Analogy between Tides and A.C. Electricity

By Dr. JOHAN VAN VEEN*

No. I

Mathematical Considerations

It is generally accepted that the precise calculation of tides in new channels is difficult. In the first part of this article an attempt is made to show the truth of this statement. In the second part a simple analogy from which elementary tidal calculations can be made is explained. This leads in the third part to electrical experimental solutions. For the deduction of the formulæ see the bibliography at the end of the article.

Apart from some early attempts to solve the problem of tides in a network of channels, Professor H. A. Lorentz, of Leiden, in 1918 was the first to introduce two practical and scientific formulæ :

$$\partial s/\partial x = -B\partial h/\partial t \quad \text{(continuity equation)} \quad . \quad (1)$$

$$1/gbH \, . \, \partial s/\partial t = -\partial h/\partial x - ks/bH \quad \text{(dynamic equation)} \quad . \quad (2)$$

where

s=flow through cross-section in cubic metres.

x=distance along axis of tidal channel.

B=breadth of river at mean level=capacity or fill breadth.

h=depth of channel section at any moment.

t=time.

g=gravitational acceleration.

b=stream breadth in cross-section.

H=depth below mean level.

k=linear resistance coefficient (constant).

These equations, which have to be used together, start from the idea that two sinusoidal waves run in opposite directions at the same time. No attempt will be made to explain them here—the explanation can be found in detail in the book of the Zuiderzee Committee[1], of which Lorentz was chairman. It took eight years' mathematical calculation to solve the problem of the Zuiderzee tides with these formulæ.

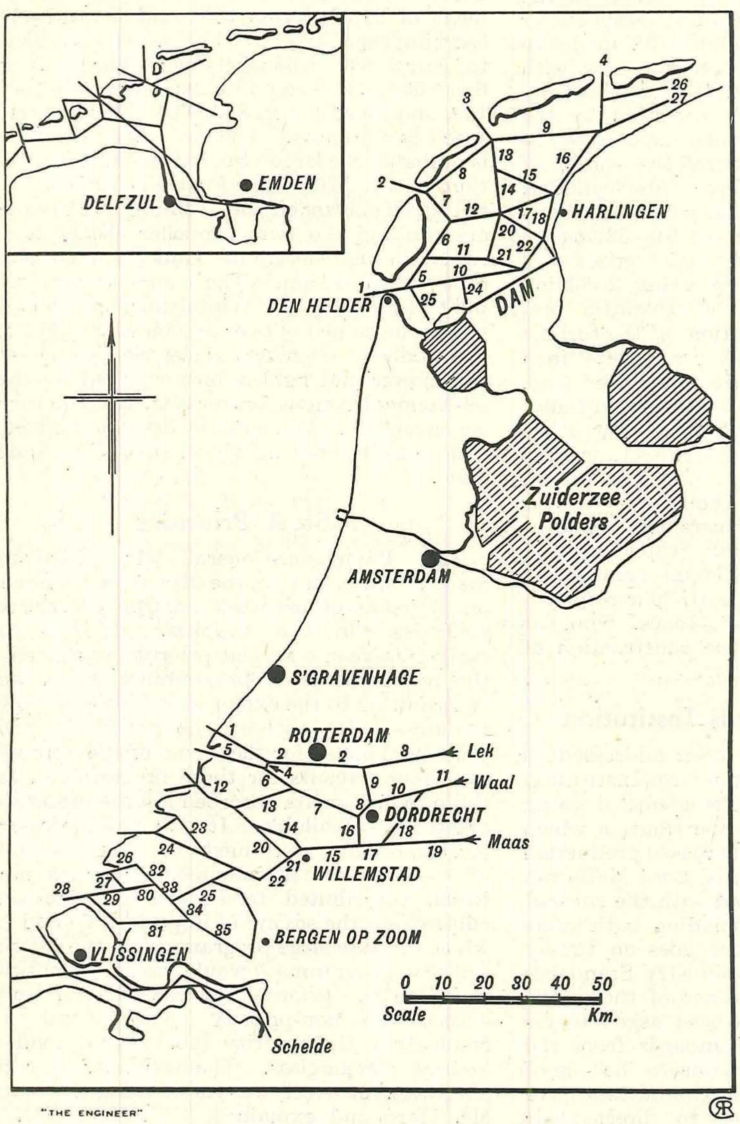

Fig. 1—Tidal Networks

The simplifications which had to be accepted were the sinusoidal tidal waves and a linear relation between stream velocity v and friction R, $R_1 \propto kv$.

The right relationship is, however :

$$R \propto v^2/C,$$

where C=Chezy's constant.

By assuming the total amount of work of both R_1 and R per tidal period to be the same, Lorentz could keep his results within bounds while keeping his equations linear and therefore solvable. In practice his ideas proved to be right. The tide after the enclosure of the Zuiderzee increased as was calculated, while the currents in the Frisian inlets increased by about 20 per cent as predicted by calculation. Superficial reasoning might have led to the faulty conclusion that these currents would have weakened after such a large area of the tidal basin had been cut off.

The Lorentz equations could not be used with exactness in tidal river mouths because of the slope which the river discharge creates in them and upon which the tidal wave travels. An extension to these equations was made by Dr. J. P. Mazure, engineer of the Zuiderzee works, which resulted in the following formulæ :—

$$\partial s/\partial x = -B\partial h/\partial t \quad . \quad . \quad . \quad (3)$$

$$\partial z/\partial x = -1/gf \, . \, \partial s/\partial t + s/gf^2[\partial f/\partial t + b\partial h/\partial t] + s^2/gf^3 \, . \, \partial f/\partial x - |s|s/C^2f^2R \quad . \quad . \quad (4)$$

where

z=height of water level above mean level.

f=area of cross-section.

$|s|$=flow regardless of direction. The other symbols have the same meaning as before.

Again, these formulæ are for purely sinusoidal tides and also with the linear relationship $R_1 akv$. Recently Mr. H. J. Stroband, of the Rijkswaterstaat improved this method so that M_2, M_4, s . . . sinusoids (the constituents of the vertical tide) can be calculated with high accuracy.

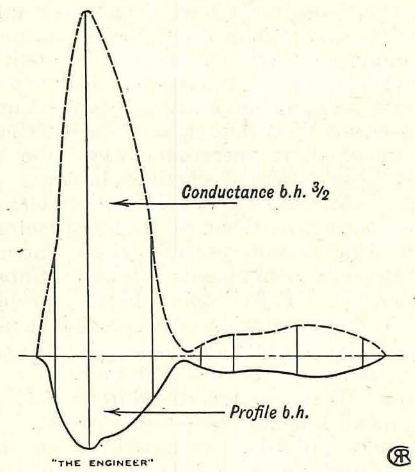

Fig. 2—Relation between Depth of Channel and Conductance

For still greater accuracy the Tidal Research Bureau of the Rijkswaterstaat developed a third set of formulæ, generally referred to as the *exact* ones. They are, of course, not quite exact. By using the Fourier series the mathematician, Dr. J. J. Dronkers, brought in the quadratical relation $RaCv^2$, and the natural non-sinusoidal tides. These exact formulæ are :

$$s_1 = s + Bx\frac{\partial h}{\partial t} \pm s\frac{x^2B}{C^2b^2h_o^3} \cdot \frac{\partial s}{\partial t} + \frac{x^2B}{2bgh_o}\,\frac{\partial^2 s}{\partial t^2} \qquad (5)$$

$$h^1 = h + \frac{|s|sx}{C^2b^2h_o^3} + \frac{x}{gbh_o} \cdot \frac{\partial s}{\partial t} + |s|\frac{\partial h_o}{\partial t} \cdot \frac{x^2B}{C^2b^2h_o^3} - s\frac{\partial h}{\partial t} \cdot \frac{(b+B)x}{b\,gh_o^2} + \frac{Bx^2}{2bgh_o} \cdot \frac{\partial^2 h}{\partial t^2} \pm \frac{B^2x^3}{3C^2b^2h_o^3}\left(\frac{\partial h}{\partial t}\right)^2 \quad . \quad . \quad . \quad . \quad . \quad (6)$$

where

h_o=depth at beginning of channel section.

and

h^1=depth at end of channel section.

