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IRREGULAR SEAS --
A NEW TOWING TANK PROBLEM

by

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SUMMARY

In 1905, R.E. Froude stated that an irregular sea surface can be represented by the summation of many regular waves and that the summation of ship responses to these regular waves will represent the correspondingly irregular ship motions. In 1933, R. Legendre actually used this technique in an investigation of ship rolling, without having all the necessary physical and mathematical tools. In an SNAME paper by St. Denis and Pierson in 1953, these ideas were first put on a sound foundation by utilising Neumann's sea observations, and the mathematical techniques of Rice, Tukey, and Lee. E.V. Lewis immediately applied them to towing tank technique. The use of irregular seas in towing tank testing requires a much closer connection between towing tank experiments and analysis than has ever been required before. This paper is devoted mainly to a simple description of the methods needed and used for the analysis and interpretation of irregular sea test data. It is hoped that it will give at least the minimum of necessary information to towing tank personnel, and will serve as a guide and an introduction to those who want to study the matter further.

INTRODUCTION

In May 1933, R. Legendre presented a paper [1]* at the meeting of a French Society in which he described his research on the rolling of two cruisers, which was conducted using a theory, towing tank tests, and observations on ships at sea. At the end, he summarized his findings as follows:

1. The rolling computed on the basis of the mean observed wave does not have the same character as the observed rolling, and is much smaller.

2. The summation of roll angles, computed on the basis of the observed wave, decomposed into its sinusoidal components, has for its period the natural period of the ship. It is of the same order of magnitude as the observed rolling.

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* Numbers in brackets refer to the list of references on page 26.

3. In order to predict the rolling of a projected ship, it is necessary to investigate the general characteristics of the actual wave.

Twenty-eight years earlier, in 1905, R.E. Froude [2] wrote*: "Irregular waves such as those commonly met with at sea ... are only a compound of a number of regular systems (individually of a comparatively small magnitude) of various periods, ranging through the whole gamut (so to speak) represented by our diagrams [of behavior in regular waves], and more. And the effect of such a compound wave series on the models would be more or less a compound of the effects proper to the individual units composing it."

Kent [4], in 1922, produced irregular waves in a towing tank in order to develop the slamming of a ship model, which did not occur in regular waves. The study of actual conditions existing at sea, and of actual ship motion at sea also occupied an important place in German literature, as illustrated in References 5, 6, and 7, taken at random. Although the irregularity of waves and of ship motions at sea was well recognized, it did not appear to be taken into account in towing tank activity.

Finally, in 1952, A.J. Williams [8] presented at an INA meeting a very comprehensive statistical study of the distribution of the rolling periods and amplitudes on three instrumented ships at sea. The word "finally" is used advisedly to indicate the end of what can be called the "pre-dawn period." Whilst the irregularity of the actual sea was recognized, and the ideas or even partial attempts to deal with it occurred, the necessary mathematical and physical tools needed to put the ideas on a firm and practical basis were lacking. The whole outlook on the subject was drastically changed in 1953 by the appearance of the papers by St. Denis and Pierson [9] of DTMB and New York University, respectively, and of R.A. Fuchs and his associates [10,11,12] of the University of California. In these papers it was shown how the actual irregular and ever-changing sea surface can be described mathematically, and how, on the basis of this description, the motions of a ship can be computed. While basically representing a theoretical development, these papers have had a direct effect

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* The quotation is actually taken from the discussion section of Reference 3, p. 201.

on the activity of towing tanks, and on the interpretation of towing tank tests and ship observations at sea alike, as will be shown later.

The practical value of the above papers was enhanced by the fact that the necessary mathematical material for their practical utilization had been developed within a few preceding years. The material can be divided into two groups: The first is formed by the work of G. Neuman [13,14], who, by summarizing the previous oceanographic data and by adding his own sea observations and deductions, derived the form of distribution of the amplitudes and frequencies of the sinusoidal waves into which the actual sea can be decomposed. The second group consists of the development of the necessary mathematical methods, which are essentially based on the work of Wiener [15], further developed by Rice [16], Lee [17], and Tukey [18] in connection with the analysis of noise in communication systems. Subsequently, aeronautical engineers found that atmospheric turbulence follows the same statistical law as communication noise and immediately applied similar methods to aircraft problems [19,20,21].

For several years now, similar statistical methods have been applied to the analysis of ocean wave records by the National Oceanographic Institute in England [22]. The application of these ideas and methods to towing tank work on ships has been undertaken and vigorously pursued by E.V. Lewis [3,23,24,25,26], and to the observations on ships at sea by D.E. Cartwright [27,28]. The latter application was made possible by the recent development of the ship-borne wave-height recorder by M.J. Tucker, described in References 29 and 30.

As can be seen from the numerous references cited above, the literature on the subject is quite extensive. However, it is very difficult to follow, because the subject was and is in the state of rapid development. The largely unrelated publications deal mostly with the development and theoretical foundation of the methods used, the terminology and symbols are not uniform, and the statistical mathematics involved is of the type not generally familiar to engineers and naval architects. The present paper is written, therefore, as an attempt to provide a brief and simple summary of the current status of the "irregular sea problem," with particular emphasis on its practical application and on the resultant interpretation of towing tank and ship observation data, suppressing all but the most elementary derivations, and concentrating on the end results. Such a summary may conceivably be sufficient for administrative personnel of towing tanks and it is hoped that it will serve as an introduction and a guide to further study for the working personnel.

It should be understood from the beginning that towing tank (as well as ship observations) work with irregular seas requires a much closer connection between experimental and theoretical activity than has ever been required in the past. The regular (or at least intended to be regular) waves heretofore used in towing tanks have been of essentially trochoidal shape and, with the relatively low height of 1/60 to 1/20 of the length, differ little from the sinusoidal (or often called harmonic) wave in its action on a ship model. In the case of a carefully conducted test, the pitching and heaving motions of a model have also been sinusoidal, with essentially uniform amplitude and period. An experimenter could limit his final report to plots of amplitudes vs. wave length and ship speed or to plots of magnification factor vs. frequency of wave encounter. Both of these plots are familiar to naval architects and are easily understood. Indeed, in the past, theoretical analyses of such a motion proceeded independently of experimental work with only a few attempts at cross checking (Kent [4], and Korvin-Kroukovsky and Lewis [23,31]).

An entirely different situation exists in the case of irregular waves in which the apparent periods and amplitudes vary continuously and irregularly. One may be tempted to report the average values and let it go at that. However, as Legendre [1] already pointed out in 1933, such information will be unrealistic, and in fact outright misleading. A typical record of irregular waves and a ship model pitching and heaving is shown on Figure 1. Each curve, i.e., of wave height, angle of pitch, and amount of heave, is generally an irregular wiggly line from which very little can be derived simply by direct examination; little purpose would be served by submitting such a record as the final result of experiments, without any idea as to its meaning. In order to derive a meaning from such a record, a mathematical analysis is needed and will necessarily have to accompany any submittal of experimental data. The nature of the analysis will depend on the problem, and this will be discussed later, after defining the "irregular sea."

IRREGULAR SEAS IN THE OCEAN AND IN THE TOWING TANK

The term "confused sea" was used in the title of Reference 9, but has since been largely abandoned. Although it has been said by no lesser authority than Lord Rayleigh that the "basic law of the seaway is the apparent lack of any law," it has recently been demonstrated by oceanographers that this lack is only apparent and that, in reality, certain laws exist. These laws are statistical, however, and from the point of view of the history of science the discovery of laws of ocean waves is merely another step in the discoveries of the statistical laws of nature. This method of discovery started over one hundred years ago with the discovery of the molecular motion in gases and has since found wide application, including the more recent studies of fluid turbulence, and of dynamic loads in airplanes flying through atmospheric gusts. The world seems to be so created that a very large number of entirely unconnected random events produces in aggregate an action which can be well defined by a law. It is important to realize, therefore, that the irregular sea to be produced in a towing tank must not merely have a disorderly confused appearance, but must conform to the statistical characteristics which are found to exist in the actual sea. The word "must" is used because it has been shown that ship motions also obey the same form of statistical laws. Hence, a ship motion is related in a definite manner to the seaway. The use of an unrealistic (in its significant aspects) sea will therefore yield unrealistic aspects of ship behavior. The statistical characteristics that are significant can be found only by a mathematical analysis of wave records.

