Roles of Illustrations in Propositional Logic Textbooks

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Abstract. Illustrations are widely used in propositional logic education, yet little is known about their role in contemporary textbooks. This study investigates the pedagogical and communicative functions of illustrations in three contemporary propositional logic textbooks through qualitative thematic analysis. A hybrid deductive–inductive coding scheme was developed to classify illustration types, instructional contexts, and communicative roles.
Two recurring themes were identified. First, illustrations function as representational translations and definitional tools, re-expressing symbolic expressions through more inspectable visual forms such as truth tables, parse trees, and logic circuits. Second, illustration choices are systematically associated with different forms of logical activity, including computation, structural analysis, and formal proof. The findings suggest that illustrations are integrated components of propositional logic exposition rather than standalone explanatory devices. This study provides an exploratory qualitative analysis of illustration practices in contemporary propositional logic textbooks and highlights the communicative and pedagogical functions of visual representations in logic education.

Keywords: logic education · propositional logic · textbook analysis · illustrations · thematic analysis

1 Introduction

Propositional logic forms a foundational component of mathematics, computer science, and digital computation. It provides the formal basis for reasoning about algorithms, digital circuits, program correctness, and mathematical proof. As a result, introductory courses in computer science and mathematics commonly include propositional logic as one of the first encounters students have with formal symbolic reasoning.

Because propositional logic deals with abstract structures and symbolic notation, educators have long employed visual representations to support explanation and understanding [19, 16]. Common examples of illustrations include truth tables, parse trees, logic circuits, and other diagrammatic representations. More generally, visual representations are often used in education to organise information and to highlight relationships [5, 2, 4].

Despite the growing literature described in Section 2, relatively little attention has been paid to the illustrations that students actually encounter in contemporary propositional logic textbooks. Existing studies primarily focus on the theoretical properties of diagrammatic systems [2, 3, 4] or propose new visual notations for teaching logic [8, 17, 7, 1]. Consequently, it remains unclear how contemporary textbook authors currently employ illustrations and what pedagogical and communicative functions these illustrations fulfil.

This paper addresses that gap through a qualitative thematic analysis of illustrations in three contemporary propositional logic textbooks. The study investigates how illustrations are used, what communicative and pedagogical roles they perform, and which aspects of propositional logic are supported through different representational choices.

The research is guided by the following question: *How do illustrations function pedagogically in propositional logic textbooks?*

2 Related Work

2.1 Illustrations and Diagrammatic Reasoning in Logic

Although formal logic is often associated with symbolic manipulation, a parallel tradition has employed diagrams to communicate logical relations, support reasoning, and make abstract structures visually accessible. Recent work argues that diagrams should be regarded as representational systems capable of conveying logical information through spatial structure and visual organization [2, 3].

Bernard argues that diagrammatic representations can make logical relations directly perceptible, providing what he describes as a form of logical justification through spatialization [2]. Similarly, Bhattacharjee characterizes logic diagrams as cognitive tools that offload reasoning onto visual perception, while highlighting the recurring tension between visual clarity and expressive power in diagrammatic systems [3]. Together, these studies suggest that different representational choices emphasise different aspects of logical reasoning.

Recent work has also examined logic diagrams from the perspective of visualization theory. Bolz argues that logical diagrams can be understood as specialised forms of data visualization whose effectiveness depends on properties such as completeness, correctness, lack of distortion, and legibility [4].

2.2 Research on Improving Logic Illustrations

A substantial body of work has proposed alternative visual representations intended to improve the teaching and learning of propositional logic. These novel approaches seek to make logical structure more accessible through diagrammatic representations.

Early examples include Clarke's Possible Models Diagrams, which replace truth tables with graph-based representations of possible interpretations and logical relations [8]. San Ginés similarly advocates the use of genealogical trees and colour-coded matrices to support visual reasoning and reduce reliance on purely symbolic manipulation [17]. More recent proposals include Cheng's Truth Diagrams, a formally developed visual notation intended to make logical structure and semantic relations directly perceptible [7], and Aznar's Marlo Diagrams, which aim to improve logical competence through visually guided deduction grounded in common-sense reasoning [1].

Existing research has largely focused on three areas: the theoretical and epistemic properties of diagrammatic reasoning [2, 3], the design and evaluation of alternative visual notations for logic [8, 17, 7, 1], and the communicative qualities of logic diagrams as visualizations [4]. Comparatively little attention has been paid to the illustrations that students actually encounter in contemporary propositional logic textbooks.

