Topology Optimization for Computational Fabrication

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Topology Optimization for Computational Fabrication

Jun Wu, Niels Aage, Sylvain Lefebvre, Charlie Wang

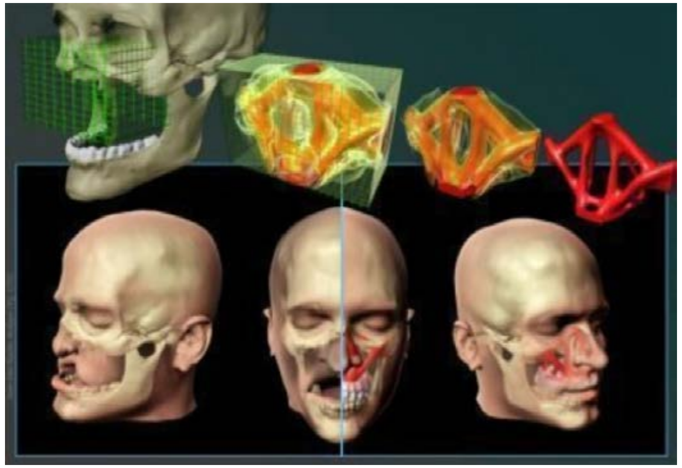
TU Delft
DTU Technical University of Denmark
Inria INVENTEURS DU MONDE NUMÉRIQUE

Topology Optimization: Applications

- Lightweight: Engineering
- Customization: Medicine
- Organic appearance: Art & Archeticture
- ...

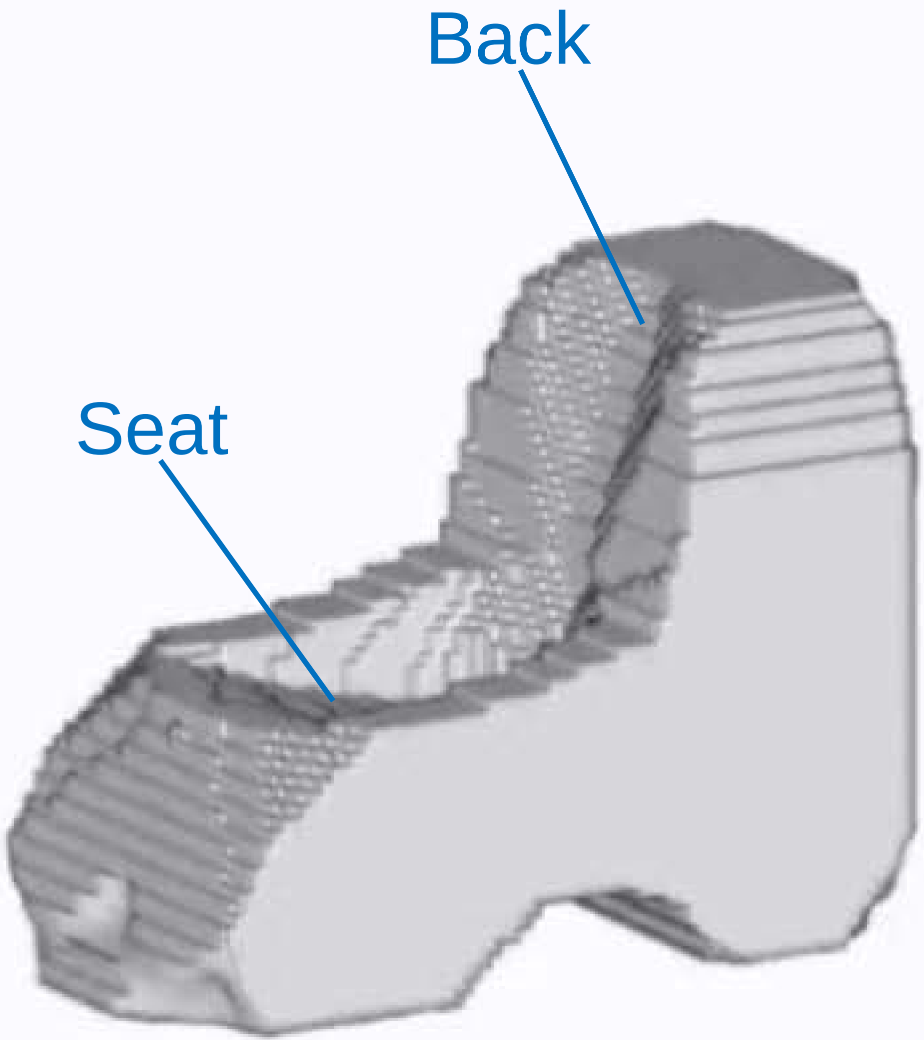
Bone Chair
by Joris Laarman

Airbus & EOS, 2014



Reconstructive surgery
Glaucio H. Paulino @ UIUC

Qatar national convention

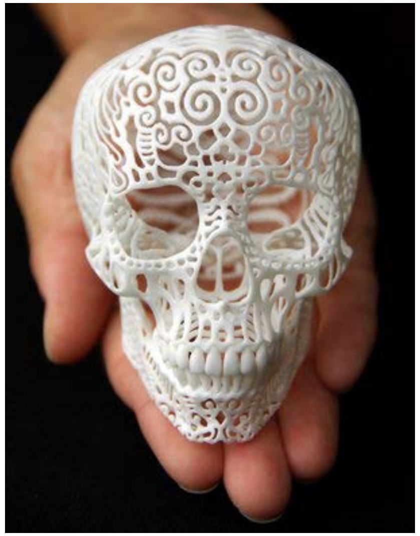


Optimization of Bone Chair
by Lothar Harzheim & Opel GmbH

Additive Manufacturing

- “Geometric complexity is (almost) free”

TU Delft & MX3D, 2015



Joshua Harker

Scott Summit

Topology Optimization

- Lightweight
- Customization
- Organic shape

Additive Manufacturing

- Geometric complexity
- Customization

Schedule

- Fundamentals of
  - Advanced Manufacturing (Charlie Wang)
  - Topology Optimization (Niels Aage)

- Coffee break, and Exhibition of 3D prints

- Controllable Topology Optimization for
  - Geometric Features (Jun Wu)
  - Appearance and Structure Synthesis (Sylvain Lefebvre)

Topology Optimization for Computational Fabrication

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Part One: Advanced Manufacturing

Charlie C. L. Wang

Delft University of Technology

April 24, 2017

Conventional Manufacturing Processes

- Net Shape Processes
  - Forging, drawing, extrusion, rolling
  - Sheet metal forming, bending
  - Die casting, investment casting
  - Injection modeling
- Subtractive Processes
  - Lathing, milling, grinding, drilling,
  - Water jetting, laser cutting, etc.

Challenges for Designers (An Example)

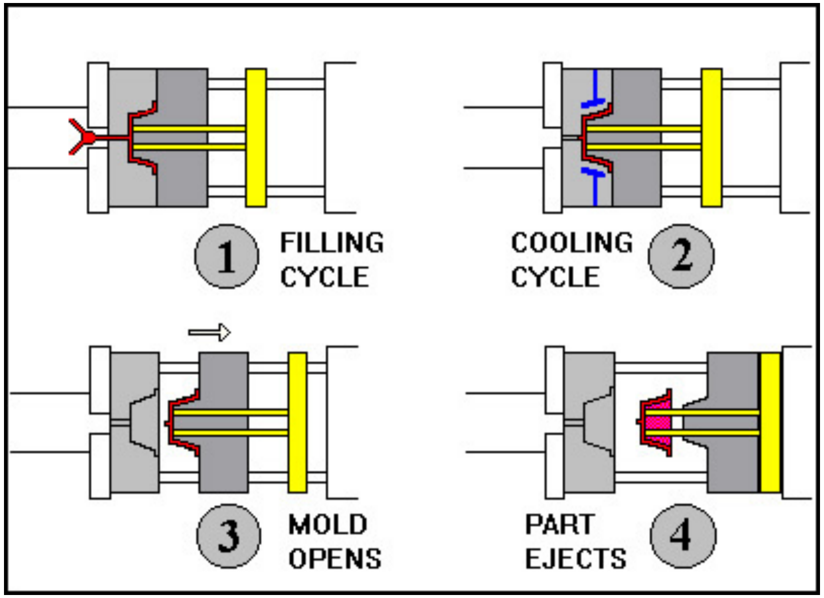
http://www.pyottdesign.com/

https://en.wikipedia.org/wiki/Apple_Mouse

Challenges for Designers (Cont.)

- Conventional Mouse – produced by Injection Molding
- Problems:
  - Complex shape? No
  - Moldability? Important
  - Flexibility? No
  - Customization? No

http://www.imould.com



http://mold-technology4all.blogspot.nl/

- Process to make a mold
  - Mold design (professional)
  - CNC machining (expensive)

Challenges for Designers

- Design a product
  - Cannot be fabricated
  - Shape limitation
  - Cannot have too many parts
  - Otherwise, having a high cost
- Design for manufacturing [1]
  - Rule 1: Reduce the total number of parts
  - Rule 2: Design for easy-to-fabrication
  - Rule 3: Use of standard components
- Main Problem:
  - Conventional manufacturing lacks of flexibility

[1] *Computer-Aided Manufacturing*, 2nd Ed., T.-C. chang, R.A. Wysk, and H.-P. Wang, Prentice Hall, 1998.

Additive Manufacturing

- Defined by ASTM as:
  - Process of joining materials to make objects from 3D model data, usually layer upon layer
- Six Different Types of AM:
  - Lasers: Stereolithography Apparatus (SLA), Selective Laser Sintering (SLS)
  - Nozzles: Fused Deposition Modeling (FDM)
  - Print-heads: Multi-jet Modeling (MJM), Binder-jet Printing (3DP)
  - Cutters: Laminated Object Modeling (LOM)
- Mainly used for Rapid Prototyping (Past)
- More and More used for 'Mass'-Production (Present)

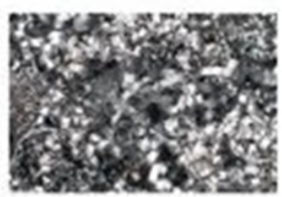
Benefit of Additive Manufacturing

- Very flexible: direct digital fabrication from CAD models
- Rapid fabrication
- Excellent for customization
- Manufacturing is responsible for 33% of the world's carbon footprint – AM has minimal material waste

Subtractive Machining:

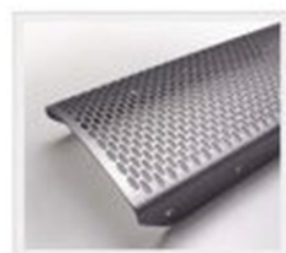
5000 lb.
forged billet

–



4750 lb.
chips

=



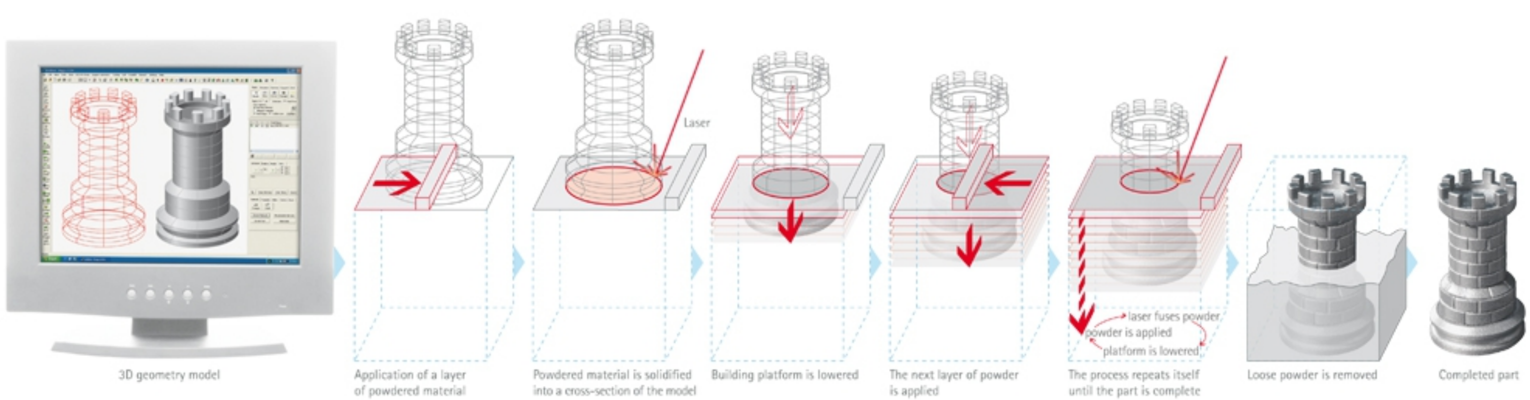
250 lb. finish
machined part

20 : 1
Buy to fly

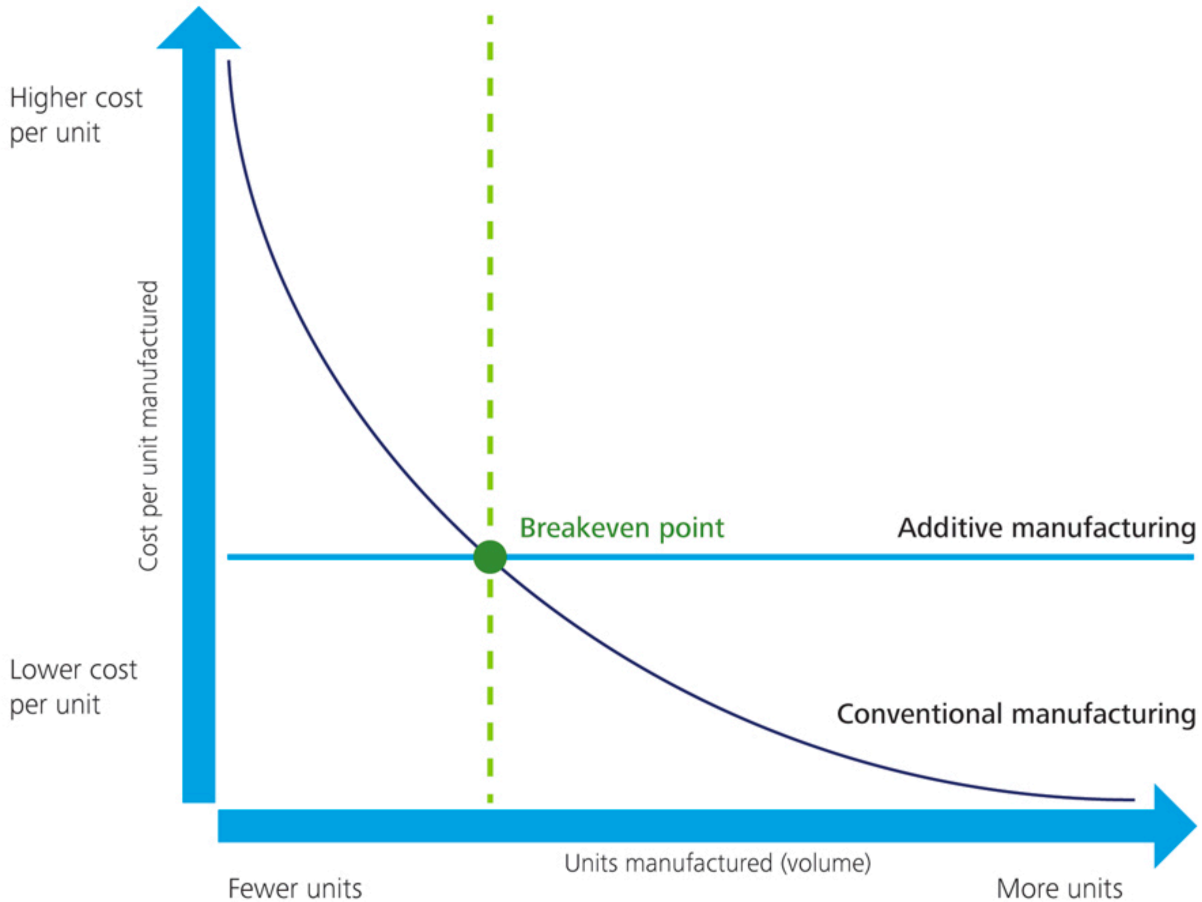
Limitations / Challenges

- Limited part sizes
- Limited fabrication speed
- Limited materials (20k vs. 200 materials)
- Poor surface finish / low accuracy
- Inconsistent part quality
- High cost (machine, material, pre- and post-processing)

General functional principle of laser-sintering



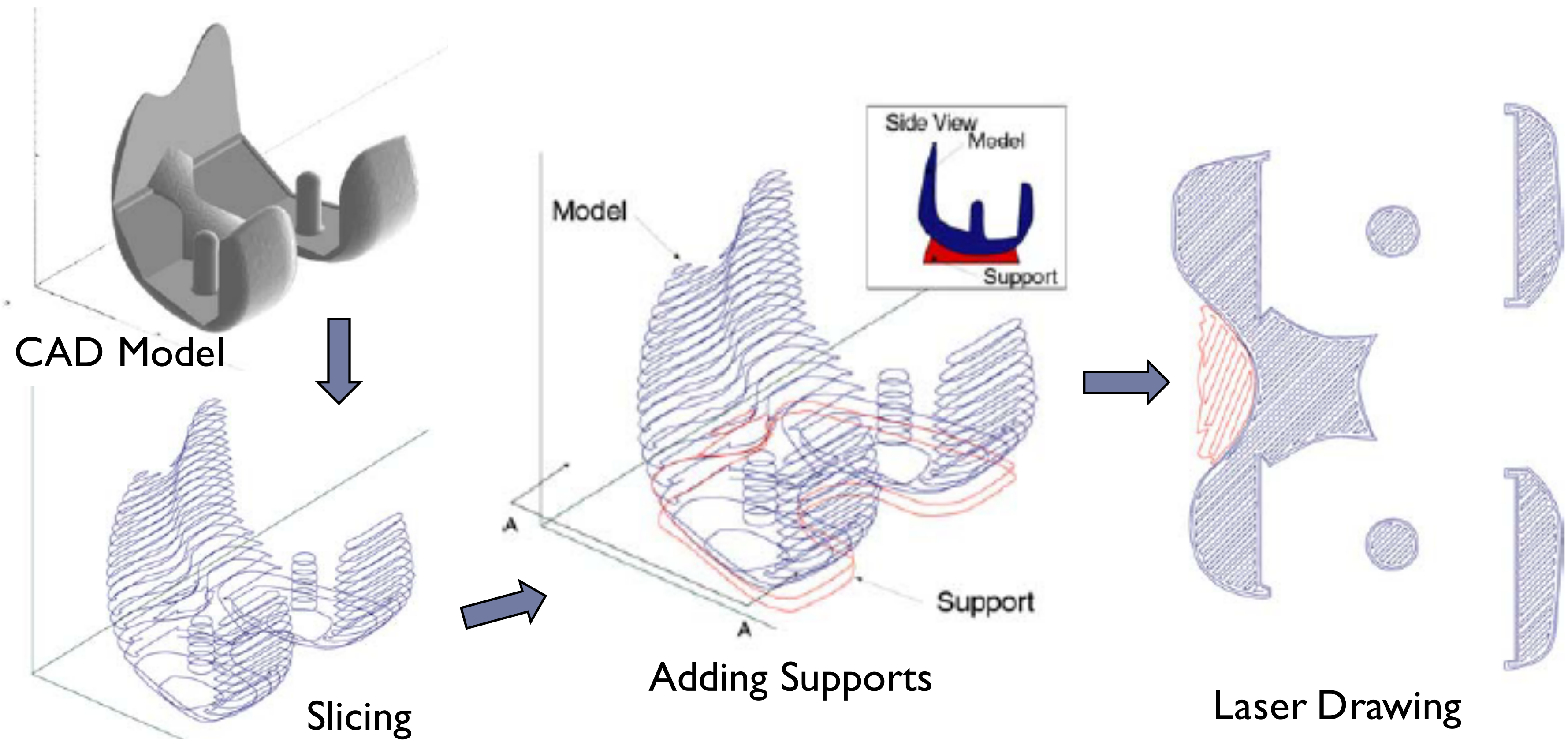
Break-even Analysis of Conventional Manufacturing and 3D Printing



Source: Mark Cotteleer and Jim Joyce, *3D opportunity: Additive manufacturing paths to performance, innovation, and growth*, Deloitte University Press, http://dupress.com/articles/dr14-3d-opportunity/, accessed March 17, 2015.

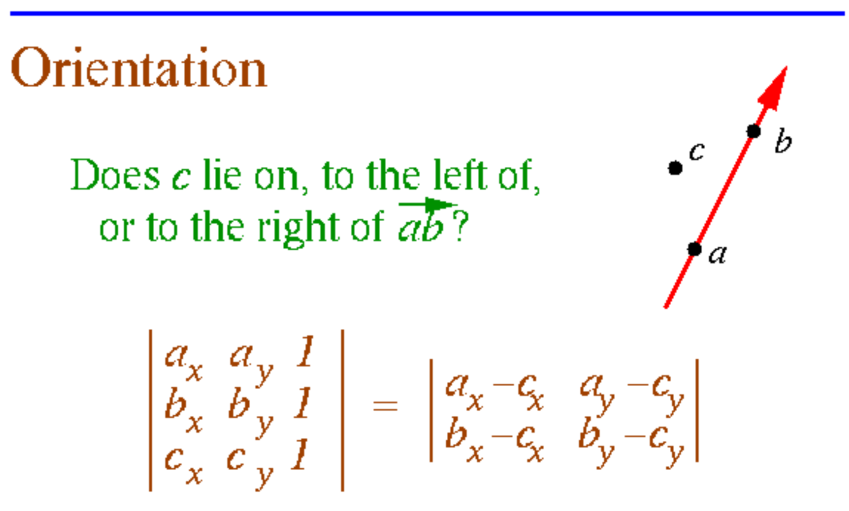
Graphic: Deloitte University Press | DUPress.com

Main Computation Steps in AM

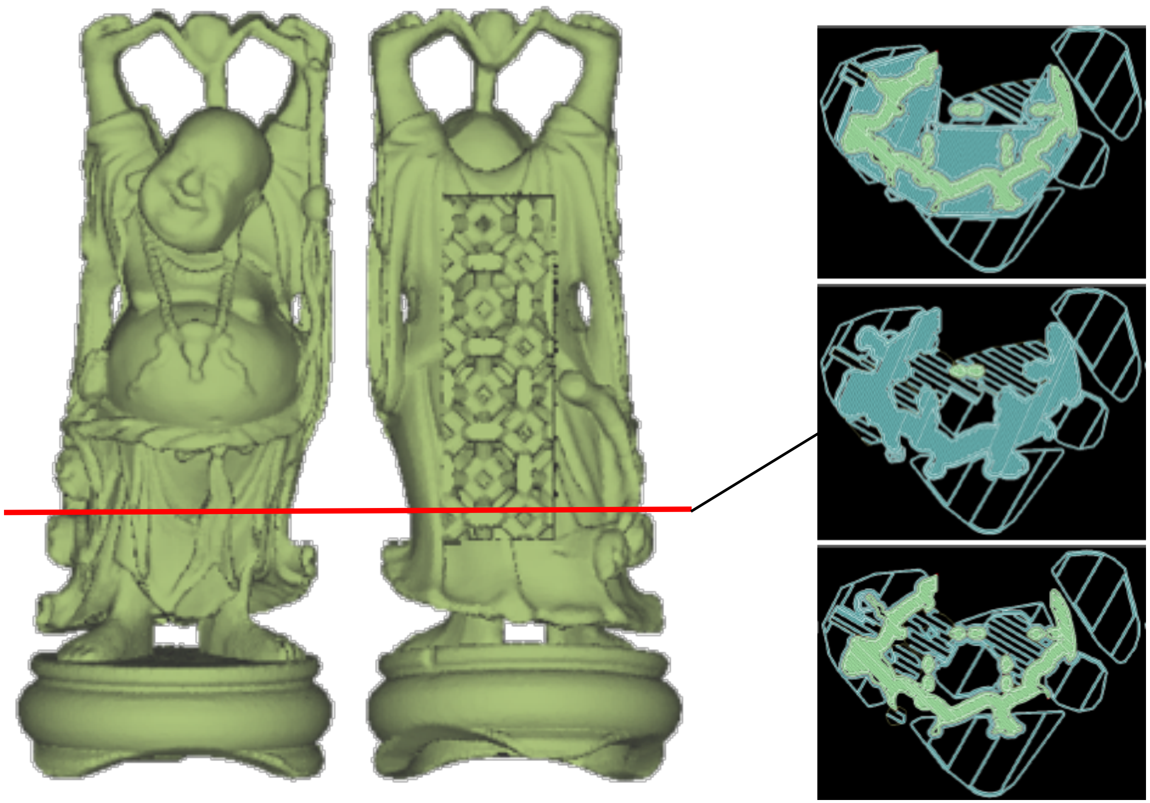


Numerical Robustness

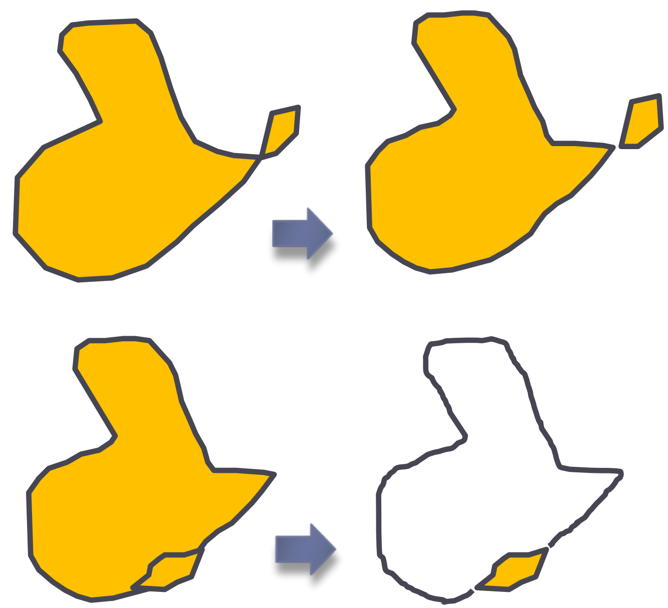
- Computation in IEEE arithmetic
  - Limited precision of floating-point arithmetic
- Geometry becomes inexact after intersection
- Geometric predicates
  - Correct?
  - Self-intersected models?



Problem of Inexact B-rep

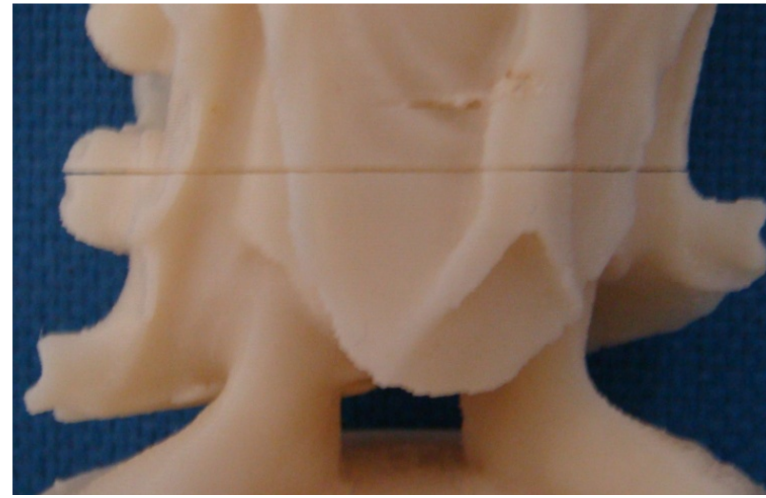


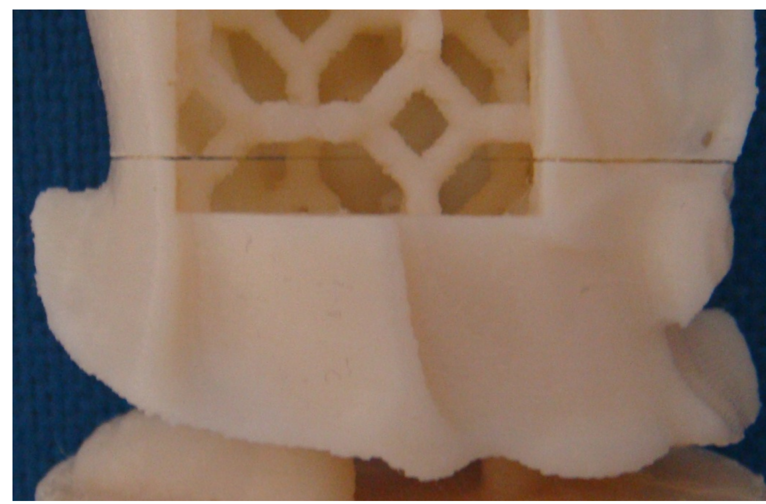
Can it be easily repaired?



P. Huang, C.C.L. Wang, and Y. Chen, "Intersection-free and topologically faithful slicing of implicit solid", ASME Transactions - Journal of Computing and Information Science in Engineering, vol.13, no.2, 2013.

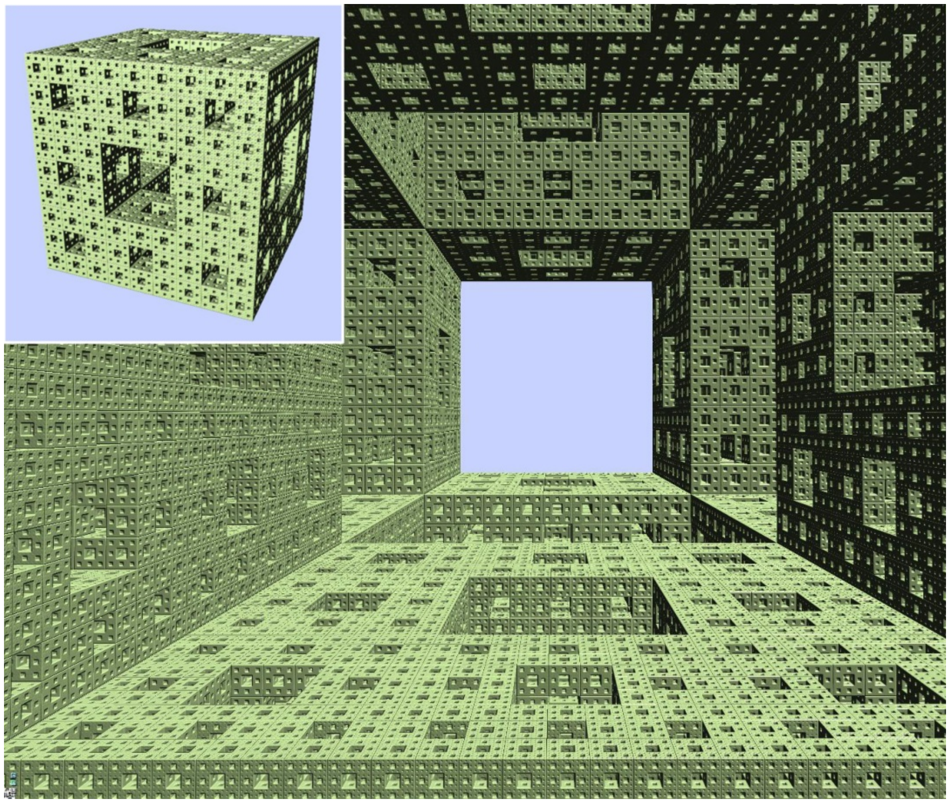
Problem of Inexact B-rep (Cont.)

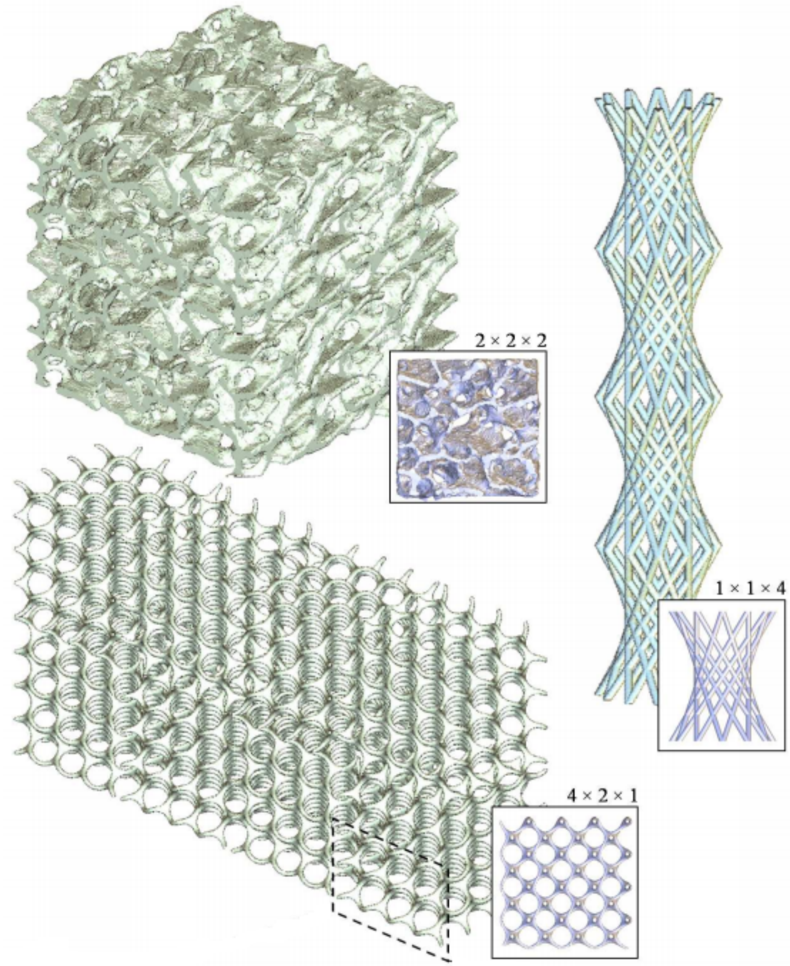




Robust Computation in Image Space

- Voxel representation



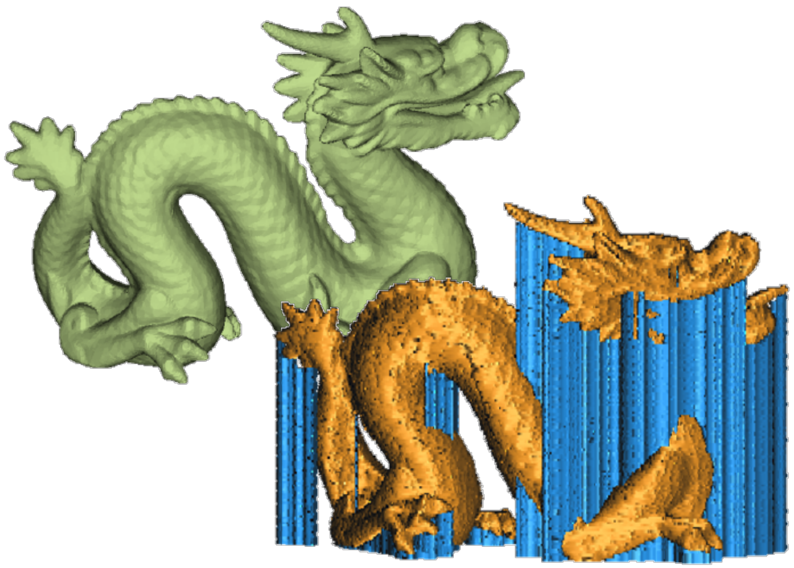


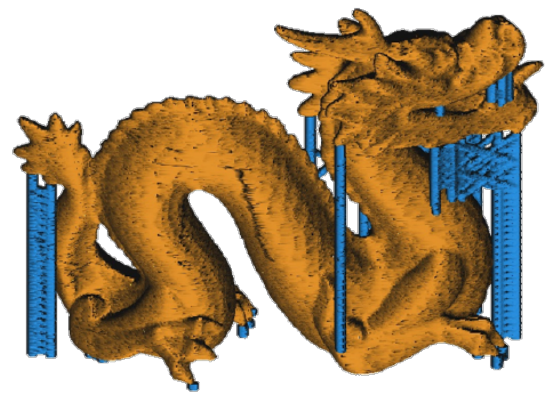
Problem: Memory Cost is extremely high

Yuen-Shan Leung, and Charlie C.L. Wang, "Conservative sampling of solids in image space", IEEE Computer Graphics and Applications, vol.33, no.1, pp.14-25, January/February, 2013.

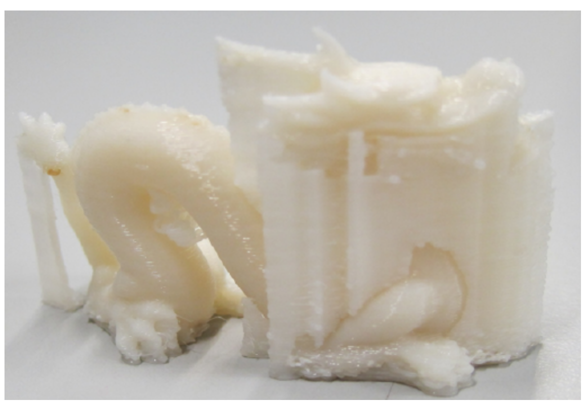
Supporting Structure?

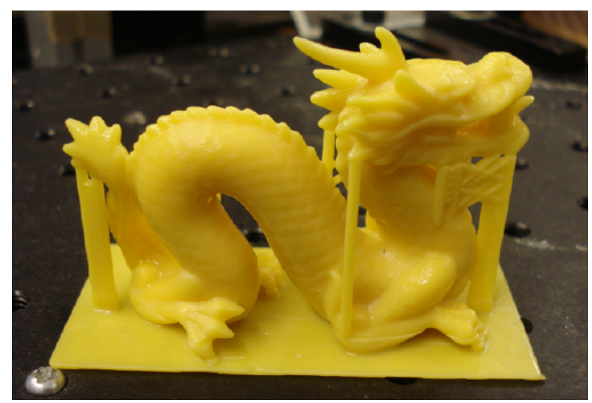
Difference? Why and how?





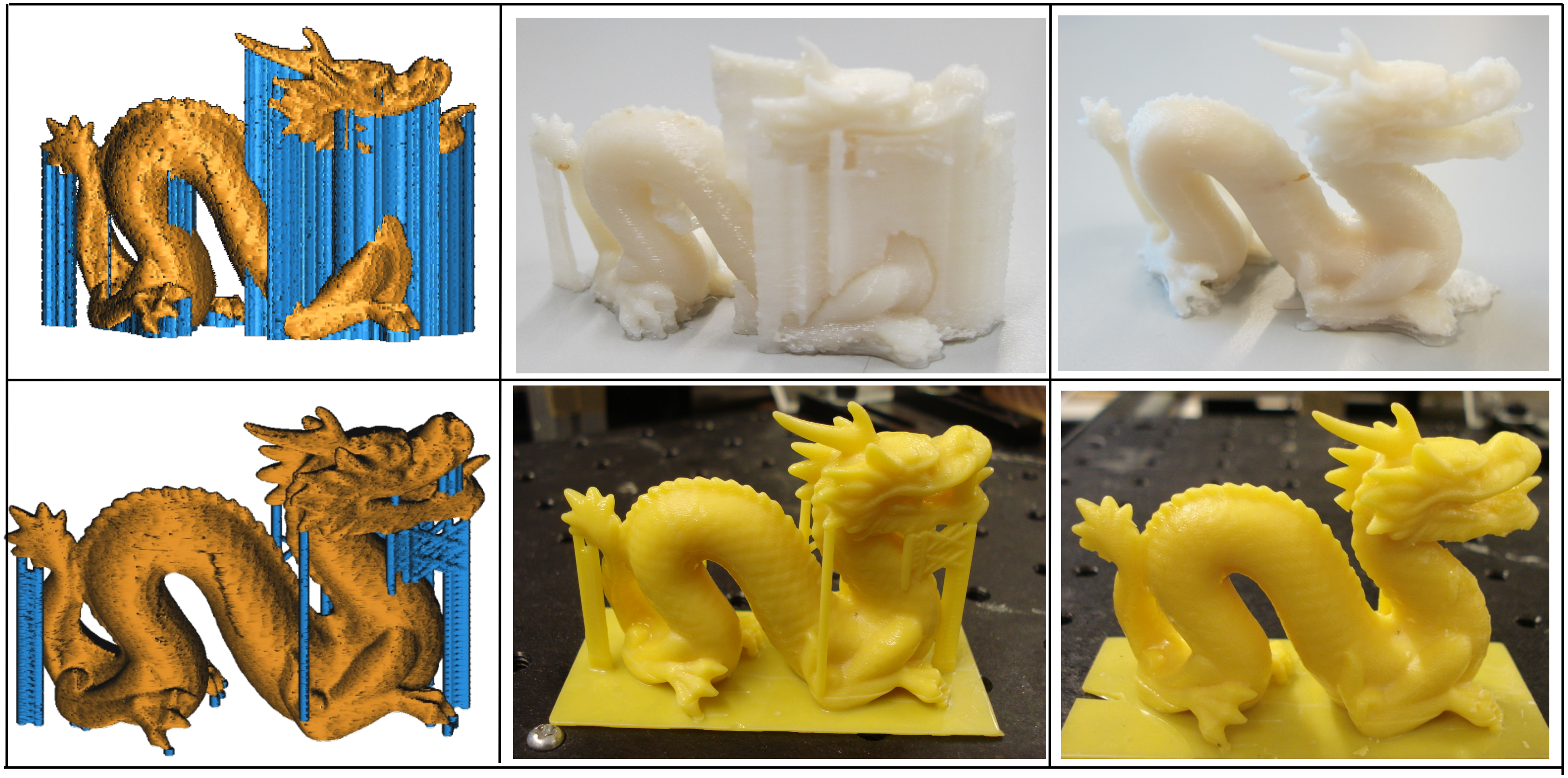
Multi-Materials: Resolvable materials for supporting structure





Single Material: Using structures to support

Support Structure Generation



Direct slicing and support generation resultant contour | Fabricated part with support | Fabricated part after removing support

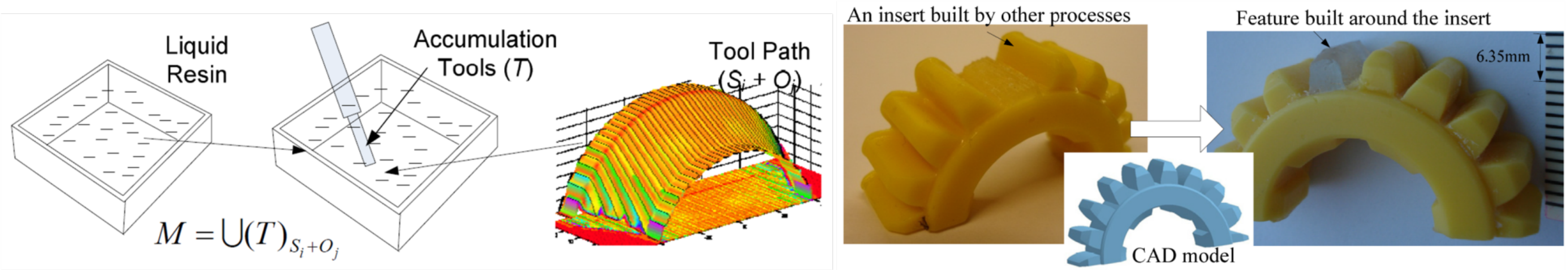
Pu Huang, Charlie C.L. Wang, and Yong Chen, "Algorithms for layered manufacturing in image space", Book Chapter, ASME Advances in Computers and Information in Engineering Research, vol.1, pp.377-410, 2014.

