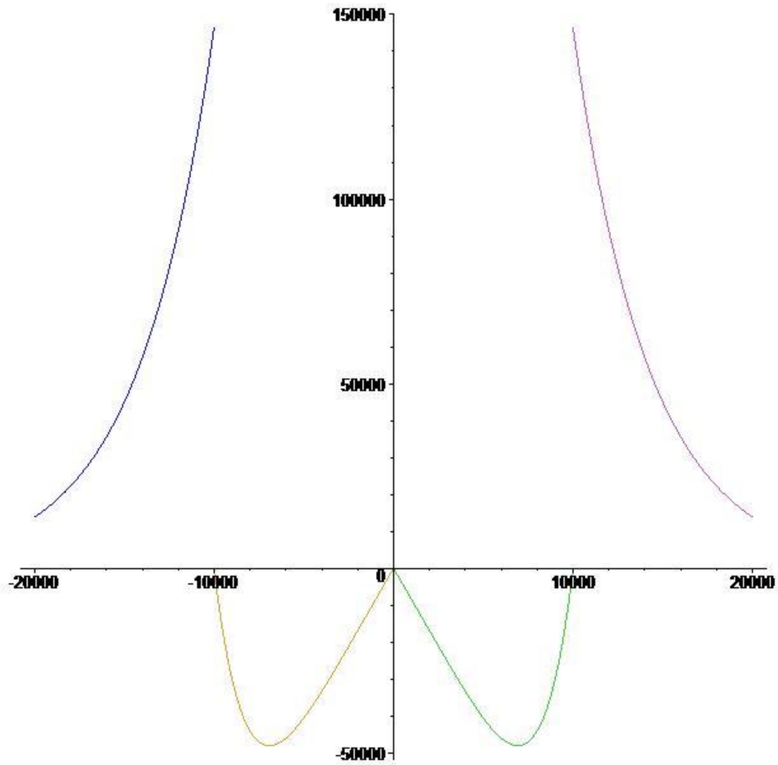
```
> # moment distribution in the plate
> M:=-EI*diff(w(x),x,x):
> plot({[x,0,x=0..l/2],[x,-M,x=0..l/2],[-x,-M,x=0..l/2]});
```




```
> # shear force in the plate
> V:=diff(M,x):
> plot({[x,-V,x=0..l/2],[-x,-V,x=0..l/2]});
```



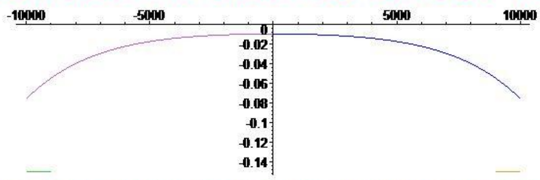

```
> # shear force in the soil shear layer
> plot({[x,0,x=0..l/2],[x,-G[p]*diff(w(x),x),x=0..l/2],[-x,-G[p]*diff(w(x),x),x=0..l/2],
      [x+l/2,-G[p]*diff(ws,x),x=0..r],[-x-l/2,-G[p]*diff(ws,x),x=0..r]});
```




```
> # contact stress N/mm2
> s1:=-(k*w(x)-G[p]*diff(w(x),x,x)-C[phi]*diff(w(x),x))/b:
> s2:=-150000/(l/2-0.9*l/2)/b; # This number needs to be manually adjusted. It is the shear force at the edge of the plate
```

$s2 := -0.1500000000$

```
> plot({[x,0,x=0..l/2],[x,s1,x=0..l/2],[-x,s1,x=0..l/2],[x,s2,x=0.9*l/2..l/2],[x,s2,x=-l/2..-0.9*l/2]});
```




```
> # The stress due to the concentrated force entering the plate edge is displayed as horizontal lines.
```

