Humanitarian Fleet Planning and Weekly Scheduling Optimisation

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by

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Contents

List of Figures

List of Tables

Introduction

The United Nations Humanitarian Air Service (UNHAS) is a branch of the World Food Programme (WFP) aviation department and currently the largest humanitarian air operator in the world. With a fleet of 90 aircraft and helicopters in 2019, UNHAS can be compared to a large commercial airline with the exception that it does not strive to maximise profit, but to accomplish it's humanitarian mandate: "to provide safe, reliable, cost-efficient and effective passenger and light cargo transport for the humanitarian community". This contrast makes it difficult for UNHAS to use the traditional optimisation models developed for commercial airlines and their planning cycle. Currently, predominant planning stages such as network design, fleet planning and aircraft routing and scheduling are created from experience and most often manually, with little decision support tools available. UNHAS has much to gain in developing mathematical optimisation models, tailored to the unique requirements of humanitarian stakeholders, which can help improve its air operations and processes. This study fits within a larger research framework made up of 4 main pillars with the overall objective to improve humanitarian air operations and create data driven decision support tools for humanitarian air operators and their users. The 4 pillars are:

1. Humanitarian Value Quantification
2. Humanitarian Air Operations Value Optimisation
3. **Humanitarian Fleet Optimisation**
4. Humanitarian Flight Routing and Scheduling Optimisation

While humanitarian daily flight routing and scheduling optimisation has already been explored by 2 previous research papers from authors S.P. Niemansburg (2019) and Y. Mekking (2020), the research presented in this report will take a step back and investigate fleet planning and weekly flight scheduling [43][45]. In consultation with UNHAS expert flight planners and the 2019 French Cour des Comptes audit of WFP aviation and UNHAS, the following main shortcomings have been identified [14]. Firstly, no optimisation model exists to help WFP contracting officers in selecting the best air assets for a mission, nor to preemptively analyse the effects of fleeting decisions on aircraft routing and scheduling, network design and operational costs. Secondly, UNHAS manually creates weekly preliminary flight schedules which drive the humanitarian booking requests without the use of decision support tools to optimise aircraft routing. There is therefore an opportunity for UNHAS to create mathematical optimisation models which can help improve it's fleet planning and weekly scheduling processes. This is expected to increase the effectiveness (demand satisfaction) and efficiency (costs minimisation) of it's overall air operations. Implementing such a decision support tool would help UNHAS and other humanitarian air operators to decrease their aircraft operational and contracting costs while increasing user satisfaction by providing a fleet more adapted to humanitarian missions. This would also benefit all stakeholders by increasing the accountability on the decisions taken by contracting officers and flight planners and on operational performance, an important consideration when reporting back to donors, stakeholders and the wider humanitarian community.

This thesis report is organized as follows. In Part I, the scientific paper is presented. Part II contains the Literature Study carried out at the start of the project which explored the background, context and technical aspects related to the problem at hand. Finally, in Part III, additional supporting work is presented.

I

Scientific Paper

Humanitarian Fleet Planning and Weekly Scheduling Optimisation

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Abstract

Fleet planning is a complex problem which consists of selecting an optimal fleet of vehicles at the right moment in time to serve a specific demand while subject to different operational constraints. While Operations Research (OR) has mainly focused on long-term strategical fleet planning optimisation, certain operators such as humanitarian air services must make shorter-term decisions due to their operating environments and mission specifications. This research proposes a novel methodology for combining fleet planning and flight scheduling on a tactical time frame, weeks to months, using the United Nations Humanitarian Service's (UNHAS) South Sudan mission as a case study. The scientific approach consists of using two different linear programming models sequentially to divide the decision making process and reduce the computational complexity of the problem at hand. A multi-commodity network flow model is first used to size an initial fleet and investigate the transshipment of passengers throughout a network. The outputs are then used by a Fleet Size and Mix Vehicle Routing Problem (FSMVRP) model to further increase the accuracy of vehicle routes and passenger operations. The main results show that the methodology proposed can reduce the weekly routing costs by 40% compared to expert flight planners who schedule and route humanitarian requests on a daily basis, and reduce the fleet size by 60% from 14 air assets to 6. The research demonstrates that the developed optimisation framework can effectively be used as a decision support tool for both aircraft contracting, and flight routing and scheduling with the main objective to increase the efficiency and effectiveness of humanitarian air operations.

1 Introduction

Logistics and transportation have a direct impact on the effectiveness and efficiency of humanitarian missions, amounting to the second largest expenditure for international humanitarian organisations (Van Wassenhove and Pedraza Martinez, 2012). However, Operations Research (OR) is rarely applied in the Humanitarian sector, mostly due to the unpredictable, last-minute and dangerous aspects of its operations. With humanitarian crises on the rise and funding for humanitarian operations stagnating, there is an increasing need for improvements in effectiveness (demand satisfaction) and efficiency (cost minimisation) of humanitarian air operations (OCHA, 2019). In 2019, the United Nations Humanitarian Air Service (UNHAS) transported approximately $412,000$ passengers to and from areas affected by crises. In South Sudan alone, UNHAS operated its biggest fleet with 10 passenger aircraft and 4 helicopters (WFP, 2019). Despite this, most planning processes from fleet planning to the creation of weekly flight schedules and daily routings are still created manually based on previous missions and UNHAS staff experience. There is an opportunity for UNHAS and other humanitarian operators to implement more data-driven decision support tools which could help in improving the effectiveness and efficiency of their overall air operations and planning processes.

Fleet planning is a common problem addressed in industry and academia for all transportation modes. It consists of selecting an optimal fleet of vehicles by addressing the following three questions (1) Which type of vehicle should I buy? (2) how many of each do I need? and (3) When should I acquire them? Multi-Commodity Network Flow (MCNF) models and general Mixed-Integer Linear Programming (MILP) formulations are most commonly used to solve this class of optimisation problems (Díaz-Parra et al., 2014; Hoff et al., 2010). However, most applications of fleet planning and weekly scheduling models focus on multi-year strategical time horizons. The types of decisions and the level of detail incorporated are limited by a significant amount of uncertainty such as demand fluctuations, evolution of the aviation industry and the availability of newer, more efficient aircraft. This paper proposes a novel methodology to approach fleet planning and scheduling for humanitarian missions on a shorter tactical time-frame, from weeks to months. The difficulty lies in incorporating the

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amount of detail necessary to simulate weekly humanitarian air operations and fleeting decisions, while keeping the size of the optimisation problem solvable in a reasonable amount of time. The decision making process is divided into two stages. A MCNF model is first created to investigate the network effects and transshipment of passengers for humanitarian missions. The outputs are then used in a FSMVRP model to further increase the accuracy of the routing, scheduling and fleet sizing decisions. The objective of the research lies in contributing to the improved accountability and optimisation of the humanitarian planning cycle by creating a fleet sizing and weekly flight scheduling decision support tool which can help flight planners increase the efficiency and effectiveness of humanitarian air operation on a tactical time-frame.

The research paper is structured as followed: Section 2 will cover humanitarian air operations, fleet planning in literature and the main motivation for the research. This is followed by Section 3 which contains the scientific approach and the mathematical formulations used for the study. The results of the South Sudan UNHAS mission case study are then presented in Section 4, followed by Section 5 which covers the verification and validation of the models and a discussion of the results. Finally, Section 6 closes the report by summarising the main conclusions and recommendations for future research.

2 Literature Review

2.1 UNHAS and humanitarian air operations

The World Food Programme (WFP) is the largest humanitarian organization in the world actively fighting malnutrition and undernourishment. WFP aviation plays a crucial role in this mission, facilitating access to remote areas not accessible by other means of transport. UNHAS is the branch of the WFP aviation service focusing exclusively on passenger and light cargo transport. Their goal is to provide "*safe, efficient, responsive, and cost-effective*" air transport for the wider humanitarian community (WFP, 2020). This mandate and the context in which UNHAS operates sets it apart from the traditional commercial airlines. Minimizing costs and maximizing demand satisfaction is substituted to profit maximisation. Instead of the seasonal cycles, humanitarian crises lead to unexpected demand peaks, requiring air operators to be flexible and able to adapt rapidly to different operating environments. Humanitarian fleets are consequently mostly operated under monthly wet lease contracts in comparison with traditional airlines which purchase their fleets years in advance. The planning cycle for humanitarian operations also takes place on a much shorter time-frame than its commercial counterpart, with aircraft contracting happening a few weeks to 3 months before the deployment of an air asset. Choosing the best vehicles to serve a specific demand and network therefore becomes a crucial decision which directly impacts the effectiveness and efficiency of weekly and daily air trasnport operations. In 2019, the average load factor on a UNHAS flight leg was 50% compared to 82.5% for commercial airlines (IATA (2020) ; Cour des Comptes France (2019)). The same sources reveal that the cost per passenger per km are on average 1.40 USD for WFP aviation, considerably expensive compared to 0.13 USD for other commercial airlines.

In 2019, the French Cour des Comptes audited WFP aviation and UNHAS, revealing several shortcomings and processes in need of improvement (Cour des Comptes France, 2019). The audit document states that requests for UNHAS services have risen in the years 2017 to 2019. The number of countries in which UNHAS operates has risen by a factor 4, and passengers transported by 26.4%, implying that there is an increasing need for its services. This contrasts with the fact that the WFP aviation's contracted fleet is ageing. In a fleet made up of 90 air assets, the production for 6 of the aircraft types used has been discontinued (accounting for 45% of all chartered air assets, put in service circa 1980's). This implies that UNHAS will need to re-evaluate their fleet capabilities in the near future. UNHAS has access to backward looking data driven decision support tools such as the Performance Monitoring Tool (PMT) which helps in evaluating the impact on the effectiveness and efficiency of historical fleet / scheduling decisions. However to our knowledge, it has limited if any forward looking decision support power to proactively improve future aircraft contracting and scheduling decisions. The auditors also observed that contract durations are poorly justified. Aircraft are contracted under Minimum Guaranteed Hours (MGH) agreements representing the amount of block hours that are expected to be flown by an asset at the corresponding monthly leasing price. These are often miscalculated, leading to higher costs due to an underestimation or overestimation of hours to be flown. Specifically to the case of South Sudan, two of the contracted aircraft flew 19% and 35% less than their originally agreed MGH (Cour des Comptes France, 2019).

2.2 Fleet planning and weekly scheduling

Mathematical programming is an optimisation technique which attempts to translate complex real-world problems into mathematical formulations. In order to analyse fleet planning and weekly scheduling for UNHAS, it is essential to translate the service, the humanitarian context and its users into a model that can simulate accurately real-world operations. Fleet planning is an optimisation problem commonly addressed in the transport industry and in Operations Research (OR) (Bielli et al., 2011). It consists of selecting an optimal fleet for an operator/mission by choosing the best type and corresponding amount of vehicles needed to satisfy a number of requirements, and the timing of their acquisition. Although this study focuses on aircraft fleets, many parallels exist between the fleet planning of different transportation modes, from road and rail to maritime and air (Baykasoğlu et al., 2018). Fleet planning has a significant impact on most of the decisions taken further down the planning cycle and therefore is usually combined with other processes in order to increase the consistency and reliability of its output. A common approach is to combine fleet planning with the design of transportation networks. A scenario similar to the "*who came first, the chicken or the egg?*" dilemma, deciding which routes to operate and assigning them a service frequency depends on the type of fleet available, and vice-versa. The modelling of such networks can be achieved through the use of Minimum Cost Flow Problem (MCFP) formulations, usually combined with a route-frequency dependent term. The general objective lies in designing an optimal network and aircraft routing policy which minimizes costs and maximises demand satisfaction, given an Origin-Destination (OD) demand matrix and operational constraints. Crainic (2000) presents a review on tactical freight transportation and service network design where the author classifies existing models based on their functionality instead of the transportation mode, providing a more comprehensive view on their applications. Marsten and Muller (1980) created a deterministic MILP model in the form of a minimum cost network flow problem for air cargo fleet planning while Jaillet et al. (1996) tackle a capacitated network design problem for airlines and draw parallels with existing Hub Location Problems (HLP). The formulation uses fractional flows as decision variables to represent direct or transfer passengers and correlates them to aircraft flows. Similarly, Sa et al. (2019) formulate an integer linear programming optimisation model using fractional flows to simulate passenger transfers at hubs. They combine fleet planning and quantitative demand forecasting by first using an econometric model and Monte Carlo simulations to reproduce demand fluctuations over time. The results are used as inputs to their optimisation model which allocates aircraft types and corresponding flight frequencies to OD pairs with the objective to maximize the overall profit of an airline and satisfy the total demand.

While the aforementioned authors approached fleet planning decisions on a strategical (long-term) time horizon, the humanitarian aviation planning cycle happens on a much shorter time frame. Since aircraft contracting typically happens weeks to 3 month before flight departure, much more information is available to help WFP decision makers in choosing the most suited air assets for a mission. Aspects such as vehicle routing and passenger pick-up and deliveries should therefore also be taken into account, a consideration which is difficult to implement in long term fleet planning and network design models due to high levels of uncertainty in predicting future demand. Fleet planning models combined with routing formulations are therefore mainly used in tactical and operational time-horizons to simultaneously size an optimal fleet and find an optimal set of routes, while satisfying a number of supply, demand and operational constraints. Hoff et al. (2010) presents an extensive literature review on fleet composition and routing problems in maritime and road transport. He chooses the Fleet Size and Mix Vehicle Routing Problem (FSMVRP) as the most representative mathematical formulation for this class of problem. First introduced by Golden et al. (1984), the FSMVRP differs from the Heterogeneous Fleet VRP by considering an unlimited number of vehicles with fixed acquisition costs, as opposed to a fixed fleet. Hoff et al. (2010) state in their review that out of 95 papers analysed, over 50% of the FSMVRP do not target a specific transportation mode. This highlights the fact that this mathematical formulation is flexible and can be adapted to a humanitarian airline routing problem. More recently, Koç et al. (2014) solved the FSM pollution-routing problem, a VRP which combines CO2 emissions and vehicle acquisition. Hiermann et al. (2016) combine FSMVRP and time windows with recharging stations for electric vehicles. Pasha et al. (2016) tackle the Multi-period FSMVRP and develop a heuristic which allows to determine an optimal fleet for two different periods with stochastic demand.

The FSMVRP is an extension of the VRP, an NP hard, combinatorial optimisation problem extensively covered in OR. They are a generalisation of the Travelling Salesman Problem (TSP), but with multiple vehicles and capacity constraints. Eksioglu et al. (2009) provide a state-of-the-art taxonomic review of VRP problems and their applications for different transportation modes. VRP's have previously been used to tackle humanitarian routing problems. Two research papers have explored humanitarian passenger flight routing and scheduling for the South Sudan UNHAS mission. Niemansburg (2019) created the Humanitarian Flight Optimization Model (HFOM), an adaptation of the heterogeneous pickup and delivery problem with time windows. The problem is formulated as a three-index MILP model with objective to minimize the sum of the routing costs and maximize demand satisfaction. The HFOM is able to produce routings with reduced costs between 2.2% and 7.8% compared to expert planners, and up to 5 times faster. Building upon this, Mekking (2020) extends the HFOM by modifying the node generation and allocation formulation in order to reduce the problem size. Another significant addition lies in the calculation of MGH which are an important consideration for UNHAS monthly operations. The author investigates how to distribute the aircraft utilisation and routing amongst each asset over a month to best match the MGH contracted. The model achieves up to 10.5% in cost savings when compared to expert flight planner solutions and improved passenger request satisfactions by 1.2%. Both papers do not consider fleeting decisions or weekly routing in their formulations.

2.3 Literature gap and motivation for research

The humanitarian sector is in a need of more data-driven decision support tools in order to help optimise aviation planning processes. With a shorter planning horizon for humanitarian missions, there is a greater opportunity to evaluate how different steps of the planning process impact each other. As aircraft contracting happens from several weeks to 3 months prior to the deployment of an air asset, choosing the best fleet to serve the humanitarian demand for a specific mission can lead to a immediate impact in effectiveness (demand satisfaction) and efficiency (cost minimisation) of weekly and daily air operations. While Mekking (2020) and Niemansburg (2019) investigated the optimisation of daily routing and scheduling of humanitarian air operations, it is important to take a step back and look at a longer time frame. A different paradigm is approached where instead of optimising air operations on a daily basis, a detailed weekly schedule can be created to satisfy the humanitarian demand for that week and size an aircraft fleet accordingly. Assuming the weekly humanitarian flight requests do not vary significantly over a month, fixing a detailed weekly flight schedule would increase the accountability and optimisation of air operations. This would in turn provide UNHAS contracting officers valuable insight on the best air assets suited to a mission as well as corresponding future operational data (block hours, MGH, load factors, fuel requirements etc.).

Fleet planning combined with network design models mostly tackle long-term time horizons and do not incorporate the level of detail needed to represent humanitarian passenger flows and operations. Considerations such as correlating passenger flows with aircraft flows and modelling pick-up and delivery operations are of utmost importance when building a weekly schedule for humanitarian operators. On the other hand, while VRP formulations can incorporate such a high level of detail, they are NP hard. Large instances cannot be solved in a reasonable amount of time without dividing the problem into smaller manageable ones, or through the use of heuristics and meta-heuristics. For VRPs, the amount of variables and the size of the problem not only depends on the amount of nodes, commodities and vehicles used, but also on the types of choices that are enabled. For example, splitting passenger requests or the use of time windows. This research proposes the combination of the aforementioned models in a sequential approach in order to reduce the amount of possible decisions the FSMVRP model will need to incorporate, leading to savings in computational complexity and time. The models will focus on a uni-modal, heterogeneous aircraft fleet with passengers as commodities.

The following questions are addressed in this research:

1. **To what extent can a minimum-cost, multi-commodity network flow model simulate humanitarian flight networks and passenger flows based on a forecasted demand and corresponding origin and destination pairs?**
2. **Can such a model be combined with an FSMVRP model to further increase the accuracy of aircraft routing and scheduling?**
3. **To what extent can this combination be used to simultaneously size an aircraft fleet and determine a feasible weekly preliminary flight schedule for humanitarian air operations?**
4. **Can such a model be used as a decision support tool to help flight planners improve the efficiency and effectiveness of humanitarian air operations and aircraft contracting?**

3 Methodology

3.1 Problem definition

The previous sections have revealed that humanitarian air transport operations can be modelled using linear programming. A FSMVRP model was considered most appropriate and accurate to simulate the complexity of UNHAS missions, the intricacies of weekly aircraft routing and scheduling, and fleet sizing decisions on a tactical time frame. From now on, the model will be referenced as the Humanitarian FSMVRP (HFSMVRP). The following main attributes characterize its mathematical formulation :

1. **Split Load**: Passenger requests can be split over multiple routes and different aircraft
2. **Simultaneous Pick-up and Delivery**: Passengers can be picked-up and delivered simultaneously as long as the associated nodes correspond to their origin or final destination

3. **Multi-Depot**: Vehicles are assigned to different depot nodes and must start and end their routes at that exact location

4. **Multi-trip**: Individual vehicles are able to leave and return to a depot multiple times as long as the total range and block hour constraints are not violated

Although Split Loads and Simultaneous Pick-up and Delivery are enabled, the possibility of transferring passengers between different aircraft and nodes after their departure is not implemented in the formulation. This decision was taken based on the anticipation that such a model would lead to much longer solving times. Adding passenger transshipment implies the use of time-windows and therefore an exponential increase in decision variables and computational complexity. This aspect however remains a crucial part of humanitarian passenger air operations, mainly due to aircraft limitations such as range and runway requirements. In order to take into account passenger transshipment, a multi-commodity, minimum-cost network flow formulation is used prior to the HFSMVRP optimisation model. Fleet planning has often been approached using multi-commodity flows in order to represent the movement of passengers across a network, subject to multiple constraints such as vehicle capacity, range, operational time and more. Passenger transfers can be simulated by the use of fractional flows as decision variables. The model is given the freedom to route passengers through different nodes of the network, as long as this minimizes overall system costs and passengers reach their final destination. A multi-commodity network flow (MCNF) formulation contains fewer variables than an FSMVRP model and should be able to be solved optimally within several minutes. Within the scope of this research, the objective of using a MCNF model is to modify the original O-D demand matrix by splitting requests between direct flows and transshipment flows through major hubs. The modified passenger requests are then used as input to the HFSMVRP model. The multi-commodity network flow problem is limited to simulating hub-and-spoke flights while the HFSMVRP is tasked to create detailed routings between the spokes and output detailed flight paths.

For large humanitarian missions such as the UNHAS one in South Sudan, the amount of weekly flight requests to and from different destinations can easily surpass 100. This makes the problem impossible to solve with an exact VRP formulation in a reasonable amount of time. It was therefore decided to divide the problem into smaller sub-problems based on the aircraft flows resulting from the MCNF model. Each sub-problem consists of the modified requests serviced by the corresponding aircraft type. Each instance is run with all the available vehicle types available at the corresponding hub. In this manner, the choice of switching aircraft types is still available depending on the additional routing considerations the HFSMVRP can take into account. Although this division reduces the size of the problem significantly, some sub-problems may still be too large to solve at once. An intra-airport, distance based clustering algorithm is used to separate requests into different regions, where aircraft routings are most probable to happen. In order to find an optimal fleet and weekly schedule for humanitarian air operations, a sequential approach is therefore taken, which is presented in more detail in Figure 1. Section 3.2 presents the FSMVRP formulation, followed by section 3.3 describing the MCNF problem and finally section 3.4 goes over the K-Means Clustering algorithm.

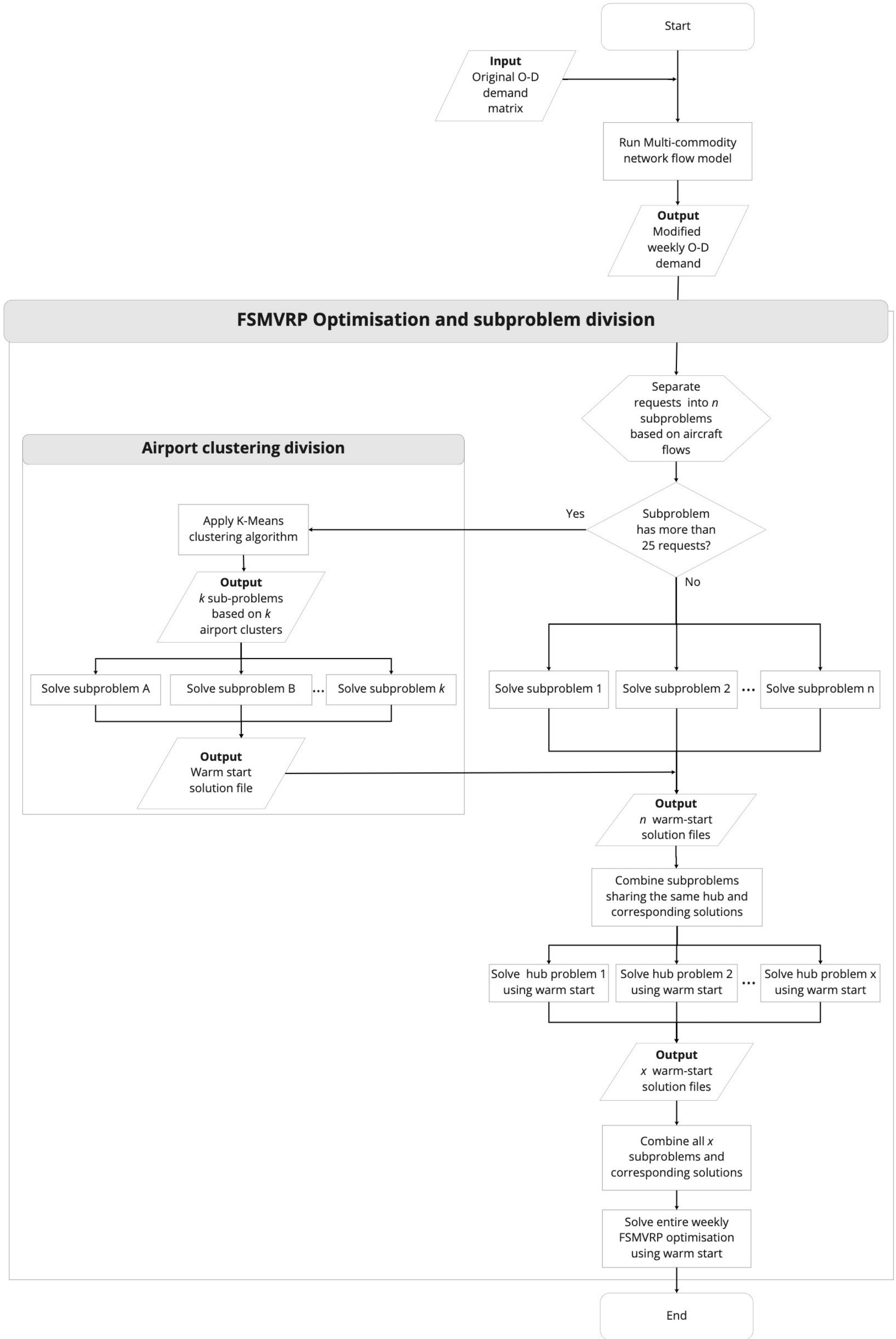


Figure 1: Humanitarian fleet and weekly scheduling optimisation methodology flowchart

3.2 A novel FSMVRP formulation

3.2.1 HFSMVRP formulation

A novel formulation for the Fleet Size and Mix VRP is proposed, described by Equations 1a till 1p. A short description of the formulation is provided below, followed by a more extensive analysis of each component in Appendix C where the verification of each constraint is performed. The objective function of the linear program is described by Equation 1a. The first term describes the fixed leasing costs which must be taken into account when starting a new lease or the acquisition of a vehicle. The fixed costs include painting, positioning/repositioning, war insurance, aircraft maintenance and safety factors. They can be found accumulated under the lease cost row in Appendix A, Table 17. The second term represents the variable routing costs which occur when a vehicle k uses the leg between nodes i and j. Equation 1b and Equation 1c ensure all vehicles start and end their routes at their corresponding depots. Equation 1d is the flow conservation constraint, stating that each vehicle arriving at a node must leave that same node. Equation 1e ensures multi-trip consistency between the previous three constraints . Equation 1f allows vehicles to visit a node more than once, allowing passengers from one request to be picked-up and delivered over different paths. Equation 1g and Equation 1h are the loading and unloading conservation constraints. The request coefficient α_j^r is equal to 1 if node j is the pick-up node of request r, -1 if it is the delivery node of latter, and 0 otherwise. Equation 1i and Equation 1j ensure that all the passengers from each requests are picked up and delivered to their final destination. Equation 1k is the capacity constraint, limiting the amount of passengers per flight leg to the maximum amount of seats of a vehicle k. Equation 1l prevents vehicles visiting nodes where passengers are not delivered or picked-up. Equation 1m and Equation 1n ensure that each aircraft does not surpass its weekly and daily maximum operational time. Equation 1o ensures that each aircraft path is not longer than its maximum range. The second term in the left-hand side accounts for increased range after an aircraft chooses to refuel. Finally, Equation 1p is added in order to limit the amount of refueling stops each aircraft make in a path to 1.

Table 1: Nomenclature for the HFSMVRP model

Sets		*Parameters*	
N	Set of airports	c_{ij}^k	Cost of arc ij for vehicle k
R	Set of requests	C_{lease}^k	Lease cost of vehicle k
K	Set of vehicle types	n^k	Max. number of trips for vehicle k
P	Set of paths per vehicle types	d_{ij}	Distance of arc ij
D^k	Set of depots available to aircraft k, $D^k \in$ N	Q^k	Amount of seats on vehicle k
F	Set of airports available for refueling, F $\in$ N	α_i^r	Request coefficient
		P_j^r	Total pax of request r to be picked-up at node j
Decision variables		D_j^r	Total pax of request r to be delivered at node j
		TAT^k	Turn-around-time of vehicle k
x_{ij}^{kp}	1 if arc ij is used by vehicle k on path p, 0 otherwise	BT_{weekly}^k	Weekly block hours of vehicle k
q_i^{krp}	Amount of pax of request r picked-up/delivered at i by vehicle k on path p	BT_{daily}^k	Daily block hours of vehicle k
u_{ij}^{krp}	Amount of pax of request r on arc ij in vehicle k on path p	$range_{max}^k$	Range of vehicle k
ac^k	Amount of aircrafts of type k	sp^k	Cruise speed of vehicle k

$$\textbf{minimize} \quad \sum_{k\in K} C_{lease}^k ac^k + \sum_{p\in P}\sum_{k\in K}\sum_{i\in N}\sum_{j\in N} c_{ij}^k x_{ijp}^k \tag{1a}$$

$$\text{subject to:} \quad \sum_{j\in N\setminus\{d^k\}}\sum_{p\in P} x_{d^kj}^{kp} \le n^k ac^k \qquad \forall k\in K \tag{1b}$$

$$\sum_{j\in N\setminus\{d^k\}}\sum_{p\in P} x_{jd^k}^{kp} \le n^k ac^k \qquad \forall k\in K \tag{1c}$$

$$\sum_{i\in N} x_{ij}^{kp} - \sum_{i\in N} x_{ji}^{kp} = 0 \qquad \forall k\in K, \forall p\in P, \forall j\in N \tag{1d}$$

$$\sum_{i\in N}\sum_{j\in N} x_{ij}^{kp} - \Big(\sum_{j\in N\setminus\{d^k\}} x_{d^kj}^{kp}\Big) * M = 0 \qquad \forall k\in K, p\in P \tag{1e}$$

$$\sum_{i\in N}\sum_{k\in K}\sum_{p\in P} x_{ij}^{kp} \ge 1 \qquad \forall j\in N \tag{1f}$$

$$\sum_{i\in N} u_{ij}^{krp} + q_j^{krp}\alpha_j^r - \sum_{i\in N} u_{ji}^{krp} - Q^k \sum_{i\in N} x_{ij}^{kp} \ge -Q^k \qquad \forall j\in N, r\in R, k\in K, p\in P \tag{1g}$$

$$\sum_{i\in N} u_{ij}^{krp} + q_j^{krp}\alpha_j^r - \sum_{i\in N} u_{ji}^{krp} + Q^k \sum_{i\in N} x_{ij}^{kp} \le Q^k \qquad \forall j\in N, r\in R, k\in K, p\in P \tag{1h}$$

$$\sum_{k \in K} \sum_{p \in P} q_j^{krp} = P_j^r \qquad \forall j \in N, r \in R \tag{1i}$$

$$\sum_{k \in K} \sum_{p \in P} q_j^{krp} = D_j^r \qquad \forall j \in N, r \in R \tag{1j}$$

$$\sum_{r \in R} u_{ij}^{krp} \leq Q^k x_{ij}^k \qquad \forall i \in N, j \in N, k \in K, p \in P \tag{1k}$$

$$Q^k \sum_{i \in N} x_{ij}^{kp} \geq q_j^{krp} \qquad \forall j \in N, r \in R, k \in K, p \in P \tag{1l}$$

$$\sum_{i \in N} \sum_{j \in N} \sum_{p \in P} x_{ij}^{kp} (\frac{d_{ij}}{sp^k} + TAT^k) \leq ac^k BT_{weekly}^k \qquad \forall k \in K \tag{1m}$$

$$\sum_{i \in N} \sum_{j \in N} x_{ij}^{kp} (\frac{d_{ij}}{sp^k} + TAT^k) \leq BT_{daily}^k \qquad \forall k \in K, p \in P \tag{1n}$$

$$\sum_{i \in N} \sum_{j \in N} (x_{ij}^{kp} d_{ij} - \sum_{f \in F} x_{fj}^{kp} (range_{max}^k)) \leq range_{max}^k \qquad \forall k \in K, p \in P \tag{1o}$$

$$\sum_{f \in F} \sum_{j \in N} x_{fj}^{kp} \leq 2 \qquad \forall k \in K, p \in P \tag{1p}$$

In order to reduce the amount of variables that the model must take into account, all redundant and unfeasible variables are deleted in a pre-processing step. Per aircraft, the following arcs and associated decision variables are filtered from the search space:

1. Arcs between nodes of which the distance is larger than the aircraft's maximum range
2. Arcs between nodes of which one of the runway lengths is smaller than the aircraft's required runway
3. Arcs between nodes of which no passenger operations are required and no refueling is possible
4. Arcs between the same nodes
5. Loading variables u_{ij}^{krp} if request r is to be delivered at node i
6. Loading variables u_{ij}^{krp} if request r is to be pick-up at node j
7. Variables q_i^{krp} if request r is does not originate or terminate at node i

3.2.2 Lazy constraints

The choice of not implementing time windows or servicing constraints implied that Subtour Elimination Constraints (SEC) needed to be added. Because of the significant size of the problem at hand, it was decided to implement them through a lazy constraint approach. Lazy constraints remain inactive until a feasible solution is found by the solver's branch-and-bound tree. This solution is then cross-checked with all lazy constraints. If one of the constraints is violated, the solution is removed and that specific constraint pulled into the active model. This incremental approach allows to reduce the order-of-magnitude of the constraints needed, speeding up model run time and reducing memory requirements. Implementing this is done through the use of *Callback* functions which allow the solver to inspect and query a feasible solution while the optimisation is still running, and modify the state of the optimisation. Lazy constraints were added to the model for two different uses, SEC and refuelling.

Subtour Elimination Constraint

For a VRP solution to be valid, all routes employed by different vehicles must start and end at the same node, namely its assigned depot, and all nodes within that route must be connected by an arc. If SECs are not added to the model formulation, certain solutions may contain sub-tours: multiple paths within an aircraft route which are not connected with each other. Preventing such solutions from being accepted as feasible was achieved by implementing the lazy constraint in Equation 2 for each aircraft route in the solution. In the following constraint, set T contains all the x_{ij}^{kp} variables forming the entire solution found by the solver, ordered from the first visited node to the last, per route p and aircraft k. Set S contains the solutions of a single path p within the T Set.

$$\sum_{i,j,k,p \in S} x_{ij}^{kp} \leq |S| - 1 \qquad \forall S \in T \tag{2}$$

Refuelling-Range Constraint

The FSMVRP formulation allows aircraft to refuel at specific hubs within a route to extend their range if it can reduce the objective function value. In order to implement refuelling stops, fictitious refueling nodes are created at the same coordinates as the desired refuelling hubs. In this way, an aircraft can choose to visit a location for normal operations, for refuelling, or both, without having to increase its flight time or distance. However refuelling should only take place at a specific point in time within a route, when the aircraft has used most of its fuel and needs to fulfill the last requests and return to base. Without adding a constraint related to this property, the model can choose to refuel the aircraft at the start of the route, increasing its total range, without taking into account that fuel tanks can only take in a maximum amount of fuel. In order to prevent this and taking into account that adding a constraint for each possible path would be inefficient, the lazy constraints in Equation 3 and Equation 4 are implemented. The sets R_1 and R_2 refer to the arcs ij used by aircraft k on path p during the flight segment before after refuelling respectively.

$$\sum_{i,j,k,p \in R1} x_{ij}^{kp} d_{ij} \leq range_{max}^{k} \qquad \forall S \in T \tag{3}$$

$$\sum_{i,j,k,p \in R2} x_{ij}^{kp} d_{ij} \leq range_{max}^{k} \qquad \forall S \in T \tag{4}$$

3.3 Multi-commodity network flow model

A linear multi-commodity network flow model is used to simulate passenger transfers throughout the network before using its results as input to the FSMVRP model in the form of a modified demand matrix. Two different types of passenger movements are allowed: direct flows between a request's origin and destination, or a transfer flow which is divided into an Origin-Hub and Hub-Destination flow. Passengers are only allowed to make one stop, and can transfer only at major hubs. This decision is taken based on the fact that a multi-commodity network flow formulation is unable to simulate accurately aircraft routing decisions as aircraft flows are not correlated with each other and only define capacity per flight leg. According to flight planner experts and the South Sudan humanitarian demand matrix, humanitarian flight requests usually replicate a hub and spoke pattern, as passengers mostly arrive and depart from international hubs to local destinations and back, rarely traveling between spokes. The network created is therefore similar to a hub and spoke model with connections only between main hubs.

The mathematical formulation for the multi commodity network flow is presented from Equation 5a to Equation 5h. The objective function in Equation 5a is made up of a first term accounting for fixed aircraft lease costs, and a second term accounting for variable routing costs. The first constraint in Equation 5b ensures that all flows departing from an airport are either transfer passengers flows towards a hub, or direct flows. Equation 5c ensures all flows arriving at an airport is either a direct flow or a transfer passenger flow from a hub to a spoke airport. Equation 5d is the arc capacity constraint and Equation 5e is a passenger flow conservation constraint ensuring consistency between the first and second leg of passengers transferring at a hub. Equation 5f ensures outbound aircraft flows are matched with inbound flows. Finally, Equation 5g ensures that if passengers are transferred at hubs without any aircraft, that an additional arc is added to the solution accounting for the fact that another aircraft must fly to that hub to pick up the passengers. Finally, Equation 5h ensures the weekly utilisation of aircraft are not exceeded.

Table 2: Nomenclature for the multi-commodity network flow model

Sets		*Parameters*	
N	Set of airports	c_{ij}^{k}	Cost of arc ij for vehicle k
H	Set of hubs, H ∈ N	d_{ij}	Distance of arc ij
K	Set of vehicle types	P_i	Amount of pax to be picked up at node i
O	Set of hubs which are not aircraft depots, O ∈ H	D_{ij}	Amount of pax wishing to travel from i to j
D^k	Set of depots for aircraft k, D ∈ H	$Seats^k$	Amount of seats on vehicle k
		sp^k	Speed of vehicle k
Decision variables		C_{lease}^{k}	Lease cost of vehicle k
		BT_{week}^{k}	Weekly block hours of vehicle k
x_{ij}	Direct passenger flow from i to j	TAT^k	Turn-around-time of vehicle k
y_{ij}	Transfer passenger flow from origin i to j		
w_{mj}^{i}	Transfer passenger flow from m to j, originating from i		
z_{ij}^{k}	Route frequency between i and j with aircraft k		
ac^k	Amount of aircraft of type k		

$$\textbf{minimize} \quad \sum_{k \in K} C^k_{lease} ac^k + \sum_{k \in K} \sum_{i \in N} \sum_{j \in N} c^k_{ij} z^k_{ij} \tag{5a}$$

$$\text{subject to :} \quad \sum_{j \in N} x_{ij} + \sum_{h \in H} y_{ih} = P_i \qquad \forall i \in N \tag{5b}$$

$$x_{ij} + \sum_{h \in H} w^i_{hj} = D_{ij} \qquad \forall i, j \in N \tag{5c}$$

$$x_{ij} + y_{ih} + \sum_{h \in H} w^h_{ij} \leq z^k_{ij} Seats^k \qquad \forall i, j \in N \tag{5d}$$

$$y_{ih} = \sum_{j \in N} w^i_{hj} \qquad \forall i \in N, h \in H \tag{5e}$$

$$\sum_{j \in N} z^k_{ij} = \sum_{j \in N} z^k_{ji} \qquad \forall i \in N, \forall k \in K \tag{5f}$$

$$\sum_{j \in N} z^k_{oj} - \sum_{h \in D^k} z^k_{ho} \leq 0 \qquad \forall k \in K, \forall o \in O \tag{5g}$$

$$\sum_{i \in N} \sum_{j \in N} z^k_{ij} \left(\frac{d_{ij}}{sp^k} + TAT^k\right) \leq BT^k_{week} ac^k \qquad \forall k \in K, \tag{5h}$$

3.4 K-Means Clustering

3.4.1 K-Means clustering algorithm

A K-means clustering algorithm was used to subdivide larger sub-problems (more than 25 requests) into smaller manageable ones. K-Means clustering is an unsupervised machine learning algorithm which partitions a data set into k clusters by assigning each sample to the cluster with the nearest mean (centroid). The algorithm can be applied to a data set made up of geographical coordinates, resulting in k clusters made up of airports closest to each other. Each cluster is then used as its own sub-problem and solved with the FSMVRP formulation. The reasoning behind this decision is the assumption that aircraft routing will most probably happen between airports which are in vicinity of each other. The algorithm is described below in Algorithm 1.

Algorithm 1 K-Means Clustering Algorithm

1: **Input** Airport coordinates (x,y) number of clusters k, number of iterations x
2: **Initialise** k random centroids
3: **Repeat**
4: Assignment step: Assign each airport to the closest cluster centroid (euclidean distance)
5: Update step : Compute and update new cluster centroid coordinates based on intra-cluster airport mean distance
6: **Until** cluster centroid coordinates do not change for x iterations
7: **Output** k clusters, k cluster centroid coordinates, list of airports per cluster

3.4.2 Elbow method heuristic

The K-Means clustering algorithm by itself does not indicate to the user how many clusters a specific data-set should be divided into. Choosing the optimal amount of clusters to partition a set can be done through the Elbow method. This heuristic allows to find the point of "diminishing return" by plotting the average Within-Cluster Sum of Squared Errors (WCSSE) for each cluster with respect to their cluster mean, versus a number k of clusters. The WCSSE is a measure of the variability of the observations within each of their respective clusters. The optimal number of clusters lies at the inflection point where the WCSSE starts decreasing linearly with respect to the amount of clusters and tends towards 0. This "elbow point" or point of "diminishing return" indicates the lowest amount of clusters needed before a change in the quality of the clustering is not improved by a significant amount. This heuristic is used every time a clustering is needed to reduce the size of the problem in order to decide in how many different clusters the airports in question must be divided into.

4 South Sudan UNHAS mission: Fleet sizing and Weekly scheduling - Results

4.1 Multi-commodity Network flow results

The multi-commodity Network Flow model was applied to the South Sudan UNHAS mission for the weekly demand of the the 30/09/2019 to 04/10/2019. UNHAS flight planners schedule humanitarian flights on a daily basis, therefore only daily request data is known, presented in Appendix A, Table 18. In order to obtain the weekly demand, all requests to and from the same destinations over the week were combined and incorporated into a single demand matrix as can be seen in Appendix A, Table 19. The airport and fleet data are presented in Appendix A, Table 16 and Table 17 respectively. Figure 2 illustrates the results of the MCNF in the form of passenger flows throughout the South Sudan network, split up into direct and transfer passenger flows. A more detailed breakdown of the passenger and aircraft flows can be found in Appendix B, in Figures 10 to 18. The Hub and Spoke effect emerges clearly from these figures, with passenger transfers occurring in the main hubs of Rumbek, Juba, Wau and Bor. Table 3 presents the data related to passenger flows.

