

Optimal CO₂ pressure for a pneumatic system

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Abstract

CO₂ powered actuators are a viable alternative in prostheses and robotics. The gas-supply is generally limited in such applications, and calls for an optimal system pressure to minimize gas-consumption. Previous work which discusses the optimal pressure was not verifiable, or strictly theoretical. The theoretical work was unintuitive, but did result in an optimal pressure level of 1.2 MPa, irrespective of the required output force. This research presents a simplified theoretical background, and verifies the optimum by measurements. The simplified theory offers improved insight into the existence of an optimal supply pressure and locates it at 1.42 MPa. Compensation for atmospheric pressure strongly affects cylinder size at low system pressures and above the optimum, as CO₂ approaches its critical point, gas consumption increases with the density of the gas. The experiments show an optimum at 1.12 MPa, with a bound of -0.10 MPa and +0.30 MPa. The uncertainty in the location of the minimum gas-consumption reflects the challenges of accurately performing the measurements, as well as the shallow curve where the minimum is found. The presented results support the optimal supply pressure found in literature, and when optimizing a pneumatic system for CO₂ use, 1.2 MPa is therefore a good choice.

Keywords — pneumatics, CO₂, gas-consumption

1 Introduction

When using compressed gas as a power source, minimizing gas-consumption is important, considering the limited amount of gas stored in a bottle. Applications for such a power source could be in systems that are ‘stand alone’, or mobile. These systems include portable systems, robots and prostheses.

Starting in the late 1950’s research on prosthetics intensified due to the thalidomide tragedy [8]. Research was ‘split’ between electronic and pneumatic fields [6]. In the pneumatic field, it was already recognised that the system pressure should be optimised ‘for gas economy’s sake’ [5]. However, eventually electronic prosthesis gained the most popularity. Difficulty in control [2], practical problems and poor quality valve technology at that time [3], all but stopped development of pneumatic prostheses [10].

The advantages of the use of a pneumatic system still remain: it can be built light, it is fast, it is reliable and it can be built small, according to Plettenburg [10]. For a powered prosthesis, it was decided pneumatics needed to be re-assessed. And so, in his thesis, the question on how to attain a minimal gas-consumption re-arose when developing a pneumatically powered hand prosthesis for a toddler.

In short there is the following problem: A system could be made using either a small cylinder with a high operating pressure, or a larger cylinder with a lower operating pressure. This leaves the designer with a problem for which there might be an optimal solution.

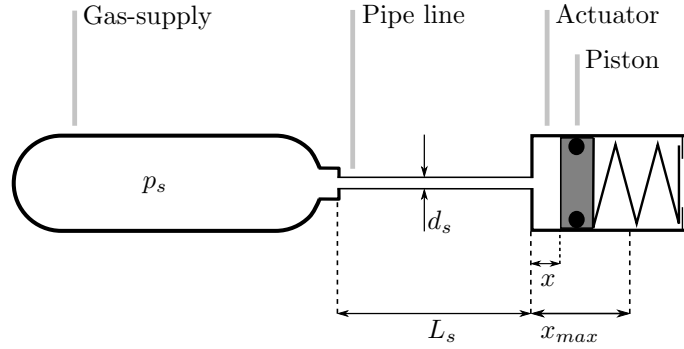


Figure 1: The system modelled by Plettenburg [10]. The pressure in the Gas-supply is assumed to be constant. The piston is moved by the pressure and forces the spring to compress. Springs with different degrees of stiffness represent different load cases, as they need different amounts of energy to be compressed.

In 1967, Lambert [6] reported a level of pressure where the optimum can be found, within a range of 150 to 200 psi, or 1.0 to 1.4 MPa, unfortunately without explaining the measurement method, nor the theoretical background. Plettenburg deduced the optimum at 1.2 MPa, based on a seemingly simple model, for which the mathematics soon became unintuitive and complex.

For this deduction, he modelled a pneumatic system, see Figure 1. This system has a defined load, represented by a spring in the model. To compress the spring a pneumatic cylinder was used, connected via a pipeline to the gas-supply bottle at a certain system pressure p_s . The pipeline, cylinder size and the system pressure were optimized for minimal gas-consumption m_c .

Except for very low energy load-cases, the optimum was the same regardless of pipeline length L_s or cycle time. In Figure 2 the relation between supply pressure and gas-consumption for a particular load case is shown. The results further indicated that the *shape* of this graph would stay the same, except for a different offset in gas-consumption, depending on specific parameters.

The objective of this paper is to explain why an optimal pressure should exist and show what the optimal pressure level is for CO₂ powered systems. To achieve this, the model in [10] was studied and simplified, and a set-up was built to find the optimal pressure level by experimentation.

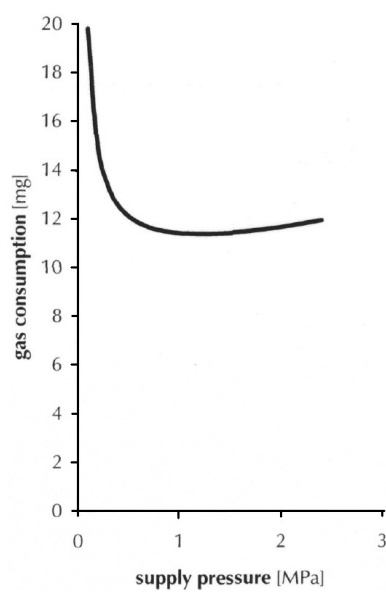


Figure 2: The result of the model by Plettenburg, with the optimum at 1.2 MPa. The shape of this graph is typical for load-cases in his model, except that for different load-cases there could be a different offset in gas-consumption.

2 Method

To determine an optimal pressure for CO₂ powered systems two approaches are taken. Therefore this section is split into two parts. Part A explains the underlying theory, part B is focussed on the measurements.

A Theory

In [10], Plettenburg mentions previous research and the fact that, regarding the optimal pressure, they neither explain their theories, nor their measurement methods. In a recent literature review [4], no new research was found which would confirm an optimal pressure level, either based on the model posed in [10], or anything similar.

First a review of the model in [10] is presented here. The model defines a system consisting of a gas supply bottle, a pipeline and an actuator (see Figure 1). For this system the model calculates the amount of gas used, m_{hc} , as a function of gas supply pressure [10, eq. 2.20],

$$m_{hc} = \rho \left[\frac{\pi}{4} d_s^2 L_s + x A_c \right] \quad (1)$$

in which ρ , d_s and A_c are dependent on the pressure. Respectively they represent density of the gas, pipeline diameter and the cylinder's effective cross-sectional area. x is the stroke of the piston. A_c depends on p_s because the load is given by the spring and the piston needs to be adapted to the pressure to be able to compress the spring properly. d_s depends on p_s due to the use of fluid dynamics equations in the model, as will be discussed shortly.

To find the relation between pressure and gas-consumption, Equation 1 is differentiated with respect to the system pressure p_s . When the differential is equal to 0, the pressure is found where there is a minimal gas-consumption, [10, eq. A2.10.3],

$$\frac{dm_{hc}}{dp_s} = \frac{d\rho}{dp_s} \cdot \frac{\pi}{4} \cdot d_s^2 \cdot L_s + \rho \cdot \frac{\pi}{4} \cdot L_s \cdot 2 \cdot d_s \cdot \frac{dd_s}{dp_s} + \frac{d\rho}{dp_s} \cdot x \cdot A_c + \rho \cdot x \cdot \frac{dA_c}{dp_s} = 0 \quad (2)$$

The differentials within this equation are solved by making use of the following equations:

First, a modification of the ideal gas law by Plank and Kuprianoff [9], converted to SI units,

$$\frac{V}{m} = \frac{RT}{p} - \frac{82.5 \cdot 10^{-3} + 12.4915 \cdot 10^{-9} p}{\left(\frac{T}{100}\right)^{10/3}} \quad (3)$$

Where V , p and T represent the volume, pressure and temperature of the gas and R represents the gas constant for CO₂. This equation accounts for the real behaviour of CO₂, especially near the critical point, a phenomenon not described by the ideal gas law ($PV = mRT$).

And second, a relation from White [13, eq. 9.73],

$$\left(\frac{\dot{m}}{\frac{1}{4}\pi d_s^2} \right)^2 = \frac{p_s^2 - p_c^2}{RT[\bar{f}L_s/d_s + 2\ln(p_s/p_c)]} \quad (4)$$

where \bar{f} is the average Darcy friction factor over the pipe length, \dot{m} is the mass flow through the pipe. In this equation the pressure drops from the source pressure p_s to the pressure in the cylinder p_c . And so the relation between the gas' mass flow to the pressure drop in the pipeline is established. d_s could be optimised when separating the time variable, present in the mass flow, from the geometric variables, including the pipe line's diameter (starting from [10, e.q. A2.7.6]).

This resulted in Equation [10, A2.10.8], written here in the following form,

$$\frac{dm(p_s)}{dp_s} = \frac{A \cdot [B \cdot \{[C] \cdot \ln[D] + [E] \cdot \ln[F]\} - P \cdot \{G \cdot \ln[D] + H \cdot \ln[F]\} \cdot [K]]}{[K]^2 \cdot \sqrt{J \cdot \{C \cdot \ln[D] + E \cdot \ln[F]\}}} + N \cdot \left[\frac{Q}{[K]^2} \right] = 0 \quad (5)$$

Where the following substitutions still need to be made,

$$\begin{aligned} A &= \frac{32\eta L_s^2}{t} (0.01T)^{3.333} & G &= \frac{F_{top}(2F_{top} - F_0)}{c \cdot p_s^3} \\ B &= R \cdot T(0.01T)^{3.333} + 12.4915 \cdot P^2 & H &= \frac{F_{top}(2F_{top} + F_0)}{c \cdot p_s^3} \\ C &= \frac{F_{top}(2F_{top} - F_0)}{c \cdot p_s^2} + \frac{107.483}{(0.01T)^{3.333}} & J &= \frac{128\eta L_s}{\pi t} \\ D &= \frac{F_0 - F_{top}}{c \cdot x + F_0 - F_{top}} & K &= R \cdot T(0.01T)^{3.333} - 82.5 \cdot P - 12.4915 \cdot P^2 \\ E &= \frac{F_{top}(2F_{top} + F_0)}{c \cdot p_s^2} - \frac{57.517}{(0.01T)^{3.333}} & N &= \frac{x F_{top}}{p_s^2} \\ F &= \frac{F_0 + F_{top}}{c \cdot x + F_0 + F_{top}} \\ Q &= 12.4915 \cdot P^3 (0.01T)^{3.333} + P^2 (0.01T)^{3.333} \cdot (82.5 + 12.4915 p_s) - p_a R T ((0.01T)^{3.333})^2 \end{aligned}$$

And, also within the substitutions above,

$$P = (p_s + p_a)$$

Where, c denotes the spring stiffness, η the viscosity of CO₂, F_{top} the maximum force that the actuator can generate at its design pressure, F_0 the pre-load on the spring, p_s and p_a the supply and atmospheric pressures respectively. The main point is, it has become a complex description. One goal in this paper is to make a more intuitive approximation of the graph which results from the model.

To describe the system in more simple terms, an analysis was done starting from Equation (1) and (3). Looking only at the force balance on the piston, and the properties of CO₂ a simplified model was made to estimate the optimal gas pressure for minimal gas-consumption. Ignoring Equation (4) means that the effects of d_s and L_s in Equation (1) are greatly reduced. The supply line volume, determined by those variables, will only behave as ‘dead space’. This is the space that does change pressure during a cycle of use, but it does not aid in compressing the spring, only in raising the gas-consumption. That is why these parameters will not be included in the model presented in section 3-A.

B Measurement set-up

Regarding the results of the model in [10], they show that the optimal pressure does not depend on either the supply line length or its diameter. Those parameters change the amount of gas used, but do not affect at which pressure the optimum is found, except for cases where the load would be so low, using it in a prosthesis would make it underpowered. What is more, the cycle time appeared not to influence the optimal pressure level. The cycle time meaning: the time it takes to move the piston to x_{\max} and back. Based on these results, the cycle time was not considered an important parameter in the measurements. And the whole volume leading up to the valve-block was considered as the gas-supply.

$D_c \pm 0.02 \text{ mm}$ [mm]	A_c [mm ²]	V_c [mm ³]	P_d [MPa]
6	28	565	3.64
8	50	1005	2.09
10	79	1571	1.37
12	113	2262	0.99
15	177	3534	0.67

Table 1: Design dimensions of the cylinders, with diameter D_c , the length of the stroke L_c given as 20 mm, which lead to piston area A_c and volume V_c . For the same power output, the cylinders should operate at design pressure P_d .

Measurements were done using a custom built set-up (Figure 3). The set-up was based on the test-bench presented in [12], enabling a force-displacement curve to be measured. Additionally a pressure sensor and a scale were included, to monitor pressure and the weight of the gas-supply. A valve-block was made to separate the supply from the cylinder and be able to vent the cylinder to the atmosphere. The measurements were done on a series of 5 cylinders (see table 1). These were designed following the guidelines in [10, p.49]. For each cylinder, measurements were done on a range of pressures, since it proved difficult to set the regulator at a specific pressure. The range of pressures made it possible to estimate ‘sensitivity’ of the cylinders’ gas-consumption to the pressure level. From this sensitivity the gas-consumption could be estimated at the design pressure.

Hardware

The gas-supply in the model (Figure 1) is represented by a single CO₂ bottle. However, in the measurement set-up (Figure 3), it comprised a bottle with a regulator and the pipeline leading up to the valve block. The weight of the bottle was monitored by placing it on a scale with μg accuracy.

The bottle was screwed into the regulator, and both were weighed at the same time. Assuming there were no leaks in the system, the reduction in weight should equal the amount of gas drawn from the bottle. The pressure regulator was designed and built by Rob [11]. With practice, it can be set to a pressure with varying accuracy; $\pm 0.05 \text{ MPa}$ for pressures up to about 0.8 MPa , increasing to $\pm 0.15 \text{ MPa}$ for pressures up to 4 MPa .

