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Observation of the Josephson-Anderson relation in experiments on hydrodynamic drag

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We verify a recent prediction [Eq. (3.50) in Eyink, *Phys. Rev. X* **11**, 031054 (2021)] for the drag on an object moving through a fluid. In this prediction the velocity field is decomposed into a nonvortical (potential) and vortical contribution, and so is the associated drag force. In the Josephson-Anderson relation the vortical contribution of the drag force follows from the flux of vorticity traversing the streamlines of the corresponding potential flow. The potential component is directly determined by the plate acceleration and its added mass. The Josephson-Anderson relation is derived from the quantum description of superfluids, but remarkably applies to the classical fluid in our experiment. In our experiment a flat plate is accelerated through water using a robotic arm. This geometry is simple enough to allow analytic potential flow streamlines. The monitored plate position shows an oscillatory component of the acceleration, which adds an additional test of the Josephson-Anderson relation. The instantaneous velocity field is measured using particle image velocimetry. It enables us to evaluate Eq. 3.50 from the above-mentioned paper and compare its prediction to the measured drag force. We find excellent agreement and, most remarkably, find that the added mass contribution to the drag force still stands out after the flow has turned vortical. We finally comment on the requirements for the experimental techniques for evaluating the Josephson-Anderson relation.

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I. INTRODUCTION

In a recent paper Eyink [1] explains the drag on an object moving through a fluid by the flux of vorticity across streamlines of the corresponding potential flow [2,3]. The equation relating drag to vorticity, the Josephson-Anderson relation, derives from the quantum description of superfluids, but remarkably applies to classical fluids. It provides a quantitative exact and instantaneous link between drag force and the velocity field. In the present context drag would be absent if there is no flux of vorticity across streamlines. It is an interesting corollary to d'Alembert's paradox in which a moving object experiences no drag in stationary potential flow.

The idea that the drag force on an object is related to the fate of vorticity as it is “washed away” from the object may be traced back to Burgers [5,6]. This idea has spawned recent research on the connection between vorticity dynamics and forces, for example, to quantify and manipulate forces

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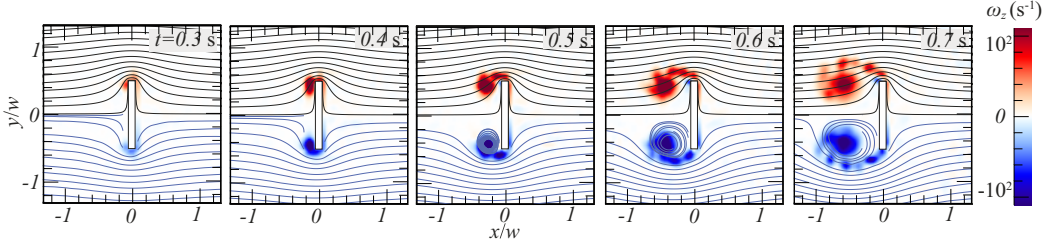


FIG. 1. Evolution of the vorticity during acceleration with maximum ($a_{\max} = 0.8 \text{ m s}^{-2}$) of the plate. The final velocity ($U = 0.3 \text{ m s}^{-1}$) is reached at $t = 0.6 \text{ s}$. The upper half of all panels shows the streamlines of the potential flow, which are symmetric in y . The lower half shows the true experimental streamlines (in blue). The essence of the Josephson-Anderson relation is that drag ensues when vorticity, carried by the true flow, crosses the streamlines of the potential flow. Animation is in the Supplemental Material [4].

and associate force events to vorticity events [7–10]. Establishing this connection became possible using recent advances in precisely mapping the velocity field using particle image velocimetry (PIV) and robotized object manipulation [11,12].

In our experiment we drag a plate through still fluid, vary its acceleration, measure the drag force, and take snapshots of the velocity field. The question is whether the Josephson-Anderson relation can predict the drag force in an experiment using a measured velocity field.

A series of snapshots of the vorticity in our experiment is shown in Fig. 1. At time $t \approx 0.5 \text{ s}$ a vortex pair detaches from the plate edges. It appears that at this time the drag force reaches a maximum.