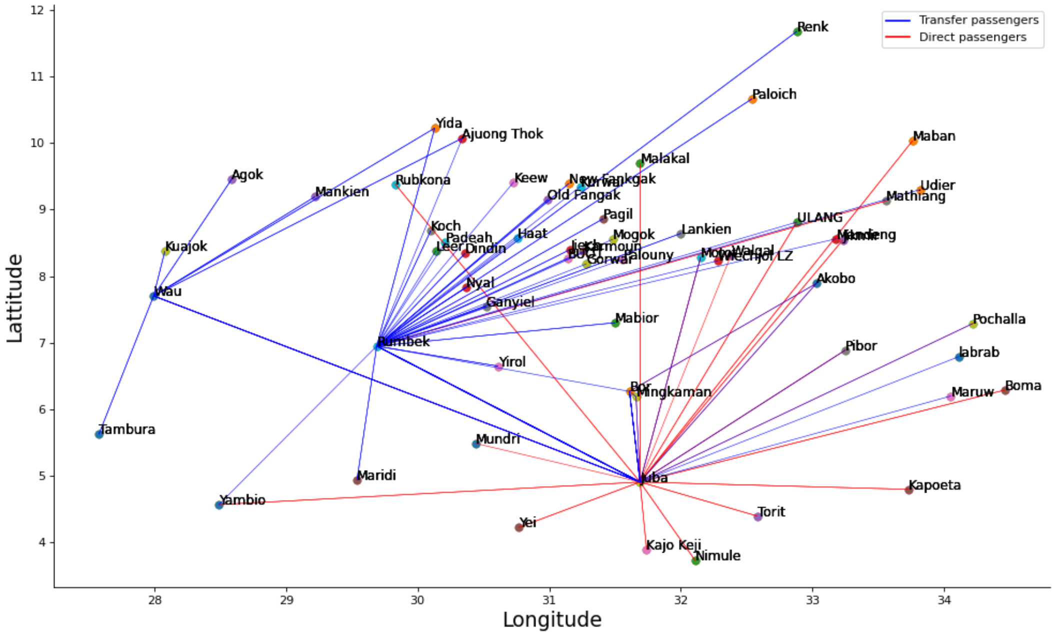


Figure 2: Passenger flows for the South Sudan UNHAS weekly demand (30/09/2019-04/10/2019)

Table 3: Passenger flows results for the South Sudan UNHAS weekly demand (30/09/2019 - 04/10/2019)

	Amount of passengers
Total direct passengers	1,115
Total transfer passengers	495
Passengers transfering through Rumbek	267
Passengers transfering through Wau	183
Passengers transfering through Bor	30
Passengers transfering through Juba	15

Table 4 summarizes the main fleet and routing results from the MCNF model. While "block hours" refer to the actual flight time of each aircraft on a weekly basis, the "operational hours" include flight times and the turn-around-times of 30 minutes per stop, assumed fixed in the model to realistically model humanitarian operations. In order to replicate current humanitarian operations, daily aircraft operational time is limited to 8.5 hours, and a week is considered to be 5 days (42.5 hours), leaving the week-ends for special flights such as emergency and special flights. According to the MCNF model, the entire weekly demand of Table 19 can be transported using six air assets and a total routing cost of 325,810 US$. The passenger flow data is extracted from the outputs and used to recreate a new demand matrix, presented in Table 20. This is done splitting the original "non-direct" requests into two, one from the origin to the transfer hub and the other from the transfer hub to the passengers final destination.

Table 4: Results of the MCNF model for the South Sudan UNHAS weekly demand (30/09/2019-04/10/2019)

	DCH8-3	Cessna 208B	Cessna 208B	Cessna 208B	MI8-T	MI8-T
Number of aircraft	1	1	1	1	1	1
Aircraft base	Juba	Juba	Rumbek	Wau	Juba	Rumbek
Weekly block hours [h]	24.4	26.8	27.6	27.9	23.3	27.8
Weekly operational hours [h]	39.4	42.3	42.1	41.9	34.3	41.8
Weekly distance flown [km]	12,962	9,228	9,494	9,630	5,170	6,174
Weekly load factor [%]	76.39	61.43	58.66	53.23	38.12	34.78
Weekly routing costs [$]	80,735.6	33,520.3	34,487.5	31,031.9	66,556.2	79,479.2
Total weekly routing costs [$]			325,810			
Total monthly lease costs [$]			719,717			

4.2 Multi-commodity network flow combined with the HFSMVRP

Using the modified weekly demand matrix containing the new transfer passenger data, the requests are split up into separate sub-problems as can be seen in Table 5. Each sub-problem is made up of the requests associated to the different aircraft selected by the MCNF model. The HFSMVRP is run for each sub-problem with all the aircraft available at the corresponding hub.

Table 5: Sub-problem division for combined MCNF and FSMVRP problem

Sub-problems	Requests	Airports	Passengers
Sub-problem 1: MI8 - Juba	13	9	165
Sub-problem 2: MI8 - Rumbek	18	14	154
Sub-problem 3: Cessna 208B - Juba	22	15	249
Sub-problem 4: Cessna 208B - Rumbek	15	11	154
Sub-problem 5: Cessna 208B - Wau	17	11	183
Sub-problem 6: DCH8 106 - Juba	14	8	1,140
Total problem	101	61	2,045

Each sub-problem, highlighted in red in Figure 3 is first solved individually. The solutions of the ones sharing the same hub are then combined and used as warm start in order to improve the quality of the solution. Finally, the solutions are combined and the entire problem is solved. The routing costs and objective function values can be found in Table 6 along with the corresponding improvements obtained after the warm starts. When using a warm-start on the entire problem, no improvements are found due to the instance being extremely large and the average performance of the processor used.

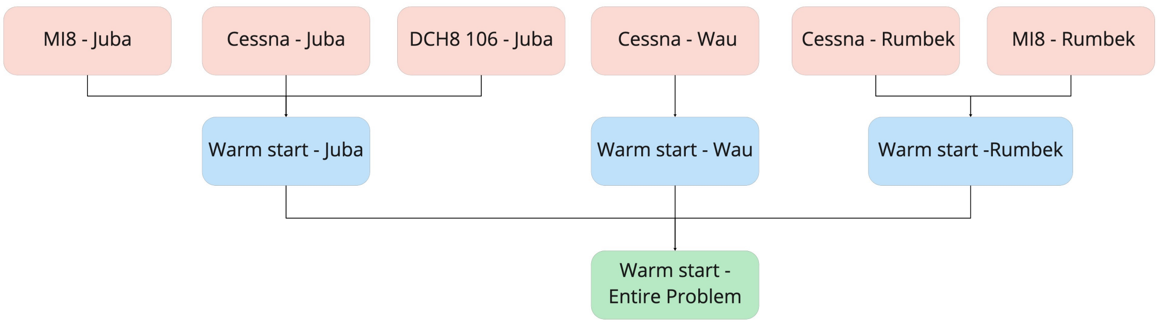


Figure 3: Sub-problem combinations using warm start

Table 6: Results for sub-problems in Figure 3 before and after using warm starts

	Objective function [-]	**Routing costs [$]**	**Decrease in objective function**	**Decrease in routing costs**
Sub-problem 1: MI8 - Juba	289,425	42,115.5	-	-
Sub-problem 2: MI8 - Rumbek	303,085	40,808.2	-	-
Sub-problem 3: Cessna 208B - Juba	91,126	28,545.5	-	-
Sub-problem 4: Cessna 208B - Rumbek	85,893	23,312.4	-	-
Sub-problem 5: Cessna 208B - Wau	83,693	21,112.0	-	-
Sub-problem 6: DCH8 3 - Juba	151,132	75,678.9	-	-
Sub-problem 7 : Warm start - Juba	518,423	139,428.6	2.5 %	5 %
Sub-problem 8 : Warm start - Rumbek	384,316	59,457.6	1.2 %	7.2 %
Entire problem : Warm start	986,432	219,998.2	0 %	0 %

The results for the modified weekly demand after having run the HFSMVRP can be found in Table 7. Figure 4 compares the routing costs before and after applying the HFSMVRP to the MCNF solution. The improvements show a 29.5% decrease of the total routing costs while keeping the same fleet. The main cost reduction can be attributed to the routing of the MI8-T helicopters, accounting for 70 % of all cost savings. This can be explained by the fact that helicopters have the highest operational costs compared to other air assets. Unlike the MCNF problem, the HFSMVRP allows the MI8s to combine small requests and different airfields together, providing a much more realistic modelling of their operations and resulting in much cheaper routing costs. The routing and combining of multiple different requests and airports also explains why load factors are higher for each aircraft, especially for the MI8-T's with a 30% to 45% increase. A full schedule of the resulting flight paths is shown in Appendix E, Table 30.

Table 7: Results of the combined MCNF and HFSMVRP models for the South Sudan UNHAS weekly demand (30/09/2019 - 04/10/2019)

	DCH8-3	**Cessna 208B**	**Cessna 208B**	**Cessna 208B**	**MI8-T**	**MI8-T**
Number of aircraft	1	1	1	1	1	1
Aircraft base	Juba	Juba	Rumbek	Wau	Juba	Rumbek
Weekly block hours [h]	22.9	24.3	19.3	19.0	11.7	12.4
Weekly operational hours [h]	36.9	42.3	34.8	31.5	18.7	23.4
Weekly distance flown [km]	12,150	8,357	6,637	6,552	2,594	2,746
Weekly load factor [%]	86.91	76.53	76.05	75.92	80.04	65.51
Weekly routing costs [$]	75,678.9	30,358.6	24,109.2	21,112.0	33,391.1	35,348.4
Total weekly routing costs [$]			219,998.2			
Total monthly lease costs [$]			719,717			

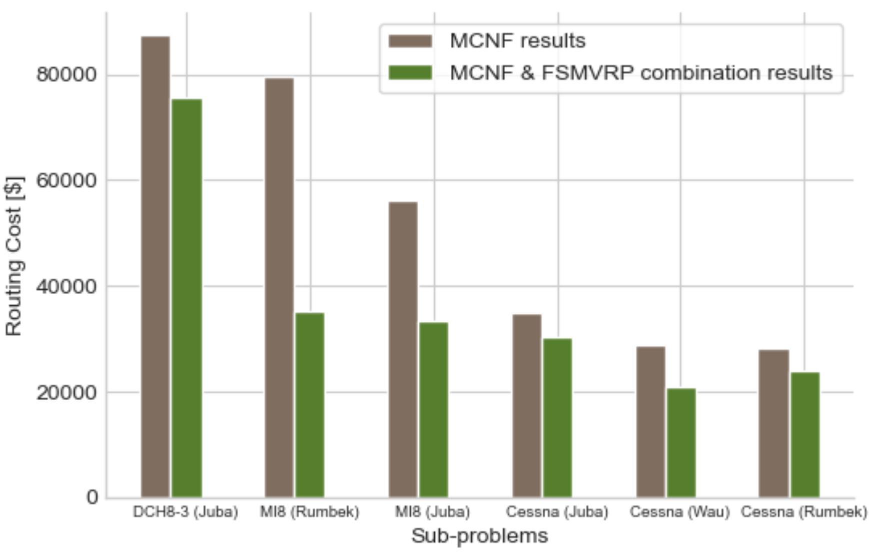


Figure 4: Routing cost comparison between the MCNF model and the MCNF & FSMVRP combination

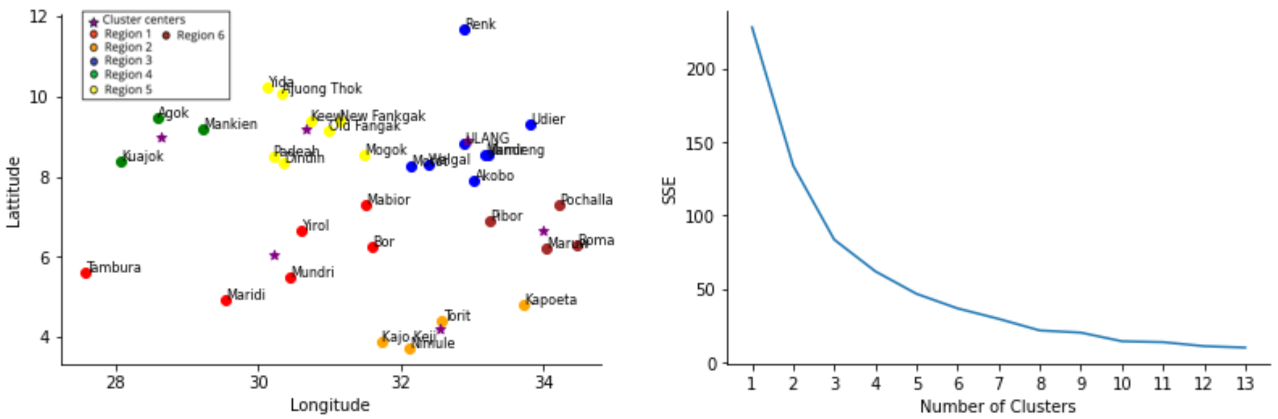
4.3 HFSMVRP

In order to understand the added value of using the MCNF model to simulate transfer passengers throughout the network, the HFSMVRP model was run by itself with the original demand matrix. As the entire problem is too large to solve in one instance, it was divided into three separate sub-problems as can be seen in Table 8. The logic remains almost the same as the one presented in the flow chart in Figure 1, with the exception that the MCNF model is not used as a pre-processing step for the original demand matrix, and the sub-problem division is based on runway lengths and not aircraft flows.

Table 8: Sub-problem division for the HFSMVRP for South Sudan UNHAS weekly demand (30/09/2019 - 04/10/2019

Sub-problems	Requests	Airports	Passengers	Available aircraft types
Sub-problem 1 : Helipad airports	25	19	191	- MI8 (Juba, Rumbek)
Sub-problem 2 : 1000m runways	61	41	714	- Cessna 208B (Juba, Rumbek, Wau) - Dornier (Juba, Rumbek) - LET(Juba) - MI8 (Rumbek, Juba)
Sub-problem 3 : $\geq$ 2000m runways	14	9	1140	- Fokker 50 (Juba), DCH8Q-400 (Juba) - DCH8-202 (Juba), DCH8-106 (Juba) - Cessna 208B (Juba, Rumbek, Wau) - Dornier (Juba, Rumbek) - LET (Juba) - MI8 (Rumbek, Juba)
Total problem	100	69	2045	- Fokker 50 (Juba), DCHQ-400 (Juba) - DCH8-2 (Juba), DCH8-3 (Juba) - Cessna 208B (Juba, Rumbek, Wau) - Dornier (Juba, Rumbek) - LET (Juba) - MI8 (Rumbek, Juba)

Sub-problem 2 contains too many requests to be solved at once by the FSMVRP model within a reasonable amount of time. The clustering algorithm is used to partition the airports into separate regions based on their distance to each other and therefore most probable routing areas. Figure 5(a) displays the result of the clustering algorithm applied to all 1000m runway airports, divided into 6 clusters, with the star symbols representing the cluster centroids. The associated Elbow graph is presented in 5(b). 6 clusters were chosen based on the trade-off between cluster sizes, which affect the solve time (amount of requests and airports per cluster), and amount of clusters, which affects how global each solution is compared to the entire sub-problem.



(a) Clustering algorithm applied to airports with 1000m runways (b) "Elbow" graph for the clustering of sub-problem 2 in Table 8

Figure 5: Clustering algorithm applied to sub-problem 2 in Table 8 for the South Sudan UNHAS mission

The results of the HFSMVRP model can be found in Table 9. The aircraft types selected are the same as the ones when passenger transfers were enabled in section 4.2, however this time the aircraft are all based out of the Juba hub as this is where the majority of requests originate from. A decrease in routing costs can be observed for the DCH8-3 due to the fact that no additional demand needs to be transshipped at other hubs. The opposite is seen for the MI8 helicopters and the Cessna-208Bs which now must travel longer distances to reach passenger destinations. A full schedule of the resulting flight paths is shown in Appendix E, Table 29.

Table 9: Results of the HFSMVRP model for the South Sudan UNHAS weekly demand (30/09/2019-04/10/2019)

	DCH8-3	**Cessna 208B**	**MI8T**
Number of aircraft	1	3	2
Aircraft base	Juba	Juba	Juba
Weekly block hours [h]	18.9	79.9	30.3
Weekly operational hours [h]	31.4	126.4	47.3
Weekly distance flown [km]	10,059	27,521	6,741
Weekly load factor [%]	81.73	84.18	59.65
Weekly routing costs [$]	62,654.0	99,966.8	86,775.5
Total weekly routing costs [$]		249,396.3	
Total monthy lease costs [$]		719,717	

Table 10 compares the routing and leasing costs for the three different models, each having been run with the same inputs. The combination of the MCNF and HFSMVRP models outputs the best results in terms of routing costs. Although all 3 models find the same leasing costs and select the same aircraft types, the base of the vehicles vary depending on whether passenger transshipment is enabled or not. The HFSMVRP model by itself must route all requests from their origin to destination without transferring them in different locations or transshipping them onto other aircraft. Since most requests originate or have as destination the Juba hub, all aircraft selected are stationed at that depot. The MCNF formulation, on the other hand, provides the option to transship passengers onto different vehicle at different hubs in the network. In this case, Cessna 208B's in Wau and Rumbek are selected as well as an MI8 in Rumbek.

Table 10: Comparison of the routing and leasing costs of the different models for the South Sudan UNHAS mission

Model	**Routing costs [$]**	**Leasing costs [$]**	**Total costs [$]**
Multi-commodity network flow	325,810.7	719,717	1,041,263.7
HFSMVRP	249,396.3	719,717	969,113.3
Multi-commodity network flow & HFSMVRP	219,998.2	719,717	939,715.2

5 Verification & Validation

5.1 Verification

The verification of the linear programming optimisation model can be found in Appendix C along with a more in depth explanation of the formulation.

5.1.1 Clustering algorithm sensitivity analysis

A sensitivity analysis was performed on the clustering algorithm in order to understand how dividing the problem into smaller instances based on intra-cluster distance affected the final solutions. The sensitivity analysis was performed on the Cessna 208B sub-problem of the HFSMVRP optimisation run presented in Table 8, as it is the largest in terms of requests and airports. For every iteration of the sensitivity analysis, the amount of clusters is increased, from 4 to 11. For each problem, all clusters are first run by themselves, the solutions are then saved and merged to be used as warm start. The entire sub-problem is finally run for three hours and the resulting final routing cost is saved. Figure 6 presents the evolution of routing costs as a function of the number of clusters before and after the warm-starts. The geographical division of the clusters can be found in Appendix D, Figures 25(a) to 25(h).

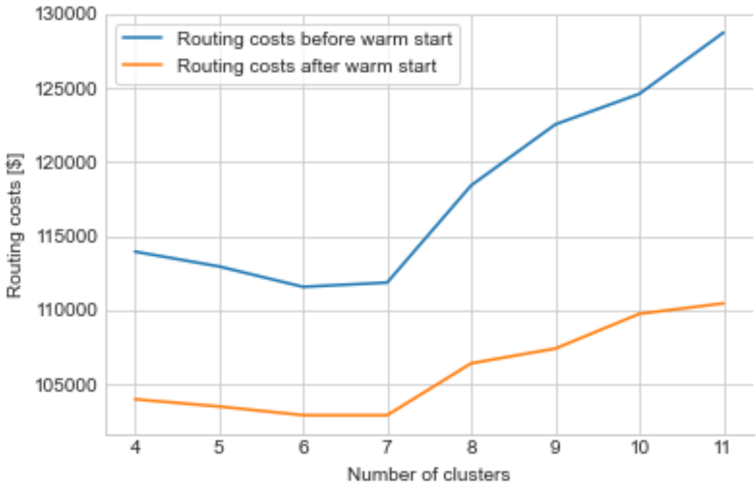


Figure 6: Effects of increasing number of clusters k on routing costs before and after warm starts

While dividing the airports of the sub-problem in 6 different clusters offers the best results, the improvements in routing costs between 4 and 7 clusters remain relatively insignificant, with a decrease in routing costs of less than 3%, as can be seen in Figure 6. As the number of clusters increases above 7, a deterioration of the solution can be observed. This is explained by the fact that certain airports such as *Renk*, *Tambura* and *Kapoeta* are assigned to their own cluster without being combined with other airports as can be seen in Appendix D, Figures 25(d) to 25(h). Figure 6 also illustrates the effects of using a warm-start with the separate cluster solutions. A reduction in total routing costs can be observed as well as a decrease in cost difference between each run. It is expected that if the warm-starts are allowed to run for an extended amount of time, the same solutions would be found in each case. Because each warm-start is only run for three hours, the solver is not able to reconstruct the best routes and re-combine the different clusters in the available time.

5.2 Warm start procedure verification

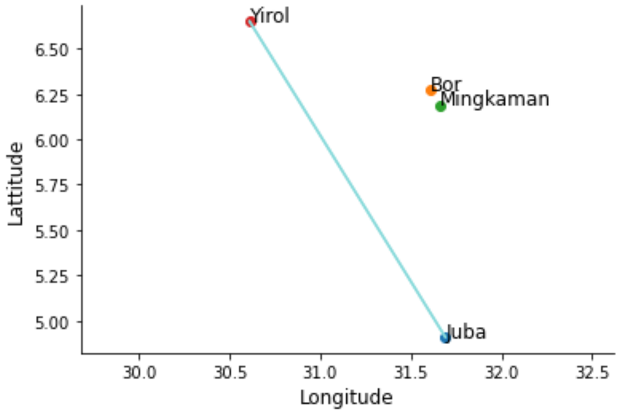
The warm start procedure was verified by using a simple real-world case study based on the same UNHAS South Sudan mission. The objective is to first run two separate problems, then combine them into one and use a warm start in order to observe whether the procedure is able to modify and improve the aircraft selection and routings. The verification case is presented in Table 11, where the airports Bor and Yirol are accessible by both a Cessna 208B and an MI8-T, whereas Mingkaman is a helicopter only airport. When run separately, it is expected that Sub-problem A will use the Cessna 208B to pick-up and deliver request $r0$, whereas Sub-problem B will use an MI8 to deliver request, $r1$. When both solutions are combined and used as a warm start for the entire problem, it is expected that the best solution is to use the helicopter to service both requests. This is due to the airports' proximity and the cost savings of using only one air asset instead of two. The results of the verification are presented in Table 12 and the flight paths in Figures 7(a) to 7(c).

Table 11: Warm start verification scenario

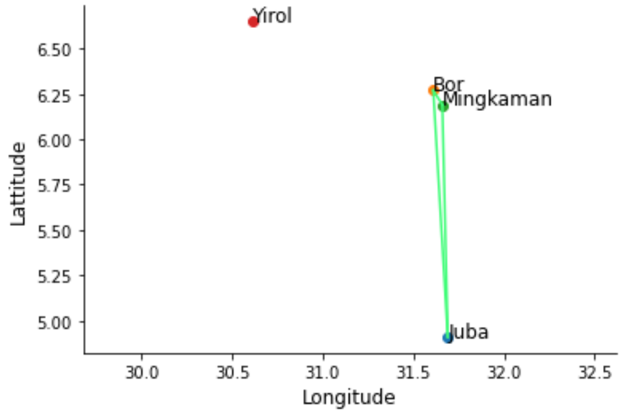
Instance	**Request**	**From**	**To**	**Pax**	**Available aircraft**
Entire problem	r0 r1	Juba Bor	Yirol Migkaman	1 1	Cessna 208B (Juba) MI8-T (Juba)
Sub-problem A	r0	Juba	Yirol	1	Cessna 208B (Juba) MI8-T (Juba)
Sub-problem B	r1	Bor	Mingkaman	1	Cessna 208B (Juba) MI8-T(Juba)

Table 12: Results for the warm start verification case

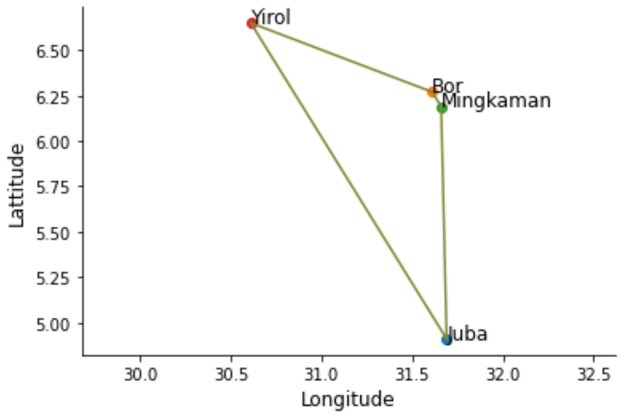
Instance	Aircraft	Leg	From	To	Pick-up	Delivery	Pax	Leg dist. [km]	Leg cost [$]
Sub-problem A	Cessna 208B	1	Juba	Yirol	r0(1)	r0(1)	1	227.4	826
		2	Yirol	Juba	-	-	0	227.4	826
Objective function cost					64233				
Sub-problem B	MI8-T	1	Jub	Bor	-	-	0	151.48	1950
		2	Bor	Mingkaman	r1(1)	r1(1)	1	10.47	135
		3	Mingkaman	Juba	-	-	0	142.37	1833
Objective function cost					232177				
Entire problem (post warm start)	MI8-T	1	Jub	Yirol	r0(1)	r0(1)	1	227.4	2927
		2	Yirol	Bor	-	-	0	118.29	1523
		3	Bor	Migkaman	r1(1)	r1(1)	1	10.47	135
		4	Mingkaman	Juba	-	-	0	142.37	1833
Objective function cost					234677				



(a) Fight path of Cessna 208B for Sub-problem A



(b) Fight path of MI8-T for Sub-problem B



(c) Fight path of MI8-T for the entire case study post- warm start

Figure 7: Warm start verification path results

5.3 Validation

5.3.1 Expert validation of the weekly schedules for the South Sudan demand (30/09/2019-04/10/2019)

The weekly schedules in Appendix E, Table 29 and Table 30, were validated by a UNHAS expert flight planner and were reported as feasible. Figure 8 displays the main inputs, outputs and decisions modeled during the optimisation framework. While all the schedules and flight paths are reported correct, the model is unable to simulate evry considerations that flight planners and contracting officers must take into account. The main characteristics that were left out are highlighted in brown in the breakdown tree. The reason for them not being modelled are either because they were not acknowledged at the start of the modeling exercise (1,5), because they were considered outside of the scope (2,3,8,9) or because they were assumed not to impact the objective function significantly (4,6,7).

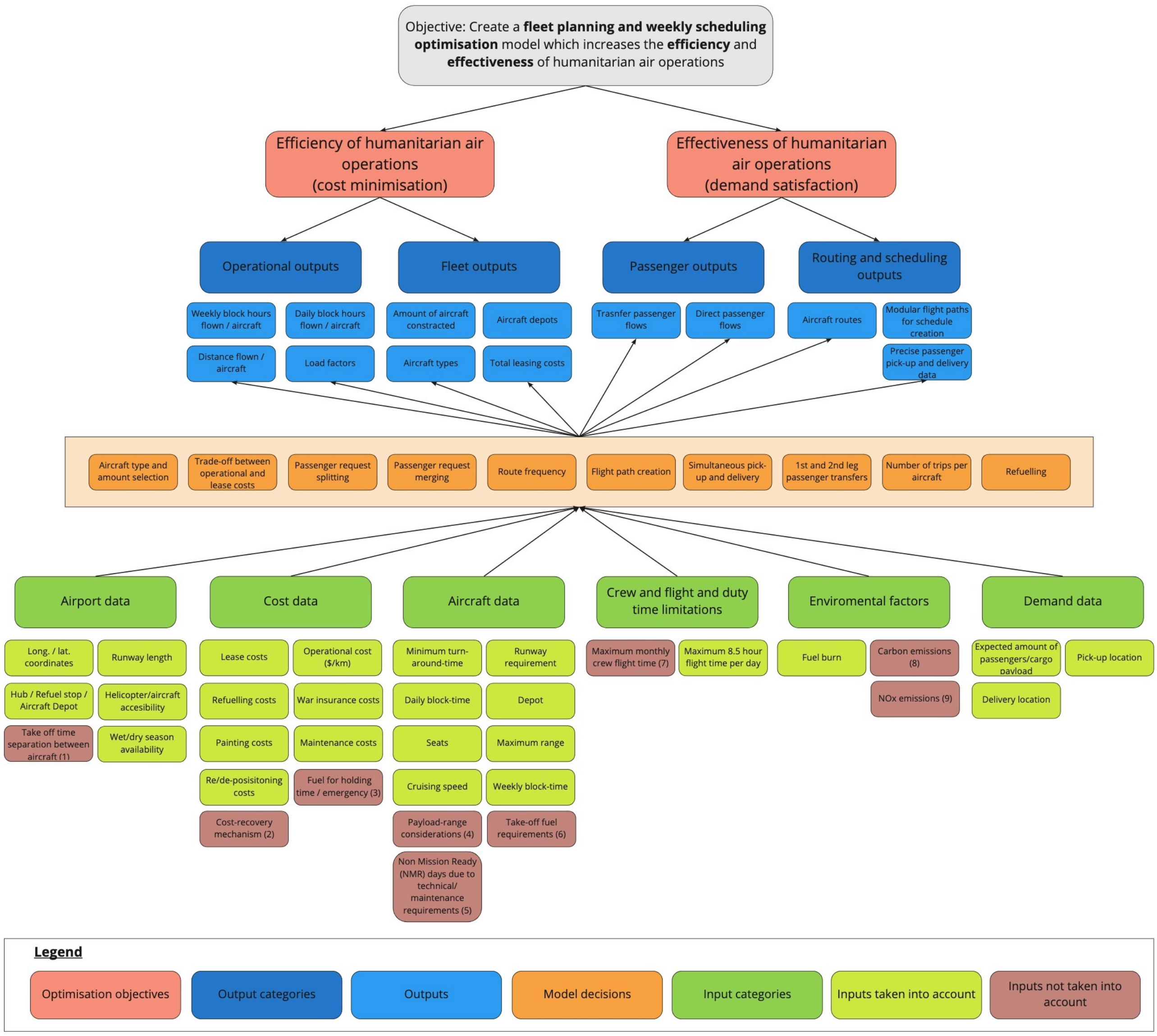


Figure 8: Breakdown tree of the inputs, outputs and main decisions contributing to the efficiency and effectiveness of humanitarian air operations

Furthermore, some comments were made regarding choices made by the models which, in reality, either never or rarely occur in the real-world mission. The following points take into account the flight planner's comments and provide an explanation based on the model's behaviour.

1. Out of the 18 helicopter flight paths, 3 of them choose to refuel in the Malakal hub. This does not happen in real-world operations due to the fact that the cost of fuel in Malakal is 8 times more expensive than in other locations. Although this price difference is incorporated in the model, the MI8s choose to refuel in Malakal because it is their only feasible option to reach certain destinations due to their maximum

range constraint of 750 kilometers. An example of this is the Monday MI8 flight path in Table 29 where the destination Mathiang is serviced by a refuel stop in Malakal. In practice, flight planners would refuel the helicopter in Bor twice, once on the way to Mathiang and once on the way back, as follows: $Juba - Bor(refuel) - Mathiang - Bor(refuel) - Juba$. The model isn't able to reproduce this as the back and forth between Bor and Mathiang is 766 kilometers.

2. Helicopters choose to service certain destinations which are accessible by other aircraft such as Cessna 208Bs and other cheaper turbo-prop aircraft. This can be observed in the MI8-Juba flight paths on Monday and Tuesday in Table 30, where the Torit and Pibor airports both have a runway of 1000m but are serviced by an MI8. While routing a helicopter through these destinations is very expensive and would most likely not happen in practice, this choice is a good example of the trade-off the model makes between routing costs, aircraft utilisation and fleet sizing. It can be observed in Table 7 that the operational hours used by the Cessna 208B in Juba is of 42.25 hours, very close to the maximum amount of weekly operational time of 42.5 hours. The model therefore chooses to use the MI8s with spare block hours to route passengers to those destinations instead of adding a new aircraft to the fleet.

3. Out of 102 flight paths, 3% of them have more than 6 stops. Having 7 stops in one route rarely happens in practice and should be avoided. However when looking at the schedules into more detail, the routes with this many stops are between destinations which are very close to each other, sometimes within 10 minutes flight time such as Jiech - Buot (4 min), Ajuong Thok - Yida (5 min), Padeah - Leer (4 min), Ulang - Mandeng (8 min), etc. In the real world mission, these destinations are combined into one, therefore reducing the amount of stops within a route. The same simplifications could be incorporated in the model without impacting the routing costs by implementing a hard constraint on the number of stops per route or combining certain destinations as one.

4. The model does not take into account a maximum amount of rotations per day. On certain days, an aircraft will return to its hub 3 times to pick up and drop off passengers. According to flight planners, two rotations a day is more realistic. However because the flight paths are modular and interchangeable, both of these issues can easily be remedied by switching flight paths around during the week while respecting the daily operational time of 8.5 hours.

5. The model does not incorporate a maximum monthly flight time per aircraft. In practice, this is limited to 100 block hours per month. However according to the operational results presented in Tables 7 and 9, all aircraft respect this limitation. This is an aspect that can easily be incorporated in the model by setting a maximum constraint on the monthly block time.

6. The model does not take into account crew duty times. These are governed by EASA, ICAO or the aircraft's country of registration regulatory body. Incorporating crew duty times could affect the aircraft selection, routing and scheduling over the weekly and monthly planning horizon as UNHAS and other operators must make time for crew rest periods and rotations. A solution could also be to hire more crew members, however this would also come at an additional cost.

7. While the model is able to route all passengers with a fleet of six aircraft, this is an idealised scenario. It does not take into account the fact that certain assets may need to go through maintenance or that on certain days unexpected emergency situations such as medical / security evacuations may require more aircraft to be available. It is therefore noted that additional aircraft should be contracted to take these considerations into account. While adding reserve aircraft to the model would increase the leasing costs, it may decrease the operational costs because more air assets would be available to transport passengers, increasing the routing possibilities.

5.3.2 Comparison with previous models

One of the advantages of the HFSMVRP model is that it can adapt to any time frame, whether it be a day, a week or a month. Five different variations of the model were run for the daily demands of the South Sudan UNHAS mission from the 30/09/2019 to the 04/10/2019 presented in Table 13. The results can be compared with the results of Humanitarian Flight Optimisation Model (HFOM) created by Mekking (2020), and the daily schedules created by expert flight planners. The exact same aircraft types, demand and airport inputs are used in order to ensure consistency when comparing results between each other. When run with the "Fixed fleet" variation, only the fleet used by UNHAS in South Sudan is allowed to be selected, the model is not allowed to add additional aircraft to the fleet. The "Variable fleet" variation does not have a limitation on the amount of aircraft of each type is allowed. Finally, the "Fixed fleet (no leasing)" variation is the same as the "Fixed fleet" variation but the cost associated to aircraft leasing are not taken into account.

Table 13: Overview of the test cases for the daily routing optimisation

		HFSMVRP	MCNF & HFSMVRP combination	Fixed fleet	Variable fleet	Fixed fleet (no leasing)
Test cases	Test case 1	x		x		
	Test case 2	x			x	
	Test case 3		x	x		
	Test case 4		x		x	
	Test case 5		x			x

While the HFSMVRP model by itself does not investigate transshipment of passengers and therefore passenger connections and corresponding time windows, this consideration must be taken into account when combining it with the multi-commodity network flow for the daily cases. In order to ensure all passengers can connect within the same day, a trunk route is implemented when needed between the hubs Juba, Rumbek and Wau, and taken into account in the routing and objective function costs. The DashQ-400 is the aircraft with the highest capacity (71 seats) in the fleet. It is responsible for flying connecting passengers in the morning before all connecting flights depart, as well as in the evening after all connecting flights have returned. This decision is validated by UNHAS expert flight planners and can also be observed in published UNHAS schedules as seen in Appendix G, Figure 27. The results for the daily demands are presented in section F.1, Tables 31 to 35. All model runs are cut off one hour after the start of the optimisation in order to use the same run time as Mekking (2020).

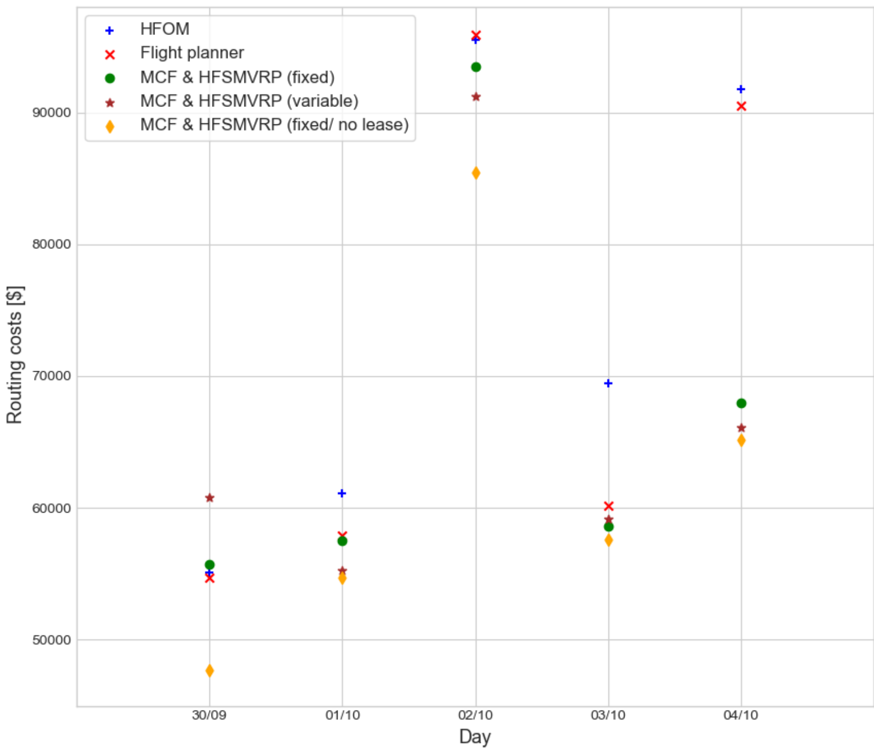


Figure 9: Routing cost comparison between different days and model variants

Figure 9 presents the daily routing cost results from running the MCNF & HFSMVRP model variations. They outperform both the human flight planner and the HFOM model four days out of five. The combined models are able to improve the routing costs between 2% to 26%, while in some cases, also reducing the fleet size by up to three air assets. The HFOM uses time windows and does not incorporate fleet planning elements in its formulation. It can be assumed that by not taking into account time windows and splitting the decision making process in two steps (transshipment and routing), the MCNF & HFSMVRP combination is able to find better results by reducing the computational complexity and search space of the problem. The results of test cases 1 and 2 are presented in section F.1, Tables 31 to 35. These two cases are run only with the HFSMVRP model and do not consider passenger transshipments. They display higher routing costs than all other models. It can be concluded that transferring passengers in the main hubs of the network is an essential part of the decision process and allows a much greater reduction in routing costs.

Table 14: Operational results for Test case 5 compared to the HFOM and flight planner

Days	Model	Routing costs [$]	Load factor [%]	Number of aircraft	Trunk route
30/Sep	HFOM	55,085.3	47.34	10	-
	Flight planner	54,700.8	53.63	-	-
	Test case 5	47,222.6	57.23	8	No
01/Oct	HFOM	61,077.2	59.54	9	-
	Flight planner	57,900.6	57.75	-	-
	Test case 5	54,732.6	66.41	9	DashQ-400
02/Oct	HFOM	95,531.1	58.64	12	-
	Flight planner	95,600.7	61.10	-	-
	MTest case 5	85,427.7	62.18	9	DashQ-400
03/Oct	HFOM	69,466.5	57.40	9	-
	Flight planner	60,200.9	61.31	-	-
	Test case 5	57,613.2	65.89	7	DashQ-400
04/Oct	HFOM	91,766.8	65.78	10	-
	Flight planner	90,500.3	60.00	-	-
	Test case 5	65,189.7	70.46	9	Dash8-2

Finally, Table 14 presents the results for Test case 5 (MCNF and HFSMVRP combination with fixed fleet and no leasing costs taken into account). It can be seen that the load factors found by the optimisation framework throughout the week are all higher than for the HFOM and the flight planner. This increase can be attributed to the fact that passenger requests are merged at two different points in the optimisation framework: firstly during the merging of daily demands into weekly demands, and secondly during the MCNF optimisation when passenger flows are consolidated on certain arcs. A trunk route needed to be implemented four out of the five days in order to ensure passenger could make their transfers within the same day. The corresponding precise daily schedules can be found in section F.2, Table 36 to 40.

5.4 Discussion

This study demonstrates that mathematical optimisation models are able to realistically simulate humanitarian air operations at various levels of detail, from a high-level network perspective to the routing of individual aircraft and passenger requests. The contrast in the different levels of detail is precisely what allows the models to make fleet planning and weekly scheduling decisions tailored to the humanitarian operating environment. As previously mentioned, the humanitarian aviation planning cycle spans from a day to a few months before the deployment of the fleet/air assets. The high inter-dependency between the decisions defining the different planning stages implies that a large amount of choices and variables must be taken into account. The methodology proposed in this study was able to solve a large instance by dividing the decision making process in two steps. Table 15 presents a comparison between the different models applied in the research and the routing created by a UNHAS flight planner. It can be seen that by using the proposed optimisation framework, weekly routing costs can be reduced by up to 40% compared to flight planners, who only consider daily routing and scheduling of humanitarian passengers and use a predetermined fleet by the contracting unit. The decision support tool is also able to solve the problem by reducing the fleet size by more than half, from 14 air assets to only 6. This highlights the fact that while contracting officers and flight planners tackle the fleet selection and routing problem separately, the MCNF and FSMVRP models combine both, resulting in better solutions for both planning stages in terms of lower routing cost, higher load factors and smaller fleet sizes. This is partly attributed to the fact that the optimisation model is able to take into account many decisions and variables at the same time, but also to the paradigm shift introduced in the study which aims to combine daily requests into a weekly demand matrix and create a detailed weekly schedule and routing. While the best results are obtained by the combination of the MCNF and the HFSMVRP models, these can be used separately in a consistent fashion as decision support tools depending on the information available to the operator, the size of the problem or the time horizon under consideration. A cost comparison between the models presented in this study is made in Table 15. Although the results of the research are promising, certain limitations related to each formulation must be acknowledged. They are explored in the rest of the discussion.