The pressure regulator was connected to the rest of the pipelines via a thin hose, with dimensions $0.75 \text{ mm} \times 1 \text{ mm}$ of about 300 mm length¹. The flexibility of the hose ensured the reading of the scale was not affected by the stiffness of the hose. Since this part of the set-up is practically static, there were no movements to change the configuration of the hose and thus its force on the scale remained constant.

The pressure sensor was connected approximately midway along the supply line. The pressure measurement was done with two pressure sensors, one for the range of 0 MPa to 1.6 MPa with an accuracy of $\pm 8 \text{ kPa}$ and the other for 1.6 MPa to 4.0 MPa with an accuracy of $\pm 20 \text{ kPa}$. The supply line connecting the sensor with the valve-block was a $2 \text{ mm} \times 4 \text{ mm}$ copper tube of about 1 m in length.

Reducing the amount of dead space was deemed important to reduce the gas-consumption. In the model, this is the volume of the pipeline connecting the bottle to the actuator. The

¹Inner Diameter (ID) \times Outer Diameter (OD)

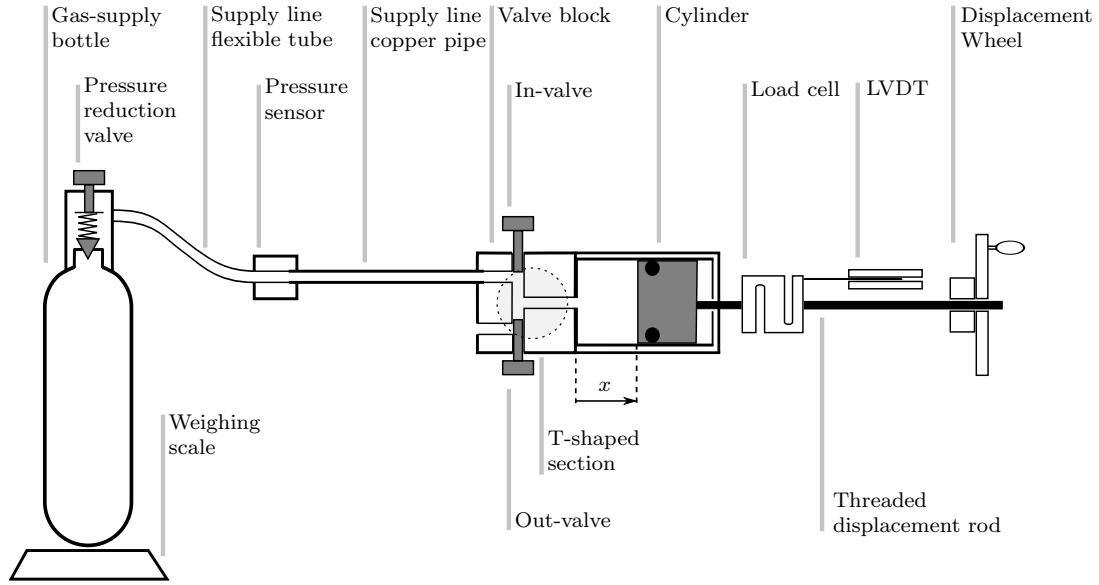


Figure 3: The measurement set-up. The gas-supply bottle contains 16 g of CO_2 , a generic model. Pressure sensors: (for pressures up to 1.6 MPa) DRTR-AL-20MA-R16B by B+B Thermo-Technik GmbH, and (for pressures up to 4.0 MPa) DMP 331; 0-40 bar; 0-10V/3-wire by BD Sensors GmbH. Weighing scale: PE 360 DeltaRange by Mettler. Load cell (Force sensor for up to 500 N): FLB3G-C3-50kg-6B by Zemic. Displacement sensor (a linear variable differential transformer (LVDT)): LCIT 2000 by Schaevitz. Computer interface (not shown): NI USB-6008 by National Instruments.

corresponding section in the set-up was therefore kept as small as possible, and was contained inside the valve block. It comprised a T-shaped bore, of 1 mm diameter and 10 mm overall length. The ends of the T-shape connect to the inflow line, the cylinder and the outflow line. The cylinder was filled and evacuated through this T-shape, the gas flow was controlled by opening and closing the valves.

Measurement routine

At the start of a measurement, the following need to be true:

- The bottle is connected.
- The Pressure reducer is set according to the currently installed cylinder.
- The piston is moved to the initial position, i.e. no displacement.
- The scale is set to 0 μg .
- The output valve is closed.

Then, the following actions are repeated until enough data is collected to estimate the gas-usage or an error is encountered. (If the start-situation is numbered 0, the numbers of the actions correspond to the numbers in Figure 4.)

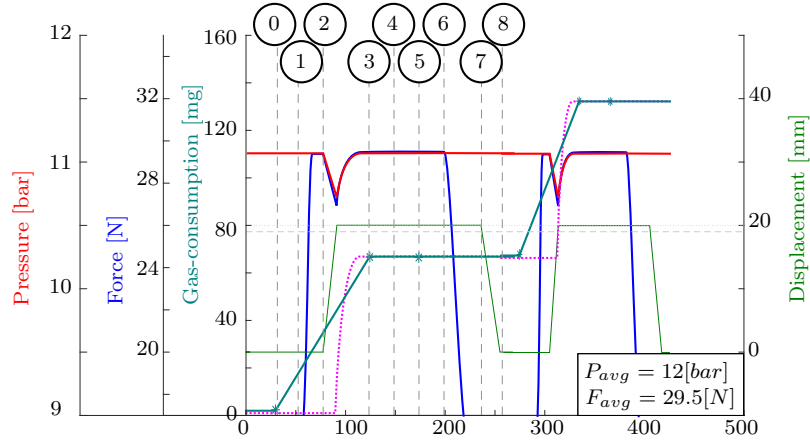


Figure 4: The numbers in the Figure correspond to the actions described in the measurement routine. Two full cycles are displayed. On the horizontal axis are the number of samples, the sampling frequency was 5 Hz. The first one executed more slowly than the last. The dotted (magenta) line, represents the estimated 'real-time' gas-consumption, based on the increase in pressure in the supply-line and it is scaled to match the gas-consumption. The forcegraph is scaled to match the pressure level. Note that the vertical axes do not necessarily start at 0, where it crosses the horizontal axis. Furthermore, the pressure is presented in bar, 10 bar equals 1.0 MPa

1. Open the input valve. This will let the supply pressure enter the cylinder, which is indicated by the force exerted on the piston.
2. Displace the piston to the end point 20 mm away. This will let the gas flow into the piston, resulting in a drop in supply pressure. Then, depending on the response of the reduction valve, the pressure in the system will remain lowered or rise back to the set point. As the pressure in the cylinder changes, so will the force on the piston.
3. Record the value of the weight. As the reduction valve sometimes responds poorly, gas might continue to be released from the bottle.
4. Close the input valve. System pressure should remain constant or increase, depending on the response of the reduction valve.
5. Record the value of the weight. When the input valve is closed, the cylinder is effectively cut-off from the supply. A measurement at this point should reflect all changes in the supply after one cycle.
6. Open the output valve. The gas in the cylinder can now escape to the atmosphere. This can also be seen in the measured force dropping to 0.
7. Displace the piston to the start point, i.e. 0 mm. This pushes the last volume of gas out of the cylinder (but not out of the dead space).
8. Close the output valve.

Data processing

The raw data files contain time-series measurements of pressure, position and force as well as the weight of the gas-bottle recorded at the corresponding times. The data files were examined and sample ranges were chosen which contained useful measurements, i.e. they contain the least amount accountable errors.

These ranges were then processed by a script which calculates the average pressure and force. The pressure was only averaged when the system pressure was present in the cylinder. This was determined by assuming that the cylinder was pressurised if the force was higher than a chosen lower limit of the force level. To estimate the gas-consumption for a single measurement, the average pressure was used.

The gas-consumption at various pressures was used to determine the sensitivity of the gas-consumption to the varying pressure. Based on the sensitivity, it was estimated what the gas-consumption would have been at the design pressure.

A non-linear model was then fitted to the estimations²,

$$m_f = \frac{b_1}{p_f} + b_2 p_f + b_3 \quad (6)$$

The non-linear model should have similar properties to the theoretical model. It combines a hyperbola, a gain and an offset part, corresponding to parameters b_1 , b_2 and b_3 . This model was then used to estimate the optimal pressure.

²using MATLAB's `nlinfit` function [1].

3 Results

This section shows the theoretical analysis. Also it contains the summary of the measurements. All the graphs of the data can be found in the appendix.

A Theoretical results

Based on the model in [10], a new model is made. This simplified model calculates how much gas it would take to fill the volume of the cylinder given the required power output, for any given pressure. The gas is modelled using Equation (3) which describes the behaviour of CO₂ better than the ideal gas law.

When a required power output is given, the size of the cylinder depends on the used system pressure p_s . For comparison purposes a power output of 2000 N mm was chosen, and the standard stroke of the pistons is set at 20 mm. This means, that the cylinders have a net output force, F_{net} , of 100 N.

The system pressure p_s pushes the piston out of the cylinder and on the other side of the piston, atmospheric pressure p_a pushes it into the cylinder. So the net force should be calculated over the pressure difference between the system and the atmosphere. The area of the piston A_c , can then be calculated using,

$$A_c = \frac{F_{\text{net}}}{p_s - p_a} \quad (7)$$

where an atmospheric pressure of $p_a = 101\,325$ Pa is chosen³.

The second step is to calculate the amount of gas required to fill the cylinder at the correct pressure. With a known A_c and stroke x , the volume V_c is easily calculated. This step includes part of Equation (1), namely,

$$m_c = \rho [xA_c] \quad (8)$$

where ρ is calculated with Equation (3), and the relation, $\rho = m/V$.

With these equations combined,

$$m_c = \frac{1}{\frac{RT}{p_s} - \frac{82.5 \cdot 10^{-3} + 12.4915 \cdot 10^{-9} p_s}{\left(\frac{T}{100}\right)^{10/3}}} \left[x \frac{F_{\text{net}}}{p_s - p_a} \right] \quad (9)$$

an optimum is found at 1.42 MPa, see Figure 5.

With 5 evenly spaced points on the result of the model, the non-linear fit finds an optimum at 1.38 MPa.

³This value is the average standard sea-level atmospheric pressure according to ISO 2533:1975

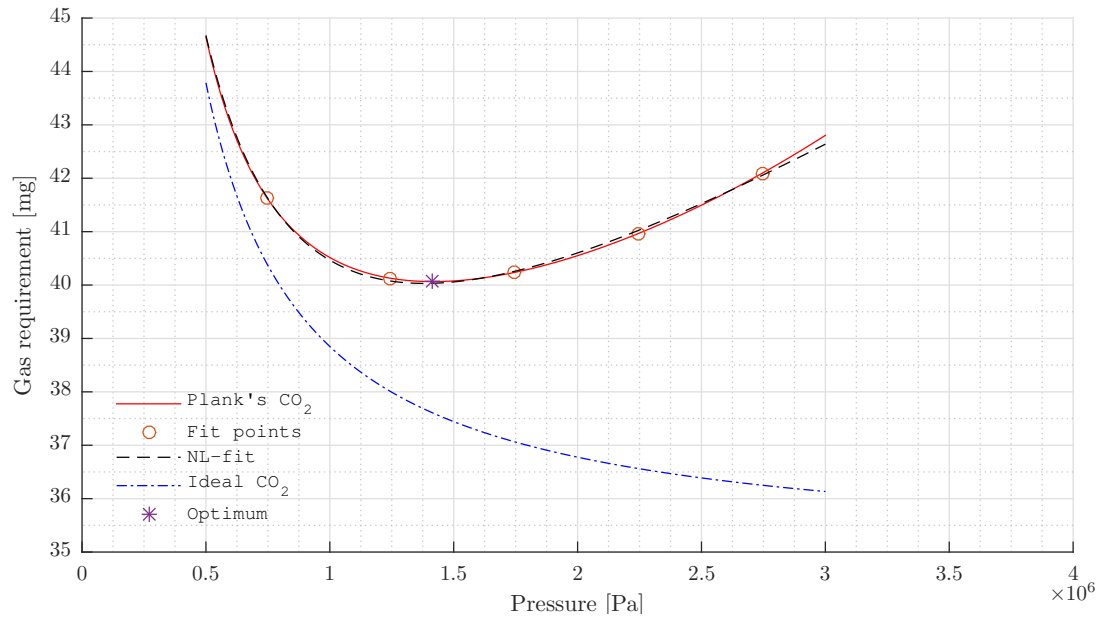


Figure 5: Gas requirement to fill a cylinder with CO₂ according to the Ideal Gas Law, and using Plank's law. The graphs are calculated for cylinders with a stroke of 20 mm and power output of 2000 Nm, Where the cylinder size depends on the pressure. To check how well the non-linear model from section A would fit to 5 selected points, this is also shown here. The non-linear fit finds the optimum at 1.38 MPa, whereas the simplified model predicts the optimal pressure at 1.42 MPa.

B Measurement results

B.1 Measurements

The average gas-consumption per stroke by each cylinder and the average pressure at which it was measured are shown in table 2. Figure 6, shows the same results, and additionally the estimated gas-consumption at the pressure the cylinders are designed for, P_d .

