On the Estimation of Complex Speech DFT Coefficients Without Assuming Independent Real and Imaginary Parts

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***Abstract*—This letter considers the estimation of speech signals contaminated by additive noise in the discrete Fourier transform (DFT) domain. Existing complex-DFT estimators assume independency of the real and imaginary parts of the speech DFT coefficients, although this is not in line with measurements. In this letter, we derive some general results on these estimators, under more realistic assumptions. Assuming that speech and noise are independent, speech DFT coefficients have uniform phase, and that noise DFT coefficients have a Gaussian density, we show theoretically that the spectral gain function for speech DFT estimation is real and upper-bounded by the corresponding gain function for spectral magnitude estimation. We also show that the minimum mean-square error (MMSE) estimator of the speech phase equals the noisy phase. No assumptions are made about the distribution of the speech spectral magnitudes. Recently, speech spectral amplitude estimators have been derived under a generalized-Gamma amplitude distribution. As an example, we will derive the corresponding complex-DFT estimators, without making the independence assumption.**

***Index Terms*—Complex-discrete Fourier transform (DFT) estimators, independence assumption, minimum mean-square error estimation.**

I. Introduction

DISCRETE Fourier transform (DFT)-domain-based methods are often employed to estimate a speech signal in additive noise, see, e.g., [1]–[4]. These methods estimate either the complex-DFT coefficients or their amplitudes. Common assumptions in the derivation of almost all complex-DFT and amplitude estimators are that noise and speech processes are additive and independent, and that the noise DFT coefficients follow a complex Gaussian distribution. These assumptions are valid for many applications. Another assumption that is often made is that the real and imaginary parts of the speech DFT coefficients are independent. This assumption, however, is not in line with measurements on speech data [2]–[4]. In this letter, we will derive some general results for the minimum mean-square error (MMSE) estimators of complex-DFT coefficients for uniformly distributed speech phase, without assuming the independency of real and imaginary parts. We will show that the estimators leave the phase of the noisy coefficient unaltered and that their magnitude is always less than or equal to the corresponding amplitude estimator.

We also consider estimation of the clean speech phase. Ephraim and Malah [1] showed, for a Gaussian speech and noise model, that different estimators lead to the noisy phase as an optimal estimator. In this letter, we will extend these results by showing that MMSE estimation of the speech phase generally leads to the noisy phase as an optimal estimator, for any amplitude distribution of the speech.

II. MMSE Estimation of Complex-DFT Coefficients

A. Definitions and Assumptions

Suppose we have a noisy speech signal that we process in short analysis windows in the Fourier domain. This leads to a sequence of short-time Fourier transforms $Y(k,m)$, where k is a frequency index and m is a time-frame index, as follows:

$$Y(k,m) = S(k,m) + N(k,m) \tag{1}$$

where $S(k,m)$ are the DFT coefficients of the clean speech signal and $N(k,m)$ that of the noise. We will assume the coefficients to be independent in time and frequency, which allows us to leave out the frequency and time-frame indices in the following for ease of notation. Our problem is to estimate S, given Y. We will use the following notations for the amplitudes and phases of the coefficients: $Y = Re^{j\Theta}$, $S = Ae^{j\Phi}$, and $N = De^{j\Delta}$. We use capitals for random variables and their corresponding lowercase symbols for realizations.

We will derive some results about the MMSE estimator of S under the following assumptions.

1) Speech and noise are independent.
2) The speech phase is uniformly distributed.
3) The noise DFT coefficients follow a complex Gaussian distribution with i.i.d. real and imaginary parts.

It is important to realize that, although we have speech signals in mind as the primary application, these assumptions may be valid for many other types of signals. Assumption 2) means that A and ϕ are independent and that their joint pdf satisfies $f_{A,\Phi}(a,\phi) = f_A(a)/2\pi$. Assumption 2) is motivated by measurements on real speech data [2], [4]. Assumption 3) is motivated by the central limit theorem. It means that the noise amplitude is independent of the uniformly distributed noise phase. Assumptions 1) and 3) determine the conditional pdf $f_{Y|S}(y|s)$, which follows from

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substituting $|y-s|$ for d in the pdf of the noise DFT coefficient as follows:

$$f_{Y|S}(y|s) = \frac{1}{\pi\lambda_N}\exp\left\{-\frac{|y-s|^2}{\lambda_N}\right\} = \frac{1}{\pi\lambda_N}\exp\left\{\frac{2ar\cos(\phi-\theta)-r^2-a^2}{\lambda_N}\right\} \quad (2)$$

where $\lambda_N = E\{|N|^2\}$ is the noise spectral variance. E is the expectation operator. The variance of S will be denoted as λ_S.