The calculations with these formulæ proceed from hour to hour throughout the tidal phase. A " starter " is needed to begin the calculations. Usually the results of calculation by formulæ (1) and (2) or (3) and (4) are used as a " starter."

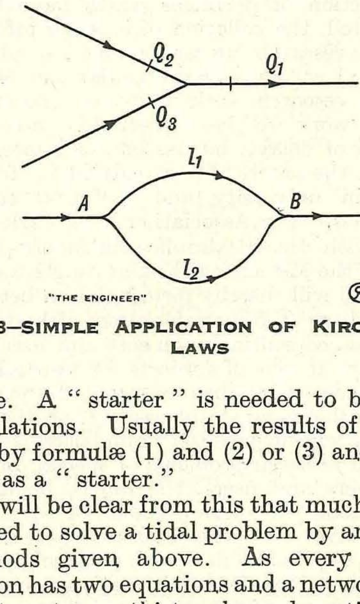

Fig. 3—Simple Application of Kirchhoff's Laws

It will be clear from this that much work is needed to solve a tidal problem by any of the methods given above. As every channel section has two equations and a network often has twenty or thirty channel sections (see Fig. 1), for each problem forty or sixty

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[1] H. A. Lorentz, *Verslag van de Staatscommissie* 1918 *ter aftsluiting van de Zuiderzee, Den Haag, Algemeene Landsdrukkerij*, 1926 ; 336 pp.; 61 fig.

equations have to be solved. Each scheme may need thirty different calculations, so that for some projects 300,000 man-hours of calculation for tides alone must be faced. Thirty years of solving such calculations has shown that the theory of tides can be mastered, but that the application of such methods is very arduous. Readers who may be interested in this problem should refer to the bibliography at the end of this article.

The Analogy

In the latest development there is a return to simplicity by the use of electrical methods.

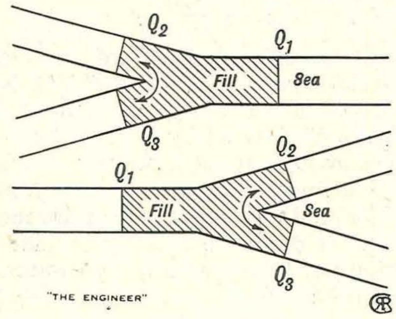

Fig. 4—Fill Area

Perhaps this should have been the original approach. The laws of Nature are often simple if solutions of extreme exactness are not sought. Intricate formulæ are not for planners. Civil engineers should apply easy and quick formulæ for tides. If ultimate exactness is needed for the final plan a trained mathematician can do the job, or electrical imitation can be made.

The main conceptions of the analogy useful for gaining fundamental tidal information are as follows :—

(1) *Direct Currents*.—A non-tidal river, flowing in one direction only, can be compared to an electrical conductor through which direct current is fed.

(2) *Alternating Current*.—A tidal channel in which the ebb and flood streams go to and fro resembles an electrical conductor through which alternating current pulsates.

(3) *Tidal River Mouth*.—A channel serving the outflow of a river as well as the tide is analogous to a conductor fed with direct current at one end and alternating current at the other.

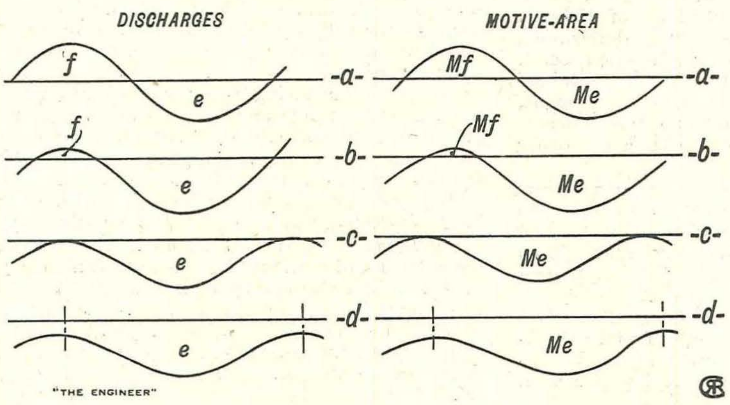

Fig. 5—Discharge and Motive Area

(4) *Conductance*.—As a thick wire has more conductance than a thin wire, so a river or channel with a large cross section has more conductance than a smaller river or channel. The hydraulic conductance for a rectangular cross section is proportional to the 3/2 power of the depth h, and is directly proportional to the breadth b. For such a cross section engineers use the expression conductance$=\Sigma bh^{3/2}$. Conductance is influenced more by depth than by breadth. A cross section of 800×1m has the same conductance as a cross section 100×4m, though the area of the first is double that of the second. When a channel is deepened to twice the original depth the conductance becomes three times greater (see Fig. 2).

(5) *Resistance*.—A channel has a resistance in the same way as a conductor. This is proportional to the length of the channel or conductor and is reciprocal to the conductance. For a river section of 1m length engineers use the expression $R=1/bh^{3/2}$.

(6) *Potential*.—In electrical science potential means the energy above some initial datum—the earth. In hydraulics we might call the height of a water level above the initial plane—the mean sea level, for instance —the potential. The expression " potential difference " is the hydraulic " head," *i.e.*, the difference in height of the water surface at two different points of a river. The term " potential gradient " means slope.

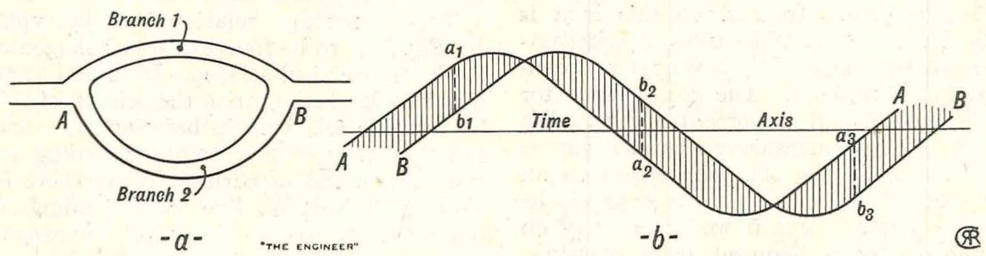

Fig. 6—Kirchhoff's Laws and Motive Area

(7) *Motive Power*.—The motive power of river streams is gravitational, expressed in terms of " slope " or hydraulic gradient. Also, the tidal streams of tidal inlets are promoted by the force of gravity. The moon and sun create the rise and fall of the tides in the ocean but we can accept the movement of the tides at the river entrances as the starting point for our considerations. All that need concern us in this matter is the fact that the vertical movement of the sea level at the river entrance creates changing slopes and these slopes cause the current to run in or out. A theory exists which considers tides as the resultant of waves proceeding and then being cast back ; that is, as two waves moving in opposite directions. This theory is often used in mathematics and is also valid. There is no essential difference between this view of the problem and the more simple consideration of a single wave entering the channel mouth and creating movement of the water in or out of the river by reason of the changing gradients imposed.

(8) *Ohm's Law*.—The simple relation which exists between potential, resistance and a continuous current is electrically expressed in Ohm's Law : $I=E/R$. That is, for a conductor of unit length, the current is proportional to the potential drop and the conductance. In hydraulics the law is similar except for the root sign.

$$\text{Current}=\frac{\sqrt{\text{hydraulic gradient}}}{\text{resistance}},\ \text{referred to}$$

as Chezy's Law.

