Investigation of Practical Flight Envelope Protection Systems for Small Aircraft

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Personal air transportation utilizing small aircraft is a market that is expected to grow significantly in the future. For this segment, "stick and rudder" related accidents should be mitigated to guide this process in a safe manner. Instead of downscaling advanced and expensive fly-by-wire platforms that incorporate flight envelope protection found in commercial aircraft, a low cost solution should be considered. This paper focuses on a flight envelope protection system for small aircraft, to allow carefree maneuvering for the less experienced pilot. Preliminary results are obtained from an empirical comparison study in the time domain, between a PID based control limiting approach, a command limiting approach and a constrained Flight Control Law (FCL) approach using Model-based Predictive Control (MPC), with and without parametric model uncertainties. Investigation of the results reveals that, for this study, command limiting and MPC should be preferred over control limiting and that the practicality of command limiting outweighs the small performance increase of MPC.

I. Introduction

In the future airspace, a growth in small aircraft movements is to be expected according to the US Small Aircraft Transportation System (SATS) and the European Personal Air Transportation System (EPATS) programs. The main reason for this growth is due to an increasing demand for people to access more communities in less time. With the introduction of improved and cost efficient technologies, it is even expected to become an attractive alternative to road transportation. In the small aircraft segment, however, fatal and non-fatal accidents are not rare.¹⁻³ Currently, an average number of 6 accidents per 100,000 flight hours dominates the small aircraft segment. As this market is expected to grow significantly in future years, measures must be taken to guide this growth in a safe manner.

By looking more closely at accident analyses, frequent causes can be traced back to poor aircraft handling and pilot decision-making errors.^{2,3} Simultaneously performing the tasks of aircraft handling, communication, navigation, and planning can be rather difficult, especially for less experienced pilots. In terms of aircraft handling, misjudging the coupling of aircraft states and the effects of external disturbances can put pilots in unsafe regions of the flight envelope. In terms of decision-making, ambiguous and conflicting information from the airborne systems can result in poor pilot "situation awareness" (SA) and decision-making.⁴ Controlled Flight Into Terrain (CFIT) is the leading type of fatal accidents in the general aviation sector in 2004.⁵ To resolve these issues, control augmentation techniques can be used to create easy and safe aircraft handling characteristics and new ways of using and presenting information on flight displays can be explored to improve SA and decision-making.^{6–8} This paper, however, only deals with improving flight safety by means of a Flight Envelope Protection (FEP) control system that allows for a "carefree maneuvering" concept for small aircraft.

Commercial aviation has a long history of using control systems to shape ideal aircraft responses. To increase safety, modern commercial aircraft, such as a Boeing 777 and an Airbus A380, are also equipped with a FEP system to protect for stall, exceeding overspeed, limit angle of attack and load factors. However, simply downscaling these advanced FBW platforms for small GA aircraft is not an option as it would significantly increase the cost of such an aircraft. In the Small Aircraft Future Avionics Architecture (SAFAR) program, an ongoing European project, a low cost FBW platform will be developed

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for small aircraft by using technologies originating from the automotive industry.⁹ The work in this paper focuses on the design of a FEP control system by trading off several control strategies adopted by the industry and academia. As such, it provides a preliminary comparison study between these control strategies and their implications on system requirements, performance, cost, and certification.

This paper is structured as follows. First, a review of FEP strategies and their benefits and pitfalls are described. Second, the role of the aircraft model fidelity and accuracy on the FEP design will be dealt with. Then, a test case will be presented utilizing a classic and an advanced control strategy on a low fidelity aircraft model. Finally, discussion of the results is provided, followed by the conclusions.

II. Review of Flight Envelope Protection Strategies

Modern FBW control systems have flight envelope protections that prevent the pilots from entering stateand control regions outside the safe flight regime of the aircraft. Determination of the safe flight regime can be done a priori or during the flight. When the limits are known, there are three different strategies in which the aircraft can be protected against exceeding its safe flight envelope: control limiting, command limiting and using a constrained Flight Control Law (FCL).¹⁰ Irrespective of the protection method, the pilot authority at the boundary needs to be defined. For this two options exist, soft limits and hard limits. In this section a review is given on all these aspects of FEP.