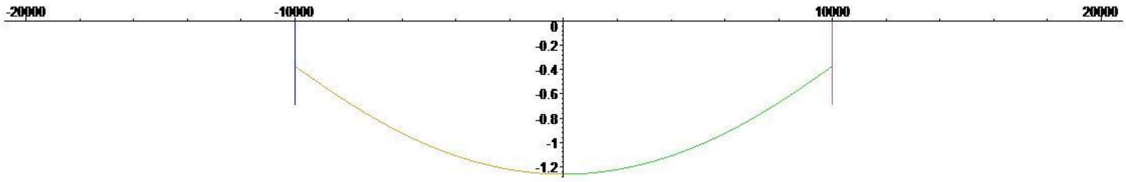
B.3. Gradient Foundation Model

B.3.1. Free-free boundary

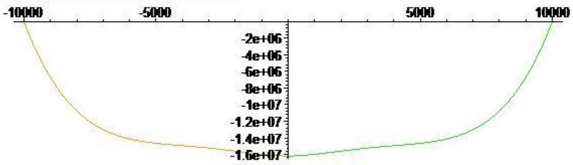
```
[> restart:
[> # Differental equation of the surrounding soil
[> DE:= k*w(x) - G[p]*diff(w(x),x,x) - C[phi]*diff(w(x),x) = 0:
[> # Solution of the differential equation with two constants C1 and C2
[> dsolve(DE,w(x)):
[> # The constant C1 must be zero because otherwise the solution explodes for x going to infinity
[> ws:=C2*exp(-1/2*(C[phi]+(C[phi]^2+4*k*G[p])^(1/2))/G[p]*x):
[> # The shear force V in the shear layer is Gp times slope.
[> V:=-G[p]*diff(ws,x):
[> # The surrounding soil can be replaced by a spring with stiffness K
[> x:=0: K:=V/ws:
[> restart:
[> l:=20000:        # mm    plate length
[> h:=1000:         # mm    plate thickness
[> b:=1000:         # mm    strip width (has no influence on the results)
[> Es:=45:          # N/mm2 Young's modulus soil
[> Ec:=0.1e5:       # N/mm2 Young's modulus concrete
[> d:=6000:         # mm    depth of the soil layer
[> p:=0.01:         # N/mm2 load on the plate  (10 kN/m2 = 0.01 N/mm2)
[> r:=10000:        # mm    width of the surrounding soil (has no influence)
[>
[> Ix:=1/12*b*h^3: # mm4   moment of inertia
[> EI:=Ec*Ix:       # Nmm2  bending stiffness
[> k:=Es*b/d:       # N/mm2 modulus of subgrade reaction
[> G[p]:=0.00001:   # N     stiffness of the soil shear layer
[> C[phi]:=Es/2*b:  # N/mm  gradient stiffness
[> q:=p*b:          # N/mm  strip load
[> K:=1/2*C[phi]+1/2*sqrt(C[phi]^2+4*k*G[p]):
[> # Differential equation of the plate with soil below without surrounding soil.
[> DE:= EI*diff(w(x),x,x,x,x) + k*w(x) - G[p]*diff(w(x),x,x) - C[phi]*diff(w(x),x) = q:
[> RV:= D(w)(0)=0,                  # in the middle no slope
       (D@@3)(w)(0)=0,              # in the middle no shear force
       (D@@2)(w)(l/2)=0,            # at the edge no moment in the plate
       -EI*(D@@3)(w)(l/2)+G[p]*D(w)(l/2)+K*w(l/2)=0:
                                    # shear force in plate plus shear layer continues into the surrounding soil
[> OP:=dsolve({DE,RV},w(x)):
[> assign(OP):
[> C2:=0.69; # This number needs to be manually adjusted such that the deflection figure is continues.
```

$$C2 := 0.69$$

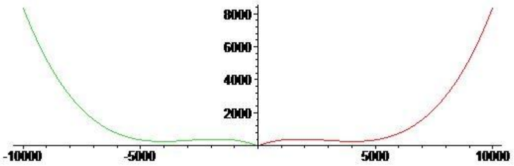
```
[> ws:=C2*exp(-1/2*(C[phi]+(C[phi]^2+4*k*G[p])^(1/2))/G[p]*x):
[> # deflection [mm]
[> plot({[x,0,x=0..l/2],[x,-w(x),x=0..l/2],[-x,-w(x),x=0..l/2],[x+l/2,-ws,x=0..r],[-x-l/2,-ws,x=0..r]});
```