HISTOGRAMIC FORM OF ANALYSIS

There exist two basically different methods of statistical analysis, used for different objectives. For the purpose of the present paper, the first will be referred to as "statistical" or "histogramic," and the second, as "time series" or "generalized harmonic" analysis. The first one conforms in fact to the simple and most popular concept of statistics, and does not require a profound mathematical knowledge. It is illustrated in Figure 2 by an example taken from Reference 14, which shows a section of a long record of wave height vs. time. The record is clearly not sinusoidal, but one can nevertheless speak of the distance between successive crests as an "apparent period," in order to distinguish it from the true period of a sinusoidal wave. Likewise, the vertical distance from a trough to the neighboring peak is called "apparent height." These are habitually designated by $\tilde{T}$ and $\tilde{H}$. Both of these quantities are highly variable; a number, say 200, of consecutive apparent periods can be measured and collected in a table. The entire content of this table can be divided into groups falling within the range of successive spreads of periods say from 2 to 2.5 sec., 2.5 to 3 sec., 3 to 3.5 sec., etc. The number of measurements found in each group is plotted then as the ordinate vs. $\tilde{T}$ as abscissa. Instead of plotting the actual quantity found in each group, the ratio of this quantity to the total quantity in percent is plotted. This percentage is usually referred to as the "frequency distribution." The resulting diagrams, which are often called "histograms," are shown on Figure 3. These illustrate the conditions found at sea, but without any direct reference to the causes of these conditions. By preparing separate diagrams from records taken at different wind strengths, a certain idea regarding the dependence of waves on the wind can be gained. Another example is given on Figure 4, taken from Reference 8. It is a plot of 200 measurements of the amplitude of roll taken on a ship at sea. The intervals into which the groups of measurements are subdivided in this case are of 0.25°. Since this form of analysis is simple and generally well known, the matter will not be pursued further.

AUTO-CORRELATION ANALYSIS & general harmonic analysis.

Generally speaking, "histograms" illustrate well "what is" without explaining "why." They indicate the range and nature of the variation of a single quantity, such as the apparent period, apparent amplitude, the level of bending stress, etc., as found by observations. They provide no means of establishing the functional relationships among sea conditions, ship geometry, and ship motions. These relationships can be obtained by a more complicated approach described as a "time series" analysis or a "generalized harmonic" analysis. This will be considered in two sections: first, the case when only one quantity (i.e., one-trace record) at a time is to be analyzed, such as the wave profile, or the pitching motion of a ship; and, second, when the mutual relationship of two simultaneously considered quantities is desired from a two-trace record, say the pitching of a ship in connection with the waves causing it. If the mutual relationship of a greater number of quantities is desired, the analysis is made in pairs, always referring to one independent quantity, usually to the wave profile.

The initial information to be analyzed is in the form of an irregular record such as the wave record of Figure 2, or the upper part of Figure 1. This record is assumed to be the result of the superposition of an infinite number of waves of all possible lengths (periods or frequencies) and all possible phase angles varying in a random fashion. In the actual sea, the variation in the direction of propagation of various wave components causes short crestedness, which is an important factor in ship motions. The consideration of short crestedness will become important in the new tanks now being constructed for model tests in oblique waves. In the present simple exposition, however, the common, long and narrow towing tank is visualized, and so randomness is limited to component wave lengths and phase angles, and all wave components propagate in the same direction. This is generally termed a "long-crested irregular sea."

It will be recollected that, as simple sinusoidal waves pass a certain point, say a measuring pole, the wetted height of the pole is given by

$$y(t) = A \cos(\omega t + \epsilon) \quad , \qquad (1)$$

where A is the amplitude (i.e., the maximum value of y); ω is the circular frequency, and ϵ the phase lag with respect to the arbitrarily assumed origin of time $t = 0$.* $y(t)$ indicates that y is a function of time. It is now assumed that the amplitude A, the frequency ω, and lags ϵ vary randomly for an infinite number of superposed waves, but that A and ω are functionally interrelated, i.e., $A = f(\omega)$. The problem is to evaluate this relationship. Why is this evaluation needed? On the one hand, this function represents a permanent characteristic of the ever-changing surface of an ocean. It remains fixed as long as the length of

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* A complete list of symbols and definitions is given on page 30.

the fetch and the strength of the wind remain unchanged while the actual succession of waves changes continuously. On the other hand, it gives means for obtaining ship motions. As R.E. Froude [2] already expressed it, if it is known how a ship reacts to a simple sinusoidal wave, and if it is known out of which sinusoidal waves the actual sea is composed, then the final motion of a ship can be obtained. So the first part of the problem is to find those simple waves out of which the irregular sea is composed, which is synonymous with evaluating the relationship $A = f(\omega)$. Actually, it has been found that amplitude squared, which is proportional to the energy content of a wave, is more significant and is at the same time more easily obtained in computations. The relation sought is therefore $A^2 = \varphi(\omega)$, which is referred to as a "power density spectrum" or "energy density spectrum." The analysis of ocean wave records will have to be made by oceanographers or by people making ship observations at sea. It will also have to be made in towing tanks in order to verify the extent to which the tank sea is realistic, and in order to interpret model motions.

At sea, the water surface changes continuously, but for a reasonable length of time the statistical characteristics, such as the power spectrum, remain unchanged. In practice, it becomes necessary to take a sample -- a finite length of the ever-changing wave record. If the sample is too short, the permanent statistical characteristics will not be accurately evaluated; if it is too long, the weather conditions may change. The most practical length of the record at sea appears to be from 12 to 20 min. or from 120 to 200 apparent wave lengths. The adjective "apparent" is used for the wave period, wave length, and wave height as directly observed at sea (where the waves are not in fact periodic), to distinguish them from the period, length, and height exactly defined for the sinusoidal components of the wave. In a towing tank, for the same number of waves, the period is scaled in proportion to the square root of the model length ratio, making the desired sample of say 2 min. for a 5-ft. long model. In reality, with a normal forward model speed, a possible period of observation may be only 15 to 20 sec. long. At the Experimental Towing Tank of Stevens Institute of Technology, for instance, the equipment is provided to produce a standard wave pattern, changing continuously for 2 min. [26]. In order to provide a good sampling, the ship model is run in five sections of this wave pattern, and the mean of the final analyzed results is then taken. Whether in the towing tank or at sea, therefore, the practical problem is to analyze a section of the record of a finite length.

The well-known method, of course, would be to apply the Fourier Harmonic Analysis. With the record containing possibly up to 200 waves, and with the desired resolution up to, say, the 150th harmonic, the labor involved becomes prohibitive. A simpler method, known as "auto-correlation," has been found, but even with this the whole procedure is made practical only by the availability of automatic computing machines. The principles of the auto-correlation procedure will be clear from the following simple derivation taken from Rice [16, p. 166].

It is well known that, in many problems of heat transfer or in electricity, a function of irregular form can be approximated by the summation of harmonic waves of various amplitudes and frequencies, i.e., in a Fourier series. Thus, let the sample wave record, such as is shown on Figure 2, be expanded in a Fourier series. The value $y(t)$ of the function represented by the record at any time t will be

$$y(t) = \frac{a_o}{2} + \sum_{n=1}^{\infty} (a_n \cos n\omega t + b_n \sin n\omega t) \quad , \qquad (2)$$

where a_n and b_n are the amplitudes of the n^{th} harmonic cosine and sine waves. These amplitudes are evaluated in the theory of Fourier series by

$$\left. \begin{aligned} a_n &= \frac{2}{T}\int_0^T y(t) \cos n\omega t \, dt \\ b_n &= \frac{2}{T}\int_0^T y(t) \sin n\omega t \, dt \quad , \end{aligned} \right\} \qquad (3)$$

where T is the total length of the sample. Let the record now be shifted by an amount of time τ . Equation (2) then becomes

$$y(t+\tau) = \frac{a_o}{2} + \sum_{n=1}^{\infty} \left[a_n \cos n\omega(t+\tau) + b_n \sin n\omega(t+\tau)\right] \quad . \qquad (4)$$

The shifting of the record evidently did not modify any of its statistical properties. However, a new relationship now appears -- that of the dependence of the measured $y(t)$ on the sum $(t+\tau)$. In order to find the characteristics of this dependence, series (2) and (4) are multiplied, and integrated with respect to t, yielding

$$\frac{1}{T}\int_0^T y(t)\, y(t+\tau)\, dt = \frac{a_o^2}{4} + \sum_{n=1}^{\infty} \frac{1}{2}(a_n^2 + b_n^2) \cos n\omega\tau + K \quad . \qquad (5)$$

The shift τ is accomplished m times so that the integral on the left-hand side, which is now called the "auto-correlation function," becomes a function of τ and is designated by $\psi(\tau)$. It is readily evaluated from a record by dividing its length T into n uniformly spaced parts of length Δt, and reading the ordinates of the wave record trace y at each of these divisions corresponding to the times t_1, t_2, t_3 ... etc., as well as at a number of points $t_1+\Delta t$, $t_1+2\Delta t, \ldots, t_1+m\Delta t$; $t_2+\Delta t$, $t_2+2\Delta t, \ldots, t_2+m\Delta t$, etc. until all n number of time points and m number of increments Δt forming τ are covered. The multiplications and the integration indicated by the left-hand side of equation (5) are then accomplished on an automatic computing machine.