As a result, it remains unclear how modern textbooks currently employ illustrations and what communicative and pedagogical functions these illustrations serve. This study addresses that gap through a qualitative thematic analysis of illustrations in contemporary propositional logic textbooks.

3 Methodology

This study employs a qualitative research design based on thematic analysis [9] to investigate how illustrations function within propositional logic textbooks. The focus is on communicative and pedagogical roles of visual representations. This approach is appropriate because it offers a theoretically flexible tool for identifying both manifest and latent patterns of meaning across diverse textbook formats [9]. Furthermore, thematic analysis acknowledges the active role of the researcher in constructing themes, which facilitates a more reflexive interpretation of how illustrations interact with surrounding textual discourse [9].

3.1 Corpus Selection and Sampling

The corpus was constructed using purposive sampling [18]. Textbooks were selected according to three criteria: (i) inclusion of introductory propositional logic content, (ii) use in mathematics or computer science education, and (iii) substantial use of visual or spatial representations.

An initial sampling phase included both historical and contemporary textbooks. However, preliminary analysis revealed substantial variation in notation, terminology, and pedagogical framing across historical sources, limiting comparability within the scope of this study. The final corpus therefore focuses on contemporary textbooks.

The selected textbooks are *Logic in Computer Science* by Huth and Ryan [13] (from now referred to as Huth), *Introduction to Mathematical Logic* by Mendelson [15] (from now referred to as Mendelson), and *Delftse Foundations of Computation* by Hugtenburg and Yorke-Smith [12] (from now referred to as Hugtenburg). These texts represent distinct educational contexts (computer science logic, mathematical logic, and computational foundations) while sharing coverage of propositional logic. This variation allows comparison across differing representational traditions within a shared formal domain.

3.2 Operational Definition of Illustration

A working operational definition of "illustration" was developed to ensure consistency during coding. An element was classified as an illustration if it:

1. is spatially organised,
2. encodes logical or mathematical structure,
3. and can be interpreted visually rather than purely sequentially.

This includes representations such as truth tables, parse trees, circuit and logic diagrams. Inline symbolic expressions and purely textual proofs were excluded.

Only *instructionally relevant* illustrations were included. An illustration was considered relevant if it contributed to explanation, interpretation, or conceptual structure beyond what is trivially recoverable from surrounding prose. Purely decorative, biographical, or historical images (e.g., portraits or publisher graphics) were excluded.

3.3 Codebook Development and Coding Procedure

The coding process followed a hybrid deductive–inductive approach. Before formal coding began, the selected textbooks were reviewed to identify potentially relevant forms of visual representation and to develop preliminary analytical observations.

Initial codes were derived from literature on multimedia learning, educational illustrations, and diagrammatic representation (for definitions see the overview in Table 1). Multimedia learning theory informed the categories `signaling` and `spatial contiguity` [14]. Educational illustration research informed communicative-role codes such as `role:explanatory`, `role:organizational`, and `role:decorative` [5]. Work on diagrammatic representation and semiotics informed categories relating to abstraction (`abstraction`) and relationships between visual and symbolic representations (`ref`) [6, 10, 11].

Illustrations and their surrounding explanatory text were imported into ATLAS.ti and coded iteratively. Codes were refined throughout analysis through merging, subdivision, renaming, and removal where necessary. Analytical memos were maintained to document coding decisions and emerging interpretations. New categories were developed inductively when existing categories proved insufficient to describe recurring patterns within the data. Coding continued until the codebook stabilised and no additional categories were required to describe recurring patterns in the corpus.

The final codebook comprised eight categories which are summarised in Table 1. Three categories were central to the analysis. The `role` category captures the communicative function performed by an illustration, while the `type` and `context` categories were developed inductively to capture patterns not addressed by other categories

The `type` category records the representational form of an illustration (e.g., truth table, parse tree, logic diagram, or directed acyclic graph). The `context`

Table 1: Overview of the final code categories.