Safe Flight Envelope Definition

The safe flight regime is commonly described in terms of limitations on airspeed, pitch and roll angles, angle of attack, and load factor. Predefining this regime has the advantage that the limits can easily be interpreted and pose little or no sensor requirements. Given the system dynamics $\dot{x} = f(x, u)$, where $x \in \mathbb{R}^n$ denotes the state of the system, $u \in \mathbb{R}^m$ the input and $T \ge 0$ some time horizon. The fixed flight envelope limits, which are limitations on the aircraft state, can be written as:

$$\underline{x} \le x(t) \le \overline{x} \quad \forall t \in [0, T] \tag{1}$$

The largest drawback of using a predefined flight envelope is that in case of failures the limits are not accurate anymore. Due to a failure (e.g. actuator hard over, sensor malfunction, structural damage of the aircraft, etc.) the safe flight envelope will shrink and limits will become tighter. When the old boundaries are still used, it cannot be guaranteed that the aircraft remains in the safe flight regime. Tang *et al.*¹¹ suggest that it should be possible to use multiple predefined flight envelope sets and let the sensor information determine which set is currently used by the FEP system. The flight envelope limits then become a function of time:

$$\underline{x}(t) \le x(t) \le \overline{x}(t) \quad \forall t \in [0, T]$$

$$\tag{2}$$

Tang *et al.* also use a different approach where the safe flight envelope is not defined on forehand, but online through reachability and viability calculations. Let $\mathcal{U}_{[t|t']}$ denote the set of Lebesgue measurable functions from the interval [t, t'] to the set of possible inputs $U \in \mathbb{R}^m$. Given the set of states $K \in \mathbb{R}^n$, the flight envelope can be estimated by gathering the initial states for which:

- There exists a $u(\cdot) \in \mathcal{U}_{[t|T]}$ for which the trajectory $x(\cdot)$ satisfies $x(\cdot) \in K \ \forall \ t \in [0,T]$.
- There exists a $u(\cdot) \in \mathcal{U}_{[t|T]}$ and a $t \in [0,T]$ such that the trajectory satisfies $x(t) \in K$.

Online model identification is used to predict the effect of control surface deflections¹² and guarantee that the aircraft will remain in a safe flight regime. The constraints calculated from the reachable set are then used in the FEP module.

Flight Envelope Protection

When the safe flight envelope limits are known, either through online or offline determination, the FCL can be changed and/or extended in order to ensure that the aircraft stays within these boundaries. This section provides three possibilities on how FEP can be achieved, called: control limiting, command limiting and using a constrained FCL.

In a control limiting setup, an additional block is placed between the flight control law (FCL) and the aircraft as shown in Figure 1(a). It performs two tasks. First, the envelope limits are mapped onto control surface deflection limits. This mapping is non-trivial and can be done in many ways, such as: inverting the input-output relation, using physical functions like a force equilibrium function or approximating the control surface deflection limits by using safe flight envelope margins (i.e. when the aircraft is far away from its limits, large control surface deflections are allowed and when close to the limits control surface deflections is required for all FCLs, i.e. only from the state limits onto control surface deflection limits irrespective of which FCL is selected. The second task of the FEP module is to keep the output of the FCL between the calculated control surface limits,

$$\underline{u}(t) \le u(t) \le \overline{u}(t) \quad \forall t \in [0, T]$$
(3)

where $\underline{u}(t) = m_1(x, \underline{x}, t)$ and $\overline{u}(t) = m_2(x, \overline{x}, t)$ with m_i denoting different mapping functions for each control surface. Reaching these control surface limits is similar to reaching actuator limitations and should be included in anti-integrator-windup schemes or Pseudo Control Hedging¹³ schemes of the FCL.