Cylinder Diameter [mm]		6		8		10		12		15	
Dataset		m_c	p_s	m_c	p_s	m_c	p_s	m_c	p_s	m_c	p_s
		[mg]	[bar]	[mg]	[bar]	[mg]	[bar]	[mg]	[bar]	[mg]	[bar]
	a)	14.44	11.0	23.40	11.4	38.00	11.4	15.44	3.0	91.67	11.7
	b)	13.50	9.8	19.09	9.9	33.25	9.7	25.73	5.4	66.33	9.3
	c)	13.13	7.6	14.40	7.5	23.40	7.5	34.71	7.2	49.33	5.9
	d)	66.67	27.0	25.00	11.7	44.00	11.6	52.75	9.7	62.00	7.9
	e)	60.00	36.3	45.03	17.9	-	-	59.36	11.2	83.67	9.9
	f)	-	-	-	-	-	-	55.60	11.8	92.67	12.1
	g)	-	-	-	-	-	-	-	-	40.33	4.9
	h)	-	-	-	-	-	-	-	-	45.25	6.0

Table 2: Measured gas-consumption m_c per stroke for each cylinder at different system pressures p_s . The datasets correspond to the measurements in the appendix.

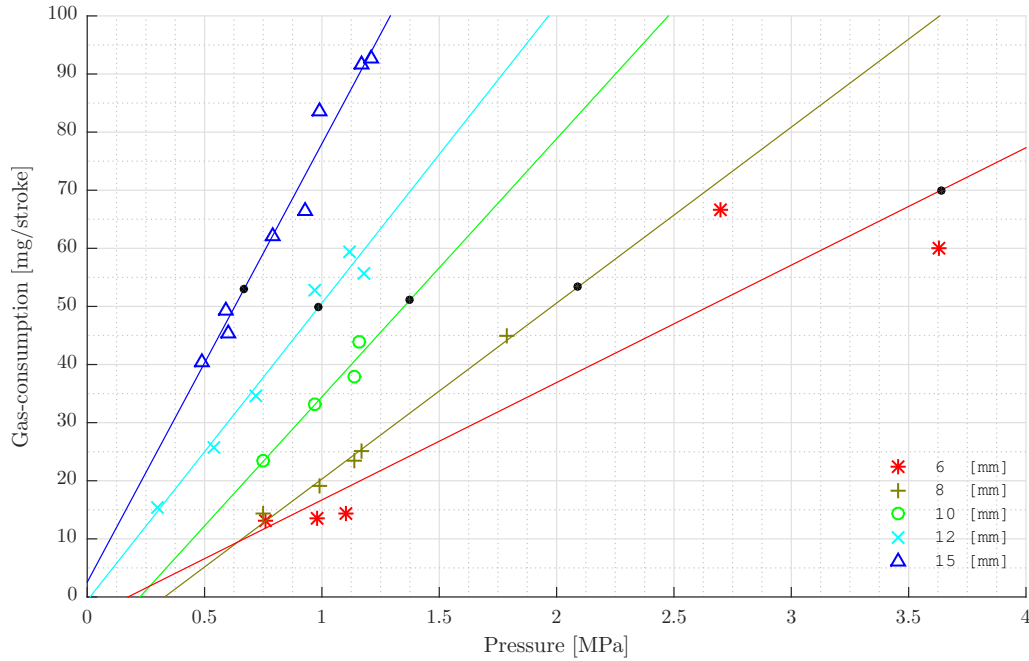


Figure 6: For each cylinder, a gas-consumption was measurement at several different system pressures. Linear regression through the measurements allow for a ‘sensitivity’ to system pressure to be estimated for each of the five cylinders. The single dots on the regression lines denote the gas-consumption estimation at the design pressures P_d .

B.2 Finding the optimum

In Figure 7 three graphs are presented, two theoretical results and the fit of the measurement results. The theoretical results used the model from subsection A, using both the ideal gas law and the adapted version by Plank. The graph of the measurement results features the estimated gas-consumptions, found in B.1 along with their 95% confidence margin. The values at the design pressures are presented in Table 3. The non-linear model, see Equation (6), was fitted to estimated gas-consumptions and with that fit, an optimum is estimated at 1.12 MPa. The fit through the estimated points is quite sensitive to the actual variation of the estimations.

D_c [mm]	m_{Ideal}	m_{Plank}	m_{Lower} 95% confidence	$m_{\text{Estimates}}$	m_{Upper} 95% confidence
6	36.52	46.61	40.31	70.00	99.69
8	37.31	41.71	47.68	53.33	58.97
10	38.32	40.93	37.54	51.08	64.61
12	39.57	41.37	45.57	49.84	54.11
15	41.86	43.07	48.66	52.90	57.14

Table 3: The second and third column show the gas-consumption according the simplified model, with different gas laws. The last three columns show the 95% confidence interval and the estimated values for the gas-consumption based on the data. For pressures corresponding with a 2000 N mm Output.

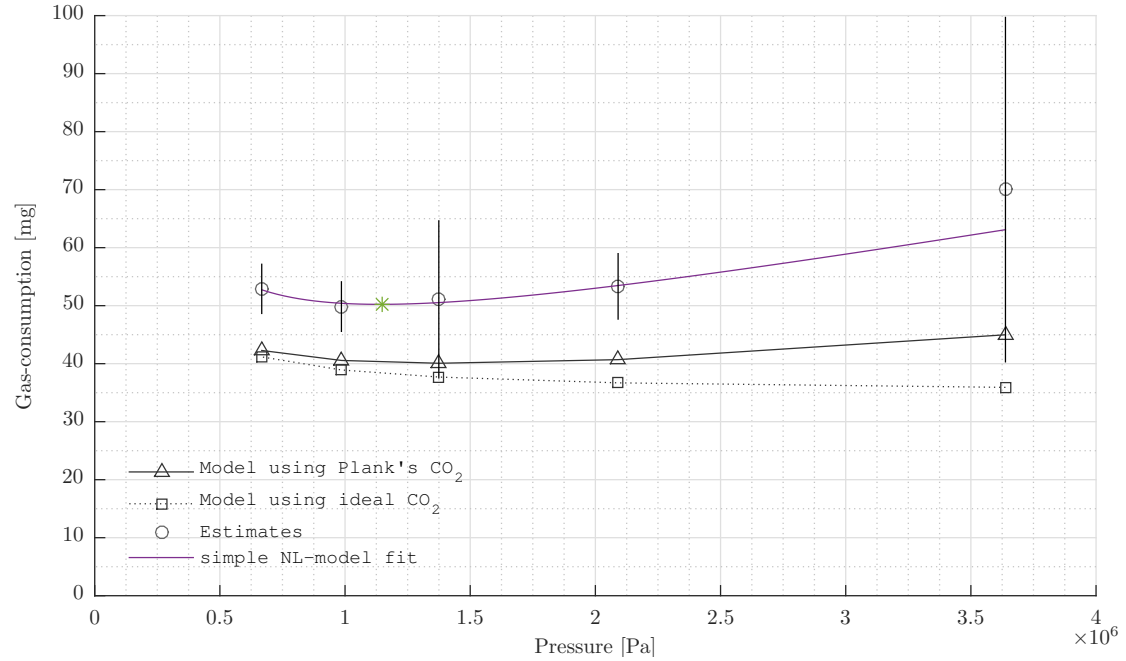


Figure 7: The gas-consumption of 5 different sized cylinders compared in three ways: Two models for different gas laws, and the estimated values and their 95% confidence intervals based on measurements. The estimations are found through regressions of the measurement data for each cylinder, and evaluated at the design pressures. A non-linear model was fit through the estimations to find an optimum at 1.12 MPa, indicated by the asterisk.

4 Discussion

The discussion comprises several subsections: A, on the theory; B, on the measurement set-up; C, on the data in general and D, on the results.

A The models

A.1 The model by Plettenburg [10]

Here are some remarkable aspects that were found when analysing this model.

On the ‘mass flow to pressure drop’ relation When Equation (4) was introduced by Plettenburg [10, eq. A2.5.4], it has changed somewhat. From

$$\left(\frac{\dot{m}}{\frac{1}{4}\pi d_s^2}\right)^2 = \frac{p_s^2 - p_c^2}{RT[fL_s/d_s + 2\ln(p_s/p_c)]}$$

by equating RT to $\frac{p_s}{\rho_s}$ and $\left(\frac{\dot{m}}{\frac{1}{4}\pi d_s^2}\right)$ to φ , to

$$p_c^2 - p_s^2 - 2 \cdot \varphi_m^2 \cdot \frac{p_s}{\rho_s} \cdot \left[\ln \frac{p_c}{\rho_c} - 2f \frac{L}{d} \right] = 0 \quad (10)$$

How this was done exactly is unclear. Especially how the logarithm of the ratio of the two pressures changed into a ratio with density and pressure. The author went on assuming that the logarithm of this ratio would be negligible, compared to the $f(L/d)$ term, therefore Equation (10) was simplified to

$$p_c^2 - p_s^2 = -4f \frac{p_s}{\rho_s} \varphi_m^2 \frac{L}{d} \quad (11)$$

This lead to a more simple differential equation, which was still complicated to solve.

No indication is made here as to what the impact would be on the model and its results, if the term in the square brackets was kept in tact.

On the Darcy friction factor f Taking for example a $\varnothing 10$ mm cylinder, with piston area $A_p = 78.5 \text{ mm}^2$ and a stroke of 6 mm, at the end of the stroke a space is filled of 471 mm^3 .

If the filling is done in, say, 0.5 s then, the gas flow through the supply line, with an inner diameter of 0.125 mm, would have a velocity of $471/((\frac{\pi}{4} \cdot 0.125^2)/0.5) = 7.68 \cdot 10^4 \text{ mm/s}$, or 76.8 m/s. This would lead to a Reynolds number, $Re = \rho v d / \eta = (23.2 \text{ kg m}^{-3} \cdot 76.8 \text{ m s}^{-1} \cdot 125 \times 10^{-6} \text{ m}) / 15.1 \times 10^{-6} \text{ Pa s} = 1.47 \times 10^4$, which is the turbulent flow regime. This would result in a friction factor according to the Moody chart [13, p.349] of $f = 2.9 \cdot 10^{-2}$ and not $f = \frac{64}{Re} = 4.3 \cdot 10^{-3}$ as predicted by the laminar flow theory.

On the other hand, the difference is not that big, and for a first approximation a laminar flow assumption might give a reasonable value. Furthermore, the supply line length did not seem to affect the optimal pressure much, according to [10], therefore the effect of flow friction is probably not significant.

A.2 The simplified model

One of the objectives of this paper was to simplify the model mentioned above. The result is a simple explanation of the cause of an optimal gas-consumption, as well as a model to calculate the optimum.

There are two main reasons for the existence of an optimal gas-consumption.

First, atmospheric back-pressure, p_a , adds to the load, and so a system, running at p_s , requires a larger cylinder than without the extra load. When p_s would be at or below p_a , it is even impossible to generate a net force pushing out the piston. Thus the cylinder size required to generate F_{net} tends to go to infinity when approaching p_a from higher pressures. Obviously, infinitely big cylinders have a large gas-consumption.

Second, density of the gas increases with pressure, more than proportionally. This means that at higher pressures less work is performed by the gas as it expands, therefore more gas is consumed by a cylinder. In other words, Boyle's law ($PV = k$) does not hold at elevated pressures, there the volume is lower than expected. Plank formulated an empirical law to compensate for this, in [9], in essence a modification of the ideal gas law. In Figure 5 the effect of different CO₂ models shows, as the two graphs diverge with increasing pressure.

Temperature effects Note that neither Plettenburg's model, nor the simplified model, include temperature effects, which generally are significant in expanding gasses. For the intended application the absolute gas-consumption is estimated at 3.5 g of CO₂ per day [10]. The amount of gas used per cycle is in the order of 10 to 20 μg . It is reasoned that the amount of energy required to heat this amount of gas to ambient temperature is not a lot, and should easily and quickly be provided by the surroundings.

Flow losses The simplified model also does not include any other loss effect, such as flow friction losses through any length of piping or bends. This is mainly because it does not include cycle time. As long as no leaks are present it is reasoned that all of the gas will end up in the cylinder (or dead space) and gas-consumption is not increased from this point of view. If time is part of a model, flow losses would play a role.

B The set-up

The set-up consisted of the test-bench to control and measure displacement and pressure levels as well as measure force levels and a scale, to measure gas-consumption. Five cylinders were designed for specific working pressures. They were supplied by a small bottle of CO₂ through an adjustable reduction valve, connected by pipes and tubes.

A difference in pipes The model includes a pipeline, but in the theory [10, p. 59] its length did not seem to affect the optimal pressure level. In the set-up the corresponding part is the T-shaped space in the valve-block (Figure 3). It has been designed quite small to reduce the amount of dead-space. Its volume in the valve-block is relatively small, 8 mm³, compared to the volume in any one of the used cylinders. Although the effect of this dead space is greater in the smallest cylinder, it was neglected as the extra gas-consumption is only 1.5%. The tubes and pipes in the set-up correspond to the supply part in the model. It is in this part that the supply pressure is regulated.

On the pressure regulator, or reduction valve The reduction valve relies on a force balance between, the spring force on the valve, which can be modified by hand, and the pressure difference over the valve. The pressure difference comes from the vapour pressure in the bottle, and, on the other side of the valve, the system pressure. When the in-valve in the valve block, opens to fill the cylinder, a pressure drop in the supply part of the system occurs. The regulator responds by releasing CO₂ through its valve.

However, this response is not 100% accurate, the valve doesn't always open at the same pressure and the CO₂ flow slows down the closer it gets to the set-point. At times, it would take several strokes of the piston to even get the opening response from the reduction valve, see e.g. Figure 11d). Due to the slow response, a new cycle was started before the pressure had come back up to the set-point.

The valve's response is pressure dependent, more specific, pressure difference dependent. The probable reason is the O-ring that seals this valve. It is not linearly elastic and doesn't close sharply. The average pressure over the cycles was therefore chosen as the pressure level for the measurement.

To increase the accuracy of setting the valve to the correct pressure, an extension has been made. The extension used a longer spring, but retained the spring force. The screw that was used to set the spring tension, could then move twice the distance for the same pressure range, giving a finer control. This was especially important at pressures above 1 MPa, because at higher pressures, the set-point became very sensitive to the spring-force. Unfortunately, it did not solve the slow response issue.