GPU-based Implementation

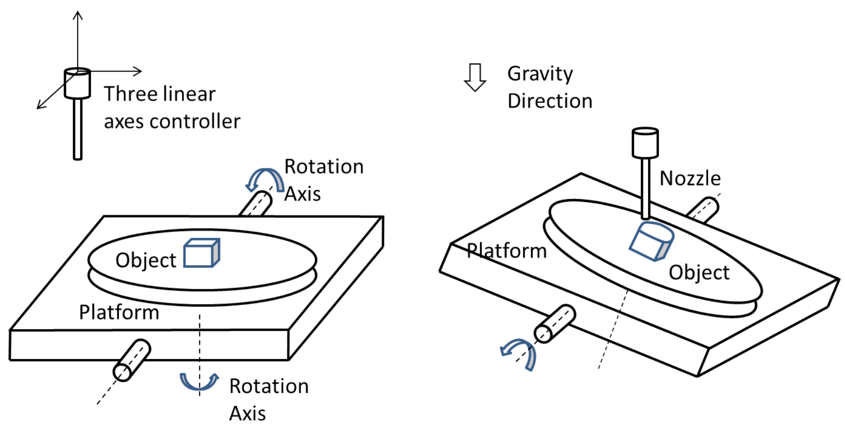
https://youtu.be/G75mS1VGqx0

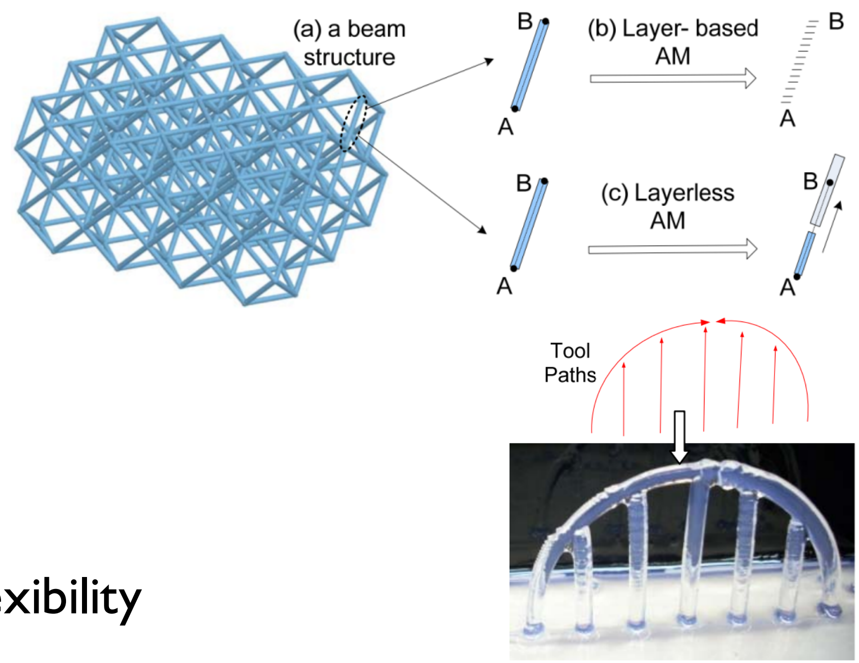
http://ldnibasedsolidmodeling.sourceforge.net/

2.5D vs 3D Printing



CNC accumulation for build-insert-around





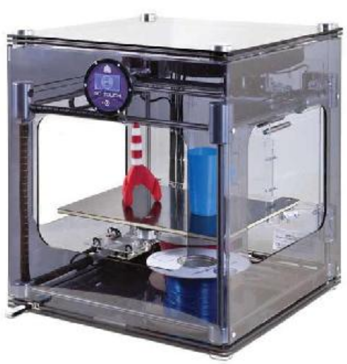
Develop a new non-layered AM

- Fused Deposition Modeling (FDM)
- Multi-axis motion introducing more flexibility

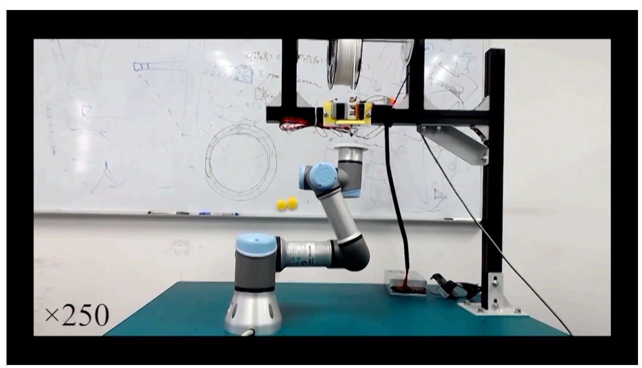
X. Zhao, Y. Pan, C. Zhou, Y. Chen, and C.C.L. Wang, "An integrated CNC accumulation system for automatic building-around-inserts", Journal of Manufacturing Processes, vol.15, no.4, pp.432-443, 2013.

Robot-Assisted Additive Manufacturing

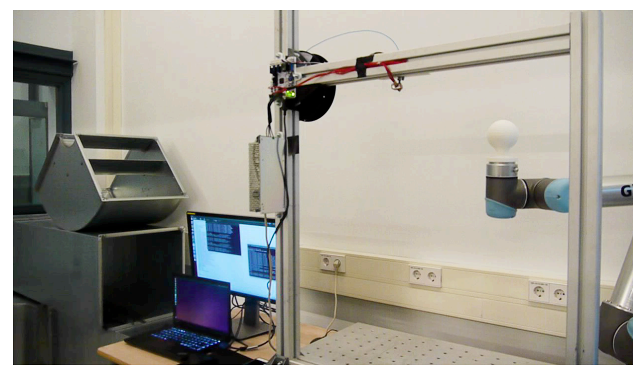
- Using robot arms as device for motion control in AM
- Collaborative operations on two arms – More DoFs to fabricate curved regions / layers
- Challenges:
  - Model decomposition
  - Collision-free tool path generation
  - Configurations in joint-angle space



vs



https://youtu.be/mrR7lKpHo9k



https://youtu.be/5B37oz4cw9s

C. Wu, C. Dai, G. Fang, Y.-J. Liu, and C.C.L. Wang, "RoboFDM: a robotic system for support-free fabrication using FDM", IEEE International Conference on Robotics and Automation (ICRA 2017).

From 3D to 4D Printing

- 3D Printed Self-Assembly Structures
- How to predict the shape of fabricated model?
- Pattern Design / Process Optimization / New Triggers

https://youtu.be/vQB49vNFu14

T.-H. Kwok, C.C.L. Wang, D. Deng, Y. Zhang, and Y. Chen, "Four-dimensional printing for freeform surfaces: design optimization of Origami and Kirigami structures", ASME Journal of Mechanical Design, 2015.

Summary Remarks

- Conventional Manufacturing vs. Additive Manufacturing
- Reduce the challenges for designers
- Slicing and support generation
- Numerical robustness
- Multi-axis 3D printing
- Robot-assisted 3D printing
- 3D printed self-assembly structures (4D printing)

Thanks for Your Questions

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TU Delft   DTU Technical University of Denmark   Inria

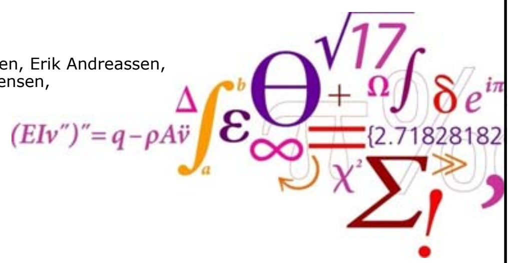
Topology optimization: Basics tools and methods

by Niels Aage

@Eurographics 2017

Mechanical Engineering
Center for Acoustic-Mechanical Micro Systems (CAMM)
Technical University of Denmark (DTU)

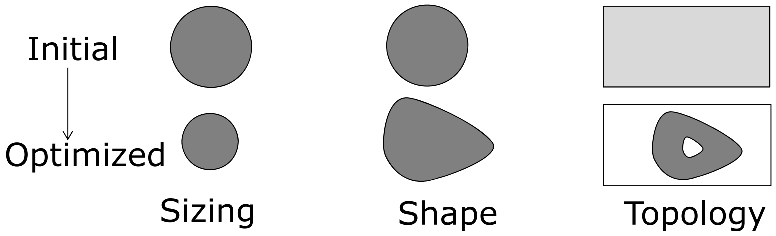
Contributing members of the DTU-TopOpt-group:
Ole Sigmund, Joe Alexandersen, Casper S. Andreasen, Erik Andreassen,
Anders Clausen, Boyan Lazarov, Morten Nobel-Jørgensen,
AT Lightning, Jun Wu.



DTU Mekanik
Institut for Mekanisk Teknologi

Classes of structural optimization

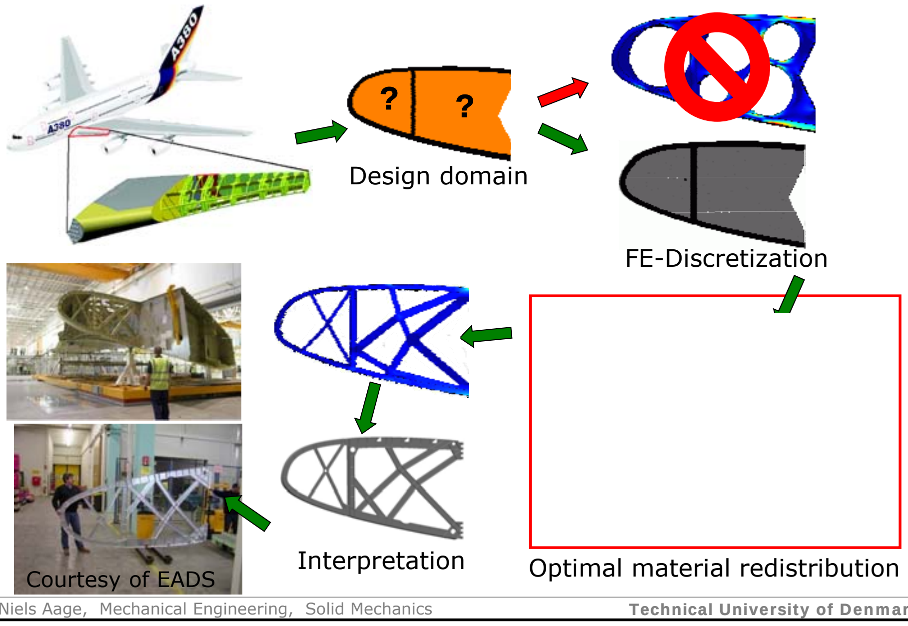
Classes of structural optimization methods:



Niels Aage, Mechanical Engineering, Solid Mechanics — Technical University of Denmark

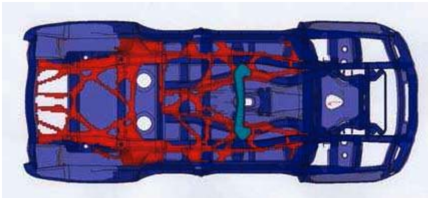
Topology Optimization in Aerospace

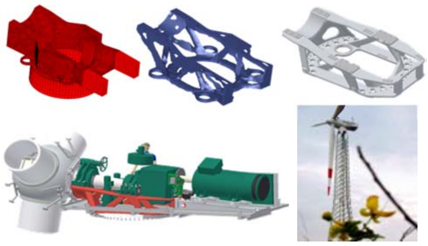
Bendsøe and Kikuchi (1988)

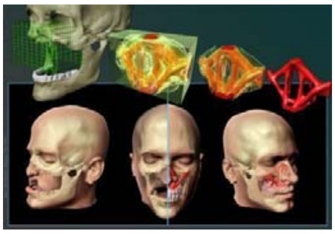


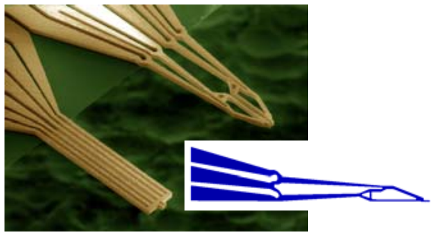
Niels Aage, Mechanical Engineering, Solid Mechanics — Technical University of Denmark

Topology Optimization Applications

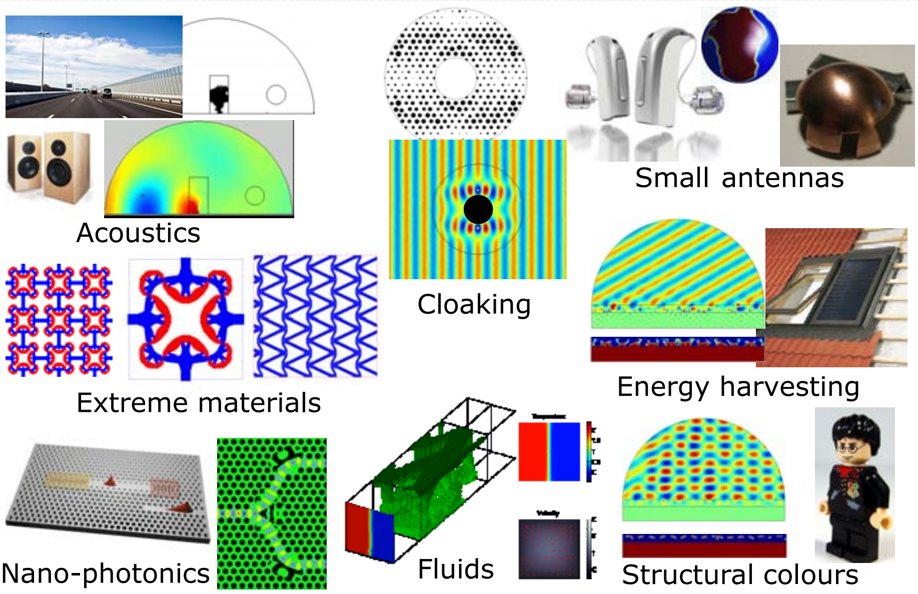

Automotive industry (Fabian Duddeck)


Wind turbines (SUZLON and FE-Design GmbH)


Reconstructive surgery (Paulino/Sinn-Hanlon)


Micromachines (DTU Nanotech)

Topology Optimization Applications



Before we get startet …

- TopOpt falls into the catagory of PDE constrained optimization:

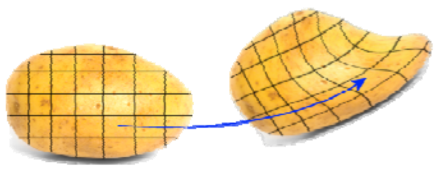
$$\begin{aligned} &\min J(y,u) \\ &\text{s.t. } c(y,u)=0, \\ &\qquad g(y,u)=0, \\ &\qquad h(y,u)\in -K \\ &\qquad y\in \mathcal{Y}_{ad}, u\in \mathcal{U}_{ad}. \end{aligned}$$

u: state variables
y: control/design variables
J: Objective function
c: PDE
g: equality constraints
h: Inequality constraints
&
Y_{ad},U_{ad}: admissible sets

- PDE – Partial Differential Equation:
  Often arise from conservation laws in physics.

Basic continuum mechanics

It starts with observations…

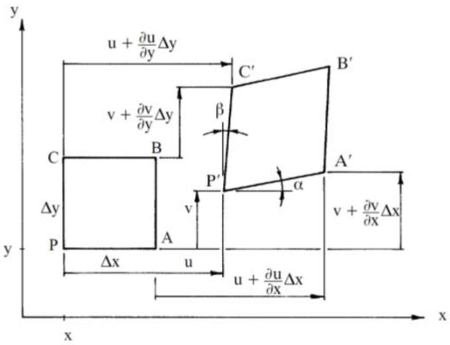


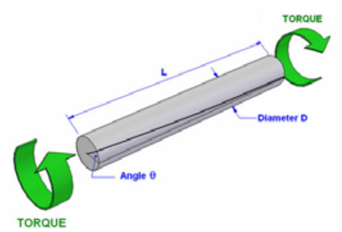
- **Deformations** (displacement)
  - Vector function that maps a material point into its new coordinate, i.e.

$$\boldsymbol{u} = [u(x,y,z), v(x,y,z), w(x,y,z)]^T$$

Basic continuum mechanics

- **Strains** (measurable) - relative deformation





- Def.: $\epsilon := \dfrac{\Delta L}{L}$ - general: (Linear!)

$$\epsilon_x = \frac{\partial u}{\partial x}, \qquad \epsilon_{xy} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$$

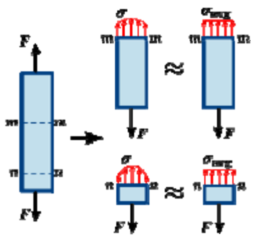
$$\epsilon_y = \frac{\partial v}{\partial y}, \qquad \epsilon_{xz} = \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)$$

$$\epsilon_z = \frac{\partial w}{\partial z}, \qquad \epsilon_{yz} = \frac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right)$$

(elongations - rotations)

Basic continuum mechanics

- **Stresses** (NOT measurable):



Important – the stress depends on the point (position) AND the orientation of cut-surface.

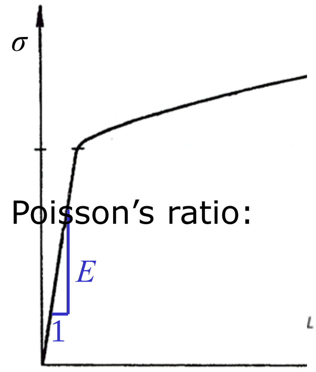
- Def.: $\sigma_{avg} := \dfrac{F}{A}$ or $\sigma = \lim\limits_{A \to 0} \dfrac{F}{A}$

- General stress state: (similar to strains)

$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_x & \sigma_{xy} & \sigma_{xz} \\ \sigma_{xy} & \sigma_y & \sigma_{yz} \\ \sigma_{xz} & \sigma_{yz} & \sigma_z \end{bmatrix}$$

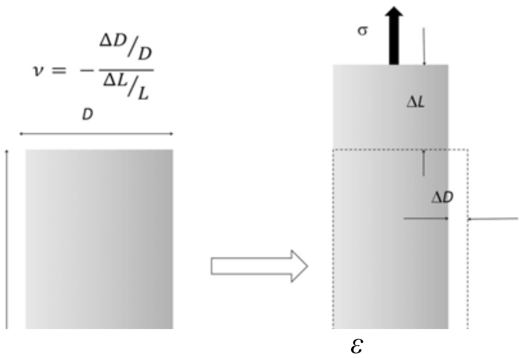
Basic continuum mechanics

- Hooke's law – linear, isotropic materials:
  Just two independent material parameters
- Stiffness: $\sigma = E\epsilon$ (E in [Pa])



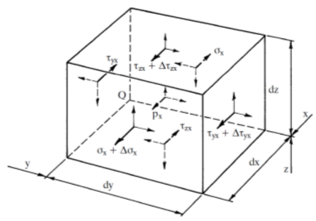
- Poisson's ratio:

$$\nu = -\frac{\Delta D / D}{\Delta L / L}$$



Basic continuum mechanics and FEM

Governing equations (using Newton's 2nd law)



The linear system of partial differential equations:

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} + p_x = 0$$
$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} + p_y = 0$$
$$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_z}{\partial z} + p_z = 0$$

or

$$(\lambda + \mu)\nabla(\nabla \cdot \boldsymbol{u}) + \mu^2 \nabla^2 \boldsymbol{u} + \boldsymbol{p} = \boldsymbol{0}$$
$$\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)}$$
$$\mu = \frac{E}{2(1+\nu)}$$

Constitutive parameters and TopOpt

- Essential since it allows us to interpolate, e.g. stiffness, density, conductivity, …

$$E(\rho) = E_{\min} + \rho^p (E_{\max} - E_{\min})$$

Different problems need different interpolations

- Principle of virtual work

$$\int_\Omega \delta\boldsymbol{\epsilon}^T \boldsymbol{E}(\rho)\boldsymbol{\epsilon}\mathrm{d}\Omega - \int_\Omega \delta\boldsymbol{u}^T \boldsymbol{P}\mathrm{d}\Omega + \int_{\Gamma_T} \delta\boldsymbol{u}^T \boldsymbol{t}\mathrm{d}\Gamma_T = 0$$

- The finite element method (FEM)

$$\boldsymbol{K}(\boldsymbol{\rho})\boldsymbol{U} = \boldsymbol{F}$$

Important mechanical quantities

- The von Mises stress (or equivalent tensile stress):

$\sigma_{vM} = \sqrt{3J_2}$ or

$$\sigma_{vM}^2 = \frac{1}{2}\left[(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\sigma_{xy}^2 + \sigma_{xz}^2 + \sigma_{yz}^2)\right]$$

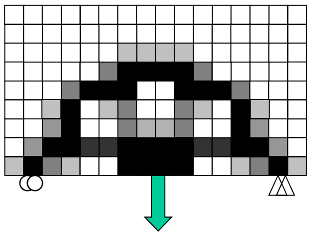
- The strain energy and compliance:

$U = \frac{1}{2}\int_\Omega \boldsymbol{\sigma}^T \boldsymbol{\epsilon}\mathrm{d}\Omega$ and $C = \boldsymbol{u}^T\boldsymbol{F} = \boldsymbol{u}^T\boldsymbol{K}\boldsymbol{u}$

- Stiffness vs compliance: $E = \frac{\partial\sigma}{\partial\epsilon}$ vs $C = \frac{\partial\epsilon}{\partial\sigma}$

Discretized SIMP-approach

Bendsøe (1989), Zhou and Rozvany (1991), Mlejnek (1992)



$$\min_{\boldsymbol{\rho}} : \quad \Phi(\boldsymbol{\rho}, \mathbf{U}(\boldsymbol{\rho}))$$

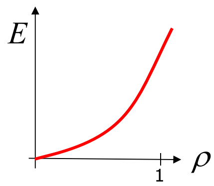
$$s.t. : \quad \sum_{e=1}^{N} v_e \rho_e = \mathbf{v}^T \boldsymbol{\rho} \leq V^*$$

$$: \quad g_i(\boldsymbol{\rho}, \mathbf{U}(\boldsymbol{\rho})) \leq g_i^*, \quad i = 1, ..., M$$

$$: \quad \mathbf{0} \leq \boldsymbol{\rho} \leq \mathbf{1}$$

$$(: \quad \mathbf{K}(\boldsymbol{\rho})\mathbf{U} = \mathbf{F} \ )$$

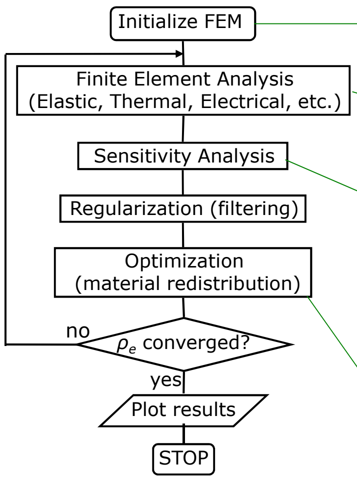
Stiffness interpolation:

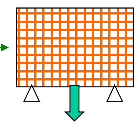


$$E(\rho_e) = E_1 + \rho_e^p (E_2 - E_1)$$

$$p > 1$$

The Topology Optimization Process





$$\boldsymbol{KU} = \boldsymbol{F} \text{ or } \boldsymbol{R}(\boldsymbol{U}) = 0$$

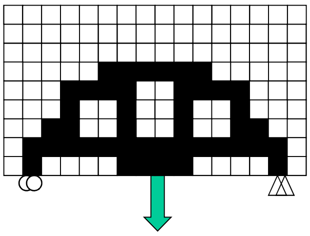
Sensitivity analysis by adjoint method

$$\frac{d\Phi}{d\rho_e} = \frac{\partial\Phi}{\partial\rho_e} + \boldsymbol{\lambda}^T\left(\frac{\partial\boldsymbol{K}}{\partial\rho_e}\boldsymbol{U} + \frac{\partial\boldsymbol{F}}{\partial\rho_e}\right)$$

$$\boldsymbol{K}^T\boldsymbol{\lambda} = -\frac{\partial\Phi}{\partial\boldsymbol{U}}$$

Mathematical Programming,
Method of Moving Asymptotes (MMA)
by Svanberg (1987)

Why gradient based methods ?



$$\min_{\boldsymbol{\rho}} : \quad \Phi(\boldsymbol{\rho}, \mathbf{U}(\boldsymbol{\rho}))$$

$$s.t. : \quad \sum_{e=1}^{N} v_e \rho_e = \mathbf{v}^T \boldsymbol{\rho} \leq V^*$$

$$: \quad g_i(\boldsymbol{\rho}, \mathbf{U}(\boldsymbol{\rho})) \leq g_i^*, \quad i = 1, ..., M$$

$$: \quad \rho_e = \begin{cases} 0 \text{ (void)} \\ 1 \text{ (material)} \end{cases}, \quad e = 1, ..., N$$

$$: \quad \mathbf{K}(\boldsymbol{\rho})\mathbf{U} = \mathbf{F}$$

0/1 Integer problem

- Combinations: $\frac{N!}{(N-M)!\,M!}$

N=10, M=5 => 252
N=20, M=10 => 185.000
N=40, M=20 => $1.4 \cdot 10^9$
N=100, M=50 => 10^{29}

Adjoint method for sensitivities - discrete

- A general function and a general residual:

$$\Phi = \Phi(\boldsymbol{\rho}, \boldsymbol{u}(\boldsymbol{\rho})), \quad \boldsymbol{R}(\boldsymbol{\rho}, \boldsymbol{u}(\boldsymbol{\rho})) = \mathbf{0}$$

- Step 1: differentiate using the chainrule

$$\frac{\mathrm{d}\Phi}{\mathrm{d}\rho_e} = \frac{\partial \Phi}{\partial \rho_e} + \frac{\partial \Phi}{\partial \boldsymbol{u}} \boxed{\frac{\partial \boldsymbol{u}}{\partial \rho_e}} \qquad \frac{\mathrm{d}\boldsymbol{R}}{\mathrm{d}\rho_e} = \frac{\partial \boldsymbol{R}}{\partial \rho_e} + \frac{\partial \boldsymbol{R}}{\partial \boldsymbol{u}} \boxed{\frac{\partial \boldsymbol{u}}{\partial \rho_e}} = \mathbf{0}$$

- Problem term – must be eliminated!

- Use the residual eqs.: $\frac{\partial \boldsymbol{u}}{\partial \rho_e} = -\left(\frac{\partial \boldsymbol{R}}{\partial \boldsymbol{u}}\right)^{-1} \frac{\partial \boldsymbol{R}}{\partial \rho_e}$

Adjoint method for sensitivities - discrete

- Step 2: Insert trouble term into derivative

$$\frac{\mathrm{d}\Phi}{\mathrm{d}\rho_e} = \frac{\partial \Phi}{\partial \rho_e} + \underbrace{\frac{\partial \Phi}{\partial \boldsymbol{u}}\left(-\frac{\partial \boldsymbol{R}}{\partial \boldsymbol{u}}\right)^{-1}}_{\boldsymbol{\lambda}^T}\frac{\partial \boldsymbol{R}}{\partial \rho_e}$$

- Step 3: Adjoint problem

$$\boldsymbol{\lambda}^T = -\frac{\partial \Phi}{\partial \boldsymbol{u}}\left(\frac{\partial \boldsymbol{R}}{\partial \boldsymbol{u}}\right)^{-1} \Rightarrow \frac{\partial \boldsymbol{R}}{\partial \boldsymbol{u}}^T \boldsymbol{\lambda} = -\frac{\partial \Phi}{\partial \boldsymbol{u}}$$

- Final sensitivity

$$\frac{\mathrm{d}\Phi}{\mathrm{d}\rho_e} = \frac{\partial \Phi}{\partial \rho_e} + \boldsymbol{\lambda}^T \frac{\partial \boldsymbol{R}}{\partial \rho_e}$$

Adjoint method for sensitivities - discrete

- Example problem – Linear compliance

$$\Phi = \boldsymbol{F}^T\boldsymbol{u} = \boldsymbol{u}^T\boldsymbol{K}\boldsymbol{u}, \quad \boldsymbol{R} = \boldsymbol{K}(\boldsymbol{\rho})\boldsymbol{u} - \boldsymbol{F} = \boldsymbol{0}$$

- The 4 required terms become

$$\frac{\partial \Phi}{\partial \rho_e} = \boldsymbol{u}^T\frac{\partial \boldsymbol{K}}{\partial \rho_e}\boldsymbol{u} \qquad \frac{\partial \Phi}{\partial \boldsymbol{u}} = 2\boldsymbol{F}$$

$$\frac{\partial \boldsymbol{R}}{\partial \rho_e} = \frac{\partial \boldsymbol{K}}{\partial \rho_e}\boldsymbol{u} \qquad \frac{\partial \boldsymbol{R}}{\partial \boldsymbol{u}} = \boldsymbol{K} = \boldsymbol{K}^T$$

- The adjoint becomes (so-called self-adjoint!):

$$\boldsymbol{K}(\boldsymbol{\rho})\boldsymbol{\lambda} = -2\boldsymbol{F} \Rightarrow \boldsymbol{\lambda} = -2\boldsymbol{u}$$

Adjoint method for sensitivities - discrete

- Example problem – Linear compliance

$$\Phi = \boldsymbol{F}^T \boldsymbol{u} = \boldsymbol{u}^T \boldsymbol{K} \boldsymbol{u}, \quad \boldsymbol{R} = \boldsymbol{K}(\boldsymbol{\rho})\boldsymbol{u} - \boldsymbol{F} = \boldsymbol{0}$$

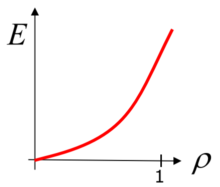
- The sensitivity now reads

$$\frac{\mathrm{d}\Phi}{\mathrm{d}\rho_e} = \boldsymbol{u}^T \frac{\partial \boldsymbol{K}}{\partial \rho_e} \boldsymbol{u} - 2\boldsymbol{u}^T \frac{\partial \boldsymbol{K}}{\partial \rho_e} \boldsymbol{u} = -\boldsymbol{u}^T \frac{\partial \boldsymbol{K}}{\partial \rho_e} \boldsymbol{u}$$

with: $$\frac{\partial \boldsymbol{K}}{\partial \rho_e} = p\rho^{p-1}(E_{\max} - E_{\min})\boldsymbol{K}_0$$

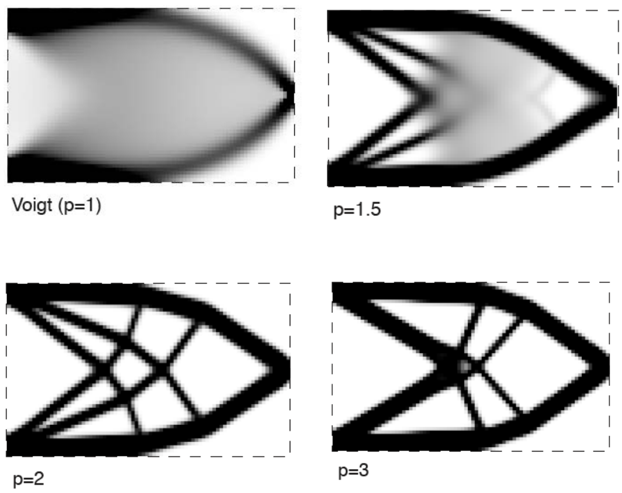
- Note: this is a negative scaled strain energy

SIMP (Simplified Isotropic Material with Penalization)



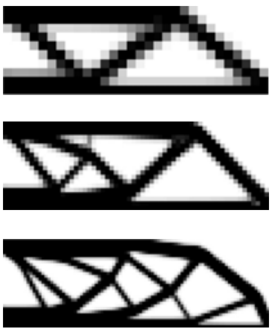
$$E(\rho_e) = \rho_e^p E_0$$

$$p \geq 1$$

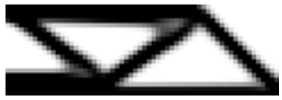


Physical motivation for SIMP in Bendsøe and Sigmund, *AAM*, **1999**, *69*, 635-654

Mesh-dependence



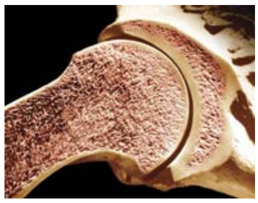
Mesh refinement

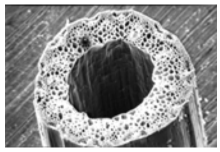


Mesh-independency

Mesh refinement

Mesh-dependency





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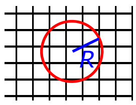
Regularization by sensitivity filtering

Neighborhood:

$$N_e = \{i \mid ||\mathbf{x}_i - \mathbf{x}_e|| \leq R\}$$



Checkerboards

Density filtering: (Bruns/Bourdin 2001)

$$E_e(\boldsymbol{\rho}) = \tilde{\rho}_e^p E_0, \quad \tilde{\rho}_e = \frac{\sum_{i \in N_e} H(\mathbf{x}_i)\rho_i}{\sum_{i \in N_e} H(\mathbf{x}_i)}$$

Mesh refinement

PDE-based filtering: (Lazarov&Sigmund, 2011)

$$\widehat{w} - r^2 \widehat{w}_{,mm} = \overline{w}$$

$$\overline{w} = \rho \frac{\partial \Phi}{\partial \rho} \; (= -p\rho^p C^0_{ijkl}\varepsilon_{ij}\varepsilon_{kl} = -SED)$$

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Alternative regularizations

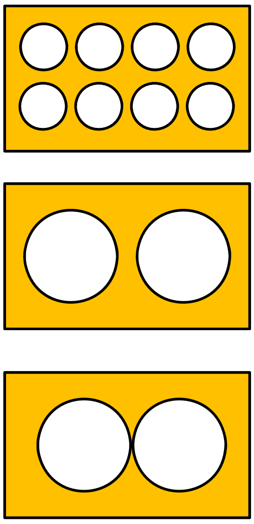
Tikhonov / phase-field regularization

$$\tilde{\Phi}(\boldsymbol{\rho}) = \Phi(\boldsymbol{\rho}) + \int_{\Omega} \left(\frac{1}{\varepsilon}\rho(1-\rho) + \varepsilon ||\nabla \rho||^2 \right) dV$$

Global regularization schemes

Perimeter control

$$TV = \int_{\Omega} ||\nabla \rho|| d\Omega \leq P^*$$

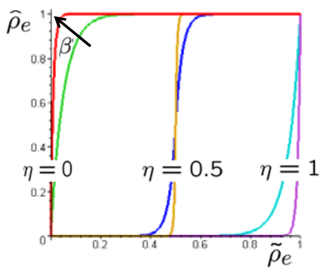


Heaviside projection methods

$$\rho \rightarrow \tilde{\rho}(\rho) \rightarrow \hat{\rho}(\tilde{\rho}(\rho))$$

Design variables — Density filter — Projection

$$\hat{\rho} = \frac{\tanh(\beta\eta) + \tanh(\beta(\tilde{\rho} - \eta))}{\tanh(\beta\eta) + \tanh(\beta(1 - \eta))}$$



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Projection method Guest *et al* (2004)

Design variables	Density filtering	Projection
ρ $\rightarrow$	$\tilde{\rho}(\rho)$ $\rightarrow$	$\hat{\rho}(\tilde{\rho}(\rho))$
	$-r^2\Delta\tilde{\rho} + \tilde{\rho} = \rho$	$\hat{\rho} = \frac{\tanh(\beta\eta) + \tanh(\beta(\tilde{\rho} - \eta))}{\tanh(\beta\eta) + \tanh(\beta(1 - \eta))}$

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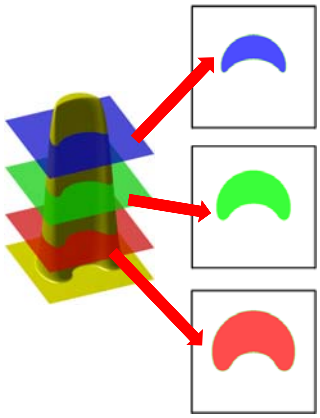
”Robust” design formulation

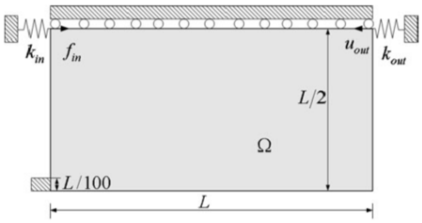


$$\min_{\rho} : \max \quad \left(\mathbf{f}(\bar{\tilde{\rho}}^{e}(\rho)), \mathbf{f}(\bar{\tilde{\boldsymbol{\rho}}}^{i}(\rho)), \mathbf{f}(\bar{\tilde{\boldsymbol{\rho}}}^{d}(\rho))\right)$$

$$\begin{aligned} s.t. : & \ \mathbf{K}(\bar{\tilde{\rho}}^{e})\mathbf{u}^{e} = \mathbf{f} \\ : & \ \mathbf{K}(\bar{\tilde{\rho}}^{i})\mathbf{u}^{i} = \mathbf{f} \\ : & \ \mathbf{K}(\bar{\tilde{\rho}}^{d})\mathbf{u}^{d} = \mathbf{f} \\ : & \ f_v(\rho) = \frac{\sum_i \bar{\tilde{\rho}}_i^{d} v_i}{V} \leq V_d^{*} \\ : & \ 0 \leq \rho \leq 1 \end{aligned}$$

”Robust” design formulation

- Force inverter – hinges in standard formulation



- Robust formulation - no hinges☺

(d) Eroded design (e) Intermediate design (f) Dilated design

Weapon of choice in TopOpt - MMA

The Method of Moving Asymptotes (Svanberg 1987).

- Problem you want to solve

$$\min_{x \in \mathbb{R}^n} \quad g_0(x)$$
$$\text{s.t.} \quad g_i(x) \leq 0, \qquad i = 1, m$$
$$x_{\min} \leq x_j \leq x_{\max}, \quad j = 1, n$$

Problem that MMA solves

$$\min_{x \in \mathbb{R}^n, y \in \mathbb{R}^m, z \in \mathbb{R}} \quad f_0(x) + z + \frac{1}{2}z^2 + \sum_{i=1}^{m} \left(y_i c_i + \frac{1}{2} y_i^2 \right)$$
$$\text{s.t.} \quad f_i(x) - a_i z - y_i \leq 0, \quad i = 1, m$$
$$\alpha_j \leq x_j \leq \beta_j, \quad j = 1, n$$
$$y_i \geq 0, \quad i = 1, m$$
$$z \geq 0$$

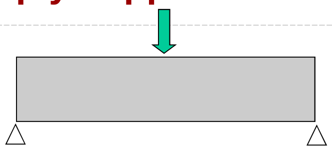
- Using first order convex seperable approximations:

$$f_i(x) = \sum_{j=1}^{n} \left(\frac{p_{ij}}{U_j - x_j} + \frac{q_{ij}}{x_j - L_j} \right) + r_i$$

Understanding the principles of TopOpt

Influence of number of load cases
and boundary conditions

TopOpt for a simply supported beam



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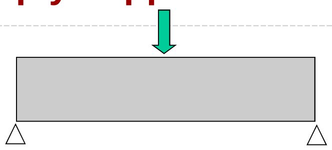
TopOpt for a simply supported beam



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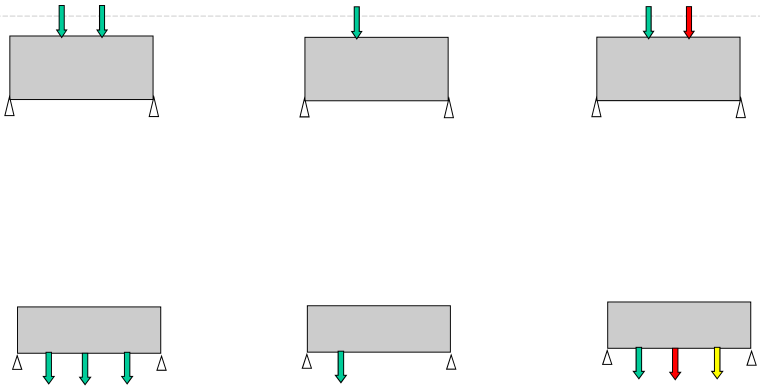
One or more load cases?



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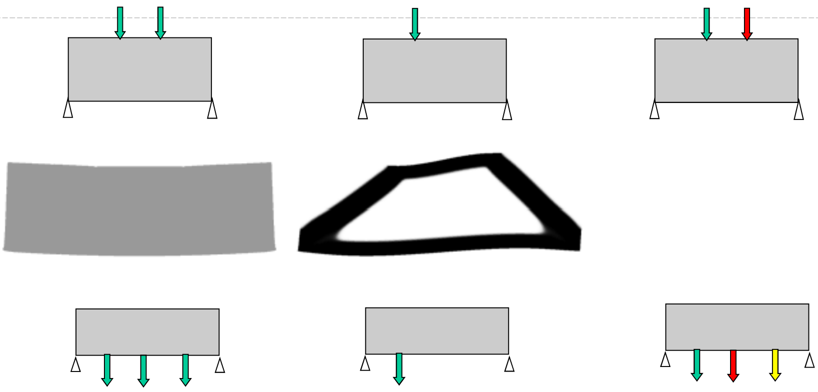
One or more load cases?



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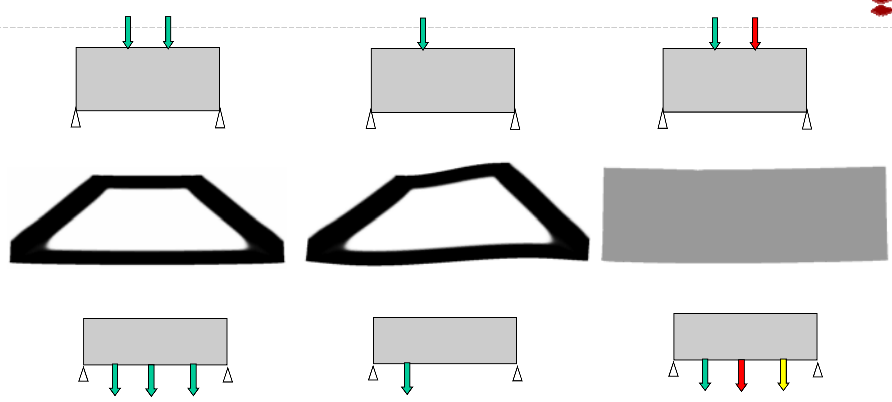
One or more load cases?



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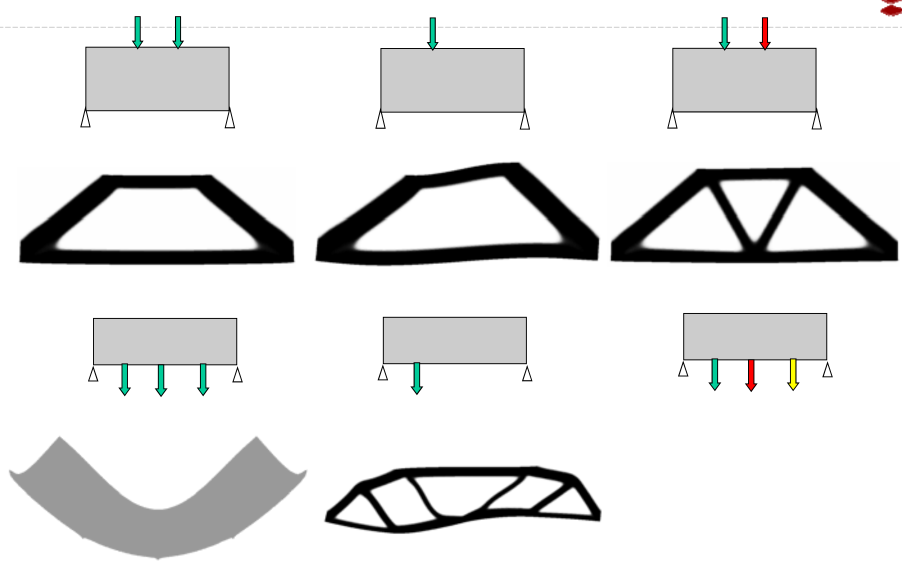
One or more load cases?



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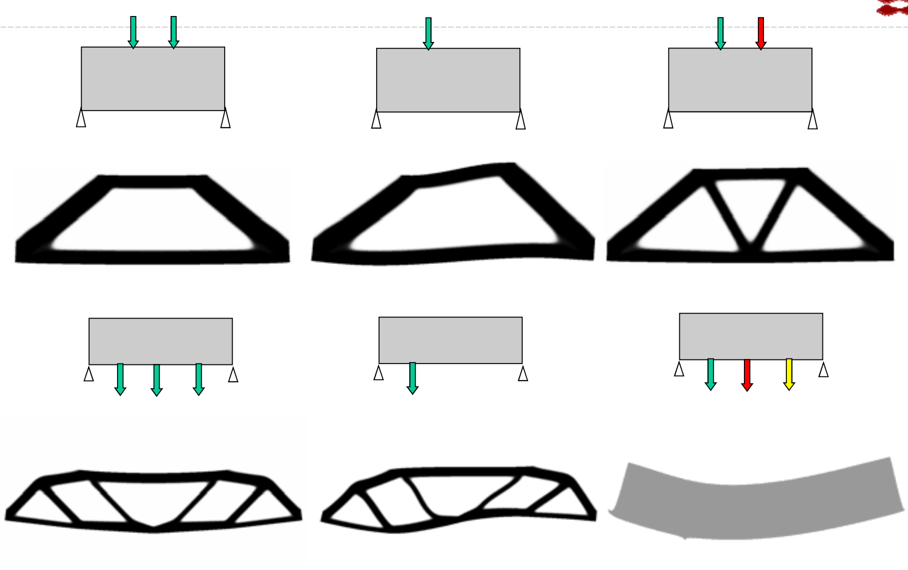
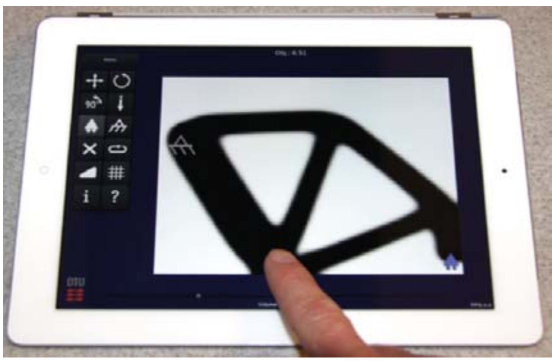
One or more load cases?