With a current Q (cubic metres per second), a gradient α, and a resistance $1/Cbh^{3/2}$, we get the well-known Chezy formula for wide channels :

$$Q=Cbh^{3/2}\sqrt{\alpha}.$$

This formula applies to simple rectangular channels. For trapezoidal channels the formula reads :

$$Q=C\theta\sqrt{R\alpha}$$

where θ is the area and R is the hydraulic radius of the cross-section. C is the well-known constant, with a value of about 40 or 60 in metric units.

(9) *The Laws of Kirchhoff for Direct Currents*.—At a junction of two ordinary rivers (Fig. 3) the total flow per second in the three channels must obey the equation $Q_1=Q_2+Q_3$.

When there is an island in a river the potentials along both branches must be the same. Therefore, $a_1l_1=a_2l_2$, if a_1 and a_2 are the slopes and l_1 and l_2 are the lengths of the two branches. It is quite easy to calculate the currents in these branches with the two formulæ of Kirchhoff and Chezy (or Ohm's) Law.

(10) *Kirchhoff Laws for Alternating Currents*.—In electricity the simple laws of Kirchhoff hardly change when alternating currents are considered in place of direct currents. The same is true in hydraulics, but it must be noted that measurements never can be made at the exact junctional point. A certain area Z (the fill area) remains between the lines of measurement ; this area is shown hatched in Fig. 4. For each tidal period our first law of Kirchhoff therefore becomes $Q_1=Q_2+Q_3+2ZA$, when the tide comes from the right and A is the tidal amplitude. When the tide comes from the left we get $Q_1=Q_2+Q_3-2ZA$. The Q's are the sums of ebb and flood per cycle. The factor 2 is for filling plus emptying.

If we try to check these simple formulæ we find they differ from nature, because of the currents going from branch 2 into branch 3 and *vice versâ* when the tide turns. This is due to the difference in phase of the two streams in the two branches. The same occurs with electrical currents. When the phase differences are large we must not neglect these " corner " currents.

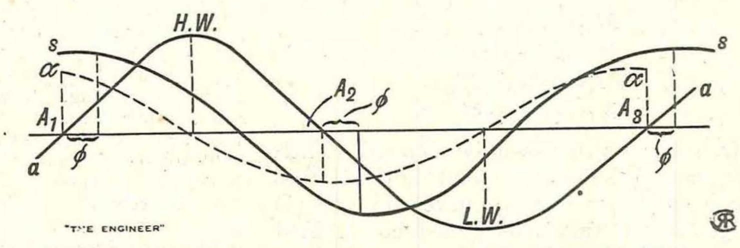

Fig. 7—Hydraulic Analogue of Self Induction

Kirchhoff's second law for alternating currents means that there is one tidal wave at A and also one tidal wave at B (see Fig. 6*a*). The propagation of the tide is such that the *time of propagation* along both sides of an island is the same. The theory that the propagation is proportional to $\sqrt{gh}$ is not considered here, since it is important to remember that this theory is only true when two opposing sets of tides are considered.

If the tidal graphs A and B are drawn on the same time axis (Fig. 6*b*), the vertical distance at a given time is a_1b_1, and this is the head between A and B if the distance between these two points is not too great. Later on the head is negative, *i.e.*, b_2a_2. The negative is the ebb-head while the positive head a_1b_1 is the flood-head. Near high and low water the heads become zero temporarily. When the tidal line B is subtracted from the tidal line A and both are sinusoids, there remains a new sinusoid, differing about 90 deg. in phase with the tidal lines. This sinusoid can be referred to as the "head graph" between A and B.

The area of the "head graph" is called the *motive area.* In Fig. 6*b* this area is hatched vertically. With tides (or alternating currents) the motive power is not constant, but sinusoidal. The motive area for any channel is an important factor. We can obtain it by measuring the tidal curves at both ends of the channel section on an exact time basis. The tide-recorder clocks must keep exact time if we are to rely on the motive areas deduced from measurements taken by such tide gauges; a minute's discrepancy may be too much.

It will be apparent that there is a very close relationship between the velocity of propagation of a tide and the motive area. It should be understood that our velocity of propagation is *not* proportional to $\sqrt{gh}$, but that it is, by our definition, the natural propagation velocity obtained from tidal measurements.

If the total flood running through a section of tidal channel is f and the total ebb is e, it follows that the total flow per tide through the cross-section is $Q=f+e$ (Fig. 5). The same is the case with the motive areas,

$$M_{AB}=M_f+M_e.$$

If there is a river discharge we obtain figures as indicated by the curves (b, c and d) in Fig. 5. The ebb quantity increases while the flood quantity decreases, when the river brings down more upper water. Making P the river discharge per tidal period (44,700 seconds) the following relationship holds:

$$Q=e+f$$
$$P=e-f$$
$$e=\tfrac{1}{2}(P+Q)$$
$$f=\tfrac{1}{2}(Q-P)$$

In the same way:

$$M_q=M_e+M_f$$
$$M_p=M_e-M_f$$
$$M_e=\tfrac{1}{2}(M_p+M_q)$$
$$M_f=\tfrac{1}{2}(M_q-M_p)$$

Far from the sea there will be no flood. Then the curves of the currents and motive forces become as shown by the curves (c) and (d) in Fig. 5. The tidal cycle must be retained in all definitions.

(11) *Self-Induction.*—In the theory of a.c. electricity the conception of self-induction and power factor is important. If I_m represents mean current, E_m represents mean electromotive force and R_m resistance (the current and voltage wave-forms being sinusoidal), then

$$I_m=E_m \cos \phi/R_m.$$

In this expression ϕ denotes the angle of lag and $\cos \phi$ (the power factor) takes into account the effect of self-induction. A typical value for $\cos \phi$ might be 0·9.

What is the hydraulic analogue to self-induction? It is the inertia of the water masses which prevent these masses ceasing to flow as soon as the slope reaches zero. Also the maximum flow is not attained when the maximum slope occurs, but after a phase delay of ϕ. In the science of tides there is the same angle of lag as there is in electricity. Usually the value of the factor $\sqrt{\cos \phi}$ is also about 0·9 in channels. In open seas it is much less, because of the huge masses and low friction.

The relation between the tidal curves (vertical tide a), the curve of gradient or slope (motive curve α), and the stream curve (horizontal curve s) in a particular cross-section of a river mouth is shown in Fig. 7.

The slope line α passes the axis or zero line at or near high and low water and reaches its maximum at A_1, A_2 and A_3—where the a line (or vertical tide) is zero. The stream line s lags ϕ behind the α line.

This important relationship, as typified by Fig. 7, is to be found in every elementary book on electrical science. It should appear in all books dealing with the science of tides. The simple relationship between a_1, α and s must be clear to any engineer dealing with tidal rivers and estuaries. When there is a slope in the H.W. line in a channel, the difference in phase of the α and a lines is not exactly 90 deg.

(12) *Ohm's Law for Tidal Currents.*—The formula $I_m=E_m \cos \phi/R_m$ for a complete cycle of electrical current has the following formula as its equivalent for a cycle of tidal currents:

$$Q=C_mb_mh_m^{3/2}\sqrt{\alpha_m T}\cos \phi. \quad . \quad (7)$$

The deduction is analogous to the electrical equivalent when the Lorentz constant is used. Here the use of C necessitates deduction, partly by analogy. This is not merely introducing a factor of self-induction into the Chezy formula; the new formula is meant for a complete cycle of T=44,700 seconds. Therefore the quantity $\alpha_m T$ is not a certain angle or slope but a motive area, or rather a mean motive gradient. If the motive area between two stations is M and the distance is l, then the motive gradient per cycle is M/l and

$$\alpha_m=\frac{M}{44{,}700l}$$

Further, the mean breadth of the wetted area of the channel is b_m, while h_m is the mean depth during the cycle. If an attempt is made to check this new formula by means of total flow measurement and by measuring the motive gradient as accurately as possible, it will be found that it gives accurate results. The value of ϕ can be roughly obtained by measuring the time between high water and the following slack water, or the time between the point of low water and the following slack water. It should be noted that the formula is for a single profile only, not for a channel section.