One option that was not explored, which could help to improve the response, is to reduce the volume between the regulator and the valve-block. This would cause a faster response of the valve, because the pressure in the supply-line would drop faster when the 'buffer' is smaller. Also it should cause a flow rate that changes faster and has a higher peak value, because of the increased pressure difference. This greater pressure difference should help close the valve more quickly near the set-point.

On the O-ring friction F_f In the Plettenburg's model, as well as the simplified model, the friction of the O-ring in the piston against the cylinder's inner surface is regarded as part of the piston load. In other words is not modelled and the friction does not deduct from the force calculated at the cylinder output. This does not say there is no friction in the set-up. According to the following analysis the effect of the friction is around 5% of the net force. This, at least partly, explains the discrepancy between model and the measured values. Across the entire pressure range, the measured gas-consumption appears to be roughly 20% above the modelled values. This is an important reason the friction has not been measured independently.

In [10, p.45] Equation (12) was given, it is the SI-version of the same equation by Martini [7, p.124]. Although this equation uses the exact area sealed by the O-ring, A_r , in stead of the approximation by Martini, who used $A_r = W\pi D_m$ where W is the O-ring thickness and D_m the O-ring mean diameter. The friction is calculated in two parts, F_h the force due to the pressure over the O-ring, and F_c the force due to O-ring compression. The compression, s , occurs because the space between between the perimeter of the seal, L_r , and the piston groove, is smaller than the O-ring thickness. With this formula the friction force F_f can be calculated,

$$\begin{aligned} F_f &= F_h + F_c = A_r \cdot f_h + L_r \cdot f_c \\ &= \frac{\pi}{4}(D_c^2 - D_p^2) \cdot 0.078p^{0.61} + 0.175\pi D_c \cdot s (-0.884 + 0.0206H_s - 0.0001H_s^2) \end{aligned} \quad (12)$$

Here D_p is the piston groove diameter, f_h and f_c are the friction factors.

To calculate the friction, the guidelines [10, p.49] for O-ring seal were followed. The compression of the O-rings is 8%, and the hardness of the O-rings is 70° Shore A. Applying the formula to the sizes of the cylinders that were built, gives a friction of about 4 to 7 N at the pressures they were designed for. These values can be seen in Table 4 and Figure 8.

It should be noted that the formula was based on O-rings on pistons running at at least 5 mm s⁻¹, in $R_a = 0.4 \mu\text{m}$ finished chrome plated pistons, lubricated with [MIL-H-5606] hydraulic oil at room temperature. It means the calculated friction is only an approximation for the friction

P_d [MPa]	F_f	F_h	F_c
3.64	4.32	2.53	1.80
2.09	4.90	2.51	2.39
1.37	5.49	2.49	2.99
0.99	6.07	2.48	3.59
0.67	6.98	2.49	4.49

Table 4: The cylinder pressures and friction forces. The total friction F_f is the sum of the friction due to the pressure over the O-ring, F_h , and the friction due to the compression of the O-ring, F_c .

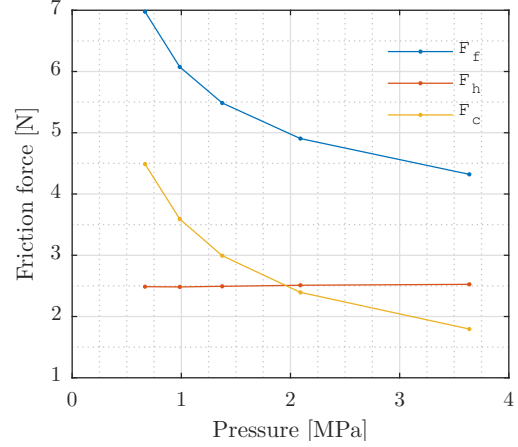


Figure 8: The graph shows the data from Table 4. The pressure related friction F_h seems nearly constant when the cylinder size is adapted according to the pressure. The friction due to O-ring compression decreases with the cylinder diameter.

in the pistons in this set-up. However, the calculations do make it plausible that increasing the pressure, while decreasing the cylinder size accordingly, will decrease the friction of O-ring.

C The data in general

To demonstrate the existence and location of the optimal pressure, the initial assumption was that measuring the gas-consumption at five chosen pressures would suffice. For each cylinder the sensitivity to the pressure variation was determined first, because practically it proved difficult to set the reduction valve to an accurate value. For the 6 mm cylinder few measurements were done successfully at high pressure. The measurement at 2.7 MPa was especially abnormal, taking 12 full cycles and a pressure drop of nearly 1.5 MPa for the pressure reducer to respond.

The internal cylinder of the test bench made sure the force level could not exceed a certain limit. That is why for a pressure level of 2 MPa and up, only the two smallest cylinders were used and for pressures exceeding 3 MPa, only the smallest cylinder was used.

On suspected leakages The gas-consumption in the measurements, seem to be about 20% above the model values over the entire pressure range, see Figure 7. One important cause of increased gas-consumption in general, is leakage. Leakage would cause a bigger gas-consumption than would be calculated by the effect of only the piston displacement. There are some points in the data where leakage is suspected. Take for instance Figure 12d). After the first two cycles, the pressure comes back up to the same level for another three cycles. The force level however, drops from 110 N down to 100 N, and for cycles 6, 7 and 8 even to 95 N. Even though no change had been made to the reducer, and similar displacements have been applied. Although for cycles 6, 7 and 8, it might be that the reduction valve simply did not respond because the lower pressure level was not reached by the supply volume.

D The results

D.1 The theoretical results

The model by Plettenburg combined a lot of factors dealing with the gas-consumption. An important part of that model, that of the supply-line geometry, was shown not to affect the result in terms of optimal pressure. Those factors have been left out in this research. A simplified model was made that produced a similarly shape graph, see Figure 2 and 5. Although, the exact location of the optimum was found at a different pressure, 1.42 MPa, with the simplified model, instead of 1.2 MPa by Plettenburg.

The optimum at 1.38 MPa found by the non-linear fit shows that the non-linear approximation fitted to 5 points can give a close approximation of the optimum with a given dataset.

Sensitivity According to the simplified model, the optimal pressure is insensitive to the net force level the cylinder is supposed to deliver. It scales the gas-consumption linearly with the force. Also changing the stroke length, does not change the optimal pressure level. It scales the gas-consumption linearly with the volume.

A change in atmospheric pressure does affect the optimum. Values for atmospheric pressure typically⁴ vary between 94.0 and 106.0 kPa, a range of 12 kPa. Within this range, the optimum varies between 1.385 and 1.445 MPa a range of 60 kPa. The gas-consumption varies from 39.9 to 40.2 mg, a range of 0.3 mg.

Explaining the optimum Even though the model by Plettenburg, Equation (5), shows the optimum, it is not trivial to see why. The new model, Equation (9), more clearly shows why there is an optimum,

At system pressures near atmospheric pressure, the piston has to be very large in order to produce a net force F_{net} (of e.g. 100 N). With the piston becoming larger, also the volume of the cylinder becomes large. This accounts for the rise in gas-consumption, when the system pressure is reduced from the optimum.

If one was to calculate m_c using the ideal gas law, the optimal pressure would always be at the highest pressure of the chosen pressure range. If there is no atmospheric pressure, gas-consumption would be the same for the whole pressure range.

When correcting for actual CO₂ behaviour, as is done with Equation (3) the increased gas-consumption at a higher pressure is explained. At pressures approaching that of saturated vapour at room temperature (i.e. 6.0 MPa), the density becomes higher than the Ideal Gas Law would predict. This is why, the graph shows an increase in gas needed to fill the void, even though less volume needs to be filled.

D.2 The measurement results

The results show an optimum at 1.12 MPa. This is in line with previous research, there are differences though.

First, the gas-consumption prediction was about 20% lower over the whole pressure range and second, the optimal pressure was slightly lower as well. The 20% discrepancy can partly be explained by O-ring friction, since this was not taken into account in the model, but it is present in the set-up. No measurements were done to confirm the level of friction, but the consistently higher gas-consumption does agree with the O-ring friction analysis in subsection B. Given the uncertainty of the regression of the measurement data, the optimal pressure could still be in a range 1.0 to 1.4 MPa.

⁴<http://www.knmi.nl/cms/content/36213/luchtdruk>, retrieved 05-03-2015, in Dutch

Fitting Equation (6) through the upper 95% certainty level of the regression data, the optimal pressure would be 1.00 MPa and at the lower 95% certainty level, the optimal pressure would be 3.03 MPa. At 80% certainty, they would be 1.06 and 1.30 MPa respectively. This shows that the uncertainty of the regression of the smallest cylinder affects the estimation of the optimum strongly.

Without the estimation for the largest cylinder, the optimum would be at 1.21 MPa. Without the estimation for the smallest cylinder (with the largest uncertainty), the optimum would be at 1.15 MPa.

The non-linear fit probably does not yield the exact value for the optimum pressure, even if the estimations are spot on, as can be seen in the theoretical results. The uncertainty of using the fit is likely to be around ± 0.05 MPa, still making it a good approximation.

Adding to the explanation for the 20% discrepancy in gas-consumption between the simplified model and the measurements, is the simplicity of the model. The most obvious omission are temperature effects. Perhaps the purity of the gas has a large effect on the density of the gas. These are only guesses, and no estimation can be given here.

5 Conclusion

This paper set out to answer the question of whether there is an optimal pressure one could use for CO₂ powered ‘stand alone’ systems and also determine this pressure. The pressure should be optimal for minimal gas-consumption, as these systems are assumed to have a limited gas-supply.

To do so, a previous model was analysed, and found to be too complex to give a clear explanation. Thus a simplified model was made, which showed the same behaviour, yet is more explicit in the origins of the behaviour. A set-up was made, based on the earlier model, to measure the optimum.

The goals of the paper are met. A straight forward model to calculate the gas-consumption of a CO₂ powered cylinder has been presented. It explains in simple terms why there is an optimal pressure, for cylinders with the same power output:

- Due to atmospheric pressure on the back of the piston, the size of the piston must increase, and with it, so will the volume. This larger volume causes a greater gas-consumption especially at pressures below the optimum.
- Due to the nature of CO₂, with increasing pressure, the density of the substance increases more than it would have according to the ideal gas law. Therefore more gas is consumed at higher than optimal pressure.
- The simplified model predicts an optimum at 1.42 MPa.

The measurements that were done agree with earlier research:

- The optimal pressure was found at 1.12 MPa. A bound to this optimum is given between 1.0 MPa and 1.4 MPa.

The bound reflects the challenges of performing accurate measurements and the flatness of the curve where the optimum is found. The earlier results, of Lambert [6] and Plettenburg [10], combined with the results presented here, lead to the conclusion that the optimum found by Plettenburg, at 1.2 MPa, is an efficient choice in system pressure to minimize gas-consumption in a CO₂ powered pneumatic system.

6 Recommendations

In this section some recommendations are presented which could improve the theory, or the way the measurements were done.

To expand the theory

- The model presented by Plettenburg, or the simplified model presented in this paper, could be expanded with thermal effects of the gas.
- Models could be made for other gasses

For performing measurements

- It will be valuable to do more measurements for all cylinders, at various pressures, to determine a more accurate sensitivity for the cylinders.
- Reduce the amount of volume between the reduction valve and valve-block. This will improve the response of the reduction valve.
- Use a scale that can handle large weights, yet retain accuracy, so it would be possible to use accurate but bulky standard reduction valves, and still measure the weight.
- Use a digitally controlled reduction valve, e.g. a piezo actuated one, which uses feedback to set the pressure.

On future applications

It is wonderful that electrically powered systems are so easy to control. With the right programming the applications seem only limited by the imagination. However, things that can be built are *not* limited by the imagination, only by physics. Herein lie the opportunities in engineering. For mechatronic engineering it is the challenge build a controllable system using all that physics has to offer.

New technologies allow for manufacturing of pneumatic systems with a better design. The main recommendation is to further explore the possibilities (and limitations) of the use of CO₂ in pneumatics, specifically:

- Build and work with thin walled cylinders, to maximize the power to weight ratio of cylinders. Current commercial cylinders are usually designed for industrial applications, they are generally not optimized for lightweight construction, durability usually outweighs mass, nor minimal gas-consumption, as an infinite amount of air is usually supplied by a compressor nearby.
- Recover energy when reducing the gas pressure. CO₂ is stored in liquid form at the equilibrium pressure of 6.0 MPa at room temperature. In the reduction valve used in this research, energy is wasted when reducing the pressure to the system pressure of 1.2 MPa. Use, e.g. multiple stages of pressure for different parts of the pneumatic system or
- Use lightweight storage of liquid CO₂, using e.g. carbon fibre wound containers. Three quarters of the energy density in the gas supply is lost in the weight of the steel bottle, which is three times the weight of the gas itself, for the 16 g supply bottles used in these measurements.