II. JOSEPHSON ANDERSON RELATION

The essence of the Josephson-Anderson (JA) relation is the separation of the velocity field $\mathbf{u}$ into a potential velocity $\mathbf{u}_\phi$ and a rotational component $\mathbf{u}_\omega$; $\mathbf{u} = \mathbf{u}_\phi + \mathbf{u}_\omega$. The irrotational (potential) part $\mathbf{u}_\phi$ derives from a potential ϕ , $\mathbf{u} = \nabla\phi$, and satisfies the no-penetration condition at the plate boundary, and in- and outflow conditions at the boundaries of the experimental domain, which are assumed far away from the plate. This guarantees a unique solution of the Laplace equation for $\phi(\mathbf{x})$.

Although d’Alembert’s paradox precludes drag in stationary potential flow, an accelerated object experiences a drag force $F_\phi = -m_a dU/dt$, with $U(t)$ the instantaneous velocity of the object, and m_a the added mass. The added mass represents the equivalent mass of the fluid that is set into motion when the plate is accelerated from rest [8]. Its context is potential flow; its value is determined by the shape of the object and the direction of motion.

When an object is accelerated from rest, and vorticity has not yet transpired, $\mathbf{u} \approx \mathbf{u}_\phi$ at the initial stage of the motion. One would naively think that the concept of added mass no longer makes sense for flow with massively separated vorticity. The Josephson-Anderson relation suggests that the decomposition of the velocity field $\mathbf{u} = \mathbf{u}_\phi + \mathbf{u}_\omega$, with the associated decomposition of the drag force, $\mathbf{F} = \mathbf{F}_\phi + \mathbf{F}_\omega$, remains relevant irrespective of the flow development away from potential flow. Below we present striking evidence for this.

The Josephson-Anderson relation expresses the vortex drag $\mathbf{F}_\omega$ on an object in terms of the flux of vorticity across streamlines of the potential flow field $\mathbf{u}_\phi$. It was derived from the change of energies in the potential and rotational flow in a channel by Huggins [13], and rederived for a moving body by Eyink [1]. The associated transfer of energy between both components is the work done by an object against drag forces. In our case, we apply it to a plate that moves horizontally with velocity $U(t)$ through still water. We then consider the plate’s reference frame, such that the plate appears to be stationary and submerged in a flow with velocity $\mathbf{V}(t)$ far from the plate, so that

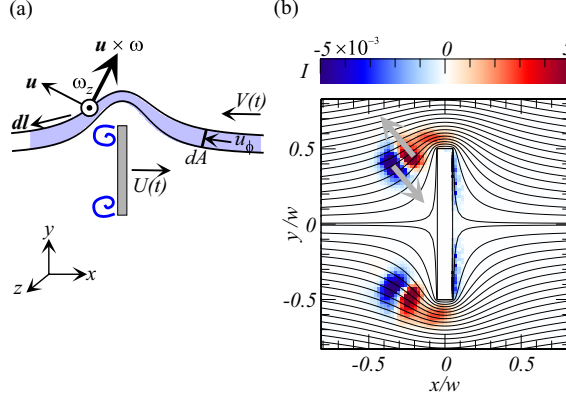


FIG. 2. (a) Illustration of the JA relation as applied to our experiment in which a plate moving to the right with velocity $U(t)$ sheds vortices. The velocity $V(t)$ in Eq. (1) is $V(t) = -U(t)$. Shown is a streamline with tangent vector $d\mathbf{l}$. At the upper half of the plate the vorticity is positive; then for velocities directed outward through a streamline, $\mathbf{u} \times \boldsymbol{\omega}$ points upward against the direction of $d\mathbf{l}$, with a positive contribution to the drag force. The flux $u_\phi dA$ of potential flow between streamlines is the difference between the values of the associated stream functions. (b) Integrand $I = dJ (\mathbf{u} \times \boldsymbol{\omega} - \nu \nabla \times \boldsymbol{\omega}) \cdot d\mathbf{l}$ in the JA relation [Eq. (1)] at time $t = 0.5$ s. Its sign is such that positive values add to drag, whereas negative values diminish drag. There is no contribution to drag when $\mathbf{u}$ is directed along a streamline. The two arrows indicate the flux of vorticity across streamlines. Equation (1) is evaluated using four times as many streamlines as shown in panel (b). Animation is in the Supplemental Material [4].