B. General MMSE Expressions

In MMSE estimation of the complex DFT-coefficients, we minimize the following *Bayesian risk* $\mathcal{R}$:

$$\mathcal{R} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} |s-\hat{s}|^2 f_{S|Y}(s_r, s_i|y)ds_r ds_i \quad (3)$$

where s is the clean speech DFT coefficient, $\hat{s}$ its estimate, and $f_{S|Y}(s_r, s_i|y)$ the joint conditional pdf of the real S_r and imaginary S_i parts of S, given the noisy DFT coefficient Y. Minimizing (3) w.r.t. $\hat{s}_r$ and $\hat{s}_i$ leads to the following expression for the estimator:

$$\hat{s} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} (s_r + js_i) f_{S|Y}(s_r, s_i|y)ds_r ds_i. \quad (4)$$

$\hat{s}$ is often expressed as y multiplied by a spectral gain function g_S, i.e., $\hat{s} = g_S y$. The spectral gain function depends on λ_N, λ_S, and y. Using Bayes' rule, it can be expressed as follows:

$$g_S = \frac{e^{-j\theta}}{r} \cdot \frac{\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}(s_r + js_i) f_{Y|S}(y|s_r, s_i) f_S(s_r, s_i)ds_r ds_i}{\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f_{Y|S}(y|s_r, s_i) f_S(s_r, s_i)ds_r ds_i}. \quad (5)$$

The transformation to polar coordinates will be useful in the following. Making use of Assumption 2), (5) turns into

$$g_S = \frac{1}{r}\frac{\int_0^\infty\int_{-\pi}^{+\pi} a e^{j(\phi-\theta)} f_{Y|S}(y|a,\phi) f_A(a)d\phi da}{\int_0^\infty\int_{-\pi}^{+\pi} f_{Y|S}(y|a,\phi) f_A(a)d\phi da}. \quad (6)$$

C. Proof That g_S Is Real and Positive

The denominator of (6) is real and positive. To prove that g_S is real and positive, we only have to consider the numerator. Consider the integral over ϕ as follows:

$$\int_{-\pi}^{+\pi} e^{j(\phi-\theta)} f_{Y|S}(y|a,\phi)d\phi = \int_{-\pi}^{+\pi}\{\cos(\phi-\theta) + j\sin(\phi-\theta)\} f_{Y|S}(y|a,\phi)d\phi. \quad (7)$$

From Assumptions 1) and 3) in the previous section, it follows that $f_{Y|S}(y|a,\phi)$ depends only on the *distance* $|y-s|$ between the noisy DFT coefficient and the clean DFT coefficient. See (2) and the graphical explanation in Fig. 1. When considered as a function of ϕ, $f_{Y|S}(y|a,\phi)$ is *symmetric*, i.e., an even function, around $\phi=\theta$ and is periodic in ϕ with period 2π. Since $\sin(\phi-\theta)$ is an odd function around $\phi=\theta$, the imaginary part of the integral (7) will vanish and we are left with

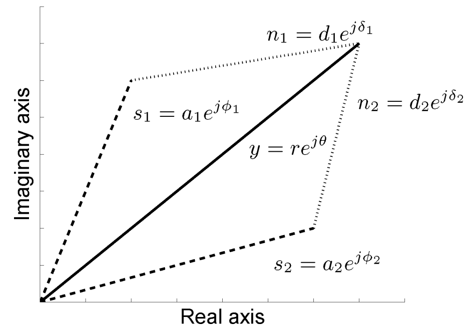

Fig. 1. Consider y which could result from many different speech and noise DFT-coefficient pairs, for example, s_1+n_1 or s_2+n_2. Under Assumptions 1) and 3), $f_{Y|S}(y|s)$ is given by (2) and depends only on the magnitude $d=|y-s|$ of the noise DFT coefficient n. Therefore, $f_{Y|S}(y|s_1)$ equals $f_{Y|S}(y|s_2)$ when $d_1 = d_2$.

$$\int_{-\pi}^{+\pi}\cos(\phi-\theta) f_{Y|S}(y|a,\phi)d\phi. \quad (8)$$

Therefore, g_S (6) is real. Since $f_{Y|S}(y|a,\phi)$, seen as a function of ϕ, has its largest values where $\cos(\phi-\theta) > 0$, i.e., between $\phi = \theta - \pi/2$ and $\phi = \theta + \pi/2$, (8) is positive and g_S as well.