In a command limiting setup, an additional block is placed between the stick and the FCL as shown in Figure 1(b). The FEP module basically performs the same tasks as for control limiting. First, the flight envelope limits are mapped onto command limits. For each control mode a different set of mapping functions, from state limits onto command limits, is needed. This is a drawback of command limiting. Second, the stick commands are limited before they are fed to the FCL. This has the advantage that no additional integrator windups will occur in the FCLs.

A third way to keep the aircraft within the safe flight envelope is to use a (state) constrained FCL as shown in Figure 1(c). Model-based Predictive Control (MPC) is a perfect candidate for this task, due to its explicit constraint handling capabilities.¹⁴ Originating from the process industry, MPC is capable of keeping multi-variable systems within explicitly defined boundaries while tracking a desired trajectory with high performance.¹⁵ MPC is a collective term for several control algorithms in which a dynamic model of the system is used to predict and optimize future states and needed inputs of the system. At each control interval, the MPC algorithm computes an open-loop sequence of the manipulated variables in such a way to optimize the future behavior of the system. The first value in this optimal sequence is applied as input to the system, and the optimization process is repeated at the subsequent control intervals.¹⁴ This principle is called the receding horizon principle and is presented graphically in Figure 2.

Often MPC is based on a discrete linear system. For these kinds of systems, future predictions can easily be made by simple matrix multiplications and additions. Considering the discrete linear system,

$$x_{k+1} = A(k)x_k + B(k)u_k,$$
(4)

predictions of this system can be written as,

$$\begin{bmatrix} \hat{x}_{k+1|k} \\ \vdots \\ \hat{x}_{k+Hp|k} \end{bmatrix} = \Psi x_k + \Upsilon u_{k-1} + \Theta \Delta U_k,$$
(5)

where

$$\Psi = \begin{bmatrix} A \\ \vdots \\ A^{H_p} \end{bmatrix}, \ \Upsilon = \begin{bmatrix} B \\ \vdots \\ \sum_{i=0}^{H_p-1} A^i B \end{bmatrix}, \ \Theta = \begin{bmatrix} B & \cdots & 0 \\ \vdots & \vdots \\ \sum_{i=0}^{H_p-1} A^i B & \cdots & \sum_{i=0}^{H_p-H_u} A^i B \end{bmatrix}$$
(6)

The optimal sequence of manipulated variables can be found by solving the QP-problem,

$$\min_{\Delta U_k} \Delta U_k^T \left(\Theta^T Q \Theta + R\right) \Delta U_k - \left(2\Theta^T Q E_k\right)^T \Delta U_k \quad \text{subject to: } F \Delta U_k \le c \tag{7}$$

in which Q and R are weighing matrices, $E(k) = T_k - \Psi x_k - \Upsilon u_{k-1}$ represents the tracking error and F and c describe the state, input and incremental input constraints of the system. The outcome of the QP-solver is either infeasibility or the optimal sequence of manipulated variables of which the first



Figure 1. Different strategies for keeping the aircraft within the safe flight envelope.



Figure 2. The Receding Horizon principle of Model Predictive Control. At time k an optimal control sequence, $u_{k+i|k}$, is calculated over a prediction horizon, H_p , in which the control is assumed to be constant after the input horizon, H_u , as shown in the upper part of the figure. Only the first element of this sequence is used as an input to the system, then the horizon is receded over one sample to k+1. The optimization is repeated with new measurements from the system, giving a new and probably different optimal control sequence as shown in the bottom part of the figure.

element can be applied to the system. Depending on the number of states of the system and the number of prediction steps used, the matrices may become very large and solving the QP-problem could be time consuming.