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7 Appendices

Measurements

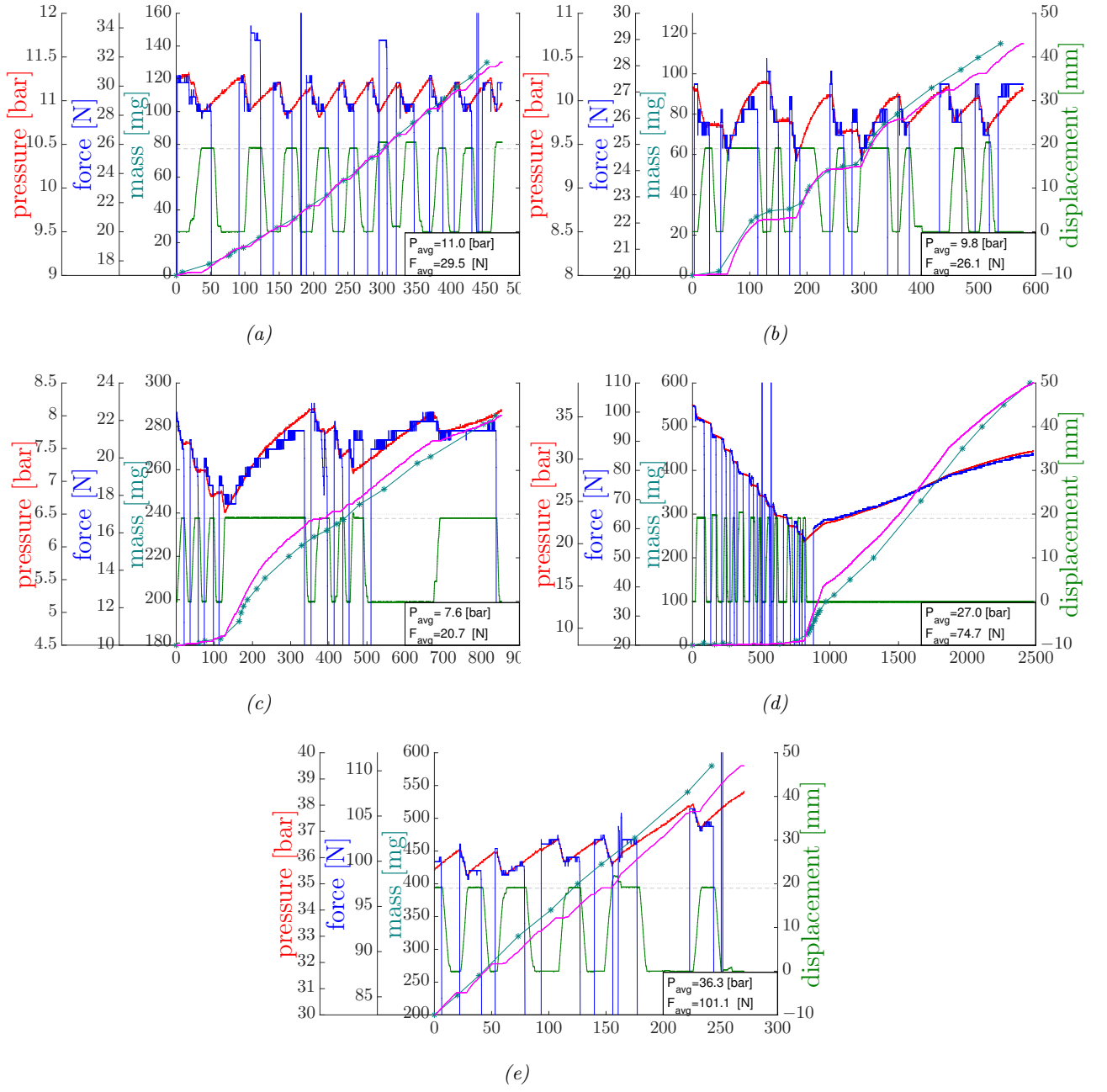
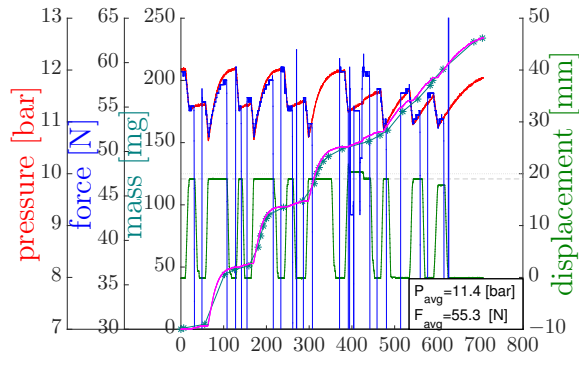
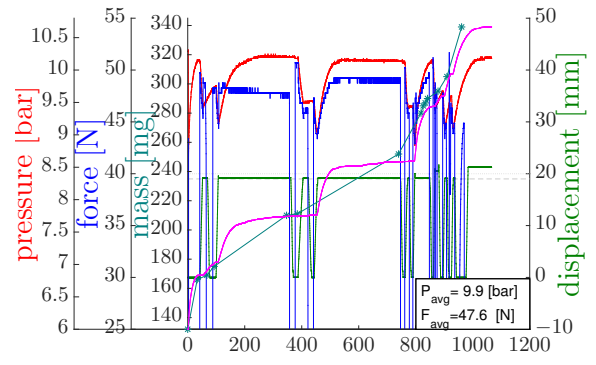


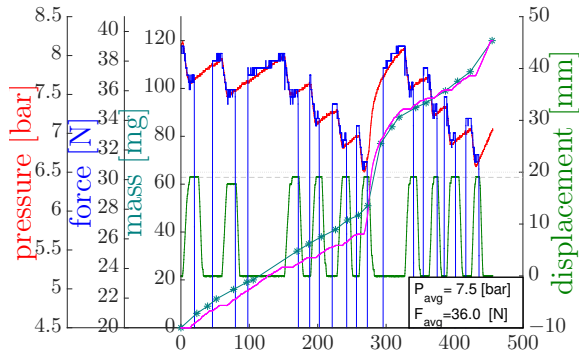
Figure 9: Figures for cylinders with $D_c = 6.0$ mm



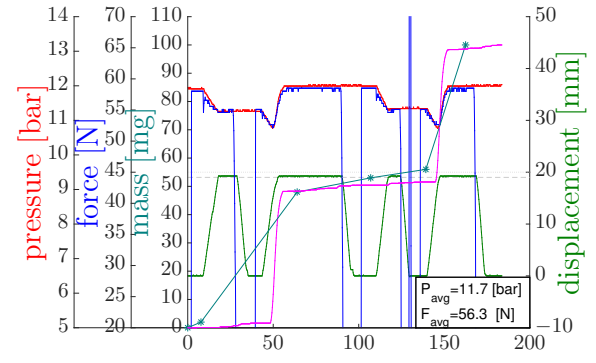
(a)



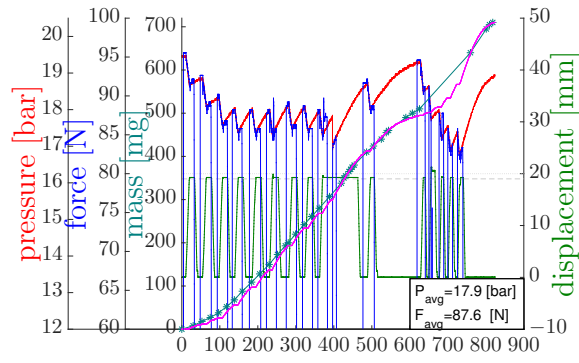
(b)



(c)

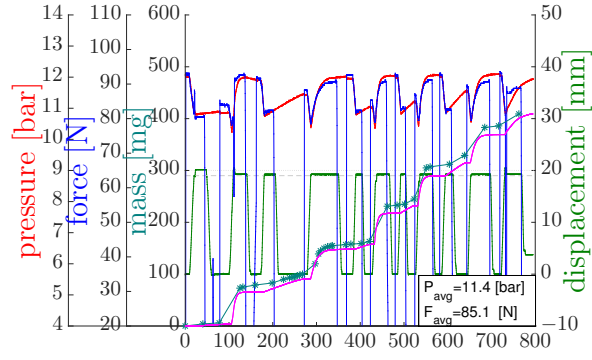


(d)

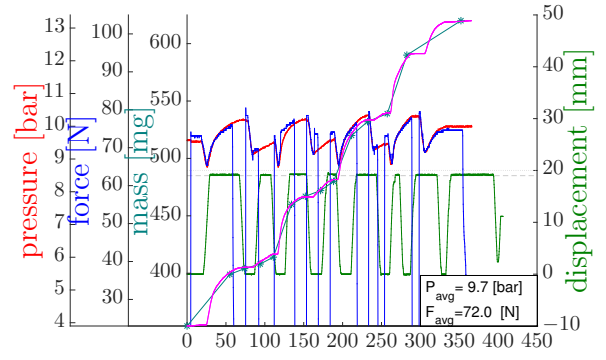


(e)

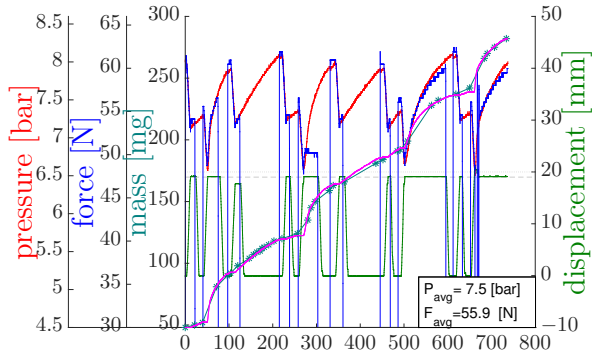
Figure 10: Figures for cylinders with $D_c = 8.0$ mm



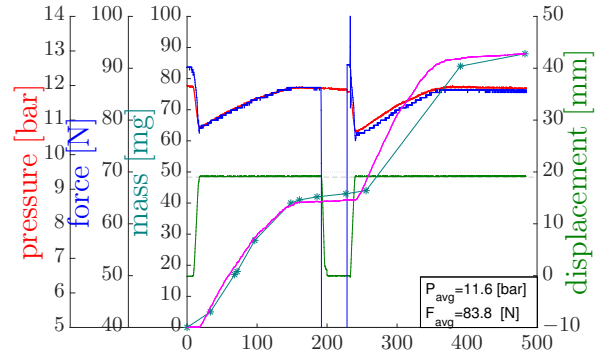
(a)



(b)

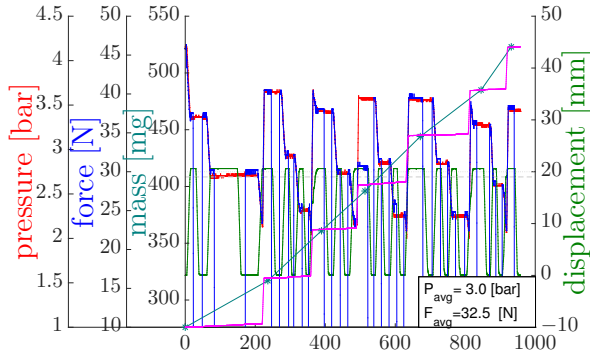


(c)

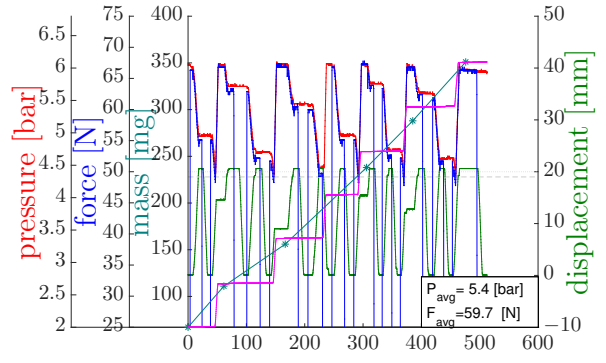


(d)

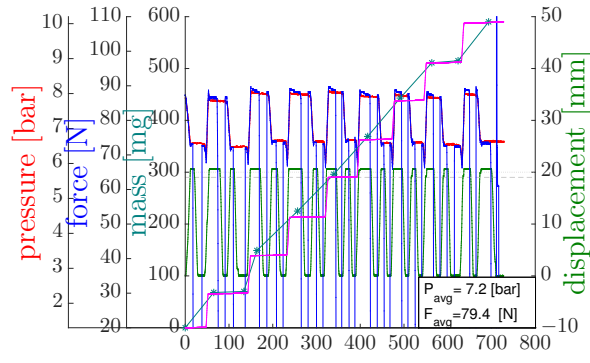
Figure 11: Figures for cylinders with $D_c = 10.0$ mm



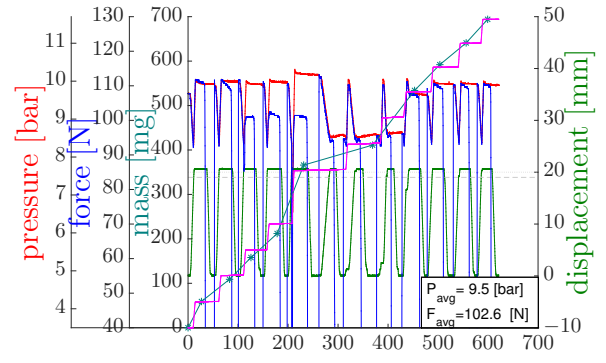
(a)



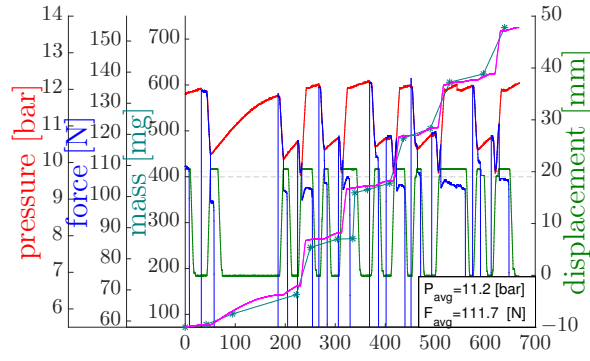
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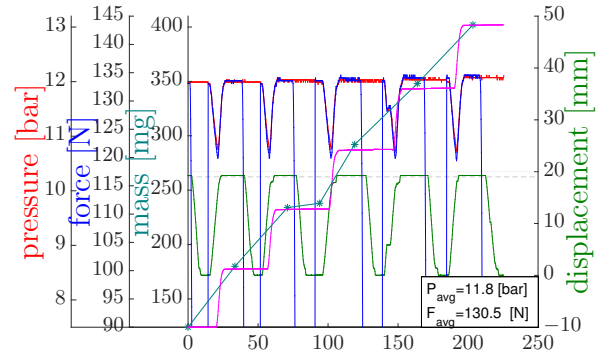
(c)