(To be continued)

520 544

The Iron and Steel Institute

No. II—*(Continued from page 488, November 21st)*

THE remainder of Wednesday morning's session was devoted to a joint discussion on the following papers:—

BRITTLE FRACTURE IN MILD STEEL PLATES

By W. Barr, A.R.T.C., F.I.M., and Constance F. Tipper, M.A., D.Sc.

Synopsis

The temperature range of transition from tough to brittle fracture of mild steel plates of different carbon and manganese contents was determined by means of notched-bar impact, notched bend, and notched tensile tests. The results obtained by each of these tests were in good agreement, except that for very soft steels the notched tensile test gave a lower transition range than the other two tests.

It was found that the transition range is raised by an increase in the ferritic grain size, by an increase in plate thickness, and by slow cooling after normalising. It was also found that a high notched-bar impact value may be accompanied by a fracture which is mainly cleavage.

Tentative conclusions, subject to confirmation, have been reached that the effects of plate thickness and slow rates of cooling in raising the transition range are reduced in mild steel plates with higher manganese contents.

EFFECT OF THE MANGANESE/CARBON RATIO ON THE BRITTLE FRACTURE OF MILD STEEL

By W. Barr, A.R.T.C., F.I.M., and A. J. K. Honeyman, B.Sc., F.I.M.

Synopsis

A series of four mild steels was made in which the only significant variable was the manganese/carbon ratio. The residual elements were low and the tensile strengths were approximately the same for each steel. The notched-bar impact properties of these steels in the annealed and in the normalised conditions have been determined. It was found that increasing the manganese/carbon ratio lowers the range of transition from tough to brittle fracture, increases the impact values at all temperatures, and tends to result in finer McQuaid-Ehn and ferritic grain sizes. A practical recommendation is made that for structural steels for shipbuilding purposes, the manganese/carbon ratio should be not less than 3·0.

SOME FACTORS AFFECTING THE NOTCHED-BAR IMPACT PROPERTIES OF MILD STEEL

By W. Barr, A.R.T.C., F.I.M., and A. J. K. Honeyman, B.Sc., F.I.M.

Synopsis

Work carried out to confirm and extend conclusions stated previously by the authors on the influence of carbon and manganese on the notch sensitivity of mild steel is described. Steels were made to carefully controlled compositions and their structures and impact properties in the normalised and annealed states were determined. It was shown that increasing the carbon content raises the transition range and lowers the impact values of steels in the normalised condition, while increase of manganese content has the opposite effect. It was confirmed that an increased manganese/carbon ratio tends to give a steel with a finer McQuaid-Ehn grain size and a finer structure in the normalised condition. A comparison of steels made with and without a grain-controlling addition of aluminium showed that the manganese/carbon ratio may be as effective as grain-size control in reducing notch sensitivity. A study of the relationship between the notched-bar impact value and the degree of cleavage in the fracture showed that the latter is not the best criterion by which to judge notch sensitivity. Material which gives a predominantly cleavage fracture will be satisfactory in service, provided that an appreciable amount of energy is absorbed before a crack is initiated.

Introducing the papers, Mr. Barr explained that the origin of all this work was the failures that were experienced with the American "Liberty" ships during the war.

Discussion

Mr. H. H. Burton (English Steel Corporation) remarked that the only heat treatment that had been considered—and this might be for reasons with which he was not familiar—was normalising. Those who had done a lot of work on some of the higher manganese pearlitic steels in which the manganese

Analogy between Tides and A.C. Electricity

By Dr. JOHAN VAN VEEN*

No. II—(*Continued from page 500, November 28th*)

(13) *Condenser and Capacity.*—An open harbour with an area Z can store an amount ZA of water if A is the amplitude of the tidal rise. Per cycle the amount of water flowing through its mouth is $2ZA$ cubic metres. An open harbour is a condenser and ZA is its capacity. A tidal channel has a capacity of its own in the same way as an electrical wire has a capacitance of its own—" spread " capacitance, in electrical parlance. For a

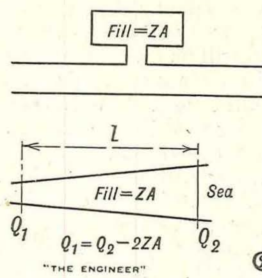

FIG. 8—CAPACITY AND FILL FORMULA

stretch of tidal channel l metres long the following relationship exists per cycle (Fig. 8):

$$Q_1 = Q_2 \pm 2AB\,l\cos\phi. \quad . \quad . \quad (8)$$

This is the " fill formula " in which B is the mean breadth of the channel-fill. This B is always larger than the stream breadth b, as may be seen from Fig. 9. The factor $\cos\phi$ means that the fill of a tidal channel is not to be taken between the levels of high water and low water, but between the levels at the time of slack water. However, our measurements are usually not sufficiently accurate to check this formula in such a way that the influence of $\cos\phi$ will be noticeable. $\cos\phi$ is usually about $0{\cdot}8$ and $\sqrt{\cos\phi} = 0{\cdot}9$, as previously mentioned. For open harbours the value becomes 1. For very deep channels or seas $\cos\phi$ is about 0, when the capacity does not count at all.

The simple fill formula follows from the so-called continuity equation, when inte-

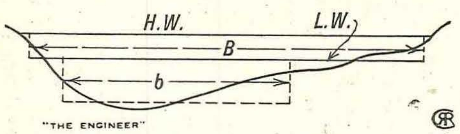

FIG. 9—MEAN BREADTH AND STREAM BREADTH

grated over a whole cycle. Electrical engineers cannot neglect the " spread " capacities of long conductors like telegraph or telephone cables. They have to use the " telegraph equation," which is analogous to our Lorentz equation (see page 498).

(14) *Wheatstone Bridge.*—For ordinary rivers or direct currents we may consider a river with two mouths as shown in Fig. 10. When an open channel is made between two points A and B having the same potential, or the same level, there will be no flow in that channel AB; it is a balanced Wheatstone bridge.

For alternating currents in tidal channels we may consider a case of two neighbouring channels, AA_1 and BB_1 (Fig. 11), with two stations C and D having the same tidal amplitude and the same phase. No motive force exists between A and B; there are therefore no streams except the filling ones of CD. We often meet with channels in tidal networks which approach the Wheatstone bridge principle. These channels are more or less parallel with the coast. Improvements at any place in such a network always act severely on the streams in these " bridges " which then become liable to many changes and to silting. Thus, if the channel CA is improved (Fig. 11) then channel CD may become almost devoid of currents, or, alternatively, its currents may become much larger, according to the change in tidal phase at station C. We never find a purely balanced Wheatstone bridge with the vertical tides at the ends exactly alike and in phase, but there are many channels which come near to the dangerous condition where silting is likely to occur.