Due to the nonlinearities at the boundaries of the flight envelope, the predictions made using such a linear model are most likely incorrect. Nonlinear MPC exists, but unfortunately it is currently too slow for application in aircraft control.^{16,17} An alternative approach often taken uses Nonlinear Dynamic Inversion^{18,19} (NDI) to obtain almost full linearity of the controllable system, on which a MPC controller can then be applied. NDI is a type of feedback linearization that continuously linearizes the aircraft model by inverting the nonlinear dynamics, causing the combination of NDI and aircraft dynamics to result in a mere chain of integrators. For demonstration, suppose a nonlinear SISO system can be described by^a:

$$\dot{x} = f(x) + g(x)u$$

$$y = h(x)$$
(8)

The control variable y is differentiated with respect to time until it becomes an explicit function of the input. Thus,

$$\dot{y} = \frac{\partial h}{\partial x}\dot{x} = \frac{\partial h}{\partial x}f(x) + \frac{\partial h}{\partial x}g(x)u \tag{9}$$

Feedback linearization is achieved using the control input,

$$u = M^{-1} \left(\nu - l \right) \tag{10}$$

where $M = \frac{\partial h}{\partial x}g(x)$ and $l = \frac{\partial h}{\partial x}f(x)$. An integrator response results from the external input ν to the output y. This linear system can easily be discretized and then used to predict the future behavior of the system in the MPC controller. Figure 3 shows the principle of NDI in a block diagram.



Figure 3. Feedback linearization using Nonlinear Dynamic Inversion

For a low fidelity aircraft model, inverting the nonlinear dynamics may be problematic, since they are simply not known very accurately. Using NDI with a linear fractional representation (LFR) of the uncertainty, it becomes clear that modeling errors can result in incomplete linearization²⁰ and therefore poor performance of the MPC controller. When NDI on the incremental input of the system (also known as Incremental Nonlinear Dynamic Inversion) is used, a large part of the uncertain parameters may drop out, thereby decreasing the sensitivity to parametric uncertainty.²¹ Consider the moment equation,

$$M_{aero} + M_{control} = J\dot{\omega} + \omega \times J\omega \tag{11}$$

Now suppose that a change in control moment has a far greater effect on the angular acceleration than on the angular rate, or in other words that time scale separation may be applied.²² The incremental moment equation can then be written as,

$$\partial M_{control} = J \left(\dot{\omega}_{new} - \dot{\omega}_{cur} \right) \tag{12}$$

In this equation $\dot{\omega}_{cur}$ can be measured and the parametric uncertain M_{aero} has dropped out. Using Equation (12) as a basis for NDI will therefore result in a control law that is less sensitive to parametric uncertainty.

^aMIMO systems can be dealt with similarly.

Pilot Authority at the Boundary

In commercial aviation control systems, there is a very important distinction between the approaches to FEP being taken by Boeing & Airbus. The Boeing 777 has so called "soft" protections, meaning that the crew can override them by using excess force on the control column. So, the protection system will make it more difficult to do something it thinks should not be done, but will always leave the final decision to the crew. The main advantage of soft limits is that pilots can always operate the aircraft at its full capability whenever required. The disadvantage is that less-experienced pilots and not-well-trained pilots can always control the aircraft in unsafe flight conditions. In other words, pilots have to be fully aware of the aircraft limitations. Otherwise, they still have the authority to make things worse.

In contrast, the protections on the A320 are so called "hard" limits that cannot be overridden. That is, you either get switched into an alternate control mode, or your inputs will be ignored. The advantage of hard limits is that the protection system will always keep the aircraft in safe flight regimes and therefore controllable, irrespective of pilot control actions. The main disadvantage is that it prevents the aircraft to be operated at its full capacity, which can also have some serious consequences, however. For example, in the China Airlines B 747 incident 300 nm northwest of San Francisco in 1985,²³ the crew was forced to overstress (and structurally damage) the horizontal tail surfaces to recover from a roll and near vertical dive following an automatic disconnect of the autopilot. At the time of disconnect, full rudder was engaged to one side and the crew was unaware of this. The crew recovered control with about 10,000 ft of altitude left from an original high altitude cruise. It is very likely that if the aircraft had prevented the crew from initiating control commands that would lead to aircraft damage, the aircraft (and passengers) would have been lost.