$V_x(t) = -U(t)$. The contribution $\mathbf{F}_\omega$ to the drag force due to vorticity is then [1]

$$\mathbf{F}_\omega \cdot \mathbf{V}(t) = \int dJ \int (\mathbf{u} \times \boldsymbol{\omega} - \nu \nabla \times \boldsymbol{\omega}) \cdot d\mathbf{l}, \quad (1)$$

with $d\mathbf{l}$ pointing along a streamline. The inner integration is along a streamline, while in the outer integration dJ is the mass flux within streamtubes, $dJ = \rho \mathbf{u}_\phi \cdot d\mathbf{A}$, so that Eq. (1) represents the flux of vorticity across potential mass current [1,2]. The Josephson-Anderson relation holds for flow that is described by the incompressible Navier-Stokes equation in a domain that extends to infinity. The Josephson-Anderson relation [Eq. (1)] is illustrated in Fig. 2(a) for the term $\mathbf{u} \times \boldsymbol{\omega}$ in Eq. (1). The integrand in Eq. (1), measured in our experiment, is shown in Fig. 2(b).

The unique and beautiful Eq. (1) is *instantaneous*, providing an instantaneous prediction of the drag force $\mathbf{F}_\omega$ from a measured velocity field.

The classical method to infer forces from a measured velocity field uses the momentum balance for a control volume, and needs both the pressure and inertial terms in the Navier-Stokes equation, one of which involves a time derivative [14,15], with an extensive review in [16]. The computation of the pressure field from the Poisson equation is now part of commercial PIV packages.

The vortex impulse method computes forces from measured velocity fields but it depends on measured time derivatives [6,9,10,17,18]. Closely related to the Josephson-Anderson method is the vortex force map [19–23] that was applied for the first time to an experiment in [24]. The vortex force map involves an “auxiliary” potential which differs from the true flow potential in the Josephson-Anderson relation. It lacks the intuitive appealing notion of drag as vorticity traversing the streamlines of the potential flow.

The decomposition $\mathbf{u} = \mathbf{u}_\phi + \mathbf{u}_\omega$ was also discussed by Lighthill [18] in the context of drag in ocean waves and by Graham [7], who stresses that the irrotational velocity field $\mathbf{u}_\phi$ acts upon the vortical forces $\mathbf{F}_\omega$ and suggests to derive this interaction using the analysis in [19,25,26].

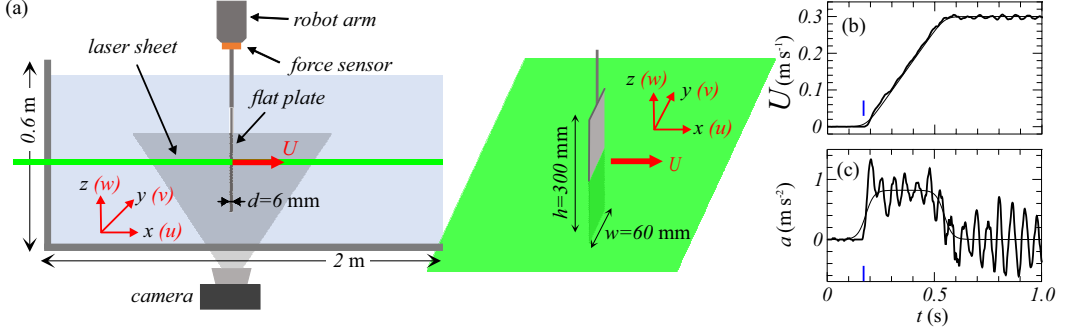


FIG. 3. Experimental setup (not to scale). (a) A rectangular plate is accelerated through the center of a large square tank (2 m $\times$ 2 m, depth 0.6 m) filled with water. A laser sheet in the (x, y) plane intersects the center of the plate, and illuminates tracer particles. A camera (frame rate 1200 Hz) registers particle images. Particle image velocimetry (PIV) is used to map the planar velocity field. (b), (c) Time trace of the plate velocity and acceleration, respectively. They were measured from optical tracking of the plate—thin lines: nominal (as supplied to the robot). The vertical blue lines in frames (b), (c) indicate the beginning of the motion (where $a > 0$ for the first time).

