

The transformation of GPS into NAP heights

Combining NAP, GPS and geoid height to compute a height reference surface for the Netherlands

Master of Science Thesis

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Preface

This report is written in the framework of a MSc. graduation project in the MSc. of Geomatics. I did my research at the Physical and Space Geodesy (PSG) group at the Institute of Earth Observation and Satellite System (DEOS) at the Faculty of Aerospace Engineering, Delft University of Technology.

People interested at the differences between EGM96 and EIGEN-CG03C for the Dutch area are recommended to read chapter 3. The possibilities of computing a height reference surface out of GPS/levelling data only is described in chapter 5. Variance component estimation is discussed in chapters 6 and 7.

First of all I would like to thank Prof. R. Klees for giving me the possibility to do my MSc. research at the PSG group and for being my supervisor. Other words of thank go to ir. Jasper van Loon for supporting me with the theory and programs of the variance component estimation and for discussing the results. And of course thanks to all my colleagues at the PSG group for helping with all possible problems in Linux, L^AT_EX, GMT, Matlab and FORTRAN and for giving me a good time working at my MSc. research.

I would like to thank ir. Joop van Buren and Gé Lobé of the Kadaster for providing me with GPS/levelling data of the GPS Kernnet. For providing me with data of the GPS/levelling heights of the 5th accuracy levelling and for reviewing my thesis, I would like to thank ir. Anton Kösters of the Adviesdienst Geo-informatie en ICT. Also thanks to my other co-reviewer dr. Rossen Grebenitcharsky.

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Summary

The use of GPS as a measurement device is growing, not only for horizontal positioning but also for determination of heights. This means ellipsoidal heights are used, which do not give information about how the water flows. Therefore these heights are transformed into traditional height systems. In the Netherlands this is the Normaal Amsterdam Peil (NAP), a levelled height system.

To perform this transformation, a geoid model is used, defined as the equipotential surface of the Earth's gravity field at mean sea level. Unfortunately, this geoid model does not coincide with the difference between GPS and NAP heights. This is due to systematic errors in the three height types. First there is the definition of tide system for the gravimetric geoid and the levelled heights, which can be different for both systems. Second, the gravimetric geoid contains a long wavelength error and approximations have to be made in the computation of a gravimetric geoid. Third point is the fact that height is the worst determined component of GPS measurements. And fourth and last are the instability of the levelling benchmarks and the atmospheric refraction in levelling.

Out of this the following research question is formulated:

How can NAP heights, ellipsoidal heights and gravimetric heights be combined to realise a height reference surface for the Netherlands, which allows the transformation of GPS heights into NAP height?

This question makes the research consist of three parts:

- The comparison of EGM96 and EIGEN-CG03C for the Dutch geoid model.
- The computation of a height reference surface out of only GPS/levelling data.
- The use of variance component estimation to compute a height reference surface out of NAP, GPS and gravimetric heights.

This research question is answered by performing a literature study and computations done in Matlab and FORTRAN. For the computation of the gravimetric geoid (GRAVGEO2006) and the corrected geoid for the Netherlands (NLGEO2006) use is made of the programs dr. ir. Erik de Min wrote for his PhD. thesis. Variance component estimation is done by using the programs written by ir. Jasper van Loon for his PhD. thesis. The GPS/levelling data used in this research are the 5th accuracy levelling of the Adviesdienst Geo-informatie en ICT of Rijkswaterstaat (RWS-AGI) and the GPS Kernnet of the Kadaster.

The comparison between EGM96 and EIGEN-CG03C for the Dutch geoid model

The Dutch geoid model is based on a gravimetric geoid with an additional correction surface. The gravimetric geoid is computed by combining a global geopotential model and local gravity data. The current Dutch geoid model, NLGEO2004, is based on EGM96. In this research a geoid model based on EIGEN-CG03C was computed, NLGEO2006. The difference between the gravimetric models is of centimetre level and of short wavelength structure. The differences between NLGEO2004 and NLGEO2006 are at centimetre level and relative height differs -7 to +7 millimetre.

The computation of a height reference surface out of only GPS/levelling data

The computation of a height reference surface based on only GPS/levelling data is done by multiquadric interpolation. This method works with radial base functions on every data point. The performance of this method has the same accuracy as thin plate splines. It is possible to compute a height reference surface out of only GPS/levelling data which has a difference with NLGEO2006 of 5 to 10 centimetres.

The use of variance component estimation to compute a height reference surface out of NAP, GPS and gravimetric heights.

In the least-squares adjustment of height measurements, mostly attention is paid to the functional model only. In this research the main focus is on the stochastic information. It is chosen to work with only one functional model, a 2-dimensional Fourier representation of degree 1. Based on this model a variance component estimation is performed where a variance component is estimated for every type of height data.

The variance/covariance matrix for GRAVGEO2006 is computed with a Monte Carlo simulation. Out of this simulation a correlation function was computed, which was used to compute the covariances. Unfortunately there is no covariance information available for the GPS/levelling data. Therefore all the covariance information has to come from GRAVGEO2006. This makes the best fitting correlation function give too small covariances. Therefore they are artificially raised. The lack of covariances for NAP and GPS heights also makes the structure of the variance matrices similar, which makes that no separate variance factor can be estimated for NAP and GPS heights.

These results make that variance component estimations gives no improvements to the height references surface at this moment.

Out of these results follows the most important recommendation of this research:

The addition of covariance information would make the results of variance component estimation more reliable. Therefore it is important that good variance/covariance information becomes available for NAP and GPS heights.

Samenvatting

GPS wordt steeds vaker gebruikt in de landmeetkunde, niet alleen voor x- en y-coördinaten, maar ook in hoogtebepaling. Hierbij worden ellipsoïdische hoogtes gebruikt. Die geven echter niet aan in welke richting het water stroomt. Daarom worden hoogtes getransformeerd naar traditionele hoogtesystemen. In Nederland is dit het Normaal Amsterdam Peil (NAP), een gewaterpast hoogtesysteem.

Deze transformatie tussen GPS en NAP hoogtes wordt uitgevoerd door middel van een geoïdemodel. De geoïde is gedefinieerd als het equipotentiaalvlak van het aardse zwaartekrachtsveld op gemiddeld zeeniveau. Dit model komt echter niet geheel overeen met het verschil tussen NAP en GPS hoogtes. Oorzaak zijn de systematische fouten in de drie meetmethoden. Ten eerste ontstaan aan verschillen doordat de definitie van het getijdensysteem voor de gravimetrische geoïde en de gewaterpaste hoogtes van elkaar kunnen verschillen. Ten tweede heeft de gravimetrische geoïde een langgolvige fout en worden benaderingen gebruikt in de berekening. Ten derde is de hoogtecomponent in GPS metingen het minst nauwkeurig. Tot slot zijn de peilmerken van de NAP-punten niet stabiel en treedt er bij waterpassing atmosferische breking op.

Vanuit deze achtergrond is de volgende onderzoeksvraag geformuleerd:

Hoe kunnen NAP hoogtes, ellipsoïdische hoogtes en gravimetrische hoogtes worden gecombineerd om een hoogterefereentievak voor Nederland te berekenen waarmee GPS hoogtes in NAP hoogtes kunnen worden getransformeerd?

Deze vraag verdeelt dit onderzoek in drie delen:

- De vergelijking tussen EGM96 en EIGEN-CG03C voor het Nederlandse geoïdemodel.
- De berekening van een hoogterefereentievak uit alleen NAP en GPS hoogtes.
- Het gebruik van variantie componenten schatting voor de berekeningen van een hoogterefereentievak uit NAP, GPS en gravimetrische hoogtes.

De onderzoeksvraag is beantwoord door middel van een literatuurstudie en berekeningen in Matlab en FORTRAN. De berekeningen van een gravimetrische geode (GRAVGEO2006) en de gecorrigeerde geoïde voor Nederland (NLGEO2006) zijn gedaan met de programma's van dr. ir. Erik de Min, welke hij schreef voor zijn promotieonderzoek. De variantie componenten schatting is uitgevoerd met de programma's van ir. Jasper van Loon die hij schreef voor zijn promotieonderzoek. De NAP en GPS informatie is afkomstig uit de 5^e nauwkeurigheidswaterpassing van de Adviesdienst Geoinformatie en ICT van Rijkswaterstaat en van het GPS Kernnet van het Kadaster.

De vergelijking tussen EGM96 en EIGEN-CG03C voor het Nederlandse geoïdemodel

Het Nederlandse geoïdemodel is gebaseerd op een gravimetrische geoïde waarbij een correctievlak is opgeteld. De gravimetrische geoïde is berekend door een globaal geopotentiaal model te combineren met lokale zwaartekrachtmetingen. Het huidige Nederlandse geoïdemodel, NLGEO2004, is gebaseerd op EGM96. Het model dat berekend is in dit onderzoek, NLGEO2006, op EIGEN-CG03C. Het verschil tussen beide gravimetrische geoïdes ligt op centimeterniveau en heeft een kortgolvlige structuur. Het verschil tussen NLGEO2004 en NLGEO2006 ligt op centimeterniveau en de relatieve hoogteverschillen tussen -7 en +7 millimeter.

De berekeningen van een hoogtereferentievlak uit alleen NAP en GPS hoogtes

De berekening van een hoogtereferentievlak gebaseerd op alleen NAP en GPS hoogtes is uitgevoerd met multikwadratische interpolatie. Deze methode werkt met radiale basisfuncties voor elk datapunt. Deze methode werkt even goed als 'thin plate splines'. Het is mogelijk om op deze manier een vlak te berekenen dat een verschil heeft van 5 tot 10 cm met NLGEO2006.

Het gebruik van variantiecomponenten schatting voor de berekening van een hoogtereferentievlak uit NAP, GPS en gravimetrische hoogtes

De meeste aandacht bij de kleinste kwadraten schatting van hoogtemetingen wordt vaak besteed aan de keuze van het functiemodel. In dit onderzoek ligt de aandacht vooral op het stochastische model. Daarom is er gekozen voor slechts één functiemodel, een 2-dimensionaal Fouriervlak van graad 1. Gebaseerd op dit model is een variantie componenten schatting uitgevoerd. Hierbij wordt een variantie component geschat voor elk type hoogtedata.

De variantie/covariantiematrix van GRAVGEO2006 is berekend met een Monte Carlo simulatie. Hieruit is een correlatiefunctie berekend. Voor de NAP en GPS hoogtes zijn geen covarianties bekend. Daarom moet alle covariantie-informatie komen uit GRAVGEO2006 en zijn de covarianties te klein. Om dit op te lossen zijn de covarianties kunstmatig verhoogd. Het gebrek aan covariantie-informatie voor de NAP en GPS hoogtes zorgt er ook voor dat de structuur van variantiematrices op elkaar lijken, waardoor geen afzonderlijk variantie componenten geschat kunnen worden.

Deze resultaten zorgen ervoor dat een variantie componenten schatting geen verbetering van het hoogtereferentievlak op dit moment oplevert.

Uit deze resultaten volgt de belangrijkste aanbeveling:

Het toevoegen van covariantie-informatie zorgt ervoor dat de resultaten van de variantie componenten schatting betrouwbaarder wordt. Daarom is het belangrijk dat deze informatie beschikbaar wordt.

Abbreviations

AGRS.NL	Actief GPS Referentie Systeem Nederland
BIQUE	Best Invariant Quadratic Unbiased Estimator
CHAMP	CHAllenging Minisatellite Payload
DEOS	Institute of Earth Observation and Satellite Systems
EGM96	Earth Gravity Model 1996
EIGEN-CG03C	EIGEN CHAMP/GRACE-03 geoid model, Combined (GFZ)
ERS-1	European Remote-Sensing Satellite 1
GEOSAT	Geodetic Satellite
GFZ	GeoForschungsZentrum
ggm	global geopotential model
GMT	Generic Mapping Tool
GPS	Global Positioning System
GRACE	Gravity Recovery and Climate Experiment
GRS80	Geodetic Reference System 1980
HRS	Height Reference Surface
IGS	International GPS Service
IMLE	Iterative Maximum Likelihood Estimator
ITRF2000	International Terrestrial Reference Frame 2000
ITRS	International Terrestrial Reference System
KN	Kernnet
MINQUE	Minimum Norm Quadratic Unbiased Estimator
MSL	Mean Sea Level
MWG	Meissl/Wong&Gore
MQ	multiquadric interpolation
NAP	Normaal Amsterdams Peil
NCF	Stichting Nationale Computerfaciliteiten
NLGE02004	current Dutch geoid model
NWP5	5 th accuracy levelling
OSU91a	Ohio State University geoid model 1991
PSG	Physical and Space Geodesy
ref. ell.	reference ellipsoid
REML	Restricted Maximum Likelihood Estimator
RMQ	reciprocal multiquadric interpolation

RMS	Root Mean Square
RWS-AGI	Adviesdienst Geo-informatie en ICT of Rijkswaterstaat
SLR	Satellite Laser Ranging
VC matrix	variance/covariance matrix
VCE	Variance Component Estimation

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Chapter 1

Introduction

The use of GPS in surveying is growing. Not only in horizontal positioning but also in height determination. GPS uses ellipsoidal coordinates. But they do not have a physical meaning, i.e. they do not give information about the direction water flows. Traditional height systems fulfil this purpose and are already available. In the Netherlands the Normaal Amsterdam Peil (NAP) is used, which is determined by levelling. Therefore GPS heights have to be transformed to NAP heights.

The transformation between GPS heights and NAP heights is done by a geoid model. A geoid is usually defined as the equipotential surface of the Earth's gravity field which coincides with mean sea level (MSL) and is computed based on gravity information. But there are different ways to define a geoid, dependent on the tide system used [Ekman, 1995]. Due to the used tide system, the difference between GPS/levelling heights and the gravimetric geoid can get large when not the same definition is chosen for NAP and the gravimetric geoid.

Furthermore all three height types have their systematic errors [Featherstone, 1998]. For the gravimetric geoid these are long wavelength errors and the approximations and assumptions that have to be made to compute a geoid out of gravity data. For GPS it is known that the height component is the part with the lowest accuracy. For levelling there is the influence of instrumental calibration and atmospheric refraction. This means the NAP surface does not coincide exactly with the geoid model. From this follows that a correction surface has to be added to the gravimetric geoid to make it a model that transforms GPS heights into NAP heights and vice versa.

The difference between the geoid and the NAP surface is not the only problem when facing the transformation between GPS heights and NAP heights. During the years, the markers of the NAP are proved not to be stable [Brand, 2004]. This means the NAP surface marked by the markers cannot be referred to as a real surface but is more or less a point cloud with a thickness of 1 to 2 centimetres.

The magnitude of those differences is not known in the Dutch case. Figure 1.1 gives an overview of the different surfaces and their relations used in this research. It states also a possible unknown transformation between the reference ellipsoids for GPS and NAP. In the Netherlands this is no problem, because parameters of both ellipsoids are known.

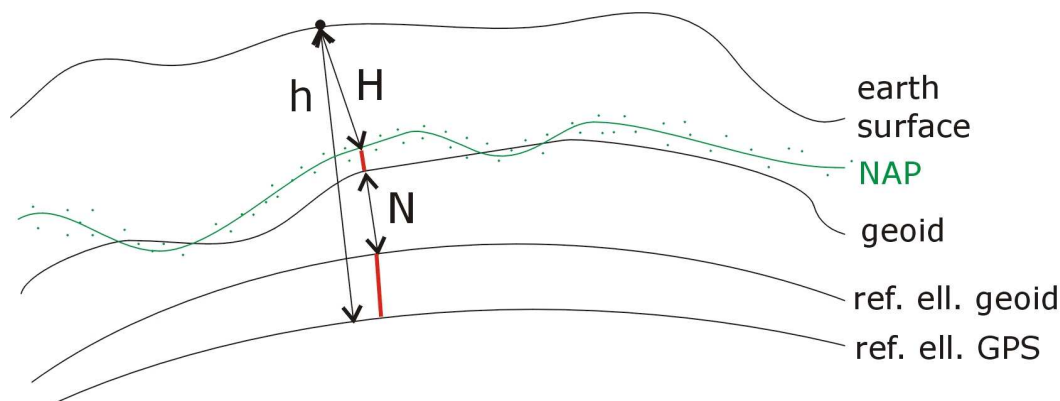


Figure 1.1: The different surfaces used as height references and the possible discrepancies in a transformation between different height reference surfaces

The before mentioned problems are the reason for searching a way to transform GPS heights into NAP heights. Therefore use is made of the gravimetric geoid, but that is not our main interest. Therefore this information is no longer chosen to be the basis of a correction surface as is done at this moment. Instead use is made of GPS/levelling heights and with the later addition of geoid heights.

From this, the following research question is formulated:

How can NAP heights, ellipsoidal heights and gravimetric heights be combined to realise a height reference surface for the Netherlands, which allows the transformation of GPS heights into NAP heights?

To answer this question, the research is started by a literature and practical study of NLGEO2004. This geoid model is recomputed with another global geopotential model. By doing this insight is obtained in the way the current model is computed, so a comparison between different computation methods can be made. Out of GPS and levelling heights from several locations a purely 'geometric geoid' is computed. In addition, to come to an improved model, gravitational geoid information is added. To improve the stochastic information, variance component estimation is used.

This research focusses on the Netherlands. In this area, 490 GPS/levelling points are available at this moment by the Adviesdienst Geoinformatie en ICT and the Kadaster. Better results may be reached when even more GPS/levelling points are available. There are also different ways to combine different kinds of height data in one adjustment procedure. The focus of this research is on the variance component estimation based on only one parametric model chosen beforehand.

The structure of this report is as follows. Chapter 2 states the research question and gives an overview of how this question is answered. Chapter 3 gives an overview of the computation of the Dutch geoid at the moment and shows the influence of a new global geopotential model, EIGEN-CG03C, on the Dutch geoid. The used data sets with GPS/levelling data are described in chapter 4 with a special focus on the variance/covariance information. Chapter 5 treats the computation of a height reference surface (HRS), based on only GPS/levelling data and computation of the variance/covariance information for the gravimetric geoid. The way geoid heights are introduced and the theory of variance component estimation are described in chapter 6. The result of the variance component estimation is discussed in chapter 7. The conclusions and recommendations can be found in chapter 8.

Chapter 2

Research questions and methods

In the previous chapter, the background of this research is given. The research question stated is explained in more detail in this chapter.

2.1 Research questions

The basis of this research is the following research question:

How can NAP heights, ellipsoidal heights and gravimetric heights be combined to realise a height reference surface for the Netherlands, which allows the transformation of GPS heights into NAP heights?

There are many ways to combine different types of height measurements with different complexity and accuracy into one adjustment procedure. In this research one of the method of Schwarz & Sideris [1997] is tested for the area of the Netherlands. Special attention is paid on the variance/covariance information.

To make the research more structured, partial questions are formulated.

1. *What is the difference in the contribution of the two geopotential models, EGM96 and EIGEN-CG03C, to the Dutch geoid?*

The computation of the present Dutch geoid, NLGEO2004, is computed based on EGM96. Nowadays there are more recent global models available which are partly derived from data from the CHAMP and GRACE satellite missions. One of the most recent models is EIGEN-CG03C. How does the Dutch geoid change if EIGEN-CG03C is used instead of EGM96? The answer to this question can be found in chapter 3.

2. When computing a height reference surface from GPS/levelling data only, what accuracy can be reached in the Netherlands?

Nowadays, the Dutch geoid is computed gravimetrically and then fitted with the NAP ‘surface’ by the use of GPS/levelling points. What will happen when no gravimetric data is used, but only GPS and NAP heights? What is the difference between this solution and NLGEO2006? This question is answered in chapter 5.

3. How can NAP heights, GPS heights and gravimetric heights be combined?

NAP, GPS and gravimetric heights are three different height systems. These system can be combined into one height reference surface. How can this be done and how accurate is this? Chapters 6 and 7 give the answer to this question.

2.2 Research methods and available software

During this research a couple of methods are used. The research is started by a literature study of the method currently used for the computation of the Dutch geoid. Also the FORTRAN programs written by Erik de Min for this geoid computation are studied and used to find the answer on the first partial question.

For the second part, the actual combination of different height types to compute one unique height reference surface, is also started by a literature study. This literature study was to get insight in the possibilities of combining the different kinds of heights. The combination of different height types is performed with Matlab. Variance component estimation is done by using Fortran programs written by Jasper van Loon for his PhD research.

Chapter 3

EGM96 vs. EIGEN-CG03C for the Dutch geoid

In 1985 the ‘van Willigen geoid’ was computed, followed in 1996 by the ‘de Min geoid’ [de Min, 1996]. An update of this geoid model has been done recently by the Adviesdienst Geo-informatie en ICT of Rijkswaterstaat (RWS-AGI) [Crombaghs, 2004]. This new geoid model is called NLGEO2004 and is the official Dutch geoid since January 1, 2005.

The difference between the ‘De Min geoid’ and NLGEO2004 can be found in the addition of terrestrial gravity measurements in Belgium and Germany and the use of a different global geopotential model [Crombaghs, 2004]. Where the ‘De Min geoid’ is based on the global geopotential model OSU91a, NLGEO2004 is computed based on the global geopotential model EGM96. Today there are more recent global geopotential models available, partly computed out of CHAMP and GRACE data, with a higher accuracy. One of the latest is EIGEN-CG03C, which is computed by the GFZ Potsdam and became available in May 2005 [Förste et al, 2005].

In this chapter, the differences between EGM96 and EIGEN-CG03C for the area of the Dutch geoid are computed. How local geoid models are computed in general and in the case of the Netherlands is described in section 3.1. The actual difference and analysis can be found in section 3.2. The influence of the local gravity data on the Dutch geoid is the topic of section 3.3. Section 3.4 is about the computation of a correction surface to make the gravimetric geoid usable for the transformation from GPS to NAP heights. Section 3.5 concludes the consequences of the difference between both geopotential models for the Dutch geoid model.

For the computations in this research use has been made of the software written by De Min. Appendix A gives an overview of this software and the computation of a Dutch geoid model with this software.

3.1 Combination of global and local gravity data

Most local geoid models are computed using two types of information: a global geopotential model and local gravity measurements. When using local data only, the first part on the right hand side of equation 3.1, we neglect the fact that gravity information of the whole Earth is needed when using the Stokes formula. When we only use the global model, we neglect the high resolution part of the geoid, because this information is not available from the global model. At this moment the global geopotential models

consist of spherical harmonic coefficients up to degree and order 360. This means the spatial resolution is approximately 110 kilometres (full wave length). This is not detailed enough for a high accuracy local geoid model.

$$\begin{aligned}
N(P) &= \frac{R}{4\pi\gamma} \int_{\lambda=0}^{2\pi} \int_{\varphi=-\pi/2}^{\pi/2} St(\psi_{PQ}) \Delta g(Q) \cos \varphi_Q d\varphi_Q d\lambda_Q \\
&= \frac{R}{4\pi\gamma} \iint_{\sigma} St(\psi) \Delta g d\sigma \\
&= \frac{R}{4\pi\gamma} \iint_{\sigma_P} St(\psi) \Delta g d\sigma + \frac{R}{4\pi\gamma} \iint_{\sigma-\sigma_P} St(\psi) \Delta g d\sigma
\end{aligned} \tag{3.1}$$

Combination solution

A local gravity data set has to be combined with a global model for high accuracy local geoid modelling. Equation 3.1 is split into parts, an inner (σ_P) and an outer ($\sigma - \sigma_P$) region. Both data parts on the right hand side of this equation can be split into high and low frequency information. But in the outer region, no high frequency information is available, what causes errors:

$$\begin{aligned}
\iint_{\sigma-\sigma_P} St(\psi) \Delta g d\sigma &= \iint_{\sigma-\sigma_P} St(\psi) \Delta g_L d\sigma + \iint_{\sigma-\sigma_P} St(\psi) \Delta g_H d\sigma \\
&= \iint_{\sigma-\sigma_P} St(\psi) \Delta g_L d\sigma + \epsilon_1
\end{aligned} \tag{3.2}$$

For the inner region this is not the case, high as well as low frequency data is available here. This makes it possible to use data out of both data sets, resulting in equation 3.3. The unknown here, is the lack of high frequency data in the first data set, the global geopotential model. This lack of information causes the truncation error, discussed later in this section.

$$\iint_{\sigma_P} St(\psi) \Delta g d\sigma = \iint_{\sigma_P} St_1(\psi) \Delta g_L d\sigma + \iint_{\sigma_P} St_1(\psi) \Delta g_H d\sigma \tag{3.3}$$

$$\begin{aligned}
&+ \iint_{\sigma_P} St_2(\psi) \Delta g_L d\sigma + \iint_{\sigma_P} St_2(\psi) \Delta g_H d\sigma \\
&= \iint_{\sigma_P} St_1(\psi) \Delta g_L d\sigma + \iint_{\sigma_P} St_2(\psi) \Delta g_L d\sigma \\
&+ \iint_{\sigma_P} St_2(\psi) \Delta g_H d\sigma + \epsilon_2
\end{aligned} \tag{3.4}$$

The two utmost cases, method A and B in which one of the data sets is used in total, are described here and made visible in figure 3.1.