III. Aircraft Model Requirements

This section provides some remarks on aircraft model requirements related to FEP system design. Why the model should be nonlinear, how a nonlinear model can be obtained and the influence of the mapping functions on the required model accuracy.

Importance of Modeling Nonlinearities

In general much effort is directed into creating an aircraft model suitable for controller design. This is quite logical since a good aircraft model will require a less robust control law and may therefore lead to a better performing controller. It is common practice that controller design is done using linearized aircraft models and classical control theory. A number of operating points are selected within the flight envelope around which the linear approximation is valid. These linearized models are then used in analysis and design tools, such as Root-Locus, Bode, Nyquist, etc. For the design of a FEP system this procedure will probably fail to provide satisfactory results. FEP plays its role at the limits of the flight envelope, where most nonlinear effects are present. Therefore, a full nonlinear model is preferred.

How to Obtain a Nonlinear Model

Linear parameters of a Piper Seneca II model, obtained using flight tests during the cruise phase, were available during this study. In order to create a nonlinear model different methods may be applied, for instance wind tunnel tests, CFD computations or handbook methods based on empirical data. In the 70's the United States Air Force combined a lot of handbook methods into a data companion called DATCOM. Using this program the stability and control derivatives can be estimated based solely on the geometric data of the aircraft. Many small aircraft have quite conventional shapes and fly at low subsonic velocity, which are precisely the conditions for which DATCOM is known to have good results.²⁴ In more recent years DATCOM+ has been created by Bill Galbraith. Instead of the Fortran based output files produced by DATCOM, this program produces xml files. For this paper a MATLAB tool has been created that converts these xml files into SIMULINK blocks embedded in the six degree of freedom nonlinear model setup shown in Figure 4. DATCOM+ provides the parameters for the following aerodynamic model^b:

^bDATCOM+ actually provides the aerodynamic forces in the aerodynamic reference frame, but for clarity both the aerodynamic forces as well as the moments are presented here in the body fixed reference frame. Transformation of the parameters is straight forward and therefore omitted in this text.

$$C_{X} = C_{X_{0}} + C_{X_{\alpha}}\alpha + C_{X_{\delta_{e}}}\delta_{e} + C_{X_{\delta_{f}}}\delta_{f}$$

$$C_{Y} = C_{Y_{\beta}}\beta + C_{Y_{r}}\frac{rb}{2V}$$

$$C_{Z} = C_{Z_{0}} + C_{Z_{\alpha}}\alpha + C_{Z_{\dot{\alpha}}}\dot{\alpha} + C_{Z_{q}}\frac{q\bar{c}}{2V} + C_{Z_{\delta_{e}}}\delta_{e} + C_{Z_{\delta_{f}}}\delta_{f}$$

$$C_{l} = C_{l_{\beta}}\beta + C_{l_{p}}\frac{pb}{2V} + C_{l_{r}}\frac{rb}{2V} + C_{l_{\delta_{a}}}\delta_{a}$$

$$C_{m} = C_{m_{0}} + C_{m_{\alpha}}\alpha + C_{m_{\dot{\alpha}}}\dot{\alpha} + C_{m_{q}}\frac{q\bar{c}}{2V} + C_{m_{\delta_{e}}}\delta_{e} + C_{m_{\delta_{f}}}\delta_{f}$$

$$C_{n} = C_{n_{\beta}}\beta + C_{n_{p}}\frac{pb}{2V} + C_{n_{r}}\frac{rb}{2V} + C_{n_{\delta_{a}}}\delta_{a}$$
(13)

Note that the rudder influence on roll $(C_{l_{\delta_r}})$ and yaw motion $(C_{n_{\delta_r}})$ are not provided by DATCOM and should be added from an alternative source. Figure 5 shows a comparison between the DATCOM results and the linear parameters obtained from flight tests for some of the more important parameters. Note that $C_{X_{\alpha}}$ was reconstructed from the induced drag coefficient C_{X_i} , since C_{X_i} is not provided by DATCOM. It is clear that most parameters match very well, only $C_{m_{\alpha}}$ and $C_{m_{\delta_r}}$ differ by about 50 %.