In this paper we compare the measured drag force on an accelerated plate to the force $\mathbf{F} = \mathbf{F}_\phi + \mathbf{F}_\omega$, with $\mathbf{F}_\phi$ the added mass force and $\mathbf{F}_\omega$ given by Eq. (1). The terms in Eq. (1) were computed from the measured velocity field. The potential flow streamlines for the rectangular cross section of the plate and boundary condition $u_\phi = V(t)$ at the left edge of the measurement domain are computed using the Schwarz-Christoffel transform [27]. The potential flow streamlines form the natural integration grid in Eq. (1). With 512 streamlines spanning the experimental domain [streamline spacing 6.5×10^{-4} m ($10^{-2} w$) at $x = -0.4$ m], the resulting integration grid is finer than the discrete velocity grid from PIV (grid cell size 2.66×10^{-3} m).

We analyze two experiments, where the plate is accelerated to the same final velocity, $U = 0.3 \text{ m s}^{-1}$ with maximum acceleration of $dU/dt = 0.8$ and 1.6 m s^{-2} , respectively. For these two experiments we compare the measured hydrodynamic force on the plate with the prediction from the Josephson-Anderson relation.

III. EXPERIMENTS

We accelerate plates (size $h \times w \times d = 0.3 \times 0.06 \times 0.006 \text{ m}^3$) through initially quiescent water. The time-dependent velocity field (u, v) is measured using particle image velocimetry. This velocity field is accurate enough to compute the vorticity $\boldsymbol{\omega}(\mathbf{x}, t)$ and diffusion of vorticity $-\nu \nabla \times \boldsymbol{\omega}(\mathbf{x}, t)$, with ν the kinematic viscosity. At our Reynolds numbers $\text{Re} \approx 2 \times 10^4$, as based on the final velocity and the width w of the plate, the contribution of the diffusion term is insignificant.

The setup is sketched in Fig. 3(a). The aspect ratio of the plate $h/w = 5$, and we measure the velocity field in a plane bisecting the plate along its height h . Hence, plate end effects are sufficiently removed from the measurement plane, and the flow can be considered as practically two-dimensional in the measurement plane.

The PIV system consists of a high-speed camera (Phantom Veo 640, 4 megapixels) with a frame rate of 1200 Hz and a laser (Litron Nd:YLF, 150 W, thickness of laser sheet 3 mm) that illuminates fluorescent tracer particles (Cospheric, 65 μm diameter) in a $0.5 \times 0.5 \text{ m}^2$ field of view.

The plate is moved by a gantry robot (Reis Robotics RL50); the drag force is registered by a force/torque sensor (Schunk FTD-Gamma SI-32-2.5) with $6 \times 10^{-3} \text{ N}$ force resolution. It is corrected for the mass of the plate and the drag of the supporting strut. The time-dependent velocity of the plate in the first experiment ($a = 0.8 \text{ m s}^{-2}$) is shown in Fig. 3(b). The plate reaches its final

velocity $U = 0.3 \text{ m s}^{-1}$ at time $t = 0.6 \text{ s}$. In the frame of reference considered here, the plate is stationary with its center at $\mathbf{x} = 0$, while the fluid moves to the left with velocity $V(t) = -U(t)$.

Using subpixel interpolation and a pinhole camera model, the actual plate position could be precisely traced from the PIV images, as the plate is always in view. The first derivative of the position trace gives the plate velocity while the second derivative provides the plate acceleration. As Figs. 3(b) and 3(c) demonstrate, the plate acceleration has an oscillatory component which could just be discerned from the velocity signal. Be it the consequence of the finite stiffness of the plate mount, or the influence of the robot (controller), we do not view it as an artifact, but as a fact. It allows us to study the response of the velocity and drag force to (periodic) modulation.

