INTERNATIONAL INSTITUTE FOR HYDRAULIC AND ENVIRONMENTAL ENGINEERING

DELFT NETHE

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Engineering potamology

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E N G I N E E R I N G   P O T A M O L O G Y

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Contents

1. Introduction

1.1. General

Percipitation (rain or snow) leads to run-off. It also leads to soil erosion. Water and sediment are transported down-hill to the sea or ocean. This transport takes place via rivers of various sizes and shapes. Knowledge of the natural processes in rivers (*river hydraulics or potamology*) is essential to understand and predict changes that will occur due to natural causes or due to human interference by *river engineering works*.

The combined transport of water and sediment is a three dimensional time depending phenomenon, which is of a complex nature. A complete *deterministic* description fails due to the *stochastic character* of the morphological processes. At best a rather schematic approach can be used starting from the equations of motion and continuity of two phases: water and sediment. This, however, is only possible when an *alluvial channel* is involved i.e. the river flowing to its own non-cohesive sediment. The picture can be completely different if the natural river differs from this idealized case. This is for instance the case when the alluvial bed contains resistant spots (clay or rock).

Another example that makes morphological prediction for rivers extremely difficult is the occurence of extremely rare high discharges that cannot be predicted. Their influence on the fluvial processes, however, can be extremely large. This is due to the strongly non-linear relationship between water movement and sediment movement.

- El Niño, a yearly dislocation in one of the world's largest weather systems over the Pacific Ocean had a large global impact in 1982-1983.
  It has enormous consequences in terms of floods and droughts (Canby, 1984). Among other things the usual flood of the Chira River (Peru) in the beginning of the year became so extremely large early 1983, that the river changed its downstream course over many kilometers due to this single flood.
  Early 1984 the tropical cyclone Demoina stayed long time over the Southern part of Mozambique. De rivers Maputo, Umbeluzi and Incomati obtained extremely large discharges as locally 700 mm of rainfall occured in a few days. Estimated discharges were about ten times higher than the recorded maximums.

These examples should be kept in mind when morphological forecasts have to be made: the predictions can be based on the statistical properties of the discharge. However, one extreme non-predictable flood can change the whole situation.

The characteristics of rivers can vary largely due to the properties of the rainfall, the characteristics of the catchment area (elevation, soil properties, vegetation, etc.) and the influence of men in the river system.

These aspects will only be treated briefly in the following Sections of Chapter 1. In Chapter 2 some essential river characteristics are treated, whereas Chapter 3 is dealing with fluvial processes due to the combined transport of water and sediment. Finally the principle of morphological predictions is discussed in Chapter 4. These predictions are necessary to forecast the morphological changes due to river engineering works.

1.2. Hydrological aspects

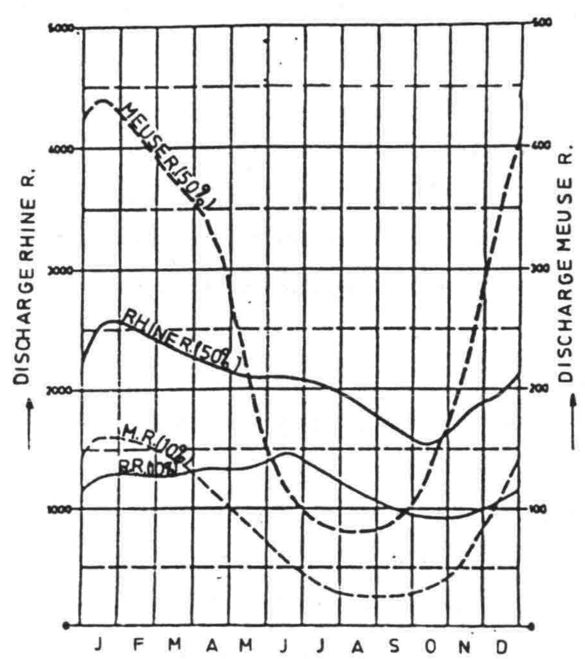


Fig. 1.1. Discharge Rivers Rhine and Meuse

Many aspects govern the shape of the discharge curve Q(t) of a river. In Fig. 1.1 one aspect has been demonstrated. For the River Rhine a substantial part of the precipitation is in the form of snow. The run-off is then retarded. Part of the discharge comes from the glaciers in Switzerland. The course of Q(t) is therefore rather regular. This is contrary to the River Meuse, directly fed by rain on the catchment area which has little storage. Another example of a river with relatively small differences between the yearly maximum and minimum discharges is formed by the Congo River near Brazzaville (Fig. 1.2). The values for 1983 were at Brazzaville Q_{max} = 77 400 m³/s and Q_{min} = 23 000 m³/s. Thus Q_{max}/Q_{min} = 3.5.

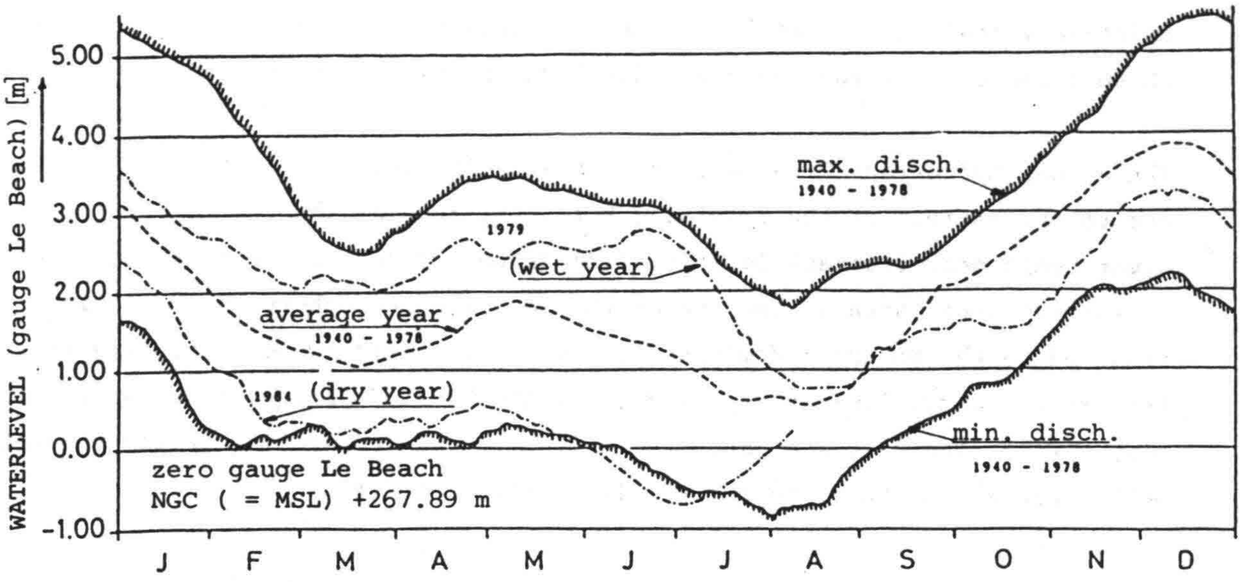


Fig. 1.2. Congo River near Brazzaville

The above given examples regard *perennial rivers*: there is a substantial discharge throughout the year.

On the other hand there are *ephemeral rivers*: during a large period of the year there is little or no discharge. Discharge takes place during a short period in the rainy season. An example is the Choshui River on Taiwan Island. (Fig. 1.3).

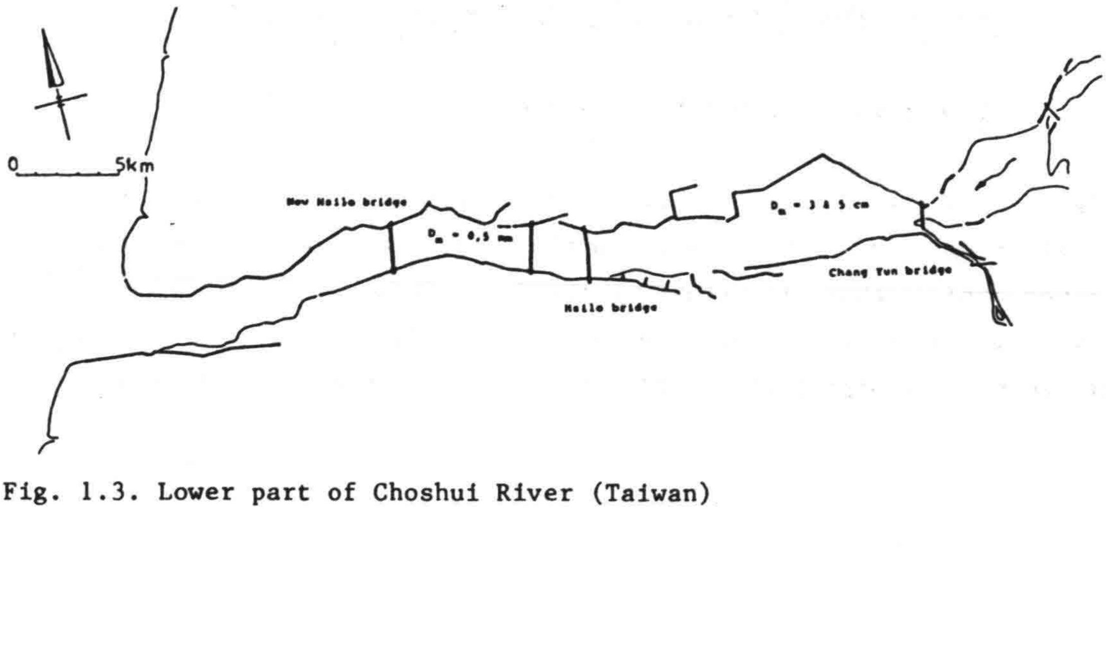


Fig. 1.3. Lower part of Choshui River (Taiwan)

During the typhoon period (July-September) this river carries a substantial discharge. During the rest of the year the river bed is almost dry. There is a substantial rainfall in the catchment area (2 555 mm/a) but the rain is concentrated. Therefore in spite of the fact that the catchment area is relatively small (3 155 km^2) substantial discharges can occur. For the River Choshui the once-in-hundred years discharge in Hsi-Lo amounts to 24 000 m^3/s.

The annual hydrograph Q(t) (or, if expressed in water levels h(t)) is partly due to the pattern of the rainfall, R(t). The regular shape h(t) for the River Congo near Brazzaville (Fig. 1.2) is partly due to the fact that 2/3 of the catchment area is located on the Southern Hemisphere while 1/3 is situated on the Northern Hemisphere. Therefore yearly two monsoons are present. This leads to two low-water periods. For the River Congo two other aspects play a role. The catchment area is not very ondulated and is heavily vegetated. Both aspects contribute to the regular pattern of h(t).

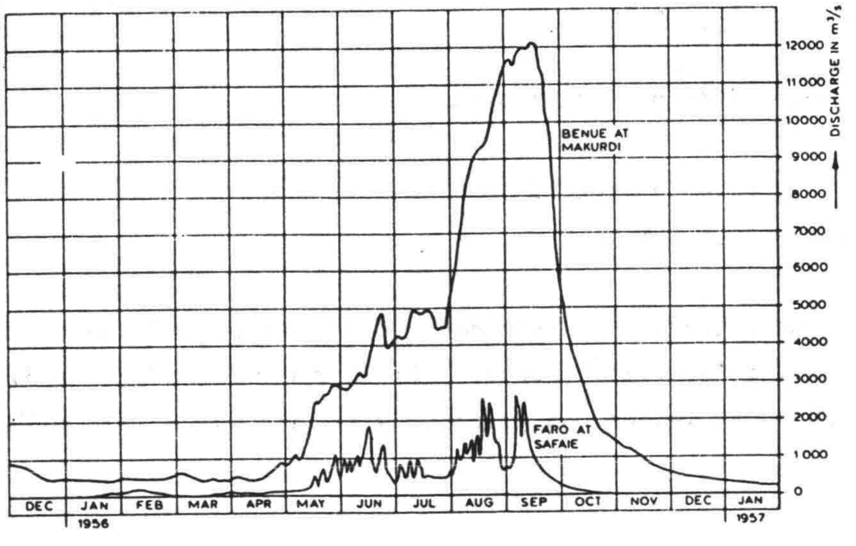


Fig. 1.4. Discharges for River Benue and River Faro.

In Fig. 1.4 the hydrograph Q(t) of the River Benue is given. This is the main tributary of the River Niger. In this figure also the hydrograph of the River Faro, a small tributary of the River Benue is given. The River Faro with its small catchment area is much more flushy than the much larger River Benue. Discharge peaks of tributaries are demped in the main river.

1.3. Geological aspects

The geology of the river basin is an important factor in the appearance. The discharge and sediment transport are characterized by the the catchment area (variation in elevation, erodibility, vegetation, etc.). Consequently a large variation in rivers is present. Table 1.1 gives some general information.

			Discharge				
			Water		Sediment		
River	Station	Catchment area 10^6 km^2	m^3 s^{-1}	mm yr.$^{-1}$	10^6 ton yr.$^{-1}$	10^{-3} mm yr.$^{-1}$	Sediment as ppm of discharge (mg l^{-1})
Amazon	mouth	7.0	100 000	450	900	90	290
Mississippi	mouth	3.9	18 000	150	300	55	530
Congo	mouth	3.7	44 000	370	70	15	50
La Plata/Parana	mouth	3.0	19 000	200	90	20	150
Ob	mouth	3.0	12 000	130	16	4	40
Nile	delta	2.9	3 000	30	80	15	630
Yenissei	mouth	2.6	17 000	210	11	3	20
Lena	mouth	2.4	16 000	210	12	4	25
Amur	mouth	2.1	11 000	160	52	15	150
Yangtse	mouth	1.8	22 000	390	500	200	1 400
Wolga	mouth	1.5	8 400	180	25	10	100
Missouri	mouth	1.4	2 000	50	200	100	3 200
Zambesi	mouth	1.3	16 000	390	100	50	200
St Lawrence	mouth	1.3	14 000	340	3	2	7
Niger	mouth	1.1	5 700	160	40	25	220
Murray-Darling	mouth	1.1	400	10	30	20	2 500
Ganges	delta	1.0	14 000	440	1 500	1 000	3 600
Indus	mouth	0.96	6 400	210	400	300	2 000
Orinoco	mouth	0.95	25 000	830	90	65	110
Orange River	mouth	0.83	2 900	110	150	130	1 600
Danube	mouth	0.82	6 400	250	67	60	330
Mekong	mouth	0.80	15 000	590	80	70	170
Hwang Ho	mouth	0.77	4 000	160	1 900	1 750	15 000
Brahmaputra	Bahadurabad	0.64	19 000	940	730	800	1 200
Dnjepr	mouth	0.46	1 600	110	1.2	2	25
Irrawaddi	mouth	0.41	13 000	1 000	300	500	750
Rhine	delta	0.36	2 200	190	0.72	1	10
Magdalena (Colombia)	Calamar	0.28	7 000	790	220	550	1 000
Vistula (Poland)	mouth	0.19	1 000	160	1.5	5	50
Kura (USSR)	mouth	0.18	580	100	37	150	2 000
Chao Phya (Thailand)	mouth	0.16	960	190	11	50	350
Oder (Germany/Poland)	mouth	0.11	530	150	0.13	1	10
Rhone (France)	mouth	0.096	1 700	560	10	75	200
Po (Italy)	mouth	0.070	1 500	670	15	150	300
Tiber (Italy)	mouth	0.016	230	450	6	270	850
Ishikari (Japan)	mouth	0.013	420	1 000	1.8	100	140
Tone (Japan)	Matsudo	0.012	480	1 250	3	180	200
Waipapa (New-Zealand)	Kanakanala	0.0016	46	900	11	5 000	7 500

Table 1.1. Some basic data of rivers (after Jansen, 1979)

More information on sediment production in river basins is provided by Fournier (1969). In Table 1.1 the rivers are listed by the length of the main stem.

The difference in average sediment concentration is large. The champion is the Yellow River (Huang He) in China. This river is flowing through a loess-area leading to a substantial transport of fine material.

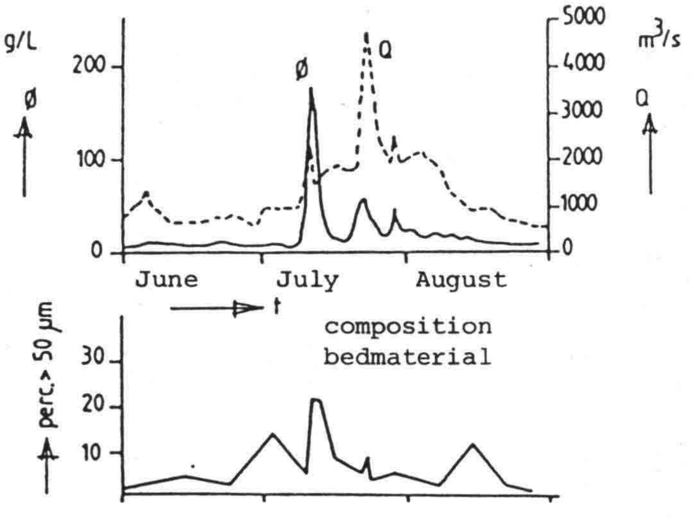


Fig. 1.5. Example sediment transport
Yellow River (Long & Xiong, 1981)

Figure 1.5 shows some transport measurements at the station Tungkuan. Concentrations upto 175 g/liter do occur.
The Yehe River, a tributary upstream of this station flows through a hilly loess-area with quite some *gully erosion*. There the mean concentration is even more than 300 g/liter. The Yehe River has a catchment area of only 3208 km^2; the sediment yield is above 14400 t/km^2.a. On the other hand the St. Lawrence River (Canada) is carrying very little sediment; this river flows through a number of lakes.

The present geological processes can still influence a river basin. Near the confluence of the River Magdalena and the River Cauca (Columbia) the Island Mompos is situated. This area is due to subsidence caused by the tectonics in the Andes. Under natural conditions the subsidence is balanced by the yearly sedimentation during floods.

Another example is reported by Murty (1973). Due to earthquakes in the Himalayas slidings occur which bring suddenly and locally large amounts of sediment in the Brahmaputra River. This causes the low water levels and the high water levels to rise (Fig. 1.6).

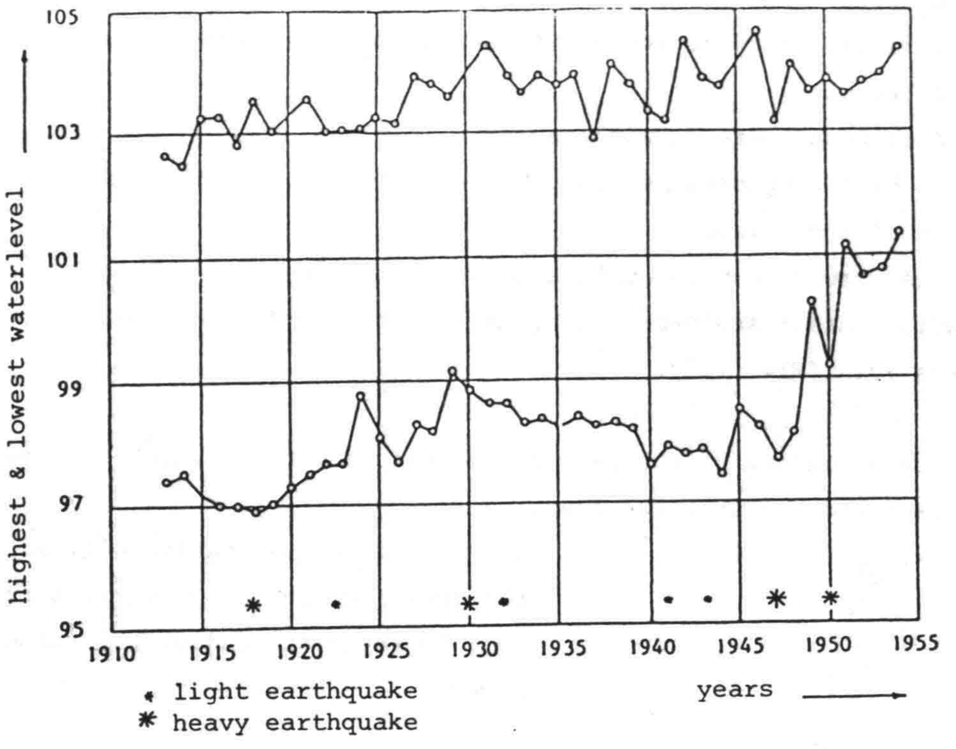


Fig. 1.6. Water level rises Brahmaputra River indirectly due to earthquakes (Murty, 1973)

The composition of the rock that is the source of the sediments (= erosion products) determines the morphological processes.

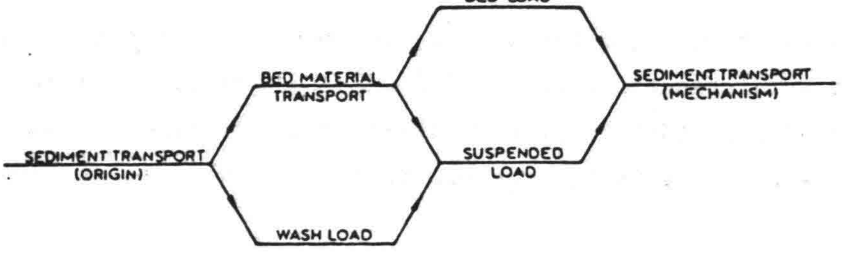


Fig. 1.7. Classification of transport.

In Fig. 1.7 the usual (qualitative!) definitions of the various modes of transport are given:

Bed material transport is the transport of the size fractions that are present in the bed material of the river.

Washload is the transport of the fine particles that are not found in appreciable quantities in the bed.

The bed material transport is determined by the composition of the bed and by the hydraulic characteristics of the stream. It can be determined by transport formulae.
On the other hand there is washload. The amount of washload in a reach is only determined by the upstream supply. Hence it is not determined by the hydraulic parameters of the stream.
This brings forward a problem because a sediment-water sample taken from the stream will contain sediment belonging to the bed material load as well as to the washload (Fig. 1.7).

In practice depending of the geological features of the catchment area rivers can be distinguished into two types.

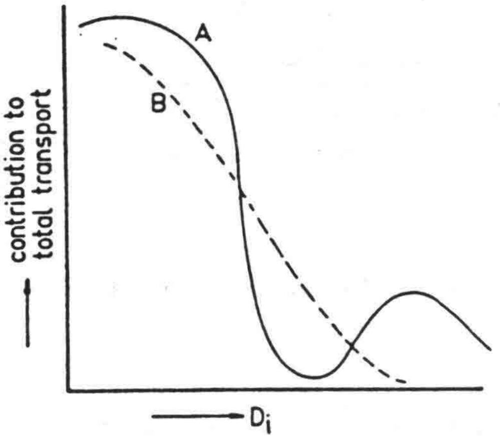


Fig. 1.8. Definition of washload.

In Fig. 1.8 a qualitative plot is given of the contribution of the various fractions (D_i) to the total sediment transport in a river.

River A indicates rivers like the River Rhine, the River Niger and the River Magdalena. A certain range of grain-sizes is hardly present.

On the other hand, however, other rivers like the Serang River on Java do not have such a clear distinction. For type A washload can be characterised by a single grain diameter (50-60 μm). For rivers of type B the grain-size alone cannot be a criterion for the distinction between the two types of transport.

Sieve opening D_i (μm)	150	105	75	62	50	42	35	25	0
$P\{D_i\}$ (%)	0.9	2.4	4.4	6.9	9.1	11.5	14.4	21.9	100

Table 1.2. Grain-sizes of suspended sedidment (Serang River)

As an example Table 1.2 gives a grain-size analysis of a sediment-water samples taken from the Serang River.

A possible way of distinction seems to be the one indicated independently by Vlugter (1941, 1962) and Bagnold (1962). The energy balance for particles in the stream is considered. Particles require energy to remain in suspension. On the other hand while floating downstream particles deliver potential energy to the stream.

According to this hypothesis the transport of particles with fall velocity W_c becomes unrestricted if

$$\frac{\rho_s - \rho}{\rho_s} W_c \leqslant u \cdot i \tag{1-1}$$

For quartz (ρ_s = 2650 kg/m³) this criterion becomes

$$W_c \leqslant 1.6 \, ui \tag{1-2}$$

This *Vlugter-Bagnold criterion* does not only contain the characteristics of the sediment (W_c) but also of the flow (ui). This seems logical: washload is by definition not taking part in morphological processes. If in a river a dam is built with a reservoir then the value of W_c is decreasing in the direction of the dam, according to Eq. (1-2). If the reservoir is large then eventually almost all sediment is trapped, even what was washload in the undisturbed river.