Category	Purpose
`type`	Visual form of the illustration (e.g., truth table, parse tree, logic diagram)
`role`	Communicative function performed by the illustration
`abstraction`	Degree of abstraction (abstract, concrete or hybrid)
`context`	Instructional context in which the illustration appears (e.g., definition, proof, exercise)
`reference`	How the illustration is referenced within surrounding text (e.g. implicit, text, caption)
`signaling`	Strength and type of guidance for interpreting the illustration provided in the caption, surrounding text or illustration itself
`spatial contiguity`	Physical proximity between illustration and explanatory/referencing text

category records the instructional setting in which an illustration appears, such as a definition, proof, exercise, or SAT-related discussion. These categories were introduced to enable comparison of illustration practices across textbooks and to identify recurring associations between particular illustration types and instructional contexts. Existing categories could not capture such patterns.

The `role` category distinguishes between illustrations that clarify existing information (`role:clarifying`), explain processes or relationships (`role:explanatory`), provide examples (`role:exemplary`), organise information (`role:organizational`), guide interpretation (`role:directive`), or connect multiple representational systems (`role:integrative`). Multiple role codes could be assigned when an illustration performed several functions simultaneously.

3.4 Thematic Analysis

After coding, related codes were grouped into higher-order conceptual clusters. These clusters were iteratively compared across textbooks and refined into themes describing recurring pedagogical and communicative functions of illustrations. Themes were required to exhibit internal coherence and to recur across multiple textbooks rather than isolated examples. The resulting themes form the basis of the findings presented in the next section.

4 Results

Thematic analysis identified two recurring themes in the communicative and pedagogical functions of illustrations in propositional logic textbooks. The following sections present these themes with examples from the corpus.

4.1 Illustrations as translations and definitions of propositional logic

Across the corpus, illustrations are systematically used as alternative representational systems for symbolic logical expressions. Rather than merely accompanying notation, they function as translations that reorganises linear symbolic structure into spatial, graphical, or tabular formats, thereby exposing structural or semantic properties that are less transparent in symbolic form.

1. Illustrations as translations of symbolic expressions A primary function of illustrations is to provide structurally faithful but perceptually distinct renderings of formal expressions. This includes parse trees, truth tables, and logic diagrams that preserve logical equivalence but change representational modality.

For instance, Huth uses parse trees to visualise the syntactic structure of complex propositional formulas (see Figure 1). This representation makes the hierarchical organisation of operators more visible compared to the linear and nested symbolic notation from which it is derived.

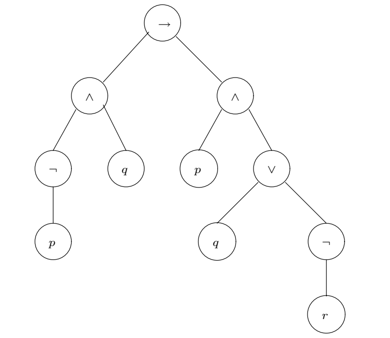

Figure 1.3. A parse tree representing a well-formed formula.

Fig. 1: Parse tree representation of the propositional formula $((((\neg p) \wedge q) \rightarrow (p \wedge (q \vee (\neg r))))$. Taken from Huth [13, p. 34].

Similarly, Hugtenburg [12] presents a stepwise construction of a logic circuit corresponding to $(A \vee B) \wedge \neg(A \wedge B)$ (Figure 2). The figure presents a translation process from formula to circuit, decomposing the transformation into three intermediate stages that expose how subformulas map onto circuit components.

In these cases, illustrations primarily serve a `role:clarifying` function: they isolate structural information already present in the notation and re-express it in a format that supports inspection from a different perspective. This can make relationships that remain difficult to see in symbolic notation, such as hierarchical structure, component dependencies, or patterns of evaluation, more immediately visible to the reader.

2. Translation is externally guided through strong textual signaling The interpretability of these alternative representations is rarely self-contained. Instead, it is enabled through explicit signaling distributed across captions, surrounding text, and occasionally within the illustration itself. These signals guide readers in establishing correspondences between visual elements and symbolic expressions, thereby helping readers learn how information encoded in one representational format maps onto another.

In a large proportion of cases, signaling is classified as `signaling:strong`, with heavy reliance on `signaling:textual` cues in the surrounding discourse (41/68 illustrations). Authors frequently use adjacent prose to specify how elements of a diagram correspond to symbolic components, often proceeding step-by-step or row-by-row in the case of truth tables.

For example, in Figures 3a and 3b, Mendelson and Huth explicitly guide interpretation of truth tables through textual explanation placed directly above and below the figure (`spatial contiguity:integrated`), structuring the reading process column-wise and row-wise.