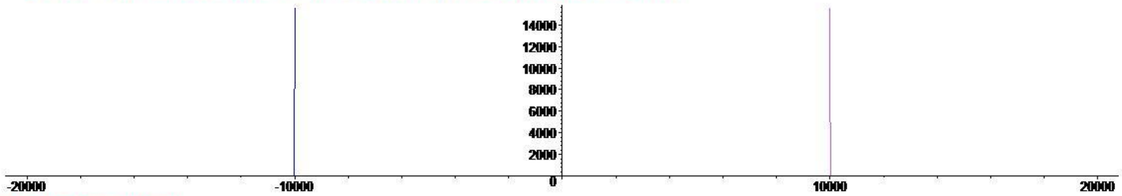

```
[> # moment distribution in the plate
[> M:=-EI*diff(w(x),x,x):
[> plot({[x,0,x=0..l/2],[x,-M,x=0..l/2],[-x,-M,x=0..l/2]});
```



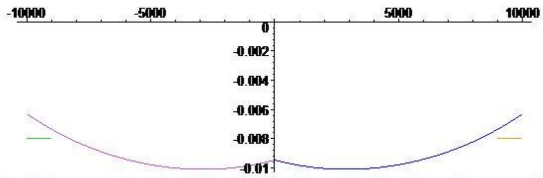
```
> # shear force in the plate
> V:=diff(M,x):
> plot({[x,-V,x=0..l/2],[-x,-V,x=0..l/2]});
```




```
> # shear force in the soil shear layer
> plot({[x,0,x=0..l/2],[x,-G[p]*diff(w(x),x),x=0..l/2],[-x,-G[p]*diff(w(x),x),x=0..l/2],
       [x+l/2,-G[p]*diff(ws,x),x=0..r],[-x-l/2,-G[p]*diff(ws,x),x=0..r]});
```




```
> # contact stress N/mm2
> s1:=-(k*w(x)-G[p]*diff(w(x),x,x)-C[phi]*diff(w(x),x))/b:
> s2:=-8000/(l/2-0.9*l/2)/b; # This number needs to be manually adjusted. It is the shear force at the edge of the plate
                              s2 := -0.008000000000
> plot({[x,0,x=0..l/2],[x,s1,x=0..l/2],[-x,s1,x=0..l/2],[x,s2,x=0.9*l/2..l/2],[x,s2,x=-l/2..-0.9*l/2]});
```




```
> # The stress due to the concentrated force entering the plate edge is displayed as vertical lines.
```

