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Can sources and receivers be interchanged for imaging?

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***Abstract*—The design of ultrasound scanning systems for applications such as breast cancer detection is a challenging task, especially when the number of sources and receivers increases and they become spread over a large surface. In order to determine the number of transducers and above all their position, several approaches could be followed. A simple and straightforward approach is to compute the energy distribution in the region of interest for several configurations, and treat each receiver as if it is a source. Here the underlying assumption is that the number of A-scans, i.e. number of sources times receivers, determines the image quality. This assumption is mainly based on reciprocity; the observation that the response R_{AB} measured by a receiver located at B and a source at A is identical to the response R_{BA} obtained after interchanging the source and receiver locations. This is valid for linear imaging methods. However, the question arises whether this is also the case for non-linear inversion methods. In this work, we evaluate the SAFT, CG and CSI imaging and inversion methods for different configurations of sources and receivers located uniformly distributed over a circular array, surrounding an heterogeneous medium. The obtained results show that with the linear SAFT and CG methods, the sources and receivers can be interchanged, and the resulting images are the same. However, for the non-linear CSI method, the results show that it is preferred to have more receivers than sources, despite the fact the measured signals are reciprocal.**

I. Introduction

Acoustic wave fields are frequently used to image the interior of earth or the human body. For biomedical applications, the contrasts are typically weak and often the Born-approximation is valid. Consequently, during imaging with linearized imaging methods – such as Synthetic Aperture Focussing Technique (SAFT) (or Delay and Sum) or Born Inversion (BI) [1] – sources and receivers can easily be exchanged. Especially as the measured signals are reciprocal. Hence, for these methods, the number of source-receiver combinations ($N_S \times N_R$; with N_S and N_R the number of independent source and receiver positions respectively) determines the accuracy of the resulting image and not how the numbers are divided amongst the sources and receivers (*e.g.* one may use more sources than receivers). However, the question arises if this is also the case when a non-linear inversion method, such as Contrast Source Inversion (CSI) [1]–[3], is applied.

To investigate the effect of exchanging sources for receivers and vice-versa, we start in section II (Theory) with the formulation of the integral equation that is used for our imaging and inversion methods. Next, we explain, in short, the principle of reciprocity followed by a small explanation of the three imaging methods applied (SAFT, BI, and CSI). In section III (Experiment Configuration), we present the two-dimensional (2-D) circular cylindrical configuration we use to test our methods. In Section IV (Results), we present the results obtained with the three methods and the source-receivers combinations. Finally, in section V (Discussion and Conclusion), these results are discussed and some conclusions are drawn.

II. Theory

The pressure field $p(\vec{x},t)$ at location $\vec{x}$ and time instant t satisfies the scalar wave equation in lossless heterogeneous media with speed of sound $c(\vec{x})$, given by

$$\nabla^2 p(\vec{x},t) - \frac{1}{c^2(\vec{x})}\frac{\partial^2 p(\vec{x},t)}{\partial t^2} = -\rho_0 \frac{\partial q(\vec{x},t)}{\partial t}, \quad (1)$$

where ∇ is the nabla operator, ρ_0 is the volume density of mass and $q(\vec{x},t)$ is the volume source density of injection rate generating the wave field. In the frequency domain with angular frequency ω, the wave equation given in Eq. (1) may be cast into the following integral equation [5], [6]

$$\hat{p}(\vec{x}) = \hat{p}^{inc}(\vec{x}) + \omega^2 \int_{\vec{x}' \in \mathbb{D}} \hat{G}(\vec{x}-\vec{x}')\chi(\vec{x}')\hat{p}(\vec{x}')dV(\vec{x}'), \quad (2)$$

with non-zero speed-of-sound contrast function $\chi(\vec{x})$ in the spatial domain $\mathbb{D}$

$$\chi(\vec{x}) = \frac{1}{c^2(\vec{x})} - \frac{1}{c_0^2}, \quad (3)$$

where c_0 is the sound speed of the homogeneous embedding, and with Green's function $\hat{G}(\vec{x})$

$$\hat{G}(\vec{x}) = \frac{\exp[-ik_0 |\vec{x}|]}{4\pi |\vec{x}|}, \quad (4)$$

with wave number $k_0 = \omega/c_0$ and $|\vec{x}|$ the Euclidean distance of the vector $\vec{x}$. The Green's function $\hat{G}(\vec{x})$ describes the