(d)



(e)



(f)

Figure 12: Figures for cylinders with $D_c = 12.0$ mm

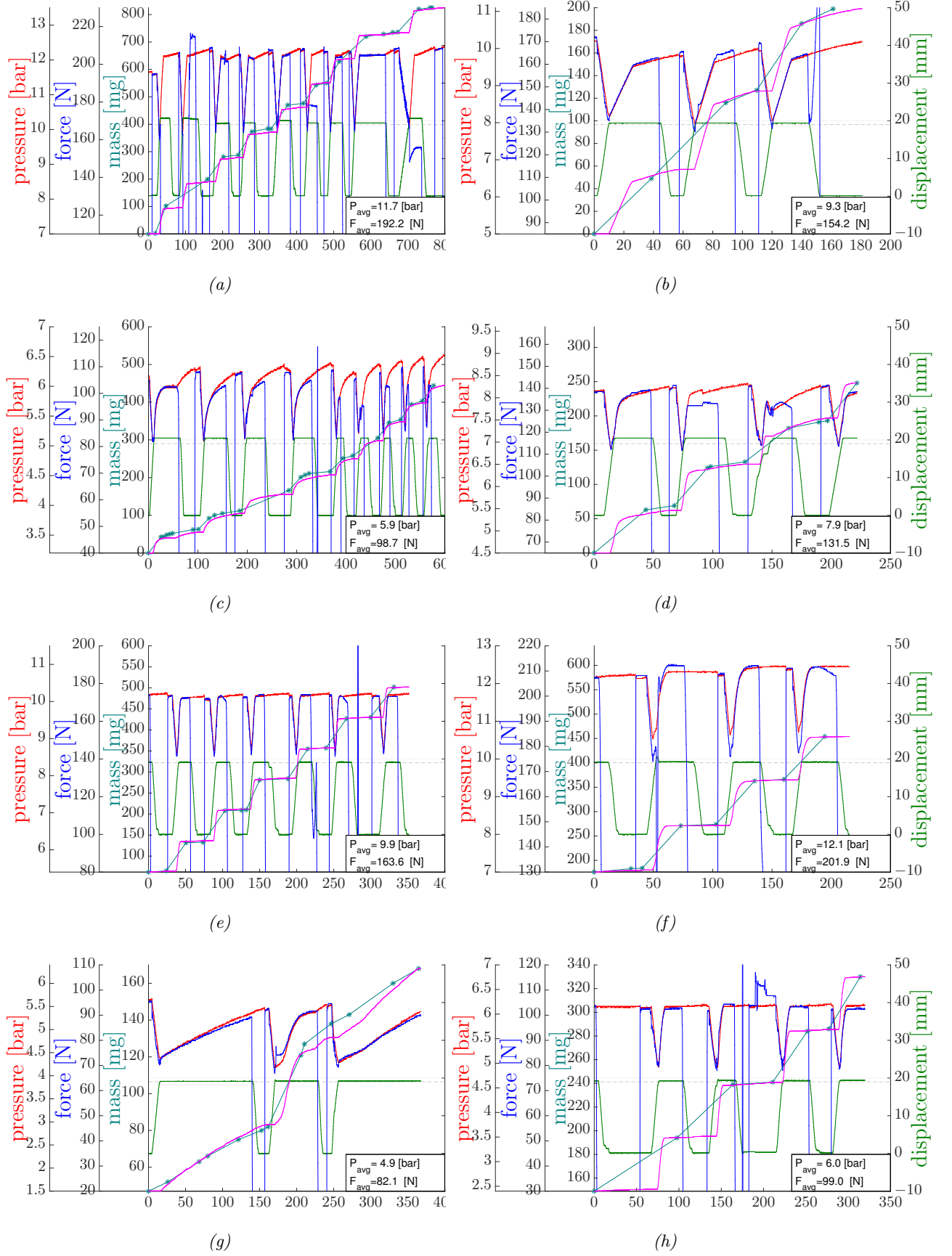


Figure 13: Figures for cylinders with $D_c = 15.0$ mm

Scripts

A variables.m

Contents

- Variables which can be manipulated
- Constants
- Material properties
- properties of the Plank gas law
- Define geometries

```
% VARIABLES is where all other scripts get their common variables,  
%           which ensures they all use the same data  
%  
% REMARKS   : SI-properties are written with a capital letter.  
%           subscripts are preceded by an underscore if they are global  
% CREATED BY: D.C.Doedens  
% DATE      : 13-02-2015
```

Variables which can be manipulated

```
global T_a F_c P_a P_s  
T_a = 273.15+30;    %[K] Atmospheric temperature  
F_c = 100;          %[N] The desired force  
P_a = 100000+1325; %[N/m2] Atmosferische pressure  
P_s= 5E5:.5E4:30E5; %[N/m2] The absolute pressure range  
%           for the system
```

Constants

Several universal variables. These should be comparable no matter what model one uses

```
global R;  
NA=6.0221417*10^23; %[ #/mol   ] Avogadro's constant  
k=1.3806488*10^-23; %[ J/K     ] Boltzman's constant  
R.ugc=NA*k;         %[ J/mol/K ] = [N m /mol /K] =[Pa m3 /(mol K)]  
% 8.3145 [Pa m3 /(mol K)], to be clear  
% Universal Gas Constante  
  
clear NA k;
```

Material properties

Rplank is the approximation of Rgiven which was published by Plank and Kuprianoff. Rugc, is the universal gas constant.

```
Mco2=44.0*10^-3;    %[ kg/mol   ] Molaire mass of CO2  
R.co2=R.ugc/Mco2;   %[N/m2  * m3]/[kg*K]  
R.plank=R.co2/9.80665;    %[kp/m2 * m3]/[kg*K] by N = 1/9.80665 kp  
R.given=19.273; %should be [kp/m2 * m3]/[kg*K]  
clear Mco2;
```

properties of the Plank gas law

```
global PLANK
PLANK.a=0.0825; % [K^(10/3)*{m3/kg}]
PLANK.b=1.225e-7; % [K^(10/3)*{m3/kg} / {kp/m2}]
PLANK.pA=PLANK.a*1e9/1e6; % [K^(10/3)*{mm3/mg}]
PLANK.pB=PLANK.b*1e9/1e6*1e6/9.80665; % [K^(10/3)*{mm3/mg}/{N/mm2}]
PLANK.A=PLANK.a; % [K^(10/3)*{m3/kg}]
PLANK.B=PLANK.b/9.80665; % [K^(10/3)*{m3/kg} / {N/m2}]
```

Define geometries

These are the basic geometries for the setup.

```
global D_c L_c A_c V_c
dc=[6 8 10 12 15]'; % [mm] cylinder diameters
lc=20; % [mm] stroke of the piston
D_c=dc*1e-3; % [m]
L_c=lc*1e-3; % [m]
A_c=.25*pi()*D_c.^2; % [m2]
V_c=A_c*L_c; % [m3]
clear dc lc
```

B model.m

Contents

- calculate densities
- Calculate required volumes for the pressure range
- Gas mass in the cylinder
- fit a non-linear model to selected points
- Plotting

```
% MODEL Builds a simplified version of the model built by Plettenburg
%
% REMARKS    : Calculate gas consumption via
%              M = rho * V
%              rho is calculated once with the ideal gas law,
%              and once with the law adapted to the behaviour of CO2,
%              using the adaptions by Plank and Kuprianoff. Furthermore, a
%              correction is made for the atmospheric pressure.
%              Then, some points are chosen, and parameters found to fit a
%              non-linear model through those points.
%
% CREATED BY: D.C.Doedens
% DATE      : 01-12-2014

% Initialise, to make sure the workspace is clean. And there are no
% unexpected figure windows open and get global variables.
clearvars;close all;clc;
run('variables.m');
```

calculate densities

```
[sV,sV_i]=gaslaw(P_s);
Rho_i = 1./sV_i; %[kg/m3] using the Ideal gas law
Rho    = 1./sV   ; % using Plank's law.
```

Calculate required volumes for the pressure range

$V = A_c * X_c$; A_c is found via $F = P * A$ Atmospheric pressure can be corrected for here.

```
A = F_c ./ (P_s);          % uncorrected
V = A .* L_c;
Ac = F_c ./ (P_s-P_a); % corrected, giving a larger A
Vc = Ac .* L_c;
```

Gas mass in the cylinder

```
Mc  = Rho    .* V; % using Plank
Mci = Rho_i  .* V; % using Ideal gas law
McC = Rho    .* Vc; % corrected for atmospheric pressure
Mcic= Rho_i  .* Vc;
```

fit a non-linear model to selected points

nlmodel has the form $m_c = b_1x^{-1} + b_2 + b_3 * x$ b has the parameters b_1, b_2 and b_3 nlinfit starts with b_init and returns b_fitd, b_fitd are the fitted parameters, these are then used by nlmodel return a gas-usage for the entire range of P_s.

```
nlmodel=@(b,x) b(1)*x.^(-1)+b(2)+b(3)*x;
b_init=[1;1;1];%some random variables to start with
selection=50:100:size(Mcc,2);
mcc=1e6*Mcc(selection);    %[mg]
ps=P_s(selection)./1e6;    %[MPa]
b_fitd=nlinfit(ps,mcc,nlmodel,b_init); %[mg] and [MPa]
mfitd=nlmodel(b_fitd,P_s/1e6);

h.range=find(P_s);
h.min=find(Mcc==min(Mcc(h.range)));
fprintf('\nBack pressure: %4.0f    hPa\n',P_a/100);
fprintf('Mass consumed: %3.0f    mg\n',Mcc(h.min)*1e6);
fprintf('Optimale druk: %6.2f MPa\n\n',P_s(h.min)*1e-6);
```

Plotting

```
h.figure=figure(1);
set(gcf,'WindowStyle','docked');hold on;grid on;
h.ax=gca;h.ax.XLim=[0 4e6];h.ax.YLim=[35 45];
h.pc=plot(P_s(h.range),1e6*Mcc(h.range),'LineStyle','-', 'color',[1 0 0]*1);
plot(ps*1e6,mcc,'o'); plot(P_s,mfitd,'--k');
h.ic=plot(P_s(h.range),1e6*Mcc(h.range),'LineStyle','-.', 'color',[0 0 1]*1);
h.m1=plot(P_s(h.min),1e6*Mcc(h.min),'*');
xlabel('Pressure [Pa]');
ylabel('Gas requirement [mg]');
legend('Plank''s CO_2','Fit points','NL-fit',...
      'Ideal CO_2','Optimum',...
      'Location','SouthWest');
%mlf2pdf(h.figure,'models_2c');
```

C bekijkdata.m

Contents

- Determine relevant files
- Acquire data from those files
- Set the limits for the observation window
- Retrieve data
- Metadata
- Creating plots in several steps
- Build the observation windows
- output images and print the values for verbruik.m

```
%BEKIJKDATA displays measured data graphically
%
% REMARKS      : This script is used in order to be able to estimate gas-usage
%                from the chosen 'observation window'
%
% CREATED BY: D.C.Doedens
% DATE       : 15-08-2014

clc;close all;clear all;

cilinder=1;
meting=0;%0 returns all measurements
maat={'06','08','10','12','15'};
```

Determine relevant files

```
%meetpad='C:\Users\Dirk\Documents\01_Studie\01_afstuderen\01_prosthesis\02_B_measureing\';
%meetpad='/media/MICROFLOWN/Dirk/01_prosthesis/02_B_measureing/Data';
meetpad='02_Data/';
%datapad='Data/Rubbish';
datapad=[maat{cilinder}];
pad=[meetpad,datapad];
if pad(end)~='/'
pad=[pad '/']; %pad should always end in \
end
inhoud=dir(pad);
nodes=size(inhoud,1);filecount=0;

for i = 1:nodes
    if inhoud(i).isdir == 0 %than it's a file
        filecount=filecount+1;
        naam{filecount,1}= inhoud(i).name;%#ok
    end
end
disp(pad);
fprintf('aantal bestanden in deze map: %i\n',filecount);
%alle relevante data is in 'pad', 'naam' and 'filecount'
```



```
clear meetpad datapad inhouid nodes i
clear fileToRead
```

```
if meting;
    looprange=meting;
else
    looprange=1:filecount;
end
```