Method A

By the use of method A, the global model is used in the inner region. This means St_1 is equal to St in σ_P and St_2 is equal to zero in σ_P . The low frequency data is represented by the global model and the high frequency data by the difference of the two data sets. This means equations 3.2 and 3.3 can be written as:

$$\iint_{\sigma-\sigma_P} St(\psi) \Delta g d\sigma = \iint_{\sigma-\sigma_P} St(\psi) \Delta g^{ggm} d\sigma + \epsilon_1 \tag{3.5}$$

$$\iint_{\sigma_P} St(\psi) \Delta g d\sigma = \iint_{\sigma_P} St(\psi) \Delta g^{ggm} d\sigma + \iint_{\sigma_P} St(\psi) (\Delta g^{loc} - \Delta g^{ggm}) d\sigma + \epsilon_2 \quad (3.6)$$

and combined forms the solution (see for the relation between method A and B appendix B.2):

$$N(P) = \frac{R}{4\pi\gamma} \left(\iint_{\sigma} St(\psi) \Delta g^{ggm} d\sigma + \iint_{\sigma_P} St(\psi) (\Delta g^{loc} - \Delta g^{ggm}) d\sigma \right) + \epsilon_1 + \epsilon_2 \quad (3.7)$$

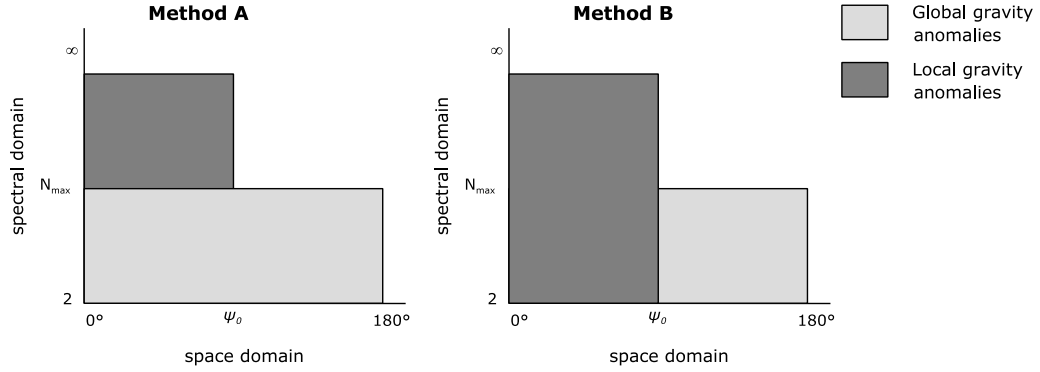


Figure 3.1: Combination of global and local gravity data in a local geoid model

Method B

When method B is used for the computation of the local geoid, in the inner region only the local gravity information is used, what makes St_1 there equal to 0 and St_2 equal to St . This also means Δg_L and Δg_H in the inner region both come from the local data, what makes the following hold true for method B:

$$\iint_{\sigma-\sigma_P} St(\psi) \Delta g d\sigma = \iint_{\sigma-\sigma_P} St(\psi) \Delta g^{ggm} d\sigma + \epsilon_1 \quad (3.8)$$

$$\iint_{\sigma_P} St(\psi) \Delta g = \iint_{\sigma_P} St(\psi) \Delta g^{loc} d\sigma + \epsilon_2 \quad (3.9)$$

which form together

$$N(P) = \frac{R}{4\pi\gamma} \left(\iint_{\sigma-\sigma_P} St \Delta g^{ggm} d\sigma + \iint_{\sigma_P} St \Delta g^{loc} d\sigma \right) + \epsilon_1 + \epsilon_2 \quad (3.10)$$

In theory, methods A and B have the same solution. In practice, there are small differences due to measurement errors and errors in the geopotential coefficients.

Kernel modification

There are more advanced ways to combine the two data sets. This is done by the use of kernel modification. As shown in equations 3.2 and 3.3 the Stokes function can be split. With this it is possible to weight the data sets differently. An important property of the Stokes function is that the parts added together have to define the total Stokes function:

$$St(\psi) = St_1(\psi) + St_2(\psi) \quad (3.11)$$

which is defined as:

$$St(\psi) = \sum_{n=2}^{\infty} \frac{2n+1}{n-1} P_n(\cos \psi) \quad (3.12)$$

and can be approximated by

$$St(\psi) = \frac{1}{s} - 6s - 4 + 10s^2 - (3 - 6s^2) \ln(s + s^2) \quad \text{with } s = \sin\left(\frac{\psi}{2}\right) \quad (3.13)$$

Well known kernel modifications are Meissl, Wong&Gore and the combination of both: Meissl/Wong&Gore. The last mentioned modification is the one used for the computation of the Dutch geoid with $L = 32$ [de Min, 1996]. The partial Stokes functions are described for $0 \leq \psi \leq \psi_0$ in equations 3.14 and 3.15. $\psi_0 = 5^\circ$ in the Dutch case.

$$St_1^{MWG}(\psi) = \sum_{n=2}^L \frac{2n+1}{n-1} P_n(\cos \psi) + St(\psi_0) - \sum_{n=2}^L \frac{2n+1}{n-1} P_n(\cos \psi_0) \quad (3.14)$$

$$St_2^{MWG}(\psi) = St(\psi) - St_1^{MWG}(\psi) \quad (3.15)$$

As said before, only the global geopotential model is available in the outer region. The Stokes functions for $\psi > \psi_0$ are described in equations 3.16 and 3.17:

$$St_1^{MWG}(\psi) = St(\psi) \quad (3.16)$$

$$St_2^{MWG}(\psi) = 0 \quad (3.17)$$

When using kernel modification instead of method A or B, the input data of the global geopotential model stays the same, only the weights of single degrees to the total contribution of the geoid model change. When method A is used, the influence of the global geopotential model is the largest possible. This contribution is made visible in figure 3.2(a). The Meissl/Wong&Gore modification makes sure that the lower degrees of the global geopotential model get a higher weight than the higher degrees and that the higher degree contribution mostly comes from the local data (fig. 3.2(b)).

Truncation error

The errors ϵ_1 and ϵ_2 are the so called truncation errors. These errors are due to the truncation of the global data (ϵ_1) and the local data (ϵ_2) This is made clear in figure 3.3.

As we do not know the values of the geopotential coefficients above degree and order 360 of the global geopotential model, estimates are made for this, which are used to compute the truncation error. These values are based on the degree variances computed by two different models: the model of Tscherning & Rapp [1974] and the model of Jekeli [1978]. Both models are based of the same equation developed by Tscherning & Rapp:

$$c_n = s^{n+2} \frac{A(n-1)}{(n-2)(n+B)} \quad (3.18)$$

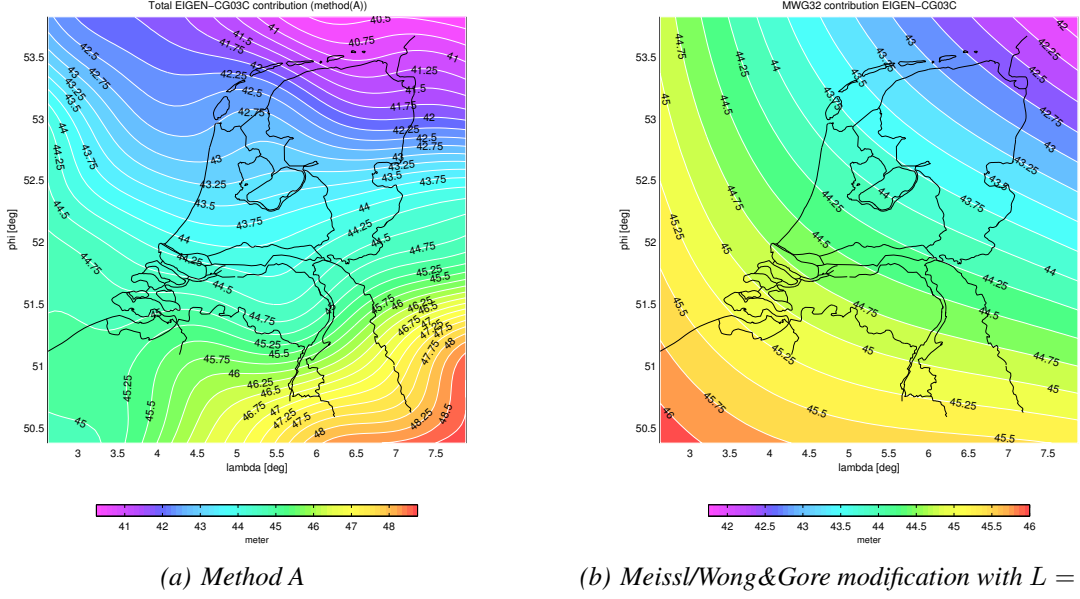


Figure 3.2: The total contribution of the EIGEN-CG03C global geopotential model

Only the parameters are different. The values are shown in table 3.1. The degree variances of those two models and of the EIGEN-CG03C model are plotted in figure 3.4. The degree variance of the EIGEN-CG03C model are computed by:

$$c_n = \gamma^2 (n-1)^2 \sum_{m=0}^{360} (\bar{\sigma}_{nm}^2 + \bar{s}_{nm}^2) \quad (3.19)$$

The truncation error is computed out of the degree variances by:

$$\epsilon_{1,N} = \left(\frac{R}{2\gamma} \right) \sum_{n=n_{max}+1}^{\infty} (Q_n^{MWG}(\psi_0))^2 c_n \quad (3.20)$$

with Q_n^{MWG} the Meissl/Wong&Gore weights. The result, when using the normal Molodensky truncation for the Tscherning/Rapp model is an error of 2.3 cm and with the Jekeli model 1.7 cm. When using the Meissl/Wong&Gore modification, this error becomes considerably smaller: 0.5 mm for the Tscherning/Rapp model and 0.4 mm for the Jekeli model.

The truncation error of the local data, $\epsilon_{2,N}$, can be described by:

$$\epsilon_{2,N} = \sum_{i=1}^I (w_i^{MWG})^2 \sigma_{Deltag}^2 \quad (3.21)$$

with I the number of block mean gravity values and

$$w_i^{MWG} = \frac{i-1}{2} Q_i^{MWG}(\psi_0) \quad (3.22)$$

Table 3.1: Parameters of degree variance models (eq. 3.18)

	Tscherning & Rapp	Jekeli
s [dimensionless]	0.999617	0.9988961
A [mgal ²]	425.28	343.3408
B [dimensionless]	24	24

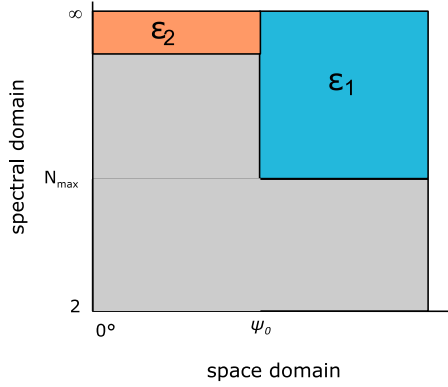


Figure 3.3: The lack of information causing the truncation error ϵ_1 and ϵ_2

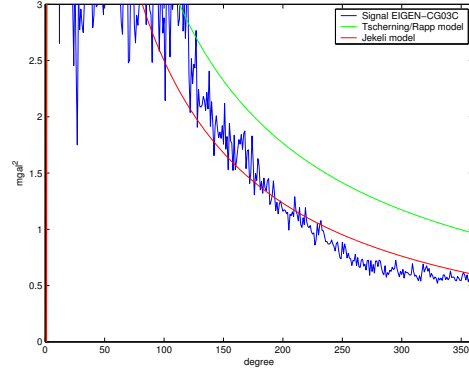


Figure 3.4: Degree variances EIGEN-CG03C, the Tscherning&Rapp model and the Jekeli model

but is not taken into account in this research.

3.2 Comparison of EGM96 and EIGEN-CG03C

EGM96 and EIGEN-CG03C are global geopotential models. The difference between these models lies in the data used for the computation of the spherical harmonic coefficients till degree and order 120. For EGM96 use is made of direct satellite altimetry by TOPEX/POSEIDON, ERS-1 and GEOSAT and satellite tracking with SLR. The lower degree and order coefficients of EIGEN-CG03C are computed with data from the satellite gravity missions CHAMP and GRACE. For the computation of the geopotential coefficients of degree and order 121 till 360 the same terrestrial data is used for both models. This does not mean the spherical coefficients for these degrees are the same in both model. For EIGEN-CG03C these coefficients are computed again and not adopted from EGM96. Further details about EGM96 can be found in [Lemoine et al, 1998] and about EIGEN-CG03C in [Förste et al, 2005].

Degree variances

Degree variances are used to show the influence of the separate degrees of the geopotential model in the final computation of, for example, geoid heights. This can also be done to see the difference between two global geopotential models by:

$$\sigma_n(N) = \left(\frac{GM}{a\gamma} \right) \sqrt{\sum_{m=0}^n (\overline{\Delta c_{nm}^2} + \overline{\Delta s_{nm}^2})} \quad (3.23)$$

The result is shown in figure 3.5.

When doing this, the geopotential coefficients have to refer to the same reference ellipsoid. With EGM96 and EIGEN-CG03C this was not the case (see table 3.2). Because EGM96, referenced GRS80, is used for the computation of NLGEO2004, these parameters are used as reference for EIGEN-CG03C too. The transformation for the EIGEN-CG03C coefficients becomes:

$$c_{nm}^{NEW} = \frac{GM_{EIGEN}}{GM_{EGM}} \left(\frac{a_{EIGEN}}{a_{EGM}} \right)^n c_{nm}^{OLD} \quad . \quad (3.24)$$

With ‘OLD’ the original coefficients of the EIGEN-CG03C geopotential model are meant, ‘NEW’ are the geopotential coefficients when the reference ellipsoid of EIGEN-CG03C is scaled to GRS80.

Table 3.2: Ellipsoidal parameters EGM96 and EIGEN-CG03C

	EGM96 (GRS80)	EIGEN-CG03C
GM	$3.986005 \cdot 10^{14}$	$3.986004415 \cdot 10^{14}$
a	6378137 m	6378136.46 m

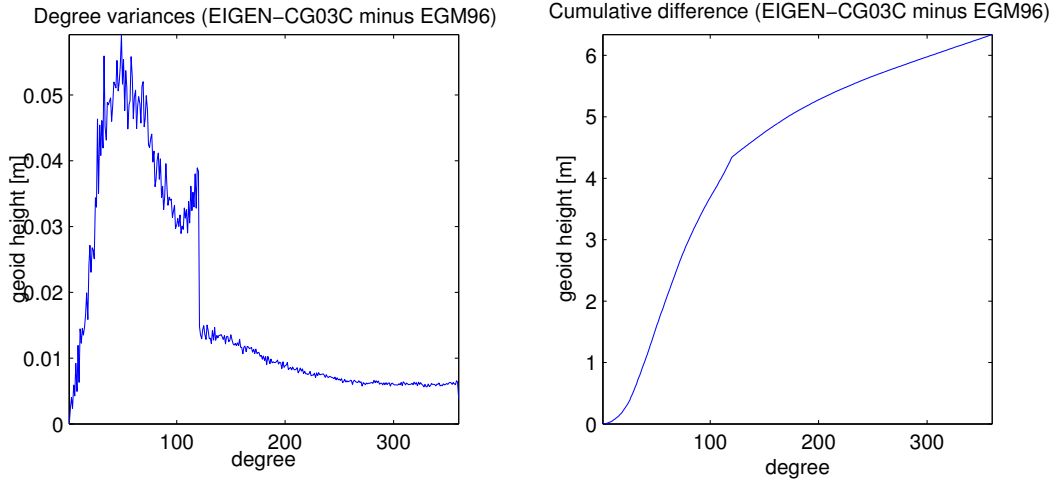


Figure 3.5: Difference between EGM96 and EIGEN-CG03C in terms of degree variances a) Maximum difference in geoid height per degree. b) Cumulative maximum difference in geoid height.

When looking at the difference between the contribution of EGM96 and EIGEN-CG03C on the final Dutch gravimetric geoid, use is made of equation 3.26. In practice, for both geopotential models the contribution to the Dutch gravimetric geoid model is computed with the Meissl/Wong&Gore modification with $L = 32$ (eq. 3.25). These two solutions are extracted of which the result can be seen in figure 3.6.

$$N(P)_{ggm} = \frac{R}{4\pi\gamma} \iint_{\sigma_P} St_1^{MWG32}(\psi) \Delta g^{ggm} d\sigma + \frac{R}{4\pi\gamma} \iint_{\sigma-\sigma_P} St(\psi) \Delta g^{ggm} d\sigma \quad (3.25)$$

$$\begin{aligned}
N(P)_{EIGEN} - N(P)_{EGM96} &= \frac{R}{4\pi\gamma} \iint_{\sigma_P} St_1(\psi) (\Delta g^{EIGEN} - \Delta g^{EGM96}) d\sigma \\
&+ \frac{R}{4\pi\gamma} \iint_{\sigma-\sigma_P} St(\psi) (\Delta g^{EIGEN} - \Delta g^{EGM96}) d\sigma \\
&= \frac{R}{4\pi\gamma} \iint_{\sigma} St_1(\psi) \delta\Delta g_1 d\sigma
\end{aligned} \tag{3.26}$$

In appendix B.1 it is described how to come from the integral relation in the space domain of equation 3.26 to the spherical harmonic representation shown in equation 3.27, which is used in the computations.

$$N(P)_{EIGEN} - N(P)_{EGM96} = R \sum_{n=2}^{360} \sum_{m=-n}^n w^{MWG}(\psi_0) \delta \bar{c}_{nm} \bar{Y}_{nm} \tag{3.27}$$

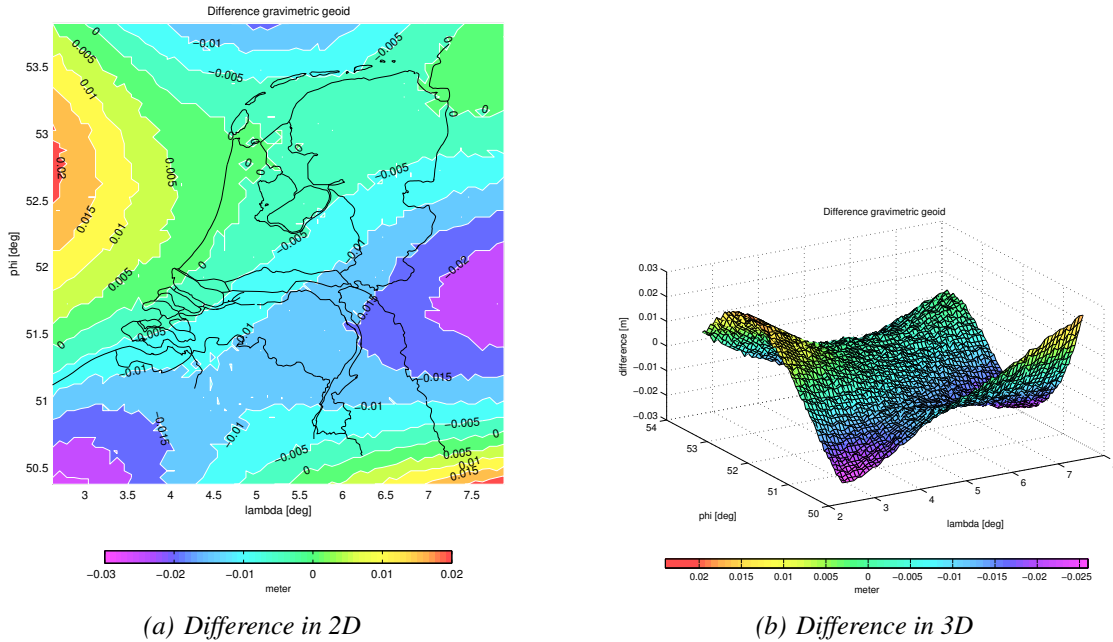


Figure 3.6: Difference between the contributions of EGM96 and EIGEN-CG03C (MWG, $L = 32$ is used) to the total gravimetric geoid (max = 2.4 cm, min = -2.6 cm, RMS = 1,05 cm)

3.3 Local gravity data

Although the only change in gravimetric data in this geoid in comparison with NLGEO2004 is the global geopotential model used, the difference shown in figure 3.6 is not the difference between both total geoid models. This is because NLGEO2004 is not a purely gravimetric geoid. To make NLGEO2004 usable for the transformation from GPS heights into NAP heights, a correction surface was computed by fitting the gravimetric geoid through 84 GPS/levelling points [Crombaghs, 2004]. For the gravimetric geoid computed with EIGEN-CG03C this also will be done in section 3.4.

Before a correction surface can be computed the total gravimetric geoid must be available. This means that the contribution of the local gravity data has to be added to the contribution of the global geopotential model as shown in figure 3.2(b). The local gravity data is the same as used for NLGEO2004 and its description can be found in [Crombaghs, 2004].

The short wavelength contribution of the local data can be found in figure 3.7(a). This is the contribution when method A is used. As said in section 3.1, the Meissl/Wong&Gore kernel modification with $L = 32$ is used in the computation of a Dutch geoid. This means that total contribution of the local data is not the same as in figure 3.7(a). The contribution in this case contains more long wavelength information and is shown in figure 3.7(b).

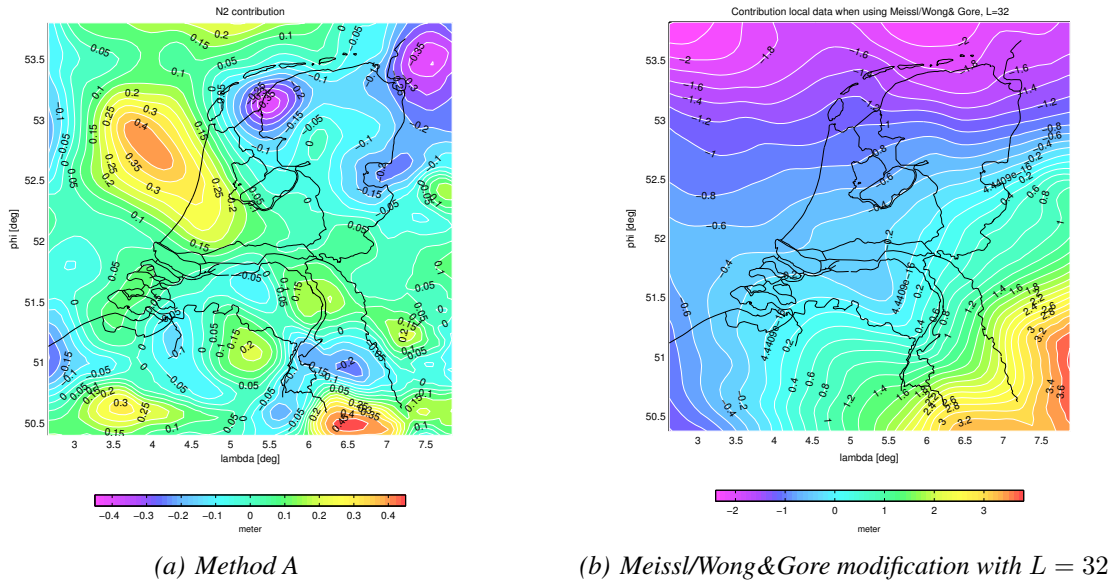


Figure 3.7: The contribution of the local gravity data to the Dutch geoid

Ellipsoidal correction

When using the Stokes function for the computation of geoid heights out of local gravity anomalies, one assumes that the Earth is a perfect sphere instead of an ellipsoid. To take this effect into account, an ellipsoidal correction is used. In this case it was computed with:

$$N_{\varepsilon}(p) = \frac{R}{2\gamma} \sum_{n=2}^{180} (1 - Q_n^{MWG}(\psi_0)) \sum_{m=-n}^n \varepsilon_{nm} \bar{Y}_{nm}(P) \quad (3.28)$$

The coefficients ε_{nm} are computed by Rapp et al [1991] for OSU91a. Details can also be found in de Min [1996, section 7.4]. By the computation of NLGEO2004, with EGM96, the same ellipsoidal correction was used, due to the almost similar parameters for both models. The same holds true for EIGEN-CG03C. The influence of this correction can be seen in figure 3.8.

