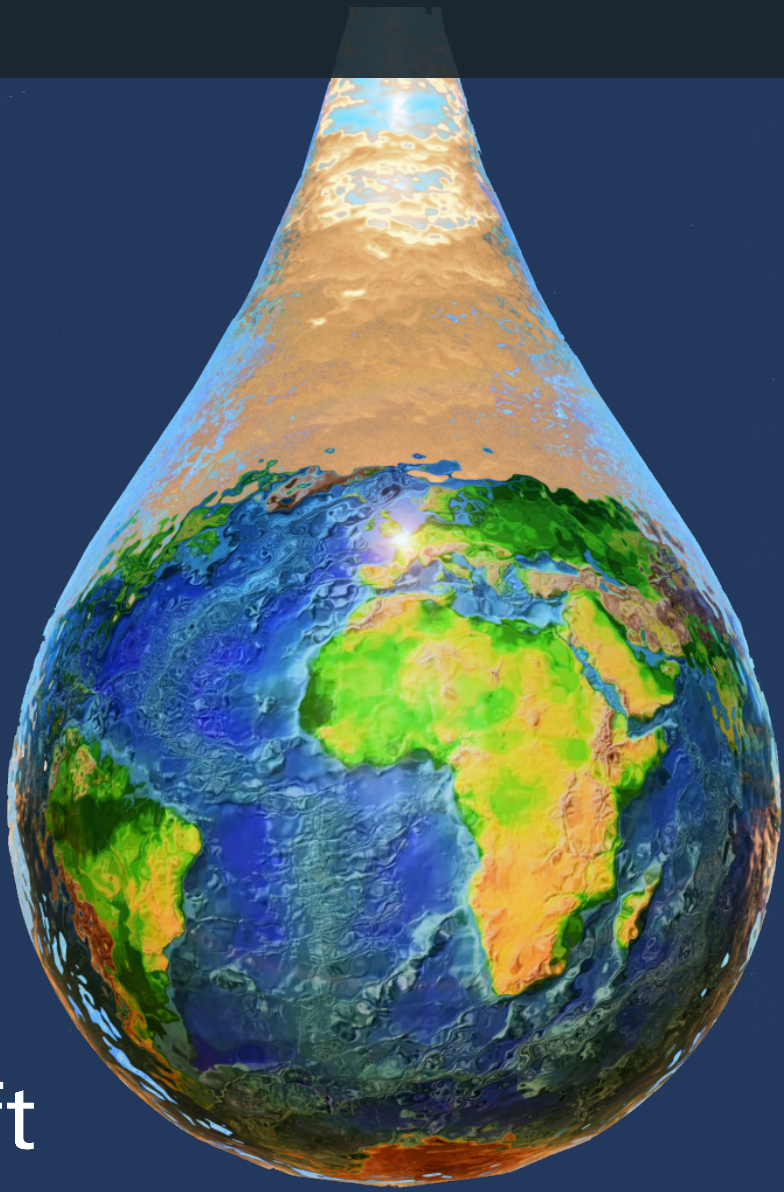
Exploring distributive justice in water resource allocation

A rival framings approach on the operationalization of equality in multi-objective optimization models for water systems

Farley Rimon



Exploring distributive justice in water resource allocation

A rival framings approach on the operationalization of equality in multi-objective optimization models for water systems

Thesis report

by

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The code associated to the work of this thesis can be found at https://github.com/farleynitro/MUSEH2O.

Faculty of Technology, Policy & Management · Delft University of Technology

Preface

To this day I am happy my family believed in that I could make it further than they could with their full support. After finishing my BSc. in Electrical Engineering, I felt a void. It became clear that efficiency from innovations wasn't enough – I yearned to ensure broader societal and ecological benefits from technological advancements. Against doubts, I transitioned to an MSc. in Engineering and Policy Analysis (EPA), aiming to tackle issues that impact all corners of society.

This thesis started as an idea that led to a different result. This subject, deeply connected to EPA, is an exemplary case on how to consider justice in future decision-support models, as well as for future situations I would like to solve in my own region. In hindsight, I gained from all the deviations I have had.

Benefitting from these deviations would not have been possible without my family. Especially my mom, who always believed in me with the kind and simple words.Thank you dad for teaching me the value of caring for others, and never for recognition. Thank you to my grandparents for always sharing messages of love and moral foundation. Thank you to my childhood swimming coaches for teaching me the art of discipline at all times of a journey. Thank you to my girlfriend for being the most supportive and empathetic woman. Thank you my friends for the laughter, and truthful honesty that led to lots of self-reflection that translates to my work, self-esteem, and perception of the world.

None of this would have been possible without Jazmin, Juan, Damla and Jan. Thank you Jazmin for being the first person during my search for a supervisor to open her heart and mind to what I wanted to research. At all times, I felt that you wanted the best for me, even when I was confused. Thank you for the kind encouragement to believe in my research, and show great excitement on understanding the problem. When all of this was still a messy ball of knowledge, we untangled it together. Together, we synthesized the results into something that is understandable for a greater audience. Thank you Juan for being someone I could always combine what felt like 'light-hearted' conversations, with serious 'let-us-get-to-business' key decisions. You taught me how to structure myself, which permanently changed my work ethic where I now combine rationality with a gut feeling, unmissable in dealing with complex problems like these. Thank you Damla for showing me how much fun it can be to do research as a job. Your workflow for the PhD. makes me want to pursue, whatever I do in life, with that same drive. Work, passion, and happiness can be combined, and you are an example of this. It yielded me novel ideas, key directions to tackle justice, and self-esteem to value my work. Whatever is in my thesis, is part of brainstorming sessions with you. Thank you Jan for being someone straight-to-the-point with key concepts I needed to jump in. From the beginning, you showed me how much you cared, which translated into direct and honest feedback, which felt invaluable to realizing a thesis where I can reflect on what I am putting on paper. Researchers like you make me reflect and think outside of the box. I believe only in doing so, we can make the advancements we need in decision-support for reliable decision-making. I could have never asked for a better committee. I realize the importance of research in justice. The problem of justice predates and will outlast me.

Farley Rimon
Delft, 2023

Executive Summary

Water has a multi-faceted purpose - whether it serves the consumption of environmental flows to protect the ecosystem, potable water for urban areas, energy production for communities, agricultural food production, or recreational purposes - water is essential to our existence in this world. Hidden behind the veil of what is visible, we are confronted with a stark reality of high water dependency amid an ever-increasing scarcity of water resources. Potable water is a finite resource, and yet facing a growing demand for which long-term sustainable water provision is in many regions not guaranteed. By 2030, the need for energy production, foremostly from water, is expected to grow by 57 %. Additionally, growing trends indicate food production to grow by 68% by 2050 (Marr, 2022). To deal with the uncertainty on the water demand satisfaction for multiple objectives, in 2010 the UN adopted historical resolution acts to declare water to be a *(human) right*. While such an act by the UN creates awareness of the right to water on a global scale, an important aspect remains vaguely defined for situations on smaller geographical scales. Consequently, the right to water, when a limited supply is distributed locally, is undefined. Hence, '*How must this notion be applied in a context where water supply is limited, everyone has the right to water, and demand is unmet (and dynamically changing) across several purposes?*' The issue of what is a right reaches in this sense a deeper level of understanding. Inherently, water management faces a trade-off (among others) between efficient, and equitable, use of water (Lévite & Sally, 2002; Wegerich, 2007). With growing stress for the multi-faceted use of limited water supply, one of the biggest challenges facing policymakers is how they can efficiently and equitably distribute water from existing reservoirs to ensure sustainable water use.

This thesis transcends the definition provided by the UN by creating a path for the inclusion of equity when a reservoir is being used for multiple purposes, and thus for multiple actors. Decision-support models for water systems that simultaneously optimize multiple objectives, also known as *Multi-Objective Optimization* (MOO) models, are essential when society encounters complex situations where due to a limited availability of water it is not possible to meet the multiple water objectives at all times. While previous studies focus on using MOO to increase the efficiency of objectives, and in this manner maximize the aggregated benefits, including objectives that ensure distributive justice on the management of water resources are as important. Each study on the '*distributive justice*' of MOO implements this justice distinctively, i.e. their notions for justice are different across studies, their (operationalization) formulations of justice are different, and their implementation is different. Hence, the problem formulations are often designed such that they fit the modelling approach, with little consideration of the implications drawn solutions considered '*optimal*'. This leads to one of the most difficult burdens a modeller has to deal with, introducing normative bias into the model, leading to normative uncertainty in the implications drawn from such decision-support model outcomes (Taebi et al., 2020).

This thesis is the first approach to understanding how the operationalization of distributive justice shapes the implications drawn from the '*optimal*' outcomes of decision-support MOO-models.This approach considers multiple high-level goals, from the traditional efficiency goal (of maximizing several objectives), to a complementary distributive justice principle goal (of including a justice objective). A *rival framings approach* acknowledges diversity in perspectives, for which it is suitable to contrast the operationalization formulation of the same distributive justice principle. The rival framing focused on the *inequality metric* and the *aggregation method over time for this metric*, both used for the formulation of inequality in the objective formulation. The distributive justice principle that stayed fixed was *egalitarianism*, with the aim to minimize the relative distance among objectives (Ciullo et al., 2020). Equality was studied in this thesis by rival framing the operationalization of the inequality formulation calculated across objectives. The Pareto front from MOO served as the reference to study the implications of the rival framings. By determining how the Pareto front shifted, it was understood how the solution space shifts from a difference in objective formulation. The research of this thesis is important because there are rising attempts to include justice in optimization models, but in doing so make use of a pragmatic approach for the formulation. Therefore, before studies continue this important journey to consider more (complex components of) justice in decision-support models, light should be shed on the choices modellers make to include justice (Fletcher et al., 2022). This leads us to the following research question:

Research Question : Main Question

How do different operationalization formulations for inequality in existing multi-objective optimization models shift the Pareto front?

In order to address the research question on inequality, the chosen case-study was the Lower Susquehanna River Basin, a subbasin of one of the oldest and largest rivers in the world which uses the Conowingo Reservoir as a water management reservoir system. The Conowingo Reservoir serves multiple purposes for which multiple stakeholders are involved with high interest. The management of the Conowingo is a complex case due to its demand for truly diverging purposes such as hydropower revenue, atomic power plant cooling, environmental flow requirements, and more. In this thesis, the simulation-based Evolutionary Multi-Object Direct Policy Search (EMODPS)-model, designed by Giuliani et al. (2014) and further improved by Zatarain-Salazar et al. (2016) was used. The model parametrized the operating release decisions of the Conowingo using Radial Basis Functions and used Multi-Objective Evolutionary Algorithms to optimize the operating policies. To the traditional six efficiency objectives, a seventh equality objective that considered the minimization of inequality among the traditional six objectives was added. For this inequality formulation, the *inequality formulation* is split in two components for rival framings. First, rival framings focused on the *inequality metric* by changing the aggregated *Gini-coefficient* to the aggregated *Euclidean distance*. Moreover, because of the lack in studies to defend the level of aggregation over time, alternative aggregation methods were compared when measuring equality. The *aggregation method over time* was alternated in three distinctive manners, namely the *daily-based yearly mean*, *monthly-based yearly standard deviation*, and the *ratio of the standard deviation and mean*. Summarizing the main key findings:

1. Equality is gained when complementing the traditional maximization formulation with the equality objective. The more equality is gained, the larger the shift of the Pareto front.
2. The chosen inequality metric has a significant influence on shifting the trade-off across objectives. The aggregation method has a weaker influence on the shift in trade-offs.
3. The Euclidean formulations yield smaller inequalities across objectives (and thus more equality) than the Gini formulations because of its quadratic formulation as contrary to the Gini. Conversely, Gini formulations lead to higher efficiency than the Euclidean formulations, while improving the equality compared to the traditional formulation.
4. The mean aggregation shows the largest shift in efficiency and equality of objectives, the deviation aggregation yields lower equality and forms exceptions depending on the indicator, and the ratio of both does not improve equality nor efficiency.
5. There is a non-linear relationship between the chosen inequality metric or aggregation method and the shift in the trade-off between efficiency and equality.

Modellers need to acknowledge the bias introduced by the choice of operationalization for equality. I suggest looking at multiple points that create a broader understanding of justice in MOO and create a deeper understanding of justice in MOO. First, it is time to consider relative (distribution) injustices from a disaggregated perspective, i.e. choosing over which objectives inequality *should be* optimized. Moreover, for future research, I suggest improving the methods to consider positive and negative variance. In doing so, considering in which direction the inequality is moving, will improve the combined satisfaction of efficiency and distributive justice. In terms of social implications, for reliable use of decision-support models, models need justice while continuing with the extraction of advice from these models. Simulation-based MOO-models such as the EMODPS-model used in this study are a way of dealing with the complexity of our world. To have higher usefulness from MOO-justice-included models, society must further reflect what their stance is on justice. For example, it needs to address how this justice formulation is related to risk aversion. Only through the elaboration of justice formulations, a future is reachable where justice is not only considered but reached. We need to collect views, unify views, and unify equitable distribution.

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Nomenclature

List of Abbreviations

CBA Cost-Benefit Analysis

DPS Direct Policy Search

EMA Exploratory Modelling Analysis

EMODPS Evolutionary Multi-Objective Direct Policy Search

MOEA Multi-Objective Evolutionary Algorithms

MOO Multi-Objective Optimization

List of Symbols

ϵ Epsilon Progress

η Efficiency

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Part I

Introduction

1
Introduction

1.1. General introduction: Injustice in water distribution

Water, an essential resource for every human being on Earth, plays a fundamental role in the development and social and economic growth of society. It fulfills a multi-faceted role in supplying and meeting the diverse demands of various sectors. Notably, water is indispensable for irrigation purposes, industrial processes, energy production, and urban water supply, as well as providing drinking water and supporting agricultural food production. This makes water crucial for the existence and well-being of humankind.

Society heavily relies on water in these areas to ensure food security and energy production, underscoring the need for a reliable and sufficient water supply. To put this into perspective, the global annual demand for water is approximately 4 trillion m^3 (Our World in Data, 2023), with an average of 70% used for agriculture, 19% for industrial purposes, and 11% for municipal consumption. All of these sectors are interconnected and essential for societal functions (Quinn, Reed, & Keller, 2017). It is important to note that this annual demand is a considerable fraction compared to the volume of water in the sea, which is approximately 18 trillion m^3 (US Geological Survey, 2023). This stark contrast not only underscores society's high dependency on water but also highlights the vulnerability and complexity of the water supply system if any changes were to occur.

Hidden behind the veil of what is visible, we are confronted with a stark reality of high water dependency amid an ever-increasing scarcity of water resources. Potable water being a finite resource, is facing a growing demand, leading to uncertain long-term sustainable water provision for many regions. Climate change is further increasing the vulnerability, as 2.3 billion people already live under water stress [1] (United Nations Water, 2023), and more are to come due to population growth and exploitation of resources.

By 2050 the injustices of water stress will further be exacerbated as global demand will double, and with this the amount of people under water stress (Cominelli et al., 2009). Some studies suggest that up to 75 % of the global population will need to cope with freshwater scarcity (Hightower & Pierce, 2008). Moreover, water is crucial to satisfy the energy transition worldwide, as it nearly supplies 16% of our current energy demand (Association, 2021). Considering there is an existing hydropower infrastructure, that would need to be rebuilt and expanded, the International Energy Agency suggests an extra 850 GW hydropower to meet climate targets and stay below 2 degrees °C (International Renewable Energy Agency, 2020). By 2030, the need for energy production from sustainable resources is expected to grow by 57 %. Hydropower is in high demand to ensure this energy transition. Additionally, growing trends indicate food production to grow by 68% by 2050 (Marr, 2022). Finding ways to solve the short- and long-term supply of water for a multi-faceted use under a gradual increase in water scarcity, raises questions on how decision-makers can ensure that society benefits from the distribution of water.

The UN responded to the water stress situation in 2010 where it adopted historical resolution acts to declare water to be a *human right* [2].While such an act by the UN creates awareness of the right to

[1] Water stress is defined by the United Nations as when annual water availability drops below 1,700 cubic meters per person. This ratio is used to determine the level of freshwater availability in a region or country compared to water demand. This categorization helps identify areas that are experiencing water stress and require urgent intervention to ensure sustainable access to safe drinking water.

[2] The human right was declared, but not the environmental right to preserve the ecosystem that depends on a specific water body. Nonetheless, many governments have implemented a type of environmental right in the decision-making process for water distribution.

water on a global and general scale, an important aspect remains vaguely defined on smaller geographical scales, namely the right to water when distributing water on a local scale. On a local scale, it remains ambiguous how to distribute water over several objectives, or over several actors. A *right* for access to clean water, but how must this notion be applied in a context where water supply is limited, several actors have the right to water, and demand is unmet (and dynamically changing) across several purposes? Or when the environment has *rights* allocated as well, forming additional criteria - that is equally important for the human *rights* of water - to the water supply system? Or more generically, in a context where water serves multiple purposes (drinking, food production, energy, environment, industry, etc.), all of which are essential to the *human*, *environment* and *industry*, indicating multiple levels of *right* to its access?

Regardless of the collective efforts of governmental institutions and research, there is a *lack of consent* on how to manage the multi-faceted use of water, specifically freshwater. Since each actor and usage has a certain *right*, and the growing *lack of consent* on the distribution, nowadays, the conversation is shifting areas with 'water stress' to areas with 'water conflicts', with over 800 official water conflicts globally since 2010 (World Water Council, 2023). The issue of what is a right reaches in this sense a deeper level of understanding, namely on how these rights should be distributed.

In order to solve the issue of how to distribute water, another problem is faced when we must consider that the distribution of water has multi-sectoral effects, and satisfies multiple purposes (Quinn, Reed, Giuliani, et al., 2017). Moreover, the burdens of the challenges in water provision, and the benefits of the same water provision, are not equally distributed among the affected population. Therefore, *'When is the distribution of water just?'*. It is a question with a non-trivial answer. In order to ensure the *right* to water is equally distributed, the topics of equity [3], distributive justice, ethics, sustainability and efficiency need to be included in a discussion regarding the management of water.

Inherently, water management also faces a trade-off between efficient, equitable, and sustainable water use (Lévite & Sally, 2002; Wegerich, 2007). With growing stress for the multi-faceted use of water, one of the biggest challenges facing policymakers is how to equitably and efficiently [4] distribute water from existing reservoirs to ensure sustainable water use.

1.2. Thesis scope

This thesis transcends the definition provided by the UN by creating a path for the inclusion of equity when a reservoir is being used for multiple purposes, and thus for multiple actors. To deal with the complexity of reservoir management decision-support models are used, giving rise to ample research to provide reliable decision-making recommendations. Therefore, the inclusion of equity is achieved through its implementation in decision-support models.

The most popular method is the Cost-Benefit Analysis (CBA) method, widely adopted for any large infrastructure project (Ciullo et al., 2020). Nevertheless, CBA's focus is on maximizing the aggregated benefits of resources such as water, neglecting the distribution among it, and therefore not ensuring equitable access to this *basic human right*. Moreover, according to Fletcher et al. (2022) what communities need is socially engaged water management, and yet, CBA's purpose is solely justifying the economic investment in a project when looking at the full spectrum of consequences.

In response to the limitations, another branch of research focuses on aiding decision-making through *optimization models* with single-objective problem formulations, but this is not enough to solve the problem of inequitable, or even inefficient, water allocation. As Zatarain-Salazar et al. (2016) debates such formulations are at the risk of simplifying the complex problem this is. Current decision-support models relying on a utilitarian perspective are therefore criticized (Fletcher et al., 2022). Under complex situations where due to a limited availability of water it is not possible to meet the multiple objectives at all times. This leads to a lack of consent on how to distribute water on a local level and causes balancing conflicts between water actors (Quinn, Reed, & Keller, 2017). Water is a multi-faceted dynamic resource, with multiple stakeholders, and should thus be explored accordingly. Decision-support models, with a focus on optimization models, need to be improved.

[3] Equity is concerned with the idea of fairness and making sure that actors are treated equally in light of their particular needs, circumstances, or contributions. Equity can be a goal in multi-objective optimization if the goal is to promote equal chances or lessen discrepancies.

[4] Leaving sustainability out of the challenge since this concerns the future use of water.

1.2.1. Knowledge gap

An improvement to previous models is the *Multi-Objective Optimization* (MOO) models which simultaneously optimize multiple objectives. Previous studies shed light on how the problem for MOO needs to be formulated to maximize the aggregated benefits. Less frequently, studies have investigated how to implement distributive justice [5] in MOO for the management of water resources.

Nonetheless, the studies considering distributive justice in MOO were pragmatic at best. Each study on the '*distributive justice*' of MOO implemented this justice distinctively, where their notion of justice was different across studies, their operationalization of justice was different, and their implementation was different. Hence, the problem formulations were often designed from the perspective of the modeller that decided how it best fits the modelling approach, with little consideration of the consequences from which solutions were considered '*optimal*'.

This leads to one of the most difficult burdens a modeller has to deal with, introducing normative bias into the model, leading to normative uncertainty in the implications stemming from such decision-support models (Taebi et al., 2020). Therefore, to elicit the implications of using alternative distributive justice principles as the goal of distribution for the model's objective formulation, studies shed light through contrasting ethical viewpoints. This leads to a significant adaptation of the problem formulation or model, incomparable results across formulations, and does not address the normative uncertainty from the modelling choice for a specific notion of distributive justice.

To deal with the latter problem, in this thesis focus lies on how the *shape* of distributive justice is implemented in decision-support models. The *shape* determines what a just distribution for water is over several actor objectives. Hence, it entails which notion is chosen for justice, how justice is operationalized, and how it is evaluated.

Moreover, the shape of distributive justice can be studied in several ways. In the first method, ethical principles that form a set of rules for the optimization can be implemented *a-priori*. Thereafter, the solutions from the optimization with differing ethical principles are compared (Ciullo et al., 2020; Reddel, 2022). In the second method, the formulation of inequality depends on the specifics of the case-study such as its time horizon, for which its implementation and mathematical formulation can be investigated through a series of frameworks on how justice should be applied in the models (Jafino et al., 2021; Xu et al., 2019). Additionally, the formulation of inequality can be studied through the operationalization method of inequality (often the *Gini-coefficient* with different variable inputs) (Hu, Chen, et al., 2016; Hu, Wei, Yao, Li, et al., 2016). Lastly, studies also look if the operationalized inequality should be implemented in the model as a constraint, or objective (Dong et al., 2022).

In the last method, ethical principles can be used to evaluate the inequality from the optimization *as-is*, and hence, is used as a post-processing method of the model results. This post-processing can be done by using decision-making game theory to investigate if actors find the solutions stable and fair (Alizadeh et al., 2017; Farhadi et al., 2016; Fu et al., 2021; Naghdi et al., 2021; Rădulescu et al., 2020; Sarva, 2021). The problem with this approach is that while it is suitable to form consent among actors, it bases its values on given weights for actor preferences. In many MOO-problems the actor preferences and weights are unknown, making it better to not *assume* weights (Rădulescu et al., 2020). Hence, recent research *a-posteriori* rank-ordered solutions according to the utility achieved from a specific ethical principle (Jafino et al., 2022). This is extremely useful since similarities and differences in solution preferences can be observed across contrasting and combined ethical principles.

While useful, this thesis supports the claim of Yang et al. (2023) where normative studies (on how resources *should be* distributed), offer the only possibility to include both *a-priori* and *a-posteriori* of decision-making. Subsequently, this is only possible if the ethical principle is implemented directly in the objective formulation. Building on the statement of Yang et al., the suggested method for combining an *a-priori* justice formulation and *a-posteriori* formulations is by adapting the objective formulation (Fletcher et al., 2022; Jafino et al., 2022).

Foremostly, the operationalization of equity for water system models (specifically MOO models) remains largely understudied compared to other analyses on the use of decision-support models. Especially,

[5] Distributive justice concerns the fair allocation of goods, benefits, or burdens within a society or system. It encompasses theories and principles about what constitutes a just distribution. In MOO, distributive justice as an objective may involve optimizing solutions that align with specific theories of justice, such as maximizing equality, rewarding merit, or prioritizing need. Hence, it does not look at the background of actors beyond the distribution. This falls beyond the scope of distributive justice, and within the scope of equity.

understudied fields are in a) the definition of justice, b) the mathematical formulation, and c) the performance (inequality) metric. Without determining what the influence is of what is actually being optimized, there is a lack of knowledge of the modeller's influence on the shape of a distribution.

Investigating distributive justice principles for the optimization of water resources is crucial for ensuring an equitable distribution (Fletcher et al., 2022). In this context, it is crucial to understand the shape of justice by understanding the implications from which the metric is being used and how aggregating this metric over time will shift the Pareto front. If it remains unknown how this Pareto front is shifting due to the justice formulation implemented, I argue that it remains unknown what trade-off is being made between efficiency and the distributive justice of a water system.

1.2.2. Research method: Rival framings approach

This thesis is the first approach to understanding how the operationalization of distributive justice shapes the implications drawn from the 'optimal' outcomes of decision-support MOO models. Rather than comparing contrasting views, a step is taken back in this research field of justice. The thesis used a *rival framings approach* to contrast the operationalization formulation of a chosen distributive justice principle, instead of over multiple principles. Rival framings acknowledges diversity in perspectives, even at this smaller formulation scale. Instead of seeking optimal solutions from one formulation chosen by the modeller for justice, rival framings leverage the existence of multiple notions of that same principle to create a better understanding of MOO.

A rival framings approach in itself is not new as Quinn, Reed, Giuliani, et al. (2017) attempted to discover the implications of deeply uncertain scenario choices. Herman et al. (2015) attempted to discover the implications of different problem formulations for decision-support models. However, to my knowledge, a rival framings approach that attempts at discovering the implications of different objective formulations where equity is included has not yet been done. Novelty is found in *rival framing* the distributive justice principle's operationalization. Consequently, the normative uncertainty resulting from the implementation of distributive justice in existing MOO models is reduced.

Distributive justice was defined using the *egalitarian* principle and hence looked at the *equality* achieved over objectives. Moreover, the core of the rival framings approach lies in the alternatives to operationalize the *inequality* [6] across objectives in the objective formulation of a MOO model for a reservoir (water system). Thereafter, the *inequality* across these objectives was minimized, aligning with definitions of previous MOO studies (Ciullo et al., 2020).

The rival framings approach used multiple high-level goals, from the baseline (traditional) efficiency goal (of maximizing utility) to a complementary distributive justice principle goal (of including a justice objective). The baseline optimization maximized the utility of objectives and the rival optimizations maximized the utility of objectives while the inequality across objectives was minimized. It is believed that this baseline (without distributive justice) was the best reference for optimal outcomes across framings of distributive justice since it does not create a dependence on the specific choice of operationalization for the chosen distributive justice principle.

The implications from the rival framings outcomes were studied by analyzing the shifting of the Pareto-optimal solutions, that together form the Pareto-front. Pareto-optimality is often used in multi-objective optimization because it allows the identification of solutions that are efficient in terms of multiple objectives. Regardless of whether distributive justice was considered, previous MOO studies used the Pareto-front - either directly and indirectly - to study the implications from the optimization (Giuliani et al., 2014; Herman et al., 2015; Quinn, Reed, & Keller, 2017). By determining how the Pareto front shifts, it was understood how the solution set changes from a difference in operationalization.

The rival framings focused on the *inequality operationalization*, which consisted of two main components. Consequently, the *inequality metric* and the *aggregation method over time for this metric* were alternated over the optimization of rival framings. Moreover, justice in MOO studies will often adopt the a) *Gini-coefficient*, a metric for inequality in distribution, which is here contrasted with the b) *Euclidean distance*, a metric for relative distances across reference points (in this case objective values). Both metrics aim to quantify the same, inequalities or relative distances across objective values, but stem from a different (not

[6] Inequality is operationalized because it acknowledges that objectives do not have the same starting point. Moreover, it is the most direct, and quantifiable method for the striving of justice under the equality principle.

contrasting) logic, making it a perfect example of how a subtle difference in logic can affect the implications. Moreover, in order to reduce computational expenses in MOO, an aggregation method for the objective formulation is chosen for the time horizon. McPhail et al. (2018) finds that this can significantly change the findings from the model. Therefore, while a similar comparison research has been done for robustness metrics, this was studied using the rival framing for the aggregation method over time of the inequality metric by formulating it as a) the intra-timely standard deviation, b) aggregated mean over the time horizon, and c) the ratio between the deviation and the mean.