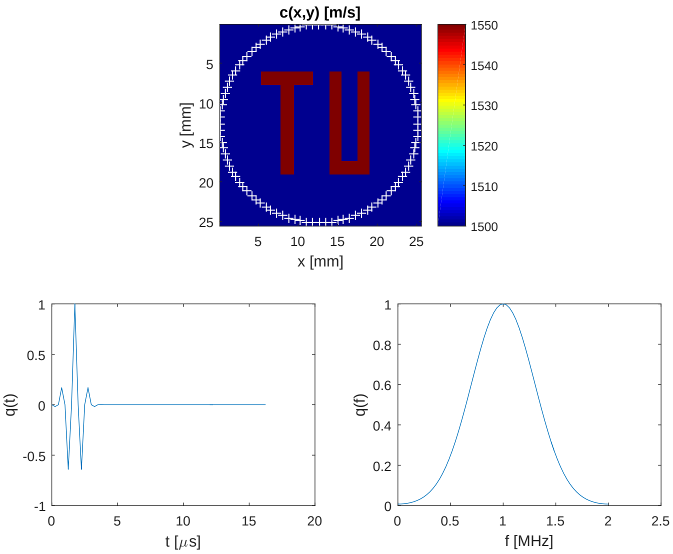

Fig. 1. The 2-D configuration (top) where the 100 sources and receivers, indicated by the white crosses, are positioned on a circle in the background medium with speed of sound $c_0 = 1500$ m/s. The circular array encloses the contrast, shaped as the characters T and U, with speed of sound $c = 1550$ m/s. The sources are excited with a Gaussian pulse (bottom) with center frequency $f_0 = 1$ MHz.

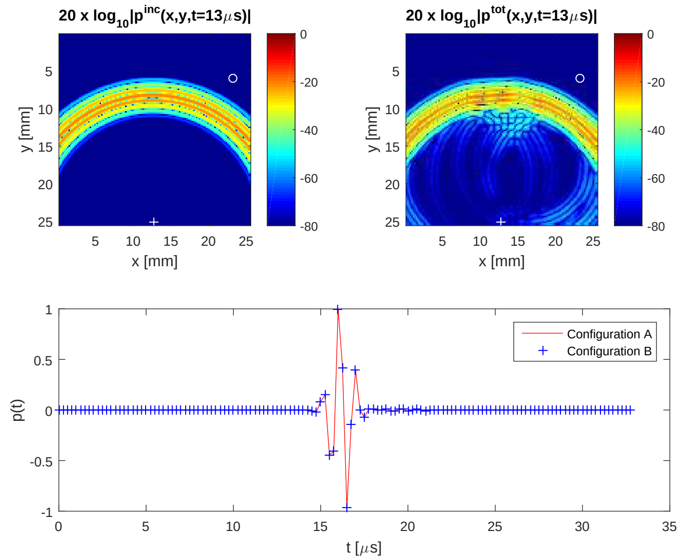

Fig. 2. A snapshot of the incident (top-left) and actual wave field (top-right) for configuration A is shown. In configuration A the source location is indicated by the white cross $+$; the receiver location by the white circle $\circ$. In configuration B, source and receiver are exchanged. The responses measured with both configurations are shown in the bottom image.

impulse response of a medium, *i.e.* the pressure field generated by a "Dirac-delta source" in the homogeneous background medium, whereas $p^{inc}(\vec{x}, t)$ is the pressure field in absence of the contrast. Note that we use the symbol $\hat{}$ to denote functions in the temporal Fourier domain.

Clearly, in the absence of contrast and for identical point source and point receiver located at respectively $\vec{x}^A$ and $\vec{x}^B$, the response measured by the receiver is identical to the response measured when the transducers are interchanged; *i.e.* with the receiver at $\vec{x}^A$ and the source at $\vec{x}^B$. In literature this is referred to as reciprocity. For heterogeneous media, although less obvious, identical sources and receivers can be interchanged as well, without showing any change in measured response. A prove of this can be found here [4], [5].

III. Experiment Configuration

In order to test the effect of exchanging sources and receivers during imaging and inversion, we use the 2-D circular cylindrical set-up shown in figure 1. The selected parameters are typical for medical ultrasound; the embedding has a speed of sound similar to water, hence $c_0 = 1500$ m/s, whereas the contrast has a sound speed $c = 1550$ m/s. The field is probed using a injection rate source which is excited with Gaussian pulse with center frequency $f_0 = 1$ MHz.