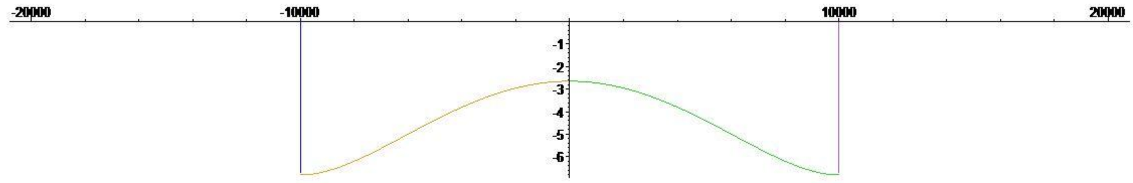
B.3.2. Very stiff boundary

```
> restart:
> # Differental equation of the surrounding soil
> DE:= k*w(x) - G[p]*diff(w(x),x,x) - C[phi]*diff(w(x),x) = 0:
> # Solution of the differential equation with two constants C1 and C2
> dsolve(DE,w(x)):
> # The constant C1 must be zero because otherwise the solution explodes for x going to infinity
> ws:=C2*exp(-1/2*(C[phi]+(C[phi]^2+4*k*G[p])^(1/2))/G[p]*x):
> # The shear force V in the shear layer is Gp times slope.
> V:=-G[p]*diff(ws,x):
> # The surrounding soil can be replaced by a spring with stiffness K
> x:=0: K:=V/ws:
> restart:
> l:=20000:        # mm     plate length
> h:=1000:         # mm     plate thickness
> b:=1000:         # mm     strip width (has no influence on the results)
> Es:=45:          # N/mm2  Young's modulus soil
> Ec:=0.1e5:       # N/mm2  Young's modulus concrete
> d:=6000:         # mm     depth of the soil layer
> p:=0.01:         # N/mm2  load on the plate (10 kN/m2 = 0.01 N/mm2)
> r:=10000:        # mm     width of the surrounding soil (has no influence)
> f:=300:          # kN/m   line load = N/mm
>
> Ix:=1/12*b*h^3:  # mm4    moment of inertia
> EI:=Ec*Ix:       # Nmm2   bending stiffness
> k:=Es*b/d:       # N/mm2  modulus of subgrade reaction
> G[p]:=0.00001:   # N      stiffness of the soil shear layer
> C[phi]:=Es/2*b:  # N/mm   gradient stiffness
> q:=p*b:          # N/mm   strip load
> P:=f*b:          # N      end load
```

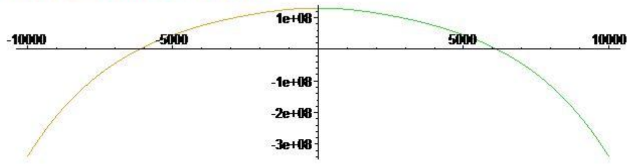
```
> K:=1/2*C[phi]+1/2*sqrt(C[phi]^2+4*k*G[p]):
> # Differential equation of the plate with soil below without surrounding soil.
> DE:= EI*diff(w(x),x,x,x,x) + k*w(x) - G[p]*diff(w(x),x,x) - C[phi]*diff(w(x),x) = q:
> RV:= D(w)(0)=0,                    # in the middle no slope
      (D@@3)(w)(0)=0,               # in the middle no shear force
      (D)(w)(l/2)=0,                # at the edge no rotation of the plate
      -EI*(D@@3)(w)(l/2) + G[p]*D(w)(l/2) + K*w(l/2) - P =0:
                                    # shear force in plate plus shear layer continues into the surrounding soil
> OP:=dsolve({DE,RV},w(x)):
> assign(OP):
> C2:=6.7; # This number needs to be manually adjusted such that the deflection figure is continues.
```

$$C2 := 6.7$$

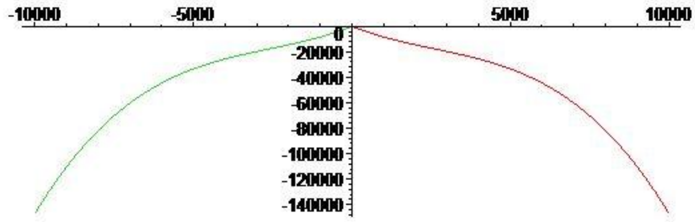
```
> ws:=C2*exp(-1/2*(C[phi]+(C[phi]^2+4*k*G[p])^(1/2))/G[p]*x):
> # deflection [mm]
> plot({[x,0,x=0..l/2],[x,-w(x),x=0..l/2],[-x,-w(x),x=0..l/2],[x+l/2,-ws,x=0..r],[-x-l/2,-ws,x=0..r]});
```