Using the knowledge gap and the rival framings approach, it became possible to formulate what will be contrasted in the rival framings to understand the effect of different operationalization methods on the model outcomes. The rival framings focused on the rival framings operationalization when it is implemented in the objective formulation of a MOO-problem. Equation 1.1 shows the components that are part of this operationalization. Here $J^{equality}$ is the objective formulation for inequality that needs to be *minimized*, and measures the relative difference between the performance of objectives. Maximizing equality is the high-level objective, that is measured through inequality, hence the name $J^{equality}$. $f(J_{i,j})$ represents the chosen inequality metric to operationalize the relative difference between objectives J_i and J_j. Moreover, the objective formulation can only have one objective that needs to be minimized at the end of each model run. This is why $f(J_{i,j})$ needs to be aggregated over time. The aggregation method over time influences what aspect of inequality is being minimized. For example, the aggregation method can measure inequalities between objectives over smaller time steps that form the time horizon, or it can measure the inequalities between objectives at the end of the time horizon. $\triangle$ represents the mathematical formulations possible to aggregate inequality over time.

$$J_{inequality} = \triangle(f(J_{i,j})) \tag{1.1}$$

1.2.3. Motivation of contribution

As modellers, there is a moral responsibility to address inequalities in communities from water systems (Fletcher et al., 2022). The responsibility, therefore, lies in:

- *What to operationalize as distributive justice*
- *How to operationalize distributive justice*
- *How to evaluate distributive justice*

The research of this thesis is important because there are rising attempts to include justice in optimization models, but in doing so make use of a pragmatic formulation. Studies a) evaluate justice (and not optimize), and b) highlight trade-offs for optimization systems without understanding if this trade-off is dependent on their formulation for normative modelling choices. Using the *Gini-coefficient* highlights inequalities, but there are ample other metrics to calculate inequality if its concept is changed instead to *relative differences*. Hence, it is truly unknown if inequality is best measured by the *Gini-coefficient*, or other metric formulations. Also, studies frequently title the '*trade-off between efficiency and equity*' but fail to form a unified view on how to implement equity in MOO-problems. Ciullo et al. (2020), Dong et al. (2022), Hu, Wei, Yao, Li, et al. (2016), and Xu et al. (2019) are examples of studies with each their own approach to understanding the trade-off. But which one is correct? I argue that the lack of a unified approach makes the trade-off dependent on the formulation for equity (specifically the distributive justice principle).

Therefore, before studies continue this important journey to create more justice in decision-support models, light should be shed on the choices modellers make to include justice (Fletcher et al., 2022). Moreover, the trade-off across objectives changes based on the chosen inequality operationalization. Subsequently, the *trade-off between efficiency and equity* will vary significantly, making the outcomes less reliable. If we (as a society) want to benefit from justice in decision-support models in a reliable manner, it is time to acknowledge that even the same justice principle will change our achieved trade-off and justice implication. This thesis is a first step in assessing how important these changes (in the objective formulation) are to the implications drawn from them.

1.3. Research question

Summarizing the knowledge gap of the Literature Review (Chapter 2), key components were found to make up the Research Question:

1. The existing objective formulation should be complemented with a justice principle (equality) for water allocation. Doing so improves understanding of implementing distributive justice without further adaptation of a model. The model keeps its original (efficiency-principle) purpose with an additional water allocation principle.
2. The operationalization of equality is ambiguous as ample studies have their own distinctive formulation. Specifically, ambiguity lies in the choice of inequality metric, and the aggregation method over time.
3. The Pareto front is used to evaluate the trade-offs between objectives from optimal solutions. Hence, how the Pareto front shifts across operationalization formulations for inequality yields how trade-offs change across these formulations. First, in the trade-off across the performance gained for objectives. Second, in the trade-off between achieved aggregated (efficiency) performance of objectives and the equality in performance gained across these objectives.

This leads us to the following research question:

Research Question : Main Question

How do different operationalization formulations for inequality in existing multi-objective optimization models shift the Pareto front?

In order to deal with the spectrum of uncertainty from the choice of performance metrics, several operationalization metrics were tested over the same case-study (McPhail et al., 2018). In this thesis, performance referred to the level of equality gained. Using the Pareto front, it was possible to determine how the trade-offs among objectives and among principles shift. Hence, the Pareto front determined the implications of the model outcome.

While the Pareto front represents optimal aggregated outcomes, on a disaggregated actor-level, there are situations where it is unable to find optimal solutions without causing conflicting water allocation. Hence, the solutions are not optimal for all objectives. Hence, I argue that after introducing the equality objective, the Pareto concept will assign higher importance to solutions that do not leverage aggregated optimality over the large disadvantage of some actor objectives.

1.3.1. Sub research questions

Research Question : Sub Question 1

How do varying formulations for inequality shift the existing (baseline) trade-offs across objectives?

The first sub question compares the trade-off among objectives, every time in reference to the existing formulation which was used as the baseline for these comparisons. Using the Parallel Axes Plot, trade-offs across solutions are compared. Since each MOO found a significant amount of Pareto-optimal solutions, it became impossible to compare several solutions over several formulations. Therefore, it was decided to use the solutions from the formulations with the equality objective that yield the lowest, highest, and median values for equality. These solutions were compared to the lowest, highest, and median value of the hydropower objective, since for the chosen case-study, this objective has the highest relevance for the efficiency principle. Hence, by comparing equality with efficiency, it became clear how varying formulations shift the trade-offs between the two.

Research Question : Sub Question 2

What is the role of the inequality metrics on shifting the solution space of the Pareto front?

In the second sub question, the distribution of objective values was studied for each formulation. This led to an understanding of how varying formulations yield varying subsets in the objective space. The objective space is the multi-dimensional space of *n x 1*, with *n* objectives, where objectives are able to obtain any value between its pre-determined ranges. The solution space constraints this multi-dimensional space to objective values yielding Pareto-optimal solutions. Hence, rather than observing trade-offs, the distribution indicated how varying formulations constrain the objective space distinctively. In terms of implications, each formulation will tell a different story of how inequality can be minimized within the pre-determined ranges of objectives.

This means that the solution space was constrained differently since the objective formulation decided which subset of the objective space was feasible as solution space. Moreover, Reddel (2022) found that using different ethical premises as principles for the objective formulation of Integrated Assessment Models significantly shifted the Pareto front across changing principles. In changing such principles, the formulation for justice was similarly to this thesis, adapted. Because of this, I expected that the distribution of solutions differ across inequality formulations used for the equality justice objective. Determining the shifting effect from the inequality metric is novel and relevant since similar studies indicate its importance in finding the final Pareto front.

Research Question : Sub Question 3

What is the role of the aggregation method over time on shifting the solution space of the Pareto front?

The third sub question looked at the aggregation method since previous studies found that the model implications were most dependent on how results are aggregated (McPhail et al., 2018). Moreover, when discussing alternatives to aggregate inequality it is found that minimizing inequality over time steps does not guarantee inequality is minimized when aggregated over the entire time horizon. Hence, the level of distributive justice achieved, aggregated and disaggregated, is sensitive to the time dimension (Jafino et al., 2021). Thus, untangling its effects yielded an understanding of the sensitivity in the formulation for cases of water allocation problems that require a specific goal of distributive justice over time.

1.4. Thesis organization

The next Chapters will structure themselves to answer the research question. Figure 1.1 shows how this will take place from the *Define* phase, to the *Structuring* phase, and finally the *Analyze* phase. Chapter 2 delves into the literature to understand the context of equity in decision-support models. At the end of this Chapter, the knowledge gaps are synthesized into precise concepts to be part of the studied core of the rival framings approach. Chapter 3 explains the approach (using Figure 3.5) to study this gap. Chapter 4 makes clear how the experiments are set up. Chapter 5 will jump into the analysis of the results using analysis for each objective and each formulation. Chapter 6 reflects on key findings and key limitations. 7 answers the research questions and draws the main implications for science and society. Moreover, in Appendix B an elaboration can be found on the formulations of the existing objectives from the used case-study, Appendix C includes supplementary figures and analyses that contribute to the main research question.

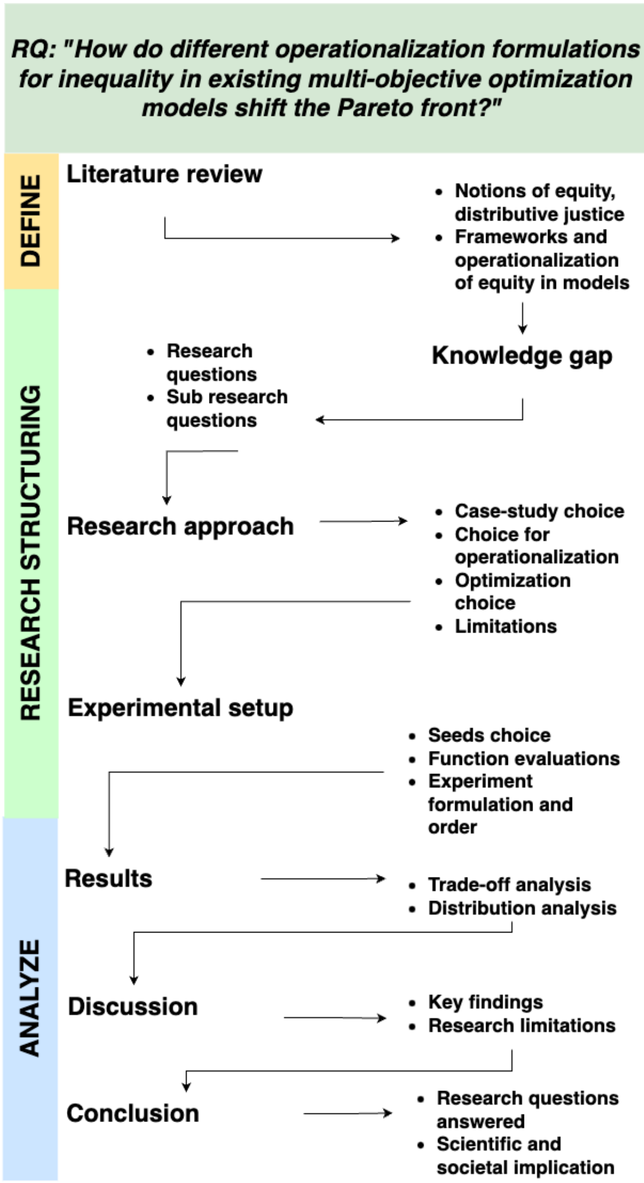

Figure 1.1: Structure of the thesis structured around the answering of the main research question.

2
Literature review

To contribute to the academic field, one must understand the advancements in research of equity incorporated into water models. It is essential to understand the entire spectrum of research. Therefore, the literature review has a broader scope than the context of this thesis. For obvious reasons, the purpose of this Literature review goes beyond water resources since many other common pool resources nowadays face similar complex multi-objective injustices.

2.1. An introduction to equity

In the following, the main themes to understand how equity was operationalized and evaluated for decision-support models were identified. In this thesis distributive justice is discussed (instead of equity), a smaller notion of equity. Since the literature on equity in decision-support models often does not draw a distinction between equity and distributive justice, it is important to highlight this before continuing.

2.1.1. Inherent trade-off of water distribution

Lévite and Sally (2002) underscored three principles that serve as constraints to the management of water distribution: efficient (beneficial) use, equity, and sustainability. Nonetheless, equity can have different notions, which is a reason why it is difficult to define equity for the distribution of water. In concrete terms, equity solely refers to the notion applied to define a fair exchange of resources when several aspects of the actor's background are taken into account such as historical disadvantages. It may consider factors such as socioeconomic status, abilities, or other relevant characteristics. Because of the diversity in factors possible to consider, at the least, equity is ambiguous (Jafino et al., 2021; Syme et al., 1999; Wang et al., 2015; Wegerich, 2007; Wolf, 1999; Young, 1994).

On the other hand, there is efficiency which is seen as the fairness-based implementation of utilitarianism [1] (M. D. Adler, 2019). Stemming from the utilitarian perspective, improving performance is equivalent to attempting to produce higher benefits for everyone, and hence treating everyone equally. Nonetheless, it does so by aggregating the benefits of actors, not caring how it is distributed among them. This results in some actors gaining high benefits at the expense of low benefits for other actors, i.e. yielding an uneven distribution of resources. This is why equity needs to be viewed as a separate concept that compensates for inequalities caused by this aggregation.

In response to this, there has been a growing amount of literature considering equity from a perspective setting emphasis on *the equal concern for each actor's benefits or costs* (M. D. Adler, 2019). Shortly, it encompasses theories and principles about what constitutes a just distribution. Hence, the goal to satisfy the demand of actors must now be considered through the *relative* distribution (of water) (Driver, 2009), and therefore in a disaggregated manner to ensure each actor receives equal rights (to water under fluctuating conditions). This branch of studies falls under the term of *distributive justice principles*. Hence, the equity of actions is solely defined by the distribution, and not the actor's background (to be compensated). Therefore, distributive justice is a sub notion of equity. Furthermore, equity is in water allocation problems defined as distributive justice which is further defined through the logic of an ethical

[1] Utilitarianism is an ethical principle where the goal is to maximize the aggregated benefits for all stakeholders. It does not look at the *relative* distribution.

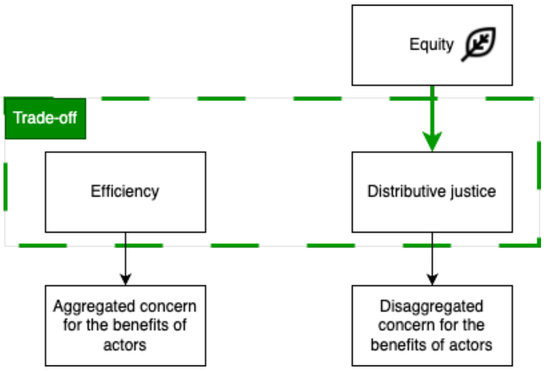

Figure 2.1: Inherent trade-off between the two main notions of equity: efficiency and distributive justice.

principle. In this literature review equity refers to distributive justice, instead of equity, in all its references. Such paradigm is shown in Figure 2.1. Also shown is the trade-off with the efficiency of a water system.

Such a distinction between efficiency and equity (i.e. distributive justice) leads to a trade-off in how water is distributed. Trade-off analysis allows decision-makers to systematically examine the impacts of different policy options by identifying and assessing the potential trade-offs between different objectives or outcomes to inform decision-making. As mentioned previously, policymakers face a difficult trade-off between equity, efficiency, and sustainability. Even when leaving out future (intergenerational) effects, the trade-off remains in the present situation for the distribution of water, between equity and efficiency. From the state-of-art analysis, there is the identification of three themes to further study this for models, as shown below. Subsequently, in Section 2.3.5, with the insights gained from analyzing these themes, the information is synthesized to determine what is most relevant to compare for the normative modelling choices when operationalizing equity.

- Frameworks for equity in models (Section 2.2): What frameworks and guidelines have previously been designed for the implementation of equity in models?
- Operationalization of equity in models (Section 2.3): How have previous studies in the water management literature, and environmental modelling literature operationalized equity after implementing its formulation in the model?

2.2. Frameworks for equity in models

The goal of implementing distributive justice is to distribute benefits and risks in an equitable manner. As per Jafino et al. (2021) distributive justice could be translated to models by building on the domains of justice in decision-support models using the *XLRM-framework*, a model-based decision-support framework by Lempert et al. (2003). The *XLRM* represents a methodology for evaluating MOO-problems that incorporates exploratory modelling and robust decision-making. Hereby, this framework can be applied to any existing problem making use of MOO-model. The model is encapsuled by this framework and explained in terms of the four domains, *X, L, R, and M*. Hereby, 'X' refers to exogenous uncertainties, 'L' refers to levers, 'R' refers to relationships in the system, and 'M' refers to performance metrics. It is imperative to consider on which domain utility [2] is being measured. Since this literature review is meant to understand the objective formulation for distributive justice, the focus lies on the *M*, performance metrics indicators, or in other words, the criteria for the MOO model.

Distributive justice studies can be further split into explorative studies, assessing how resources *will be* distributed, and normative studies, assessing how resources *should be* distributed. As argued in the Introduction, studies suggest adapting the objective formulation. Hereby, the objective formulation determines how resources (may that be water) are distributed among actors. Therefore, studies assessed are on how resources *should be* distributed, the *normative studies*.

Using the indicator to use the *XLRM-framework* to determine which domain to delve in to consider distributive justice in MOO, and using the indicator to look at *normative studies*, a step is taken further in

[2] Utility is an abstract unit of measurement to determine the benefit or risk experienced.

what subdomains can be explored for equity-based MOO-problems.

2.2.1. Normative studies: Steps of distributive justice

As was mentioned before, normative studies do research on how resource *should be* distributed. In this branch, Jafino et al. (2021) draw out the main aspects for modelling distributive justice. First, are the steps over which distributive justice is looked at, over the *unit* of justice, the unit to measure the distribution, the *scope* of justice, the objectives over which to assess the distribution disaggregated, or the *shape* the desired distribution that is used to determine the (just) performance of the distribution. Second, is over which intertemporal aspect this distributive justice is determined. Third, is the formulation of distributive justice that aims to form a measuring principle for the aforementioned aspects.

Unit and Scope

Unit of distribution In order to assess the distributive justice of water, one is required to have a collective unit of measurement across several objectives. Having a comparable and collective unit of measurement over actors is useful for understanding the allocation of resources. There are three ways of defining a comparable unit of distribution: a) volumetric unit, b) utility, and c) social welfare.

Water is according to scientific literature measured when it is in motion, in volumetric unit per time unit, or when it is being stored, in volumetric unit. In social sciences, there are other ways of measuring water, since stakeholders may opt to measure the personal benefit being gained from water. Proven to be useful for resource management, utility is quantified by the benefit gained or the costs generated to each stakeholder (Fishburn, 1968; Von Winterfelt, 1975). The advantage of utility is its possibility to apply stakeholder preferences, therefore bringing *equal right* of preferences. Therefore no actors will be disadvantaged in the procedure to distribute resources in a just manner (Jafino et al., 2021). In addition to utility, *social welfare* is another established method to measure the benefits gained or lost (Lombard, 2008). Social welfare measures the overall benefits and costs to society. Studies aggregate positive utility (benefits) and negative utility (costs) together and aggregate over stakeholders.

Although utility and social welfare are both methods that improve the trade-off analysis over outcomes for actors and between actors, they require more information on how the stakeholder values the resource. Most case-studies do not have stakeholder-consulted weights for utility available. Moreover, choosing the weights leads to a normative bias introduced in the model, making the implications drawn from the model outcomes less objective (Rădulescu et al., 2020). Also, aggregating benefits and costs cover hidden injustices of water allocation, leading to the problem of hidden consequences that are not show in model outcomes as defined under Arrow's Paradox (Kasprzyk et al., 2016). Consultation with actual stakeholders is required for well-researched weights (Taebi et al., 2020).

Scope of distribution The *scope of distribution* looks to the actors to be included in multi-actor problems for the distribution of resources. While I defend in this thesis that distributive justice is essential to consider in serving the multiple purposes of water, to whom these allocation rights can be attributed is equally important. Syme et al. (1999) showed that water should be managed for the community as a whole. Moreover, the environment has allocation rights, justifying the need for environmental flows. Additionally, self-interest and economic arguments should not contribute to the determination of whether to continue or uphold environmental flows (Syme et al., 1999). Conclusively, both the *environment* and communities require their own objective in MOO-studies.

Shape of distribution The concept of distributive justice in water resource management centers on the shape of the distribution, which directly impacts perceptions of fairness (Jafino et al., 2022; Jafino et al., 2021). Defining this shape involves two interconnected lines of inquiry: equity in resource exchange and the ethical principle serving as the moral foundation for water distribution. These aspects are interdependent, as the notion of exchange is clarified by ethical considerations, and vice versa. Ignoring one without the other is inaccurate, making it vital to explore both when determining fairness in water allocation. The contextual nature of fairness further complicates matters, with some advocating equal water allocation for all and others emphasizing allocation based on essential needs. Therefore, understanding both perspectives is crucial to comprehensively describe the distribution's shape in water resource management.

Notion of exchange Equity [3] is an ambitious concept where an attempt is made to reach a consent among several parties in terms of one definition (Wegerich, 2007). The aim of a notion of equity in this way, sets foot for determining when an action is leading to *inequality* among actor objectives. Firstly, equity is defined as *'horizontal'*. Similar to the egalitarian principle, where the resources should consistently be distributed equally among actors (Syme et al., 1999), regardless of their individual demand. This aligns with the *equal inputs* of equity theory (Savas, 1978). The limitation of 'horizontal equity' is that it disregards the demand of actors. Hence, what may satisfy the needs of one actor, may be insufficient for the other. While equality is reached in terms of allocation, this type of equity will not satisfy justice according to the demand of actors.

Therefore, it forms a contrasting view to the second notion of equity, 'vertical' equity, where demand satisfaction is part of what is deemed as justice. Here equity is defined as proportional to the demand, where resources are distributed based on the actual gain from water allocation. Proportionality implies redistributing until allocation is deemed *'fair'* by the actor (Wegerich, 2007). In this thesis, I argue that using this notion of equity for water management purposes is correct since it includes actor preferences without requiring more information than a baseline reference demand.

Ethical principle The equity of distribution heavily relies on what our view is of the world, our ethical principle. Choosing an ethical principle is the beginning point of choosing a desired distribution (Jafino et al., 2021). Figure 2.2 shows equity is subdivided between efficiency, and distributive justice, and distributive justice is subdivided in shape, scope, and unit.

Equity, as previously mentioned, encompasses the concept of a 'just notion of exchange'. However, the perception of what is considered 'just' in terms of equity varies depending on the ethical principles adopted as a foundational standpoint. Distributive justice principles therefore rely on ethical principles, and in this way represent the same. Ethical principles are the only way in which water management can be more '*effective, efficient, and ethically acceptable*' Rossi (2015) and Sohail and Sue (2006). These principles are the *lens* used to determine the equity of distribution. In water management, the most important ones are *utilitarianism*, *sufficientarianism*, *egalitarianism*, *prioritarianism*, *envy-free*, and the *Rawlsian difference principle* (Ciullo et al., 2020; Jafino et al., 2022; Sarva, 2021). Defining equity from an ethical lens obscures the implications on actors as seen from another lens, raising issues of moral uncertainty in the system assessed (Taebi et al., 2020). Therefore, studies suggest exploring multiple ethical principles simultaneously to avoid introducing normative uncertainty (Jafino et al., 2022; Jafino et al., 2021). To showcase the differences, below is a brief description of how ethical principles drive fairness. Later in this Chapter (Chapter 2.3.5, one principle is chosen as the ethical foundation for the research of distributive justice.

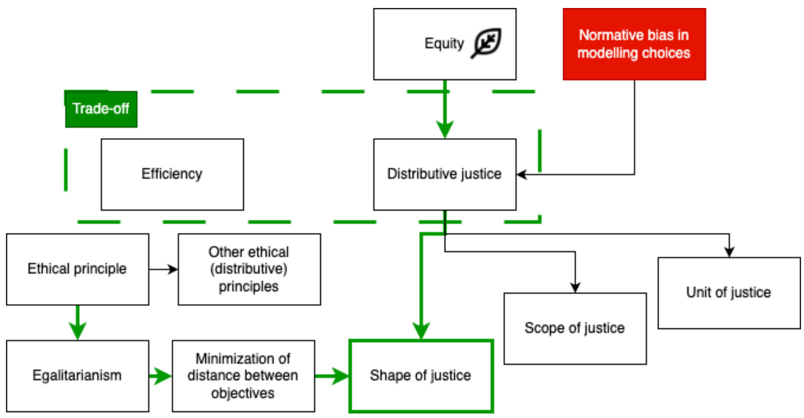

Figure 2.2: Thesis scope on the alternative notions for equity. This tree will grow throughout the thesis. In green, are the concepts considered in the rival framings approach. Green boxes are modelling concepts that are directly studied in this thesis. Green arrows point to modelling concepts that are included in this thesis, but not studied on their implications.

Utilitarianism deems equity as a situation where the aggregated benefits between actor objectives

[3] Studies talk about *equity*, but in this thesis I assume it is the same as *distributive justice*.

are maximized without considering the *relative* distribution between them (Driver, 2009). Hence, its focus is efficiency as it focuses solely on maximizing the overall social benefit, making it a popular method for the standard (yet very limited) CBA method (Ciullo et al., 2020). **Sufficientarianism** deems equity as a situation where it is satisfied only when a stakeholder gains a minimum threshold value, or *sufficient* amount of benefit (Doorn, 2019). Studies introduce this in models in different ways. It can be introduced as a maximization objective of the difference between the minimum and the objective value (Jafino et al., 2022), another conceptualization can focus on minimizing the positive difference distance between the objective value and the minimum threshold, i.e. ensuring *sufficiently* of the resource is available for the subsequent time step or time horizon. How one comes to this minimum threshold remains non-trivial. **Envy-free** deems equity as the situation where each (actor) objective does not create '*envy*' between stakeholders, i.e. the policy solution provides stability. This is however a principle whose conceptualization in optimization is computationally expensive due to the recursive comparison of objectives. Moreover, this can only be done if the conversion factor to utility is unequal to one. Lastly, this principle is suitable for the post-analysis of optimization models (Sarva, 2021). **Prioritarianism** deems equity as the situation where the level of benefit of the marginalized surpasses a certain threshold which is deemed as unfair (Ciullo et al., 2020). In response to this, studies suggest prioritarianism to be introduced in models where the worse-off actor is leveraged such that when inequalities rise it is only because these worse-off actors are compensated (M. Adler et al., 2017). **Rawlsian difference principle**, similar to prioritarianism, the Rawlsian difference principle deems equity as finding policy inputs that only bring benefits to the worse-off actors (Jafino et al., 2022). **Egalitarianism** deems equity as a situation where the available benefits are *equally* distributed among (actor) objectives. Studies will often model this as the minimization of *relative* difference between objectives. Because of this it can be seen the foundation of more elaborate ethical principles, as described below.

2.2.2. Normative studies: Intertemporal aspect

Several studies have analyzed how one should implement distributive justice principles into decision-support models. Jafino et al. (2021) built a framework for the implementation of distributive justice. For this, first, the disaggregation of the intertemporal aspect is required. A distinction is drawn between the *intra-generational justice dimension* and the *intergenerational justice dimension* because decision-making has a longstanding effect on the availability of resources and hence affects both current and future generations. Xu et al. (2019) attempts to integrate both intertemporal aspects, along with sustainability as an objective principle [4], where intra-generational equity is measured through the *Gini-coefficient* (Equation 2.1) of water allocation. Interestingly, Xu et al. (2019) finds that inequality increases when intergenerational justice aspects are not considered, while the efficiency remains the same. Moreover, Xu et al. (2019) finds that if distributive justice is not included in the objective formulation, water resources will drop over time. This results in an inefficient (detrimental) effect for all actors involved. Thus, including distributive justice, is essential for efficient and sustainable water use.

Fletcher et al. (2022) proposes several other requirements to determine how resources *should be* distributed. More importantly, system modelling-specific requirements are to explore multiple metrics for the operationalization of equity as decision-makers improve the weighting on trade-offs. They continue by giving an illustrative example of how the operationalization of inequality leads to different conclusions from MOO-models. In this example, they showcase a) optimizing the mean of inequality across the time horizon, or b) optimizing the standard deviation of inequality between time steps. Nonetheless, regardless of the intertemporal aspect, and how it is quantified MOO-justice studies agree on the trade-off between efficiency and equity (Ciullo et al., 2020; Dai et al., 2018; Hu, Chen, et al., 2016).

Fletcher et al. (2022) continues by explaining how to improve the trade-off between distributive justice and efficiency. They propose contrasting the operationalization formulation for justice, i.e. a rival framings approach. While Xu et al. (2019) tried to solve several issues at once and drew some useful implications on how to design equity-MOO models, there are undiscussed parts of the parametric uncertainty (e.g. the NPV is often arbitrary and used to skew the implications drawn from a model), and structural uncertainty on how the optimization problem is framed (e.g. ecology should also be included in the trade-offs, or that sustainability already partially satisfies intergenerational justice). To avoid the use of models that are not fit for its purpose, jumping in on the rival framings approach of the performance metrics is key in determining

[4] Lévite and Sally (2002) states the triangle of principles for water allocation: efficiency, equity, and sustainability. The triangle creates a trade-off and bounds the water system in what goal is important.

where the normative drives the outcomes. This is discussed in the next Section.

2.3. Operationalization of equity in models (explorative studies)

Explorative studies look at how resources *will be* distributed. Therefore operationalizing distributive justice is an important component of these studies. Fletcher et al. (2022) advocates understanding the effects performance (inequality) metrics have on the policy implications. In addition to this, Jafino et al. (2021) discusses the requirement of having value-based disaggregated metrics. The following Section therefore synthesize how state-of-art literature is operationalizing distributive justice in decision-support models. This synthesis is split into the four most important components to formulate distributive justice in a mathematical formulation. Emphasis is set on the last two since this will be part of what is researched in this thesis.

1. First lies in having a model that is able to capture trade-offs, either across objectives or across principles.
2. Second is the solution concept, used for finding optimal solutions in MOO models.
3. Third is the inequality metric that parametrizes how I define differences in distributions among actor objectives.
4. Fourth how to aggregate over time this objective. The latter is important to synthesize as its specific formulation is often left out of the explanation and studies, and even more its implications for distributive justice studies.

2.3.1. Direct Policy Search models to capture trade-offs

The latest research improves on the limitations of previous approaches (of CBA, and water simulation models (Single-Objective Optimization)) by making use of simulation-based MOO-problem formulation of release (water allocation) policies (Giuliani, Castelletti, et al., 2016; Kasprzyk et al., 2016; Mason et al., 2018; Quinn, Reed, & Keller, 2017; Rădulescu et al., 2020; Sari, 2022; Wild et al., 2019; Zatarain-Salazar et al., 2016). MOO sheds light on trade-offs between multiple objectives and finds optimal solutions, balancing conflicting objectives, which would lead to the desired improved efficiency and sustainability of water management (Giuliani, Anghileri, et al., 2016). In the field of MOO, there is the Evolutionary Multi-Objective Direct Policy Search (EMODPS) approach that identifies optimal water management policies through the use of evolutionary optimization, and whilst considering system constraints (Giuliani, Castelletti, et al., 2016). Moreover, it is a flexible method of conceptualizing the problem, providing a satisfactory model as a foundation for the rival framings approach. EMODPS consists of two components, the Direct Policy Search (DPS) method, and the Multi-Objective Evolutionary Algorithms (MOEAs).