The ”TopOpt App”

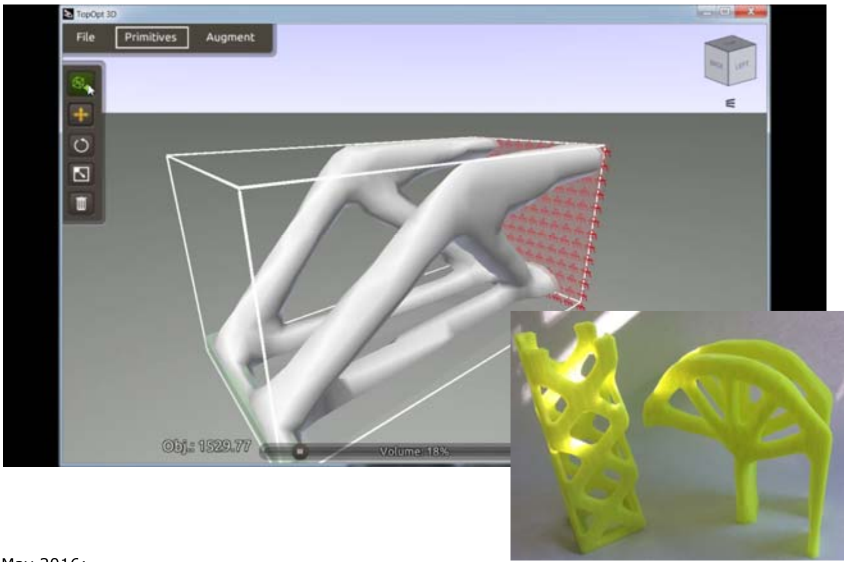


The ”TopOpt App”: AppStore (iOS)
Google Play (Android)
Web-version: www.topopt.dtu.dk

Stats: May 2016:
Android: 5380, iOS: 9450

See **www.topopt.dtu.dk** for more

TopOpt3D App



Stats: May 2016:
iOS: 4100, web: 1500

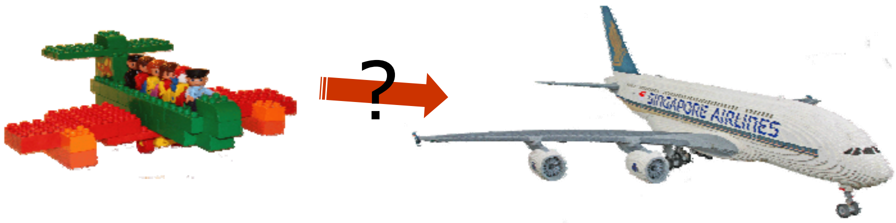
(NB! Only iOS, OSX and PC – see **www.topopt.dtu.dk**)

www.topopt.dtu.dk

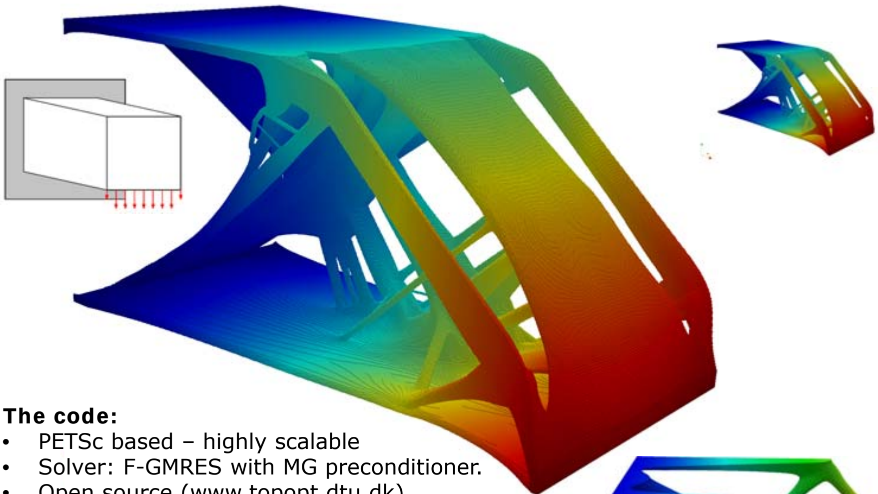
Code refs and image of topopt site

High resolution TopOpt

(overcoming the Duplo problem)

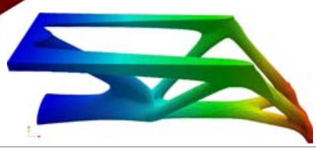


+100M design variables



The code:

- PETSc based – highly scalable
- Solver: F-GMRES with MG preconditioner.
- Open source (www.topopt.dtu.dk)
- Include: Filters, MMA, IO.
- Comes with minimum compliance example.
- Aage; Andreassen & Lazarov, **2015,** *SMO.*



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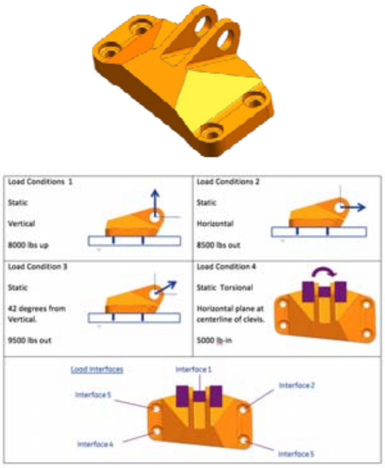
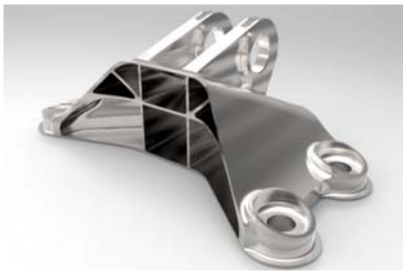
GrabCAD Challenge 2013 (640 entries)

Minimize weight of additive manufactured jet engine bracket

Design problem

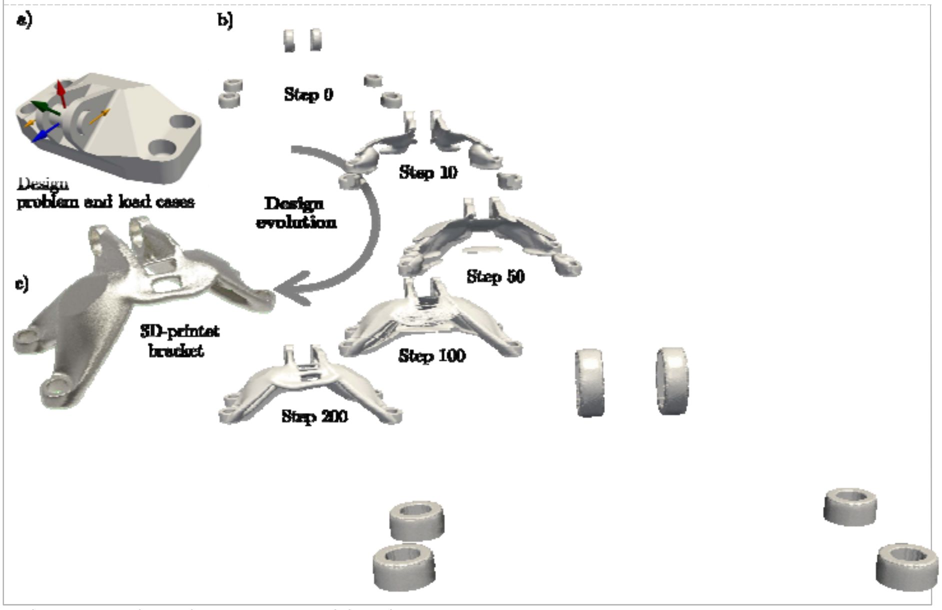


Winner – 340 g
16 % volume fraction



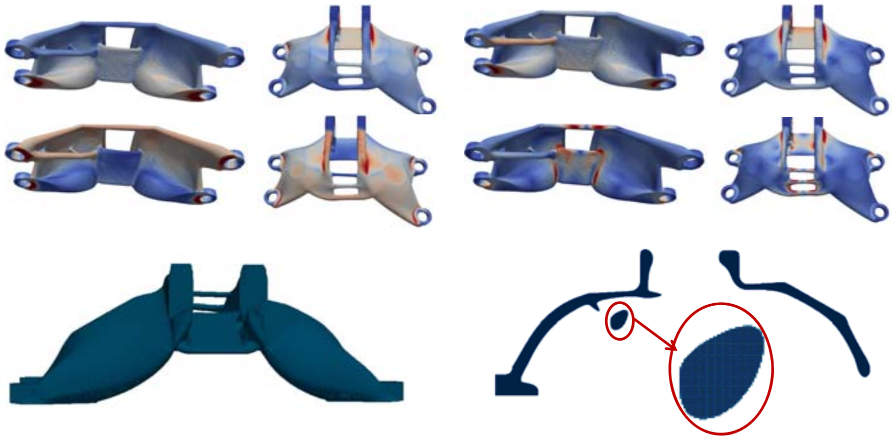
From: GrabCAD.com, by M. Kurniawan

Design history

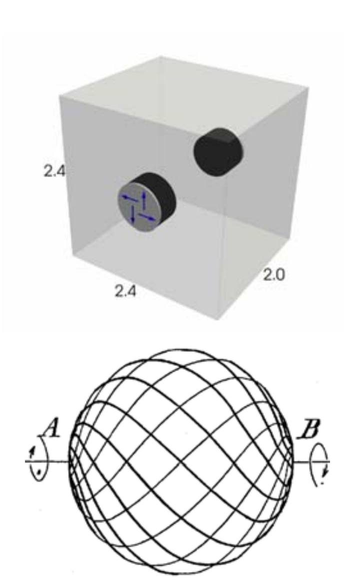


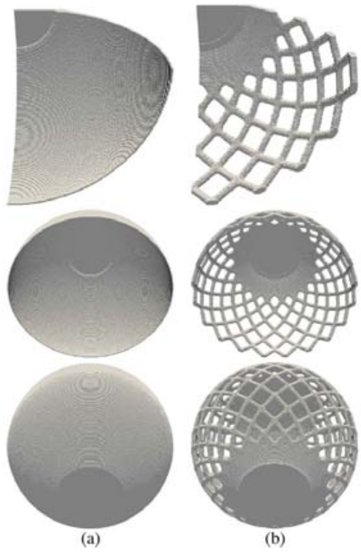
Optimized bracket

- 35M cubic elements (size 0.6mm)
- Result obtained in approximately 12,000 CPU hours
- Target weight 300 g (10% lighter than challenge winner)
- Max. von Mises stress around 700 MPa (yield stress >900 MPa)



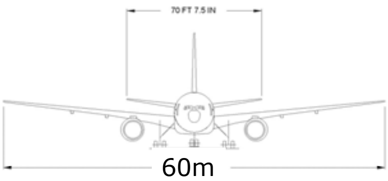
Rediscovering optimality - Michell

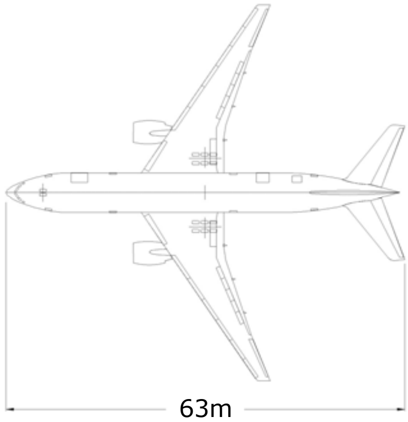


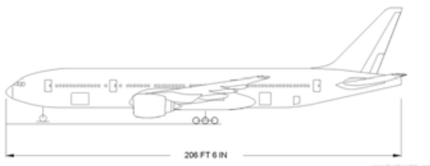


C=184T^2 vs C=255T^2

Boing 777 dimensions



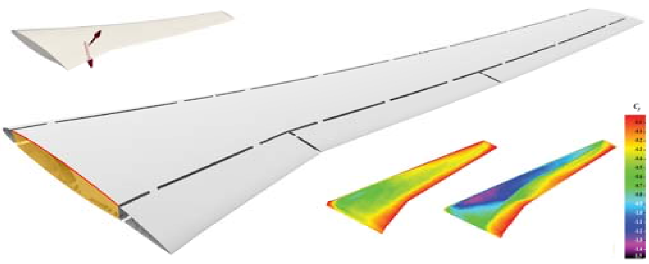




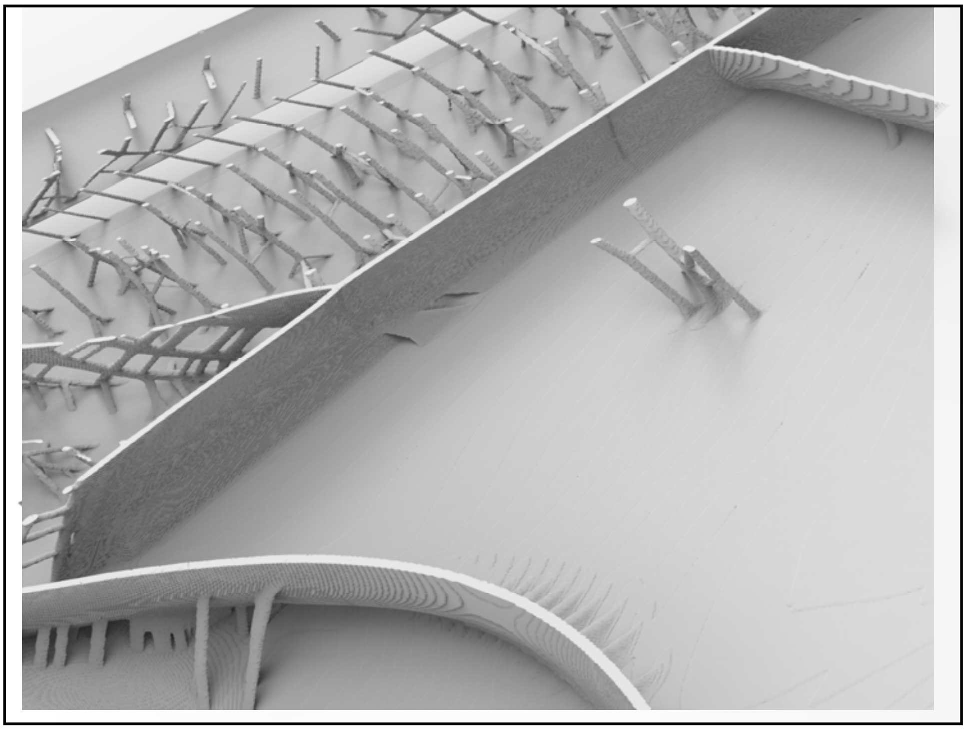
NASA Common Research Model

Geometry and pressure load data from NASA:

Discritized including supports and loads

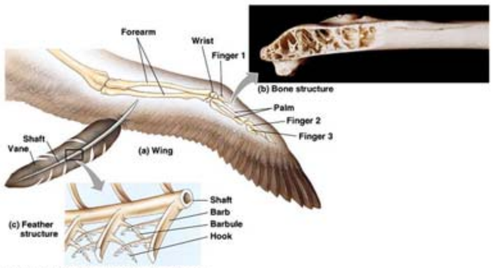


Mesh with ~**1.1 billion** elements (1216 x 256 x 3456)…
… largest element side **0.8 cm** (wing is ~ 26.5m x 11.5m x 2m)



Mimics nature

Copyright Natural History Museum, London, UK.



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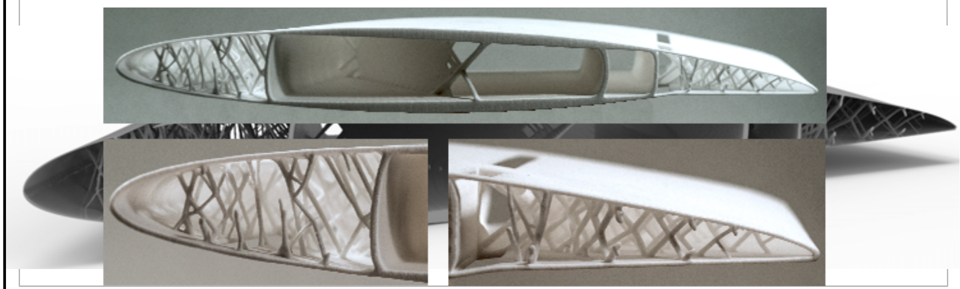
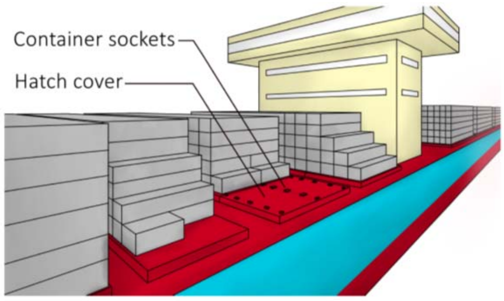


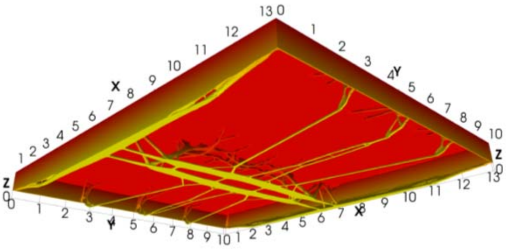
Designing containership components

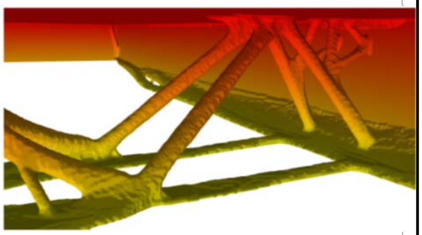
Study with Mærsk Line with the goal to reduce costs.



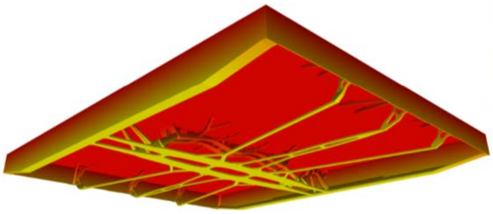
Designing containership components

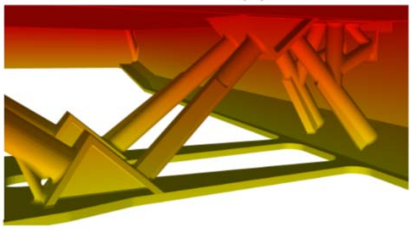
Parameterizing the optimized design (manually!)





Optimized

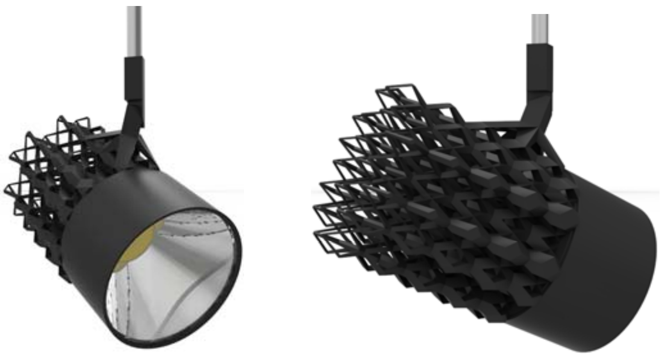




Interpreted

Cooling fins for LED lamps

HYPERCOOL – Cool Danish Design



Innovationsfonden

DET FRIE FORSKNINGSRÅD
DANISH COUNCIL FOR
INDEPENDENT RESEARCH

AT • LIGHTING

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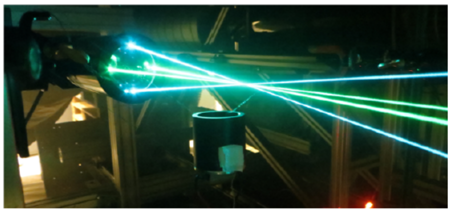
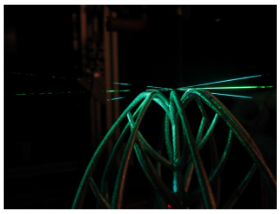
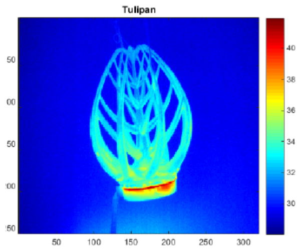
DTU Mechanical Engineering
Department of Mechanical Engineering

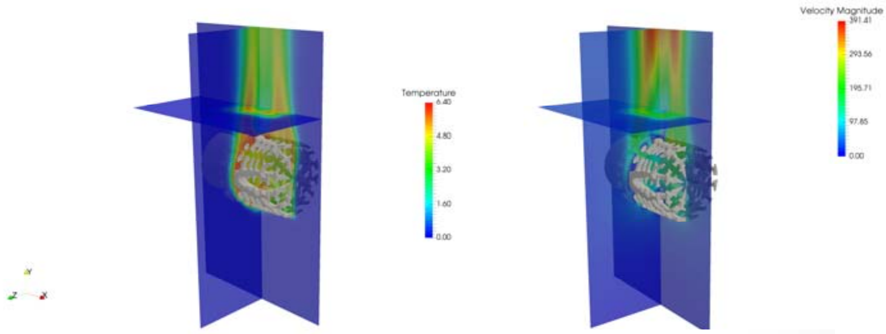
Coolers for LEDs: HyperCool

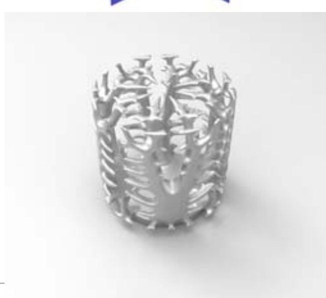




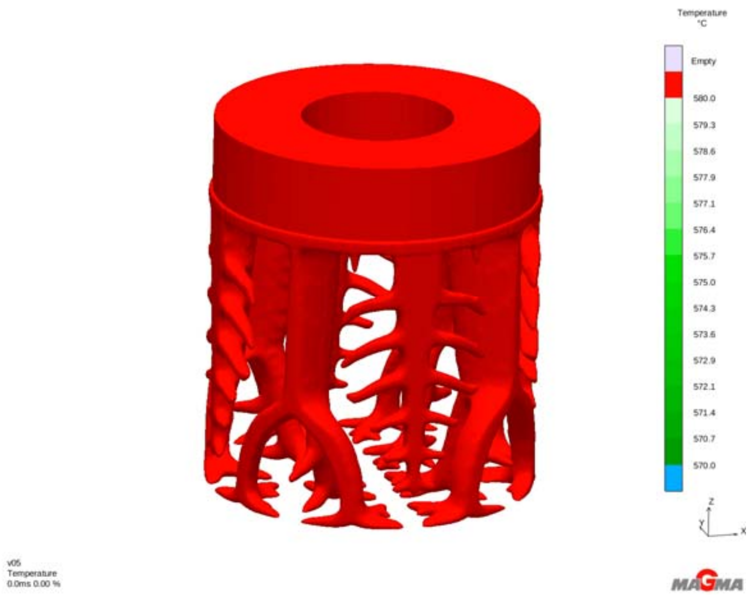


Coolers for LEDs: HyperCool



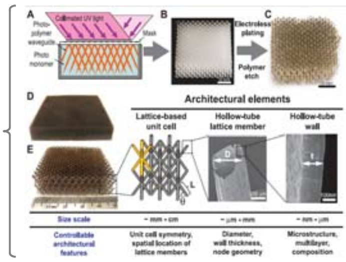


Optimal casting?

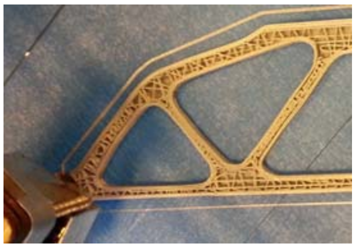


Integration with AM
and design of "shell structures"

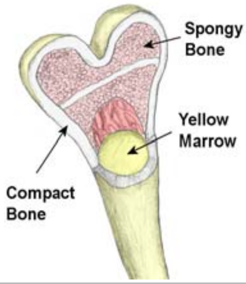
Coating and stiff interface structures



Schaedler et al., Science 334 (6058): 962-965, **2011**



Infill printed by FDM

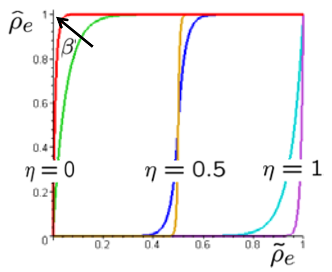


Repeated filtering and projection

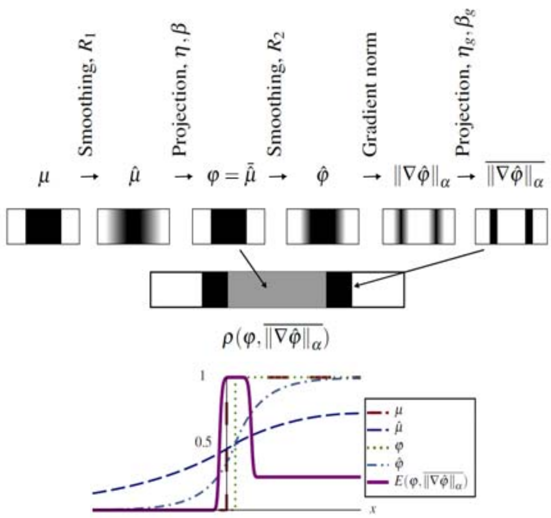
$$\rho \rightarrow \tilde{\rho}(\rho) \rightarrow \hat{\rho}(\tilde{\rho}(\rho))$$

Design variables → Density filter → Projection

$$\hat{\rho} = \frac{\tanh(\beta\eta) + \tanh(\beta(\tilde{\rho} - \eta))}{\tanh(\beta\eta) + \tanh(\beta(1 - \eta))}$$

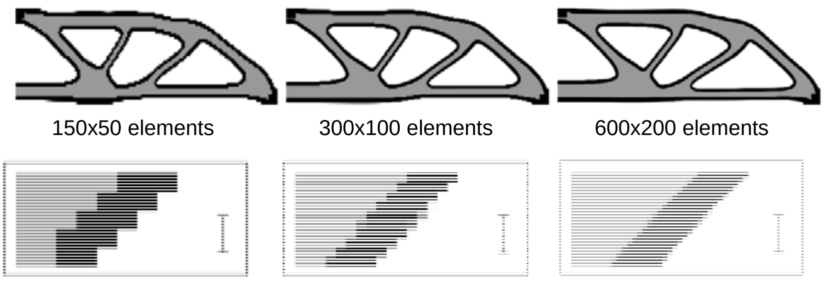


Material interpolation model



$$E(\varphi, \overline{\|\nabla\hat{\varphi}\|_\alpha}) = E^0\left[\lambda_E\varphi^p + (1-\lambda_E\varphi^p)(\overline{\|\nabla\hat{\varphi}\|_\alpha})^p\right]$$

Results and convergence

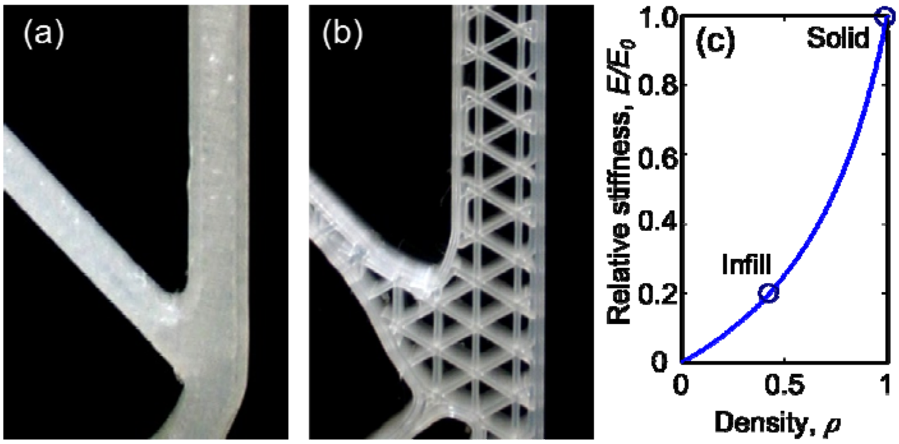


150x50 elements 300x100 elements 600x200 elements

$R_2 = 2.5$ $R_2 = 5.0$ $R_2 = 7.5$

Clausen at al., *CMAME*, **2015**, *290*, 524-541

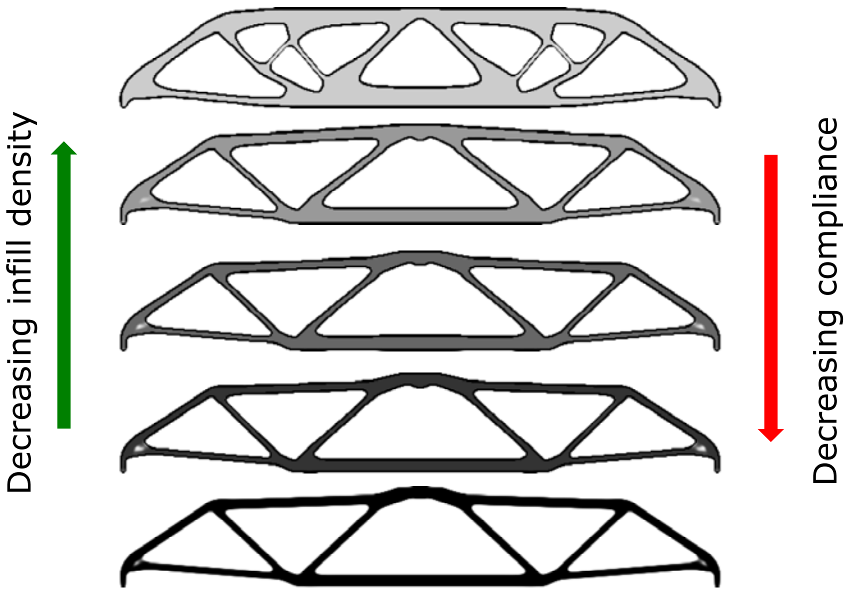
TopOpt formulation for coating and infill



Standard | Coating | Infill properties

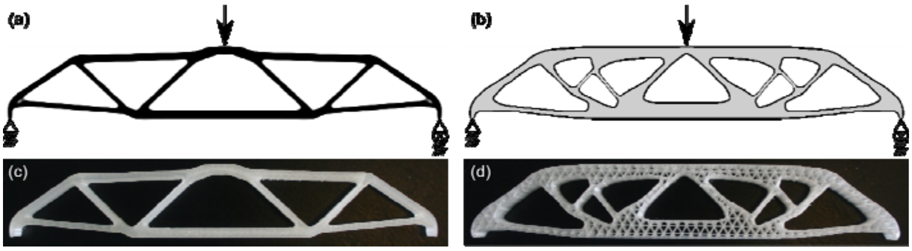
Clausen; Aage & OS, *CMAME*, **2015**, *290*, 524-541

Dependence on infill stiffness



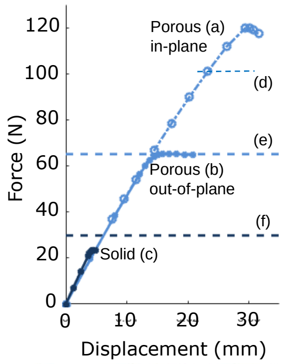
Mechanical tests on MBB beam

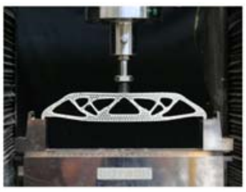
Print material: SEBS (Styrene-Etylene-Butylene-Styrene)

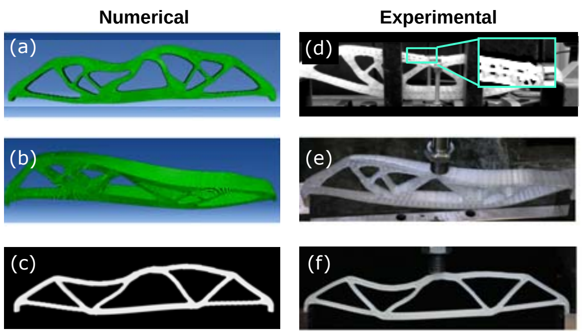


Solid (standard) **Porous (coating)**

Buckling load improved >5 times



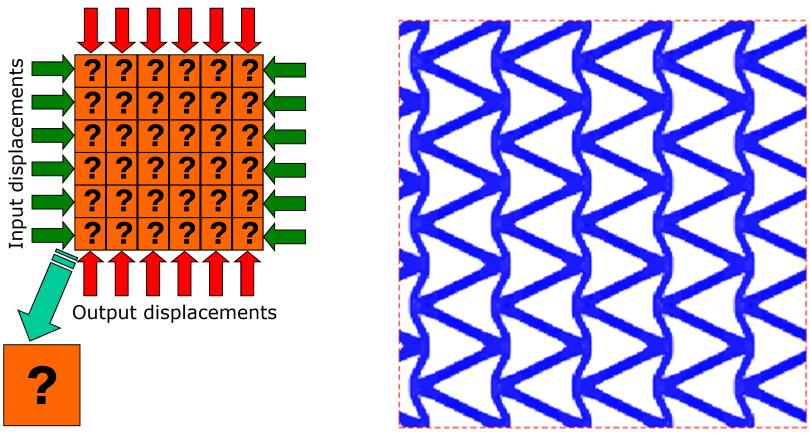




Clausen; Aage & Sigmund, **2016**, *Submitted*

Material design problems

Material with negative Poisson's ratio

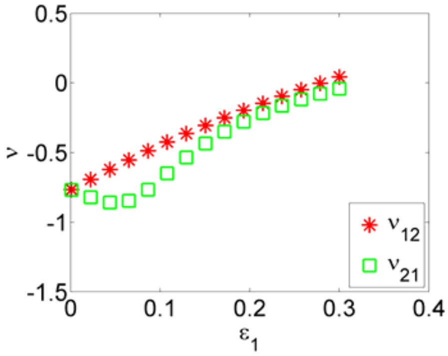


- FE on one cell with periodic B.C.
- Minimize Poisson's ratio
- Constraint on bulk modulus and symmetry

Sigmund (1995)

Non-linear material modelling

$\nu_{12} = -0.766$
$\nu_{21} = -0.770$

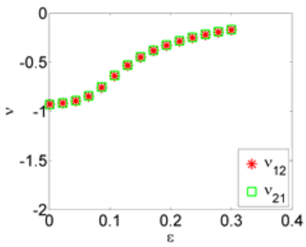


Wang et.al., *JMPS,* **2014**

Negative Poisson's ratio design

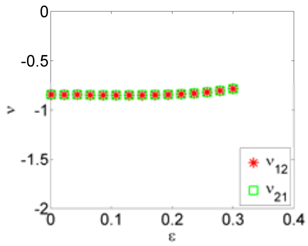
Linear case

$\nu_{12} = -0.931$
$\nu_{21} = -0.929$



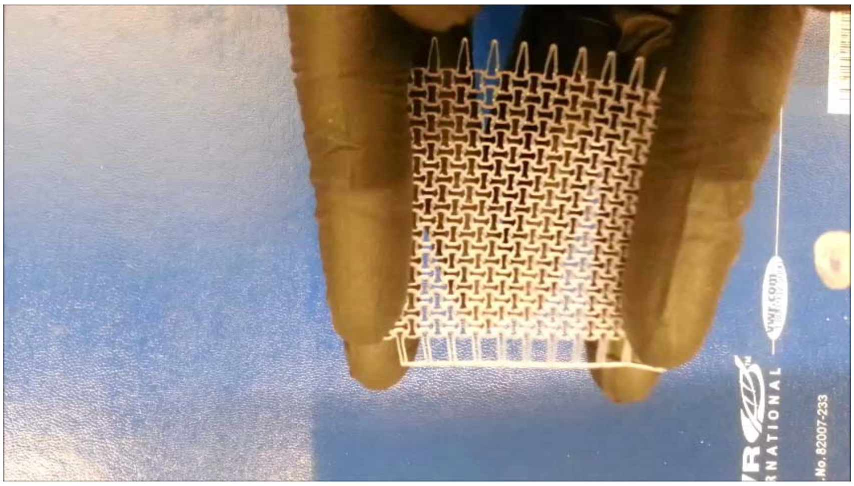
Nonlinear case

$\overline{\nu}_{12} = -0.838$
$\overline{\nu}_{21} = -0.838$



Wang et.al., *JMPS,* **2014**

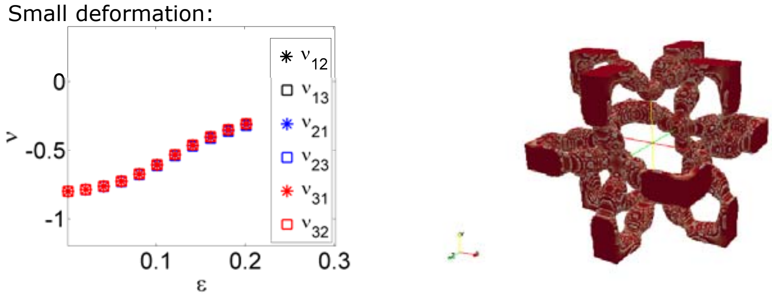
Experimental verifications



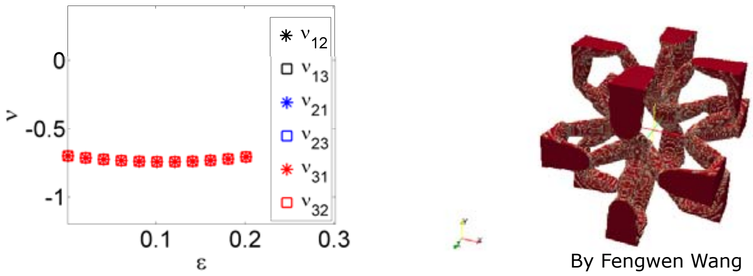
Clausen at al., *Advanced Materials*, **2015**, *27*, 5523-5527

3D Poisson's ratio -0.8

Small deformation:

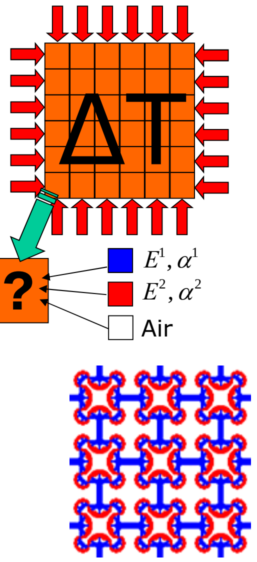


Finite deformation:



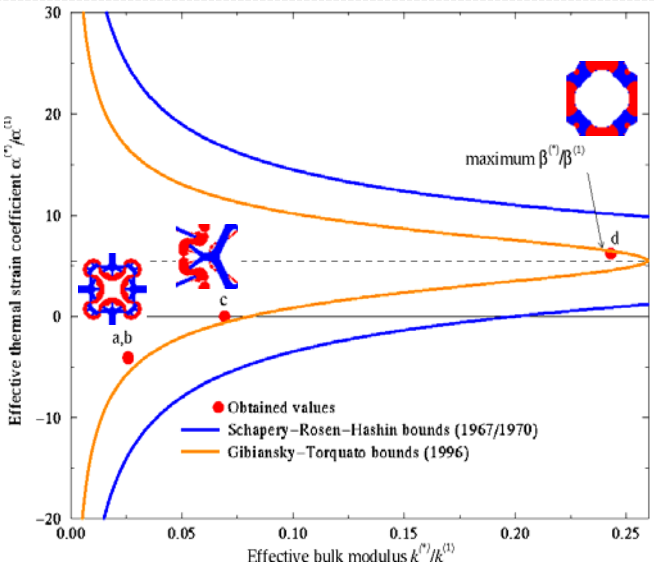
By Fengwen Wang

Negative thermal expansion coefficient



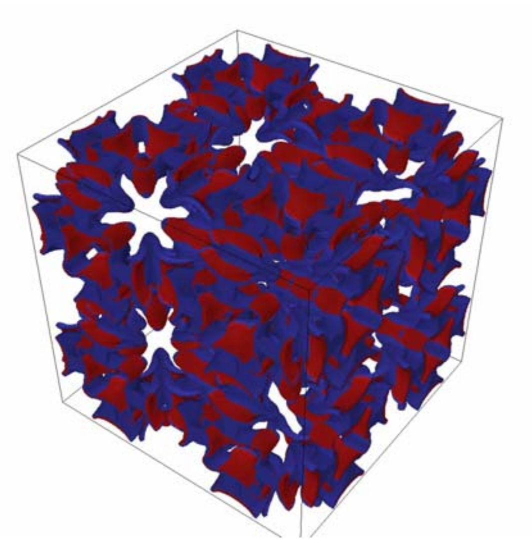
$\alpha^* = -4.02$

Comparisons with bounds for thermal expansion



Sigmund and Torquato, 1996/1997

3d negative thermal expansion



$$\alpha_{red} = 3.5$$
$$\alpha_{blue} = 1$$
$$E_{red} = 1$$
$$E_{blue} = 3.5$$

$$\nu^H = 0.18$$
$$E^H = 0.0016$$
$$\alpha^H = -5.4$$

Produced by Erik Andreassen

Papers and references

Klarbring book on structural optimization

Bendsøe + Sigmund book on TopOpt

On multigrid-CG for efficient topology optimization
Amir, O.; Aage, N. & Lazarov, B.S., *SMO, 49, 815-829,* **2014**.

Topology optimization using PETSc:
An easy-to-use, fully parallel, open-source topology optimization framework
Aage, N; Andreassen, E. & Lazarov, B.S., 51(3):565-572, **2015**.

Interactive TopOpt on hand-held devices
Aage; Nobel-Jørgensen; Andreasen & *OS,*, *SMO,* **2013**, *47*, 1-6

TopOpt with Flexible Void Area
Clausen, A.; Aage, N. & *OS,*, *SMO,* 50:927-943, **2014**.

TopOpt of interface problems and coated structures
Clausen, A.; Aage, N. & *OS,*, *CMAME*, 290:524-541,**2015.**

Large scale three-dimensional TopOpt of heat sinks cooled by natural convection
Alexandersen, J., Sigmund, O., Aage, N., IJHMT, 100:876-891, **2016**.

Parallel framework for TopOpt using the Method of Moving Asymptotes
Aage, N. Lazarov, B.S, SMO, 47:493-505, **2013**.