The practical significance of the auto-correlation function $\psi(\tau)$ is demonstrated by consideration of the right-hand side of equation (5). The summation of the squares of the amplitudes a_n^2 and b_n^2 for each harmonic component n represents the square of the total amplitude of the wave, which will be designated A_n^2. One-half of the amplitude squared, $A_n^2/2$, multiplied by the weight of water per cubic foot represents the energy contained in the n^{th} harmonic component. The summation of the energies contained in each harmonic component over the total number of components n gives the total energy content of the sea wave record under consideration. Making n infinitely large and Δt infinitely small permits the relationship (5) to be expressed in integral form, i.e.,

$$\psi(\tau) = \int_0^{\infty} G_{yy} \cos\omega\tau \, d\omega \qquad (6)$$

where the symbol G_{yy} is used to represent the energy in the wave component of the circular frequency ω = (wave period $2\pi/\omega$). The constant term in a_o is eliminated here by assuming the ordinates y to be measured from the mean line of the record. The symbol K designates a certain error resulting from the ends of the record at which the time $t+\tau$ cannot be carried to the full value of $\tau = m\Delta t$. This error is assumed to be made small by a sufficiently large number of subdivisions n with respect to m.

Equation (5) can be solved for G_{yy} by using a Fourier transform, yielding

$$G_{yy} = \frac{2}{\pi}\int_0^\infty \psi(\tau) \cos \omega\tau d\tau \quad . \tag{7}$$

The auto-correlation function $\psi(\tau)$ can be readily obtained from the measurements of the wave record and from the integration indicated by the left-hand side of equation (5). The second integration indicated by the right-hand side of equation (7) yields then the value of G_{yy} or the wave energy for each circular frequency ω. With a continuous distribution of freqnencies, one speaks of the energy $G_{yy}(\omega)\, d\omega$ contained in a band of frequencies between ω and $\omega + \Delta\omega$, rather than of the energy at a given frequency.

In fact, with the total energy in the wave system designated as E, the energy density distribution is defined as

$$G_{yy} = \frac{dE}{d\omega} \quad .$$

The mass or weight of water is usually not directly included, and so the energy E and the energy density distribbtion G_{yy} are given merely as half of the squares of the wave amplitudes. The effect of the water density is included implicitly in the "frequency response function" discussed later.

A plot of $G_{yy}(\omega)$ vs. ω, as measured from a wave record obtained in a towing tank, is shown on Figure 5a. A similar plot, obtained by the same procedure from a record of the pitching motion of a ship model, is shown on Figure 5c. The symbol $z(t)$ will be used to designate ship motion ordinates measured at uniform time intervals of Δt, and $G_{zz}(\omega)$ will be used to denote the power spectrum of the ship motion. The spectral density $G_{zz}(\omega)$

In the second case, it is visualized that power spectra for the sea, G_{yy}, and for the ship (or model), $G_{zz}(\omega)$, were obtained at sea or in irregular waves in a towing tank, and it is desired to analyze the ship ...
can be considered as being obtained from a record of the ship model pitching in sinusoidal waves at a number of frequencies ω (i.e., periods $2\pi/\omega$ as obtained from a series of tests in a towing tank). The abscissa of this plot is the same as on the two others, representing the circular frequency of encounter of the ship by waves. The ordinates are the ratios of the amplitude of model motions, say the maximum angle of pitch in regular waves, to the wave height at a given frequency. In the dynamics of rigid bodies, the "magnification" or "resonance" factor (or, alternatively, 1/impedance) is defined as the ratio of the amplitude of the body swing to the displacement which would have resulted from a steady application of the exciting force or moment. In the present case, however, the motions are referred not to the true exciting force, but to the wave height, which is the only quantity present in the wave record of Figure 5a. The ordinates of the curve of Figure 5b are then expressed as "pitching angle per unit amplitude of wave height," "amount of heave per unit of wave height," etc. The entire curve is referred to as the "frequency response function," and is designated here by $Y(\omega)$. St. Denis and Pierson [9] referred to it as the "response amplitude operator."

The value of the entire development outlined above hinges on the fact, derived in the theory of time series, that three parts of Figure 5 are so interconnected that if any two are known, the third is thereby uniquely determined. Symbolically expressed, the two important practical cases are

$$G_{zz}(\omega) = G_{yy}(\omega)\,[Y(\omega)]^2 \qquad (8)$$

and

$$[Y(\omega)]^2 = \frac{G_{zz}(\omega)}{G_{yy}(\omega)} \qquad (9)$$

In the first case, it is visualized that the ship response $Y(\omega)$ has been previously determined in a towing tank experiment or by calculation. The ship behavior characterized by $G_{zz}(\omega)$ can then be computed for any sea state described by G_{yy}.

the wave is twice the potential one. With regard to this, letting $\tau = 0$ in equation (6) gives

$$\frac{1}{T}\int_0^T y^2(t)\, dt = \int_0^\infty G_{yy}(\omega)\, d\omega \quad , \tag{11}$$

i.e., the mean energy of the actual waves is equal to the area of the energy spectrum. Often, however, instead of expressing G_{yy} as $A_n^2/2$, the amplitude squared, A_n^2, is used. In such a case, the mean energy is given as half of the area of the spectrum.

Now let $\tilde{a}_n$ designate the apparent wave amplitudes observed at sea. The "root-mean-square" amplitude is then defined as

$$\bar{a} = \sqrt{\frac{1}{N}\left(\tilde{a}_1^2 + \tilde{a}_2^2 + \ldots \tilde{a}_n^2\right)} \; . \tag{12}$$

It has been shown by Longuet-Higgins [32] that $\bar{a}$ is connected with the area of the energy density spectrum. Hence,

$$\bar{a} = \sqrt{\text{Area of Energy Density Spectrum}} \quad . \tag{13}$$

It has also been shown by Longuet-Higgins [32] that various heights of the actual waves resulting from the energy spectrum can be defined in terms of $\bar{a}$ as follows:

Mean wave amplitude	$= 0.886\ \bar{a}$	For the wave height defined as the total distance from crest to trough, these figures should be doubled.
Mean amplitude of 1/3 highest waves	$= 1.416\ \bar{a}$	
Mean amplitude of 1/10 highest waves	$= 1.800\ \bar{a}$	
Mean amplitude of 1/100 highest waves	$= 2.359\ \bar{a}$	

These theoretically derived relationships appear to be well confirmed by observations at sea. The mean of 1/3 highest waves is given the name "significant wave height," and is usually reported in all wave observations. It appears that the mean wave observed visually at sea corresponds approximately to the "significant height," since the observer tends to overlook the small waves and ripples, which would enter into the definition of the true mean. Thus the "significant wave height" provides the link between practical sea observations and the theoretical concept of the energy spectrum.

Similarly, when the power spectrum of the ship motion is found, the root-mean-square $\bar{a}$ found from the spectrum will define the mean amplitudes of the motions to be expected on the basis of the Longuet-Higgins relationships tabulated above.

It is emphasized that the individual amplitudes A of the component waves represent essentially a mathematical concept and have no direct connection with the visually observed sea wave. In the observed sea, all of the components are present simultaneously, and therefore only the area of the spectrum, or the root-mean-square amplitude $\bar{a}$, has a physical significance in defining the sea state. This is particularly important in connection with the ship motion power spectrum $G_{zz}(\omega)$ shown in Figure 5c. It may often be found to have unexpected undulations, mostly because of the defects of the analysis in using relationship (8). These undulations have little significance, however, since the final significance of this spectrum depends only on its area. This statement is not necessarily true for any possible form of the energy spectrum, but it is essentially true for the spectra resulting from sea wave records and from ship motion records.