D. Proof That g_S Is Less Than or Equal to g_A

The expression for the gain function g_A for MMSE estimation of the amplitude is (see, e.g., [1])

$$g_A = \frac{1}{r}\frac{\int_0^\infty\int_{-\pi}^{+\pi} a f_{Y|S}(y|a,\phi) f_A(a)d\phi da}{\int_0^\infty\int_{-\pi}^{+\pi} f_{Y|S}(y|a,\phi) f_A(a)d\phi da}. \quad (9)$$

The only difference with (6) is the complex exponential $e^{j(\phi-\theta)}$ in the numerator of (6) of which only the real part $\cos(\phi-\theta)$ contributes to the integral. Since $|\cos(\phi-\theta)| \le 1$, it follows immediately that $g_S \le g_A$, since $f_{Y|S}(y|a,\phi)$ and $f_A(a)$ are greater than or equal to 0. Note that we have not assumed any specific distribution $f_A(a)$ for the speech amplitudes.

III. MMSE Estimation of the Speech Phase

Ephraim and Malah [1] considered estimation of the speech phase under the assumption of a complex Gaussian distribution for the DFT coefficients of noise and speech. They showed that the MMSE estimator of the complex exponential of the speech phase has an argument equal to the noisy phase, but it has a magnitude smaller than one. The MMSE estimator of the complex exponential, constrained to have unity magnitude, was shown to be equal to the complex exponential of the noisy phase. Furthermore, they showed that the estimator minimizing the following cosine criterion

$$E\left\{1-\cos(\phi-\hat{\phi})\right\} \quad (10)$$

again leads to the noisy phase as the optimal estimator for the clean speech phase. We will extend these results in the following way: under the fairly general conditions of Section II-A, we will show that the MMSE estimator of the speech phase itself equals the noisy phase. We will not make any assumptions about the speech amplitude distribution $f_A(a)$.

We will use in the following that $f_{\Phi|Y}(\phi|y)$ is symmetric around the noisy phase θ. This can be shown as follows. Using Bayes' rule and Assumption 2), we can express $f_{\Phi|Y}(\phi|y)$ as

$$f_{\Phi|Y}(\phi|y) = \frac{\int_0^\infty f_{Y|S}(y|a,\phi) f_A(a) da}{\int_{-\pi}^{+\pi} \int_0^\infty f_{Y|S}(y|a,\phi) f_A(a) da d\phi}. \tag{11}$$

$f_{Y|S}(y|a,\phi)$ is periodic in ϕ with period 2π and depends on $\cos(\phi-\theta)$. The integral over ϕ in the denominator is over one whole period and therefore does not depend on θ. In the numerator, only $f_{Y|S}(y|a,\phi)$ depends on θ and is symmetric around $\phi=\theta$. Therefore, $f_{\Phi|Y}(\phi|y)$ is symmetric around θ. $f_{\Phi|Y}(\phi|y)$ is also periodic in ϕ with period 2π, because $f_{Y|S}(y|a,\phi)$ is periodic in ϕ and $f_A(a)$ does not depend on ϕ [Assumption 2)].

The MMSE estimator of the clean phase follows from minimizing

$$MSE(\hat{\phi}) = \int \left\{\phi - \hat{\phi}\right\}^2 f_{\Phi|Y}(\phi|y) d\phi \tag{12}$$

w.r.t. $\hat{\phi}$. Note that we have not specified the boundaries of the integral yet. $f_{\Phi|Y}(\phi|y)$ integrates to 1 for any 2π-interval of ϕ. Still, the value of $MSE(\hat{\phi})$ depends on the integration boundaries, because of the term $\left\{\phi - \hat{\phi}\right\}^2$. For a given value of $\hat{\phi}$, (12) will be minimized w.r.t. the integration boundaries if we integrate from $\hat{\phi}-\pi$ to $\hat{\phi}+\pi$. The MMSE estimator of ϕ is therefore the value of $\hat{\phi}$ that minimizes

$$MSE(\hat{\phi}) = \int_{\hat{\phi}-\pi}^{\hat{\phi}+\pi} \left\{\phi - \hat{\phi}\right\}^2 f_{\Phi|Y}(\phi|y) d\phi. \tag{13}$$