Remark: The data of the Serang River is Table 1.2 show that all (fine) grain-sizes are present. It is a typical example of river type B in Fig. 1.8. The Serang River gets its sediment from the erosion of limestone.

The geological features of the river basin influence the character of a river. The following examples can be given:

- Some rivers have their origin in a lake. For the Nile River the origin of the White Nile is Lake Victoria, whereas the Blue Nile comes from Lake Tana (Ethiopia). Moreover, the discharge of the White Nile is influenced by the swampy area (the Sudds) where much water is lost due to evaporation. The Shire River (Malawi), a tributary of the Zambezi River originates from Lake Malawi.

• Some rivers have a 'rocky section' in their alluvial course. This is for instance the case with the Orinoco River. The Rufiji River in Tanzania has a rocky section at Stiegler's Gorge. At those reaches (nearly) all sediment is transported as washload.

Example: knowledge of the geology of a river is essential for the understanding of the character of the river and the use of a river. A typical example is reported by Neill (1973) as given in Fig. 1.9.

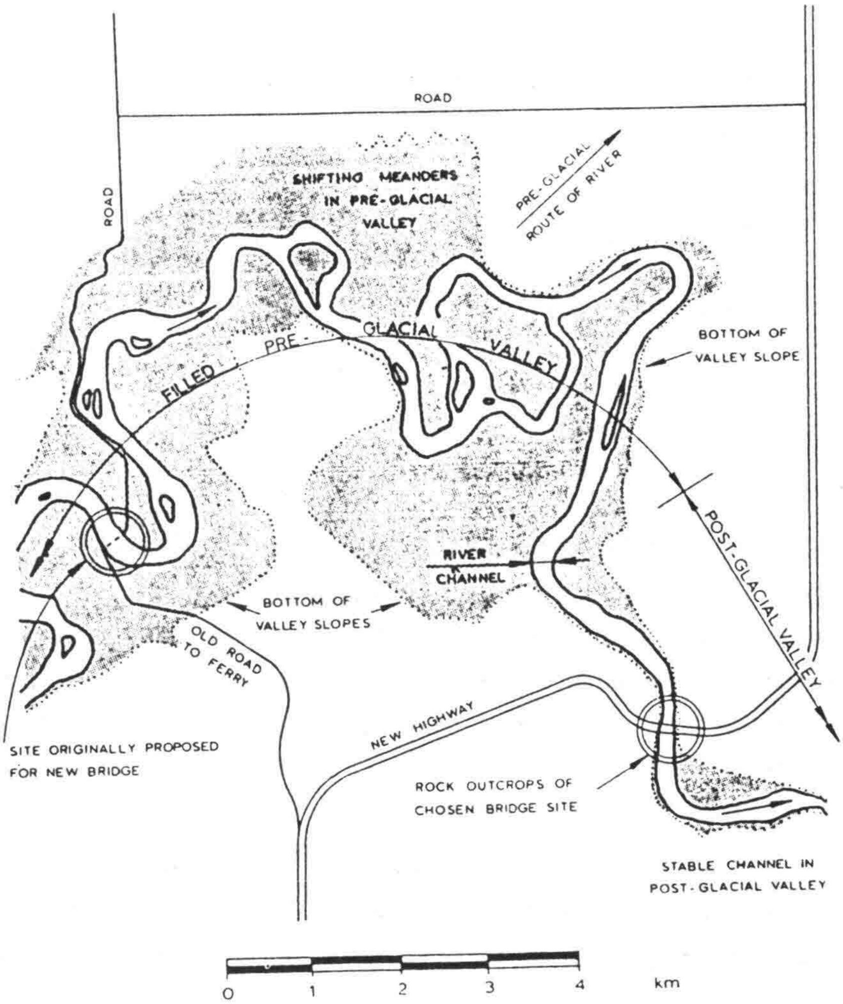


Fig. 1.9. Geology influencing the selection of bridge site (after Neill, 1973).

By studying the geological characteristics of the river valley a bridge site could be selected where it is unlikely that in future the river bed will shift.

1.4. Literature

There exists an abundant amount of literature on potamology and river engineering. Most of it is scattered in articles. Part of it is from a geological (sedimentological) nature, others are directed to river engineering. A few handbooks exist. Scheidegger (1970) has tried from a geomorphological point of view to describe some river processes in a quantitative sense. A mathematical approach to various hydrological aspects of rivers is given by Eagleson (1970). Schumm (1972) and Leopold *et al* (1964) describe some morphological problems.

For the direction of river engineering Shen (1971) and Jansen (1979) can be mentioned. The first book contains a number of separate contributions whereas in the second book an integrated approach is offered. Much information on sedimentation engineering is offered in Vanoni (1975).

Obviously books do not give during a long time the state-of-the-art. For recent finding articles are the appropriate source of information. For instance Jansen (1979) contains material that has mostly been compiled a decade ago from now (1985). The effort spent in the Netherlands on the *research project on rivers* in a close co-operation between Rijkswaterstaat, the Delft Hydraulics Laboratory and the Delft University of Technology during more than one decade has brought forward results that have not yet been incorporated in handbooks. In these lecture notes part of the results are treated.

2. River characteristics

2.1. General

Given from upstream a discharge Q(t) and attached sediment transport S(t) of a grain-size D through a valley with a slope i, a river can have many shapes. Human interference can have altered the shape by major river training (*normalization or canalization*) or by smaller works like local bank protection.This all influences the appearance of a river. Moreover, the river may change its shape as a function of time.
Some general characteristics are treated in this Chapter.

2.2. Planform

In Fig. 2.1 the idealized course of a river is demonstrated. From the *head waters* the river reaches the middle course as a *braided river* gradualy becoming a *meandering river* until in the lower course a delta formation may take place. In the case of a sea (or ocean) the influence of the tides is present in the delta.

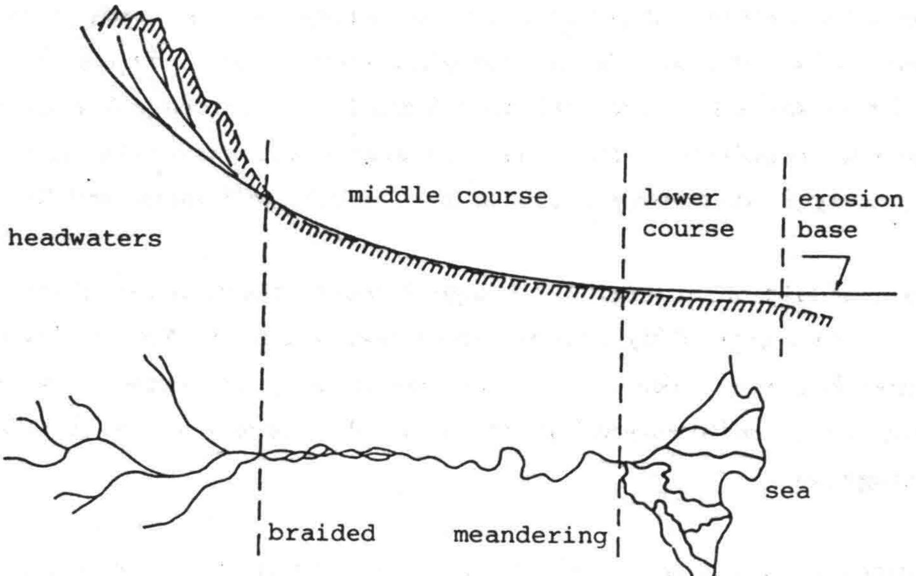


Fig. 2.1. Idealized river

A meandering river is characterized by a single channel whereas a braided river has a number of channels. Leopold and Wolman (1957) have made clear that slope and discharge characterize the planform.

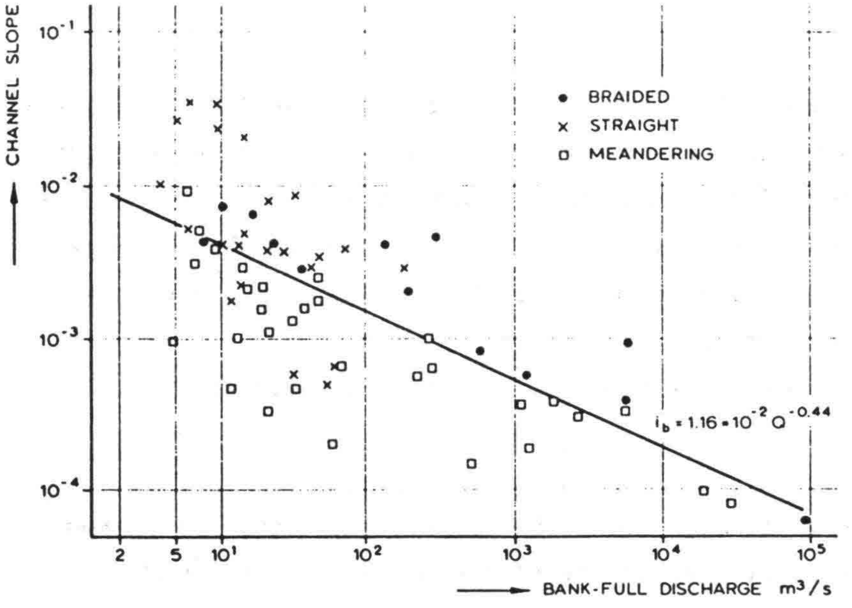


Fig. 2.2. Planform types (after Leopold and Wolman, 1957)

They also mention *straight rivers* as a type of planform. However, this form seems to be unstable. In a straight river there is a tendency to meander in the river bed. There appear *alternate bars* propagating downstream slower than the normal bedforms. These alternate bars have also been noticed in straight laboratory flumes with a mobile bed (Wang and Klaassen, 1981).

The composition of Fig. 2.2 brings forward the problem of schematization of the discharge Q(t) into a single discharge. In Fig. 2.2 the *bankfull discharge* has been taken. It is the discharge just large enough to fill the *low water bed*. Roughly speaking it is the discharge that occurs once or twice in an average year.

Figure 2.3 gives an example of a river in which part is braided and part is meandering. There are indications that the braided part of the Tigris River (the reach upstream of Balad) has an *armoured* bed. Armouring is a result of a degradated bed composed of different grain sizes. Sorting processes are responsible for the fact that finally the toplayer of the bed consists of coarse grains (thickness 1 to 2 D) above the original sediment mixture.

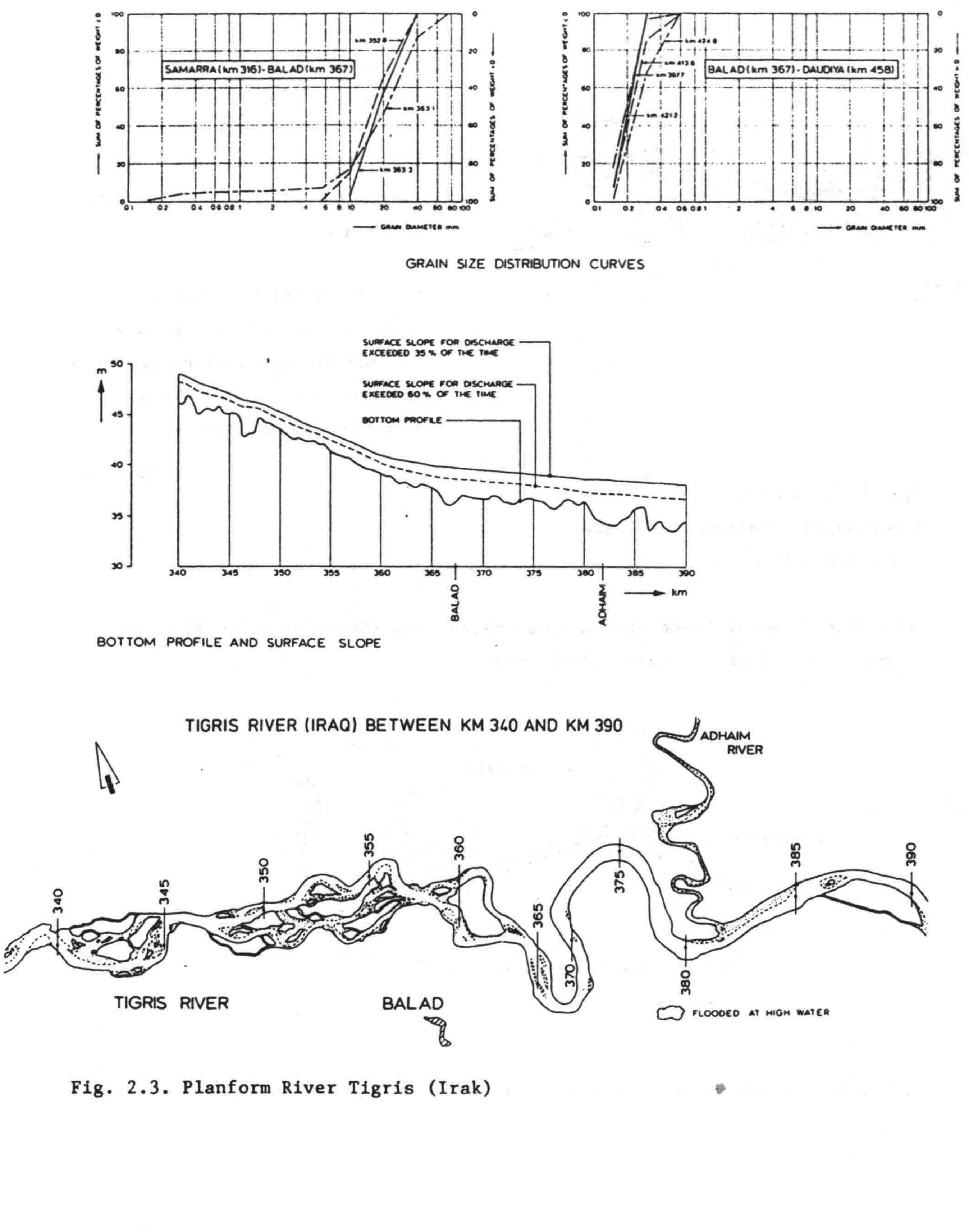


Fig. 2.3. Planform River Tigris (Irak)

Human interference can transform a braided river into a meandering one. An example is given in Fig. 2.4. Normalization works in the 19th century in the River Rhine downstream of Basle (Switzerland) have changed the planform.

The artificial new meanders have fixed banks. If the new course is made straight, alternate bars are likely to occur. This a nuisance for navigation.

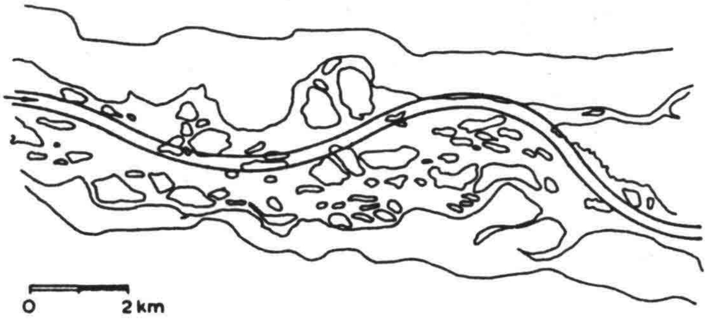


Fig. 2.4. Normalization of the River Rhine downstream of Basle (19th century).

More downstream of Basle the original meandering River Rhine has also been normalized. Figure 2.5 gives an example.

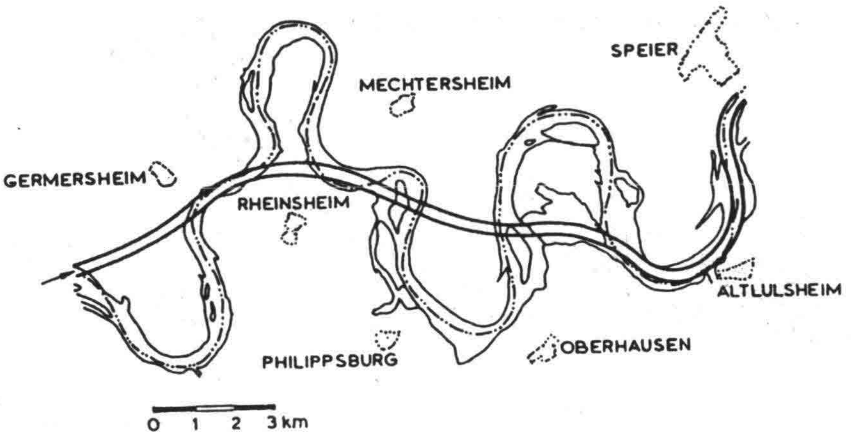


Fig. 2.5. Normalization of the River Rhine upstream of Mannheim (19th century)

In normalized rivers the natural appearance can hardly be recognized. In Fig. 2.6 the change of the River Waal (the main branch of the River Rhine in the Netherlands) in the course of time is represented.

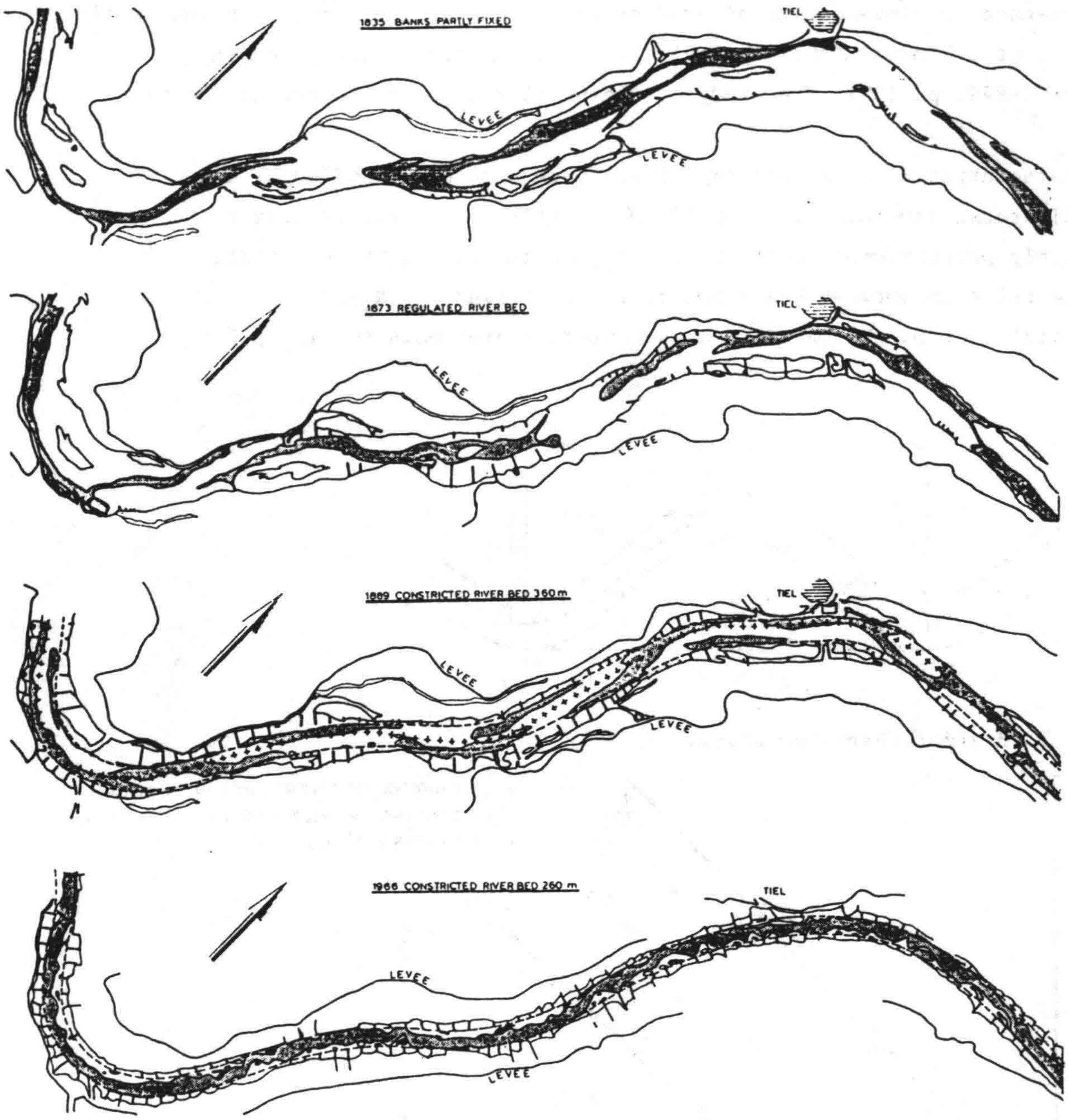


Fig. 2.6. Development of the River Waal near Tiel (km 910 - 930)

In this case the original reason for river training was the prevention of jamming of flooting ice. Icejams caused flood problems. The later works were carried out to reduce the width to obtain more depth for navigation.

The presence of a meandering or braided rivers has been examined mathematically by means of a linear stability analysis. Some references are given in Janssen (1979, p. 133). The study of Olesen (1983) can be quoted in addition.

The characteristics of meandering rivers have also been studied by many investigators. According to Leopold *et al* (1964) the meander lenght (λ) is roughly proportional to the width (B_s) of the river. The same holds for the relation between λ and the radius of curvature (R_m).
The definitions of the meander characteristics are shown in Fig. 2.7.

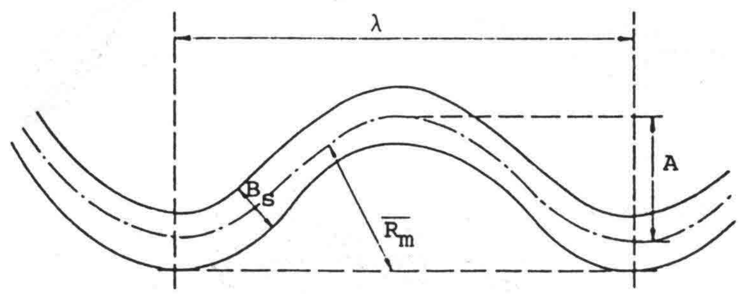


Fig. 2.7. Meander characteristics.

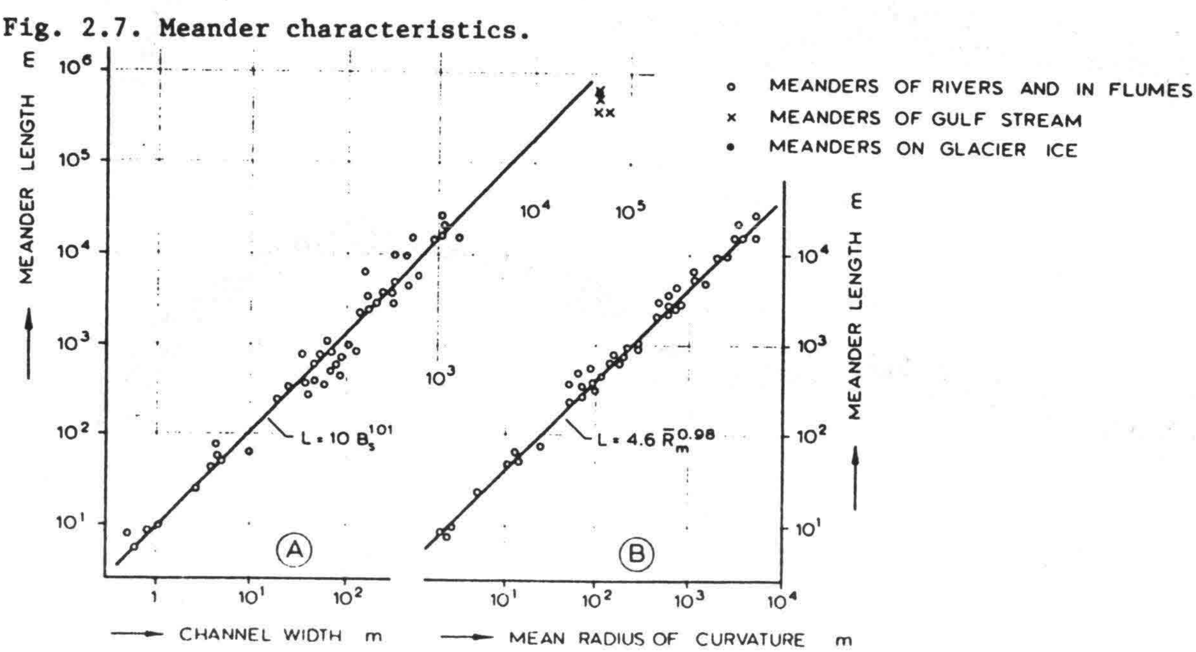


Fig. 2.8. Meander characteristics (after Leopold *et al*, 1964).

The findings of Leopold *et al* (1964) are represented in Fig. 2.8.