(15) *The Vitality Factor.*—Because of the " spread " capacity of tidal channels the cross-sections of these must become larger towards the sea. The shores of tidal channels often have—or need to have—a flare. Equation (7) is useful when considering a single cross-section, but in order to be able to deal with a channel length with varying cross-sections we have to introduce the term " vitality factor," F given by

$$F = Q/bh^{3/2}$$

For normalising a tidal channel this factor F should be a constant for all profiles of the channel. It means that the relation between

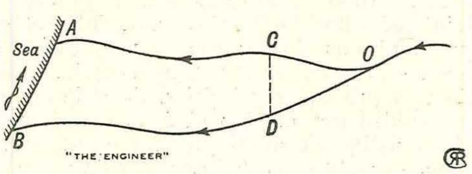

FIG. 10—WHEATSTONE BRIDGE

the tidal total flow per cycle and the conductance should be constant in that particular channel. From equation (7) it follows that:

$$F = C\sqrt{\alpha_m}\,T\cos\phi. \quad . \quad . \quad (9)$$

so that a constant F in a channel (Fig. 12) implies a constant mean gradient α_m (or a constant velocity of propagation), since C and ϕ vary little.

Measurement in any tidal network of channels shows that nature has made F constant, or nearly so, when the bottom sand varies little and the channel is not too sinuous. There are, of course, large differences from one profile or cross-section to another, but as a whole the value of F remains constant in any stretch of the channel (with metric units, about 7000 to 12,000). A high value of F shows tendency to scour and a low value is a sign that there may be silting. Yet there is an exception to this rule: sand streams can upset scouring tendencies, and generally F is highest on a bar.

The three graphs—total flow, conductance and vitality—are usually as shown in Fig. 12. The first two increase while going downstream, but the vitality remains more or less constant. In a network of tidal or non-tidal channels we cannot change the value of F in any one channel without affecting the values of F in all the other channels. It is, of course, important to have a network with good vitality factors because silting and scouring are dependent upon it. Bars cause local troubles near the knots in such networks, due to irregular sandstreams.

(16) *The Flare Formula.*—Dr. Herbert Chatley gave in No. 71 of the Selected Engineering papers of the Institution of Civil Engineers the following formula:—

$$\lambda = 2000B^2A/Q,$$

in which λ is the flare per kilometre and Q the total amount of flow in a channel profile per cycle. This formula is for a constant mean depth h over the whole length of the channel and can be obtained from the fill formula, thus:

$$Q_1 = Q_2 + 2AB\,l\cos\phi$$

This may be written

$$C_1B_1h_1^{3/2}\sqrt{\alpha_1\cos\phi_1} - C_2B_2h_2^{3/2}\sqrt{\alpha_2\cos\phi_2} = 2ABl\cos\phi.$$

In this B is taken to be the same as b (see Fig. 9). Assuming that $C_1 = C_2 = C$, and

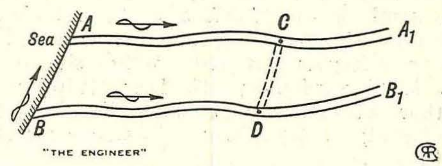

FIG. 11—WHEATSTONE BRIDGE—TIDAL CHANNELS

$\alpha_1 = \alpha_2 = \alpha$, and $\cos\phi_1 = \cos\phi_2 = \cos\phi = 1$ and $h_1 = h_2 = h$, we obtain

$$(B_1 - B_2)Ch^{3/2}\sqrt{\alpha} = 2AB\,l,$$

or $\lambda = B_1 - B_2/l = 2000 \quad B^2A/Q$ metres per kilometre. This formula is often too simple, since we have to remember that b differs from B and h_1 from h_2. Generally we need a constant $F = Q/bh^{3/2}$, and we get

$$b_1h_1^{3/2} - b_2h_2^{3/2} = (2000\,AB\cos\phi)/F \text{ metres per kilometre} \quad . \quad . \quad . \quad (10)$$

This formula only differs from Chatley's inasmuch as it works with conductivity and not with a constant depth $h_1 = h_2$. Equation (10) means that the growth in conductance must be proportional to the mean fill breadth B and to the double tidal rise × $\cos\phi$ while being inversely proportional to the vitality

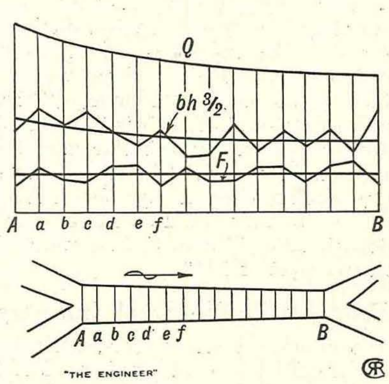

FIG. 12—VITALITY FACTOR

factor. The flare can therefore be taken according to wish or circumstance, the depth being calculated accordingly.

(17) *Propagation of the Tidal Wave.*—As has been said before, the propagation velocity of the tidal wave has a close relationship to the motive area and therefore to the currents. This may need further explanation.

If there is uniform propagation (Fig. 13), the tide line in A has exactly the same form as the tide line in B, but not the same phase. In that case the horizontal lines $a_1b_1 = a_2b_2 = a_3b_3$, &c. The time of propagation T is the same for all points of the vertical tide. If the distance between the two stations

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A and B is l, then the velocity of propagation is

$$V = l/T \text{ metres per second.}$$

If the propagation is not uniform the velocity of propagation is not a constant for all points of the tide. We may, however, take an average (Fig. 14).

V (average) $=(A_1+A_2)l/M$ metres per second, since the total area between the tidal curves of station A and station B is the motive area M and A_1 and A_2 are the tidal rises therein.

The relation between the motive force and

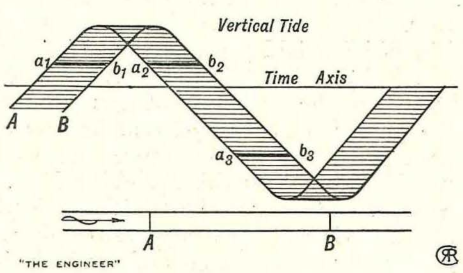

Fig. 13—Uniformly Propagated Tidal Wave

the propagation of the vertical tide is very close indeed. It is:

Propagation area AB = motive area AB.

When the propagation of the tidal wave is rapid then the slopes are small and the currents are relatively weak. If the propagation of the tide is difficult and slow the slopes are steep and therefore the currents rapid. We can see, therefore, on any map showing the co-tidal lines the points where fierce currents will occur. They are where the co-tidal lines are close together.

(18) *Propagation Governed by Ohm's and Kirchhoff's Laws.*—The formula $v=\mu\sqrt{gh}$, which assumes that the velocity of pro-

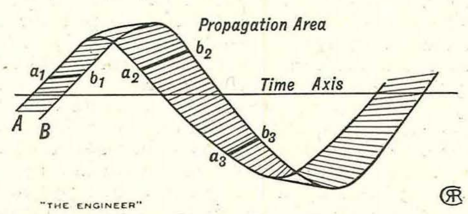

Fig. 14—Non-Uniform Propagation

pagation is proportional to the root of the depth and the root of the gravitational acceleration, is not a suitable formula for use in our engineering calculations. This formula should never be used unless we are dealing with the difficult theory of waves going in either direction at the same time, as considered in higher mathematics.

It can easily be shown that in some cases the velocity of propagation may be quite independent of the depth of the channel; for instance, when a purely balanced Wheatstone bridge is considered. In that case, the velocity of propagation is infinitely great, while the influence of the depth is practically immaterial. In any network of tidal channels there can be but one vertical tide in any knot or junction. Therefore, around a mesh as that shown by Fig. 15 the following simple relation exists:

$$T_1+T_2=T_3+T_4,$$

where the symbol T denotes the times of propagation. The velocities of propagation in each of the four channels are such that there is only one vertical tide in A and one in B. We can also say $M_1+M_2=M_3+M_4$, because the motive areas are comprised between the tidal curves of A and B.