This equation means that we will minimize the mean-square error w.r.t. $\hat{\phi}$ *and* the integration boundaries. To make the minimization of (13) easier, we make the substitution $\psi = \phi - \hat{\phi}$, leading to

$$MSE(\hat{\phi}) = \int_{-\pi}^{+\pi} \psi^2 f_{\Phi|Y}(\psi + \hat{\phi}|y) d\psi \tag{14}$$

where $f_{\Phi|Y}(\psi+\hat{\phi}|y)$ is symmetric around $\psi = \theta - \hat{\phi}$. Furthermore, it will have most of its probability mass near that value since $f_A(a)$ in (11) does not depend on ϕ. $MSE(\hat{\phi})$ will therefore be minimized if we choose $\hat{\phi}$ such that the maximum of $f_{\Phi|Y}(\psi+\hat{\phi}|y)$ lies at $\psi = 0$, i.e., for $\hat{\phi}=\theta$, where there is minimal weighting from the ψ^2 term. This leads to the noisy phase θ as the MMSE estimator of the clean phase.

A. *Relaxation of Assumption 3)*

From the explanations, it should be clear that all results of Sections II and III remain valid if we relax Assumption 3) of Section II-A to: the noise phase is uniformly distributed, i.e., $f_\Delta(\delta) = 1/2\pi$, and $f_N(n)$ has most of its probability mass near the origin.

IV. Complex-DFT Estimators Under Generalized-Gamma Amplitude Priors

In this section, we derive MMSE complex-DFT estimators under a generalized-Gamma density of the speech DFT magnitudes with the assumptions of Section II-A. The corresponding MMSE DFT-magnitude estimators have been published in [4]. The generalized-Gamma density is defined as

$$f_A(a) = \frac{\gamma \beta^\nu}{\Gamma(\nu)} a^{\gamma\nu-1} \exp(-\beta a^\gamma)$$
$$\beta > 0,\ \gamma > 0,\ \nu > 0,\ a \geq 0. \tag{15}$$

The parameter β depends on γ, ν, and λ_S. We consider here the cases $\gamma = 1$ and $\gamma = 2$. For $\gamma = 1$, $\beta = \sqrt{\nu(\nu+1)/\lambda_S}$ and for $\gamma = 2$, $\beta = \nu/\lambda_S$ [4].

We are interested in $E[S|Y]$. From Section II, we know that the corresponding gain function g_S is given by

$$g_S(\xi,\zeta) = \frac{1}{r} \frac{\int_0^\infty \int_{-\pi}^{+\pi} a\cos(\phi-\theta) f_{Y|S}(y|a,\phi) f_A(a) d\phi da}{\int_0^\infty \int_{-\pi}^{+\pi} f_{Y|S}(y|a,\phi) f_A(a) d\phi da}. \tag{16}$$

This gain function can always be written as a function of two dimensionless parameters, the *a priori* SNR ξ and the *a posteriori* SNR ζ, defined as $\xi = \lambda_S/\lambda_N$ and $\zeta = r^2/\lambda_N$, respectively. The density $f_{Y|S}(y|a,\phi)$ was given in (2).

A. *Gain Function for $\gamma = 2$*

Inserting (15) with $\gamma = 2$ and (2) into (16) and using [5, Eqs. 8.431.5, 6.643.2, 9.210.1, 9.220.2] leads, for $\nu > 0$, to

$$g_S^{(2)}(\xi,\zeta) = \frac{\nu\xi}{\nu+\xi} \frac{\mathcal{M}\left(\nu+1; 2; \frac{\zeta\xi}{\nu+\xi}\right)}{\mathcal{M}\left(\nu; 1; \frac{\zeta\xi}{\nu+\xi}\right)} \tag{17}$$

where $\mathcal{M}(\cdot)$ is the confluent hypergeometric function. The superscript (2) indicates that $\gamma = 2$. Equation (17) reduces to the Wiener filter for $\nu = 1$.

B. *Gain Function for $\gamma = 1$*

Substituting (15) with $\gamma = 1$ and (2) into (16) and using [5, Eq. 8.431.5] leads to

$$g_S^{(1)}(\xi,\zeta) = \frac{1}{r} \frac{\int_0^\infty a^\nu \mathcal{I}_1\left(\frac{2ar}{\lambda_N}\right) \exp\left\{-\frac{a^2}{\lambda_N} - \beta a\right\} da}{\int_0^\infty a^{\nu-1} \mathcal{I}_0\left(\frac{2ar}{\lambda_N}\right) \exp\left\{-\frac{a^2}{\lambda_N} - \beta a\right\} da} \tag{18}$$