DPS is a closed-loop control method [5], parameterize the operating policies using a context-specific family of mathematical functions to find policy combinations that optimize the set of objectives (Quinn, Reed, & Keller, 2017). The non-linear approximations, either Artificial Neural Networks (ANNs) or Radial Basis Functions (RBFs), map the system states to time-varying decisions. In water management, this enables finding optimal decisions for every system state, and therefore finding non-linear relationships among release decisions. Subsequently, the operating policy is a set of release decisions. Finally, to conclude on the optimal use, RBFs are proven to be suitable non-linear approximators for complex water management problems (Giuliani, Castelletti, et al., 2016).

Additionally, DPS can be combined with evolutionary optimization (using MOEAs) to directly optimize multiple objectives, and finally approximate the Pareto set of solutions for MOO-problems (Quinn, Reed, & Keller, 2017; Zatarain-Salazar et al., 2016). Thus, DPS ensures the parametrization of release decisions into system states. Evidently, this reduces the curse of dimensionality (Quinn, Reed, & Keller, 2017). To this, the evolutionary search algorithms will approximate Pareto-optimal solutions for the multiple objectives. From this, reliable representations of trade-offs can be found. Hence, its solutions are considered mathematically stable [6] across objectives.

While the EMODPS captures trade-offs, it is often along the lines of the same utilitarian goal, which seeks to evaluate and compare optimal policies by solely looking at the best-aggregated consequences,

[5] The adaptive problem formulation using closed loops between the policies and the system states are proven to deal with complex and uncertain water demands (Soncini-Sessa et al., 2007).

[6] according to the Pareto solution concept

i.e. maximizing the overall benefits and minimizing the overall costs of model outcomes. Again, such formulations, oversimplify the problem (Ciullo et al., 2020), hiding policy outcomes that are otherwise not seen by the model abstracting the MOO-problem (Kasprzyk et al., 2016), and lead to poor system performances (Ciullo et al., 2020). Currently, studies on the EMODPS approach have not unified a methodology to include distributive justice in the optimization formulation of a MOO-model. Therefore, I argue that EMODPS requires the exploration of the sensitivity of Pareto-optimal solutions and the approximation of the Pareto front, from alternative methodologies to implement and operationalize distributive justice.

2.3.2. Solution concept

The problem with water ethics lies in the fact that they can be difficult to implement in models. Water ethics are similar to the UN's definition of water rights, of qualitative quality for the analysis. In response to this limitation, solution concepts are a useful tool for implementing distributive justice in decision-support models because they provide a framework for evaluating and comparing different solutions based on how well they satisfy different fairness criteria. Decision-support models with the aim of optimizing solutions are designed to identify optimal solutions that balance multiple competing objectives. There may be several different feasible solutions that are all optimal in terms of satisfying the objectives, but they may not be equally just or equitable for all actors. Solution concepts provide a framework for implementing distributive justice. While there are many solution concepts available such as the egalitarian, utilitarian, Rawlsian or Nash bargaining solution, the most widely adopted solution concept for optimization models is the Pareto dominance (Giuliani et al., 2014; Quinn, Reed, & Keller, 2017; Zatarain-Salazar et al., 2016).

In using the Pareto dominance for distributive justice, there are two possibilities to evaluate how well solutions perform. First, one can adapt the objective formulation. This serves as a pre-defined criterion for the Pareto-search for solutions. Several studies have used such pre-defined formulations (Ciullo et al., 2020; Hu, Wei, Yao, Li, et al., 2016; Xu et al., 2019). Second, the Pareto-search can run as-is, and only after evaluated based on pre-defined criterion for fairness (Jafino et al., 2022; Sarva, 2021). The latter is suitable to compare a set solutions and rank them according to each pre-defined criterion, which may be in the form of the operationalization of an ethical principle.

Pareto-optimal solutions aim for aggregated system-wide efficiency where no actors are disadvantaged by the gain acquired from other actors, and therefore inherently one is considering a utilitarian approach which obscures marginalized objectives or stakeholders (Rădulescu et al., 2020). In this context, **how can distributive justice be reached if marginalized stakeholders are underrepresented in the Pareto front?**. Nonetheless, Pareto is among the few solution concepts that can easily be implemented in MOO models. Hence, which is why the pre-defined objective formulation for the Pareto solution concept was used in this thesis.

2.3.3. Inequality metric

Examples of the inequality metric are the *Gini-coefficient*, *Theil-index*, *Atkinson-index*, *Palma ratio*, and many more. Each metric conceptualizes inequality differently, but most of these have data-specific requirements to be able to implement. Some of these metrics (e.g., Theil-index, Atkinson-index) describe inequality not across objectives, but within objectives, i.e. its spatial distribution. The Theil-index would yield the dispersion rate of inequality, or the Atkinson index would need data on how actors prefer redistribution of resources. Hence, the most widely adopted metric in MOO-studies, is the one with least requirements to implement and most direct to describe relative differences over objectives. Therefore, the *Gini-coefficient* is used in many optimization problems for water allocation (Ciullo et al., 2020; Deng et al., 2022; Farhadi et al., 2016; Y. Guo et al., 2020; Hu, Wei, Yao, Li, et al., 2016; Lopes et al., 2015; Nishi et al., 2015; Xu et al., 2019; Yang et al., 2023). The *Gini-coefficient*, a distance-based measure, is the popular inequality measure (Gini, 1921). Its validity is based on for examples studies such as Cullis and Van Koppen (2007), who confirm that the *Gini-coefficient* is a valid approach to quantify and measure inequality in water allocation.

To integrate inequalities, as already mentioned, studies use the *Gini-coefficient* as shown in Equation 2.1. Here, x_i is objective i, x_j is objective j, n is the number of objectives, and $\bar{x}$ is the average across all objectives. Also, in each study, the *Gini-coefficient* has the *x* (in Equation 2.1) defined differently. In some studies *x* will be the ratio of net economic benefit and water allocation, the water allocation, the ratio between water demand and allocation, or in studies it is defined as the risk. This is understandable seeing that each context is different.

$$Gini = \frac{\sum_{i=1}^{n}\sum_{j\neq i}^{n}(x_i - x_j)}{2n^2\bar{x}} \tag{2.1}$$

Another branch of studies examined how to consider equity in MOO models. This can be either through a redefinition of the original objectives to represent a distributive or ethical principle such as equity through the maximization for the economic benefit of the water management (Hu, Chen, et al., 2016). Or by studying the trade-off between efficiency and equity in water allocation. The latter may come in different ways. Dai et al. (2018) studied this for water allocation, S. Guo et al. (2022) studied this for agricultural purposes on food-water-energy nexus, and contrastingly Xu et al. (2019) added an intergenerational component, by also examining the trade-off between intra-generational and intergenerational equity optimization. Finally, Ciullo et al. (2020) studied the trade-off by changing the ethical principle to which distributive justice is viewed. In doing so, the implications of four ethical principles for flood risk management are assessed, which is similar to the recent work of Yang et al. (2023) where the study focused on the effect of alternative objectives formulation when implementing distributive principles.

Implementation of the inequality metric

Tabs. 2.1, 2.2 form an overview of the most relevant papers addressing the operationalization of distributive justice in the different forms of multi-objective optimization models. This provides us with the foundation on how to operationalize ethical principles, and foremostly, distributive justice which is more ambiguous. Most papers analyze the implications of having different ethical principles as the beginning point. Others will study the trade-off between equity and efficiency.

As Yang et al. (2023) mentioned, and is also clear from the Tables 2.1, 2.2, most of the objective formulations set to focus on Pareto-optimal solutions. Therefore, studies will try to counteract the limitation of the Pareto regarding the underrepresentation of marginalized actors by changing the problem formulation, which is used to give a direction for the optimization that finds the (Pareto-) optimal solutions. In doing so, the Pareto concept has no other option than to consider inequalities between objective values.

1. Distributive justice is implemented in the MOO-problem formulation.
2. Distributive justice is analyzed by using the Pareto-optimal solutions as input for cooperative game theory (Sarva, 2021).
3. Distributive justice principle values are inputted to social welfare functions (Jafino et al., 2021).

The implementation of distributive justice in MOO-problem formulations gains definition from the use of an ethical principle. Table 2.3 shows the advantages and disadvantages of using a specified ethical principle as the notion to define a just exchange of resources. Utilitarianism is often the most straightforward to consider, but as seen will fail to consider distributional aspects. Other principles will introduce a new level of complexity, namely on how to quantify these concepts. Moreover, because of the trade-off between efficiency and distributive justice, other principles will lead to sub-optimal Pareto-outcomes. Within these principles, besides utilitarianism, egalitarianism is the most straightforward ethical (distributive justice) principle that will handle inequalities among actor (objectives). Regardless of the ethical principle, there is still a missing problem with the implementation: *the operationalization of inequality*.

Examples of operationalization of distributive justice

There are several ways in which the operationalization of distributive justice (and thus inequality) was studied. As Tables 2.1, 2.2 show Yang et al. (2023) tackled operationalizing distributive justice by performing sensitivity analysis on the distributive justice formulation parameters. They changed the existing objective formulation for the MOO-model while adding the distributive justice principle formulation with alternative weights to penalize relative differences among objectives. Here, inequity is defined as the ratio between the standard deviation between objectives and the mean of the aggregated objectives. During this process, Yang et al. (2023) found firstly that this objective formulation ensures more distributive justice between the objectives, i.e. the ratio is smaller. Secondly, it shows that water levels improve when including distributive justice relative to multi-objective optimization formulation that has an objective aimed solely at the individual maximization of objectives. Moreover, by applying value functions (linear and non-linear) to mitigate the bias of aggregating values (Kasprzyk et al., 2016), it is found that the trade-offs for objectives on the Pareto front change.

Table 2.1: Part 1 (continued): Overview of papers that have operationalized equity in Multi-Objective Optimization-models.

Paper	Optimization method	Unit of resource	Ethical principles
Hu, Chen, et al. (2016)	Compromise Programming (CP)	Water (allocation and mean economic benefit)	Aggregated utilitarian, Egalitarian
Deng et al. (2022)	MOEA	Water (consumption and GDP)	Aggregated utilitarian, Egalitarian
Ciullo et al. (2020)	MOEA	Currency (Expected annual damage costs)	Cost-benefit analysis, Constrained Cost-Benefit Analysis, Egalitarian, Prioritarian
Yang et al. (2023)	EMODPS	Water (volumetric reliability)	Disaggregated utilitarian, Absolute-value egalitarian
Tjallingii (2021)	IAM and DMDU	Social welfare (aggregated benefits and costs)	Aggregated utilitarian, Prioritarian, Sufficientarian, Egalitarian
Farhadi et al. (2016)	MODFLOW using MOEA	Water (deficit, allocation)	Aggregated - utilitarian, Egalitarian
Kazemi et al. (2022)	MOO (Undefined)	Currency, water (allocation), and utility	Aggregated utilitarian
S. Guo et al. (2022)	MOO (ELI and FMO)	Water, emissions, currency	Aggregated utilitarian, Egalitarian
Reddel (2022)	IAM and DMDU	Social welfare (aggregated benefits and costs)	(Dis)aggregated Utilitarian, Prioritarian Sufficientarian, Egalitarian
Xu et al. (2019)	Mass-balanced MOO	Water (allocation and mean economic benefit)	Aggregated utilitarian, Egalitarian
Sarva (2021)	EMODPS and Game-theory	Utility (based on water allocation)	Aggregated utilitarian, Prioritarian, Sufficientarian, EgalitarianEnvy-free
Jafino et al. (2022)	IAM with robustness analysis	Utility	Utilitarian, Strict-egalitarian, Rawlsian difference principle, Prioritarian, Sufficientarian, Envy-free, Composite principles (Utilitarian & Strict-egalitarian)

On another example, Ciullo et al. (2020) claims that distributive justice should have the aim of minimizing the relative distance between actors. This is supported by Fletcher et al. (2022) who argues for the trade-off between efficiency and *equality* Here, equality refers to the relative difference among the existing (baseline) objective values. Therefore, distributive justice is reached through the reduction of absolute distances between gains. To this, they approach this minimization from three distinctive ethical principles, each giving a different weight to relative differences across actor objectives. Prioritarianism, relative to egalitarianism, resulted in a more fair distribution of risk while minimizing the aggregated costs, hence addressing inequalities better (Ciullo et al., 2020). Moreover, not considering these ethical principles resulted in some marginalized areas experiencing a significant increase in risks (negative benefits) while the overall benefits were maximized. Reddel (2022) built on ethical principles by investigating aggregation/disaggregation effects of utility, and finding that while disaggregation of egalitarianism and prioritarianism leads to a more uniform distribution of welfare loss, welfare gain follows an inverse proportional trend, making a trade-off to be made when having to choose the problem formulation.

Limitations of the inequality metrics

The limitation in the *Gini-coefficient* lies in how *equality* is defined using the ratio between deviation and mean of Equation 2.1. The relative variability can increase or decrease faster than the mean, leading to disparities in this inequality metric (Fletcher et al., 2022). As a result, situations may arise where inequality is gained in one metric while lost in another, highlighting the need to reveal implications from the outcomes. To address this, a disaggregated approach between standard deviation and mean is necessary to uncover these implications. Consequently, the *Gini-coefficient* does not ensure distributive justice is satisfied on its

Table 2.2: Part 2 (continued): Overview of papers that have operationalized equity in Multi-Objective Optimization-models.

Paired multiple ethical principle objectives?	Solution concept	Inclusion of inequality in objective formulation?	Evaluation method for distribution?
Yes	Pareto	Yes, relative Gini-coefficient, Ratio water allocation and economic benefit	Data analysis
Yes	Pareto	Yes, 3 relative Gini-coefficients, Ratio water allocation and population, Ratio water allocation and water availability, Ratio water allocation and GDP	Relative Gini-coefficient and data analysis
No	Pareto	Yes, relative Gini-coefficient	Relative Gini-coefficient
Yes	Pareto	Yes, aggregated mean deviation with value function	Trade-off and data analysis
No	Pareto	Yes, relative Gini-coefficient	Relative Gini-coefficient
Yes	Nash-bargaining	Yes, volumetric reliability-based Gini-coefficient	Data analysis
Yes	Pareto	Yes, relative Gini-coefficient, Ratio water allocation and economic benefit	Data analysis
Yes	Undefined	Yes, relative Gini-coefficient, Ratio water allocation and land resources	Data analysis
No	Pareto	Yes, relative Gini-coefficient	Relative median-based outcome score
Yes	Undefined	Yes, relative Gini-coefficient, Ratio water allocation and economic benefit	Data analysis
No	Pareto	Yes, relative Gini-coefficient	Relative Gini-coefficient
Yes	Pareto	Yes, defined per moral principle	Ranking of suggested policies

own. To achieve distributive justice in terms of relative equality across all objectives, multiple formulations are needed in distance-based measures like the *Gini-coefficient* (Fletcher et al., 2022).

As the *Gini-coefficient* was chosen as a reference for the limitations of widely adopted inequality metrics, the last limitation discussed of the *Gini-coefficient* lies in its formulation, which treats all actor objectives equally. Yet, it overlooks the sensitivity to variations in benefits demanded across objectives. This means that if one actor objective requires significantly more benefits than others, the *Gini-coefficient* may yield a high inequality value, even though the actors themselves are unequal. To address this, the distribution assessment should consider the actor background through *actor-based disaggregation* (Jafino et al., 2021).

Moreover, most studies treat the *Gini-coefficient* as an additional objective, represented as the sum of pairwise distances. As a result, it obscures larger distribution differences between two or more objectives, hindering a comprehensive assessment of inequality. Ciullo et al. (2020) resolved this by introducing several extra objectives for distributive justice, one for each actor objective. These new objectives measure the relative distance between each objective and the one receiving the highest allocation difference. By doing so, they ensure fairness between actors with the greatest disparities.

2.3.4. Aggregation method over time for the inequality metric

When decision-support models are optimized, a formulation is needed to determine how one deals with the intermediate results of the model between time steps. This is because of the '*curse of computational expenses*' (Giuliani, Castelletti, et al., 2016). Moreover, it would not be logical to optimize between time steps if the aggregate objective results are unsatisfactory. For example, when looking at the water allocation over a year, optimizing the water allocation for a day can negatively influence the optimal allocation over a year. This is why, each study aggregated its objectives over time using its own indicator such as the mean, 99^{th} percentile, standard deviation, etc. Hu, Wei, Yao, Li, et al. (2016) minimized the covariance of the matrix of objectives matrix throughout the time horizon, Zatarain-Salazar et al. (2016) optimized the daily-based mean of each objective singularly over the time horizon. Interestingly, it is recurrent that MOO studies do not specify how the objectives are aggregated.

The aggregation method over time influences the outcomes for MOO whenever the performance needs to be measured for a social aspect, i.e. not only distributive justice. McPhail et al. (2018) acknowledges this and built a framework for robustness metrics of deep uncertainty problems. Here they conduct a rigorous sensitivity analysis on the performance metrics transformation (for us this would be the inequality metric), and the performance metric calculation (for us this would be the aggregation method over time) for the robustness. While testing an extensive combination of the metric, its calculation, and the subset scenario choice, they find that the performance calculation has the largest effect on the robustness of policies.

Table 2.3: Advantages and disadvantages of using varying popular (in MOO) ethical principles as the notion for distributive justice. Utilitarianism will be oppositional to the other ethical principles.

Ethical principle	Notion of equity	Advantages	Disadvantages
Utilitarianism	Fairness over the aggregation of outcome	Pareto-optimal outcomes	Neglects inequalities (distribution) in the outcomes
Egalitarianism	Fairness through equal satisfaction of outcomes	Justice favoured over efficiency	Sacrifices Pareto optimality
Sufficientarianism	Minimum benefit must be reached from outcome	Justice favoured over efficiency	Sacrifices Pareto optimality Non-trivial minimum threshold
Envy-free	Actor must not 'prefer' the outcome of another objective	Justice favoured over efficiency	Increases complexity of the model Sacrificed Pareto optimality
Prioritarianism	Benefit of the marginalized is always most important	Justice favoured over efficiency	Complex to define and measure the least advantaged population while also optimizing variations in the least advantaged Sacrificed Pareto optimality
Rawlsian difference principle	Inequalities caused are valid if the marginalized are better off	Justice favoured over efficiency	Complex to define and measure the least advantaged population while also optimizing variations in the least advantaged Sacrificed Pareto optimality

Therefore, they identified that an optimization model is sensitive to the aggregation method (descriptive calculation) being used.

I argue that by acknowledging the influence this has on the generic use of a performance metric, regardless of the social aspect, the aggregation over time is also important for distributive justice. Considering distributive justice problems also deal with the need to aggregate the objectives, a knowledge gap lies in the role of aggregation over time as it is left unspecified. Paraphrasing McPhail et al. (2018) to the distributive justice context, they find that the mean and sum of objectives assign equal weight to every time step.In contrast, rather than the mean or sum, Yang et al. (2023) used the ratio between standard deviation and mean as a balancing method for variability over time steps. Therefore, the differing use from studies needs to be investigated in the same rival framings way Fletcher et al. (2022) argues it for the use of inequality metrics. Together the rival framings then consist of the alternating operationalization for the inequality metrics, and the aggregation method separately.

2.3.5. Synthesis of research approach

This Section summarizes the relevant concepts and knowledge gaps found in the Literature review. So how do we structure what was found in the Literature?

Figure 2.3 shows the different options for the modelling of distributive justice stemming from the Literature Review. At the core lies how the *distributive justice principle* will be operationalized in its *objective formulation*. The core of what was studied is indicated by the green boxes. Green arrows refer to the modelling concepts used in this thesis, but these are not further studied. The knowledge gaps in defining the *shape of justice* for a MOO-problem, the green boxes, as well as the suggested methods in literature to deal with these gaps are discussed below.

1. Inequality metric.
2. Aggregation method over time.
3. Objective formulations with combined efficiency and equity objectives.

Inequality metric: For water problems, it is often already possible to express everything in the same volumetric unit without requiring a transformation calculation for inequality. In response to metrics where assumptions are made and gains are aggregated, distributive justice is implemented using mathematical functions that calculate the relative distance between the unweighted objective values, i.e. the inequality among objectives. It remains heavily understudied if the way this inequality is formulated correctly quantifies relative differences. Therefore, this thesis used the mathematical formulations of inequality metrics as a comparison in the rival framings approach as they provide flexibility in application across different water allocation problems.

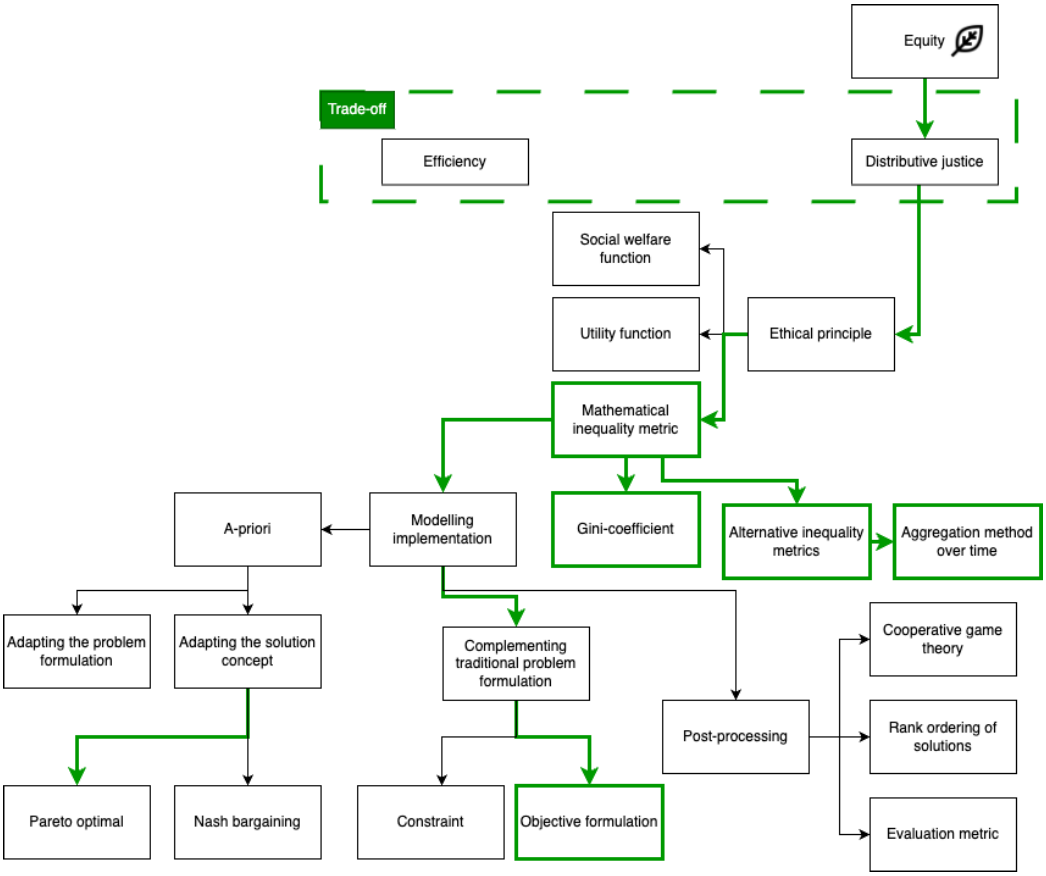

Figure 2.3: Thesis scope on the alternative implementations of equity in decision-support models. This tree will grow throughout the thesis. In green, are the concepts considered in the rival framings approach. Green boxes are modelling concepts that are directly studied in this thesis. Green arrows point to modelling concepts that are included in this thesis, but not studied on their implications.

Furthermore, the *Gini-coefficient* served as a useful starting point for the rival framings approach. While widely adopted, the *Gini-coefficient* falls short when defining inequality since its aggregated pairwise definition, ignores differences between the direction of the deviation, and the direction of the mean, and also the singular pairwise differences between objectives are hidden. As defended in the Literature review, a method of dealing with this hidden uncertainty is by contrasting the *Gini-coefficient* with other distance-based measures that do not include such ratios in the formulation.

Aggregation over time: Furthermore, to the best of my knowledge, the role of aggregation over time has not been thoroughly assessed in the existing literature. In many studies, the choice of aggregation level for the objective formulation is often assumed by the modeller without adequate justification. The objectives are commonly aggregated over the time horizon to manage computational complexity and align with the overall problem formulation, such as maximizing water allocation gains over the entire duration. However, even for deep uncertainty Multi-Objective Optimization (MOO), which shares similarities with distributive justice-MOO in terms of requiring performance metrics, the choice of aggregation method has been found to significantly influence the implications (McPhail et al., 2018). The aggregation method is also an integral part of defining distributive justice operationalization.

To address this gap, Fletcher et al. (2022) suggests examining the influence of modelling choices on distributive justice operationalization, including the subcomponents that have the most substantial impact on the implications. In this thesis, a rival framings approach will be adopted to investigate different formulations for the aggregation method over time. The robustness metric framework of McPhail et al. (2018) will serve as the starting point for exploring these various formulations.

Combined principle for objective formulations: Studies so far have understudied the combination of efficiency and distributive justice principles in the objective formulation for water system models with varying operationalization formulations for the same notion of distributive justice. Instead of forming contrasting objective formulations, as is done for studies where ethical principles are implemented in the formulation and assessed, understanding how the Pareto front of the existing baseline formulations - where objectives are optimized without looking at their relative differences - shifts is useful because efficiency and distributive justice are equally important (Lévite & Sally, 2002). Hence, arguing that both are equally important, both aspects should be represented in the objective formulation. The baseline (traditional) formulation is therefore for each optimization formulation complemented with an alternative formulation for the distributive justice principle. Additionally, using the existing efficiency-focused problem formulation serves as a reference across different rival framings formulations when the distributive justice formulations are changed across the framings, i.e. yielding the *net* effect of the specific formulation assessed.

Part II

Research design

3
Research Approach

In this Chapter an explanation is given on how to answer the research question. By delving in the literature, the most suitable approaches are found for this. Also, limitations of this research approach are discussed, hence providing the research and conclusion suitability, are also discussed.

Firstly, a choice is made for the case-study where optimization experiments were run. Secondly, the chosen rival inequality metrics will be discussed. Thirdly, an explanation is given on the chosen rival aggregation methods. Fourthly, it is discussed how this will be implemented in the model, and assumptions and limitations are specified.

3.1. Focus area: Conowingo Reservoir System

3.1.1. Case-study: Multi-faceted use of the Conowingo

The chosen case-study is the Lower Susquehanna River Basin, - crossing multiple states in the United States of America - a subbasin of one of the oldest and largest rivers in the world. Its spatial overview is shown in Figure 3.1. Built in 1926, originally for serving solely hydropower purposes, the Conowingo Reservoir is a water body used for water allocation management and distribution from the upstream Susquehanna River Basin. The Conowingo Reservoir needs to satisfy the six objectives of Figure 3.2 at all times. Each of these objectives yields a gain to one of the local stakeholders. Additional to the Conowingo Reservoir, the Muddy Run Reservoir (MR) is a pumped-hydropower plant that leverages intra-daily cycles by pumping water from the Conowingo to the Muddy Run when energy prices are low, and during peak energy prices it is released back to maximize hydropower profit (Zatarain-Salazar et al., 2016). The more water passes through the turbines of both hydropower plants, the more power generated, and the higher the revenue.

In Table 3.1 one can see who benefits from objectives. The management of the Conowingo is also

Figure 3.1: Spatial overview of how the Lower Susquehanna River Basin, together with its Conowingo Reservoir, forms part of the larger Susquehanna River Basin.

a complex case due to its demand for truly diverging purposes such as hydropower revenue, atomic power plant cooling, and environmental flow requirements. Additionally, from Figure 3.2 I infer that the demand difference between actors is high, obscuring marginalized actors in the regular Pareto set found. Due to this fact, uncovering distributive justice in such a system is especially relevant for actors who are marginalized in the Pareto-optimal solutions. All in all, the case-study is appropriate to analyze what the effects are from introducing distributive justice in alternative mathematical formulations.

Due to the complexity of satisfying six objectives, there is an existing EMODPS-model for the case-study. Giuliani et al. (2014) designed an EMODPS-model in collaboration with stakeholders to assess trade-offs, and Zatarain-Salazar et al. (2016) tested the performance of alternative MOEAs in terms of generational distance, ϵ-progress, and hypervolume convergence. Sarva (2021) analyzed the fairness and stability of the Pareto-optimal set of solutions obtained from the EMODPS-model through cooperative game theory. Most recently, Zatarain-Salazar et al. (2022) determined the performance to obtain the Pareto optimal control policies from a set of alternative Radial Basis Functions (RBFs). Hence, examining the implications of multiple inequality principles adds to the understanding of the Conowingo Reservoir's optimal release policies. Furthermore, building on previous research, results in a better understanding of the system behaviour, since previous limitations of the model have already been identified.