Topology Optimization for Computational Fabrication

Jun Wu, Niels Aage, Sylvain Lefebvre, Charlie Wang

EUROGRAPHICS 2017
The 38th annual conference of the
EUROPEAN ASSOCIATION FOR COMPUTER GRAPHICS

TU Delft
DTU Technical University of Denmark
Inria INVENTEURS DU MONDE NUMÉRIQUE

Topology Optimization for Computational Fabrication

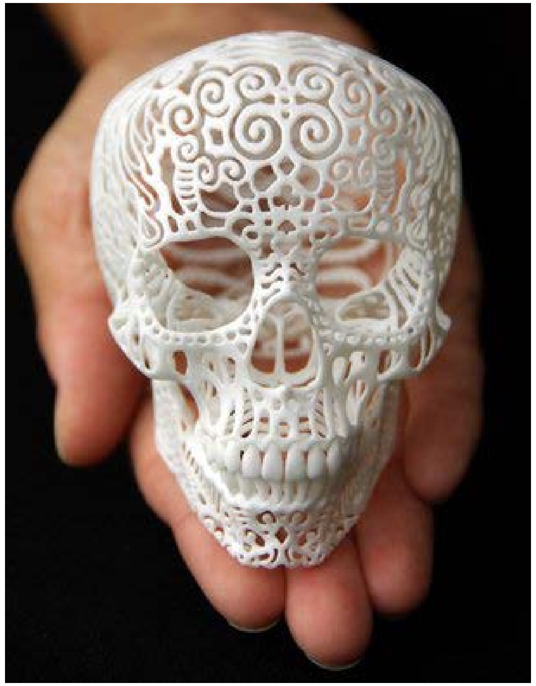
Part 3: Controllable Topology Optimization – Geometric Features

Dr. Jun Wu

TU Delft

Complexity is free

TU Delft & MX3D, 2015

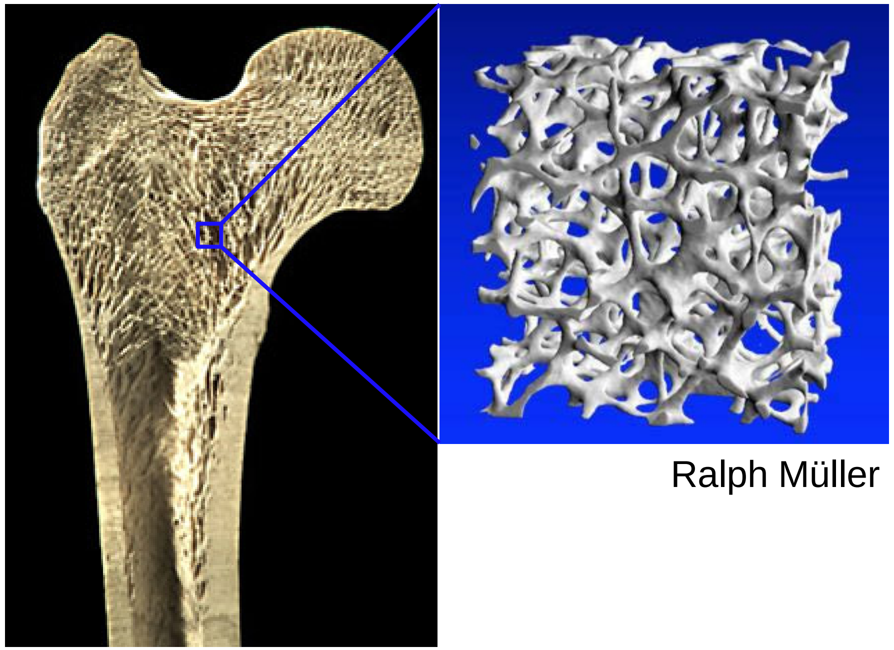


Joshua Harker

Scott Summit

Complexity is free? … Not really!

Tiny details



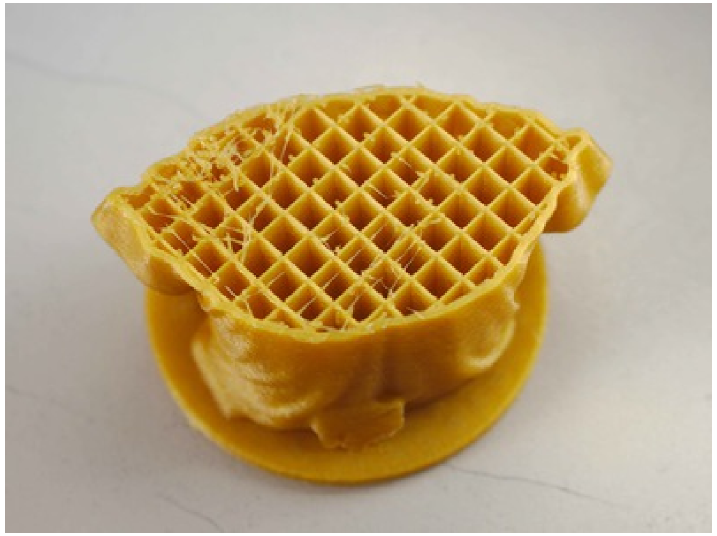
Ralph Müller

Paul Crompton

Supports

Concept Laser GmhH

Infill



mpi.fs.tum.de

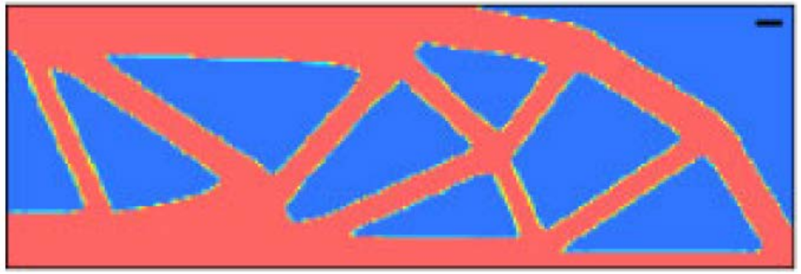
Outline

- Geometric feature control by **density filters**
- Geometric feature control by **alternative parameterizations**

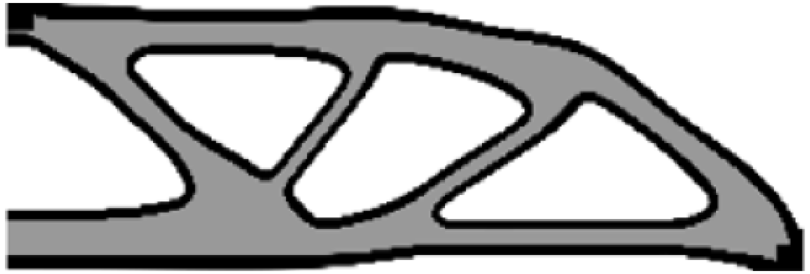
Geometric feature control by density filters
(An incomplete list)

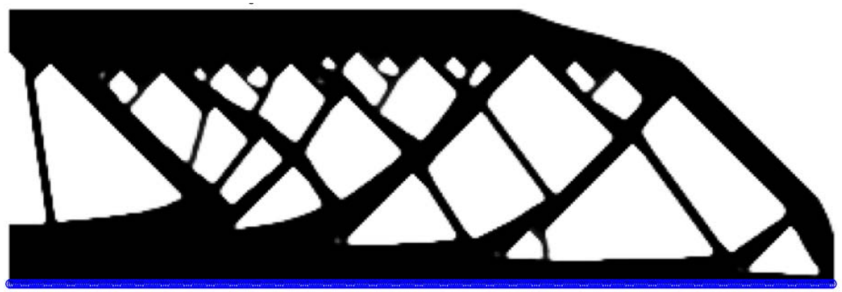
Reference

Minimum feature size, Guest'04

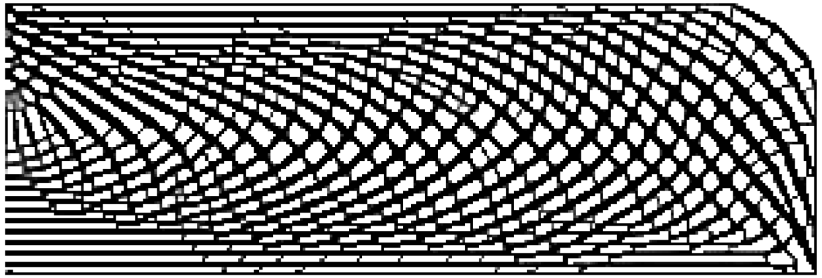


Coating structure, Clausen'15



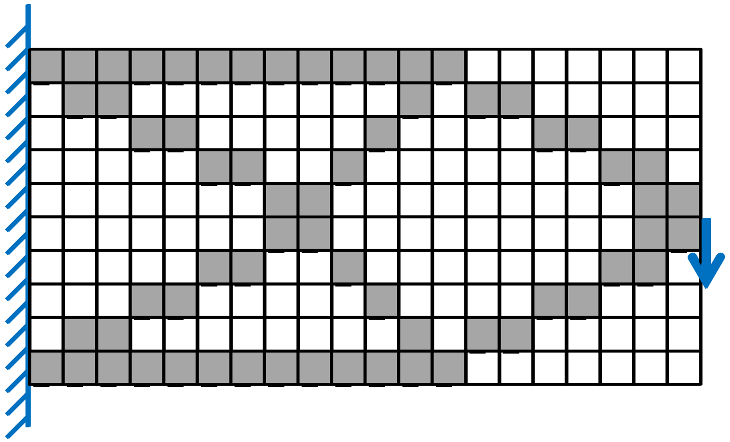


Self-supporting design, Langelaar'16



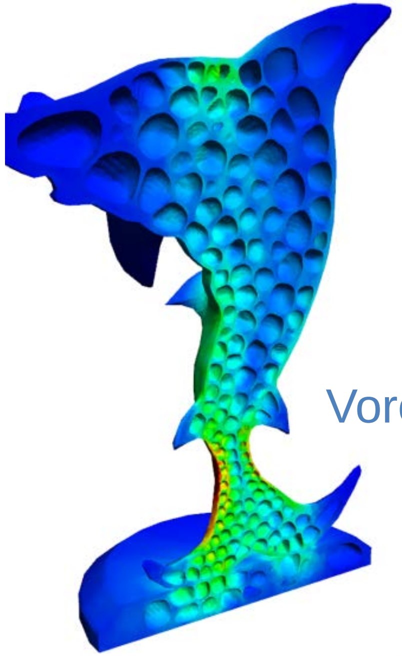
Porous infill, Wu'16

Geometric feature control by alternative parameterizations (An incomplete list)

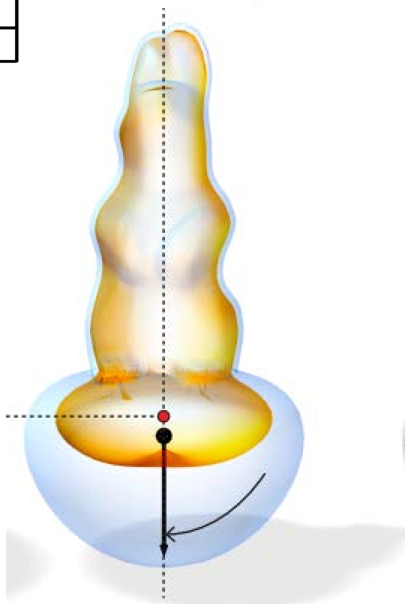


Reference: Voxel discretization

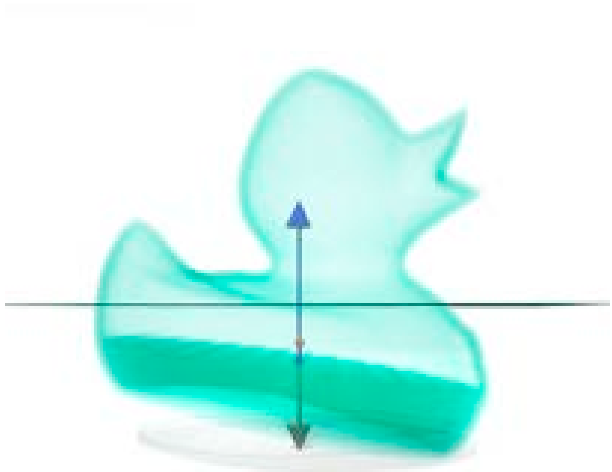
Skin-frame, Wang'13



Voronoi cells, Lu'14



Offset surfaces, Musialski'15



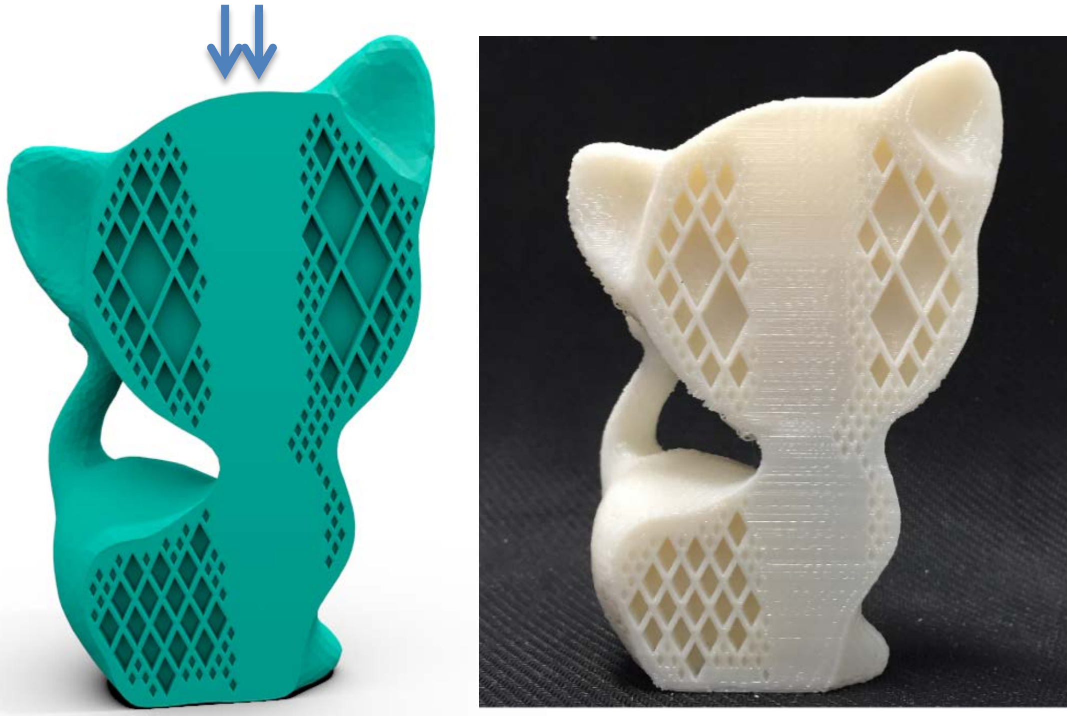
Ray representation, Wu'16

Adaptive rhombic, Wu'16

Outline

- Geometric feature control by **density filters**
- Geometric feature control by **alternative parameterizations**

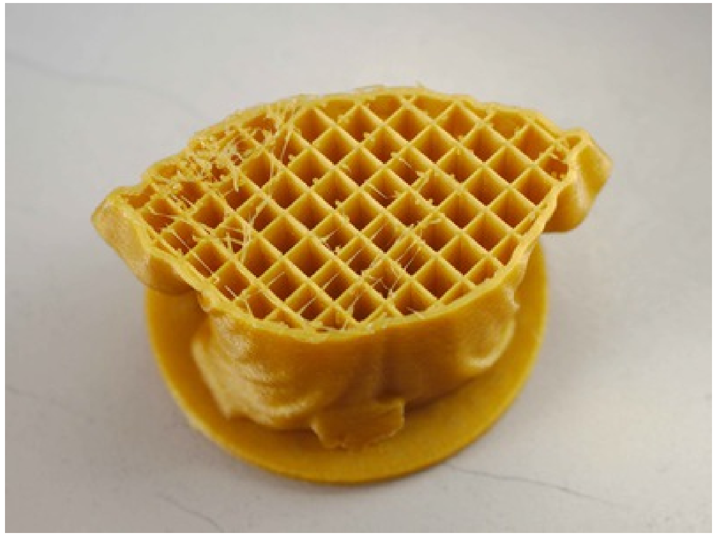
Bone-inspired infill



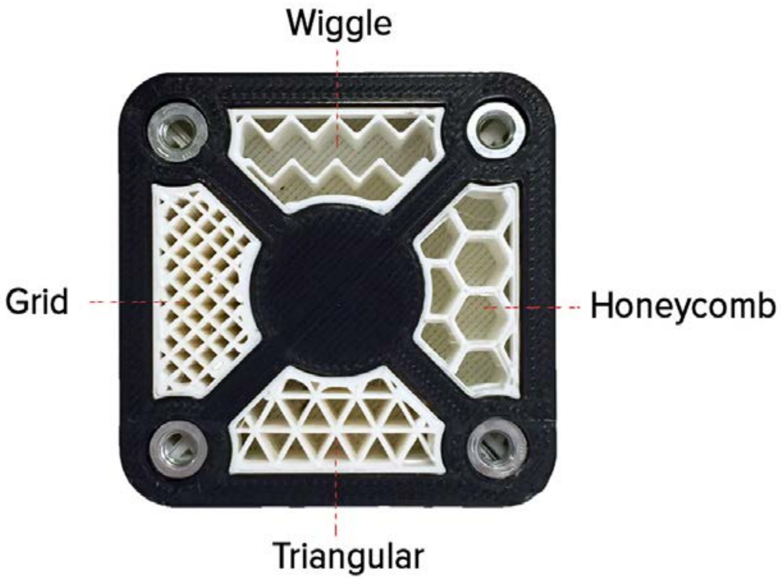
Self-supporting infill

Infill in 3D Printing

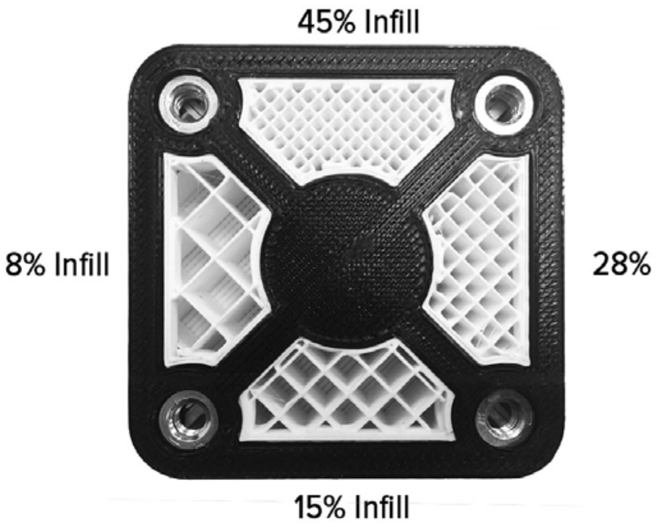
- A user-selected regular pattern, with a volume percentage
- A rough balance between
  - Physical properties (mass, strength), and
  - Cost (material usage, print time)



Infill



Different infill patterns

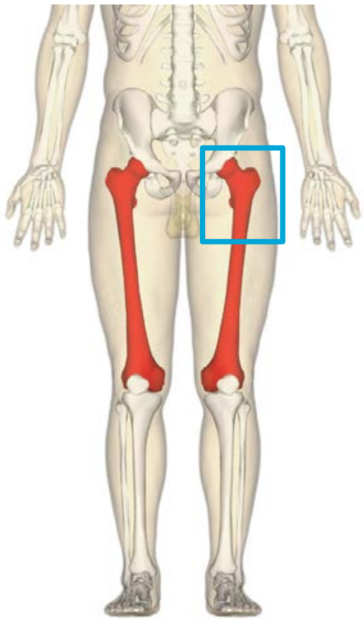


Different infill percentages

https://3dplatform.com/3d-printing-tips-infill-percentage-and-pattern-explained/

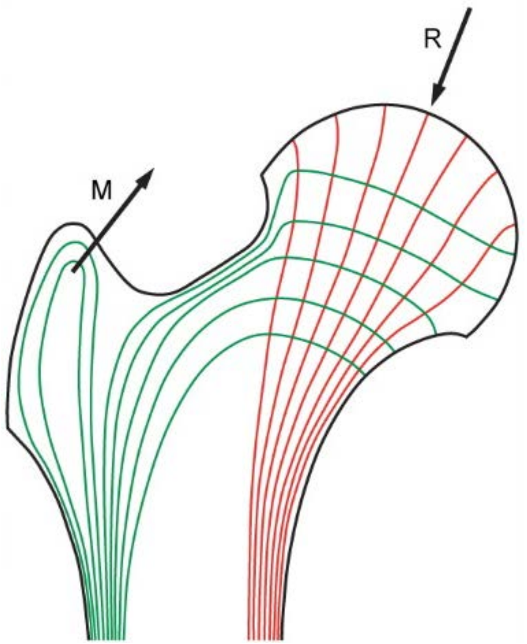
Infill in Nature

- Trabecular bone
  - Porous structures, oriented with the principle stress direction
  - Resulted from a natural optimization process
  - Light-weight-high-resistant



wikipedia.org

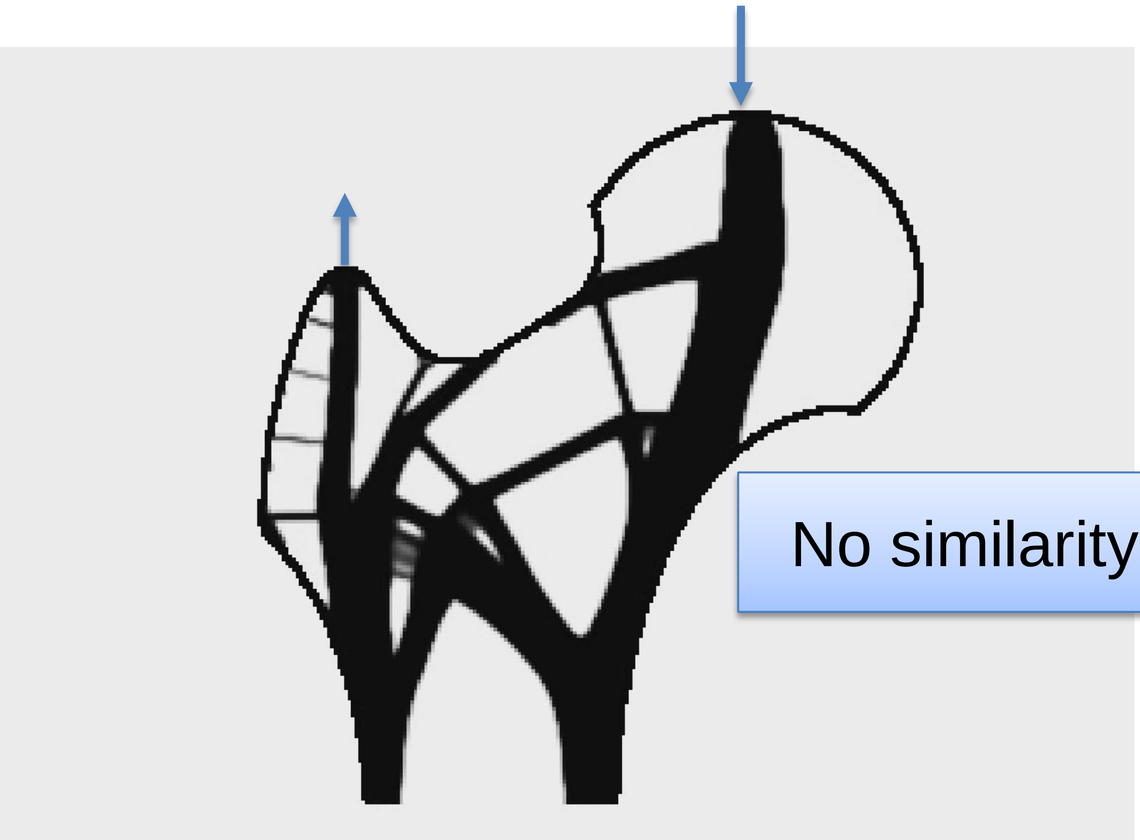
Cross-section of a human femur



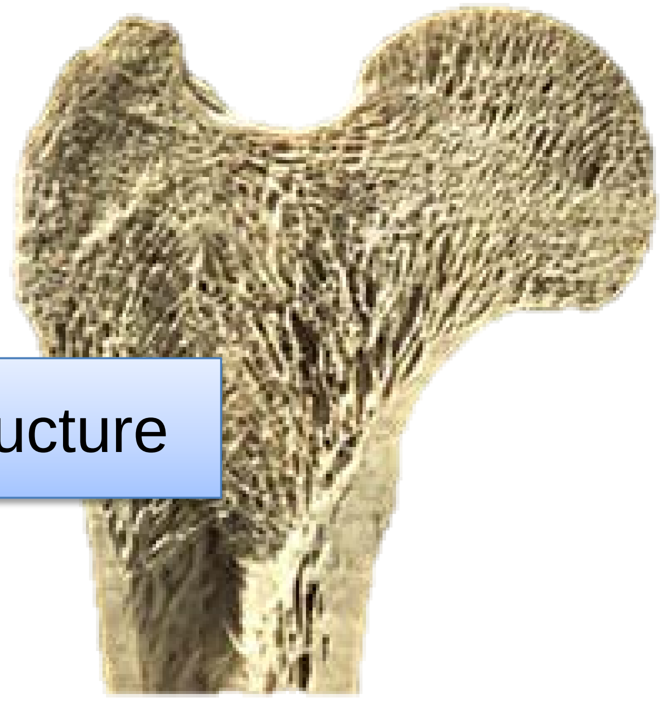
Principle stress directions

Optimize bone-like structures as infill for AM?

Topology Optimization Applied to Design Infill



Topology optimization

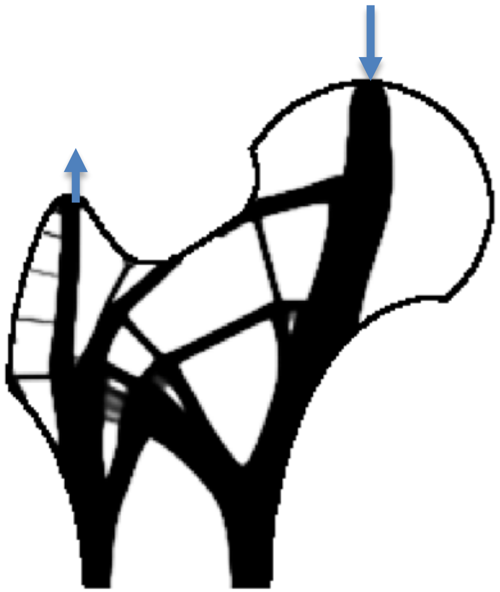


Infill in the bone

No similarity in structure

Topology Optimization Applied to Design Infill

- Materials accumulate to “important” regions
- The total volume $\sum_i \rho_i v_i \leq V_0$ does not restrict local material distribution



Infill by standard topology optimization

Infill in the bone

Approaching Bone-like Structures: The Idea

- Impose **local constraints** to avoid fully solid regions

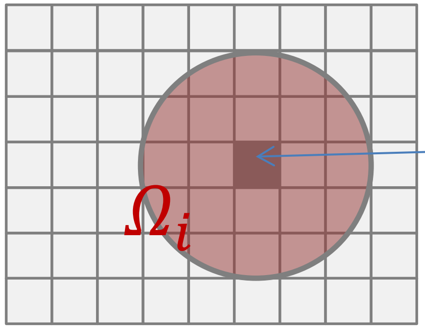
Min: $c = \frac{1}{2} U^T K U$

s.t. : $KU = F$

$\rho_i \in [0,1], \forall i$

~~$\sum_i \rho_i \leq V_0$~~

$\hat{\rho}_i \leq \alpha, \forall i$



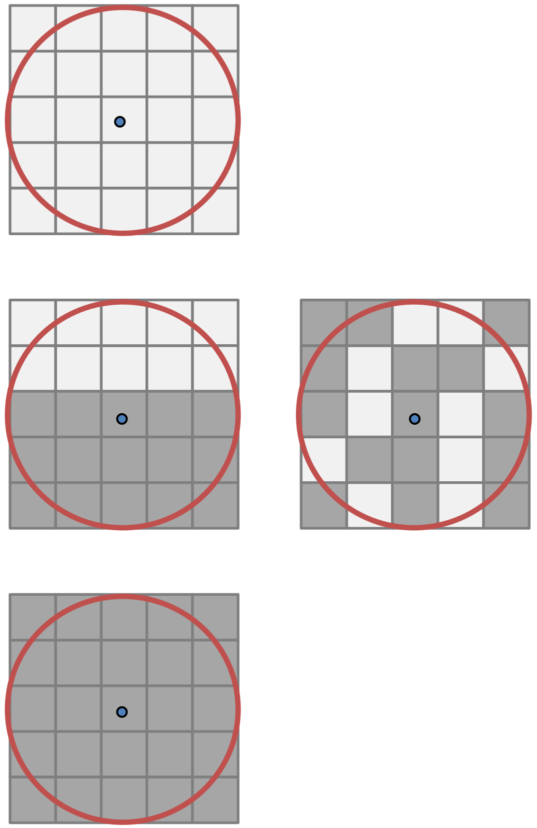
$$\hat{\rho}_i = \frac{\sum_{j \in \Omega_i} \rho_j}{\sum_{j \in \Omega_i} 1}$$

Local-volume measure

$\hat{\rho}_i = 0.0$

$\hat{\rho}_i = 0.6$

$\hat{\rho}_i = 1.0$



Constraints Aggregation (Reduce the Number of Constraints)

$\hat{\rho}_i \leq \alpha, \forall i$ ⟺ $\max_{i=1,\ldots,n} |\hat{\rho}_i| \leq \alpha$ ⟺ $\lim_{p\to\infty} \|\rho\|_p = \left(\sum_i (\hat{\rho}_i)^p\right)^{\frac{1}{p}} \leq \alpha$

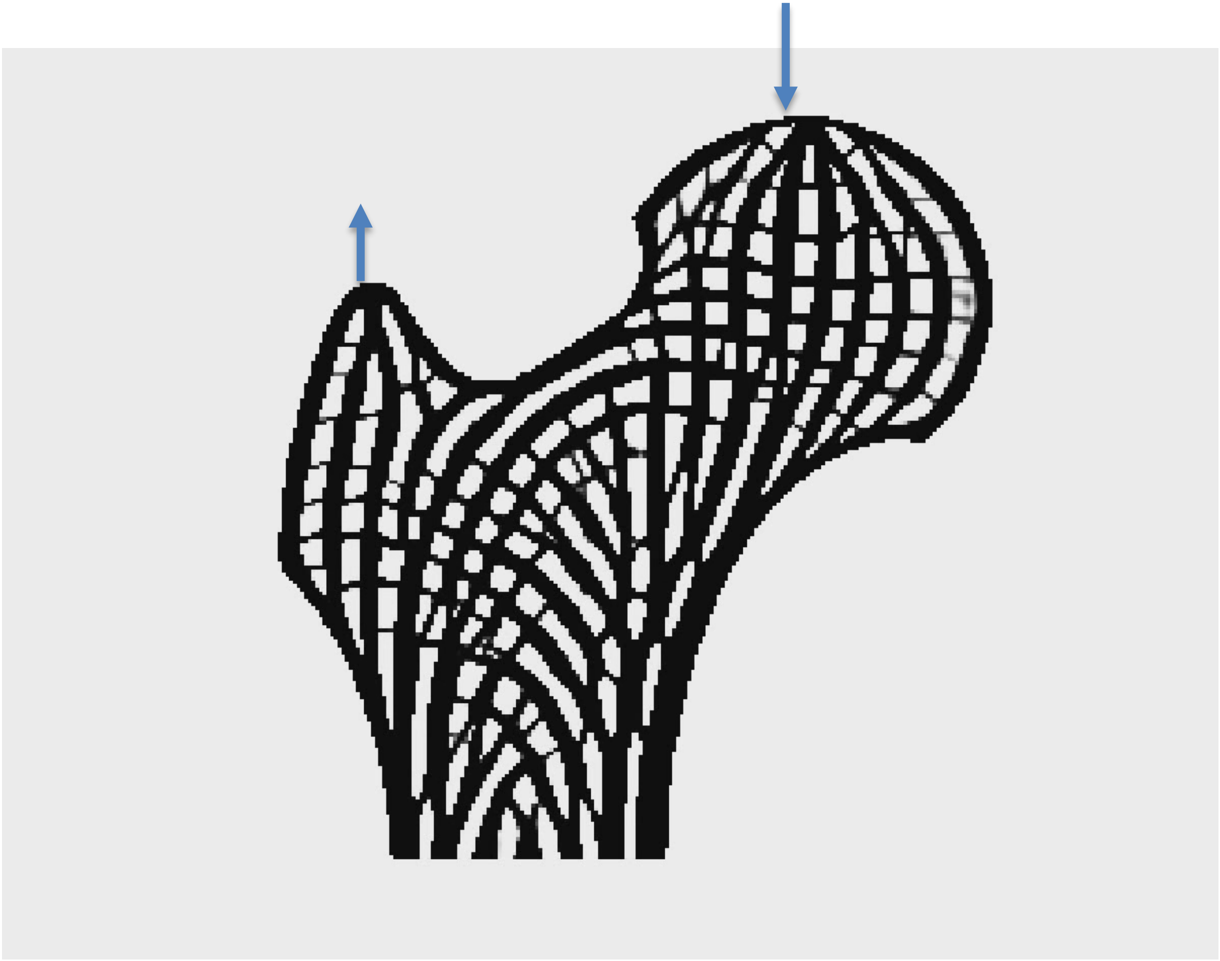
Too many constraints!

A single constraint
But non-differentiable

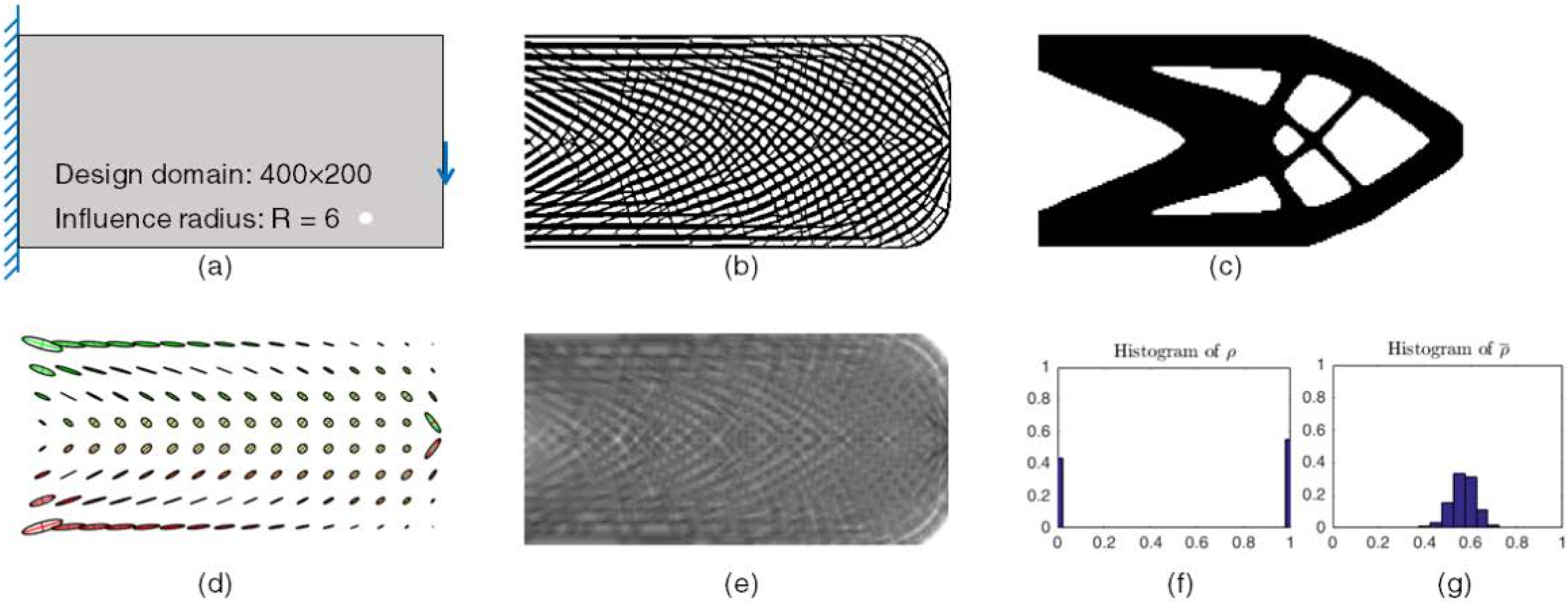
A single constraint
and differentiable
Approximated with p =16

Bone-like Infill in 2D

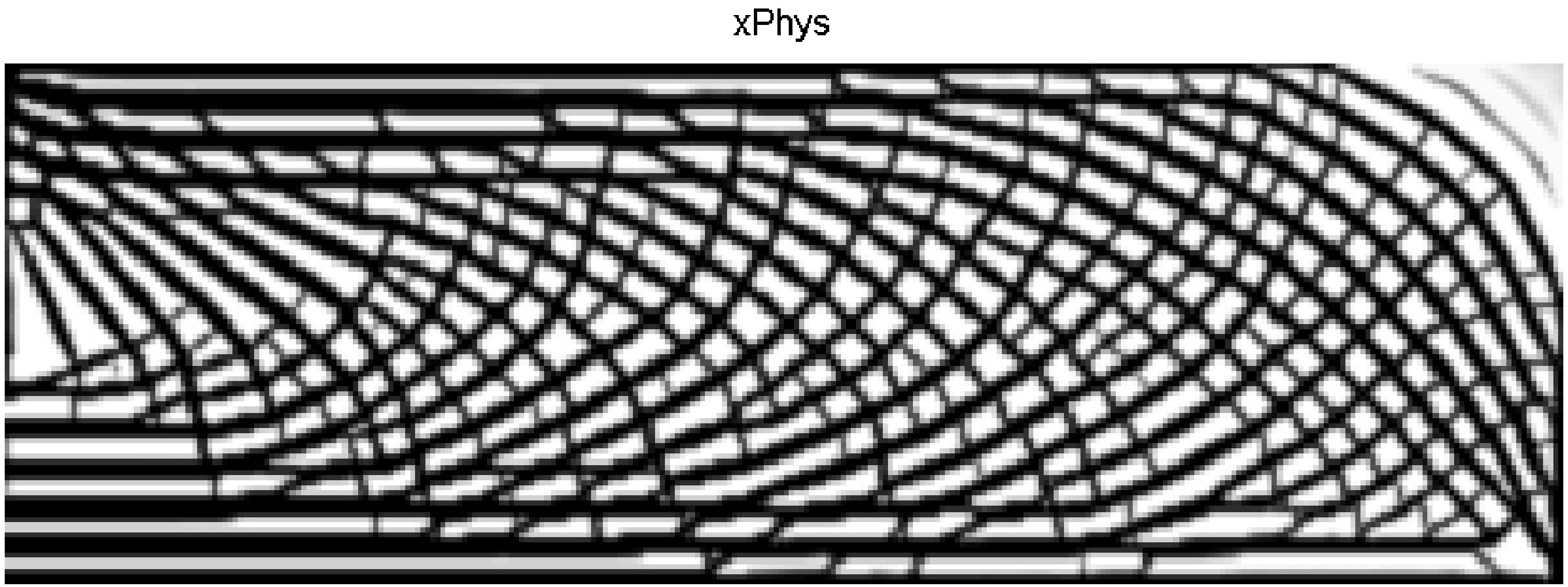
Cross-section of a human femur



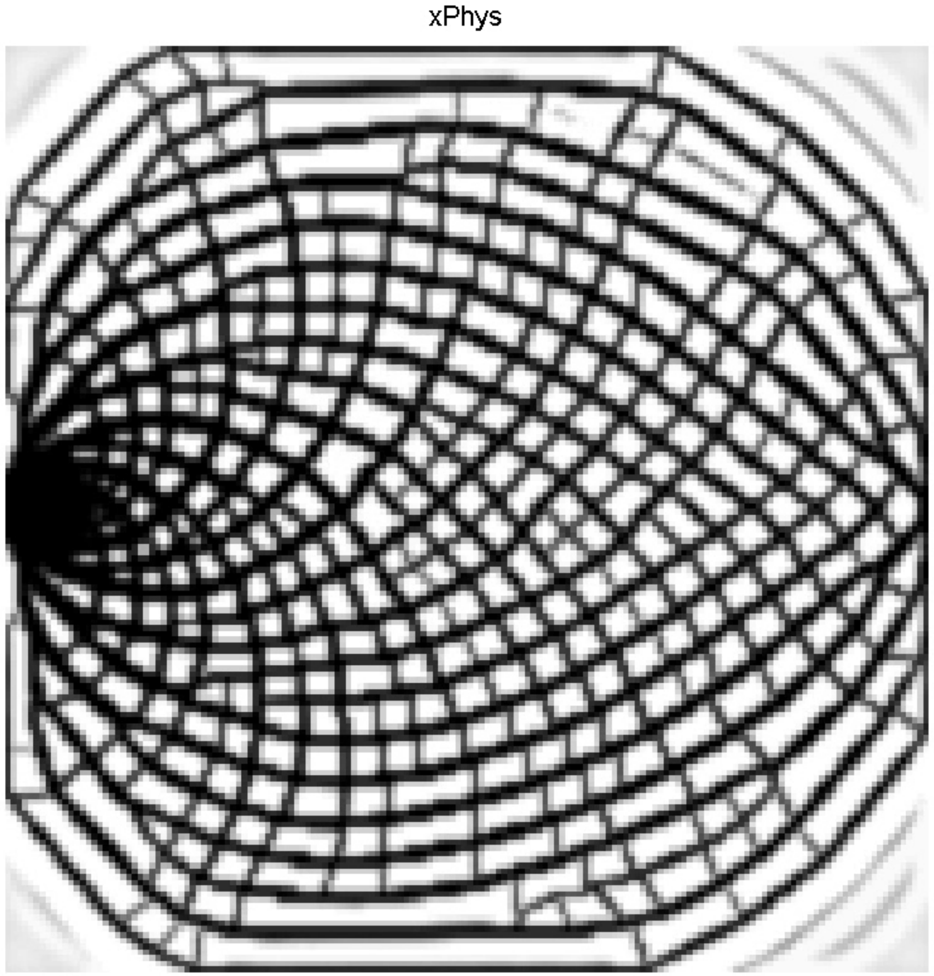
A Test Example



Result: 2D Animation



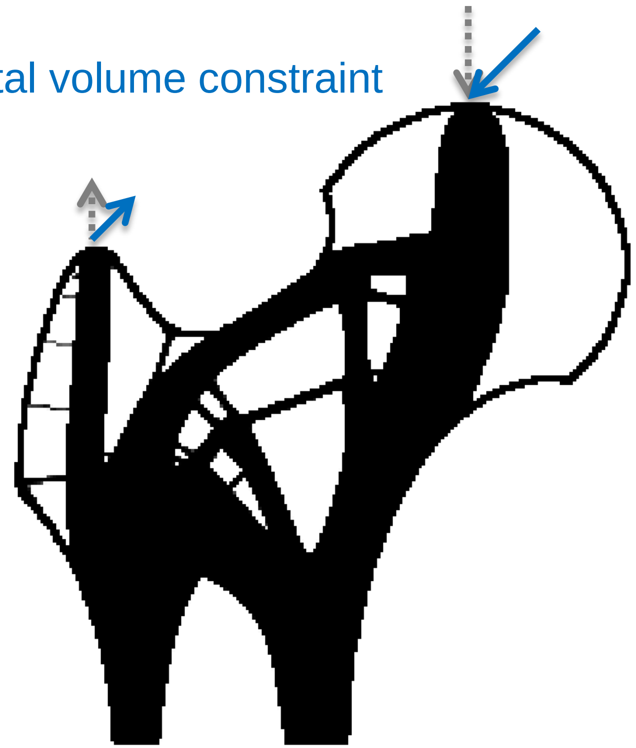
Result: 2D Animation



Robustness wrt. Force Variations

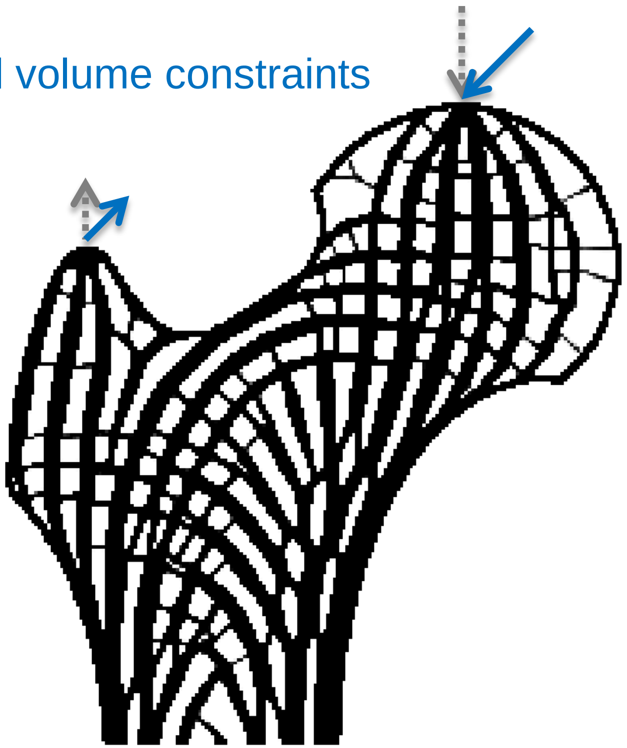
- Bone-like structures are significantly stiffer (126%) in case of force variations