The above remark about the unimportance of the shape of the energy spectrum refers only to the final result of the analysis. In the earlier stages, say when using relationship (8), it is important to know the shape of the sea spectrum $G_{yy}(\omega)$, since each particular ordinate $G_{yy}(\omega_1)$ must be multiplied by the corresponding ordinate $[Y(\omega_1)]^2$ and the result is evidently affected by the nature of the variation of G_{yy} with ω. The form of the sea power spectrum $G_{yy}(\omega)$ resulting from a wind blowing in one direction for a sufficient time and over sufficient fetch has been established by Neumann [13] on the basis of theoretical considerations together with empirical observations at sea. It is defined as

$$[A(\omega)]^2 = \frac{C}{\omega^6} e^{-2g^2/\omega^2 v^2} , \qquad (14)$$

where ω = circular frequency in radians/sec. In CGS units with $g = 981$ cm/sec.^{2} and wind strength v in cm/sec., Neumann gives the value of $C = 8.27 \times 10^{-4}$ sec.$^{-1}$. The form of the spectrum for different wind strengths is shown on Figure 6. The dotted-line curves on Figure 3 show the distribution of "apparent periods" resulting from the above spectrum with the histogramic summary of direct observations at sea. Independent analyses [33,34] have confirmed the frequency distribution found by Neumann, but a question has been raised as to whether the constant C is not too large.

The Neumann spectrum refers to the "fully developed sea," i.e., the sea having reached the state of equilibrium, so that it does not grow with further increases in wind duration or length of the fetch. This condition will be often fulfilled in light winds, but extremely seldom in winds of gale force. Furthermore, in reality, the sea generated by local winds will often be accompanied by swells from distant storms. The spectrum of the swell can be predicted by methods given by Pierson and Neumann [35], and superimposed on the local storm sea. For the time being, then, it appears that the most practical procedure for towing tank organizations is to accept Neumann's spectrum in routine work, keeping in mind the possibility of reducing the constant C if the correlation between the behavior of a model in a towing tank and the ship at sea can thereby be improved.

In the case of a regular sinusoidal or trochoidal wave, the wave length, period, and celerity are defined by well-known relations:

Wave Length $$\lambda = gT_0^2/2\pi \qquad (15)$$

Period $$T_0 = \sqrt{2\pi\lambda/g} \qquad (16)$$

Celerity $$c = \sqrt{g\lambda/2\pi} \qquad (17)$$

These relationships are valid for the harmonic components of the sea wave, but observable $\tilde{L}$, $\tilde{T}_0$, and $\tilde{c}$ will result from the superposition of an infinite number of components and will have different values. From statistical considerations, Pierson [36] deduces that the relationship for the average distance between wave crests $\tilde{\tilde{L}}$ and the average time interval between crests $\tilde{\tilde{T}}$ is

$$\tilde{\tilde{L}} = \frac{2}{3} g \frac{\tilde{\tilde{T}}^2}{2\pi} \qquad (18)$$

Thus, the "apparent wave length" for a given "apparent period" is 2/3 of what it would have been for the true length and period in a regular sea.

This relationship is particularly valuable for sea observations, since the apparent periods can be observed with much greater accuracy than the apparent wave lengths.

CORRELATION OR CROSS-SPECTRAL ANALYSIS

Returning now to equation (2), it is noted that the Fourier expansion can be made in an alternate form (disregarding the constant and error terms):

$$y(t) = \sum_{n=1}^{\infty} C_n \cos(n\omega t - \epsilon_n) \quad . \qquad (19)$$

In the form of equation (2), $y(t)$ can be thought of as being a vector of magnitude $C_n = (a_n^2 + b_n^2)^{1/2}$, the direction of which is defined by the relationship of the cosine and sine terms. In the form of equation (19), this direction is given explicitly by the phase lag angle ϵ. Both C_n and ϵ_n are taken as varying independently and randomly. When only one record is available, and the procedure used to obtain equation (5) is followed, the phase angle ϵ is lost. As a result it is possible to obtain from the original record the auto-correlation function $\psi(\tau)$ and the power spectrum $G_{yy}(\omega)$ but it is not possible to reverse the process and to obtain the original record from the power spectrum. In this procedure, the information which can be found from relationship (9) in order to construct the diagram of Figure 5b, is not complete, in that no information regarding the phase relationship between the wave and ship motions is given. In many cases this phase relationship will be no less important than the amplitude relationship. An example is the case of slamming which, according to Szebehely [37] and Lewis [38], occurs when the ship's forefoot emerges from the water and subsequently, in descending motion, contact with water is made at the near maximum vertical velocity of the bow. The (highly undesirable) fulfillment of both of these conditions depends on a certain phase relationship between the ship and wave motions. Another often important problem is to find the amount (i.e., phase) by which the heaving motion lags after pitching, or in other words the apparent axis about which the ship pitches. This defines the relative amounts of bow and stern motions and accelerations, and different criteria may be found to be desirable for different types of ships. The phase relationship for regular waves can be readily obtained; the problem is to evaluate it statistically for irregular waves. In other words, the problem is to establish in all details the way in which the ship motion follows that of the irregular wave. Conversely, in the case defined by equation (9), the problem is to obtain the ship response described by its magnitude and its phase lag; the latter is directly connected with such important hydrodynamic parameters as damping.

The basic information for the analysis consists now of two records, say those of the wave height and of the pitching motion of the ship made simultaneously, usually in the form of two traces on the same recording tape. A sample length of the tape, corresponding to T sec., is chosen and is subdivided into n uniform divisions as before. The ordinates of the two record traces are measured, giving a series of numbers $y(t)$ for the wave record, and $z(t)$ for the pitching record. The measurements are repeated at $t+\tau$, where τ is the lag taken from 1 to m increments Δt after each value of t. The "cross correlation" function is formed by analogy with equation (5):

$$\gamma(\tau) = \frac{1}{T}\int_0^T y(t)\, z(t+\tau)\, dt \qquad (20)$$

The "cross-spectral density" G_{yz} is calculated again by the Fourier transform, this time in complex form:

$$G_{yz}(\omega) = \frac{2}{\pi}\int_0^\infty \gamma(\tau)e^{-i\omega\tau}\, d\tau \qquad (21)$$

It is necessary to resort to complex notation in order to handle two quantities -- the magnitude and the phase angle -- in one compact mathematical expression. After all of the integrations are completed (on an automatic computing machine), the result takes the general form of

$$G_{yz}(\omega) = \xi + i\eta \qquad (22)$$

The real quantity ξ is called the "co-power spectrum," and indicates the product of the in-phase frequency components of two individual spectra,

$y(t)$ and $z(t)$. The imaginary part η is called "quadrature spectrum" or "quad-spectrum," and indicates the product of 90° out of phase frequency components of these two functions.

The ship frequency response function is now evaluated [17a] as

$$Y_i(\omega) = \frac{G_{yz}(\omega)}{G_{yy}(\omega)} , \tag{23}$$

i.e., (quoting from F.B. Smith [21]) the transfer function $Y_i(\omega)$ equals the input-output (in the present case, wave-ship motion) cross spectrum $G_{yz}(\omega)$ divided by the power spectrum of input (i.e., wave) $G_{yy}(\omega)$. This is obviously also a complex quantity containing simultaneous values of two characteristics -- the amplitude of ship motion and the phase lag of ship motion with respect to wave motion for each simple wave component corresponding to a frequency ω, i.e.,

$$Y_i(\omega) = a + ib = Ae^{i(\omega t+\epsilon)} , \tag{24}$$

where $A = (a^2 + b^2)^{1/2}$ and $\tan\alpha = b/a$.

It will be recollected that when the ship response to regular waves is calculated theoretically, as shown in Reference 31, the final answer is given in a complex form identical with equation (24). Also, when a model is tested in a towing tank in regular waves, the amplitude A and the phase angle α are directly measured. When these data are obtained for a number of wave frequencies ω, the resulting graph of A and α vs. frequency ω is given the name of "frequency response function," designated here by $Y_i(\omega)$. The subscript i is added to remind the reader that this is a complex quantity, and to relate it to a similar function $Y(\omega)$ used in equation (9) in connection with the auto-correlation analysis, which is a real quantity. Actually, $Y(\omega)$ is the modulus of $Y_i(\omega)$.