The study of meander characteristics is hampered by the fact that not all meanders of the same river are equal. Spectral analysis has been applied by Speight (1965) on the meanders of the Angabunga River (Papua-New Guinea). Fig. 2.9 shows that there are two peaks in the spectra.

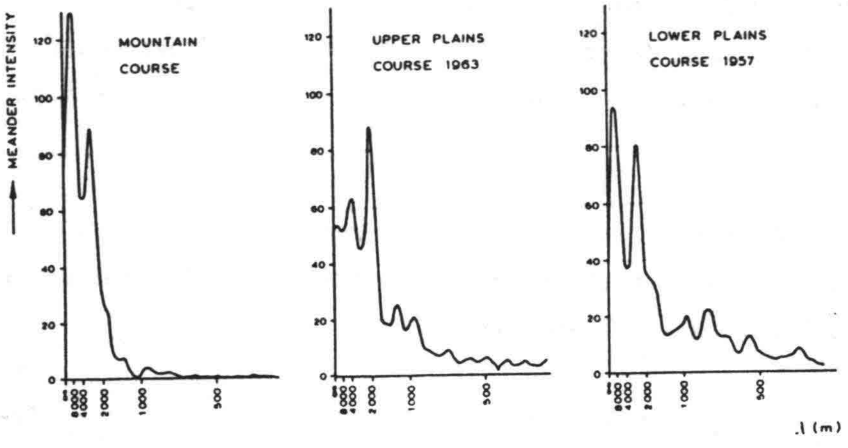


Fig. 2.9. Meander spectra for Angabunga River (after Speight, 1965)

This in accordance with Schumm (1963) who suggests that two characteristic meander lengths may be present for the same stream at the same time.

Another problem is the (varying) discharge. In most cases bankfull discharge has been used to find relation with the mean meander length. A summary given in Jansen (1979, p. 137) suggest $\lambda \simeq Q^{\alpha}$ with α = 0.4 to 0.5 if the bankfull discharge is taken. Ackers and Charlton (1970) have studied the influence of the hydrograph on the meander length. They studied the River Kaduna (tributary of the Niger River). They tried to reproduce the meanders by means of a scale model and found that reproduction was possible with a constant discharge 13% higher than bankfull discharge.

In freely meandering rivers in time meanders propagate downstream and/or increase their amplitude. If the amplitude becomes very large, the river may during flood cut-off the bend, leaving the original meander loops as *oxbow lakes* in the river valley. Gradually these oxbow lakes get filled up with fine material. This causes inhomogeneities of the sediment composition of the high water bed. Therefore for the Mississippi River the local alignment of the channel depends largely on the local variation of the composition of the bank material (Leopold *et al*, 1964, p. 298).

Hence it is not easy to predict the time depending behaviour of the planform of freely meandering rivers. However, attempts are being made (Ikeda *et al*, 1981; Parker *et al*, 1982 and Chang, 1984).

2.3. Longitudinal profile

The idealized river presented in Fig. 2.1 shows that the bedslope becomes slower in the downstream direction. This is the general tendency found. Moreover, the mean grain-size decreases in the downstream direction. As early as 1875 Sternberg describes this phenomenon mathematically (*see* Leliavski, 1955).

The mass reduction (dM) of the grain during the transport process is supposed to be proportional with the mass (M) of the grains and the distance (dx) over which the grains are transported.
Hence,

$$dM = -\alpha M dx \tag{2-1}$$

in which α is a coefficient describing the properties of the grains and the river.
Integration gives

$$M = M_o \exp\{-\alpha x\} \tag{2-2}$$

in which the integration constant represents the mass at x = o

For the grain-size D this can be transformed into

$$D = D_o \exp\{-\alpha' x\} \tag{2-3}$$

The variation of D(x) seems to be due to wearing and sorting. The process has not yet been analysed quantitatively. Leliavski (1955) reports on some data of M{x} for European rivers. Note that in principle the α-value of Eq. (2-2) can have quite different values for rivers. Some times the grain-size can decrease over small distances. This is for instance the case for the Choshui River (Taiwan) as can be noticed from Fig. 1.3.

Also the longitudinal profile can be approached by an experimental function. For the Rio Grande (USA) the relation for the bed slope $i_b = 0.0022 \exp\{-5.8\ 10^{-3}\ x\}$ has been reported. As $i_b = -\partial z_b/\partial x$ also z_b will be changing exponentially with the distance (*see* Jansen, 1979, p. 141).

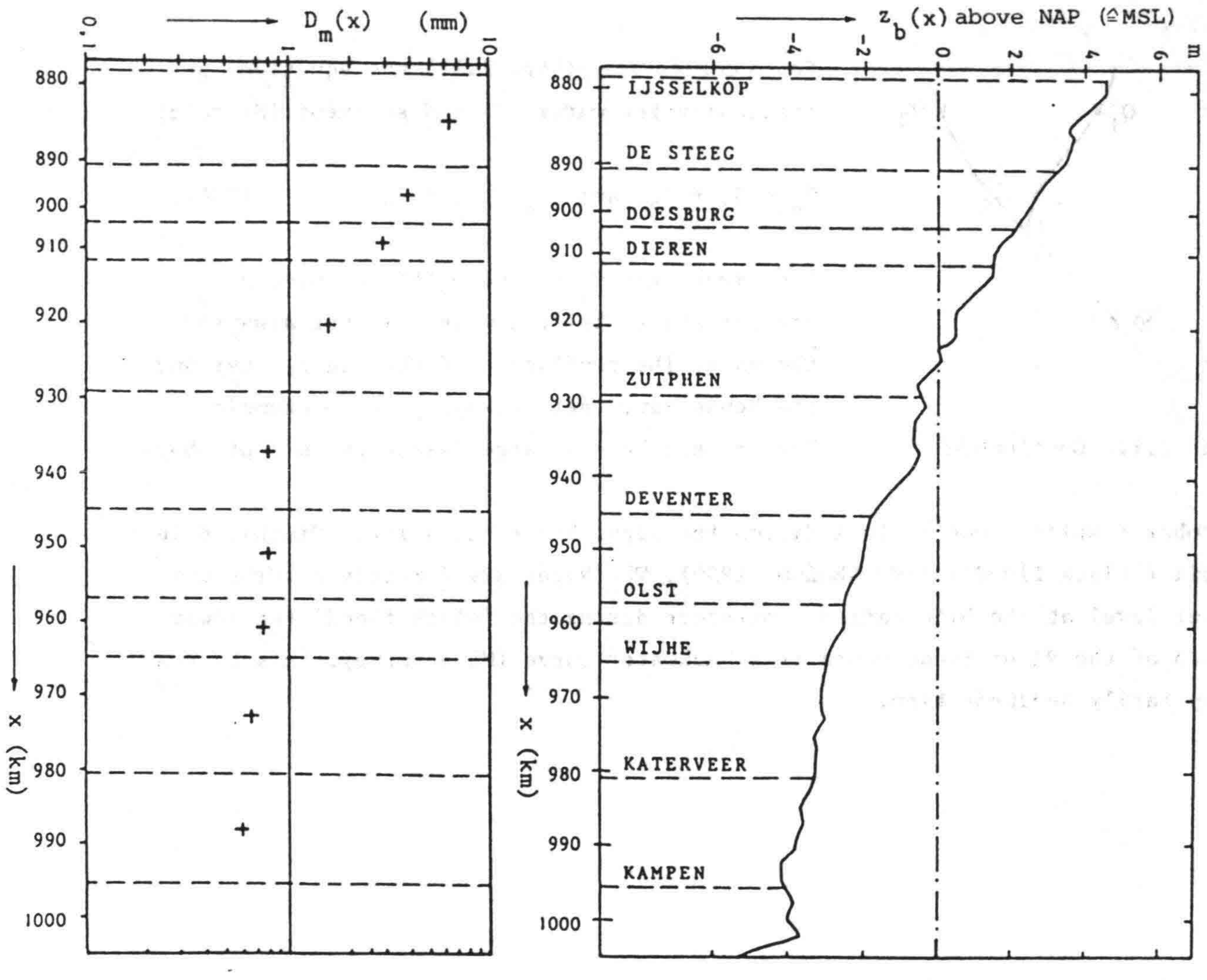


Fig. 2.10. Longitudinal profile IJssel River (after Zeekant, 1983)

As an example of $z_b(x)$ and $\bar{D}(x)$, Fig. 2.10 shows the variation along the axis of the River IJssel, the minor branch of the River Rhine in the Netherlands. Downstream of Kampen the River IJssel is discharging into the IJssel Lake. Hence the downstream part of this river branch is not influenced by tides.

2.4. Confluences and bifurcations

Confluences are mainly present in the upper reach of a river whereas bifurcations are usually present in the lower reach (Fig. 2.1)

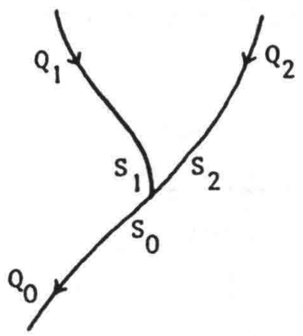


Fig. 2.11. Confluence

For a *confluence* (Fig. 2.11) the equations of continuity for water (Q) and sediment (S) hold.

$$Q_o = Q_1 + Q_2 \quad \text{and} \quad S_o = S_1 + S_2 \qquad (2\text{-}4)$$

The discharges $Q_1(t)$ and $Q_2(t)$ may have a similar shape. This, however, is not always the case. The confluence of the Niger River and the Benue River near Lokoja give an example. Both rivers have a large discharge in September-October ('white flood'). In addition the Niger River has a large discharge in April ('black flood') (*see* NEDECO, 1959). The Niger River mainly governs the water level at the bifurcation. Therefore during the 'black flood' the lower reach of the River Benue contains a backwater curve (M1 - curve). This causes temporarily sedimentation.

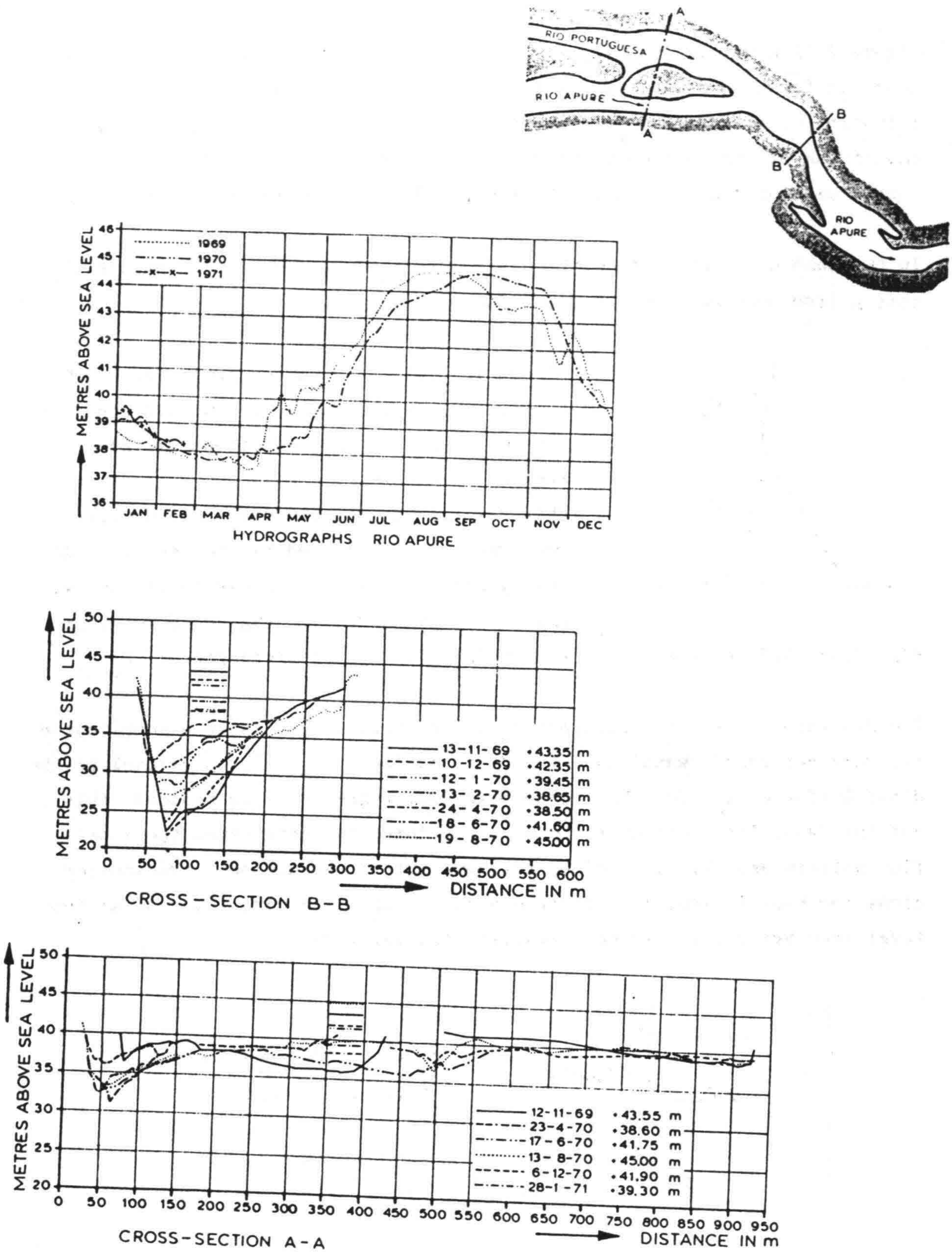


Fig. 2.12. Confluence Apure River and Portugesa River (after DHL, 1971)

Figure 2.12 gives an example of a confluence in Venezuela. The Apure River, near San Fernando de Apure, has a rather regular hydrograph. It regards a tributary of the Orinoco River . Note the large variation of the bed level downstream of the confluence with the Portugesa River, especially in the narrow section B-B. It regards here natural i.e. non-trained rivers.

In the case of a *bifurcation* the discharge (Q_o) and sediment transport (S_o) coming from upstream are devided. (Fig. 2.13).

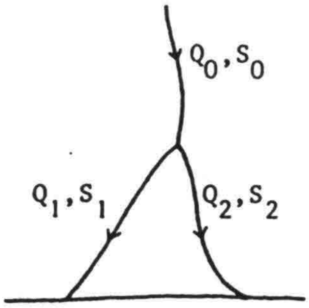


Fig. 2.13. Bifurcation

Of course here also the continuity equations of Eq. (2-4) hold. However, now each equation has two unknowns. Thus additional information is necessary.
This distribution of the discharge Q_o over two branches is governed by the fact that *at* the bifurcation only one water level can exist. Hence the *conveyances* of the two downstream rivers determine the distribution of Q_o.

The distribution of the transport S_o at the bifurcation is more complicated. For some sediment ('washload') the distribution of S_o is proportional to the distribution of Q_o. For the coarse material, transported as bedload, this is not the case. The *local geometry* of the bifurcation determines the local flow pattern and this determines the movement of the sediment transported along the bed. In general a river branching off in an outer bend of another river receives relatively more sediment (Bulle, 1926).

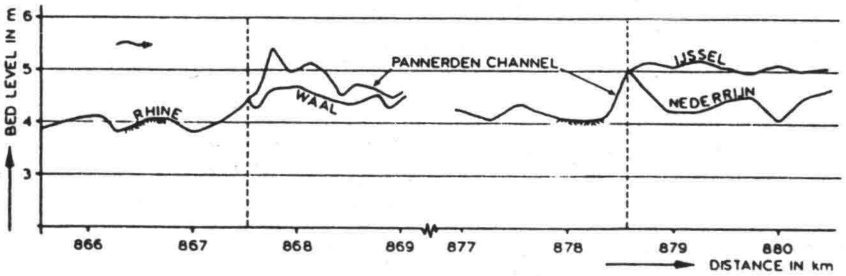


Fig. 2.14. Bed profile near the bifurcations of the River Rhine in the Netherlands

Given the distribution of Q_o and S_o as well as the continuity of the water level at the bifurcation, it is not surprising that the bedlevel can show *discontinuities* (see also Section 4.5). Figure 2.14 shows these discontinuities at the bifurcations of the River Rhine in the Netherlands. The situation of the bifurcations is given in Fig. 2.15.

MEAN VALUES	Q m^3/s	S m^3/day	D_m mm
RHINE	2140	550	6
WAAL	1490	700	4
PANN. CHANNEL	650	300	4
NEDERRIJN	390	270	2
IJSSEL	260	130	5

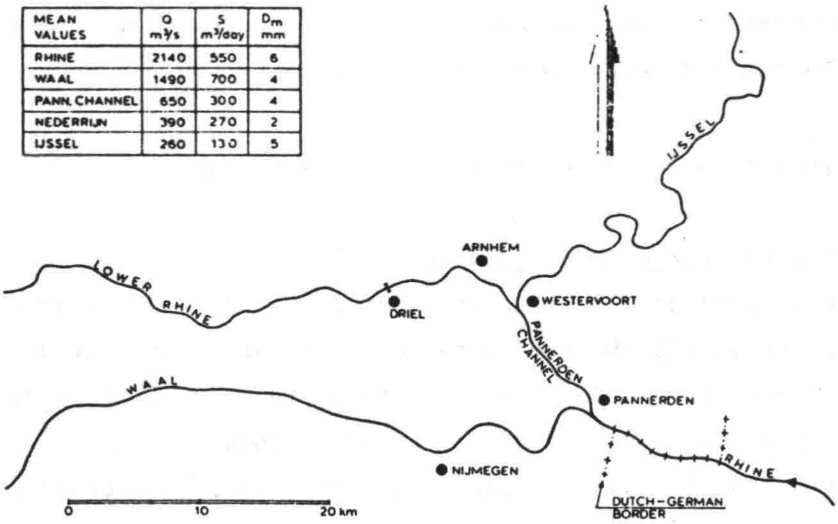


Fig. 2.15. Bifurcations of the River Rhine in the Netherlands

As the sediment coming from upstream is usually non-uniform, grain-sorting is likely to occur at a bifurcation. Figure 2.6 gives an example. It requires some care in sampling to show this phenomenon as other causes of grainsorting (river bends, bedforms) are present at the same time.

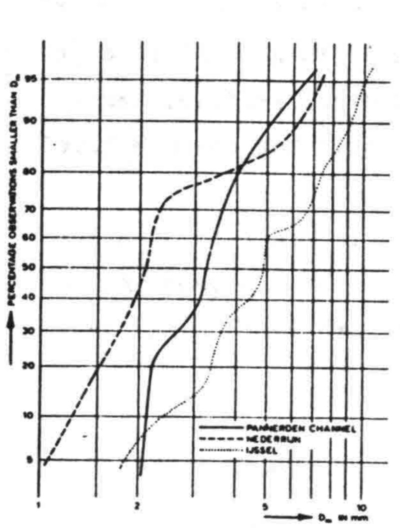


Fig. 2.16. Grain sorting at the bifurcation Westervoort.

To obtain Fig. 2.16 over some kilometers in each branch samples have been taken along the river axis and at distances ± ¼ B from the axis. Each sample had a sufficiently large size to get a good estimation of the mean grain size (D_m) on the particular location (de Vries, 1970).

2.5. River mouths

A river discharges into another river (like the River Benue into the River Niger), into a lake (like the River IJssel into the IJssel Lake) or in a sea. To a large extent the water level at the mouth is not governed by the river, it is therefore an independent boundary condition. At a far distance upstream of the mouth the water movement and sediment movement are independent of the boundary condition. Naturally the bed level there is influenced by the presence of the mouth.

An elementary analysis of some schematic cases is given below:

- River with constant discharge entering a deep lake

  The most simple case regards a river with constant width and discharge. Upstream of the mouth ($x > 0$) the same S and Q are transported. For uniform bed material the bottom slope (i_b) and the waterdepth (a) will be constant. Waterlevel and bedlevel are then parallel straight lines.

  Due to sedimentation in the lake the mouth will gradually move downstream. The process is governed by the yearly sediment transport and the depth of the lake.

- River with varying discharge entering a deep lake

  At the mouth ($x = 0$) the water level $h(0,t)$ = constant. If again the width (B) and the grain size are supposed to be constant only the variation of Q has to be considered in addition. It can be stated in general that it takes much time to change the slope of the longitudinal profile. Hence the water level upstream of the mouth may vary in time but the bed level hardly does.

  The bed slope (i_b) can now be found from the reasoning that the yearly sediment transport through each cross-section has to be the same. As an approximation the transport formulae for this case are represented by $s = m\,u^n$ with m and n being constant.

  The transport can now be expressed with Q and i_b as parameters.

$$S = B\,m\,u^n = B\,m\,\left\{\frac{Q}{Ba}\right\}^n = B\,m\left\{\frac{Q}{B}\cdot\left[\frac{Q}{BCi_b^{1/2}}\right]^{-2/3}\right\}^n \qquad (2\text{-}5)$$

in which Chezy equation $Q = BC\sqrt{a i_b}$ is used

Hence

$$S \sim B^{1-n/3} . Q^{n/3} . i_b^{n/3} \tag{2-6}$$

If f{Q} is the probability density of the discharge then the yearly sediment transport for each section amounts to

$$\int_0^\infty S(Q).f\{Q\}dQ = \text{constant} \tag{2-7}$$

As i_b does not change with the discharge Eqs. (2-6) and (2-7) can be combined.

$$B^{1-n/3} . i_b^{n/3} . \int_0^\infty Q^{n/3} . f\{Q\} . dQ = \text{constant} \tag{2-8}$$

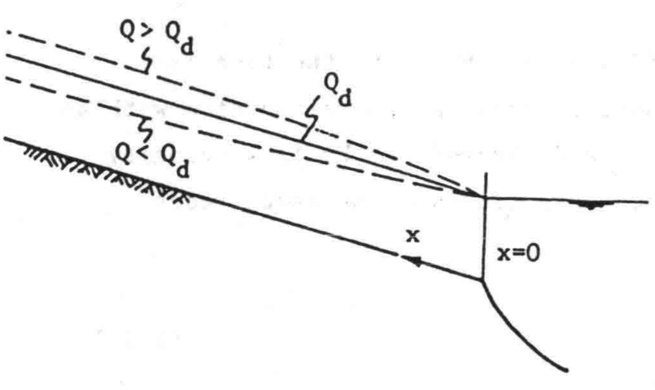


Fig. 2.17. River discharging into a lake

Also in this case i_b is constant (if B is). There is one discharge (Q_d) for which the flow is uniform. For $Q \neq Q_d$ backwater curves are present (Fig. 2.17).

For mild (positive) slopes the back-water curve will be of the M_1-type if $Q<Q_d$ and of the M_2-type for $Q>Q_d$.

It is interesting to look in this case at the depth (a_o) in the mouth. Therefore transport formulae must now be expressed with Q and a as parameters. Combination of Eqs. (2-5) and (2-7) gives for the yearly transport

$$\int_0^\infty Bm \{ \frac{Q}{Ba_0} \}^n . f\{Q\} .dQ = \text{constant} \tag{2-9}$$

If now a_o is supposed not to vary with Q (which is less likely than for i_b) then Eq. (2-9) can be written as

$$B^{-n+1} a_o^{-n} \int_0^\infty Q^n . f\{Q\} .dQ = \text{constant} \tag{2-10}$$

The comparison of Eqs. (2-8) and (2-10) will be given further attention in Section 2.6.

- River discharging into a sea

A river discharging into a sea is near the mouth under the influence of the tidal movement. The tidal movement enlarges the sediment transporting capacity. Therefore the cross-section will generally increase in the direction of the mouth. The principle can be explained from the non-linear relationship between flow velocity and sediment transport.

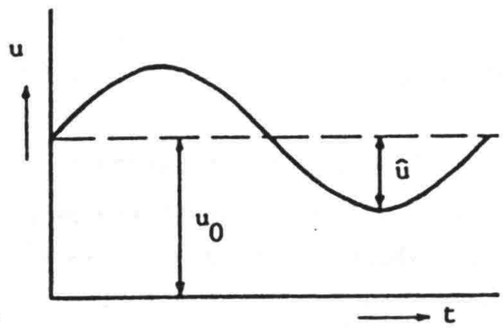


Fig. 2.18. Tidal influence

In Fig. 2.18 the variation of the flow velocity is given. Due to upstream discharge (Q_o) there is the flow velocity u_o.