Total volume constraint



c = 30.54
c’= 45.83

Local volume constraints

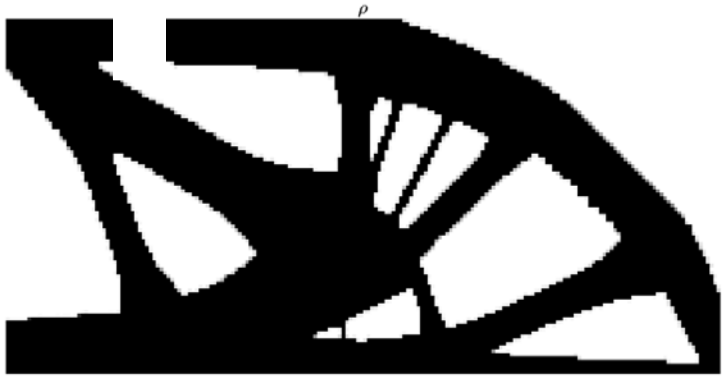


c = 36.72
c’ =36.23

Robustness wrt. Material Deficiency

- Bone-like structures are significantly stiffer (180%) in case of material deficiency

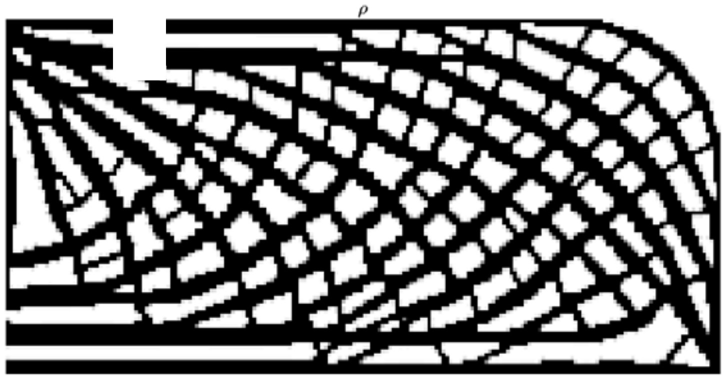
Total volume constraint



c = 76.83

c' =242.77

Local volume constraints

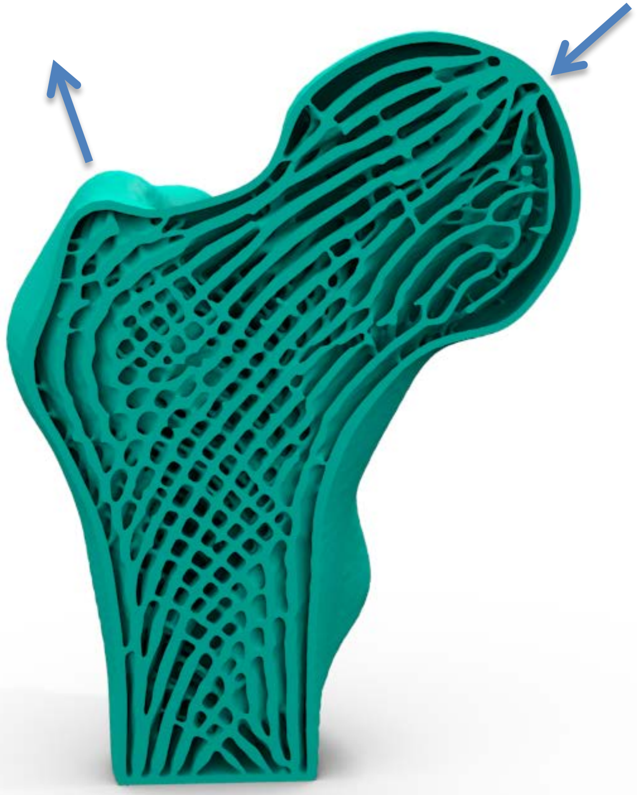


c = 93.48

c'= 134.84

Bone-like Infill in 3D

Infill in the bone

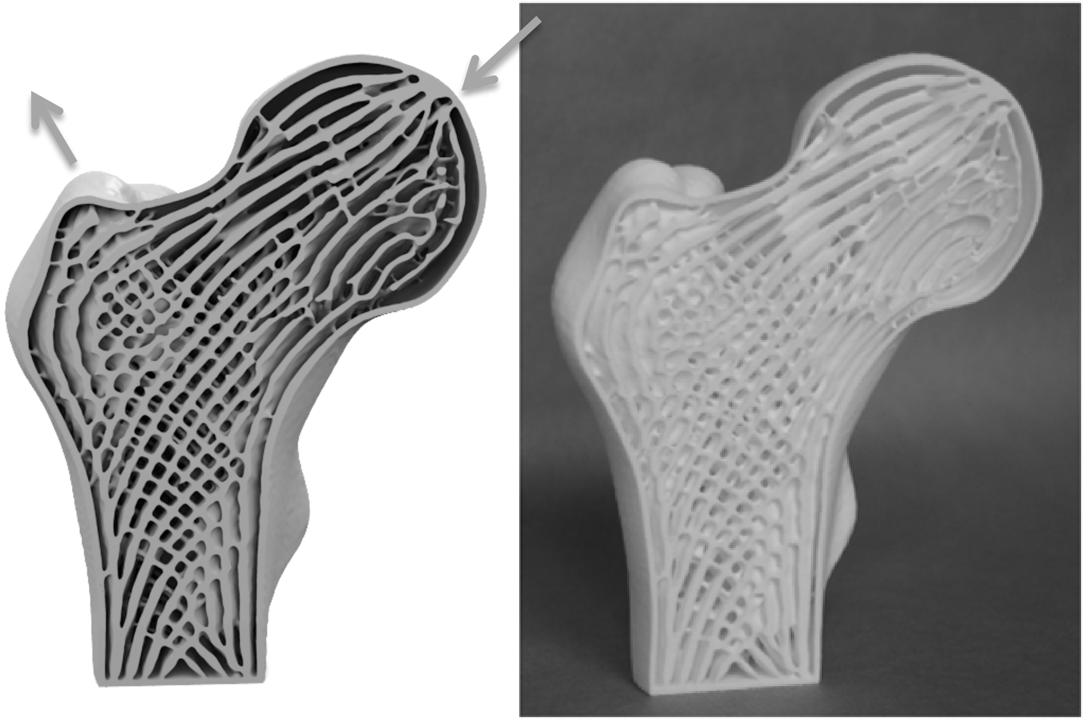


Optimized bone-like infill

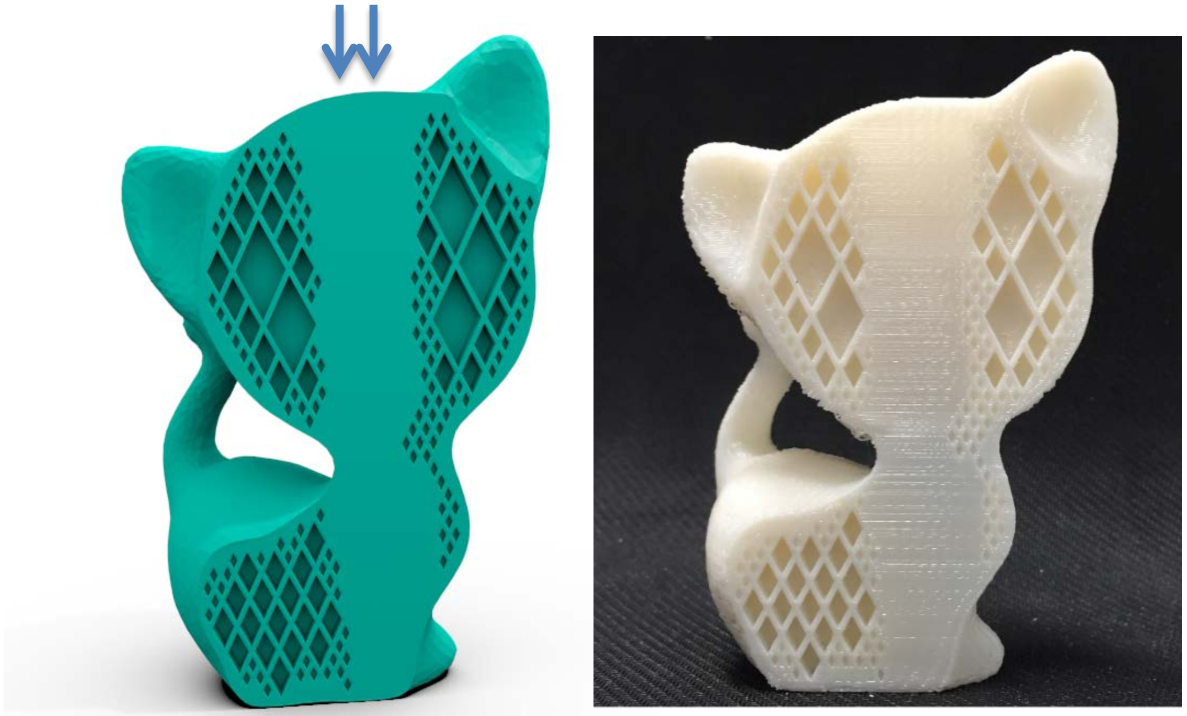
Wu et al., TVCG’2017

Outline

- Geometric feature control by **density filters**
- Geometric feature control by **alternative parameterizations**



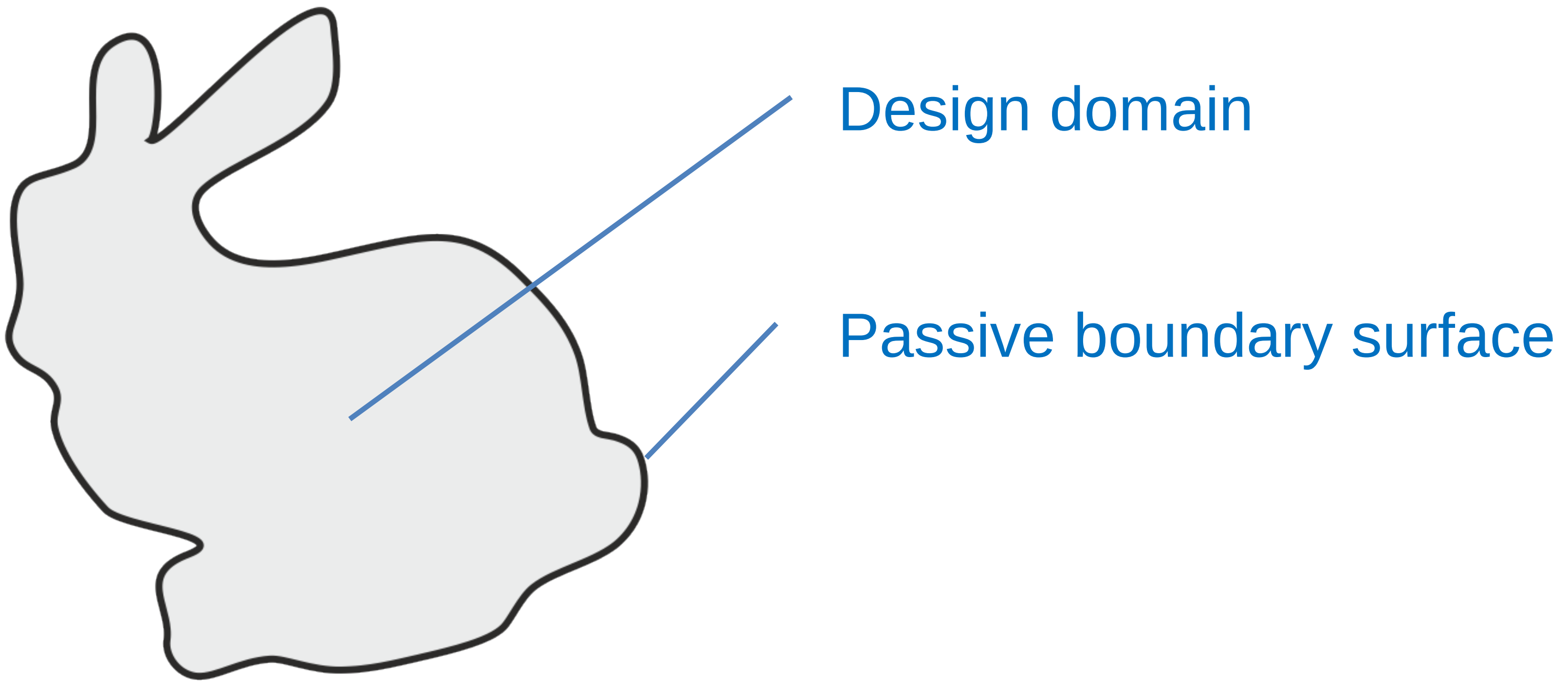
Bone-inspired infill



Self-supporting infill

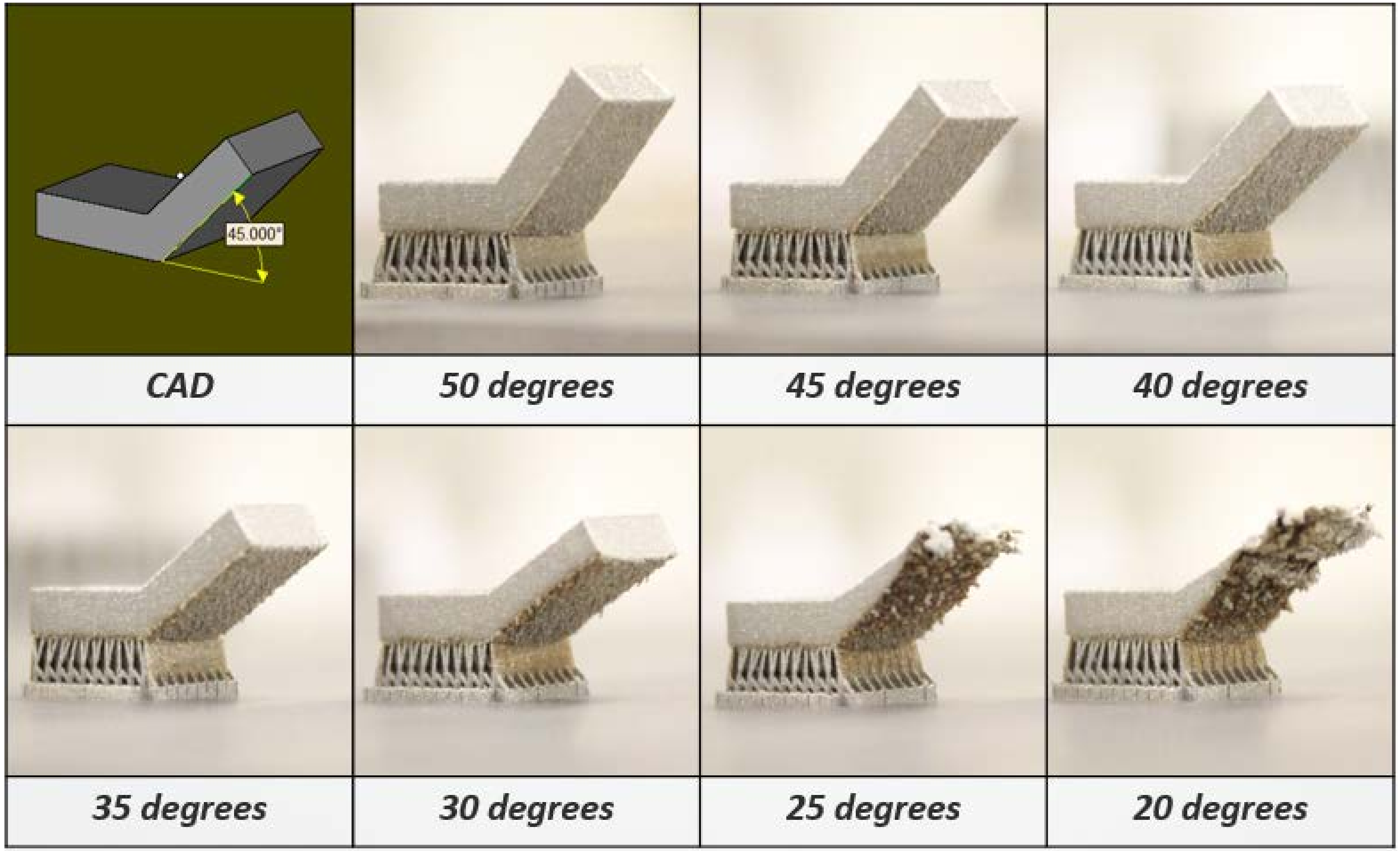
Infill Optimization

- To find the optimal material distribution in the interior of a given shape



Overhang in Additive Manufacturing

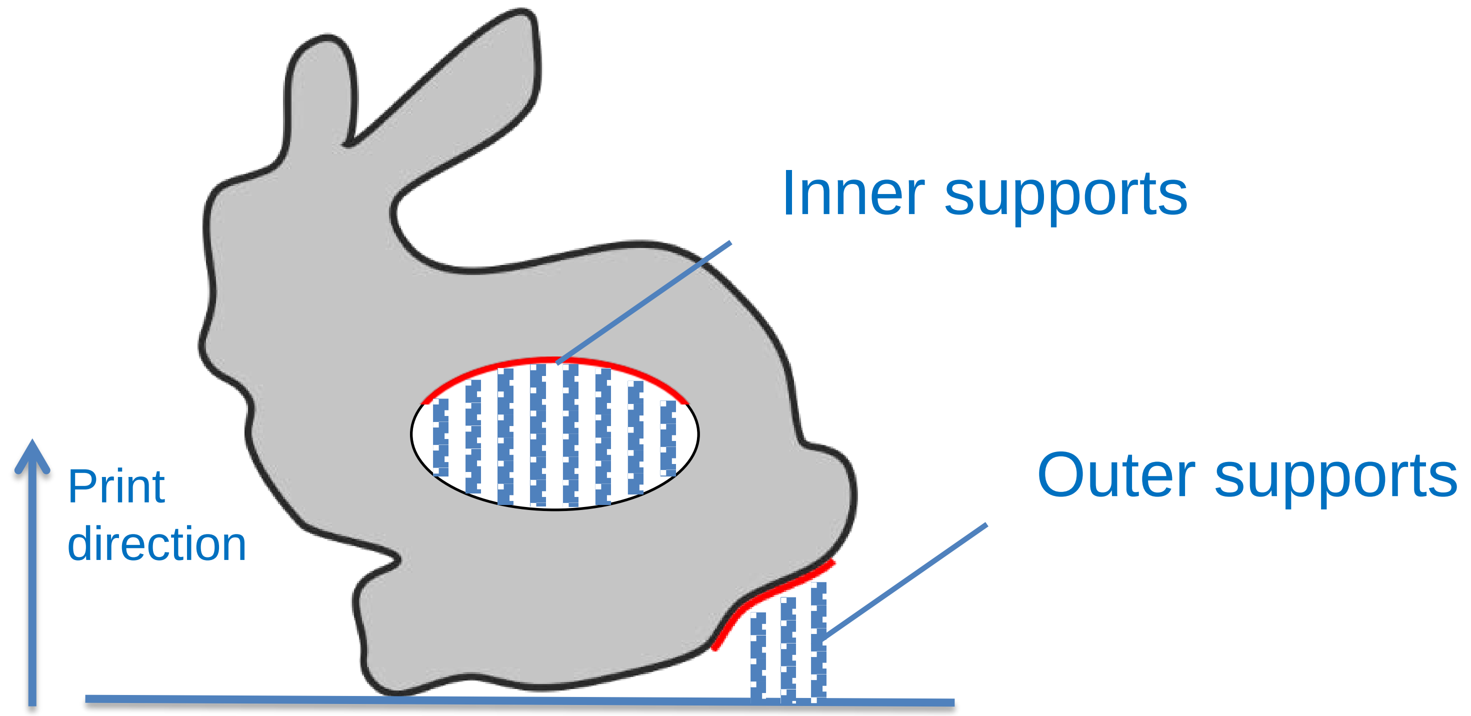
- Support structures are needed beneath overhang surfaces



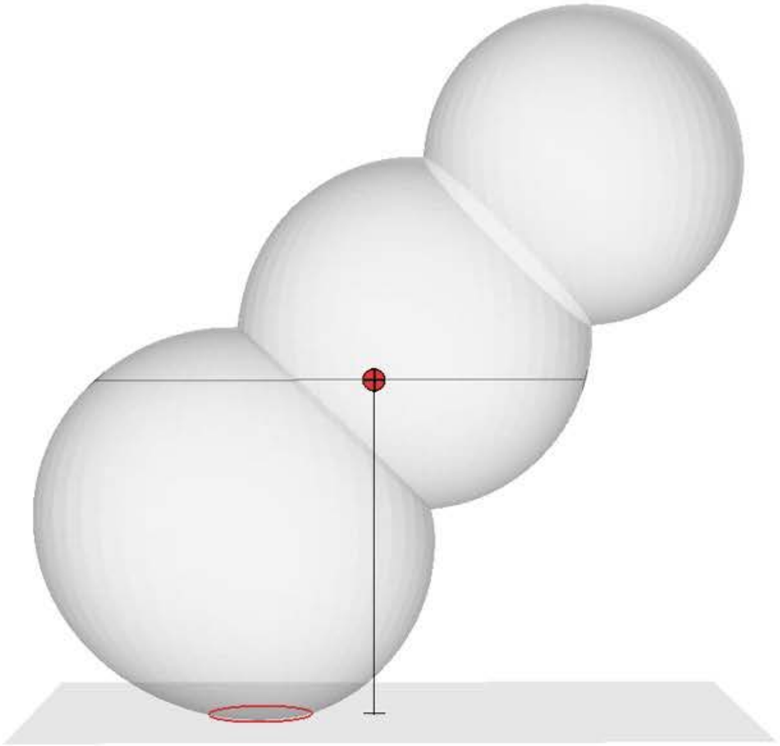
https://www.protolabs.com/blog/tag/direct-metal-laser-sintering/

Support Structures in Cavities

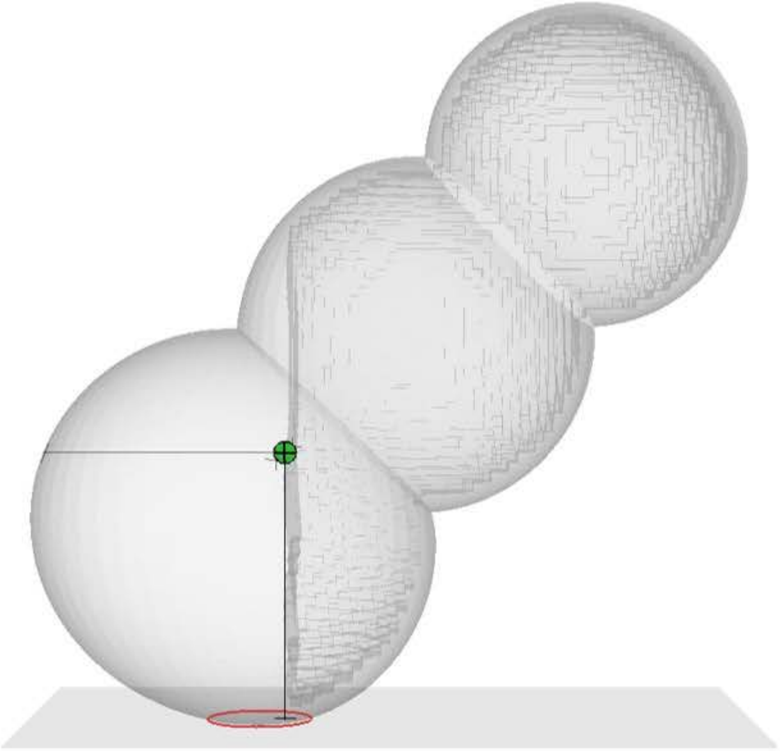
- Post-processing of inner supports is problematic



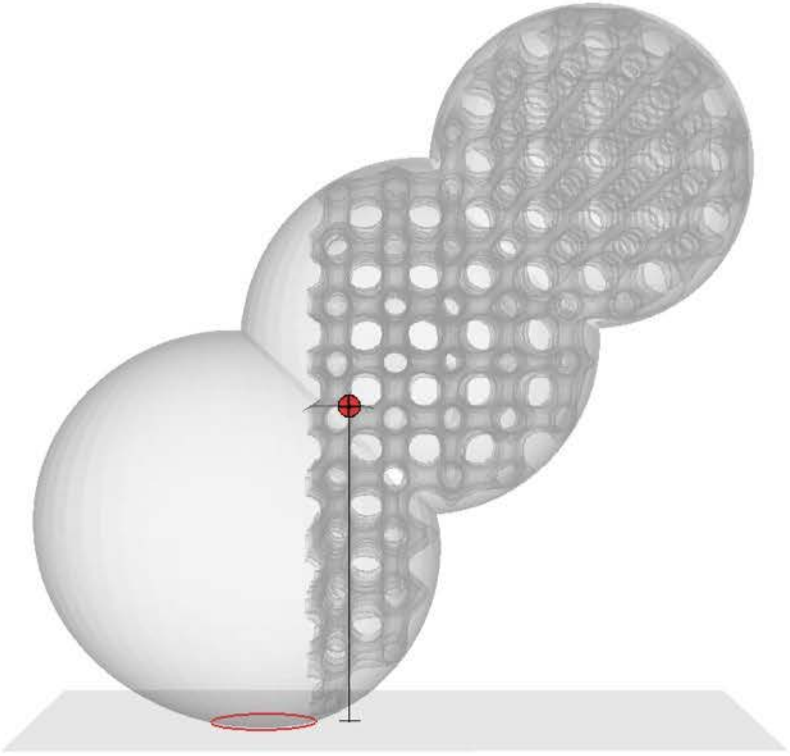
Infill & Optimization Shall Integrate



Solid,
Unbalanced



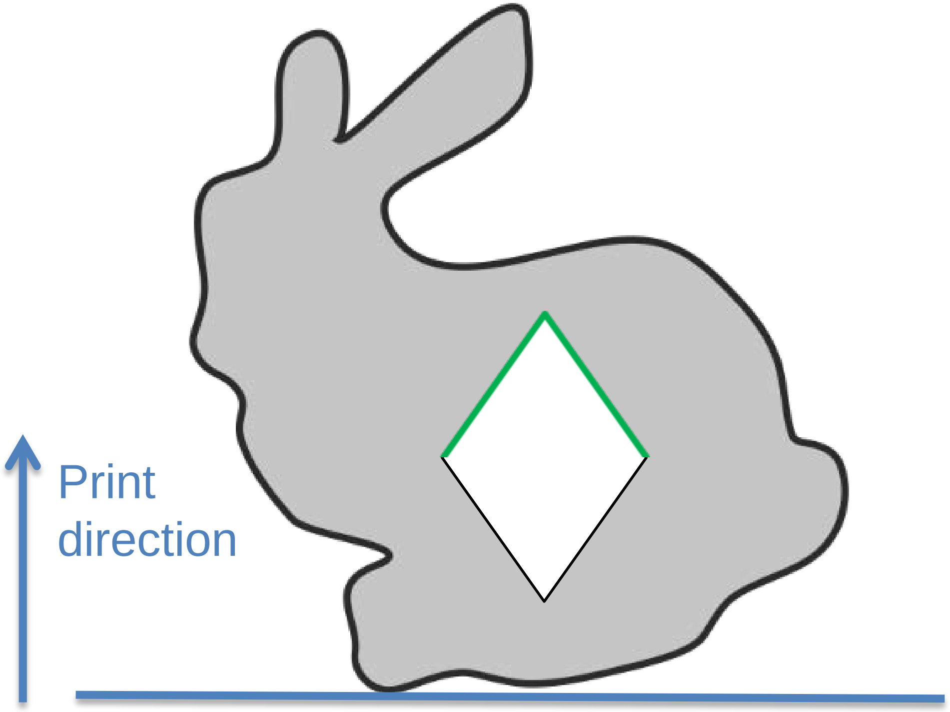
Optimized,
Balanced



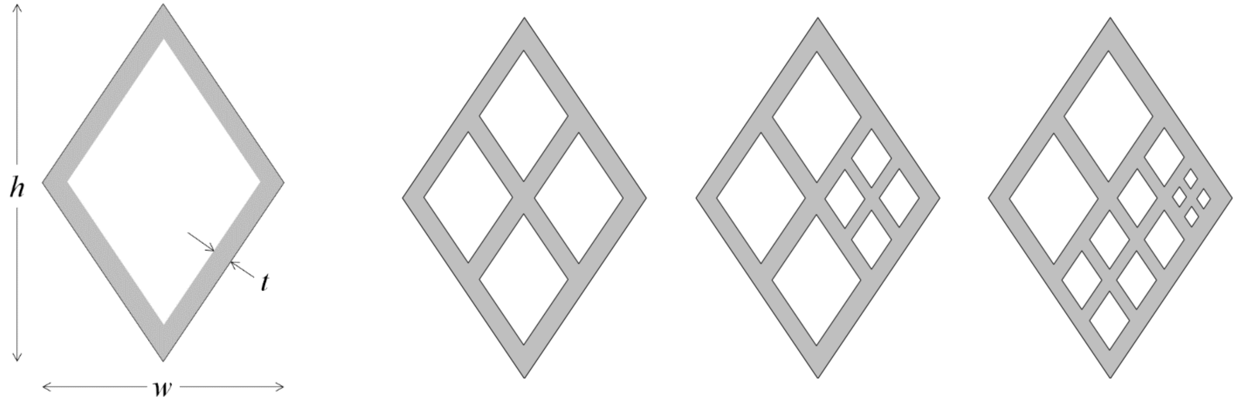
With infill,
Unbalanced

The Idea

- Rhombic cell: to ensure self-supporting
- Adaptive subdivision: as design variable in optimization

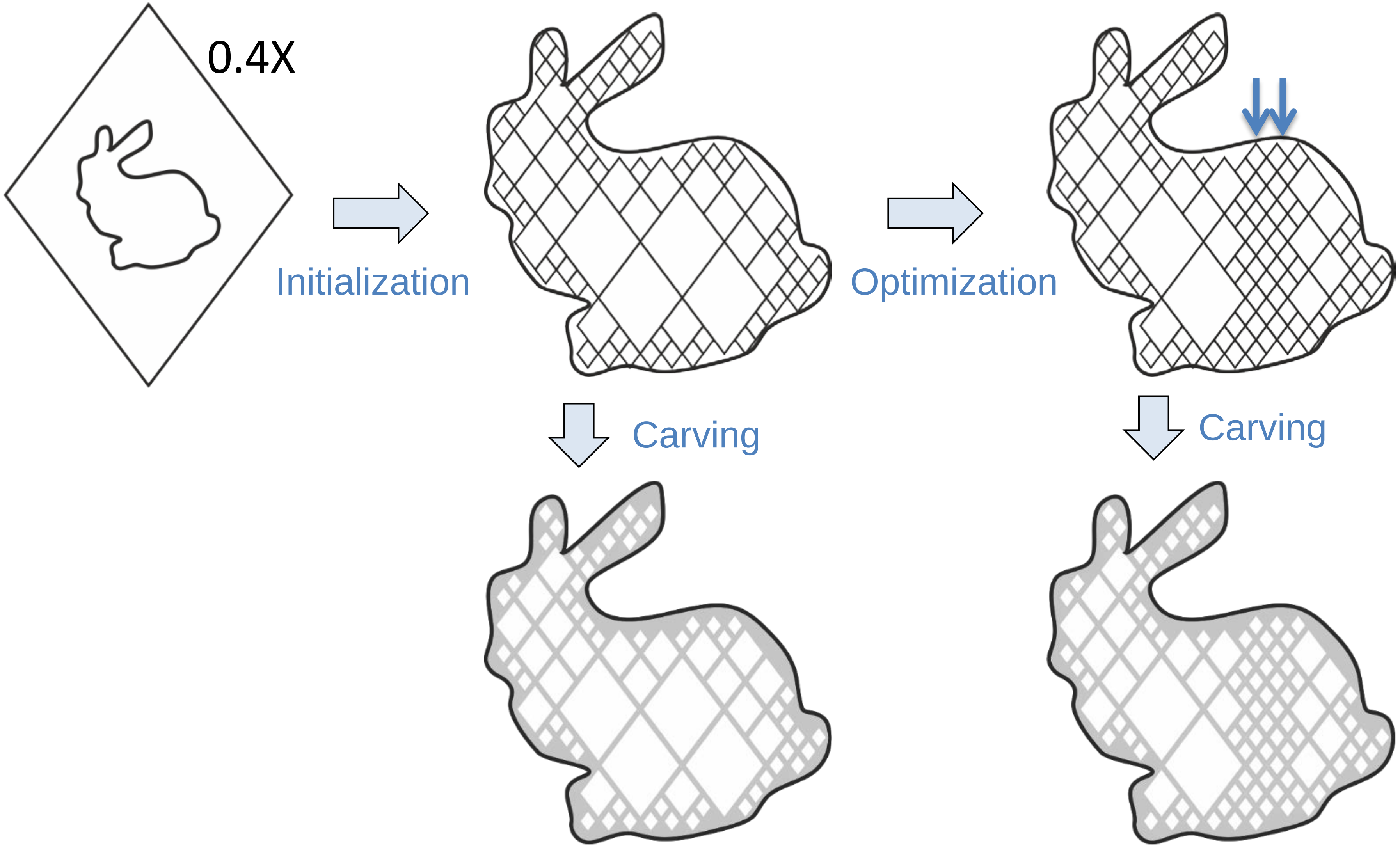


Rhombic cell



Adaptive subdivision

Self-Supporting Rhombic Infill: Workflow



Self-Supporting Rhombic Infill: Subdivision Criteria

- Min: $c = \frac{1}{2} U^T K U$ Subject to: $KU = F; V = \sum_i \rho_i \leq V_0$

Voxel-wise topology optimization

Per-voxel density as variable

$$\rho_i \in \{0.0, 1.0\}, \forall i$$

Per-voxel sensitivity: $G_i = -\frac{\partial c / \partial \rho_i}{\partial V / \partial \rho_i}$

Subdivision-based topology optimization

Per-subdivision as variable

$$\beta_c \in \{0, 1\}, \forall c$$

Per-voxel density assigned by subdivision

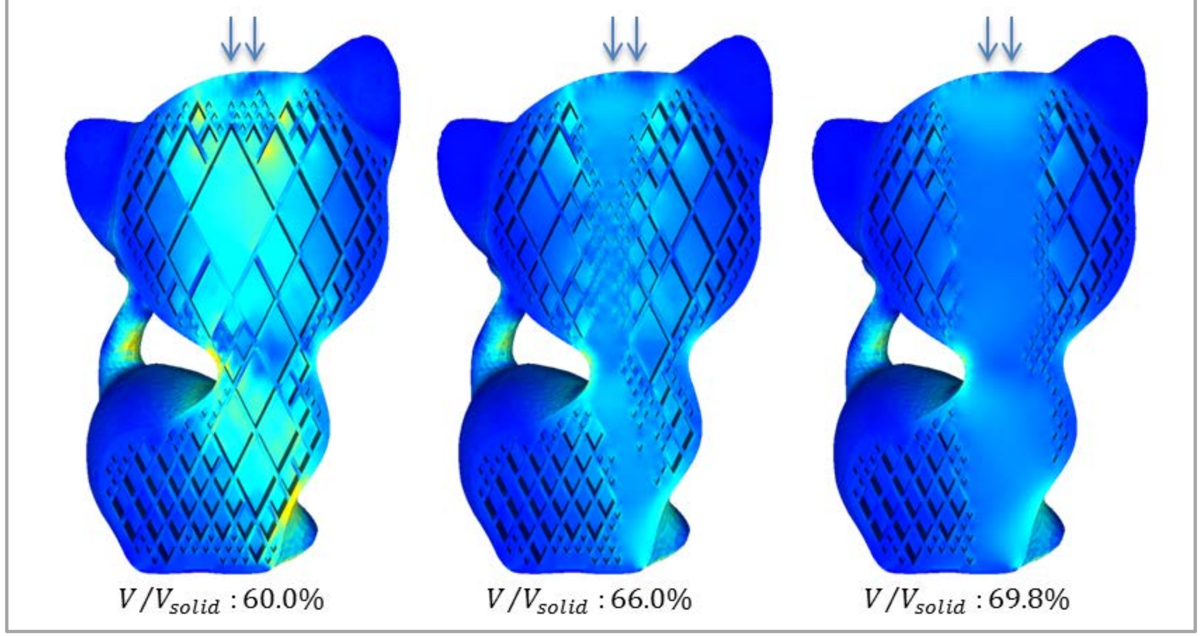
$$\rho_i(\beta) = \begin{cases} 1.0 & i \text{ covered by walls} \\ 0.0 & \text{otherwise} \end{cases}$$

Per-subdivision sensitivity: $G_c = -\frac{\partial c / \partial \beta_c}{\partial V / \partial \beta_c}$

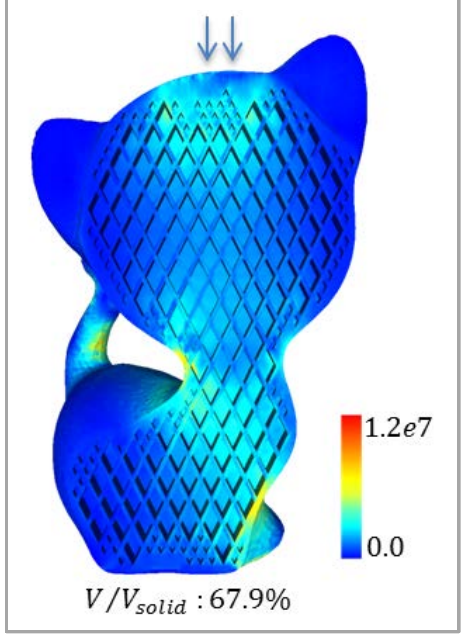
Self-Supporting Rhombic Infill: Results

- Optimized mechanical properties, compared to regular infill
- No additional inner supports needed

Optimization process



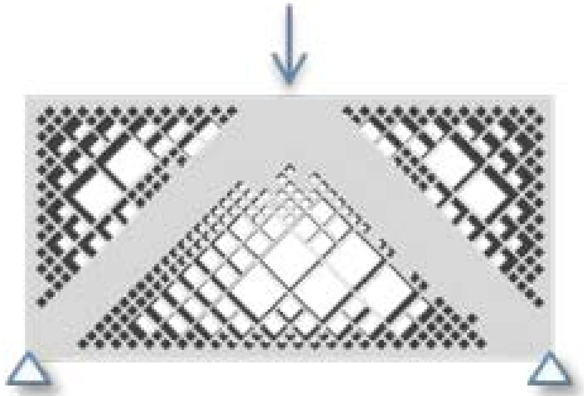
Reference

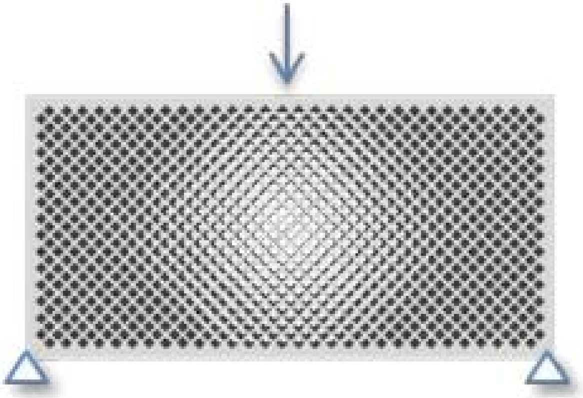


Print

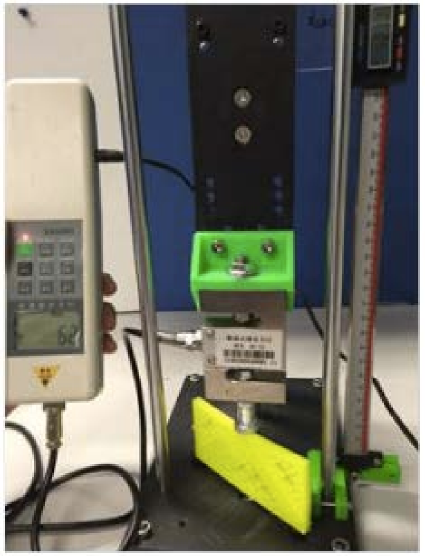
Wu et al., CAD’2016

Mechanical Tests





Under same force (62 N)



Dis.
2.11 mm

Dis.
4.08 mm

Under same displacement (3.0 mm)

Force
90 N

Force
58 N

Summary

- Geometric feature control by **density filters**
- Geometric feature control by **alternative parameterizations**

Thank you for your attention!

Questions?

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Depart. of Design Engineering, TU Delft

Incomplete references: Density filters

- Guest, James K., Jean H. Prévost, and T. Belytschko. "Achieving minimum length scale in topology optimization using nodal design variables and projection functions." International journal for numerical methods in engineering 61, no. 2 (2004): 238-254.
- Wang, Fengwen, Boyan Stefanov Lazarov, and Ole Sigmund. "On projection methods, convergence and robust formulations in topology optimization." Structural and Multidisciplinary Optimization 43, no. 6 (2011): 767-784.
- Clausen, Anders, Niels Aage, and Ole Sigmund. "Topology optimization of coated structures and material interface problems." Computer Methods in Applied Mechanics and Engineering 290 (2015): 524-541.
- Langelaar, Matthijs. "An additive manufacturing filter for topology optimization of print-ready designs." Structural and Multidisciplinary Optimization (2016): 1-13.
- Wu, Jun, Niels Aage, Ruediger Westermann, and Ole Sigmund. "Infill Optimization for Additive Manufacturing--Approaching Bone-like Porous Structures." IEEE Transactions on Visualization and Computer Graphics, 2016.

Incomplete references: Alternative parameterizations

- Wang, Weiming, Tuanfeng Y. Wang, Zhouwang Yang, Ligang Liu, Xin Tong, Weihua Tong, Jiansong Deng, Falai Chen, and Xiuping Liu. "Cost-effective printing of 3D objects with skin-frame structures." ACM Transactions on Graphics (TOG) 32, no. 6 (2013): 177.
- Lu, Lin, Andrei Sharf, Haisen Zhao, Yuan Wei, Qingnan Fan, Xuelin Chen, Yann Savoye, Changhe Tu, Daniel Cohen-Or, and Baoquan Chen. "Build-to-last: Strength to weight 3d printed objects." ACM Transactions on Graphics (TOG) 33, no. 4 (2014): 97.
- Musialski, Przemyslaw, Thomas Auzinger, Michael Birsak, Michael Wimmer, and Leif Kobbelt. "Reduced-order shape optimization using offset surfaces." ACM Trans. Graph. 34, no. 4 (2015): 102.
- Wu, Jun, Lou Kramer, and Rüdiger Westermann. "Shape interior modeling and mass property optimization using ray-reps." Computers & Graphics 58 (2016): 66-72.
- Wu, Jun, Charlie CL Wang, Xiaoting Zhang, and Rüdiger Westermann. "Self-supporting rhombic infill structures for additive manufacturing." Computer-Aided Design 80 (2016): 32-42.

