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Impulse voltage distribution on disk winding: calculation of disk series capacitance using analytical method

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Abstract—Nonlinear voltage distribution along the transformer winding during a lightning overvoltage can cause severe electric stress specially to the area of the winding close to the high voltage terminals. For high voltage transformers different methods such as interleaved winding, electrostatic shield wires, and wound-in shield wires are implemented on the first few disks to improve such nonlinear voltage distribution. Through application of those methods, the series capacitance of the first few disks is increased which results in reduction of electric stress on the highest stress region close to the high voltage terminal. In this paper disk series capacitances are calculated using the energy summation method and a simplified analytical formula is derived for different configuration of disk winding construction.

Keywords—disk winding, disk series capacitance, lightning over voltage, transformer

I. INTRODUCTION

During lightning strikes in a power grid, transient voltages occur which can enter transformers through the high voltage connections. These lead to high dielectric stress inside and between the windings. Especially between the turns, layers or discs of the windings high voltages can occur locally, mainly at or near the high voltage connections of the windings. For the highest voltage levels disk windings are used mainly with radial spacers. Detailed models such as multi conductor transmission line (MTL) model and lumped RLC ladder network model have been introduced and investigated in a number of references [1, 2] to study very fast transient overvoltage on transformer winding. In many cases however, a capacitance equivalent model of the winding can represent the voltage distribution along the winding sufficiently accurate during the initial stage (rise time) of the lightning impulse. The results obtained from a capacitance equivalent model represents the voltage distribution for ultimate fast excitation (step voltage). For a slower front wave, when the peak voltage is reached the effect of inductances also come into play and hence the voltage distribution is less severe than what is obtained by the capacitance equivalent model [3, 4]. Based on the voltage distribution calculated from the equivalent capacitance model, and together with some extra safety margins, transformer designers can decide upon an adequate insulation requirement between the discs of the winding. A representation of the simplified capacitance equivalent model of a winding is shown in Fig. 1. Assuming that C_s is the total series capacitance of the winding and C_g is the total ground capacitance of the winding, it can be shown [3] that if the winding is exited with an step function of magnitude V_0, the initial voltage distribution along this capacitive network is determined by (1) when the winding end is grounded and by (2) when the winding end is floating.

$$V = V_0 \frac{\sinh \alpha \frac{x}{l}}{\sinh \alpha} \quad (1)$$

$$V = V_0 \frac{\cosh \alpha \frac{x}{l}}{\cosh \alpha} \quad (2)$$

In which $\alpha = \sqrt{\frac{C_g}{C_s}}$, x is the position along the winding and l is the length of the winding. Fig. 2 illustrates the initial and final voltage distribution along the winding for different values of α when the end of the HV winding is grounded. It can be seen that for large value of α, the part of the winding close to the HV terminal is highly stressed. The smaller the α, the more uniform initial voltage distribution along the winding could be achieved.

One way to decrease the value of α is to increase the value of C_s. For the layer winding represented in Fig. 1, assuming that the winding has n turns in total, we have equation (3) between C_s and C_{s1}, and between C_g and C_{g1}:
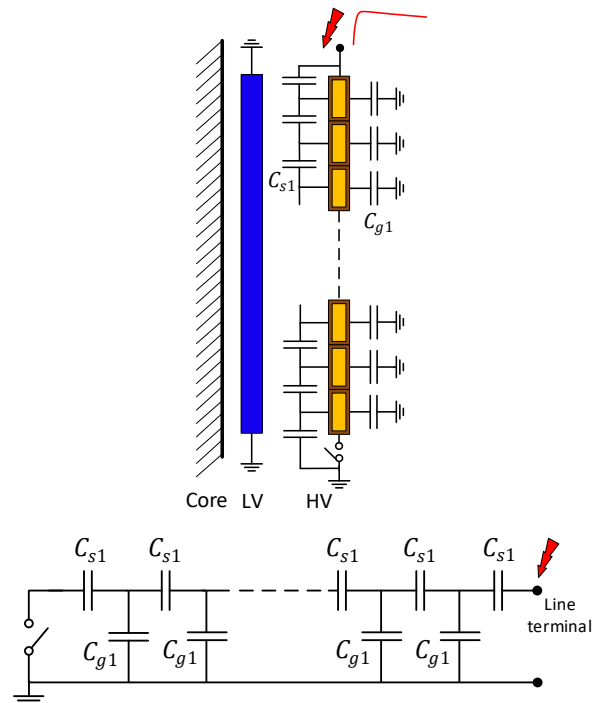
$$C_s = \frac{C_{s1}}{n-1} \qquad C_g = nC_{g1} \quad (3)$$