The final gravimetric geoid, from now called GRAVGEO2006, is the combination of the global geopotential model, the local gravity data and the ellipsoidal correction. The result is shown in figure 3.9.

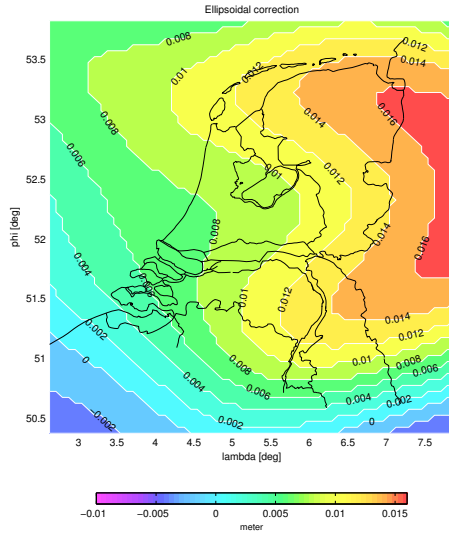


Figure 3.8: The influence of the ellipsoidal correction on the gravimetric geoid

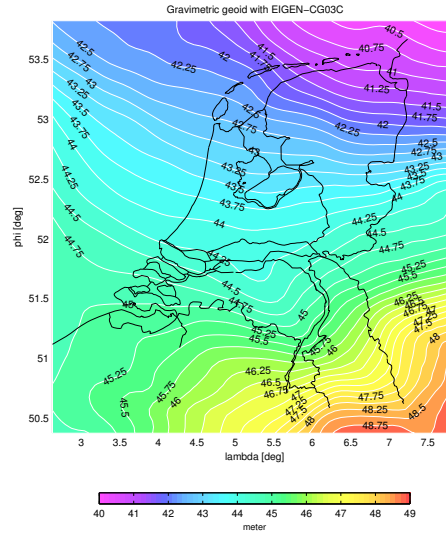


Figure 3.9: The total Dutch gravimetric geoid with EIGEN-CG03

3.4 Correction surface

As said in the previous section, the Dutch geoid is not a purely gravimetric geoid. This is because surveyors need a geoid to transform GPS heights into NAP heights. The Dutch NAP system is not equal to the geoid, due to systematic errors in all height types, unstability of the NAP markers and the choice of the tidal system. This makes the gravimetric geoid not a suitable model to transform GPS heights into NAP heights.

To fit the gravimetric geoid so that the difference between GPS and NAP heights becomes available, GPS/levelling points are used. For NLGEO2004 use is made of the 84 points of the 5th accuracy levelling. This also is done with GRAVGEO2006. The correction surface will also be computed when use is made of de points out of the 5th accuracy levelling as well as the GPS Kernnet of the Kadaster. An overview of the locations of the points of the different sets can be found in figure 3.10.

The computation is done by the steps described in Hilferink [2001] where also the details of the method can be found:

- Determination of the gravimetric geoid heights at GPS/levelling points by Overhauser spline interpolation, N .
- Computation of the differences between gravimetric geoid heights and geometric geoid heights at the GPS/levelling points, $N - N_{GPS/NAP}$.
- Estimation of a surface through the differences computed in the previous step, $N - N_{GPS/NAP} = f(p)$.
- Test of the significance of the surface parameters, test of residual differences, carrying out the global test.

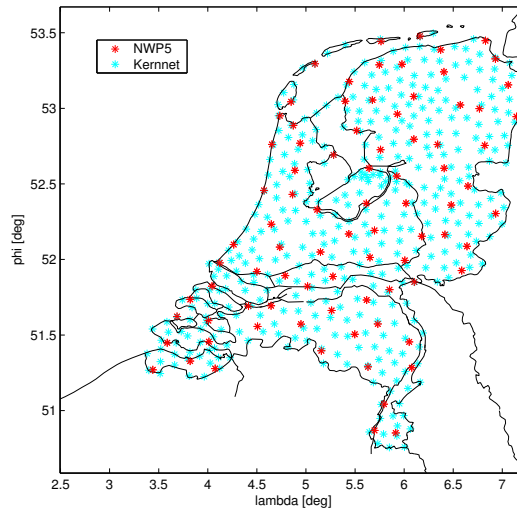


Figure 3.10: The locations of the available GPS/levelling heights

- Computation of the adjusted geoid.

The correction surface is represented as a 6 parameter surface. This means a bivariate polynomial of degree 2 is used:

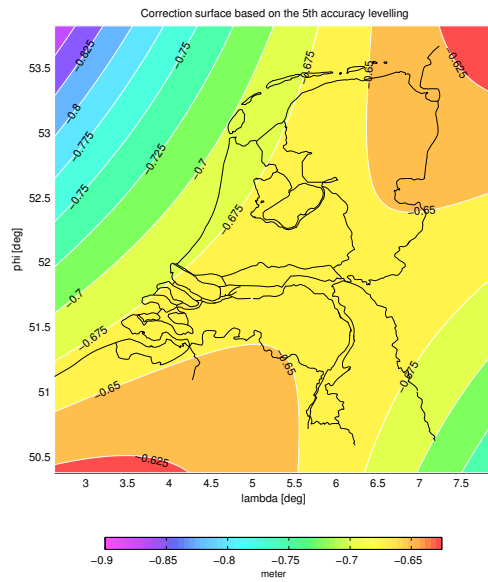
$$y = a + b(\lambda - 5.0) + c(\varphi - 52.0) + d(\lambda - 5.0)^2 + e(\varphi - 52.0)^2 + f(\lambda - 5.0)(\varphi - 52.0) \quad (3.29)$$

where $(\lambda = 5.0, \varphi = 52.0)$ is the midpoint of the area.

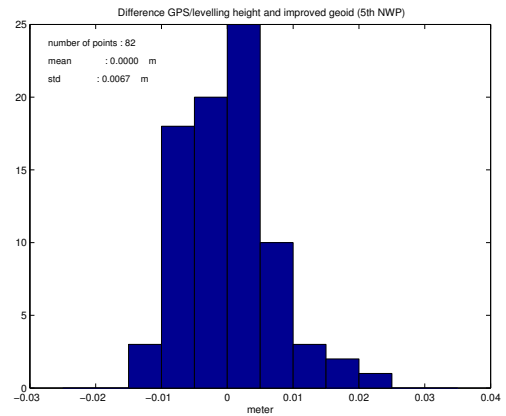
The results of the correction surface computed for GRAVGEO2006 based on different GPS/levelling data sets are shown in table 3.3. The difference surfaces, their statistics and the differences between them can be found in figures 3.11 - 3.15. The surface which is used in the further computation is based on the 5th accuracy levelling only. The total Dutch geoid with EIGEN-CG03C is shown in figure 3.16 and will be called NLGEO2006.

Table 3.3: Parameters correction surface with 5th accuracy levelling

Used GPS/levelling points	a	b	c	d	e	f
5th NWP, diagonal VC matrix	-0.6619	0.0138	-0.0211	-0.0058	-0.0036	0.0211
Kernnet, identity VC matrix	-0.6598	0.0167	-0.0256	-0.0059	-0.0013	0.0171
5th NWP + Kernnet, identity VC	-0.6639	-0.0148	-0.0268	-0.0033	-0.0006	0.0169

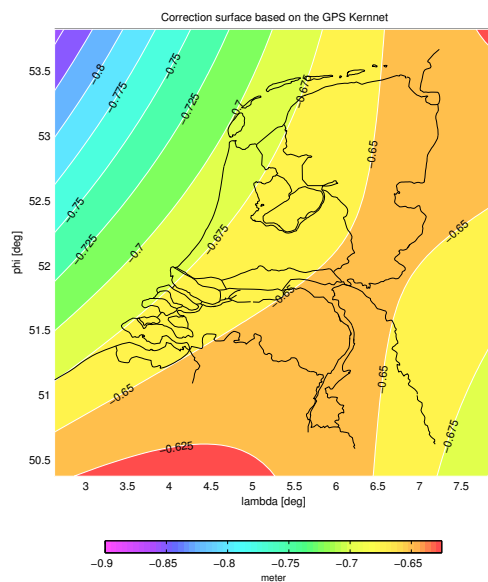


(a) Correction surface

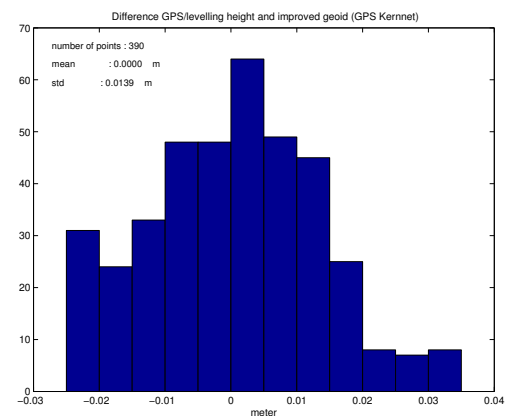


(b) Difference with NLGEO2006

Figure 3.11: Correction surface based on the 5th accuracy levelling

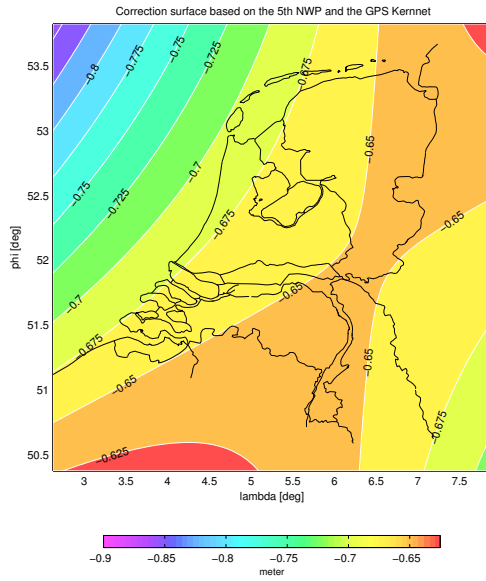


(a) Correction surface

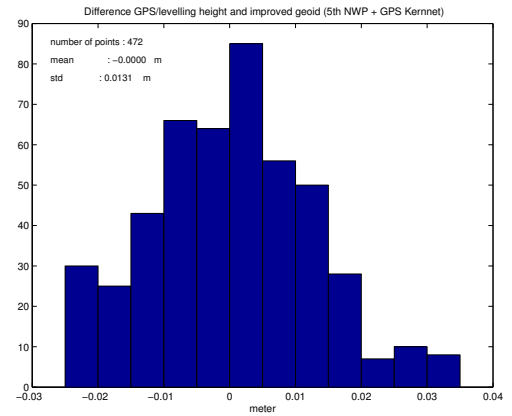


(b) Difference with NLGEO2006

Figure 3.12: Correction surface based on the GPS Kernnet



(a) Correction surface



(b) Difference with NLGEO2006

Figure 3.13: Correction surface based on the 5th accuracy levelling and the GPS Kernnet

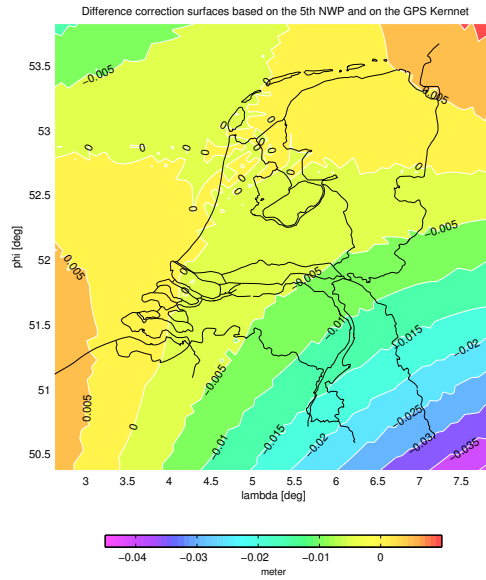


Figure 3.14: Difference between correction surfaces based on the 5th accuracy levelling (fig. 3.11) and the GPS Kernnet (fig. 3.12)

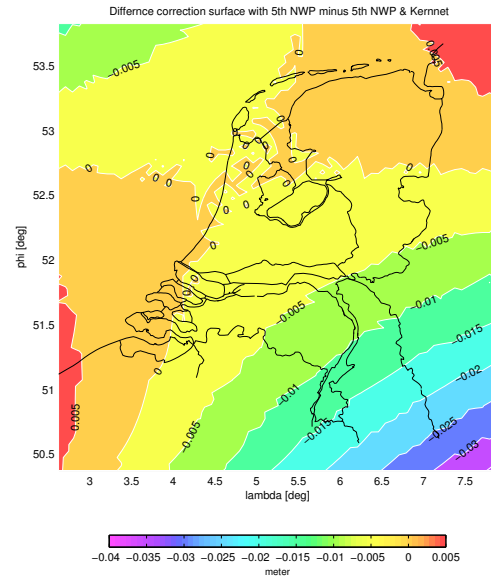


Figure 3.15: Difference between correction surfaces based on the 5th accuracy levelling (fig. 3.11) and the 5th accuracy levelling + the GPS Kernnet (fig. 3.13)

The reason for the computation of the correction surface is to make a good comparison between NLGEO2006 and NLGEO2004. The difference is shown in figure 3.17. The size of this difference is only slightly smaller than the difference between NLGEO2004 and the earlier computed geoid of ‘de Min’.

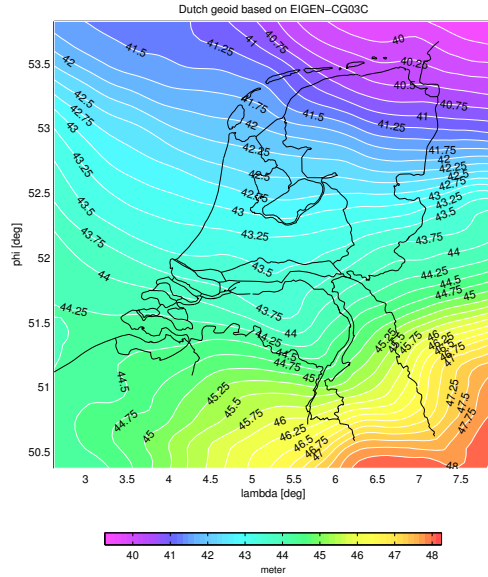


Figure 3.16: NLGEO2006

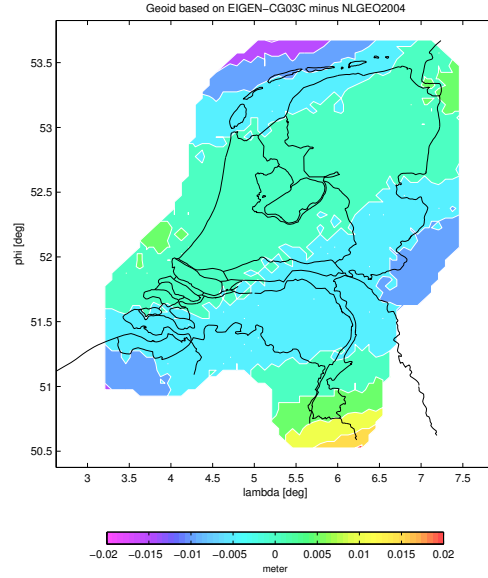


Figure 3.17: Difference between NLGEO2006 and NLGEO2004

But we are not really interested in the absolute differences of the geoid heights. Most of the time surveyors work with height differences between points. For 5 profiles in the Netherlands the height differences between starting and ending points are compared for NLGEO2004 and the geoid model computed in this chapter. The results can be seen in table 3.4 and more detailed information in Appendix C. The numbers show that there is still a difference between -7 and 7 millimetres.

Table 3.4: Difference between the geoid model based on EIGEN-CG03C and NLGEO2004 when looking at height differences

Profile	NLGEO2006 - NLGEO2004 [cm]
Den Helder – Maastricht	0.7
Delfzijl – Middelburg	-0.5
Oost-Vlieland – Terneuzen	0.2
Pieterburen – Maastricht	0.7
Delft – Enschede	-0.7

3.5 Conclusions

As can be seen in the previous sections, the computation of a Dutch geoid model with EIGEN-CG03C instead of EGM96 gives some interesting results:

- First of all the difference between the contribution of both geopotential models is of short wave-length nature, due to the Meissl/Wong&Gore modification.
- The difference between using the GPS/levelling heights of both the 5th accuracy levelling and the GPS Kernnet instead of the 5th accuracy levelling only is -5 to +15 mm in the final Dutch geoid. The biggest difference can be found in south of the Netherlands. It would be interesting to have a closer look at the computation of the correction surface. There may be better possibilities than the use of a 6 parameter biharmonic surface.
- When looking at the differences between NLGEO2006 and NLGEO2004, differences of approximately half the difference between NLGEO2004 and its predecessor, the ‘De Min geoid’, are found. The differences are of centimetre level. Most striking is the difference in the southern part of the Netherlands the small amount of parameters in the correction surface.
- Because absolute heights are not much used by surveyors but relative heights, also some relative geoid heights are determined. The results are shown in table 3.4. Still a difference between both models of -7 to 7 mm can be found.

It is interesting to have a look at the difference between both geopotential models with and without correction surface. This shows the influence of the correction surface. For comparison, figures 3.6 and 3.17 are placed again in figures 3.18 and 3.19, this time with the same colour scheme. As can be seen, the difference over the largest part of the Netherlands becomes smaller by using a correction surface. In the southern part of the Netherlands a strange effect can be found: the negative difference of 1 cm becomes with the correction surface a positive difference of 0.5 cm. Most likely this is due the small amount of GPS/levelling in the area.

So it can be concluded that the difference between NLGEO2006 and NLGEO2004 is half as large as the difference between NLGEO2004 and the ‘De Min geoid’: sub-cm level. This means a significant difference. But at this moment it is too soon to adopt NLGEO2006 as a new standard. There are possibilities to improve the biharmonic correction surface.

In the following chapters other possibilities to combine GPS/levelling data are discussed. Chapter 5 shows the results of using GPS/levelling data only. The use of variance component estimation is described in chapters 6 and 7.

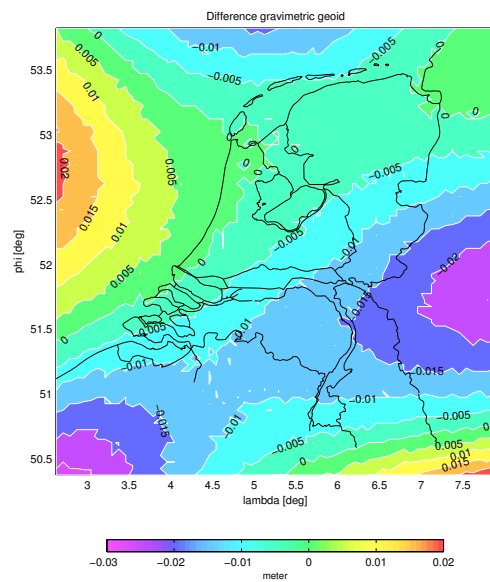


Figure 3.18: Difference between EGM96 and EIGEN-CG03C

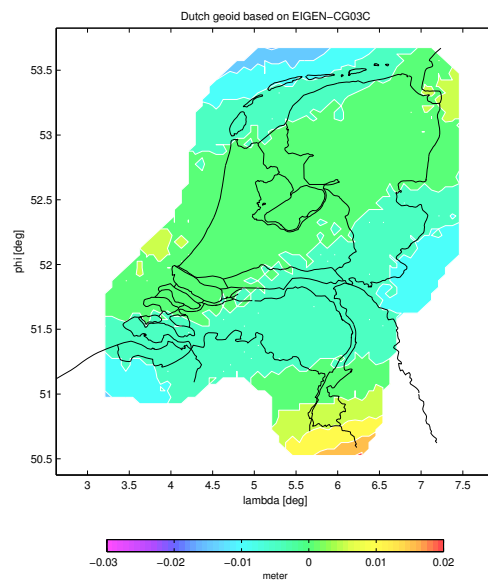


Figure 3.19: Difference between NLGEO2006 and NLGEO2004

Chapter 4

Variance/covariance information of the height measurements

During this research, different kinds of height data from different sources have been used. In the previous chapter, the different sources already were shortly mentioned with the computation of the correction surface to the gravimetric geoid. This chapter gives a detailed overview of the data sets, focussing mainly on the variance/covariance information.

Levelling is done with respect to the national Dutch height system, the Normaal Amsterdams Peil (NAP). This system is founded by levelling measurements. Because of the instability of the underground and the foundation of the markers, some of these markers are declined. Furthermore the Netherlands are tilting. The difference in declination of the NAP points causes the NAP not to be a real surface anymore, but more or less a point cloud with a thickness of a few centimetres.

GPS measurements are performed in the ETRS89 system. This system is established in the Netherlands by AGRS.NL, which consists of 5 reference stations: Delft, Apeldoorn, Westerbork, Terschelling and Eijsden. In 2004, a new solution for the AGRS.NL coordinates was computed, based on the year 2002 solution and realisation ITRF2000, known as ETRS89 based on realisation ITRF2000 of ITRS on epoch 2002.5(ITRS). The height difference with the 1997 solution is between 4 and 7 mm [Pol and Kösters, 2004].

The first two sections of this chapter describe the data sets containing GPS/levelling information. These are the 5th accuracy levelling by the Adviesdienst Geoinformatie en ICT (AGI) of Rijkswaterstaat (4.1) and the GPS-Kernnet of the Dutch Kadaster (4.2). Section 4.3 describes the way the structure of the variance/covariance matrix of NLGEO2006 is determined.

4.1 The 5th accuracy levelling

The 5th accuracy levelling has taken place from 1996 until 2000. Based on this levelling campaign, the heights of the NAP markers were corrected and published on January 1, 2005. At the same moment NLGEO2004 became the official Dutch geoid. The reason for the correction of the NAP heights was the difference between the official heights and the result of the 5th accuracy levelling of -4 to +2 cm. Moreover, the reference point in Amsterdam (000A2350, $\lambda = 4^{\circ}53'31''$, $\varphi = 52^{\circ}22'23''$) was not stable

enough. This caused a group of 6 underground markers in the middle of the Netherlands to be chosen as a new stable reference to which the height at Amsterdam was adjusted. With these new heights, the following goals were reached [Brand et al, 2004]:

- Stable fixation of the zero point of the NAP, where a connection to the historical definition of the zero level is matched as good as possible.
- Computation of new heights for the points of the primary network, where precision and reliability are sufficient for practical use. This means disturbances may not be noticed in large scale levelling projects and by the use of satellite navigation.
- Adjustment of the new geoid model (NLGEO2004) the new heights of the first order NAP network.

These corrected NAP heights were also the basis of the correction surface of NLGEO2004 and during this research also the new heights are used. The points of the 5th accuracy levelling are shown in figure 4.1.

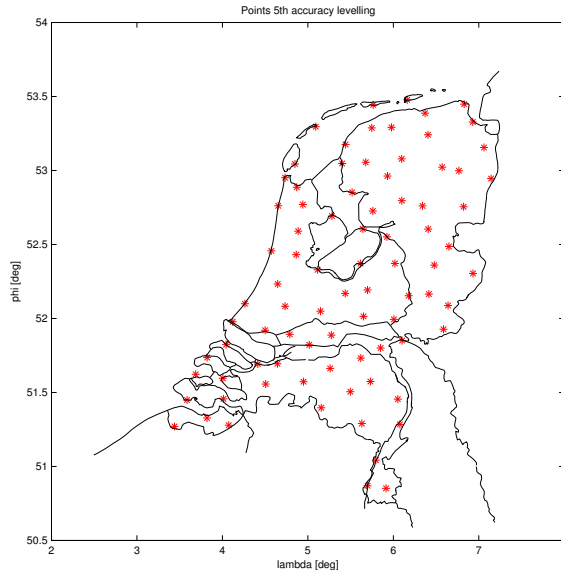


Figure 4.1: Locations of the 84 points of the 5th accuracy levelling

Variances and covariances of NAP heights

The primary NAP network exists of the points of the 5th accuracy levelling. Although these 84 points were part of one large adjustment procedure, the real variances and covariances are not available anymore. For these points, the relative accuracy with respect to Amsterdam was determined as the standard deviation in and expressed in terms of mm of a point [Brand, 2004], [Kösters, 2006, personal communication]:

$$\sigma_{rel} = 2 + 0.2 \cdot \sqrt{L} \quad (4.1)$$

with L the distance in kilometres to the zero point of the NAP in Amsterdam . These standard deviations were already computed by the AGI and can be found in appendix D.

Covariance information is not available anymore. Although equation 4.1 is also used to compute the relative accuracy between two first order NAP points, this is not enough information to compute the covariances between the points. This means only a diagonal variance matrix can be derived for the NAP heights at the points of the 5th accuracy levelling. These variances vary between 3.2 and 20.6 mm².