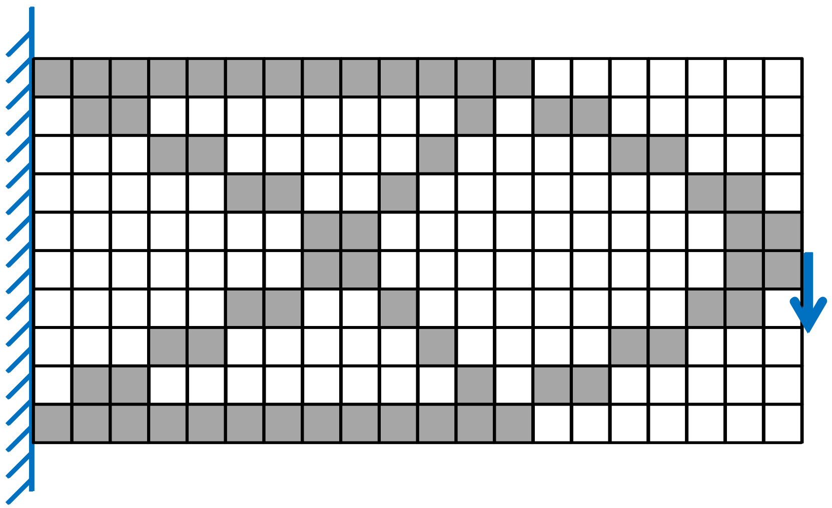
Topology Optimization

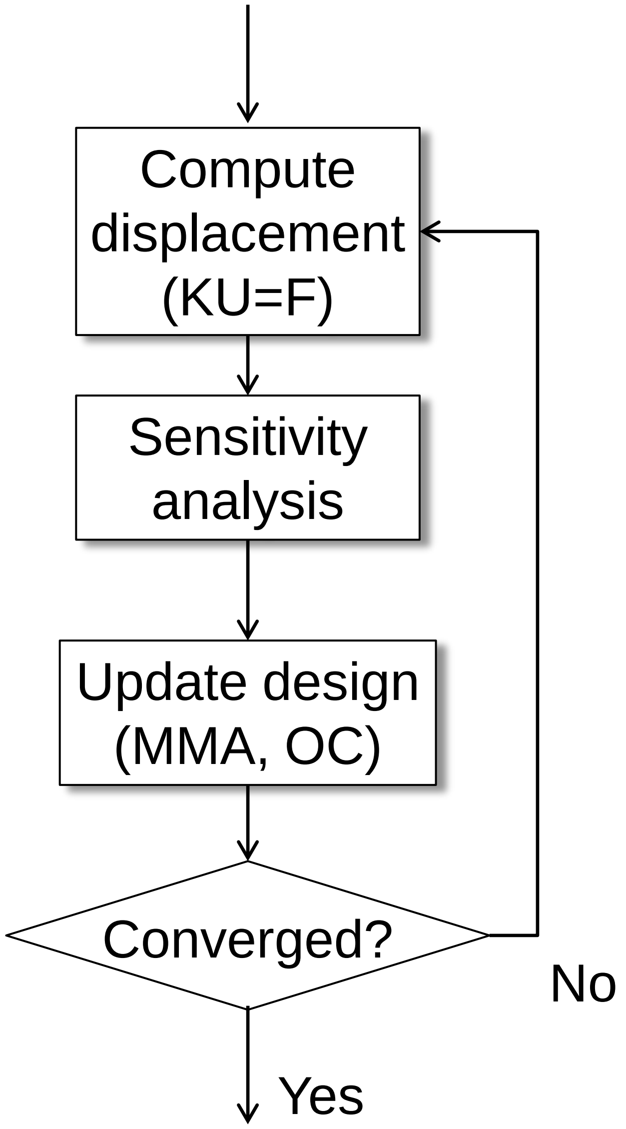
Minimize: $c = \frac{1}{2} U^T K U$

Subject to: $KU = F$

$\rho_i \in [0,1], \forall i$

$\sum_i \rho_i \leq V_0$





EUROGRAPHICS2017
The 38th annual conference of the
EUROPEAN ASSOCIATION FOR COMPUTER GRAPHICS

Topology Optimization for Computational Fabrication

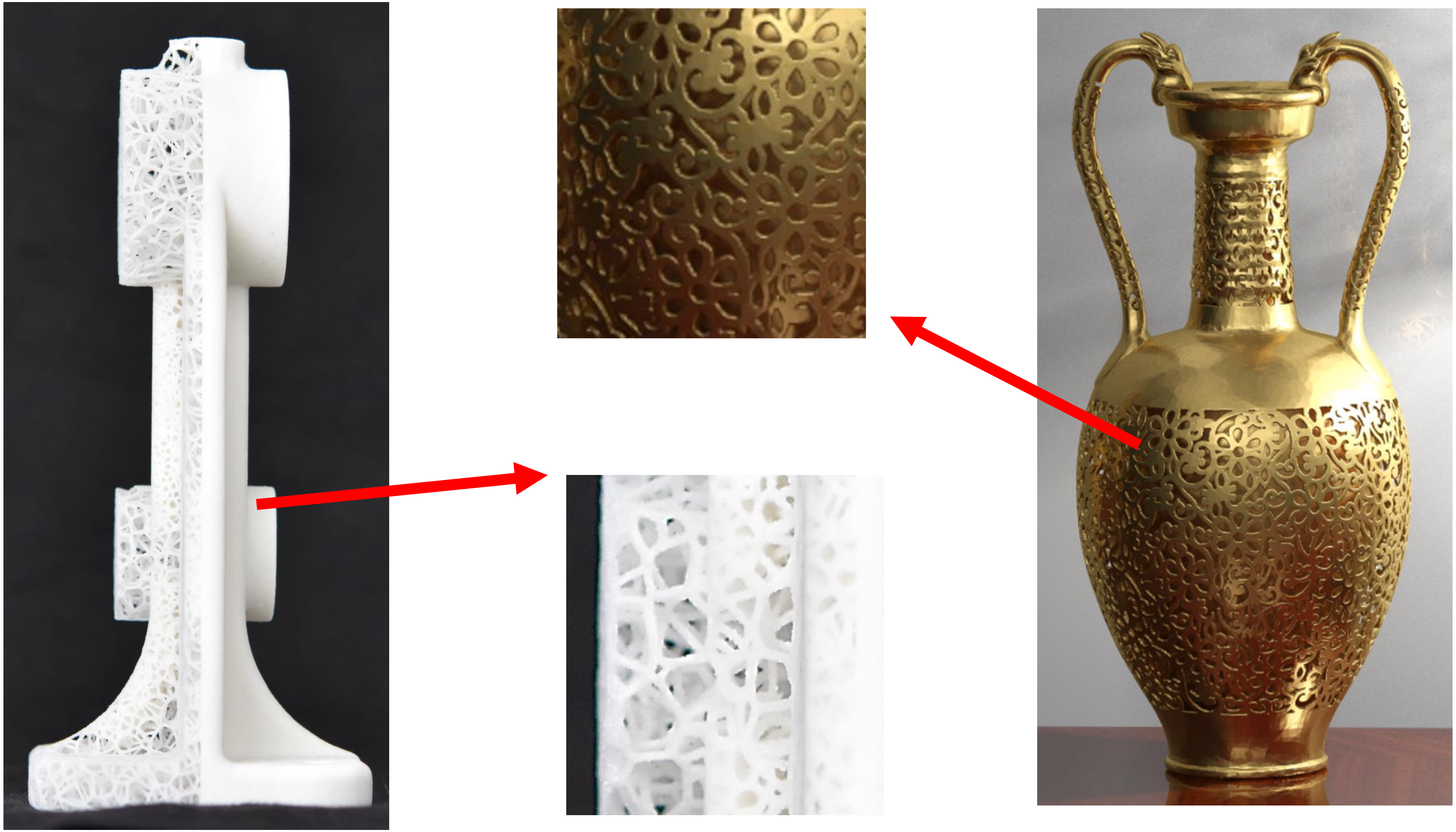
Jun Wu, Niels Aage, Sylvain Lefebvre, Charlie Wang

TU Delft
DTU Technical University of Denmark
Inria INVENTEURS DU MONDE NUMÉRIQUE

Topology Optimization for Computational Fabrication

Part 4: Topology Optimization for Appearance and Structure Synthesis

Sylvain Lefebvre
Inria



Textures in Computer Graphics

Authoring textures

Forza Horizon 3, Microsoft Studios *https://www.forzamotorsport.net/en-us/games/fh3*

Authoring textures

Too much content to be done entirely manually

Forza Horizon 3, Microsoft Studios *https://www.forzamotorsport.net/en-us/games/fh3*

Texture Synthesis

- Three main directions
  - By-example synthesis
  - Procedural synthesis
  - Simulation (e.g. erosion)

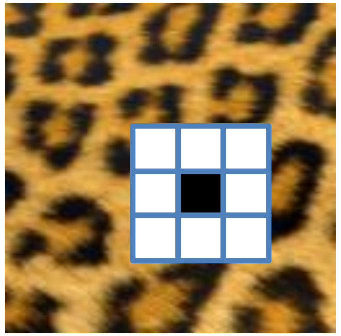
We will see both in the context of fabrication

Texture Synthesis

- Three main directions
  - By-example synthesis
  - Procedural synthesis
  - Simulation (e.g. erosion)

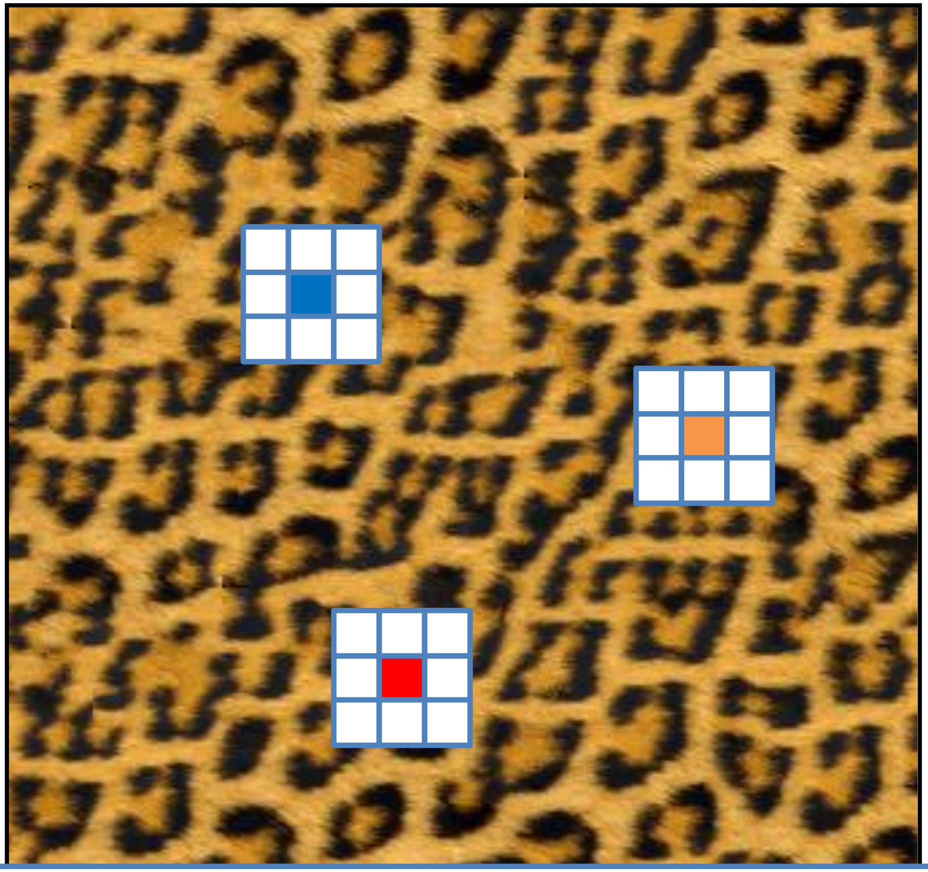
We will see both in the context of fabrication

Texture synthesis: color formulation



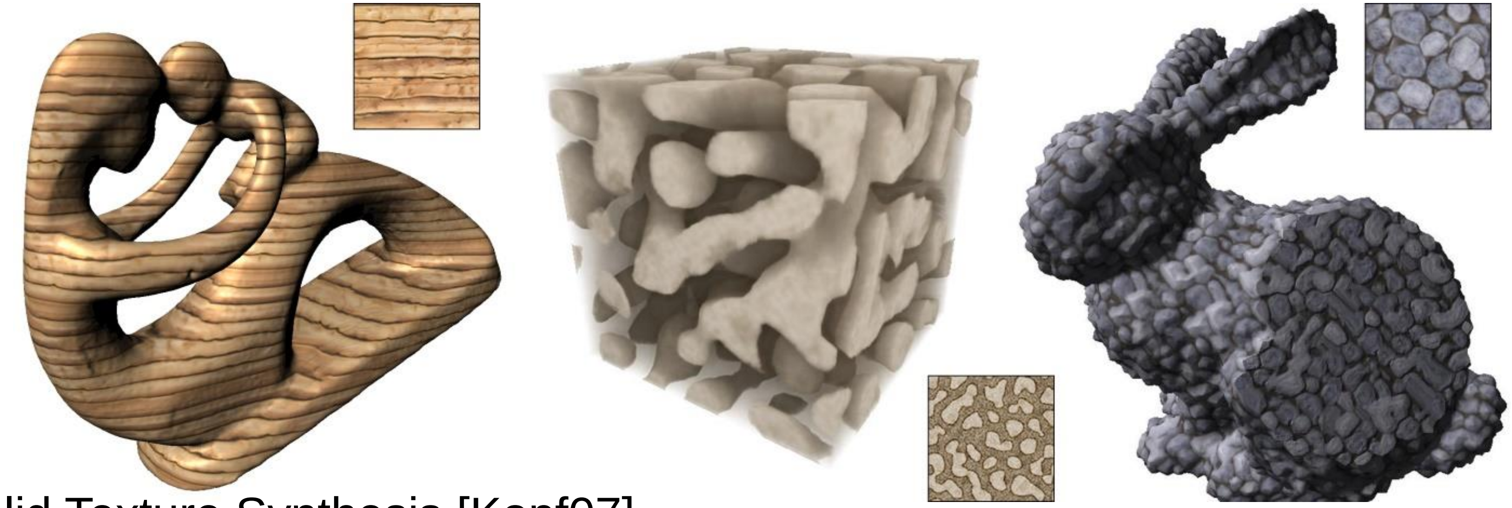
Exemplar

(color field)

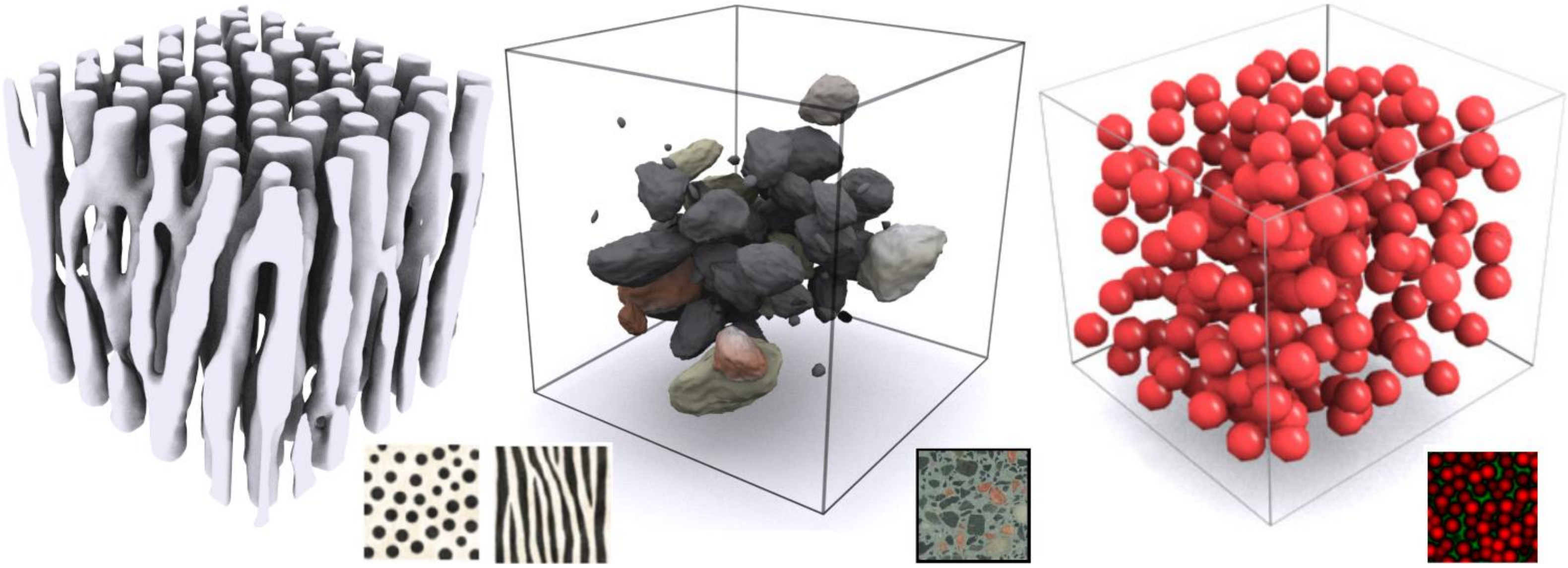


Assumption (MRF):
Same neighborhoods at all scales → Same visual content

Volume Texture Synthesis

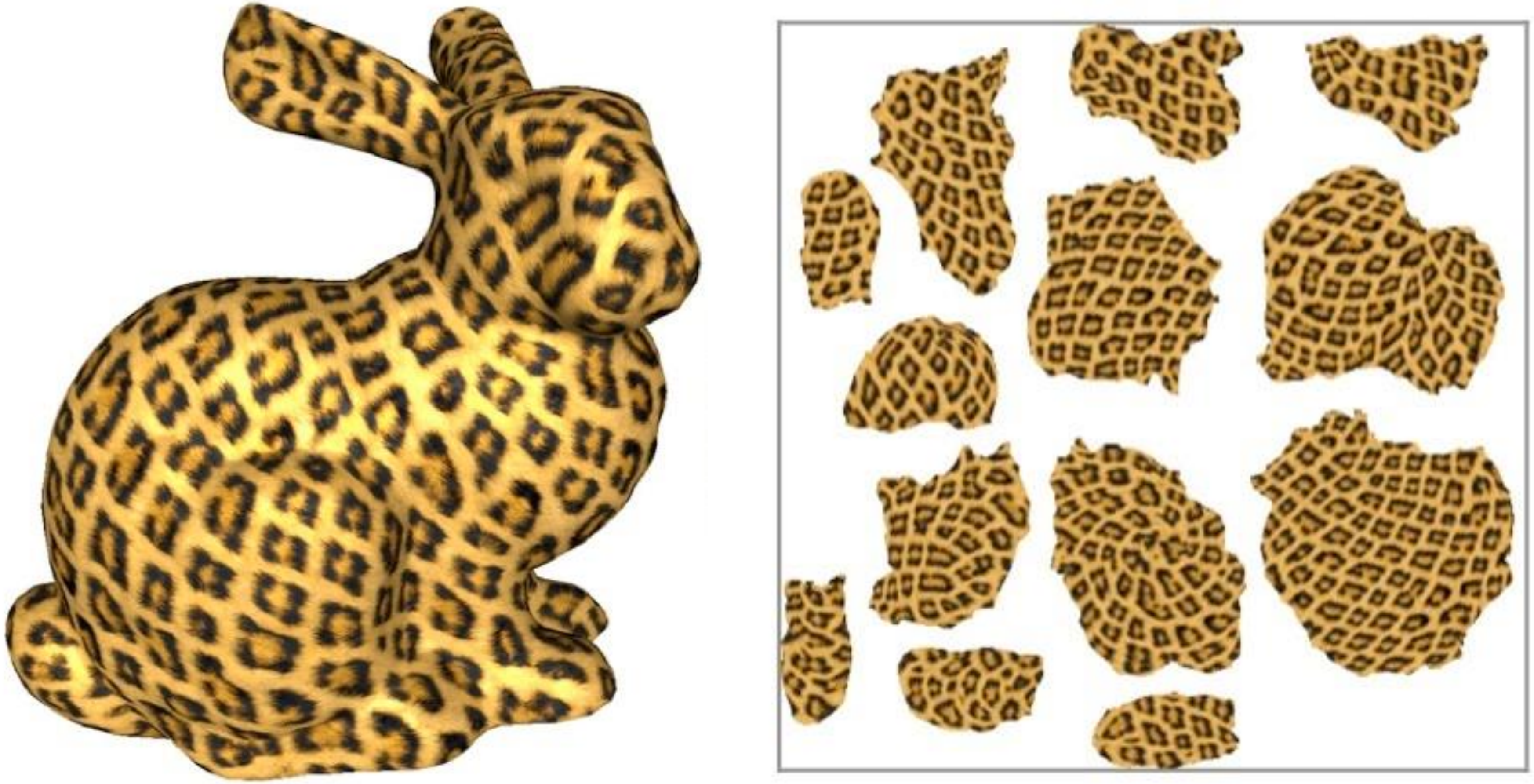


Solid Texture Synthesis [Kopf07]



Lazy Solid Texture Synthesis [Dong08]

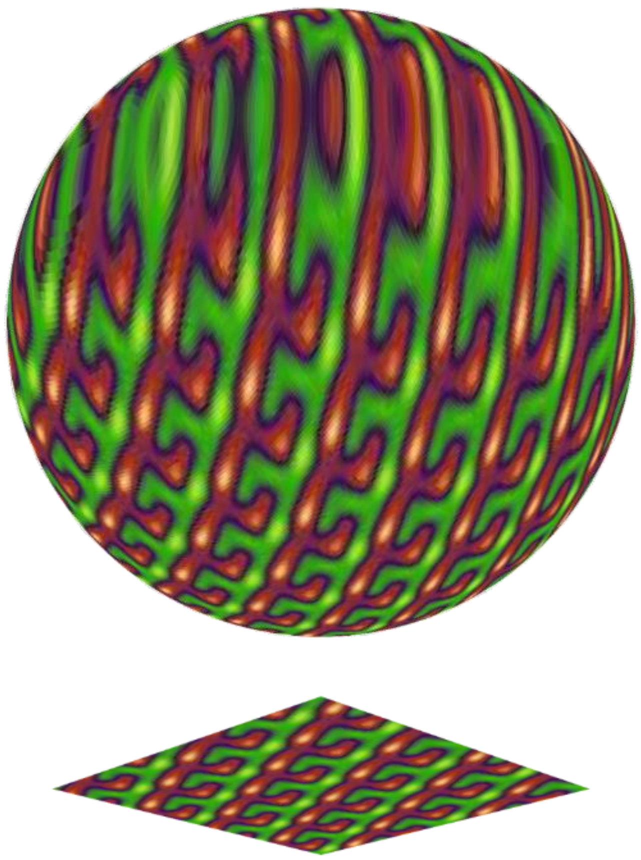
On-surface texture synthesis



[Lefebvre and Hoppe 2006]

On-surface texture synthesis, the easier way

On-surface texture synthesis, the easier way



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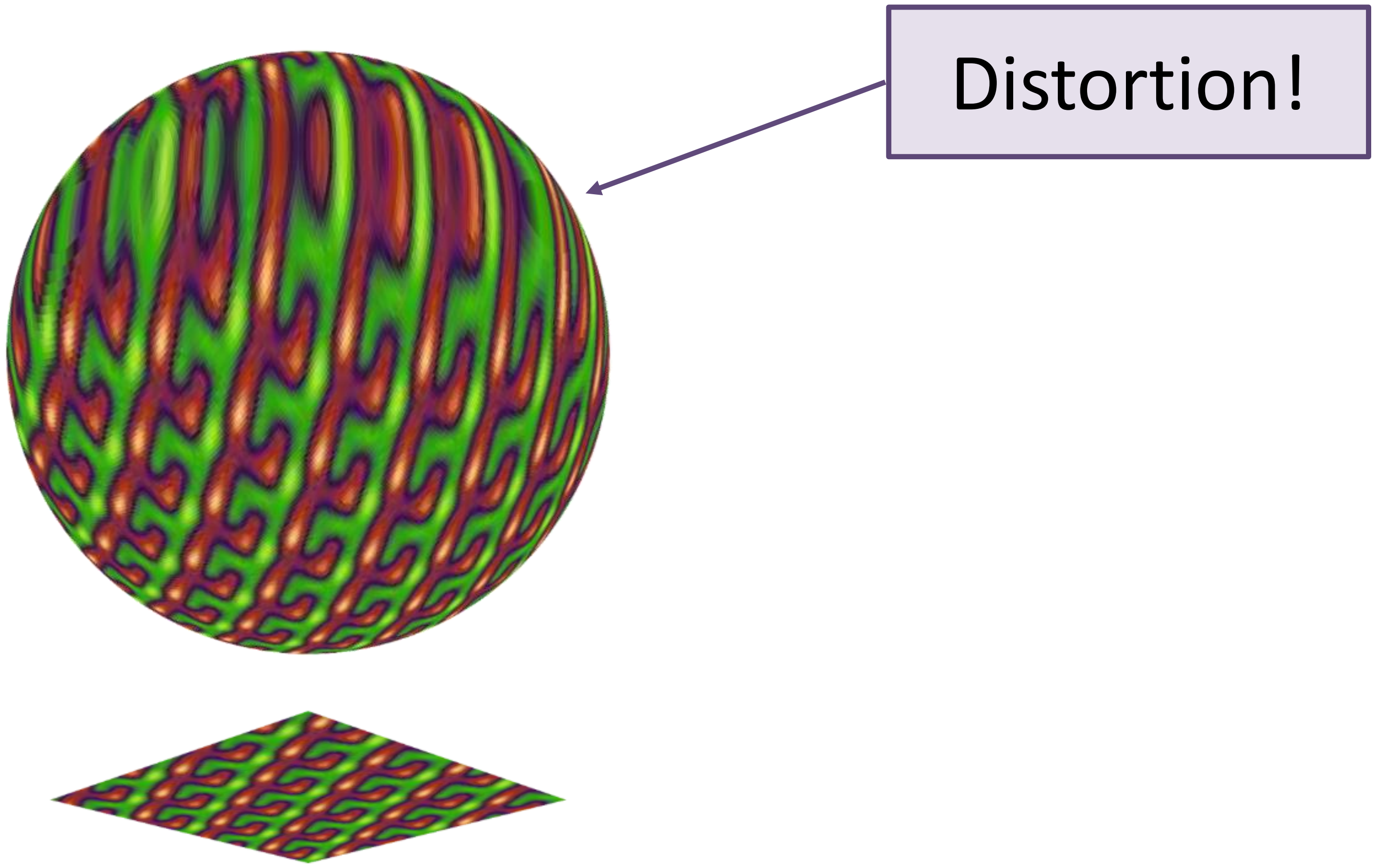
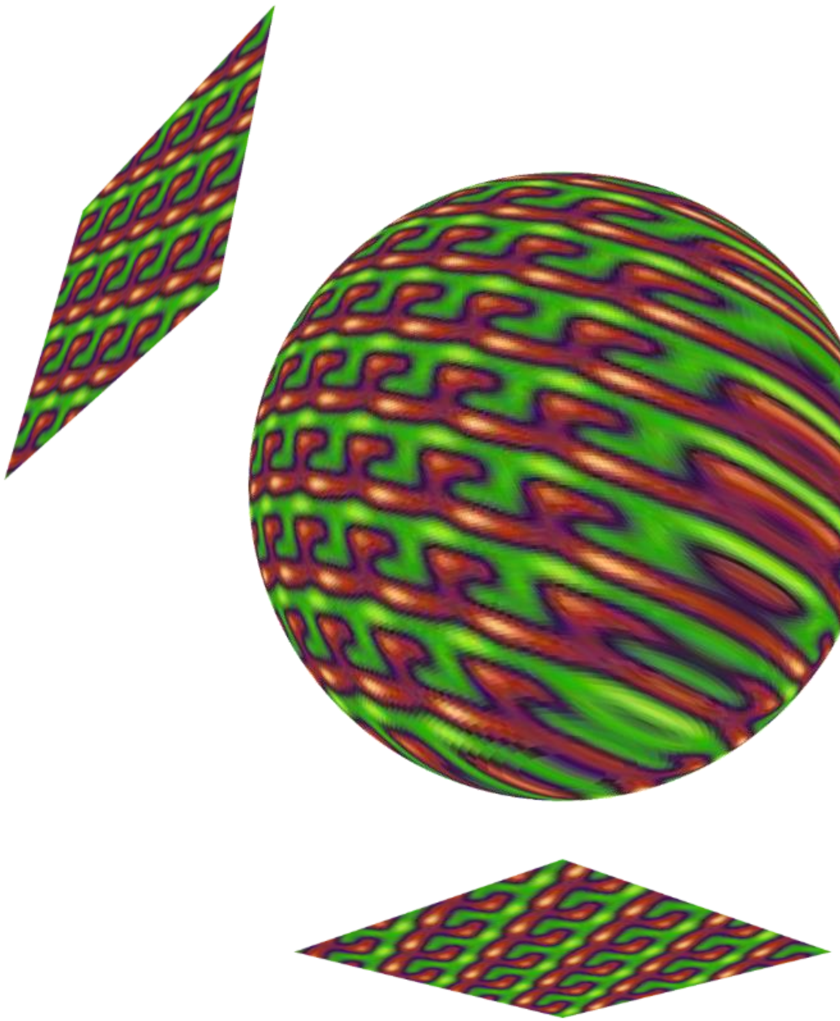


On-surface texture synthesis, the easier way



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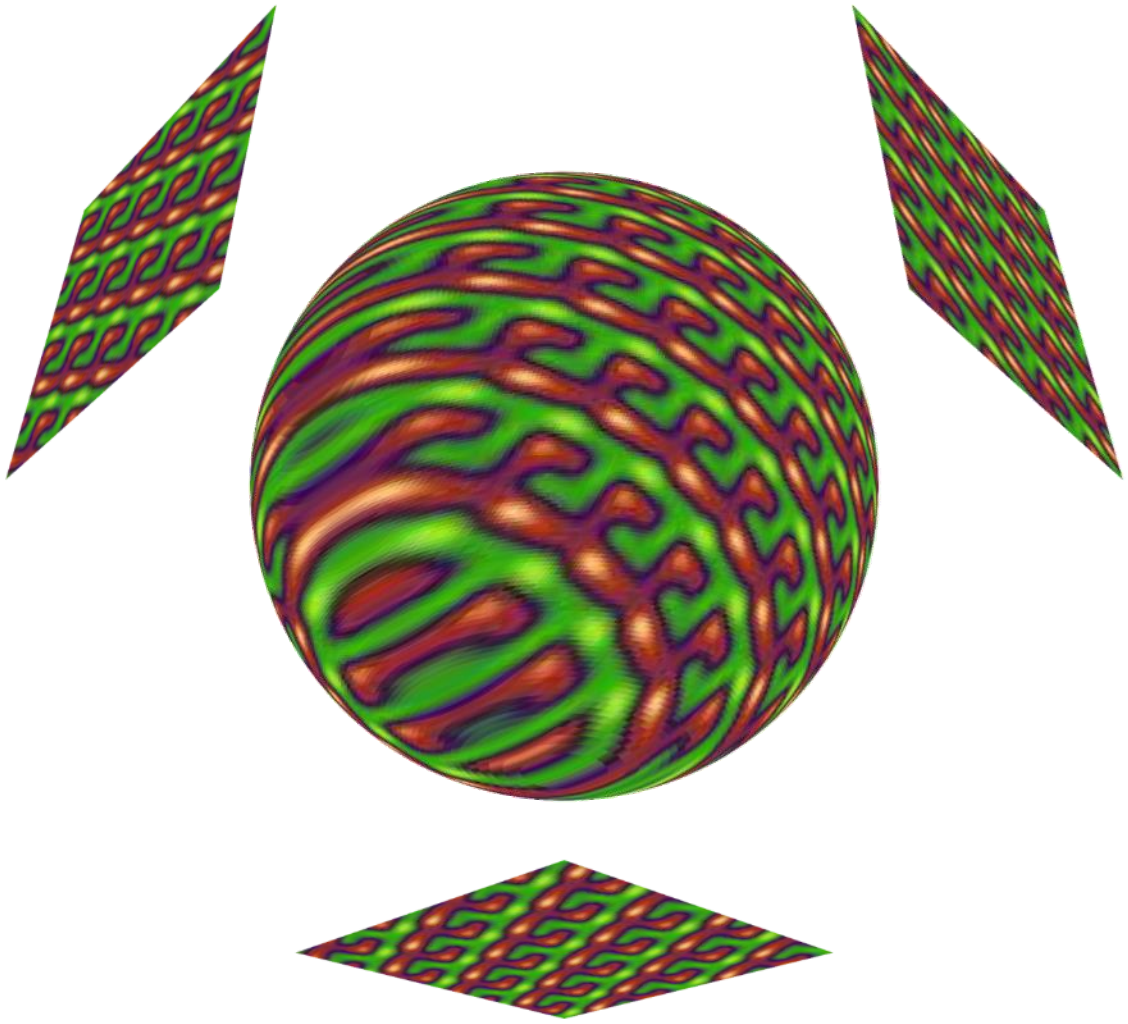
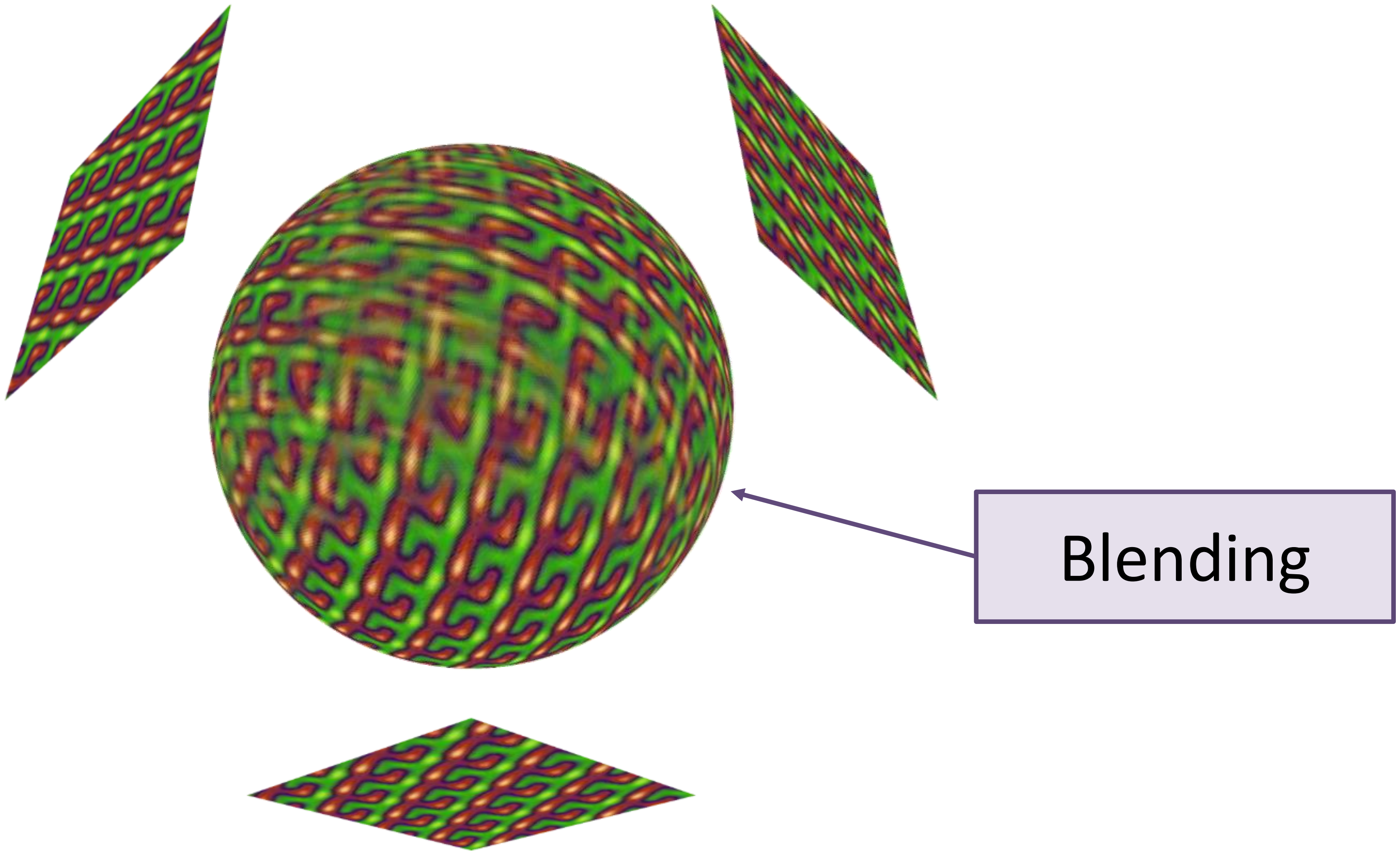
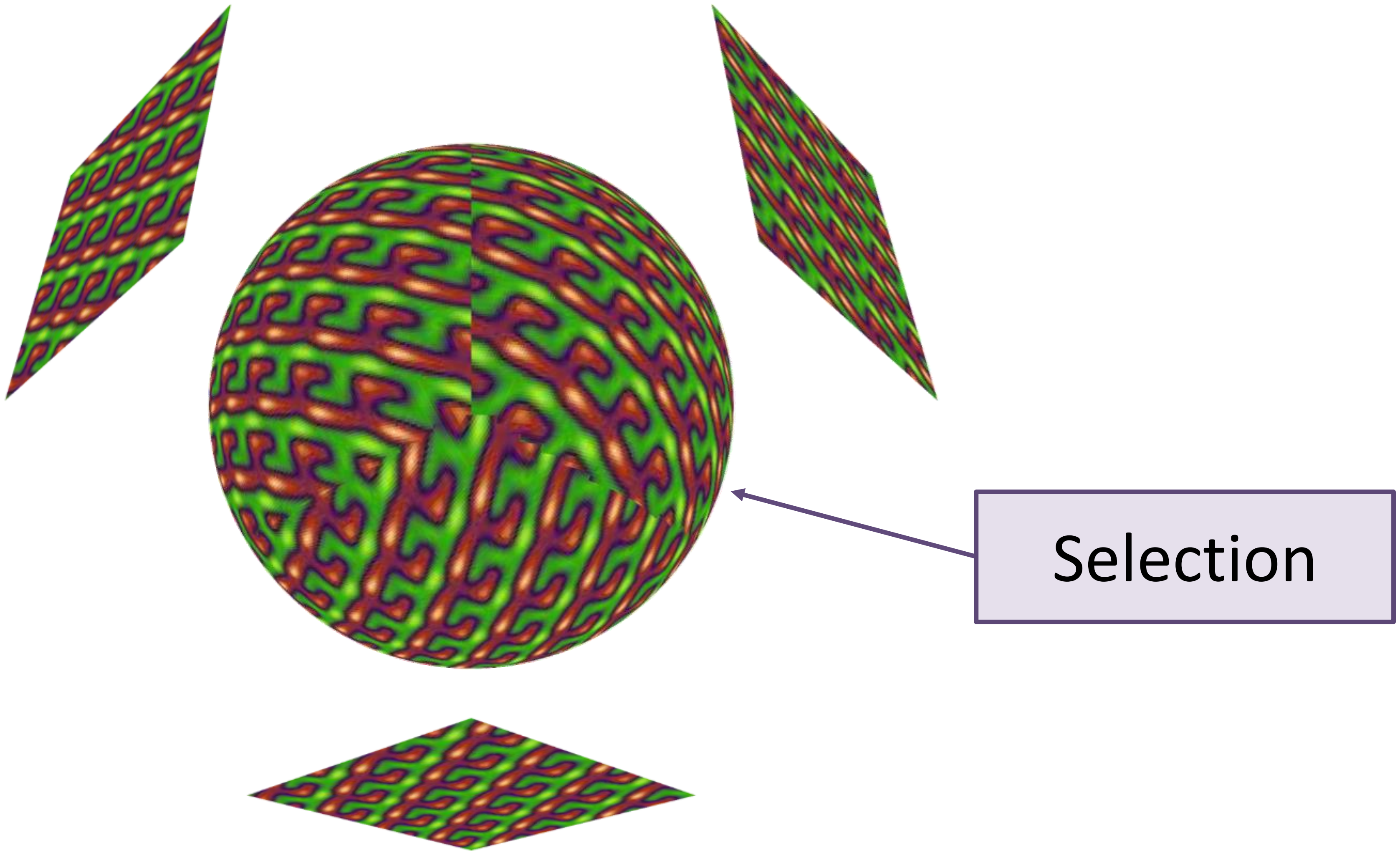


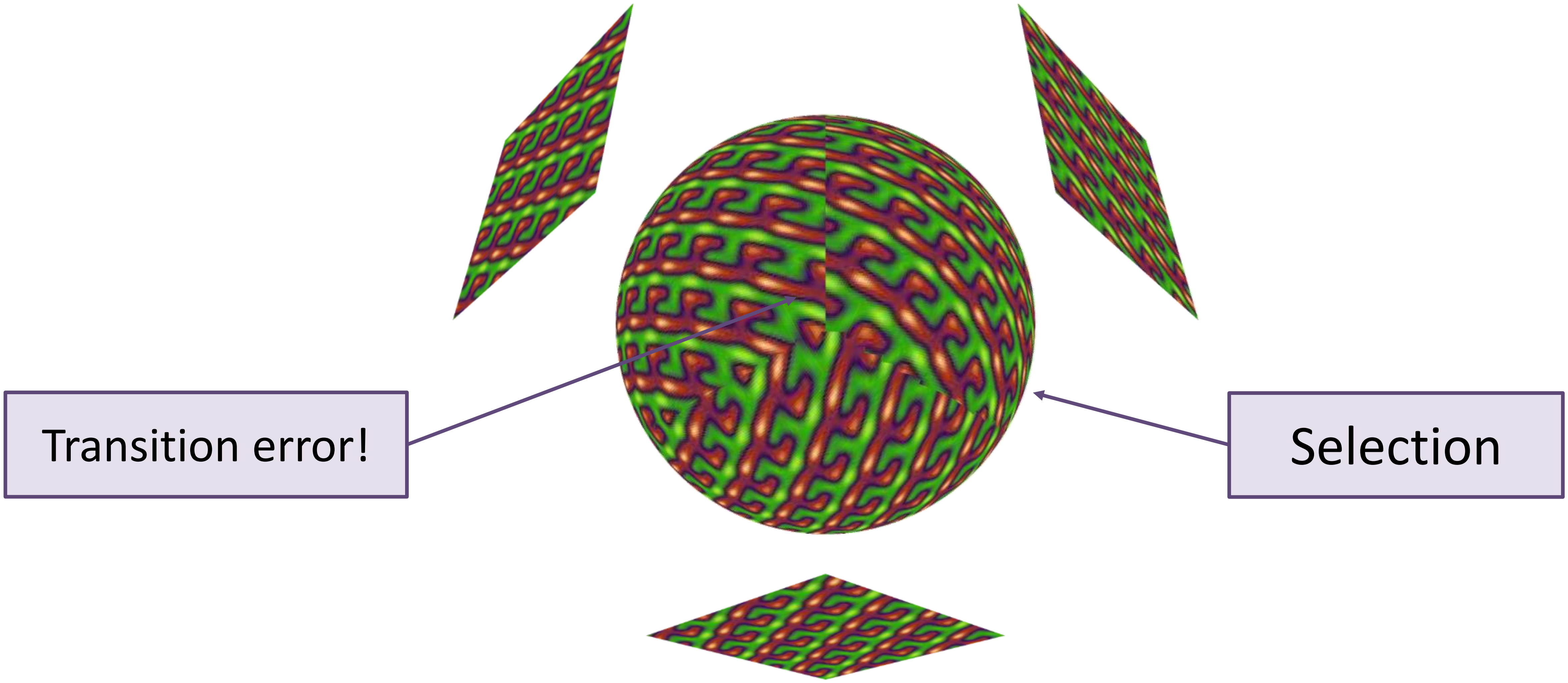
On-surface texture synthesis, the easier way



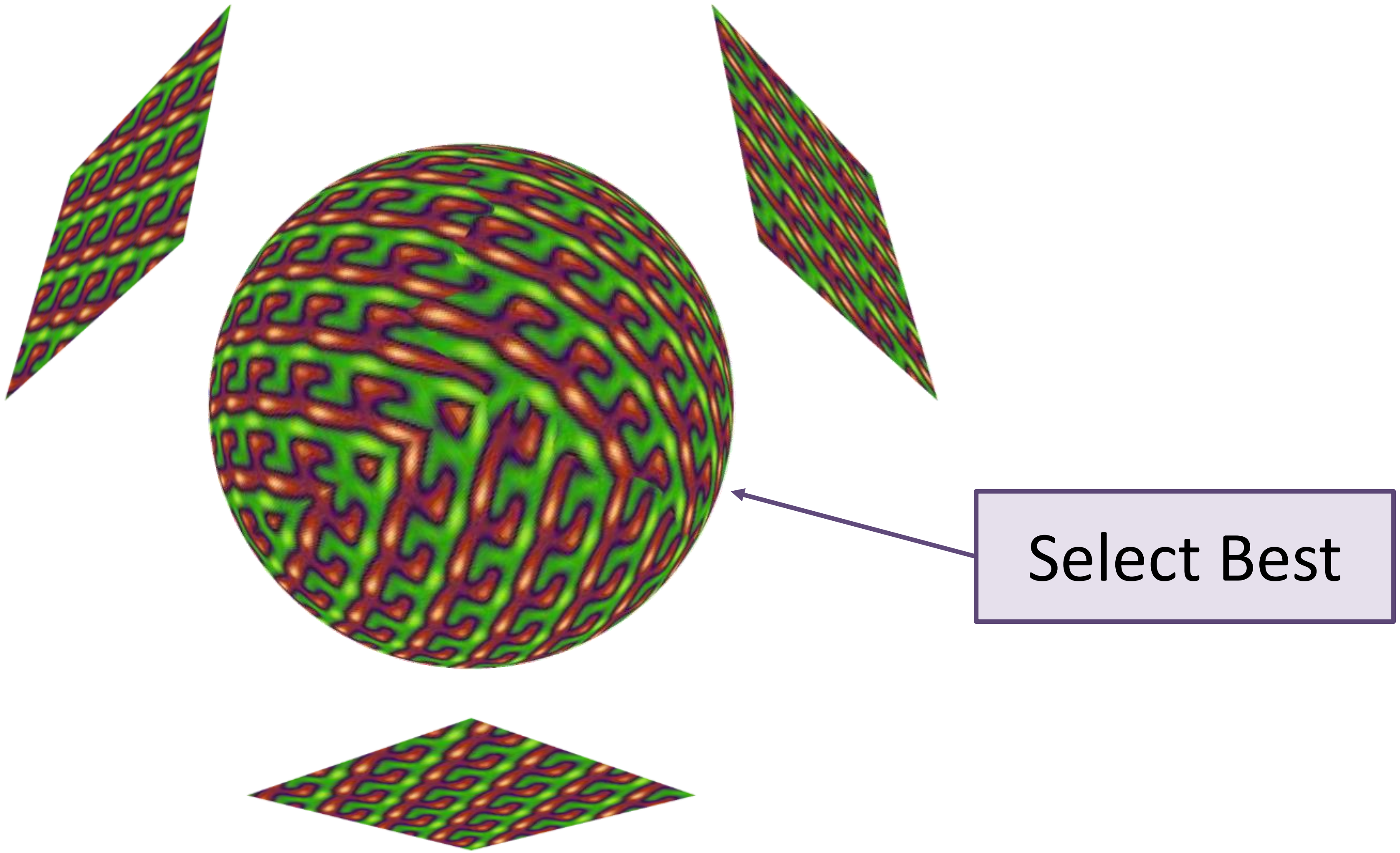
On-surface texture synthesis, the easier way