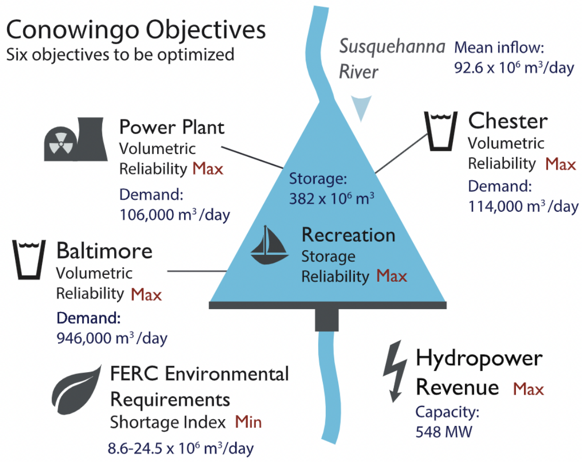

Figure 3.2: Overview of the Conowingo River System. The objectives are measured in volumetric water reliability. For Hydropower the revenue is measured. Adapted from Zatarain-Salazar et al. (2016).

Table 3.1: The stakeholders matched with the objective included in the water system optimization problem.

Actor	Objective
Hydrodam owner	Hydropower revenue
Nuclear reactor owner	Atomic power plant cooling supply-demand reliability
Chester municipality	Domestic supply-demand reliability of Chester
Baltimore municipality	Domestic supply-demand reliability of Baltimore
Visitors of the Lower Susquehanna River Basin	Supply-demand reliability of the Conowingo storage
Federal Energy Regulatory Comission	Supply-demand reliability of the environment

3.1.2. Evolutionary Multi-Objective Direct Policy Search: Conowingo Reservoir Model

So how does this EMODPS-model of the Conowingo Reservoir System work?

As mentioned in Chapter 2, EMODPS is a simulation-based Multi-Objective Optimization method where optimal policies are found through a closed-loop control problem. This method can deal with the modelling of complex water systems, and it is easily adaptable to problem contexts, making it possible to complement existing EMODPS models with distributive justice objectives. Hence, EMODPS is a widely applied MOO method to approximate the Pareto front of a problem (Giuliani, Castelletti, et al., 2016; Quinn, Reed, & Keller, 2017). In this thesis, the EMODPS model designed by Giuliani et al. (2014) and further improved by Zatarain-Salazar et al. (2016) was used. The model parametrizes the operating policies [1] and will make use of Radial Basis Functions (RBFs) to optimize the operating policies. Radial Basis Functions (RBFs) are non-linear approximators responsible for mapping the system state through a vector of decision variables.

The model makes use of MOEA, a meta-heuristic approach responsible for finding the optimal policy sets over the policy search space. Hence, the MOEA is the sampling method for the extensive policy space [2]. Specifically, the ϵ-NSGA-II sampling method was used as it was found that this MOEA is most suitable for water allocation case-studies (Zatarain-Salazar et al., 2016). For more explanation on how this 'black-box' approach works please refer to Giuliani, Castelletti, et al. (2016) and Quinn, Reed, and Keller (2017), and Zatarain-Salazar et al. (2016) for how it is set up for this focus area.

But how are variables interacting the model?

The EMODPS-model uses the *XLRM*-framework [3], where input parameters interact with the model and on every time step t the model outcomes are calculated. This is shown in Figure 3.3. The water flows mainly from the Susquehanna River Basin, as well as some lateral flows from other River Basin into the Lower Susquehanna River Basin until it reaches the Conowingo Reservoir System, the Reservoir responsible for the local water distribution. Policy levers (L) are described as the set of release decisions, for every objective, during each time step. Model outcomes (M) are the objectives of the traditional formulation. The following input specifications are given.

- **Input:** Inflow trajectories, evaporation losses, energy prices of the year 1999, as this represent a year with droughts. (Zatarain-Salazar et al., 2016), and thus limited water resources, making the case for distributive justice under water stress relevant.
- **Time horizon:** The year of 1999. Using the data of 1999, the model will simulate the optimal water allocation for that year.
- **Time step:** A daily time step with release decisions every four hours.

The mass-balance equations are used to describe the flow of water through the Lower Susquehanna River Basin, with the various inputs, outputs, and transformations that occur. In its simplest form, the mass-balance equation states that the rate of change of the total amount of water in a system is equal to the difference between the total inflow and outflow rates. In this case-study, Equations 3.1, 3.2 are used to describe the mass-balance equations where t is the time-index in (days), s_i is the volume at the reservoirs (i = Conowingo (CO), or Muddy Run (MR)), q_i^{t+1} are the main and later flows at $[t, t+1]$, r_i^{t+1} are the releases to MR or to the four release options of the XLMR-framework, E_i^{t+1} are the evaporation losses of i at $[t, t+1]$, q_p^{t+1} is the water pumped from CO to MR at $[t, t+1]$, and finally $q_i^{t+1} + q_{i,L}^{t+1}$ are the mainstem measured at the Marietta Gauging Station.

$$s_{CO}^{t+1} = s_{CO}^{t} + q_{CO}^{t+1} + q_{CO,L}^{t+1} - r_{CO}^{t+1} - E_{CO}^{t+1} - q_p^{t+1} + r_{MR}^{t+1} \tag{3.1}$$

$$s_{MR}^{t+1} = s_{MR}^{t} + q_{MR}^{t+1} - r_{MR}^{t+1} - E_{MR}^{t+1} - q_p^{t+1} \tag{3.2}$$

3.1.3. Conowingo Reservoir system objectives

This Subsection explains how the objective formulation was adapted in order to have a baseline (reference) of the model outcomes.

[1] The operating policies are the release decisions. The release decisions are parametrized for this case-study within a given class of functions.

[2] The policy space refers to the set of possible policies that an optimization algorithm can choose from in order to solve a given problem.

[3] The XLRM-framework is a policy analysis approach that focuses on understanding and addressing the complexities of policy issues by examining the levers available for intervention, the relationships between various actors and factors, the models used to represent the system, and the uncertainties associated with the policy problem, aiming to provide more comprehensive and effective policy recommendations.

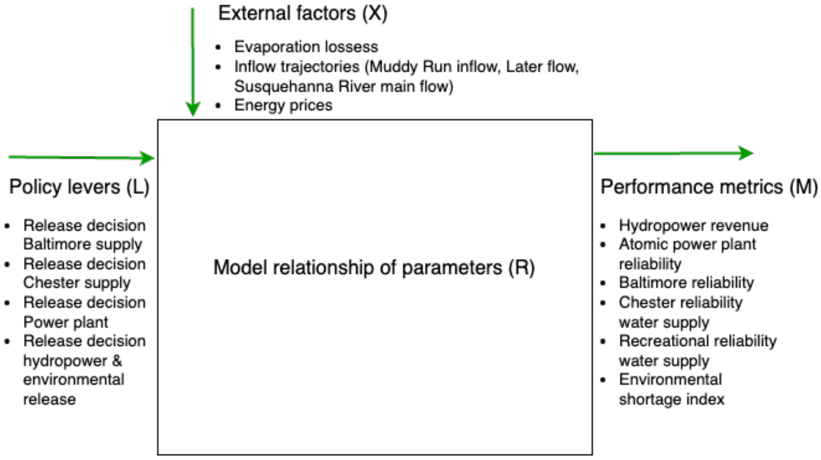

Figure 3.3: Simplified XLRM-framework for the Lower Susquehanna River Basin, the exemplary focus area of this thesis. On the left, the policy levers (L), our decision variables. On the right, the performance metrics (M), the objectives. On the top, the external (X), the data fed into the model for the year 1999. In the middle, the model relationships (R), how the model simulates the interactions as per the mass-balance equations.

Traditional objective formulation

As indicated, the model has six objectives as described below. These are the model outcomes. Equation 3.3 shows the objective function to maximize all objectives, except for the environmental shortage index (seen by the negative sign). The formulation, objective, and description can be found in Zatarain-Salazar et al. (2016) or in the Appendix B.

$$J^{Traditional} = ArgMax(f(J^{Hydro}, J^{Atomic}, J^{Baltimore}, J^{Chester}, -J^{Environmental}, J^{Recreational})) \quad (3.3)$$

3.2. Model implementation of distributive justice

3.2.1. Objective formulation with distributive justice principle complemented

The above objectives could be seen as the disaggregated utilitarian approach, aiming to maximize gains for objectives without considering their relative distribution (Driver, 2009; Sen, 2018). In the case study of the Lower Susquehanna River Basin, the lack of consensus on water distribution has led to issues like insufficient funding for the Conowingo System projects (Hicks et al., 2008) and deteriorating water quality (Ain et al., 2014). To meet objectives amid uncertainty about the distribution, the utilitarian lens is commonly employed as it assigns equal effort to optimizing all objectives, satisfying the efficiency concept (Hu, Chen, et al., 2016). This formulation is referred to as the *traditional formulation*.

However, utilitarianism is criticized for overlooking lower-level factors and prioritizing large-scale infrastructure projects under the CBA method (Ciullo et al., 2020). It may favor stakeholders with relatively high gains while obscuring the disadvantages of marginalized actors since performance is based on overall gains. This *interpersonal aggregation* assumes that it is morally acceptable for some actors not to benefit from the allocation as long as others do (Hansson, 2007).

To complement the traditional formulation with distributive justice, I propose considering the *egalitarian principle* by Ciullo et al. (2020), aiming to minimize relative differences between all objectives (Doorn, 2019). Two options are presented: the *maximum disaggregated approach*, minimizing relative differences between objectives with the largest difference, and the *aggregated approach*, minimizing relative differences between all objectives, regardless of their size. The latter aligns with *vertical egalitarianism*, proportional to demand (Doorn, 2019; Jafino et al., 2022). This approach addresses the limitations of both utilitarianism and egalitarianism, forming the *combined principle formulation* (Jafino et al., 2022).

The new objective formulation with *distributive justice* (referred to as *equity*) added is shown in Equation 3.4.

$$J^{Combined} = ArgMax(f(J^{Hydro}, J^{Atomic}, J^{Baltimore}, J^{Chester}, -J^{Environmental}, J^{Recreational}, -J^{Equality})) \tag{3.4}$$

3.2.2. Inequality metric

Always using the *Gini-coefficient* leaves an important normative uncertainty open on how the metric affects the solution space. This thesis intended to assess the performance of the water model optimization when it undergoes changes in its mathematical formulation, i.e. which inequality metric is used. I argue it is important to observe that distance-based measures are the most straightforward-to-implement method to accurately shed light on inequalities. Due to the limited time availability, the scope is bounded to comparing the aggregated *Gini-coefficient* with at least one other metric. In this research, the other chosen method is the widely multidisciplinary adopted aggregated *Euclidean distance* metric. This metric is popular for any type of problem where the variance in a set of points must be determined (in this case objectives). It is the reason why D'Agostino and Dardanoni (2009) and Dokmanic et al. (2015) argue for its applicability in a variety of problem contexts. Comparing a standardized method for relative distance, to the standardized method for inequalities brings light to the two outermost extremes on how implications are shaped by the mathematical formulation applied. Shown in Equation 3.5, the sum of pairwise differences is considered, similar to Equation 2.1, with the difference that as advocated previously, there is not a fraction between deviation and mean. Instead, deviation is looked at using the *Euclidean distance*. Here again, x_i refers to the value of objective i, and x_j refers to the value of objective j. Additionally, large relative distances between x_i and x_j will gain higher weight in the calculation with the Euclidean from its quadratic formulation.

$$Euclidean = \sqrt{\sum_{i=1}^{n} \sum_{j \neq i}^{n} (x_i - x_j)^2} \tag{3.5}$$

3.2.3. Aggregation method over time

The focus area, the Lower Susquehanna River Basin, has seen previous EMODPS-studies optimizing over yearly mean water objectives without providing a rationale for this level of aggregation (Giuliani et al., 2014; Zatarain-Salazar et al., 2016). To address this lack of justification, this thesis compared alternative aggregation methods when measuring distributive justice to better understand their implications.

The original yearly mean aggregated objective, used in the traditional Pareto front comparison (Zatarain-Salazar et al., 2016), is retained. Additionally, three distinctive formulations are employed to address distributive justice aggregation over time, following the framework of McPhail et al. (2018) and insights from previous MOO-studies. One formulation by Yang et al. (2023) used the ratio between standard deviation and mean of objective values. However, to address the issues of relative variability, the formulation is decomposed as suggested by Fletcher et al. (2022), examining distributive justice from the perspective of *mean*, *standard deviation*, and the *ratio of both* for better comparability with previous findings.

To measure distributive justice, the objective values were first transformed into *performance values* using an inequality metric. Subsequently, the aggregation method's value over time was converted into the final value used in the optimization formulation. Figure 3.4 illustrates this process for each experiment.

Aggregation method 1: Mean

The mean refers to the yearly (daily-based) mean of the *inequality metric* assessed. This formulation is similar to the one for the other objectives of the traditional formulation. Moreover, the mean assigns equal weight to inequalities reached for each day of the year. In Equation 3.6, T is the number of days in a year (T = 365 days), and x_{day} is the inequality value of the specific day.

$$Mean = \frac{\sum_{day=1}^{T} x_{day}}{T} \tag{3.6}$$

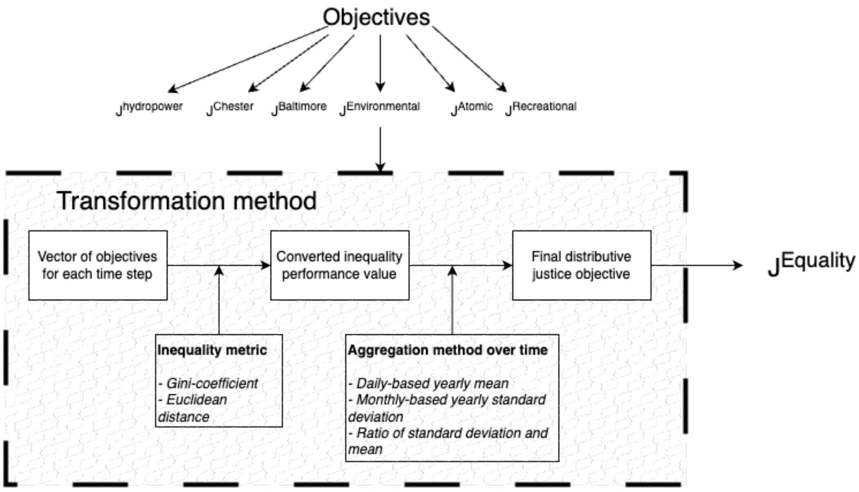

Figure 3.4: Diagramatic overview of the transformation method to arrive at the distributive justice value ($J^{equality}$) which is implemented as seventh objective in the optimization formulation of Equation 3.4.

Aggregation method 2: Standard deviation
The standard deviation refers to the yearly (monthly-based) standard deviation of the *inequality metric* [4]. This method considers intra-monthly differences to ensure the variability in inequality between months is minimized. This method is a simplified, yet useful, manner to consider another type of intertemporal aspect of distributive justice. Moreover, Giuliani et al. (2014) finds that the demand throughout the year between objectives can vary significantly. During certain months the trade-off between objectives can be so high that critical thresholds are met (Sheer & Dehoff, 2009). This formulation deals with such effects over time. In Equation 3.7, n is the number of months in a year (n = 12 months), and x_{month} is the inequality value of the specific $month$, $x_{average}$ is the yearly average of inequality values for the n months.

$$Std = \sqrt{\frac{|(\sum_{month=1}^{n}(x_{month} - \bar{x}_{average})|^2}{n}} \tag{3.7}$$

Aggregation method 3: Ratio between standard deviation and mean
The disaggregated methods are combined into one method as defined by the literature. This is similar to previous studies with a similar scope. (Yang et al., 2023). Moreover, a combination ensures both types of justice formulations are considered.

3.3. Final thesis tree

Now that the approach is demarcated, the modelling concepts are merged with the notions of equity. This is shown in Figure 3.5. In this way, the final research focus for this thesis is reached. Subsequently, it also forms the basis for the Experimental design which will be explained further in Chapter 4.

3.3.1. Assumptions

In order to have a working EMODPS-model that would include the distributive justice objective in the optimization formulation, assumptions needed to be made. In the next Subsection the most important assumptions made in this research are discussed.

Two assumptions were made to enable the distributive justice objective. Firstly, all objectives were expressed in the same unit, namely volumetric reliability. Most objectives were already converted, except for hydropower revenue. To address this, *hydropower reliability* was introduced, representing the energy generated by the Conowingo and Muddy Run dam divided by the desired maximum energy output uniformly distributed for release decisions, every four hours. The desired maximum power output was obtained

[4] It was assumed here that a monthly scale is a satisfactory intertemporally disaggregated time step while ensuring computational capabilities are manageable by the CPU.

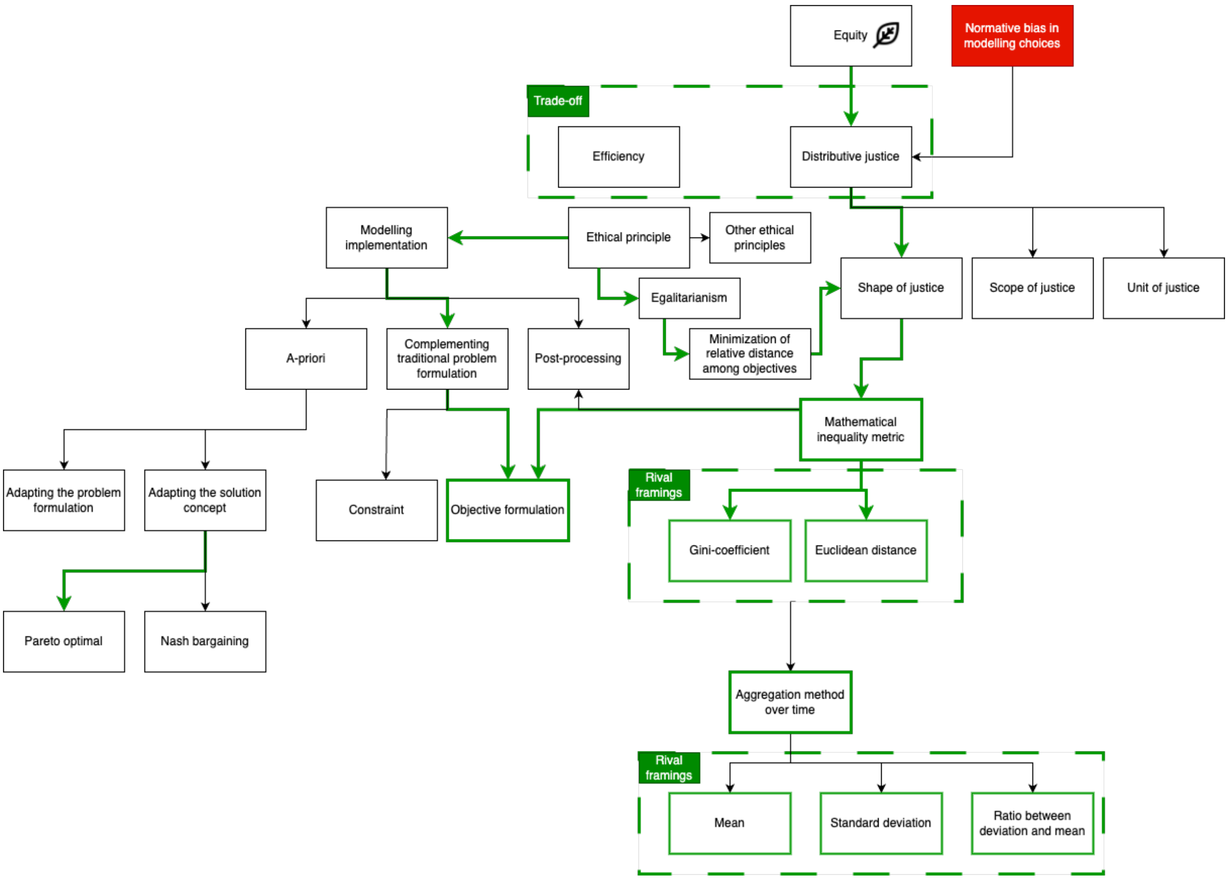

Figure 3.5: Thesis scope on the alternative implementations of equity in decision-support models. Equity is defined as equality. Green boxes are modeling concepts that are directly studied in the rival framings approach. Green arrows point to modeling concepts that require normative choices, but are not studied on their implications.

from Exelon's online data, with the dam's current contribution in 2023 averaging 1,600 MWatt-Hours annually. It was converted to a desired energy output for 1999 using a conversion factor to ensure that a *hydropower reliability* of 1 equals the maximum hydropower revenue achieved in Giuliani et al. (2014)'s previous case-study.

Secondly, the yearly recreational reliability objective was duplicated for every month due to the model's previous formulation. This decision aimed to maintain consistency with the local community's specific recreation season on the Conowingo Reservoir. While this duplication might slightly underestimate inequality between objectives in some months, it was deemed negligible as only one of the six objectives was assumed constant for the monthly inequality calculation.

3.4. Limitations

Foremostly, the egalitarian idea for distributive justice was criticized by the fact that it will not ensure the maximum benefit to be gained for each actor, in this case the reliability for each objective. While this is not the goal of optimizing relative distribution, it does not ensure the reliability of marginalized objectives is considered. Moreover, other studies even suggested that prioritarianism should be the main ethical principle of policy evaluation (Lamont, 2017). However, they are more difficult to implement (Reddel, 2022; Sarva, 2021). M. Adler (2011) proposes the application of a concave function that decreases the value of the objective as the gain increases. Similarly, Yang et al. (2023) applies a concave function to the distributive justice formulation that adds higher weights to marginalized actor objectives. These studies are each aiming at reducing inequality between the outermost extreme values of actor objectives, while this study as mentioned in Subsection 3.2.1 focuses on considering distributive justice for all actor objectives. Hence, these limitations depend on the formulation of distributive justice one is chasing.

The inequality metrics still used the aggregated distributive justice perspective. While it was possible to uncover the pairwise inequality between actor objectives, this would come at the cost of an increase in objectives. Namely for each combination of pairwise comparisons of six objectives, minimally $36 (= 6^2)$ objectives would be added. This would be extremely computationally expensive as Giuliani, Castelletti, et al. (2016) finds. Another solution was evaluating which pairwise comparisons are most relevant to include in the objectives, similar to Ciullo et al. (2020)'s comparison between objectives with the largest difference. This would require a more experimental phase in the research to determine what is most suitable for the focus area, and due to time constraints for this thesis, it was impractical to realize this in a timely manner. Also, due to the computational expensiveness, the aggregation of time was performed in months, which was considered to be an appropriate disaggregated time scale to consider intertemporal variability. Future research should test if there are significant differences if the time scale would be increased in granularity. Other limitations are briefly described below.

- The approximation of the Pareto front found using the MOEA is highly dependent on the initial conditions of the sampling search. This increases the risk of not converging to the actual Pareto front (Laumanns et al., 2002). Zatarain-Salazar et al. (2016) validates that the ϵ-NSGA-II is among the most suitable MOEAs to approximate the Pareto front of this case-study.
- Lastly, the RBFs (neural networks) are a 'black-box' as there is little explainability on how the EMODPS model came to its final solution set. Therefore, the limitation of this thesis, and many other research was the interpretability and explainability of MOO models. To future research, I suggest improving the explainability of such complex models.

4
Experimental setup

The goal was to run the EMODPS model for different formulations where the traditional formulation was complemented with equality formulations in the optimization function. The simulation setup made it possible to determine how to run the experiments and why. Therefore, the simulation setup was determined to remove [1] dependencies from this setup to the model outcomes. Lastly, a validation process known as the *convergence of solution* was chosen to determine if the solutions found were indeed an accurate approximation of the Pareto front. If not, the simulation setup was adapted.

4.1. Simulation setup

4.1.1. Seeds

As mentioned in the previous Chapter the ϵ-NSGA-II algorithm was used to run the EMODPS simulation for the Susquehanna River Basin water system. The algorithm uses stochasticity to sample candidate solutions, and this made the solutions found sensitive to the used seed. The seeds used in the simulation aim to choose random initial points of the system state to begin the simulation. Having random seeds is important to reduce the dependency of solutions found on the chosen seed.

Nonetheless, random seeds used in simulations are pseudo-random generated, which are a deterministic (non-random) sequence. Hence, solutions found depend on the pseudo-random sequence. To reduce this dependency, random seed analysis controls the unprecedented effects of variability to find the consistency among several seeds runs (Reed et al., 2013). Therefore, the more seeds are run for one operationalization, the less dependent the Pareto front is on the pseudo-random behaviour.

This research used five *seeds* using five pseudo-random generators. While previous studies on the Susquehanna case used ten seeds of ten pseudo-random generators, this research was bounded to computational expenses, resulting in the assumption that running five seeds per operationalization formulation is satisfactory.

4.1.2. Function evaluations

The number of function evaluations refers to the number of times the EMODPS model is run. The search process is updated after every function evaluation. Therefore, the number of function evaluations will track the convergence [2] of the optimization, i.e. whether the found solutions are stable. To recall, the EMODPS using ϵ-NSGA-II algorithm *approximates* the Pareto front. Hence, stability again refers to when the Pareto front approximation does not significantly change after subsequent model runs.

The more function evaluations ran, the more likely it is that the solutions converge to the *real Pareto front*. Previous studies used *100,000* function evaluations and found that the solutions converge (Zatarain-Salazar et al., 2022). Since the existing (baseline) objective formulation was complemented (Equation 3.3) with an equality objective, I infer that the complexity of the model increases. This is due to the calculation

[1] Ideally, one should remove all dependencies. However, due to the burdens of large-scale complex simulation systems, the implications will always have a degree of dependence on the chosen experimental setup (Giuliani, Castelletti, et al., 2016; Reed et al., 2013).

[2] Convergence refers to the phenomena where after each function evaluation there are not new solutions found. The search algorithm has already found every potential solution there is, and hence the best approximation of the Pareto front.

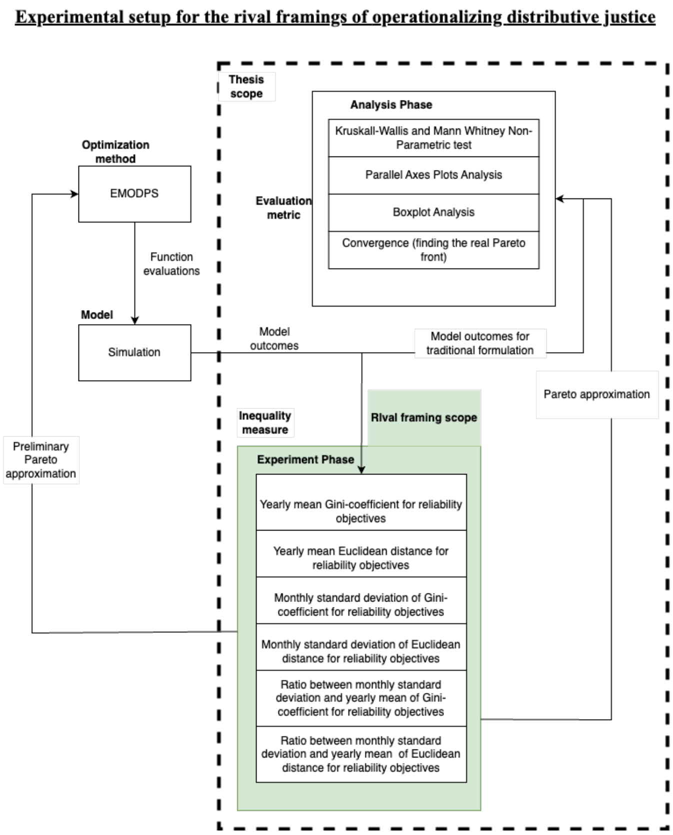

Figure 4.1: Experimental design for the research.

of differences between the baseline objectives. To find solutions that are Pareto-optimal and converge, more function evaluations were run. After an experiment with *250,000* function evaluations, it was found that these are sufficient to reach convergence of solutions (see Appendix C.1).

4.2. Experimental setup

The experimental setup is described in Figure 4.1 where the rival framings approach is shown, as well as how it is linked to the simulation-based MOO (EMODPS) approach, and how the results will be analyzed. At core of the rival framings approach are the combinations for operationalizing the objective formulation for equality. Both the original model outcomes and the operationalization for equality (together forming Equation 3.4) were fed into the simulations. The operationalization formulation was changed for each experiment. Finally, once all function evaluations were run, non-dominated solutions were filtered in the analysis, using the $epsilon - sort$ function designed by Woodruff and Herman (2013) [3]. These non-dominated solutions were fed into the *Analysis phase*. Note that the solutions for the baseline formulation were also part of this analysis since the rival framing formulations used the traditional formulation as a reference.

Finally, the non-dominated solutions were tested on convergence. In other words, the solutions were tested if they are stable in approximating the Pareto front, across the progress of function evaluations. If this was not the case, then the solutions found only point to local optima, i.e. a specific subset of the solution space. The global optima solutions refer to the strongest trade-off and therefore the real Pareto front. Hence, solutions stuck in local optima will not capture the strongest trade-offs. That is why the convergence analysis is important and has an entire Section devoted in Appendix C.1.