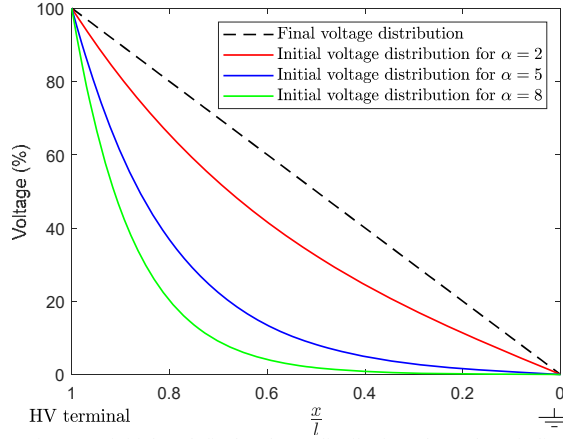Figure1. equivalent capacitance network of a winding

Figure 2. initial and final voltage distribution along the winding for different value of α

For both layer and disk winding, capacitance to ground can be calculated using the coaxial capacitance formula [5, 6]. For the case of a layer winding, calculation of C_{s1} is straight forward and can be done using the parallel plate capacitor equation [5, 6].

II. ANALYTICAL CALCULATION OF DISK SERIES CAPACITANCE

In case of a disk winding, C_{s1} is the capacitance between two adjacent disks. The energy method can be used to derive the series capacitance of a disk pair. A disk pair with N turns on each disk is shown in Fig. 3. In reality, there are stray capacitances between any two turns in a disk as well as between each turn of one disk to each turn of the other disk. To calculate the total energy stored in this system, for simplicity reason, only the most influential capacitances of this system is considered, as shown in Fig. 3. These capacitances are turn-to-turn capacitances in one disk, i.e. C_ts, and capacitances between each turn in one disk and the opposite turn in the other disk, i.e. C_ds.

A. Continus disk winding

Assuming that voltage is distributed uniformly along each disk and the voltage of each turn is concentrated at the midpoint of that turn, the total energy stored in C_ds and C_ts can be calculated. The energy stored in C_ds can be calculated using the micro-element method, referring to Fig. 4. The voltage difference between point 1 in the upper disk and the opposite point in the lower disk (point 2) is given by (4). The total energy stored in C_ds can be calculated using (5). In (4) and (5), d is the outer radius minus inner radius of the winding. In (5) d can be replaced with N (number of turns in each disk) and get (6). The equivalent capacitance of all C_ds is given by (7). Notice that derivation of (6) from (5) is accurate when N is large, e.g. $N > 20$.
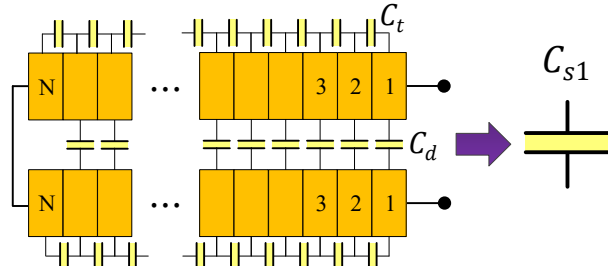

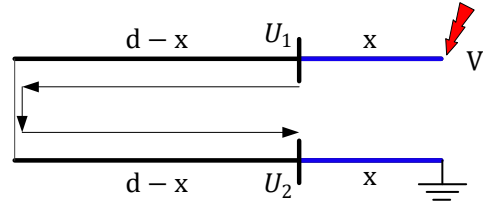Figure 3. equivalent series capacitance of a continuous disk pair

Figure 4. micro-element method to calculate total energy stored in all C_d

$$\Delta U = \frac{V}{d}(d - x) \tag{4}$$

$$E_{tot\_d} = \int_0^d \frac{1}{2} C_d (\Delta U)^2 \, dx = \frac{d}{6} C_d V^2 \tag{5}$$

$$E_{tot\_d} = \frac{1}{2} C_{tot\_d} V^2 = \frac{N}{6} C_d V^2 \tag{6}$$

$$C_{tot\_d} = \frac{N}{3} C_d \tag{7}$$

In which, C_{tot_d} is the total capacitance between the two disks and E_{tot_d} is the total energy stored in all C_ds. There are in total $2N - 2$ turn-to-turn capacitances. The voltage difference between adjacent turns, ΔV, is given by (8). In (8) is true if it is assumed that voltage of each turn is concentrated at the midpoint of the turn. The total energy stored in C_ts can be calculated with (9). In equation (9) C_t is the capacitance between the two middle turns of the top disk which is considered as an approximation for all C_ts. The equivalent capacitance of all C_ts is given by (10).