Table 15: Comparison of the routing costs of different models for the weekly demand of the South Sudan UNHAS mission (30/09/2019 - 04/10/2019)

Model	Weekly routing costs [$]	Avg. load factor [%]	Air assets
HFOM (combined daily results, (Mekking, 2020))	372,925	57.22	14
Expert UNHAS flight planner (combined daily results)	359,200	59.77	14
MCNF (weekly demand)	325,810	60.56	6
HFSMVRP (weekly demand)	249,396	75.18	6
MCNF & HFSMVRP (weekly demand)	219,998	76.82	6

5.4.1 MCNF

The MCNF model was able to accurately replicate the South Sudan UNHAS hub-and-spoke network based on humanitarian O-D request pairs and decide how to divide the weekly demand into direct and transfer passengers flows. The fleet chosen by the model based on a trade-off between aircraft routing and leasing costs was the same as the one chosen by the HFSMVRP. This implies that using a network flow model is consistent enough to use as a stand alone decision support tool for aircraft contracting processes. However, as routing between spokes is not allowed, passengers wishing to travel between secondary destinations are always transshipped through a hub even though it might be cheaper to fly them directly between spokes. This not only affects the network structure, but also the fleeting and routing decisions. Fortunately, as a majority of the humanitarian requests involve a main hubs, this limitation does not significantly affect the final solutions.

The fact that routing is not taken into account in the MCNF model also influences the block hours flown by vehicles and their operational performance. The choice of aircraft type and corresponding amount is therefore also affected. As requests are often smaller than the capacity of a vehicle, the aforementioned metrics are mostly overestimated. The MCNF model is however still able to predict the same fleet as the other models while using a simplified and faster formulation which leads to the conclusion that it can be used realistically to size an initial fleet.

Time windows are not taken into account with respect to passenger transshipment. This implies certain passengers may not be able to transfer on the same day between their first and second flight leg and have to layover for a few days or over the week-end. Assuming weekly schedules are periodic and repeat over a month, this does not pose an issue with respect to the integrity and feasibility of the solution, as enough capacity is provided for each request. It does however make the model unfeasible for daily routing and scheduling. A remedy to this problem is to add an aircraft to the fleet and implement a trunk route between the main transfer hubs, with a rotation in the morning and in the evening. Adding this additional aircraft and route still results in cheaper routing than the HFOM and the flight planners as was shown in Table 14.

5.4.2 HFSMVRP

With respect to the HFSMVRP model, limitations are also observed. Passenger transshipments are not taken into account, which does not accurately represent the real world situation. Transferring passengers at the main hubs is a common strategy used to transport requests with larger and cheaper aircraft as close to their final destination as possible, and finish off their route with smaller but more appropriate aircraft. This also has an impact on the positioning of the aircraft. As most humanitarian requests originate or terminate in the international hub of a certain country, the fleet chosen when not considering passenger transfers is almost always based at the corresponding airport.

Choosing not to model passenger transfers, however, eliminates the need for time windows and trunk routes. All passengers who leave their their origin will reach their final destination within the same aircraft route. For the South Sudan case, the HFSMVRP chooses the same aircraft types as the MCNF, however they are all positioned in Juba. The results for weekly routing are still 23.4% cheaper than when using the MCNF by itself but 12% more expensive than when combining both models and considering transfer passengers. This confirms the expectation that modelling passenger transshipments and network structure can significantly decrease routing costs while representing real-world operations. Finally, an important consideration to take into account is the fact that the HFSMVRP formulation is NP hard and the computational complexity increases exponentially as requests and aircraft are added. The solvers used cannot find an optimal solution within a reasonable amount of time as the problem size increases, but do find feasible, sub-optimal solutions with varying MIP gaps depending on the amount of variables considered.

5.4.3 MCNF & HFSMVRP

Using the MCNF & HFSMVRP problem sequentially produced the best results in terms of weekly routing costs when using the same fleet as when both models were run separately. The combination of both models allowed for decreased routing costs of 32% and 11% with respect to the MCNF and HFSMVRP. These cost savings can be attributed to the fact that both models are able to complement each other by taking up different parts of the decision process and avoiding the exponential increase in computational complexity. It is however important to note that once the modified passenger transfer matrix has been created, the HFSMVRP is unable to modify it and cancel transfers if it would prove beneficial to the final solution. The sequential nature of the methodology implies that certain parts of the solution space are deleted and a truly optimal solution can never be found. However, this does not take away from the fact that better solutions are still found when running the combination of the MCNF and HFSMVRP model for daily demands in section 5.3.2 compared to the previous humanitarian oriented VRP models of Mekking (2020) and Niemansburg (2019). Using the same exact case study, inputs and run time, a decrease in routing costs between 2% and 26% can be observed as well as a decrease in the number of aircraft used. It is assumed that by splitting up the decision making process in two steps and therefore reducing the amount of variables needed, the computational complexity is reduced and the models are able to find better solutions faster. Another reason lies in the fact that less sub-problems are used to divide the search space and therefore larger instances can be solved at once, leading to a more global solution.

A novel way of dividing a geographical network into sub-regions with the aim of creating "most-probable" routing areas was analysed in the study. This was implemented due to the fact the HFSMVRP formulation struggles to find feasible solutions to an instance with more than 25 requests and 8 aircraft types within a reasonable amount of time. The clustering algorithm in combination with the Elbow graph heuristic allowed for a more structured way to divide the geographical area into "most probable" routing regions instead of only using flight planner experience. This implies that this technique can be used when setting up new missions where no previous expertise is available. The algorithm and the elbow graph heuristic is however, slightly sensitive to outliers. When comparing the Elbow graphs of the 1000m runway airports in 5(b) and the one of the helipads in 26(b), the latter shows a much clearer inflection point for the optimal "point of diminishing return". This must be kept in mind when selecting the optimal amount of clusters. Finally, it must be noted that although grouping destinations in clusters based on geographical distance is a good approximation on how flight planners usually combine pick-up and delivery operations, en-route airports should also be taken into account.

Passenger requests are merged at two separate moments in the optimisation framework. Firstly in the preprocessing step where daily passenger requests are merged into weekly requests, and secondly after running the MCNF model when certain requests are merged into transfer requests between hubs. While this effect has clear benefits on the routing costs and fleet sizing by consolidating passenger flows on certain arcs, it must be noted that this modelling perspective strays from what happens in the real-world. It implies that certain destinations are visited less often, in some cases once a week, and that therefore humanitarian passengers will need to stay a longer period of time in that location. Secondly, it also implies that the models try to maximize the load factors of certain flight legs, sometimes to 100% full capacity. Based on flight planner input, this is rarely the case and aircraft flight legs are usually associated with load factors around 63%. Although not taken into account in this study as the aim was to create a model applicable to general humanitarian and short-term fleet planning applications, these two limitations can easily be remedied by slight modifications of the HFSMVRP formulation. Load factors can be taken into account by multiplying the right hand side of the loading constraints in Equation 1g and Equation 1h by a factor between 0 and 1. Furthermore, increasing the right hand side of the split request constraint in Equation 1f from 1 to 2 would force the vehicles to visit each destination at least twice a week.

The models approximate real-life operations and do not incorporate all the details that flight planners and contracting officers must consider when making decisions. The aircraft and their operations must respect UNHAS and WFP guidelines as well as regulations from the country of registration. Considerations such as crew duty times and rotations are not incorporated. Requirements regarding additional fuel for holding time or flight to alternate airport are not integrated in the models. In South Sudan, UNHAS operates a partial cost recovery scheme taking the form of ticket sales to humanitarian passengers at a fixed price (Cour des Comptes France, 2019). This aspect is not taken into account in the cost section of the model, however it could influence the outcome of the fleet selection and weekly routing costs by favoring the use of aircraft with a higher amount of seats. Aircraft range is a fixed parameter and does not vary based on the amount of passengers taken on board. Payload-range diagrams should be used to realistically model the effects of aircraft load factors on range.

6 Conclusions

6.1 Research

The study has revealed that a MCNF model and an FSMVRP model can be combined sequentially to be used as a tactical decision support tool for aircraft fleet planning and weekly flight scheduling. The MCNF model is able to accurately re-create a humanitarian Hub and Spoke network while optimising passenger transshipment flows and sizing an initial fleet. The HFSMVRP uses the outputs of the MCNF to further increase the accuracy of aircraft routing and passenger operations, while finalising the fleeting choices. This produced the best results in terms of weekly routing costs while selecting the same fleet when both models were run separately. The combination of both models allowed decreased routing costs of 32% and 11% compared to the MCNF and HFSMVRP respectively. It achieves up to 40% cost savings compared to the cumulative costs of the daily flight schedules created by expert planners for the same week, and uses only 6 aircraft instead of 14. For the South Sudan UNHAS mission, the models suggests that the best fleet to route all weekly requests is composed of one Dash8-3 (Juba), three Cessna 208Bs (Juba, Rumbek, Wau) and two MI8s (Juba, Rumbek). The flight paths resulting from the optimisation are modular/interchangeable and can be used to create precise weekly schedules, providing enough capacity to route all passengers from their desired origin to destination.

This research has proposed a novel methodology which can be used as a decision support tool for humanitarian air operators. Not only does it provide the humanitarian community with more information and power to plan trips in advance, but it also increases the robustness of the humanitarian fleet contracting process. The results of such a model can be used to inform humanitarian air operators on the air assets most adapted to a certain mission and give them an estimate of the amount of MGH needed per aircraft, as well as corresponding operational data. The MGH are an essential aspect of humanitarian aircraft contracting as it affects the future utilisation and costs of the asset. In turn this increases the accountability in their planning cycle, an important consideration when reporting to users, donors and other stakeholders. Although the research focused on humanitarian passengers, the model can also be applied to a variety of short-term tactical missions, including cargo operations. The research also showed that a clustering algorithm can successfully be used to segregate a humanitarian flight network into "most probable" routing areas. This allows for a more reliable and automated way to divide a large humanitarian routing optimisation problem into smaller solvable instances. This would be all the more useful when starting new missions where flight planners and contracting officers do not yet have information on previous fleet, routing and scheduling operations.

6.2 Recommendations

The following recommendations are made in order to encourage future work to further improve the methodology proposed in the research and the general humanitarian planning cycle.

- **Tabu search algorithm**: Heuristics or meta-heuristics were not implemented in the research. Using a Tabu Search heuristic could be beneficial in order to explore the solution space more effectively than the current exact formulation of the HFSMVRP proposed.
- **Fleet Assignment Model**: In order to increase the accuracy and feasibility of the MCNF and HFSMVRP models, using a Fleet Assignment Model once the preliminary schedules are created would allow the optimisation of passenger connections and reduce layovers while re-evaluating fleeting choices.
- **Multi-period fleet planning**: Fleet planning on a larger time horizon and subject to demand uncertainty has not been covered in this report. It would be of interest to analyse the robustness of humanitarian fleets over multiple years / seasons, while taking into account demand evaluations.
- **Hub Location Problem**: While the current model is able to choose between aircraft at different airports, the hubs are fixed and increasing the number of depots will increase the computational complexity exponentially. Combining humanitarian fleet planning and a Hub Location Problem would be beneficial to decision makers when planning new humanitarian missions.
- **Clustering algorithm**: The clustering algorithm only takes into account intra-cluster distance between airports. It would be interesting to analyse the effects of including other metrics such as amount of passengers travelling to and from a cluster. It would also be interesting to take into account en-route airports between the clusters and aircraft hubs.

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Appendices

Table 16 presents the airport data used in the South Sudan mission. The runway lengths are approximations and rounded values are used to separate airports accessible by different aircraft types. Runways of 100m are only accessible by helicopters while runways of 3000m are accessible by all aircraft types. Table 17 presents the aircraft types available and Table 19 the weekly South Sudan demand for the week of the 30/09/2019.

A Appendix 1 - South Sudan UNHAS mission input data

Table 16: South Sudan airport data (* indicate runways only accessible by helicopter during rainy season)

Airport	Lat [°]	Long [°]	Runway [m]	Airport	Lat [°]	Long [°]	Runway [m]
Juba	4,91	31,69	3000	Kurwai	9,35	31,24	100
Rumbek	6,95	29,69	3000	labrab	6,79	34,11	100
Wau	7,70	28,00	3000	Maban	10,03	33,76	2000
Bor	6,27	31,61	1000	Mabior	7,30	31,50	1000*
Malakal	9,70	31,69	3000	Mandeng	8,56	33,18	1000*
Ajuong Thok	10,07	30,34	1000	Mankien	9,19	29,22	1000
Torit	4,40	32,58	1000	Maridi	4,94	29,54	1000
Kapoeta	4,80	33,73	1000	Maruw	6,20	34,05	1000
Keew	9,41	30,73	1000*	Mathiang	9,13	33,56	100
Lankien	8,64	32,00	100	Mayendit	8,28	30,10	1000*
Mingkaman	6,19	31,66	100	Mogok	8,55	31,48	1000*
Padeah	8,50	30,21	1000	Motot	8,28	32,15	1000*
Yambio	4,57	28,49	2000	Mundri	5,49	30,44	1000
Yida	10,22	30,13	1000	New Fankgak	9,39	31,15	1000*
Leer	8,38	30,14	100	Nimule	3,73	32,11	1000
Dindin	8,34	30,36	1000*	Nyal	7,83	30,37	100
Agok	9,46	28,59	1000	Old Fangak	9,15	30,99	1000*
Alek	8,81	28,19	1000	Pagil	8,86	31,41	100
Aweil	8,88	27,45	3000	Palouny	8,25	31,56	100
Boma	6,29	34,46	1000	Pibor	6,89	33,25	1000*
Buot	8,27	31,14	100	Pieri	8,09	32,15	1000*
Ganyiel	7,54	30,52	100	Pochalla	7,29	34,22	1000*
Gorwai	8,18	31,28	100	Raja	8,60	25,80	1000
Haat	8,57	30,76	100	Rubkona	9,38	29,83	2000
Akobo	7,89	33,03	1000*	Tambura	5,63	27,58	1000
Paloich	10,66	32,54	2000	Touch Riak	8,22	30,31	100
Renk	11,67	32,88	1000	Udier	9,30	33,82	1000*
Jiech	8,40	31,16	100	Ulang	8,81	32,88	1000*
Jikmir	8,54	33,23	1000*	Wai	8,39	31,35	100
Karam	8,32	31,88	1000	Wiechjol	8,23	32,28	100
Karmoun	8,37	31,26	100	Wanding	8,19	33,11	1000
Kajo Keji	3,90	31,74	1000	Walgal	8,31	32,38	1000*
Koch	8,68	30,10	100	Yei	4,23	30,77	2000
Kuajok	8,37	28,08	1000	Yirol	6,65	30,61	1000
Kuron	5,70	35,52	1000				

Table 17: South Sudan UNHAS aircraft fleet composition

Aircraft	DCH8-Q	DCH8-3	DCH8-2	Fokker-50	Cessna-208B	Cessna-208B	Cessna-208B	Dornier-D228	LET-410	MI8T	MI8T
Depot	Juba	Juba	Juba	Juba	Juba	Rumbek	Wau	Juba	Juba	Juba	Rumbek
Speed $[\frac{km}{h}]$	555,6	531,5	535,2	530,0	344,5	344,5	344,5	314,8	314,8	222,2	222,2
Seats	71	49	37	50	12	12	12	18	17	19	19
TAT [hr]	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5
Range [km]	2037	1711	2084	2000	1982	1982	1982	1111	769	750	750
Runway [m]	2500	1500	1500	1500	500	500	500	500	500	50	50
Lease costs [$]	65910	75454	97350	116250	62581	62581	62581	59305	96831	228260	228260
Operational costs $[\frac{\$}{km}]$	12,36	6,23	5,13	6,50	3,63	3,63	3,22	7,54	7,21	12,88	12,88
Daily block time [hr]	8,5	8,5	8,5	8,5	8,5	8,5	8,5	8,5	8,5	8,5	8,5
Weekly block time [hr]	42,5	42,5	42,5	42,5	42,5	42,5	42,5	42,5	42,5	42,5	42,5
Fuel burn $[\frac{L}{hr}]$	860	480	540	640	234	234	234	340	400	700	700
Fuel cost $[\frac{\$}{km}]$	2,48	1,44	1,61	1,93	1,09	1,22	1,22	1,73	2,03	5,04	5,67
Amount available	1	1	1	1	1	1	1	2	2	2	2

Table 18: Daily request data for the UNHAS South Sudan mission from the 30/09 - 04/10

30-09

From	To	Pax
AJUON	JUB	13
BOR	JUB	15
HSTR	JUB	7
JUB	AJUON	12
JUB	BOR	20
JUB	HSTR	12
JUB	KAPO	11
JUB	KEEW	3
JUB	LKEN	12
JUB	MAK	29
JUB	MINGK	8
JUB	PADEA	6
JUB	RUM	24
JUB	WAU	22
JUB	YAM	29
JUB	YIDA	4
KAPO	JUB	6
LER	DINDI	9
LKEN	JUB	4
MAK	JUB	13
MINGK	JUB	2
RUM	JUB	8
WAU	JUB	17
YAM	JUB	15
YIDA	JUB	1

01-10

From	To	Pax
AGOK	JUB	8
BOMA	JUB	3
BOR	MABR	2
BUOT	JUB	3
BUOT	KARMO	15
GANY	JUB	4
HSPA	JUB	1
HSRN	JUB	6
HSTR	JUB	3
JCH	JUB	1
JUB	AGOK	10
JUB	BOMA	1
JUB	GANY	5
JUB	HSPA	4
JUB	HSRN	6
JUB	HSTR	6
JUB	JCH	5
JUB	KUAJK	9
JUB	MABR	5
JUB	MARID	4
JUB	MOGOK	1
JUB	MUNDR	1
JUB	NYAL	2
JUB	PAGL	1
JUB	PIBR	14
JUB	POCAL	7
JUB	RUM	24
JUB	WAU	13
JUB	YET	16
KUAJK	JUB	12
MABR	BOR	1
MABR	JUB	7
MARD	JUB	2
NYAL	JUB	4
PAGL	JUB	1
PIBR	BOR	8
PIBR	JUB	10
POCAL	BOR	1
POCAL	JUB	2
RUM	JUB	7
WAU	JUB	16
YEI	JUB	8

02-10

From	To	Pax
AJUON	JUB	12
BOR	JUB	9
GORWA	HAAT	17
HSAK	JUB	11
HSRN	JUB	10
JUB	AJUON	11
JUB	BOR	15
JUB	HSAK	6
JUB	HSRN	10
JUB	KURWA	1
JUB	MABAN	63
JUB	MAK	17
JUB	MINGK	3
JUB	MOTO	3
JUB	NEWFG	8
JUB	NIMU	1
JUB	NYAL	4
JUB	OLDFG	10
JUB	RUB	46
JUB	RUM	4
JUB	TAMBU	3
JUB	UI-ANG	20
JUB	WAU	7
JUB	YAM	19
JUB	YEI	1
JUB	YIM	3
MABAN	JUB	32
MAK	JUB	19
MINGK	BOR	1
MINGK	JUB	1
MOTO	JUB	3
NEWFG	JUB	10
NYAL	JUB	2
OLDFG	JUB	1
RUB	JUB	29
RUM	JUB	6
TAMBU	JUB	2
ULANG	JUB	10
WAU	JUB	7
YAM	JUB	9
YEI	JUB	1
YIDA	JUB	3

03-10

From	To	Pax
AGOK	JUB	3
BOR	JUB	5
HSAK	JUB	4
HSPA	JUB	2
HSRN	JUB	9
HSTR	JUB	19
JIKMI	JUB	5
JUB	AGOK	10
JUB	BOR	3
JUB	DINDI	9
JUB	HSAK	4
JUB	HSPA	3
JUB	HSRN	4
JUB	HSTR	6
JUB	JIKMI	11
JUB	KJK	5
JUB	KOCH	8
JUB	KUAJK	3
JUB	MANK	7
JUB	MATHI	9
JUB	NIMU	5
JUB	RUM	7
JUB	WAU	6
JUB	WGK	18
JUB	YEI	5
JUB	VIROL	3
KJK	JUB	1
KOCH	JUB	6
KUAJK	JUB	11
MANK	JUB	7
MATHI	JUB	2
NIMU	JUB	3
RUM	JUB	15
WAU	JUB	13
YEI	JUB	15
YIROL	JUB	5

04-10

From	To	Pax
AJUON	JUB	17
BOR	JUB	8
BOR	LABR	6
BOR	MARUW	6
JUB	AJUON	12
JUB	BOR	16
JUB	JUB	71
JUB	MA BAN	28
JUB	MAK	13
JUB	MATHI	3
JUB	MINGK	1
JUB	PALON	10
JUB	PIBR	19
JUB	RUB	18
JUB	UDR	1
JUB	YAM	10
JUB	YIM	7
MABAN	JUB	20
MAK	JUB	25
MANDE	JUB	8
MINGK	JUB	3
PIBR	JUB	12
RUB	JUB	28
WAU	JUB	9
WCHJL	JUB	3
YAM	JUB	12
YEI	JUB	4
YIDA	JUB	9

Table 19: Original weekly demand matrix for South Sudan UNHAS mission from the 30/09/2019 - 04/10/2019

From	*To*	*Pax*
Rumbek	Juba	36
Wau	Juba	63
Bor	Juba	37
Malakal	Juba	57
Ajuong Thok	Juba	42
Torit	Juba	29
Kapoeta	Juba	6
Lankien	Juba	4
Mingkaman	Juba	6
Yambio	Juba	36
Yida	Juba	13
Agok	Juba	11
Boma	Juba	3
BUOT	Juba	3
Ganyiel	Juba	4
Akobo	Juba	15
Paloich	Juba	3
Renk	Juba	25
Jiech	Juba	1
Jikmir	Juba	5
Kajo Keji	Juba	1
Koch	Juba	6
Kuajok	Juba	23
Maban	Juba	52
Mabior	Juba	7
Mandeng	Juba	8
Mankien	Juba	7
Maridi	Juba	2
Motot	Juba	3
New Fankgak	Juba	10
Nimule	Juba	3
Nyal	Juba	6
Old Fangak	Juba	1
Pagil	Juba	1
Pibor	Juba	22

From	*To*	*Pax*
Pochalla	Juba	2
Rubkona	Juba	57
Tambura	Juba	6
Ulang	Juba	10
Wiechjol LZ	Juba	3
Yei	Juba	28
Yirol	Juba	5
Juba	Rumbek	60
Juba	Wau	48
Juba	Bor	54
Juba	Malakal	59
Juba	Ajuong Thok	35
Juba	Torit	24
Juba	Kapoeta	11
Juba	Keew	3
Juba	Lankien	12
Juba	Mingkaman	12
Juba	Padeah	6
Juba	Yambio	58
Juba	Yida	14
Juba	Dindin	18
Juba	Agok	20
Juba	Mundri	1
Juba	Nyal	6
Juba	Old Fangak	10
Juba	Ganyiel	5
Juba	Mundri	1
Juba	New Fankgak	8
Juba	Boma	1
Juba	Akobo	10
Juba	Paloich	7
Juba	Renk	20
Juba	Jiech	5
Juba	Jikmir	11
Juba	Kajo Keji	5

From	*To*	*Pax*
Juba	Koch	8
Juba	Kuajok	12
Juba	Kurwai	1
Juba	Maban	91
Juba	Mabior	5
Juba	Mankien	7
Juba	Maridi	4
Juba	Mogok	1
Juba	Motot	3
Juba	Nimule	6
Juba	Pagil	1
Juba	Yirol	3
Juba	Palouny	10
Juba	Pibor	33
Juba	Rubkona	64
Juba	Tambura	3
Juba	Udier	1
Juba	Ulang	20
Juba	Walgal	18
Juba	Yei	22
Rumbek	Mathiang	12
Mathiang	Rumbek	2
Bor	Mabior	1
Bor	Akobo	1
Bor	Labrab	6
Bor	Maruw	6
Akobo	Bor	2
Mabior	Bor	1
Motot	Bor	1
Pibor	Bor	8
Pochalla	Bor	1
Mingkaman	Bor	1
Leer	Dindin	9
Gorwai	Haat	17
Buot	Karmoun	15

B Appendix 2 - Multi-commodity network flow results

B.1 Multi-commodity aircraft flows

Weekly passenger flows for the South Sudan UNHAS mission can be found in Figures 10 to 12 and the related aircraft flows are presented in Figures 13 to 18. They illustrate how a Hub and Spoke model can be used to transfer passengers throughout the network. Certain airports are very close together (ex. Yida and Ajuong Thok) but are modelled as two different spokes. Servicing requests to these destinations without considering routing between them is far from real-world operations, which entails the use of a more precise routing model. Because the Multi-commodity network flow model cannot take refueling into account, the range of MI8 helicopters were fictitiously increased in order to account for helipad only destinations which can only be reached with a fuel stop.

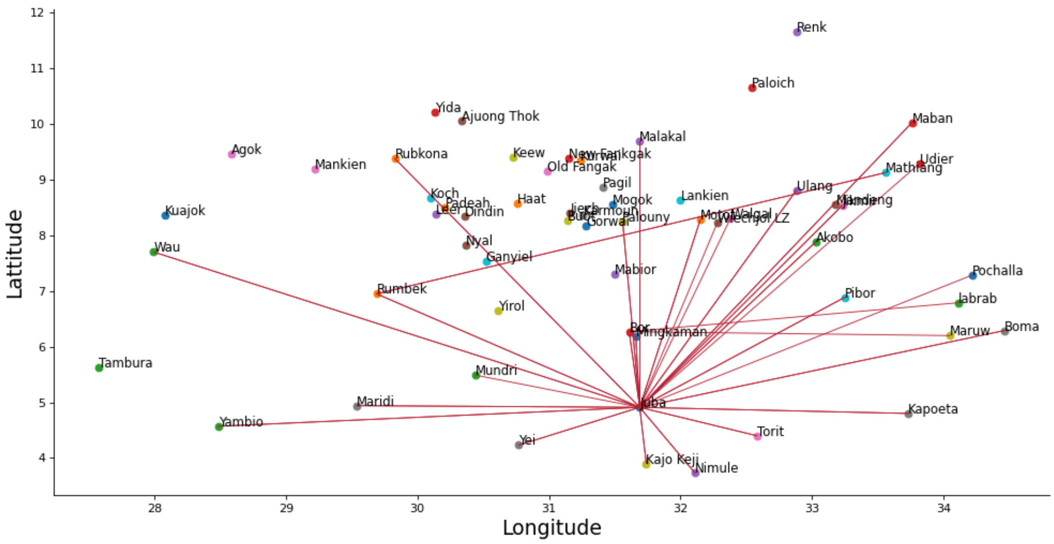


Figure 10: Direct passenger flows for South Sudan UNHAS weekly demand (30/09/2019 - 04/10/2019)

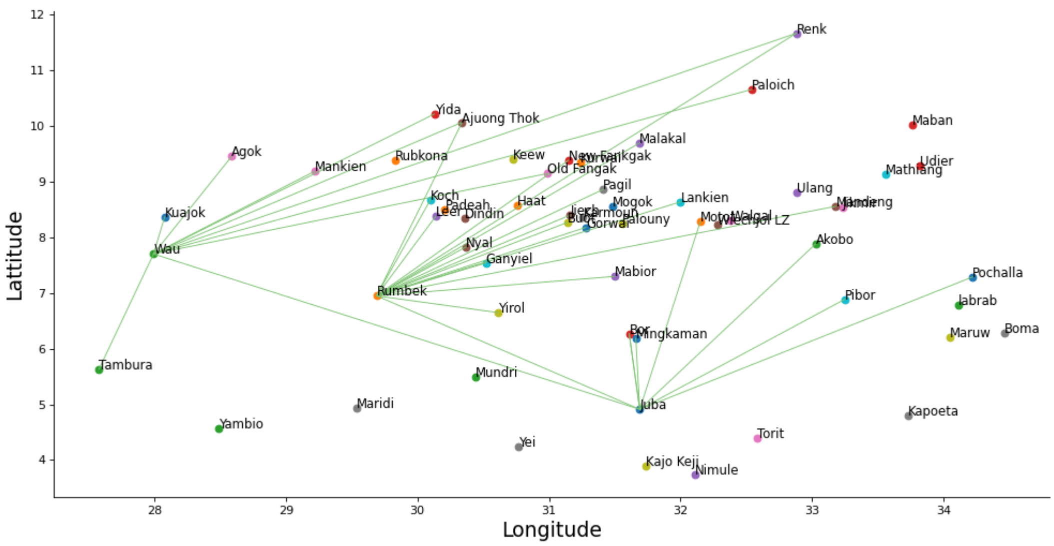


Figure 11: 1^{st} leg transfer Passenger flows for South Sudan UNHAS weekly demand (30/09/2019 - 04/10/2019)

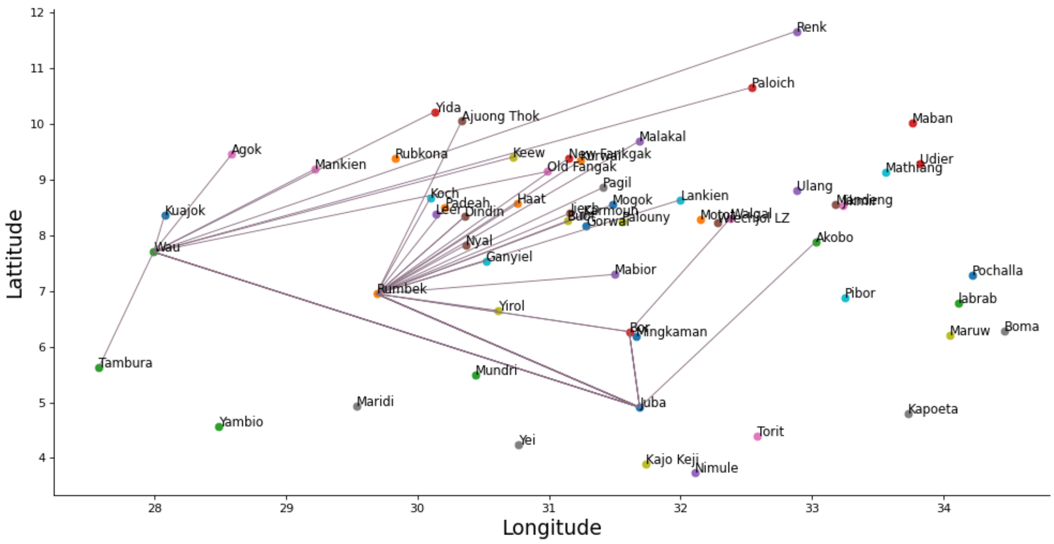


Figure 12: 2^{nd} leg transfer Passenger flows for South Sudan UNHAS weekly demand (30/09/2019 - 04/10/2019)

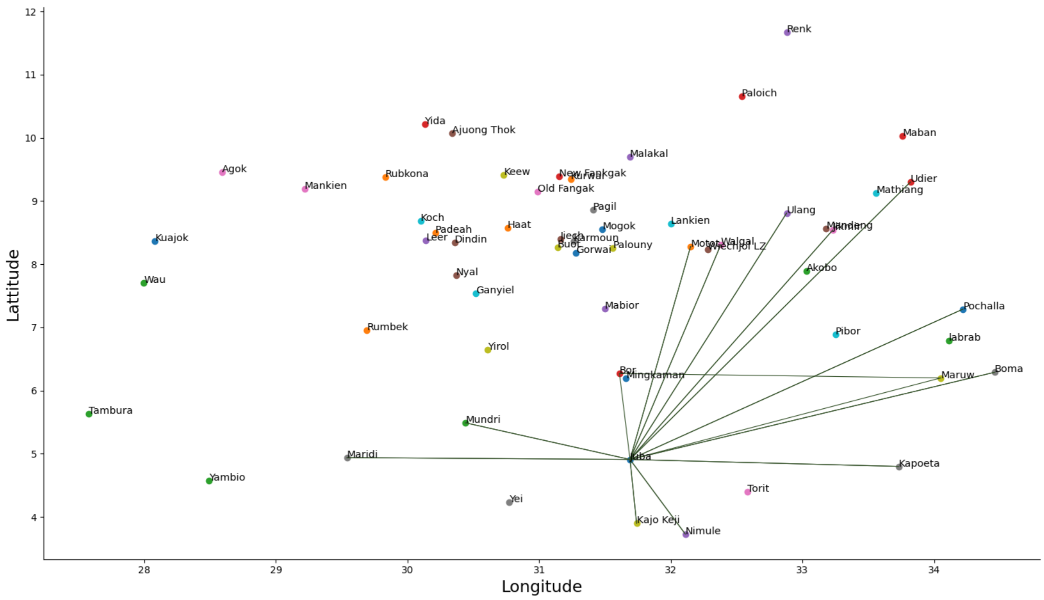


Figure 13: Cessna 208B-Juba weekly vehicle flows

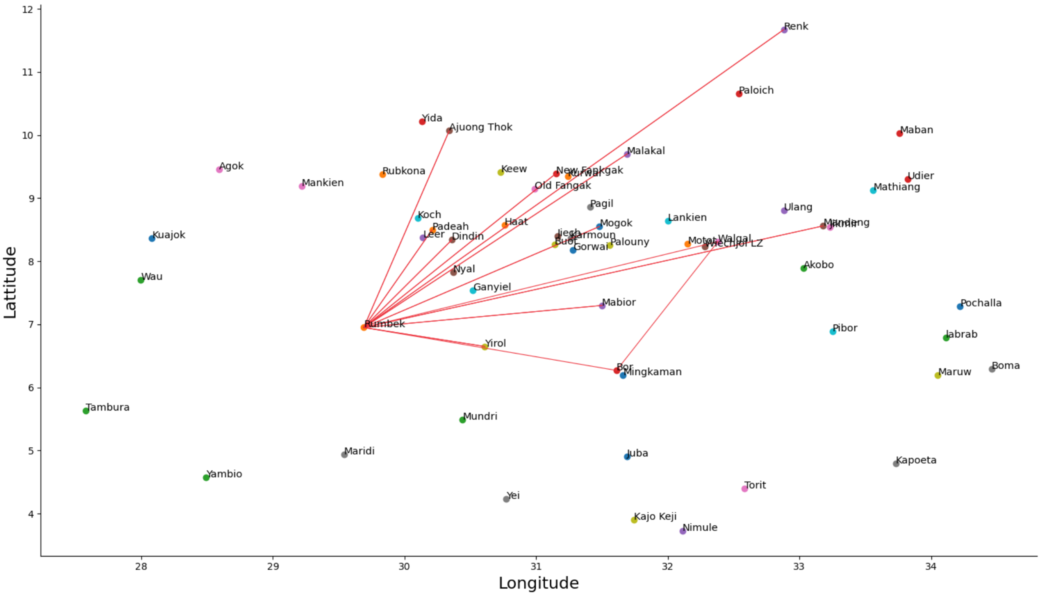


Figure 14: Cessna 208B-Rumbek weekly vehicle flows

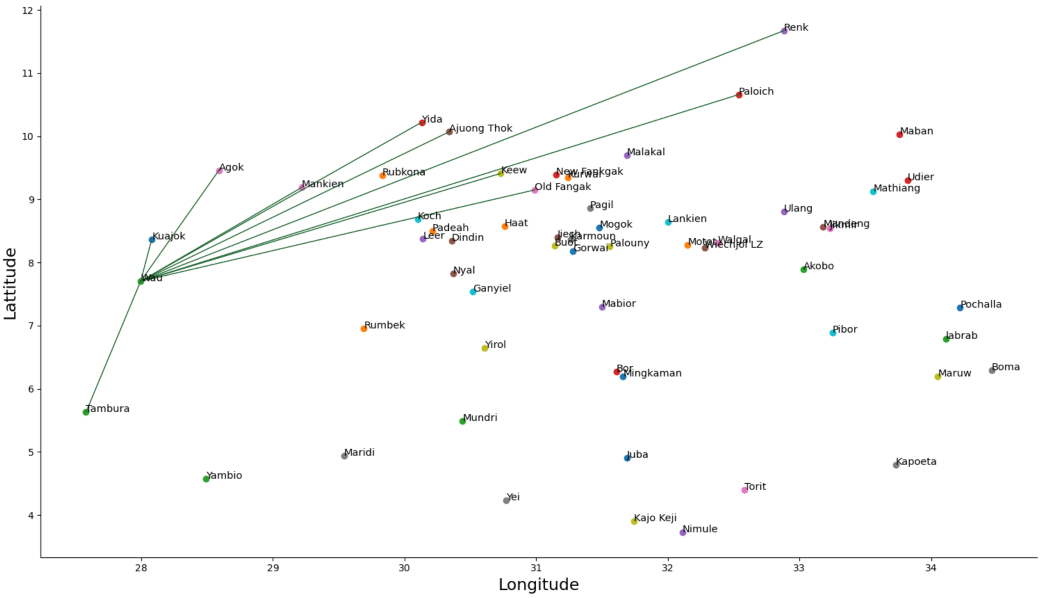


Figure 15: Cessna 208B-Wau weekly vehicle flows

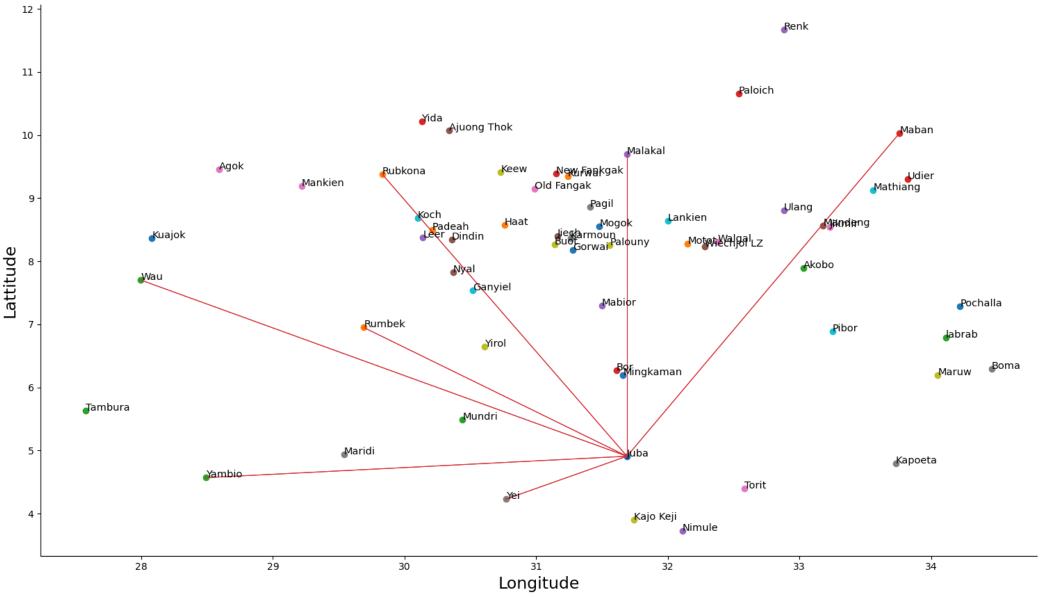


Figure 16: DCH8-3 - Juba weekly vehicle flows

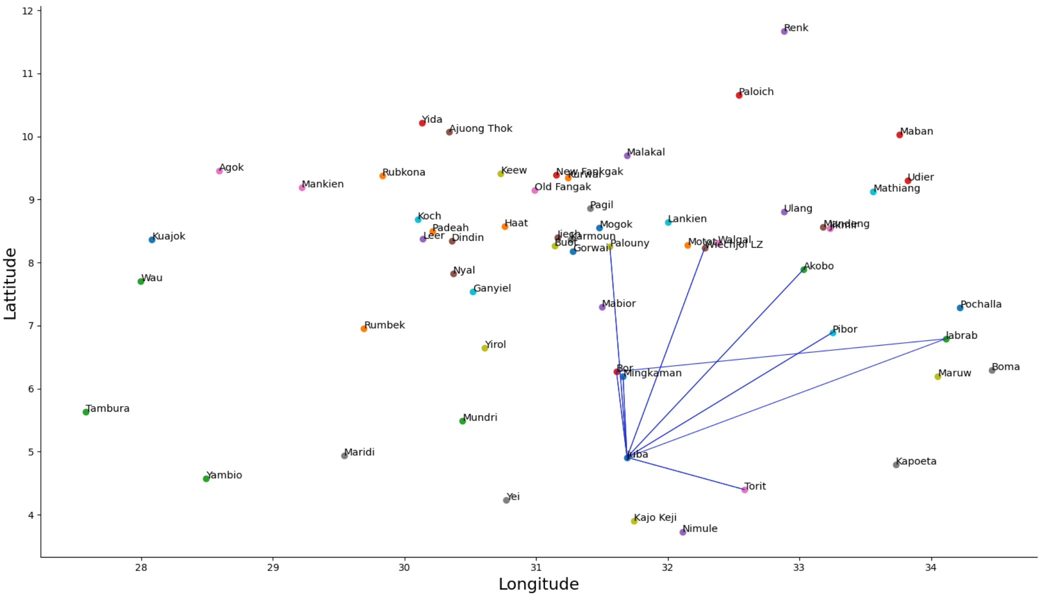


Figure 17: MI8-Juba weekly vehicle flows

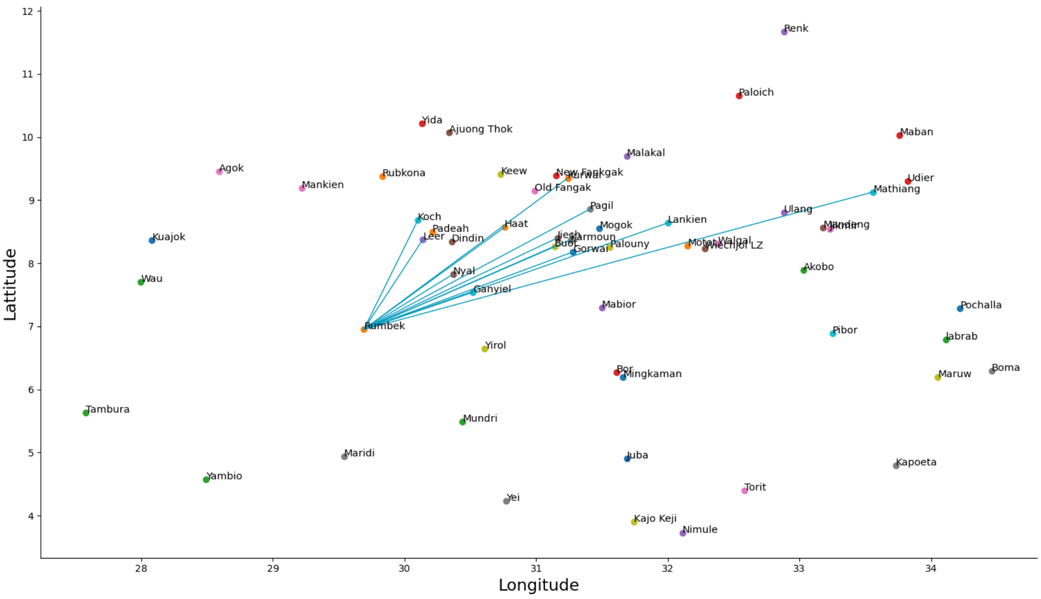


Figure 18: MI8-Rumbek weekly vehicle flows

From	*To*	*Pax*
Juba	Rumbek	181
Juba	Wau	144
Juba	Bor	84
Juba	Malakal	49
Juba	Torit	24
Juba	Kapoeta	11
Juba	Lankien	12
Juba	Mingkaman	12
Juba	Yambio	58
Juba	Boma	1
Juba	Akobo	11
Juba	Jikmir	11
Juba	Kajo Keji	5
Juba	Maban	91
Juba	Maridi	4
Juba	Mundri	1
Juba	Nimule	6
Juba	Pibor	33
Juba	Rubkona	64
Juba	Udier	1
Juba	Ulang	12
Juba	Walgal	12
Juba	Yei	22
Rumbek	Juba	145
Rumbek	Bor	2
Rumbek	Malakal	10
Rumbek	Ajuong Thok	23
Rumbek	Padeah	6
Rumbek	Dindin	27
Rumbek	Ganyiel	5
Rumbek	Haat	17
Rumbek	Renk	12
Rumbek	Jiech	5
Rumbek	Karmoun	15
Rumbek	Koch	8
Rumbek	Kurwai	1

From	*To*	*Pax*
Rumbek	Mabior	5
Rumbek	Mathiang	12
Rumbek	Mogok	1
Rumbek	Motot	3
Rumbek	New Fankgak	8
Rumbek	Nyal	6
Rumbek	Pagil	1
Rumbek	Yirol	3
Rumebk	Walgal	6
Wau	Ajuong Thok	12
Wau	Keew	3
Wau	Yida	14
Wau	Agok	20
Wau	Paloich	7
Wau	Renk	8
Wau	Kuajok	12
Wau	Mankien	7
Wau	Old Fangak	10
Wau	Tambura	3
Bor	Juba	38
Bor	labrab	6
Bor	Maruw	6
Bor	Palouny	10
Bor	Ulang	8
Akobo	Juba	17
Boma	Juba	3
Jikmir	Juba	5
Kajo Keji	Juba	1
Kapoeta	Juba	6
Lankien	Juba	4
Maban	Juba	52
Malakal	Juba	49
Maridi	Juba	2
Mingkaman	Juba	7
Nimule	Juba	3
Pibor	Juba	30

From	*To*	*Pax*
Pochalla	Juba	3
Rubkona	Juba	57
Torit	Juba	29
Ulang	Juba	10
Wau	Juba	147
Wiechjol LZ	Juba	3
Yambio	Juba	36
Yei	Juba	28
Ajuong Thok	Rumbek	36
BUOT	Rumbek	18
Ganyiel	Rumbek	4
Gorwai	Rumbek	17
Jiech	Rumbek	1
Koch	Rumbek	6
Leer	Rumbek	9
Mabior	Rumbek	8
Malakal	Rumbek	8
Mandeng	Rumbek	8
Mathiang	Rumbek	2
Motot	Rumbek	4
New Fankgak	Rumbek	10
Nyal	Rumbek	6
Pagil	Rumbek	1
Renk	Rumbek	11
Yirol	Rumbek	5
Agok	Wau	11
Ajuong Thok	Wau	6
Kuajok	Wau	23
Mankien	Wau	7
Old Fangak	Wau	1
Paloich	Wau	3
Renk	Wau	14
Tambura	Wau	6
Yida	Wau	13

Table 20: Modified weekly demand for South Sudan UNHAS mission from the 30/09/2019 - 04/10/2019

C Appendix 3 - Verification

C.1 Verification of the HFSMVRP model

Verifying the models was done through the use of a straightforward and simple case study with the objective being to ensure the constraints behave according to expectations. In order to verify the HFSMVRP model, the unit-less input data is used presented in Table 21 and Table 22. The different scenarios that are tested along with the aspects of the model they verify are shown in Table 23.