It should be clear from the above discussion that the frequency response function $Y_i(\omega)$, or in simple words the ratio of amplitude of ship motion to wave height, and the phase lag plotted against frequency, which are characteristics of the ship form and mass distribution, can be obtained by four different methods:

1. By computations such as are given in Reference 31.

2. By a series of model tests in a towing tank, in regular waves at a number of frequencies.

3. By a cross-spectral analysis of records of wave and model motion in irregular waves in a towing tank.

4. By the same process using the records obtained on a ship at sea.

Since the function $Y_1(\omega)$ is a characteristic common to all of the above methods, all attempts to correlate the results of the different approaches should be made on the basis of $Y_1(\omega)$. Furthermore, $Y_1(\omega)$ is a specific characteristic of the form and mass distribution of a ship. Methods (1) and (2) are based on the same conditions of model operation, and therefore close agreement between the results can reasonably be expected. The correlation of (3) with (1) and (2) can be expected to be good in waves of low height and small ship motions, when the various forces involved are linearly connected with the ship motions. A certain discrepancy must necessarily arise in the case of more severe motions, when the force and motion relationships strongly deviate from the assumed linearity. The most important feature of the cross-spectral analysis, however, is that it permits the data of the model and the ship -- items (3) and (4) -- to be correlated on an equal basis. The ship test data will always refer to a different sea state; the sea is ever-changing and never repeats itself. The application of the cross-spectral analysis, however, permits a permanent part, i.e., the frequency response $Y_1(\omega)$, to be extracted from the ever-changing record. Because of the ever-changing nature of the sea, it will not be practical to attempt to duplicate in a towing tank every condition actually met by a ship; this would be extremely costly and not particularly useful. A towing tank should preferably standardize on a certain typical irregular wave pattern, for instance, such as that described by Neumann's power spectrum, concentrating its effort on having this pattern accurately reproducible. The frequency response function $Y_1(\omega)$ is a property of the model form and mass distribution, it is basically independent of the details of the wave power spectrum of the tank, and it is directly comparable to that obtained at sea, provided the significant wave heights do not differ materially.

Towing tank personnel are generally conscious of geometric accuracy in the reproduction of a ship in the model and in having the correct lead waterline. In tests involving only pitching and heaving in head or following seas, the moment of inertia about the transverse axis of the ship must be reproduced accurately. In tests involving rolling and yawing, not only the moment of inertia, but also the products of inertia must be reproduced accurately. Generally, even a pitching record on a ship at sea is merely a record abstracted from the motions in all six degrees of freedom. In the correlation of such test data with towing tank test data, the correct products of inertia should be provided. It is necessary, therefore, that any ship observation data to be analyzed be accompanied by sufficiently complete weight distribution data for the calculation of the moments of inertia and of the products of inertia.

FILTERING TECHNIQUE OF ANALYSIS

The description of the analysis process given above was based on manual measurements of a paper tape record, with subsequent processing on a digital type of automatic computing machine. This method was given first because it appears to be more directly connected with the basic theory of the "time series." In practice, the application of an electronic filtering technique permits a much more rapid and less costly method of analysis, when the necessary special equipment is available. The input into the analyzer proper is usually in the form of a frequency-modulated electric carrier wave. Physically, it is produced by scanning the record made on a magnetic tape, or scanning a paper or film tape with photocells. In the latter case, the paper or film on one side of the trace is blackened, as is described in References 22 and 39. A suitable length of the record tape is spliced into a loop, and the loop is run continuously through the analyser. The output of the reading heads, after suitable electronic manipulations, is applied to an electric filter, through which only a certain narrow band of frequencies can pass. The energy contained in this narrow band is then measured and plotted, The position of this band on the entire frequency scale is then varied, either by varying the filter characteristics, or by using a fixed filter and varying the speed at which the record tape moves past the reading head. As a result of applying this filter, and measuring the power which passed through it at various frequencies, the entire power spectrum is directly plotted. A simple and clear description of the principles of this process is given in a paper by Francis B. Smith [21], which is attached as an Appendix to the present memorandum with the kind permission of the "Aeronautical Engineering Review." More detailed descriptions of the elctronics involved are presented in Reference 40, and a comprehensive analysis of the theory, an evaluation of the unavoidable errors, and the accuracy to be expected are given in Reference 41.

Only one aspect of the above process needs to be discussed further. When a finite-length sample of a continuous record of the wave or ship motion is taken and is spliced into a loop, the continuous distribution of frequencies is in fact replaced by a series of discrete harmonics of the periods obtained by dividing the longest possible period, equal to the length of the record T, by the whole numbers 1, 2, 3,...,n (Reference 30). If the filter used were capable of a very fine resolution of frequencies, the power spectrum would be represented by a series of sharp spikes, since the energy would be shown to be contained only at these discrete frequencies. This would represent not a true natural phenomenon, but a distortion of it by the properties of the analyzing instrument. This difficulty is avoided by using a filter capable of passing a certain band width of $\pm\Delta\omega$ at any particular frequency ω. A diagram showing the percentage of energy passed vs. the interval $\Delta\omega$ is shown on Figure 7. Only a typical form of the diagram is shown, but the magnitude of $\Delta\omega$ in terms of the number of discrete frequency peaks it can cover can be varied to suit the conditions. By passing the energy distributed over the range of $\pm\Delta\omega$, such a filter acts as an integrating and averaging device. Hence, displacing or sliding the filter over the entire frequency range ω gives a smooth plot of the power spectrum instead of a series of discrete peaks. The terms "smoothing" and "averaging" used in the schematic descriptions of such analyzers cover the various devices that are used to alleviate the drawbacks resulting from the properties of an analyzing instrument. The width $\pm\Delta\omega$, however, affects the results of the analysis. If $\pm\Delta\omega$ covers too few of the discrete Fourier frequencies, the resulting record may be too irregular or jumpy, because various accidental events are not statistically averaged. If $\pm\Delta\omega$ covers too many Fourier frequencies, the record may be too smooth in the sense that it may have obliterated some significant undulations of the record. The proper choice of a band width is a very important question, further elucidation of which may be found in References 18 and 41.

Although the above remarks have been given in connection with the record spliced into a loop and filtered, they are to a large extent applicable as well to the digital analysis discussed earlier. The plot of the auto-correlation function $\psi(\tau)$ generally resembles the curve of an electronic filter. One could actually think of $\psi(\tau)$ as the filter by which the length T of the record is scanned; in fact, the words "filter" or "computational filter" are used in this connection by Tukey in Reference 18. The considerations that are involved in the choice of the suitable band width of an electric filter are also involved in the choice of the number of subdivisions n of the record, and in the maximum number m of such subdivisions used for the record displacement τ. The procedure used to optimize the results is given on pages 51-56 of Reference 18.

EXPERIMENTAL PROCEDURE

In the present paper, attention has been concentrated on the analytical procedure, because it is in the understanding and application of this procedure that the whole scope of the problem of the irregular sea essentially lies. The experimental techniques applied in using irregular waves do not differ much from those employed for tests in regular waves. The apparatus for towing models must allow the model to be free to pitch, heave, and surge, and must record these motions on a tape. Descriptions of suitable apparatus are given by Dr. F.H. Todd and by the present author in References 42 and 43, respectively. An added feature is the method of making a reproducible irregular wave. The method employed at the Experimental Towing Tank of Stevens Institute of Technology is described by E.V. Lewis in Reference 44. The wavemaker used is of the vertical plunger type, with the motor speed controlled by a rheostat with 25 control points. A 100-contact rotary stepping switch is driven by the same motor through a reduction gearing such as to make it advance one step for each stroke of the plunger. The contacts of the stepping switch are randomly connected to the rheostat points so that a wide range of voltages can be randomly applied to the motor. The motor is shunt wound, and a high magnetic flux is maintained in the field. Thus, the armature reacts lively either to the application of the accelerating voltage, or to the regenerative braking when the voltage is reduced. Since the stroke of the plunger remains fixed, the wavemaker produces waves that are too high at the higher frequencies. These are attenuated to the height commensurable with the height of the low-frequency waves by means of a float 3 ft. wide, extending the width of the tank just in front of the wavemaker. The diagrams given in Reference 44 indicate that the resultant wave pattern of about 2-min. duration is satisfactorily reproduced at will, and that its power spectrum corresponds approximately to Neumann's spectrum for a 40-knot wind, which has not blown quite long enough to produce a "fully arisen sea." A fully arisen sea at a 40-knot wind would be a rare occurrence in nature. To the author's knowledge, E. V. Lewis' work described in References 23, 24, 25, and 44 is the only example to date of the use of irregular seas in connection with ship models in a towing tank in which effort is made to represent the statistical properties of a sea spectrum, and the results are being continuously checked by the methods of extended harmonic analysis.