Mapping Functions and Aircraft Model Fidelity

Another requirement on the aircraft model is posed by the choice in mapping function of the FEP module. When physical functions are used, or when input-output relations are inverted the accuracy of the model needs to be high. Consider the transfer function from elevator deflection angle to pitch angle,

$$H(s) = \frac{\theta(s)}{\delta_e(s)} \tag{14}$$

This relation can be used to calculate the elevator deflection corresponding to the upper pitch angle limit,

$$\delta_e(s) = H^{-1}(s)\overline{\theta}(s) \tag{15}$$

Clearly H should be invertible and it should also be accurate, since there is no correctional term in the equation. Would an inaccurate model be used, the mapping may become inexact leading to the possibility of exceeding the safe flight envelope. Using approximation functions, the mapping is more likely to be robust against small model uncertainties and therefore the fidelity of the aircraft model used may be lower. For example, suppose an elevator deflection limit can be related to the upper pitch angle limit as follows,

$$\overline{\delta_e} = K_p(\theta - \overline{\theta}) \tag{16}$$

This relation is not dependent on model information and is therefore less sensitive to model uncertainty.

IV. Test Case Definition

Using the knowledge on the aircraft model and the benefits and pitfalls of FEP strategies, a comparison study will be done in the remainder of this paper. The performance, defined as the ability to keep the aircraft in the safe flight envelope, of three different FEP systems will be compared in the time domain. In this section the test case for this study is defined. First the aircraft model, FCL and input signal are chosen. Next, the three FEP options are presented and in the end the results of the test case are given.

Aircraft Model, Flight Control Law and Input Signal

For this test case, the dynamic behavior of a Piper Seneca II is captured in a multi-model six degrees of freedom nonlinear mathematical model. The aerodynamic model of the aircraft is created using DATCOM+ as mentioned in the previous section. This model is combined with a mass model and a thrust model supplied by the manufacturer. The aircraft will be trimmed at cruise condition, 120 kts and 6000 ft.

Direct control of this model (and the aircraft) is not very easy. Part of this is caused by the coupling of aircraft states, for example banking to the left will also pitch down the nose of the aircraft. An attitude rate command attitude hold (RCAH) control law, as shown in Figure 6, will be used as FCL. The benefit of using such a control law is that it rejects turbulence and decouples the control of the aircraft states. This allows for easy handling characteristics and may be suitable for less experienced pilots.²⁵ The drawback of this FCL is that it is not stable. This means keeping a certain stick deflection will not result in a new equilibrium situation, but a constantly changing attitude angle. Even keeping the stick centered is not necessarily safe, because speed



Figure 4. DATCOM2SIMULINK, a 6 Degree of Freedom Nonlinear Model Structure



Figure 5. Comparison of important aerodynamic model parameters for a Piper Seneca II

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Figure 6. Attitude Rate Command Attitude Hold Control Law

stability is removed from the dynamic behavior of the aircraft. Without application of FEP this behavior is very dangerous. It should therefore be a good control law for this test case.

In order to test the performance of the FEP systems, a sufficiently large step is given as pitch rate command. This will cause a sharp pull up maneuver during which the angle of attack limit is reached followed by a pitch angle limit. The aircraft will decelerate in this condition until the stall speed limit is reached. After 20 seconds the input will be reversed to check for integrator windups of the control system.

Flight Envelope Protection

The selected control law requires the use of FEP for safe flying. However, before the flight envelope can be protected, it must first be defined. When using low cost sensors, the use of online model identification is effectively prohibited. Therefore a predefined flight envelope limit set will be used in the test case.