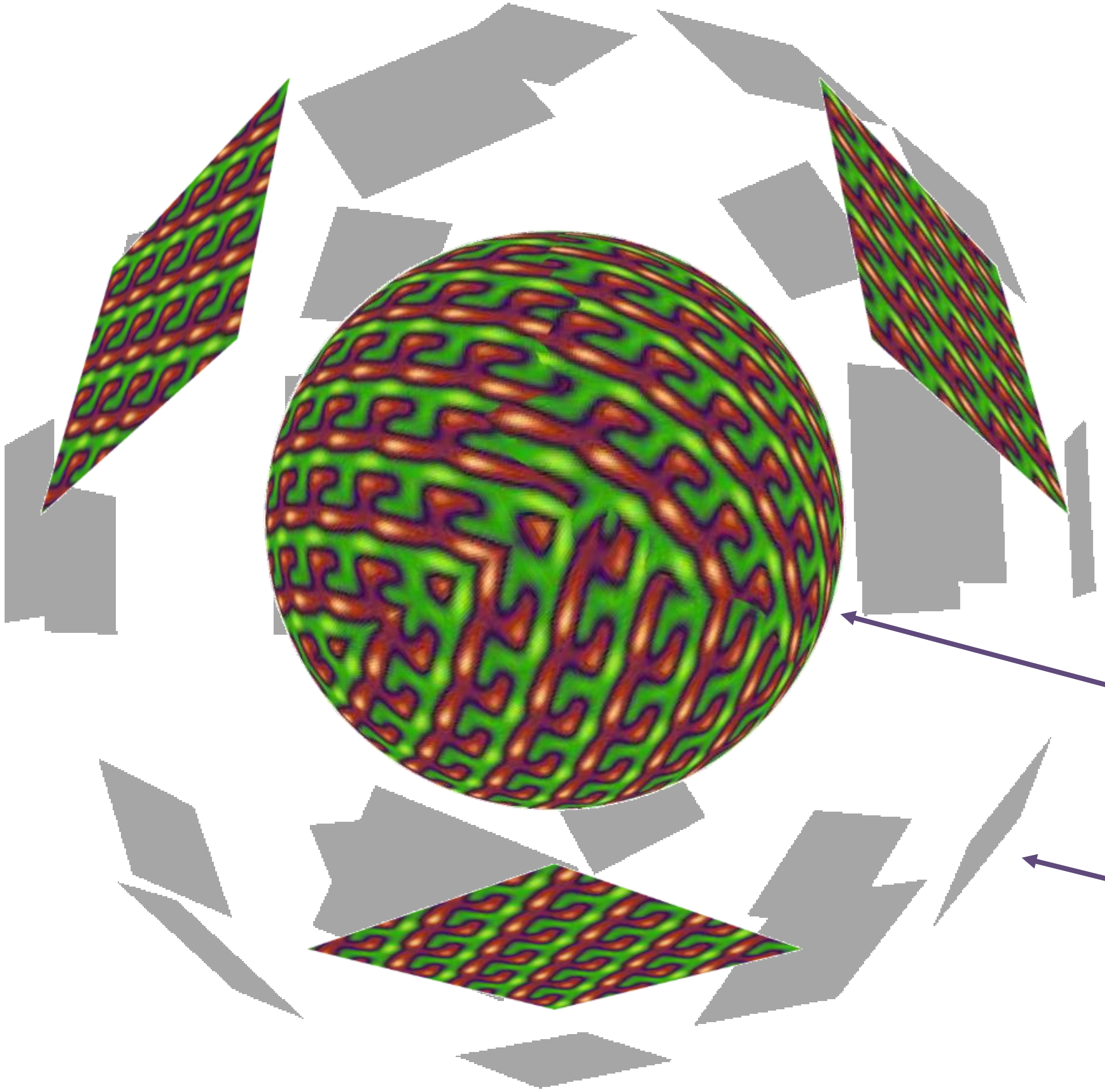
On-surface texture synthesis, the easier way



On-surface texture synthesis, the easier way



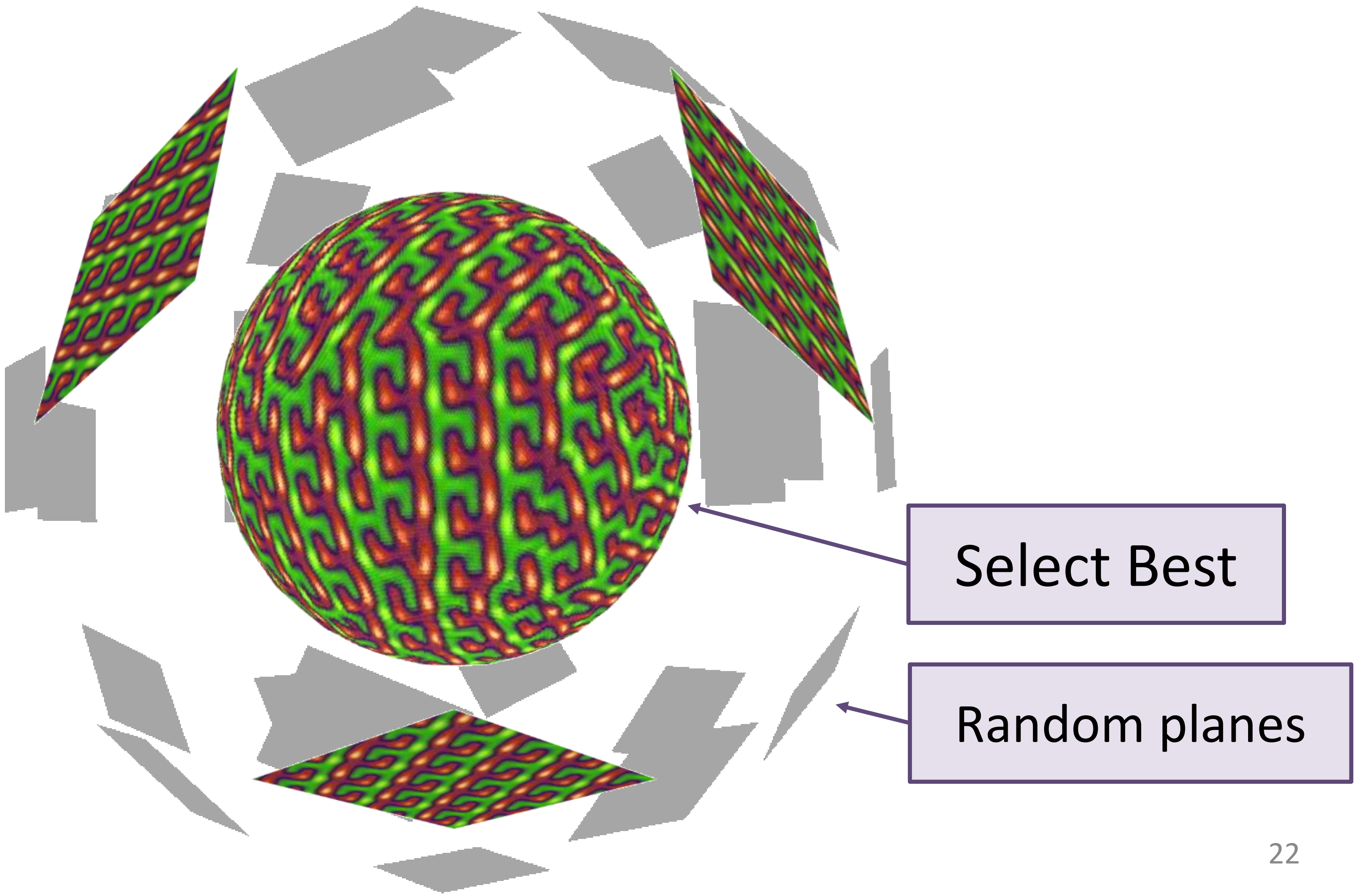
On-surface texture synthesis, the easier way



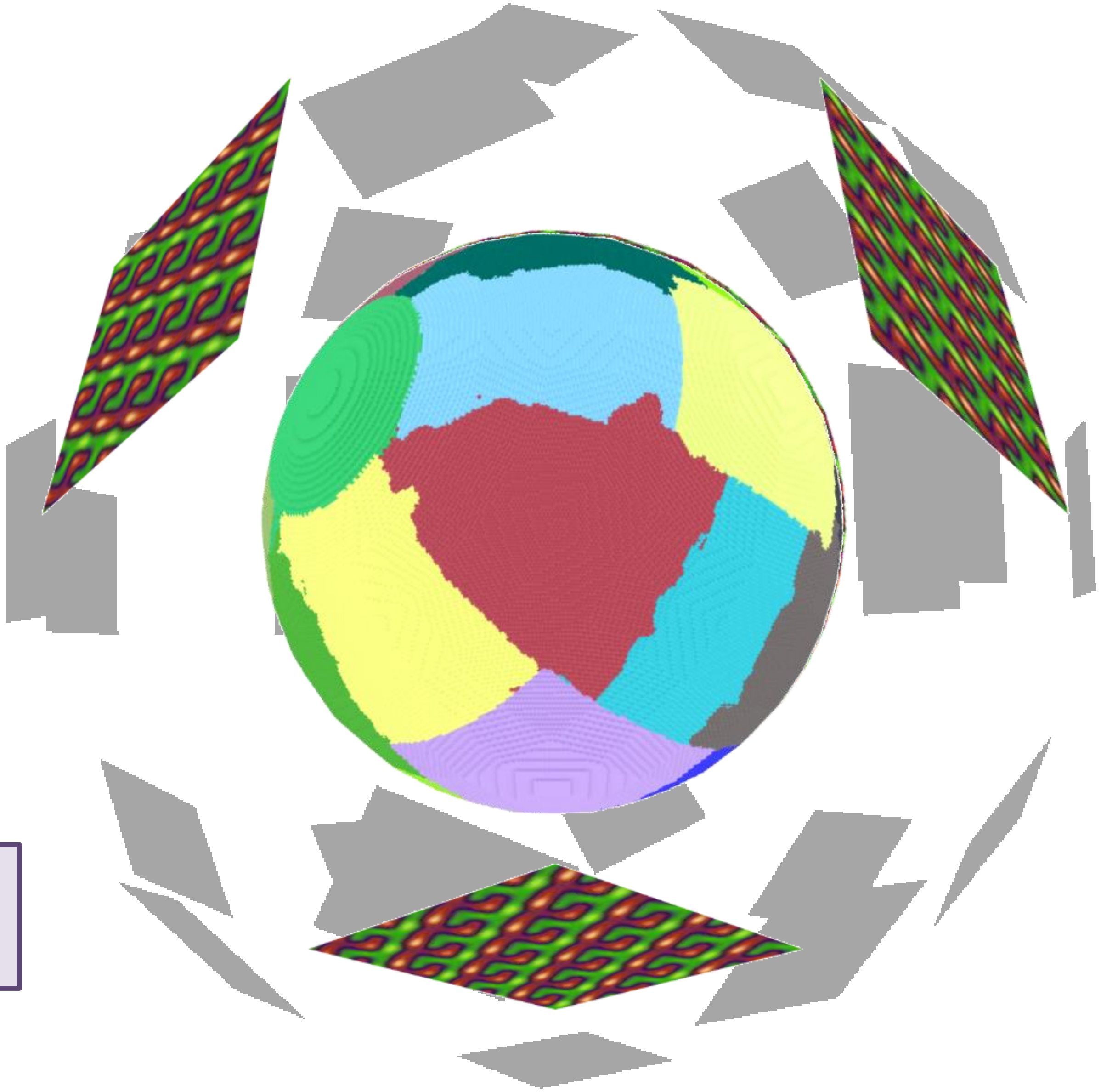
Select Best

Random planes

On-surface texture synthesis, the easier way

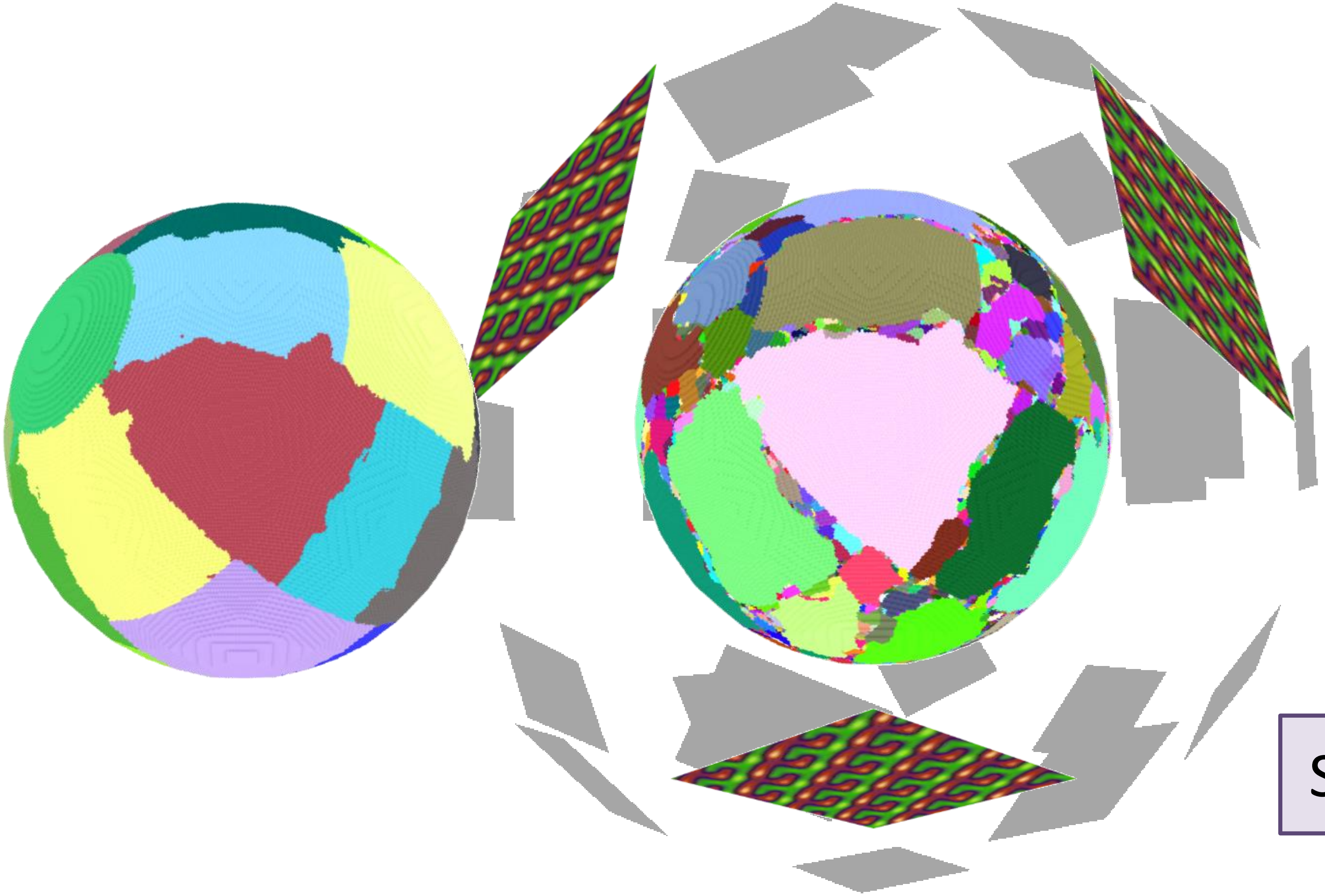


On-surface texture synthesis, the easier way



Plane choices

On-surface texture synthesis, the easier way



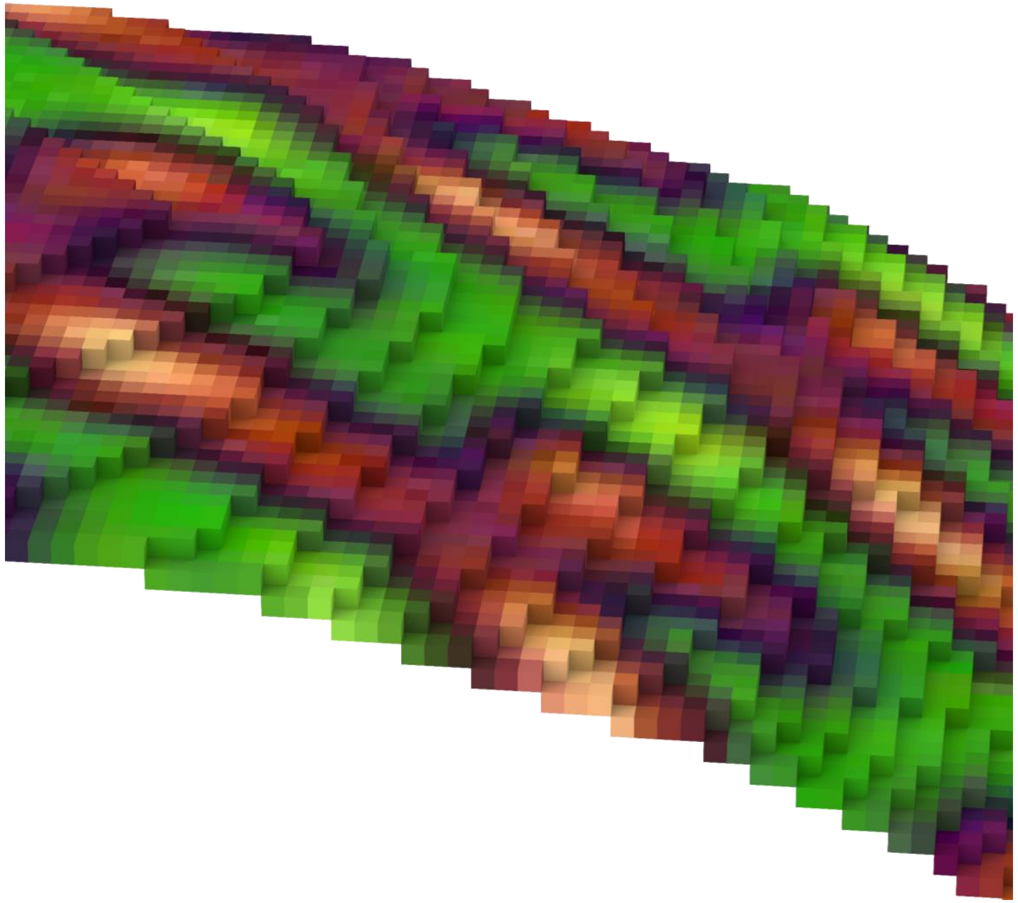
Shifts + Rotations

Labelling Problem

- Surface neighborhood (2D)

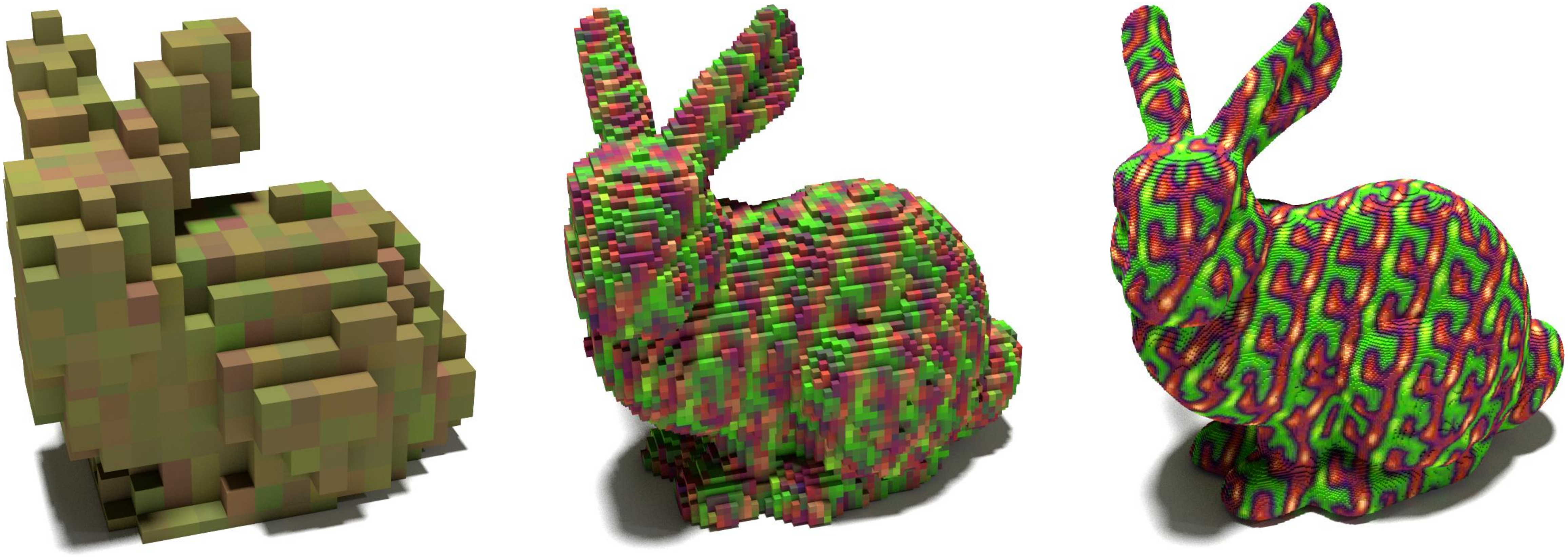
Transition error

Distortion error

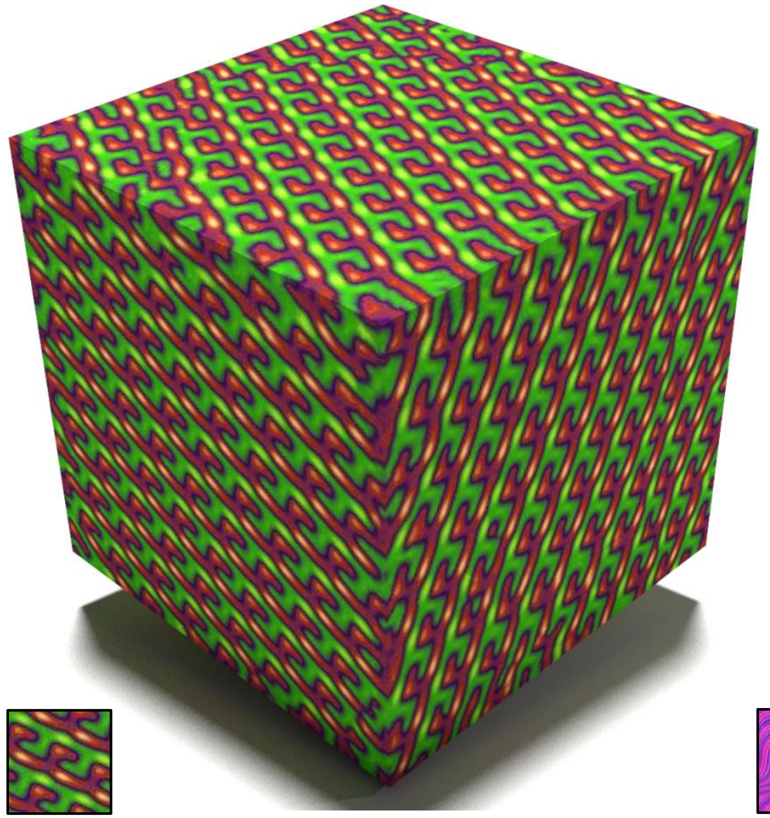


Multiresolution Synthesis

- Upsample, jitter, correction [Lefebvre and Hoppe 2005]

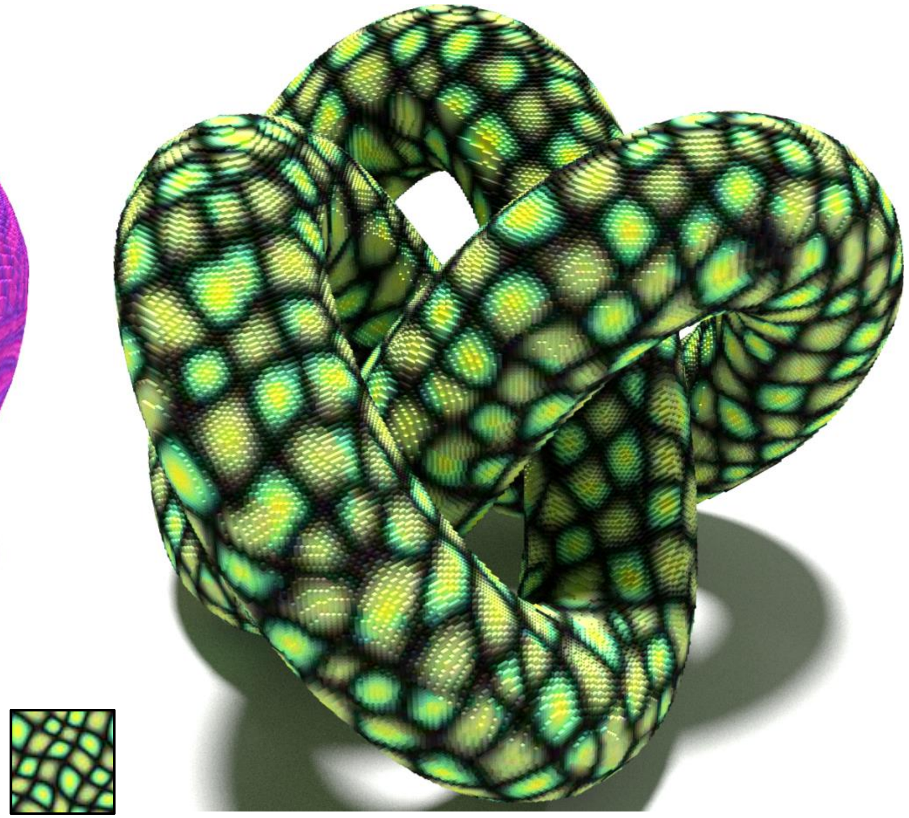


Results



Time 28.6s

Time 14.7s
thing:168602 (Steelyd)



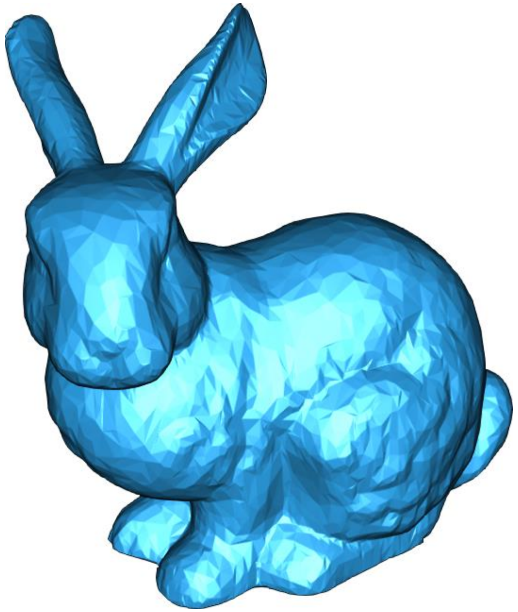
Time 18.7s
thing:5506 (chylld)

Texture as structure?

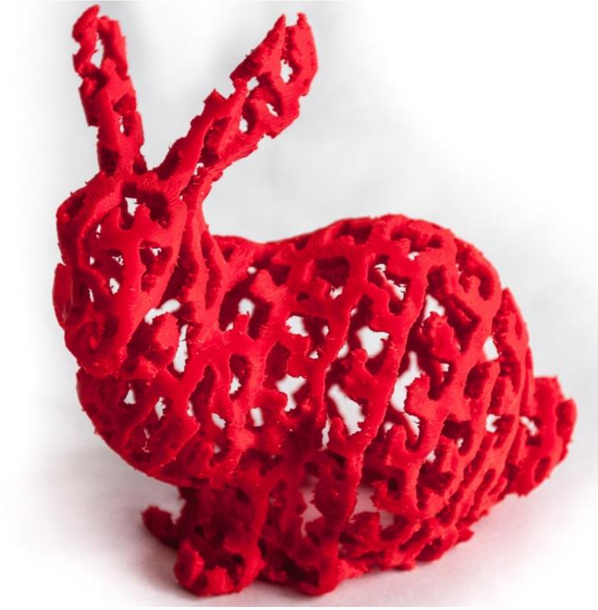
Model

+ appearance 

+ structure 

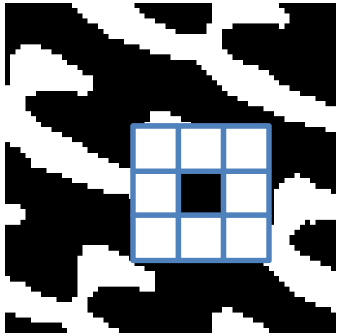


Texture Synthesis?

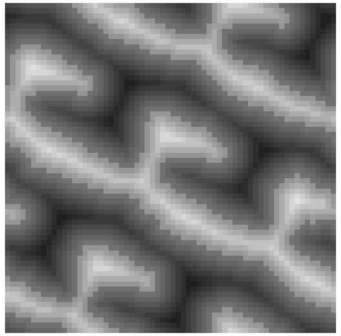


???

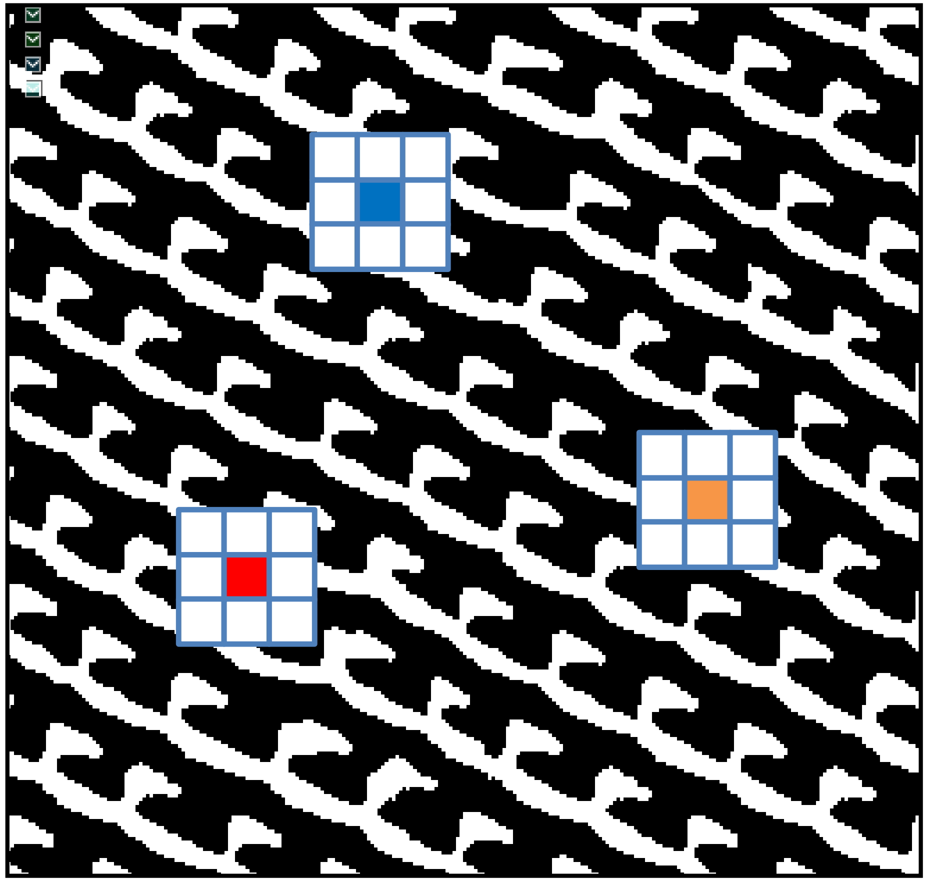
Texture synthesis: structure formulation



Exemplar



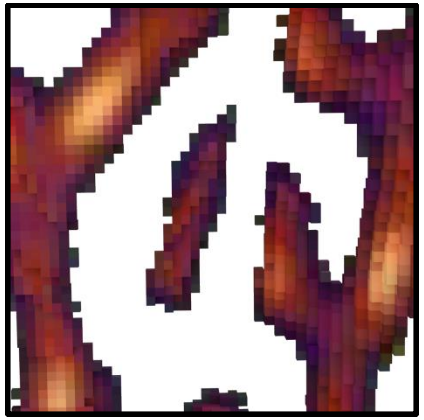
(density field)

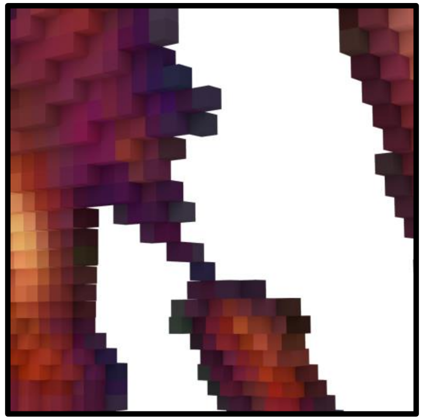


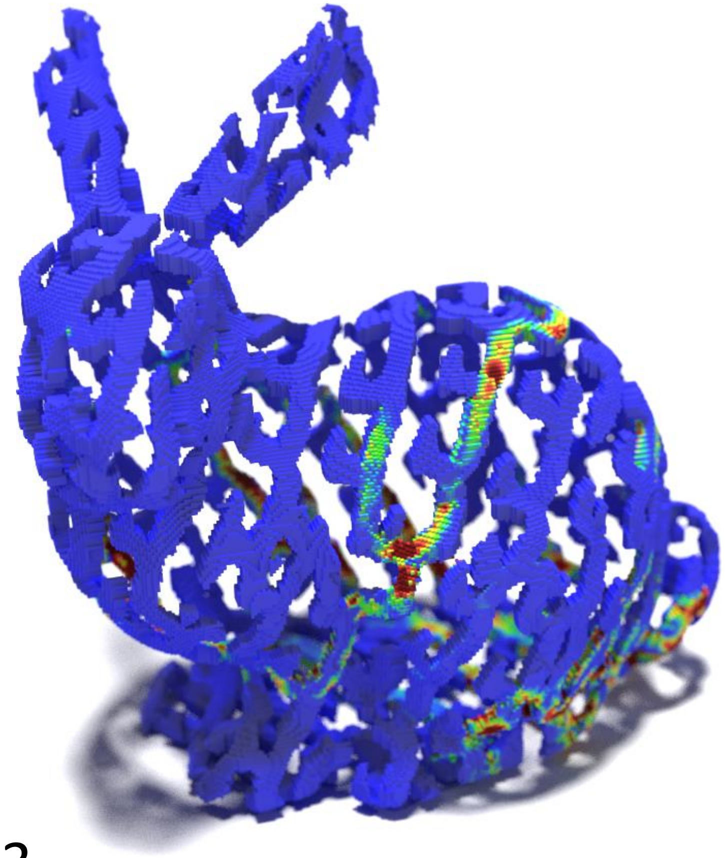
Neighborhoods capture ***local geometry*** accross scales

Printability

1. Connected components
2. Minimum thickness
3. No weak part (rigidity)

1. 

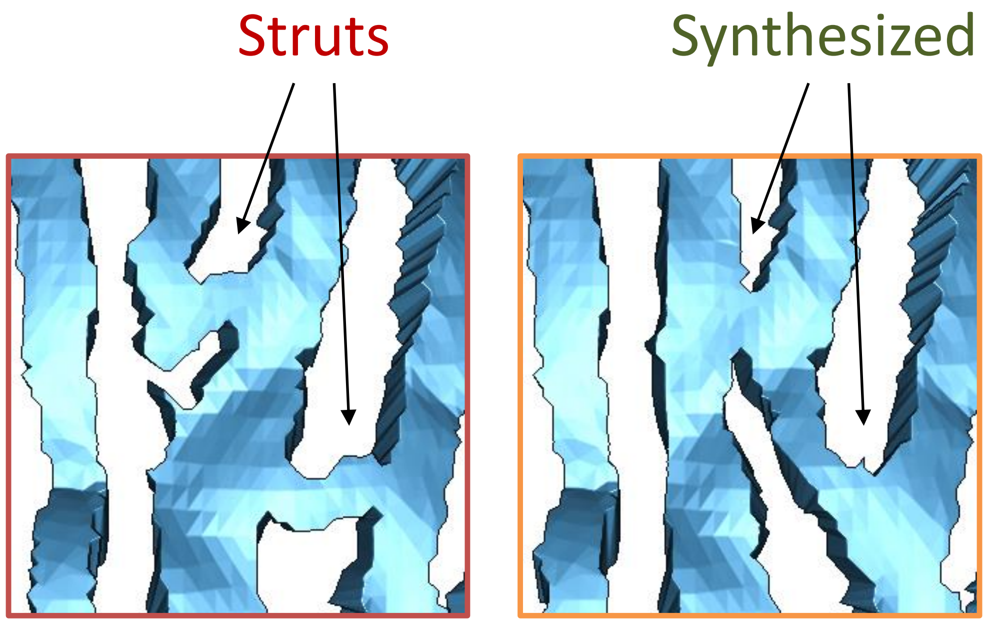
2. 

3. 