Immediately next in importance to the method of wave generation is the problem of wave absorption by a suitable beach at the end of the tank. This problem has not appeared to be critical in tests in regular waves, because it has been possible to generate a certain number of regular waves and to take a record of the model motions for a few seconds before any reflected wave makes its appearance. In the case of an irregular sea, however, it is usually necessary to run the wavemaker continuously for longer periods, and a good wave-absorbing beach is necessary to avoid the standing wave system. It should be noted that the standing wave system, resulting from the reflection from a poor beach, in no way resembles the progressive system of irregular waves at sea. The reader is warned again not to take any disorderly water motion as being a realistic sea wave. The irregular sea waves form a progressive wave system definitely characterized by a certain power spectrum, which apparently conforms closely in form to the one derived by Neumann [13,14].

CONCLUDING REMARKS

In the above exposition an attempt has been made to present as simply as possible the information on the irregular sea as found in nature, and as it is reproduced (in its essential features) in a towing tank. The "essential features" are defined simply at first, in a limited form by means of "histograms," and then in a more useful form by the "power density spectrum" derived by the auto-correlation technique in generalized harmonic analysis. Any recorded feature of the motion of a ship or a ship model can be analyzed in a similar way. From the relation of the sea and ship power spectra, the characteristics of the ship (i.e., its frequency response function) can be obtained. By forming an energy cross spectrum of the wave and ship records, the ship characteristics can be described not only by the values of the magnification factor, but by the accompanying phase angle lags as well. This complete form of ship response function serves then as the link common to model tests in irregular seas in the tank, observations on a ship at sea, model tests in regular waves, and the calculations of model motions in regular waves. These latter form in turn the link between the observable complex motion of a ship and its fundamental dynamic and geometric properties.

In an attempt to provide a simple and vivid exposition of the fundamental relationships, many details have had to be suppressed. Various statements have been made in definite positive forms, suppressing the "if's" and "but's" which are usually considered to be necessary in a learned treatise. In particular, the positive statement made in regard to the relationships among the three parts of Figure 5 and the relationships (8) and (9) probably should be softened. The positive assertion refers to an ideal case, whereas, in practice, certain errors and deviations certainly occur. However, the work of Lewis and Numata [26] indicates that these relationships hold reasonably well.

In conclusion, the following reading list is suggested to those wishing to gain a deeper understanding of the techniques employed in connection with irregular seas, either in a towing tank or at sea:

a. On the composition of ocean waves: References 13a, 35, 45, and 46, in the order given.

b. Introduction to the generalized harmonic analysis and its applications: References 21 (attached here as an Appendix), 19, and 20.

c. Advanced time series (generalized harmonic) analysis: References 16, 17, 47, and 48.

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LIST OF SYMBOLS

A	The amplitude, i.e., maximum value of $y(t)$, in sinusoidal waves; used also for amplitude of ship motions
$\bar{a}$	Root-mean-square amplitude, i.e., $(a_n{}^2/n)^{1/2}$
a_n, b_n	Amplitudes of individual harmonic wave components
c	Wave celerity
$\tilde{c}$	Apparent wave celerity
f	Frequency of occurrence
$G_{yy}(\omega)$	Power spectral density of wave record $y(t)$
$G_{zz}(\omega)$	Power spectral density of ship motion record $z(t)$
$G_{yz}(\omega)$	Cross-spectral density of wave and ship motion
g	Acceleration of gravity
$\tilde{H}$	Apparent wave height of irregular waves
$\bar{\tilde{H}}$	Mean apparent period
$\tilde{L}$	Apparent wave length
$\bar{\tilde{L}}$	Mean apparent wave length
m	Maximum number of time subdivisions Δt forming T
n	Number of subdivision of the length of the record T
T	Length of a record in seconds
$\tilde{T}$	Apparent period of irregular waves
$\bar{\tilde{T}}$	Mean apparent period
T_o	Period of a harmonic wave
t	Time
v	Wind velocity
$Y(\omega)$	Frequency response function (modulus of)
$Y_1(\omega)$	Complex frequency response function
$y(t)$	An ordinate of a wave record at time t
$z(t)$	An ordinate of a ship motion record at time t

α	Phase lag of ship motion with respect to t
$\gamma(\tau)$	Cross-correlation function
ϵ	Phase of a harmonic wave component with respect to ωt
λ	Wave length
ξ, η	Real and imaginary parts of $C_{yz}(\omega)$
τ	Time displacement of a record
$\psi(\tau)$	Auto-correlation function
ω	Circular frequency

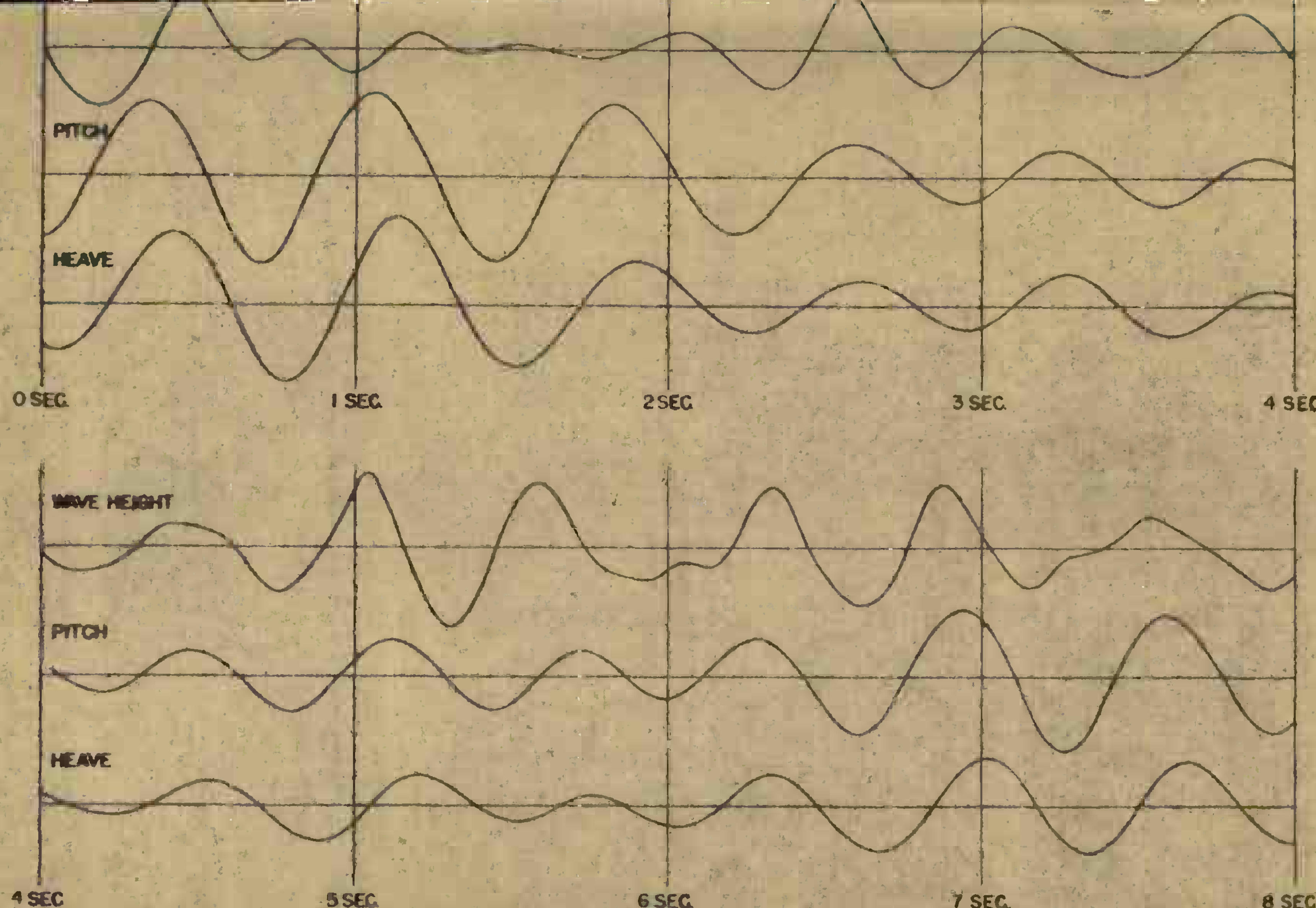

Fig.1-- SIMULTANEOUS RECORD OF IRREGULAR WAVES ENCOUNTERED BY A SHIP MODEL AND OF THE RESULTANT HEAVING AND PITCHING MOTIONS. SERIES 60, 0.60 BLOCK COEFFICIENT MODEL 5 FT. LONG AT 2 FT./SEC. (REFERENCE 26).