A more explicit form of guidance appears in Hugtenburg [12], where Figure 2 itself contains text (`signaling:labels`) and is organised as a procedural sequence (`signaling:stepwise`). The illustration thus functions not only as a representation but also as a self-contained guide for performing the translation.

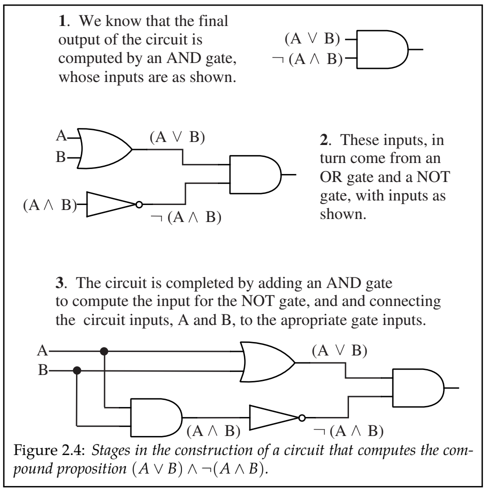

Figure 2.4: *Stages in the construction of a circuit that computes the compound proposition* $(A \vee B) \wedge \neg(A \wedge B)$.

Fig. 2: Stepwise construction of a logic circuit implementing the proposition $(A \vee B) \wedge \neg(A \wedge B)$. Taken from Hugtenburg [12, p. 32].

3. Illustrations as a way to define propositional logic Illustrations also function as primary definitional mechanisms for propositional logic concepts. In these cases, meaning is not merely illustrated but constructed through the visual format itself, with the illustration serving a `role:explanatory` within a `context:definition`.

Truth tables are the dominant example of this pattern. Across all analysed textbooks, they are repeatedly used to introduce the semantics of connectives such as negation, conjunction, disjunction, implication, biconditional, and exclusive disjunction. Here, the truth table does not only accompany a definition, it *is* the definition, specifying meaning through exhaustive valuation of truth conditions.

In Figure 3, Mendelson [15] and Huth [13] both introduce conjunction via truth tables. These truth tables are useful since they explicitly show the truth conditions of conjunction, which are then reinforced in subsequent textual explanations.

A	B	$A \wedge B$
T	T	T
F	T	F
T	F	F
F	F	F

a Taken from Mendelson [15, p. 1].

ϕ	ψ	$\phi \wedge \psi$
T	T	T
T	F	F
F	T	F
F	F	F

b Taken from Huth [13, p. 37].

Fig. 3: Truth tables defining the semantics of conjunction by exhaustively specifying truth conditions for $\wedge$.

A similar definitional strategy appears in Hugtenburg [12], where logic gate symbols are introduced in a figure containing labelled (`signaling:labels`) logic gates, an example logic circuit, and its translation to a symbolic expression (Figure 4). The illustration thereby functions as a hybrid definitional space, simultaneously specifying syntax, semantics, and usage context.

Taken together, these findings suggest that the selected textbooks teach propositional logic through a coordinated set of symbolic and visual languages, including truth tables, parse trees and logic circuits. Illustrations function both as translations between these representational systems and as vehicles for defining logical concepts. The strong integration of text and image observed throughout the corpus supports readers in learning how these representations correspond, which enables movement between symbolic and visual forms of reasoning.

4.2 Consistent illustration choices support different aspects of propositional logic

Although the selected textbooks shared many representational conventions, substantial differences were observed in the illustration types they employed and the contexts in which those illustrations appeared. These choices were not distributed randomly. Instead, each textbook exhibited recurring combinations of illustration

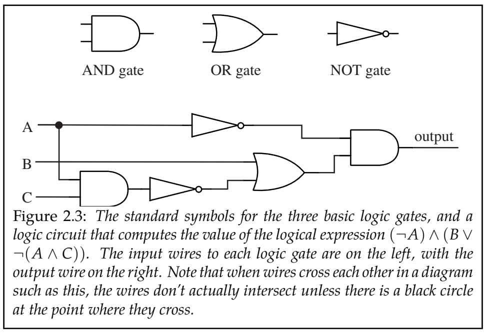

Fig. 4: Definition of logic gate symbols and example usage with translation to propositional formula. Taken from Hugtenburg [12, p. 30].

types and instructional contexts, suggesting that different visual representations were used to support different aspects of propositional logic.