In Tables 4.1, 4.2 the abbreviations used during the experiments are shown. The distribution goal

[3]The $epsilon - sort$ non-dominated solution search algorithm can be found in the pareto.py of the Github repository

is specified for each formulation in Table 4.1. The baseline formulation will from now on be named the *traditional formulation* as done for the baseline formulation in Yang et al. (2023). The traditional formulation aligns with Equation 3.3 where the goal is to optimize the desired direction for the objectives. For each formulation, a new distribution goal was added which aligns with the distributive justice principle. For example, when the goal was to minimize the variability over the time horizon (one year), the goal was to minimize the standard deviation of monthly-means (the chosen time step to determine variability over). In Table 4.2 the combination of inequality metric and aggregation method was indicated for each experiment.

Table 4.1: Specifying the abbreviated experiment name for each formulation. Additionally, the distribution goal is indicated for every formulation. Note that in each formulation, the traditional formulation is part of the goal, since this traditional formulation is complemented with distinctive alternative goals for the equality of water allocation for these objectives.

Experiment name	Formulation	Distribution goal
F1	Traditional formulation	Maximize daily-based annual mean of objectives
F2	Combined traditional & Gini-mean	Maximize daily-based annual mean of objectives & Minimize Gini of annual mean of objectives
F3	Combined traditional & Gini-standard deviation	Maximize daily-based annual mean of objectives Minimize annual standard deviation of monthly means
F4	Combined traditional & Gini ratio	Maximize daily-based annual mean of objectives & Minimize Gini of annual mean of objectives & Minimize annual standard deviation of monthly means
F5	Combined traditional & Euclidean-mean	Maximize daily-based annual mean reliability & Minimize Euclidean of annual mean of objectives
F6	Combined traditional & Euclidean standard deviation	Maximize daily-based annual mean reliability & Minimize annual standard deviation of monthly means
F7	Combined traditional & Euclidean ratio	Maximize daily-based annual mean of objectives & Minimize Euclidean of annual mean of objectives & Minimize annual standard deviation of monthly means

Table 4.2: For each experiment, an indication is given on the chosen inequality metric, and chosen aggregation method over time. Together, they satisfy the distribution goal of Table 4.1.

Experiment name	Inequality metric	Aggregation method over time	Papers applying similar formulation
F1	None	Mean (central tendency of outcomes)	Traditional MOO studies
F2	Gini	Mean (central tendency of outcomes)	(Hu, Chen, et al., 2016)
F3	Gini	Standard Deviation (variability)	None
F4	Gini	Ratio standard deviation & Mean	(Siddiqi et al., 2018; Yang et al., 2023)
F5	Euclidean	Mean (central tendency of outcomes)	(Hu, Chen, et al., 2016)
F6	Euclidean	Standard Deviation (variability)	None
F7	Euclidean	Ratio standard Deviation & Mean	(Siddiqi et al., 2018; Yang et al., 2023)

Part III

Analysis

5
Results

5.1. Convergence and statistical test

5.1.1. Choice of convergence metrics

To determine if solutions converge three popular convergence metrics were used, namely the generational distance (GD), the ϵ-indicator (EI) and the ϵ-progress. For each of these convergence metrics, the solution set is compared to the reference set. The reference set is the set of solution that are non-dominated Pareto-optimal, i.e. the best approximation to the real Pareto front. An elaboration on the discussion of convergence can be found in Appendix C.1. Furthermore, the code can be found in the notebook for convergence tests.

To give a quick elaboration on what each convergence metric calculates: the GD calculates the distance between the solution set and the reference set. A smaller distance indicates the solution set is converging. In addition, the EI measures the convergence and diversity of solutions. Since the GD determines only the distance between the reference set and the solution set, it is not sufficient to determine how dominant the solution is compared to the rest of the solution space. The EI takes this into account and calculates the distance the approximation sets needs to be translated in order to dominate the reference set. However, this metric is sensitive to gaps in the Pareto front. That is why the GD and EI complement each other's findings for which its discussion was left in the Appendix C.1. Doing so yields the coverage of the real Pareto front. Lastly, the ϵ-progress determines how many new solutions have been added over the progress of function evaluations (Reed et al., 2013). It does this by comparing solutions in ϵ-box and only making a choice out of the non-dominated solutions (Zatarain-Salazar et al., 2022). Hence, the ϵ progresses as it escapes local optima in the objective space. Convergence is determined by the flatness of the curve. In Figure 5.1 the results are discussed for the ϵ-progress.

ϵ-Progress

ϵ-progress indicates the ability to escape local optima and to find continued improvements to the non-dominated archive. Specifically, the epsilon value indicates the user-specified threshold for which the search algorithm needs to produce at least one solution above this threshold at a certain frequency to avoid stagnation.

In summary, the formulations are able to escape their local optima. Nevertheless, two observations were made. First, for the Gini Mean, Euclidean Deviation, and Euclidean Ratio some of the seeds progress more than others. Hence, the convergence is dependent on the seed run. Second, although they escape local optima, none of the formulations seems to have reached a point of convergence. Especially when comparing this to *1,000,000* function evaluations (in the notebook for validation one can see that the epsilon can still progress. Hence, ideal convergence is far from being reached but would be extremely computationally expensive to reach.

To deal with this limitation, I infer that when the $\epsilon - progress$ is not growing linearly, a satisfactory level of convergence is satisfied. Linear growth indicates it has not converged. In light of this context, most formulations have reached some convergence since they stopped growing linearly after approximately 150,000 function evaluations. The only formulation that does show a higher linear behaviour compared to other formulations is the Gini Ratio. Hence, I conclude that the solutions converge to the Pareto front for most formulations, except for the Gini Ratio formulation.

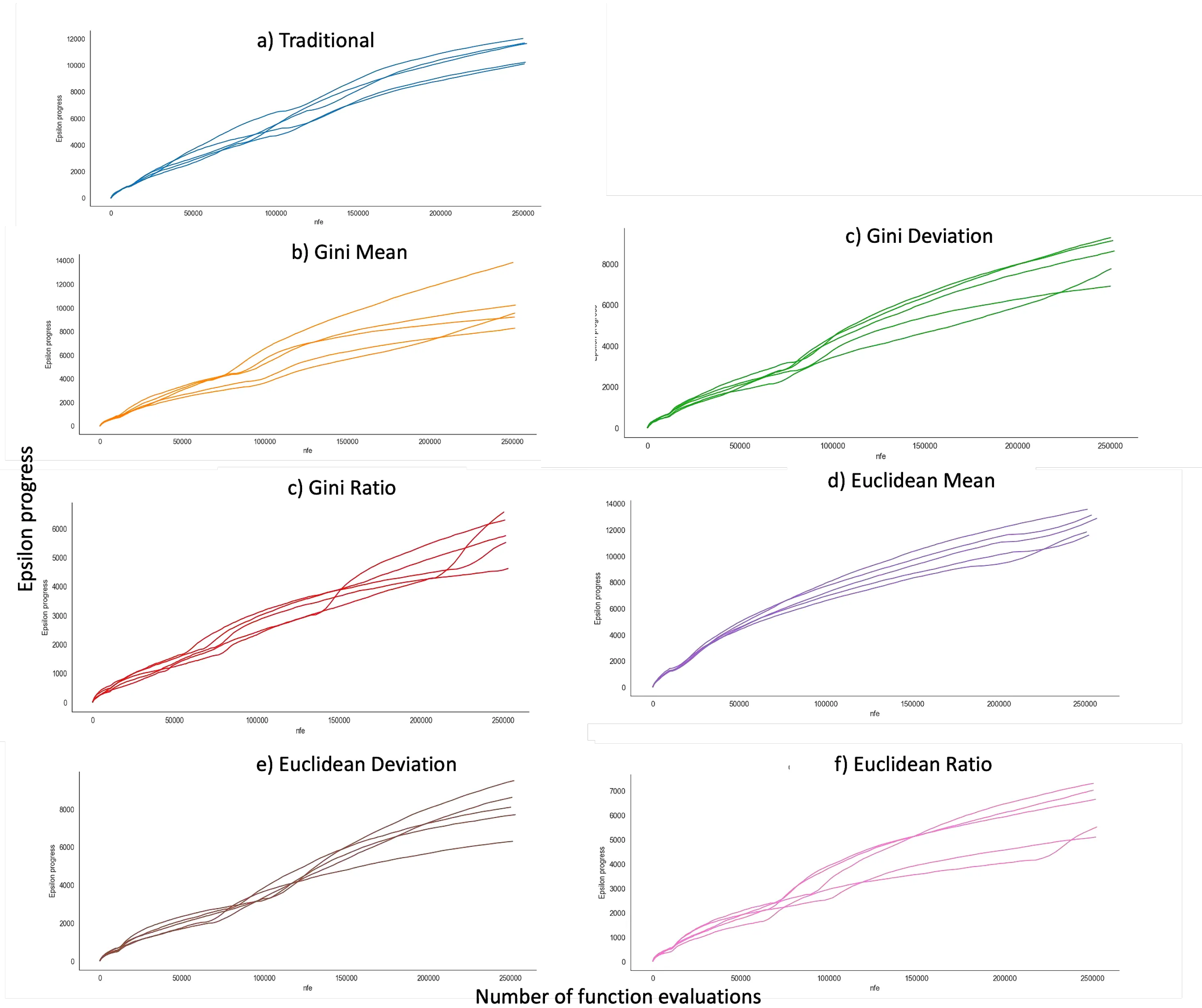

Figure 5.1: Epsilon Progress for each formulation in Tables 4.1, 4.2.

5.1.2. Statistical Analysis

In order to determine if differences in the objective values across formulations are due to randomness, or from an actual difference in the objective formulation, statistical tests were performed. Using the non-parametric tests, Kruskall- Wallis H test and Mann-Whitney U test, two main observations were made. First, the distribution of the Traditional formulation has a significant statistical difference (p-value < 0.05) among its objectives compared to the other formulations, except for the Atomic PP. Second, the Deviation and Ratio formulations (F3, F4, F6, and F7) share a similar distribution within the same inequality metric formulation for the objectives of Chester, and the Atomic PP. The common factor here is the inclusion of the standard deviation in the optimization. Hence, seen every formulation with the Deviation (Equation 3.7) was affected, I could infer that the standard deviation has a higher influence than the Mean on the Ratio objective formulation. It's important to consider these observations when forming a conclusion about formulations, since its distribution is not unique to its formulation. Further explanation can be found in Appendix C.2.

5.2. Trade-off analysis

In this Section, the discussion focus is on the trade-offs across objectives for the solution set from each formulation. First, an explanation on how to interpret the Parallel Axes Plot, the visualization for the trade-offs, can be found in Appendix A. Next, the trade-offs for the solutions of the Traditional formulation (Equation 3.3) are shown. Last, using this baseline (traditional) trade-off the trade-offs from formulations with a complementing inequality objective formulation are compared to the Traditional formulation.

One should note that the axes were reversed for the Environment to have a coherent direction of preference relative to the other objectives. The *Environmental shortage index* ought to be minimized. To visually represent this, the *Environmental reliability value* ought to be maximized. To achieve this, the optimized (minimized) value was subtracted from 1.

Moreover, equal satisfaction of objectives was desired to achieve distributive justice. Hence, the solutions ideally represent a horizontal solution, i.e. no trade-off across objectives. Since this is not always easy to read from graphs, additionally, inequalities were mathematically represented. An alternative was using the operationalized inequalities as performance indicators for the level of inequality. However, it was found that this would skew the results to the specific operationalization formulation. Hence, dependency would increase from a normative choice made. How these solutions are skewed from the choice of inequality metric can be found in Appendix C.3.

To counteract this, I infer that it is best to compare the level of inequality across traditional objectives (Equation 3.3) is by using the standard deviation, *Deviation score*, between objectives for each solution found. This served as a performance indicator of the equality across objectives. A low deviation is equal to high equality, and a high deviation is equal to high inequality. Hence, the *Deviation score* indicated how strong the trade-offs are across objectives. *This Deviation score is a global measure of inequality that can be related to all formulations.*

5.2.1. Baseline: Traditional formulation

In Figure C.3 the trade-off rising from the optimization of the Traditional objective formulation is shown, and further described by Equation 3.3. Due to the controversy around the Hydropower objective for the case-study it was decided to use the maximum, minimum, and median value of the Hydropower revenue as a reference to what the trade-offs are of the Traditional Formulation. Moreover, Giuliani et al. (2014) color grades the solutions based on how much Hydropower revenue they produce, affirming its relevance to use as comparison for solutions. Hence, the Hydropower objective represents the objective where focus is set on its efficiency.

The reference set spans over the entire objective space since objectives lie within two outermost extremes from high values to low values. For the Hydropower revenue, this lied between 36.25 M$ and 81.08 M$. Across the objective space, I identified a pattern of trade-offs where solutions yielded a revenue for Hydropower of approximately 60 M$. This subset of solutions yielded a strong trade-off where also high values for the Atomic PP, Recreation, Environment, were obtained and low values for Chester and Baltimore. For example, one trade-off for the solution is when the Hydropower Revenue is around 60 M$ (80% of the maximum revenue attainable), Atomic PP 100%, Baltimore 0%, Chester 95%, the Environment 90%, and the Recreation close to 100%. Hence, objectives will be close to satisfactory values at the

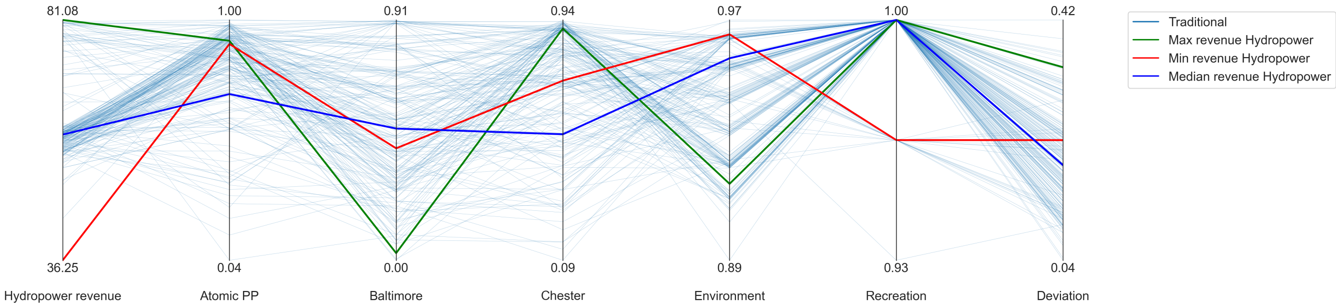

Figure 5.2: Parallel Axes Plot: solutions of the Traditional Formulation, with emphasis on the maximum, minimum, median solution of the Hydropower revenue objective. The direction of preference is up for the objectives except for the Deivation score. Besides the optimization of the Traditional formulation, the deviation score. The direction of preference for the Deviation score is down. The maximum solution shows the strongest trade-off.

expense of the efficiency performance of the Hydropower Revenue objective, and even stronger for the Baltimore objective.

Furthermore, I infer that low values for Baltimore were because the demand of Baltimore is the highest across the six objectives as shown in Figure 3.2. Hence, the reliability, which made of this demand as a reference value to calculate the reliability value leads to the same water allocation across objectives, representing large differences in the reliability objective obtained. In such a case, having a complementing inequality formulation enables solutions with such large relative differences across objectives to not be considered in the solution space.

The solution with the maximum value of Hydropower revenue yielded a maximum value of 81 M$. This led to a strong trade-off with the Baltimore objective at 0%, and the Environment at 90%. Conversely, the median value showed more horizontality across objectives, with objective values around 50-55% of their maximum attainable. The Environment and Recreation benefited from the median solution as they were both close to maximum performance values. The solution with minimum Hydropower revenue increased the objective value of Baltimore relative to the maximum Hydropower solution, but stayed below the value achieved by the median Hydropower solution.

The higher Deviation value also showed that the maximum Hydropower revenue leads to the strongest trade-off across objectives. Conversely, the minimum and median solutions of Hydropower revenue lead to more equality across the objectives. The median has the lowest inequality across objectives. This indicates that the mean aggregation of solutions for the Traditional formulation has weaker trade-offs.

Pattern observed for the Traditional formulation: *The maximum revenue led to the strongest trade-offs across objectives. Baltimore gained very low objective values while Recreation, Atomic PP, and Hydropower gained high values. The median revenue led to more water allocation for other objectives, and thus more equality across other objectives.*

5.2.2. Complementing formulations: Median evaluation

The median solution of the Traditional formulation has high values for the Hydropower revenue. Conversely, Baltimore had no water allocated in this case. Other objectives gained reliability between 60-90%, with the Atomic PP receiving the highest value out of the remaining objectives. The strongest trade-off lay with high values for the Hydropower revenue, Environment and Recreation, and low values for Baltimore. Chester gained a reliability close to 60%, leading to a weaker trade-off than the trade-off between Hydropower revenue and Baltimore. In Figure 5.3 one can already see that the Traditional formulation led to the highest inequality across objectives.

In comparison, the *Gini Mean* shows that the trade-off between high performance values for the Hydropower revenue, Atomic PP, Chester, and low values for Baltimore, are now weaker (reduced) compared to the trade-off observed for the Traditional formulation. This is observed in the lowering of the Deviation value. Moreover, the trade-off between objectives on the Pareto front changes. For example, the

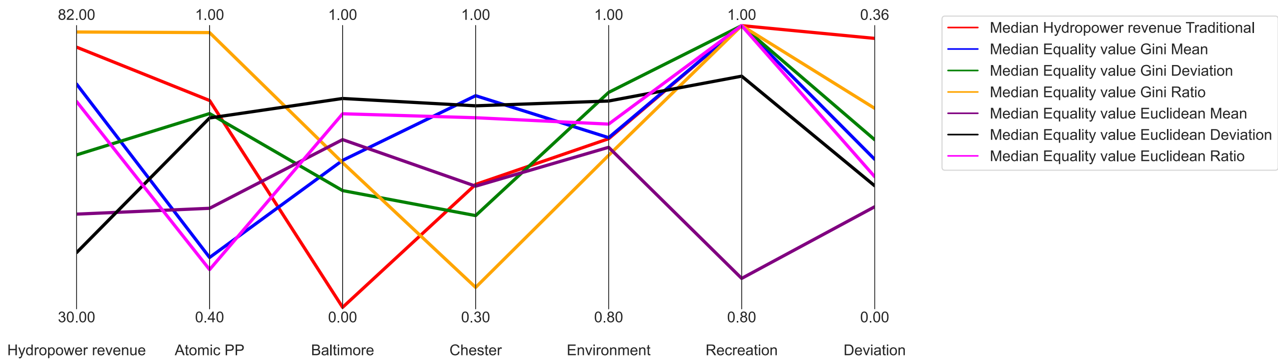

Figure 5.3: Parallel Axes Plot: Median solution of all formulations. The median is calculated according to each formulation's equality objective value. For the traditional formulation, this is calculated from the median of the Hydropower revenue. The equality score is left out due to a lack of overlapping comparisons. This is shown in Appendix C.

Atomic PP drops around 30% relative to the pattern of solutions observed for the Traditional formulation. Chester and the Environment benefit from this drop in the Atomic PP. The Environment and Recreation gain a similar value.

Conversely, the *Gini Deviation* formulation yielded less equality across objectives. The Hydropower revenue dropped even more than for the Gini Mean, while the Atomic PP stayed approximately at the same value as the Traditional formulation. Baltimore's values dropped again below Chester's objective values for the Gini Mean, but were significantly higher compared to the Traditional formulation. Other objectives seem to lie relatively around the same value. The increase in Baltimore and Chester for the *Gini Mean* relative to the *Gini Deviation* means that the low values are constant across months (i.e. less inequality across months), for which the *Gini Mean* leads to higher values for these objectives that previously had very low values.

The *Gini Ratio* created stronger trade-offs than for the previous Gini formulations, observed in the trade-offs, but also in the Deviation value which is closest to the Traditional. Across the Gini Formulations, this formulation will also yield the lowest value for the Environment, and Chester. Hence, the trade-off across objectives is strong, but weaker relative to the Traditional formulation. I infer that this drop in equality is because it is difficult to find solutions in the objective space that can find Pareto-optimal values for both the disaggregated (monthly standard deviation) and aggregated (yearly mean) scale, in one solution. Thus when combined, there is a shift of the traditional Pareto front, but the equality effect of both operationalization formulations cancelled each other out. From Appendix C.11 it was seen that indeed it is not possible to reach high values for one of the operationalized equality objectives without leading to lower values for the other operationalized equality objective.

The trade-offs of the *Euclidean formulations* are significantly weak compared to previous formulations (including the Traditional). The Deviation shows that the lowest inequality was reached for the *Euclidean formulations*, with the *Euclidean mean* leading to the smallest Deviation (inequality) score. Yet, it seems to be the formulation that led to the lowest objective values. A good explanation is that in the *Euclidean formulation* in Equation 3.5, large relative distances are penalized because of the quadratic formulation.

While the objective value of Hydropower drops, other objectives gained from this difference in water allocation. For the *Euclidean Mean*, the Recreation objective dropped to around 80% which is still deemed satisfactory when considering performance criteria (i.e. to maximize the objective).

Moreover, the *Euclidean Deviation* showed a similar behaviour across objectives. When the Hydropower revenue further drops for the *Euclidean Deviation*, significantly higher, and horizontal values can be achieved for the other objectives. I infer that the *Euclidean Deviation* reaches an even lower inequality score than depicted if the Hydropower revenue was left out. Nonetheless, even with this objective that had a large relative difference compared to other objectives, the Deviation was relatively low compared to other formulations.

The *Euclidean Ratio* was the first to show a trade-off between relatively high Hydropower values and low Atomic PP values. Yet, it is the first formulation that yielded high values for both the Baltimore and Chester objectives at an equal level. The Environment dropped to approximately 94%, and Recreation stayed at 100%. In the Deviation, it is less notable if the inequality has increased relative to the Traditional formulation.

Moreover, I can infer from this that the *Euclidean formulations* have a stronger penalizing effect than the *Gini formulations* on large relative distances between objectives due to the quadratic formulation. This is why objectives such as the Hydropower revenue, and the Recreation which would have the highest value have dropped to a relatively lower value.

Pattern observed for the median evaluation: *The Euclidean formulations led to a more egalitarian behaviour across objectives. The Euclidean Mean had the highest equality, at the expense of lower objective values. The Ratio formulation led for both inequality metrics (Gini, and Euclidean) to the lowest equality. The Mean formulations led for both inequality metrics to the highest equality.*

Trade-off for the median evaluation: *The complementing formulations yielded weaker trade-offs. Objectives with previously low values such as Baltimore received higher water allocations, while objectives with low values had now less water allocated.*

5.2.3. Complementing formulations: Maximum evaluation

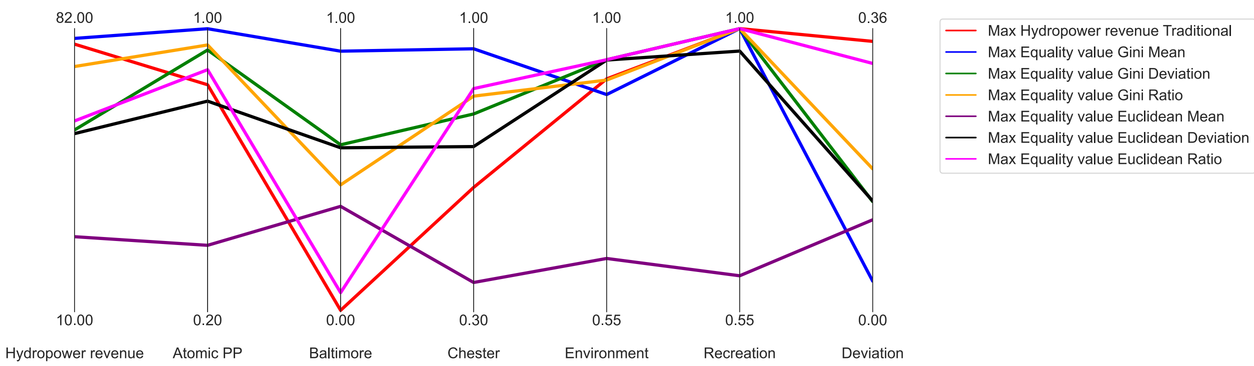

Figure 5.4: Parallel Axes Plot: Maximum solution of all formulations. The maximum is calculated according to each formulation's equality objective value. For the traditional formulation, this is calculated from the maximum of the Hydropower revenue. The equality score is left out due to a lack of overlapping comparisons. This is shown in Appendix C.

Recall that the Traditional Formulation led to very strong trade-offs. Conversely, the *Gini Mean* is now the formulation leading to the highest equality, as well as the highest performance in terms of efficiency. All objectives were close to 100% of the values they can reach. This is similar to the small subset of solution observed for the Traditional formulation in Fig. C.3. Only the environment reached a reliability of 90%, i.e. still high.

In comparison with the *Gini Deviation*, Hydropower revenue dropped by around 30% of its maximum. The objectives of the Atomic PP, Baltimore, Chester, obtain lower values compared to *Traditional* (Baltimore 25% lower), but for the Environment this again rose to 95%. Hence, the *Gini Deviation* formulation shifted the traditional Pareto front to higher values for the Environment.

The *Gini Ratio* showed stronger trade-offs than other *Gini formulations*. Baltimore dropped more (around 50 % relative to the *Gini Mean*), the Environment to 90%, and Hydropower revenue was close to its maximum value. For the Atomic PP, Baltimore, Chester, the Environment, and Recreation, the difference in value compared to the *Gini Deviation* formulation was negligible. This aligned with the findings during the Mann-Whitney analysis that the pattern of distribution of the *Gini Deviation* was similar to the *Gini Ratio*. This was due to the dominant influence of the Deviation in the formulation.

The *Euclidean Mean* achieved similarly to the median evaluation, very high equality across objectives. However, this was heavily at the cost of the efficiency of objectives, because all objective values experience a significant drop. Hydropower revenue is close to 14 M$ which was the lowest across all objectives, the Atomic PP had a value close to 15%, Baltimore around 2%, Chester around 30%, and for the first time, the Environment was around 60 %, and the Recreation objective dropped close to 60%. Nonetheless, trade-offs across objectives were not apparent. In comparison, Baltimore had the highest relative objective value. This points to what was mentioned before where objectives with a high gain receive less, and objectives with less gain receive more under the *Euclidean formulation*.

The *Euclidean Deviation* provided high equality, similar to the *Gini Mean*, but with a lower performance. The efficiency performance is still higher across objectives than for the Euclidean Mean.

The *Euclidean Ratio* showed trade-offs similar to the Traditional formulation. The Hydropower revenue drops, but the Environment increases by around 5%. Hence, what is lost in Hydropower revenue, goes mostly to the Environment, while the values for other objectives become more equal across objectives. It seemed that the Euclidean Ratio cancelled out the equality effects achieved from the optimization of individual aggregation methods.

Pattern observed for the maximum evaluation: *The Gini Mean led to both high equality and high efficiency performance. Similarly to the median evaluation, the Euclidean Mean led to high equality, but now at a higher cost for efficiency. Euclidean formulations led to a more egalitarian behaviour across objectives. Within inequality metrics, Ratio formulations led to the highest inequality across objectives.*

Trade-off for the maximum evaluation: *The complementing formulations yielded weaker trade-offs, except for the Ratio formulations. Trade-offs across objectives became weaker, but will not change in terms of what objectives have higher and lower values.*

5.2.4. Complementing formulations: Minimum evaluation

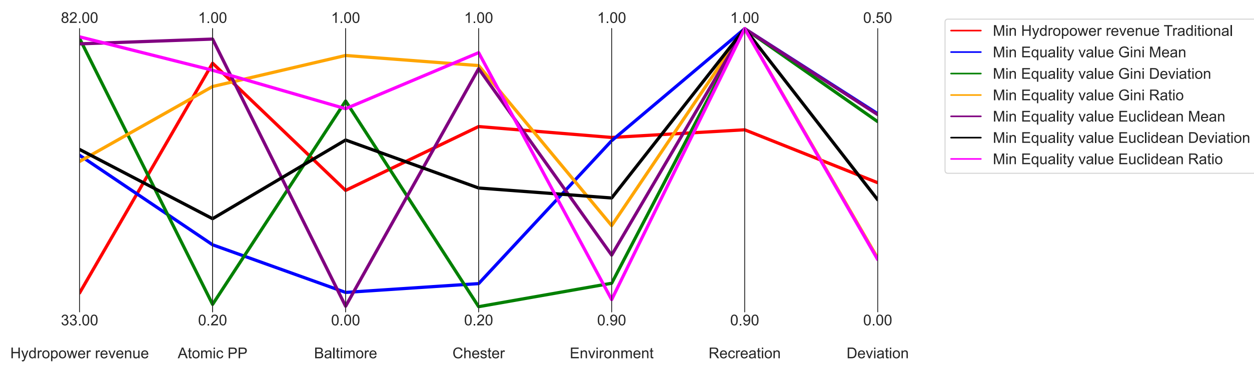

Figure 5.5: Parallel Axes Plot: Minimum solution of all formulations. The minimum is calculated according to each formulation's equality objective value. For the traditional formulation this is calculated from the minimum of the Hydropower revenue. The equality score is left out due to a lack of overlapping comparison. This is shown in Appendix C.