In the case of a huge mesh, like the one around the coast of England and Scotland, the times of tidal propagation along the two paths—the coast of Scotland and the English channel—will be different, but nature ensures phase coincidence where the two paths meet off the coast of Kent by allowing the Northern wave to arrive exactly one cycle later. It therefore does not meet its

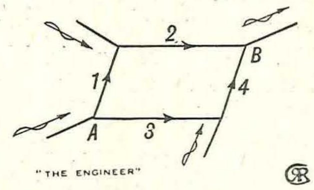

Fig. 15—Simple Network

"brother" at the Kentish coast but its "uncle." Amphidromic points (Fig. 16) are, as it were, small islands near the centre of some coastal bend which allow the vertical tide to travel around them in a whole cycle along that particular coast.

(19) *The Left (or Right) Tendencies of Sea Mouths.*—The deepest and largest channel in the estuary will be found in the direction from which the tide comes. For instance, along the Dutch and German North-Sea coasts the outflow of all main channels is towards the south-west or the west. This preference can be easily explained by theory. Let us take the case of a forked mouth of a tidal river (Fig. 17) and the tide in the sea

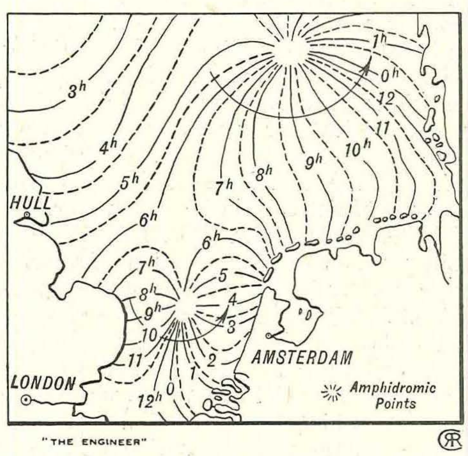

Fig. 16—Amphidromic Points

going in the direction from A towards B along the coast. In that case channel AC will have scoured a larger profile than channel BC. This can be shown by drawing the lines of the vertical tide for the stations A, B and C on the same time-axis. At flood tide the "head" $a_1c_1 > b_1c_1$ and at ebb-tide also $a_2c_2 > b_2c_2$.

We can also say $M_{ac} > M_{bc}$. There is a bigger motive force in channel AC than

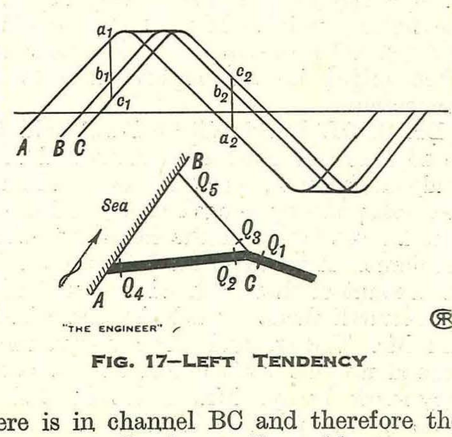

Fig. 17—Left Tendency

there is in channel BC and therefore there are stronger flood as well as ebb streams in the first channel, creating larger profiles.

Slight increases or decreases in tidal amplitude from A to B or C will not alter the "left" tendency, as may be noticed when drawing figures resembling those of Fig. 17. On the German North Sea coast the increase of amplitude is towards the East, but it has very little influence on the left-tendency of outflow. The force of Coriolis must also be considered; in the northern hemisphere this force would push the water to the right (ebb streams as well as flood streams), but generally it is not very influential when the channels are not too wide.

The main factors to be considered are the motive gradients, that is the motive areas per kilometre of the channel. The larger the motive area along the coast from A to B the larger the "left" tendency. When there are three or more sea-mouths of a single river and the head of the delta O is in full tidal swing, the proportions of the water in the various

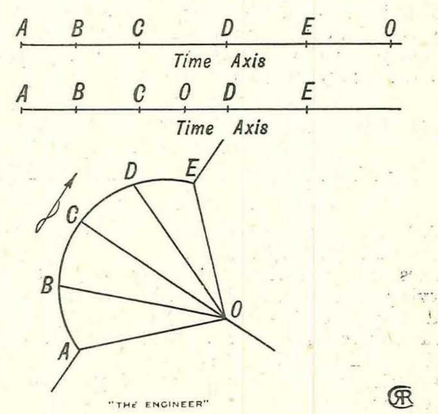

Fig. 18—Tidal Delta

mouths will be more or less as indicated by the motive gradients and the conductances of those mouths.

If the vertical tide does not change much in the mouths we can indicate the phases simply by marking on the same time-axis a point to represent each knot or junction (Fig. 18). Supposing that the phase at the head O of the delta comes last, the motive areas in the five different mouths are proportional to the times AO, BO, CO, DO and EO. Supposing that the phase of the vertical tide in O comes between C and D, the motive areas are also proportional to the same AO, BO and CO, &c., but we see that the flood runs towards the sea in the channels DO and EO, while it runs towards the land in the channels AO, BO and CO.

(To be continued.)

New Canadian Plant

Four new units costing a total of 1,680,000 dollars are under construction at the plants of Shawinigan Chemicals, Ltd., at Shawinigan Falls, Quebec. First to be completed will be an additional unit to the acetylene hydration process. It is to be housed in a building 54ft square and 70ft high, with concrete foundations and steel superstructure. Scheduled for completion in February, 1948, is a monochloracetic acid plant, housing a new process developed by the Shawinigan Company. The building, L-shaped, will have one wing 18ft by 20ft and 60ft high, and the other wing 54ft by 55ft and 20ft high. The third unit to be completed under the scheme will be an extension to the present vinyl acetate plant. It is to be ready in March, 1948, and will produce vinyl acetate by the vapour phase process. The last of the units now under way is to be an addition to the present butanol plant to produce butyraldehyde. Most of this unit will be housed in the present plant, but it requires a new pump-house, 20ft by 23ft and 15ft high, built of concrete and brick, and a tank farm, 35ft by 72ft of concrete design for ten containers.

Analogy between Tides and A.C. Electricity

By Dr. JOHAN VAN VEEN*

No. III—(*Continued from page 521, December 5th*)

Practical Examples

As stated in the first part of this article, there are two formulæ to work with: the dynamic equation and the continuity equation. The simplest equations to which these can be reduced are "Ohm's Law" (7) and the "fill equation" (8). They are both for a whole cycle and are not intrinsically interwoven as they should be.

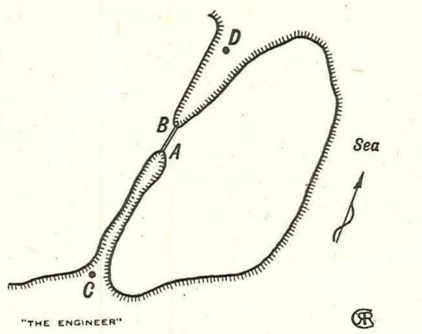

Fig. 19—Peninsular and Single Channel

Therefore only simple questions can be tackled with them. In cases where our works will not alter the amplitude of the vertical tide these simple formulæ can be used but they should not be used in the larger cases when the amplitude does change considerably.

For a first approach it is instructive to use the simplest method. Generally the vertical tides are well developed in tidal networks and any change in the profile of one or more channels will hardly affect the amplitude. When the amplitude is affected this change can be estimated to start with. The main changes which occur are in the currents, the friction and the propagation, not in the vertical amplitude. We should start by measuring C (Chezy constant) in all our channels, using very good chronometers to determine the exact phases of the vertical tide lines.