Hugtenburg [12] relied heavily on computational representations. Computational representations refer to illustrations that depict how propositional formulae can be implemented and executed within computational systems. Of the thirteen selected illustrations, six were coded as `type:logic diagram`. These illustrations frequently appeared in explanatory and procedural contexts, such as the stepwise construction of logic circuits from propositional formulae (Figure 2). In addition, truth tables were occasionally used in computational contexts such as binary addition (Figure 5). These recurring choices consistently connect propositional formulae to computational artefacts and hardware-oriented procedures. Logic diagrams are used not merely to represent logical expressions, but to demonstrate how those expressions can be realised as digital circuits and employed in low-level computational tasks. The visual emphasis therefore falls on the implementation and execution of logical operations.

A	*B*	*C*	output
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

A	*B*	*C*	output
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

Figure 2.8: *Input/output tables for the addition of three binary digits, A, B, and C.*

Fig. 5: Truth tables representing binary addition. Taken from Hugtenburg [12, p. 38].

Huth [13] placed greater emphasis on structural and algorithmic representations. Ten selected illustrations were coded as `type:parse tree` (as seen above in Figure 1), while eight were coded as `type:dag`. Notably, all DAG representations occurred in SAT-related contexts (`context:sat`), including the representation shown in Figure 6. Compared with the other textbooks, which include neither DAGs nor parse trees, Huth devotes substantially more visual attention to syntactic structure and algorithmic procedures. The recurring use of parse trees and DAGs supports reasoning about formula structure and algorithmic problem-solving, particularly in SAT-related contexts. Rather than illustrating how logical expressions can be physically implemented, these representations focus on how they can be manipulated, analysed, and processed within higher-level computational procedures.

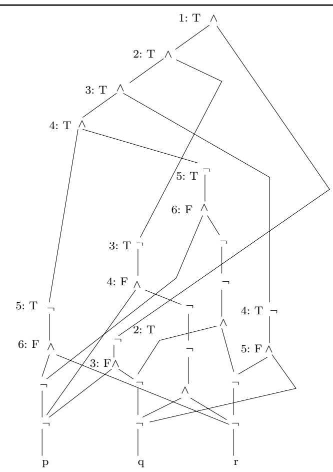

Figure 1.17. The DAG for the translation of the formula in (1.11). It has a ∧-spine of length 4 as it is a conjunction of five clauses. Its linear analysis gets stuck: all forced constraints are consistent with each other but several nodes, including all atoms, are unconstrained.

Fig. 6: Directed acyclic graph representation of a propositional formula used in SAT-solving procedures. Taken from Huth [13, p. 74].

Mendelson [15] exhibited a different distribution of illustration types and contexts. Eleven selected illustrations were coded as `type:truth table`, with many appearing in definitional (8/27 coded `context:definition`) or proof-related (11/27 coded `context:proof`) settings. For example, the three-valued truth tables shown in Figure 7 form part of a proof concerning axiom independence. Compared with the other textbooks, visual representations in Mendelson more frequently functioned as components of formal mathematical arguments and never did as computational or structural visualisations. This places greater visual emphasis on logical justification and proof than on computation or syntactic analysis. In Mendelson, illustrations primarily function as tools for establishing, verifying, and communicating formal logical arguments.

Proof

To prove the independence of axiom schema (A1), consider the following tables:

A	$\neg A$
0	1
1	1
2	0

A	B	$A \Rightarrow B$
0	0	0
1	0	2
2	0	0
0	1	2
1	1	2
2	1	0
0	2	2
1	2	0
2	2	0

For any assignment of the values 0, 1, and 2 to the statement letters of a wf $\mathscr{B}$, these tables determine a corresponding value of $\mathscr{B}$. If $\mathscr{B}$ always takes the value 0, $\mathscr{B}$ is called *select*. Modus ponens preserves selectness, since it is easy to check that, if $\mathscr{B}$ and $\mathscr{B} \Rightarrow \mathscr{C}$ are select, so is $\mathscr{C}$. One can also verify that all

Fig. 7: Three-valued truth tables used in a proof of axiom independence. Taken from Mendelson [15, p. 37].

The emphasis observed in Hugtenburg's textbook [12] is consistent with its educational context. At TU Delft, Reasoning and Logic is taught alongside Computer Organisation, making logic diagrams and binary addition examples a natural connection between propositional logic and computer hardware. Although the educational contexts of Huth [13] and Mendelson [15] were not examined, the consistent pairing of illustration types and instructional contexts across all three textbooks suggests that these choices serve specific pedagogical purposes.