The flow velocity due to the tide is supposed to vary as a sine-function with an amplitude û. Therefore the flow velocity in the cross-section considered reads

$$u = u_o + \hat{u} \sin \omega t \tag{2-11}$$

The transport per unit of width is s and using $s = m\,u^n$ gives for the average transport $\bar{s}$ during the tidal period (T):

$$\bar{s} = T^{-1} \int_o^T m \{u_o + \hat{u} \sin \omega t\}^n \, dt \tag{2-12}$$

If the parameters m and n do not change too much this gives with $\phi = \hat{u}/u_o$

$$\bar{s} = m u_o^{\,n} \cdot T^{-1} \int_o^T \{1 - \phi \sin \omega t\}^n \, dt \tag{2-13}$$

Due to the upper discharge Q_o the transport would be $s_o = m\,u_o^{\,n}$.

From Eq. (2-13) follows with $\omega t = 2\pi$ and $\omega t = y$ or $dy = \omega dt$:

$$\bar{s} = \bar{s}_o \cdot \frac{1}{2\pi} \int_0^{2\pi} \{\phi \sin y + 1\}^n \, dy = \beta s_o \tag{2-14}$$

with

$$\beta = \frac{1}{2\pi} \int_0^{2\pi} \{\phi \sin y + 1\}^n \, dy = \beta(\phi,n) \qquad (2\text{-}15)$$

For instance the following functions for $\beta(\phi)$ can be found analytically:

$n = 1 \quad \beta = 1$

$n = 3 \quad \beta = 3/2\,\phi^2 + 1$

$n = 5 \quad \beta = 15/8\,\phi^4 + 5\phi^2 + 1$

These relations are given in Fig. 2.19 for $0 \leqslant \phi \leqslant 1$.

Due to above mentioned non-linear relationship $n > 1$. Thus $\beta > 1$ or $\bar{s} > s_o$.

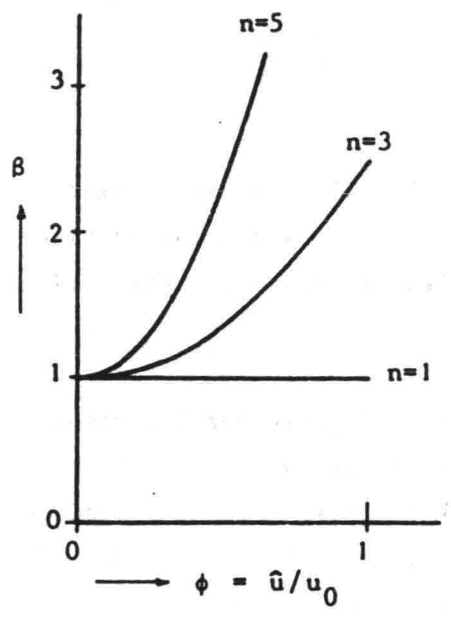


Fig. 2.19. $\beta = f(\phi,n)$

Consider now two cross-sections. The one upstream of the tidal influence (subscript o) has the characteristics $u = u_o$; $\phi_o = 0$ and $B = B_o$. The cross-section under the tidal influence has the subscript 1. So here the characteristics are $\phi = \phi_1$ and $B = B_1$.

For a constant upper discharge Q_o the mass balance has to express that both cross-sections have to have the same total transport as $S_o = S_1$ or

$$B_1\,\beta_1 \cdot s_{o1} = B_o \cdot s_{oo} \qquad (2\text{-}16)$$

in which s_{o1} is the transport unit width in the cross-sectional area A_1 with $Q_o = u_{o1} \cdot A_1$

From Eq. (2-16) follows:

$$B_1 \cdot \beta_1 \cdot m \left[\frac{Q_o}{a_1 B_1}\right]^n = B_o \cdot m \left[\frac{Q_o}{a_o B_o}\right]^n \tag{2-17}$$

Or

$$\beta_1 = \frac{B_1^{n-1} \cdot a_1^n}{B_o^{n-1} \cdot a_o^n} = \frac{B_o}{B_1} \left[\frac{A_1}{A_o}\right]^n \tag{2-18}$$

As $\beta_1 > 1$ for a constant width ($B_1 = B_o$) it follows from Eq. (2-18) that $a_1 > a_o$. In general it will follow from Eq. (2-18) that $A_1 > A_o$.

The above given analysis is only of a qualitative nature. Near the mouth the analysis will not hold due to the fact that density currents will be present and the flow direction will reverse.

2.6. Schematization of the regime

The main characteristic of a river discharge is that this varies in time. As a consequence the morphological parameters of a river will also be time-depended. Therefore if morphological forecasts have to be made, this variation in time has to be taken into consideration.

At present (1985) it has become possible to carry out these morphological computations with a varying discharge Q(t). However, it is then still questionable which (recorded) Q(t) has to be taken. There will be a tendency to use an average year and if possible also wet years will be used. No systematic research as yet seems to have been carried out.

Instead of a time depending prediction it is possible to study the change of an equilibrium situation into a new one, leaving the time depending predictions of the transient from one equilibrium into another for a second approximation. In this steady approach the probability distribution f(Q) is used. This method has been used in Section 2.5 to find the equilibrium bed slope of a river discharging into a lake. In Eq. (2-8) the right hand side represents the yearly sediment transport. Hence this equation can be used to study the change of the slope if the width of the river is changed (see also Section 4.4).

It has to be remarked that in the above quoted analysis the transport function is approached by a power law $s = m\,u^n$. This has been done to make the analysis sufficiently transparent. For practical problems it is quite possible to use a real transport formula, i.e. one adequate to the river which is studied.

In literature frequently the river regime is drastically schematized into one single discharge (*'dominant discharge'*). The use of bankfull discharge for the study of meander characteristics is an example (Section 2.2).
It can easily be shown that such a dominant discharge does not exists. In other words one single discharge cannot describe more than one morphological parameter of a river.

The proof of this statement can be obtained from the example of a river discharging into a lake (see Fig. 2.17). Two parameters are considered *viz* the bed slope i_b upstream of the mouth and depth a_o at the mouth.
Following the procedure usually applied with the concept of 'dominant discharge' Eq. (2-8) would lead to a discharge Q_{d_i} for the slope i_b according to

$$B^{1-n/3} \cdot i_b^{n/3} \int_0^\infty Q^{n/3} \cdot f\{Q\}\, dQ = B^{1-n/3} \cdot i_b^{n/3} \cdot Q_{d_i} \tag{2-19}$$

or

$$Q_{d_i}^{n/3} = \int_0^\infty Q^{n/3} \cdot f\{Q\} \cdot dQ \tag{2-20}$$

A similar approach for the depth a_o would lead with Eq. (2-10) to a 'dominant' discharge Q_{d_a} with

$$Q_{d_a}^{n} = \int_0^\infty Q^{n} \cdot f\{Q\} \cdot dQ \tag{2-21}$$

Equations (2-20) and (2-21) show that always $Q_{d_i} \neq Q_{d_a}$. In other words one single discharge *cannot* lead to correct answers for both i_b *and* a_o.

Two more remarks can be made in this respect.

(i) The definitions applied to find the 'dominant' discharge use the characteristics of the *existing* river. Obviously a different discharge has to be applied to forecast the response of the river on man-made changes in the river system.

(ii) The above given examples for i_b and a_o show that there is no need to define such a thing as a 'dominant' discharge. In principle the problem of finding i_b and a_o can be solved by means of Eqs. (2-8) and (2-10).

In summary the schematization of the regime of a river can be two-fold.

- For time-depending prediction the 'real' Q(t) has to be used.
- For studying new equilibrium situations it is advised to use the probability density f{Q} of the discharge.

In practice both f{Q} and Q(t) will be approximated. For instance

$$\int_0^{\infty} f\{Q\}\, dQ \simeq \sum_{i=1}^{n} Q_i \qquad (2\text{-}22)$$

As an example it can be tested whether a continuous probability density f{Q} based on daily discharges can be approximated by a histogram based on monthly averaged discharges. As the sediment transport plays a key role in the morphological predictions it is logical to test this approximation via S. This can be done by some test computations of the factor α with n being the number of days in a month and

$$\alpha = \frac{n * S\{\bar{Q}_{month}\}}{\sum_{i=1}^{n} S\{\bar{Q}_{day}\}} \qquad (2\text{-}23)$$

For flushy upper rivers due to the non-linear relationship between Q and S the value will be $\alpha \ll 1$. For lower rivers, however, the discharge usually does not change rapidly. Then $\alpha \simeq 1$, which means that Eq. (2-22) can be used thus the computations can be based on monthly averaged discharges.

A similar approach may be used for time-depending morphological computations with Q(t). As will be shown later (see Chapter 3) in morphological computations often time steps larger than one day can be used. Hence also in that case discretization is adopted, this time of Q(t).

3. Fluvial processes

3.1. General

The combined transport of water and sediment in rivers is a complex process because there is an interaction between the transports of the two phases. The problem is time-depending, dealing with three space dimensions. It requires a great deal of schematization in order to be able to describe the problems in a mathematical sense, leading to mathematical models that can be used for morphological forecasts.

In this chapter the mathematical description is treated. In the first place Section 3.2 deals with the one-dimensional approach. Here the average values of the morphological parameters for each cross-section are considered as function of time and place. In this approach there is only one space dimension left, the coordinate x along the river axis.

In Section 3.3 two-dimensional approaches are treated. The two space dimensions are in the first place the x and y coordinate in the horizontal plane. Also two-dimensional approaches in the x-z plane are considered (Sub-Section 3.3.3). These approaches are necessary when the transport of sediment in suspension varies considerably in the longitudinal direction.

The basic parameters are indicated in the definition sketch of Fig. 3.1.

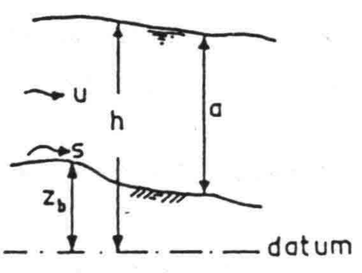


Fig. 3.1. Definition sketch.

- The *water depth* (a) is mainly of importance for navigation. Prediction of a (x,y,t) is anticipated.
- The *water level* (h) is of interest for the possibility to withdraw water for irrigation or with regard to flood problems.
- The *bed level* (z_b) is important to know when bank protection works or bridge piers have to be designed. Obviously $z_b(x,y,t)$ has to be predicted.

3.2. One-dimensional approach

3.2.1. Analysis of basic equations

In the one-dimensional approach the average values of a, h, and z_b are considered for the cross-sections. With $h = a + z_b$ according to the definition (Fig. 3.1) this means that a and z_b can be considered as dependent variables for which relevant basic equations have to be found. Moreover the flow velocity u(x,t) and the transport s(x,t) are dependent variables. This means that *four* basic equations are required.

The equations are:

momentum water $$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + g\frac{\partial a}{\partial x} + g\frac{\partial z_b}{\partial x} = -g\frac{u|u|}{C^2 a} \qquad (3\text{-}1)$$

continuity water $$\frac{\partial a}{\partial t} + u\frac{\partial a}{\partial x} + a\frac{\partial u}{\partial x} = 0 \qquad (3\text{-}2)$$

transport formula $$s = f\{u, \Delta, D, C \text{ etc}\} \qquad (3\text{-}3)$$

continuity sediment $$\frac{\partial z_b}{\partial t} + \frac{\partial s}{\partial x} = 0 \qquad (3\text{-}4)$$

The following remarks have to be made:

(i) The equations are valid for a wide river with constant width B. The banks are supposed to be fixed or less erodible than the river bed. For erodible banks also B(x,t) would have to be considered as a dependent variable. This would require an additional equation, which is not readily available.

(ii) The equations are valid for $s/q \ll 1$; i.e. small mean sediment concentrations.

(iii) Any suitable transport formula can be used in principle. In this elementary analysis all parameters except u(x,t) are supposed not to vary with x and t.

(iv) Equation (3-3) implies that the sediment transport is a function of the *local* hydraulic parameters. Hence this model is not applicable if there is a change in suspended load over short distances (see Sub-Section 3.3.3).

It has been shown (de Vries, 1959, 1965) that Eqs. (3-1) through (3-4) form a hyperbolic system with the characteristic celerities $dx/dt = c$. The three celerities $c_{1,2,3}$ are the roots of the cubic equation:

$$c^3 - 2uc^2 - \{ga - u^2 + gdf/du\}c + ugdf/du = 0 \qquad (3\text{-}5)$$

An analysis can for instance be found in Jansen (1979, p. 94).
Equation (3-5) can be modified using the following three dimensionless parameters.

- relative celerity $\phi = c/u$
- Froude number $Fr = u/\sqrt{ga}$ (3-6)
- Transport parameters $\psi = a^{-1} df/du$

The dimensionless form of Eq. (3-5) becomes then:

$$\phi^3 - 2\phi^2 + \{1 - Fr^{-2} - \psi Fr^{-2}\} + \psi Fr^{-2} = 0 \qquad (3\text{-}7)$$

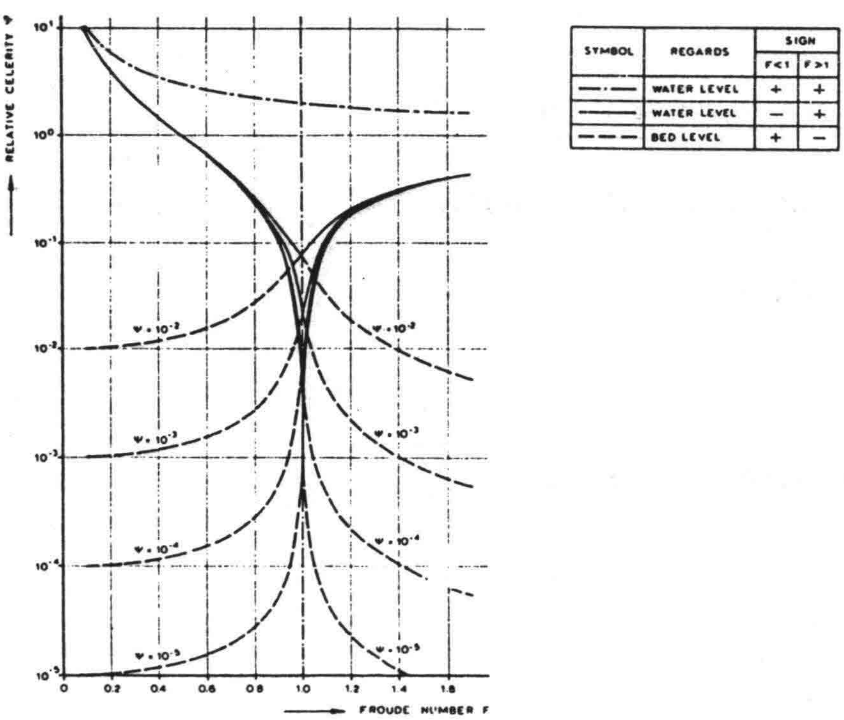


Fig. 3.2. Relative celerity of disturbance (after de Vries, 1969)

In Fig. 3.2 the three roots c_i ($i = 1,2,3$) have been represented graphically as functions of the Froude number and ψ.

Before analysing this figure it is of importance to pay some attention to the parameter ψ.
Using as an approximation $s = m\, u^n$ it follows

$$\psi = \frac{df/du}{a} = \frac{m\, n\, u^{n-1}}{a} = n\,\frac{s}{q} \tag{3-8}$$

Hence $\psi \sim s/q$, a value which is usually much smaller than unity. Note that $O(n)=O(5)$. Therefore in Fig. 3.2 only values $\psi \ll 1$ are sketched. The figure shows that two celerities $c_{1,2} = u \pm \sqrt{ga}$ or $\phi_{1,2} = 1 \pm Fr^{-1}$ are apparently for $Fr < 0.6$ not influenced by the mobility of the bed (thus by ψ).

Inserting for this case the known roots $\phi_{1,2}$ in Eq. (3-7) lead to an expression for ϕ_3. This can be done as follows. Equation (3-7) can be written as:

$$(\phi - \phi_1)(\phi - \phi_2)(\phi - \phi_3) = 0 \tag{3-9}$$

So comparing Eqs. (3-7) and (3-9) gives

$$\psi\, Fr^{-2} = -\,\phi_1 \phi_2 \phi_3 \tag{3-10}$$

or

$$\psi\, Fr^{-2} = -(1+Fr^{-1})(1-Fr^{-1})\,\phi_3 \tag{3-11}$$

thus

$$\phi_3 = \frac{\psi}{1 - Fr^2} \tag{3-12}$$

Note that in Fig. 3.2 for $Fr < 0.6$ it holds $|\phi_{1,2}| \gg \phi_3$ Hence if we are interested in changes of the bed the Eq. (3-12) can be used for ϕ_3, moreover $|\phi_{1,2}| \to \infty$ can be concluded. This implies that the flow can be assumed to be quasi steady. Thus for this case Eqs. (3-1) through (3-4) can be simplified to

$$u \frac{\partial u}{\partial x} + g \frac{\partial a}{\partial x} + g \frac{\partial z_b}{\partial x} = -g \frac{u|u|}{C^2 a} \tag{3-13}$$

$$a \frac{\partial u}{\partial x} + u \frac{\partial a}{\partial x} = 0 \quad \text{or} \quad q = q(t) \tag{3-14}$$

$$s = f\{u, \text{parameters}\} \tag{3-15}$$

$$\frac{\partial z_b}{\partial t} + \frac{\partial s}{\partial x} = 0 \tag{3-16}$$

Thus for Fr < 0.6 the system of equations can be decoupled. Equations (3-13) and (3-14) can be combined to the equation for the backwater curve

$$\frac{\partial u}{\partial x} \left[u - \frac{gq}{u^2} \right] + g \frac{\partial z_b}{\partial x} = - g \frac{u^3}{C^2 q} \tag{3-17}$$

For a given discharge q and known bed level z_b the flow velocity u can be computed for specific boundary conditions.

Moreover Eqs. (3-15) and (3-16) can be combined into:

$$\frac{\partial z_b}{\partial t} + \frac{df(u)}{du} \cdot \frac{\partial u}{\partial x} = 0 \tag{3-18}$$

Thus for known velocities u the bed level in future can be computed with Eq. (3-18) if the appropriate boundary conditions are applied.
Hence Eqs. (3-17) and (3-18) in principle can be used for the description of morphological processes in rivers.
Two additional assumptions can lead to a further simplification of Eqs. (3-17) and (3-18). This leads to two mathematical models that can be used for analysing morphological phenomena.

(i) For *small values* of x and t the *friction term* (right hand side) in Eq. (3-17) can be neglected with respect to the other terms. This gives the *simple wave model*.
(ii) For *large values* of x and t the *backwater effects* (first terms in Eq. (3-17)) can be neglected. This leads to the *parabolic model*.

ad (i) The characteristics of the *simple wave model* can be demonstrated easily when in addition the assumption $Fr \ll 1$ is made. The momentum equation (Eq.(3-13)) simplifies then into

$$\frac{\partial a}{\partial x} + \frac{\partial z_b}{\partial x} = \frac{\partial h}{\partial x} = 0 \quad \text{thus } h = \text{const.} \tag{3-19}$$

This means that the water level is horizontal. (*'rigid lid approximation'*)

As $q = u.a$ = constant, it can be written $u\,a/\partial x + a\partial u/\partial x = 0$
Combining this with Eq. (3-18) yields

$$\frac{\partial z_b}{\partial t} + \frac{df(u)}{du} \cdot \left[- \frac{u\partial a}{\partial x} \right] = 0 \tag{3-20}$$

Considering in addition Eqs. (3-8) and (3-12) for $Fr^2 \ll 1$ gives

$$\frac{\partial z_b}{\partial t} - c\frac{\partial a}{\partial x} = 0 \tag{3-21}$$

From Eq. (3-19), however, follows $\partial a/\partial t = -\partial z_b/\partial t$. Hence

$$\frac{\partial a}{\partial t} + c\frac{\partial a}{\partial x} = 0 \tag{3-22}$$

An application of this simple-wave equation is given in Sub-Section 3.2.2.

ad (ii) The *parabolic model* is obtained from Eq. (3-17) if the first term (responsible for the backwater effects) is neglected.
Differentiation with respect to x gives

$$\frac{\partial^2 z_b}{\partial x^2} + 3\frac{u^2}{C^2 q}\frac{\partial u}{\partial x} = 0 \tag{3-23}$$

Eliminating $\partial u/\partial x$ from Eqs. (3-18) and (3-23) gives

$$\frac{\partial z_b}{\partial t} - \left[\frac{df(u)}{du} \cdot \frac{1}{3} \cdot \frac{Cu^2}{q} \right] \frac{\partial^2 z_b}{\partial x^2} = 0 \tag{3-24}$$

Linearization yields

$$\frac{df(u)}{du} \cdot \frac{1}{3} \cdot \frac{Cu^2}{q} = \frac{df(u)/du}{di/du} \approx \frac{ds_0/du}{di_0/du} = K \tag{3-25}$$

The parabolic model gives therefore the following morphological equation to describe the changes of the bedlevel

$$\frac{\partial z_b}{\partial t} - K \frac{\partial^2 z_b}{\partial x^2} = 0 \tag{3-26}$$

with

$$K = \frac{ds_o/du}{di_o/du} \simeq \frac{mnu^{n-1}}{1/3u^2/C^2q} = \frac{3ns_o}{i_o} \tag{3-27}$$

in which the subscript o refers to the orginal uniform situation for which changes are considered.

Remarks:

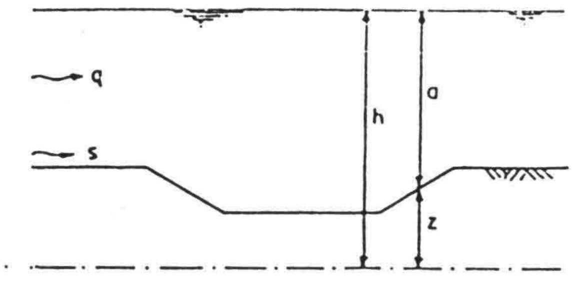
- In the above derivation only differentiation with respect to x has been applied. Therefore the derivation remains valid for $q = q(t)$ and $s = s(t)$. For a river with varying discharge $K = K(t)$. Hence the general equation reads

$$\frac{\partial z_b}{\partial t} - K(t) \frac{\partial^2 z_b}{\partial x^2} = 0 \tag{3-28}$$

- In the derivation backwater effects have been neglected. This is only valid if relatively large values of x are considered. Comparison with the complete equation leads to the conclusion that the assumption implies the condition $\Lambda > 2$ to 3 with $\Lambda = xi_o/a_o$. The 'length scale' Λ is a characteristic parameter for the river considered. Note that $\Lambda = 1$ is obtained for a river reach with a length over which the difference in piezometric head is equal to the normal depth.

An application of the use of Eq. (3-18) is given in Sub-Section 3.2.2.

3.2.2. Example: Deformation of a dredged trench



In Fig. 3.3 a trench is represented dredged across a river (t=0).
How will the trench be deformed if only bedload transport is present?
It has to be noted that relatively small values of x are concerned. Hence the *simple wave equation* can be applied.

Fig. 3.3. Dredged trench.

The variation in the depth a is so large that the celerity c cannot be considered constant.
If the variation in depth is considered it follows from Eq. (3-20) with $s = m\,u^n$

$$\frac{\partial z_b}{\partial t} - \left[m\ n\ u^n \right] a^{-1} \frac{\partial a}{\partial x} = 0 \tag{3-29}$$

Or, for a constant discharge and a horizontal water level $\partial z_b/\partial t = -\partial a/\partial t$ thus

$$\frac{\partial a}{\partial t} + \left[\frac{m\ n\ q^n}{a^{n+1}} \right] \frac{\partial a}{\partial x} = 0 \tag{3-30}$$

Thus

$$\frac{\partial a}{\partial t} + c(a) \frac{\partial a}{\partial x} = 0 \tag{3-31}$$

Now the deformation of the trench can be estimated qualitatively for $t > 0$.
Three parts can be considered:

- The *horizontal bed* will not deform as $\partial a/\partial x = 0$, hence $\partial u/\partial x = 0$ thus $\partial s/\partial x = 0$ and therefore $\partial z/\partial t = 0$.
- The *downstream slope* will become flatter because $\partial a/\partial x < 0$ thus $\partial u/\partial x > 0$ or $\partial s/\partial x > 0$ and $\partial c/\partial x > 0$. A point of the slope with depth a will move in the time Δt downstream over a distance $\Delta x = c(a)\,\Delta t$.
- The *upstream slope* will for $t > 0$ get steeper. This will continue until the slope will be under the angle of repose.

In Fig. 3.4 the situation is sketched for t = 0 for t = Δ t.