Synthetic measurement data is obtained by solving the forward problem as defined via Eq. (2) using a conjugate gradient inversion method. [5], [6] A snapshot of the obtained wave fields are shown in the top row of figure 2. Both the incident and the actual wave field, $p^{inc}(\vec{x}, t)$ and $p(\vec{x}, t)$ respectively, are shown. To test reciprocity, two configurations A and B are compared. For configuration A, the location of the source is in figure 2 indicated by the white cross $+$ and the receiver by the white circle $\circ$; in configuration B the source and receiver locations are exchanged. For both configurations, the measured responses are plotted in the same graph, see the bottom row of figure 2. Clearly, both signals are identical as may be expected from reciprocity.

IV. Results

To test the effect of interchanging sources and receivers for imaging, three configurations are considered; for the configuration I, we use all 100 sources and all 100 receivers. In configuration II, we selected 10 sources and use all 100 receivers; and in configuration III, we use all 100 sources and selected 10 receivers. In all cases, the sources and the receivers are uniformly distributed over the circular array.

Figure 3 depicts the images obtained using SAFT method, and figure 4 shows the results obtained with CG and CSI methods; for each method, the three configurations of sources and receivers are tested. For CG and CSI, the results after 32 iterations are displayed. Note in both figures that for SAFT and CG, the images obtained with each method for configurations II and III are visually the same. But for CSI, the images obtained with configuration II and III are not the same. In fact, the image obtained with a smaller number of receivers, i.e. configuration III, is far from the correct solution.

Finally, figure 5 depicts the images obtained with CSI only for configurations I, II and III, at five different number of iterations; i.e. $n_{it} = 4, 16, 64, 256$ and 1024. Note that for configuration I, correct values of the speed of sound are already obtained at several spatial positions at iteration 32 and for configuration II at iteration 64. However, for configuration III the image remains blurred for many iterations and at

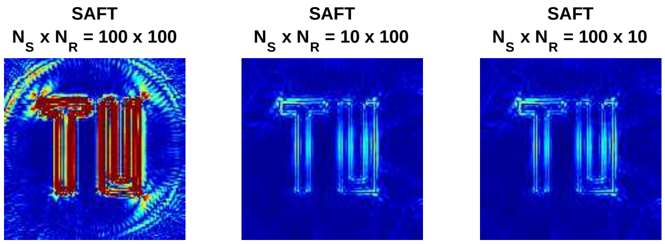

Fig. 3. Images obtained using SAFT for configurations I, II, and III. All three images are displayed using the same colour scale.

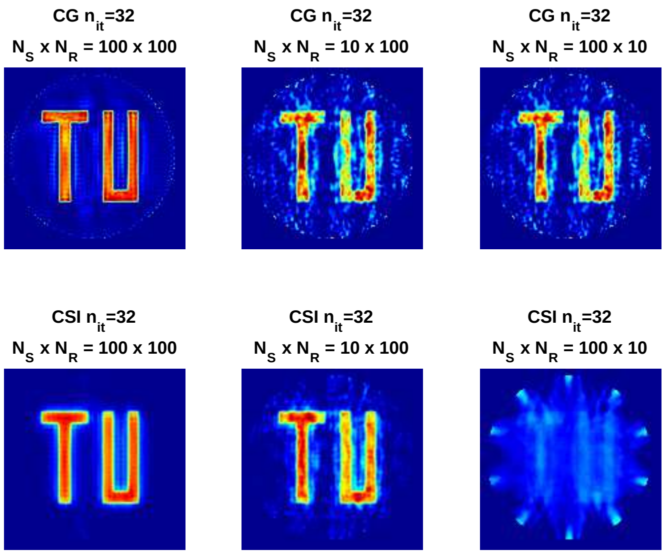

Fig. 4. Images obtained using CG (top row) and CSI (bottom row) for configurations I, II, and III. In all cases cases, the number of iterations is $n_{it} = 32$. All images are displayed using the same colour scale as being used for figure 1.

iteration 1024 the CSI method still estimates incorrect speed of sound values at many locations. This result indicates that in non-linear inversion, the number of receivers plays an important role in the quality of the resulting images. In fact, a minimum number of receivers should be used such that the method will converge to the correct solution. By increasing the number of receivers, the CSI method converges to the solution more rapidly.

V. Discussion and Conclusion

In this work, we evaluated the performance of three imaging and inversion methods, when different configurations of sources and receivers are used. In particular, for linear inversion methods such as SAFT and BI, the reconstruction of the contrast or speed of sound profile of an heterogeneous medium is the same when the sources and receivers are exchanged. This suggests that similar images are obtained when the same number of transducers are used, no matter the configuration of the transducers.