We also can measure the value of $\cos\phi$ in all channels. Average values are $\phi=35°$ and

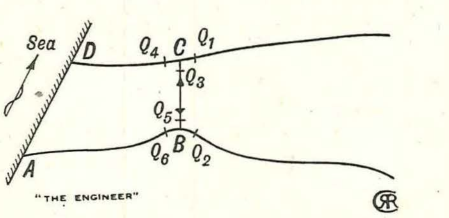

Fig. 20—Open Channel Connecting Tidal Rivers

$\sqrt{\cos\phi}=0{\cdot}9$. This last figure varies between $0{\cdot}87$ and $0{\cdot}94$ for different Dutch channels. Because C and $\cos\phi$ vary so little in the whole network we can combine them with the constant $\sqrt{44{,}700}$, which enters into every calculation. We then get the convenient figure of $52\times\sqrt{44{,}700}\times0{\cdot}9=10{,}000$.

The following simple formula can therefore be used: $F=10{,}000\sqrt{M/l}$ (dynamic equation), and $Q_1=Q_2\pm1{\cdot}8\,ABl$ (continuity or fill equation). But it is not certain that all estuaries will show the same convenient figure 10,000. Only flow measurements and exact gauge readings can check this.

It is not advisable to take the river sections too long. For rivers about 400m wide the length of the sections should not exceed 15km; for wider rivers greater length can safely be taken; the stylisation should be done with care.

For the simplest primary calculations we have therefore to consider four formulæ, the so-called Ohm's Law, the fill formula, and the two laws of Kirchhoff. These give a good insight into the principle of the conditions in any estuary or other network of tidal or non-tidal channels, for instance, irrigation networks. But the results they give are not exact. For larger and more important works other means of calculation must be sought—to which reference is made later—but an example of the approximate method is given below.

Calculation of Single Channel Cutting off Peninsula.—The simplest case for calculation is when a single channel cuts off a peninsula around which there is a tidal sea (Fig. 19). The calculation of an open channel like the Panama canal poses the same simple problem. Let us suppose that the new channel streams will not affect the vertical tides in the seas near A and B. Then the motive area M_{AB} can be measured accurately and it will not change when the new channel is made. The total flow can, therefore, be found immediately by applying the formula:

$$Q=10{,}000\,bh^{3/2}\sqrt{M_{AB}/l}$$

For sinusoidal curves the max. current is:

$$S_{max}=\frac{Q\pi}{2\times44{,}700}$$

Supposing that the tides in A and B do change after the channel is made then two more sections, AD and BC, must be calculated (Fig. 19). Then M_{CD} is our fixed "border condition." The quantities required are M_{AD}, M_{AB}, M_{BC}, Q_A. For these four the following equations have to be solved:

$$M_{AD}+M_{AB}+M_{BC}=M_{DC}$$
$$F_{AB}=Q_A/b_1h_1^{3/2}=10{,}000\sqrt{M_{AB}/l_{AB}}$$
$$F_{AD}=10{,}000\sqrt{M_{AD}/l_{AD}}$$
$$F_{DC}=10{,}000\sqrt{M_{DC}/l_{DC}}$$

The velocities in the new channel can be

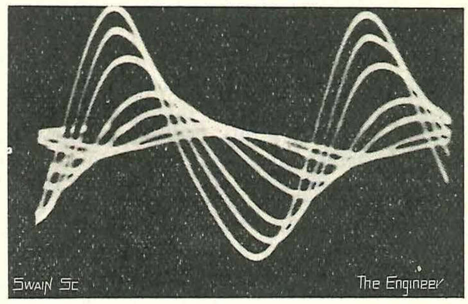

Fig. 22—Oscillograms of Vertical Tide—River Lek

estimated by using the sinusoidal formula mentioned above.

The Calculation of the "Left" Tendency.—In a tidal river with two mouths (Fig. 17), the border conditions are M_{AB}, the motive area in the sea along the coast, and the discharge of the upper river Q. The following formulæ can be deduced.

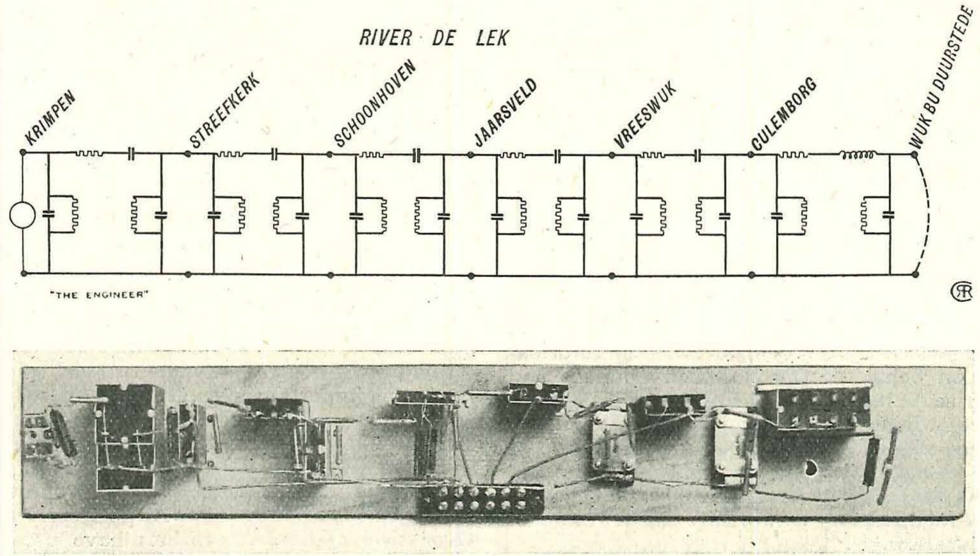

Fig. 21—Electrical Equivalent of Tidal Rivers

We suppose that the vertical tide will not change when we fix the channel profiles in AC and BC to known proportions. Then Q_1 just above the dividing point C remains the same and is known. To begin with:

$$Q_2+Q_3=Q_1$$
$$M_{AC}+M_{BC}=M_{AB}$$
$$F_{AC}=10{,}000\sqrt{M_{AC}/l_{AC}}$$
$$F_{BC}=10{,}000\sqrt{M_{BC}/l_{BC}}$$

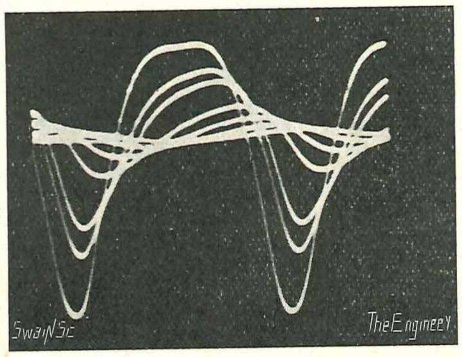

Fig. 23—Oscillograms of Horizontal Flow—River Lek

Supposing that the profiles are known we

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can calculate the streams or, alternatively, if we suppose that the maximum velocity in both mouths must be x metres per second, then the dimensions of the profiles can be calculated.

Calculation of a New Open River Connecting Two Others.—For a new channel connecting two existing parallel tidal rivers (Fig. 20) we can proceed as follows: We know Q_1, Q_2 and M_{AD}, and can assume that the vertical tidal amplitude will not change because of the digging of the new open river.

Then

$$Q_1=Q_3+Q_4$$
$$Q_2=Q_5+Q_6$$
$$M_{AD}=M_{DC}+M_{BC}+M_{AB}$$
$$F_{AB}=10{,}000\sqrt{M_{AB}/l_{AB}}$$
$$F_{BC}=10{,}000\sqrt{M_{BC}/l_{BC}}$$
$$F_{CD}=10{,}000\sqrt{M_{CD}/l_{CD}}$$

The fill formulas must be used for obtaining the flows in all profiles, in AB, BC and DC.