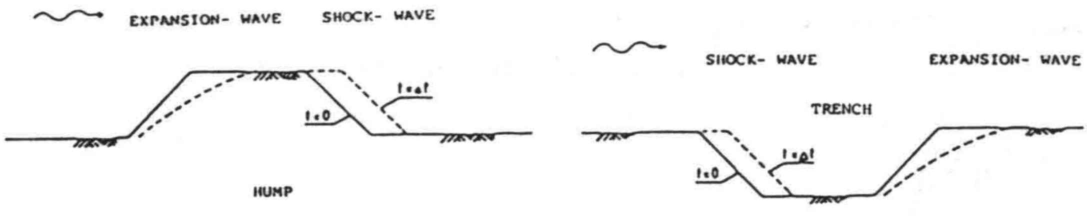


Fig. 3.4. Deformation of a hump and a trench.

In this figure also the deformation of a hump instead of a trench is given. Note the similarity and the difference. A gradual flattening of a slope is similar to what is called in gasdynamics an *expansion wave*. The opposite (the slope becomes steeper) is called a *shock wave*.

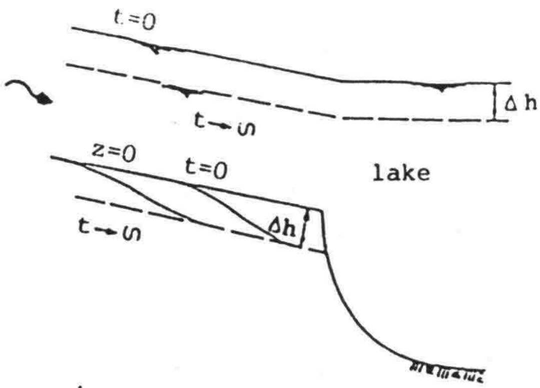
Remarks:

(i) For the above given considerations it is essential that s = f(u) holds. For small distances this implies that bedload transport is postulated. In the case of suspended load the picture can be completely different.

(ii) The information on the deformation of a hump will be used to explain the morphological phenomena occuring due to closure of one branch of a river flowing around an island (Section 4.6).

3.2.3. Example: Morphological time-scale for rivers

The parabolic model derived in Sub-Section 3.2.1 can be used to obtain some information on the speed at which morphological processes in rivers take place. A *morphological time-scale* can be defined (de Vries, 1975).



It is assumed that the river considered is discharging into a lake. At t=0 the water level of this hypothetical lake is lowered over a distance h. This leads to a degradation of the riverbed which ends at $t \to \infty$. When also the bed level is lowered over a distance Δh.

For the mathematical solution of the problem reference is made to the original publication (de Vries 1975). See also Janssen (1979, p. 123).

Fig. 3.5. Definition sketch.

If the x-axis is taken along the original bed level upstream from the mouth (x = 0) then the bed level variation $z_b(x,t)$ is described by

$$z_b(x,t) = -\Delta h \text{ erfc} \left[\frac{x}{2\sqrt{Kt}} \right] \qquad (3\text{-}32)$$

in which the '*complementary error-function*' is described by

$$\text{erfc } y = \frac{2}{\sqrt{\pi}} \int_y^{\infty} \exp\{-u^2\}.\, du \qquad (3\text{-}33)$$

y	-1.0	-0.5	-0.2	-0.1	0	0.1	0.2	0.5	1.0	2.0
erfc y	1.84	1.52	1.22	1.11	1.00	0.89	0.78	0.48	0.16	0.005

Table 3.1 Complementary error-function.

In the first place it will be assumed that the discharge is constant. This facilities understanding. The solution of Eq. (3-32) can be used to answer the following question:

If a station $x = L_m$ is selected, at what time $t = T_m$ will the degradation have reached 50% of the final value?

From Eq. (3-32) is follows with Table 3.1

$$\text{erfc}\left[\frac{L_m}{2\sqrt{KT_m}}\right] = \tfrac{1}{2} \quad \text{or} \quad L_m \simeq \sqrt{KT_m} \qquad (3\text{-}34)$$

Hence the parameter K plays a key role in the description of this morphological process.

In practice the river discharge will vary in time. It can be shown (de Vries, 1975) that for K = K (t) the solution of the problem is given by

$$z(x,T) = -\,\Delta h \;\text{erfc}\,\frac{x}{2\sqrt{\int^{T} K(t)\,dt}} \qquad (3\text{-}35)$$

Hence the 50% degradation is reached at $t = T_m$ for $x = L_m$ if

$$L_m = \sqrt{\int^{T_m} K(t)\,dt} \qquad (3\text{-}36)$$

Using Eq. (3-27) one may define the parameter Y with

$$Y = \int_0^{1\ \text{year}} K(t)\,dt = \frac{1}{3}\,\frac{n}{Bi} \int_0^{1\ \text{year}} S(t)\,dt \qquad (3\text{-}37)$$

The integral of Eq. (3-37) denotes the average yearly transport of the river.

Hence at $x = L_m$ the 50% degradation is reached after N_m years with

$$N_m = \frac{L_m^2}{Y} \qquad (3\text{-}38)$$

For a number of rivers Table 3.2 gives the value of N_m. For L_m the large value L_m = 200 km has been chosen to fullfil the requirements for the parabolic model $\Lambda > 2$ to 3.

RIVER	STATION (approx. distance to sea)	D mm	i $*10^{-4}$	3a/i km	N_m centuries
Rhine (Netherlands)	Zaltbommel (100 km)	2	1.2	100	20
Magdalena (Colombia	Puerto Berrío (730 km)	0.33	5	30	2
Danube (Hungary)	Dunaremete (1826 km)	2	3.5	40	10
	Nagymaros (1695 km)	0.35	0.8	180	1.5
	Dunaujvaros (1581 km)	0.35	0.8	180	1.5
	Baja (1480 km)	0.26	0.7	210	0.6
Tana (Kenya)	Bura (230 km)	0.32	3.5	50	2
Apure (Venezuela)	San Fernando	0.35	0.7	200	4.4
Mekong (Thailand)	Pa Mong	0.32	1.1	270	1.3
Serang (Indonesia)	Godong	0.25	2.5	50	2.0
Rufiji (Tanzania)	Stiegler's Gorge	0.4	3.2	20	4.0

Table 3.2. Morphological time-scale (after de Vries, 1975) for L_m = 200 km.

3.2.4. Comparing equilibrium situations

In this Sub-Section for a number of standard problems the steady state solutions due to morphological changes will be discussed. An equilibrium situation in a river is changed into a new equilibrium. This means that the basic equations are considered for $\partial z_b/\partial t = 0$. In the most simple cases with a constant width it means that also $\partial s/\partial x = 0$ and $\partial a/\partial x = 0$.

Hence the analysis can be carried out with

$$Q = C\ B\ a^{3/2}\ i_b^{\frac{1}{2}} \tag{3-39}$$

and

$$S = B\ m\ u^n \tag{3-40}$$

In this elementary analysis m and n are supposed to be constant. The same holds for C or if the Strickler formula with $C = 25(a/k_N)^{1/6}$ is used the value of k_N is supposed to be constant.

The examples given below are essential for the understanding of the time-depending morphological predictions treated in Chapter 4.

Case I: Withdrawal of water from a river

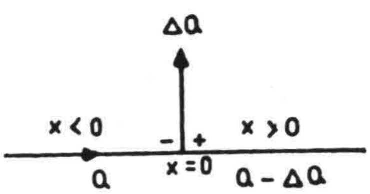


Fig. 3.6. Withdrawing water

From a river with a constant width (B) a part ΔQ of the constant discharge Q is withdrawn. What changes take place eventually (i.e. for $t \to \infty$)?

At the intake (Fig. 3.6) the width is continuous. Hence s has to be same at the upstream side (minus sign) and the downstream side (plus sign). Thus $s_- = s_+$.

Apparently

$$m\ u_-^{\ n} = m\ u_+^{\ n} \tag{3-41}$$

or

$$\frac{q}{a_-} = \frac{q - \Delta q}{a_+} \tag{3-42}$$

or with $a_+ = a_- - \Delta a$

$$\frac{\Delta a}{a_-} = \frac{\Delta q}{q} = \frac{\Delta Q}{Q} \tag{3-43}$$

The sudden change Δa at the intake has to be reflected by a step Δz_b in the bed: $\Delta z_b = \Delta a$ because the water level is continuous ($h_- = h_+$).

In this case the smaller discharge can only carry the original transport if the downstream bedslope increases eventually. With the assumption $C_- = C_+$ it follows

$$q - \Delta q = C\{a - \Delta a\}^{3/2}\{i + \Delta i^{\frac{1}{2}}\} \tag{3-44}$$

in which i is the original bedslope. Equations (3-39) and (3-44) give for the relative change of the slope

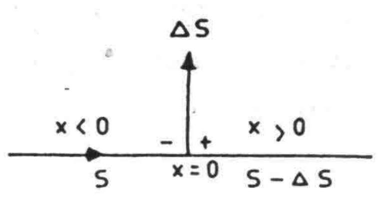
$$\frac{\Delta i}{i} = \frac{\Delta Q/Q}{1 - \Delta Q/Q} \simeq \frac{\Delta Q}{Q} \quad \text{if } \Delta Q << Q \tag{3-45}$$

If not the assumption $C_- = C_+$ but $k_{N-} = k_{N+}$ is made then the result is

$$\frac{\Delta i}{i} = \frac{1}{\left[1 - (\Delta Q/Q)\right]^{4/3}} - 1 \tag{3-46}$$

The proof is left to the reader.

Case II: Withdrawing sediment from a river



From a river with a constant width B and a constant discharge Q, sediment is withdrawn at a constant rate ΔS from t = 0. The sediment is used for building purposes.

Fig. 3.7. Withdrawing sediment

The new equilibrium situation ($t \rightarrow \infty$) can be estimated as follows. Using again $s = m\,u^n$ it follows

$$Q = B.a.u = B.a.\{s/m\}^{n^{-1}} \qquad (3\text{-}47)$$

Hence (see Fig. 3.7):

$$a_{-}.s^{n^{-1}} = (a + \Delta a)(s - \Delta s)^{n^{-1}} \qquad (3\text{-}48)$$

if Δa is the increase of the depth for $x > 0$.

From Eq. (3-48) follows with $a = a_{-}$

$$\frac{\Delta a}{a} = \left[\frac{1}{1-(\Delta S/S)}\right]^{n^{-1}} - 1 \qquad (3\text{-}49)$$

In the downstream reach the slope has to decrease. This can be estimated by means of Chézy formula and the assumption $C_{-} = C_{+}$.

From $Q_{-} = Q_{+}$ and $B_{-} = B_{+}$ follows

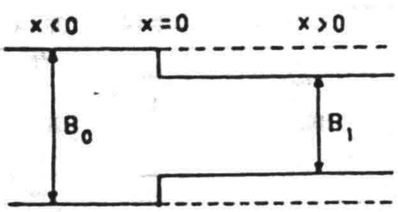
$$a_{-}^{3/2}\, i_{-}^{1/2} = (a_{-} + \Delta a)^{3/2}\, (i_{-} - \Delta i)^{1/2} \qquad (3\text{-}50)$$

thus

$$\frac{\Delta i}{i} = 1 - \left[1 - (\Delta S/S)\right]^{3/n} \qquad (3\text{-}51)$$

The exponent $3/n$ becomes $10/3n$ if $k_{N-} = k_{N+}$ is assumed instead of $C_{-} = C_{+}$

<u>Case III: Change of width</u>



A river with fixed banks and constant width B_o is narrowed for $x > 0$ to a new width B_1. The discharge Q is constant. Instead of the old depth (a_o) for $x > 0$ and $t \rightarrow \infty$ the new depth becomes a_1.

Fig. 3.8. Change of width

From $S_o = S_1$ and $Q_o = Q_1$ follows with $s = m\,u^n$.

$$\frac{a_1}{a_o} = \left[\frac{B_o}{B_1}\right]^{\frac{n-1}{n}} \qquad (3\text{-}52)$$

For $x = 0$ there is a bottom step because the water level is continuous. The bottom step is $\Delta z_b = a_1 - a_o$.

In this case the new slope (i_1) is smaller than the old slope (i_0). With the assumption $C_o = C_1$ it follows easily from the Chézy equation

$$\frac{i_1}{i_o} = \left[\frac{B_1}{B_o}\right]^{\frac{n-3}{n}} \tag{3-53}$$

If one assumes $k_{No} = k_{N1}$ instead of $C_o = C_1$ then the exponent in Eq. (3-53) becomes: $(4n - 10)/3n$.

Remarks:

(i) For a varying discharge and a width that does not vary too much the above given analysis can also be given, using f{Q} . This is the analysis given in Section 2.5 resulting in Eq. (2-8). The estimate for the new slope according to Eq. (3-53) therefore also holds for a varying discharge. This equation is graphically represented in Fig. 3.9.

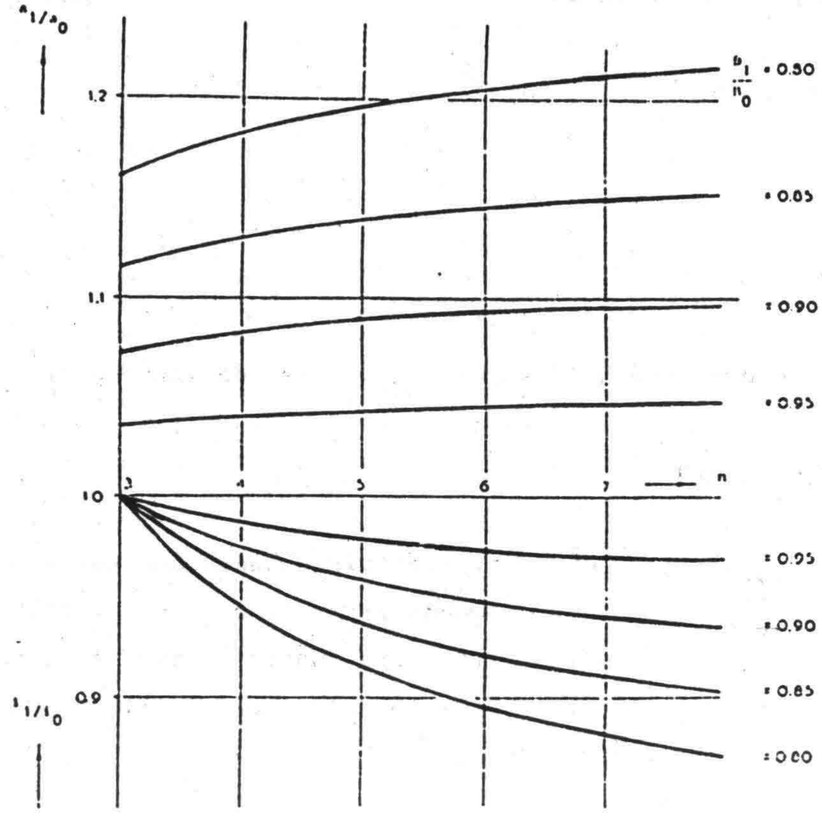


Fig. 3.9. Consequences of constriction of river width.

(ii) According to Eq. (3-53) as also can be seen from Fig. 3.9 the new slope is equal to the old slope if n = 3. According to the Meyer-Peter and Mueller (1948) formula this is the case for a large transport of coarse bed material. It should be remarked that the above given analysis is an elementary one. In a practical problem a more in-depth-approach is advised taking into account the change in roughness that may occur.

(iii) In the three cases described above a discontinuity in Q, S and B respectively lead at x = o to a discontinuity in the depth. As the water level is continuous (no hydraulic jump) this will result in a step Δz_b in the bed. The bed *level* has then not yet been obtained. This is because the bed level is governed by the water level (boundary condition) at the downstream end of the channel. The bottom*level* Δz_b is reached only for $t \rightarrow \infty$.

3.2.5. Influence of suspended load transport

So far the morphological processes have been described by using a transport formula s = f(u), thus assuming that the sediment transport can be described by the *local* hydraulic conditions. This is true when *bedload transport* is present. In the case of *suspended load transport*, however, it is only true for steady uniform flow. This also the situation for which transport formulae are developed and tested experimentally.

The first question is how transport can be estimated for *non-uniform flow*. It is customary and justified to introduce the friction term in the differential equation for the backwater curve (equation of Bélanger) as if the flow were uniform. Similarly this can be done for bedload. The local shear stress can by means of the Chézy formula be transformed into the flow velocity. Hence $ai = u^2/C^2$.

For non-uniform flow with suspended load this cannot be done without restrictions. This is related to question what *local* means in this respect. For non-uniform flow and suspended load a distinction has to be made between *transport* and *transport capacity*:

transport (s') = $\int^{a} u \phi \, dz$

transport capacity s = f(u) i.e. following from a formula

in which ϕ denotes the sediment concentration.
Generally in the case of suspended load

- $\partial u/\partial x > 0$ gives $s' < s$
- $\partial u/\partial x < 0$ gives $s' > s$

The explanation can be found from the following rather extreme example. Suppose a steady uniform flow in a laboratory flume with a fixed bed for $x \leqslant 0$ and a mobile bed of fine sediment for $x > 0$. The mean concentration over the vertical is $\overline{\phi}$. For large values of x the average equilibrium concentration $\overline{\phi}_c$ is reached. It requires a certain *adaption length* to reach the equilibrium concentration over the entire depth. This equilibrium concentration belongs to the flow conditions present and the sediment characteristics.

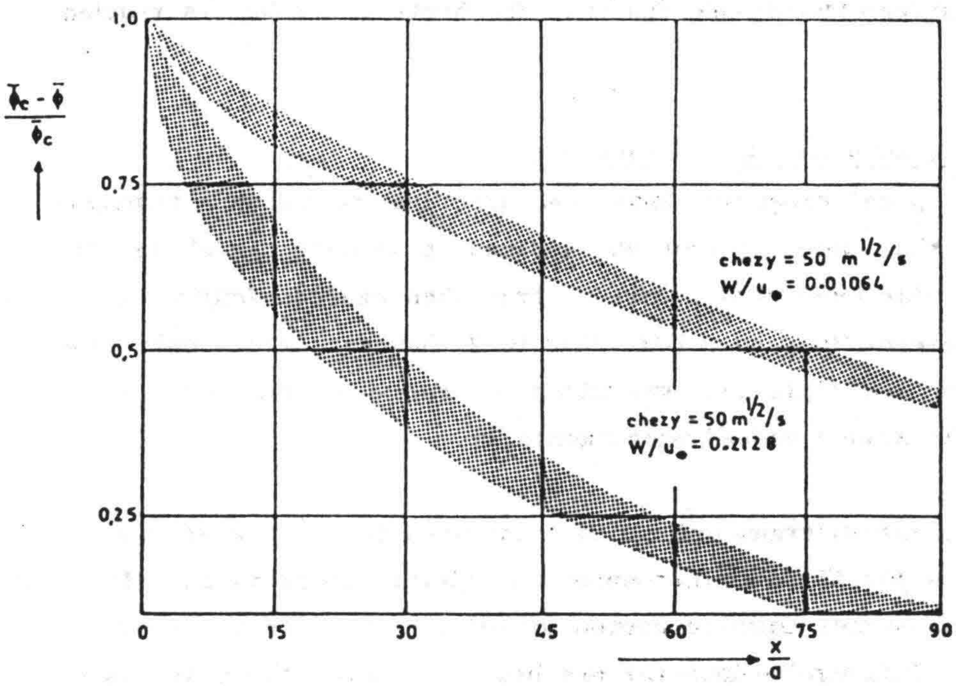


Fig. 3.10. Adaptation concentration vertical (after Galappatti, 1983)

In Fig. 3.10 the theoretical adaptation of the concentrations determined by Galappatti (1983) is given. It has been assumed that at a distance z = 0.0125 a the concentration is instantaneously equal to the equilibrium concentration. (here for $x > 0$).

It appears that the *adaptation length* depends on the parameter W/u_* and the roughness (C-value). In this example no sediment supply at x=0 is present. Hence erosion will follow for $x > 0$. In Fig. 3.10 the situation is considered for which no erosion has yet taken place (thus small value of t).

In practice morphological computations are carried out with numerical models with discrete values Δx and Δt. If L is the adaptation length than two cases can be distinguished.

- $\Delta x > L$. In this case the one-dimensional morphological model can be used for suspended load as for bed load.
- $\Delta x < L$. Now in fact a two dimensional (vertical) model has to be used. The concentration $\phi(x,z,t)$ has to be calculated and the transport $s' = \int u \phi \, dz$ has to be determined before bed-level variations by means of the equation of continuity for the sediment can be computed.

Between these two cases Galappatti (1983) has developed an asymptotic approach which extends the region for which a one-dimensional approach can be used. (See Sub-Section 3.3).

3.3. Two-dimensional approaches

3.3.1. Flow in river bends

To understand the bed level in river bends some attention has to be paid to the details of the flow in bends of open channels.

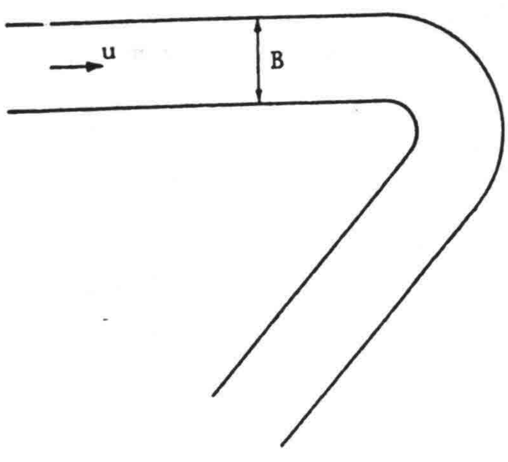


Fig. 3.11. Circular bend.

In Fig. 3.11 a circular bend in a laboratory flume is sketched. The up-stream and downstream parts of the flume are straight. The (fixed) bed is horizontal.

In the first approach friction is neglected. This implies that potential flow can be postulated. In this problem a natural coordinate system is appropriate. Here s is the coordinate along the streamline and n the direction normal to the flow line (thus s-n is the horizontal plane). The b-axis is perpendicular to the s-n plane. The components of the velocity vector are u_s, u_n and u_b respectively. According to the definitions of s,n and b it holds $u_s \neq 0$; $u_n = 0$ and $u_b = 0$. However, derivatives of u_n and u_b exist.

For steady flow the momentum equations in the natural coordinate system read:

$$u_s \frac{\partial u_s}{\partial s} = -\frac{1}{\rho}\frac{\partial p}{\partial s} + a'_s \tag{3-54}$$

$$\frac{u_s^2}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial n} + a'_n \tag{3-55}$$

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial b} + a'_b \tag{3-56}$$

in which p is the pressure. The right-hand sides contain the components of the acceleration $\vec{a}'$ that is present in addition (e.g. due to gravity and/or friction). In Eq. (3-55) the radius of curvature of the streamline is indicated by the parameter r.

If friction is neglected only g is present in $\vec{a}'$. This implies $a'_s=0$; $a'_n = 0$ and $a'_b = g$. Hence Eq. (3-56) indicates that in the (vertical) b-direction the *hydrostatic pressure distribution* is present.

If the piezometric head is measured from the bed level, then the Bernoulli equation along the streamline gives

$$\frac{u_s^2}{2g} + a = \text{constant} \tag{3-57}$$

The absence of friction means that potential flow can be postulated. This means that here is one Bernoulli-constant for the entire flow field.

In the n-direction holds

$$\frac{u_s^2}{r} = -\frac{\partial}{\partial n}\{g(a-z)\} \tag{3-58}$$

if z is the distance to the bottom.

Hence

$$\frac{u_s^2}{gr} = - \frac{\partial a}{\partial n} \tag{3-59}$$

Differentiation of Eq. (3-57) with respect to r gives

$$\frac{u_s}{g} \frac{\partial u_s}{\partial r} + \frac{\partial a}{\partial r} = 0 \tag{3-60}$$

The coordinate n is taken positive in the direction of the centre of curvature. Hence $\partial/\partial r = -\partial/\partial n$. Combining Eqs. (3-59) and (3-60) gives

$$\frac{u_s^2}{gr} = - \frac{\partial a}{\partial n} = \frac{\partial a}{\partial r} = - \frac{u_s}{g} \frac{\partial u_s}{\partial r} \tag{3-61}$$

or

$$u_s \simeq r^{-1} \tag{3-62}$$

Hence the assumption of potential flow leads to the conclusion that the largest flow velocities are found in the inner bend.

Now the influence of friction can be taken into consideration. In a vertical there is only one value of da/dr. As u_s is due to friction larger at the water surface than near the bed it holds

- water surface $\frac{u_s}{g} \frac{du_s}{dr} > \frac{da}{dr}$
- bed level $\frac{u_s}{g} \frac{du_s}{dr} < \frac{da}{dr}$

Hence the water particles near the water surface are slightly deviating outward whereas the water particles near the bed are slightly deviating inward ($u_s g^{-1} du_s/dr$ relatively small). Thus in the bend there exists a *helicoidal flow*. This helicoidal flow is composed of a main current in the direction of the channel axis and a circulation in the cross-section (Fig. 3.12).