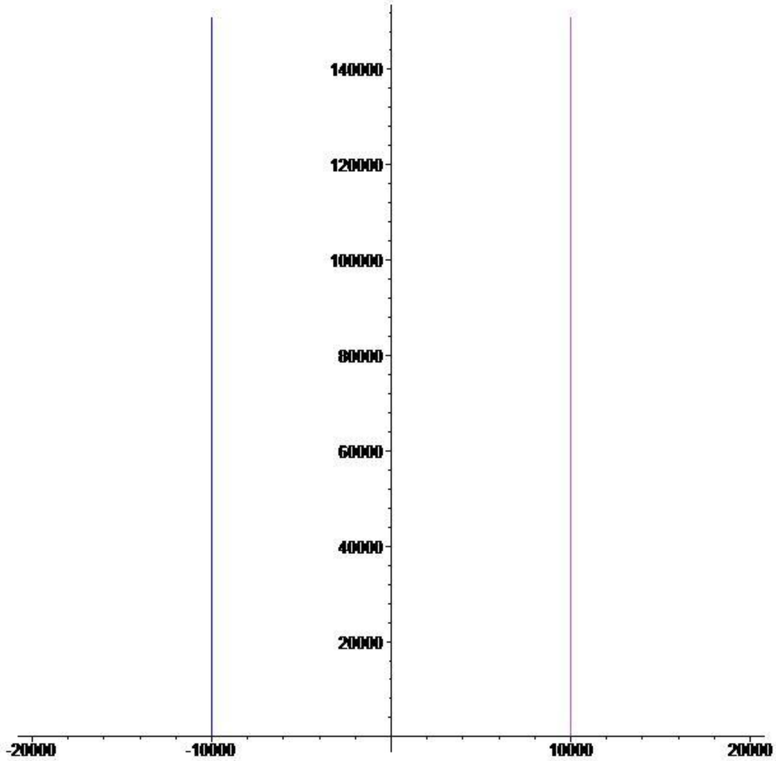

```
> # moment distribution in the plate
> M:=-EI*diff(w(x),x,x):
> plot({[x,0,x=0..l/2],[x,-M,x=0..l/2],[-x,-M,x=0..l/2]});
```




```
> # shear force in the plate
> V:=diff(M,x):
> plot({[x,-V,x=0..l/2],[-x,-V,x=0..l/2]});
```



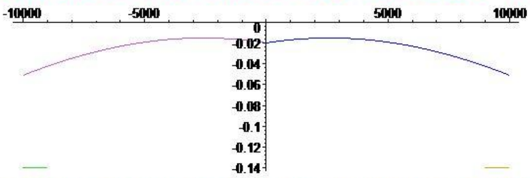
```
> # shear force in the soil shear layer
> plot({[x,0,x=0..l/2],[x,-G[p]*diff(w(x),x),x=0..l/2],[-x,-G[p]*diff(w(x),x),x=0..l/2],
       [x+l/2,-G[p]*diff(ws,x),x=0..r],[-x-l/2,-G[p]*diff(ws,x),x=0..r]});
```




```
> # contact stress N/mm2
> s1:=-(k*w(x)-G[p]*diff(w(x),x,x)-C[phi]*diff(w(x),x))/b:
> s2:=-140000/(l/2-0.9*l/2)/b; # This number needs to be manually adjusted. It is the shear force at the edge of the plate
```

$$s2 := -0.1400000000$$

```
> plot({[x,0,x=0..l/2],[x,s1,x=0..l/2],[-x,s1,x=0..l/2],[x,s2,x=0.9*l/2..l/2],[x,s2,x=-l/2..-0.9*l/2]});
```




```
> # The stress due to the concentrated force entering the plate edge is displayed as horizontal lines.
```

C. Determination Soil Input Parameters Using SM

The input parameters which need to be determined when using the SM are:

- Height of the soil layer
- Young's modules
- Poisson ratio
- Unsaturated specific soil weight
- Saturated specific soil weight
- Structural strength coefficient
- Ground water level

The height of the soil layer must be determined on the basis of geological data. From a sounding graph the cone resistance and friction can be determined. From these two values the friction number (in percentages) can be determined. This number is the friction divided by the cone resistance (q_c) and the answer multiplied by 100%. If the values are 1% à 2% it indicates that there is sand present. If the values are 3% à 5% there is clay and for 8% à 10% indicates that there is veen [9]. With the help of this method the height of the soil layer can be determined. A sounding graph in combination with Table C-1 can help in determining the height of a soil layer.