However, in the non-linear CSI inversion method, the configuration of the transducers is of significant importance for obtaining a correct image of the speed of sound profile of an

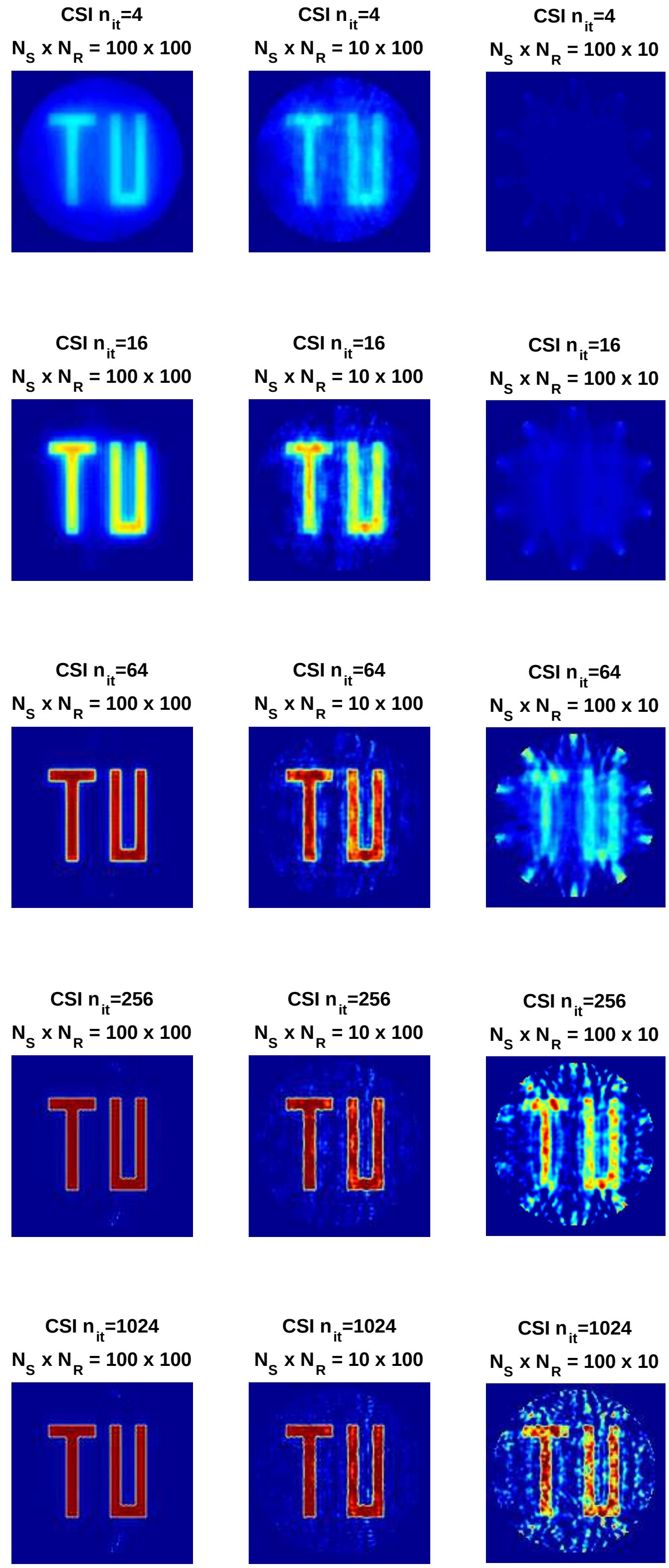

Fig. 5. Images obtained using CSI at different number of iterations ($n_{it} = 4, 16, 64, 256, 1024$) using configurations I (left column), II (middle column), and III (right column). All images are displayed using the same colour scale as being used for figure 1.

heterogeneous medium. In particular, for the tested configuration we conclude that increasing the number receivers has a far more positive effect on the resulting images (i.e. obtaining higher quality images in less iterations) than increasing the number of sources. Note that also from a computational point of view (memory load and computing time) it is for CSI preferable to have more receivers than sources, than the other way around.

To conclude, the obtained results show that with the linear SAFT and CG methods, sources and receivers can be interchanged without affecting the resulting images. However, for the non-linear CSI method, the results show that it is preferred to have more receivers than sources, despite the fact the measured signals are reciprocal.

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