Variances and covariances of GPS heights

On the points of the 5th accuracy levelling also GPS measurements are collected. This was done to connect the levelled NAP data to the gravimetric geoid by the computation of a correction surface for the gravimetric geoid (as described in section 3.4). The GPS stations are connected to 4 AGRS.NL stations (Delft, Eijssden, Terschelling and Westerbork) and to 4 IGS stations: Brussels (B), Onsala (S), Potsdam (D) and Wettzell/Koetzing (D). The variances of the GPS heights vary between 74.0 and 278.9 mm². A total list of standard deviations per point of the 5th accuracy levelling was provided by the AGI and can be found in appendix D.

It is not possible to reconstruct the covariances of the GPS heights, therefore also for the GPS heights is worked with a diagonal variance matrix.

4.2 GPS Kernnet

The GPS Kernnet is maintained by the Kadaster and is called Kernnet for short. This network consists of 406 points all over the Netherlands with a mutual distance of 10 to 15 kilometres. The network was set up to create a possibility for users to connect their GPS measurements easily to the RD's system of coordinates. The locations are chosen in a way that they have unobstructed view and are easily accessible by car. They are shown in figure 4.2. The network is connected to the EUREF, so the coordinates are expressed in ETRS89 [de Bruijne et al, 2005].

During the computation of a correction surface for the gravimetric geoid, it was already noticed there were blunders in the heights of the Kernnet points. These points are removed from the data set. This is done by comparing the values of $h_{GPS} - H_{NAP}$ of the Kernnet by computed values. These computed values are determined by a multiquadric interpolation through the points of the 5th accuracy levelling. The largest difference was 70 centimetres (point 1250323_11). The histogram of the differences can be found in figure 4.4. In this histogram only differences of 10 cm or less are taken into account. Thresholds of 5, 4 and 3 centimetres were tested, causing 40, 54 and 90 points to be removed from the original data set respectively. Based on these values it was decided to work with a threshold of 3 centimetres, so 316 points are left. These are shown in figure 4.3.

As little is known about the trajectories of the levelling heights and the measurement setup of GPS height, no covariances are taken into account for the Kernnet.

Variances NAP heights

The Kernnet points are connected to 2 points of the NAP network with the requisites of method 2B of the NAP network. This means the maximum tolerance in millimetres between forward-backward levelling of two points is $2.5 \sqrt{L} + 0.5 L$, with L in kilometres and the tolerance in millimetres [AGI,

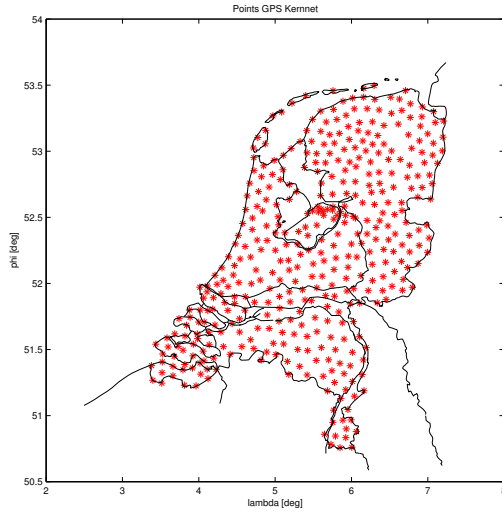


Figure 4.2: Points of the GPS-kernet

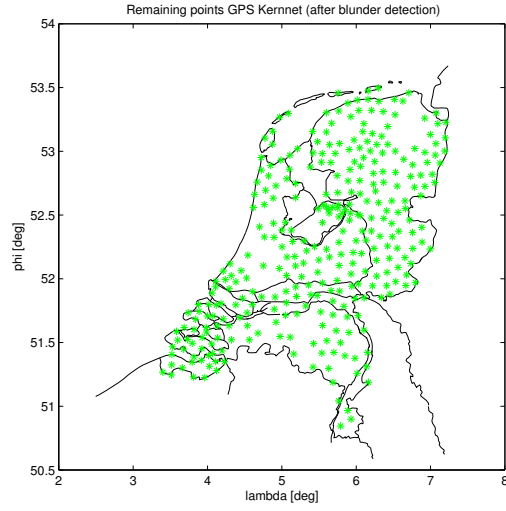


Figure 4.3: The 316 points of the Kernnet left after blunder detection.

2004]. Assuming the measurements to be normally distributed, the maximum tolerance is set equal to 3σ ($\sim 99.7\%$ confidence interval). It also can be stated that the largest levelling distance to the nearest NAP point (L) is 2.5 km. This distance is assumed for all points (worst case scenario), so the standard deviation of a connection to the NAP network, $\sigma_{NAP,KN}$, is equal to 1.7 mm:

$$3\sigma = 2.5 \cdot \sqrt{2.5} + 0.5 \cdot 2.5 \text{ mm} \Rightarrow \sigma = 1.7 \text{ mm} \quad (4.2)$$

The variance of a NAP point is computed by equation 4.1. This can also be done for the points to which the Kernnet is connected. It is not known to which 2 NAP points a Kernnet point is connected, but their distance to Amsterdam can be taken equal to the distance from the Kernnet point to Amsterdam. This distance is used to compute the standard deviation of the NAP points, σ_{NAP} by equation 4.1. This

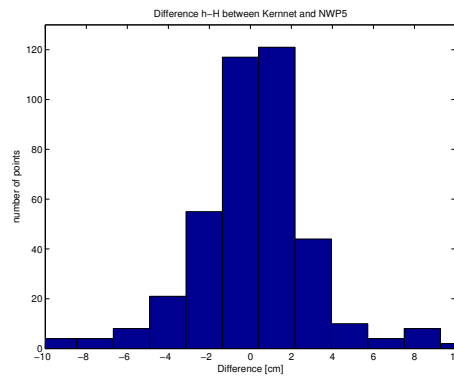


Figure 4.4: Histogram of the difference between $h - H$ of the Kernnet and NWP5

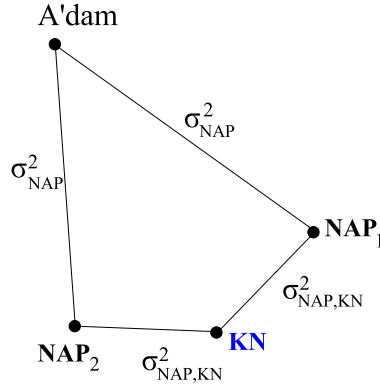


Figure 4.5: Connection of Kernnet points to the NAP network

means that the total variance of the NAP heights of the Kernnet can be determined by:

$$\sigma_{H_{NAP,KN}}^2 = \sigma_{NAP}^2 + \sigma_{NAP,KN}^2 \quad (4.3)$$

This results in variances between 4.55 and 12.55 mm².

Variances GPS heights

About the accuracy of the GPS heights less is known. The measurements are done with respect to ETRS89. The measurements are done in two periods of 90 minutes and only the difference between 2 or 3 measurements per point is known. Unfortunately, this is not known for all points and in some cases the difference is more than 1 meter. Therefore it was chosen to work with an identity variance matrix for the GPS heights of the Kernnet points. It is possible to scale this matrix with an a-priori variance, based on the mean difference, d , between 2 measurements, assuming $\sigma \sim \frac{1}{\sqrt{(2)}}d$. The computed standard deviation is equal to 18.7 mm.

4.3 GRAVGEO2006

The geoid information used during this research are the geoid heights of GRAVGEO2006, described in the previous chapter. The result of the geoid computation is a 3'x5' grid over the area of the Netherlands. Gravimetric geoid heights at GPS/levelling points are interpolated by the use of Overhauser splines.

The computation of a total variance/covariance model for a geoid model is beyond the scope of this research, although the result would be more realistic. This is because a lot of different data is used for geoid determination, as shown in chapter 3.

The total gravimetric geoid consists out of a global, N_1 and a local geoid part, N_2 :

$$N = N_1 + N_2 \quad (4.4)$$

Therefore the variance/covariance matrix is an addition of the matrix for the local part and the global part:

$$Q_N = Q_{N_1} + Q_{N_2} \quad (4.5)$$

First the global part of the geoid. This part is computed out of the global geopotential coefficients of the EIGEN-CG03C by:

$$N_1(P) = \frac{GM}{R\gamma} \sum_{n=2}^{\infty} \sum_{m=-n}^n Q_n^{MWG} \Delta \bar{c}_{nm} \bar{Y}_{nm} \quad (4.6)$$

with R the mean radius of the Earth, Q_n^{MWG} the Meissl/Wong&Gore coefficients, $\Delta \bar{c}_{nm}$ the global coefficients and $\bar{Y}_{nm}$ normalised spherical harmonics.

As we have only variances and no covariances for the global geopotential coefficients, the variances of the geoid heights can be computed by:

$$\sigma_{N_1}^2 = \left(\frac{GM}{R\gamma} \right)^2 \sum_{n=2}^{\infty} \sum_{m=-n}^n (Q_n^{MWG})^2 \sigma_{\Delta \bar{c}_{nm}}^2 \bar{Y}_{nm}^2 \quad (4.7)$$

The local part of the geoid is computed by

$$N_2(P) = \sum_{i=1}^I w_i^{MWG} \Delta \bar{g} \quad (4.8)$$

and so the variances by:

$$\sigma_{N_2}^2 = \sum_{i=1}^I (w_i^{MWG})^2 \sigma_{\Delta \bar{g}}^2 \quad (4.9)$$

with

$$w_i^{MWG} = \frac{i-1}{2} Q_i^{MWG}(\psi_0) \quad (4.10)$$

and I the number of grid cells. Only noise of the block mean gravity values is assumed. This noise is assumed to be normally distributed with a standard deviation of 0.15 mgal.

Due to the complexity of the geoid computation and by that a complex error propagation, it was chosen to perform a Monte Carlo simulation. The simulation is done by computing 170 realisations of the geoid model with adding random noise to the global geopotential coefficients, based on the error information of the GFZ Potsdam, and the block mean gravity values. Due to the high computational load this is done on the SGI Altix 3700 super-computer in the framework of the grant SG-027, which was provided by “Stichting Nationale Computerfaciliteiten” (NCF).

Out of these 170 realisations the variances and covariances of the geoid heights for the grid cells are computed by

$$\hat{\sigma}_j^2 = \frac{1}{m-1} \sum_{i=1}^m (N_{i,j} - \bar{N}_j)^2 \quad j = 1, 4480 \quad (4.11)$$

$$\hat{\sigma}_{j,k} = \frac{1}{m-2} \sum_{i=1}^m (N_{i,j} - \bar{N}_j) (N_{i,k} - \bar{N}_k) \quad j, k = 1, 4480 \quad (4.12)$$

with m the number of realisations and j and k the indexes of grid cells.

Unfortunately this variance/covariance matrix is not positive definite, which is necessary in the used variance component estimation to achieve positive variance components [Rao and Kleffe, 1988]. Most

likely this is due to the small amount of realisations, as we have 4480 grid points and only 170 realisations. Per realisation we can get one independent estimated variance:

$$Q_N \approx \frac{1}{I} \sum_{i=1}^I \underline{x}_i \underline{x}_i^T \quad (4.13)$$

with $\underline{x}_i$ the measurements reduced by the mean value. In this case holds $I \ll \dim(\underline{x}_i)$, which causes the variance/covariance matrix to be rank deficient.

Due to time restrictions it was decided to work with the result of the 170 realisation. Instead of working with the computed covariances it was decided to compute a correlation function. Correlations are computed out of covariances by:

$$\rho(i, j) = \frac{\sigma_{i,j}}{\sigma_i \sigma_j} \quad (4.14)$$

with i, j being two different points. For the variance the estimated values are used.

The correlation function is determined by using the grid points, making 4480 point in total which results in 10,032,960 point pairs. For these pairs correlation is plotted against the mutual distance (fig. 4.6). Next the empirical correlation function is computed by taking the average correlation per interval of 10 kilometres. There are several ways to compute estimate a function through this empirical function. Two possibilities are discussed in this section and shown in figure 4.7.

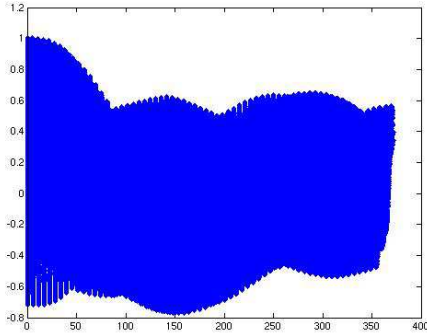


Figure 4.6: Scatterplot of the correlations

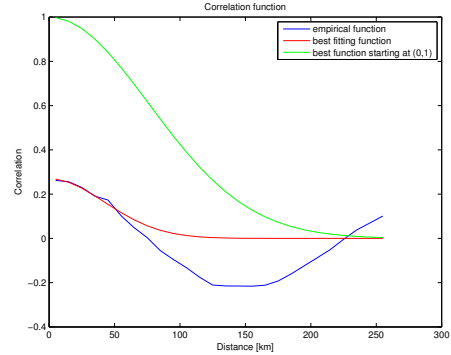


Figure 4.7: Correlation function GRAV-GEO2006

The first possibility is to fit a positive definite function as good as possible through the data. The described covariance functions in [Moritz, 1980, ch. 22] are used. Thereby the function of the following form was chosen:

$$\sigma^2(x) = C_0 e^{-A^2 x^2} \quad (4.15)$$

with:

$$A^2 = \frac{\ln(2)}{\xi^2} \quad (4.16)$$

where ξ is the correlation length in kilometres and C_0 the empirical correlation at zero-distance, resulting in:

$$\rho(x) = 0.27 e^{-\frac{\ln(2)}{50^2} x^2} \quad (4.17)$$

with x the distance between two points.

The (co)variances are computed for the points of the 5th accuracy levelling and the GPS-Kernet by using the propagation law of variances and covariances:

$$Q_{N,\text{pnt}} = A Q_{N,\text{grid}} A^T \quad (4.18)$$

where A contains the 16 coefficients of the Overhauser spline interpolation per interpolation point.

In section 7.1 the use of the correlation function of equation 4.17 is discussed because it causes problems in the variance component estimation. Therefore a correlation function which starts with a correlation of 1 at zero-distance was used, which has the same shape as the empirical function. This results in:

$$\rho(x) = e\left(-\frac{\ln(2)}{90^2}x^2\right) \quad (4.19)$$

The resulting variance/covariance matrices for the correlation functions described by equations 4.17 and 4.19 are shown in figures 4.8 and 4.9.

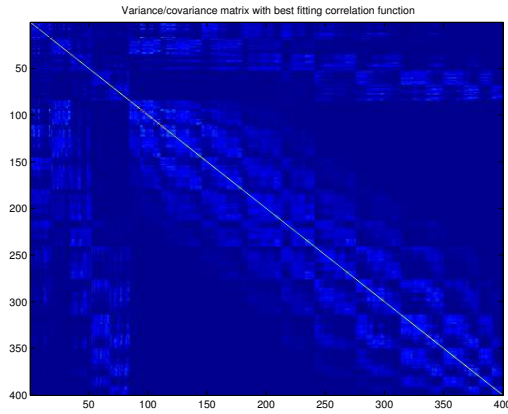


Figure 4.8: Variance/covariance matrix for the GPS/levelling points based on equation 4.17

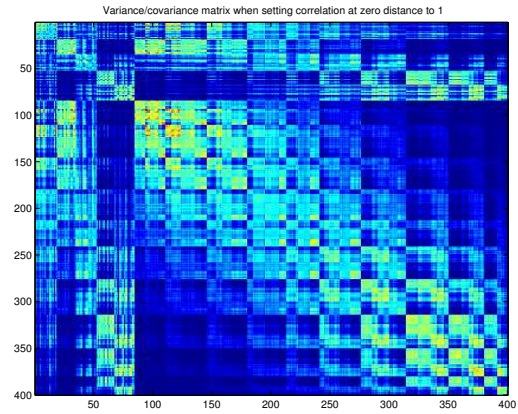


Figure 4.9: Variance/covariance matrix for the GPS/levelling points based on equation 4.19

Difference in correlation function means a different structure of the a-priori variance/covariance matrix of GRAVGEO2006. It cannot be said which structure is the most realistic, but the structure plays an important role in the variance component estimation. This role is discussed in section 7.1.

The data sets described in this chapter are the input later on. First we look at the possibilities of working with NAP and GPS heights only, without taking the variance/covariance information into account. This is the subject of chapter 5. Next geoid information is added and a variance component estimation is performed to get an idea of the correctness of the a-priori variance information. This is used to compute a height reference surface for the Netherlands. This is described in chapter 6.

Chapter 5

Height reference surface

The way the geoid is computed at this moment was described in chapter 3. It is also explained that the main focus of this research is on the transformation between GPS and NAP heights, not on getting the optimal gravimetric geoid. As shown, nowadays use is made of the gravimetric geoid which is fitted through 84 GPS/levelling points to get a transformation surface between GPS and NAP heights. While we are not interested in gravity but in a height transformation, it is also possible to make the differences between GPS and levelling heights the most important input data. This is done in this chapter with the GPS/levelling data described in section 4.1.

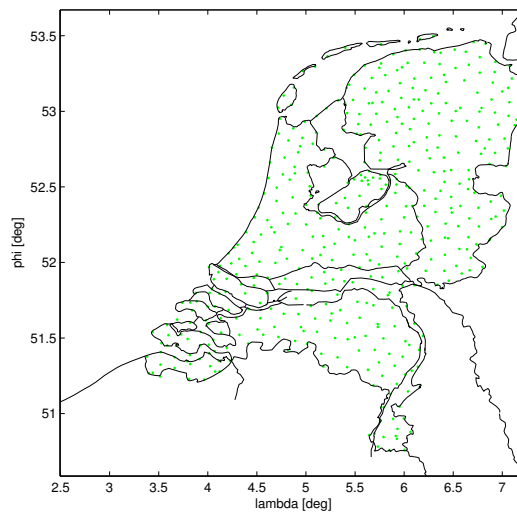


Figure 5.1: Location of the 472 input points, 18 points out of the original data set are removed because of the small distance to a neighbouring point

For all 490 points of the 5th accuracy levelling and the GPS Kernnet the difference between GPS height and NAP height is computed. This means the bad points as described in section 4.2 were not removed at this stage of the research. In this data set with 490 points, 18 points with a mutual distance of less than 2 km to another point were removed (fig. 5.1) to avoid a singular matrix in the computation. The remaining 472 points are the input points for the computation of a surface which gives a fairly good transformation

between GPS and NAP heights. This surface is computed by a multiquadric interpolation through the 472 GPS/levelling points. This method is explained in section 5.1. Multiquadric interpolation involves the choice of a smoothing parameter. This parameter is not known a priori but has to be determined from the data. The discussion to find the best smoothing parameter can be found in section 5.2. To give a comparison between different interpolation methods, also use is made of thin plate splines at the end of that section. Conclusions are drawn in section 5.3.

5.1 Multiquadric interpolation

The multiquadric interpolation method was introduced by Hardy [1971]. The purpose was to represent topography from a small amount of data points.

Hardy [1971] describes three different approaches of the multiquadric interpolation method. These are the collocation, the least squares and the osculating approach. In the collocation approach the given data points are exactly represented by the interpolated surface. This is contrary with the least squares approach, where it is possible to add additional information like slopes at certain data points and perform a least squares fit where the slopes are fitted exactly. The third approach is the osculating approach and is a combination of the first two approaches. This means that the data points and the slopes are fitted exactly. As we do not have other information than the heights at the data points, in this research the collocation approach is used. An overview of the development of the multiquadric interpolation method over the years can be found in Hardy [1990].

A combination of the multiquadric interpolation and a polynomial is used. A low-order polynomial is usually added to a radial base function when otherwise the coefficient are not uniquely defined. In this case, with a base function at every entry point, this was not necessary, but does not influence the final results. The interpolation is represented by the following series expansion:

$$z(\lambda, \varphi) = \sum_{i=1}^n \alpha_i K(\lambda, \varphi, \lambda_i, \varphi_i) + \sum_{k=1}^4 \beta_k q_k(\lambda, \varphi) \quad (5.1)$$

In this equation, K_i represents the base functions per data point, also called kernel. This kernel is based on the distance (MQ) or the reciprocal distance (RMQ) between points and represented by equations 5.2 and 5.3.

$$K^{MQ} = [\ell^2 + \Delta^2]^{1/2} \quad (5.2)$$

$$K^{RMQ} = [\ell^2 + \Delta^2]^{-1/2} \quad (5.3)$$

with ℓ the Euclidean distance is computed by the Haversine Formula:

$$\ell_{P_i, P} = 2R \sin^{-1} \left(\sqrt{\sin^2 \left(\frac{\varphi_P - \varphi_{P_i}}{2} \right) + \cos \varphi_{P_i} \cos \varphi_P \sin^2 \left(\frac{\lambda_P - \lambda_{P_i}}{2} \right)} \right), \quad (5.4)$$

where R is the semi-major axis of the Earth and P are the given data points. In this research the MQ kernel is used.

The second part on the right hand side of equation 5.1 represents a polynomial fit of which the coefficients are defined as follows:

$$q_1 = 1, \quad q_2 = \lambda, \quad q_3 = \varphi, \quad q_4 = \lambda \cdot \varphi \quad (5.5)$$

The model in equation 5.1 is not solvable in this way, because there are more unknowns than observations. To solve this constraints are added. Because the first and second term are not allowed to model signal from the same suspect, the following holds:

$$\sum_{i=1}^n \alpha_i q_k(\lambda_i, \varphi_i) = 0, \quad k = 1, \dots, 4 \quad (5.6)$$

The coefficients α_i and β_k in equation 5.1 are determined by solving:

$$z(\lambda_j, \varphi_j) = \sum_{i=1}^n \alpha_i K(\lambda_j, \varphi_j, \lambda_i, \varphi_i) + \sum_{k=1}^4 \beta_k q_k(\lambda_j, \varphi_j); \quad j = 1, \dots, n \quad (5.7)$$

This equation can be written in matrix notation

$$\mathbf{A}\boldsymbol{\alpha} = \mathbf{z} \quad , \quad (5.8)$$

with the model matrix $\mathbf{A}$ represented by:

$$\left[\begin{array}{ccc|cccc} K_{11} & \cdots & K_{1n} & 1 & \lambda_1 & \varphi_1 & \lambda_1 \varphi_1 \\ \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\ K_{n1} & \cdots & K_{nn} & 1 & \lambda_n & \varphi_n & \lambda_n \varphi_n \\ \hline 1 & \cdots & 1 & 0 & \cdots & \cdots & 0 \\ \lambda_1 & \cdots & \lambda_n & 0 & \cdots & \cdots & 0 \\ \varphi_1 & \cdots & \varphi_n & 0 & \cdots & \cdots & 0 \\ \lambda_1 \varphi_1 & \cdots & \lambda_n \varphi_n & 0 & \cdots & \cdots & 0 \end{array} \right] \quad (5.9)$$

The upper left square of this matrix consists of the base functions for the multiquadric interpolation, represented by:

$$K_{ij} = \sqrt{\ell_{P_i, P_j}^2 + \Delta^2}; \quad i, j = 1, \dots, n, \quad (5.10)$$

Because matrix $\mathbf{A}$ is a square matrix of full rank it can be inverted. This means that the coefficient vector $\boldsymbol{\alpha}$ can be computed by:

$$\boldsymbol{\alpha} = \mathbf{A}^{-1} \mathbf{z} \quad . \quad (5.11)$$

About the determination of the parameter Δ in multiquadric interpolation much discussion can be found in literature. Therefore, a couple of methods are investigated in this research which are explained and discussed in the next section.

5.2 The choice of Δ

The multiquadric interpolation kernel has one parameter to be selected by the user, the smoothing parameter Δ . Most literature says Δ to depend on the data distribution. But also the option of dependence on the data values itself is mentioned [Carlson, 1991]. In this section three methods are used and the results are compared.

Depending on residuals at the control points

The choice of Δ is data dependent. To see the effect of different smoothing parameters on our data set, the data set is split into two parts: a test data set which will be used to compute the coefficients α_i and β_k and a set of control points where the heights are computed by the use of the computed coefficients. This is done 100 times, with a test set with 422 points and 50 control points. For these data sets the Δ with the lowest RMS is determined and the result is shown in figure 5.2.