Aquire data from those files

```
for loopnr=looprange
```

Set the limits for the observation window

These are set to determine the 'observaion window' from which the measurements are taken. They are manually determined for each file. The limits of the ranges are paired in columns of two. The limits are set for mass **mass**, force **F**, pressure **P**, displacement **x** and time **t**.

```
% mass | F | P | x | t ;
limits06=[...
    0, 160 , 17,35 , 9,12 , -10, 50 , 1,1900;...1
    0, 130 , 20,30 , 8,11 , -10, 50 , 1,0 ;...2
    179, 300 , 10,24 , 4.5,8.5 , -10, 50 , 1,0 ;...3
    0, 600 , 20,110 , 8,39 , -10, 50 , 1,0 ;...4
    200, 600 , 83,112 , 30,40 , -10, 50 , 300,0 ;...5
];% Manually edited for each plot
% mass | F | P | x | t ;
limits08=[...
    0, 250 , 30,65 , 7,13 , -10,50 , 50,0 ;...1
    132, 345 , 25,55 , 6,10.8 , -10,50 , 1,0 ;...2
    0, 130 , 20,41 , 4.5,8.5 , -10,50 , 60,471*(1e3/250);...3
    0, 110 , 20,70 , 5,14 , -10,50 , 1,0 ;...4
    0, 720 , 60,100 , 12,20.5 , -10,50 , 1,0 ;...5
];% Manually edited for each plot
% mass | F | P | x | t ;
limits10=[...
    0, 600 , 20,110 , 4,14 , -10,50 , 1,0 ;...1
    354,625 , 23,105 , 3.9,13.4 , -10,50 , 30,0 ;...2
    48, 300 , 30,66 , 4.5,8.6 , -10,50 ,455,3400;...3
    0, 100 , 40,100 , 5,14 , -10,50 , 1,0 ;...4
];% Manually edited for each plot
% mass | F | P | x | t ;
limits12=[...
% 317, 550 , 10,50 , 1,4.5 , -10, 50,(2+435.8*5),0 ;...1
    276, 550 , 10,50 , 1,4.5 , -10, 50,(2+199.6*5),0 ;...1
    67, 400 , 25,75 , 2,6.8 , -10, 50,1031,0 ;...2
    0, 600 , 20,110 , 1.3,10.2, -10, 50, 1,0 ;...3
    0, 700 , 40,130 , 3.5,11.7, -10, 50, 1,0 ;...4 for all of the file
%411, 700 , 40,130 , 3.5,11.7, -10, 50,1847,0 ;... for the last half of the file
```

```

% 1, 500 , 40,130 , 3.5,11.7, -10, 50, 1,1000;... for the second half
% 0, 250 , 58,158 , 5.5,14 , -10, 50,198,1200;... don't trust the first bit...
72, 750 , 58,158 , 5.5,14 , -10, 50,1810,0 ;...5 Dunno if I should use this.
124, 410 , 90,145 , 7.5,13.2, -10, 50,351,0 ;...6
];% Manually edited for each plot
% mass | F | P | x | t ;
limits15=[...
0, 825 ,110,230 , 7,13.5 , -10, 50 , 1,0 ;...1
0, 200 , 85,185 , 5,11.1 , -10, 50 , 1,0 ;...2
0, 600 , 40,125 ,3.2,7 , -10, 50 , 1,0 ;...3
% 0, 330 , 65,168 ,4.5,9.6 , -10, 50 , 1,0 ;...4
0, 330 , 65,168 ,4.5,9.6 , -10, 50 , 1,888 ;...4
62, 600 , 80,200 ,5.4,11.5 , -10, 50 ,174,0 ;...5
176, 640 ,130,220 , 7,13 , -10, 50 ,439,1300;...6
47, 170 , 20,110 ,1.5,6.4 , -10, 50 , 1,0 ;...7
149, 340 , 30,120 ,2.4,7 , -10, 50 ,885,0 ;...8
];% Manually edited for each plot
% mass | F | P | x | t ;
limits=eval(['limits' maat{cilinder}]);
tofset=limits(loopnr,9);tend=limits(loopnr,10);

```

Clean up before a new plotset, to prevent data from mixing

```

clear limits06 limits08 limits10 limits12 limits15;
clear newData vars; %tijdelijke data
clear colheaders data textdata; %tijdelijke data
clear time massa Druk Druk40 kracht verplaatsing; %te verwachten data
clear drukrise;

```

Retreive data

This part should return the measured data: time, kracht, verplaatsing, Druk (Druk40), massa and imassa most of these variablesnames are defined by the headers in the measurement file.

```

fileToRead=[pad naam{loopnr}];
fprintf('\n ***** \ndata uit %i: %s\n',[loopnr,naam{loopnr}]);
[newData,~,~] = importdata(fileToRead,'\t'); % get colheaders, data, textdata
%disp(sprintf('number of header lines: %i', nhead))
vars = fieldnames(newData); % Create new variables in workspace.
for i = 1:length(vars) %make variables out of header
    assignin('base', vars{i}, newData.(vars{i}));
end
if tend==0
    tend=length(data);
end
assignin('base', sscanf(eval([vars{end} '{1}']),'%c%s*'),data(tofset:tend,1));%time data
%assignin('base',sscanf(colheaders{2},'%c%s*'),data(:,2));% raw data in volt
%assignin('base',sscanf(colheaders{3},'%c%s*'),data(:,3));%
%assignin('base',sscanf(colheaders{4},'%c%s*'),data(:,4));%
assignin('base', sscanf(eval([vars{end} '{5}']),'%c%s*'),data(tofset:tend,5));% Druk
assignin('base', sscanf(eval([vars{end} '{6}']),'%c%s*'),data(tofset:tend,6));% Kracht

```

```

assignin('base', sscanf(eval([vars{end} '{7}']), '%c%s*'), data(tofset:tend,7)); % Verplaatsing
assignin('base', sscanf(eval([vars{end} '{8}']), '%c%s*'), data(tofset:tend,8)); % massa
highpressure=0;
if size(colheaders,2)>8
assignin('base', sscanf(eval([vars{end} '{9}']), '%c%s*'), data(tofset:tend,9)); % Druk40
highpressure=1;
end %check voor number of pressure sensors
if (highpressure && max(Druk40)<0.3)
    highpressure=0;
end % A hack for a particular file with both pressure sensor data columns
    % but only one sensor actually attached.

```

Metadata

Here the data is being processed for metadata, which can be used in e.g., minimum and maximum values, or average values in the 'observation window'

```

time=(time-min(time))/1000;
ndata=length(time); %number of datapoints
fprintf('Aantal kolommen data: %i\n',size(colheaders,2));
ival=find(kracht<-limits(loopnr,3)&kracht>-limits(loopnr,4));
nmeting=size(ival,1);
fgem=-sum(kracht(ival))/nmeting;
if highpressure
fprintf('Er is dus met de 40bar sensor gemeten\n');
dgem=sum(Druk40(ival))/nmeting;
else % low pressure
dgem=sum(Druk(ival))/nmeting;
end % get dgem
dia=sqrt(4*fgem/(pi()*(dgem/10)));
fprintf([
    'Er zitten %i nuttige meetmomenten in het bestand.\n' ...
    'Uit %i metingen, bij gemiddeld \n%2.2f bar en %2.2f N \n' ...
    'Wordt een gemiddelde diameter teruggerekend: \n'...
    '%2.1f\n'],[ndata nmeting dgem fgem dia]);
imass=int32(find(diff(massa))); %indices of mass changepoints
mass=massa([imass;end]); %include end-value
if mass(end-1)==8925 %
    mass(end-1)=825;
end % Very quick and dirty hack for a single false entry

    Clean up after extraction

clear i ival %counters
clear fileToRead nmeting;
%clear nmeting dgem fgem dia;
clear newData nhead delim vars; %tijdelijke data
clear data txtdata colheaders rowheaders; %tijdelijke data
clear massa tofset tend; %no longer needed

```

Creating plots in several steps

- Create axes
- Plot data one by one

1. mass
2. pressurerise
3. pressure
4. force
5. displacement

- Build the 'observation window'

1. Change limits
2. Re-place the axes, so the graphs overlap
3. Add some final touches before finishing the plot

Create axes

```
h.figure=figure(loopnr);      %create a new figure
set(gcf,'WindowStyle','docked');
h.ax_mass=gca;
h.ax_force=copyobj(h.ax_mass,h.figure);
h.ax_pressure=copyobj(h.ax_mass,h.figure);
h.ax_displace=copyobj(h.ax_mass,h.figure);
linkaxes([h.ax_mass,h.ax_force,h.ax_pressure,h.ax_displace],'off');
set(h.ax_pressure,'Color','none');
set(h.ax_force,'Color','none');
set(h.ax_displace,'Color','none');
set(h.ax_mass,'Color','white');
axfont=15;%fontsize for axes
```

plot mass data

```
axes(h.ax_mass);
ylabel('mass [mg]','color',[0 .5 .5],'FontSize',axfont);
h.mass=line([0;time(imass)],mass,'Marker','*','Color',[0 .5 .5]); %plot at changepoints
%h.masslegend=legend(h.mass,'massa');
%clear imass
```

plot pressurerise/massaverbruik

- massa contains mass data,
- imass are the indices where mass has increased
- Druk/|Druk40| contain pressure data
- ndata are the number of datapoints in this window. The data for pressure is filtered to extract the data which show when pressure rises. The argument being that only when pressure rises, gas is drawn from the supply. Thus the pressure rise should coincide with the weight drop on the scale.

```
if highpressure
    %smooth out data
    wts = [1/20;repmat(1/10,9,1);1/20];
    Ds = conv(Druk40,wts,'valid');
```

```

        %plot(time(5:end-6),Ds*coef.drk(loopnr))
        %find rises
        idrukriise=(find(diff(Ds)>0.001));%drukriise
%       idrukriise=(find(diff(Ds)~=0));%drukriise
        drukriise(ndata)=0;%preallocate
        for i=5:ndata
            if find(idrukriise==i)
                drukriise(i)=drukriise(i-1)+Ds(i)-Ds(i-1);
            else
                drukriise(i)=drukriise(i-1);
            end
        end
    else %low pressure
        % smoothout data
        wts = [1/20;repmat(1/10,9,1);1/20];
        Ds = conv(Druk,wts,'valid');
        %plot(time(5:end-6),Ds*coef.drk(loopnr))
        idrukriise=(find(diff(Ds)>0.001));%drukriise
        drukriise(ndata)=0;%#ok<*SAGROW> %preallocate
        for i=5:ndata
            if find(idrukriise==i)
                drukriise(i)=drukriise(i-1)+Ds(i)-Ds(i-1);
            else
                drukriise(i)=drukriise(i-1);
            end
        end
    end
end

set(h.figure,'CurrentAxes',h.ax_mass);
scaledruk=(max(mass)-min(mass))/max(drukriise);
h.rise=line(time,drukriise*scaledruk+min(mass),'color','m');

clear i idrukriise Ds wts scaledruk;
clear ndata

    plot pressure data

%axes(h.ax_pressure);
set(h.figure,'CurrentAxes',h.ax_pressure);
ylabel('pressure [bar]','color','r','FontSize',axfont);
if highpressure
    h.pres=line(time, Druk40,'color','r');
    %h.pressurelegend=legend('Druk40');
    %set(h.pressurelegend,'Location','NorthEast','Color','white');
else % er is geen Druk40, low pressure
    h.pres=line(time,Druk,'color','r');
    %h.pressurelegend=legend('Druk');
    %set(h.pressurelegend,'Location','Southeast','Color','white');
end %check voor aantal druksensoren

```

```
clear highpressure;
```

plot force data

```
axes(h.ax_force);
set(h.figure,'CurrentAxes',h.ax_force);
ylabel('force [N]','color','b','FontSize',axfont);
h.forc=line(time,-kracht,'color','b');
%h.forcelegend=legend('force');
```

plot displacement data

```
%axes(h.ax_displace);
set(h.figure,'CurrentAxes',h.ax_displace);
ylabel('displacement [mm]','color',[0 .5 0],'FontSize',axfont);
h.disp=line(time,verplaatsing,'color',[0 .5 0]);
line([h.ax_displace.XLim],[20 20],'LineStyle',':','color',[.8 .8 .8]);
line([h.ax_displace.XLim],[19 19],'LineStyle','--','color',[.8 .8 .8]);
fprintf('mean displacement: %4.1f\n',...
        mean(verplaatsing(verplaatsing>18)));
verpl(loopnr)=mean(verplaatsing(verplaatsing>18));%#ok
%h.placelegend=legend('displacement');
```

Build the observation windows

change y-limits

```
set(h.ax_mass,'Ylim', [limits(loopnr,1) limits(loopnr,2)]);
set(h.ax_force,'Ylim', [limits(loopnr,3) limits(loopnr,4)]);
set(h.ax_pressure,'Ylim', [limits(loopnr,5) limits(loopnr,6)]);
set(h.ax_displace,'Ylim', [limits(loopnr,7) limits(loopnr,8)]);
```

```
clear limits;
```

separate axes for final plot

```
%set(h.ax_force,'color','none');
set(h.ax_mass,'Position', [.3 0.1 .6 .8]);
set(h.ax_force,'Position', [.2 0.1 .7 .8]);
set(h.ax_pressure,'Position', [.1 0.1 .8 .8]);
set(h.ax_displace,'Position', [.3 0.1 .6 .8]);
set(h.ax_displace,'YAxisLocation','right');
% Run the part below, after the first plot
axlimx=get(h.ax_displace,'XLim');
range=diff(axlimx);
rangstep=range/.6;
bxlimx=[axlimx(1)-.1*rangstep axlimx(2)];
cxlimx=[axlimx(1)-.2*rangstep axlimx(2)];
set(h.ax_force,'Xlim',bxlimx);
set(h.ax_pressure,'Xlim',cxlimx,'color','white');
set(h.ax_mass,'Xlim',axlimx,'color','none');
set(h.ax_force,'Xtick',[]);
```

```

set(h.ax_pressure,'Xtick',[]);
set(h.ax_mass,'Xtick',[]);
hold on;
uistack(h.ax_mass,'top');

```

```

clear range rangstep;
clear axlimx bxlimx cxlimx;

```

details of plot

```

detail=sprintf('P_{avg}=%4.1f [bar]\nF_{avg}=%4.1f [N]',...
               [dgem;                               fgem]);
h.annotation=annotation(h.figure,...
    'textbox',[.7 .1 .2 .13],...
    ...%'String',detail,...
    'EdgeColor','black',...
    'BackgroundColor','white',...
    'FontSize',14);
h.annotation=annotation(h.figure,...
    'textbox',[.7 .1 .2 .13],...
    'String',detail,...
    'FontSize',14);
ftitle=sprintf('r%02i - P%4.2f MPa - %4.1f N',[ceil(dia) dgem/10 fgem]);
%title(ftitle,'FontSize',10);%, 'FontWeight', 'bold');
set(h.figure,'units','centimeters');
set(h.figure,'WindowStyle','normal');
set(h.figure,'WindowStyle','docked');
%pos = get(h.figure,'position');
%set(h.figure,'position',[0,0,8.5,6]);
%print('-depsc','-tiff','-r300',[pad '././04_Output/' ftitle(ftitle~=' ') '.eps']);
%matlabfrag([pad '././04_Output/'...
%           num2str(cilinder) '- ' num2str(loopnr) '.eps'],...
%           'handle',h.figure);
dlist(loopnr)=dgem;

clear dgem fgem dia;
clear detail;

end