where $\mathcal{I}_n(\cdot)$ is the modified Bessel function of the first kind and order n. The superscript (1) means that $\gamma = 1$. If we make the transformation $x = 2ar/\lambda_N$ and define $\mu \equiv \sqrt{\nu(\nu+1)}$, (18) can be written as

$$g_S^{(1)}(\xi,\zeta) = \frac{1}{2\zeta} \frac{\int_0^\infty x^\nu \mathcal{I}_1(x) \exp\left\{-\frac{x^2}{4\zeta} - \frac{\mu x}{2\sqrt{\xi\zeta}}\right\} dx}{\int_0^\infty x^{\nu-1} \mathcal{I}_0(x) \exp\left\{-\frac{x^2}{4\zeta} - \frac{\mu x}{2\sqrt{\xi\zeta}}\right\} dx}. \tag{19}$$

The derivative of $\mathcal{I}_0(x)$ is $\mathcal{I}_1(x)$. Using partial integration of the numerator, (19) can alternatively be written as

$$g_S^{(1)}(\xi,\zeta) = \left[\frac{\int_0^\infty \left(\frac{x^{\nu+1}}{2\zeta} + \frac{\mu x^\nu}{2\sqrt{\xi\zeta}}\right) \mathcal{I}_0(x) \exp\left\{-\frac{x^2}{4\zeta} - \frac{\mu x}{2\sqrt{\xi\zeta}}\right\} dx}{2\zeta \int_0^\infty x^{\nu-1} \mathcal{I}_0(x) \exp\left\{-\frac{x^2}{4\zeta} - \frac{\mu x}{2\sqrt{\xi\zeta}}\right\} dx} - \frac{\nu}{2\zeta}\right]. \tag{20}$$

The integrals cannot be solved analytically. Similar integrals appear in the equations for the amplitude gain functions under the same $f_A(a)$ [4]. Approximation of the Bessel functions leads to integrals that can be solved analytically. Using Taylor expansions in (19) leads to approximations that are accurate at low SNRs. The Taylor expansions of $\mathcal{I}_0(x)$ and $\mathcal{I}_1(x)$, truncated after K terms, are given by [6, Eq. 9.6.10]

$$\mathcal{I}_0(x;K) = \sum_{j=0}^{K-1} \left(\frac{x}{2}\right)^{2j} \frac{1}{(j!)^2}$$
$$\mathcal{I}_1(x;K) = \sum_{j=0}^{K-1} \left(\frac{x}{2}\right)^{2j+1} \frac{1}{j!(j+1)!}. \quad (21)$$

Substitution of (21) into (19) and using [5, Eq. 3.462.1], we find for $\nu > 0$ the gain function for $\gamma = 1$ for low SNRs as follows:

$$g^{(1)}_{S,\ll,K}(\xi,\zeta) = \frac{1}{2}\frac{\sum_{k=0}^{K-1} \frac{1}{k!(k+1)!}\left(\frac{\zeta}{2}\right)^k \Gamma(\nu+2k+2)\mathcal{D}_{-(\nu+2k+2)}(\chi)}{\sum_{k=0}^{K-1} \left(\frac{1}{k!}\right)^2 \left(\frac{\zeta}{2}\right)^k \Gamma(\nu+2k)\mathcal{D}_{-(\nu+2k)}(\chi)} \quad (22)$$

where $\chi = \sqrt{\nu(\nu+1)/(2\xi)}$, $\Gamma(\cdot)$ is the gamma function, and $\mathcal{D}_{\nu'}(\cdot)$ is a parabolic cylinder function of order ν'. Notice that one could also derive a gain function for low SNRs by using the Taylor expansion of $\mathcal{I}_0(x)$ in (20). However, it can be shown that this will lead to a less accurate approximation than (22).

Using the large-argument approximation [6, Eq. 9.7.1]

$$\mathcal{I}_0(x) \sim \frac{1}{\sqrt{2\pi x}}\exp(x) \quad (23)$$

in (20) leads to an approximation that is most accurate at high SNRs. We find the gain function, valid for $\nu > 0.5$, by substitution of (23) into (20) and using [5, Eq. 3.462.1]

$$g^{(1)}_{S,\gg}(\xi,\zeta) = \left[\frac{\left(\nu-\frac{1}{2}\right)\left\{\chi\mathcal{D}_{-(\nu+1/2)}(\chi')+\left(\nu+\frac{1}{2}\right)\mathcal{D}_{-(\nu+3/2)}(\chi')\right\}}{2\zeta\mathcal{D}_{-(\nu-1/2)}(\chi')} - \frac{\nu}{2\zeta}\right] \quad (24)$$

where $\chi' = \chi - \sqrt{2\zeta}$.