$$\Delta V = \frac{V}{2N} \tag{8}$$

$$E_{tot\_t} = \frac{1}{2} C_{tot\_t} V^2 = (2N - 2) \frac{1}{2} C_t \left(\frac{V}{2N} \right)^2 \tag{9}$$

$$C_{tot\_t} = \frac{N - 1}{2N^2} C_t \tag{10}$$

The total energy stored in the system is the sum of energy stored in C_ds and C_ts, (10), from which the total series capacitance of the continuously wounded disk pair can be calculated with (12).

$$E_{tot} = \frac{1}{2} C_{s1} V^2 = E_{tot\_d} + E_{tot\_t} \tag{11}$$

$$C_{s1} = \frac{N}{3} C_d + \frac{N - 1}{2N^2} C_t \tag{12}$$

B. Interleaved disk winding

Similarly for an interleaved disk pair, the total series capacitance of a disk pair can be calculated using the energy summation method. In this case the capacitances between two disks is negligible compared to capacitances between the turns (due to the larger gap between the disks in comparison with the gap between the consecutive turns). With the assumption of equal voltage share per individual turn, the total energy stored in C_{t1}s is:

$$E_{tot\_t1} = \sum_{i=1 \, (i \neq \frac{N+1}{2})}^{N} \frac{1}{2} C_{t1} \left((N + i - i) \frac{V}{2N} \right)^2 \tag{13}$$
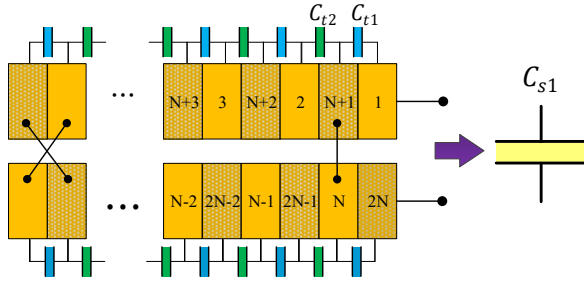
$$= \frac{(N - 1) C_{t1}}{8} V^2$$

Figure 5. equivalent series capacitance of an interleaved disk pair

Similarly the total energy stored in C_{t2}s is:

$$E_{tot\_t2} = \sum_{i=1}^{N-1} \frac{1}{2} C_{t2} \left((N+i-(i+1)) \frac{V}{2N} \right)^2$$
$$= \frac{(N-1)^3 C_{t2}}{8N^2} V^2 \qquad (14)$$

Equation (13) and (14) C_{t1} and C_{t2} are the capacitance between the two middle turns of the top disk and is considered as an approximation for all C_{t1}s and C_{t2}s (blue and green capacitors shown in Fig. 5). E_{tot_t1} is the total energy stored in all C_{t1}s and E_{tot_t2} is the total energy stored in C_{t2}s. The total energy stored in a disk pair is given in (15) and the equivalent series capacitance of an interleaved disk winding is given in (16).

$$E_{tot} = \frac{1}{2} C_{s1} V^2 = E_{tot\_t1} + E_{tot\_t2} \qquad (15)$$

$$C_{s1} = \frac{C_t(N-1)}{4} \left(1 + \frac{(N-1)^2}{N^2} \right) \qquad (16)$$

In (16) C_t is the capacitance between the two middle turns and $C_{t1} = C_{t2} = C_t$. Notice that in (13), (14) and (16) it is assumed that N is an odd number, if N is an even number (13) and (14) can be written as (17) and (16) has to be adjusted accordingly.

$$E_{tot\_t1} = \frac{N C_{t1}}{8} V^2$$
$$E_{tot\_t2} = \frac{(N-2)(N-1)^2 C_{t2}}{8N^2} V^2 \qquad (17)$$

C. Continious disk winding with floated electrostatic shield wires

Consider k electrostatic pairs of shield wires in a disk pair. By assumption that voltage is linearly distributed along the disks, the voltage of each pair of floating electrostatic shields is equal to $\frac{V}{2}$. The total energy in this system can be calculated by (18).

$$E_{tot} = E_{tot_{turn\_turn}} + E_{tot_{turn\_shield}} + E_{tot_{disk-disk}} \qquad (18)$$