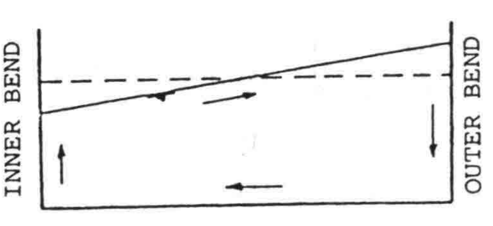


Fig. 3.12. Circulation in the cross-section

The water level in the bend has a slope perpendicular to the river axis. The value of the difference in head (ΔH) accross the river can be estimated using Eq. (3-59).
If r_o is the radius of curvature of the inner bend then

$$\Delta H = \int_{r_o}^{r_o+B} \frac{u_s^2}{gr} dr \qquad (3\text{-}63)$$

Example:
For the River Waal at a certain place r_o = 2 km and B = 260 m. The average flow velocity is $\bar{u}_s$ = 1.2 m/s. The Eq. (3-63) can be approached by

$$\Delta H \simeq \frac{\bar{u}_s^2}{g(r_o + \tfrac{1}{2}B)} \cdot B = \frac{(1.2)^2 \cdot 260}{9.8(2000 + 130)} = 0.02 \text{ m} \qquad (3\text{-}64)$$

So far it has been assumed that the bed level in the bend is horizontal. However, when the bed is mobile, this is not a stable situation. Due to the helicoidal flow near the bed the transport will have a direction to the inner bend. The means that the transport vector has a component in the direction transverse to the channel axis. The inner bend becomes shallower while the outer bend gets a greater depth. The cross slope becomes so steep that the perpendicular transport component is compensated by a transport downwards due to gravity. The change of the bed level implies that in a natural river Eq. (3-62) will not hold anymore. The velocity u_s is relatively large then in the *outer* bend.

By Van Bendegom (1947) and Rosovskii (1957) the magnitude of the secondary velocity has been studied theoretically assuming for the velocity distribution in the vertical a power law and a logarithmic law respectively. More details can be found in Jansen (1959, p. 59).

In Fig. 3.13 the two theoretical profiles are compared with measurements.

It has to be noted that these measurements are not easy to be carried out accurately. This is because the velocity vector $\vec{u}$ has the component u in the direction of the river axis and the radial component v. Generally u >> v. This means that a small error in the measurement of u leads to a large error in v.

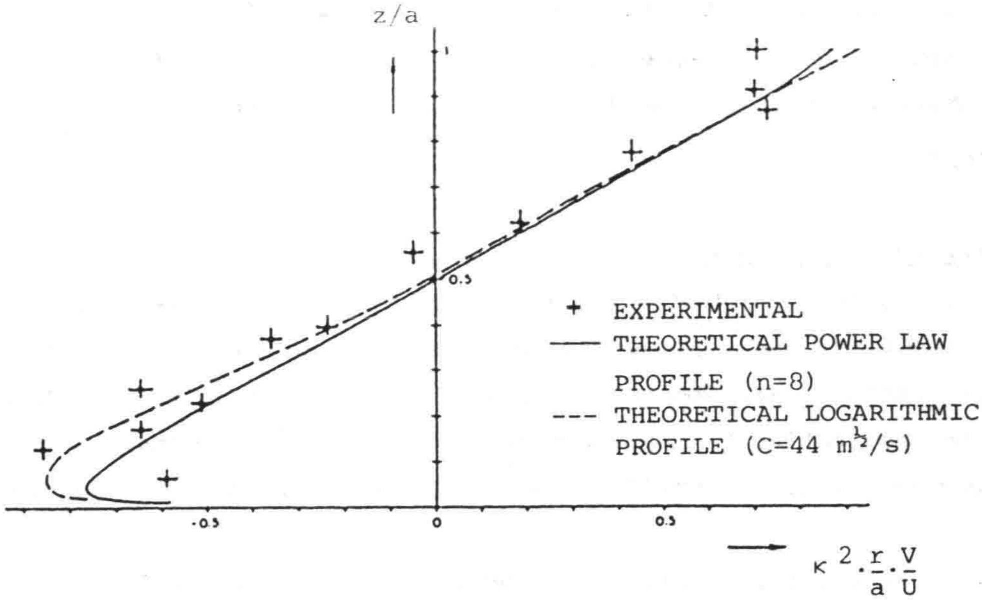


Fig. 3.13. Secondary current (v) in a riverbend: comparison of theory and measurement (after Kondrat'ev *et al*, 1959)

As $v \ll u$, the total shear stress at the bed makes only a small angle δ with the river axis (order of magnitude δ = 1 to 2°).

The flow through bends in open channels will not be discussed here in detail. For a thorough investigation on this topic reference is made to de Vriend (1981).

The studies by Van Bendegom (1947) and Rosovskii (1959) assume that locally the velocity field is adapted to the local radius of curvature. According to de Vriend and Struiksma (1983) this can only be the case if there is much friction! In general there will be phase lag. The velocity field lags behind the change in geometry (expressed e.g. in the radius of curvature). It seems that this lag is essential in explaining the bed topography in river bends.

Experimentally it has been shown (de Vries, 1961) that in the branches of the Rhine in the Netherlands phase lags do exist in the bends between some morphological parameters. If along a line parallel to the river axis the flow velocity $u(s,r)$, the depth $a(s,r)$ and the mean grain size $D_m(s,r)$ are measured, then these morphological parameters vary in the s-direction. By treating the experimental data statistically it was shown that u reacts more downstream than a on a variation in the radius of curvature and D_m reacts more downstream than u. These tendencies were also found in a scale model of a reach of the Lower Rhine. This demonstrates the complexity of the flow in a curved alluvial channel especially if the bed material is not uniform, as usually is the case in nature.

3.3.2. Bed configurations in bends

The bed configuration in river bends are of paramount importance in river engineering. In the bends the depth at the outerbend is of importance in the design of bankprotections. Also the availabe width in the bend with a certain required depth for navigation is important to know. Between the bends i.e. at the *river crossings* the available depth is of importance to navigation.

An early statistic research was carried out by Lely (1922) for the Rhine branches in the Netherlands. The research was carried out for rivers reaches with constant width (B) and fixed banks. Lely's conclusions were

- The mean depth across the river in the bends is about the same as at the crossings.
- The change in the curvature (r) of the bend leads to a change in the transverse slope (β) of the bed. The transverse slope reacts about 1½ B downstream of a change in r (phase lag).
- The magnitude of β depends on r with (metric units):

$$\beta = \frac{22}{r} \qquad (3\text{-}65)$$

Based on the work of van Bendegom (1947) in NEDECO (1959) a method is given to compute for a hypothetical river with fixed banks the depth accross the river for an infinitely long circular bend. Hence this approach does not consider a variation in the s-direction. Using the expression for the bed shear stress it was derived:

$$\frac{da}{a^2} = \frac{3}{2}\alpha\, i_x \, \frac{dr}{r\Delta D} \qquad (3\text{-}66)$$

in which

r = radius of curvature of the circular bend

D = mean grain size

i_x = longitudinal slope

and

$$\alpha = \frac{2n^2}{\kappa^2 (n+2)(n+3)} \tag{3-67}$$

The parameter n is here the exponent of the powerlaw for the vertical velocity distribution ($u(z) \approx z^{n-1}$)

The additional hypothesis

$$i_x \cdot r = i_o \cdot r_o \tag{3-68}$$

(in which the subscript o stands for the outer bend) is questionable. Equations (3-66) and (3-68) lead to

$$\frac{1}{a} - \frac{1}{a_o} = \left\{ \frac{1}{r} - \frac{1}{r_o} \right\} \frac{1.5\alpha i_o r_o}{\Delta D} \tag{3-69}$$

This equation sometimes gives good results especially as the coefficient can be used to tune the equation, this in fact also holds for D.

Apmann (1972) studies the same problem. He argues that

$$\frac{da}{dr} = m \frac{a}{r} \tag{3-70}$$

in which the coefficient m depends on the *flow parameter* $\Delta D/ai$. The maximum depth a_{max} in the outer bend follows then from

$$\frac{a_{max}}{a} = \frac{(m+1)(1-r_i/r_o)}{1 - \{r_i/r_o\}^{m+1}} \tag{3-71}$$

From measurements of the Buffalo Creek a value m = 2.5 is deduced by Apmann. For the Rhine branches in the Netherlands m $\approx$ 7 is found.

Odgaard (1981) concludes that for this axial-symmetrical approach the expression

$$\left[\frac{da/dr}{a/r} \right]_{axis} = K \tag{3-72}$$

is used by various authors albeit that the expression for K differs. According to Odgaard the expression $K = 2F_D^2$ is used by van Bendegom (1947) with $F_D = \bar{u}/\sqrt{g\Delta D}$. Odgaard uses laboratory measurements from Zimmerman & Kennedy (1978) and his own data from the Sacramento River. He concludes that his expression for K has to be preferred above the ones used by others. Odgaard does not discuss the work of Apmann (1972).

The above indicated approaches all considered axial-symmetrical flow and therefore neglect phase lags between u and a. Moreover uniform bed material is postulated (no grain sorting in the bends).

A completely different approach to predict the bed topography is river bends is indicated by Einstein. This is a statistical analysis and synthesis based on the following assumptions. The basic idea is that for a river reach with (nearly) constant width

- The regime Q(t) is the same.
- The composition of the bed material is the same.
- The influences of the banks can be neglected if B/a >> 1.

For each cross-section therefore only the *geometry* in the horizontal plan is different. The geometry is expressed by the radius of curvature only.

For the river considered part of the reach is used to deduce the statistical parameters and they are used to predict the bed morphology downstream. Therefore measured bed level $z_b(y)$ are matched with a linear series of (orthogonal) legendre polynomials $P_r(y)$. Here y is measured perpendicular to the river axis.

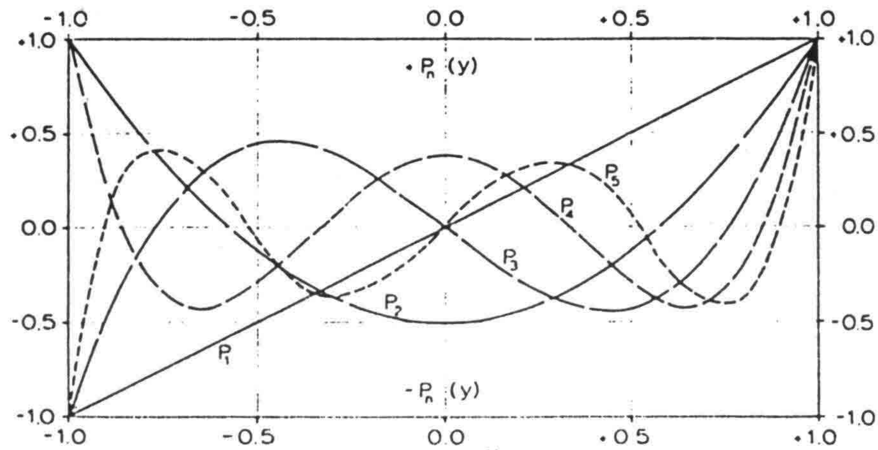


Fig. 3.14. Legendre polynomials

The functions $P_r(y)$ are given in Fig. 3.14.

$$z_b(y) = a_o P_o(y) + a_1 P_1(y) + \ldots\ldots + a_r P_r(y) + \ldots\ldots + a_N P_N(y) \qquad (3\text{-}73)$$

The parameters a_r for any cross-section n are related to the curvature C of the cross-section p upstream of n according to

$$a_{r,n} = A_{o,r} + \sum_{p=1}^{p_o} A_{p,r} \cdot C_{n-p+1} \qquad (3\text{-}74)$$

The parameters a_r belong to a certain cross-section whereas the parameters A belong to the entire river.

The procedure is now as follows:

Analysis:

(i) Use the measured bed levels $z_b(y)$ to determine the coefficient a_r from Eq. (3-73).

(ii) For the river with known geometry (C) determine the parameter A from Eq. (3-74).

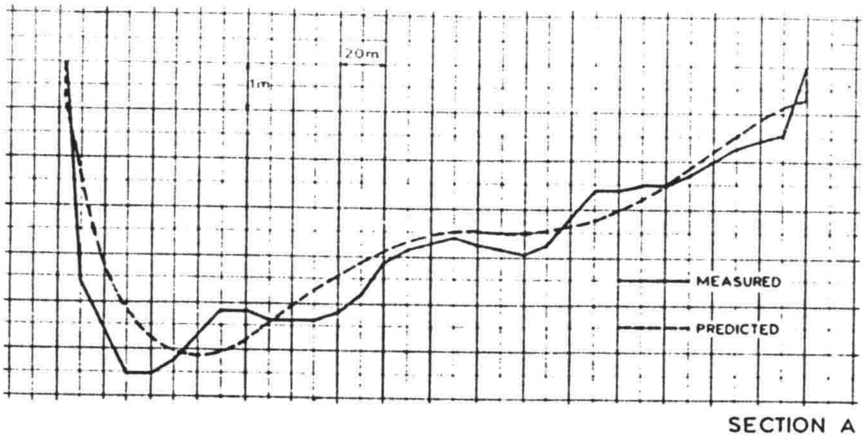
Synthesis:

(i) In Eq. (3-74) the parameters A are now known. For another part of the river C is known thus $a_{r,n}$ can be computed from Eq. (3-74).

(ii) With known parameters $a_{r,n}$ in Eq. (3-73) the bed level $z_b(y)$ can be computed.

It remains to be seen how many (N) polynomials have to be used. Moreover the number (p_o) of cross-sections upstream of the cross-sections considered has to be selected.
Einstein (1971) used in his application to the Missouri River N = 6 and he took p_o = 15 which means a distance of about 2 km.

Nijdam (1973) applied the method to the Waal River. He also took N = 6 but selected p_o = 30 which means that the geometry upto a distance of 3¾ km upstream of the cross-section considered is responsible for the bed level in that cross-section.



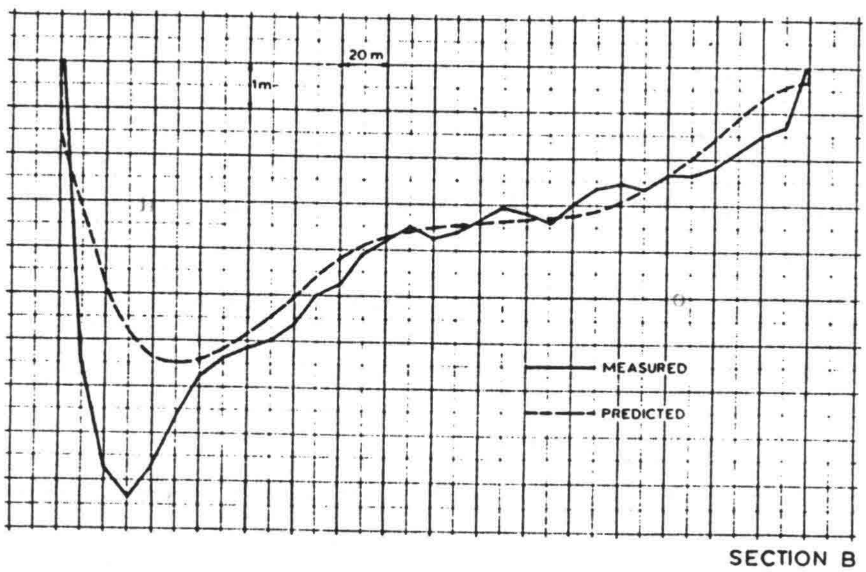


Fig. 3.15. Statistical prediction of bed levels (Waal River after Nijdam, 1973)

Some results are given in Fig. 3.16. It has to be remarked that the result is sensitive to the selection of p_o. Moreover Nijdam reports that the results are less good at the crossings. With respect to Fig. 3.16 it has to be remarked that the disagreement for section B between measured and predicted bed level at the left hand bank seems to be due to the fact that locally a groin is present. Hence the assumption of absence of wall-influences is there not justified.

The statistical method treated have has another essential underlying assumption *viz* the presence of an *alluvial* bed. If part of the bed is not erodible (rock, clay-layers, armoured areas) then the prediction fails. Obviously the method also fails in the case for the Meuse River near Roermond in the Netherlands. The bed consists there of gravel that only is transported at large discharges. At moderate discharges some sand is transported over the gravel bed. The bed profile of the bends in this river differs from an alluvial river (Fig. 3.16). There is no sand deposition in the outer bend. Via the crossing the sand is transported to the inner bend. There sand deposits are present. In this river morphological predictions are difficult, because transport formulae do not apply. Hence the transport of sand in a reach is governed by the *supply* at the upstream end. This is a similar situation as can be present in an alluvial river with suspended load. For this part of the Meuse River the sediment transport is *smaller* than the sediment transport capacity. The gravel layers prevent erosion.

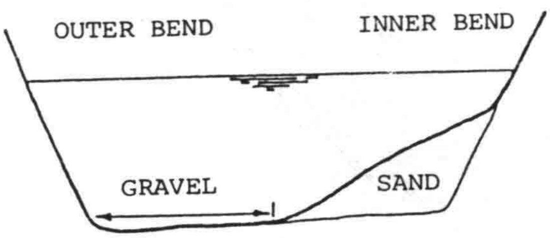


Fig. 3.16. Cross-section Meuse River near Roermond

Gradually it becomes possible to compute bed levels in alluvial bends based on the hydrodynamic equations. In the two-dimensional (horizontal) water equations the effect of the helicoidal flow has to be incorporated. The sediment equations have also to be taken for the two-dimensional case. Moreover some formula has to be adopted for the direction of the transport.

In Fig. 3.17 some results are given of the two-dimensional model SEDIBO being developed by the Delft Hydraulics Laboratory (Schilperoort *et al*, 1984). The results look promissing in spite of the fact that the presence of uniform sediment was assumed. The computations have been carried out for a constant discharge.

The development of these types of morphological models is in the direction of also including the case of a sediment mixture and/or the presence of suspended load.

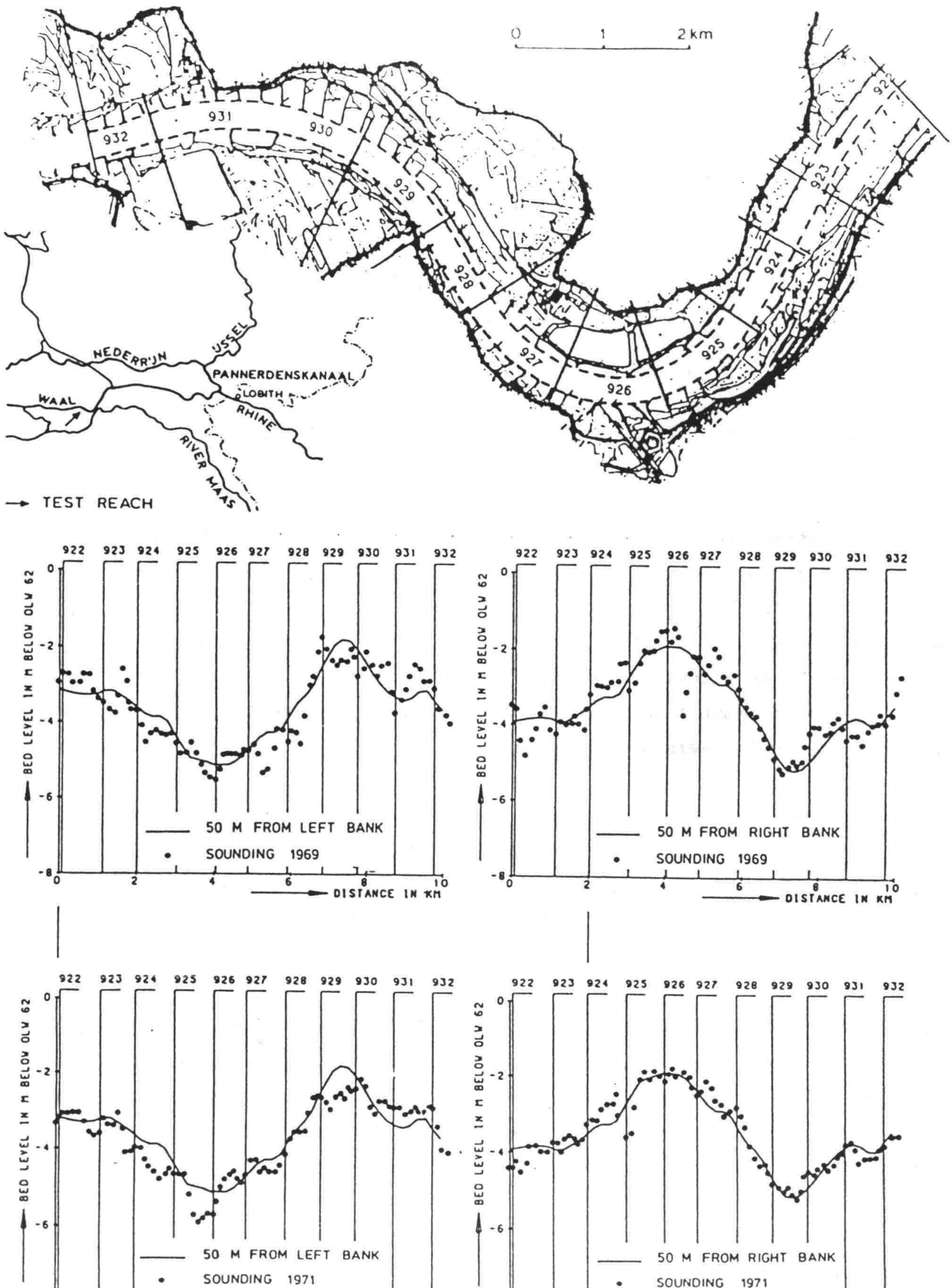


Fig. 3.17. Prediction of bed level in the Waal River with the SEDIBO-model (after Schilperoort *et al*, 1984)

3.3.3 Two-dimensional vertical

In Sub-Section 3.2.5 it has already been indicated that in the case of suspended load changes in the geometry over relatively short distances may cause that the local sediment transport is not equal to the local sediment transport capacity.

The mathematical models with one space dimension can then not be used. In stead a two-dimensional model (vertical) is applied to determine the sediment concentration $\phi(x,z)$. In that case no sediment transport formula is applied but the transport is determined by the integral $\int \phi(z)\, u(z)\, dz$ over the depth.

For a first introduction in this type of models reference can be made to Kerssens *et al*(1979).
The concentration (x,z) is determined by solving the two-dimensional convection-diffusion equation

$$u \frac{\partial \phi}{\partial x} - \frac{\partial}{\partial z} \{ W\phi + \varepsilon \frac{\partial \phi}{\partial z} \} = 0 \qquad (3\text{-}75)$$

in which

W = fall velocity
$\varepsilon = \varepsilon(z)$ = eddy viscosity

Note that if the flow is steady uniform for both water and sediment, than the first term of Eq. (3-75) disappears. Equation (3-75) then leads to the well-known Rouse-distribution for uniform flow. The solution of Eq. (3-75) requires a boundary condition at the bed. The discussion of these type of models is outside the scope of these lecture notes.

4. Morphological predictions

4.1. General

Artificial interference in a river system by engineering works (discharge regulation, water level regulation, normalization, canalization etc.) will lead to a response of the river. It is required to predict the response to the artificial interference. However, only part of the response can be predicted.

For instance if the discharge of a river is controled by installing a large dam (like in the River Nile and the Zambezi River) it can be expected that major changes can take place downstream in the characteristics of the river. To the writer's knowledge no prediction techniques have been successful in the respect. At best the *regime theory* can be used to predict some tendencies. This 'theory' (see Graf, 1971, p. 243-272) has little to do with a theory in the usual sense. River characteristics are related statistically. However, to use this statistical relations to predict changes when the boundary conditions (e.g. by discharge control) are altered is questionable. No rivers have been analysed thoroughly in this respect. This also due to the fact that changing of the appearance of a river takes time and the regime 'theory' does not contain time as a parameter.

The best predictions of changes can be carried out when the morphological problems involved can be described in a deterministic way. This is why in this Chapter the predictions are restriced to cases in which the banks are fixed or when the mobility of the banks is much smaller than that of the (alluvial) bed.

The morphological predictions can in principle be obtained by scale models and mathematical models. However, some problems are too complicated to give a reliable prediction.

As an example the morphological problems present at the access to the ports of Brazzaville (Congo) can be mentioned. Figure 4.1 gives the situation of the 'Port à Grumes' (Timberport) along the Congolese branch of the Congo River.