1) Combining $g^{(1)}_{S,\ll,k}$ and $g^{(1)}_{S,\gg}$: In order to combine the two gain functions $g^{(1)}_{S,\ll,K}(\xi,\zeta)$ and $g^{(1)}_{S,\gg}(\xi,\zeta)$ into one estimator, we adopt the procedure used in [4], where the maximum of both gain functions was found to be an accurate approximation of the exact MMSE gain for a wide range of the parameters. The combined gain function is then defined as

$$g^{(1)}_{S,C,K}(\xi,\zeta) = \max\left[g^{(1)}_{S,\ll,K}, g^{(1)}_{S,\gg}\right] \quad (25)$$

valid for $\nu > 0.5$. Numerical computations have shown that the maximum relative errors in (25) w.r.t. (18) lie between -1.5 dB and 0.7 dB for $K = 10$, and between -0.1 dB and 0.7 dB for $K = 20$, where a positive error means (25) is larger than (18).

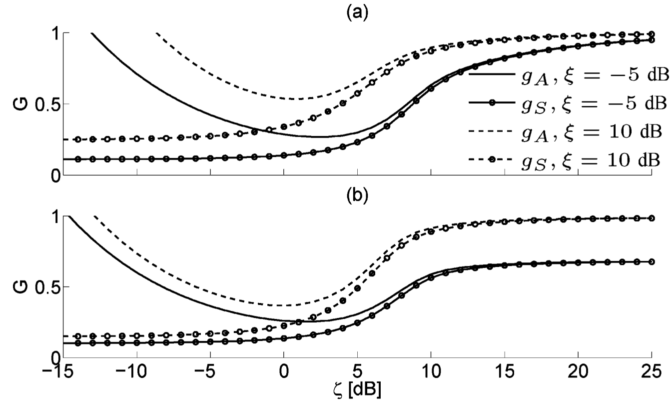

Fig. 2. Complex-DFT and DFT-magnitude gain curves for $\xi = -5$ dB and $\xi = 10$ dB, for (a) $\gamma = 1$, $\nu = 0.6$, $K = 20$ and (b) $\gamma = 2$, $\nu = 0.15$.

C. Illustration of Complex-DFT Estimators

We have computed the gain functions $g^{(2)}_S$ and $g^{(1)}_{S,C,K}$ for a wide range of the parameters, compared them to their amplitude counterparts in [4], and verified that the complex-DFT gain functions are always the smallest. This remains true for the approximations applied in the $\gamma = 1$ case. The complex-DFT and amplitude gain functions are illustrated in Fig. 2 for $\gamma = 1$, $\nu = 0.6$, and $K = 20$, and for $\gamma = 2$, $\nu = 0.15$. All gain functions are plotted for the *a priori* SNR values $\xi = -5$ dB and $\xi = 10$ dB, as a function of ζ.

V. Concluding Remarks

Existing complex-DFT estimators of speech generally assume the real and imaginary parts to be independent, although this assumption is not valid. We have shown some interesting general relations between the complex-DFT estimators and their magnitude counterparts, without making the independence assumption. For example, the complex-DFT estimators always suppress more noise than the corresponding magnitude estimators. This is a consequence of minimizing a different error criterion. It depends on the application which type of estimator is to be preferred.

References

[1] Y. Ephraim and D. Malah, “Speech enhancement using a minimum mean-square error short-time spectral amplitude estimator,” *IEEE Trans. Acoust., Speech, Signal Process.*, vol. ASSP-32, no. 6, pp. 1109–1121, Dec. 1984.

[2] T. Lotter and P. Vary, “Speech enhancement by MAP spectral amplitude estimation using a super-Gaussian speech model,” *EURASIP J. Appl. Signal Process.*, vol. 7, pp. 1110–1126, 2005.

[3] R. Martin, “Speech enhancement based on minimum mean-square error estimation and supergaussian priors,” *IEEE Trans. Speech Audio Process.*, vol. 13, no. 5, pp. 845–856, Sep. 2005.

[4] J. S. Erkelens, R. C. Hendriks, R. Heusdens, and J. Jensen, “Minimum mean-square error estimation of discrete Fourier coefficients with generalized-Gamma priors,” *IEEE Trans. Acoust., Speech, Lang. Process.*, vol. 15, no. 6, pp. 1741–1752, Aug. 2007.

[5] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products*, 6th ed. New York: Academic, 2000.

[6] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, 9th ed. New York: Dover, 1964.