The energy stored in the capacitances between the disks is the same as the case of continuous winding. The total energy stored in the turn-turn capacitance can be calculated similar to the case of a continuous winding, with the difference that now instead of $2(N-1)$ turn-to-turn capacitances, there are $2(N-k-1)$ turn-to-turn capacitances present. The equivalent capacitance of all C_ds and C_ts is given by (19). For the pair of electrostatic shield wire number m, the energy stored in the adjacent capacitances is given by (20).

$$C_{tot\_d} = \frac{N}{3} C_d \qquad C_{tot\_t} = \frac{N-k-1}{2N^2} C_t \qquad (19)$$

$$E_{turn\_shield_m} = \qquad (20)$$

$$\frac{1}{2} C_{ts\_m} \left\{ \left(V_{sh} - \left(V - \frac{(m-1)V}{2N} \right) \right)^2 + \left(V_{sh} - \left(V - \frac{mV}{2N} \right) \right)^2 \right.$$
$$\left. + \left(V_{sh} - \frac{(m-1)V}{2N} \right)^2 + \left(V_{sh} - \frac{mV}{2N} \right)^2 \right\}$$

In which the first two terms are the energy stored in the capacitances between the shield wire number m located in the top disk and the adjacent winding turns and the second two terms are the energy stored in the capacitances between the shield wire number m located in the bottom disk and the adjacent winding turns. The total energy stored in the capacitances between electrostatic shield wires and winding turns is given in (21). The total equivalent capacitance between the shield wires and winding turns is given in (22).

$$E_{tot_{turn\_shield}} = \sum_{m=1}^{k} E_{turn\_shield_m} \qquad (21)$$

$$C_{tot\_ts} = \frac{C_{ts}}{2N^2} \sum_{m=1}^{k} \{ 2(N-m)(N-m+1) + 1 \} \qquad (22)$$

In (22), C_{ts} is the capacitance between the middle shield wire and the adjacent turn in the top disk which is considered as an approximation for all C_{ts}s. The total series capacitance of a disk pair with k electrostatic shield wires is given by (23). For k=1 and k=2, C_{s1} is shown in table 1. In derivation of the formula given in table 1, it is assumed that $C_{ts} = C_t$.

$$C_{s1} = \frac{N}{3} C_d + \frac{N-k-1}{2N^2} C_t \qquad (23)$$
$$+ \frac{C_{ts}}{2N^2} \sum_{m=1}^{k} \{ 2(N-m)(N-m+1) + 1 \}$$

D. Continus disk winding with wound-in shield wires

For the case of wound-in shield wires, in which the shield wires in the two disks are bound together at one point (for example in Fig. 7, the first shield wires are connected and are at floating potential. As an alternative, this cross-point can be connected to the input lead of the top disk. Other configurations are also possible [5] which are not included in the following analysis). The shield wires are connected in the top disk and voltage induced on them is considered to be the same as voltage induced on each turn of the winding shown in (8).
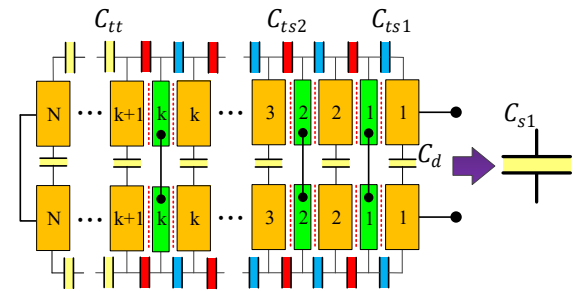

Figure 6. equivalent series capacitance of a pair of disks with k pairs of electrostatic shield wires

TABLE I. C_{s1} FOR K PAIRS OF FLOATED ELECTROSTATIC SHIELD WIRES

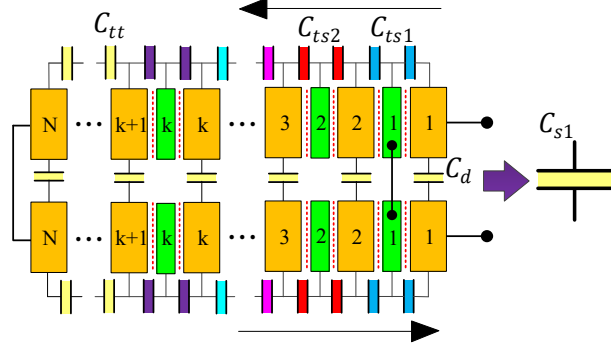| $k=1$	$\dfrac{N}{3}C_d + \dfrac{2N^2 - N - 1}{2N^2}C_t$
$k=2$	$\dfrac{N}{3}C_d + \dfrac{4N^2 - 7N + 3}{2N^2}C_t$

Figure 7. equivalent series capacitance of a pair of disks with k wound-in shield wires in each disk

The same is applicable to the shield wires in the bottom disk, with the difference that for the bottom disk when the disks are made the winding turns and shield wires are wrapped from left to right.