In this figure can be seen that different splitting of data into test and control points give a different optimal Δ . It can immediately be seen that the choice of Δ heavily depends on the choice of control points. From figure 5.3 also can be seen that the influence of Δ on the height of the control points is small. This figure shows the height difference on the control points for $\Delta = 0$ m and $\Delta = 4000$ m. This difference is on millimetre level, which is below measurement accuracy. Although the differences are small, most times the best interpolation result is obtained by the use of $\Delta = 0$ m. The result of the multiquadric interpolation with this smoothing parameter is shown in figure 5.4. In figure 5.4(a) some points with a large difference between interpolated and given value can be seen. These points most likely are blunders in the data set.

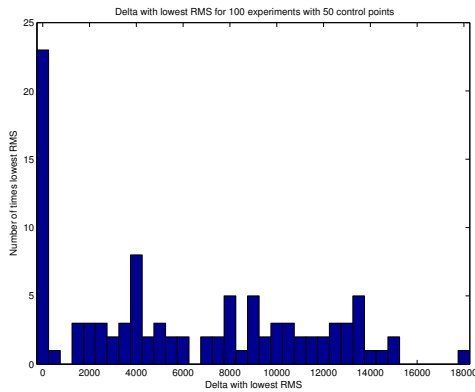


Figure 5.2: The optimal Δ for 100 experiments with 422 test points and 50 control points.

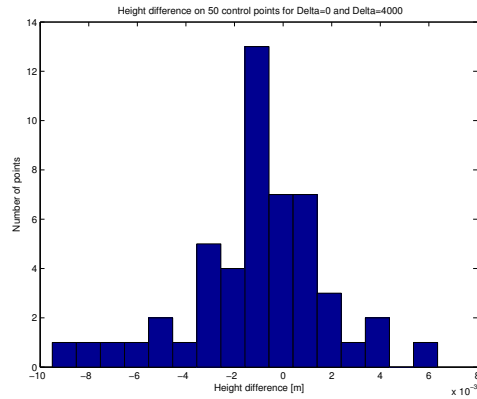


Figure 5.3: The height difference on 50 control points by using $\Delta = 0$ m and $\Delta = 4000$ m.

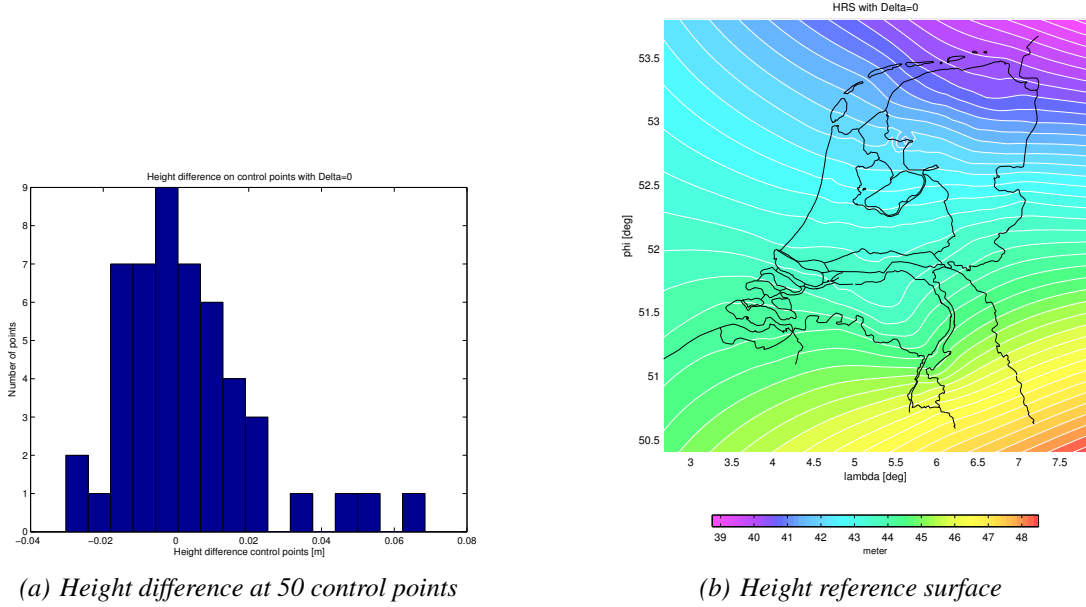


Figure 5.4: The height reference surface computed with $\Delta = 0$ m

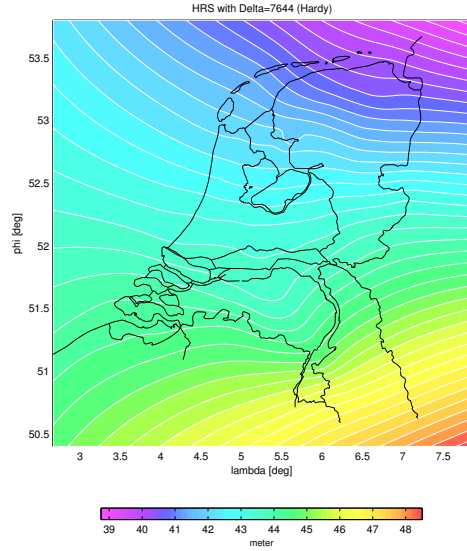
Depending on point density

In most literature the choice of Δ depends on the mean distance from each point to each nearest neighbour. This distance, d , is computed by

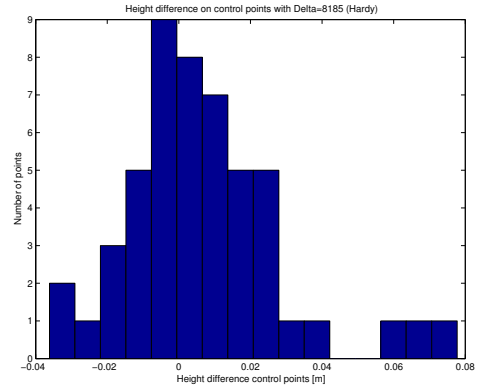
$$d = \sqrt{\frac{\text{surface of data}}{\text{number of points}}} \quad (5.12)$$

The surface of the Netherlands is equal to 41526 km² [www.wikipedia.nl]. According to Hardy [1990], Δ is equal to $0.815d$, Carlson says it to be $1.25d$ [Carlson, 1991]. In this research 472 points are used, which causes d to be equal to 9380 m. This means that with Hardy's method $\Delta = 7644$ m and with the method of Carlson $\Delta = 11725$ m. Because, in order to look at the RMS, also control points are needed, only 422 points are used in the actual interpolation. This means that for the methods of Hardy and Carlson Δ is equal to 8185 and 12400 respectively. The results for the data sets as a whole are shown in figures 5.5(a) and 5.6(a). The difference between the given heights and the interpolated heights at the control points is shown in figures 5.5(b) and 5.6(b).

The difference between the solutions for both values of Δ is made clear in the figure 5.7. This difference is on centimetre level.

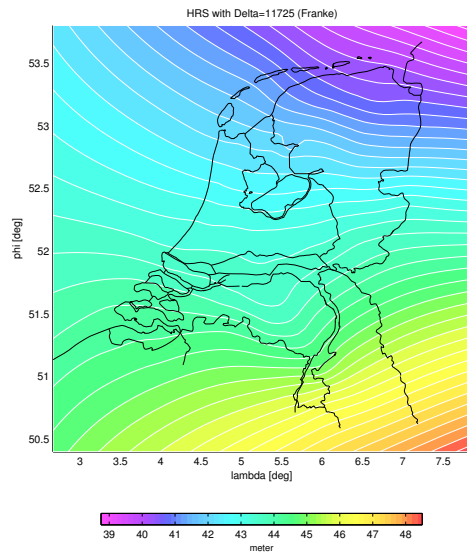


(a) Height reference surface

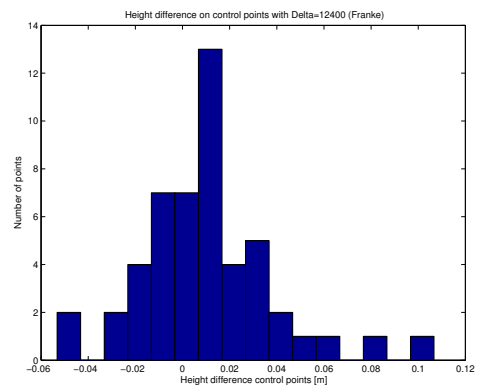


(b) Height difference at 50 control points

Figure 5.5: The height reference surface computed with $\Delta = 7644$ m

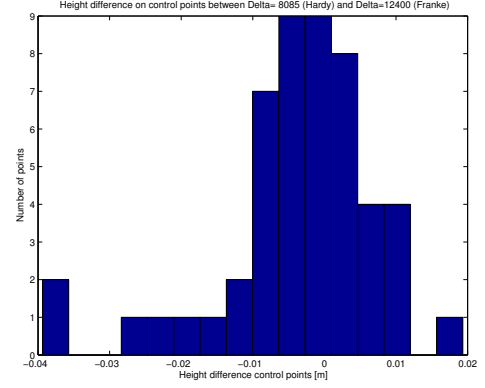
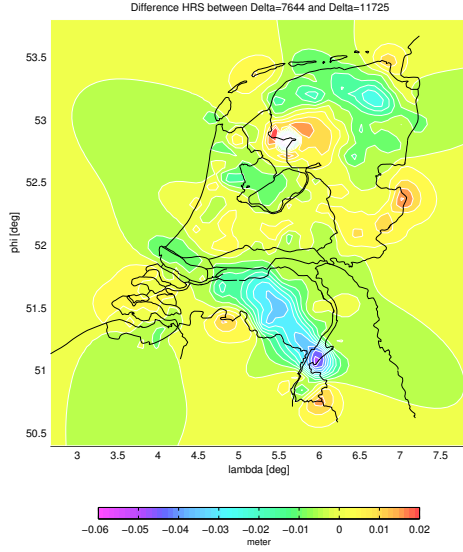


(a) Height reference surface



(b) Height difference at 50 control points

Figure 5.6: The height reference surface computed with $\Delta = 11725$ m



(a) Difference between height reference surfaces

(b) Height difference between height reference surfaces at 50 control points

Figure 5.7: The difference between the height reference surface computed with $\Delta = 7644$ m and $\Delta = 11725$ m

Depending on quadratic polynomial fit through scaled data

Another method of Carlson [1991] is not based on the point density but on the data values z_i to be interpolated. He discovered the observed residual values of a quadratic least squares fit are roughly inversely proportional to an effective choice for Δ by experimenting with different data sets. For this method, the data is scaled to a unit cube. This is done since the error of the quadratic fit, $q(\bar{x}, \bar{y})$, is affected by the scale of the z_i values. For the scaled coordinates holds:

$$\begin{aligned}\bar{x}_i &= \frac{x_i - x_{min}}{x_{max} - x_{min}} \\ \bar{y}_i &= \frac{y_i - y_{min}}{y_{max} - y_{min}} \\ \bar{z}_i &= \frac{z_i - z_{min}}{z_{max} - z_{min}}\end{aligned}$$

Through this scaled data, a quadratic polynomial with 6 unknown parameters is fitted.

$$q(\bar{x}_i, \bar{y}_i) = a + b\bar{x}_i + c\bar{y}_i + d\bar{x}_i^2 + e\bar{y}_i^2 + f\bar{x}_i\bar{y}_i \quad (5.13)$$

The average residual is then computed by

$$V = \sum_{i=1}^n \frac{(\bar{z}_i - q(\bar{x}_i, \bar{y}_i))^2}{n} \text{ m}^2 \quad (5.14)$$

From this $\overline{\Delta}$ is defined.

$$\overline{\Delta} \doteq \sqrt{\frac{1}{1 + 120V}} \text{m} \quad (5.15)$$

While this value for $\overline{\Delta}$ is based on scaled data, the multiquadric interpolation also has to be performed on the scaled data. The result then must be transformed to the original domain again.

This procedure is followed with a resulting $\overline{\Delta} = 0.92492$ m. The value of $\overline{\Delta}$ is large when looking at the size of the scaled interpolation area. This means that the interpolation becomes very smooth which causes important details to be lost. The interpolated height reference surface can be seen in figure 5.8(a). The difference of the heights at the control points are shown in 5.8(b). Because this is up to several tens of centimetres, this method of determining Δ is not appropriate for this data set.

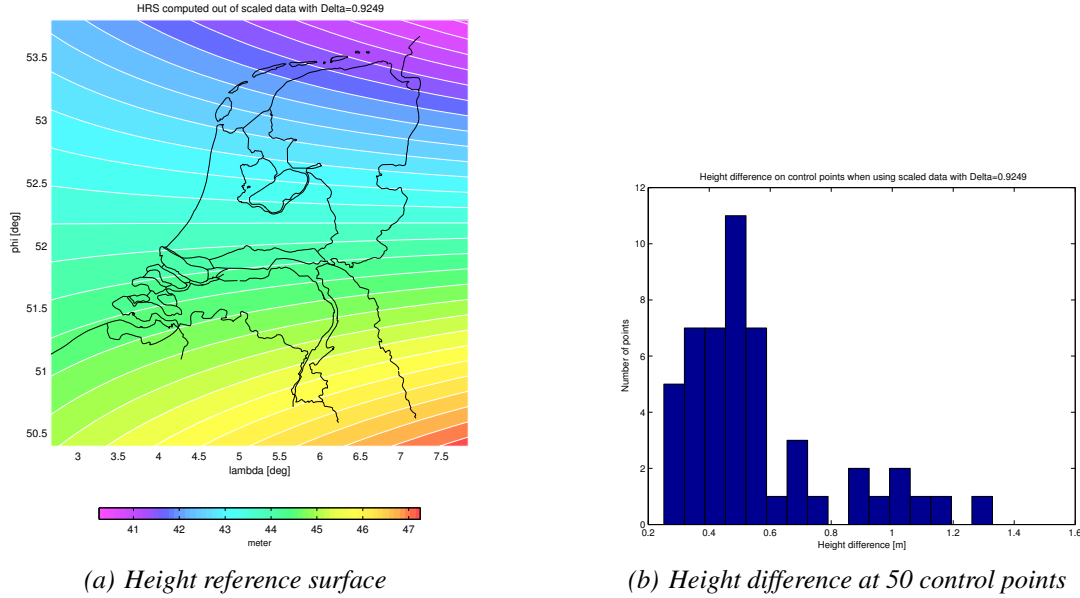


Figure 5.8: The HRS computed by the method of [Carlson, 1991]

Thin plate splines

In order to check the results of the multiquadric interpolation use is made of thin plate splines under tension. This is a generalisation of the minimum curvature interpolation. The minimum curvature interpolation gives the smoothest possible interpolation. A disadvantage is a misfit between the interpolation points. To decrease this misfit, tension is included by means of a tension factor T , with a value between 0 and 1. When $T = 0$ the thin plate spline interpolation is equal to the minimum curvature interpolation. When $T = 1$ the curvature is restricted to the data points, i.e. the minima and maxima are located on the data points [Smith and Wessel, 1990].

For the data set with 422 GPS/levelling points a thin plate splines interpolation is performed with different values of T with the Generic Mapping Tool (GMT). More information about this tool can be

found on www.soest.hawaii.edu/GMT. The RMS on the control points is computed (fig. 5.9) from which can be concluded that the best value for the tension factor is 1.

The interpolation result for the thin plate splines with $T = 1$ is shown in figure 5.10(a). The difference between the given and the interpolated heights at the control points is demonstrated in figure 5.10(b). From this difference can be concluded that the performance of multiquadric interpolation is as good as the performance of thin plate splines.

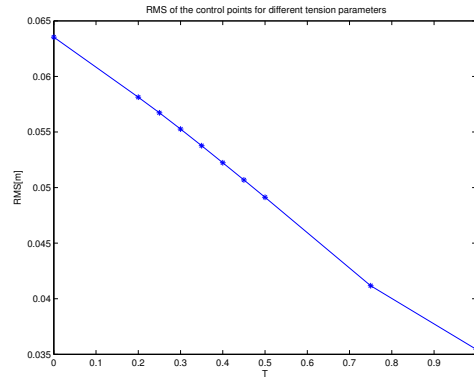
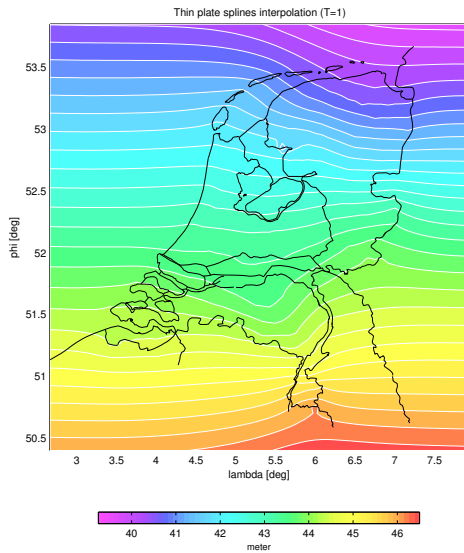
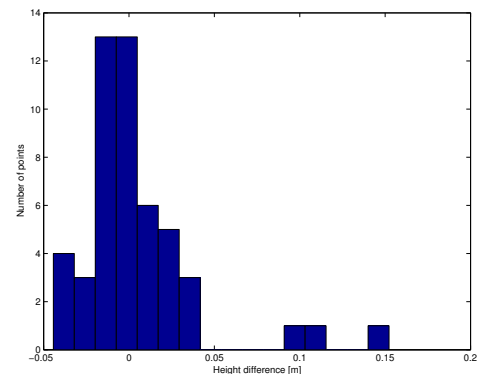


Figure 5.9: RMS at the control points. The smallest RMS is obtained when $T = 1$.



(a) Height reference surface



(b) Height difference at 50 control points

Figure 5.10: The HRS computed by the use thin plate splines with $T = 1$

5.3 Conclusions

In the previous section, the results of the multiquadric interpolation are shown. This section gives an overview of what can be concluded out of these results.

- For the multiquadric interpolation, several smoothing parameters Δ are used. The methods to find Δ give significant different results, so a method based on comparing the RMS for different values of Δ is the best choice to find the best parameter. Unfortunately with this data set the best choice for Δ heavily depends on the choice of the control points (fig. 5.2). It is shown that the choice of Δ does not have much influence on the final interpolation result (fig. 5.3).
- A comparison between multiquadric interpolation and thin plate splines is performed. The interpolation result for both methods is equal when looking at the height difference between computed and given heights at the control points (fig. 5.4(a) and 5.10(b)).
- All the histograms in this chapter contain points with a large difference between computed and given heights. Most likely these points are blunders and are to be removed in further research, as described in section 4.2.

It can be concluded that the multiquadric interpolation gives promising results and that $\Delta = 0$ m is the best choice with this data set. Interesting is to look at the difference between the computed height reference surface of figure 5.4 and NLGEO2006. This difference is shown in figure 5.11. The height difference at the GPS/levelling points is visualised in figure 5.12.

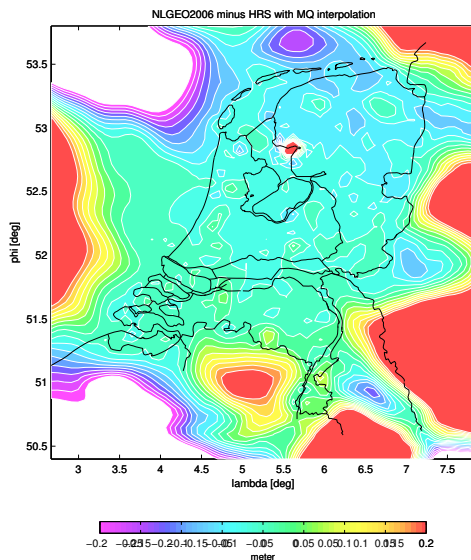


Figure 5.11: The difference between NLGEO2006 and the height reference surface computed by a multiquadric interpolation with $\Delta = 0$.

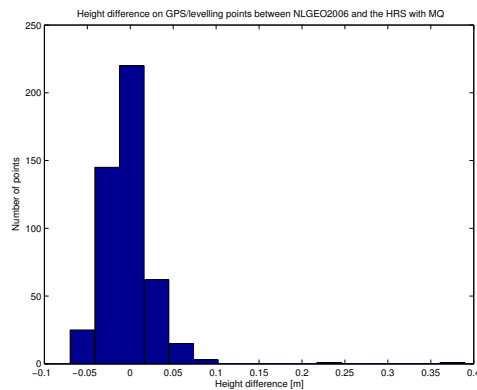


Figure 5.12: Histogram of the difference between NLGEO2006 and the HRS with multiquadric interpolation on the 472 GPS/levelling points

Figure 5.11 shows that the difference between NLGEO2006 and the HRS is 5 to 10 cm. This is a significant difference which is expected to be improved by adding gravimetric geoid information. This is the subject of the chapters 6 and 7.

Chapter 6

Variance component estimation

In the previous chapter a surface was computed by the use of only GPS and NAP heights at 490 GPS/levelling points. Neither gravimetric information nor variance/covariance information was taken into account. In this chapter a possible method to combine of all three types of heights – GPS, NAP and geoid – and the use of variance/covariance information is described.

The problem faced in this research is the transformation between GPS and NAP heights, because GPS still is coming up as measurement method and NAP is the height reference surface of the Netherlands, established by levelling. In theory, the difference between those height systems can be described by a geoid model, in practice it is not that easy. This is due to several reasons, of which the most important are:

- establishment of the NAP heights by the use of levelling, without taking gravity into account;
- subsidence of NAP markers over time, which causes the NAP surface to be more or less a point cloud;
- different reference ellipsoids for geoid computation and GPS measurements;
- measurement errors in NAP, GPS and geoid heights;
- different tidal systems.

An overview of the different reference errors and the possible errors is made clear in figure 6.1. Expressed in equations this means that $h_{GPS} - H_{NAP} - N$ does not hold, but instead the following is true:

$$h_{GPS} - \underline{H}_{NAP} - \underline{N} = A\underline{x} + \underline{e} \quad (6.1)$$

The purpose of this research is to find a suitable model A to cope with these problems and get better insight in the variance/covariance information of all three kinds of heights. Recent developments in this kind of research in other parts of the world can be found in for example Kotsakis and Sideris [1999], Fotopoulos [2003, 2005] and Jäger[1999].

The choice of the parametric model to describe the difference between the gravimetric geoid and the difference between NAP and GPS heights is the subject of section 6.1. Section 6.2 explains the method of variance component estimation (VCE). This method is used to compute a height reference surface for the Netherlands, which results can be found in the next chapter.

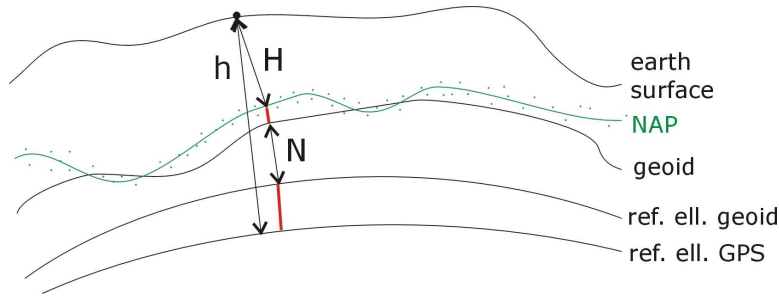


Figure 6.1: Overview of different reference surfaces

6.1 Choice of the parametric model

The first choice to be made when computing the height reference surface is the way it should be represented. This choice is due to the amount of data available, the computational load, the expected shape of the surface and the existence of residuals. This representation can be computed in total at once or in several parts and then be combined to one smooth surface as is done by Jäger [1999]. Although this splitting can give better results in the small areas, extra unknown parameters occur because the different parts have to be continuous differential on their boundaries. Looking at the size of the area, the number of GPS/levelling points and the expected pattern it was chosen to work with only one representation for the whole area and not with different patches.

There are a lot of possibilities to describe a surface, of which only a few are discussed here. No actual tests were performed before choosing a representation model, the choice was only based on theoretic advantages and disadvantages and the expected shape of the correction surface.

Multiquadric interpolation

In determining the surface based on GPS and NAP heights only was worked with multiquadric interpolation and this method gave good results. The method is described by:

$$z(\lambda, \varphi) = \sum_{i=1}^n \alpha_i [\ell^2 + \Delta^2]^{1/2} + \sum_{k=1}^4 \beta_k q(\lambda, \varphi) \quad (6.2)$$

and explained in detail in section 5.1. The resulting surface can be seen in figure 6.2. On all data points a base function is computed. This means that the input data is exactly reproduced, so no redundancy is left. While variance component estimation is based on the residuals at the input points, multiquadric interpolation is not a suitable representation in this case.