Table 21: Airport data for the verification case

Airport	**Index**	**Depot**	**Latitude**	**Longitude**	**Runway [m]**	**Refuel coefficient**
A	0	0	0	0	1000	0
B	1	1	0	3	1000	0
C	2	0	4	0	1000	0
Fuel	3	0	0,5	1	1000	2

Table 23: Verification scenarios

Scenario	Constraint verification	Eq.	**Aircraft**	**Max. trips per ac**	**Request**	**To**	**From**	**Pax**
1	Multi-trip Flow conservation	1d	AC 2, AC 3	2	0 1	A B	C C	2 2
2	Multi-depot Objective function	1b, 1c, 1e 1a	AC 1, AC 2	2	0 1	A B	C C	2 3
3	Split requests Capacity Request satisfaction	1f 1k 1j, 1i	AC 1, AC 2	2	0	A	C	5
4	Loading and unloading	1g,1h,1l	AC 2	1	0 1 2 3	B B A C	C A C B	2 1 2 3
5	Daily utilisation Weekly utilisation	1n 1m	AC 1, AC 2	10	0	A	C	0-25
6	Range Refueling	1o 1p, 3,4	AC 1	1	0	B	C	2

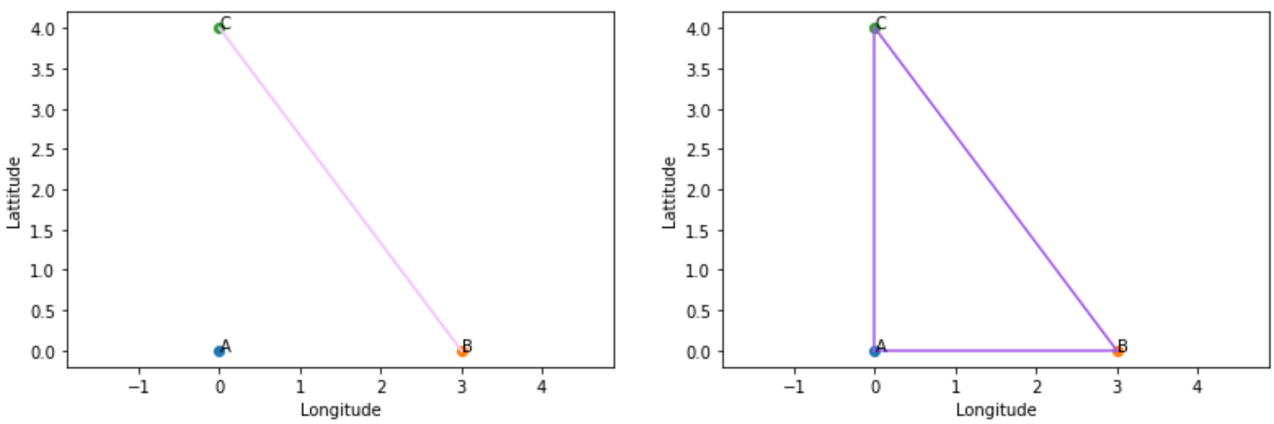
Table 22: Aircraft data for the verification case

Aircraft	**Depot**	**Speed**	**Seats**	**Operational costs**	**Lease costs**	**Range**	**Block time day**	**Block time week**	**TAT**	**Runway requirement**	**Fuel cost**
AC 1	A	1	2	4	1	10	15	30	0,5	100	1
AC 2	B	1	3	3	2	12	20	30	0,5	100	1
AC 3	A	1	3	1	1	12	15	30	0,5	3000	1

Multi-depot, Multi-trip constraints

Equation 1b, Equation 1c and Equation 1e represent the multi-depot aspect of the formulation and constrain all vehicles to start and end their trips at their corresponding base. While the first two limit the amount of trips allowed per aircraft from their respective depots and bound the ac^k decision variable (amount of aircraft per types), the third constraint ensures that each trip ends and begins at the aircraft type's hub. Finally, these 3 constraints must be combined with the aircraft flow conservation constraint in Equation 1d in order to create a fully consistent multi-depot, multi-trip formulation.

Scenario 1 is used to verify the Multi-trip property of the model and the flow conservation constraint. Two requests are created which in total have more passengers the maximum capacity of Aircraft 2 or 3. In this way it is expected that two trips will have to be made, leaving and ending at the vehicle's depot B. Due to the fact Request 1 originates in A and ends in C, the aircraft will have to first fly empty to A and pick up passengers, then fly to C to deliver them, and finally fly back to the depot empty. Although Aircraft 3 is available and cheaper to operate in every aspect, it is not chosen as its runway requirements are not met. 19(a) and 19(b) presents the aircraft flows and verify that multiple trips by the same vehicle are possible, and that the vehicle flow conservation is ensured.



(a) Aircraft 2, trip 1

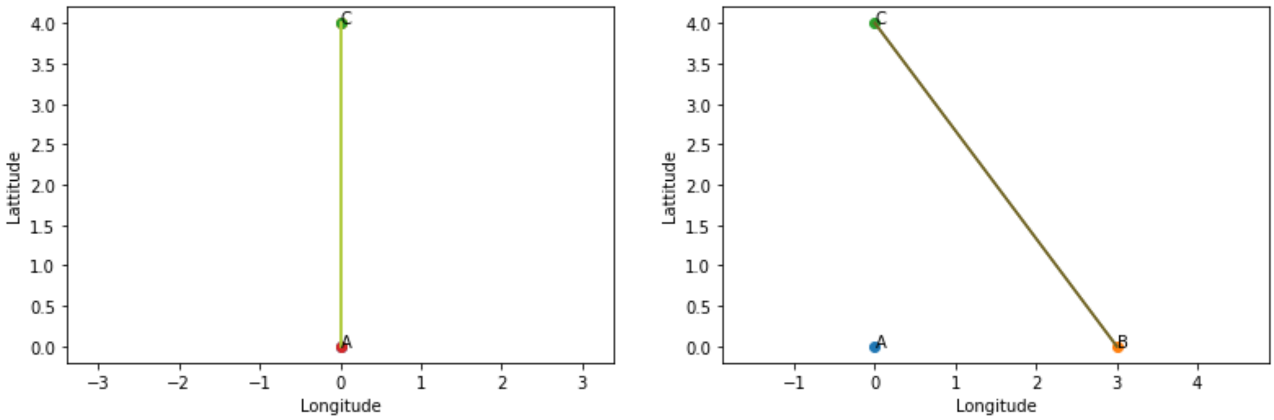
(b) Aircraft 2, trip 2

Figure 19: Verification of Scenario 1, Aircraft flows

Table 24: Results for the verification of Scenario 1

Aircraft	Trip	Leg	From	To	Pick-up	Delivery	Pax	Leg dist.	Leg cost
AC 2	1	1	B	C	r1(1)	r1(1)	1	5	15
		2	C	B	-	-	0	5	15
	2	1	B	A	r1(1)	-	1	3	9
		2	A	C	r2(2)	r1(1), r2(2)	3	4	12
		3	C	B	-	-	0	5	15
Aircraft lease costs						2			
Routing costs						66			
Objective function						68			

Scenario 2 is used to verify the Multi-depot property of the model and the objective function. Two aircraft are available at two different depots and the model is prompted to choose the best fleet to serve two different requests. It is expected that the trade off between aircraft leasing costs and routing costs results in a combination of both vehicles, satisfying the requests originating from their respective depots. This is confirmed as can be seen in 20(a) and 20(b).



(a) Aircraft 1, trip 1

(b) Aircraft 2, trip 1

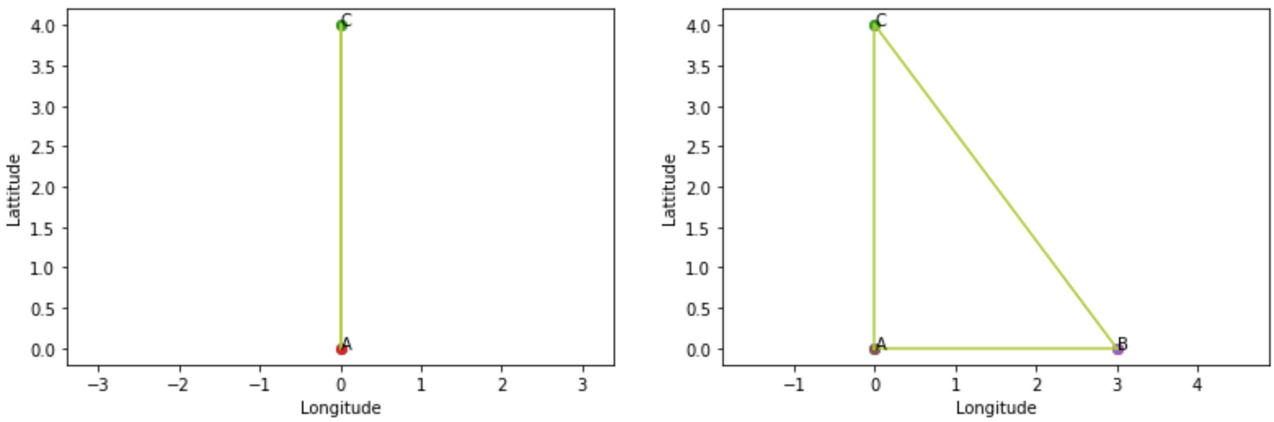
Figure 20: Verification of Scenario 2, Aircraft flows

Table 25: Results for the verification of Scenario 2

Aircraft	Trip	Leg	From	To	Pick-up	Delivery	Pax	Leg dist.	Leg cost
AC 1	1	1	A	C	r0(2)	r0(2)	2	4	16
		2	C	A	-	-	0	4	16
AC 2	1	1	B	C	r1(2)	r1(2)	2	5	15
		2	C	B	-	-	0	5	15
Aircraft lease costs						3			
Routing costs						62			
Objective function						65			

Split request, request satisfaction and capacity constraint
Splitting requests between different aircraft and trips is ensured by Equation 1f. Requests can be split between different aircraft and different trips. This is ensured by allowing vehicles to visit nodes more than once. Furthermore, the constraints in Equation 1j and Equation 1i are needed to specify that the sum of all passengers from a request over all vehicle trips must be equal to the total amount of passengers to be picked-up and delivered from the respective request. Finally, Equation 1k ensures that the amount of passengers on a flight leg does not surpass the amount of seats.
These three constraints are verified by scenario 3 where a request of 5 passengers is made, larger than both aircraft capacities. As expected, the passengers are split up between different paths and vehicles, and the total

request is satisfied. This is illustrated by 21(a) and 21(b).



(a) Aircraft 1, trip 1

(b) Aircraft 2, trip 1

Figure 21: Verification of Scenario 3, Aircraft flows

Table 26: Results for the verification of Scenario 3

Aircraft	**Trip**	**Leg**	**From**	**To**	**Pick-up**	**Delivery**	**Pax**	**Leg dist.**	**Leg cost**
AC 1	1	1	A	C	r0(2)	r0(2)	2	4	16
		2	C	A	-	-	0	4	16
AC 2	1	1	B	A	-	-	0	3	9
		2	A	C	r2(3)	r2(3)	3	4	12
		3	C	B	-	-	0	5	15
Aircraft lease costs						3			
Routing costs						68			
Objective function						71			

Loading constraints

Equation 1g and Equation 1h account for the simultaneous pick up and delivery of passengers while ensuring the loading on a flight leg is lower than the maximum amount of seats on the selected aircraft type. Both constraints originate from Equation 6, but were linearised in order to be used consistently in the formulation. The equation states that if a node j is visited by an aircraft, the loading on the flight leg before and after the visit must be balanced out by the amount of passengers picked-up or delivered at that same node. The term α_j^r is either equal to -1 if passengers from request r are dropped off at node j, equal to 1 if passengers are picked up, or 0 if no passenger movement is made.

$$x_{ij}^{kp}(u_{ij}^{krp} + q_j^{krp}\alpha_j^r - u_{ji}^{krp}) = 0 \qquad \forall i \in N, j \in N, r \in R, k \in K, p \in P \tag{6}$$

Equation 6 is the product of the binary variable x_{ij}^{kp} and an expression made up of continuous variables ($u_{ij}^{krp} + q_j^{krp}\alpha_j^r - u_{ji}^{krp}$). It is therefore a non-linear, non-convex constraint of the form $z = x \cdot y$ with y having a lower bound of $-Q^k$ and an upper bound of $+Q^k$, the capacity of the vehicle in question. McCormick Envelopes can thus be used to recreate a convex set of constraints by the following four expressions in Equations 7, 8, 9 and 10.

$$z \geq y^L x + yx^L - y^L x^L \tag{7}$$
$$z \leq y^U x + yx^L - y^U x^L \tag{8}$$
$$z \geq y^U x + yx^U - y^U x^U \tag{9}$$
$$z \geq y^U x + yx^L - y^L x^U \tag{10}$$
$$\tag{11}$$

The results of the convex relaxation and the new loading constraints can be seen in Equations 12, 13,14 and 15.

$$0 \geq -Q^k x_{ij}^{kp} \qquad \forall i \in N, j \in N, k \in K, p \in P \tag{12}$$
$$0 \leq Q^k x_{ij}^{kp} \qquad \forall i \in N, j \in N, k \in K, p \in P \tag{13}$$
$$0 \geq (u_{ij}^{krp} + q_j^{krp} \alpha_j^r - u_{ji}^{krp}) + x_{ij}^{kp} Q^k - Q^k \qquad \forall i \in N, j \in N, r \in R, k \in K, p \in P \tag{14}$$
$$0 \leq (u_{ij}^{krp} + q_j^{krp} \alpha_j^r - u_{ji}^{krp}) - x_{ij}^{kp} Q^k - Q^k \qquad \forall i \in N, j \in N, r \in R, k \in K, p \in P \tag{15}$$

Equation 12 and Equation 13 are redundant and therefore left aside. The loading and unloading constraints must be combined with Equation 1l to ensure that passenger pick-up and deliveries can only made if an aircraft visits that airport. The loading constraints are verified using scenario 4 where different requests are to be picked up and delivered at different nodes of the network. Aircraft 2 is selected as it has enough capacity to mix and match all different requests using one single route.

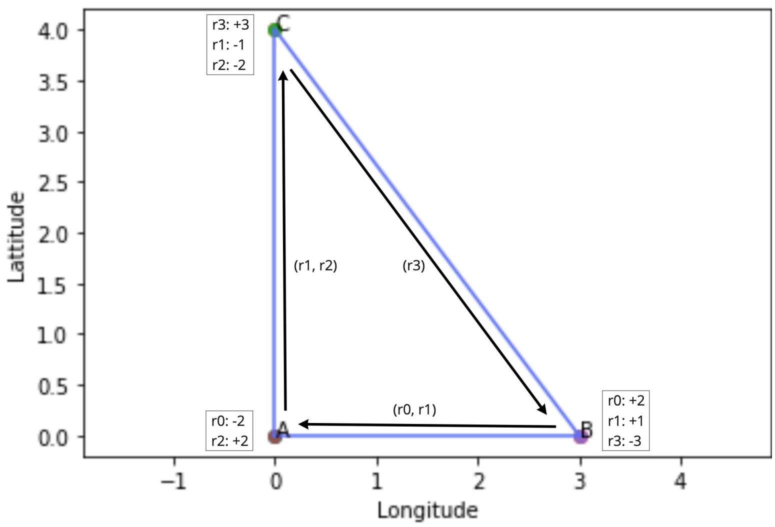


Figure 22: Verification of Scenario 4, Aircraft flows

Table 27: Results for the verification of Scenario 4

Aircraft	Trip	Leg	From	To	Pick-up	Delivery	Pax	Leg dist.	Leg cost
AC 2	1	1	B	A	r0(2), r1(1)	r0(2)	3	3	9
		2	A	C	r2(2)	r2(2), r1(1)	3	4	12
		3	C	B	r3(3)	r3(3)	3	5	15
Aircraft lease costs					2				
Routing costs					36				
Objective function					38				

Utilisation constraints
Equation 1m and Equation 1n ensure that individual aircraft do not fly more than they are allowed to on a daily and weekly basis. The weekly utilisation constraint ensures that the total amount of flight and turn-around time over different days and trips per aircraft does not surpass 42.5 hours. If this is the case, the decision variable ac^k is increased to account for adding an additional aircraft to the fleet. The daily utilisation constraint ensures the aircraft utilisation per day does not surpass 8.5 hours over all trips. Scenario 5 is used to to verify the utilisation constraints. An increasing demand is created iteratively, from 0 passengers to 25, all originating from A and terminating at C. When Aircraft 2 surpasses its weekly utilisation, the decision variable ac^k increases by one indicating that another aircraft is needed to carry out the requests. Figure 23 shows the interaction between passenger requests, block hours and aircraft selection. As the number of passenger increases, so do the required block hours and therefore the number of Aircraft 2 needed to transport the total demand withing the weekly time frame. Aircraft 1 is also chosen for certain passenger numbers as the model trades off between leg costs

and aircraft leasing costs.

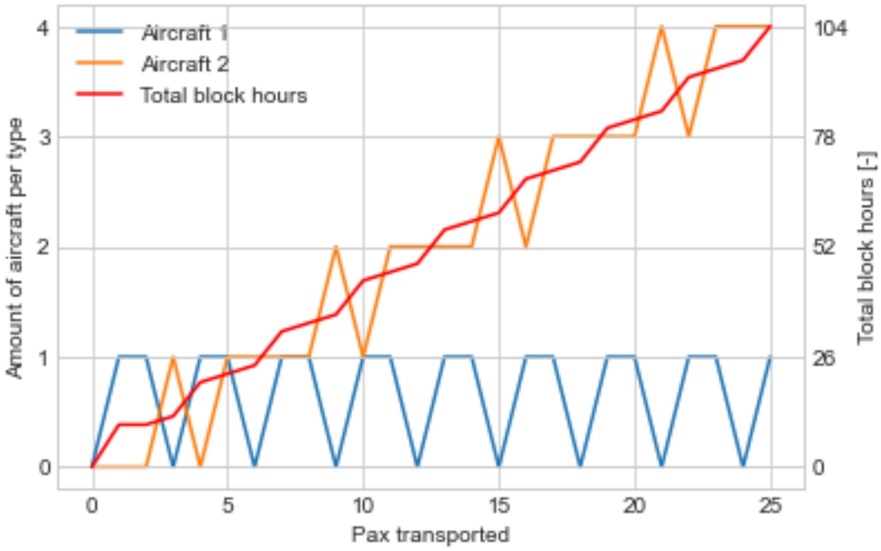
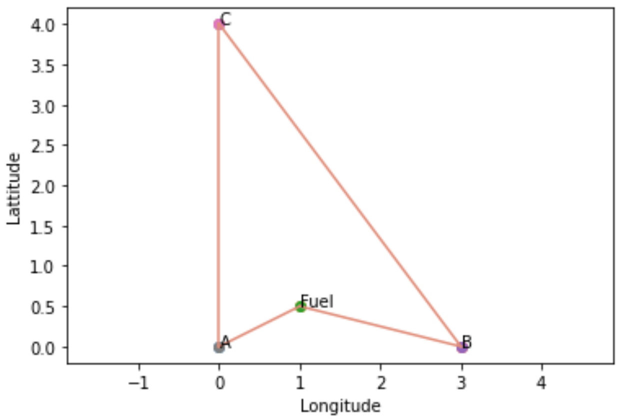
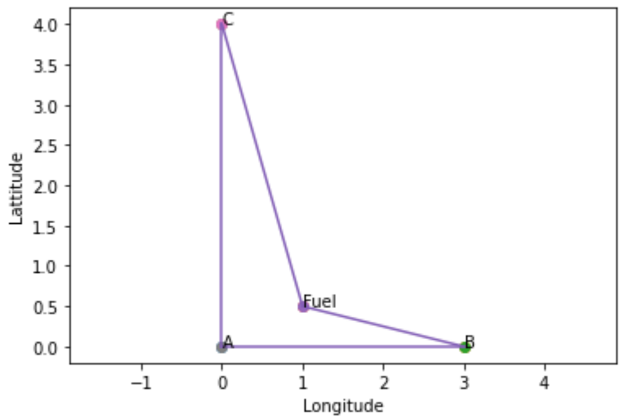


Figure 23: Verification of Scenario 5, Evolution of the amount of aircraft needed per type and block hours in function of passengers transported

Range and refueling constraints
Range and refuelling are taken into account in Equation 1o and Equation 1p, in combination with the lazy constraints in section 3.2.2. The choice of refuelling is added to the range constraint by extending the aircraft's maximum range by two in a specific trip if the vehicle visits a refueling airport. Depending on the cost of fuel, the objective function is increased accordingly in order to penalize refueling stops. The costs of each arcs leaving a refuelling node includes the costs of that specific arc and an extra-term associated to the cost of refuelling. The cost of refuelling is approximated by multiplying half the maximum range of an aircraft k by its cost of fuel per km, and a coefficient associated to the price of fuel at the specific airport. The lazy constraints in Equation 3 and Equation 4 ensure that an aircraft refuels at an appropriate point in its route. Scenario 6A and 6B are used to verify these constraints. Aircraft 1 having only a range of 10 units and the path A-B-C-A having a length of 12 units, if refueling is not used, the requests cannot be satisfied. Once the option of refueling is added, Aircraft 1 is able to fulfill the requests. The verification scenario 6a is first run without the lazy constraint. The result can be seen in 24(a). Although Aircraft 1 has refueled and its total range has been increased, the length of the path between the refuel stop and the depot is 11,5 units, still larger than its maximum range. Once the lazy constraint is implemented in scenario 6b, the correct path can be observed in 24(b).



(a) Aircraft 1, trip 1, (infeasible refuelling)



(b) Aircraft 1, trip 1 (feasible refuelling)

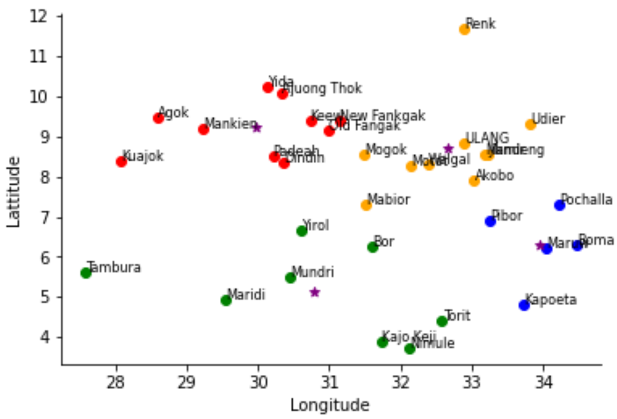
Figure 24: Verification of Scenario 6, Aircraft flows

Table 28: Results for the verification of Scenario 6 (feasible refuelling

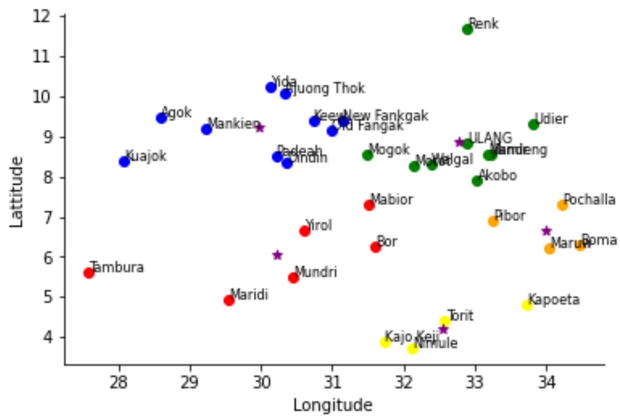
Aircraft	Trip	Leg	From	To	Pick-up	Delivery	Pax	Leg dist.	Leg cost
AC 1	1	1	A	Fuel	-	-	0	1,12	4,48
		2	Fuel	B	-	-	0	2.06	8.24
		3	B	C	r0(2)	r0(2)	2	5	20
		4	C	A	-	-	0	4	16
Aircraft lease costs				1					
Routing costs				48.72					
Refueling costs				21					
Objective function				69.72					

D Appendix 4 - Clustering Algorithm results

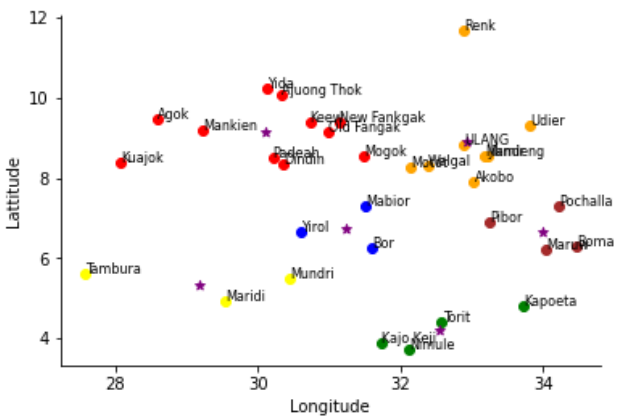
D.1 Appendix 4 - Clustering Algorithm sensitivity analysis



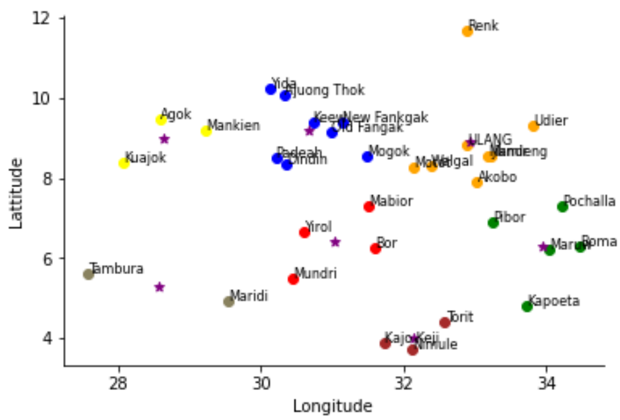
(a) 1000m runway airport clustering (k=4)



(b) 1000m runway airport clustering (k=5) flows

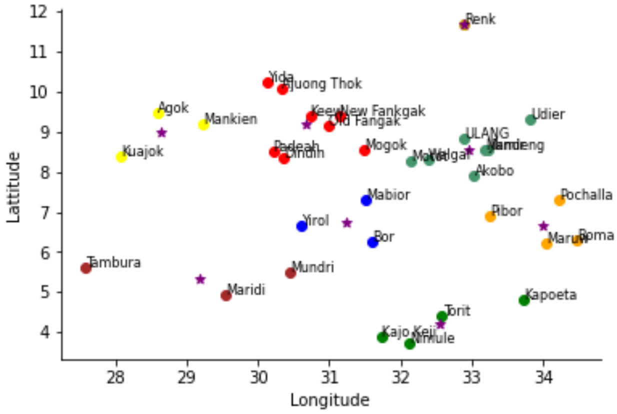


(c) 1000m runway airport clustering (k=6)

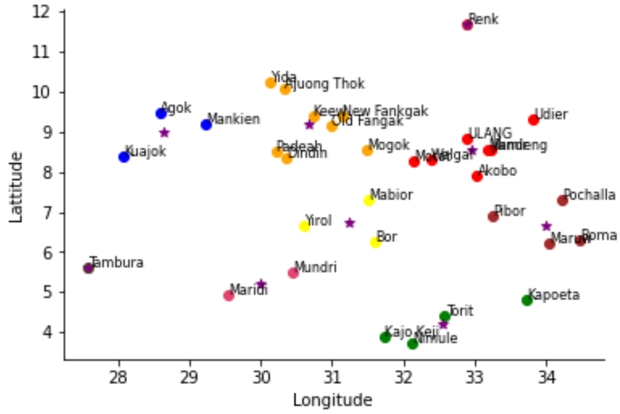


(d) 1000m runway airport clustering (k=7)

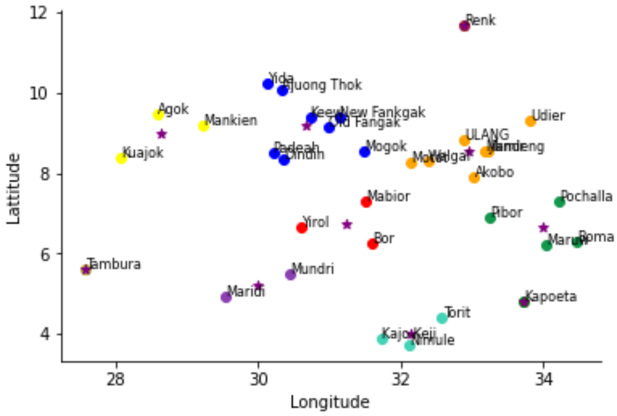
7



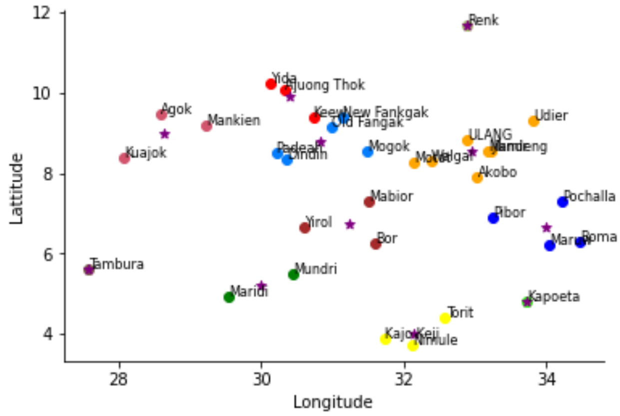
(e) 1000m runway airport clustering (k=8)



(f) 1000m runway airport clustering (k=9)



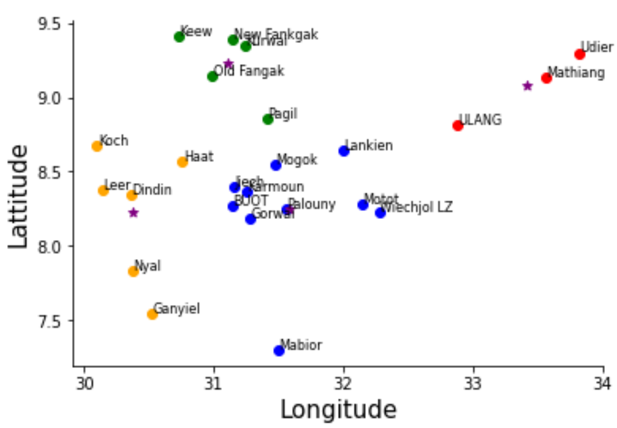
(g) 1000m runway airport clustering (k=10)



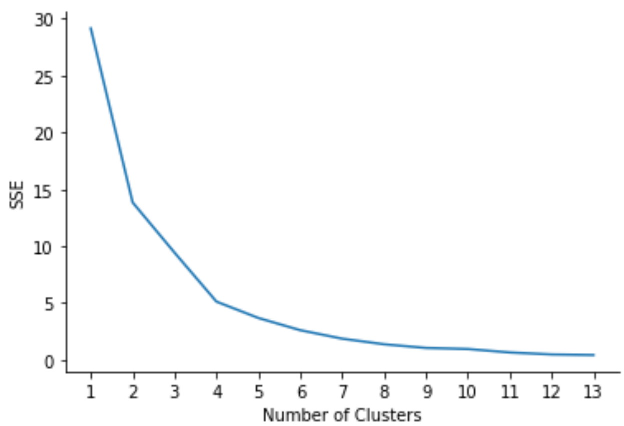
(h) 1000m runway airport clustering (k=11)

Figure 25: 1000m runway airport clustering for k clusters between 4 and 11

D.2 Appendix 4 - Clustering Algorithm for helipads in South Sudan during the rainy season



(a) Clustering algorithm results for helicopter accessible airports during the rainy season



(b) "Elbow" graph corresponding to the clustering of Figure 26(a)

Figure 26: Clustering algorithm applied to helipads in South Sudan during the rainy season

E Appendix 5 - Weekly schedules

Table 29: Proposed optimised weekly schedule for the UNHAS South Sudan mission (30/09/2019-04/10/2019) as a result of the HFSMVRP model

	Monday				**Tuesday**				**Wednesday**				**Thursday**				**Friday**			
MI8- Juba	08:30	10:54	Juba	Malakal	08:30	09:08	Juba	Migkaman												
	11:24	12:22	Malakal	Mathiang	09:38	09:41	Mingkaman	Bor												
	12:52	15:10	Mathiang	Juba	10:11	10:52	Bor	Juba												
MI8-Juba	08:30	09:11	Juba	Bor	08:30	10:29	Juba	Pagil	08:30	10:11	Juba	Weichjol LZ	08:30	09:56	Juba	Ganyiel	08:30	09:56	Juba	Ganyiel
	09:41	10:40	Bor	Palouny	10:59	11:26	Pagil	Malakal	10:41	10:56	Weichjol LZ	Lankien	10:26	10:36	Ganyiel	Nyal	10:26	10:54	Ganyiel	Leer
	11:10	11:19	Palouny	Gorwai	11:56	12:13	Malakal	Kurwai	11:26	12:38	Lankien	Bor	11:06	11:27	Nyal	Padeah	11:24	11:33	Leer	Koch
	11:49	12:08	Gorwai	Haat	12:43	13:12	Kurwai	Jiech	13:08	14:24	Bor	Labrab	11:57	12:01	Padeah	Leer	12:03	12:56	Koch	Rumbek
	12:38	14:31	Haat	Juba	13:42	13:46	Jiech	Buot	14:54	16:26	Labrab	Juba	12:31	12:38	Leer	Dindin	13:26	14:18	Rumbek	Juba
					14:16	15:58	Buot	Juba					13:08	14:20	Dindin	Bor				
													14:50	15:31	Bor	Juba				
Cessna 208-Juba	08:30	09:19	Juba	Pibor	08:30	09:19	Juba	Pibor	08:30	09:19	Juba	Pibor	08:30	08:50	Juba	Kajo Keji	08:30	09:37	Juba	Walagal
	09:49	10:38	Pibor	Juba	09:49	10:38	Pibor	Juba	09:49	10:09	Pibor	Pochalla	09:20	09:28	Kajo Kej	Nimule	10:07	11:13	Walagal	Renk
	11:08	13:21	Juba	Renk	11:08	12:15	Juba	Walagal	10:39	11:33	Pochalla	Bor	09:58	10:22	Nimule	Juba	11:43	12:32	Renk	Udier
	13:51	16:04	Renk	Juba	12:45	12:59	Walagal	Ulang	12:03	12:50	Bor	Maruw	10:52	11:31	Juba	Kapoeta	13:02	13:21	Udier	Mandeng
					13:29	14:48	Ulang	Juba	13:20	13:28	Maruw	Boma	12:01	12:25	Kapoeta	Torit	13:51	15:07	Mandeng	Juba
					15:18	15:44	Juba	Bor	13:58	14:58	Boma	Juba	12:55	13:15	Torit	Juba	15:37	16:03	Juba	Bor
					16:14	16:40	Bor	Juba	15:28	15:48	Juba	Torit	13:45	15:04	Juba	Ulang	16:33	16:59	Bor	Juba
									16:18	16:38	Torit	Juba	15:34	16:53	Ulang	Juba				
Cessna 208- Juba	08:30	09:46	Juba	Jikmir	08:30	10:05	Juba	Mankien	08:30	08:56	Juba	Bor	08:30	10:16	Juba	Agok	08:30	10:43	Juba	Renk
	10:16	10:17	Jikmir	Mandeng	10:35	11:01	Mankien	Yida	09:26	09:46	Bor	Mabior	10:46	11:09	Agok	Kuajok	11:13	12:08	Renk	New Fankgak
	10:47	12:03	Mandeng	Juba	11:31	11:36	Yida	Ajuong Thok	10:16	11:13	Mabior	Mankien	11:39	13:15	Kuajok	Juba	12:38	13:19	New Fankgak	Mabior
	12:33	14:16	Juba	Ajuong Thok	12:06	13:49	Ajuong Thok	Juba	11:43	11:56	Mankien	Agok	13:45	14:00	Juba	Padeah	13:49	14:35	Mabior	Juba
	14:46	16:29	Ajuong Thok	Juba	14:19	14:39	Juba	Torit	12:26	14:06	Agok	Juba	14:30	14:34	Padeah	Dindin	15:05	15:31	Juba	Bor
					15:09	15:29	Torit	Juba	14:36	15:02	Juba	Bor	15:04	16:15	Dindin	Juba	16:01	16:27	Bor	Juba
									15:32	15:53	Bor	Yirol								
									16:23	17:03	Yirol	Juba								
Cessna 208 - Juba	08:30	09:53	Juba	Old Fangak	08:30	08:56	Juba	Bor	08:30	10:17	Juba	Yida	08:30	09:41	Juba	Dindin	08:30	09:16	Juba	Mabior
	10:23	10:29	Old Fangak	New Fangak	09:26	10:08	Bor	Akobo	10:47	12:34	Yida	Juba	10:11	10:33	Dindin	Keew	09:46	10:10	Mabior	Mogok
	10:59	12:26	New Fangak	Juba	10:38	11:41	Akobo	Juba	13:04	14:47	Juba	Ajuong Thok	11:03	11:18	Keew	Ajuong Thok	10:40	10:54	Mogok	Motot
	12:56	14:32	Juba	Kuajok	12:11	12:38	Juba	Muncri	15:17	17:00	Ajuong Thok	Juba	11:48	13:31	Ajuong Thko	Juba	11:24	11:28	Motot	Walagal
	15:02	16:38	Kuajok	Juba	13:08	14:03	Mundri	Tambura									11:58	12:13	Walagal	Akobo
					14:30	15:10	Tambura	Maridi									12:43	13:25	Akobo	Bor
					15:40	16:21	Maridi	Juba									13:55	14:21	Bor	Juba
DCH8 106 - Juba	07:30	08:06	Juba	Rumbek	07:30	08:06	Juba	Rumbek	08:30	09:28	Juba	Wau	07:30	08:06	Juba	Rumbek	08:30	09:31	Juba	Rubkona
	08:36	09:12	Rumbek	Juba	08:36	09:12	Rumbek	Juba	09:58	10:29	Wau	Rubkona	08:36	08:59	Rumbek	Wau	10:01	10:24	Rubkona	Malakal
	09:42	10:51	Juba	Maban	09:42	10:42	Rumbek	Malakal	10:59	12:00	Rubkona	Juba	09:29	10:27	Wau	Juba	10:54	11:54	Malakal	Juba
	11:21	12:30	Maban	Juba	11:12	12:30	Malakal	Rumbek	12:30	13:43	Juba	Paloich	10:57	11:37	Juba	Yambio	12:24	12:38	Juba	Yei
									13:13	13:30	Paloich	Maban	12:07	12:47	Yambio	Juba	13:08	13:37	Yei	Yambio
									14:00	15:40	Maban	Juba					14:07	14:47	Yambio	Juba