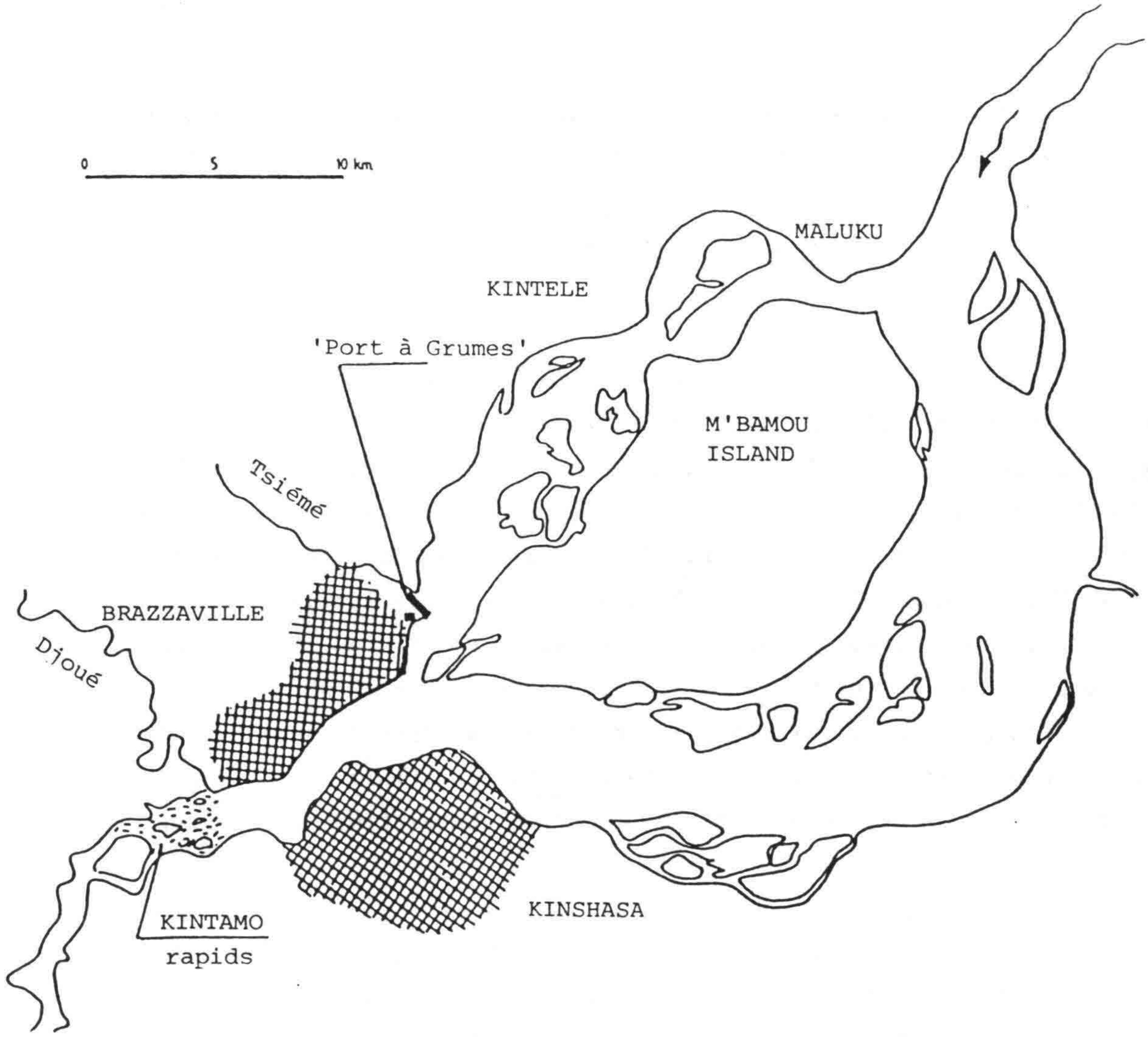


Fig. 4.1. Situation 'Port à Grumes', Brazzaville

The main branch of the river flows along the other side of the M'bamou Island. (Zairese branch). Just downstream of Brazzaville and Kinshasa the Kintamo rapids are situated. This is why at the port the logs transported along the river are loaded on trains to be transported to the coast (Pointe Noire).
The Congolese branch is characterized by islands and bars. Although the sediment transport of the Congo River is relatively small (see Table 1.1) nevertheless serious morphological problems can occur ($\overline{D} \approx \frac{1}{4}$ mm).

The 'Port à Grumes' has been designed in the sixties when the deep channel of the Congolese branch had a favourite position with respect to the river bank near Brazzaville. In 1972 the port was completed.
In 1977 the access to the port became difficult. In 1982 the deep channel of the Congolese branch was located at a large distance from the river bank. (Fig. 4.2).

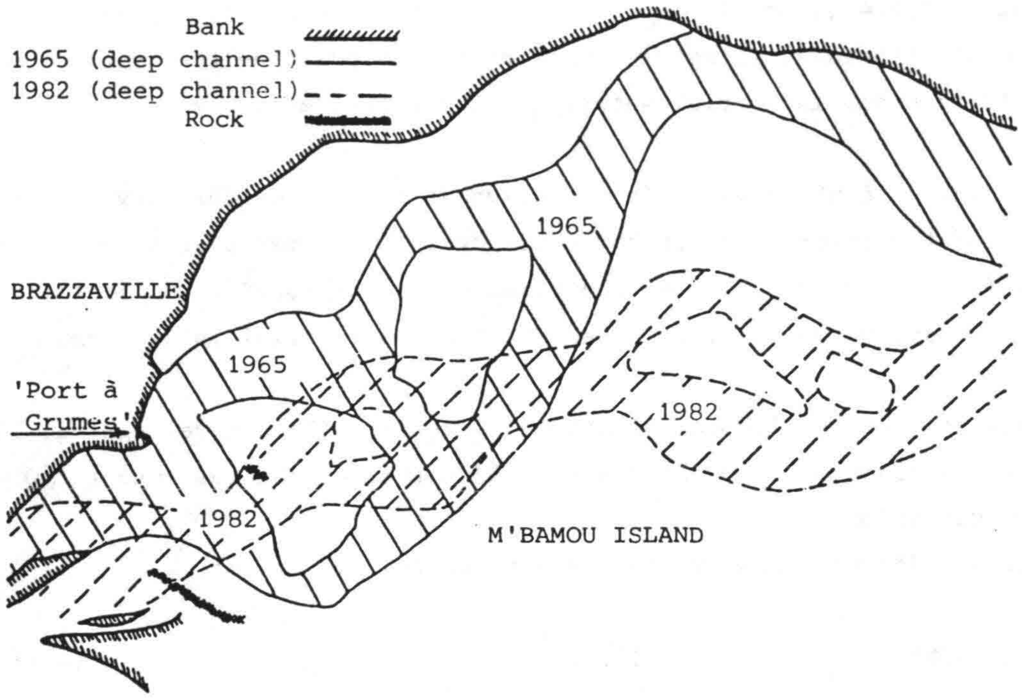


Fig. 4.2. Channel changes of the Congo River near Brazzaville

The prediction of these changes in the position of the channel(s) by means of a scale model is not possible because a large area has to be reproduced. At present (1985) also a mathematical model is not available for quantitative forecasts. This is because the flow pattern has a strong three-dimensional character governing the distribution of sediment upstream of the bars and the islands.

Given the size of the problem it is questionable whether river engineering works (perhaps except maintenance dredging) can improve the situation. Logically the problems are most serious for navigation in dry years (see also Fig. 1.2).

In the following sections the possible morphological prediction for some cases are treated. It will become clear that the river has to be schematized considerably to make morphological computations possible. The predictions are only carried out for problems that can be schematized in one space dimension.

4.2. Withdrawal of water

4.2.1. Principle

In Sub-Section 3.2.4 (Case I) the basic principle is considered on treating the the problem of withdrawal of water from a river. In practice the water is withdrawn for irrigation, industrial water, cooling water, etc.

The case is taken in which water is withdrawn at $x = L$ from the varying discharge $Q(t)$ of the river. The discharge withdrawn (Q) may also vary in time. Downstream (x L) the reaction of the river has to be considered. In this case the banks are supposed to be fixed and the bed material is uniform.

First of all the new equilibrium situation (Subscript 1) is investigated. The probability distribution $f_o\{Q\}$ of the the old situation changes into $f_1\{Q\}$ for the new situation.
As the yearly sediment transport is the same in the two cases:

$$\int_0^\infty S(Q) \,.\, f_o\{Q\}dQ = \int_0^\infty S(Q) \,.\, f_1\{Q\}\,dQ \tag{4-1}$$

Using as an approximation a powerlaw for the sediment transport, i.e. using Eq. (2-6) for B = constant gives

$$\frac{i_{b1}}{i_{b0}} = \left[\frac{\int_0^\infty Q^{n/3} f_0\{Q\}dQ}{\int_0^\infty Q^{n/3} f_1\{Q\}dQ}\right]^{3/n} \tag{4-2}$$

As less water has to transport the same amount of sediment, i_b i_{bo}.
At the mouth for the same reason the depth will decrease.
For a constant width B the ratio for the depths in the mouth (a_o / a_{oo}) can be deduced from Eq. (2-10)

$$\frac{a_{01}}{a_{00}} = \left[\frac{\int_0^\infty Q^n f_1 Q\{dQ\}}{\int_0^\infty Q^n f_0 Q\{dQ\}}\right]^{1/n} \tag{4-3}$$

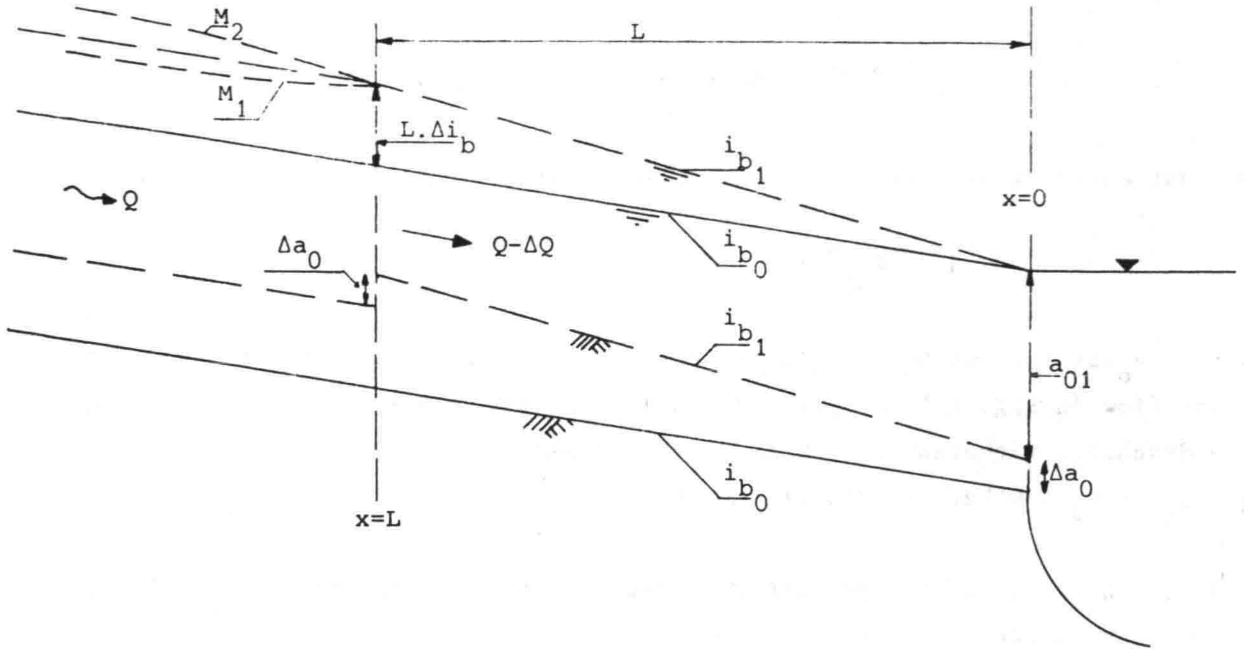


Fig. 4.3. Withdrawal of water at x = L

In Fig. 4.3 the changes of the bed are drawn for a constant discharge. The water level at the intake rises with an amount $L \cdot \Delta i_b$. At the mouth the depth reduces with $\Delta a_o = a_{oo} - a_{oi}$.
Note that the same depth reduction is present at the intake!
Just downstream of the intake the bed level rises eventually with $\Delta a_o + L \cdot i_{bi}$

Remark:
It requires some additional analysis to see how Fig. 4.3 would look like for a regime with $f_o\{Q\}$ which is transformed into $f_1\{Q\}$ due to the withdrawal of water. The bed levels remain the same but the water levels have to be given further attention.

Downstream of the intake Fig. 4.3 can be interpreted as the situation for a discharge Q_{d1} for which just uniform flow exists with a normal depth just equal to a_1. This discharge is found from

$$a_0^{-n} \int_0^\infty Q^n f_1\{Q\} dQ = a_1^{-n} \cdot Q_{d1}^n \qquad (4\text{-}4)$$

Upstream of the intake uniform flow with normal depth equal to a is present for a discharge Q_{do} with

$$a_o^{-n} \int_0^\infty Q^n f_0\{Q\} dQ = a_o^{-n} Q_{do}^n \qquad (4-5)$$

The continuity of *sediment* in the new equilibrium situation requires

$$a_1^{-n} Q_{d_1}^n = a_o^{-n} Q_{do}^n \qquad (4-6)$$

As $a_1 < a_o$ it follows $Q_{d_1} < Q_{do}$. The character of the backwater curve for the steady flow in Fig. 4.3 upstream of the intake depends now in the value of ΔQ (the discharge withdrawn). Upstream of the intake the flow is uniform if just $\Delta Q = Q_{do} - Q_{dl}$. Other possibilities are:

(i) $\Delta Q > Q_{do} - Q_{d_1}$. The upstream discharge is now $Q_o = Q_{d_1} + \Delta Q > Q_{do}$. There will be a backwater curve of the M_2 type.

(ii) $\Delta Q < Q_{do} - Q_{d_1}$. The upstream discharge is $Q_o = Q_{d_1} + \Delta Q < Q_{do}$. Now upstream of the intake the backwater curve will be of the M_1-type.

4.2.2. Application of fixed weir

In order to obtain a sufficiently high water level at the intake sometimes a fixed weir is installed. This is common practice on Java to withdraw irrigation water from a river. In this case the water level at the intake is discontinuous.

The bed level upstream of the weir is obviously influenced by the presence of the weir. In case all sediment passes eventually the weir ($t \to \infty$) the bed level can be estimated. This is the case if the sediment passes over the top of the weir or when it is flushed through the weir by special gates.

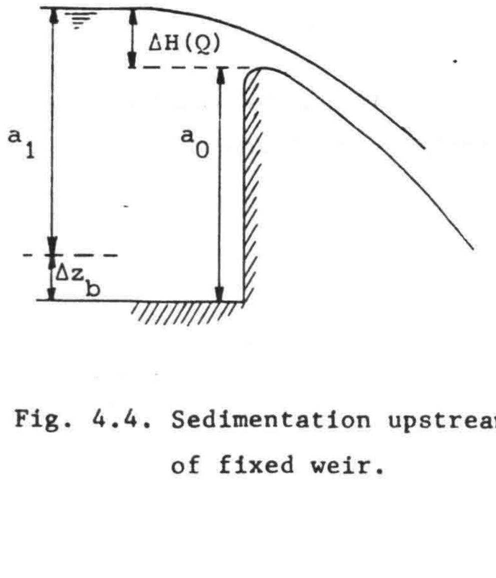


Fig. 4.4. Sedimentation upstream of fixed weir.

The yearly sediment transport in the old equilibrium situation amount to

$$V = \int_0^\infty S(Q) \cdot f_o\{Q\} \cdot dQ \qquad (4-7)$$

When this yearly amount passes the weir then the eventual average bed level rise (Δz_b) upstream of the weir can be found by expressing the transport formula as a function of the depth.

If a_o is the height of the crest of the weir above the original bed level then the depth (a_1) upstream of the weir follows from (Fig. 4.4):

$$a_1 = a_o + \Delta H - \Delta z_b \tag{4-8}$$

in which the depth of flow (Δ H) over the weir is a (known) functions of the discharge (Q). Hence with $S = B\ m\ u^n$

$$V = \int_0^\infty B\ m\ \{ \frac{Q}{B(a_o + \Delta H - \Delta z_b)} \}^n . f_o\{Q\}.dQ \tag{4-9}$$

Combination of Eqs. (4-7) and (4-9) shows that Δz_b is the only unknown. Solution is possible e.g. by means of the *regula falsi*. Note that in this case f{Q} is the same in the old and the new situation. Naturally any suitable (real) sediment transport formula can be used.

4.2.3. Example: Morphological predictions Tana River

In Jansen (1979, p.433-440) a practical example is given regarding the morphological consequences of a proposed weir in the Tana River. The water is withdrawn for irrigation purposes.

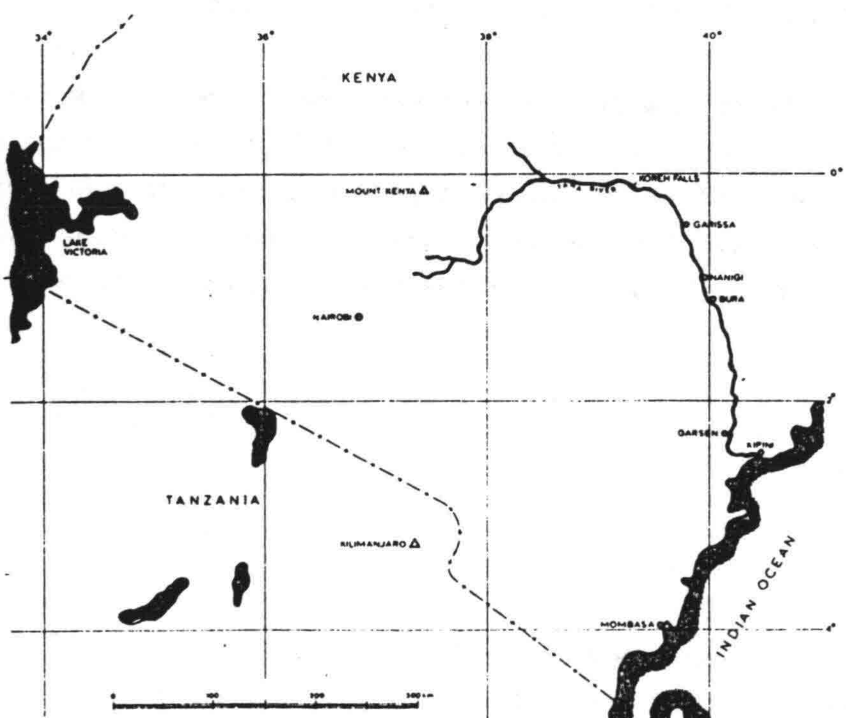


Fig. 4.5. Tana River (Kenya)

The computations were made with the Engelund-Hansen (1967) transport formula ($n = 5$) based on the availabe data ($D_{50} = 0.32$ mm; $i_o \simeq 0.35 \cdot 10^{-4}$). Using Eq. (4-9) the eventual rise of the bedlevel upstream of the weir was estimated at $\Delta z_b \simeq 2$ m for $a_o = 4$m.

Note: In Jansen (1979, p.437) the term (1 - dh/ds) in Eqs. (6.4-12), (6.4-13) and (6.4-15) has to be deleted.

As can be seen from Table 3.2 the Tana River is relatively fast. The main reason to carry out time-depending computations was the wish to get informed about $z_b(t)$ downstream of the weir. Qualitatively temporary erosion can there be expected. This is due to the fact that for small values of t a small sediment supply via the weir will be present due to the sedimentation *upstream* of the weir. The computations showed a temporary degradation of 3 to 4 m downstream of the weir to be reached after 6 months. Note that on top of this degradation *local scour* downstream of the weir may be present.

4.3. Withdrawal of sediment

In Sub-Section 3.2.4 the principle of the withdrawal of sediment from a river has been discussed with respect to the new equilibrium situation ($t \rightarrow \infty$) due to the continuous dredging of part of the sediment transported.

Now the problem will be treated in a more general sense. The problem can occur due to 'sand mining'. However, a similar problem can be present due to the subsidence of the river bed due to mining of gas, oil, coal, etc. in deep layers below the river bed. From the basic equations (see Sub-Section 3.2.1) the water equations are still valid. This holds also for the equations for the sediment transport.

The equation of continuity for the solid phase requires a change. This equation now reeds

$$\frac{\partial z_b}{\partial t} + \frac{\partial s}{\partial x} = W(x,t) \qquad (4\text{-}10)$$

The right-hand side represents a *source-term* describing the lowering of the bed due to subsidence. This term will be non-zero in a certain reach of a river.

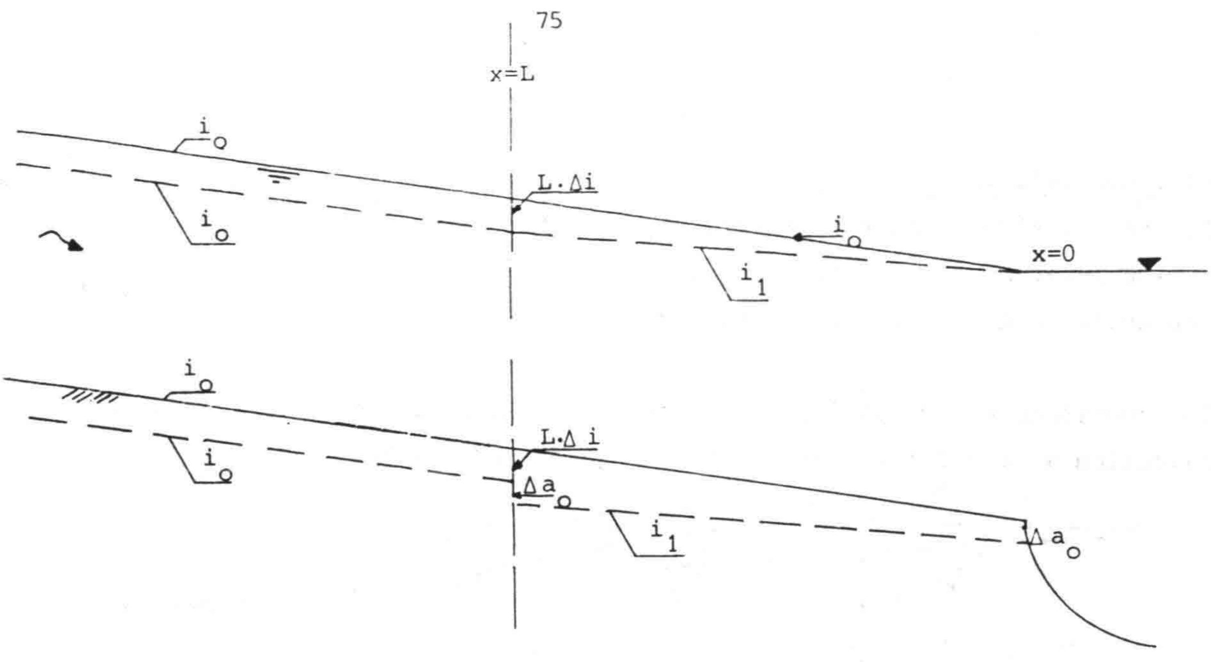


Fig. 4.6. Sediment withdrawal ($t = 0$ and $t \to \infty$)

In Fig. 4.6 the situation is sketched for continuous dredging of ΔS at $x = L$. As has been shown in Sub-Section 3.2.1 downstream of $x = L$ the slope will for $t \to \infty$ become flatter and the depth smaller. For a constant discharge the water level at $x = L$ will lower over a distance $\Delta h = L \cdot \Delta i = L(i_o - i_1)$.

In the interval $0 < x < L$ the depth will increase by Δa_o following from Eq. (3-49). Just downstream of $x = L$ the bed will finally have lowered over a distance $\Delta z_b(L,\infty) = \Delta h + \Delta a_o$. At $x = L$ the bed level is discontinuous. The bottom step is Δa_o. For all values $x > L$ both the bed levels and the water levels are lowered by Δh.

In Fig. 4.6 a mild positive bedslope has been assumed. Therefore the flow is subcritical. Note that the water level and bed level have to be drawn starting from the erosion-base (sea or lake level). This is the only point that remains the same for $t = 0$ and $t \to \infty$.

Remark:

(i) The situation of Fig. 4.6 is a rather theoretical one. In practice both Q and ΔS will be time-depending. However, to understand the results of time-depending morphological computations such a simplified case is of great help.