```

output images and print the values for verbruik.m

```

for imgloopnr=looprange
h.figure=figure(imgloopnr);
matlabfrag([pad '././04_Output/'...
            num2str(cilinder) '- ' num2str(imgloopnr) '.eps'],...
            'handle',h.figure);
end
fprintf('%4.1f ',verpl);fprintf('\n');
fprintf('%4.1f ',dlist);fprintf('\n');

```

D verbruik.m

Contents

- Check how the script is called
- Results of processed data
- calculate linear regression through the datasets.
- Plot figures
- Plot the same with the mean force level aswell.

```
% VERBRUIK Plots all data in a graph
%
% REMARKS    : This script is can be run alone, or is called upon by
%              toegepast.m
%
% CREATED BY: D.C.Doedens
% DATE      : 15-08-2014
```

Check how the script is called

When called by another script, we don't want to clear the workspace

```
[stacka,stackb]=dbstack;
if size(stacka,1)==1
    clearvars; close all; clc;
    run('variables.m');
else
    clear stacka stackb
end
```

Results of processed data

- druk average pressure measured over a number of displacement cycles This is generated by `bekijkdata.m`. The given values are in [bar]. Calculations are in [MPa].
- nstr the number of strokes used in averaging. These are counted by hand
- tgu total gas-usage in [mg], for the counted strokes.
- agu average gas-usage.
- avd average displacement. (Generated by `bekijkdata.m`)
- mfl measured average forcelevel.
- afl average forcelevel.

```
druk=[11.0  9.8  7.6 27.0 36.3  0.0  0.0  0.0;...
      11.4  9.9  7.5 11.7 17.9  0.0  0.0  0.0;...
      11.4  9.7  7.5 11.6  0    0.0  0.0  0.0;...
      3.0   5.4  7.2  9.7 11.2 11.8  0.0  0.0;...
      11.7  9.3  5.9  7.9  9.9 12.1  4.9  6.0;]; druk=druk/10;% in MPa
nstr=[ 9   8   8   9   4   1   1   1 ;... 1 % using 1 to prevent
      10  9  10   4  15   1   1   1 ;... 2 % deviation by 0
       6   8  10   2   1   1   1   1 ;... 3 % in calculating agu
      16  11  17   4  11   5   1   1 ;... 4
       9   3   9   4   6   3   3   4 ;]%; 5
tgu=[(130) (108) (284-179) (600) (470-230) (0) (0) (0) ;... 1
```



```

(234) (339-167.2) (107+37) (100) (693.5-18) (0) (0) (0);... 2
(307-79) (620-354) (282-48) (88) (0) (0) (0) (0);... 3
(523-276) (350-67) (590) (109+694-592) (725-72) (402-124) (0) (0);... 4
(825) (199) (444) (248) (502) (454-176) (168-47) (330-149)];;% 5
agu=tgu./nstr;
% The following data are not used?
avd=[...Average displacement. These data are extracted in bekijkdata.m
19.4 19.1 19.1 19.3 19.3 00.0 00.0 00.0;... cilinder 1 r= 6mm
19.1 19.4 19.0 19.2 19.3 00.0 00.0 00.0;... cilinder 2 r= 8mm
19.3 19.1 19.1 19.2 00.0 00.0 00.0 00.0;... cilinder 3 r=10mm
20.4 20.3 20.4 20.5 20.3 19.2 00.0 00.0;... cilinder 4 r=12mm
19.8 19.4 20.3 20.4 19.1 19.2 19.2 19.3]; % cilinder 5 r=15mm
mfl=[... Average measured force level
29.5 26.1 20.7 74.7 101.1 0.0 0.0 0.0;...
55.3 47.6 36.0 56.3 87.6 0.0 0.0 0.0;...
85.1 72.0 55.9 83.8 0.0 0.0 0.0 0.0;...
32.5 59.7 79.4 102.6 111.7 130.5 0.0 0.0;...
192.2 154.2 98.7 131.5 163.6 201.9 82.1 99.0]; %
afl=[... Average backcalculated force level,
%given the known size of the cylinders
31.1 27.7 21.5 76.3 102.6 0.0 0.0 0.0;...
57.3 49.8 37.7 58.8 90.0 0.0 0.0 0.0;...
89.5 76.2 58.9 91.1 0.0 0.0 0.0 0.0;...
33.9 61.1 81.4 109.7 126.7 133.5 0.0 0.0;...
206.8 164.3 104.3 139.6 174.9 213.8 86.6 106.0]; %
%clean up this section
clear nstr tgu avd mfl afl;

```

calculate linear regression through the datasets.

sets or n are the number of measurements for each cylinder beta holds the intercept and slope df the number of statistical degrees of freedom in the calculation rmsE the root of the mean squared error with respect to the regression Xbar = mean of the input parameters ssX sum of squared differences to the mean of X

```

sets=[5,5,4,6,8];%number of data points for each cylinder
%counting from small to large diameter
n(numel(sets),1)=0;beta(5,2)=0;% space allocation to
df=n;rmsE=n;Xbar=n;ssX=n;% column vectors
for i=1:5 %cylinders from small to large
X=druk(i,1:sets(i))*1e6; %[Pa]
Y=agu(i,1:sets(i))*1e-6; %[kg]
linreg=fitlm(X,Y);%use the statistics toolbox to get the model
beta(i,:)=linreg.Coefficients.Estimate(:)';
n(i)=linreg.NumObservations;
df(i)=linreg.DFE;
rmsE(i)=linreg.RMSE;
Xbar(i)=mean(linreg.Variables.x1);
ssX(i)=sum(linreg.Variables.x1.^2)-n(i)*Xbar(i).^2;%fitlm automatically called
end %the first input paramter x1

```

```
%All that is left to do is, finish the statistics calculations for the
%confidence bounds at given pressures |PDb| the design pressures
%pressures. This is done in |toegepast.m|.
clear linreg X Y sets
```

Plot figures

Here the measurement data is simply plotted into one graph.

```
h.figure=figure(1); %create a new figure
set(gcf,'WindowStyle','docked');
h.ax=gca;hold on;
h.line6=plot(druk(1,1:5),agu(1,1:5),'*','color',[1 0 0]);
h.line8=plot(druk(2,1:5),agu(2,1:5),'+','color',[.5 .5 0]);
h.line10=plot(druk(3,1:4),agu(3,1:4),'o','color',[0 1 0]);
h.line12=plot(druk(4,1:6),agu(4,1:6),'x','color',[0 1 1]);
h.line15=plot(druk(5,1:8),agu(5,1:8),'^','color',[0 0 1]);
h.line=lsline; %h.line points to the 5 lsline
ylim([0 100]);% one line for each cilinder
xlabel('Pressure [MPa]');ylabel('Gasconsumption [mg/stroke]');
legend('6 [mm]','8 [mm]','10 [mm]','12 [mm]','15 [mm]',...
'Location','SouthEast');
for i=[6,8,10,12,15]
    %make thick markers
    set(eval(sprintf('h.line%i',i)), 'markersize',6);
    set(eval(sprintf('h.line%i',i)), 'linewidth',1);
    % Clear empty datapoints
    eval(sprintf('h.line%i.XData(~h.line%i.XData)=[];', [i i]));
    eval(sprintf('h.line%i.YData(~h.line%i.YData)=[];', [i i]));
%mlf2pdf(h.figure2,'gasmeasurements');
end
clear i;
```

Plot the same with the mean force level aswell.

```
if 0 %kept for possible use.
h.figure2=figure(2); %create a new figure
set(gcf,'WindowStyle','docked');
h.ax2=gca;hold on;
h.lined6=plot3(druk(1,1:5),agu(1,1:5),mfl(1,1:5),'*','color',[1 0 0]);
h.lined8=plot3(druk(2,1:5),agu(2,1:5),mfl(2,1:5),'+','color',[.5 .5 0]);
h.lined10=plot3(druk(3,1:4),agu(3,1:4),mfl(3,1:4),'o','color',[0 1 0]);
h.lined12=plot3(druk(4,1:6),agu(4,1:6),mfl(4,1:6),'x','color',[0 1 1]);
h.lined15=plot3(druk(5,1:8),agu(5,1:8),mfl(5,1:8),'^','color',[0 0 1]);
h.lined=lsline;
ylim([0 100]);% one line for each cilinder
xlabel('Pressure [bar]');ylabel('Gasconsumption [mg/stroke]');
zlabel('Force [N]');
legend('6 [mm]','8 [mm]','10 [mm]','12 [mm]','15 [mm]','Location','SouthEast');
end
```

E toegepast.m

Contents

- Design pressures (P_d) and expected gasconsumption
- Get data measured data
- Fit a non-linear model to the data
- plot consumption lines

```
% TOEGEPAST combines the measured data with the models
%
% REMARKS    : This script calls verbruik.m to get a prediction for
%              gas consumption, and some statistics.
%              This script also calculates the amount of gas for the
%              contents of a cylinder, at different pressures,
%              for different gas laws
% REQUIRES   : variables.m, verbruik.m
%
% CREATED BY: D.C.Doedens
% DATE      : 15-08-2014
```

Initialise, to make sure the workspace is clean. And there are no unexpected figure windows open.

```
clearvars;close all;clc;
run('variables.m'); %load variables into workspace
```

Design pressures (P_d) and expected gasconsumption

All cilinders outputs were matched to 2000 Nmm so 100 N for a 20 mm stroke leads to the following designpressure: $\frac{100\text{N}}{A_c} = P_d$

```
F_a=P_a*A_c;           %[N] atmospheric counterforce
P_d=(F_a+100)./A_c;     % This adds a small amount of force.
[sV,sV_i]=gaslaw(P_d); % the specific volumes for P_d
M=V_c./sV;             %[kg]=[m3/{m3/kg}]
Mi=V_c./sV_i;          % also for the 'Ideal' case
% now something similar for the whole pressure range
[sVr,sVir]=gaslaw(P_s);
Mr(numel(sVr))=0;Mir=Mr; %pre-allocate space
for cylinder=1:numel(D_c) %5 cilinders in total
    Mr(cylinder,:)=V_c(cylinder)./sVr;
    Mir(cylinder,:)=V_c(cylinder)./sVir;
end
```

Get data measured data

run the verbruik.m file, which contains the summary of all the measured data. And puts into the workspace, the statistics data needed to calculate the confidence intervals. **studt** are the 95% confidence interval values for the first 8 DOF's according to \ref{Buijs_99_} **Beta** holds the intercept and slope **df** the number of statistical degrees of freedom in the calculation **rmse** the root of the mean squared error with respect to the regression **Xbar** = mean of the input parameters **SSX** sum of squared differences to the mean of X

```

run('verbruik.m');
studt=[12.706 4.304 3.182 2.776 2.571 2.447 2.365 2.306]';
xP=linspace(P_d(1),P_d(5),100); % a range of points to plot the fitted model
smm=rmsE.*sqrt(1./n+(P_d-Xbar).^2./ssX); %The 95% confidence margin
Yhat=beta(:,1)+beta(:,2).*P_d;
Ymm=smm.*studt(df)*[-1 0 1]+Yhat*[1 1 1];

```

Fit a non-linear model to the data

the models are fitted to mg/MPa values

```

ymm=Ymm*1e6;yhat=Yhat*1e6; %[mg]
p_d=P_d/1e6;xp=xP/1e6;      %[MPa]
%gufun=@(b,x) (b(1)*(x+b(4)).^(-1))+b(2)+b(3)*(x+b(4));
gufun=@(b,x) b(1)*x.^(-1)+b(2)+b(3)*x;
b=[1;1;1];%some variables to start with I dunno
bfit=nlinfit(p_d,ymm(:,2),gufun,b); % parameters for the data
%rfit=nlinfit(p_d,M*1e6,gufun,b);    % parameters for this model
%pfit_model =[5.7029;31.7760;2.9872];% parameters from |model.m|
yfitd=gufun(bfit,xp);              %[mg] the data fitted to xP
optP4Y=xP(yfitd==min(yfitd));% optimal pressure for Y
%rfitd=gufun(rfit,xp);              %[mg] this model fitted to xP
%pfitd=gufun(pfit_model,xp);%[mg] model.m model fitted to xP

```

plot consumption lines

```

h.figure=figure(2);      %create a new figure
set(gcf,'WindowStyle','docked');
h.ax=gca;hold on;
hp=plot(P_d,M*1e6,      '-~','color',[1 1 1]*.2);%Plank
hi=plot(P_d,Mi*1e6,      ':s','color',[1 1 1]*.2);%Ideal
hm=plot(P_d,Yhat*1e6,'*','color',[1 1 1]*.4); %Measured
hy=plot(xP,yfitd);
ho=plot(optP4Y,min(yfitd),'*');
% These are the confidence lines, one at each design pressure level
line(P_d(1)*[1 1],ymm(1,[1 3]), 'LineStyle','-', 'Color',[0 0 0]);
line(P_d(2)*[1 1],ymm(2,[1 3]), 'LineStyle','-', 'Color',[0 0 0]);
line(P_d(3)*[1 1],ymm(3,[1 3]), 'LineStyle','-', 'Color',[0 0 0]);
line(P_d(4)*[1 1],ymm(4,[1 3]), 'LineStyle','-', 'Color',[0 0 0]);
line(P_d(5)*[1 1],ymm(5,[1 3]), 'LineStyle','-', 'Color',[0 0 0]);
xlabel('Pressure [Pa]','FontSize',10);
ylabel('Gas consumption [mg]','FontSize',10);
ylim([0 100]); %same axes as verbruik.m gives.
hleg= legend(...
    'Model using Plank''s CO_2',...
    'Model using the ideal CO_2',...
    'Measurements',...
    'simple NL-model fit',...
    'Location','NorthWest');
%mlf2pdf(h.figure2,'Optimum');

```