Table 30: Proposed optimised weekly schedule for the UNHAS South Sudan mission (30/09/2019-04/10/2019) as a result of the combined MCNF and HFSMVRP models

	Monday				**Tuesday**				**Wednesday**				**Thursday**				**Friday**			
MI8-Juba	08:30	09:01	Juba	Torit	08:30	09:45	Juba	Pibor	08:30	09:11	Juba	Bor	08:30	09:11	Juba	Bor	08:30	10:11	Juba	Wiechjol LZ
	09:31	10:02	Torit	Juba	10:15	11:30	Pibor	Juba	09:41	09:44	Bor	Mingkaman	09:41	10:57	Bor	Labrab	10:41	10:56	Wiechjol LZ	Lankien
									10:14	10:52	Mingkaman	Juba	11:27	12:59	Labrab	Juba	11:26	12:38	Lankien	Bor
																	13:08	14:07	Bor	Palouny
																	14:37	16:17	Palouny	Juba
MI8-Rumbek	09:30	10:03	Rumbek	Nyal	09:30	10:32	Rumbek	Jiech	09:30	10:03	Rumbek	Nyal	09:30	10:23	Rumbek	Koch	09:30	10:03	Rumbek	Nyal
	10:33	11:06	Nyal	Rumbek	11:02	11:05	Jiench	Karmoun	10:33	11:17	Nyal	Pagil	10:53	11:02	Koch	Leer	10:33	10:58	Nyal	Haat
	11:36	12:06	Rumbek	Ganyiel	11:35	11:41	Karmoun	Gorwai	11:47	12:03	Pagil	Kurwai	11:32	12:17	Leer	Rumbek	11:28	11:41	Haat	Jiech
	12:36	13:06	Ganyiel	Rumbek	12:11	13:11	Gorwai	Rumbek	12:33	12:50	Kurwai	Malakal					12:11	12:15	Jiech	Buot
									12:20	13:18	Malakal	Mathiang					12:45	13:14	Buot	Ganyeil
									13:48	15:00	Mathiang	Rumbek					13:44	14:14	Ganyiel	Rumbek
Cessna 208-Juba	08:30	08:56	Juba	Bor	08:30	09:46	Juba	Jikmir	08:30	08:56	Juba	Bor	08:30	08:56	Juba	Bor	08:30	08:56	Juba	Bor
	09:26	09:52	Bor	Juba	10:16	10:35	Jimir	Udier	09:26	09:52	Bor	Juba	09:26	09:52	Bor	Juba	09:26	10:00	Bor	Pibor
	10:22	11:11	Juba	Pibor	11:05	11:45	Udier	Pochalla	10:22	10:42	Juba	Torit	10:22	11:41	Juba	Ulang	10:30	10:50	Pibor	Maruw
	11:41	12:30	Pibor	Juba	12:15	13:22	Pochalla	Juba	11:12	11:32	Torit	Juba	12:11	13:30	Ulang	Juba	11:20	11:28	Maruw	Boma
	13:00	14:03	Juba	Akobo	13:52	14:18	Juba	Bor	12:02	13:09	Juba	Walagal	14:00	14:41	Juba	Maridi	11:58	12:58	Boma	Juba
	14:33	15:36	Akobo	Juba	14:48	15:14	Bor	Juba	13:39	13:54	Walagal	Akobo	15:11	15:31	Maridi	Mundri	13:28	14:07	Juba	Kapoeta
					15:44	16:04	Juba	Kajo Keji	14:24	15:27	Akobo	Juba	16:01	16:28	Mundri	Juba	14:37	15:16	Kapoeta	Juba
					16:04	16:12	Kajo Keji	Nimule												
					16:42	17:06	Nimule	Juba												
Cessna 208-Rumbek	09:30	10:32	Rumbek	Ajuong Thok	09:30	10:05	Rumbek	Mabior	09:30	09:49	Rumbek	Yirol	09:30	09:49	Rumbek	Yirol	08:30	09:28	Rumbek	Walagal
	11:02	12:04	Ajuong Thok	Rumbek	10:35	10:55	Mabior	Bor	10:19	10:56	Yirol	Padeah	10:19	10:38	Yirol	Rumbek	09:58	10:27	Walagal	New Fankgak
	12:34	13:29	Rumbek	New Fankgak	11:25	12:20	Bor	Ulang	11:26	11:30	Padeah	Dindin	11:08	12:10	Rumbek	Ajuong Thok	10:57	11:52	New Fankgak	Rumbek
	13:59	14:11	New Fankgak	Malakal	12:50	12:58	Ulang	Mandeng	12:00	12:33	Dindin	Ajuong Thok	12:40	13:42	Ajuong Thok	Rumbek	12:22	14:12	Rumbek	Renk
	14:41	15:04	Malakal	Mogok	13:28	13:48	Mandeng	Motot	13:03	14:05	Ajuong Thok	Rumbek	14:12	14:42	Rumbek	Dindin	14:42	16:32	Renk	Rumbek
	15:34	15:58	Mogok	Mabior	14:18	15:12	Motot	Rumbek	14:35	15:05	Rumbek	Dindin	15:12	16:42	Dindin	Rumbek				
	16:28	17:03	Mabior	Rumbek					15:35	16:05	Dindin	Rumbek								
Cessna 208-Wau	09:30	10:11	Wau	Tambura	09:30	10:32	Wau	Keew	09:30	11:31	Wau	Renk	09:30	10:06	Wau	Agok	09:30	10:07	Wau	Mankien
	10:41	11:21	Tambura	Wau	11:02	11:44	Keew	Paloich	12:01	13:00	Renk	Yida	10:36	10:59	Agok	Kuajok	10:37	11:03	Mankien	Yida
	11:51	12:55	Wau	Old Fangak	12:14	12:35	Paloich	Renk	13:30	14:34	Yida	Wau	11:29	11:42	Kuajok	Wau	11:33	12:26	Yida	Kuajok
	13:25	14:29	Old Fangak	Wau	13:05	14:36	Renk	Wau	15:04	15:17	Wau	Kuajok	12:12	12:48	Wau	Agok	12:56	13:09	Kuajok	Wau
									15:37	15:50	Kuajok	Wau	13:18	13:31	Agok	Mankien	13:39	14:43	Wau	Ajuong Thok
													14:01	14:38	Mankien	Wau	15:13	16:17	Ajuong Thok	Wau
DCH8 106-Juba	08:30	09:06	Juba	Rumbek	08:30	09:06	Juba	Rumbek	08:30	09:28	Juba	Wau	08:30	09:28	Juba	Wau	08:30	09:39	Juba	Maban
	09:36	10:12	Rumbek	Juba	09:36	10:12	Rumbek	Juba	09:58	10:56	Wau	Juba	09:58	10:56	Wau	Juba	10:09	11:18	Maban	Juba
	10:42	11:40	Juba	Wau	10:42	11:42	Juba	Malakal	11:26	12:27	Juba	Rubkona	10:26	11:27	Juba	Rubkona	11:48	12:24	Juba	Rumbek
	12:10	13:08	Wau	Juba	12:12	13:12	Malakal	Juba	12:57	13:28	Rubkona	Rumbek	11:57	12:58	Rubkona	Juba	12:54	10:12	Rumbek	Juba
	13:38	14:47	Juba	Maban	13:42	14:22	Juba	Yambio	13:58	14:34	Rumbek	Juba	13:28	14:08	Juba	Yambio				
	15:17	16:26	Maban	Juba	14:52	15:32	Yambio	Juba					14:38	15:07	Yambio	Yei				
													15:37	15:51	Yei	Juba				

F Appendix 6 - Daily scheduling

F.1 Results for the daily test cases

Table 31: Results for the daily demand of the 30/10/2019

Model variant	Routing costs [$]	Obj.	Aircraft	Fleet
HFOM	55,085.3	-	10	DQ(1),D81(1),D82(1),Le(2),MI8J(1),MI8R(2),CR(1),CW(1)
Flight planner	54,700.8	-	-	-
HFSMVRP (fixed)	65,987.2	809,756	6	DQ(1), D8(1), CJ(1),CR(1), Do(1) MI8_R(1)
HFSMVRP (variable)	63,632.1	783,783	6	D8(1), Do(3), MI8J(1), MI8R(1)
MCNF & HFSMVRP (fixed)	55,746.7	612,913	6	DQ(1), D8(1), CJ(1),CR(1), CW(1), MI8_R(1)
MCNF & HFSMVRP (variable)	60,813.3	614,905	6	DQ(1), D8(1), Do(1), CR(1), CW(1), MI8_R(1)
MCNF & HFSMVRP (fixed/No lease costs)	47,222.6	47,222.6	10	DQ(1),D83(1),D82(1), CJ(1),CR(1),CW(1),Le(1),MI8_J(1),MI8_R(1),

Table 32: Results for the daily demand of the 01/10/2019

Model variant	Routing costs [$]	Obj.	Aircraft	Fleet
HFOM	61,077.2	-	9	DQ(1),D81(1),D82(1),Do(1),Le(2),CJ(1),CR(1),MI8R(1),
Flight planner	57,900.6	-	-	-
HFSMVRP (fixed)	69,733.3	882,280	7	D8(1), CJ(1), Do(2), Le(1), MI8J(2)
HFSMVRP (variable)	61,851.5	843,424	7	D8(1), CJ(3), Do(1), MI8J(2)
MCNF & HFSMVRP (fixed)	57,528.0	827,331	7	DQ(1), CR(1), CW(1), Do(2), MI8R(2)
MCNF & HFSMVRP (variable)	54,211.3	827,291	7	DQ(1), CJ(2), CR(1), Do(1), MI8R(2)
MCNF & HFSMVRP (fixed / No lease costs)	54,732.6	54,732.6	8	DQ(1),D82(1),CJ(1),CR(1),CW(1),Do(1),MI8R(2)

Table 33: Results for the daily demand of the 02/10/2019

Model variant	Routing costs[$]	Obj.	Aircraft	Fleet
HFOM	95,531.1	-	12	DQ(1),D81(1),D82(1),CJ(1),CW(1),Do(2),Le(1),MI8J(2),MI8R(2)
Flight planner	95,600.7	-	-	-
HFSMVRP (fixed)	102,897	1,167,394	10	DQ(1), D8(1),D82(1),CJ(1),Do(2),CR(1),Le(2),MI8J(1), MI8R(1)
HFSMVRP (Variable)	99,897	1,130,786	9	D81(2), CJ(4), Do(1), MI8J(2)
MCNF & HFSMVRP (fixed)	93,496.9	1,080,896	9	DQ(1),D81(1),D82(1),CJ(1),CR(1),CW(1),MI8J(1), MI8R(1)
MCNF & HFSMVRP (variable)	91,184.5	1,078,658	9	D8(2), CR(3), CW(1),Do(1), MI8R(2)
MCNF & HFSMVRP (fixed / No lease costs)	85,427.4	118,549	10	DQ(1),D83(1),D82(1),CJ(1),CR(1),CW(1), Do(1), Le(1),MI8R(1),MI8J(1)

Table 34: Results for the daily demand of the 03/10/2019

Model variant	Routing costs [$]	Obj.	Aircraft	Fleet
HFOM	69,466.5	-	9	DQ(1),D81(1),CJ(1),CR(1),CW(1),Do(1),Le(1),MI8R(1),MI8J(1)
Flight planner	60,200.9	-	-	
HFSMVRP (fixed)	82,289.3	869,037	8	D8(1),CJ(1),Do(2),CR(1),Le(2),MI8J(1)
HFSMVRP(Variable)	66,087.6	709,668	7	D8(1), CJ(4), Do(1), MI8J(1)
MCNF & HFSMVRP (fixed)	58,617.2	696,114	7	DQ(1), CJ(1), CR(1), CW(1), Do(2), MI8R(1)
MCNF & HFSMVRP (variable)	59,173.5	697,314	7	DQ(1), CR(1), CW(2), Do(2), MI8R(1)
MCNF & HFSMVRP (fixed / No lease costs)	57,613.7	91,701	7	DQ(1),D82(1), CJ(1),CR(1), CW(1), Do(1), Le(1), MI8R(1)

Table 35: Results for the daily demand of the 04/10/2019

Model variant	Routing costs [$]	Obj.	Aircraft	Fleet
HFOM	91,766.8	-	10	DQ(1),D81(1),D82(1), Le(1),Do(2), MI8J(2),MI8R(1), CW(1)
Flight planner	90,500.3	-	-	-
HFSMVRP (fixed)	88,976.6	915,608	7	DQ(1), D8(1), CJ(1), CR(1), Do(1), MI8J(2)
HFSMVRP(Variable)	67,000.4	899,394	7	D8(2), CJ(2), Do(1), MI8J(2)
MCF & HFSMVRP (fixed)	67,939.0	984,906	8	DQ(1), D81(1), D82(1) CJ(1), CR(1), Do(1), MI8J(1), MI8R(1)
MCF & HFSMVRP (variable)	66,089.1	892,861	7	DQ(1), D8(1), CR(2), Do(1), MI8J(1), MI8R(1)
MCF & HFSMVRP (fixed / No lease costs)	65,189.7	101,255	9	D83(1),D82(1),F50(1),CJ(1),CR(1),CW(1),Do(1),MI8J(1),MI8R(1)

F.2 Daily schedules for the MCF & HFSMVRP combination for a fixed fleet without leasing costs

Table 36: Proposed optimised daily schedule for the 30/10/2019 of the UNHAS South Sudan mission for Test case 5 (combined MCNF and HFSMVRP models without lease costs)

	Leg	Departure	Arrival	Origin	Destination	Pick-up	Delivery	Pax[-]	Load factor[%]	Block-time [h]	Cost [$]	Distance [km]	TAT [h]
	1	08:00	08:40	Juba	Yambio	r7(29)	r7(29)	29	0,59	0,7	2221	356,6	0,5
DCH8_3	2	09:10	09:50	Yambio	Juba	r25(15)	r25(15)	15	0,31	0,7	2221	356,6	0,5
Juba	3	10:20	10:56	Juba	Rumbek	r0(46)	r(0)46	46	0,94	0,6	1973	316,8	0,5
49 Seats	4	16:26	17:00	Rumbek	Juba	r8(13)	r8(13)	13	0,27	0,6	1973	316,8	-
									0,53	2,5	8388	1346,9	1,5
	1	08:00	09:00	Juba	Malakal	r3(29)	r3(29)	29	0,59	1,0	2734	532,6	0,5
DCH8-2	2	09:30	10:30	Malakal	Juba	r18(13)	r18(13)	13	0,27	1,0	2734	532,6	0,5
Juba	3	11:00	11:57	Juba	Wau	r1(34)	r1(34)	34	0,69	1,0	2633	513,0	0,5
37 Seats	4	16:03	17:00	Wau	Juba	r13(29)	r13(29)	29	0,59	1,0	2633	513,0	-
									0,54	3,9	10734	2091,2	1,5
	1	08:00	09:43	Juba	Ajuong Thok	r4(12)	r4(12)	12	1,00	1,7	2153	592,7	0,5
Cessna 208B	2	10:13	11:56	Ajuong Thok	Juba	r19(1)	r19(1)	1	0,08	1,7	2153	592,7	0,5
Juba	3	12:26	13:05	Juba	Kapoeta	r6(11)	r6(11)	11	0,92	0,7	822	226,4	0,5
12 seats	4	13:35	14:14	Kapoeta	Juba	r22(6)	r22(6)	6	0,50	0,7	822	226,4	-
									0,63	4,7	5950	1638,2	1,5
	1	11:30	12:00	Rumbek	Dindin	r9(3),r11(4),r12(5)	r12(5)	12	1,00	0,5	622	171,3	0,5
Cessna 208B	2	12:30	12:52	Dindin	Keew	-	r9(3)	7	0,58	0,4	457	125,7	0,5
Rumbek	3	13:22	13:41	Keew	Yida	-	r11(4)	3	0,25	0,3	405	111,5	0,5
12 seats	4	14:11	15:15	Yida	Rumbek	r26(1)	r26(1)	1	0,08	1,1	1332	366,8	-
									0,48	2,3	2816	775,3	1,5
	1	12:00	12:47	Wau	Dindin	r14(6), r15(6)	r15(6)	12	1,00	0,8	869	269,8	0,5
Cessna 208B	2	13:17	13:21	Dindin	Padeah	-	r14(6)	6	0,50	0,1	78	24,3	0,5
Wau	3	13:51	14:21	Padeah	Ajuong Thok	-	-	0	0,00	0,5	564	175,2	0,5
12 seats	4	14:51	15:55	Ajuong Thok	Wau	r0(12)	r0(12)	12	1,00	1,1	1187	368,3	-
									0,63	2,4	2698	837,5	1,5
	1	08:00	08:22	Juba	Torit	r5(12)	r5(12)	12	0,71	0,4	820	113,8	0,5
LET 410	2	08:52	09:14	Torit	Juba	r21(7)	r21(7)	7	0,41	0,4	820	113,8	0,5
Juba	3	09:44	10:13	Juba	Bor	r2(17)	r2(17)	17	1,00	0,5	1092	151,5	0,5
17 seats	4	10:43	11:12	Bor	Juba	r16(15)	r16(15)	15	0,88	0,5	1092	151,5	-
									0,75	1,7	3824	530,5	1,5
MI8	1	08:00	08:41	Juba	Bor	r2(11)	r2(11)	11	0,58	0,7	1950	151,5	0,5
Juba	2	09:11	09:14	Bor	Mingkaman	r17(8)	r17(8)	8	0,42	0,1	135	10,5	0,5
19 seats	3	09:44	10:22	Mingkaman	Juba	r24(2)	r24(2)	2	0,11	0,6	1833	142,4	-
									0,37	1,4	3918	304,3	1,0
	1	11:30	12:55	Rumbek	Lankien	r10(12),r12(7)	r12(12)	19	1,00	1,4	4072	316,3	0,5
MI8	2	13:25	14:14	Lankien	Dindin	r23(4)	r12(7)	11	0,58	0,8	2361	183,4	0,5
Rumbek	3	14:44	14:51	Dindin	Leer	-	-	4	0,21	0,1	317	24,6	0,5
19 seats	4	15:21	16:06	Leer	Rumbek	r27(9)	r23(4),r27(9)	13	0,68	0,8	2144	166,6	-
									0,62	3,1	8894	690,9	1,5

Table 37: Proposed optimised daily schedule for the 01/10/2019 of the UNHAS South Sudan mission for Test case 5 (combined MCNF and HFSMVRP models without lease costs)

	Leg	Departure	Arrival	Origin	Destination	Pick-up	Delivery	Pax[-]	Load factor[%]	Block-time [h]	Cost [$]	Distance [km]	TAT [h]
	1	07:00	07:34	Juba	Rumbek	r35(35), r36(36)	r35(35)	71	1,00	0,6	3927	316,8	0,5
DCHQ-400 (Trunk route)	2	08:04	08:26	Rumbek	Wau	-	r36(36)	36	0,51	0,4	2496	204,8	0,5
Juba	3	16:00	16:26	Wau	Rumbek	r37(38)	-	38	0,54	0,4	2496	204,8	0,5
71 seats	4	16:56	17:30	Rumebk	Juba	r38(26)	r37(38),r38(26)	64	0,90	0,6	3927	316,8	-
									0,74	1,9	12845	1043,3	1,5
DCH8-2	1	08:00	08:14	Juba	Yei	r7(16)	r7(16)	16	0,43	0,2	652	127,0	0,5
Juba	2	08:44	09:58	Yei	Juba	r25(8)	r25(8)	8	0,22	0,2	652	127,0	-
49 Seats									0,32	0,5	1304	253,9	0,5
	1	08:00	08:20	Juba	Torit	r2(6), r4(4)	r2(6)	10	0,83	0,3	413	113,8	0,5
	2	08:50	09:50	Torit	Maridi	r19(3)	r4(4)	7	0,58	1,0	1243	342,2	0,5
Cessna 208B	3	10:20	11:01	Maridi	Juba	r29(2)	r29(2), r19(3)	5	0,42	0,7	865	238,2	0,5
Juba	4	11:31	11:58	Juba	Mundri	r0(10), r5(1)	r5(1)	11	0,92	0,5	555	152,7	0,5
12 seats	5	12:28	13:00	Mundri	Rumbek	-	r0(10)	10	0,83	0,5	662	182,3	0,5
	6	13:30	14:25	Rumbek	Juba	-	-	0	0,00	0,9	1151	316,8	-
									0,60	3,9	4889	1346,0	2,50
Cessna 208B	1	09:00	10:50	Rumbek	Renk	r9(6)	r9(6)	6	0,50	1,8	2291	630,8	0,5
Rumbek	2	11:20	13:10	Renk	Rumbek	r25(6	r25(6)	6	0,50	1,8	2291	630,8	-
12 seats									0,50	3,7	4582	1261,6	0,5
	1	08:30	10:14	Wau	Paloich	r16(4), r17(8)	r16(4)	12	1,00	1,7	1925	597,4	0,5
	2	10:44	12:20	Paloich	Kuajok	r24(1)	r17(8)	9	0,75	1,6	1777	551,4	0,5
Cessna 208B	3	12:50	13:03	Kuajok	Wau	r27(10)	r24(1), r27(10)	11	0,92	0,2	241	74,8	0,5
Wau	4	13:33	13:46	Wau	Kuajok	r15(10), r17(1)	r17(1)	11	0,92	0,2	241	74,8	0,5
12 seats	5	14:16	14:39	Kuajok	Agok	r27(2)	r15(10)	12	1,00	0,4	430	133,5	0,5
	6	15:09	15:45	Agok	Wau	r20(8)	r27(2), r20(8)	10	0,83	0,6	664	206,0	-
									0,90	4,8	5278	1637,9	2,5
	1	08:30	09:23	Juba	Pibor	r6(14)	r(14)	14	0,78	0,9	2108	279,7	0,5
	2	09:53	10:46	Pibor	Juba	r33(18)	r33(18)	18	1,00	0,9	2108	279,7	0,5
	3	11:16	11:45	Juba	Bor	r1(15), r3(1)	r1(15)	16	0,89	0,5	1141	151,5	0,5
Dornier	4	12:15	12:37	Bor	Mabior	r18(7)	r18(7)	9	0,50	0,4	868	115,2	0,5
Juba	5	13:07	14:07	Mabior	Pochalla	r28(8)	-	8	0,44	1,0	2260	300,0	0,5
18 seats	6	14:37	14:59	Pochalla	Boma	r34(3)	r3(1)	12	0,67	0,4	861	114,3	0,5
	7	15:29	16:34	Boma	Juba	r21(3)	r21(3), r28(8), r34(3)	14	0,78	1,1	2583	342,8	-
									0,72	5,0	11929	1583,2	3,00
	1	08:30	09:59	Rumbek	Palouny	r11(15), r12(1), r14(1)	-	17	0,89	1,1	3241	251,7	0,5
	2	10:29	10:38	Palouny	Mogok	r32(1)	r12(1)	18	0,95	0,2	444	34,5	0,5
MI8	3	11:08	11:18	Mogok	Pagil	-	r14(1)	17	0,89	0,2	455	35,3	0,5
Rumbek	4	11:48	12:03	Pagil	Karmoun	r31(1)	r11(15)	17	0,89	0,3	733	56,9	0,5
19 seats	5	12:33	12:38	Karmoun	Buot	-	-	2	0,11	0,1	222	17,3	0,5
	6	13:08	14:07	Buot	Rumbek	r22(17)	r22(17), r31(1), r32(1)	19	1,00	1,0	2793	217,0	-
									0,79	2,8	7888	612,7	2,50
	1	08:30	09:00	Rumbek	Ganyiel	r8(5),r13(2),r10(5)	r8(5)	12	0,63	0,5	1450	112.63	0,5
MI8	2	09:30	09:59	Ganyiel	Buot	r223(4)	-	11	0,58	0,5	1365	106.07	0,5
Rumbek	3	10:29	10:33	Buot	Jiech	r22(1)	r10(5)	12	0,63	0,1	188	14.62	0,5
19 seats	4	11:03	11:32	Jiech	Nyal	r26(1)	r13(2)	8	0,42	0,5	1385	107.61	0,5
	5	12:02	12:35	Nyal	Rumbek	r30(4)	r22(1),r23(4),r26(1),r30(4)	10	0,53	0,6	1587	123.28	-
									0,56	2,1	5975	0,0	2,00

Table 38: Proposed optimised daily schedule for the 02/10/2019 of the UNHAS South Sudan mission for Test case 5 (combined MCNF and HFSMVRP models without lease costs)

	Leg	Departure	Arrival	Origin	Destination	Pick-up	Delivery	Pax[-]	Load factor[%]	Block-time [h]	Cost [$]	Distance [km]	TAT [h]
	1	07:00	07:34	Juba	Rumbek	r39(33),r40(20)	r39(33)	53	0,75	0,6	3927	316,8	0,5
DCHQ-400 (Trunk route)	2	08:04	08:26	Rumbek	Wau	-	r40(20)	20	0,28	0,4	2496	204,8	0,5
Juba	3	16:20	16:42	Wau	Rumbek	r41(20)	-	20	0,28	0,4	2496	204,8	0,5
71 seats	4	17:12	17:46	Rumebk	Juba	r42(27)	r41(20), r42(27)	47	0,66	0,6	3927	316,8	-
									0,49	1,9	12845	1043,3	1,5
	1	08:00	09:09	Juba	Maban	r6(43)	r6(43)	43	0,88	1,2	3820	613,3	0,5
DCH8_3	2	09:39	10:48	Maban	Juba	r30(14)	r30(14)	4	0,08	1,2	3820	613,3	0,5
Juba	3	11:18	12:19	Juba	Rubkona	r0(3),r10(46)	r10(46)	49	1,00	1,0	3349	537,7	0,5
49 seats	4	12:49	13:20	Rubkona	Rumbek	r35(29)	r0(3)	32	0,65	0,5	1686	270,6	0,5
	5	13:50	14:26	Rumbek	Juba	-	r35(29)	29	0,59	0,6	1973	316,8	-
									0,64	4,4	14648	2351,9	2
	1	08:00	08:40	Juba	Yambio	r4(19), r12(1)	r4(19)	20	0,54	0,7	1830	356,6	0,5
	2	09:10	09:39	Yambio	Yei	r25(9)	r12(1)	10	0,27	0,5	1312	255,6	0,5
DCH8_2	3	10:09	10:36	Yei	Juba	r38(1)	r28(1), r25(9)	10	0,27	0,2	652	127,0	0,5
Juba	4	11:06	12:15	Juba	Maban	r2(17),r6(20)	r6(20)	37	1,00	1,2	3148	613,3	0,5
37 seats	5	12:45	13:11	Maban	Malakal	r30(18)	r2(17)	35	0,95	0,4	1179	229,7	0,5
	6	13:41	14:41	Malakal	Juba	r22(19)	r22(19),r30(18)	37	1,00	1,0	2734	532,6	-
									0,50	4,0	10855	2114,8	2,5
	1	08:00	09:06	Juba	Motot	r7(3),r11(8)	r7(3)	11	0,92	1,1	1374	378,2	0,5
Cessna 208B	2	09:36	09:53	Motot	Ulang	r31(4)	r11(8)	12	1,00	0,3	362	99,6	0,5
Juba	3	10:23	11:42	Ulang	Juba	r37(8)	r31(4), r37(8)	12	1,00	1,3	1646	453,1	0,5
12 seats	4	12:12	13:39	Juba	New Fankgak	r8(8)	r8(8)	8	0,67	1,5	1822	501,7	0,5
	5	14:09	15:36	New Fankgak	Juba	r32(10)	r32(10)	10	0,83	1,5	1822	501,7	-
									0,88	1,1	1405	386,9	0,4
	1	08:00	09:02	Rumbek	Ajuong Thok	r13(11)	r13(11)	11	0,92	1,0	1287	354,2	0,5
Cessna 208B	2	09:32	10:04	Ajuong Thok	Rumbek	r23(12)	r23(12)	12	1,00	1,0	1287	354,2	0,5
Rumbek	3	10:34	11:23	Rumbek	Old Fangak	r17(9),r18(3)	r17(9)	12	1,00	0,8	1029	283,4	0,5
12 seats	4	11:53	13:50	Olg Fangak	Tambura	r34(1)	r18(3)	4	0,33	1,6	1971	542,7	0,5
	5	14:20	15:08	Tambura	Rumbek	r36(6)	r34(1),r36(6)	7	0,58	0,8	1001	275,6	-
									0,77	1,1	1315	362,0	0,4
	1	08:30	09:34	Wau	Yida	r19(3)	r19(3)	3	0,25	1,1	1176	365,1	0,5
Cessna 208B	2	10:04	11:08	Yida	Wau	r26(3)	r26(3)	3	0,25	1,1	1176	365,1	0,5
Wau	3	11:38	13:39	Wau	Renk	r20(10)	r20(10)	10	0,83	2,0	2235	365,1	0,5
12 seats	4	14:09	16:10	Renk	Wau	r29(10)	r29(10)	10	0,83	2,0	2235	365,1	-
									0,54	6,2	6822	1460,5	1,5
Dornier D228	1	08:00	09:26	Juba	Ulang	r5(6),r11(12)	r11(12)	18	1,00	1,4	3414	453,1	0,5
Juba	2	09:56	10:16	Ulang	Akobo	r37(2)	r5(6)	8	0,44	0,3	781	103,6	0,5
18 seats	3	10:46	11:55	Akobo	Ulang	r28(11)	r28(11),r37(2)	13	0,72	1,2	2735	362,9	-
									0,72	2,9	6930	919,7	1
	1	08:00	08:29	Juba	Bor	r1(17)	r1(17)	17	1,00	0,5	1092	151,5	0,5
LET 410	2	08:59	09:28	Bor	Juba	r21(9	r21(9)	9	0,53	0,5	1092	151,5	0,5
Juba	3	09:58	10:25	Juba	Nimule	r9(1)	r9(1)	1	0,06	0,5	1003	139,2	0,5
17 seats	4	10:55	11:22	Nimule	Juba	-	-	0	0,00	0,5	1003	139,2	-
									0,40	1,9	4190	581,4	1,5
MI8	1	08:00	08:38	Juba	Mingkaman	r3(3)	r3(3)	3	0,16	0,6	1833	142,4	0,5
Juba	2	09:08	09:46	Mingkaman	Juba	r24(11)	r24(11)	11	0,58	0,6	1833	142,4	-
19 seats									0,37	1,3	3666	284,7	0,5
	1	08:00	08:58	Rumbek	Haat	r14(17),r15(1),r17(1)	r14(17)	19	1,00	1,0	2771	215,3	0,5
	2	09:28	09:47	Haat	Old Fangak	-	r17(1)	2	0,11	0,3	892	69,3	0,5
MI8	3	10:17	10:27	Old Fangak	Kurwai	-	r15(1)	1	0,05	0,2	455	35,3	0,5
Rumbek	4	10:57	11:32	Kurwai	Gorwai	-	-	0	0,00	0,6	1676	130,2	0,5
19 seats	5	12:02	13:02	Gorwai	Rumbek	r27(17)	r27(17)	17	0,89	1,0	2862	222,3	0,5
	6	13:32	14:35	Rumbek	Nyal	r16(4)	r16(4)	4	0,21	0,6	1587	123,3	0,5
	7	15:05	15:38	Nyal	Rumbek	r33(2)	r33(2)	2	0,11	0,6	1587	123,3	-
									0,34	4,1	11830	918,9	3

Table 39: Proposed optimised daily schedule for the 03/10/2019 of the UNHAS South Sudan mission for Test case 5 (combined MCNF and HFSMVRP models without lease costs)

	Leg	Departure	Arrival	Origin	Destination	Pick-up	Delivery	Pax[-]	Load factor[%]	Block-time [h]	Cost [$]	Distance [km]	TAT [h]
	1	07:00	07:34	Juba	Rumbek	r32(40),r33(26)	r32(40)	66	0,93	0,57	3927	316,8	0,5
DCHQ-400 (Trunk route)	2	08:04	08:26	Rumbek	Wau	-	r33(26)	26	0,37	0,36	2496	204,8	0,5
Juba	3	14:30	14:52	Wau	Rumbek	r34(34)	-	34	0,48	0,36	2496	204,8	0,5
71 seats	4	15:30	16:04	Rumebk	Juba	r35(37)	r34(34),r35(37)	71	1,00	0,57	3927	316,8	-
									0,69	1,86	12845	1043,3	1,5
DCH8-2	1	08:00	08:14	Juba	Yei	r8(5)	r8(5)	5	0,14	0,23	652	127,0	0,5
Juba	2	08:44	09:58	Yei	Juba	r30(15)	r30(15)	15	0,41	0,23	652	127,0	-
49 Seats									0,27	0,47	1304	253,9	0,5
	1	08:00	08:26	Juba	Bor	r0(5),r3(3)	r0(5)	8	0,67	0,43	550	151,5	0,5
	2	08:56	10:23	Bor	Paloich	r17(6)	r3(3)	9	0,75	1,45	1812	498,7	0,5
Cessna 208	3	10:53	12:46	Paloich	Juba	r21(2)	r21(2),r17(6)	8	0,67	1,88	2347	646,2	0,5
Juba	4	13:16	13:36	Juba	Kajo keji	r5(5), r6(5)	r5(5)	10	0,83	0,33	408	112,4	0,5
12 seats	5	14:06	14:14	Kajo keji	Nimule	r24(1)	r6(5)	6	0,50	0,13	164	45,2	0,5
	6	14:44	15:00	Nimule	Torit	r29(3)	-	4	0,33	0,27	330	90,9	0,5
	7	15:30	15:50	Torit	Juba	r18(8)	r24(1),r18(8), r29(3)	12	1,00	0,33	413	113,8	-
									0,79	4,83	6024	1658,8	3
	1	08:00	08:30	Rumbek	Dindin	r9(9),r13(3)	r9(9)	12	1,00	0,50	622	171,3	0,5
Cessna 208	2	09:00	09:33	Dindin	Yirol	-	r13(3)	3	0,25	0,55	690	189,9	0,5
Rumbek	3	10:03	10:22	Yirol	Rumbek	r31(5)	r31(5)	5	0,42	0,32	388	106,9	0,5
12 seats	4	10:52	12:42	Rumbek	Renk	r10(4)	r10(4)	4	0,33	1,83	2291	630,8	0,5
	5	13:12	15:00	Renk	Rumbek	r22(9)	r22(9)	9	0,75	1,83	2291	630,8	-
									0,55	5,03	3991	1729,7	2,0
	1	09:00	09:36	Wau	Agok	r14(10)	r14(10)	10	0,83	0,60	664	206,0	0,5
	2	10:06	10:29	Agok	Kuajok	r19(3)	-	3	0,25	0,38	430	133,5	0,5
Cessna 208	3	10:59	11:12	Kuajok	Wau	r26(9)	r19(3),r26(9)	12	1,00	0,22	241	74,8	0,5
Wau	4	11:42	11:55	Wau	Kuajok	r15(3), r16(7)	r15(3)	10	0,83	0,22	241	74,8	0,5
12 seats	5	12:25	12:52	Kuajok	Mankien	r26(2)	r16(7)	9	0,75	0,45	499	154,9	0,5
	6	13:22	13:59	Mankien	Wau	r27(7)	r26(2), r27(7)	9	0,75	0,62	687	213,3	-
									0,74	2,48	2762	857,3	2,5
	1	08:00	09:13	Juba	Walagal	r7(18)	r7(18)	18	1,00	1,22	2906	385,7	0,5
Dornier	2	09:43	10:56	Walagal	Juba	-	-	0	0,00	1,22	2906	385,7	0,5
Juba	3	11:26	12:49	Juba	Jikmir	r2(5), r4(11)	r4(11)	16	0,89	1,38	3300	438,0	0,5
18 seats	4	12:19	12:33	Jikmir	Akobo	r23(5)	r2(5)	10	0,56	0,23	569	75,6	0,5
	5	13:03	14:12	Akobo	Juba	r20(6)	r20(6), r23(5)	11	0,61	1,15	2735	362,9	-
									0,61	5,20	12416	1647,8	2,0
LET 410	1	08:00	08:22	Juba	Torit	r8(5)	r8(5)	5	0,14	0,37	820	113,8	0,5
Juba	2	08:52	09:14	Torit	Juba	r30(15)	r30(15)	15	0,41	0,37	820	113,8	-
19 Seats									0,27	0,73	1640	227,6	0,5
	1	08:00	08:53	Rumbek	Koch	r11(8), r12(9)	r11(8)	17	1,00	0,88	2544	197,6	0,5
MI8	2	09:23	10:19	Koch	Malakal	r12(6)	refuel	15	0,88	0,93	2679	208,1	0,5
Rumbek	3	10:49	11:47	Malakal	Mathiang	refuel	r12(9)	15	0,88	0,97	2764	214,7	0,5
19 seats	4	12:17	14:29	Mathiang	Rumbek	r28(2)	r25(6),r28(2)	8	0,47	2,20	6310	490,2	-
									0,81	4,98	14297	1110,6	1,5

Table 40: Proposed optimised daily schedule for the 04/10/2019 of the UNHAS South Sudan mission for Test case 5 (combined MCNF and HFSMVRP models without lease costs)

	Leg	Departure	Arrival	Origin	Destination	Pick-up	Delivery	Pax[-]	Load factor[%]	Block-time [h]	Cost [$]	Distance [km]	TAT [h]
DCH8_2 (trunk route)	1	07:00	07:35	Juba	Rumbek	r27(32)	r27(32)	32	0,45	0,6	1631	171,6	0,5
Juba	2	16:00	16:35	Rumbek	Juba	r28(37)	r28(37)	37	0,52	0,6	1631	171,6	-
37 seats									0,49	1,1	3262	343,2	0,5
Fokker 50	1	08:00	09:09	Juba	Maban	r1(13), r5(28)	r5(28)	41	0,82	1,2	3987	613,3	0,5
Juba	2	09:39	10:05	Maban	Malakal	r21(20)	r1(13)	33	0,66	0,4	1493	229,7	0,5
50	3	10:35	11:35	Malakal	Juba	r16(25)	r16(25), r21(20)	45	0,90	1,0	3462	532,6	-
									0,79	2,6	8942	1375,7	1
	1	08:00	09:01	Juba	Rubkona	r3(10),r8(18)	r8(18)	28	0,57	1,0	3349	537,7	0,5
DCH8_3	2	09:31	10:34	Rubkona	Yambio	r24(28)	r3(10)	38	0,78	1,1	3456	554,9	0,5
Juba	3	11:04	11:33	Yambio	Yei	r19(12)	-	40	0,82	0,5	1592	255,6	0,5
49	4	12:03	12:17	Yei	Juba	r26(4)	r19(12),r24(28),r26(4)	44	0,90	0,2	791	127,0	-
									0,77	2,8	9188	1475,2	1,5
Cessna 208	1	08:00	08:52	Juba	Maruw	r6(6),r7(6)	r6(6)	12	1,00	0,9	1082	298,0	0,5
Juba	2	09:22	09:42	Maruw	Pibor	-	r7(6)	6	0,50	0,3	425	117,0	0,5
12 seats	3	10:12	11:01	Pibor	Juba	r23(12)	r23(12)	12	1,00	0,8	1016	279,7	-
									0,83	2,0	2523	694,7	1
	1	08:00	09:02	Rumbek	Ajuong Thok	r9(7)	r9(7)	7	0,58	1,0	1287	354,2	0,5
Cessna 208	2	09:32	10:34	Ajuong Thok	Rumbek	r17(12)	r17(12)	12	1,00	1,0	1287	354,2	0,5
Rumbek	3	11:04	12:06	Rumbek	Ajuong Thok	r9(5),r10(7)	r9(5)	12	1,00	1,0	1287	354,2	0,5
12 seats	4	12:36	12:41	Ajuong Thok	Yida	r17(3)	r10(7)	10	0,83	0,1	103	28,4	0,5
	5	13:11	14:15	Yida	Rumbek	r20(9)	r20(9), r17(3)	12	1,00	1,1	1332	366,8	-
									0,88	4,3	5296	1457,8	2
	1	08:00	09:15	Wau	Bor	r13(9)	r13(9)	9	0,75	0,6	660	204,8	0,5
	2	09:45	10:39	Bor	Mandeng	r15(1)	-	1	0,08	1,3	1384	429,6	0,5
Cessna 208	3	11:09	11:28	Mandeng	Udier	r22(8)	r15(1)	9	0,75	0,9	992	307,9	0,5
Wau	4	11:58	13:06	Udier	Ajuong Thok	-	-	8	0,67	0,3	349	108,2	0,5
12 seats	5	13:36	14:38	Ajuong Thok	Rumbek	r17(2)	r17(2),r22(8)	10	0,83	1,1	1260	390,9	0,5
	6	15:08	15:44	Rumbek	Wau	-	-	0	0,00	1,0	1141	354,2	-
									0,51	5,2	5786	1795,7	2,5
Dornier 228	1	08:00	08:53	Juba	Pibor	r0(5), r7(13)	r7(13)	18	1,00	0,9	2108	279,7	0,5
Juba	2	09:23	10:00	Pibor	Bor	-	r0(5)	5	0,28	0,6	1460	193,8	0,5
18 seats	3	10:30	10:59	Bor	Juba	r14(18)	r14(18)	18	1,00	0,5	1141	151,5	-
									0,76	2,0	4709	625,0	1
	1	08:00	08:38	Juba	Mingkaman	r0(12), r2(1)	r11(1)	13	0,68	0,6	1833	142,4	0,5
MI8	2	09:08	09:11	Mingkaman	Bor	r18(3)	r0(12)	15	0,79	0,1	135	10,5	0,5
Juba	3	09:41	10:22	Bor	Juba	r14(11)	r14(11), r18(3)	14	0,74	0,7	1950	151,5	0,5
19 seats	4	10:52	12:24	Juba	Labrab	r4(6)	r4(6)	6	0,32	1,5	4372	339,6	0,5
	5	12:54	14:26	Labrab	Juba	-	-	0	0,00	1,5	4372	339,6	-
									0,51	4,4	12662	983,6	2
	1	08:00	09:08	Rumbek	Palouny	r11(3),r12(10)	r12(10)	13	0,68	1,1	3241	251,7	0,5
MI8	2	09:38	09:59	Palouny	Weichjol LZ	-	-	3	0,16	0,4	1020	79,3	0,5
Rumbek	3	10:29	11:16	Weichjol LZ	Mathiang	r25(3)	r11(3)	6	0,32	0,8	2223	172,7	0,5
19 seats	4	11:46	12:44	Mathiang	Malakal	-	refuel	3	0,16	1,0	2764	214,7	0,5
	5	13:14	15:10	Malakal	Rumbek	refuel	r25(3)	3	0,16	1,7	4849	376,7	-
									0,29	4,9	14097	1095,1	2

G Appendix 8 - Published South Sudan UNHAS documents

G.1 UNHAS weekly schedule

WFP UNHAS Humanitarian Air Service

South Sudan
Weekly Schedule
Effective, **01 November 2020**

Depart From	Destinations	MONDAY	TUESDAY	WEDNESDAY	THURSDAY	FRIDAY	SATURDAY	SUNDAY
	Agok	R✈	R✈		R✈	R✈		
	Ajuong Thok		✈		✈	✈		
	Akobo	B✈			B✈			
	Alek		R✈		R✈			
	Aweil	✈	✈		✈	✈		
	Boma		✈					
JUBA	Bor	✈	✈ ref. #3	✈	✈ ref. #3	✈		
	Buong			B✈				
	Dablual	R✈						
	Ganyiel		R✈	R✈		R✈		
	Gorwai		R✈					
	Jiech		R✈					
	Jikmir				✈			
	Juba	✈	✈	✈	✈	✈		
	Kajo Keji				✈			
	Kapoeta	✈	✈	✈				
	Karam					✈		
	Koch	R✈			R✈			
	Kuajok		R✈		R✈			
	Kurwai			R✈				
	Lankein	B✈				B✈		
	Leer					R✈		
	Maban	✈		✈		✈		
	Mabior		B✈					
	Malakal	✈		✈		✈		
	Mankien	R✈			R✈			
	Maridi		✈					
	Mathiang				✈			
JUBA	Mayendit					✈	RECOVERY & SPECIAL FLIGHTS	STANDBY
	Mingkaman	✈		✈		✈		
	Mogok		R✈					
	Motot	✈						
	Mundri		✈					
	New Fangak			R✈				
	Nimule				✈			
	Nyal		R✈	R✈		R✈		
	Old Fangak			R✈				
	Padeah	R✈						
	Pagil		R✈					
	Paloich		✈		✈			
	Pibor		✈ ref. #4			✈ ref. #4		
	Pieri					✈		
	Pochalla		✈		✈			
Wau	Raja			W✈ ref. #6				
	Renk		✈		✈			
	Rubkona	✈	✈		✈			
	Rumbek	✈	✈	✈	✈	✈		
	Tambura			✈				
	Torit	✈	✈	✈				
	Touch Riak					R✈		
JUBA	Ulang	✈		✈				
	Wau	✈	✈	✈	✈	✈		
	Wiechjol					B✈		
	Yambio	✈		✈		✈		
	Yei		✈		✈			
	Yida	✈		✈		✈		
	Yirol				R✈			

UNHAS will remain available during "no-fly days" for emergency medical evacuations and security relocations

SAVING LIVES CHANGING LIVES

DISCLAIMER: I. UNHAS provides humanitarian air services to locations within South Sudan where the population is in need of aid in strict adherence to the humanitarian principles of neutrality, impartiality and humanity. The passenger has overall responsibility of ensuring that she/he is comfortable with assigned UNHAS flight routing before boarding the flight. UNHAS must be notified when the passenger is not comfortable with the route, for further advice.