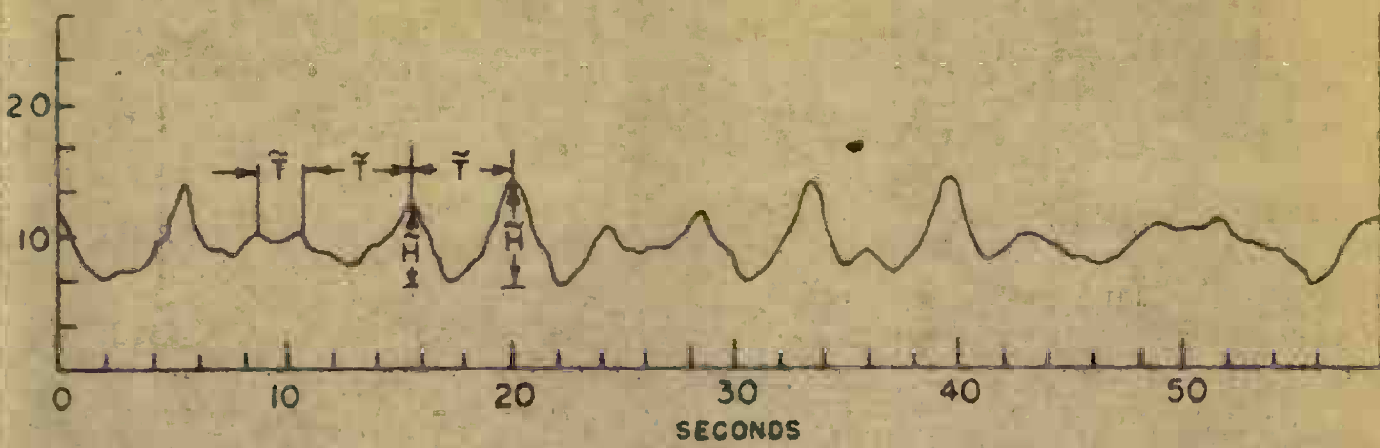

Fig.2-SECTION OF A WAVE RECORD TAKEN AT SEA, SHOWING THE NOTATION OF THE APPARENT PERIOD $\tilde{T}$ AND APPARENT WAVE HEIGHT $\tilde{H}$ (TAKEN FROM REFERENCE 14).

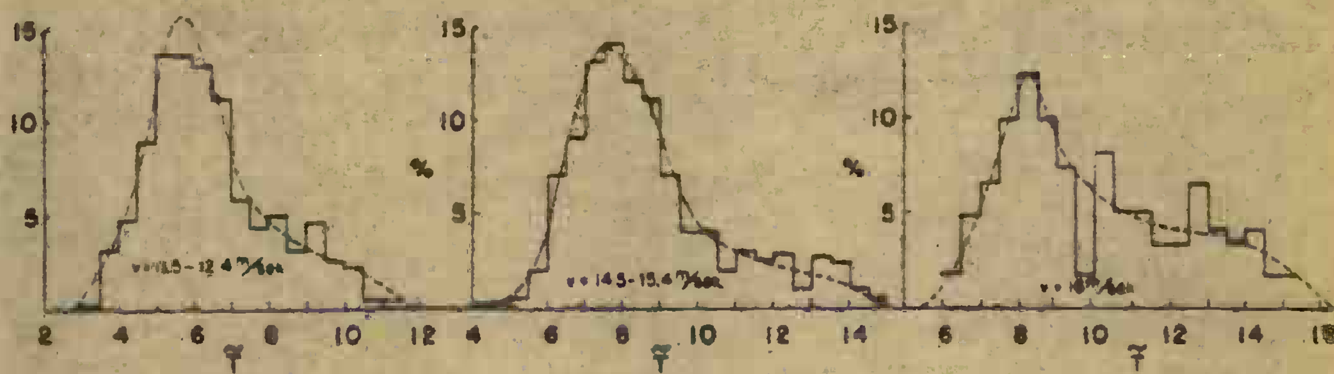

Fig.3-DISTRIBUTION OF APPARENT PERIODS $\tilde{T}$ OBSERVED AT SEA AT DIFFERENT WIND SPEEDS.

Fig.4-TYPICAL CURVE OF FREQUENCY DISTRIBUTION OF THE AMPLITUDE OF ROLL MEASURED ON A SHIP AT SEA (TAKEN FROM REFERENCE 8).

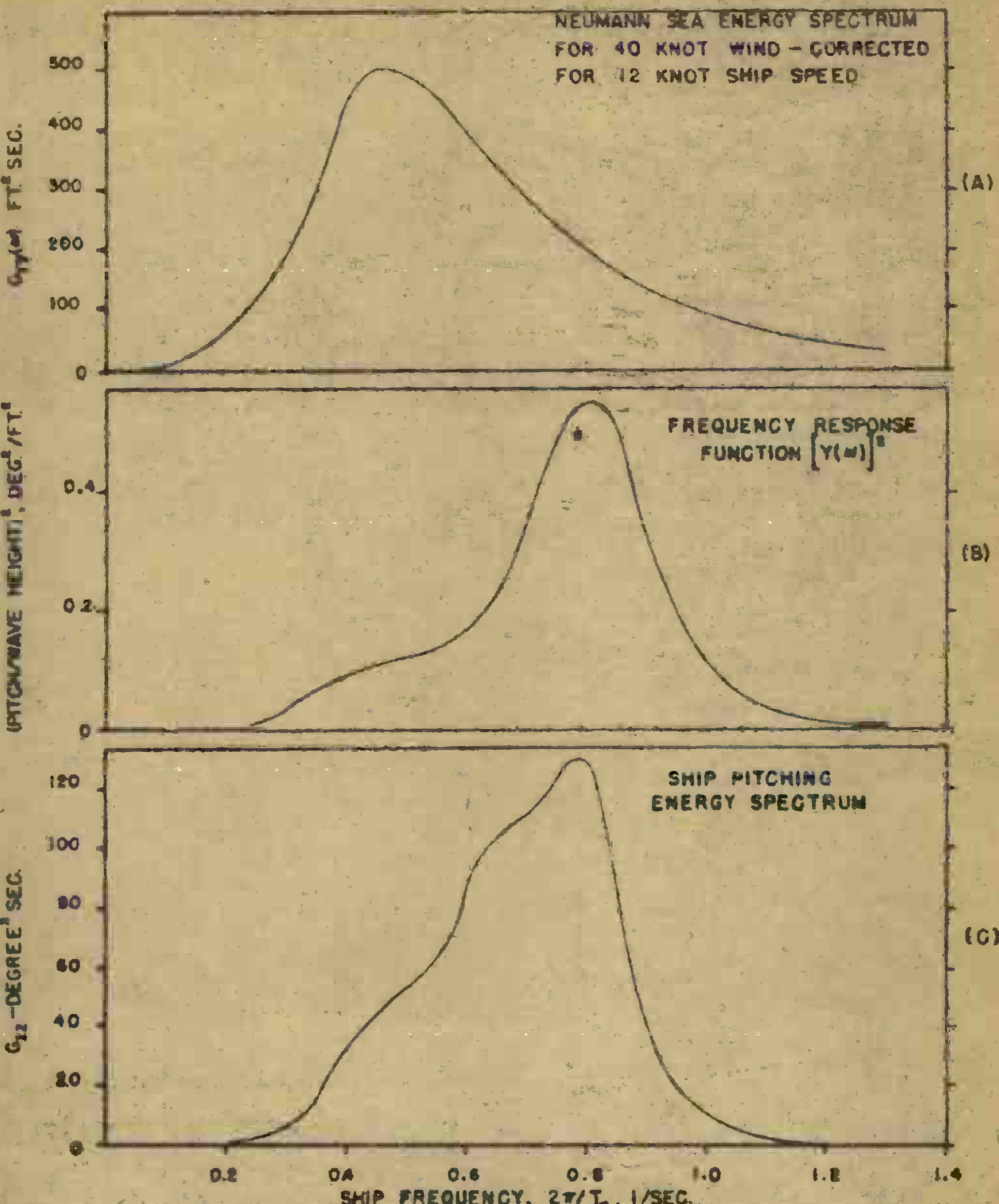

Fig. 5 - ENERGY SPECTRA AND RESPONSE OF A SHIP MODEL IN AN IRREGULAR SEA. SERIES 60, 0.60 BLOCK COEFFICIENT MODEL 5 FT. LONG, EXPANDED TO 500-FT. SHIP. (ABSTRACTED FROM REF. 26)