Added mass

The potential force contribution $F_{x,\phi} = m_a dV/dt$ needs the added mass m_a for motion in the x direction. It was computed using the Schwarz-Christoffel transform for a plate with rectangular cross section $w \times d$ that is accelerated through still fluid. The added mass per unit height h is

$$m_a = \rho \int_S \phi n_x dS = \rho \, 3.352 \, 01 \, (w/2)^2, \quad (2)$$

with ρ the density of the fluid, ϕ the unit potential field (unit velocity at far away boundaries), and $\mathbf{n}$ the unit normal on the plate surface. It is very close to the added mass per unit height of an infinitely thin plate (thickness $d = 0$) and width w , which equals the added mass of a cylinder with radius $w/2$,

$$m_a = \rho \int_S \phi n_x dS = \rho \, \pi \, (w/2)^2 = \rho \, 3.141 \, 59 \, (w/2)^2. \quad (3)$$

Still, it appears essential to use the streamlines of the thick plate in the computation of Eq. (1) as usage of the thin plate streamlines leads to a significantly smaller vortex force F_ω .

IV. RESULTS

The steps leading to the force prediction are illustrated in Fig. 2 where we show a snapshot of the integrand $I = dJ(\mathbf{u} \times \boldsymbol{\omega} - \nu \nabla \times \boldsymbol{\omega}) \cdot d\mathbf{l}$ in Eq. (1), and which can be considered as a *heat map of drag*. Notice that ω_z is positive near the top of the plate, and negative at the bottom, whereas the integrand in Eq. (1) switches sign when $\mathbf{u} \times \boldsymbol{\omega}$ turns against the direction of the potential flow velocity $\mathbf{u}_\phi$. There is no contribution to drag when $\mathbf{u}$ is directed along a streamline. The positive regions in Fig. 2(b) correspond to outward motion of vorticity across streamlines.

The total drag force $\mathbf{F}_\omega + \mathbf{F}_\phi$ is compared to the measured force in Fig. 4. The measured peak forces agree with the prediction model proposed by Reijtenbagh *et al.* [12]. A striking observation is that the sharp rise of the total drag force at the start of the acceleration at $t = 0.2 \text{ s}$ is completely due to the potential force. Until $t = 0.4 \text{ s}$ and $t = 0.3 \text{ s}$, for the cases $a_{\max} = 0.8 \text{ s}^{-2}$ and $a_{\max} = 1.6 \text{ m s}^{-2}$, respectively, the contribution of the vortical force is negligible. The maximum of the total drag force is almost entirely due to the vortical contribution F_ω and is reached *after* the added mass force F_ϕ ceases.

Figure 4(c) shows the same results as those in Fig. 4(a), but now as a function of the nondimensional “formation time,” t^* , $t^*(t) = \int_0^t U(t') dt'/w$ [28], as is commonly used in this context [12]. It compresses the information at the start of the plate motion.

The measured force oscillations follow from the oscillating acceleration and the added mass. The excellent agreement even extends to long times when the flow around the plate has turned vortical and the concept of added mass, which is based on irrotational flow, no longer applies strictly.

The modulations of the acceleration could influence $\mathbf{u}_\phi$, $\mathbf{u}_\omega$, or both, and could, via them, result in the measured drag modulation. As our results in Figs. 4(a) and 4(c) demonstrate, acceleration modulations are mainly communicated through potential forces. In Figs. 4(b) and 4(d) the computed