Table C-1 Friction number and cone resistance for differ types of soil [9]

Type of soil	Friction number	q_c
Sand, medium-rough	0,4%	
Sand, fine – medium	0,6%	5-30 MPa
Sand, fine	0,8%	
Sand, silt	1,1%	
Sand, clay	1,4%	5-10 MPa
Sand, clay or loam	1,8%	
Silt	2,2%	
Clay, silt	2,5%	
Clay	3,3%	0,5-2 MPa
Clay , humic	5,0%	
Peat	8,1%	0-1 MPa

The designer must also be aware that an equal amount of layers need to be input in the geological profile. If a certain layer is absent in the data a very thin layer of 1mm can be chosen to make the amount of layers equal to one another.

The soil young's modulus (E_s) is a soil property which describes the stiffness of the soil. It should not be confused with the modulus of sub-grade reaction (k) which is a model property which describes the soil stiffness in a foundation model. The E_s can be determined according to table 2.b from the Eurocode 7 section 2.4.5.2 [19]. For soil however the modulus can differ quite a lot due to the type of loading. Globally the young's modulus of a short load can be a factor 2 à 3 higher.

For dynamical loads the E_s can be determined with following rule of thumb [26]:

$E_{dynamic,soil}$ = 2,5*$E_{static,soil}$

The Poisson ration, named after Siméon Poisson, is a ratio which describes the amount an object deforms in one direction if it is loaded in the other direction. The ratio between these two quantities is the Poisson's ratio.

The Poisson's ration for soil (υ_s) can be related to the internal angle of soil (φ). The φ can be determined out of table 2.b from the Eurocode 7 section 2.4.5.2 [19]. The υ_s and φ has the following relation (from the CUR rapport 2003-7 [27]):

$$k_0 = 1 - sin\varphi$$

$$k_0 = \frac{\upsilon_s}{1 - \upsilon_s}$$

$$k_0 = \frac{\sigma'_h}{\sigma'_v}$$

k_0 = neutral soil compression coefficient
σ'_h = horizontal grain pressure
σ'_v = vertical grain pressure
υ_s = Poisson's ratio for soil
φ = internal angle of soil
Also with the following relation we can find the υ_s.

$$\upsilon_s = \frac{1 - sin\varphi}{2 - sin\varphi}$$

The unsaturated and saturated specific soil weight can determine according to table 2.b from the Eurocode 7 [19].

The structural strength coefficient of "*m*" depends on which norms the designer uses. If the Eurocode norms are used the m = 0,2 by default and cannot be changed in "*SCIA engineering 2012*" program. If the Czech norms are used the structural strength can be changed. This however not advice as this study has not focus on the effect the change of this value will have on the model.

D. Modulus of Sub-Grade Reaction

In the following literature (k) value can be found:

- CUR aanbeveling 36:2011 "design of concrete floors and pavements on elastic foundations" [24]
- STUVO-rapport 91 [1990] "elastisch ondersteunde bedrijfsvloeren van beton" Eindrapport van STUVO-cel 130 (table on page -7.28-) [25]
- SBR [1991] (Stichting bouw Research) tabel.1 blz. 19 [4]

CUR aanbeveling 36

The information of this table is recommended to be used for floors and pavements that are made of cast-in-structural concrete (in-situ), and which are elastically supported by means of a shallow foundation. The floors or pavements must be kept free of other components of the structure and should not form part of the building foundation. In the recommendation nothing is stated about the liquid density of the floors (please refer to the CUR/PBV-Aanbevlingen 44 and 65). This recommendation does not apply to plate structures that serve as a foundation of (heavy) industrial installations. Furthermore, the recommendation does not apply to unreinforced concrete floors where storage racks are placed which if failures occur can cause major economic damage and/or personal injury.