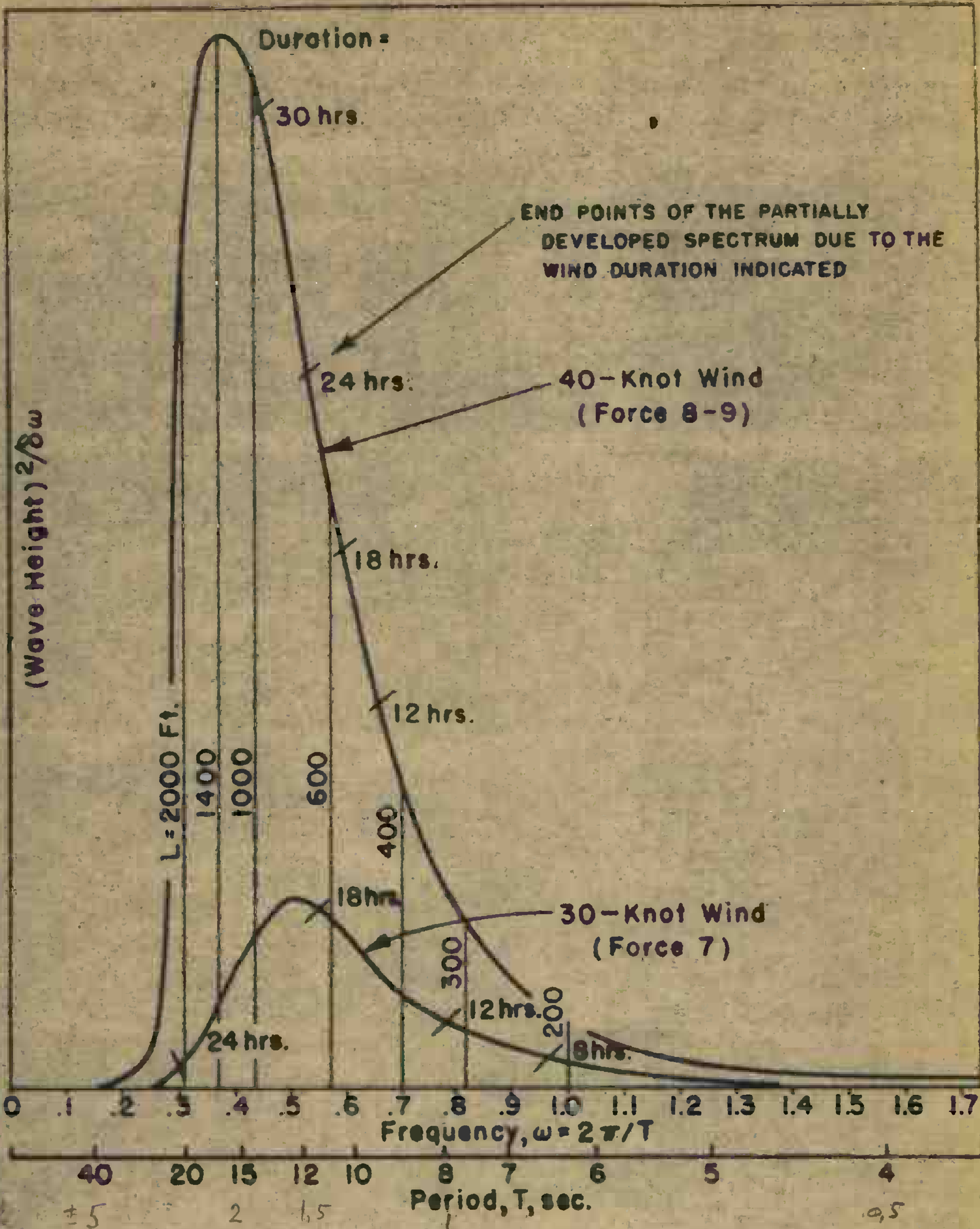

Fig.6 - NEUMANN'S FULLY ARISEN SEA SPECTRA FOR TWO WIND SPEEDS
(TAKEN FROM REFERENCE 28)

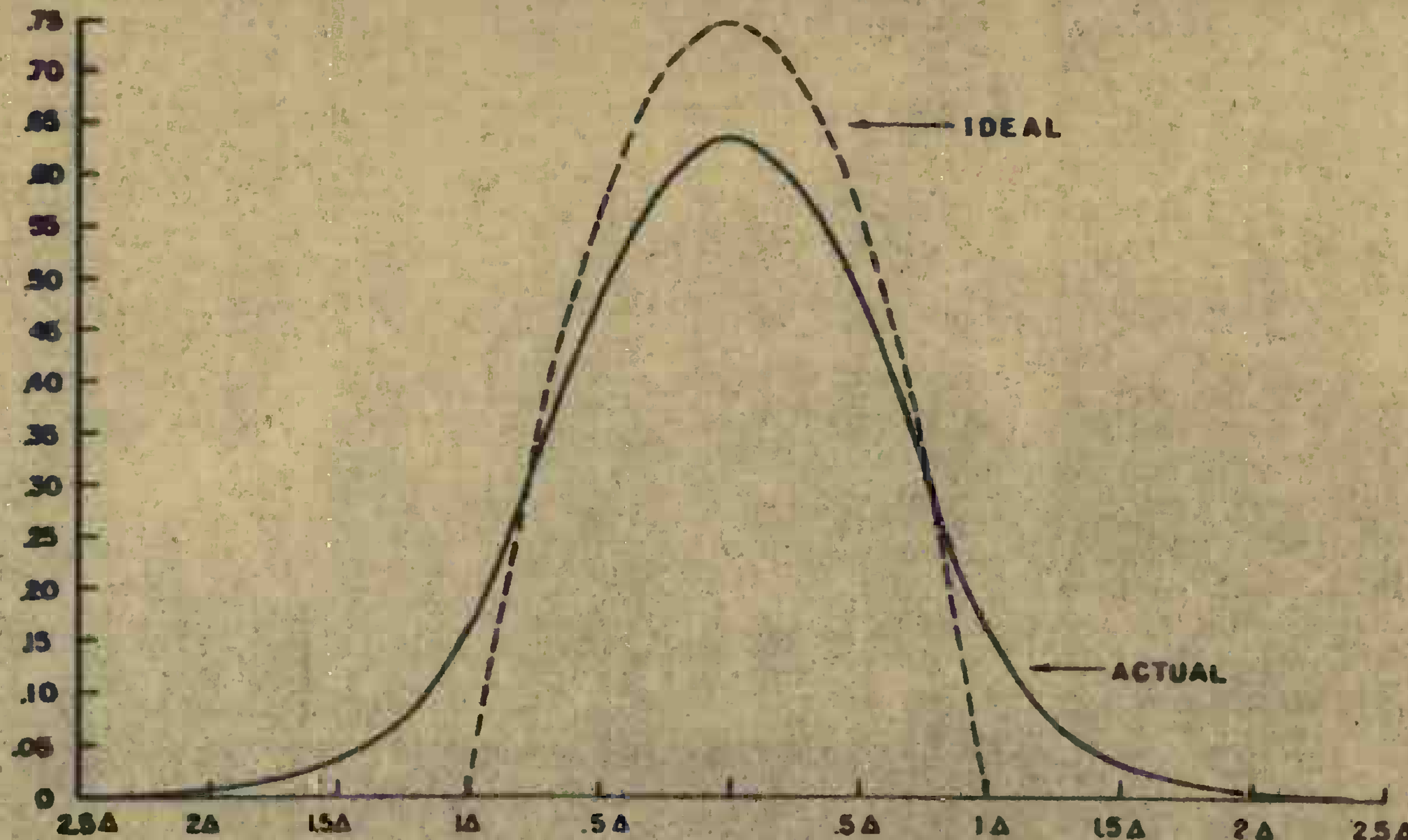

Fig.7-IDEAL AND ACTUAL ELECTRONIC FILTER
(TAKEN FROM REFERENCE 41)