(ii) The lowering of water- and bed-levels due to withdrawal of sediment may have negative side-effects to other users of the river (e.g. for a water intake).

4.4. Constriction of width

Figure 4.6 gives also the general solution for $t \to \infty$ if the width is constricted in the interval $0 < x < L$. For the smaller width the slope becomes flatter and the depth larger. (See also Fig. 3.9).

To understand the morphological phenomena it is of importance to study the situation at $t = 0$ thus before any change of the bed has taken place.

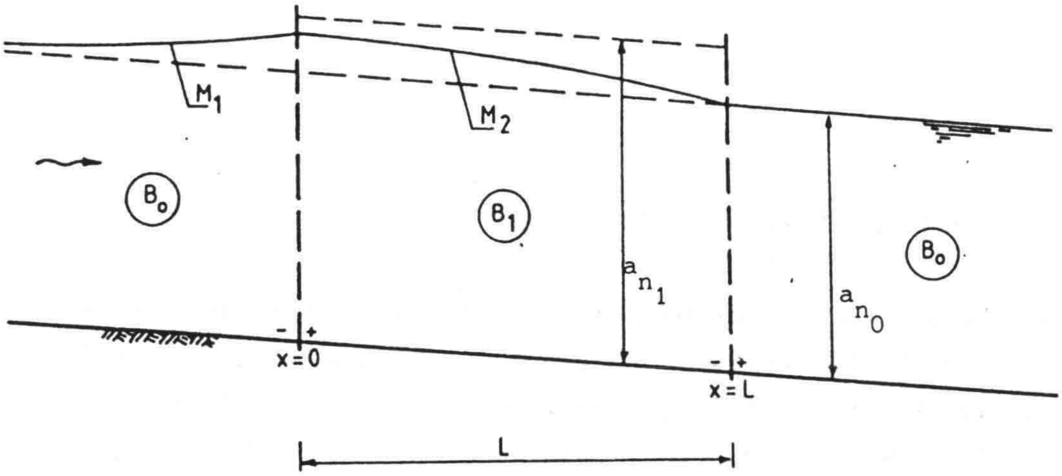


Fig. 4.7. Constriction of width: situation at t = 0.

In Fig. 4.7 the longitudinal profile is given. In the reach $0 < x < L$ the width has been reduced from B_o to B_1. Therefore the *normal depth* is there larger. For a mild positive slope backwater curve M_1 and M_2 are present as indicated in Fig. 4.7.

This gives regions of erosion and sedimentation. In Fig. 4.8 the function S(x,o) is indicated qualitatively.

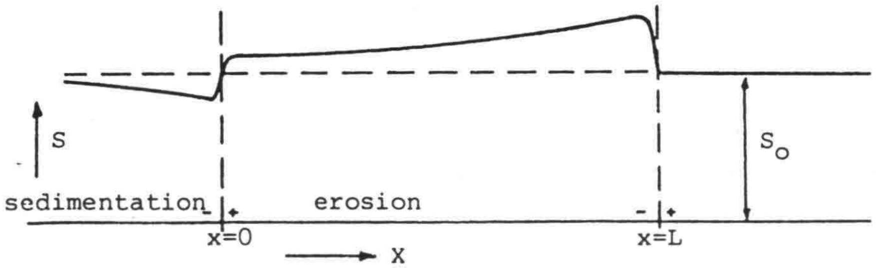


Fig. 4.8. Function S(x,o) due to constriction.

At $x = 0$ and $x = L$ there will be discontinuities in the bed level. To study these discontinuities it is of importance to realize that both the transport of water and sediment is continuous at $x = 0$ and $x = L$. However, the continuity of the sediment *cannot* be studied by

$$B \frac{\partial z_b}{\partial t} + \frac{\partial S}{\partial x} = 0 \tag{4-11}$$

because $\partial z_b / \partial t$ is not defined at the discontinuities. However, the continuity equation holds in the *integral form* namely $S_- = S_+$. Moreover ofcourse $Q_- = Q_+$. Hence with $s = m\, u^n$ and $q = u.a.$

$$\frac{a_+}{a_-} = \left[\frac{B_-}{B_+} \right]^{\frac{n-1}{n}} \tag{4-12}$$

As the water level is continuous the step in the bed level is equal to the difference $\Delta a = a_+ - a_-$

- For $x = 0$ the result is $a_-(o,t) < a_+(o,t)$. Hence there is a *downward* bottom step in the flow direction.
- For $x = L$ the result is $a_-(L,t) > a_+(L,t)$. This gives in the flow direction a step *upward* in the bottom.

The function $S(x,o)$ as indicated in Fig. 4.8 will lead to erosion and sedimentation. At $x > L$ temporary sedimentation takes place which has disappeared at $t \to \infty$. In the final situation, for a constant discharge Q_o the sediment transport follows from $S(x,\infty)$ = constant = S_o.

For $t \to \infty$ the following changes of the bed level compared to the old equilibrium are present

- $x > L$ — no change
- $0 < x < L$ — lowering = $(i_o - i_1)(L - x) + \Delta z'_b$
- $x < 0$ — lowering = $(i_o - i_1)L$

in which $\Delta z'_b$ follows from Eq. (3-52) and i_1 from Eq. (3-53).

4.5. Bend cutting

River navigation may be improved by cutting a sharp bend. In Fig. 4.9 a schematic example is given. A morphological computation has been carried out for a varying discharge, solving Eqs. (3-17) and (3-18).

In order to understand the morphological processes Fig. 4.10 indicates the principle. In this figure the water level differences are exaggerated. In the new bend (reach II) it is assumed that the bed level at t = 0 follows a straight line between the bed levels of the reaches I and III of the river. The length L_o of the original bend is shortened to L_1. Thus $i_{b_1} > i_{bo}$. As $q = Ca_n^{3/2} i_b^{\frac{1}{2}}$ it follows $a_{n_1} < a_{no}$. Hence at t = 0 backwater curves are present (Figs. 4.9 and 4.10).

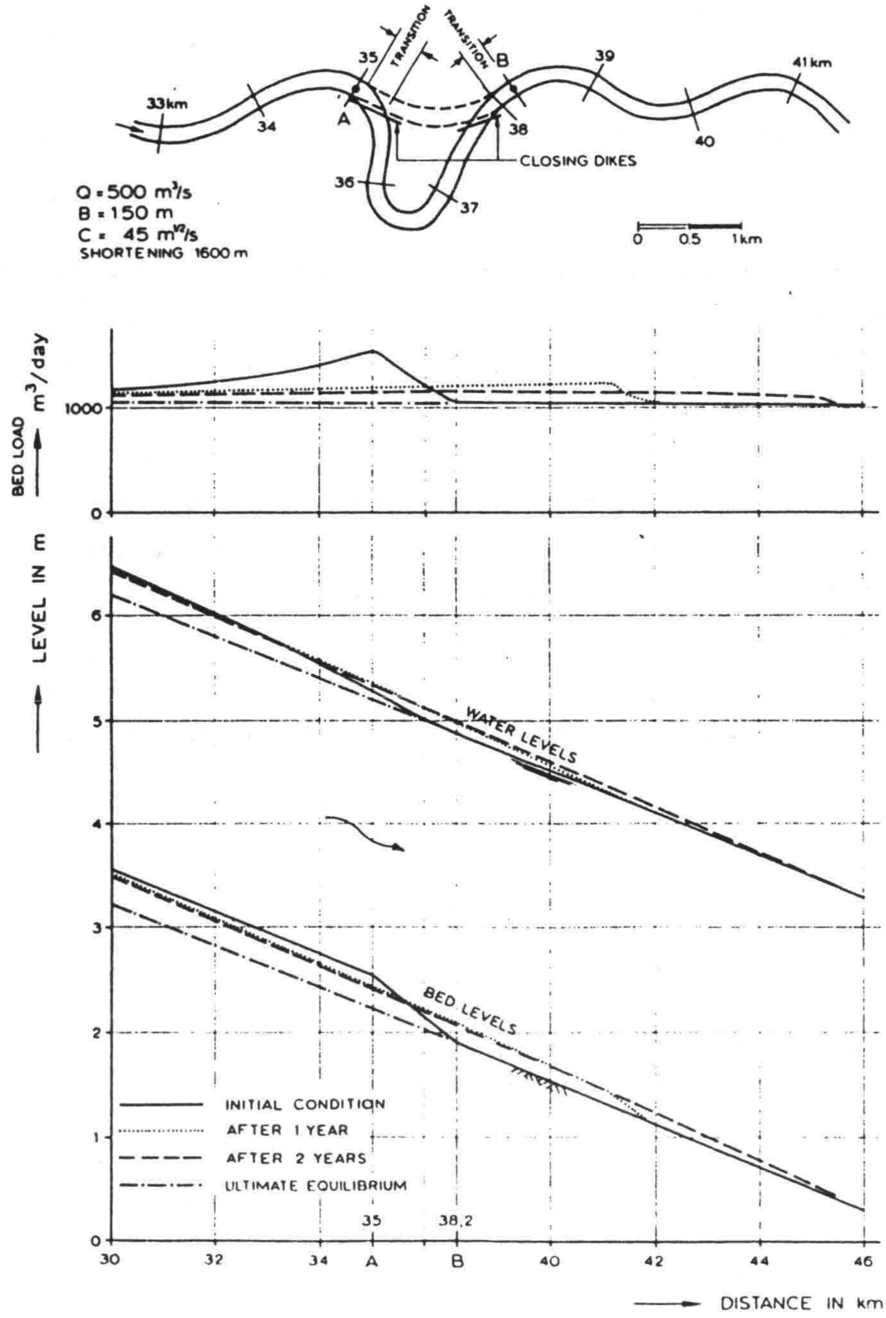


Fig. 4.9. Short-cut of a single channel (after Jansen, 1979)

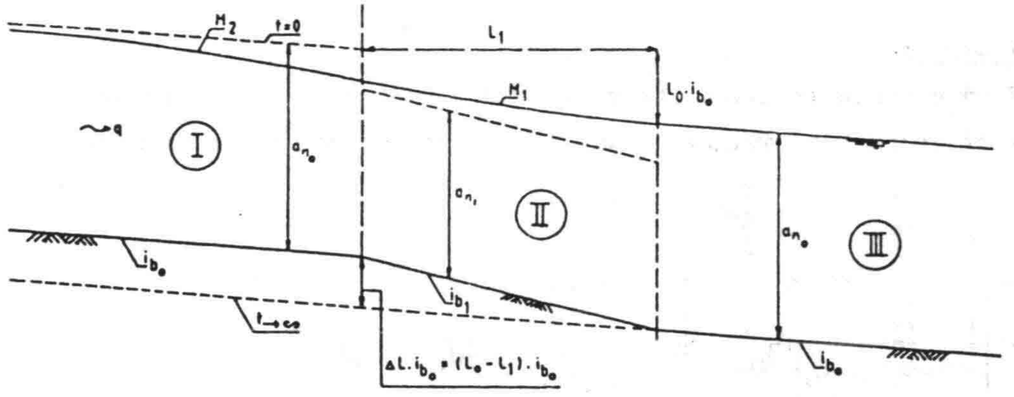


Fig. 4.10. Principle of bend-cutting (t = 0)

The situation for t = 0 cannot be an equilibrium one. The velocity gradients $(\partial u/\partial x)$ lead to transport gradients $(\partial s/\partial x)$. Hence $\partial z_b/\partial t \neq 0$.

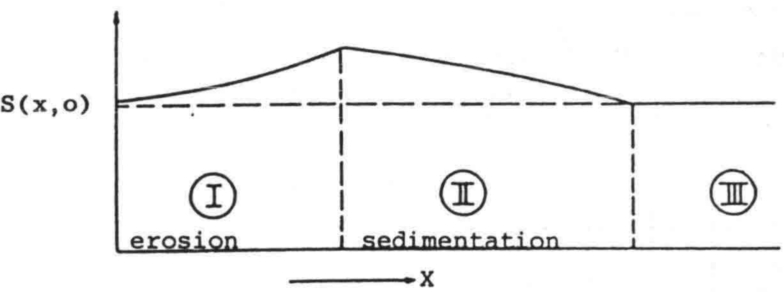


Fig. 4.11 Function S(x,o) for Fig. 4.10.

In Fig. 4.11 schematically the function S(x,0) is sketched for the case B(x) = constant. As $\partial z_b/\partial t + \partial s/\partial x = 0$ the reach I has erosion for t = 0. Sedimentation occurs in reach II (t = 0). For t > 0 temporary sedimentation occurs in reach III.

For reach II at $t \to \infty$ the original bed level is again present. For $t \to \infty$ the bed level in reach I has been lowered by $\Delta z_b = (L_o - L\ )i_b$.

Remark:

(i) For slow rivers dredging in the new bend will prevent the temporary reduction of the depth in reach III due to sedimentation. For quick rivers this is not necessary. One may even dredge along the alignment of the new bend a *pilot channel*. The river will then reach relatively soon the new situation.

(ii) Naturally the old bend will be closed by means of a dam preferably during low discharges. For Fig. 4.9 the computations have been carried out assuming that at t = 0 the dam was constructed.

4.6. Channel closure

In Fig. 4.12 an example is given, taken from Jansen (1979, p.348) regarding the increase of depth for navigation by the closure of one river branch.

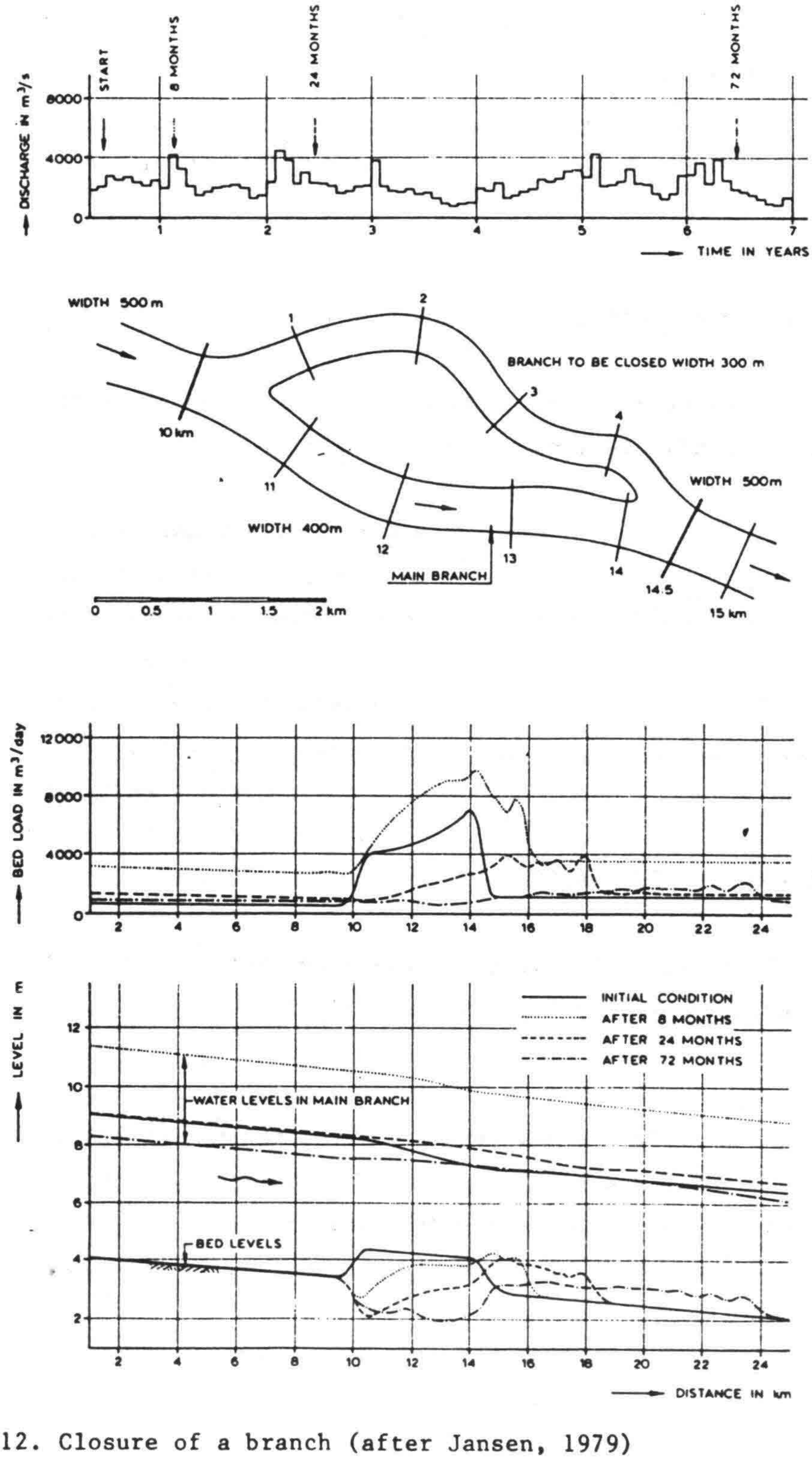


Fig. 4.12. Closure of a branch (after Jansen, 1979)

In this case the river is flowing around an island. The narrow branch is closed in order to arrive *eventually* at the situation that the other branch is deepended. In the example of Fig. 4.12 the discharge varies in time.

To understand the behaviour of $z_b(x,t)$ first the situation for a constant discharge is considered.
For $t < 0$ at $x = 0$ (the bifurcation) and at $x = L$ (the confluence) a step in the bed level Δz_o will be present. This step cannot be computed as it depends on the distribution of the sediment at the bifurcation (see Section 2.4). Also the slopes in the two branches for $t < 0$ depend in the distribution of water and sediment at the bifurcation.

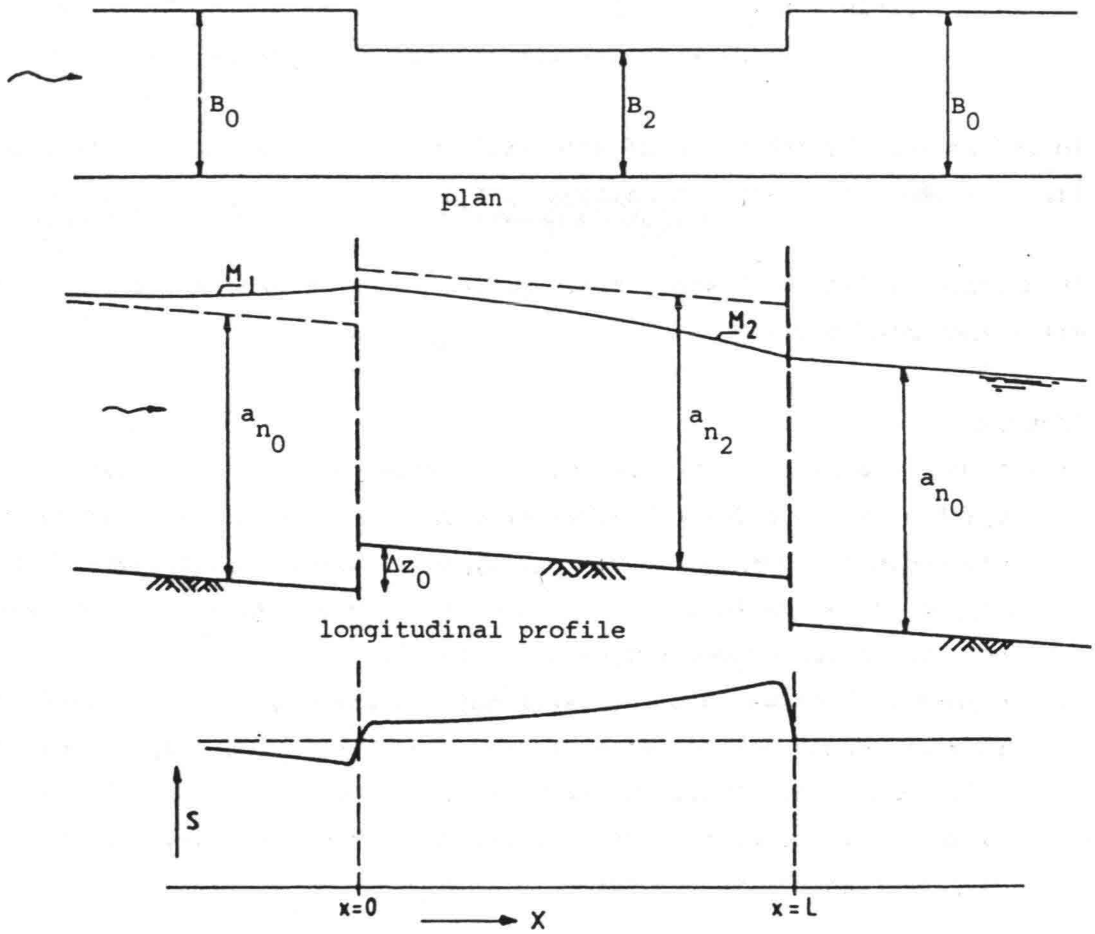


Fig. 4.13. Principle of channel closure (t = 0).

In Fig. 4.13 the situation at $t = 0$ is given; the minor branch has just been closed. The essential difference with Fig. 4.10 is that here the bottom steps Δz_o are present; they originate from the situation at $t < 0$.

Besides these two steps there are two more steps present due to the change in width at $x = 0$ and $x = L$ for $t > 0$. These are the steps treated in Sub-Section 3.2.4 (Case III).

Thus for $t > 0$ there are *four* bottom steps present:

- For $x = 0$: (i) The step because $B_o > B_1$. This step remains at $x = 0$ for $t > 0$.
  (ii) The step Δz_o originates from $t < 0$. This propagates downstream for $t > 0$. It is an *expansion wave*, so it becomes flatter for $t > 0$.
- For $x = L$: (iii) A step because $B_2 < B_o$, this step stays at $x = L$.
  (iv) A step Δz_o from the situation at $t < 0$. This discontinuity propagates downstream for $t > 0$. It is a shockwave.

To understand the behaviour of step (ii) and (iv) reference can be made to Fig. 3.4 where the deformation of a hump is sketched.

Inspection of Fig. 4.13 shows that $z_b(x,t)$ indeed contains the four bottom steps indicated above.

Remarks:

(i) As was explained in Section 4.4 the presence of discontinuities in the width at $x = 0$ and $x = L$ makes that the *differential* equation expressing the sediment continuity does not apply at the discontinuities because $\partial z_b/\partial t$ is not defined there. The continuity of sediment is expressed in *integral* form because $S(x,t)$ is continuous.

(ii) Figure 4.12 shows that the water depth downstream of the island is temporary decreased due to sedimentation. Apparently the example regards a relatively slow river. In practice it may be advisable to 'help' the river by means of dredging. Otherwise the benefit of the local river improvement will be obtained only after some time.

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Main symbols

symbol	description	dimensions
a	depth	[L]
A	area	[L^2]
B	width	[L]
c	celerity	[LT^{-1}]
C	Chézy-coefficient	[$L^{½}T^{-1}$]
D	grain diameters	[L]
Fr	Froude number	[-]
g	acceleration of gravity	[LT^{-2}]
h	water level	[L]
H	energy head	[L]
i	(energy) slope	[-]
k_N	Nikuradse sandroughness	[L]
L	length	[L]
M	mass	[M]
p	pressure	[$ML^{-1}T^{-2}$]
q	discharge per unit width	[L^2T^{-1}]
Q	discharge	[L^3T^{-1}]
r	radius of curvature	[L]
R	hydraulic radius	[L]
s	sediment transport per unit width (bulk volume)	[L^2T^{-1}]
S	sediment transport (bulk volume)	[L^3T^{-1}]
t	time	[T]
u	flow velocity in x-direction	[LT^{-1}]
v	flow velocity in y-direction	[LT^{-1}]
w	flow velocity in z-direction	[LT^{-1}]
W	fall velocity	[LT^{-1}]
x	horizontal coordinate	[L]
X	transport parameter = $s/ D^{3/2}\sqrt{g\Delta}$	[-]
y	horizontal coordinate	[L]
Y	flow parameter = $\Delta D / \mu ai$	[-]
z	vertical coordinate	[L]

$z_{(b)}$	bed level	[L]
Z	$= W/ku_*$	[-]
Δ	relative density sediment $= (\rho_s - \rho)/\rho$	[-]
ε	turbulent viscosity	$[L^2T^{-1}]$
η	$= z/a$ = relative depth	[-]
κ	von Kármán constant	[-]
Λ	$= xi/a$ = length scale river	[-]
μ	ripple factor	[-]
ν	kinematic viscosity	$[L^2T^{-1}]$
ρ	density of water	$[ML^{-3}]$
ρ_s	density of sediment	$[ML^{-3}]$
τ	shearstress	$[ML^{-1}T^{-2}]$
ϕ	sediment concentration	[-]