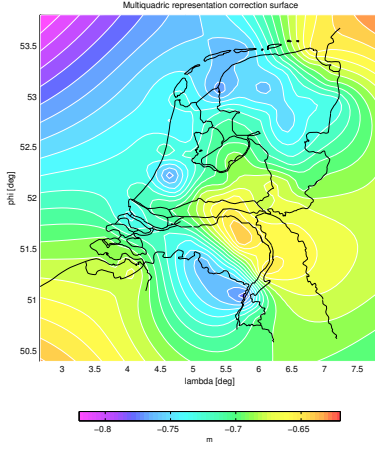


Figure 6.2: Representation of the correction surface by multiquadric interpolation

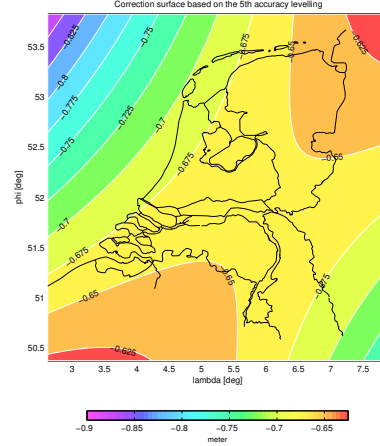


Figure 6.3: Representation of the correction surface by a quadratic polynomial

Quadratic polynomial

For the current geoid model a quadratic polynomial is fitted to compute the correction surface for the gravimetric geoid and is described in section 3.4. The advantage is that this kind of surface representation is easy to implement and only 6 parameters have to be estimated:

$$y = a + b(\lambda - 5.0) + c(\varphi - 52.0) + d(\lambda - 5.0)^2 + e(\varphi - 52.0)^2 + f(\lambda - 5.0)(\varphi - 52.0) \quad (6.3)$$

The resulting surface is shown in figure 6.3. This representation makes the correction surface inflexible, which makes the surface not to fit well on the edges of the area. It was already concluded in Hilferink [2001] that addition of more polynomial terms does not have a significant effect.

Trigonometric representation

The representation chosen in this research is the trigonometric, by the use of a 2-dimensional Fourier series. This is represented by:

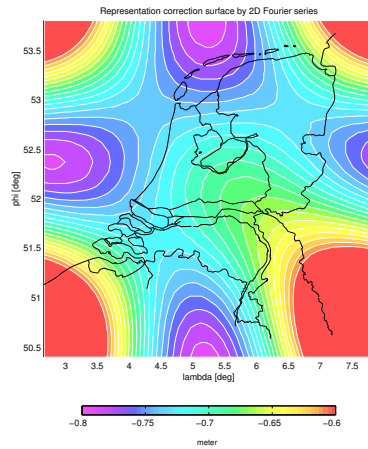
$$l = \sum_{m=-d}^d \sum_{n=-d}^d \varphi_m(x) \varphi_n(y) \quad \text{with} \quad \varphi_j = \begin{cases} \cos\left(\frac{2\pi j i}{D_i}\right) & m \geq 0 \\ \sin\left(\frac{2\pi j i}{D_i}\right) & m < 0 \end{cases} \quad (6.4)$$

where, d is degree, $j = m$ and $i = x$ or $j = n$ and $i = y$ and $D_x, D_y = 400$ km. The resulting surface is shown in figure 6.1. For these computations use is made of coordinates in a local Cartesian reference frame, x_{lc} . These are computed out of geocentric coordinates, x_{gc} , by the following transformation:

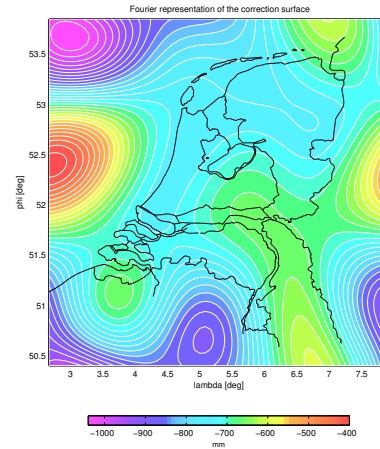
$$x_{lc} = R_Z(\lambda_c) R_Y(\varphi_c) x_{gc} + t \quad (6.5)$$

with R_Z and R_Y rotation matrices, λ_c, φ_c the coordinate centre of the region and t the translation vector. In this case $\lambda_c = 5.3^\circ$, $\varphi_c = 52.2^\circ$ and $t = (t_x, t_y, t_z) = (0, 0, 16)$ km.

This representation is chosen because it is more flexible than the biharmonic representation and the surface has the shape we expect beforehand. Also the computations are easy and higher order terms can



(a) Up to degree 1, RMS=18mm



(b) Up to degree 2, RMS=14mm

Figure 6.4: Fourier representation of the correction surface up to degree

be added easily. The representation of degree 1 is chosen, because a RMS of 1.8 cm is only slightly larger than 1.4 cm and not more realistic when looking at the measurement accuracies.

This research will focus on the stochastic model, taking the parametric model as chosen in this section. The next section describes briefly the idea and the equations of the variance component estimation.

6.2 Variance component estimation

In a least-squares adjustment of heights, mostly the focus is on the parametric model. But another important role is played by the stochastic model. The used variance/covariance information affects the results of the estimated parameters and performing variance component estimation gives deeper insight in the quality of the a-priori variances and covariances. Improving the stochastic model can be done by variance component estimation. This technique is used in for example the computation of a gravity model from CHAMP satellite data [Van Loon, accepted] and the computation of height reference surfaces [Kotsakis and Sideris, 1999], [Fotopoulos, 2003, 2005].

To estimate the variance components several different methods can be used. The differences lay in the functional model to be used, the stochastic model and the computational procedure. A complete overview goes beyond the scope of this research. Therefore this section discusses only the method used during this research. This is the Iterative Maximum Likelihood Estimator (IMLE). When this method is used with normal distributed data, the resulting variance components have minimum variance when the series are converging. This means this method gives equal results as the Minimum Norm Quadratic Unbiased Estimator (MINQUE) [Rao, 1971], the Best Invariant Quadratic Unbiased Estimator (BIQUE) [Sjöberg, 1983] and the Restricted Maximum Likelihood Estimator (REML) on which IMLE is based. The IMLE has a few advantages in comparison with the other methods, like the lower computational load and the fact that covariance factors cannot become negative for positive semidefinite co-factor matrices

[Rao and Kleffe, 1988].

In a least-squares adjustment the functional model is described by:

$$\underline{y} = A\underline{x} + \underline{e}, \quad D\{\underline{y}\} = Q_y \quad (6.6)$$

In our case $\underline{y} = \underline{h}_{GPS} - \underline{H}_{NAP} - \underline{N}$. The model A represents the trigonometric surface. The least-squares estimation for the vector of unknowns $\underline{x}$ is then computed by the well-known equation

$$\hat{\underline{x}} = (A^T Q_y^{-1} A)^{-1} A^T Q_y^{-1} \underline{y} \quad (6.7)$$

Variance component estimation shortly means that the a-priori variance/covariance matrix is improved within the least-squares adjustment procedure. The variance/covariance matrix describes the stochastic of the residuals, e :

$$Q_y = E\{(\underline{y} - A\underline{x})(\underline{y} - A\underline{x})^T\} = E\{\underline{e}\underline{e}^T\} \quad (6.8)$$

In the determination of a height reference surface out of different height types, all different types have their own stochastic errors. The total variance/covariance matrix, Q_y , is build from three co-factor matrices:

$$Q_y = \sigma_1^2 Q_1 + \sigma_2^2 Q_2 + \sigma_3^2 Q_3 = \sum_i \sigma_i^2 Q_i; \quad i = 1, 2, 3 \quad (6.9)$$

with

Q_i co-factor matrices
 σ_i^2 variance components

From now on the variance components σ_i^2 are gathered in the vector $\underline{\sigma}$.

The difference in variance components makes that the distinct observation groups are given another weight in the resulting variance/covariance matrix. This changes the structure of the total variance/covariance matrix in such a way that the weighted residual square sum, $\hat{\underline{e}}^T Q_y^{-1} \hat{\underline{e}}$ is minimised.

The REML method was first proposed by Hardville [1977] and modified by Koch [1986, 1990]. REML can only be used when the observations are normally distributed, which holds true in our case. The density function of the observations $\underline{y}$ with the unknown parameters $\underline{x}$ and $\underline{\sigma}$ is given by:

$$p(\underline{y}|\underline{x}, \underline{\sigma}) = \frac{1}{(2\pi)^{m/2} (\det Q_y)^{1/2}} \exp \left[-\frac{1}{2} (\underline{y} - A\underline{x})^T Q_y^{-1} (\underline{y} - A\underline{x}) \right] \quad (6.10)$$

The idea of REML is to maximise this density function. The vector with unknown parameters, $\underline{x}$, is assumed to exist of free parameters. These free parameters are integrated out of the likelihood function in order to maximise the part of the likelihood function independent of the unknown parameters. To achieve this, a non-informative prior is introduced, resulting in a constant value for the conditional density function:

$$p(\underline{y}|\underline{\sigma}) = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} p(\underline{y}|\underline{x}, \underline{\sigma}) p(\underline{x}|\underline{y}) d\underline{x} \quad (6.11)$$

which can be simplified, according to Koch [1990], to:

$$p(\underline{y}|\underline{\sigma}) \propto \frac{1}{(\det Q_y \det(A^T Q_y^{-1} A))^{1/2}} \exp \left(-\frac{1}{2} \underline{y}^T R \underline{y} \right) \quad (6.12)$$

where $\propto$ means proportionality and R is Rao's matrix:

$$R = Q_y^{-1} - Q_y^{-1} A (A^T Q_y^{-1} A)^{-1} A^T Q_y^{-1} \quad (6.13)$$

To find the REML the density function has to be maximised. This maximisation is simplified by looking for the maximum of the natural logarithm of the density function:

$$\ln p(\underline{y}|\underline{\sigma}) \propto C - \ln \det Q_y - \ln \det(A^T Q_y^{-1} A) - \underline{y}^T R \underline{y} \quad (6.14)$$

with C being a constant. Finding the maximum is done by setting the derivative equal to zero. Therefore this function is differentiated with respect to σ_i^2 . Use is made of the following relations [Koch, 1986]:

$$\frac{\partial \ln \det A}{\partial a_i} = \text{tr} \left(A^{-1} \frac{\partial A}{\partial a_i} \right) \quad (6.15)$$

$$\frac{A^{-1}}{\partial a_i} = -A^{-1} \frac{\partial A}{\partial a_i} A^{-1} \quad (6.16)$$

resulting in:

$$\begin{aligned} \frac{\partial \ln p(\underline{y}|\underline{\sigma})}{\partial (\sigma_i^2)} &= -\text{tr}(Q_y^{-1} Q_i) + \text{tr}[(A^T Q_y^{-1} A)^{-1} A^T Q_y^{-1} Q_i Q_y^{-1} A] - \underline{y}^T \frac{\partial R}{\partial (\sigma_i^2)} \underline{y} \\ &= -\text{tr}(Q_y^{-1} Q_i) + \underline{y}^T R Q_i R \underline{y} \end{aligned} \quad (6.17)$$

This means the maximum of this function is found by:

$$\underline{f}(\sigma) = -\text{tr}(Q_y^{-1} Q_i) + \underline{y}^T R Q_i R \underline{y} = 0 \quad (6.18)$$

This equation can be solved by using Fisher scoring, resulting in [Koch, 1986]:

$$S \underline{\hat{\sigma}} = \underline{u} \quad (6.19)$$

with

$$\begin{aligned} S_{ij} &= (R Q_i R Q_j) \\ u_j &= \underline{y}^T R Q_j R \underline{y} \end{aligned}$$

The computation of the elements of the matrix S_{ij} is costly and therefore simplified. This can be done by stating [Koch, 1986]:

$$S \underline{\bar{\sigma}} = \underline{t} \quad (6.20)$$

with

$$t_i = \text{tr}(R Q_i) \quad (6.21)$$

and $\underline{\bar{\sigma}}$ is the a-priori variance component or $\hat{\sigma}_{i-1}$. The matrix S is replaced by a diagonal matrix in a way that

$$D \underline{\bar{\sigma}} = \underline{t} \quad (6.22)$$

This means the diagonal elements can be computed by

$$d_i = \frac{t_i}{\bar{\sigma}_i} \quad (6.23)$$

Then the estimator becomes:

$$\hat{\sigma}_i = \frac{\underline{y}^T R Q_i R y}{\text{tr}(R Q_i)} \bar{\sigma}_i \quad (6.24)$$

which can be solved iteratively.

The general case of the estimator was first derived by Förstner [1979]. Therefore the IMLE is also known as Förstner's estimator.

In this chapter the equations used in variance component estimation were shortly explained. In the next chapter the result of the variance component estimation is discussed.

Chapter 7

Results of the variance component estimation

In the previous chapter the choice of the parametric surface to describe the difference between the gravimetric geoid and GPS/levelling heights was made. Also the theory of variance component estimation was explained. This chapter discusses the results of the variance component estimation for the Dutch data sets described in chapter 4. The a-priori variances are shown in table 7.1

Table 7.1: A-priori variances for the different data sets

Data set	Data type	A-priori standard deviations [mm]
5 th accuracy levelling	NAP heights	1.7 – 4.6
	GPS heights	8.6 – 16.7
	GPS/levelling	9.5 – 17.0
Kernnet	GPS/levelling	18.9 – 19.4
GRAVGEO2006	geoid heights	78 – 136

Section 7.1 analyses the results for the 5th accuracy levelling. The result of the combination of the 5th accuracy levelling and the Kernnet is discussed in section 7.2. The results are concluded in section 7.3. The surfaces out of the residuals after variance component estimation are interpolated by the use of thin plate splines. The RMS values computed in this chapter are the RMS values of the difference between the given heights and the computed heights, as this value shows how well the computed surface matches with the given heights.

7.1 5th accuracy levelling

Based on what is known beforehand about the different data sets, it is chosen to first use the variance component estimation only on the GPS/levelling data of the 5th accuracy levelling. The results of different ways to combine those data and the effect of the chosen correlation function for the gravimetric geoid data are discussed in this section. Section 7.1.1 discusses the results of working with 2 observation

groups, i.e. the GPS/levelling data and the gravimetric geoid. Section 7.1.2 describes the result when estimating variance components for levelling data and GPS data separately.

7.1.1 2 groups: Geometric and gravimetric geoid

When working with the 5th accuracy levelling, we have 84 input points. Variance component estimation is performed by stating that the data consists of 2 observation groups: GPS/levelling data ($h_{GPS} - H_{NAP}$) and gravimetric data (N). This results in the following representation of the total variance/covariance matrix:

$$Q_y = \sigma_1^2 Q_{h+H} + \sigma_2^2 Q_N \quad (7.1)$$

with Q_{h+H} a diagonal matrix and Q_N a full matrix.

The resulting variance components and the correction surface are computed twice. The difference between both models is correlation function for the gravimetric geoid. As discussed in section 4.3 different functions could be chosen.

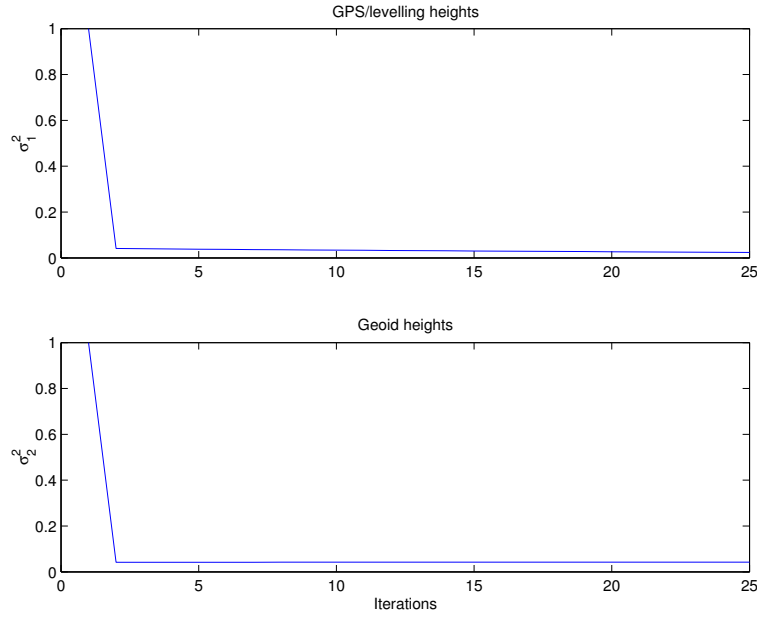


Figure 7.1: Variance components for the GPS/levelling data and the gravimetric geoid when working with the best fitting correlation function, $\sigma_1^2 = 2.4 \cdot 10^{-2}$, $\sigma_2^2 = 4.2 \cdot 10^{-2}$

The result of the best fitting correlation function (eq. 4.17) is shown in figures 7.1 and 7.2(a) with the residuals shown in figure 7.2(b). As can be seen in figure 7.1 the variance components are approaching zero. Most likely this is due to the missing covariance information for the GPS/levelling part. This means that the covariance information for all three height types comes from the gravimetric geoid only. This makes σ_2^2 larger to get the covariances right. At the same time the variances are getting larger, and therewith larger then they should be. This is compensated in Q_y by down weighting Q_{h+H} . The variance factors of never get negative for positive semi-definite co-factor matrices when using the REML method, causing σ_2^2 to approach zero.

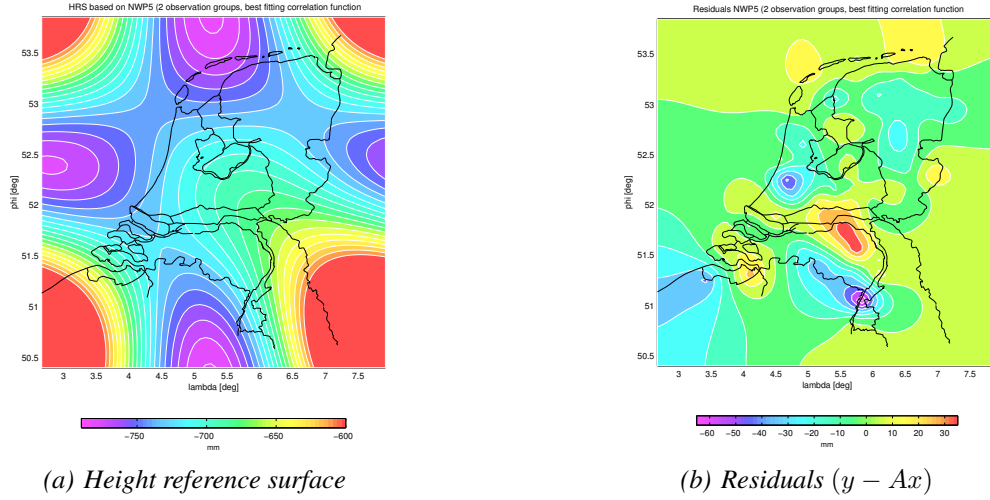


Figure 7.2: Height reference surface when working with the best fitting correlation function, $RMS = 18\text{ mm}$

To overcome this problem, another correlation function for the gravimetric geoid is chosen, as already discussed in section 4.3. This function is chosen in such way that the final covariances in the variance/covariance matrix of GRAVGEO2006 become significantly larger. This means that when the aforementioned effect is due to the lack of covariance information for the GPS/levelling information, with this function σ_2^2 is not getting that large, so also the variances of Q_{h+H} do not have to be down-weighted. As can be seen in figure 7.3 this indeed is true. The resulting correction surface and residuals are shown in figures 7.4(a) and 7.4(b) respectively.

Although the RMS is 3,5 times larger based on this result of the variance components it is chosen to use

$$\sigma^2(x) = e^{\left(-\frac{\ln(2)}{90^2}x^2\right)} \quad (7.2)$$

as the correlation function for the gravimetric geoid. This choice is made because only then variance component estimation is usefull based on our data sets. It should be noted that in the covariances computed based on this function for the gravimetric geoid, also the effect of the covariances for the GPS/levelling data is partly taken into account. This is not a desirable situation, but due to lack of covariance information for GPS/levelling this is the only possible solution at this moment. The larger RMS also is an indication that too less variance/covariance information is available to successfully use variance component estimation.

Another drawback of this choice of correlation function is that no absolute values for the real variances can be given, because the choice of the correlation function is significantly influencing the result.

7.1.2 3 groups: NAP height, GPS heights and gravimetric heights

The previous section dealt with the observations split in 2 groups: a GPS/levelling part and a gravimetric part. The GPS/levelling part can be separated into 2 different groups: a GPS part and a levelling part.

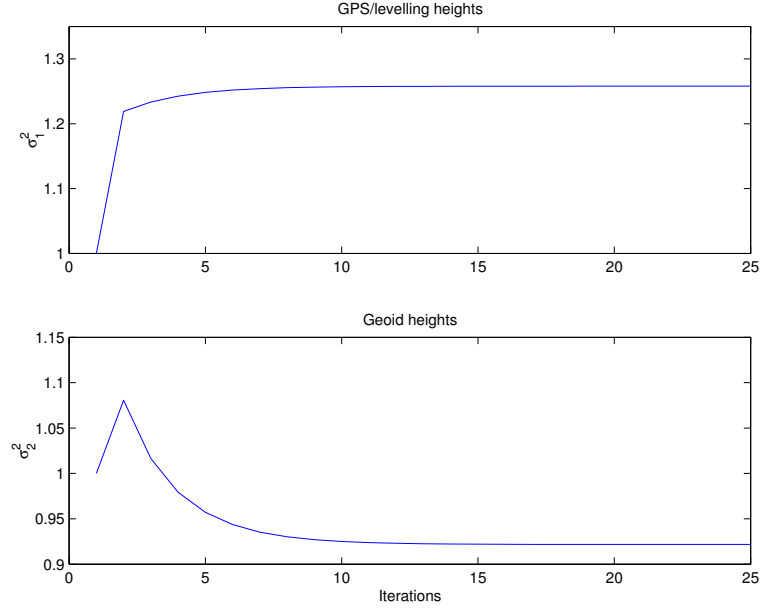


Figure 7.3: Variance components for the GPS/levelling data and the gravimetric geoid when working with a correlation function starting at $(0,1)$, $\sigma_1^2 = 1.26$, $\sigma_2^2 = 0.92$

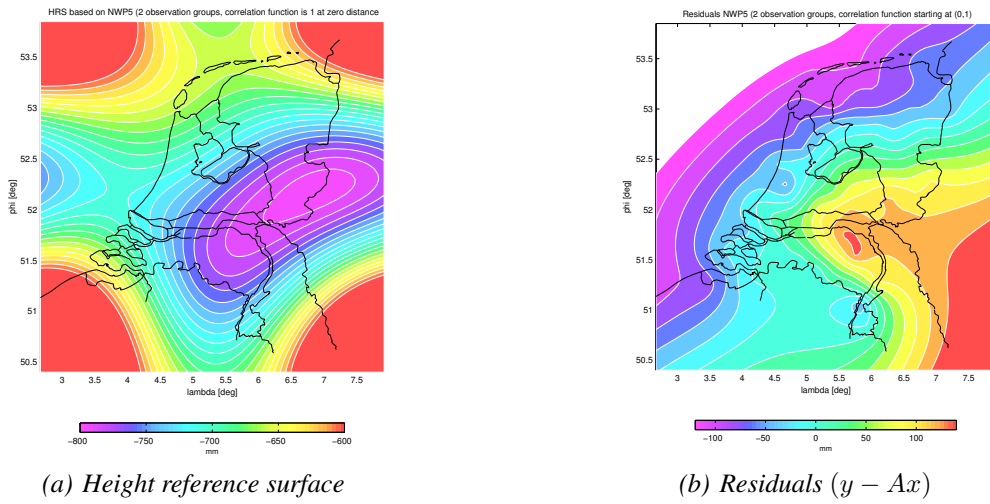


Figure 7.4: Height reference surface when working with a correlation function starting at $(0,1)$, $RMS = 68$ mm

This results in the following total variance/covariance matrix:

$$Q_y = \sigma_1^2 Q_h + \sigma_2^2 Q_H + \sigma_3^2 Q_N \quad (7.3)$$

with Q_h and Q_H diagonal matrices and Q_N a full matrix.