Upcoming UNHAS no-fly days: 25 December 2020, 26 December 2020 and 28 December 2020

II. Flights are subject to change due to operational exigencies or external factors.

REMARKS

NOMINAL FEE
All UNHAS destinations are charged
USD 275. per passenger; USD 3. per kg cargo

UNHAS Finance Focal Points Contacts:
unhasjuba.finance@wfp.org
peter.kenyi@wfp.org 211922465577
patrick.duku@wfp.org 211921666757

Special Information on Destinations
1 ✈ UNHAS flight
2 W✈ R✈ B✈ = connections from Wau, Rumbek, or Bor
3 Tuesdays & Thursdays connections out of Bor
4 Pibor served from Bor, on demand
5 Kodoc and Aburoc on demand only
6 Raja is served from Wau
7 Kajo Keji served on Thursdays
8 Pochalla and Boma served from Bor, on demand , (with a minimum of 4 passengers)
9 Dadlual, Nimule and Tambura subject to ops capability

Schedule Changes
1 ABUROC removed. RUBKONA & AJUONG THOK adjusted.

Intermittent CONNECTIONS
of paired locations served on days indicated:
WAU - AWEIL on Mondays and Thursdays
AWEIL - WAU on Tuesdays and Fridays
TORIT - KAPOETA on Tuesdays
KAPOETA - TORIT on Thursdays
GANYIEL - NYAL on Fridays
NYAL - GANYIEL on Wednesdays
YIDA - AJUONG THOK on Mondays
AJUONG THOK - YIDA on Fridays

PROHIBITED
a. UNIFORMED PERSONNEL or INDIVIDUALS wearing camouflage of military uniforms
b. Weapons and Ammunition are not authorized to be transported on board UNHAS flights

PASSENGER INFORMATION
i. Valid Photographic Agency ID Card as per the Agency/Organisation indicated on e-ticket or
ii. Valid Photographic ID card with
iii. Letter of Introduction endorsed by the Agency Head
iv. Baggage Weight Limit is 20 kgs (2 pieces only) and 5 kgs hand luggage (1 piece only) per passenger
v. Maximum Excess Luggage is 25 kgs. (2 pieces only)
Excess Lugguage - A ticket is requred for acceptance at check-in.
vi. Luggage is to consist of a passenger's personal belongings only
vii. Juba Airport check-in commences at 7:00am
viii. Juba check-in is 2 hours before departure & from field locations, 1 hour before departure
ix. Dangerous Goods (DG) must be declared and processed prior to flight date at UNHAS Cargo:

southsudan.unhascargo@wfp.org

DEADLINES: BOOKING & CANCELLATION

Travel Day	Book before:	Cancel before:
Monday	Thursday - 10:00	Friday - 10:00
Tuesday	Friday - 10:00	Monday - 10:00
Wednesday	Monday - 10:00	Tuesday - 10:00
Thursday	Tuesday - 10:00	Wednesday - 10:00
Friday	Wednesday - 10:00	Thursday - 10:00

UNHAS Booking Office
Official Work Hours - Telephone and e-mail:
Mondays to Fridays: 08:30 to 17:00
Booking Office: +211 922 465 575 / +211 922 465 240
Outside Official working hours - MEDEVAC or SECEVAC:
Heads Booking: +211 922 465 565 / +211 922 465 701
unhas.southsudan@wfp.org

Figure 27: Published UNHAS flight schedule for South Sudan, effective 01/11/2020) UNHAS, (2020)

II

Literature Study

1
Introduction

1.1. Background

Logistics and transportation have a direct impact on the effectiveness and efficiency of humanitarian missions, amounting to the second largest expenditure for international humanitarian organisations [59]. Yet, Operations Research (OR) has hardly been applied to the humanitarian sector compared to the commercial one, mostly due to the unpredictable, last-minute and dangerous nature of it's operations. With humanitarian crises on the rise and NGO funding stagnating, there is an increasing need for improvements in effectiveness and efficiency of humanitarian air operations [49]. In 2019, UNHAS transported approximately 412,000 passengers to and from areas affected by crises. In South Sudan alone, UNHAS operated it's biggest fleet with 10 passenger aircraft and 4 helicopters [61]. Despite this, most planning processes such as the creation of weekly flight schedules and daily routings are still carried out manually. An audit document released by the World Food Program (WFP) in 2020 revealed several downfalls regarding UNHAS's ageing fleet, it's aircraft contracting process and the lack of automation and accountability in their operations [14]. It brought to light the need for UNHAS to implement data-driven decision support tools to help increase the effectiveness and efficiency of it's overall air operations. Following in the footsteps of [43] and [45] on the optimisation of daily routing and scheduling for humanitarian air operations, the proposed study will focus on optimising the UNHAS fleet planning process by finding an optimal fleet and an optimal weekly flight schedule.

Fleet planning is a common problem addressed in industry and academia for all transportation modes. It consists of selecting an optimal fleet by addressing the following 3 questions: Which type of vehicle should I buy? How many of each do I need? When should I acquire them? Minimum-cost, multi-commodity network flow models and Mixed-Integer Linear Programming (MILP) models are most commonly used to solve these optimisation problems [18]. However in order to take into account routing considerations and the intricacies of the problem at hand, a VRP was deemed most appropriate. VRPs and their variants are NP-hard, combinatorial optimisation and integer programming problems, extensively studied over the past decades in the field of OR. The novelty of the proposed research problem lies in the fact that no such formulation or model have been created to tackle simultaneously aircraft fleet planning and weekly scheduling, in the humanitarian sector.

1.2. Research Objective and Context

The main objective of this study is to "**Contribute to improving accountability and automation in humanitarian fleet planning by creating a fleet sizing and weekly flight scheduling decision support tool which can increase the efficiency and effectiveness of humanitarian air operation.**" The context of this thesis will focus on UNHAS passenger transport operations, using South Sudan as cases study. The 2020 audit of the WFP aviation and UNHAS has revealed the need for more accountability and automation in it's fleet planning process and weekly flight scheduling. There is an opportunity to create a data-driven decision support tool which can help in determining an optimal fleet and weekly schedules for humanitarian air operations. This could highly benefit the service and it's users in the long term by reducing costs and increasing demand satisfaction. Fleet planning combined with network design or stochastic models cannot capture in enough detail complex air operations on a tactical to operational time-frame. They focus on the long-term planning horizon where less information is know and make fleeting decisions based flight leg frequency per aircraft type, without taking into account detailed routing considerations. A FSMVRP formulation such as the one presented by [28] incorporating Multi-depot, Split loads and Simultaneous Pick up and Delivery is deemed to be the most appropriate model to tackle fleet planning and weekly humanitarian air operations. This is because it can easily correlate passenger flows with individual aircraft flows, giving much more flexibility in modelling the specifics of humanitarian operations. Possibilities like splitting passenger requests and keeping track of individual aircraft paths are easily implemented and will allow a more comprehensive understanding of the operations. In order to take into account South Sudan's changing seasons and corresponding operational challenges, different strategies will be created to find an optimal fleet for both the dry and rainy seasons, similarly to [53]. The size of the problem will most likely lead to the implementation of heuristics or meta-heuristics such as a Tabu Search algorithm to find sub-optimal solutions.

1.3. Research Questions

To which extent can a FSMVRP model determine an optimal fleet size and composition for weekly humanitarian air operations using as input a forecasted demand and corresponding origin and destination pairs?

1. **To which extent can the Multi-Depot, Split Load and Simultaneous Delivery and Pick-Up VRP variants be combined with the FSMVRP formulation to create a consistent MILP model?** A novel formulation based on [28] FSMVRP will need to be created combining different VRP variants. This implies that the original objective functions, decision variables and constraints might not be consistent with each other and adaptations/additions will be needed

2. **Can such a model be used to simultaneously size an aircraft fleet and determine a feasible weekly preliminary flight schedule?** The UNHAS weekly preliminary schedule drives the humanitarian community's flight bookings. Weekly schedules repeat for multiple months without changes. Determining an optimal fleet for UNHAS operations can be assumed to be equivalent to determining the optimal fleet for 1 week of operations.

3. **To which extent can the model improve the efficiency and effectiveness of humanitarian air operations?** Unlike traditional airlines, humanitarian aviation efficiency is mostly defined by minimizing operational and vehicle acquisition costs. Effectiveness is mainly defined by maximizing the humanitarian demand satisfaction. Both of these parameters can be compared to current operations and indicate whether a better flight schedule and fleet can be found, along with accounting for different performance indicators. One must still determine to which extent the assumptions and modeling simplifications limit the real-life case.

4. **Can such a model determine an optimal fleet adapted to at least 2 periods with different operational constraints?** South Sudan experiences 2 different seasons over the year: the dry season and the wet season. During the wet season, certain airports are only accessible with specific aircraft. During a UNHAS mission, the fleet reviewed every 3 to 6 months: finding an optimal fleet suited to both seasons would greatly affect UNHAS's contracting and asset renewal decision. Similarly to [53], different strategies will need to be developed in order to find a multi-period optimal fleet.

1.4. Report Structure

In chapter 2, the field of humanitarian aviation is explored. The predominant stakeholders are presented along with their operating environments. A detailed comparison is made between the traditional airline and humanitarian planning processes. Finally, the main operational and organisational challenges that face UNHAS and it's users are defined, based on an audit of the 2020 World Food Program (WFP) Finance Committee. In chapter 3, an overview of common optimisation techniques used in Operations

Research is given. The chapter focuses on network flow problems along with their linear programming formulations and applications. Chapter 4 dives into fleet planning by reviewing mathematical models for different applications related to network design, demand uncertainty and routing. Aircraft routing and scheduling is covered in chapter 5 by comparing the traditional commercial airline methods to the humanitarian ones.

2

Humanitarian air operations

This chapter provides an overview of the different aspects of humanitarian aviation and defines the operating environment and challenges it is faced with. Firstly, the main operators and users are defined in Section 2.1. In Section 2.2, a comparison is made between the humanitarian and commercial aviation planning process. The South Sudan UNHAS mission is then analysed in Section 2.3.

2.1. Humanitarian aviation services

Humanitarian aviation is divided in two groups, peacekeeping air services related to military interventions, and humanitarian relief services. This literature study will focus on the latter. In Subsection 2.1.1 and 2.1.2, UNHAS and Non-Governmental Organisations (NGO) are defined respectively.

2.1.1. United Nations Humanitarian Air Services

At the time of writing, 1 in 9 people on Earth are malnourished. The WFP is the largest humanitarian organization in the world actively addressing the issue of malnutrition and has taken on the difficult mandate of ending world hunger by 2030 [62]. The WFP strives to accomplish this goal by providing food-related emergency assistance to affected communities all over the world. The WFP's aviation service is a crucial part of this process, facilitating access to remote areas that others cannot reach. When trucks, boats and other transportation means are not available, humanitarian cargo is delivered in the form of airfreight, airlifts and airdrops by a fleet of around 90 aircraft and helicopters. The WFP aviation service also facilitates the transportation of essential emergency staff. In January 2004, the UNHAS was created and became the WFP's passenger and light cargo transport service. It's mandate is to provide "safe, efficient, responsive and cost-effective" air transport for the United Nation's staff, Non-Governmental Organisation (NGOs) and the wider humanitarian community. The WFP aviation distinguishes 3 different types of services:

1. UNHAS: light cargo and passengers transport to areas not accessible by other transportation means;

2. Exceptional aviation services : airdrops and medical evacuations when no other service is available;

3. Aviation services for external clients: transport of diplomats, donors, special cargo.

The research presented will focus on passenger transport and therefore UNHAS operations. In 2019, UNHAS operated in more than 20 countries as can be seen in Figure 2.1, transporting approximately 404,000 passengers and 3,200 Mt of cargo to and from areas affected by crises. With the intention of providing equal access to aviation services to as many humanitarian actors as possible, its passengers are on average made up of 55% NGO workers, 41% UN staff and 4 % other [61].

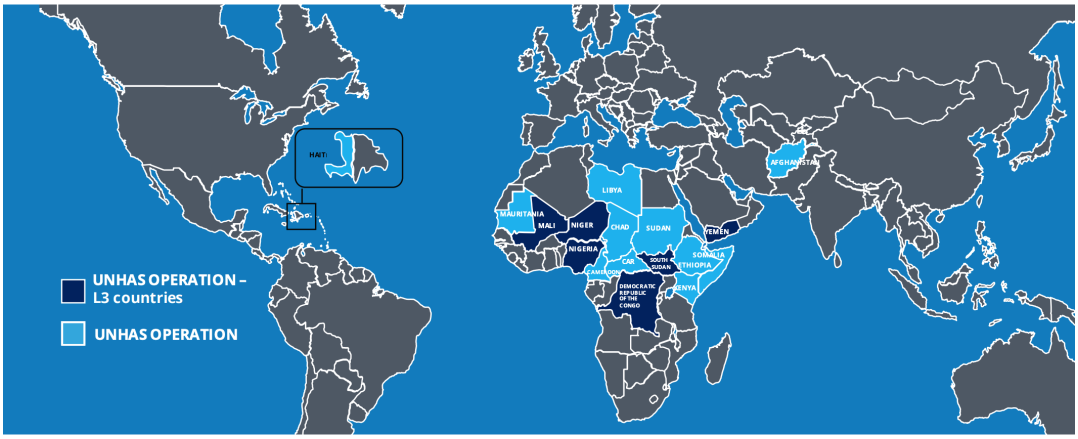


Figure 2.1: World map of UNHAS interventions [61]

2.1.2. Non-Governmental Organisations

NGO passengers are UNHAS's largest user group. In the broadest terms, NGOs are non-profit, self-funded associations functioning independently from governments. They act according to a specified mandate centered around improving development, aid and philanthropy at a community, national or international level. Different types of NGOs exists, however the focus will be on Operational NGOs. These organisations design, implement and coordinate projects with the goal of providing relief directly to endangered populations through logistical help, access to food, healthcare and more. NGOs are more than half of the passengers transported by UNHAS and drive a big part of network design. The primary use of air transport for NGOs is to provide quick and safe access for their staff to remote areas not reachable by other means. Aviation is also used to transport high value items such as cash in order to avoid targeting and theft risks. Certain NGOs have specialised themselves in Air Transport and provide an alternative to UNHAS when it is unavailable or when it cannot satisfy all the humanitarian demand. This is the case for Aviation Without Borders (ASF) whose mission is to:

> "[...] provide aeronautical expertise and resources to the humanitarian aid sector and sustainable development. Neutral, independent and not in competition with local stakeholders, ASF-International goes where others do not (or no longer go) to allow aid and development stakeholders to access isolated groups in need [1]."

Based on publications made available by ASF-Belgium, a significant discrepancy has been identified between the needs of the humanitarian community in terms of air transport, and what is being provided by UNHAS or other established operators. Using Mali and Burkina Faso as a case studies, and in close collaboration with the international NGOs operating in the respective countries, ASF Belgium performed a critical analysis of the air transport services offered by UNHAS and ECHO-Flight [2][3]. The analysis highlighted that throughout their report that in dangerous environments, NGOs are not offered enough air transport flexibility by humanitarian common services such as UNHAS and ECHO-Flight to allow their international staff to travel to and from the destinations safely. More specifically, the dangerous climate in these countries requires NGOs to be able to perform Remote Control missions. These missions should allow staff to visit certain risky destinations for a limited amount of time, ideally enabling return trips within the same day. However, the current flight rotation cycles are insufficient or badly distributed throughout the monthly schedule, making remote monitoring impossible and leaving NGO staff stranded for multiple days in locations where their safety is compromised. This in turn limits the help that NGOs can provide to affected populations, which reflects badly on donors and institutions who fund considerable amounts of these missions.

2.2. Humanitarian vs. Airline planning process

The humanitarian planning process is significantly different to the commercial airline planning process. Minimizing costs and maximizing demand satisfaction is substituted in place of profit maximisation. Humanitarian demand satisfaction drives the decision making process and network design instead of commercial revenue management. Unexpected disasters lead to unexpected demand peaks and require a high level of flexibility from the WFP aviation services. This contrasts with the cyclical / seasonal markets which drive commercial aviation. The UNHAS fleet is mostly operated under monthly wet lease contracts whereas most legacy commercial airlines purchase their fleets and operate them for multiple years. Average load factor for UNHAS flights is around 50% compared to an average of 82.5% for commercial airlines in 2020, according to IATA statistics [29]. Cost per available seat kilometer (CASK) are of around 1.43 USD, considerably high compared to commercial airlines who keep their CASK between 0.09 and 0.13 USD [14]. In Figure 2.2, a general overview of the commercial airline planning process is presented, the same is done afterwards in Figure 2.3 for the humanitarian planning cycle. A description of the main decisions made for each block is described below [8].

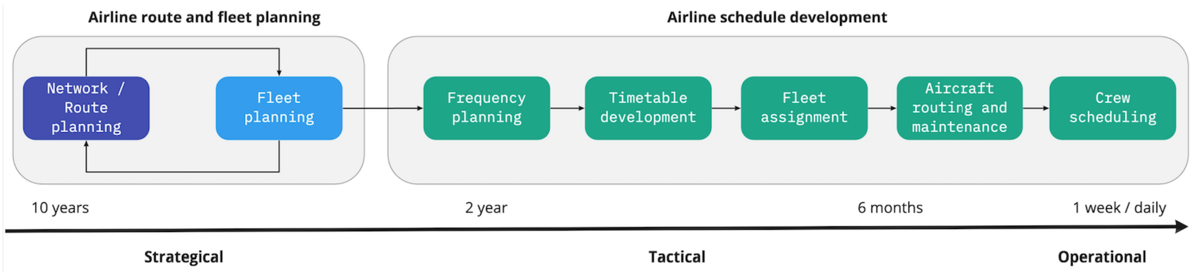


Figure 2.2: General airline planning process

- **Network and route planning**: Determines the type of network to be operated (hub & spoke / point-to-point / hybrid) and which routes to fly in order to maximize profitability. This step relies heavily on passenger forecasting and network flow models which try to simulate the behaviour of future clients and account for changes in the aviation industry.

- **Fleet planning**: Determines the amount and type of aircraft to be operated by the airline for a defined period in the future. Decisions such as aircraft acquisition and leasing over time are also incorporated in this step, which is highly correlated with the previous planning step as acquiring new aircraft may lead to new profitable routes and vice versa.

- **Frequency planning & timetable development**: Determines the amount of times a route is flown. Increasing frequency increases an airlines market share and the amount of passengers it can capture. This in turn improves it's competitive power and passenger convenience. A timetable is created to match frequencies with peak departure times and maximize aircraft utilisation.

- **Fleet assignment**: Assigns a aircraft type to a flight leg, often with the aim to minimize operational costs, maximize revenue or aircraft utilisation. The inputs to this step are a fleet plan, a departure schedule and multiple operational constraint. Time-space networks are most commonly used to simulate feasible aircraft paths for all schedule and fleet combinations .

- **Aircraft routing and maintenance** : Assigns a tail number to a flight leg. This block goes one step further by considering individual aircraft instead of aircraft types. Integrating it with other blocks of the planning process such as fleet assignment and timetable development is becoming increasingly popular in order to simulate more accurately the airline decision making process.

- **Crew scheduling**: Assigns a cockpit and cabin crew to specific flight leg or route. The objective is often to minimizes the crew costs while maximizing their satisfaction and taking into account operational constraints and legalities relating to rest times, qualifications, licensing etc.

In contrast to the long-term commercial airline planning process, humanitarian flight planning relies on medium to short-term planning decisions as shown in Figure 2.3. A description of the main operational planning decision steps can be found below based on the WFP Aviation Air Transport Manual [63].

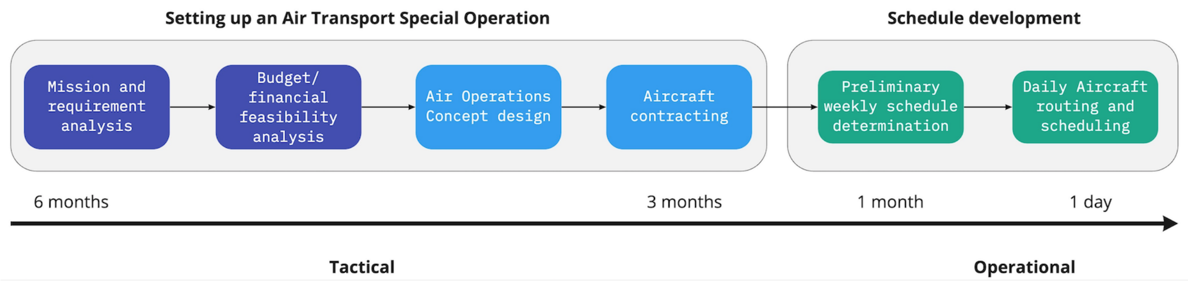


Figure 2.3: General humanitarian planning process

- **Mission and requirement analysis**: In this first step, the Chief of the Aviation Services (OSCA) determines if the use of WFP Aviation Services is possible and if so, absolutely necessary, depending on the crisis and type of mission [65]. International, regional and last mile air transport is taken into consideration and selected if no other transportation means are available.
- **Budget and financial feasibility analysis**: OSCA's Budget Unit investigates whether the air asset part of the special operation under consideration is financially feasible. It must take into account factors such as donor contributions, funding campaigns, operational budgets, future unexpected expenses etc. This step determines the total amount of money that can be allocated to air related assets and mission elements and most importantly, the available budget for aircraft leasing and operational costs.
- **Air Operation Concept design (AOC)**: Determines the type of network that will be operated by the aviation services within or close to the affected area in question. The creation of the AOC is mainly based on the affected country's or region's geographical characteristics, the humanitarian need, available infrastructure and gravity/urgency of the emergency. This step is performed by the chief of OSCA and the Chief Air Transport Officer (CATO) appointed to the mission. The CATO is able to provide input and insight obtained from the field in order to help steer decisions such as which types of aircraft and helicopters are best suited for the mission, which airports and airfields are available, whether airdrops are needed and more.
- **Aircraft contracting**: The Procurement Office of the WFP Supply Chain Division and OSCA's Air Transport Unit are responsible for aircraft contracting once the AOC has been defined or when an air charter Request For Offer (RFO) is issued by a CATO / County Office for an on-going mission. Two types of contracts exists: Air Freight Service Agreements (AFAs) and Air Charter Agreements (ACAs). AFAs only deal with contracting cargo space on pre-existing flights whereas ACAs deal with the contracting of entire aircraft in the form of ad-hoc air charter for passengers or cargo. A diagram of the main considerations taken by the WFp and UNHAS when selecting appropriate aircraft fleets can be found in Figure 1.2.
- **Preliminary weekly scheduling**: Every month, a User Group meeting is held at respective field offices [58]. Users of UNHAS in the affected county establish a monthly forecasted amount of passengers and their associated origin and destination pairs. New operational constraints and their consequences are discussed such as road closures, unavailable airstrips, increased insecurity levels etc. These inputs are used to created weekly preliminary schedules which drive passenger bookings. These schedules are usually drafted by hand from experience. The aim is to provide transport to as many passengers as possible while keeping operational costs reasonably low.
- **Daily routing and scheduling** On a daily basis, the schedule is revised and modified based on the bookings that have been made by the humanitarian community. Flights with little or no bookings are canceled before a detailed routing is created for the daily operations. The aim is to transport all passengers while minimizing the transportation costs.

2.3. South Sudan UNHAS mission

The Republic of South Sudan is the country with the largest UNHAS mission in the world, currently more than two-thirds of the population is in critical need of humanitarian assistance. UNHAS started operating in South Sudan in 2011, the year North and South Sudan officially became separate nations. Since 2005, South Sudan's population has witnessed a bloody civil war, violent inter-community clashes, heavy seasonal flooding leading to displacements, extreme famine and more recently risks of disease outbreaks [61]. This environment also creates significant challenges for humanitarian operations. The poor infrastructure and growing insecurity limits the use of road networks, which is even more restricted during the rainy season when flooding deprives access to more than half all roads in the country.

UNHAS operates a hub and spoke network in South Sudan, similar to the diagram depicted in Figure 2.4. An Airport of Entry is designated to receive and concentrate all flows of supplies, passengers and flights coming into the affected country. This airport is usually the capital international airport as it often has the largest and most convenient facilities necessary for air transport operations, easy access to critical resources like fuel and food, and provides international connections for incoming humanitarian staff. This humanitarian hub is then connected to other smaller secondary regional hubs or directly to delivery airfields. This network influences the fleet that is used and considerations such as providing helicopter flights to areas where only helipads can be used and not runways. This is often the case during the rainy season in South Sudan where floods damage roads and runways and render them unusable by ground transport or aircraft.

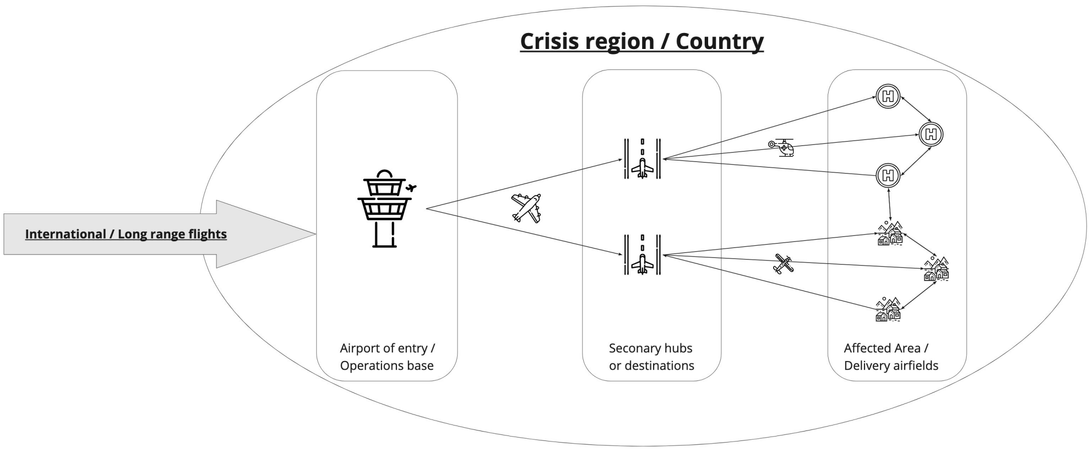


Figure 2.4: Example of the Hub & Spoke AOC model

2.4. Gap analysis

Request for UNHAS services have seen a significant rise in the years 2017 to 2019. The countries in which UNHAS operates has risen by 4, and the passengers transported has increased by 26.4%. This implies that the service needs to be able to scale rapidly in all domains and remain flexible, from aircraft contracting and funding to field operations and accountability. According to discussions with experts on the matter and a WFP audit document, the following areas of improvement have been identified [14].

Accountability and funding

The external audit recognises a lack of accountability when reporting on UNHAS operations. Donors and the wider humanitarian community would appreciate more transparency on how funding is used by UNHAS in the context of air operations. A motivation for this is the fact that UNHAS expenditures are often lower than the predicted budget for specific missions and donor funding is sometime used as cumulative carry-over. This comes as a surprise to the humanitarian community because UNHAS still charges NGOs for flight tickets in a "cost-recovery" scheme. By the end of 2019, missions in South Sudan and Somalia were already funded for 2020. Furthermore, this cost-recovery scheme is rigid and does not account for differences in the services provided by UNHAS, for example whether local or international NGOs are being transported. The 2020 WFP audit recommends that the WFP should investigate how the cost-recovery mechanism could be more suited to the users of the service by diversifying rates and taking ticket sales into account when creating budgets and incorporating funding expenditure decisions.

Performance management

Performance management is related to accountability and is crucial when making decisions for future improvements. The Performance Management Tool (PMT) is available to the UNHAS staff. It uses TakeFlite data (the Electronic Flight Management Application allowing online passenger bookings) to track the overall performance of the UNHAS fleet. The PMT evaluates performance based on multiple Key Performance Indicators (KPIs) such as aircraft occupancy rate, number of passengers served, contracted hours against hours flown, number of requests not filled, etc. It is however mostly used for monitoring and real-time assessment of operations. A data driven tool to support tactical and operational decisions has not yet been developed to our knowledge.

Aircraft contracting

Chartered air assets are often contracted under very short notice compared to commercial aviation due to the fact that UNHAS and the WFP must be flexible and able to respond to emergency situations rapidly. Aircraft procurement happen around 3 months prior to the assets deployment on the field. Renewal of air chartered contracts can happens on an even shorter time frame, sometimes up to 2 days before the expiry of the air charter agreement. No data driven decision support tool is available for aircraft chartering and contract renewal procedures which is usually based on the UN staff and CATO's field experience. The external audit also observed that the whole process is poorly documented and contract duration/extension poorly justified. Aircraft are contracted under a minimum guaranteed hours (MGH) agreement which represent the amount of block hours that are expected to be own by an aircraft and the corresponding price, paid monthly to the lessor. Unfortunately, these are often miscalculated and each hour under the MGH is lost or each hour own above the MGH is billed at a higher cost. Specifically to the case of South Sudan, two of the contracted aircraft flew 19% and 35% less than their originally agreed MGH, leading to questions regarding the robustness of the contracting process and the estimation of block hours needed to fly each asset. In other UNHAS mission (Yemen, Nigeria and Democratic Republic of Congo), the initial contracted flying hours were surpassed by at least 30 hours leading to high costs.

Ageing fleet and environmental impact

The WFP aviation fleet is ageing. Out of a fleet of approximately 90 different models of aircraft in 2019, production has been discontinued in 6. This account for 45% of all chartered aircraft (put in service circa 1980's) implying that the WFP and UNHAS will need to re-evaluate their chartered air assets in the near future. The external auditors recommend that the WFP must look 5 to 10 years in the future and try to anticipate how many and which type of air assets will be needed. Furthermore, the use of older aircraft coupled with the increase of WFP's aviation related activities in the last few years has not proven to be environmentally friendly. In Figure 2.5, it can be seen that emissions have increased drastically since 2013. These are due to the recent implication of UNHAS and WFP Aviation in larger humanitarian missions such as South Sudan and Democratic Republic of Congo, which rely heavily on air transport. The reporting on the environmental impact lacks detail, for example it does not distinguish between WFP aviation or UNHAS activities, cargo or passenger transport. When deciding to renew it's fleet, UNHAS and the WFP are incentivised to consider aircraft that are more environmentally friendly.

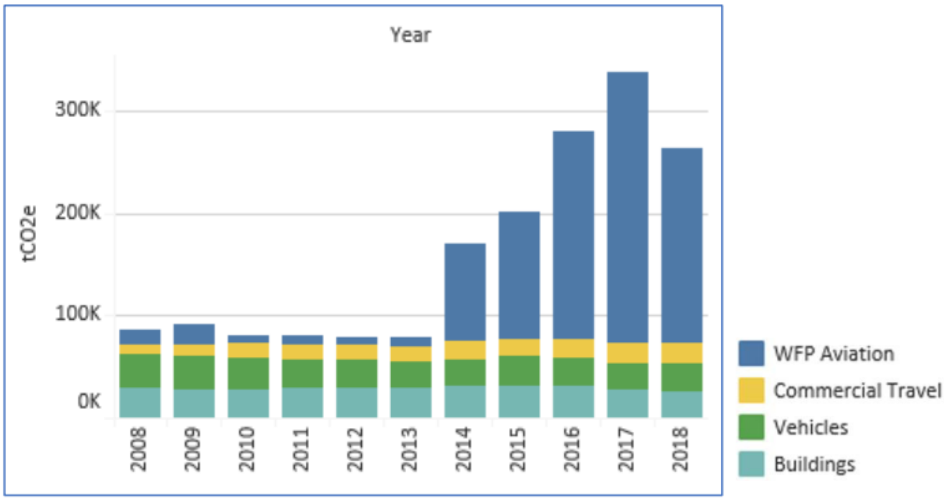


Figure 2.5: Evolution of WFP CO2 emissions per year and per sector [14]

2.5. Research focus

The focus of this thesis will lie on UNHAS passenger transport operations, using South Sudan as case study. Therefore, the network that will be under consideration is a Hub & Spoke network centered around the capital Juba and secondary hubs Wau, Rumbek and Bor. Although the whole humanitarian community will be taken into account, NGO needs will be mostly considered as they are the primary users of the service. Subsection 2.1.2 outlined the fact that NGO's need a more flexible schedule with more weekly aircraft rotations, providing opportunities for shorter stays in dangerous destinations. Section 2.2 highlights the main differences between the commercial and humanitarian planning processes, where minimizing costs and maximising demand satisfaction is a central objective. In Section 2.4, the 2020 Finance Committee audit of UNHAS has revealed the need for more accountability, transparency and automation in it's planning processes. It also highlights the fact that the UNHAS fleet is ageing and it should be incentivised to renew its air assets while taking into account their environmental impact. This first chapter therefore reveals the need for the creation of data-driven tactical and operational decision support tools which can increase robustness of humanitarian Fleet Planning processes. Applying this to aircraft contracting and fleet renewal decisions could highly benefit UNHAS and it's users in the long term by reducing costs, increasing demand satisfaction and reducing it's CO2 emissions.

3

Optimisation techniques applied to Operations Research

Mathematical optimisation is a discipline which allows to translate complex real-life problems into mathematical models in a way which they can be solved to find optimal or sub-optimal solutions in a reasonable amount of time. In this chapter, a general overview on Network Flow problems and their most common mathematical formulations is given. This is followed by a detailed description of Vehicle Routing Problems and it's applications to Operations Research.

3.1. Network flow problems

Network flow problems are a special application of graph theory widely used in logistical and transportation problems. They consist of a number of nodes (vertices) linked together by capacitated and directed arcs (edges), which together form a network (directed graph). The aim is to allocate flows to arcs in order to achieve a certain goal, all the while respecting restrictions such as arc capacity, directions and connections. Flows are measurable quantities which usually represent units of a chosen commodity such as number of passengers or tons of freight. A diagram representing a directed graph G = (V,E) can be seen in Figure 3.1 with the set V containing all nodes and set E all arcs. Nodes can be further divided into sink (A) and source (E) nodes where flow can either only originate or disappear respectively. Supply nodes (B) allow higher output of flows than they receive, while demand nodes (C) allow a greater input flow than it outputs. Finally transshipment nodes have equal input and output flows. When considering arcs, they can either be uni-directional or bi-directional. They are allocated weights which quantify the amount of flow/commodity that can be transported through them.

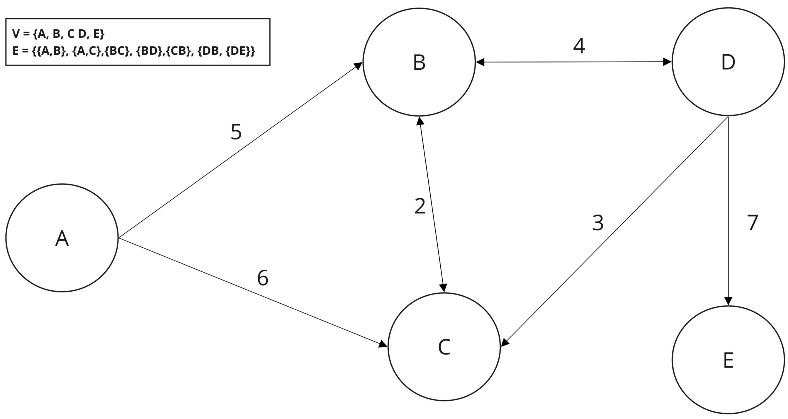


Figure 3.1: Example of a network flow diagram G=(V,E)

Although general formulations exist such as the ones enumerated in Section 3.3, network flow problems vary depending on the objectives, constraints and field they are applied to. Network flow problems can be formulated as Linear Optimization Problems (LP). This allows to solve large scale problems which are normally infeasible by hand.

3.2. Linear programming

Linear programming is a mathematical programming technique widely used in operations research to obtain optimal solutions to problems with many variables, and in industry to make processes more efficient or to serve as a quantitative decision support tool. As can be seen in Figure 3.2, it belongs to the category of deterministic optimisation techniques where historical data is accurately known. Within this category, one can differentiate between convex and non-convex optimisation. A convex optimisation problem refers to a problem in which the mathematical formulation is made up of a convex objective function and convex constraints. This implies that the feasible solution set is also a convex set and that a local extremum (maximum or minimum) is also a global extremum.

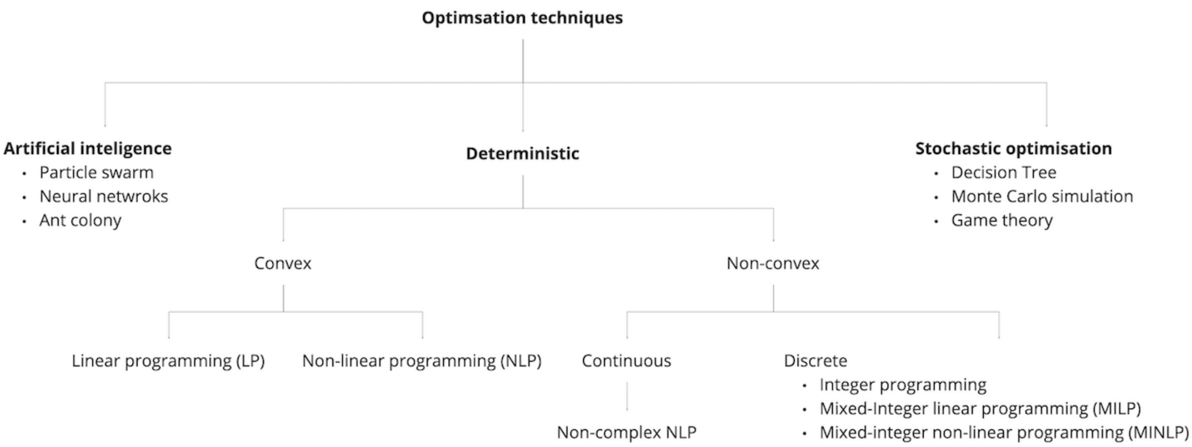


Figure 3.2: High-level classification of optimisation techniques

The aim of a linear programming model is to find an optimal value for a linear objective function by varying decision variables over their feasible and pre-determined range. The objective function is bounded by various equality or inequality constraints. All linear program formulations share the following same elements :

1. **Decision variables**: These variables define the decisions that need to be made. They are often represented as resources, quantities or levels of activity that need to be determined to solve a linear programming problem. Initially unknown, they are defined over a range of numerical value which the program iterates over. Once an optimal value for each of the variables is found, the linear problem is considered solved. Their algebraic representation is usually in the following format : $x_1, x_2, ..., x_n$ where the subscript defines the n^{th} variable. In network flow problems, it is convenient to to use x_{ij} to represent flows between 2 nodes i and j. These decision variables can be binary, integer, or a mix of both depending on the application.