These differences suggest that illustration choice is not neutral. Across the corpus, authors consistently paired particular illustration types with particular instructional contexts. Logic diagrams repeatedly appeared in computational and procedural settings, parse trees and DAGs in structural and algorithmic settings, and truth tables in definitional and proof-oriented settings. Rather than serving as interchangeable visual aids, these representations supported different forms of reasoning and directed attention toward different aspects of propositional logic.

5 Discussion

This study investigated how illustrations function pedagogically and communicatively within contemporary propositional logic textbooks. The thematic analysis identified two recurring patterns across the corpus. First, illustrations function as structured translations of symbolic expressions used to either clarify or define logic content. Second, illustration choices are systematically associated with distinct kinds of logical activity, such as computation, syntactic analysis, and formal proof. Together, these findings show that visual representations are integral to how propositional logic is communicated and structured in textbook discourse.

The first theme indicates that illustrations such as truth tables, parse trees, logic circuits, and directed graphs are translations of symbolic expressions. These representations re-express symbolic notation in forms that make particular structural or semantic properties more accessible. Across the corpus, authors actively construct these correspondences through captions, surrounding prose, and spatial alignment between text and figure. Parse trees, for example, reconfigure linear formula notation into hierarchical structures that foreground operator precedence and syntactic decomposition, while truth tables externalise semantic evaluation into exhaustive tabular form.

This finding is consistent with work on diagrammatic representation in formal reasoning. Bernard [2] argues that spatial organisation can render relational structure directly perceptible, while Bhattacharjee [3] characterises logic diagrams as cognitive tools that partially offload reasoning onto visual perception. The present findings are compatible with these accounts: within the analysed textbooks, illustrations systematically re-express symbolic structure in visual formats that make syntactic and semantic relations more directly inspectable. They function as structured translations that support interpretation of symbolic expressions.

The second theme concerns how illustration types are distributed across different instructional contexts. The analysis shows that representational choices are consistently aligned with specific forms of logical activity. Logic circuits and binary representations are primarily associated with computational interpretation, parse trees and directed acyclic graphs with syntactic and algorithmic reasoning, and truth tables with semantic definition and formal proof.

A further implication concerns the hidden pedagogical assumptions embedded within illustrations. The analysis suggests that visual representations do more than explain individual concepts; they also communicate views about what logical reasoning is and how it should be performed. Different textbooks consistently foregrounded different aspects of logic, including computation, syntactic structure, and formal mathematical proof. Illustrations therefore participate in shaping students' conceptions of the discipline itself. This observation reinforces the importance of studying visual representations not only as educational tools but also as carriers of disciplinary values and epistemological assumptions.

From this perspective, illustration choice is not neutral. It structures the conceptual entry points through which students encounter propositional logic. Hugtenburg [12] foregrounds implementation-oriented reasoning through logic circuits and hardware-level examples such as binary addition. Huth [13] emphasises structural and algorithmic reasoning through parse trees and directed acyclic graphs, particularly in SAT-related contexts. Mendelson [15] situates truth tables primarily within definitions and proofs, where they function as formal instruments for establishing logical properties. These patterns indicate that illustrations are consistently embedded within distinct explanatory regimes rather than used interchangeably.

More broadly, the study contributes to understanding how formal disciplines are mediated through mixed representational systems. Propositional logic is often characterised as purely symbolic, yet the corpus demonstrates that visual representations are systematically integrated into its exposition. Truth tables, logic diagrams, and trees are not peripheral aids but recurring components of explanation, definition, and proof. This suggests that formal reasoning in educational contexts is routinely supported by hybrid symbolic–visual structures, where meaning is distributed across multiple representational modalities.

5.1 Limitations and Future Research

Several limitations of this study also point toward promising directions for future research. First, the analysis examined only three contemporary propositional logic textbooks. Therefore, the findings cannot be assumed to generalise to all logic textbooks. Future studies could analyse larger corpora and investigate variation across educational levels, institutions, cultures, or historical periods. Preliminary observations made during corpus selection suggest that older logic textbooks may employ more concrete and naturalistic illustrations than contemporary texts, making historical comparisons a particularly promising avenue for further research.