The three FEP strategies to compare are: control limiting, command limiting and replacing the FCL by a constrained FCL. For all these strategies a mapping of the state constraints is required. Having low cost sensors affects the choice in mapping functions that can be used. The use of physical functions, such as a force equilibrium or an energy conservation function, require much knowledge on the aircraft and would therefore pose considerable sensor requirements. Using inverted input-output relations is also unwise, since the fidelity of the model used in this test case is not very high. Therefore, approximate mappings will be used.

In the control limiting setup, multiple parallel PID controllers, one for each flight envelope limit, can be used for the mapping of state constraints onto control surface deflection constraint as shown in Figure 7. The output of each PID controller is interpreted as a control surface limit. For instance:

$$\overline{\delta_e} = K_p(\theta - \overline{\theta}) + K_i \int (\theta - \overline{\theta}) dt + K_d \frac{d(\theta - \overline{\theta})}{dt}$$
(17)

Note that in order to prevent suction towards the limit while being in the safe flight envelope, the integral should be kept zero until the limit is reached. When the output of the currently selected FCL exceeds a control surface limit, control is switched to that limit hold controller. Additional switching logic is used to return control authority to the pilot on input reversal and prevent integrator windups in the FCL and in the parallel PID controllers.



Figure 7. Control Limiting using multiple PID controllers

The second candidate used is the command limiting setup shown in Figure 8. In this setup the FEP controller will limit the attitude rate commands fed to the FCL. The mapping of pitch and bank angle limits onto command limits is done using the following logical functions:

if
$$(q_{command} \ge 0 \text{ and } \theta - \theta < q_{command})$$
 then $q_{command} = \theta - \theta$
if $(q_{command} \le 0 \text{ and } \underline{\theta} - \theta > q_{command})$ then $q_{command} = \underline{\theta} - \theta$ (18)

and

if
$$(p_{command} \ge 0 \text{ and } \phi - \phi < p_{command})$$
 then $p_{command} = \phi - \phi$
if $(p_{command} \le 0 \text{ and } \phi - \phi > p_{command})$ then $p_{command} = \phi - \phi$ (19)

To prevent the aircraft from stalling, the upper pitch angle limit can be decreased on approaching the stall speed. This may be done using a hyperbolic tangent function,

$$\overline{\theta} = \overline{\theta} \cdot \tanh\left(c_1 \frac{V - \underline{V}}{c_2}\right) \tag{20}$$

Similar constructions are possible for the over speed and load factor limits. The angle of attack can be protected using the flight path angle and the pitch angle limits.



Figure 8. Command Limiting

In the third and last candidate the PID FCL is replaced by a MPC controller combined with INDI. The INDI control variables have to be chosen carefully since they have effect on the the required mapping of the flight envelope limits onto the controller constraints. Here the state and output vectors of the MPC controller are chosen as follows:

$$\zeta = \left[\begin{array}{c} \phi \\ \theta \end{array} \right], \qquad \nu = \left[\begin{array}{c} \dot{p} \\ \dot{q} \end{array} \right]$$

In this way there the pitch and bank angle limits can be fed straight to the MPC as state constraints. The other aircraft state limits can be protected using the same construction as used in the command limiting setup, i.e. hyperbolic tangent functions. Figure 9 shows the MPC command limit setup in a block diagram.



Figure 9. Replacing the FCL by MPC+INDI

Having defined the three candidates for comparison, only the pilot authority at the boundary remains to be fixed. For this test case the pilot authority at the envelope boundary is defined using hard limits. In that way the system is usable for less experienced pilots, since they are not able to structurally damage the aircraft. Furthermore the use of soft limits would require a force or force feedback stick, which is more expensive.

Results

The performance of the multiple PID control limiting, the command limiting and the MPC FEP systems is shown in Figure 10. This figure also shows the input signal to the system in green and the safe flight envelope limits in red. Figure 11 shows the performance of the same systems, but now using altered aircraft model parameters. Since the test case is defined using a longitudinal input signal, the altered parameters are: mass, longitudinal center of gravity position, some vertical and longitudinal aerodynamic force coefficients and some pitch moment coefficients. For example the mass was increased by 10% and the elevator effectiveness by 40%. The results are discussed in detail in the following section.