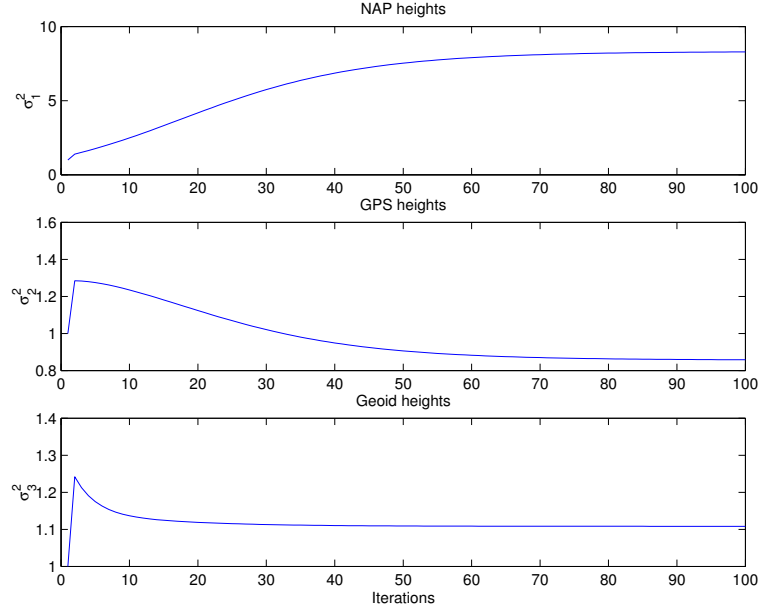


Figure 7.5: Variance components for NAP, GPS and the gravimetric geoid, $\sigma_1^2 = 8.29$, $\sigma_2^2 = 0.85$, $\sigma_3^2 = 1.11$

The results of this estimation are shown in figures 7.5, 7.6(a) and the residuals in 7.6(b). In figure 7.5 can be seen that the variance component for the NAP heights gets large and the variance component for the GPS heights gets small, what is not realistic. As Q_h and Q_H both are diagonal matrices, a high correlation may be assumed. This means that the quality description of the GPS and the levelling data cannot be separated. Therefore for the rest of this research is worked with a combination of GPS/levelling data, as in the previous section.

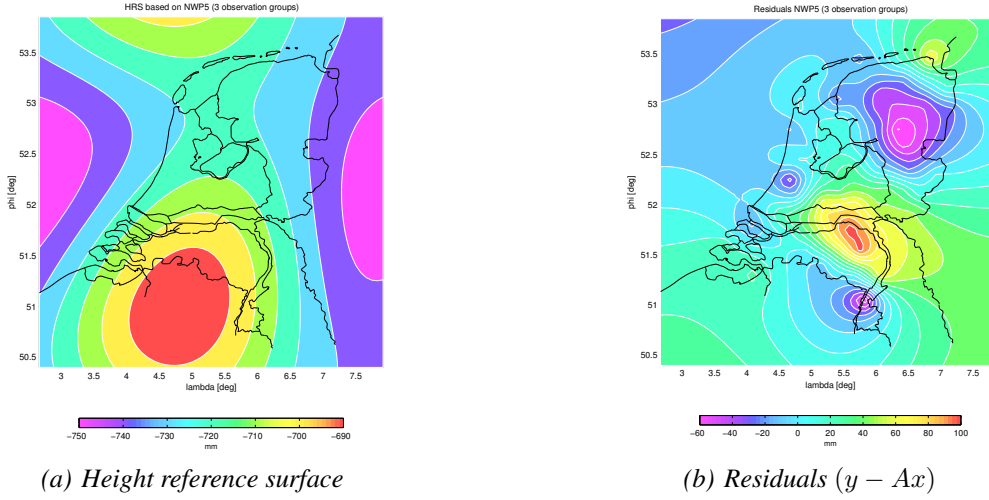


Figure 7.6: Height reference surface when splitting the observations into 3 observation groups, $RMS = 38$ mm

7.2 Combination of 5th accuracy levelling and Kernnet

To have a larger input more data points are added. These points are the 316 Kernnet points as described in section 4.2. As GPS/levelling data is treated like being only 1 dataset, we now have 2 GPS/levelling data sets and 1 data set containing geoid information, resulting in:

$$Q_y = \sigma_1^2 Q_{NWP5} + \sigma_2^2 Q_{KN} + \sigma_3^2 Q_N \quad (7.4)$$

The results of this computation are shown in figures 7.7 and 7.8(a) and the residuals in 7.8(b). In figure 7.7 can be seen that the variance component for the points of the Kernnet is low and at the same moment the gravimetric geoid is down-weighted, resulting in almost no influence of the geoid heights on the height reference surface. It also can be seen that the resulting correction surface is completely different from the one computed with the 5th accuracy levelling only.

Because of the small variance component for the Kernnet points and the large one for the gravimetric geoid, the resulting surface is mainly based on the GPS/levelling input of the Kernnet. According to what we expected beforehand of the quality of the Kernnet points, the estimated variance component does not seem to be correct. Therefore another computation is performed with the same data sets, but then the resulting standard deviation of the Kernnet remained fixed on 5 cm. The result is shown in figures 7.9, and 7.2.

As can be seen in figure 7.10(a), the resulting surface now comes close to like the surface of the 5th accuracy levelling only. Also the variance component of the gravimetric geoid is getting a more realistic value. Most likely there is a fundamental difference between the GPS/levelling points of the 5th accuracy levelling and the Kernnet. Therefore a least-squares adjustment is performed with both data sets separately, without taking any variances into account. The resulting correction surfaces are to be found in figures 7.11 and 7.12. The difference is shown in figure 7.13. It can be seen that there is systematic difference between the points of the 5th accuracy levelling and the Kernnet at centimetre level.

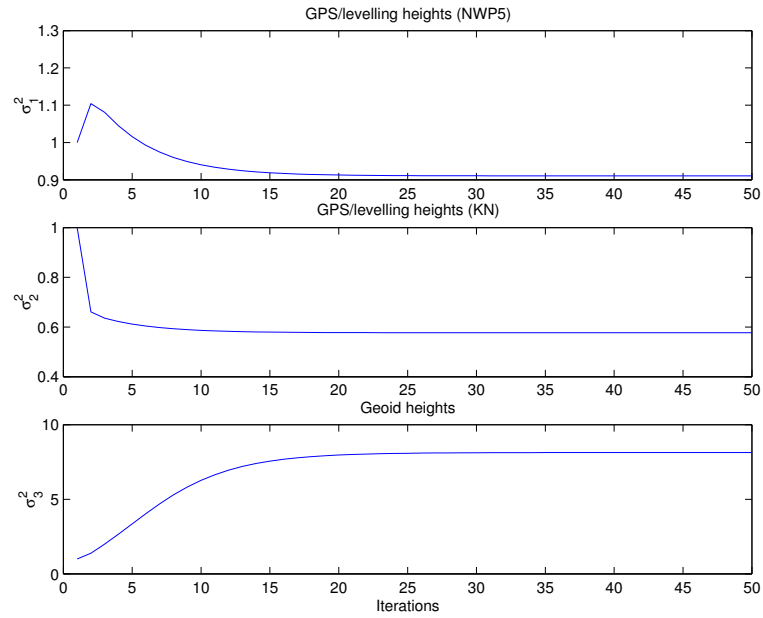
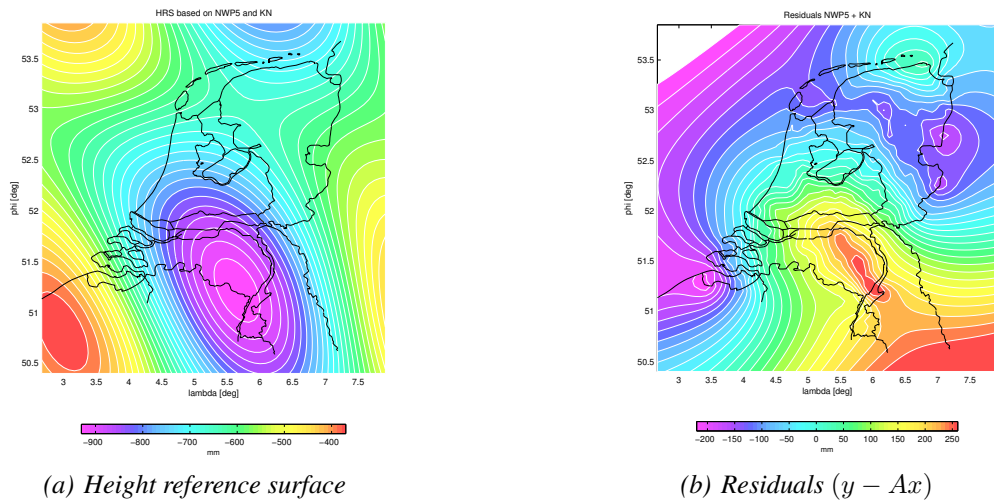


Figure 7.7: Variance components for the GPS/levelling of both data sets and the gravimetric geoid, $\sigma_1^2 = 0.91$, $\sigma_2^2 = 0.58$, $\sigma_3^2 = 8.13$



(a) Height reference surface

(b) Residuals ($y - Ax$)

Figure 7.8: Height reference surface with 2 GPS/levelling sets and the gravimetric geoid, $RMS = 112$ mm

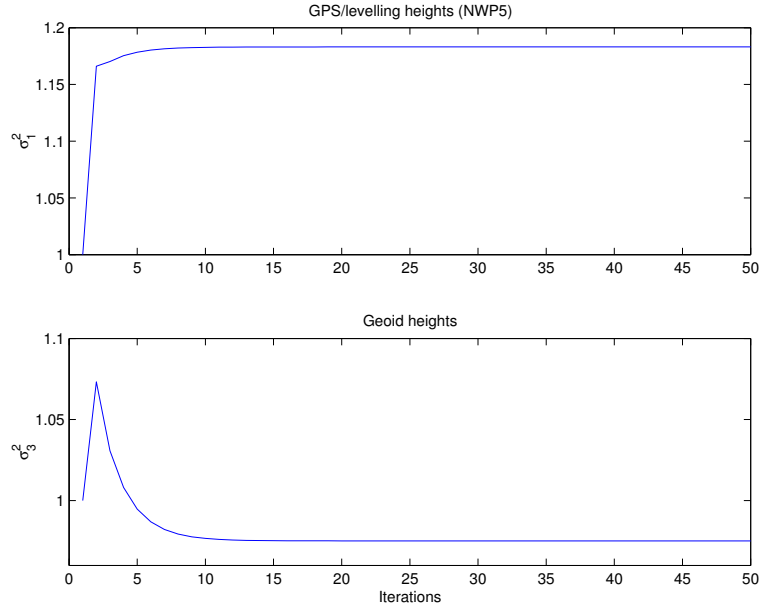


Figure 7.9: Variance components for the GPS/levelling of both data sets and the gravimetric geoid where the standard deviation for the Kernnet is fixed on 5 cm, $\sigma_1^2 = 1.18$, $\sigma_3^2 = 0.92$

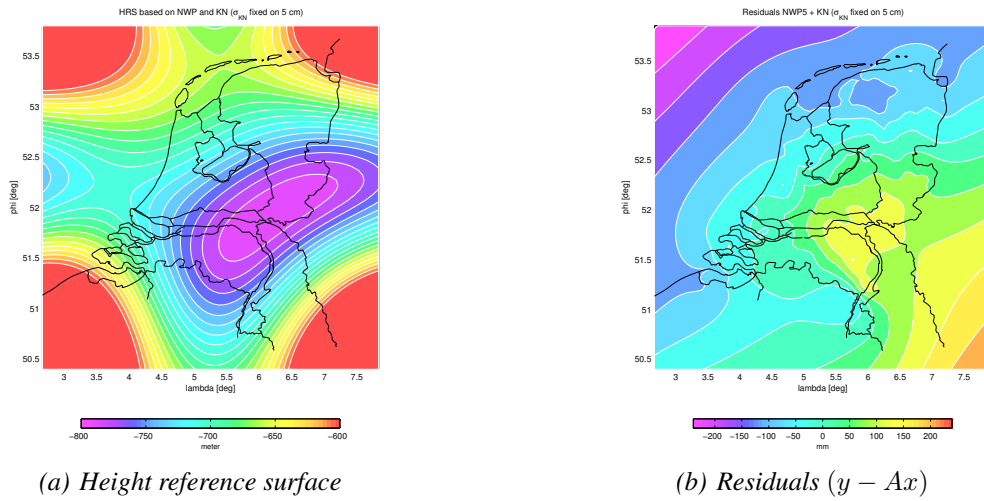


Figure 7.10: Height reference surface with 2 GPS/levelling sets and the gravimetric geoid where the standard deviation for the Kernnet is fixed on 5 cm, $RMS = 71$ mm

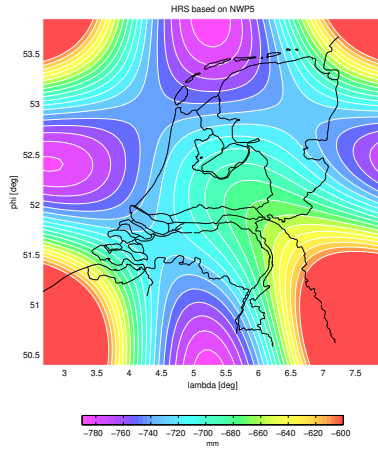


Figure 7.11: Height reference surface based on the 5th accuracy levelling, RMS=18mm

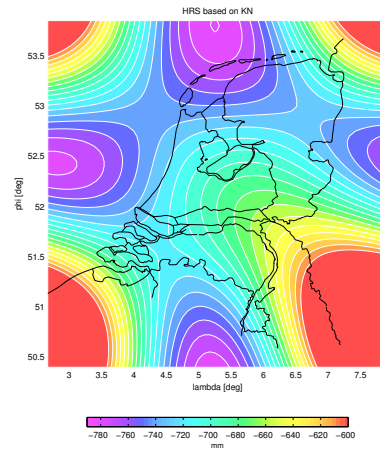


Figure 7.12: Height reference surface based on the Kernnet, RMS=19mm

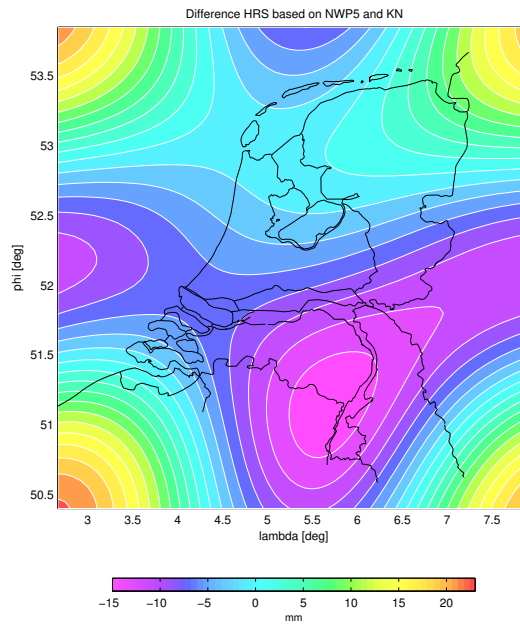


Figure 7.13: Difference between a correction surface based on the 5th accuracy levelling and the Kernnet

7.3 Conclusions

In the previous sections the results of the variance component estimation are discussed. This discussion is summarised in this section. The resulting variance components are summarised in table 7.2. The RMS of the difference between the original data and the computed surface is shown in table 7.3

Table 7.2: Resulting variance components

Observation groups	Variance components		
NWP5, 2 groups, ρ eq. 4.17	$\sigma_{h/H}^2 = 2.4 \cdot 10^{-2}$		$\sigma_N^2 = 4.2 \cdot 10^{-2}$
NWP5, 2 groups, ρ eq. 4.19	$\sigma_{h/H}^2 = 1.26$		$\sigma_N^2 = 0.92$
NWP5, 3 groups	$\sigma_H^2 = 8.29$	$\sigma_h^2 = 0.85$	$\sigma_N^2 = 1.11$
NWP5 + KN, 3 groups	$\sigma_{h/H,NWP5}^2 = 0.91$	$\sigma_{h/H,KN}^2 = 0.58$	$\sigma_N^2 = 8.13$
NWP5 + KN, 3 groups, $\sigma_{KN} = 5$ cm	$\sigma_{h/H,NWP5}^2 = 1.18$	$\sigma_{h/H,KN}^2 = 8.6$	$\sigma_N^2 = 0.97$

Out of the results of the variance component estimation it can be concluded that:

- Due to the missing covariance information for the NAP and GPS heights, the covariances of the geoid heights are weighted as if they are the total covariances. Therefore the variances of the geoid heights get too large when an empirical correlation function is used. The covariances are artificially raised to overcome this problem. As a result no absolute values for the final computed variances can be given, as the choice of the correlation function for the gravimetric heights has a significant influence on the variance components. When better quality descriptions would be available the results would be more reliable.
- For GPS and NAP heights only variances are known. This makes the variance matrices of both height types highly correlated. Therefore it is not possible to estimate variance components for both data types separately.
- There is a large difference between a height reference surface for the 5th accuracy levelling only and the Kernnet. Most likely there is a systematic difference between both data sets.

Table 7.3: RMS values for different combinations of data

Observation group	RMS [mm]
NWP5, 2 groups, ρ eq. 4.17	19
NWP5, 2 groups, ρ eq. 4.19	68
NWP5, 3 groups	38
NWP5 + KN, 3 groups	112
NWP5 + KN, 3 groups, $\sigma_{KN} = 5$ cm	71
NWP5, no variances/covariances	18
KN, no variances/covariances	19

- In table 7.3 the smallest the smallest RMS is found when no variance/covariance information is used. Most likely this is due to the lack of covariance information for GPS/levelling heights. Therefore variance component estimation gives no improvement of the final height reference surface.

This chapter discussed the results of the variance components. The following chapter contains the final conclusions and recommendations founded in this research.

Chapter 8

Conclusions and recommendations

In the previous chapter the results of the variance component estimation were discussed. This chapter gives the conclusions and recommendations for the three parts of the research:

- the comparison between EGM96 and EIGEN-CG03C for the Dutch geoid model;
- the computation of a height reference surface out of only GPS/levelling data;
- the performance of variance component estimation.

In section 8.1 the conclusions for the three parts of the research are drawn. Recommendations are given in section 8.2.

8.1 Conclusions

About the comparison between EGM96 and EIGEN-CG03C for the Dutch area, as described in chapter 3, the following can be concluded:

- There is a difference between the global parts of the gravimetric geoid models of -26 to 24 mm of short wavelength structure. Most likely this structure is due to the use of Meissl/Wong&Gore kernel modification.
- The difference between NLGEO2004 and NLGEO2006 is approximately half the size of the difference between NLGEO2004 and the ‘de Min geoid’. The largest difference can be found in the southern part of the Netherlands.
- There is a relative height difference of -7 to +7 mm between NLGEO2004 and NLGEO2006.

Looking at the computation of a height reference surface out of only GPS/levelling data it can be said that:

- When using multiquadric interpolation, the smoothing parameter has only small influence on the final results.
- The performance of multiquadric interpolation and thin plate splines have the same accuracy.

- It is possible to compute a surface with a difference of 5 to 10 centimetres with NLGEO2006.

Regarding the performance of variance component estimation can be said:

- Due to the lack of covariance information for NAP and GPS heights the covariances of the geoid heights are scaled in such a way that they take care of the total data set. This makes the variances of the geoid heights too large. To overcome this effect, the variances of the GPS/levelling data are down-weighted. Therefore an artificial high correlation function is chosen for the geoid heights.
- The choice of the correlation function for GRAVGEO2006 has a large influence on the resulting variance components. As there are many possibilities to choose a correlation function, no conclusion can be made about the absolute variances. The variance components can only be seen as relative scaling factors.
- As we have only variances for NAP and GPS heights, the variance matrices are highly correlated by their structure. Therefore it is not possible to estimate variance factors for both height types separately.
- There is a systematic difference between the GPS/levelling heights of the 5th accuracy levelling and the Kernnet.
- Out of the RMS values in table 7.3 it can be concluded that variance component estimation does not contribute to better results for the computation of a height reference data, based on the data available on this moment.

8.2 Recommendations

Looking at the performance of an estimation of a height reference surface based on GPS/levelling data only, it is recommended to add geoid heights.

According to the performance of variance component estimation it is recommended to:

- Get covariance information for the GPS/levelling data. This will give more reliable results for the variance component estimation.
- Look at the systematic difference between the GPS/levelling heights of 5th accuracy levelling and the Kernnet to find out what is the reason for this difference.
- Use a more sophisticated model to describe the difference between gravimetric and geometric geoid, by using:

$$\underline{h}_{GPS} - \underline{H}_{NAP} - (B\bar{c} + D\Delta g) = A\underline{x} + \underline{e} \quad (8.1)$$

- Look at other possible parametric models to describe the height reference surface.

Appendix A

Computation of a Dutch geoid model

This appendix gives an overview of the software used for the computation of the Dutch geoid model and the files needed in this procedure. This appendix only describes the procedure when another global geopotential model is used, not how additional local gravimetric information can be added. All the different programs used for this procedure are shortly described in table A.1.

Table A.1: Programs used to compute the Dutch geoid

cnvcoef	Conversion of the file with the global geopotential model from ASCII to binary and vice versa
gm71	Gravity field modelling out of the global geopotential model
gravdat	Combination of different local data sets into one gravity anomaly set of 3'x5'
stokes	Computation of geoid heights out of gravity anomalies from the local data set
tran	Addition/subtraction of solution grids

Gravimetric geoid

The computation of the Dutch geoid starts with a global geopotential model. During this research, use is made of the EIGEN-CG03C model, which can be found on http://www.gfz-potsdam.de/pb1/op/grace/results/grav/g004_eigen-cg03c.html. A global geopotential model file has to be build up with 4 columns: degree, order, c_{nm} and s_{nm} . Before this file can be used, it has to be made binary. This can be done with the program *cnvcoef*. Make sure that lines are read correctly, because the input file can have a different layout. This binary file is used in the further computation of the contribution of the global geopotential model to the local model.

This contribution is computed by the use of *gm71*. To use this program, the parameter file *gm70n1.par* is used. Before use, rename this file to *gm70.par*. In this file the parameters of the geopotential model are described. The things to be noted are the GM and a of the used global geopotential model. These are stated in the 3th and 4th line of the parameter file. The reference a , GM , f and ω at the end of the

parameter file, are the parameters of the ellipsoid which is used as reference. In the Dutch case this is GRS80.

Another important line in the parameter file is the line which says `kernmod`. With this option it is possible to choose between different kernel modifications: 0 means the truncation coefficients are equal to 1, 1 means they're Molodensky coefficients, with 2 it are Meissl coefficients, 3 makes them Wong&Gore coefficients and 4 gives Meissl/Wong&Gore coefficients. For the last three options the degree of the coefficients is taken equal to 32, which is used for the Dutch geoid. This number can be changed inside the program. For computing the total gravimetric Dutch geoid, `kernmod` should be equal to 0. When only looking at the differences between two global geoid models, this number should be changed. This is treated at the end of this appendix.

The other number on the same line is the size of the spherical cap in degrees, ψ_0 . For computation of the total Dutch geoid, this has to be 180d0.

Important in the parameter file is that all the exclamation marks must be in column 41. The output file is `gm71n.out`. For clearness this file is renamed to `N1_ggm.3x5` in which 'ggm' is the name of the used global geopotential model.

For the Dutch geoid model not the total contribution of the global geopotential model is used. This unused part will be subtracted later on. The preparation for this is done again by *gm*. This time the used parameter file is `gm70dg.par` (again rename to `gm70.par`). Again the used `kernmod` should be equal to 0 and the spherical capsize 180d0. The output file is `gm70dg.out`. This file is renamed to `nln5.3x5`.

These gravity anomalies have to be combined with the local gravity anomalies. To this local information, no additional information was added during the research. This means the already available `nln1.3x5`–`nln4.3x5` can be used. Make sure that these 4 files and the just computed `nln5.3x5` are in the same directory and use *gravdat* to combine these data sets into one. With this program, use option 4, 'create combined data sets'. The outcome is `nln.3x5`. Rename this file to `nln_ggm.3x5` so it is clear to which global geopotential model it belongs.

To this data the atmospheric correction has to be added. This is already computed and can be found in `atm.3x5`. Adding up can be done with *tran*, option 12. The result is `nln_tot_ggm.3x5`. Then subtract `nln5_ggm` from `nln_tot_ggm`, again by *tran*, option 11. The result is `deg_res_ggm.3x5`.

This file is the input file for *stokes*. With *stokes* the parameter file `stokes70.par` has to be used. On the 3rd line of this parameter file the name of the output is placed, on the 4th line the input file and on the 5th line the used kernel modification as in `gm70.par` but then 10 added. The program computes the geoid heights out of gravity anomalies. Please note this is very slow and will take approximately 2 hours. The result is `N2_ggm.3x5`.

To finally get the total gravimetric geoid, add up `N1_ggm.3x5`, `N2_ggm.3x5` and `ell.3x5`, of which the last one is the ellipsoidal correction. The result: `gg.3x5`

Correction surface

The correction surface is computed by the use of Matlab. A detailed description of the software can be found in Hilferink [2001]. In this appendix only the most important points are described.