These k values are derived for dynamical loads acting on the foundation. If they want to be used for uniform distributed loads they should be divided by a factor 3 (k/3). In Table D-1 the values of the modulus of sub-grade reaction values can be seen for different soil properties.

Table D-1 *k* values as presented in CUR aanbeveling 36 [24]

Tabel 2 Enkele richtwaarden voor verschillende grondeigenschappen

soort ondergrond	conuswst. q_c N/mm²	beddinggetal k 1) N/mm³	elast.mod. E_{dyn} N/mm²	CBR-waarde 2) %	a -	b -
veen	0,1-0,3	0,01-0,02	10-35	1-2	0,5	10,0
klei	0,2-2,5	0,02-0,04	15-60	3-8	1-2	7,5
leem	1,0-3,0	0,03-0,06	50-100	5-10	-	5,0
zand	3,0-25,0	0,04-0,10	70-200	8-18	4-7	2,5
grind/zand	10,0-30,0	0,08-0,13	120-300	15-40	-	1,5

1) The modulus of sub-grade reaction should be a factor 3 smaller (k/3) in the case of uniform distributed load.
2) CBR = California Bearing Ratio

STUVO-rapport 91

The moduli of sub-grade reaction of this report are global values for block, point and wheel loads.

Table D-2 *k* values as presented in STUVO-rapport 91 [25]

GRONDSOORT	BEDDINGSCONSTANTE IN MN/M^3		
	BLOKLASTEN	PUNTLASTEN	WIELLASTEN
goed verdicht zand (qc groter dan 10MPa)	10 - 20	50 - 75	50 - 75
grindhoudend zand	20 - 35	40 - 80	40 - 80
matig vast zand (qc=4 à 8 MPa)	5 - 15	40 - 60	40 - 60
leemhoudend zand	4 - 10	20 - 30	20 - 60
kleihoudend zand	3 - 8	15 - 40	15 - 50
zandhoudend klei	2 - 5	5 - 20	10 - 40
rivierklei	1 - 2	5 - 10	15 - 30
veen	0.1 - 0.5	1 - 5	4 - 20

Stichting bouw Research (SBR)

The values in Table D-3 are useable for silty soil and ground corrected soil. If no result of the ground density or sound graph is available the designer should assume in the calculation loose soil. The values of dense soil layers may only be used if a sounding graph supports it.

Table D-3 *k* values as presented in SBR

Tabel 1:
Grondeigenschappen voor onsamenhangende grondsoorten.

Grondsoort	pakking	representatieve waarden van de grondeigenschappen							
		qc Mpa	ρg kN/m^3	ρsg kN/m^3	ρ'g kN/m^3	φ graden	E MPa	C -	Kv MN/m^3
Grind:	los	15	17,0	19,0	9,0	32,5	75	500	125
zwak zandig	matig	25	18,0	20,0	10,0	35,0	125	1000	200
	vast	30	19,0	21,0	11,0	37,5	150	1200	250
sterk zandig	los	10	18,0	20,0	10,0	30,0	50	400	85
	matig	15	19,0	21,0	11,0	32,5	75	600	125
	vast	25	20,0	22,0	12,0	35,0	125	1000	200
Zand:	los	5	17,0	19,0	9,0	30,0	25	200	40
schoon	matig	15	18,0	20,0	10,0	32,5	75	600	125
	vast	25	19,0	21,0	11,0	35	125	1000	200
zwak siltig/kleiig		5	18,0	20,0	10,0	27	25	450	40
sterk siltig/kleiig		2	18,0	20,0	10,0	25	10	200	15