Figure 10. MPC vs. PID Control Limit vs. Command Limit



Figure 11. MPC vs. PID Control Limit vs. Command Limit, sensitivity analysis

V. Discussion

Performance

The aircraft responds to the earlier defined input using the different FEP systems is shown in Figure 10. It should be noticed that the PID control limiting setup slightly overshoots the envelope limits. This is quite logical since each controller becomes active after the limit is reached and acts only as a hold controller. When using control limiting the safe flight envelope must therefore be chosen conservative. In the command limiting setup the controllers already act when approaching the limit and prevent overshoots. This results in a very smooth intercept of the limits, which is preferable for ride comfort and pilot awareness of approaching flight envelope limits. There is barely any difference between the command limiting setup and MPC setup. The only difference being that MPC is slightly more aggressive, which is a tuning choice. All three controllers do not suffer from integrator windups and control is returned to the pilot immediately after input reversal.

Sensitivity

Not only good performance is required, also robustness against modeling errors is crucial for practical use of the FEP system. Figure 11 shows the performance of the controllers in the presence of modeling errors. The PID control limiting setup is reasonably robust, but does get influenced by the uncertainties. Limits are exceeded slightly further and limit transitions, such as from pitch limit to stall speed limit, oscillate more. This fact can be explained by realizing that the mapping from state limits onto control surface deflection limits is largely dependent on the control surface effectiveness. Modifying the control surface effectiveness alters the state change caused by a control surface deflection and would therefore require a change in mapping. The command limiting controller also suffers slightly from the uncertainties, which can be seen during the transient initial response. Performance of the command limiting controller is much better than that of the control limiting PID controller however. The MPC controller does not suffer at all from the uncertainties. The tracking term in the INDI controller makes sure that the system is still linearized and the MPC controller continuously changes the effective controller gain in the optimization function.

Implications for Small Aircraft

The cost of a FEP system is largely determined by the sensor requirements and certification costs. Using predefined flight envelope limits and approximate mapping functions results in low sensor requirements and therefore potential cost savings. Also from a certification point of view the predefined flight envelope limits are advisable. Certification also plays a role in the choice between application of MPC and command or control limiting. MPC has a lower functional visibility, mainly due to the QP-solver, and therefore requires a more extensive certification procedure than the other controllers.²⁶ A smaller part of the total cost is determine by flight control system hardware. Using MPC poses a much higher demand on processing power than using PID controllers, which means more expensive hardware is needed.

Future Investigations

The ultimate goal of a practical FEP system is of course to be used in real flight. In the SAFAR project, the objective is to install low cost fly-by-wire technology in a DA-42. On this platform, a RCAH FCL combined with a FEP system will aim to make flying this small aircraft easier and more safe. This paper contributes to the selection of that FEP system.

In a real flight setup, the time delays of individual sensors will affect the total time delay of the platform. It should therefore be studied what the impact of time delays is on the application of FEP. Furthermore, turbulence and sensor noise act as a disturbance on the FEP system. For successful application, the impact of these disturbances should be investigated as well.

VI. Conclusions

This paper presents a study of FEP techniques for small aircraft. A comparison is made between a PID based control limiting approach, a command limiting approach and a constrained FCL approach using MPC. All three controllers perform well when there are no model uncertainties present. Full certainty is hardly the case however when a low cost system is considered. Model parameters are altered up to 40% and the test case is rerun without adapting the controllers. Command limiting proved far less sensitive to the aircraft model changes than the control limiting setup, but MPC is influenced least of all. MPC does have more stringent hardware requirements and is more difficult to certify than the other solutions. Since the performance gain of MPC over command limiting is small, command limiting is believed to be the preferred option for the test case chosen.

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