The basis of the geoid computation is the gravimetric geoid, `gg.3x5`. The main program for the computation of the correction surface is `gpsh.m`. Here the input data and the variance/covariance information is defined. The input data had to be ordered in a `.hnn` file, consisting of 4 columns: φ , λ , h_{GPS} and H_{NAP} . In the file `corrvlak.m` on line 230, `ccc = zeros(xx,1)`, `xx` has to be equal to the number of data points defined in the `.hnn` file.

When statistical information about the data points is known, it is possible to use this too. For the 5th accuracy levelling this information is gathered in the `.napcov` and `.gpscov` files. If not known, an identity matrix is used.

To compute a geoid grid out of point information, an `.mpar` file is used. Please note that the origin of the coordinate system for a grid is the lower left corner and Matlab used the upper left corner.

The result of this computation is a `.corr` file which name is also defined in `gpsh.m`. This file contains the 6 parameters of the correction surface and their significance. Then all the points are mentioned and if they are accepted or not. The last part of the output file gives information about the global testing parameters.

Other output files are the correction surface itself and the improved geoid model, both in a 64x52 and a 70x64 grid.

Comparison of different global geopotential models

If the only thing wanted is to compare the influence of two different global geopotential models on the Dutch geoid, it is not necessary to compute both models in total. This is only necessary when also the influence of the correction surface should be taken into account.

With *gm* it is immediately possible to compute the contribution of a global model to the Dutch geoid when using the Meissl/Wong&Gore kernel modification. This can be done in the input file, `gm70n1.par`, on the line which says `kernmod`. When this is chosen to be 4 and the spherical cap on 5 degrees (the second number on that line), then the Meissl/Wong&Gore contribution is computed. This can be done for both models. These results can be subtracted which gives the difference between both geopotential models for the Netherlands.

Appendix B

Mathematical relations

This appendix gives the proves of the most important equations used in this research. The representation of the computation of geoid heights in the frequency domain is explained in section B.1. Section B.2 gives insight in how method A of data combination for computing local geoid models fits in the idea of data splitting in an inner and an outer region.

B.1 Spherical harmonic data representation

In chapter 3 is worked with the integral representation for geoid heights in the spatial domain. In programming mostly is worked with the representation in the frequency domain. This is only allowed when data is known globally. This representation is explained in this section, based on de Min [1996].

The geoid height is computed out the disturbing potential by

$$N(P) = \frac{T(P)}{\gamma} \quad (\text{B.1})$$

which gives the following representation for the computation of the geoid height:

$$N(P) = \frac{R}{4\pi\gamma} \iint_{\sigma} St^*(\psi_0) \Delta g d\sigma \quad (\text{B.2})$$

The disturbing potential fulfils the Laplace equation (Heiskanen & Moritz, 1967) and can be written as a spherical harmonic expansion. The Stokes function then becomes:

$$St^*(\psi_0) = \sum_{n=2}^{\infty} \frac{2n+1}{n-1} \frac{2}{n-1} w_n^* P_n(\cos(\psi_0)) \quad \text{with} \quad w_n^* = \frac{n-1}{2} Q_n^*(\psi_0) \quad (\text{B.3})$$

The same can be done for the gravity anomalies:

$$\Delta g(P) = \sum_{n=2}^{\infty} \sum_{m=-n}^n \Delta g_{nm} \bar{Y}_{nm}(P) \quad (\text{B.4})$$

where $\bar{Y}_{nm}$ are normalised spherical functions combined with associated Legendre polynomials with cosine and sine functions. Δg_{nm} are the gravity components of degree n and order m . These can be described out of spherical harmonic coefficients from a global geopotential model with

$$\Delta g_{nm} = \gamma(n-1)\Delta\bar{c}_{nm} \quad . \quad (\text{B.5})$$

Equations B.4 and B.5 together give:

$$\Delta g(P) = \sum_{n=2}^{\infty} \sum_{m=-n}^n \gamma(n-1)\Delta\bar{c}_{nm}\bar{Y}_{nm}(P) \quad . \quad (\text{B.6})$$

When substituting equations B.3 and B.6 in equation B.2 we get:

$$N(P) = \frac{R}{4\pi} \sum_{n=2}^{\infty} \frac{2n+1}{n-1} w_n^* \sum_{m=-n}^n \iint_{\sigma} \Delta\bar{c}_{nm}\bar{Y}_{nm}P_{nm}(\cos\psi)d\sigma \quad (\text{B.7})$$

To simplify this equation, the orthogonality relations of spherical harmonic functions,

$$\frac{1}{4\pi} \iint_{\sigma} \bar{Y}_{nm}(P)\bar{Y}_{n'm'}(P)d\sigma = \delta_{nn'}\delta_{mm'} \quad (\text{B.8})$$

$$\Rightarrow \iint_{\sigma} P_{nm}\bar{Y}_{nm}(P)d\sigma = \frac{4\pi}{2n+1}\delta_{nn'}\delta_{mm'} \quad (\text{B.9})$$

$$\text{with } P_{nm} = \frac{1}{2n+1} \sum_{m=-n}^n \bar{Y}_{nm}(P)\bar{Y}_{nm}(Q) \quad ,$$

is used, out of which follows

$$N(P) = \frac{R}{4\pi} \sum_{n=2}^{\infty} \frac{2n+1}{n-1} w_n^* \sum_{m=-n}^n \frac{4\pi}{2n+1} \delta_{nn'}\delta_{mm'} \Delta\bar{c}_{nm}\bar{Y}_{nm} \quad . \quad (\text{B.10})$$

The final simplified description of the computation of the geoid height with in the frequency domain becomes:

$$N(P) = R \sum_{n=2}^{\infty} \sum_{m=-n}^n w_n^* \Delta\bar{c}_{nm}\bar{Y}_{nm} \quad . \quad (\text{B.11})$$

B.2 Relation between method A and B of data combination

As already became clear in section 3.1 it is quite easy to describe method B when using an inner and an outer region and a global and a local gravity data set. The way to write method A as a method B is already described by de Min [1996], equation 2.84, and will be repeated here.

$$\begin{aligned}
N(P) &= \frac{R}{4\pi\gamma} \iint_1^* (\psi) \Delta g^{ggm}(P) d\sigma + \frac{R}{4\pi\gamma} \iint_\sigma (St(\psi) - St_1^*(\psi)) \Delta g(P) d\sigma \\
&= \frac{R}{2\gamma} \sum_{n=2}^{360} Q_n^*(\psi_0) \Delta g_n^{ggm}(P) + \frac{R}{2\gamma} \sum_{n=2}^{\infty} \left(\frac{2}{n-1} - Q_n^*(\psi_0) \right) \Delta g_n(P) \\
&= \frac{R}{2\gamma} \sum_{n=2}^{360} Q_n^*(\psi_0) \Delta g_n^{ggm}(P) + \frac{R}{2\gamma} \sum_{n=2}^{360} \left(\frac{2}{n-1} - Q_n^*(\psi_0) \right) \Delta g_n(P) \\
&\quad + \frac{R}{2\gamma} \sum_{n=361}^{\infty} \left(\frac{2}{n-1} - Q_n^*(\psi_0) \right) \Delta g_n(P) \\
&= \frac{R}{2\gamma} \sum_{n=2}^{360} \frac{2}{n-1} \Delta g_n^{ggm}(P) + \frac{R}{2\gamma} \sum_{n=2}^{360} \left(\frac{2}{n-1} - Q_m^*(\psi_0) \right) (\Delta g_n(P) - \Delta g_n^{ggm}(P)) \\
&\quad + \frac{R}{2\gamma} \sum_{n=361}^{\infty} \left(\frac{2}{n-1} - Q_m^*(\psi_0) \right) (\Delta g_n(P) - \Delta g_n^{ggm}(P)) \\
&= \frac{R}{2\gamma} \sum_{n=2}^{360} \frac{2}{n-1} \Delta g_n^{ggm}(P) + \sum_{n=2}^{\infty} \left(\frac{2}{n-1} - Q_m^*(\psi_0) \right) (\Delta g_n(P) - \Delta g_n^{ggm}(P)) \\
&= \frac{R}{2\gamma} \sum_{n=2}^{360} \frac{2}{n-1} \Delta g_n^{ggm}(P) + \frac{R}{4\pi\gamma} \iint_\sigma (St(\psi) - St_1^*(\psi)) (\Delta g - \Delta g^{ggm}) d\sigma \quad (B.12)
\end{aligned}$$

Appendix C

Selected profiles for relative height computation

This appendix treats in detail the profiles that are described in section 3.4 which are used to determine the difference in relative heights between the geoid model computed in chapter 3 and NLGEO2004. Table C.1 shows the locations of the end points of the profiles and their heights in both geoid models. The relative heights and their differences can be found in table C.2. Figure C.1 shows the location of the profiles.

Table C.1: Locations end point profiles and their heights

Location	φ	λ	N NLGEO2004 [m]	N new model [m]
Delft	51.98	4.38	43.528	43.5306
Delfzijl	53.23	6.89	40.523	40.5265
Den Helder	52.97	4.75	42.127	42.1261
Enschede	52.22	6.89	43.560	43.5556
Maastricht	50.85	5.69	45.718	45.7244
Middelburg	51.50	3.67	44.315	44.3138
Oost-Vlieland	53.30	5.07	41.589	41.5848
Pieterburen	53.40	6.45	40.434	40.4338
Terneuzen	51.35	3.81	44.443	44.4410

Table C.2: Height differences of the profiles

Profile	ΔN NLGEO2004 [m]	ΔN new model [m]	Difference ΔN [cm]
Den Helder – Maastricht	3.591	3.581	0.7
Delfzijl – Middelburg	3.792	3.769	−0.5
Oost-Vlieland – Terneuzen	2.854	2.845	0.2
Pieterburen – Maastricht	5.284	5.271	0.7
Delft – Enschede	0.032	0.022	−0.7

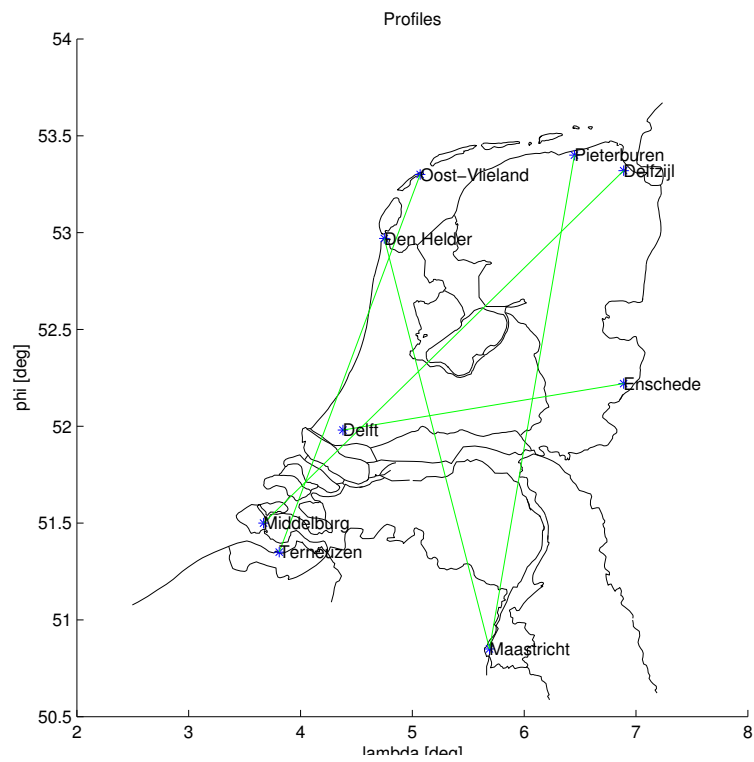


Figure C.1: The location of the profiles

Appendix D

Overview 5th accuracy levelling

The data sets with GPS/levelling points are described in section 4.1. The numerical values of the 5th accuracy levelling are given in this appendix (table D.1) The information is totally obtained by the Adviesdienst Geoinformatie and ICT of Rijkswaterstaat.

Table D.1: The points with their heights and accuracies of the 5th accuracy levelling

GPS-code	NAP-code	λ	φ	H NAP	stdev NAP	h GPS	stdev GPS
CA01	032B0317	5.437	52.169	10.2492	2.85	53.4948	15.3
CB02	014E0148	4.870	52.887	2.6145	2.91	44.8282	15.5
CC03	009D0181	4.850	53.044	4.7355	3.00	46.7553	14.2
CD04	010D0102	5.401	53.047	5.6649	3.18	47.4247	12.4
CE05	025H0210	5.114	52.329	29.0978	2.47	72.0957	12.2
CF06	014G0277	4.941	52.770	1.8677	2.65	44.2029	11.8
CG07	020A0122	5.282	52.692	4.6063	2.72	47.0077	12.5
CH08	005D0080	5.441	53.176	8.8870	2.89	50.3438	11.8
CI09	020F0127	5.644	52.604	8.8566	2.57	51.3695	16.7
CJ10	009D0182	4.733	52.951	9.9190	2.98	52.0807	13.7
CK11	024F0038	4.573	52.457	11.7201	2.15	54.6486	12.3
CL12	014C0177	4.654	52.761	4.5394	2.60	46.9636	10.8
CM13	031H0500	5.147	52.049	8.4340	2.54	51.8298	10.8
CN14	026H0221	5.615	52.371	14.1322	2.78	57.0584	11.7
CO15	019G0381	4.887	52.590	5.1026	3.08	47.7117	12.3
CP16	025B0387	4.864	52.431	3.7140	1.80	46.6111	11.3
CR17	004F0033	5.092	53.295	8.6828	3.19	50.2685	12.0
CS18	pre0007890	4.647	52.233	0.8548	2.19	44.0349	14.0
NA01	015E0176	5.518	52.852	3.9067	3.28	46.0507	12.7
NB02	008C0227	7.061	53.155	-0.0629	3.92	40.8023	12.2
NC03	012D0216	6.571	53.022	16.3780	3.13	57.6338	12.2
ND04	003C0140	6.372	53.387	4.6972	4.10	45.1915	12.3
NE05	007C0218	6.406	53.241	6.5584	3.47	47.3080	10.0

NF06	013D0159	7.144	52.945	11.3706	3.39	52.7584	12.4
NG07	006B0313	5.978	53.292	1.6691	3.16	42.5756	11.3
NH08	011D0229	5.932	52.962	3.6835	3.11	45.4949	11.1
NI09	012G0220	6.767	52.998	23.2333	3.24	64.5118	14.7
NJ10	011E0265	6.098	53.079	0.9902	3.51	42.3915	14.5
NK11	010F0223	5.677	53.055	0.1688	2.93	41.8150	11.5
NL12	002G0100	6.162	53.477	9.8965	4.10	50.3450	8.6
NM13	007F0265	6.930	53.327	8.4844	4.05	49.0037	13.6
NN14	003H0106	6.829	53.448	3.9704	4.39	44.3290	13.1
NO15	002C0127	5.764	53.444	6.2178	3.38	46.9653	9.9
NP16	005F0113	5.747	53.288	1.1293	3.26	42.1868	14.3
OA01	016H0236	6.342	52.760	11.1745	3.43	53.2368	16.6
OB02	028C0196	6.480	52.360	13.9451	3.21	57.0372	12.7
OC03	029C0209	6.932	52.304	47.8837	3.39	91.2582	12.4
OD04	027D0220	6.019	52.372	15.4589	2.94	58.4573	11.1
OE05	034A0299	6.415	52.165	17.6179	2.95	61.1607	10.2
OF06	040G0206	6.104	51.851	21.4823	3.11	65.3204	11.3
OG07	016G0240	6.099	52.796	6.5441	2.99	48.6864	12.1
OH08	017H0212	6.822	52.754	21.7630	3.31	63.7320	13.9
OI09	022G0155	6.650	52.486	20.1291	2.98	62.8909	12.0
OJ10	034G0220	6.640	52.087	29.6059	3.17	73.3731	10.9
OK11	041B0306	6.587	51.927	33.2107	3.30	77.2247	10.9
OL12	032F0173	5.698	52.192	25.8715	2.88	69.1215	12.8
OM13	039F0273	5.652	52.014	16.0851	3.05	59.5482	13.1
ON14	040B0297	6.007	51.994	17.0133	2.91	60.5997	12.1
OO15	016C0168	5.760	52.726	3.0932	3.18	45.4086	14.2
OP16	022A0204	6.408	52.604	12.5477	3.00	54.9976	11.3
OQ17	033E0353	6.180	52.153	12.4338	2.78	55.8718	12.4
OR18	021D0286	5.925	52.552	5.9988	2.66	48.6249	10.9
WA01	038D0383	4.789	51.893	8.2271	2.70	51.8060	11.2
WB02	055A0203	4.075	51.278	5.0348	4.13	49.5789	15.0
WC03	037H0598	4.500	51.920	9.3436	2.73	52.9324	11.3
WD04	048H0193	4.012	51.456	6.9292	3.55	51.2823	11.4
WE05	054E0222	3.820	51.328	12.1675	3.77	56.6256	12.7
WF06	037A0148	4.122	51.978	7.6825	2.92	51.2431	12.1
WG07	030D0120	4.264	52.099	9.8401	2.64	53.2450	12.3
WH08	048C0136	3.587	51.449	13.1572	3.86	57.5144	12.0
WI09	042D0053	3.687	51.623	8.5836	3.61	52.7815	13.0
WJ10	031D0225	4.734	52.082	2.7687	2.42	46.1640	11.0
WK11	053F0104	3.440	51.270	5.1004	4.54	49.5538	12.5
WL12	037C0173	4.043	51.823	11.1046	3.08	54.9330	11.6
WM13	043G0251	4.415	51.691	12.9389	2.86	56.8757	12.2
WN14	042E0150	3.822	51.736	6.8525	3.46	50.8824	16.6
WO15	042H0177	4.006	51.595	9.3911	3.32	53.5788	11.8

ZA01	046A0238	5.851	51.800	21.5158	2.91	65.2855	10.8
ZB02	050E0165	4.949	51.572	13.2832	2.96	57.1997	11.4
ZC03	058E0196	6.078	51.284	35.7818	3.38	80.5215	11.2
ZD04	061F0190	5.698	50.869	54.7292	4.28	100.3996	12.5
ZE05	045F0263	5.619	51.732	21.7227	2.80	65.4455	11.5
ZF06	051F0432	5.732	51.574	25.8463	2.85	69.7739	12.2
ZG07	050H0144	5.156	51.396	33.5383	3.17	77.7625	14.2
ZH08	049F0311	4.507	51.557	9.3537	3.10	53.4847	10.6
ZI09	052G0258	6.052	51.454	37.7328	3.08	82.1534	11.0
ZJ10	039C0319	5.275	51.887	8.5835	2.65	52.0868	11.4
ZK11	045C0323	5.261	51.663	8.6802	2.84	52.2883	12.9
ZL12	062B0431	5.915	50.852	178.3836	4.38	224.2546	11.7
ZM13	060A0242	5.795	51.043	34.9908	3.90	80.0492	11.3
ZN14	057F0195	5.632	51.290	35.9101	3.24	80.2136	11.7
ZO15	051E0230	5.497	51.505	20.9317	3.05	64.7368	12.7
ZP16	044C0244	4.645	51.694	6.7295	2.78	50.5843	11.5
ZR17	038G0417	5.017	51.822	8.8383	2.71	52.4066	13.9

References

- Brand, G.B.M., “*Herberekening van het primaire net van het NAP; Verbetering precisie en betrouwbaarheid ten behoeve van de nieuwe NAP publicatie*”, Adviesdienst Geoinformatie en ICT, AGI/GAP-04/004, August 2004
- Carlson, R.E., “*The parameter R^2 in multiquadric interpolation*”, Computers & Mathematics with Applications, Vol. 21, No. 9, p. 29–42, 1991
- Crombaghs, M., “*NLGeo2004 - Een nieuw geoidemodel voor Nederland*”, Adviesdienst Geoinformatie en ICT, RWS, November 2004
- Ekman, M., “*What is the geoid?*”, in: “Coordinate Systems, GPS and the geoid” (ed. M. Vermeer), Reports of the Finnish Geodetic Institute 95:4, p. 49–51, 1995
- Featherstone, W. “*Do we need a gravimetric geoid or a model of the Australian Height datum to transform GPS Heights in Australia?*”, Australian surveyor, Vol. 43, No. 4 p. 273–280, 1998
- Förste, C., F. Fletcher, R. Schmidt, U. Meyer, R. Stubenvoll, F. Barthelmes, R. König, K.-H. Neumayer, M. Rothacher, C. Reigber, R. Biancale, S. Bruinsma, J.-M. Lemoine, J.C. Raimondo, “*A new high resolution global gravity field model derived from combination of GRACE and CHAMP mission and altimetry/gravimetry surface gravity data*”, Poster presented at the European Geosciences Union, General Assembly 2005, Vienna, Austria, April 24–29, 2005
- Förstner, W., “*Ein Verfahren zur Schätzung von Varianz-Kovarianz Komponenten*”, Allgemeine Vermessungs-Nachrichten, Vol. 86, p. 446–453, 1979
- Fotopoulos, G., “*An analysis on the Optimal Combination of Geoid, Orthometric and Ellipsoidal Height Data*”, PhD thesis, University of Calgary, 2003
Can be found online on: <http://www.geomatics.ucalgary.ca/links/GradTheses.html>
- Fotopoulos, G., “*Calibration of geoid error models via a combined adjustment of ellipsoidal, orthometric and gravimetric height data*”, Journal of Geodesy, Vol. 79, p. 111–123, 2005
- Hilferink, G.W., “*Correctievlakbepaling ten behoeve van geoidberekening*” Report internship at the Meetkundige Dienst, Delft University of Technology, 2001
- Hardville, D.A., “*Maximum likelihood approaches to variance component estimation and to related problems*”, Journal of the American Statistical Association, Vol. 72, p. 320–338, 1977

- Hardy, R.L., “*Multiquadric equations of topography and other irregular surfaces*”, Journal of Geophysical Research, Vol 76, No. 8, p. 1905–1915, 1971
- Hardy, R.L., “*Theory and applications of the multiquadric-biharmonic method*”, Computers & Mathematics with Applications, Vol. 19, No. 8/9, p. 163–208, 1990
- Jäger, R., “*State of the art and present developments of a general concept for GPS-based height determination*”, First Workshop on GPS and Mathematical Geodesy in Tanzania (Kilimanjaro Expedition), October 4, 1999
- Jekeli, C. “An investigation of two models for the degree variances of global covariance functions”, OSU Report 275, 1978
- Koch, K.R., “*Maximum likelihood estimate of variance components*”, Bulletin Géodésique, Vol. 60, p. 329–338, 1986
- Kotsakis, C., M.G. Sideris, “*On the adjustment of combined GPS/levelling/geoid networks*”, Journal of Geodesy, Vol 73, p. 412–421, 1999
- Lemoine, F.G., D.E. Smith, L. Kunz, R. Smith, E.C. Pavlis, N.K. Pavlis, S.M. Klosso, D.S. Chinn, M.H. Torrence, R.G. Williamson, C.M. Cox, K.E. Rachlin, Y.M. Wang, S.C. Kenyon, R. Salman, R. Trimmer, R.H. Rapp, R.S. Nerem “*The development of the NASA GSFC and NIMA Geopotential Model EGM96*”, NASA Goddard Space Flight Center, Greenbelt, Maryland, USA, July 1998
- Min, E. de, “*De geoïde voor Nederland*”, PhD-thesis Delft University of Technology, 1996
- Rapp, R.H., Y.M. Wang, N.K. Pavlis, “*The Ohio State 1991 geopotential and sea surface topography harmonic coefficient models*”, OSU Report 410, 1991
- Rao, C.R., “*Estimation of Variance and Covariance Components - MINQUE Theory*”, Journal of Multivariate Analysis 1, p. 257–275, 1971
- Sjöberg, L.E. “*Unbiased Estimation of Variance-Covariance Components in Condition Adjustment with Unknowns - A MINQUE Approach -*”, Zeitschrift für Vermessungswesen 108:9, p. 382–387, 1983
- Smith, W.H.F, P. Wessel, “*Gridding with continuous curvature splines in tension*”, Geophysics, Vol. 55, No. 3, p. 293–305, 1990
- Tscherning, C.C., R.H. Rapp, “*Closed covariance expressions for gravity anomalies, geoid undulations, and deflections of the vertical, implied by anomaly degree variance models*”, OSU Report 208, 1974
- Van Loon, J.P., J. Kusche, “*Towards an optimal combination of satellite data and prior information*”, accepted for publication in the Proc. Dynamic Planet 2005, “Monitoring and Understanding a Dynamic Planet with Geodetic and Oceanographic Tools”, p. 345–353.