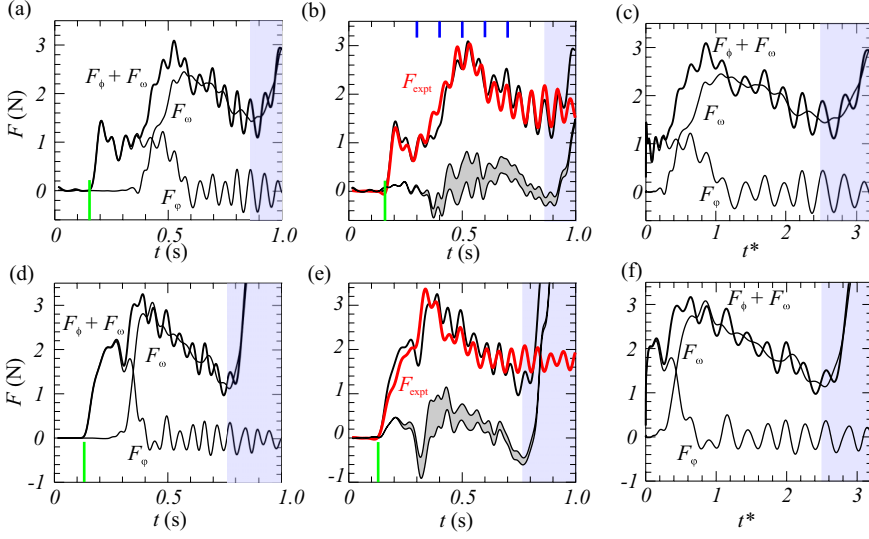


FIG. 4. Measured [red lines in (b), (e)] and computed hydrodynamic drag forces for two acceleration scenarios: in panels (a)–(c) the maximum acceleration is $a_{\text{max}} = 0.8 \text{ m s}^{-2}$, while in (d), (e), (f) $a_{\text{max}} = 1.6 \text{ m s}^{-2}$. Both scenarios end with the same final velocity, $U = 0.3 \text{ m s}^{-1}$. The vertical blue lines in frame (b) indicate the times of the frames in Fig. 1. (a), (d) Drag forces computed from the measured velocity field using the Josephson-Anderson relation equation (1). The two contributions—the potential force F_ϕ and the vortical force F_ω —as well as their sum are indicated. The computation of the drag forces uses the measured plate accelerations [see Figs. 3(b) and 3(c)]. At long times, $t^* > 2.5$ [shaded, corresponding to $t > 0.9 \text{ s}$ in (a), (b) and $t > 0.8 \text{ s}$ in (d), (e)], the flow turns three dimensional, and our approach, which is based on planar flow, no longer applies. (b), (e) Comparison of measured and computed forces. Red lines: F_{expt} ; black lines: $F_\phi + F_\omega$. Gray lines: difference $F_\phi + F_\omega - F_{\text{expt}}$ between the computed total force and experiment. The width of the line represents an estimate of the uncertainty. (c), (f) Same as panels (a), (d), respectively, but now plotted as a function of “formation time,” $t^*(t) = \int_0^t U(t') dt'/w$ [28]. The vertical green lines in panels (a), (b), (d), (e) indicate the start of the plate motion (where $a > 0$). All results (time traces) have been filtered using a binomial filter with root mean square width of 0.015 s .

and measured (F_{expt}) drag forces are compared. The overall agreement between the measured force and the prediction of the Josephson-Anderson relation is excellent, but incidentally can be as large as 30% for the experiment with the largest acceleration. The shown error estimate is discussed below.

V. APPLYING THE JOSEPHSON-ANDERSON RELATION TO EXPERIMENTAL DATA

Let us now discuss the possible errors incurred when applying Eq. (1) to our data. First, the Josephson-Anderson relation assumes an infinite domain, but the experiment domain is necessarily restricted. The influence of the domain boundaries can be learned in the experiment from evaluating Eq. (1) for the drag force on a smaller domain. Clearly, the plate, being the source of vorticity, should remain inside. No significant difference was found when reducing the linear dimensions by a factor of 3.

Measurement of the velocity fields and vorticity fields close to the plate presents an experimental challenge. Tracer particles at both sides of the plate are illuminated and used in the PIV analysis. The finite resolution of PIV (velocity grid cell $2.66 \times 10^{-3} \text{ m}$) affects the measurement of velocity gradients, which are large close to the plate edges. With a Reynolds number $\text{Re} = hU/\nu \approx 2 \times 10^4$, we estimate a boundary layer thickness $\text{Re}^{-1/2} h \approx 4 \times 10^{-4} \text{ m}$, which remains unresolved.