Interestingly, the baseline formulation now yielded higher equality across objectives than the *Gini Mean, Gini Deviation, and Euclidean Mean* formulations. It was these formulations that showed the highest equality when looking at the median and maximum solutions. An explanation could not be found for this. However, I can infer that the formulation's performance in terms of equality depends on the indicator used. If it is desired to avoid worst-case inequalities, it is better to optimize the formulations that lead to the highest equalities for the minimum solutions.

From a holistic observation, across formulations, I observe that most formulations showed very strong trade-offs across objectives. The Traditional formulation showed a strong trade-off, but an even stronger trade-off was achieved by the aforementioned best-performing (in median and maximum) formulations. The Hydropower Revenue, Atomic PP, Chester, the Environment, and Recreation gained a high value, while Baltimore and Chester were worse off. The demand of Chester was similar to the Atomic PP (in the range of 100,000 m^3/day), indicating that there was a high degree of inequality reached from this solution.

There are exceptions. The *Euclidean Ratio* and *Gini Ratio* achieved the same Deviation score, and had high objective values with a drop of approximately 5% on the Environment to compensate for this. Hence, the *Euclidean Ratio* and *Gini Ratio* were able to find a suitable balance between efficiency and equity. Moreover, this behaviour resembled the trade-off of the maximum solution of the *Gini Mean*. For other formulations, I observed that there is always at least one objective with a high value, while others remain significantly low. The *Euclidean Deviation* is the other formulation to reach more equality than the *Traditional*.

Pattern observed for the minimum evaluation: *The Gini Ratio and Euclidean Deviation managed to achieve the highest equality across objectives. Previously well-performing formulations for equality now led to stronger trade-offs.*

Trade-off for the minimum evaluation: *Most of the formulations shifted the Pareto front to a stronger trade-off between one or more objectives. Often Hydropower revenue and Recreation obtained higher values compared to other objectives.*

Main observations
When comparing formulations:

- The performance of formulation in terms of equality and efficiency depended on the indicator (median, maximum, minimum) being used for post-processing. There is a similarity in the implications from the median and maximum.
- The *Euclidean Mean* stood out for the maximum and median indicators since it led to more equality. Additionally, the *Euclidean Deviation* also yielded high equality, but also higher efficiency performance relative to the *Euclidean Mean*.
- Overall, for the maximum and median indicators, the *Euclidean* formulations performed better. The *Ratio* aggregation method over time, for both inequality metric operationalization led to the highest inequality, and a lower efficiency performance compared to the Traditional formulation.
- For the minimum solution, the *Ratio* aggregation methods over time yielded the highest equality. Also, the other formulations led to significantly higher inequality. This is in contrast to the implications from the median and maximum indicators.

When comparing the Pareto front shift from solutions:

- Each formulation had its own Pareto front shift, and in that sense not only affects over which objectives there is a trade-off, but also the strength of these trade-offs.
- The differences in objective performance values from different formulations relative to the Traditional formulation always lied in the range of 5%. Hence, the drop in efficiency is smaller than the achieved equality, making a strong case to consider both.
- An alternative is analyzing all solutions as shown in Appendix C. However, due to the number of solutions, it is more difficult to determine the Pareto front shift. One requires looking at patterns across all solutions. From this pattern, and the previous statistical analysis, I infer that the *Euclidean* formulations yield more egalitarian behaviour than the *Gini* formulations.

5.3. Distribution analysis

The second part of the results were visualized through the boxplots, an easy-to-interpret visual representation of the distribution of the solution space from the different experiments. The Parallel Axes Plots were an aggregation of the solution space through its maximum, minimum, and median indicators. To complement this the solution space for each objective were shown using the boxplot. Trade-offs are not visible, but shifts in the space are. Hence it is purposeful to look at the tendency of data and not its patterns. For each objective, the median, interquartile (IQ) range, and outliers were visualized.

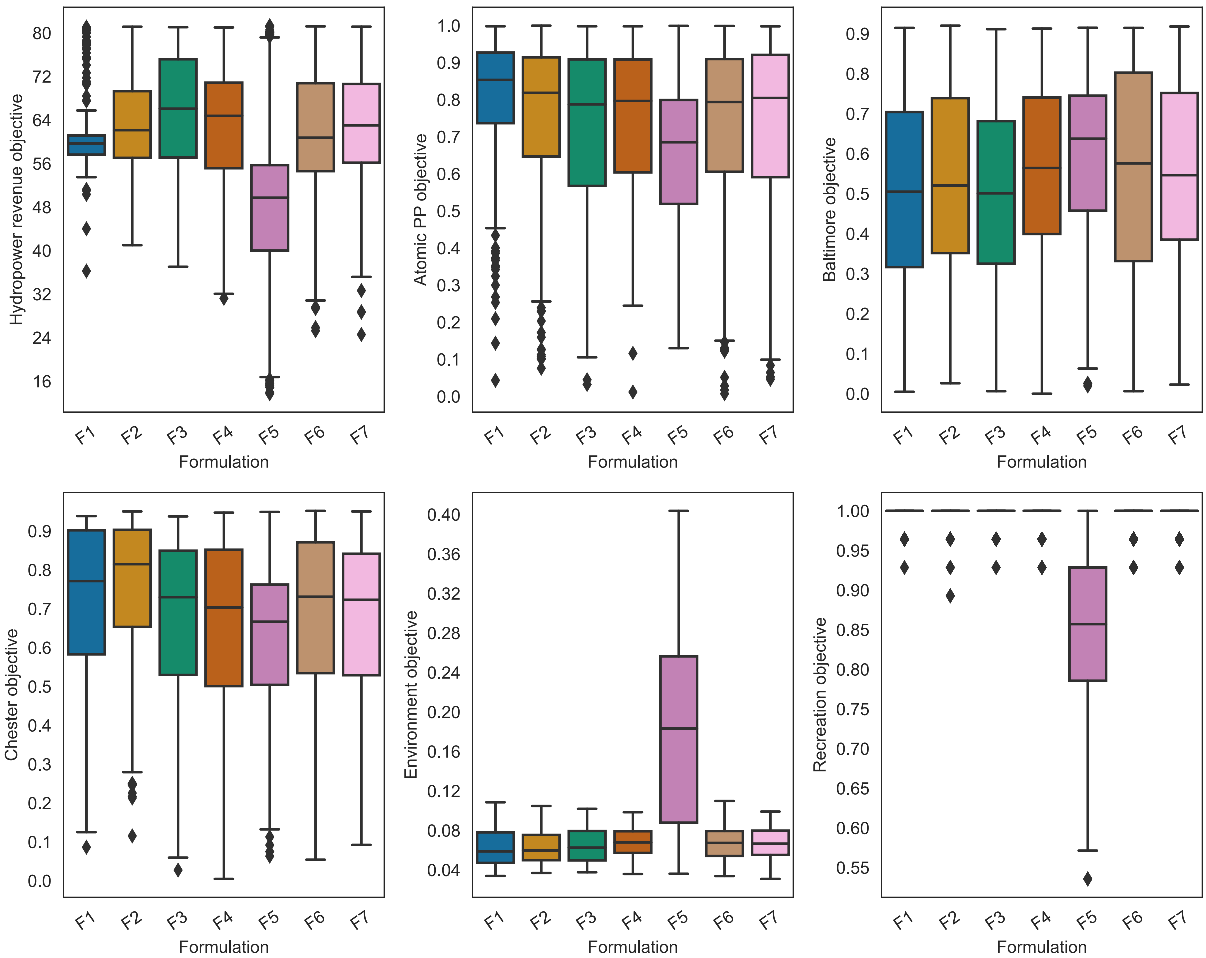

Figure 5.6: Boxplot: Distribution of the solutions from formulations in terms of each objective value. F1 is the Traditional formulation. F2 to F7 are the Traditional formulations combined with a complementing equality objective in the objective function. F2 combines the Traditional formulation (F1) with the equality objective using the Gini Mean operationalization formulation, F3 with the Gini Deviation formulation, F4 with the Gini Ratio formulation, F5 with the Euclidean Mean formulation, F6 with the Euclidean Deviation formulation, and F7 with the Euclidean Ratio formulation.

5.3.1. Hydropower revenue

The IQ range (between 25th percentile and 75th percentile) of the Traditional formulation lay between 55 M$ and 65 M$ with the median at 60 M$. In terms of Hydropower revenue, this was around 75% of what can maximally be achieved, meaning efficiency was gained for this objective. There were some upper outliers reaching 80 M$, and lower outliers at 35 M$. Most formulations had a wider IQ range, and thus a more variated set of solutions. The *Gini Mean*, *Gini Deviation*, *Gini Ratio*, *Euclidean Deviation*, and *Euclidean Ratio*, had approximately a median higher than the Traditional formulation. The *Gini Mean*, *Gini Deviation*, *Gini Ratio*, and *Euclidean Ratio* had a median between 63 M$ and 68 M$. The 75th percentile of the aforementioned formulations now reached above 70 M$, with the *Gini Deviation* yielding an even higher 75th percentile of 75 M$. The 25th percentile of each formulation except the *Euclidean Mean* lay around 55 M$, similar to the Traditional formulation. Only the Euclidean Mean led to a lower IQ range, with the 75th percentile at 55 M$, the median at 50 M$ and the 25th percentile at 40 M$. As observed during the Parallel Axes Plot, the solutions of the Euclidean Mean led to a more horizontal (egalitarian) distribution across objectives. The egalitarian reallocation of water came from the drop in Hydropower revenue, which dropped for these solutions. This was what most likely led to other objectives obtaining higher values when solutions were compared across objectives for the Parallel Axes Plot.

Most formulations had a wider and higher IQ range compared to the Traditional formulation, their outliers reached relatively low values as well. Moreover, the lower limit was quite different across formulations. The *Gini Mean* and *Gini Deviation* had the highest lower bounds, lying around 40 M$. The *Gini Ratio* dropped further to 35 M$, Euclidean Mean to 15 M$, *Euclidean Deviation* to 30 M$, and Euclidean Ratio to 35 M$. The upper bounds of the formulations lay at the same range as the outliers of the Traditional formulation, i.e., at 80 M$. I considered the differences significant since the formulation is in M$, where 5 M$ made a substantial difference.

5.3.2. Atomic PP

For the Atomic PP, the IQ range of the Traditional formulation lay between 95% and 75%, with the median at 85%. The maximum value was 100%, and the minimum value was 45%, with outliers that went all the way to 0%. Compared to this, other formulations had a wider IQ range. Moreover, the 25th percentile of complementing equality objectives were all lower than the Traditional formulation, and for the 75th percentile, the difference was smaller. The median of the formulations except for the *Euclidean Mean* were approximately 80%, 5% lower than for the Traditional formulation. For the Euclidean Mean, this was 70%, 15% lower. The IQ range was different across formulations, but I notice that the *Gini Deviation* had the widest IQ range. The extremes across formulations were attributed to the *Gini Mean* and *Euclidean Mean* where the 25th percentile lay for the Gini Mean at 65%, and for the *Euclidean Mean* at 55%, a 10% difference. In all cases, the 25th percentile dropped by 5-15% compared to the Traditional formulation. The minimum values of the *Gini* formulation were slightly above 0.2, while for the Euclidean formulations, this dropped below 0.2. Similarly, to the Traditional formulation, outliers can range between the minimum and values around 0.

5.3.3. Baltimore

For the Baltimore objective, there is a wider distribution of values compared to other objectives. Looking at the Traditional formulation, the minimum and maximum values were across formulations similar. The IQ range was between 30% and 70%, with the median at 50%. The minimum and maximum value could range for anything between 0% and 100%. The *Gini Mean* and *Gini Ratio* brought this IQ range up by approximately 5-10%, with the *Gini Ratio* showing also a higher 25th percentile. The IQ range seemed to be relatively similar across formulations with small (but significant) differences in the IQ range, namely with the 75th percentile at 70%, the median at 50%, and the 25th percentile at 35%. The *Gini Deviation* would perform quite similarly to the Traditional formulation. Again, this came from the fact that the intertemporal equality was increased but could not be seen when visualizing data on the yearly mean. The median of the Euclidean Mean outperformed other medians as it lay around 0.65, and the 25th percentile at almost 0.5, around 20% higher than for the Traditional formulation. The *Euclidean Deviation* also had a median higher than the rest, close to 0.6, and with the 75th percentile at 0.8, the highest across formulations. The 25th percentile was however similar to the Traditional formulation, showing no improvement here. The *Euclidean Ratio* kept the same IQ range width but brought this up by 5% compared to the Traditional formulation. The median for this formulation is 50%.

5.3.4. Chester

To begin with, the optimization of certain formulations yielded higher reliability Baltimore, at the cost of the reliability for Chester. This can be noticed when comparing the median of the formulations with the median of the Traditional formulation. Formulations with a higher median for Baltimore, now had a lower median for Chester, specifically seen for the *Gini Ratio*, *Euclidean Mean*, and *Euclidean Deviation*. The Traditional formulation had an IQ range between 60% and 90%, and a median close to 80%. An exception for the aforementioned statement was the *Gini Mean*, whose median was higher for both objectives. The *Gini Mean*'s increase was 5% which is relatively small. Moreover, its IQ range became smaller, with the 25th percentile at 70%, 10% lower. Hence the solutions of the *Gini Mean* satisfied higher values. Other formulations performed worse than the Traditional formulation. Also, the IQ range across formulations showed more difference than for the other objectives. The Euclidean Mean had the lowest 75th percentile and median. The Gini Deviation, *Gini Ratio*, *Euclidean Deviation*, and *Euclidean Ratio* had a 75th percentile 5% lower than the Traditional formulation. Their median was also around 5% lower. Only the *Gini Ratio* and *Euclidean Mean* had a 25th percentile close to 50%, 10% lower. Hence, the *Gini Ratio* had a wider solution space than other formulations. The differences in minimum value were only notable for the *Gini Mean* where compared to other formulations the reliability of Chester shifted from 10% (and 20%) to 30%, an increase of 10-20%.

5.3.5. Environment

One should recall that while the Environmental objective was desired to be as high as possible in the Parallel Axes Plot, it was because the Axes were reverted so the trade-off across the objective was more easily visible if the direction of preference was the same for all objectives. Hence, in this boxplot, the aim iwas to minimize the Environmental shortage index. A lower Environment objective value was desired. In this sense, I observe that the *Euclidean Mean* immediately stood out. The IQ range of this formulation was not only wider than other formulations but significantly higher. Its median value was 15% than for other formulations. A reason why the Environment was heavily jeopardized compared to other objectives remained unclear. However, from the Parallel Axes Plot, it was shown that the Euclidean Mean led to one of the highest horizontal distributions across objectives, and while the Environment gained significantly high values for other formulations, the *Euclidean Mean* penalized the relative advantage the Environment gained compared to other objectives. The operationalization formulation then ensured the water was reallocated to objectives with previously low values. This can be observed for the Baltimore objective, where most trade-offs from other formulations led to a low Baltimore reliability, while for the *Euclidean Mean*, it had the highest and most narrow IQ range. The Traditional formulation and other formulations showed a similar distribution for the Environmental shortage index, with the entire distribution lying between 4% and 12%. This distribution is close to the desired behaviour.

5.3.6. Recreation

The Recreation objective seemed to be satisfied maximally for all solutions across all formulations, except for the Euclidean Mean. The outliers were negligible, and would still lie at high values between 90% and 100%. This 10% drop in the Recreation objective was allocated to other objectives. The median of the Euclidean Mean brought the IQ range to lie between 80% and 90%. Minimum values would drop even further to 60%. This meant that overall the Euclidean Mean led to a decrease in performance of 40%. Again, a similar explanation was found as to the Environment objective. The *Euclidean Mean* penalized objectives that previously gained high relative values in the simulation. Subsequently, this water was allocated and distributed more equally across objectives, leading to a lower overall system efficiency satisfaction. The Euclidean Mean represented the strongest trade-off between distributive justice and efficiency, as for most objectives it performed less well compared to other formulations.

Distribution analysis 1: *The distribution across formulations depended on the objective. In general, the solution space for objectives became larger (wider in the objective space) when complemented with an equality objective. Among these larger solution spaces, differences across formulations for each objective were mostly apparent in the width of the IQ range. The pattern depended on the objective and on the formulation, for which there was not a fixed pattern in the shifting of the Pareto front.*

Distribution analysis 2: *The medians of complementing formulations lay within the 5-10% range from the median of the Traditional formulation. This indicates small changes in the tendency of the Pareto front in the objective space. Overall, the IQ range (incl. median) of the Atomic PP, Chester, was worse (lower) for complementing formulations. For the Hydropower this range will be better (higher), except for the Euclidean Mean. For the Environment and Recreation the range stayed the same across formulations, except for the Euclidean Mean which was again worse off. The Euclidean Mean had across objectives both median, IQ range, and extreme values at a different range of the objective space. The Euclidean Mean was also the formulation that led to a significant shift in the Pareto front of the post-processed median solution and maximum solution that also in most cases resulted in lower objective values.*

Distribution analysis 3: *Moreover, as expected, the boxplot analysis does not make it possible to look at the shifting of the solution space across time. This is why the Deviation and Ratio formulations showed smaller shifting in the Pareto front of the yearly mean objectives than the other two Mean formulations.*

Part IV

Conclusion

6

Discussion

Based on the results of Chapter 5 the key findings are discussed, as well as the limitations of this research.

6.1. Key findings

Research Question : Main Question

What is the shifting effect on the Pareto front from introducing different formulations for the equality objective of existing multi-objective optimization models?

This question set out to explore what influence normative choices, from the formulation of a distributive justice principle objective, have on the Pareto front, and subsequently on the implications from existing decision-support models for water allocation. Without understanding the implications, it remains unknown if the implementation is satisfactory. Egalitarianism was chosen as justice principle for demand satisfaction. Hence, this question was explored by implementing an equality objective in the objective formulation of a MOO-model. For this question, I argue that the solution set is sensitive to the formulation of (in)equality. To explore this question three levels of formulation were used, the notion of exchange for resources (hereby equality), the formulation for the inequality metric (Gini, and Euclidean), and the aggregation method of the inequality over time (Mean, Standard deviation, and Ratio of both).

So what do the results tell me?

It becomes apparent that the shifting effect of the Traditional Pareto front is dependent on the objective formulation for the desired equality formulation. The shift per objective is between 5-10%, indicating that the shift per objective (in terms of median) is less significant. It is the solution space over multiple dimensions that will thus change, and lead to a change in the trade-offs as observed from the Parallel Axes Plots. Therefore, I argue that the only way of establishing reliable justice methods for the decision-support using models is by a) using MOO-models with disaggregated actor objectives to assess trade-offs across objectives and high-level goals (efficiency and distributive justice), b) explaining why for the choice of justice notion (in our study it is deemed as equality), ethical premise, or any ethical choice in that sense, c) understanding what formulation is suitable for the efficiency goals and justice goals of the model.

6.1.1. Inequality metrics

First, the trade-off on the Pareto front was observed. The Deviation score tells us that the *Traditional* formulation leads to a high degree of inequality across objectives. Moreover, it leads to strong trade-offs between the water supply of Baltimore, and the Hydropower revenue. While in the model these are merely two objectives, in real life this can set out to strong conflicting situations among stakeholders. Introducing equality in the objective formulation sets out to favour the demand of everyone without causing large relative differences across this demand satisfaction. Nonetheless, how much equality is reached depends on the indicator of choice for evaluation. While considering this, the *Euclidean* formula (Equation 3.5) as the inequality metric implemented in the objective formulation led to a smaller degree of inequality across objectives when looking at the solution space of the median and maximum solution. Moreover, the quadratic formulation creates higher equality across objectives that previously had a large relative difference. For example, Baltimore and Hydropower had closer satisfaction values to each other, regardless of their difference in demand.

The *Gini* formula (Equation 2.1), frequently used in MOO-justice studies, led to less inequality across objectives, but performed less well than the *Euclidean* formula for the median solution. There was an exception for the *Gini Mean* that performed the best for the maximum solution set.

For the minimum solution, the *Euclidean Ratio* was found to yield the most equal, and efficient solution. The minimum indicator could be deemed to be the indicator for actors that want equality in the worst-case scenarios, while the median and maximum serve as indicators for actors that want equality in either the most recurrent scenario or the best-case scenario.

In the end, both formulations shift the trade-offs of the Pareto to a more equal satisfaction (i.e. higher equality) across objectives. To add to this, the *Euclidean* will generally lower the satisfaction values for all objectives, at the expense of (even) more equality. Conversely, the *Gini* is able to keep high objective values for objectives that already possessed a high value in the *Traditional* formulation. In light of this context, the shifting effect from the *Euclidean* formulation is more significant. This is also observed in the boxplot where the *Euclidean mean* significantly shifts the solution space of the Traditional Pareto front for several of the objectives, for better and worse. Additionally, in the boxplots, it becomes evident that the Pareto front shift of other formulations is quite different since the median, and its IQ range have high relative differences across objective and formulations. For the Atomic PP, Baltimore, and Chester this results in a lower and wider IQ range. For the Hydropower revenue the solution space for complementing formulations becomes also wider, but will also shift the IQ range up. An exception to this is the *Euclidean Mean* that for other objectives shows also a solution space that is less Pareto-optimal [1]. Coming to an understandable pattern is close to impossible and emphasizes the normative uncertainty introduced by the modeller.

6.1.2. Aggregation method

On a more granular scale, the aggregation method defined if the optimization was considering the inequality across the monthly time step, or on the aggregate of the yearly mean. While at the trade-off analysis, the shifting effect is stronger based on the inequality metric used, the aggregation method also played its role in shifting the trade-off. In the maximum solution, the *Gini Mean*'s high equality was closely followed by *Euclidean Deviation*, and after by the *Euclidean Mean*. This pattern was broken in the median and minimum solution where the *Deviation* (regardless of the chosen inequality metric) formulation led to lower equality than the *Mean* formulation. The *Ratio* for example also showed controversial findings. In the median and maximum solution, it led to the lowest equality of all, for both inequality metrics formulations. Moreover, it provided the best solution in terms of efficiency (defined by how much of the objective values are satisfied). The differences were seen across aggregation methods with on one side the *Standard deviation* and *Mean*, and on the other side, the *Ratio of Standard deviation and Mean* are deemed significant. Hence, across aggregation methods, the shift on the Pareto front plays a role. Depending on the indicator used, this shifting effect will yield more or less equality. Over which objectives there is a trade-off does change significantly based on the choice of aggregation.

The distribution of objective values will be (statistically) similar for the *Deviation* and *Ratio* for both *Euclidean* and *Gini*. This indicates that the *Ratio* is strongly sensitive to the intertemporal equality measured (and optimized) from the monthly *Standard Deviation*. Across inequality metrics, the *Mean* showed no similarity in pattern, since the *Gini Mean* and *Euclidean Mean* have their own distinctive distribution for each objective. For the *Euclidean Mean*, as already mentioned, the Pareto front will shift to a sub-optimal space. Taking the Environment as example, the solution space is much higher compared to other formulations that stay close to the Traditional formulation's distribution. Also, there is no apparent similarity with the other aggregation methods. Similarly to the finding for the inequality metrics, the median and its IQ range for each specific formulation are even more highly varying across objectives. Hence, making a choice on the aggregation method seems to have a less significant shifting effect in trade-offs, but in terms of distribution will play a significant role.

[1] With Pareto-optimal it is referred to solutions that would otherwise yield the highest possible values for all objectives separately.

6.2. Research limitations

The findings point to useful implications for the choice of modelling inequality. Before accepting the findings point to a fixed shifting effect of the traditional Pareto front, from the choice of inequality formulation, limitations affect the suitability of the conclusion of this research, and the research itself.

6.2.1. Limitation of the research

The problem formulation for justice

Notably, the formulation of inequality explored in this research is limited in the number of formulations but also limited in the modelling concept researched. I argued that inequality must be introduced in the objective formulation. Nonetheless, in this research, the possibility of exploring the problem formulation was left out of the reasoning of constraining the solution space. Instead of implementing it in the *objective function*, it is put in the *model constraint*. Even with a rival framings approach making use of the same notion of justice (i.e. equality), the solution set will be different. Nonetheless, to constrain the inequality of the solution sets, a higher stakeholder input is needed, to determine what is deemed unacceptable from their viewpoint. The objectives do not make use of the same reference demand, for which the value range of inequality is non-intuitive from an external point. Only with close collaboration, other modelling aspects for justice in the objective formulation can be studied.

Flavours of equality in the implementation

Moreover, the limited choice of rival framings sets out to leave an unattended bias in what is deemed as inequality. Ciullo et al. (2020)'s formulation where the relative distances of actor objectives must be minimized was the starting point of this study. This relative distance can be measured across all actor objectives and aggregated as was done here. But as Ciullio et al. themselves do, relative differences can also be measured among the objectives with the largest relative difference. Instead of optimizing the relative differences that are negligible since they could be already deemed as satisfying equality, only high inequalities are given priority. Doing so is another dimension of the notion for equality that could yield better results in terms of equality, and unknown effects to the efficiency performance of objectives. Another limitation is in the choice of objectives considered for equality from an actor's perspective. The aggregation of relative distances hides the influence of each objective. An alternative lies in backpropagating the results by leaving objectives in and out of this relative difference calculation. In doing so, a deeper sense is gained of which objectives equality is beneficial. Currently, the *Euclidean* formulation already showed that polynomial effects will prioritize large relative differences, from which I hypothesize that it is between the highest and lowest objective (values). This finding is limited to the results of the model and could stem from a different reason, making the backpropagation of equality important.

Inconsummerable comparison between efficiency and justice

Closely related to the previous limitation, in this research I never discussed how to visually interpret the relations between (increase and drops in) efficiency and distributive justice. In this sense, while the results are synthesized by looking at the shift in inequality from each formulation, similarly, the synthesis could have been based on the differences in achieved efficiency. In doing so, I can find a more objective answer to how the trade-off between efficiency and equality is changing. For example, this would highlight '*What are acceptable decreases in levels of efficiency for the increase in levels of equality?*'. Since the efficiency is defined by six objectives (Equation 3.3) and the equality by one objective (Figure 3.4), the trade-off across solutions is (objectively) incomparable. Two opposite concepts are being compared. Currently, I drove implications from a visual inspection between the objective values and relative differences (supported by the Deviation metric). In light of this context, the conclusion on the trade-off between the two principles is biased from the visual interpretation of the modeller.

Limited scope for the justice of solutions

Equity is not bound to distributive justice, nor a chosen definition such as equality for this study. In the Literature review, I already mentioned that procedural justice, the fairness of the process for how decisions are taken, is as important. Especially, since this determines if actors accept the solutions proposed by the decision-support model. This is difficult to implement in models, and that is why the fairness of solutions is determined *a-posteriori* using cooperative game theory. Here the preference of actors is considered and the stability of solutions. This was not considered, which means that while an ambitious attempt at defining what is fair is made, actors might not accept this definition. However, this is simultaneously a post-processing step of this research beyond the thesis scope.

The rank-ordering of solutions was not assessed in this research while previous studies found that this is the most useful method to determine justice from decision-support models (Jafino et al., 2022). Currently, the post-processing of solutions was limited to statistical tests, and foremost, the statistical indicators. While the maximum, minimum, and median capture a great part of the range of implications, ordering all solutions instead of their extremes or average, yields more reliable implications of the Pareto front shift.

Lastly, not all levels of justice were considered. The level of justice considered in this research was at best the top-level, which assumes a general notion of equity for all actors involved. In reality, it is up to individual actors to determine what is deemed fair. The importance becomes clear when one considers that actors will want compensation for historic events in the water allocation of a river basin. Moreover, Groenfeldt (2019)'s domains of water ethics is yet again another way of viewing that justice depends on the actor environment. For example, the environmental perspective sets priority on intertemporal justice rather than the aggregated justice, and will also have concrete constraints in the water management. The same applies to other domains. Currently, this is not part of the problem formulation since it would add a complexity layer beyond the possibility of convergence for the model. It must also be added that this top-level justice perspective at no time inspects the lower-level spatial justice. Once Baltimore or Chester has water supplied, the local distribution of water among its community is *equally* important. This variability scope is kept outside this research but can be implemented using more data and other inequality metrics such as the *Theil-index*.

6.2.2. Limitations of the model

Formulations of objectives

Inherently, the objectives that are part of the objective function have a limitation in the way they are formulated. For example, the environmental objective (Equation B.4) in its formulation dates back to 1982, and since then has been validated (Hashimoto et al., 1982). However, it does not penalize severe shortages in important months for the local flora and fauna. The assessment of justice in this study is over one year, but not penalizing this over more years, will change the ecology of the lower Susquehanna River Basin, which is exactly what is experienced right now (Ain et al., 2014; Hicks et al., 2008; Noe et al., 2020; Zhang et al., 2013). Finding more elaborate formulations that deal with this intertemporal complexity will be essential to the development of decision-support models.

Limited reality of water allocation

In this model, there is a wrong representation of reality because the allocation of water serves solely one purpose. The context of this case-study is a River Basin, so one can imagine that if water is released to generate Hydropower revenue, this water is also useful for the Environmental flow, since water is already flowing. In this sense, there are multiple benefits to the water being released. In this research, it is assumed that this effect is negligible. Since there is a lack of expert consultation and the fact that many MOO-decision-support models do not consider this, it was left out of the scope. However, what if this effect is not negligible? In this case, the optimization would prove unfeasible to capture the system's complexity, and hence the result implications are not useful. Future research should consult experts on this matter before continuing.