Second, the study focused exclusively on propositional logic. Other areas of logic, including predicate logic, modal logic, proof theory, and automated reasoning, may employ substantially different visual conventions and pedagogical strategies. Extending the present analysis to these domains would help determine whether the patterns identified here are specific to propositional logic or characteristic of logic education more broadly.

Finally, coding and interpretation were conducted by a single researcher. While coding decisions were iteratively refined through repeated review, analytical memos, and discussions with peers and supervisors, formal inter-rater reliability was not established. Future work could strengthen the robustness of the findings through collaborative coding procedures and formal measures of coding agreement.

More generally, the present study focused on how illustrations are used within textbooks rather than on how learners interact with them. Combining textbook analysis with classroom observations, interviews, eye-tracking studies, or learning assessments could provide a stronger basis for evaluating the pedagogical effectiveness of particular representational strategies. Such work would also support design-oriented research exploring alternative visual languages, concrete analogical representations, interactive diagrams, or exploratory visualisations intended to make formal logical concepts more accessible to learners. A further limitation concerns the relationship between intended and actual communicative function. The coding framework captures how illustrations appear to function within textbook discourse, but it cannot determine how students interpret or use these representations in practice.

6 Conclusion

This study investigated how illustrations function pedagogically and communicatively within contemporary propositional logic textbooks. Addressing a gap in the literature on logic visualisation and diagrammatic reasoning, a qualitative thematic analysis was conducted on illustrations drawn from three textbooks representing different educational contexts within mathematics and computer science. Using a hybrid deductive–inductive coding approach, the study examined how illustrations are integrated into explanations and what pedagogical and communicative roles they perform.

Two principal themes emerged from the analysis. First, illustrations functioned as representational translations and definitional tools. Visual representations such as truth tables, parse trees, logic circuits, and directed acyclic graphs re-expressed symbolic notation in alternative visual forms that made particular syntactic, semantic, or procedural relationships more accessible. In many cases, these illustrations did not merely accompany definitions but actively participated in constructing meaning by defining logical connectives, demonstrating structural relationships, or guiding transformations between representational systems.

Second, illustration choices were consistently associated with different instructional contexts and forms of logical activity. Logic diagrams predominantly supported computational and implementation-oriented reasoning, parse trees and directed acyclic graphs supported structural and algorithmic reasoning, and truth tables were most commonly used in definitional and proof-oriented contexts. These recurring patterns suggest that illustrations are not interchangeable visual aids. Instead, different representational forms foreground different aspects of propositional logic and support different ways of reasoning about the subject.

These findings suggest that illustrations form an integral part of the representational infrastructure through which propositional logic is taught. They help define concepts, translate between representational systems, and direct attention toward particular forms of reasoning. Illustrations therefore contribute not only to the communication of logical content but also to how the discipline itself is presented to learners.

Overall, this study provides a descriptive account of how illustrations are used in a small sample of contemporary propositional logic textbooks. By examining the visual representations that students encounter in these texts, it offers a foundation for further research into the pedagogical implications, effectiveness, and design of visual representations in logic education.

7 Responsible Research

7.1 Positionality statement

This study was motivated in part by the researcher's internship as a computer science teacher. During this work, including teaching propositional logic to secondary-school students, the researcher observed that learners often found common instructional representations such as truth tables and parse trees challenging to interpret. These observations informed the choice of research topic but did not determine the coding process or analytical outcomes.

The researcher was also familiar with one of the analysed textbooks, namely the textbook by Hugtenburg and Yorke-Smith [12], which is used in the TU Delft course *Reasoning and Logic*. This familiarity may have introduced potential bias during interpretation. To mitigate this risk, coding decisions were guided by an explicitly defined codebook and were iteratively reviewed through discussions with peers and supervisors.

7.2 Licensing and reproducibility

All analysed textbooks were accessed through legitimate channels, including the TU Delft Library and publisher-provided digital editions. Figures reproduced in this paper are used solely for academic analysis, criticism, and discussion, and are accompanied by appropriate citations to the original sources.

To support transparency and reproducibility, the coding methodology and code definitions are described in sufficient detail within this paper. The analysed textbooks are publicly available through libraries or publishers, allowing future researchers to inspect the source material. However, the full codebook and coded dataset are not currently publicly available. The author intends to make these materials openly accessible in future revisions where possible.

This study follows the principles of open and transparent research by documenting the corpus selection procedure, coding framework, and analytical process. No human participants were involved, and therefore no personal data or sensitive information were collected.