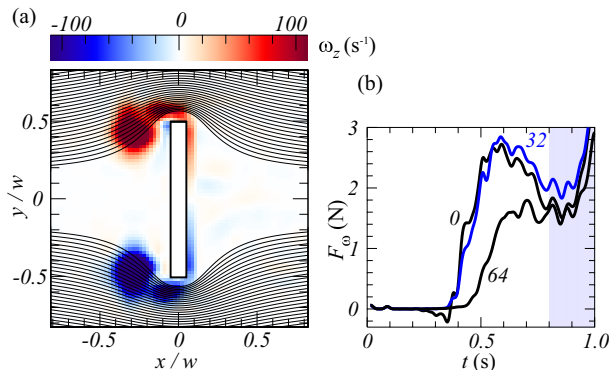


FIG. 5. Dependence of the computed vortex force F_ω at $t = 0.5$ s on the grid of streamlines, used as an integration grid in Eq. (1). (a) Omitting the 32 streamlines (out of 512) that are closest to the plate—color scale: vorticity ω_z . (b) Influence of the number of omitted streamlines on the vortical force F_ω . For the curve indicated “0,” all 512 streamlines were used in the integration; the case “32” is illustrated in panel (a).

The relevance of vorticity close to the plate in computing the drag force from Eq. (1) can be assessed by excluding vorticity in a thin layer (width 5.2 mm) from the top and bottom edges (at $y = \pm w/2$) of the plate cross section in the integration in Eq. (1). The comparison of the vortex force computed from the full experimental domain to that from the domain with this layer deleted leads to an estimate of the uncertainty in the computed force. This estimate is shown in Figs. 4(b) and 4(d). Other sources of error are the uncertainty and finite resolution of the measured velocity field.

The potential flow streamlines form the backbone of the Josephson-Anderson relation: drag ensues when vorticity crosses those streamlines. The streamlines close to the stagnation streamline (at $y = 0$) closely hug the plate cross section and the integration over them depends on the measured velocity (vorticity) field close to the plate. The question is how many of those close streamlines are needed for a faithful prediction of drag. Figure 5(a) shows a drastic deletion of streamlines, with the resulting vortical force F_ω in Fig. 5(b). Integration in Eq. (1) over the remaining streamlines captures sufficient vorticity flux to explain the drag maximum at $t = 0.6$ s.

Finally, our analysis is two-dimensional and is applied to a two-dimensional cut of the velocity field. This is suggested by the aspect ratio $h/w = 5$ of our plate. At initial times a rectangular vortex ring is formed, of which we observe a two-dimensional cut. Two-dimensionality can be checked through the measured divergence of the velocity field; it is progressively lost after $t \approx 0.8$ s. A challenge remains the application of the Josephson-Anderson relation to a full three-dimensional experimental velocity field.

VI. CONCLUSION

Both the measured and computed drag forces reach a maximum shortly, when the acceleration starts to decline. From our results it appears that this maximum owes itself to the vortical force F_ω ; it is not an added mass (potential flow) effect. Reijtenbagh *et al.* [12] find that the force at this maximum scales with the square root of the acceleration, which is explained by the diffusive growth of the boundary layer on the plate.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