6.2.3. Limitations of the method

Neglected discrimination of variability

Although it was found that the strongest influence on the findings stems from the *inequality metric* used, the influence of the *aggregation method* is clearly not negligible. Therefore, the inequality measured is sensitive to the formulation to determine variance (relative distances). In this research, the *Standard Deviation* method was used. Other (unconsidered) metrics such as the skewness methods discriminate between positive and negative variance. A good example of when the sign of variance is important is when one objective becomes negative while it needs to be maximized (e.g. Recreation) while the other objective needs to be minimized (e.g. Environment). Moreover, even when variances are calculated, the level of efficiency performance reached for this is undesirable. Hence, a baseline must be considered to determine when variance changes of sign. This again depends on how one defines the interaction between efficiency and justice. For the sake of simplicity, in this study, it was assumed that any type of positive or negative signed inequality is deemed wrong. Considering this limitation would only improve the quality of justice implementation in decision-making, but it does not invalidate the methodology used.

Post-processing of results
Another limitation lies in the fact that the Deviation and Ratio optimized intra-monthly time steps, whose concrete implications from achieved equality are not as clearly seen for the yearly mean, as is the case for the formulation when the yearly mean equality is optimized. This raises the question if the trade-offs highlight the right findings. The truth is that there are hidden implications in this aggregation method. As the aggregation is over twelve-time steps, I infer that the implications from model results are not too divergent from each other. However, to still make some sense out of this, Appendix C.5 highlights differences in achieved Equality on a yearly basis, as well on a monthly intertemporal basis. For future research, I suggest considering plotting the change in trade-off across months so it is further understood what the hidden consequences are, which can have a great impact (Kasprzyk et al., 2016).

Foremostly, the largest limitation of this research lay in how the results were evaluated. In Appendix C.4 the trade-offs sorted by the Hydropower revenue generated are visible for each formulation. Especially here one can notice how diverse and wide the solution space is. Hence, it remains debatable whether using the median, maximum, or minimum is sufficient to determine the implications of different optimization formulations. This is a limitation every MOO-study must deal with and is dependent on the context being studied. For this specific context (since the solution space is so wide it would not lead to concrete implications) this method of post-processing the results - comparing the median, maximum, and minimum of the equality solution, to the same indicators for the Hydropower revenue solution of the Traditional formulation - aligns the most with the research questions where light is shed on the trade-off rising from the implementation of equality, and the trade-off between efficiency (covered by the solution indicator using the Hydropower revenue objective) and equality (covered by the solution indicator using the equality objective).

7 Conclusion

In this thesis, the notion of a chosen distributive justice principle for its use in a multi-objective optimization (MOO)-model for a multi-purpose water reservoir system was operationalized. The chosen distributive justice principle was egalitarianism with its equality defined as the goal to minimize the relative distance between the multiple purposes of the water reservoir system. Existing MOO-models included objectives to maximize the benefits of each objective. This aligned with the efficiency principle, i.e. to gain as much as possible. Equality was as an important principle to be included in the optimization as efficiency. Therefore, its (operationalized) formulation was inserted in the existing objective formulation, and in this sense complemented the traditional formulation. This provided a reference on how the traditional (and thus existing) Pareto front shifted when an objective was inserted that considered distributive justice. For an existing MOO-model, the EMODPS-model of the real-world case-study of the Lower Susquehanna River Basin was used, which has six conflicting objectives that need to be optimized during a year of severe drought and its water allocation was managed by the release decision of its Conowingo Reservoir System.

Moreover, I point out the need for understanding how the operationalization of distributive justice shapes the implications drawn from the 'optimal' outcomes of decision-support MOO-models. Without determining what the influence is on the optimization of distributive justice, normative uncertainty is added to the outcomes.

To assess the difference in implications drawn from outcomes based on the chosen operationalization methods, this research uses a rival framings approach. In this rival framings, the traditional optimization formulation was complemented with an equality objective, yielding a total of six distinctive equality objectives. The difference lied in their operationalization formulation that consisted of a combination of changing the *inequality metric* and *aggregation method over time for the inequality metric*. Moreover, the *inequality metrics* studied are the *Gini-coefficient* and the *Euclidean distance*. Both metrics aim to quantify relative differences across objective values but have a different mathematical formulation, making it a perfect example of how a subtle difference in logic can affect the implications. In addition, studies found that aggregation methods affected model outcomes. Since the objectives require an aggregation method over time, differences here also affect implications drawn from outcomes. The aggregation methods studied are the *daily-based yearly mean* which represented the entire time horizon, the *monthly-based standard deviation over a year*, and the *ratio of the monthly standard deviation and yearly mean*. Hence, seven - 2 inequality metrics x 3 aggregation methods, including the traditional optimization - optimizations were run and compared in terms of achieved equality and efficiency. In the following Section, the sub questions are answered based on the previous Chapters, and finally the main research question for this research.

7.1. Research questions answered

Research Question : Sub Question 1

How do varying formulations for inequality shift the existing (baseline) trade-offs across objectives?

The solutions of maximum, median, and minimum are compared for the equality objective to each other and to the traditional (reference) solutions. For the median I argue that the *Euclidean Mean* and the

Euclidean Deviation lead to the least deviation across objectives, and in this sense highest equality across objectives. Trade-offs are much weaker. Nonetheless, the efficiency of objectives drops significantly. On the other hand, the *Gini Mean* and *Gini Deviation* leads to less equality (stronger trade-offs) than the *Euclidean* formulations, but will yield more equality than the *Traditional* formulation. In contrast to the *Euclidean* formulations, the efficiency of objectives will be higher. Hence, the equality objective satisfaction is less sensitive to the *Gini* formulations. Out of the three aggregation methods, the *Ratio* leads for both inequality metrics to the least inequality across aggregation methods. Additionally, the *Euclidean Mean* is the only formulation to change the trade-offs across objectives since Baltimore will now have the highest efficiency gain (in the Traditional the lowest), and for other formulations, it has the lowest. Thus, other formulations lead to more equality but maintain a similar trade-off. The trade-offs are similar to the *Traditional* formulation, except for the *Euclidean Mean*, *Gini Mean*, and *Euclidean Deviation*. The maximum solutions point to similar results with the difference that the *Gini Mean* leads to the highest equality across objectives. Moreover, it will also yield the highest efficiency for objectives, and even higher than for the Traditional formulation. The *Euclidean* formulations still lead to a high degree of inequality at the (further) cost of efficiency. The *Ratio* formulation for both inequality metrics yields again the lowest equality (largest trade-offs), however, a higher equality than the *Traditional* formulation.

The minimum solutions point to contrasting implications since the *Ratio* formulations now provide the highest equality across objectives. The high equality is followed by the *Euclidean Deviation*. Other formulations lead to a lower equality than the *Traditional* formulation, while for the median and maximum solution, a different pattern was observed. This indicates that the trade-off depends on the indicator, even if it is found that one formulation leads to a large shift in the trade-off, either for more efficiency or for more equality.

Research Question : Sub Question 2

What is the role of the inequality metrics on shifting the solution space of the Pareto front?

When neglecting the *aggregation method over time* in the analysis I conclude that the inequality metric plays a key role in the level of achieved equality, and level of achieved efficiency. Summarizing the results, the main conclusions are: a) The *Euclidean* formulations lead to more equality because of the penalizing effect the quadratic formulation has compared to the lack of this formulation in the *Gini*, b) as observed from the *Euclidean Mean*, the higher the equality, the larger the shift on the Pareto front. and c) the distribution of the solution space widens when equality is introduced in the objective formulation.

The latter is an interesting finding. I infer that the widening of the solution space stems from the aggregated formulation for equality among the existing objectives. Therefore, more combinations of redistribution of water become apparent in order to achieve higher equality. The equality objective does not discriminate against how this redistribution should take place. For example, release decisions to Baltimore with very low water allocation in the existing formulation can now have water (re-)allocated from the Hydropower revenue (with previously the highest efficiency gain), but also from any other objective in that sense. Subsequently, trade-offs point in different directions, widening the solution space. If it is desired to have a smaller solution space, one can disaggregate the equality objective per objective from the Traditional formulation in Equation 3.3, increasing the *curse of multiple objectives* (Giuliani, Castelletti, et al., 2016). Nonetheless, having disaggregated objectives improves the understanding of trade-offs rising from specific equality objectives.

Research Question : Sub Question 3

What is the role of the aggregation method over time on shifting the solution space of the Pareto front?

When observing patterns of the aggregation method, I infer that across aggregation method formulations the shift in the Pareto front is less significant than across inequality metrics. The inequality metric has a larger significance in this shift. Furthermore, in terms of consistency, for efficiency, the pattern is not consistent across aggregation methods. The *Deviation* and *Ratio* have a similar distribution for the solution space, but differ in trade-offs across objectives. The *Mean* has a different distribution and trade-off. In terms of equality, for the median and maximum solution indicators, the *Deviation* yields lower equality across objectives than the *Mean*. The *Ratio* yields the lowest equality of the three. Fletcher et al. (2022) finds that if the *Mean* and *Deviation* are optimized as a *Ratio*, chances are that they gain during the optimization in opposite directions. This could explain the worse-off behaviour of the *Ratio*. Also, I infer that the optimization is not able to find a Pareto-optimal solution space, where both the aggregated (yearly mean) and disaggregated (intertemporal, monthly deviation) aspects are considered to collectively yield higher equality in terms of the Deviation score (which is used as performance indicators for inequality). However, in the minimum solutions, this pattern breaks where the *Ratio* yields the highest equality, followed by the *Deviation*, and finally the *Mean*. Depending on the indicator used, this shifting effect yields more or less equality. The trade-off across objectives does not follow a specific pattern from the chosen aggregation method. Nonetheless, the difference between the *Ratio* on one hand, and the *Mean* and *Deviation* (among the latter two the difference is smaller) on the other, is large when considering equality. The inequality metric is of higher influence.

Research Question : Main Question

How do different operationalization formulations for inequality in existing multi-objective optimization models shift the Pareto front?

From this study, I conclude that the trade-off between equality and efficiency is highly dependent on the chosen formulation for equality. As a reference, I highlight the behaviour of the *Euclidean Mean*, which yields relatively higher equality at the expense of the efficiency gain. Moreover, if it is desired to achieve high levels of equality as for the *Euclidean Mean*, the Pareto front will shift drastically compared to other formulations. For other formulations, the shifting effect will be smaller, but all point out significant changes in the a) level of equality, b) the trade-off across objectives, and c) the strength of the trade-offs.

7.2. Scientific and societal implication

In terms of scientific implications, using the conclusion above, it must be acknowledged that there is a large bias introduced from the choice of formulation for equality. Many MOO-studies adopt formulations in the line of the *Gini*, but I find that this does certainly not guarantee maximum distributive justice is reached (at least in terms of equality). I infer that the same conclusion will hold for other distributive justice principles modelled. Rather than contrasting ethical viewpoints, the focus should lie on finding ways to unify these viewpoints. An attempt was made by combining the aim of efficiency and equality.

To further unify collective views, unifying the views under the same distributive justice (ethical) principle is as important. Future studies need to address this normative uncertainty by contrasting the chosen objective formulation for the distributive justice principle with *at least* one other formulation. The focus should lie on contrasting it with another inequality metric formulation. Here I infer that the inequality metric influences how relative distances are compared and thus optimized. Considering its influence, it is time now to open the uncertainty box. So, it is time to further explore possible inequality metrics beyond the widely adopted *Gini*, or (now) *Euclidean*. Each inequality metric will have a different formula with penalizations through quadratic, polynomials, exponential, and linear distances, and thus its own way of considering justice.

In contrast, I debate that the chosen aggregation method is of less influence, but will most certainly shift the Pareto front and thus drive the implications. Therefore, future research should critically reflect on what type of justice their aim is. If intertemporal justice is needed, consider the use of methods such as the *Standard deviation*, or other formulations to calculate intertemporal injustices. Moreover, next to having a critical reflection, the limitations of MOO-model need to be considered, and not just 'rely on' that it is doing what it should do. For example, the *Ratio* of Deviation and Mean will not yield beneficial results for equality nor justice, while its formulation is assumed to do this. The criteria posed to a MOO-study must be realistic before its complexity is impossible to handle.

If future research decides to further study the concept elaborated in this thesis, I suggest looking at multiple points that create a broader view of justice in MOO, and create a deeper understanding of justice in MOO. First, it is time to consider relative injustice from a disaggregated perspective. Rather than optimizing the sum of inequalities, it is possible to solely focus on high inequalities. It depends on the context. As it was already found (in the *Ratio* setting too many contrasting criteria for the EMODPS), the more precise the objective formulation is designed to its goal, the more fit the model will be for its purpose. In this sense, a maximum relative distance as done in Ciullo et al. (2020) would be interesting to contrast with *aggregated Euclidean Mean*. Moreover, for future research, I suggest improving the methods to consider positive and negative variance. In doing so, considering in which direction the inequality is moving (as discussed in Chapter 6, *discrimination of variance*) will improve the combined satisfaction of efficiency and distributive justice.

In terms of social implications, if our decision-making makes use of reliable decision-support models, it is time to implement justice in these models and continue with the extraction of advice from these models. Simulation-based MOO-models such as the EMODPS-model used in this study are a way of dealing with the complexity of our world by putting its most relevant components flexibly, and modularly in a model. The time has come to stop using models such as the CBA that are a snapshot of a situation with many assumptions introduced to study its future implications. Not only will it not be possible to achieve efficiency in this manner, but it will also not be possible to achieve justice.

To have higher usefulness from these justice-included models, society must further reflect what their stance is on justice. For example, it needs to address how this justice formulation is related to risk aversion. This is seen in the results where the minimum solutions imply the optimization of the *Euclidean or Gini Ratio* - a formulation that would for the maximum and median solution lead to high inequality - if high equality is desired. The optimization of formulations to minimize the inequality for the minimum solution could be seen as the aggregation of undesirable scenarios where actors (or stakeholders) want to avoid injustices at all costs. Vice versa, optimizing for the maximum solutions connects to scenarios where actors have a positive prospect about the future they want to optimize. In these statements, the probability of achieving these desired results is not part of the scope. What is part of the scope, is to include what actors want before getting to pragmatic implementations. Even with a lack of data, that is seen as the holistic purpose of this thesis. Collect views, unify views, and unify distribution.

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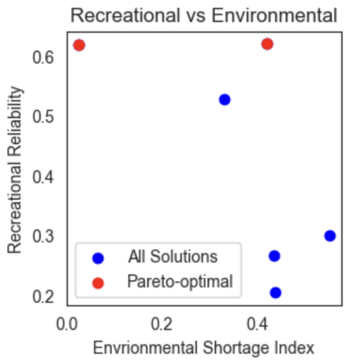
A

Parallel Axes Plot explained

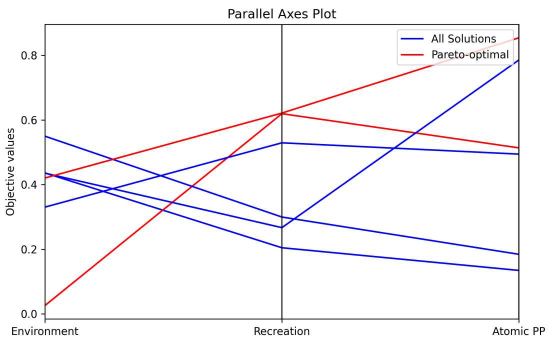
To explain how the Parallel Axes Plot works, Figure A.1 a) exemplifies solutions of an optimization problem with two objectives of the case-study. In the case-study, I want to minimize the environmental shortage index, and maximize the recreational reliability. Both objective values can lie between 0 and 1. All solutions found for this problem are plotted. All solutions are in blue, and the Pareto-dominant solutions are in red, further described as the solutions that are superior to other solutions. A solutions gains this title if the found solution is better than other solutions for (at least) one of the objectives, and at least better than another solution for the other objective. Hence, the solution cannot improve without sacrificing the performance for another objective. In Figure A.1 a), this is possible for the solution with coordinates (0, 0.6) and (0.4, 0.6). In these two Pareto-dominant solutions, the implications of a trade-off directly become apparent, where the recreation objective for coordinate (0.4, 0.6) is maximized at the cost of an increase in the environmental objective of 0.4.

Now imagine that there are not two objectives, but multiple, i.e. dimension higher than two (three or more). It becomes impossible to plot the solutions using a 2-D scatter plot in the same manner. Luckily, the Parallel Axes Plot is the solution to deal with this. Figure A.1 shows that when also the atomic power plant objective needs to be maximized, it is better to show the coordinates of a higher dimension optimization problem using a *solution line*. The solution line passess through the coordinates of each objective, and in this way sheds light on trade-offs across objectives. For example, coordinate (0.4, 0.6) is shown in Figure A.1 through the red line passing through 0.4 on the first axis (Environment), 0.6 on the second axis (Recreation), and 0.8 on the third axis (the Atomic PP).

Now that the Parallel Axes Plot has been explained which is the foundation for the trade-off analysis, I can continue with the analysis of results. The Parallel Axes Plot will be used to see how the set of solutions yields combinations in the objective space (the space of all possible objective values that can be achieved). The Parallel Axes Plot sheds lights on trade-offs between objectives, making it the most useful visualization on how the Pareto front eventually shifts relative to the Traditional formulation when being complemented with alternative operationalization formulations for the distributive justice principle.



(a) Example Scatter Plot. On the x-axis, the environmental objective, to be minimized. On the y-axis, the recreational objective, to be maximized.



(b) Example Parallel Axes Plot. On the axis 1, the environmental objective, to be minimized. On the axis 2, the recreational objective, to be maximized. On the axis 3, the atomic power plant objective, to be maximized.

Figure A.1: Example plots to explain how to convert a scatter plots to Parallel Axes Plots.

B
Formulations

B.1. Objectives

The explanation on the objectives is paraphrased from Zatarain-Salazar et al. (2016). The formulations are inspired from Hashimoto et al. (1982).

Hydropower revenue objective

$$J^{hydro} = \sum_{t=1}^{H} HP_t \cdot p_t \tag{B.1}$$

where J^{hydro} (to be *maximized*) is expressed in $\$/MWh$ and is the sum of the Hydropower generated (in MWh) multiplied by the energy price, at time step t. HP_t generated is further calculated through Eq. B.2.

$$HP_t = \eta \cdot g \cdot \gamma_w \cdot \Delta\bar{h} q_t^{turb} \cdot 10^{-6} \tag{B.2}$$

where η is the turbine efficiency, g is acceleration due to gravity, γw is the water density (1000 kg/m^3), ΔH is the net hydraulic level in metres (reservoir level - tail water level) and q_t^{turb} is the turbine flow in m^3/s at time step t.

Chester, Baltimore, Atomic power plant reliability objective

$$J_{VR,i} = \frac{1}{H} \sum_{t=1}^{H} \frac{Y_{i,t}}{D_{i,t}} \tag{B.3}$$

where $J^{VR,i}$ (to be *maximized*) is expressed in $volumetricreliability$ ranging between 0 (no reliability) to 1 (full reliability). i can be for the three actors of Tab. 3.1 - Baltimore, Chester and Atomic Power Plant. At time step t for actor i, volumetric reliability is expressed as the ratio between the water allocated $Y_{i,t}$ in m^3 and the demand $D_{i,t}$ in m^3.

Environmental shortage index objective

$$J_{SI} = \frac{1}{H} \sum_{t=1}^{H} \left(\frac{\text{max} \underset{(}{} Z_t - Y_t, 0)}{Z_t} \right)^2 \tag{B.4}$$

where J^{SI} (to be *minimized*) is expressed in the self-formulated $shortageindex$, that is relative to the FERC [1]. The $shortageindex$ ranges between 0 (high shortage) to 1 (no shortage). At time step t, volumetric reliability is expressed as the ratio between the difference of the FERC flow requirement Zt and the water allocated Y_t divided by the FERC flow requirement $Y + t$, both variables expressed in m^3

[1] Federal Energy Regulatory Commission reponsible for environmental flows of River Basins such as the Susquehanna River Basin.

Recreational objective

$$J^{SR} = 1 - \frac{n_f}{2N_{we}} \tag{B.5}$$

where J^{SR} (to be *maximized*) is expressed in $storage reliability$ in weekends of touristic periods, ranging between 0 (no reliability) to 1 (full reliability). n_f is the number of weekends the storage falls below the level, divided by the total number of weekends N_{we} of the touristic period. The target level is 32.5 m (106.5 ft), a sufficient level for boats to drive on the reservoir.

C
Remaining results

C.1. Convergence plots

To summarize the findings, the GD indicates that the solutions for all formulations converge after 50,000 function evaluations.

The EI shows that for all formulations, except the Gini Mean, the ϵ dropped below 0.3, indicating a high convergence. The Traditional formulation showed quite some diversity across seeds, as well as for one seed of the Gini Ratio formulation. Other formulations had a reasonably stable behaviour across seeds, indicating its convergence but also that the solution set across seeds is less diverse. There is the risk that solutions converge to a local optimum since the solutions converge relatively fast. However, running more function evaluations would not offer a solution. More seeds need to be run, but this would be beyond the computational load of this thesis.

C.1.1. Generational Distance

In this case, a low metric value was desired as it indicates the average distance between the global reference set and the Pareto front approximation. Generational distance was used in this study mainly to detect an absolute failure in the configurations.

After approximately 50,000 function evaluations, most solutions had converged. In conclusion, the generational distance converges to a reasonably low distance value from the global reference set, which aimed to represent the real Pareto front.

C.1.2. Epsilon Indicator

The additive epsilon indicator measured gaps in the Pareto front, hence, it was a harder metric to meet than generational distance. Similarly to generational distance, this metric was computed relative to a global reference set and a low value was desired as it measured the distance that an approximation set needs to be translated in order to dominate the global reference set.

The Gini Mean principle did not approach low distances, as well as the Euclidean Mean. Conversely, it is these two formulations that showed the least variability in solutions found across seeds. As a matter of fact, the Mean formulations were far off from an ideal distance. An explanation for this could be that the global reference set was calculated without having a methodology that standardized the distributive justice values from each formulation to a uniform value. Therefore, formulations such as the normalized Euclidean Mean with values had a different range to Gini values between 0 and 1. Other formulations converged towards a low distance from the global reference set. However, there was a significant amount of variability for the Gini Ratio, Gini Deviation, and Euclidean Deviation, while for the Euclidean Ratio, the solution was more stable.

Hence, for most of the formulations, the epsilon indicator indicates that there was quite some variability across seeds for each formulation. This may indicate that the solution space from each seed was stuck in local optima, showcasing different trade-offs from the objective space. This was without necessarily being near to the global reference (except for the Euclidean Ratio, Euclidean Deviation, and Gini Deviation).

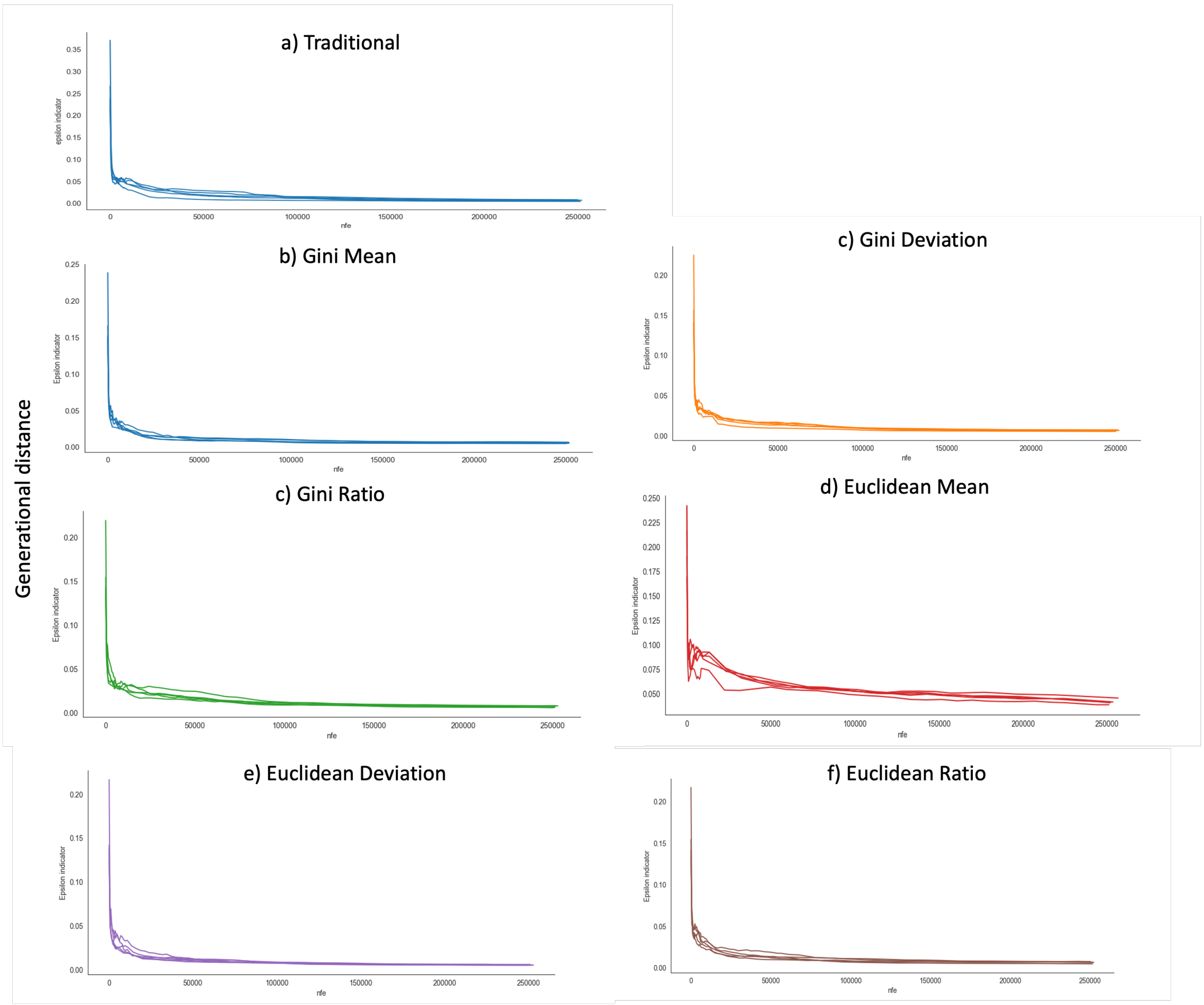


Figure C.1: Generational Distance for each formulation in Tables 4.1, 4.2.

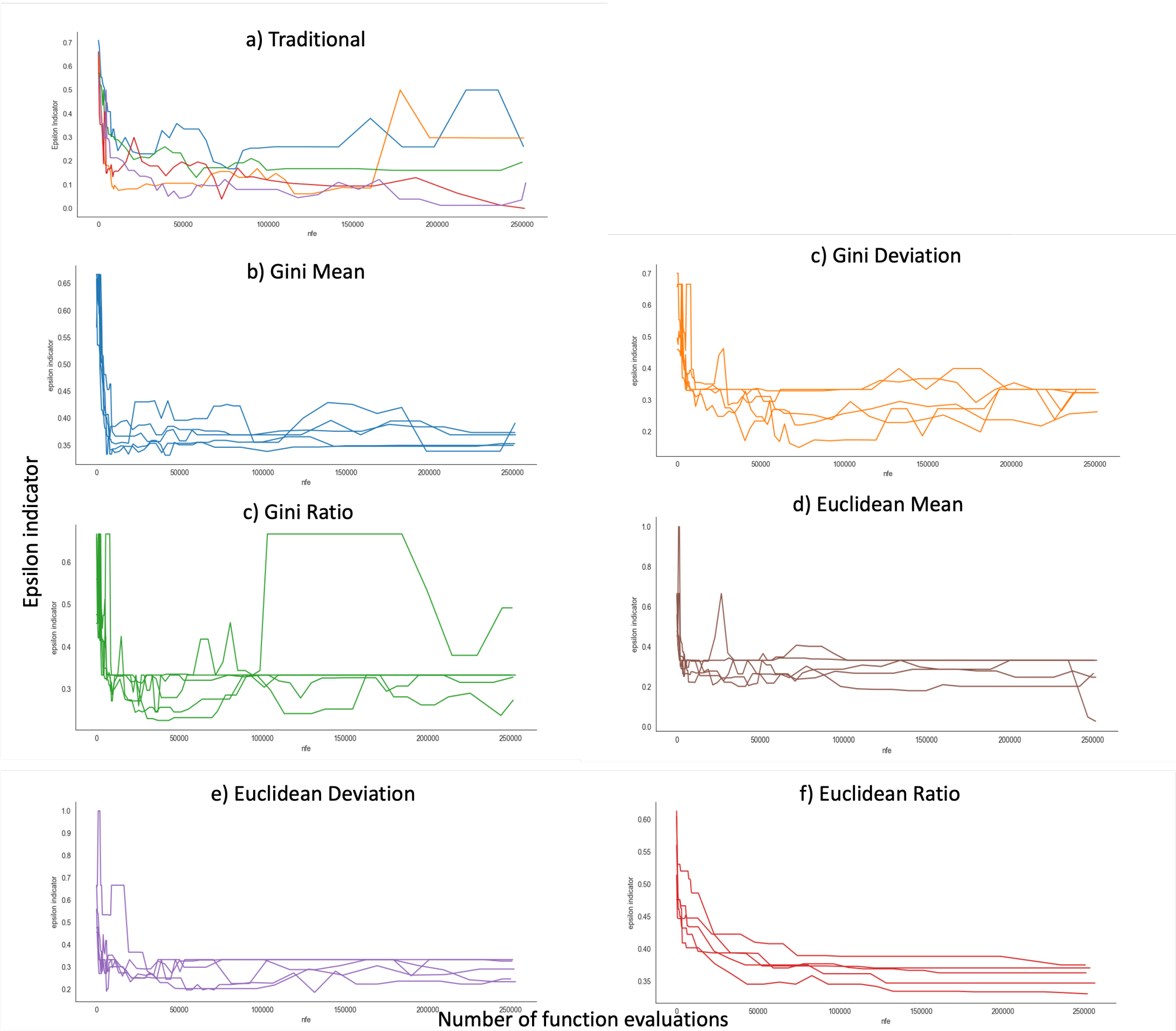


Figure C.2: Epsilon Indicator for each formulation in Tables 4.1, 4.2.