Equation (18) is used to calculate the total energy stored in a disk pair. The turn-turn capacitances and disk-disk capacitances can be calculated using the same formula given in (19). For the turn-shield capacitances let us assume that the voltage of the cross point of the first pair of shield wires is V_{cross}. Note that the voltage at the midpoint of the top shield wire would be half a turn-voltage lower than V_{cross} and for the bottom disk it would be half a turn-voltage higher than V_{cross}. The same is applicable to the winding turns, therefore when calculating the voltage difference between each shield wire and the adjacent turns, this addition/subtraction of half turn-voltage cancels out and hence we can consider that the first shield wires are at V_{cross} and the first winding turn in the top disk is at V and the first turn of the lower disk is at 0 volt (as shown in Fig. 7). The energy stored in all capacitances between shield wire number m and the adjacent winding turns can be calculated by (24). The energy stored in the capacitances between each pair of shield wires in the upper and the lower disks and the adjacent winding turns is independent of the shield wire number.

$$E_{turn\_shield_m} = \qquad (24)$$

$$\frac{1}{2}C_{ts\_m}\left\{\left(\left(V_{cross} - \frac{(m-1)V}{2N}\right) - \left(V - \frac{(m-1)V}{2N}\right)\right)^2\right.$$

$$+ \left(\left(V_{cross} - \frac{(m-1)V}{2N}\right) - \left(V - \frac{mV}{2N}\right)\right)^2$$

$$+ \left(\left(V_{cross} + \frac{(m-1)V}{2N}\right) - \frac{(m-1)V}{2N}\right)^2$$

$$\left. + \left(\left(V_{cross} + \frac{(m-1)V}{2N}\right) - \frac{mV}{2N}\right)^2\right\}$$

After simplification of (24) the total energy stored in all shield-turn capacitances can be calculated by (25). In (25) an equivalent capacitance C_{ts} between the middle shield wire and the adjacent winding turn in the top disk is considered as an approximation for all shield-turn capacitances.

$$E_{tot_{turn\_shield}} = \qquad (25)$$

$$k\frac{1}{2}C_{ts}\left\{(V_{cross} - V)^2 + \left(V_{cross} - V + \frac{V}{2N}\right)^2 \right.$$

$$\left. + (V_{cross})^2 + \left(V_{cross} - \frac{V}{2N}\right)^2\right\}$$

If the cross-over is connected to the input terminal, then $V_{cross} = V$. If the cross-over of the first pair of shield wires is floated then $V_{cross} = \frac{V}{2}$. Equation (25) can be respectively simplified to (26) and (27) for the two aforementioned cases.

$$E_{tot_{turn\_shield}} = \frac{k}{2}C_{ts}\frac{8N^2 - 4N + 2}{4N^2}V^2 \qquad (26)$$

$$E_{tot_{turn\_shield}} = \frac{k}{2}C_{ts}\frac{4N^2 - 4N + 2}{4N^2}V^2 \qquad (27)$$

Using (18), (19), (26), and (27), the total series capacitances between the two disks can be calculated for these two cases, and the results are given in table 2. In the formulas given in table 2, it is assumed that $C_{ts} = C_t$.

TABLE II. C_{s1} FOR K PAIRS OF WOUND-IN SHIELD WIRES

$V_{cross} = V$	$\dfrac{N}{3}C_d + \left(\dfrac{N - k - 1}{2N^2} + k\dfrac{8N^2 - 4N + 2}{4N^2}\right)C_t$
$V_{cross} = \dfrac{V}{2}$	$\dfrac{N}{3}C_d + \left(\dfrac{N - k - 1}{2N^2} + k\dfrac{4N^2 - 4N + 2}{4N^2}\right)C_t$

III. CONCLUSION

Analytical formulas for series capacitance of a disk pair are derived and summarized for four cases. Continuous disk winding, interleaved disk winding, continuous disk winding with k floated electrostatic shield wires, and continuous disk winding with k wound-in shield wires. In the next step these analytical formula will be compared with numerical results obtained by Finite Element Method.

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