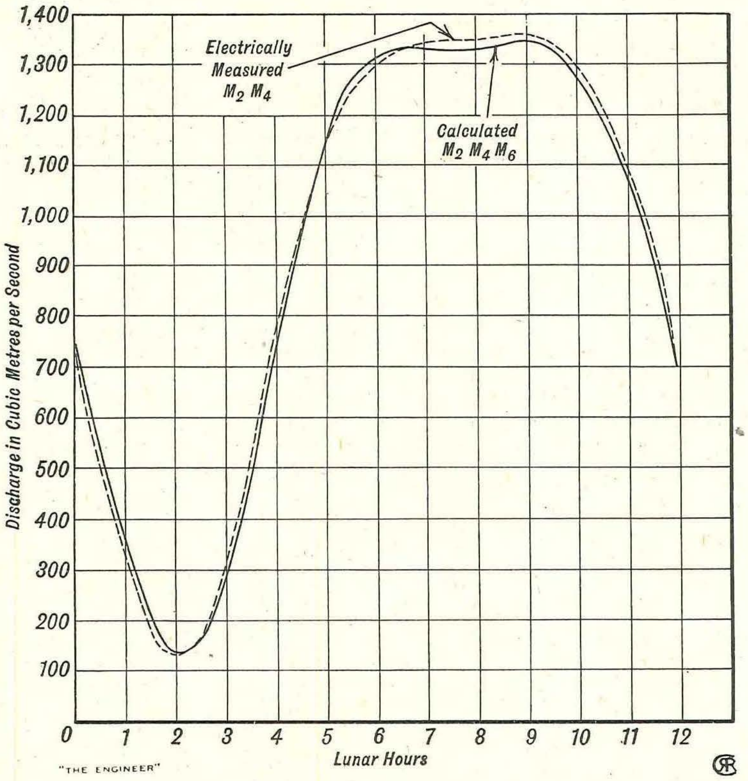
Any other example can be solved in this way provided we presume that the amplitude of the vertical tide does not change because of our works. The flow of the upper river has only a small influence upon the value of cos ϕ as has been proved in the Rhine mouths. The value of $\sqrt{\cos\phi}$ gradually changes from 0·9 into 1 when reaching the limit where the tide dies out, but in practice this small difference can hardly be noticed. The total flow, the resistance and the propagation can be calculated without giving much thought to the flow of the upper river: Q is the total flow whether tidal or not.

The Method of Electrical Simulation

To obtain greater accuracy we must use either one of the sinusoid methods or the still more tedious "exact" method. They can deal with the vertical tide as well as with the horizontal tide, but the *exact* method gives more detail and is more accurate because it takes the quadratical relation between velocity and friction into account correctly.

In order to avoid the calculation work involved we can simulate our network of tidal channels by means of copper wires, electrical resistances, condensers, &c., putting an alternating current into one end of this network while injecting a direct current into the other end. It goes without saying that the linear Lorentz equations can be simulated quite easily electrically because this method is exactly analogous to the telegraph formulæ. The photograph reproduced in Fig. 21 shows an electrical circuit which is equivalent to a tidal river, the Lek, in which the tide comes from the left and dies out towards the right, where the upper river brings its fresh water.

The accuracy of such an electrical equivalent is the same as that given by the mathematical formulæ of Lorentz. The two sets of results do not differ by 0·5 per cent. The disadvantage of the linear method is not only that it gives inadequate results when great accuracy is needed, but also that it needs much previous calculation for fixing the Lorentz constant which is proportional to the unknown current velocity. The simulation of the quadratical tides does not need these preliminary calculations, and therefore gives better and quicker results. The difficulty is to obtain the quadratical relation between current and friction. Ordinarily electrical currents give only the linear relation. However, electronic technology provides a method of reproducing this quadratical relation electrically. With the aid of a cathode ray tube we can see the new tidal curves and they can be photographed.



Tidal Stream Curves at Krimpen on River Lek.

Fig. 25—Comparison of Electrically Measured and Calculated Tidal Curves

The vertical tidal curves (quadratical resistance) for a river of 50 miles length are shown in Fig. 22 as reproduced on a cathode ray tube. The different lines are for different stations and they show the tide diminishing in amplitude from left to right when going up-river. The influence of river discharge on mean level cannot be shown by the cathode ray tube, which is, however, convenient for demonstration. Accurate measurements have to be taken electrically by other means.

Fig. 23 shows the tidal streams (quadratical resistance) on the same river and for the same stations. The deformations are due mainly to the quadratical relations between velocity and resistance. The portions of the curves above the datum line represent ebb conditions; the portions of the curves below the datum are steeper and they correspond to the river in flood. Note the phase differences, with the tide diminishing in amplitude from left to right.

In Fig. 24 the tidal curve obtained electrically is compared with one obtained by calculation, assuming quadratical friction in both instances.

Though the electrical analogy eliminates a considerable amount of difficult calculation, the theory of the tides should not be neglected. Mistakes may be made unwittingly with the decline in research and actual knowledge of the river, which may follow the application of easier methods of dealing with tidal estuaries. Even while using an electrical equivalent we should not neglect to check the results with river measurement.

There are three ways of dealing with tidal problems: river research, which comes first and last; laboratory research; and mathematical research. None of these three should be neglected. They permit us to view our problems from three different angles and we cannot ignore one of these without running the risk of committing errors. But why should electrical engineers apply mathematical means for calculating their networks and hydraulic engineers, dealing with more costly analogous problems, not do likewise?

Bibliography

[1] H. A. Lorentz, *Verslag van de Staatscommissie 1918 ter aftsluiting van de Zuiderzee, Den Haag, Algemeene Landsdrukkerij*, 1926; 336 pp.; 61 fig.

[2] J. J. Dronkers, "Een getijberekening voor Benedenrivieren," *De Ingenieur*, 1936, 6 pp., 3 fig.

[3] J. van Veen, "Getijstroomberekening met behulp van wetten analoog met die van Ohm en Kirchhoff," *De Ingenieur*, 1937, No. 19; 9 pp.; 16 fig.

[4] J. P. Mazure, *De berekening van getijden en stormvloeden op benedenrivieren. Gerritsen Den Haag*, 1937; 222 pp.; 12 fig.

[5] H. J. Stroband, "Een bijdrage tot de kennis van de getijberekening op benedenrivieren en zeearmen." *De Ingenieur*, Sept. 6, 1947; 6 pp.; 8 fig.

The Iron and Steel Institute

No. IV—(*Continued from page 523, December 5th*)

The third and final session was held on Thursday morning, November 13th, the President (Dr. C. H. Desch, F.R.S.) again being in the chair.

The first paper presented was:

THE FLUIDITY OF STEEL

By R. Jackson, D. Knowles, T. H. Middleham and R. J. Sarjant

Synopsis

The paper described experimental investigations on the fluidity/temperature relationships, as indicated by Ruff and Spiral mould tests, of four steels, viz. (1) 2 per cent Cu steel, (2) Si-Ni steel, (3) low-carbon steel, and (4) 13 per cent Mn steel, melted in high-frequency furnaces with acid and basic linings. Temperature measurements were made by optical and quick-immersion-couple methods. The Spiral mould gave more consistent results than the Ruff. Comparison of the authors' results with those of Taylor, Rominski and Briggs on similar steels showed widely differing fluidity/temperature relationships with similar moulds. Investigation of the pyrometric methods employed in the two sets of trials indicated that the differences could be mainly ascribed to the time lag in the temperature measurement employed by the American authors.

Discussion

Dr. W. C. Newell (B.I.S.R.A.) said that one of the outstanding conclusions reached by the authors was that the Spiral test was superior to the Ruff test. The paper contained ample justification for that conclusion. Another valuable conclusion was that regarding the correlation between fluidity and optical pyrometer readings and between fluidity and quick immersion pyrometer readings or true temperature readings. The paper confirmed the general knowledge that acid steel was more fluid according to the optical pyrometer, but by the immersion pyrometer there was scarcely any difference. If anything, the bias was in favour of basic steel being more fluid, but the difference was within experimental error. Comparing true temperatures against fluidity, the corre-