C.2. Statistical analysis

C.2.1. Choice for statistical tests

Non-parametric statistical tests were used to determine if there were statistically significant differences in the distribution between the objectives across formulations. This was done using the Kruskall-Wallis H test, and the Mann-Whitney U test. Both tests were used to determine if there are statistically significant differences across objectives, and across formulations. Both tests were non-parametric, meaning that they did not rely on assumptions of normality or equal variances. For this analysis, a p-value < 0.05 indicated that the H_0 could be rejected.

1. The *Kruskal-Wallis H test* was used when comparing two objectives from the same formulation, and it tested the null hypothesis that the distributions from the two objectives were drawn from equal medians. In light of this context, the Kruskal-Wallis was be tested to determine if the median of the traditional objectives in Equation 3.3 was drawn from the median of the justice objective (i.e. J^{equity}).
2. The *Mann-Whitney U test*, on the other hand, was used to compare the objectives between formulations, and it tested the null hypothesis that the two samples had the same distribution.

For the Kruskall-Wallis test the following null hypothesis was assumed:

H_0 = The samples from the distribution for objective i in Equation 3.3 originated from the same distribution as Equation 3.4. Hence if the null hypothesis was rejected it means that the mean of groups from justice levels was different across a chosen objective i, i.e. the justice levels affected (correlate with) the objective i values.

For the Mann-Whitney test the following null hypothesis was assumed:

H_0 = The distribution from objective i in Equation 3.3 and formulation j in Table 4.1 had the same distribution as another objective in Eq. 3.3 and another formulation in Table 4.1. The probability of distribution were equal.

Hence if the null hypothesis is rejected it means that the distribution of a specific objective (i) for a specific formulation (j) is independent from the distribution of another objective, or over another formulation.

C.2.2. Statistical tests results discussion

Due to the extensiveness of the results across objectives and across formulation, the results are only visible in the notebook for statistical tests. To summarize the analysis, the null hypothesis of the Kruskall-Wallis test can be rejected for all objectives. Hence, the means of the justice objective and traditional objective were independent of each other.

Mann-Whitney across formulations with the same objective: For the Mann-Whitney analysis, interesting insights are formed. The p-value of the distribution for the Atomic PP exceeded a p-value of 0.05 across several of the formulations. Between the Traditional formulation and formulations with distributive justice, the p-value would always exceed 0.05 for the Atomic PP. Subsequently, the distribution for the Atomic PP objective was not unique to the formulation observed.

Interestingly, while the Gini formulations shared a high p-value when comparing the Atomic PP, Chester, and the Environment objectives across different aggregation methods (over time), for the Euclidean formulation the case was less strong. Hence, the Gini, regardless of the aggregation method over time, provided the same distribution for the aforementioned objectives. Nonetheless, the p-value exceeded 0.05 for any Deviation or Ratio formulation, regardless of the inequality metric used. This indicates the strength of the influence the Deviation formulation of Eq. 3.7 has on the Ratio.

Mann-Whitney across formulation with a different objective compared: The distributions across different objectives from different formulations were different such that the null hypothesis can be rejected. Only between formulations with the Mean (aggregation method over time) (i.e. F2, F4, F5, and F7 of Table 4.1) I infer that the distributions of the Atomic PP and Chester are similar (p-value > 0.05). Such formulations shared the same shift in the Pareto front. Note that this does not say anything about the trade-offs across objectives since the combination of objective values to form the Pareto front is insensitive to the distribution. For other objectives, there was a statistical difference between the distribution of objectives.

C.3. Parallel Axes Plot: Traditional formulation with inequality metrics

There was already a subset of solutions with less strong trade-offs between the Hydropower revenue, Atomic PP, and Baltimore. However, this came at the cost of lower values for Chester. Additionally, there were also solutions with a strong opposite trade-off between Chester and Baltimore, where Chester had low values, and the Atomic PP and Baltimore subsequently reached higher values. Finally, there was also a small subset of solutions, that produced no trade-off between objectives and ensured high objective values for each objective. The frequency of such solutions was very low, which was almost not apparent in the objective space, as shown in Figure C.3.

When bringing inequality into the discussion, relatively high equality values were achieved. For the Gini Mean, this lied between 76% and 92%, Gini Deviation between 99% and 100%, Euclidean Deviation between 77% and 95%, the Gini Ratio and Euclidean Ratio between 96% and 99%. Conversely, due to the range of the Euclidean Mean, the metric was normalized according to its own range of values. Subsequently, the range varied between 100% and 0%, with the lowest justice value indeed leading to a strong trade-off between objectives (low Baltimore value and high Chester value, high Hydropower revenue, Atomic PP value, and Environmental value). It should be noted that across inequality metrics, the value for justice of the solutions for the extreme (maximum, median, minimum) of the Hydropower revenue, varied. For example, while the solution for max Hydropower revenue reached the lowest justice value according to the Euclidean Mean, the Gini Deviation indicated that this solution reached the highest justice value. *Similar variations in implications could be noted across other inequality formulations between minimum, maximum, and median. This indicates how dependent the justice values from solutions are on the inequality formulatuion implemented. The inequality formulation resulted in opposing levels of justice across solutions.*

When optimizing over a formulation with an equality formulation, the Traditional formulation had low equality values compared to these formulations. The optimization of other formulations ensured that the relative distance between objectives was minimized. However, each formulation changed the trade-offs observed in the Traditional formulation differently, which is discussed next.

To summarize, the solution of maximum Hydropower revenue led to the strongest trade-off across objectives, and the median solution of Hydropower revenue led to the weakest trade-off across objectives. The inequality formulations that showcased this behaviour were the Gini Mean and Euclidean Mean. Hence, these two formulations can be used as reference objectives to analyze trade-offs between achieving efficiency of the Hydropower revenue, and the according justice value. Note, that this does not mean the standard deviation formulations were not apt to determine justice. Instead, such inequality metrics optimized justice across the intertemporal scale, something that could not be seen in this plot with a different aggregation over time. Since the traditional objectives were calculated in yearly means, the behaviour of justice across objectives was difficult to put into the context of formulations optimizing over deviations across smaller time steps.

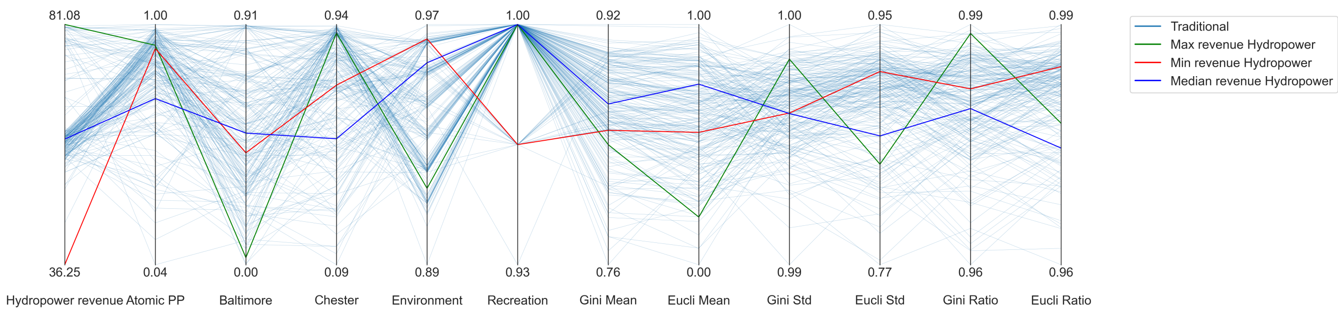


Figure C.3: Parallel Axes Plot: solutions of the Traditional Formulation, with emphasis on the maximum, minimum, median solution of the Hydropower revenue objective. Besides the optimization of the Traditional formulation, the justice score according to each distributive justice formulation is visible.

C.4. Parallel Axes Plots: All solutions sorted by Hydropower revenue formulation

The widespread of solutions was large, which puts into question if having the median, maximum, or minimum as a reference was the best method to show differences. The Hydropower revenue was kept as a reference for efficiency. Subsequently, the deviation was used how much equality is reached.

There was a significant difference between the Euclidean and Gini formulations. The Gini kept the same trade-off as observed for the Traditional formulation. When comparing the Gini Mean with the Euclidean Mean, I notice that the Euclidean Mean is the first formulation to show solutions where Hydropower revenue were favoured while also reaching high values across other objectives. In comparison, each Gini formulation showed a similar Pareto front to the Traditional formulation. The Euclidean Deviation and Euclidean Ratio will resemble the Traditional trade-off. However, baseline (traditional) Pareto-optimal solutions led to higher values for other objectives, or the same Pareto solutions shifted the trade-offs. For example, for the Euclidean Ratio, some of the solutions yielded for Baltimore higher values, while Chester had lower values.

In terms of Deviation, there was little difference to notice since the range of values seemed to be spread over the entire spectrum. A small difference observed was that complementing formulations reached lower Deviation values than the Traditional formulation.

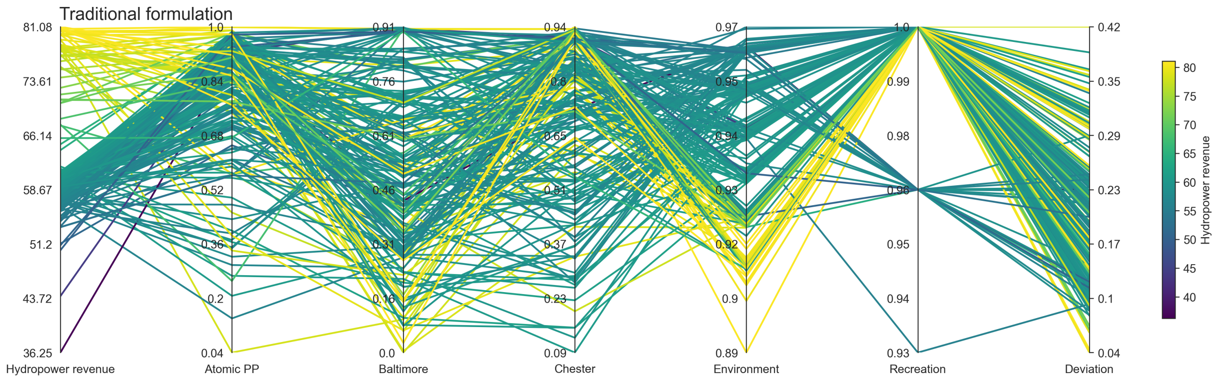


Figure C.4: Parallel Axes Plot: Solutions for the Traditional formulation, sorted by the Hydropower revenue. There is a strong trade-off between the objectives having high values, and Baltimore having low values.

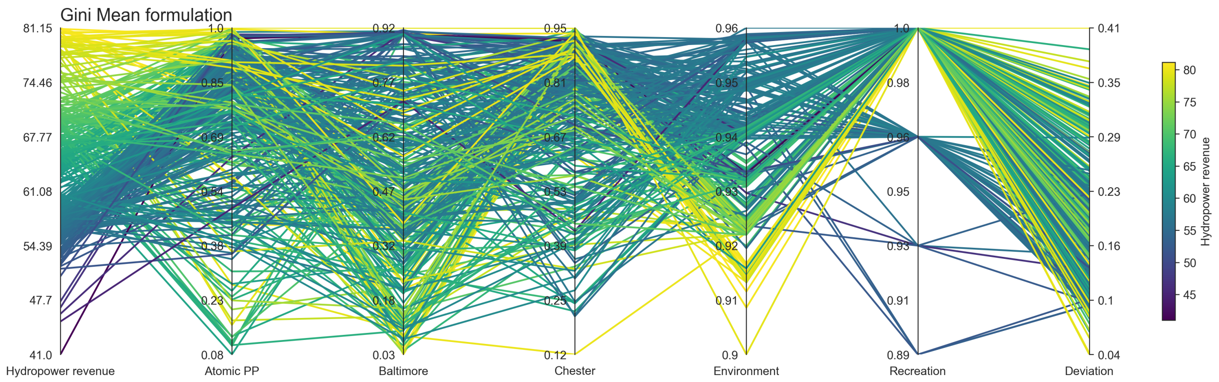


Figure C.5: Parallel Axes Plot: Solutions for the Gini Mean formulation, sorted by the Hydropower revenue. There is a strong trade-off between the objectives having high values, and Baltimore having low values.

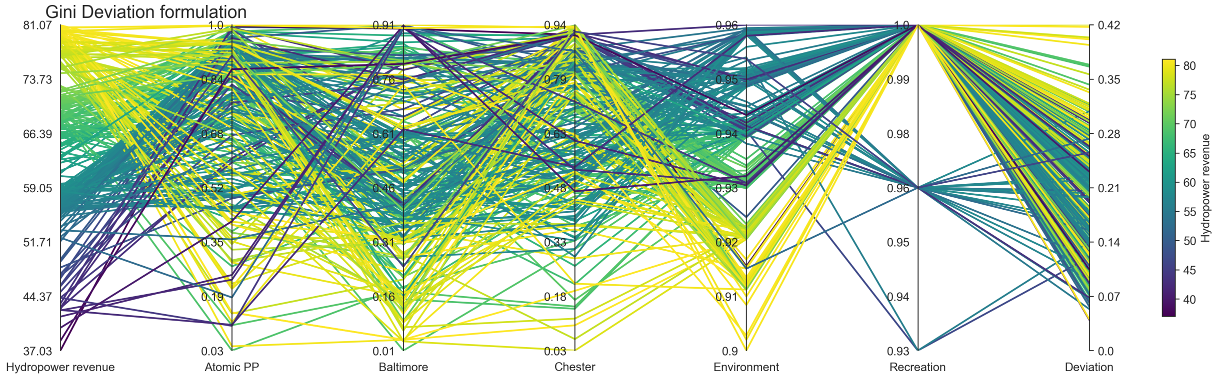


Figure C.6: Parallel Axes Plot: Solutions for the Gini Deviation formulation, sorted by the Hydropower revenue. There is a strong trade-off between the objectives having high values, and Baltimore having low values.

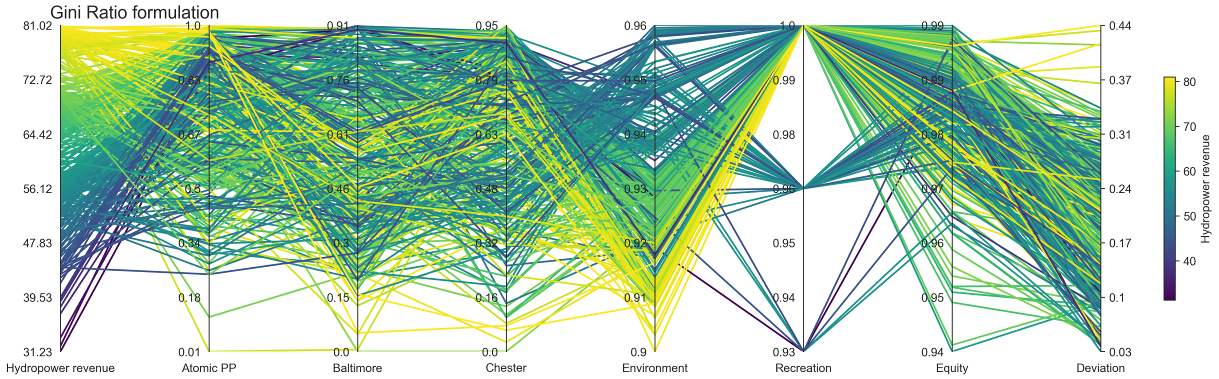


Figure C.7: Parallel Axes Plot: Solutions for the Gini Ratio formulation, sorted by the Hydropower revenue. There is a strong trade-off between the objectives having high values, and Baltimore having low values.

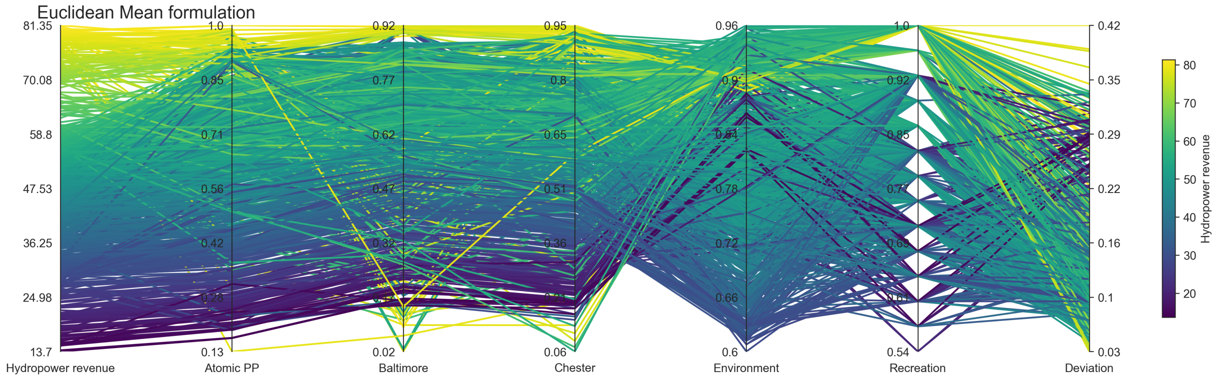


Figure C.8: Parallel Axes Plot: Solutions for the Euclidean Mean formulation, sorted by the Hydropower revenue. There is a strong trade-off between the objectives having high values, and Baltimore having low values.

C.5. Boxplot of inequality scores

In this Section, the discussion focuses on values in relative terms without looking at the values of justice, because the values are merely a measure of relative distance, i.e. inequalities. Hence, what it means to achieve a certain equity value was not representative of an exact degree of justice. The metric values gained relevance when they were compared to each other. To make this possible, while the optimization

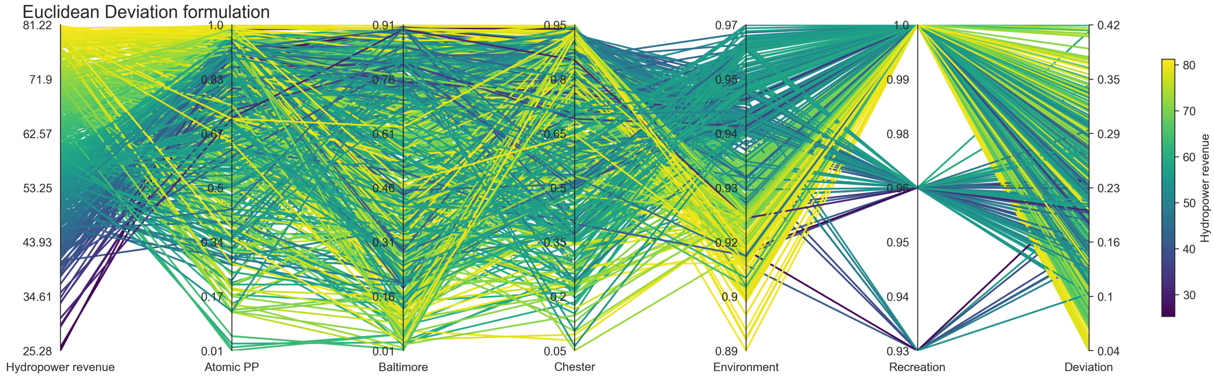


Figure C.9: Parallel Axes Plot: Solutions for the Euclidean Deviation formulation, sorted by the Hydropower revenue. There is a strong trade-off between the objectives having high values, and Baltimore having low values.

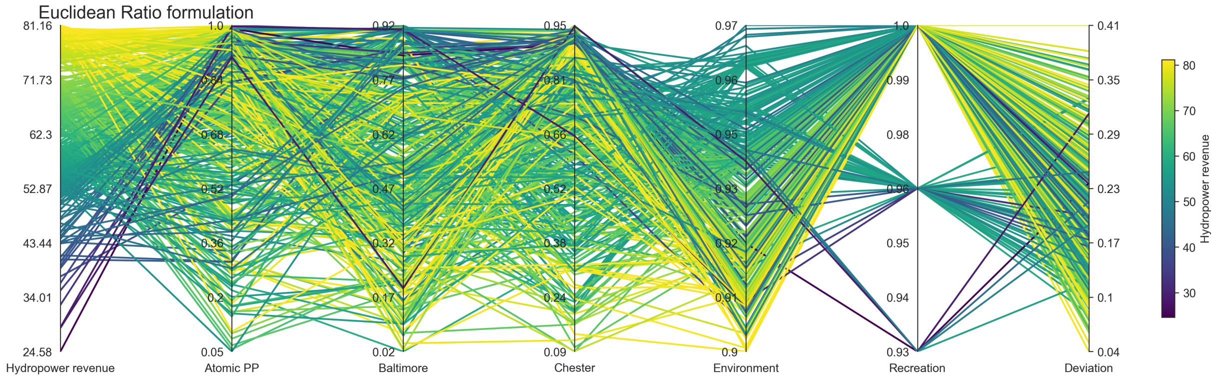


Figure C.10: Parallel Axes Plot: Solutions for the Euclidean Ratio formulation, sorted by the Hydropower revenue. There is a strong trade-off between the objectives having high values, and Baltimore having low values.

was run for a specific distributive justice formulation such as the Gini Mean complementing the Traditional formulation, for each solution found the other metrics were calculated as well, without optimizing over them. Hence, it became possible to compare what degree of justice had been achieved according to one specific formulation, over other formulations. This brought light to differences between justice values achieved across inequality metrics and across aggregation methods over time. One should recall that it was desired to minimize the justice value. Hence the distribution closest to 0 was desired.

C.5.1. Gini Mean inequality score

According to the Gini Mean, there was no difference in justice achieved between the Traditional formulation, Gini Mean, and Gini Deviation. The Gini Ratio led to higher inequality. The Euclidean Mean was the formulation reaching lower inequality. This was also seen in the Parallel Axes Plot. For other formulations, it was unclear why the Traditional formulation had a similar distribution for this inequality score. Especially, when observing the median of solutions, this indicates that the trade-offs were substantially weaker compared to the Traditional formulation.

C.5.2. Euclidean Mean formulation inequality score

Previously it was determined that the Gini Mean and Euclidean Mean best represent when a solution is just due to the horizontal line across objectives observed aligning with a higher justice value in its formulation. Since the Euclidean Mean was normalized to its own solution range space it was most suitable to show the ranges achieved across solutions.

According to the Euclidean Mean, the different formulations will lead to similar justice values. It

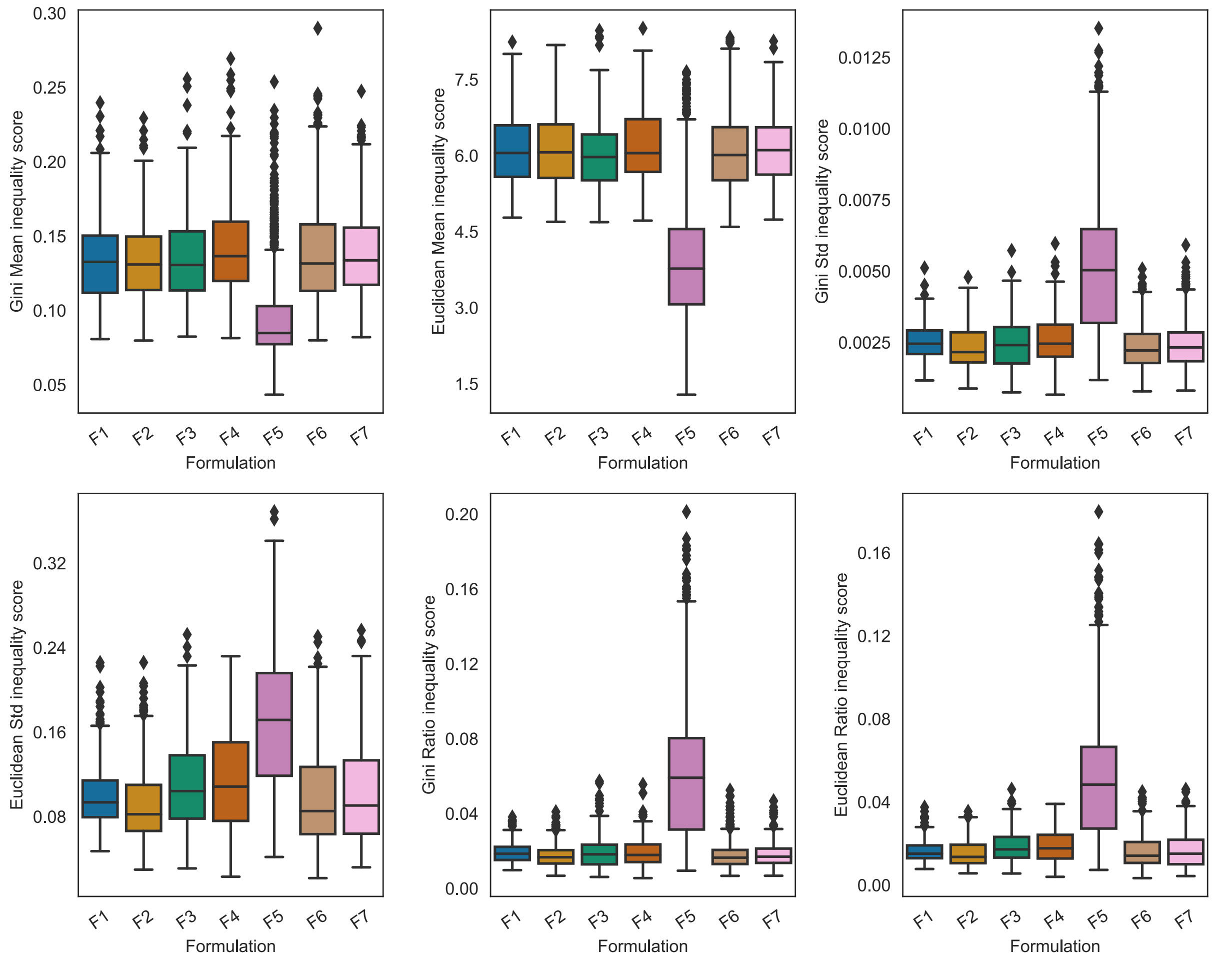


Figure C.11: Boxplot: Distribution of the solutions from formulations in terms of each objective value. F1 is the Traditional formulation. F2 to F7 are the Traditional formulations combined with a complementing equality objective in the objective function. F2 combines the Traditional formulation (F1) with the equality objective using the Gini Mean operationalization formulation, F3 with the Gini Deviation formulation, F4 with the Gini Ratio formulation, F5 with the Euclidean Mean formulation, F6 with the Euclidean Deviation formulation, and F7 with the Euclidean Ratio formulation.

is interesting to note that the justice achieved by the Traditional formulation again was similar to the formulations where distributive justice was being optimized. When zooming into relative differences, while the Gini Mean indicated that the Gini Deviation led to a slightly higher IQ range, for this inequality score, the Gini Deviation indicated a lower IQ range. Thus, there was a difference in justice across inequality metric used for optimization.

Moreover, only the Euclidean Mean reached lower justice values. This was also seen in the Parallel Axes Plot analysis. However, this will be at the cost of efficiency as observed from the distribution of the different objectives.

C.5.3. Gini Deviation, Gini Ratio, Euclidean Deviation, Euclidean Ratio inequality score

Both inequality scores indicated the same. While the Euclidean Mean achieved more justice when looking over the yearly mean objectives, across months, the Euclidean Mean was responsible for a higher inequality. This was an interesting finding, which indicates that there is a trade-off to be made between justice across time (disaggregated) and justice over the entire time span (aggregated).

Furthermore, when looking at the differences across formulations, there were small changes in inequality compared to the Traditional formulation. For this, the Euclidean Deviation was used as inequality score since it best represented the differences. In contrast to the Euclidean Mean, the Gini Mean lowers the IQ range compared to the Traditional formulation. The Euclidean Deviation, and Euclidean Ratio lower the median similarly to the Gini Mean. Conversely, the Gini Deviation and Gini Ratio bring the IQ range up (with the median). This indicates that the penalizing effect for relative distances across months was weak such that the behaviour for distributive justice relative to the traditional maximization was not dominant in the optimization.