7.3 AI disclosure

AI tools were used as research and writing aids during the project. NotebookLM was used to obtain preliminary summaries of literature (always verified against the original publications before any further use). ChatGPT was used to improve the clarity, readability, structure and academic style of the paper, including assistance with phrasing, paraphrasing and LaTeX formatting. All research decisions, analyses, interpretations, and final written content remain the responsibility of the author.

References

1. Aznar, M.B.L.: "The Marlo Diagram in the Classroom". In: Diagrammatic Representation and Inference. Cham: Springer International Publishing, 2020, pp. 490–493. ISBN: 9783030542498. https://doi.org/10.1007/978-3-030-54249-8_41.
2. Bernard, J.: "Can We "Show" the Correctness of Reasoning? On the Role of Diagrammatic Spatialization in Logical Justification". In: When Form Becomes Substance: Power of Gestures, Diagrammatical Intuition and Phenomenology of Space. Cham: Springer International Publishing, 2022, pp. 27–64. ISBN: 9783030831257. https://doi.org/10.1007/978-3-030-83125-7_2.

3. Bhattacharjee, R.: Visual Clarity and Expressiveness in Logic Diagrams. Logica Universalis **19**(2), 329–344 (2026). https://doi.org/10.1007/s11787-025-00375-5
4. Bolz, R.: Logical Diagrams, Visualization Criteria, and Boolean Algebras. In: The Exoteric Square of Opposition, pp. 195–224. Springer International Publishing, Cham (2022). https://doi.org/10.1007/978-3-030-90823-2_9
5. Carney, R.N., Levin, J.R.: Pictorial Illustrations Still Improve Students' Learning from Text. Educational Psychology Review **14**(1), 5–26 (2002). https://doi.org/10.1023/a:1013176309260
6. Carter, J.: Exploring the fruitfulness of diagrams in mathematics. Synthese **196**(10), 4011–4032 (2019). https://doi.org/10.1007/s11229-017-1635-1
7. Cheng, P.C.-H.: Truth Diagrams Versus Extant Notations for Propositional Logic. Journal of Logic, Language and Information **29**(2), 121–161 (2019). https://doi.org/10.1007/s10849-019-09299-y
8. Clarke, M.C.: Possible models diagrams: a visual alternative to truth tables. In: Proceedings of the Twenty-Fourth SIGCSE Technical Symposium on Computer Science Education. SIGCSE '93, pp. 232–236. Association for Computing Machinery, Indianapolis, Indiana, USA (1993). https://doi.org/10.1145/169070.169458
9. Clarke, V., Braun, V.: Thematic analysis. The Journal of Positive Psychology **12**(3), 297–298 (2017). https://doi.org/10.1080/17439760.2016.1262613
10. Dimmel, J.K., Herbst, P.G.: The Semiotic Structure of Geometry Diagrams: How Textbook Diagrams Convey Meaning. Journal for Research in Mathematics Education **46**(2), 147–195 (2015). https://doi.org/10.5951/jresematheduc.46.2.0147
11. Herbst, P.: Interactions with diagrams and the making of reasoned conjectures in geometry. Zentralblatt für Didaktik der Mathematik **36**(5), 129–139 (2004). https://doi.org/10.1007/bf02655665
12. Hugtenburg, S., Yorke-Smith, N.: Delftse Foundations of Computation. TU Delft OPEN Publishing (2022)
13. Huth, M., Ryan, M.: Logic in Computer Science: Modelling and Reasoning About Systems. Cambridge University Press (2004)
14. Mayer, R.: Multimedia Learning. Cambridge University Press (2020)
15. Mendelson, E.: Introduction to Mathematical Logic. CRC Press/Taylor & Francis Group (2015)
16. Ray, P.K.: A Text-book of Deductive Logic: For the Use of Students. Macmillan (1886)
17. San Ginés, A.: Visual Tools for Teaching Propositional Logic. In: Tools for Teaching Logic, pp. 215–222. Springer Berlin Heidelberg, Berlin, Heidelberg (2011). https://doi.org/10.1007/978-3-642-21350-2_26
18. Tenny, S., Brannan, J.M., Brannan, G.D.: Qualitative study. StatPearls (2017). https://pubmed.ncbi.nlm.nih.gov/29262162 (visited on 06/10/2026)
19. Wolf, A.: Textbook of Logic. Collier Books (1938)