2. **Constraints**: A constraint is a linear equation in the form of an equality or inequality which limits the range of the decision variables and therefore the feasible region for the solution of the problem. Two main different types of constraints can be identified: functional constraints and non-negativity constraints. While the first one is related to the operational aspect of the problem at hand, the second ensures that decision variables do not take values smaller than zero for coherence. The format of a functional constraint is usually represented in the following way, with a_{ij} a coefficient and b_i a numerical value which imposes the limitation:

$$\sum_{j=1}^{n} a_{ij}x_j \left\{ \begin{array}{c} \leq \\ = \\ \geq \end{array} \right\} b_i, \text{ for } i = 1 \dots m \tag{3.1}$$

3. **Objective functions**: An objective function is a linear equation made up of fixed parameters, coefficients and decision variables. It must be minimized or maximized, depending on problem

that needs to be solved. This is done by exploring the solution space, the combination of all possible decision variables bounded by constraints, and finding an optimal numerical value for the linear equation. It's algebraic representation can be generalised to the following expression with c_i a coefficient:

$$\text{Minimize or Maximize z} = c_1x_1 + ... + c_nx_n = \sum_{j=1}^{n} c_ix_i \tag{3.2}$$

During the second world war, George B. Dantzig developed the first solution method for solving large scale linear programming problems: the simplex method. The method consists of testing iteratively adjacent vertices of a bonded polygonal region, called the feasible region, defined by constraint functions and finding which vertices produce optimal or sub-optimal values for a specific objective function. These vertices are called corner point feasible solutions (CFP). The best CFP solution is the optimal solution to an optimisation problem. The simplex method and derivatives of it's algorithm are extensively used to solve network flow problems in research or in industry. Commercial mathematical programming solvers using simplex and exact algorithms are available such as GUROBI and CPLEX which facilitate solving large scale linear programming problems and provide interfaces tailored to optimisation problems [26][30].

3.3. Common network flow problem formulations

In this section, an general overview of common network flow problem formulations is given, along with their objective functions and constraints. These are used in research and industry as building blocks which are then expanded to suit a specific field or application. The nomenclature for the formulations in Section 3.3 can be found in Table 3.1.

Sets		Parameters	
A	set of nodes (i, j, k $\in A$)	c_{ij}	cost of travelling from i to j
		v	maximum amount flow in the network
Decision variables		u_{ij}	arc i,j upper capacity limit
		l_{ij}	arc i,j lower capacity limit
x_{ij}	flow of commodities from i to j	b_i	demand requirement for node i
x^b_{ij}	binary decision variable equal to 1 if solution, 0 otherwise	a_i	available commodities at node i

Table 3.1: Nomenclature for the formulations in section 3.3

3.3.1. Maximum flow problems

The maximum flow problem has for objective to find a feasible flow pattern that allow as much commodity as possible to move between a specific source to a specific sink in a capacitated network. The first instances of this problem appear in literature as early as 1930, but it is formally introduced by R.Fulkerson and G. Dantzig (1955) [16]. Two main constraints are considered in the general formulation of the maximum flow problem in 3.3. The first constraint is the flow conservation constraint which forces the sum of flows arriving at a node to be equal to the sum of flows leaving that node, apart from at the sink and source. The second constraint is the called the capacity constraint and forces the flows through an arc to be bounded, in this case by an upper limit u_{ij}.

$$\begin{aligned} &\text{maximize} \quad v \\ &\text{s.t} \quad \sum_j x_{ij} - \sum_k x_{ki} = \begin{cases} v & \text{if i is a source} \\ 0 & \text{otherwise} \\ -v & \text{if i is a sink} \end{cases} \\ &\qquad 0 \le x_{ij} \le u_{ij} \qquad (i = 1, ..., n; j = 1, ..., n) \end{aligned} \tag{3.3}$$

3.3.2. Minimum cost flow problems

In min-cost-flow problems, the objective is to minimize the total cost of a flow pattern within a network. The most general formulation of this application is found in Section 3.4. Common industry adaptations and extensions of this problem can be found in Subsections 3.3.3, 3.3.4, 3.3.5, and 3.4. The objective function minimizes the sum of all the transport costs of commodities on arcs ij multiplied by the amount of flow on these arcs. It is subject to 2 main constraints, the first being a flow balance constraint which ensures that the incoming and outgoing flows at a node satisfy the node's supply and demand requirements b_i. The second constraint is the capacity constraint which in this case ensures that the flows stay bounded with respect to arc capacities, l_{ij} being the lower bound and u_{ij} the upper bound.

$$\begin{aligned} \text{minimize} \quad & \sum_i \sum_j c_{ij} x_{ij} \\ \text{s.t} \quad & \sum_i x_{ij} - \sum_k x_{ki} = b_i, \quad i = 1, ..., n \\ & l_{ij} \le x_{ij} \le u_{ij}, \qquad j = 1, ..., m \end{aligned} \tag{3.4}$$

3.3.3. Shortest path

The shortest path problem has for objective to find the shortest/cheapest/fastest combination of arcs from an origin node to a destination node. The objective function in formulation 3.5 is the same as for the minimum cost flow problem, however the constraints differ. The first constraint ensures that there is a net supply of one unit at the source, and a net demand of one unit at the sink, without any net inflow/outflow at all other nodes. Shortest path problem has been extensively covered in literature and multiple techniques have been developed to increase solving time such as Dijkstra's algorithm and the A* search algorithm. Shortest path problems can often be found in geo- and web-mapping applications such as Google Maps, in the creation of road networks, and in routing and layout problems [33] [56].

$$\begin{aligned} \text{minimize} \quad & \sum_i \sum_j c_{ij} x_{ij} \\ \text{s.t} \quad & \sum_j x^b_{ij} - \sum_k x^b_{ki} = \begin{cases} 1 & \text{if i is a source} \\ 0 & \text{otherwise} \\ -1 & \text{if i is a sink} \end{cases} \\ & x^b_{ij} \ge 0 \qquad \forall \text{ arcs ij in the networks} \end{aligned} \tag{3.5}$$

3.3.4. Transportation problem

The transportation problem has for objective to minimize the cost of distributing a commodity from multiple origins to multiple destination. The first two constraints in formulation 3.6 ensure that the flows respect availability and demand requirements, a_i and b_j respectively. O. Díaz-Parra et al. (2014) published a comprehensive review on transportation problems [18]. They present a range of mathematical models for different transportation modes (land, sea, air) which include variants of the well known vehicle routing problem but also loading, dispatching and inventory problems. Transshipment problems are also considered to be extensions of the transportation problem. While transportation problems only allow direct flows between sink and source, transshipment problems allow flows to go thought intermediary nodes.

$$\begin{aligned} \text{minimize} \quad & \sum_i^m \sum_j^n c_{ij} x_{ij} \\ \text{s.t} \quad & \sum_j^n x_{ij} = a_i, \quad (i = 1, ..., m) \\ & \sum_i^m x_{ij} = b_j, \quad (j = 1, ..., n) \\ & x_{ij} \ge 0 \qquad (i = 1, ..., m; j = 1, ..., n) \end{aligned} \tag{3.6}$$

3.3.5. Assignment problem

Assignment problems are an extension of the transportation problem where the decision variable x_{ij} is binary and takes a value of 1 if i is assigned to j, and 0 otherwise. The first 2 constraints in formulation 3.7 indicate that all i's must be assigned exactly to one j, and vice versa. This is possible because the amount of i's is equal to the amount of j's. The Assignment problem is an integer program and can be solved efficiently through the Hungarian Algorithm [32]. It's applications are numerous, for example in the airline industry where it is often used to optimise the assignment of aircraft tail numbers and crew members to specific flights.

$$\begin{aligned}
&\text{minimize} && \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}x_{ij} && \\
&\text{s.t} && \sum_{j=1}^{n} x_{ij}^{b} = 1 && (i = 1, ..., n) \\
& && \sum_{i=1}^{m} x_{ij}^{b} = 1, && (j = 1, ..., n) \\
& && x_{ij}^{b} = 0 \text{ or } 1 && (i = 1, ..., n; j = 1, ..., n)
\end{aligned} \tag{3.7}$$

3.4. Vehicle routing problems

Vehicle Routing Problems (VRP) are extensively covered in operations research. Originally developed in 1959 by G. Dantzig and J. Ramser who applied it to petrol truck deliveries, they are a generalisation of the Travelling Salesman Problem (TSP), only with multiple vehicles and the fact that a vehicle must return to the starting position after a certain amount of locations have been visited. It's overall objective is to find an optimal set of routes for a fleet of vehicles who need to visit a number of destinations to deliver or collect commodities. Finding an optimal routing for a fleet implies either minimizing the amount of time needed, the costs involved, the length of each journey, fuel consumption or more depending on the application. VRPs can be differentiated according to 3 different aspects: their modeling approach, their mathematical formulation and their solution methods.

VRP modeling approaches

There are 3 main ways to model a VRP problems based on the type of decisions and variables used. Vehicle Flow models are integer linear programming models where the decision variables model vehicle flows over a network, usually in the form of binary decision variables. Another type of modelling approach consists of creating a set-partitioning-based model where the decision variables represent the set of feasible routes. The advantage of using such a model is that it facilitates the implementation of difficult route-based constraints such as time windows [47]. Finally,the commodity flow formulation models the flow of goods as-well as vehicles as decision variables. This often implies that a mixed-integer linear programming (MILP) formulation needs to be used.

VRP mathematical formulations

Many different VRP mathematical formulations exist depending on the field it is applied to and the characteristics of the problem at hand. For the purpose of this study, the multi-commodity flow formulation of the Classical VRP (CVRP) with a homogeneous fleet is presented below along wit hthe nomenclature in Table 3.2 [36][41].

Sets		***Parameters***	
A	set of nodes (i, j, k $\in A$)	c_{ij}	cost of travelling from i to j
		m	number of vehicles
Decision variables		K	vehicle capacity
		d_k	demand at node k
$x_{ij} = \begin{cases} 1 \\ 0 \end{cases}$	if a vehicle travels directly from i to j 0 otherwise		
y_{ij}^{k}	flow of commodities from i to j, destined for k		

Table 3.2: Nomenclature for the CVRP commodity flow based problem

Homogeneous CVRP commodity flow based formulation

$$\text{minimize} \sum_{i=0}^{n} \sum_{j=0}^{n} c_{ij} x_{ij} \tag{3.8}$$

s.t.

$$\sum_{i=0}^{n} x_{ij} = 1 \qquad (j = 1, ..., n) \tag{3.9}$$

$$\sum_{j=0}^{n} x_{ij} = 1 \qquad (i = 1, ..., n) \tag{3.10}$$

$$\sum_{j=1}^{n} x_{0j} \leq m \tag{3.11}$$

$$\sum_{i=0}^{n} y_{ij}^{k} - \sum_{i=0}^{n} y_{ji}^{k} = \begin{cases} d_k & \text{if j=k} \\ 0 & \text{if j} \neq \text{0 or j} \neq \text{k} \end{cases} \qquad \forall j, k \tag{3.12}$$

$$\sum_{k=1} y_{ij}^{k} \leq K x_{ij} \qquad i \neq j = 0, ..., n \tag{3.13}$$

$$y_{ij}^{k} \geq 0, \qquad \forall i, j, k \tag{3.14}$$

$$x_{ij} = 0 \text{ or } 1 \qquad \forall i, j \tag{3.15}$$

In the above formulation, Equation 3.8 is the objective function which has for goal to minimize the total routing costs. Equation 3.9 and 3.10 represent the continuity constraints, allowing exactly one vehicle to enter and leave each node. In Equation 3.11, the inequality constraint defines the total amount of vehicles available and Equation 3.12, the flow balance constraint, ensures that the demand in each node is satisfied. Equation 3.13 imposes a constraint on commodity flows based on vehicle capacities. Finally Equation 3.14 and 3.15 ensure that the decision variable y_{ij}^{k} is always non-negative and that x_{ij} remains binary. Applying this model to a heterogeneous fleet is easily done by modifying the decision variable x_{ij} to x_{ij}^{k} which indicates the type of vehicle k travelling on a certain arc from i to j. Equation 3.9, 3.10 and 3.11 must be modified accordingly and the vehicle capacity constraint inEquation 3.11 must be replicated as many times as vehicle types are included. The VRP problem has been extended to suit multiple different applications and problem characteristics. Amongst the most well-known extensions of the CVRP are the VRP with Time Windows (VRPTW), VRP with Pickup and Delivery (VRPPD) and the Multi-Depot VRP (MDVRP).

VRP solution methods

The VRP is a combinatorial optimisation problem, more specifically NP-hard, which implies that the set of feasible solutions is extremely large and that the computational power needed to solve the problem increases exponentially with it's size. Small or moderate instances of VRP can be solved using exact algorithms to find global optimal solutions within a limited amount of time, however these models might be over-simplified or not generalisable to larger applications. The most popular exact algorithms for VRP are Branch and Bound, Branch and Cut, other Spanning Tree techniques and Dynamic programming [21] [35]. When exact algorithms cannot be used to find an optimal solutions to a problem, heuristics and meta-heuristics can be used. Heuristics can be compared to shortcuts: practical methods used to solve NP-hard problems which do not guarantee optimal solutions but help in finding sub-optimal, satisfactory solutions by exploring a limited part of the feasible solution set in a finite amount of time. G. Laporte et al. (2000) provide an extensive review on classical and modern heuristics used for VRPs [34]. Most common found heuristics in literature are Savings and Sweep algorithms, Cluster First Route Second, and Route Improvement Methods. Finally, meta-heuristics have become recently a common method to approach large NP-hard problems. They are not problem specific, but help guide the search for optimal solutions by introducing a set of guidelines to explore more efficiently the solution space. Two main strategies are usually used: intensification and diversification. While diversification explores the search space on a global level, intensification segregates the solution space based on aggregated experience into regions with high probability of containing local or potentially global optimums

[10]. Usually, a balance between both strategies is necessary, allowing the algorithms to leave local optimums in search of better ones. In VRP, meta-heuristics can be divided into Local Search algorithms (ex. Tabu Search), Population Search (ex. Genetic algorithms) and Learning Mechanism (ex. Neural networks, Ant colony optimisation)[57].

3.5. Research Focus

This chapter has focused on how real-world problems can be translated into mathematical formulations and then solved optimally or sub-optimally. The movement of commodities and vehicles through a network can be conveniently reproduced using Network Flow formulations and models. In order to analyse the UNHAS South Sudan mission, it is necessary to translate the service, the context and its users into a mathematical model that can simulate accurately, and in a reasonable amount of time, the real-world operations. Two main objectives are highlighted in Chapter 2, namely minimizing costs and maximizing demand satisfaction. Minimizing transportation cost can be represented by a Min-Cost Multi-Commodity Network Flow Problem where different commodities are different passenger classes or types. Maximizing demand satisfaction is equivalent to minimizing a certain penalty associated to demand spillage. This implies that at least two objective functions need to be optimised simultaneously, and therefore results will most likely be presented in the form of trade-offs (for example Pareto fronts). Having identified a general direction for the type of model that will be created, the following chapters will focus on literature covering the applications of these techniques to fleet planning and the humanitarian context.

4

Fleet planning

Fleet planning is a common problem addressed in the transport industry. It consists of selecting an optimal fleet by addressing the following 3 main questions: Which type of aircraft should I buy? How many of each do I need? When should I acquire them? In this chapter, first the main fleet planning characteristics will be presented, followed by a comparison between airline and humanitarian fleet planning. Finally, 3 subsections are dedicated to fleet planning methods combined with network design, stochastic demand evolution and routing.

4.1. Fleet planning characteristics

Although the scope of this study focuses on aircraft fleets, many parallels exist between the fleet planning of different transportation modes, whether they address road, rail or maritime fleets. The high-level decisions and objectives remain similar: one searches for an optimal amount of vehicles of a specific type for a defined period in time, usually with the aim of maximizing profit or minimizing costs. The differences appear when considering specific scenarios and their operational constraints. The following subsections present the most common characteristics of fleet planning models by first reviewing the different time horizons on which it can take place. This is followed by comparing homogeneous to heterogeneous fleet planning, uni- to multi-modal fleet planning and finally single to multi-commodity fleet planning.

Fleet planning horizons

Fleet planning can be implemented at multiple different stages of the network and operational design cycle. The time frame under consideration determines which types of decisions are made and therefore which methodologies are most appropriate. Three main planning horizons are considered in fleet planning problems: strategic (long-term), tactical (medium-term) and operational (short-term) [15][7]. Despite their differences, these 3 stages are usually highly interdependent and the outputs of one are often used as input for the others.

- Strategic fleet planning takes place 2 to 5 years before the employment of an aircraft. It is often confronted to challenges related to high uncertainty and lack of robustness. This is mainly due to the difficulties in forecasting revenue, demand and operating costs so far in the future, all of which are closely inter-related. These effects can be mitigated through revenue management models, aircraft leasing schemes, network analysis (hub & spoke, point-to-point, hybrid), risk sharing etc. The primary aim is often to increase the flexibility and resilience of the airline to changes in its operating environment for the years to come. Decisions such as fleet size and fleet mix are central to this planning stage.
- Tactical fleet planning is considered to take place between a year to a few months before the start of operations. Because the time horizon has decreased, more parameters and operational constraints are known. This allows more realistic fleet planning models which deal not only with fleet composition, but integrate decisions such as vehicle allocation to routes, freight and

passenger allocation to vehicles, schedule design, fleet deployment and more. This time-frame allows decisions to be made on whether vehicles need to be added to or removed from a fleet.

- Operational fleet planning usually takes place between a month and a day before departure, more detailed and precise models are used to optimise daily operations where only very few uncertainties exist. Common problems involve empty vehicle re-positioning, inventory management, detailed daily routing and timetabling.

Homogeneous vs. heterogeneous fleet planning

More often than not, an operator's fleet consists of a same transportation mode, but of multiple different vehicle types. This allows a fleet to be more versatile, flexible and enable a range of different opportunities that a homogeneous fleet could not. Hoff et al. [28] distinguishes 3 different aspects defining heterogeneous fleets: physical dimensions, compatibility constraints, and costs. While physical dimensions often determine the possible capacity that can be offered (amount of passenger seats, size of a cargo hold), it also affects an entire range of characteristics such as vehicle weight, range, speed and fuel consumption. Size also determines which locations a vehicle can access, for example ship operators must take into account canal width and depths, airlines must take into account runway specifications and handling equipment at airports. When considering compatibility constraints, operators must be wary of the wider environment in which they evolve. For example, the fleet must respect environmental guidelines, and noise and emission restrictions. When transporting valuable or dangerous goods, certificates are needed and only specific types of vehicles are allowed to be used. Finally, operating costs must be taken into account. These play an important role when considering transportation frequencies between destinations. Determining an ideal fleet type for a network is closely linked to frequency and timetabling. It becomes equivalent to answering whether flying once a day with a 400 seat aircraft is better than flying 4 times a day with only a 100 seats.

Uni- vs. multi-modal fleet planning

Fleet planning can also be applied to uni- or multi-modal transportation problems. Multi-modal transportation networks address the complexities related to modelling multiple different types of transportation modes together, often in an international logistic context, for example a combination of rail and road services. Baykasoğlu et al. [7] provide an extensive review on existing fleet planning in uni- and multi-mode transportation problems and their characteristics. In Figure 4.1, the authors classified the most common sub-problems in inter-modal fleet planning and illustrated their relationships in time.

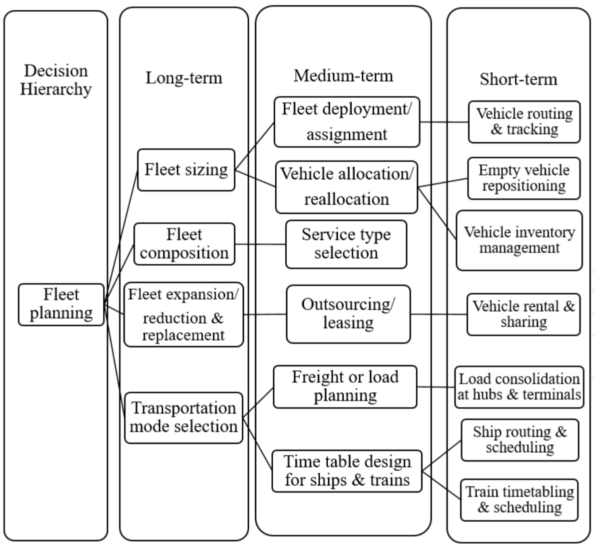


Figure 4.1: Strategic, tactical and operational level decisions on inter-modal fleet planning [7]

Single- vs multi-commodity fleet planning

Instead of considering a problem where only one type of good is being transported, it is common to model networks where multiple different commodities with distinct characteristics are being distributed from sources to destinations. Multi-commodity network flow problems (MCNF) are common in feet planning and network design problems as they allow a more realistic representation of logistical problems. Each commodity is characterised by an origin, a destination, a total demand and a flow cost. For example in maritime fleet planning problems, multi-commodity flows are used to model the transportation of different container sizes which need to be loaded or unloaded at ports with different infrastructures [37]. In commercial aviation fleet planning, commodities can represent, amongst others, different passenger classes or passengers with a different number of airport connections in their trip [55].

4.2. Airline vs. Humanitarian fleet planning

Airline fleet planning is a long-term strategical decision which significantly impacts airline operations and finances. P. Clark in his book "Buying the big jets, fleet planning for airlines" describes fleet planning as :

"the process by which an airline acquires and manages appropriate aircraft capacity in order to serve anticipated markets over a variety of defined periods of time with a view to maximising corporate wealth [12]."

Although fleet planning is one of the first steps in the airline planing process, it's implications define the airline's strategy for the decades to come. When creating a fleet plan, one must take into account factors such as the pre-existing fleet, the number and type of new aircraft to be acquired or to be disposed of, fleet age and commonality, aircraft depreciation costs, potential future demand, competition and market shares etc. These factors evolve throughout time while remaining significantly inter-related which makes it difficult to change one aspect of the process without affecting others. Another major difficulty lies in predicting what the aviation industry will look like in the next 10 to 20 years. The high uncertainty related to these decisions limits the design of strategical optimisation models and decision support tools which are much more reliable when considering tactical and operational problems. Indeed, Fleet Assignment Models (FAM), Revenue Management Models (RVM), VRPs and other tactical scheduling and routing problems are found extensively in Operation Research literature. They often stem from a baseline model which is then adapted to different specific scenarios, integrated with other stages of the planning process, or modified to account for new state-of-the-art computational techniques and heuristics. However, long-term fleet planning models vary significantly from one another due to the large differences in applications, time periods and factors taken into account.

4.3. Fleet planning and network design

Strategic and tactical fleet planning is often combined with network design due to their high interdependence. Deciding which routes to operate and assigning them a service frequency depends on the type of fleet an operator has available. Vice-versa, choosing a fleet size and composition depends on the type of network that is being operated. In 1980, authors R. Marsten and M. Muller created a deterministic mixed-integer programming model for air cargo fleet planning [42]. They explore the design of a service network and a corresponding fleet composition for the cargo-airline Flying Tiger Line. The aim is to maximize the profits of the airline while taking into account fuel, demand and capacity constraints per city pairs and analysing day and night operations.

T. Crainic (2000) presents a review on tactical freight transportation and service network design along with a general mathematical formulation and it's extensions [15]. The author classifies the models based their functionality instead of the transportation mode which provides more comprehensive view on their applications. The modelling of such networks is similar to the minimum cost flow problem formulations seen in formulation 3.4, with the exception that a frequency dependent term is present. The authors outline the main considerations that must be approached when designing such a model :

1. Service selection: route selection along with frequency and scheduling characteristics
2. Traffic distribution: defining service levels, facilities and types of operations for each itinerary and demand type

3. Terminal policies: constraints applied to specific terminals (hubs) along with consolidation strategies and allocation of tasks

4. Empty balancing strategies: repositioning policies to meet the needs of the next planning period

Jaillet et al. (1996) tackle a capacitated network design problem for airlines and draw parallels with existing Hub Location Problems (HLP) [31]. The objective is to design an optimal network and aircraft routing policy which minimizes costs and maximises demand satisfaction, given a fixed fleet with known operational costs and an OD demand matrix. The authors formulate 3 linear integer programming models where the novelty lies in the use of fractional flows as decision variables to represent passenger movements. Because no initial network is fixed, and only passenger origin and destinations are know, the models are given the freedom to route passengers through different nodes of the network, as long as this minimizes system costs and passengers reach their final destination. All 3 linear programming models differ in the amount of stops a passenger can make before reaching it's final destination: one-stop, two-stop, or all-stops. The nomenclature in Table 4.1 is used for the one-stop and two-stop models, and the mathematical formulation for the two-stop model is presented in Equation 4.1, 4.2 and 4.3.

Sets		Decision variables	
N	Set of airports	**One-stop model**	
K	Set of aircraft types	x_{ilj}	Passenger flow from airport i to j, commuting through l
F_{ij}	Set of passenger flows f_{ij} from airport i to j	y_{ij}^k	Number of aircraft k used on link i to j
Parameters		**Two-stop model**	
		x_{ilj}	Passenger flow from airport i to j, commuting through l
d_{ij}	Distance. from airport i to j	x_{iltj}	Passenger flow from airport i to j, commuting through l and t
c_k	Cost per mile of flying aircraft k	y_{ij}^k	Number of aircraft K used on link i to j
b_k	Capacity of aircraft k		

Table 4.1: Nomenclature for the one-stop and two-stop model [31]

Two-stop model formulation

$$\text{minimize} \sum_{i \neq j} \sum_{k \in K} d_{ij} c_k y_{ij}^k \tag{4.1}$$

s.t.

$$\begin{aligned} f_{ij} + \sum_{t \neq i,j} (f_{it} x_{ijt} + f_{tj} x_{tij} - f_{ij} x_{itj}) + \sum_{l,t \neq i,j} (f_{lj} x_{ltij} + f_{it} x_{ijlt} + f_{lt} x_{lijt} - f_{ij} x_{iltj}) \\ \leq \sum_{k \in K} b_k y_{ij}^k \qquad \forall i \neq j \end{aligned} \tag{4.2}$$

$$\sum_{t \neq i,j} x_{itj} + \sum_{l,t \neq i,j} x_{iltj} \leq 1 \qquad \forall i \neq t \neq j \tag{4.3}$$

$$x_{itj} \leq 0 \qquad \forall i \neq j \tag{4.4}$$

$$x_{iltj} \leq 0 \qquad \forall i \neq j \neq l \neq t \tag{4.5}$$

$$y_{ij}^k \leq 0 \qquad \forall i \neq j, k \in K \tag{4.6}$$

The objective function in Equation 4.1 is a general cost minimization equation and is identical for both models. Equation 4.2 is the capacity constraint and ensures that the flow between two city pairs is smaller than the capacity provided by the aircraft operating on that link. The first term represent the direct passengers flow between i and j, the second term represents the one-stop fractional passenger flows between i and j and the third term the two-stop fractional passenger flows between i and j. Equation 4.3 ensures that fractional flows are always smaller than 1. Finally, Equations 4.4, 4.5 and 4.6 are the non-negativity constraint. The output of the model is a set of passenger flows associated to airport pairs and specific aircraft types. When the network is analysed, one can notice concentration effects on certain links. This outlines the presence of airports more suited as hubs than others. The authors however note that the difference between all 3 policies is small. The emergent proprieties of the models remain similar for a one-stop, two-stop or all-stop policy when only considering minimizing transportation costs. The research could be furthered by considering different objectives such as maximizing passenger revenues and profit.

4.4. Fleet planning and stochastic models

Fleet planning combined with stochastic processes has become a trend for long-term fleet and network planning. Linear programming and uncertainty has also been a significant area of research since 1955 [16]. In the airline industry, forecasting future demand and trends is a crucial part of the planning process when considering robustness and resilience.

4.4.1. Fleet planning and demand forecasting

In research and industry, qualitative and quantitative demand forecasting methods are used in order to account for uncertainties linked to demand evolution, and used as input to optimisation models. Three traditional quantitative methods are most commonly used to forecast future aviation demand: time series, market share and econometric forecasting [50]. While time series are widely used by airlines, they only project into the future the trends of the past and rely heavily on historical data. The multi-dimensional character of aviation demand is best exploited by econometric/causal models which rely on a whole range of social, political, economical and supply factors (ie. Isard's Gravity Model [48], Regression analysis, machine learning techniques) [19]. These are usually much more computationally intensive than the other methods, in the order of months instead of days, but provides insight on scenarios where historical data is scarce, for example when evaluating new routes and long haul demand forecasting. A high-level classification of demand forecasting methods is shown in Figure 4.2.

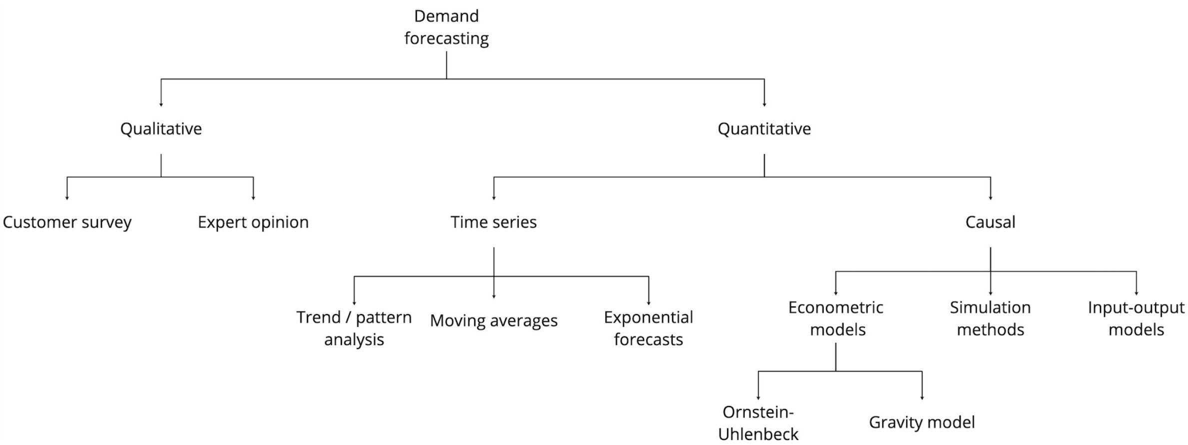


Figure 4.2: Preliminary classification of demand forecasting methods

An application of fleet planning combined with quantitative demand forecasting can be found in the paper by C.A. Sa, B.F. Santos, and J.P.B. Clarke (2019) [55]. In a first instance, the authors create a stochastic demand forecasting model using a mean-reverting Ornstein-Uhlenbeck process. The econometric model's main assumption is that variations in demand are cyclical/seasonal and correlated to GDP variations. Applying this to forecast future aviation demand growth rates and levels between different OD pairs, the authors perform Monte Carlo simulations to evaluate the possible evolution of this demand over multiple years. The Monte Carlo runs are then grouped and sampled yielding averaged OD demand matrices per year. This demand is then used as input to a fleet planning optimisation model. The authors formulate an integer linear programming optimisation model which allocates aircraft types and corresponding flight frequencies to OD pairs, with the objective to maximize the overall profit of an airline. The nomenclature for the model can be found in Table 4.2 and the mathematical formulation below. The objective function in Equation 4.7 has the aim to maximize profits through 4 different terms : operating revenue from direct and connecting passengers, ownership costs and operating costs. Equation 4.8 is the demand verification constraint, ensuring passenger flows do not exceed the demand for a particular OD pair. Equations 4.9, 4.10 and 4.11 are the capacity constraint, limiting the amount of passenger flow per flight by the capacity of the aircraft of type k used on that arc. The aircraft continuity constraint in Equation 4.12 ensures that the amount of aircraft arriving at an airport is equal to the amount leaving that same node. Equation 4.13 and 4.14 guarantee that aircraft utilisation is not exceeded and maximum range requirements are respected.

Sets		Parameters	
N	Set of airports	$Q_{o,d}$	demand between airport o and d
K	Set of aircraft types	$D_{o,d}$	distance between airport o and d
H	Set of hub airports (subset of N)	$\text{yield}_{o,d}$	yield per route for direct arcs
		$\text{yield}^h_{o,d}$	yield per route for connections at h
Decision variables		AC^k	number of aircraft of aircraft type k
		C^k_{fix}	aircraft ownership cost per aircraft type k
$x_{o,d}$	Direct passenger flow between origin o and destination d	s^k	number of seats per aircraft type k
$w^h_{o,d}$	Connecting passenger flow between origin o and destination d via hub h	C^k_{var}	aircraft operating cost per aircraft type k (i.e. CASM)
$z^k_{i,j}$	Flight frequency on arc (i,j) with aircraft type k	vc^k	cruise speed per aircraft type k
		U^k	aircraft maximum utilization per week for asset of type k
		T^{dep}_i	taxi time per departure airport
		T^{arr}_j	taxi time per arrival airport
		R^k	range per aircraft type k

Table 4.2: Nomenclature for the airline fleet planning under stochastic demand model

Airline fleet planning formulation [55]

$$\begin{aligned}\text{Maximize z } =& \sum_{o\in N}\sum_{d\in N}\left[\text{ yield }_{0,d}\cdot D_{o,d}\cdot x_{0,d}\right] + \sum_{o\in N}\sum_{d\in N}\sum_{h\in H}\left[\text{yield}^h_{o,d}\cdot D_{o,d}\cdot w^h_{0,d}\right]\\ &-\sum_{k\in K}\left[AC^k\cdot C^k_{fix}\right]-\sum_{i\in N}\sum_{j\in N}\sum_{k\in K}\left[C^k_{var}\cdot D_{i,j}\cdot s^k\cdot z^k_{i,j}\right]\end{aligned} \tag{4.7}$$

s.t.

$$x_{0,d}+\sum_{h\in H} w^h_{0,d}\leq Q_{0,d} \qquad \forall o,d\in N, o\neq d \tag{4.8}$$

$$x_{i,j}+\sum_{m\in N} w^j_{i,m}\leq \sum_{k\in K} z^k_{i,j}\cdot s^k \qquad \forall i\in N\backslash H, j\in H, i\neq j \tag{4.9}$$

$$x_{i,j}+\sum_{m\in N} w^i_{m,j}\leq \sum_{k\in K} z^k_{i,j}\cdot s^k \qquad \forall j\in N\backslash H, i\in H, i\neq j \tag{4.10}$$

$$x_{i,j}\leq \sum_{k\in K} z^k_{i,j}\cdot s^k \qquad \forall i,j\in N\backslash H, i\neq j \tag{4.11}$$

$$\sum_{j\in N} z^k_{j,i} = \sum_{j\in N} z^k_{i,j} \qquad \forall i\in N, k\in K \tag{4.12}$$

$$\sum_{i\in N}\sum_{j\in N} z^k_{i,j}\cdot\left[\frac{D_{i,j}}{vc^k}+T^{dep}_i+T^{arr}_j+TAT^k\right]\leq AC^k\cdot U^k \qquad \forall k\in K \tag{4.13}$$

$$z^k_{i,j}=0 \qquad \forall i,j\in N, i\neq j, k\in K \text{ if } R^k< D_{ij} \tag{4.14}$$

$$x_{0,d}\in\mathbb{Z}^+, w^h_{0,d}\in\mathbb{Z}^+, z^k_{i,j}\in\mathbb{Z}^+ \tag{4.15}$$

Similar to the mathematical formulation of the service design network by P. Jaillet (1996), passengers are not grouped into a single commodity. The weekly passenger flows are divided into non-stop and connecting passengers. These 2 different flows are used as decision variables. This allows the model to simulate more accurately the effects of a hub & spoke network by allowing the choice to either fly direct flights between an origin-destination pair, or fly a connecting flight through a hub. This also allows the possibility for an airline to operate less profitable routes if it means the overall network profits are maximized. A third decision variable, flight frequency per aircraft type per OD pair, is used. The objective function is made up of 4 terms which according to the authors encompass the most significant aspects of long term fleet planning: operating revenue from non-stop and connecting passenger flows, ownership costs and aircraft operating costs. The set of constraints can be divided in two categories, the first being traditional capacity and continuity constraints found in Equation 3.4, adapted to suit the set of decision variables. The second are specifically related to the aviation industry and aircraft operations.

4.4.2. Multi-stage fleet planning

Multi-stage stochastic programming is a sequential decision-making process under uncertainty, where information is progressively revealed over a number of time stages. Multi-period fleet planing is used to help operators make fleet changes over-time, increase the robustness of their fleet plans or mitigate the impacts of uncertainty on fleet size and composition decisions [39]. G. Barbarosoglu and Y. Arda develop a scenario-based stochastic programming model which aims to model the transportation of emergency commodities in a urban setting, after an earthquake has happened [4]. Two main stages are modelled where the first stage can take the form of 3 different scenarios, each representing the "early response". Each of these can then evolve in 3 sub-scenarios representing the coordinated response. Uncertainty is represented by a random allocation of capacities, supply and demand to the network at both stages. The authors create a multi-modal, multi-commodity network flow mathematical formulation, the objective being to transport as much commodities from an origin to a destination while minimizing costs. G. Pantuso, K. Fagerholt, and S.W. Wallace (2016) developed a solution technique for a specific type of MSP, Hierarchical Stochastic Programming (HSP), and applied this to the Maritime Fleet Renewal Problem (MFRP) where the goal is to make decisions on when to dispose or purchase a ship of a specific type to serve future demand [52]. M.G. Repko and B.F. Santos (2017) focused on long-term airline fleet planning decisions by developing a multi-period scenario tree model taking into account demand uncertainty and fluctuations over time. Each scenario is characterised by a set of 3 decision nodes, each anchored at 3 different points in time [54]. The decision tree therefore allows 3 different fleeting changes to happen over the planning horizon. Each decision node is connected to 3 other nodes by branches as can be seen in Figure 4.3. These branches represent the potential demand evolution from one stage to another, characterized by 3 possible demand changes (Low, High and Medium) and the associated probability of occurrence (20%, 50%, 30% respectively).

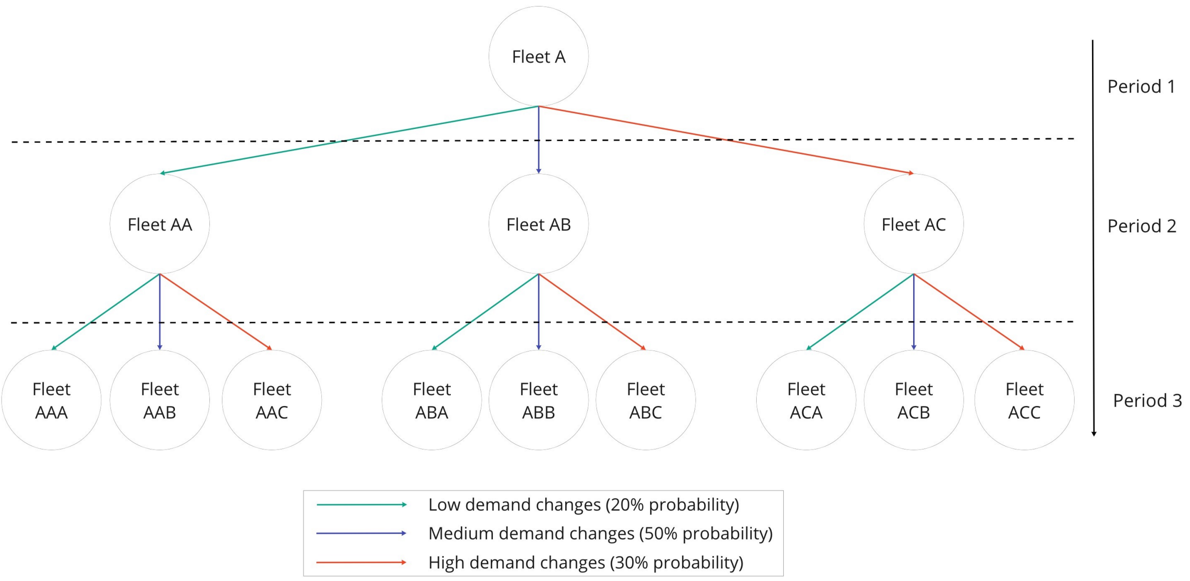


Figure 4.3: Different fleeting scenarios over 3 time periods

A mixed-integer linear programming model is then created to determine the optimal fleet for each scenario. Five decision variables are available, each fixed in time and corresponding to a specific scenario. The objective function aims to maximize profit while taking into account aircraft operational, leasing, disposal and acquisition costs. Constraints are divided in two different categories, the first one related to usual operational constraints, the second related to multi-period planning. This modeling approach addresses the issues of uncertainty in long-term airline fleet planning by allowing planners to make educated and data driven decisions on fleet sizes and compositions at multiple periods in time in order to best serve a forecasted demand.

4.5. Fleet planning and routing models

Fleet planning models combined with routing formulations are mainly used in tactical and operational time-horizons to simultaneously solve the dilemma between finding an optimal fleet size and mix, and an optimal set of routes while satisfying a number supply, demand and operational constraints. M. Bielli, A. Bielli and R.Rossi (2011) identified relevant applications of fleet management for different transportation modes and the mathematical models and computation algorithms best suited to represent them [9]. Hoff et al. (2010) presented an extensive literature review on fleet composition and routing problems in maritime and road transport [28]. He chooses the Fleet Size and Mix Vehicle Routing Problem (FSMVRP) as the most representative mathematical formulation for this class of problem. This model was first introduced by Golden et al. (1984) where the arc flow mathematical formulation is an extension of the VRP found in section 3.4 with a heterogeneous fleet of vehicles and their acquisitions costs [23]. Although heuristics are often needed to solve this formulation, the nomenclature of the problem in Table 4.3 and the mathematical formulation are presented below.