- [1] G. L. Eyink, Josephson-Anderson relation and the classical D'Alembert paradox, [Phys. Rev. X **11**, 031054 \(2021\)](#).
- [2] G. L. Eyink, Erratum: Josephson-Anderson relation and the classical D'Alembert paradox, [Phys. Rev. X **14**, 039901\(E\) \(2024\)](#).
- [3] S. Kumar and G. L. Eyink, A Josephson-Anderson relation for drag in classical channel flows with streamwise periodicity: Effects of wall roughness, [Phys. Fluids **36**, 093113 \(2024\)](#).
- [4] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/bcwy-gt4l> for animations of Figs. 1 and 2.
- [5] A. Biesheuvel and R. Hagmeijer, On the force on a body moving through a fluid, [Fluid Dyn. Res. **38**, 716 \(2006\)](#).
- [6] J. M. Burgers, On the resistance of fluids and vortex motion, *Proc. K. Akad. Wet.* **23**, 774 (1921).
- [7] W. R. Graham, Decomposition of the forces on a body moving in an incompressible fluid, [J. Fluid Mech. **881**, 1097 \(2019\)](#).
- [8] S. J. Corkery, H. Babinsky, and W. R. Graham, Quantification of added-mass effects using particle image velocimetry data for a translating and rotating flat plate, [J. Fluid Mech. **870**, 492 \(2019\)](#).
- [9] E. Limacher, J. McClure, S. Yarusevych, and C. Morton, Comparison of momentum and impulse formulations for PIV-based force estimation, [Meas. Sci. Technol. **31**, 054001 \(2020\)](#).
- [10] P. Gehlert, I. Andreu-Angulo, and H. Babinsky, Vortex force decomposition—Forces associated with individual elements of a vorticity field, [Exp. Fluids **64**, 112 \(2023\)](#).
- [11] E. J. Grift, N. B. Vijayaragavan, M. J. Tummers, and J. Westerweel, Drag force on an accelerating submerged plate, [J. Fluid Mech. **866**, 369 \(2019\)](#).
- [12] J. Reijtenbagh, M. J. Tummers, and J. Westerweel, Drag force on a starting plate scales with square root of acceleration, [Phys. Rev. Lett. **130**, 174001 \(2023\)](#).
- [13] E. R. Huggins, Energy-dissipation theorem and detailed Josephson equation for ideal incompressible fluids, [Phys. Rev. A **1**, 332 \(1970\)](#).
- [14] B. W. van Oudheusden, PIV- based pressure measurement, [Meas. Sci. Technol. **24**, 032001 \(2013\)](#).
- [15] K. N. Lucas, J. O. Dabiri, and G. V. Lauder, A pressure-based force and torque prediction technique for the study of fish-like swimming, [PLoS One **12**, e0189225 \(2017\)](#).
- [16] D. E. Rival and B. van Oudheusden, Load-estimation techniques for unsteady incompressible flows, [Exp. Fluids **58**, 20 \(2017\)](#).
- [17] J. C. Wu, Theory for aerodynamic force and moment in viscous flows, [AIAA J. **19**, 432 \(1981\)](#).
- [18] J. Lighthill, Fundamentals concerning wave loading on offshore structures, [J. Fluid Mech. **173**, 667 \(1986\)](#).
- [19] M. S. Howe, On the force and moment on a body in an incompressible fluid, with application to rigid bodies and bubbles at high and low Reynolds numbers, [Q. J. Mech. Appl. Math. **48**, 401 \(1995\)](#).
- [20] J. Li and Z. Wu, Vortex force map method for viscous flows of general airfoils, [J. Fluid Mech. **836**, 145 \(2018\)](#).
- [21] D. S. F. Noa and D. Jeon, A comparison of methods for evaluating time-dependent fluid dynamic forces on bodies, using only velocity fields and their derivatives, [J. Fluids Struct. **13**, 551 \(1999\)](#).
- [22] J. Li, Y. Wang, M. Graham, and X. Zhao, Evaluating unsteady fluid dynamic forces in viscous flows from the vorticity field, [AIAA J. **59**, 22 \(2021\)](#).

- [23] K. Menon and R. Mittal, Significance of the strain-dominated region around a vortex on induced aerodynamic loads, [J. Fluid Mech. **918**, R3 \(2021\)](#).
- [24] S. Otomo, P. Gehlert, H. Babinsky, and J. Li, Vortex map method to estimate unsteady forces from snapshot flowfield measurements, [Exp. Fluids **66**, 64 \(2025\)](#).
- [25] J. D. Eldredge, A reconciliation of viscous and inviscid approaches to computing locomotion of deforming bodies, [Exp. Mech. **50**, 1349 \(2010\)](#).
- [26] E. Limacher, C. Morton, and D. Wood, Generalized derivation of the added-mass and circulatory forces for viscous flows, [Phys. Rev. Fluids **3**, 014701 \(2018\)](#).
- [27] T. A. Driscoll and L. N. Trefethen, *Schwarz-Christoffel Mapping* (Cambridge University Press, Cambridge, 2009).
- [28] M. Gharib, E. Rambod, and K. Shariff, A universal time scale for vortex ring formation, [J. Fluid Mech. **360**, 121 \(1998\)](#).