Sets		Parameters	
N	Set of customers	n	number of customers
A	Set of possible travel arcs	K	number of vehicles of types
V	Set of vehicle types	Q_k	capacity of vehicle type k ($Q_1 < Q_2 < ... < Q_K$)
		f_k	acquisition cost for vehicle k ($f_1 < ... < f_K$)
Decision variables		q_j	demand of customer j
		c_{ij}	cost of travelling from i to j
y_{ij}	flow of goods from i to j	M_{ijk}	very small constant
$x_{ij}^k = 1$	if vehicle of type k travels from i to j		

Table 4.3: Nomenclature for the FSMVRP

FSMVRP arc flow formulation [28]

$$\text{minimize} \sum_{K \in V} \sum_{j \in N} f_k x_{0j}^k + \sum_{k \in V} \sum_{i,j \in A} c_{ij} x_{ij}^k \tag{4.16}$$

s.t.

$$\sum_{k \in V} \sum_{i \in N} x_{ij}^k = 1, \qquad \forall j \in C \tag{4.17}$$

$$\sum_{i \in N} x_{ij}^k - \sum_{i \in N} x_{ji}^k = 0, \qquad \forall k \in V, \forall j \in C \tag{4.18}$$

$$\sum_{i \in N} y_{ij} - \sum_{i \in N} x_{ji} = q_j, \qquad \forall j \in C \tag{4.19}$$

$$y_{0j} \leq \sum_{k \in V} Q_k x_{0j}^k, \qquad \forall j \in C \tag{4.20}$$

$$y_{ij} \leq \sum_{k \in V} M_{ijk} x_{ij}^k, \qquad \forall i, j \in A \tag{4.21}$$

$$y_{ij} \geq 0, \qquad \forall i, j \in A \tag{4.22}$$

$$x_{ij}^k \in (0, 1), \qquad \forall k \in V, \forall i, j \in A \tag{4.23}$$

The objection function in Equation 4.16 is split in two terms, the first one related to the fixed and acquisition costs of vehicles and the second to their operating costs. The first constraint in Equation 4.17 ensures that customers are visited exactly once. Equation 4.18 is the continuity constraint. Equation 4.19 ensures that all commodity flows satisfy the demand requirements and constraint 4.20 ensures that vehicle capacities are respected. In Equation 4.21, the inequality ensures no commodities flow on arcs without vehicles by using the variable M_{ijk} as a very small number. Finally Equations 4.22 and 4.23 are the decision variable non-negativity and binary constraints.

Hoff et al. (2010) state in their review that out of 95 papers analysed, over 50% of the FSMVRP do not target a specific transportation mode. This highlights the fact that this mathematical formulation is flexible and could be adapted to a cargo or even humanitarian airline routing problem. The rest of the reviewed paper are divided between maritime and road applications. The authors also give their critical opinion on research in fleet composition and routing problems by identifying that most of the applications are idealized and far from representing real world cases. Furthermore, uncertainty and stochasticity are not often treated and should be included in order to create more robust and risk averse models.

G. Pantuso, K. Fagerholt and L.M. Hvattum (2014) will provide another in depth survey focused solely on the Maritime Fleet Size and Mix Problem (MFSMP) and it's variants [51]. Originally developed by Evrett et al.(1972), this model uses a similar objective function as can be seen in Equation 4.24. In the formulation, V is the set containing all available ship types and R_v the set of routes r appropriate for a ship of type v [22]. The first term C_v^F is a fixed cost associated to adding a type v ship to the fleet. It is multiplied by the decision variable y_v which relates to the total amount of ships of type v needed. The second term incorporates the cost coefficient C_{vr}^V which relates to the cost of allocating a ship type to a route r, multiplied by the decision variable x_{vr} representing the sailing frequency.

$$\text{minimize} \sum_{v \in V} C_v^F y_v + \sum_{v \in V} \sum_{r \in R_v} C_{vr}^V x_{vr} \tag{4.24}$$

The objective function above is then complemented by operational constraints specific to ship operations and the operating environment.

4.6. Research focus

In Section 2.2, humanitarian fleet planning is found to take place on a tactical time-frame, 6-3 months before the deployment of the aircraft. According to Section 4.1, the overview of different fleet planning characteristics has enabled to identify the general type of fleet planning problem that will be approached. The time-horizon will be tactical due to the humanitarian context and UNHAS flight requests and schedules which are assumed to repeat on a monthly basis. The focus being on air transport operations, the model created will consider a uni-modal, heterogeneous fleet with passengers as the commodities. The fleet planning model will be combined with aspects of Network Design as presented in Section 4.3. Although all the nodes of the network are fixed and the flight requests deterministic, it would be valuable for UNHAS to analyse how they could direct their passengers within the network more effectively and efficiently and how this would affects their fleet. Useful trade-offs can be obtained from such a model such as the effect of different fleet types on flight frequencies, operating and acquisition costs, demand spillage, CO2 emissions and more. In order to capture passenger routing possibilities, the commodities will represent passengers with different amounts of possible airport connections, similarly to the formulations created by [31], [54] and [55]. The decision variables will therefore represent the amount of direct passengers, one-stop passengers and two-stop passengers which are routed on an arc. These connections can only be made at one of the 4 hubs: Juba, Rumbek, Bor and Wau. Stochastic and multi-stage fleet planning will not be considered due to the fact that most UNHAS missions do not last more than 10 years and forecasting humanitarian demand is difficult. There is however interest in analysing the fleet changes due to seasonality. In the 10 years that UNHAS has been operating in South Sudan, it has had to adapt to changes between the dry and rainy season, a change which significantly affects it's fleet and costs.

5

Aircraft routing and scheduling

This chapter will cover the last stages of the airline and humanitarian air operations planning cycle. In Section 5.1 the traditional commercial routing and scheduling techniques are presented, followed by Section 5.2, which explores the same process but applied to the humanitarian setting. The main divergences lie in the objectives and types of mathematical models used to represent the two different realities.

5.1. Commercial routing and scheduling

In this section, the Fleet Assignment and the Aircraft Routing stages of the airline planning process are reviewed. They follow the Frequency Planning and Timetable Development steps which are created on a yearly basis before flight departures. The latter two are not explored in detail due to the fact that no mathematical model can yet fully capture the complexity of the problem due to its size and many variables [25]. Furthermore, this stage is very airline specific and generalising it is difficult due to the airline's varying operating environments, competition aspects, airport regulations and more. Today, this problem is often solved by using previously existing schedules and implementing incremental changes to it over time.

5.1.1. Fleet assignment

Fleet assignment models are often used by airlines once an initial schedule has been created and the final fleet of aircraft is known. The objective is to assign aircraft types to scheduled flights in order to minimize costs and spillage or maximize profit while respecting operational constraints. It is a tactical decision made between 6 months and a year before the fights take place. Due to the fact that the fleet assignment model has both a temporal and spatial component, the problem is often represented by a time-space network. Instead of having a static formulation of the problem such as the ones in fleet planning, each node is characterised by a location and an instant in time. Time-space networks try to represent as accurately as possible a flight schedule by using 2 different types of arcs. A flight arc is a connection between 2 nodes and is characterised by an origin, a destination, a departure time and a arrival time, potentially including turn around time. Ground arcs represent aircraft that need to stay put at an airport for a limited period of time in order to be able to service another flight arc. According to the authors of The global airline industry, "finding a feasible fleet assignment is analogous to selecting a path through the time-space network for each aircraft type"[8]. Hane et al. presented a fleet assignment problem formulation called the Basic Fleet Assignment Model (FAM) which can be found in the formulation below with the nomenclature in Table 5.1 [27].

Sets		***Parameters***	
F	set of flight legs	n	number of nodes
K	set of fleet types	n^+	ground arc originating at node n
N^k	set of nodes for fleet k	n^-	ground arc terminating at node n
G^k	set of ground arcs for fleet k	c_i^k	cost of assigning a/c type k to flight leg f
O(k,n)	set of flight legs origination from node n for fleet k	M^k	number of aircraft of type k
I (k,n)	set of flight legs ending at node n for fleet k		
CL(k)	set of flight legs for fleet k		
CG(k)	set of ground arcs for fleet k		
Decision variables			
y_a^k	number of aircraft of type k on ground arc a		
$f_i^k = \begin{cases} 1, \\ 0, \end{cases}$	if flight leg f is assigned to fleet k othewise		

Table 5.1: Nomenclature for the Fleet Assignment model

Basic Fleet Assignment Model

$$\text{minimize} \sum_{i \in F} \sum_{k \in K} c_i^k f_i^k \tag{5.1}$$

s.t.

$$\sum_{k \in K} f_i^k = 1, \qquad \forall i \in F \tag{5.2}$$

$$y_{n^+}^k + \sum_{i \in O(k,n)} f_i^k - y_{n^-}^k - \sum_{i \in I(k,n)} f_i^k = 0, \qquad \forall n \in N^k, \forall k \in K \tag{5.3}$$

$$\sum_{a \in CG(k)} y_a^k + \sum_{i \in CL(k)} f_i^k \leq M^k, \qquad \forall k \in K \tag{5.4}$$

$$f_i^k \in \{0,1\}, \qquad \forall i \in F, \forall k \in K \tag{5.5}$$

$$y_a^k \geq 0, \qquad \forall a \in G^k, \forall k \in K \tag{5.6}$$

$$\tag{5.7}$$

The Basic FAM is therefore an integer, multi-commodity network flow problem. Fleet assignment happens on a tactical time frame which means that the objective is more often linked to minimizing costs than maximizing revenues and potential profit, factors which have already been taken into account during the fleet planning, frequency planning and timetable development. The objective function in Equation 5.1 minimizes the sum of the operating costs of assigning an aircraft of type k to a flight leg. Equation 5.2 ensures that each flight leg is assigned to an aircraft type. Equation 5.3 represents the balance constraint and ensures that the same number of aircraft of the same type arrive and depart from an airport. The constraint in Equation 5.4 imposes a limit on the amount of aircraft of each type assigned based on the maximum amount available in the fleet. Finally, Equation 5.5 ensures that the decision variable f_i^k remains binary and Equation 5.6 is the non-negativity constraint imposed on y_a^k.

The Basic FAM has been extended to incorporate multiple different types of operational constraints such as noise restrictions, maintenance and airport specific requirements and more. However several limitations still exist such as the fact that spillage and passenger recapture is not considered. Barnhart et al. (2002) created the Itinerary-based Fleet Assignment Model (IFAM) which incorporated the Passenger Mix Flow model (PMF) with the Basic FAM which extended the latter to include spillage and passenger recapture rates [6]. The objective function of the IFAM can be found in Equation 5.8 where the first term is the FAM component found in Equation 5.1. The second term represents the negative of the passenger revenues which must be minimized by spilling passengers that are either low fare, or that can be recaptured on alternative itineraries. The set P represents the total set of passenger itineraries, and P_p the set of itineraries which can recapture passengers from itinerary p. The parameter $fare_r$ represents the price of travelling on itinerary r, t_p^r the expected number of passengers wanting to travel on itinerary p but spilled to itinerary r and b_r^p the recapture rates.

$$\text{minimize} \sum_{i \in F} \sum_{k \in K} c_i^k f_i^k - \sum_{p \in P} \sum_{r \in P} fare_r b_p^r t_p^r \tag{5.8}$$

5.1.2. Aircraft rotation planning

The aircraft rotation or routing problem deals with assigning specific aircraft tail numbers with flight legs. This step comes after the fleet assignment problem when each flight leg has been associated with an aircraft type, assuming an initial schedule and fleet is known. The objective is usually to minimize costs while determining an optimal sequence of flight legs for each aircraft in the fleet. The main considerations taken into account when creating an aircraft routing problem can vary significantly from airline to airline depending on the network they operate, the fleet types available and their revenue sources. Factors such as service level, maintenance opportunities, aircraft utilisation, flight coverage must be balanced based on the type of operations in question (passenger transport, cargo, military) and on the operators priorities. A common problem cited in literature is the Aircraft Maintenance Routing Problem (AMRP) and its derivatives [24][38]. The goal is to assign aircraft to flights and routes while minimizing operating and maintenance costs or maximizing through-revenues. Maintenance checks at fixed intervals are an obligation for most airlines who must incorporate them in their regular schedule. For example, regulators in the USA require operators to perform routine checks every 3 to 5 days which significantly impacts aircraft operations and schedules [8]. Aircraft routing and maintenance problems need to incorporate 2 new aspects of airline operations, the first being aircraft can only undergo maintenance at specific locations where maintenance crews and facilities are available, and the second being that this needs to be performed cyclically on a weekly, monthly or yearly basis.

Another important consideration in the commercial aircraft rotation planning is the crew assignment problem which is subdivided into crew pairing and crew assignment. The main aim is to assign a cabin crew and pilots to each flight while minimizing crew costs, or maximizing personnel preferences, and respecting labor rules. To achieve this, first the crew pairing problem is solved where one searches for an optimal sequence of flights and layovers which begin and end at a same crew base and is covered exactly once. Typical constraints are maximum number of days away from base, balanced work distribution between crews, maximum number of duties, minimum and maximum layover time. Another consideration to take into account is the fact that cockpit and cabin crews are usually bound to specific aircraft families based on the licenses they posses and their training. Once the crew pairing has been solved, the crew assignment, also called crew rostering, consists of allocating schedules to crew members or crew types in order to maximize their preferences and minimize the number of crew needed. Crew types are separated by rank (Captain, first officer, flight engineer etc.) and by aircraft family. Creating a crew scheduling model is a significantly difficult task due to the fact that the problem size is extremely large, safety and work regulations are complex and vary between countries, and costs involved are non-liner and difficult to represent mathematically. Similarly to frequency planning and timetable development, a common technique used for solving the crew paring and assignment problem is to start from previous schedules and apply incremental changes over time. Integer programming models are also used to improve solutions by generating or enumerating subsets of feasible pairings and using heuristics and relaxation algorithms such as column generation to solve reduced versions of the problem.

Recent research has focused on integrating the different parts of the schedule development to create more comprehensive models, closer to reality. A. Mercier and F. Soumis (2005) create a mathematical formulation for the "integrated aircraft routing, crew scheduling and flight re-timing model". They develop a solving technique using Benders decomposition method with dynamic constraint generation [44]. C, Barnhart et al. (1998) develop a solving technique and mathematical model to approach simultaneously fleet assignment and aircraft routing problems [5]. A.M. Cohn and C. Barnhart (2003) noticed that crew costs are the second most expensive costs for an airline, and that crew scheduling possibilities are limited by the decisions made earlier in the planning process. They integrate the crew pairing problem and aircraft maintenance routing problem in order to provide as many maintenance-feasible crew pairings as possible, giving more flexibility to decision makers and reducing overall costs [13]. Because integrated models are larger and more computationally intensive than the single problem formulations, an opportunity lies in developing methods to efficiently solve them in a finite amount time.

5.2. Humanitarian routing and scheduling

Operations research and optimisation is not commonly applied to the humanitarian context although logistics plays a crucial role in its effectiveness. While decision support tools have significantly elevated the performance of private sectors by reducing costs and increasing efficiency, the humanitarian sector is still lagging behind in this field [60]. A.M. Caunhye, X. Nie and S. Pokharel (2012) provide an extensive review on optimisation models applied to emergency logistics [11]. The authors breakdown literature in 3 main groups : Facility location problems, relief distribution and casualty transportation, and other. E. Nikbakhsh and R.Z. Farahani (2011) provide an overview on common mathematical formulation for the location problem, inventory problem, VRP, the transportation and distribution problem, but adapted to humanitarian operation [46]. V. De Angelis et al. (2007) tackle the air delivery optimisation problem for the WFP mission in Angola [17]. The goal is to create a routing and schedule which maximizes demand satisfaction subject to availability and operational constraints. They extend the VRP to create the Vehicle Routing Variable Depot Full Load (VRVDFL) model, an Integer Linear Programming (ILP) model which deals with the weekly routing and scheduling of a homogeneous fleet of cargo planes between multiple depots and clients. G.Barbarosoglu and Y. Arda (2004) propose a two-stage stochastic model for emergency transportation planning where the time horizon is split between Early-response and Response after a disaster has struck. They create a multi-commodity, multi-modal network flow mathematical formulation which models the movement of aid in an urban setting while taking into account different scenarios characterised by random allocation of supply quantities, arc capacities and demand [4]. In the same effort to avoid deterministic models, S.J. Rennemo et al. (2014) create a three-stage stochastic facility routing model where they integrate the facility location problem and VRP [20].

More recently, two research papers have explored humanitarian passenger flight routing and scheduling using as case study the South Sudan UNHAS mission. S.P. Niemansburg (2019) created the Humanitarian Flight Optimization Model (HFOM), an adaptation of the multi-depot heterogeneous pickup and delivery problem with time windows [45]. The problem is formulated as three-index Mixed-Integer Linear Programming model with objective to minimize the sum of the routing costs subject to demand satisfaction, flow conservation, availability, capacity and operational constraints. Due to the size of the problem and the high number of nodes needed to represent accurately potential routings, a division heuristic is used to sub-divide the problem into smaller instances which are solved sequentially: South Sudan is divided in 6 different regions, each associated with specific daily passenger request data. Due to the multi-objective dimension of the model, the outputs are Pareto Fronts, allowing one to trade-off between costs and demand satisfaction. The model is validated by an expert flight planner who is asked to produce the routing as if in a real case scenario. This is compared to the results of the model and the author concludes that the HFOM is able to produce routings with reduced costs between 2.2% and 7.8% and output a result up to 5 times faster. Building upon this research, Y. Mekking (2020) extends the HFOM by modifying the node generation and allocation formulation in order to reduce the problem size [43]. Another significant addition lies in the incorporation of the concept of minimum guaranteed hours (mgh) which are an important benchmark for UNHAS monthly operations. A fleet order constraint is added to the formulation and dynamic pricing is used in order to model the evolution of flight hours per aircraft and keep utilisation of each asset as close to the mgh as possible, avoiding over-time costs. The author develops a similar division heuristic where dynamic elements are used to solve each sub-problem according to a region prioritisation scheme of which the results are illustrated by a map in **??**. Unnecessary decision variables are then removed and auxiliary constraints added, ensuring that the problem is bounded, converging in a finite amount of time. The model achieves between 1% and 20% in cost savings when compared to expert flight planner solutions and improved passenger request satisfactions by 1.2%. For practical reasons, multiple Pareto fronts are once again created and can be used as decision support tools for flight planners when trading off between costs and demand satisfaction.

5.3. Research focus

H. De Vries (2017) clearly identifies the need for more automation in humanitarian planning and routing applications in his review on "Evidence-based optimisation for humanitarian logistics" [60], stating that research on humanitarian schedule development and routing has been lagging behind compared to the commercial sector. Routing and scheduling models created for humanitarian aid and emergency response are usually found to be extensions of the VRP formulations, mostly due to the fact that the objective revolves around minimizing costs and maximizing demand satisfaction. Daily operations are also often considered due to the difficulty in predicting requests and changes in the operating environment which happen on a last-minute basis. It is however important to take a step back and attempt to increase robustness of humanitarian operations on a longer time frame. As previously mentioned in chapter 2, daily routings and schedules are created based on humanitarian requests submitted 72 hours before departure, which in turn are driven by a preliminary weekly schedule drafted by UNHAS. This preliminary weekly schedule has the aim to concentrate OD demand efficiently throughout the week in order to provide humanitarian staff the opportunity to travel to as many destinations as possible while keeping costs low and respecting operational constraints. Until now, no research has been found on optimising this schedule. Apart from Y. Mekking's research paper, models taking into account minimum guaranteed hours is also not present in literature [43]. Incorporating this aspect at the tactical instead of operational level would allow a decision makers to tailor a fleet and preliminary schedules to reduce costly over-time hours and balance aircraft utilisation.

III

Supporting work

1

Appendix 1

This appendix provides more information on UNHAS reporting lines and the humanitarian aircraft contracting process in order to help situate the research within the overall humanitarian context. It also presents maps of South Sudan published by the United Nations logistics cluster during 2019 providing more information on the operational aspect of the mission.

1.1. UNHAS reporting lines

Figure 1.1 shows the main hierarchy of the WFP aviation services and UNHAS. The main actors concerned by this study are the Air Transport Unit, the Chief Air Transport Officers (CATO) and the User group comity. The Air Transport Unit is partly made up of contracting officers responsible for the chartering of air assets. They are in direct contact with CATOs on the field and jointly make decisions on which air assets are most suited to a specific mission as well as when they should be contracted, based on available aircraft types, leasing costs, operational costs, and the need in the country of designation. Every month, a User Group meeting is held where the main users of UNHAS discuss air transport with the CATOs and the establish a monthly forecasted amount of passengers. These inputs are used to created weekly preliminary schedules which drive the passenger bookings and may lead to the conclusion that more or less air assets are needed. This is communicated back to the contracting officers in at headquarters.

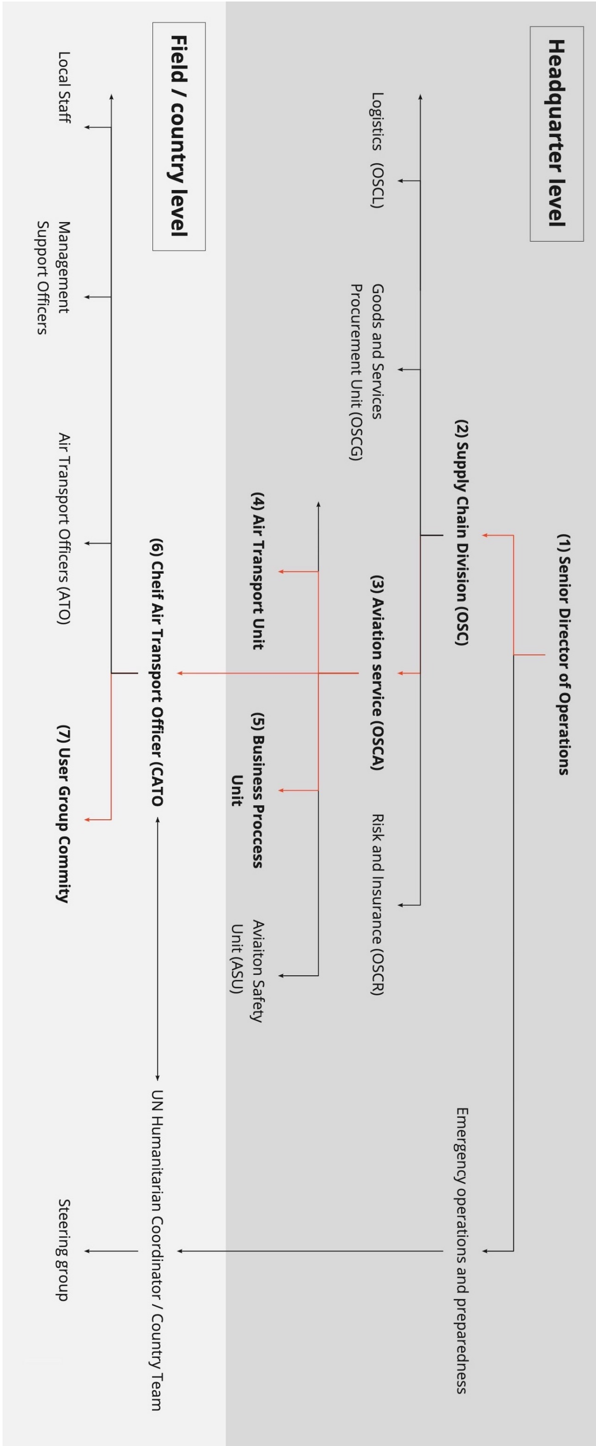


Figure 1.1: Diagram of UNHAS reporting lines (links of interest for this study in red)

1.2. UNHAS aircraft selection processes

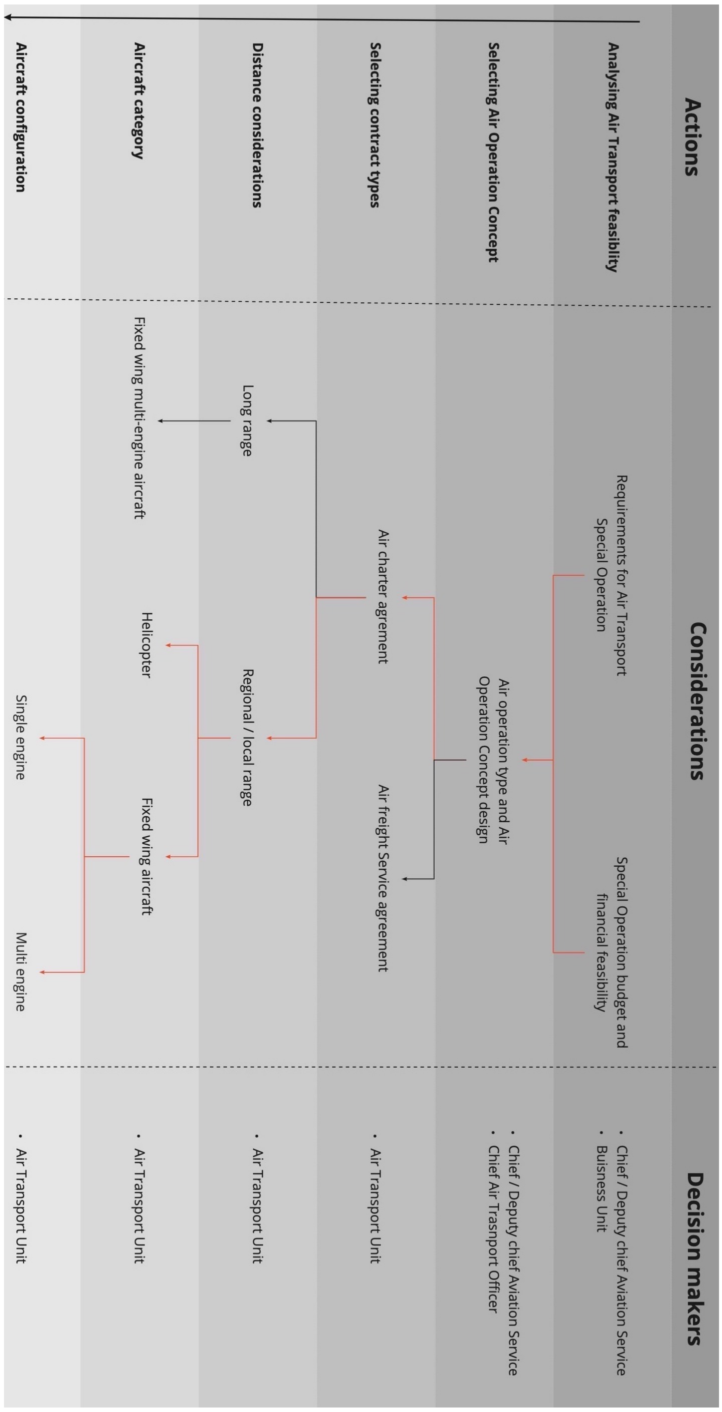


Figure 1.2: Diagram of main steps taken by the WFP aviation services and UNHAS in selecting appropriate aircraft fleets

1.3. South Sudan Road access maps

The following road access maps in Figure 1.3 and 1.4 both present the state of the transportation network in South Sudan during the dry and rainy season respectively. It can be observed that during the rainy season, a number of destinations can no longer be accessed due to flooding and multiple roads are closed off to humanitarian aid convoys. The need for air transportation is increased during this period, specially for MI8 helicopters who are able to access remote locations that small trubo-prop aircraft such as Cessna 208s can no longer use due to flooding and runway damage.

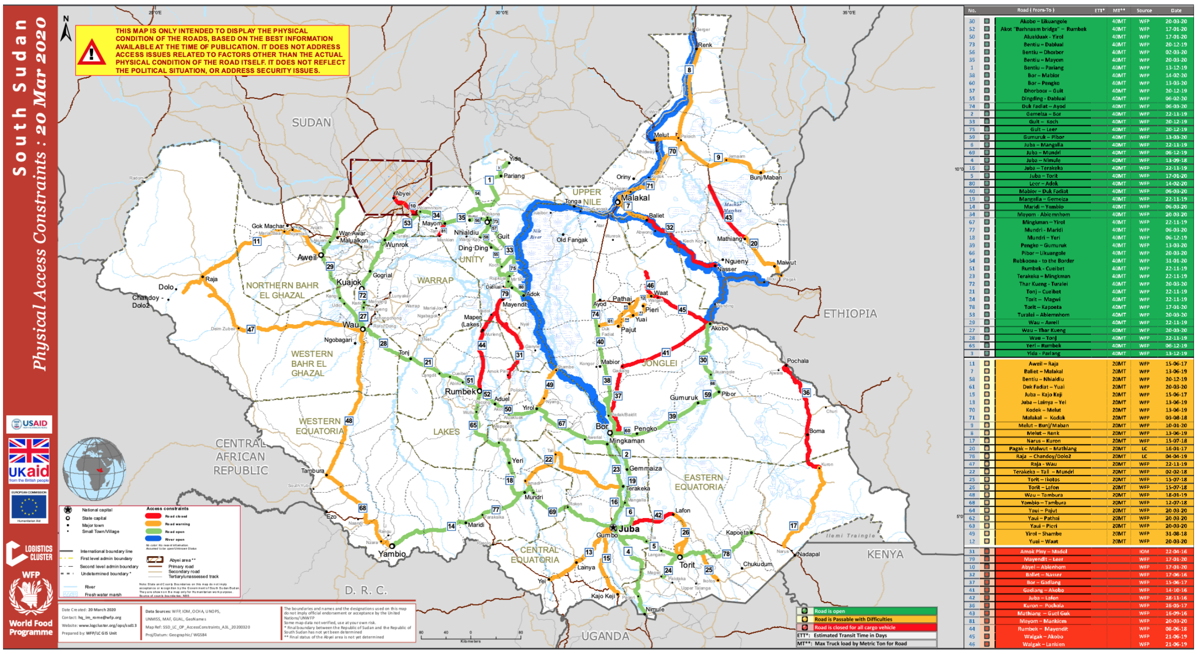


Figure 1.3: South Sudan access constraint map during the dry season (November - April) [40]

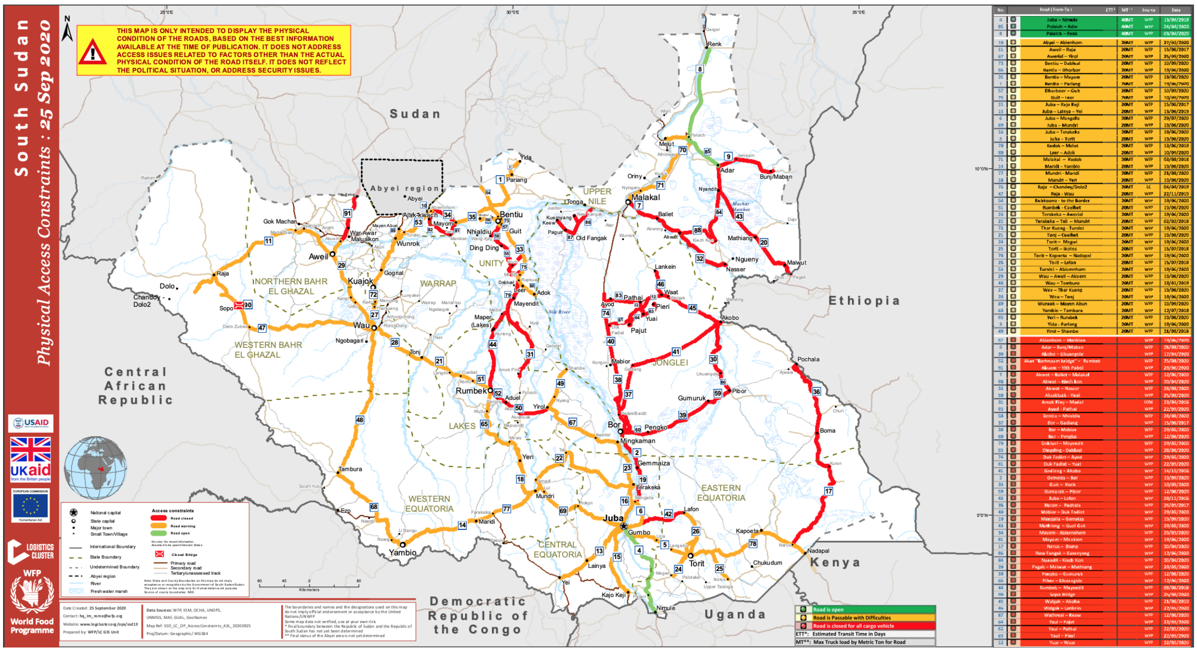


Figure 1.4: South Sudan access constraint map during the rainy season (May-October) [40]

1.4. South Sudan UNHAS routes

Figure 1.5 is a schematic representation of the destinations served by UNHAS. A hub and spoke pattern clearly emerges form the figure and supports the modelling choices made in the research paper.

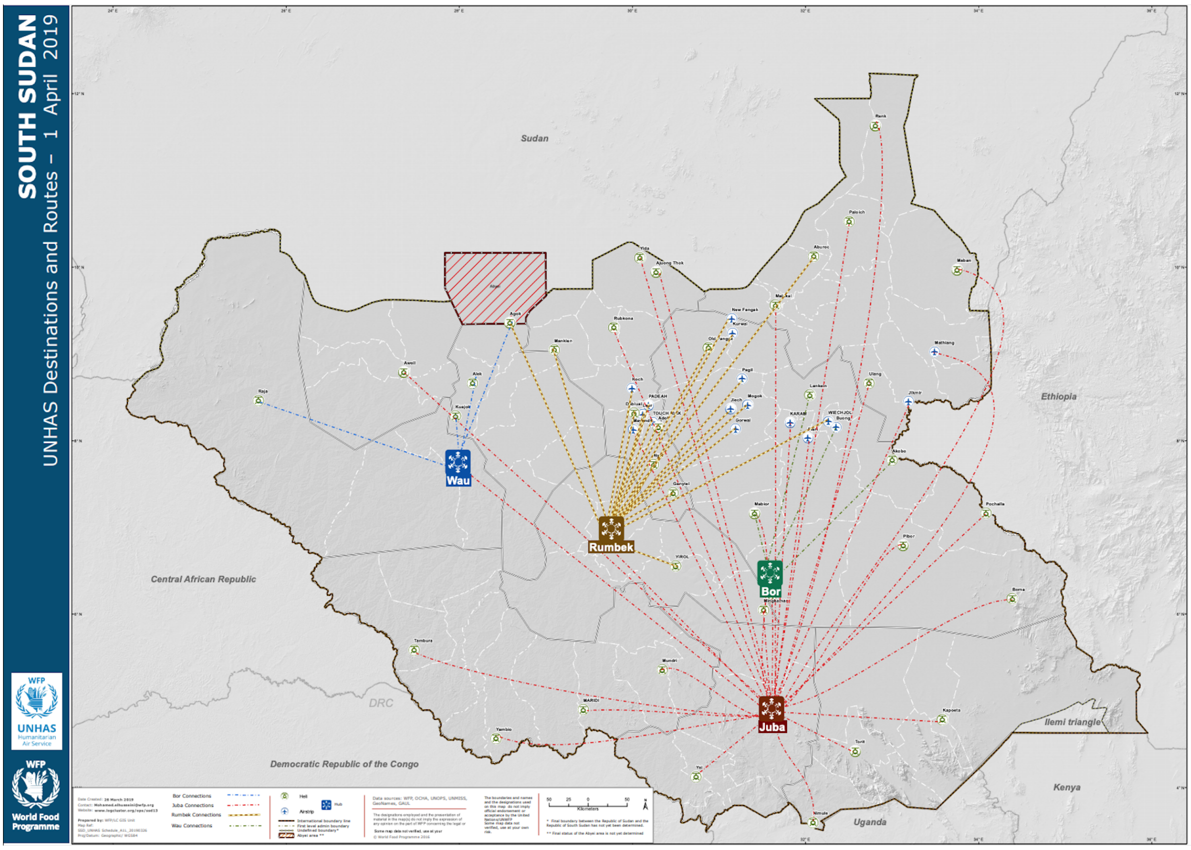


Figure 1.5: UNHAS flight destinations and connections map for April 2019 [40]

2

Appendix 2

The MCNF & HFSMVRP model were run for the South Sudan rainy season in order to observe how the fleet would change from season to season according to the changing operating.

2.1. South Sudan rainy season

South Sudan is subject to 2 different seasons, the tropical/rainy season starting end of April and lasting until October, followed by the dry season. During the rainy season, heavy floods occur, damaging roads and runways and rendering them unusable by ground transport or aircraft. The maps in Figure 1.3 and Figure 1.4 illustrate the state of the road network during these 2 different season, also taking into account road closures due to increased risks of insecurity on these links (armed conflict/explosives/kidnappings etc.). It is therefore important to evaluate if a different fleet is needed during the rainy season than the dry one. It is assumed that the demand to and from airports does not change, however certain airfields can no longer be accessed by turboprop aircraft and must be reached by helicopter. According to UNHAS expert planners and for the purpose of the case study, the airports marked with a * in the research paper's Appendix A, Table 18, are considered only accessible by MI8 helicopters during the rainy season. The MCNF model is first used to model passenger transshipment. The results can be found in Table 2.1. The fleet selected is the same one as for the dry season with the exception that 2 MI8s are needed in Rumbek instead of 1. Routing costs are also much higher for the helicopters as they must reach destinations now inaccessible to the cheaper turboprop aircraft.

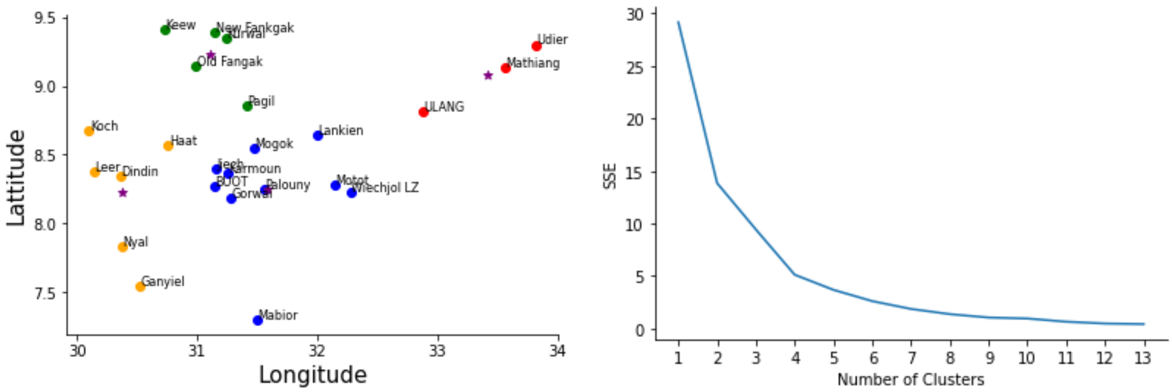
Table 2.1: Multi-commodity network flow model results for South Sudan weekly demand (30/09/2019-04/10/2019) during rainy season

	DCH8-3	Cessna 208B	Cessna 208B	Cessna 208B	MI8T	MI8T
Number of aircraft	1	1	1	1	1	2
Aircraft base	Juba	Juba	Rumbek	Wau	Juba	Rumbek
Weekly block hours [h]	20.3	24.0	24.0	11.2	31.6	59.8
Weekly operational hours [h]	33.3	40.5	37.0	18.2	42.1	84.8
Weekly distance flown [km]	10,781	8,265	8,268	3,848	7,029	13,287
Weekly routing costs [$]	67,151.4	30,024.9	30,033.5	12,399.2	90,490.5	17,1045.4
Total weekly routing costs [$]	401,144.5					
Total monthly lease costs [$]	947,977					

Table 2.2: Sub-problem division as a result of the Multi-commodity network flow model for South Sudan rainy season

Sub-problems	Requests	Airports	Passengers
Sub-problem 1: MI8 - Juba	14	10	190
Sub-problem 2: MI8 - Rumbek	35	23	245
Sub-problem 3 :DCH8 106 - Juba	14	8	1,115
Sub-problem 4: Cessna 208B - Juba	16	9	237
Sub-problem 5: Cessna 208B - Rumbek	22	12	202
Sub-problem 6: Cessna 208B - Wau	9	11	85
Total problem	110	61	2,074

The sub-problem division resulting form the MCNF problem displayed in Table 2.2 shows that sub-problem 2 contains more than 25 requests. The clustering algorithm is therefore used to divide the region into 4 different clusters as can be seen in 2.1(a), based on the "Elbow graph" in 2.1(b). Each cluster is first run separately using the HFSMVRP model. Their individual solutions are then combined and used as a warm start for the entire sub-problem 2. Prior to the warm start, the results of all individual clusters indicate that 2 MI8's would still needed, with a total routing cost of 72295.97 \$. Once the warm start is used, the HFSMVRP model is able to find a better solution, reducing the MI8 helicopters to 1 and reducing the routing costs by 11.46 %. A summary of the main results can be found in Table 2.3.



(a) Clustering algorithm applied to sub-problem 2

(b) "Elbow" graph for the clustering of sub-problem 2

Figure 2.1: Clustering algorithm applied to helipads in South Sudan during the rainy season

Table 2.3: HFSMVRP results for sub-problem 2 for the South Sudan weekly demand (30/09/2019-04/10/2019) during the rainy season

	MI8T Region 0	MI8T Region 1	MI8T Region 2	MI8T Region 3	MI8T All Regions	MI8T All Regions (Final)
Amount	1	1	1	1	2	1
Aircraft base	Rumbek	Rumbek	Rumbek	Rumbek	Rumbek	Rumbek
Weekly block hours [h]	7.9	6.6	5.8	5.1	25.2	22.4
Weekly operational hours [h]	15.4	12.1	9.2	7.5	44.1	41.9
Weekly distance flown [km]	1,750	1,464	1,262	1,139	5,616	4,972
Weekly load factor [%]	72,05	52.52	47.63	48.42	55.14	68.52
Weekly routing costs [\$]	22,532.5	18,847.9	16251.5	14,665.0	72,296.0	64004.1
Weekly routing costs decrease after warm start				11.46 %		
Monthly lease costs decrease after warm start				50 %		

The final results for the entire weekly demand during the rainy season can be found in Table 2.4. The HFSMVRP is able to reduce the total routing costs by 33 % by refining the routing of the aircraft. It also reduces the fleet leasing costs by exchanging the Cessna 208B in Juba by a Dornier 228.

Table 2.4: MCNF & HFSMVRP model results for South Sudan weekly demand (30/09/2019-04/10/2019) during rainy season

	DCH8-106	Cessna 208B	Cessna 208B	Dornier 228	MI8T	MI8T
Number of aircraft	1	1	1	1	1	1
Aircraft base	Juba	Rumbek	Wau	Juba	Juba	Rumbek
Weekly block hours [h]	21.2	22.8	8.5	13.8	22.4	22.4
Weekly operational hours [h]	34.7	36.8	15.5	25.3	32.9	41.9
Weekly distance flown [km]	11,266	7,840	2,939	4,338	4,990	4,972
Weekly load factor [%]	87.20	72.63	67.50	79.13	67.70	68.52
Weekly routing costs [$]	70,173.7	28,479.0	9,471.1	32,689.4	64,242.2	64,004,0
Total weekly routing costs [$]			269,059.5			
Total monthly lease costs [$]			716,441			

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