Key ideas for structure synthesis

Pattern is stochastic

- Exhibits degrees of freedom
- Use pattern itself to locally reinforce structure



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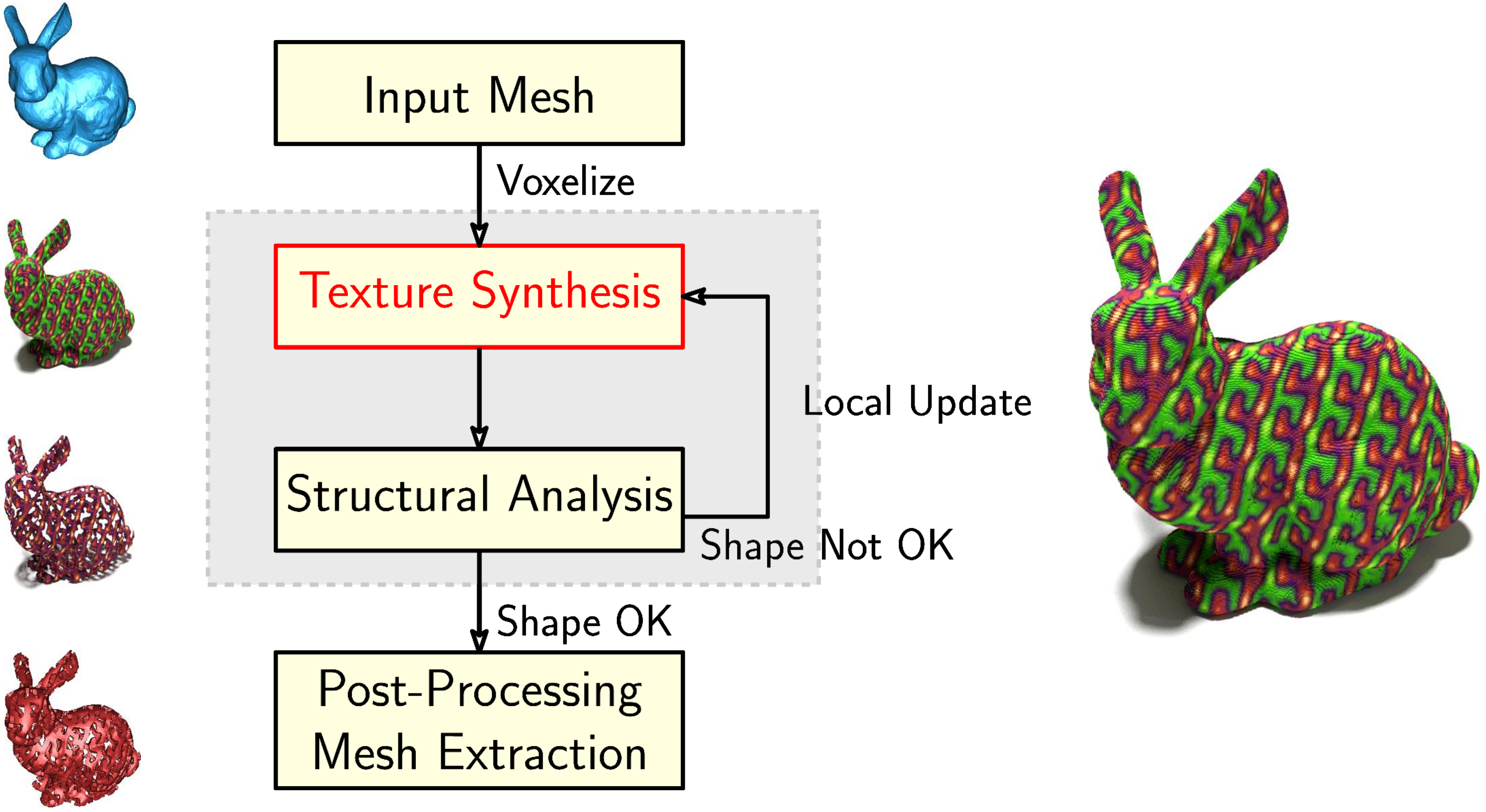
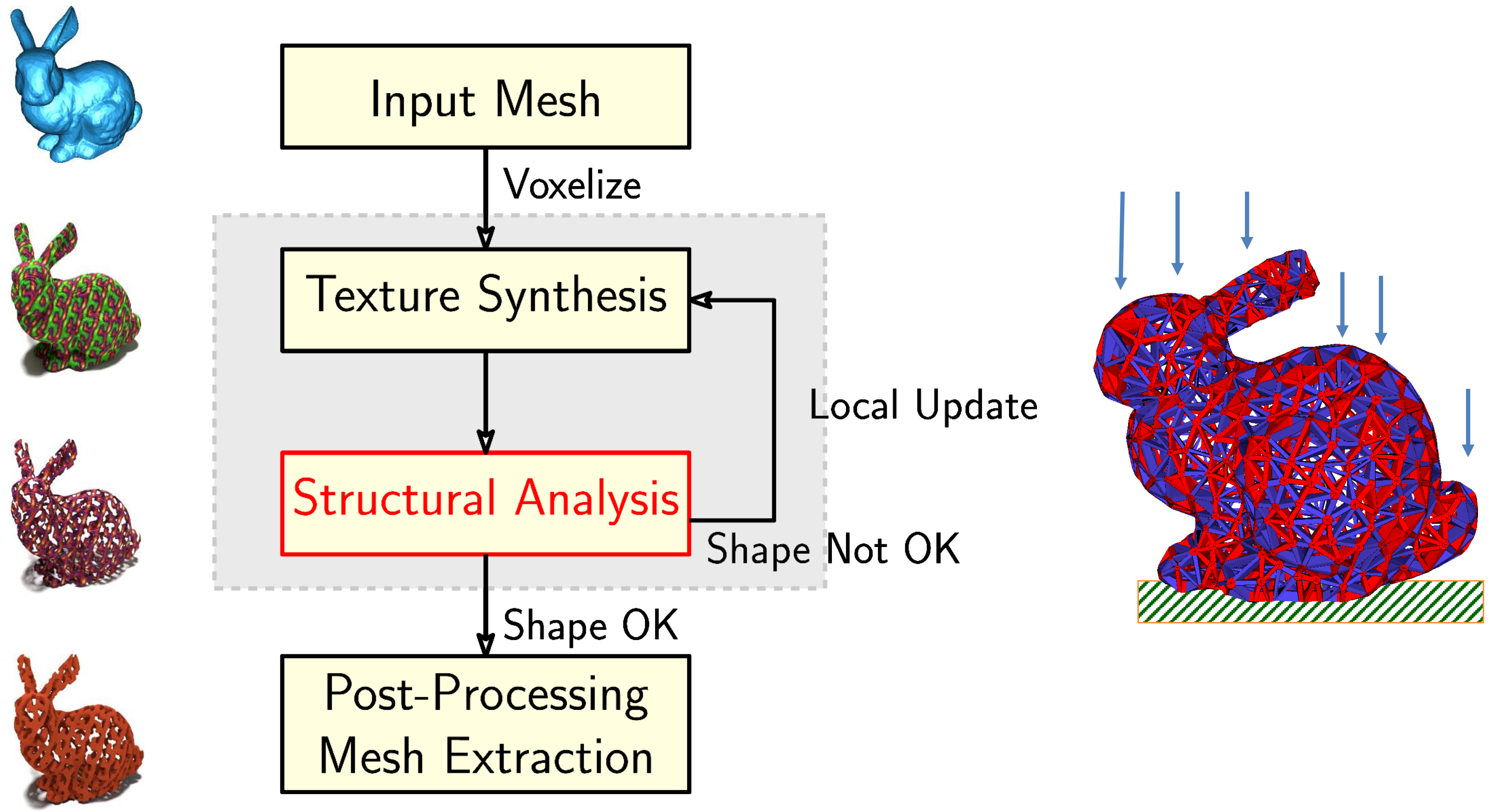
Pipeline


Input Mesh
Voxelize
Texture Synthesis
Local Update
Structural Analysis
Shape Not OK
Shape OK
Post-Processing
Mesh Extraction

Pipeline


Input Mesh
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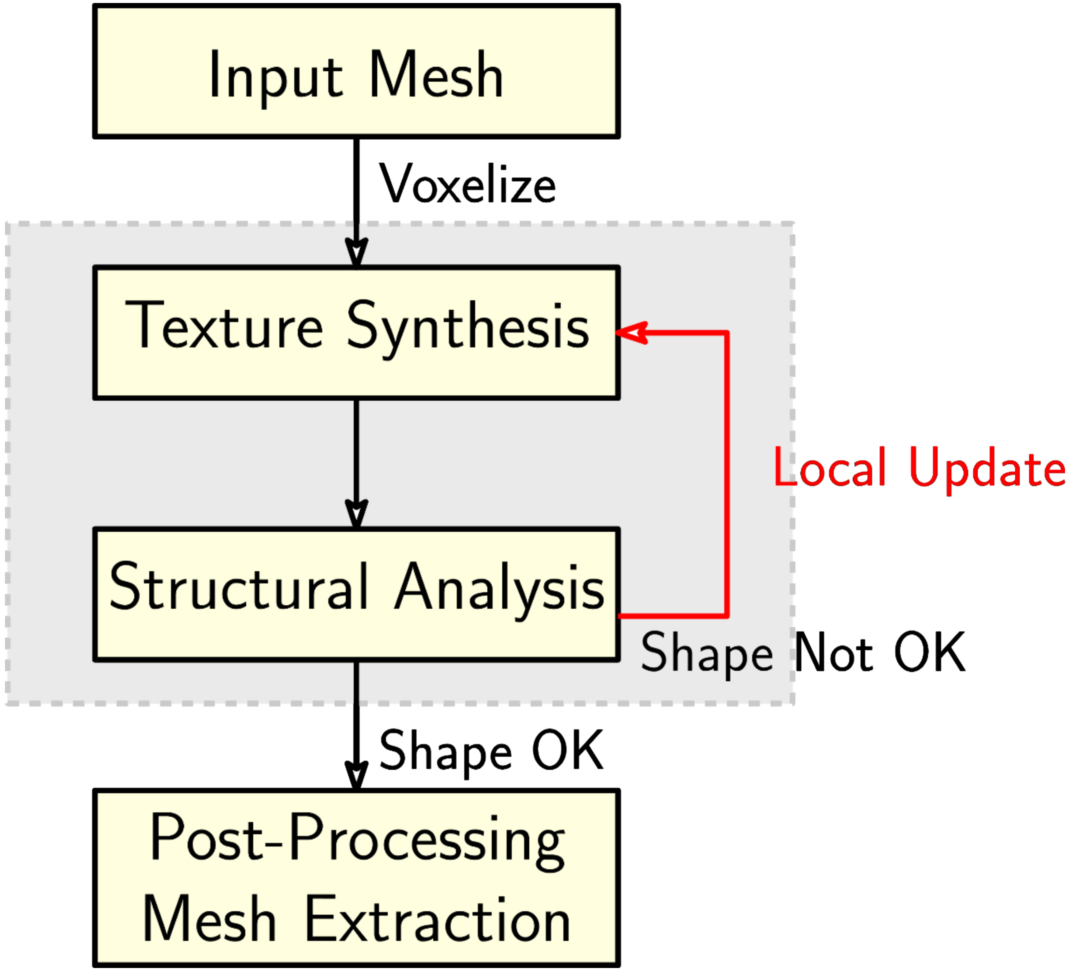
Input Mesh
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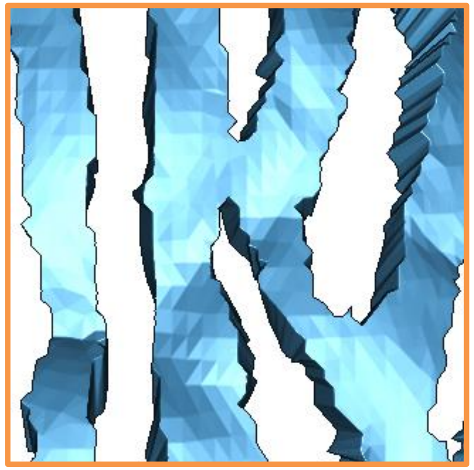
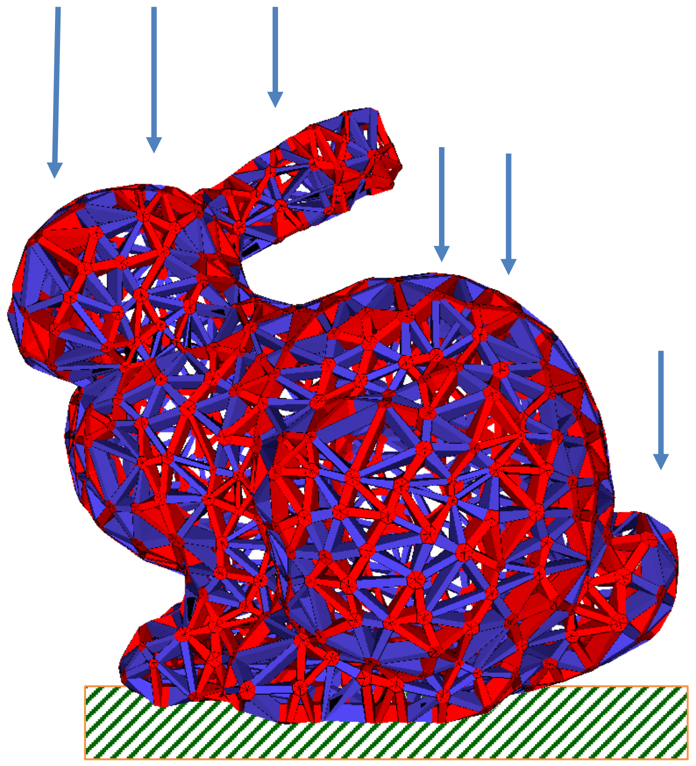


How to evaluate weak parts?

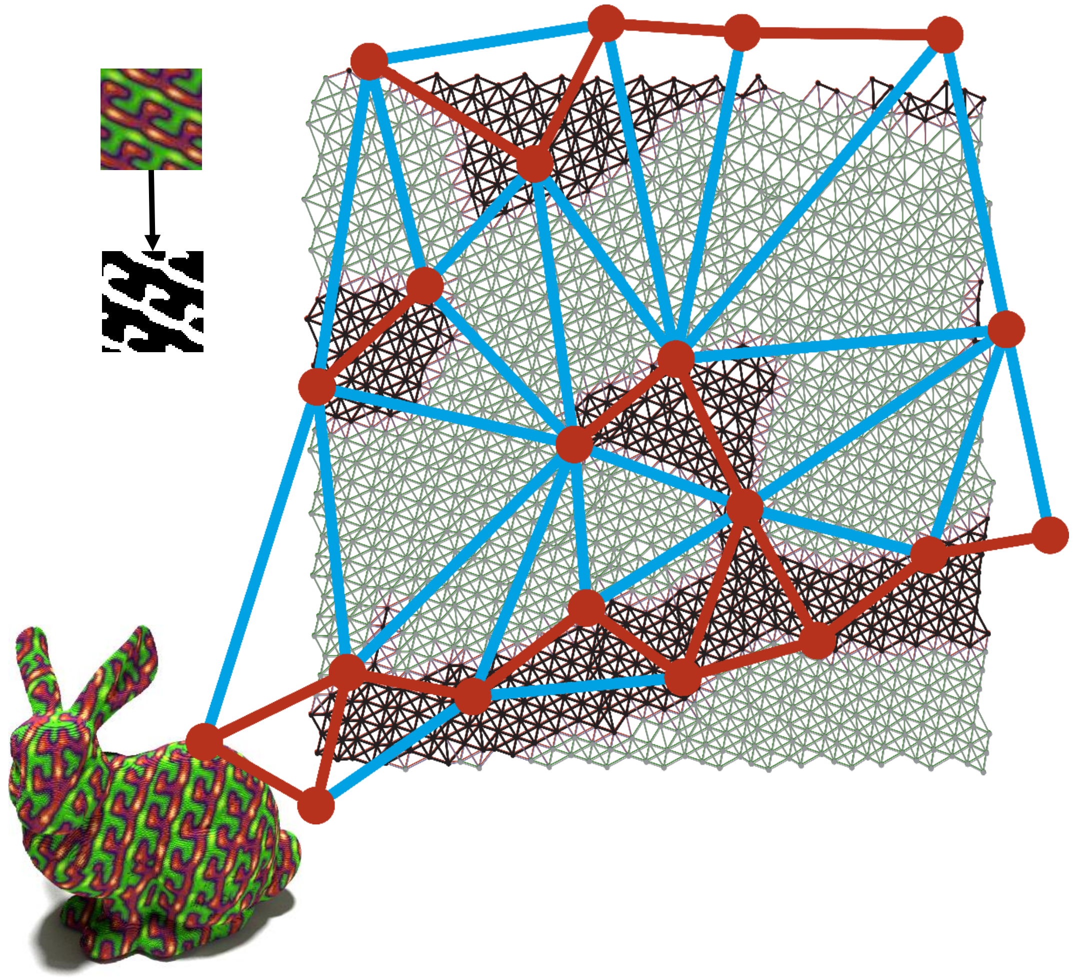
- Similar to SIMP method, we consider ‘weak’ and ‘strong’ material
- Issues:
  - Voxel grid is huge (~ 5M voxels)
  - Weak and strong ➔ hard to converge
  - We need 20-30 iterations synthesis/analysis

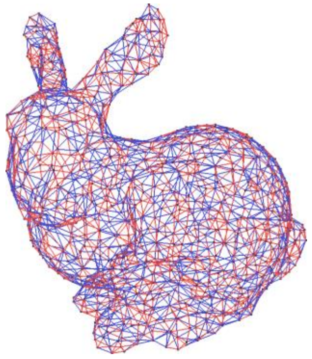
➔ Too expensive

➔ Approximate the pattern



Abstract Pattern Graph

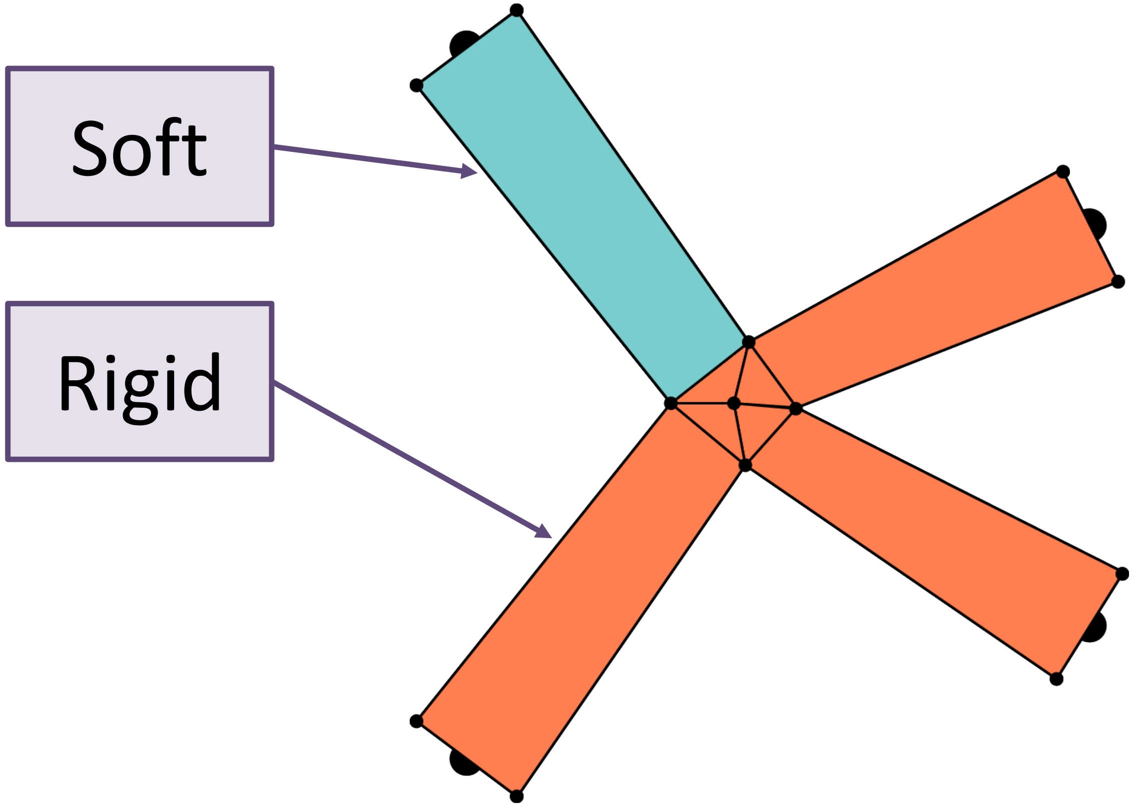




Physical Simulation

- Basic idea: replace graph by finite elements

In 2D: Quad & Triangle
In 3D: Hex & Wedge



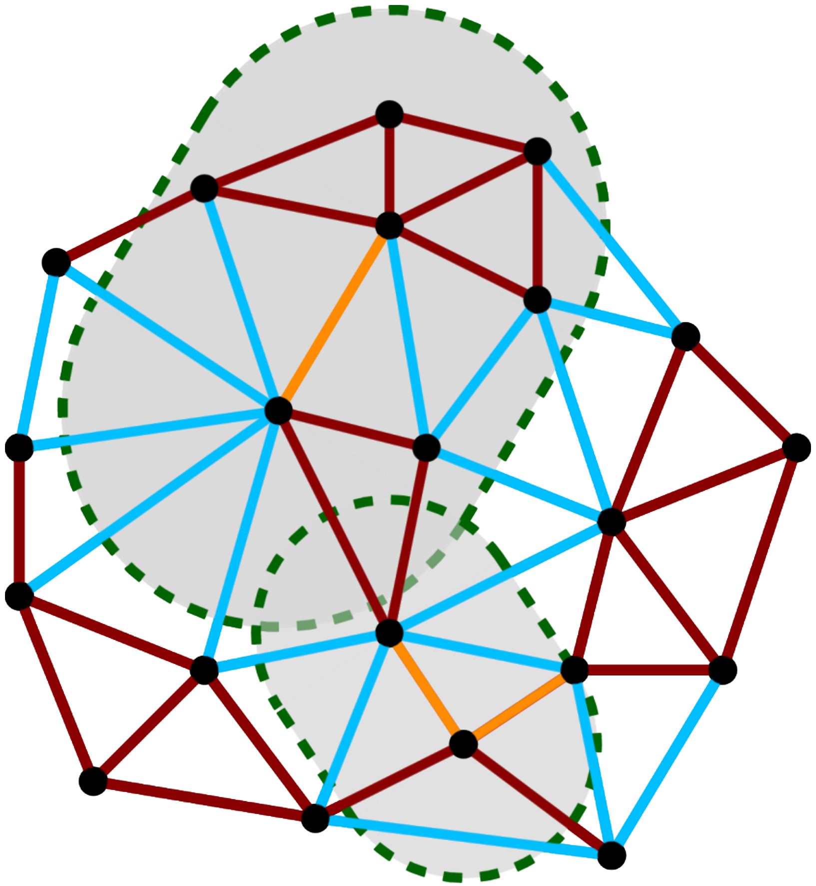
→ Local planarity assumption
→ Few elements: fast solution (1s)

Edge Selection Process

Solid

Empty

Selected

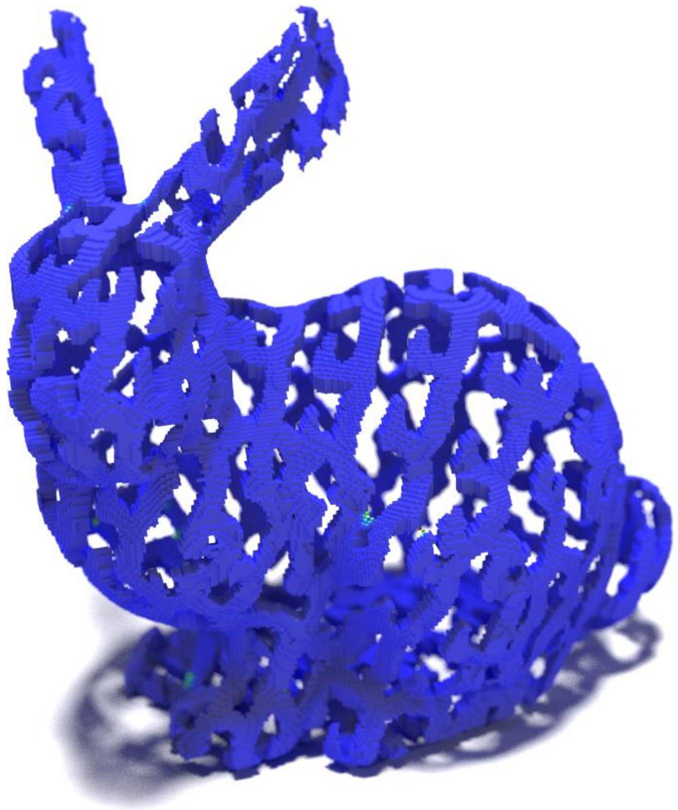


Simulation on the Final Mesh

Stress 99th%

153.9 KPa

30.5 KPa



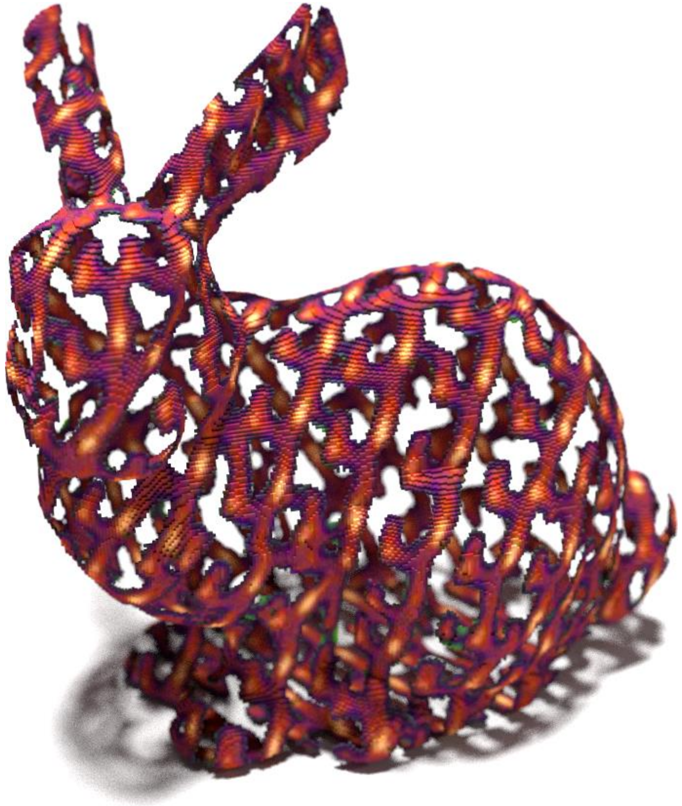
Results – Structure + Color

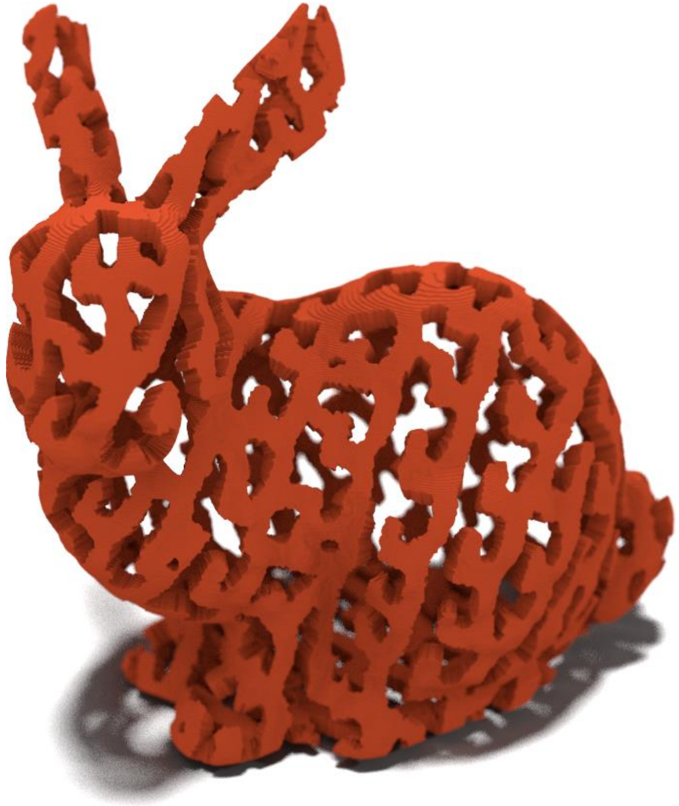
t_{total}: 34.8s

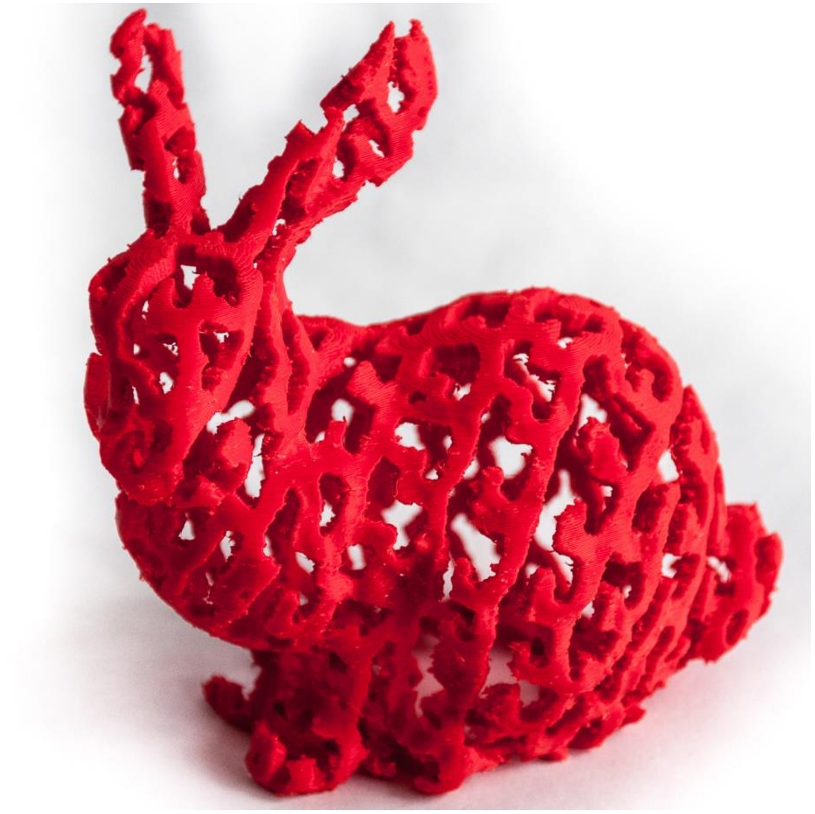
t_{total}: 40.0s

t_{total}: 14.6s

From surface structure to final mesh



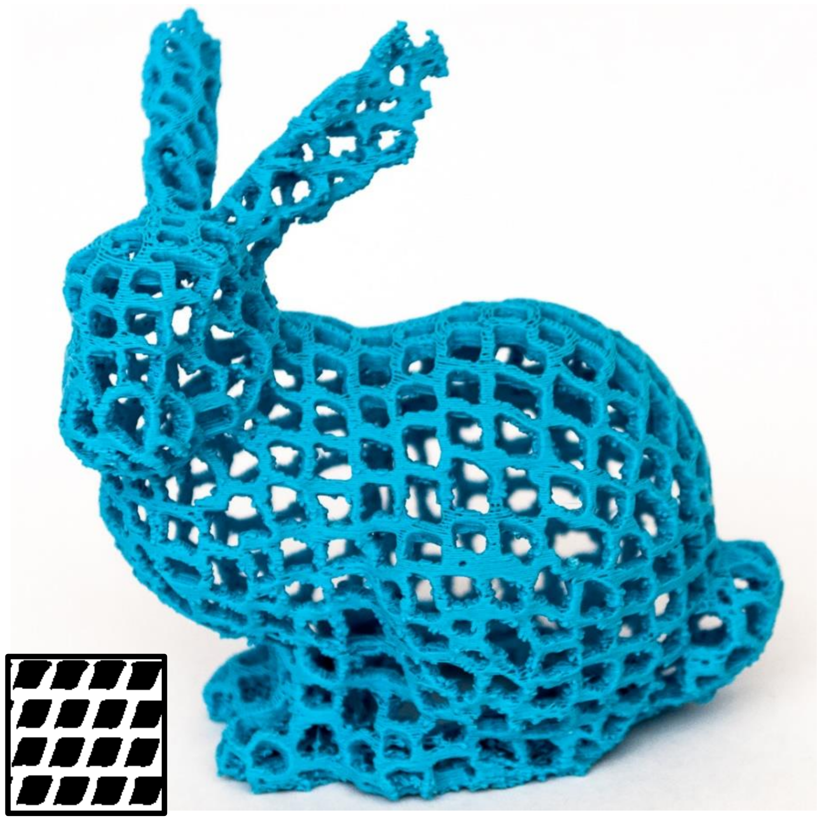




Results - Printouts

t_{total}: 52.4s

t_{total}: 11.4s



t_{total}: 14.5s

Other recent references

- **Designing Structurally-Sound Ornamental Curve Networks**
  J. Zehnder, S. Coros, B. Thomaszewski, SIGGRAPH 2016

- **Stenciling: Designing Structurally-Sound Surfaces with Decorative Patterns**
  C. Schumacher, B. Thomaszewski, M. Gross, SGP 2016

- **Synthesis of Filigrees for Digital Fabrication**
  W. Chen, X. Zhang, S. Xin, Y. Xia ,S. Lefebvre and W. Wang, SIGGRAPH 2016

All these works use a different point of view: discrete element distributions

Key ideas for structure synthesis

Pattern is stochastic

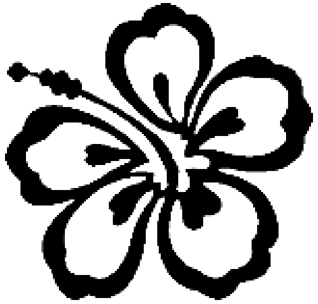
- Exhibits degrees of freedom
- Use pattern itself to locally reinforce structure

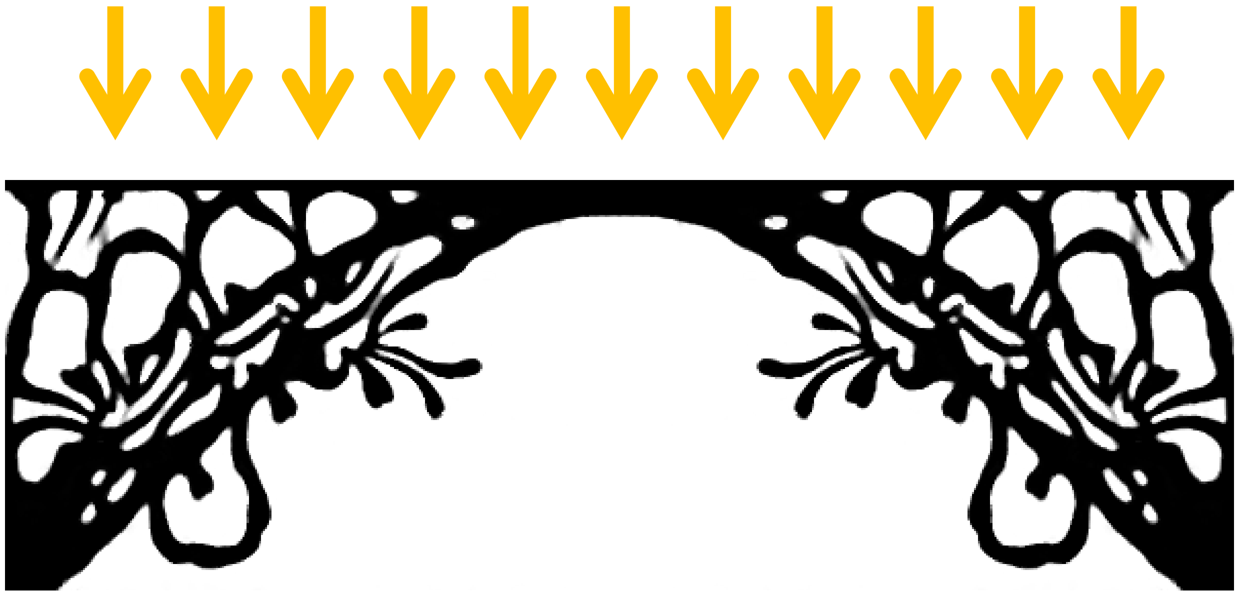
Exemplar specifies local geometry

- Large scale arrangement can be optimized ‘orthogonally’
- Combination with topology optimization?

Our Goal

Exemplar





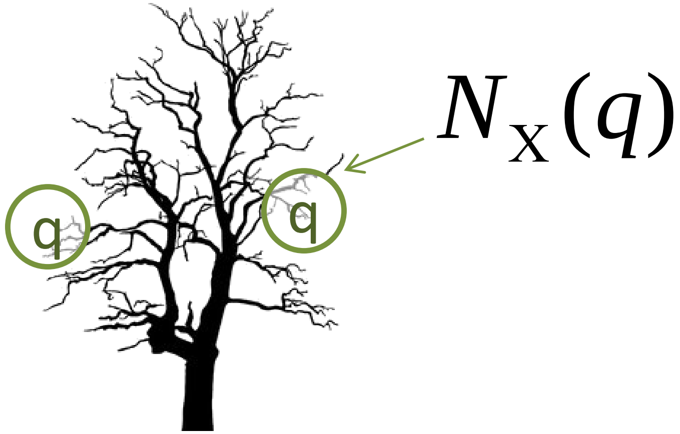
Synthesize shapes under structural and appearance objectives

Local geometry

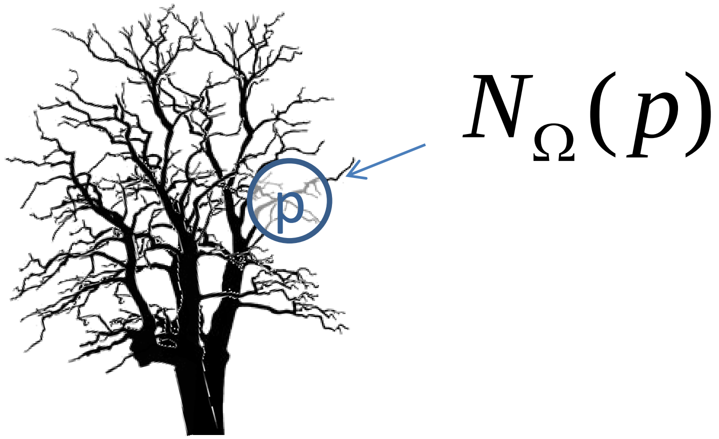
minimise

$$E(\Omega) = \int_{p\in\partial\Omega} \min_{q\in\partial X} D(N_X(q), N_\Omega(p))$$

Local geometry



X Example shape



Ω Synthesized shape

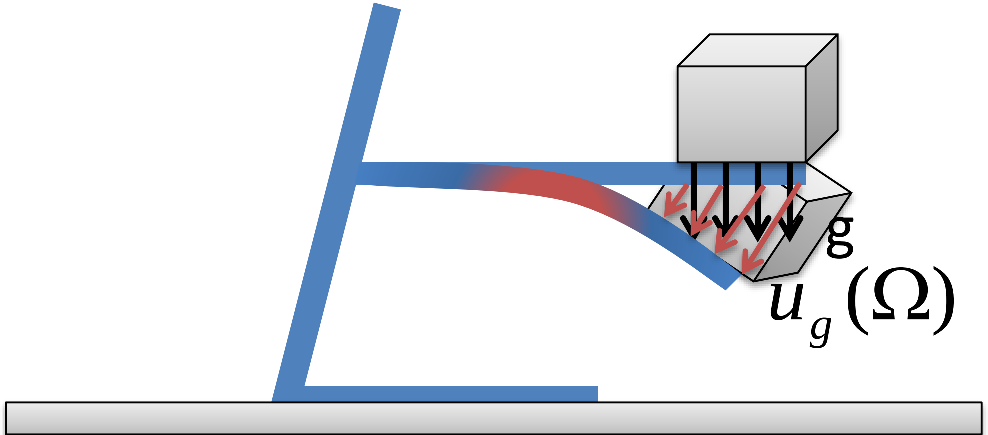
Structural properties

minimise

$$E(\Omega) = \int_{\partial\Omega} g.u_g(\Omega)dx$$

compliance

rigidity



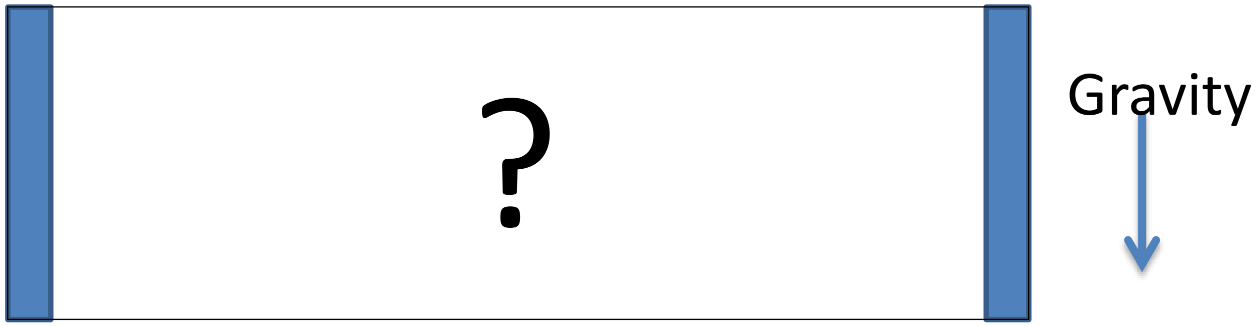
Structural properties

minimise

$$E(\Omega) = \int_{\partial\Omega} g.u_g(\Omega)dx$$

compliance

rigidity



Topology optimization [Osher, Allaire, Sigmund]

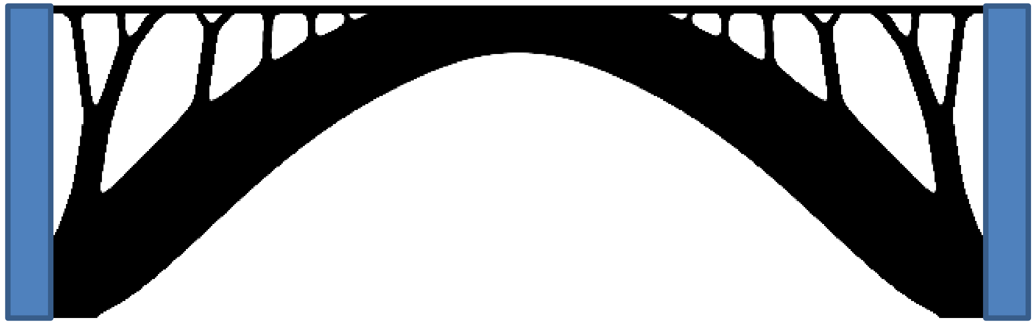
Structural properties

minimise

$$E(\Omega) = \int_{\partial\Omega} g.u_g(\Omega)dx$$

compliance

rigidity



Gravity

Topology optimization [Osher, Allaire, Sigmund]

Challenge

minimise

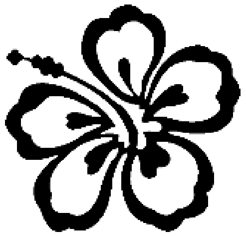
$$E_0(\Omega) = \int_{p \in \partial\Omega} \min_{q \in \partial X} D(N_\Omega(p), N_X(q))$$

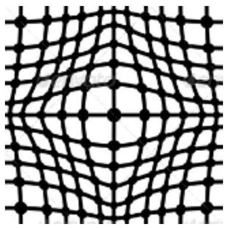
local geometry

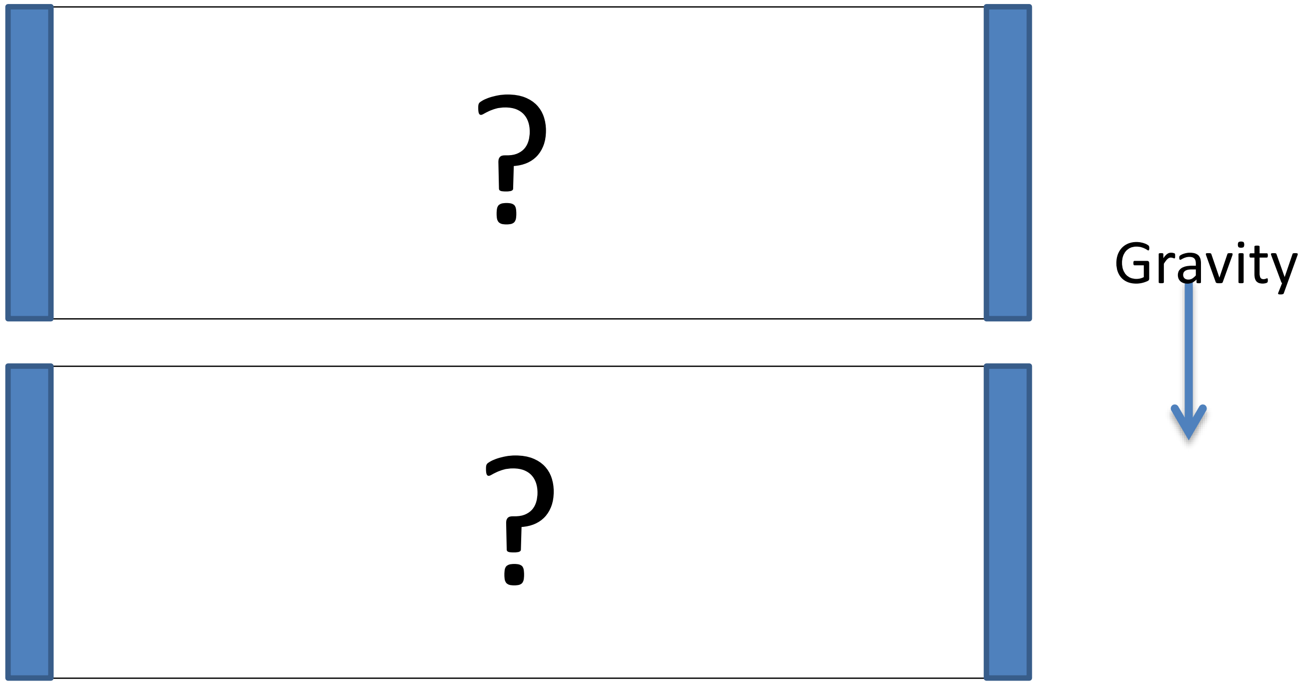
minimise

$$E_1(\Omega) = \int_{\partial\Omega} g.u_g(\Omega)dx$$

rigidity







Challenge

minimise

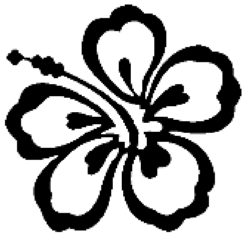
$$E_0(\Omega) = \int_{p \in \partial\Omega} \min_{q \in \partial X} D(N_\Omega(p), N_X(q))$$

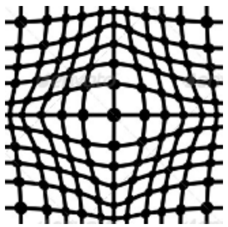
local geometry

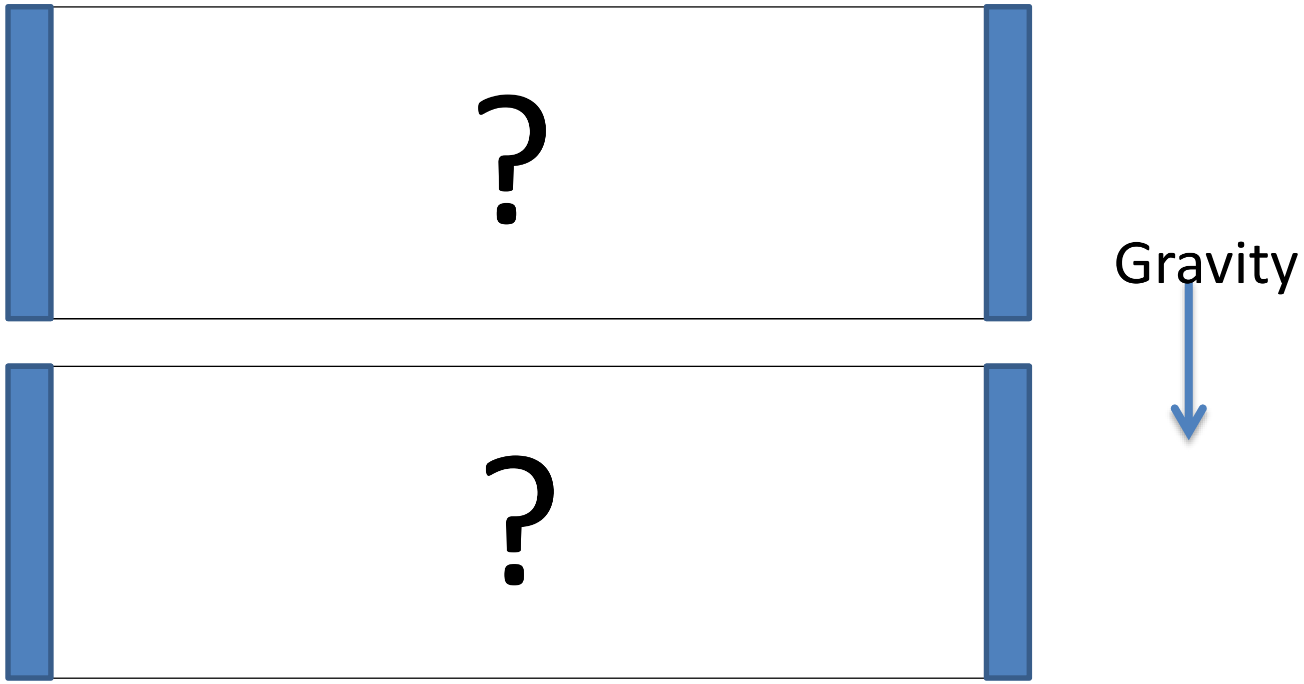
minimise

$$E_1(\Omega) = \int_{\partial\Omega} g\,H_g(\Omega)\,dx$$

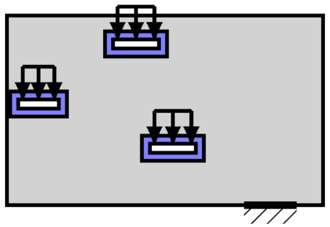
rigidity

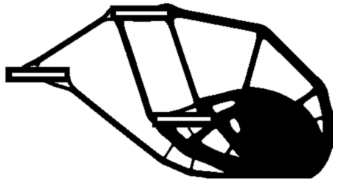




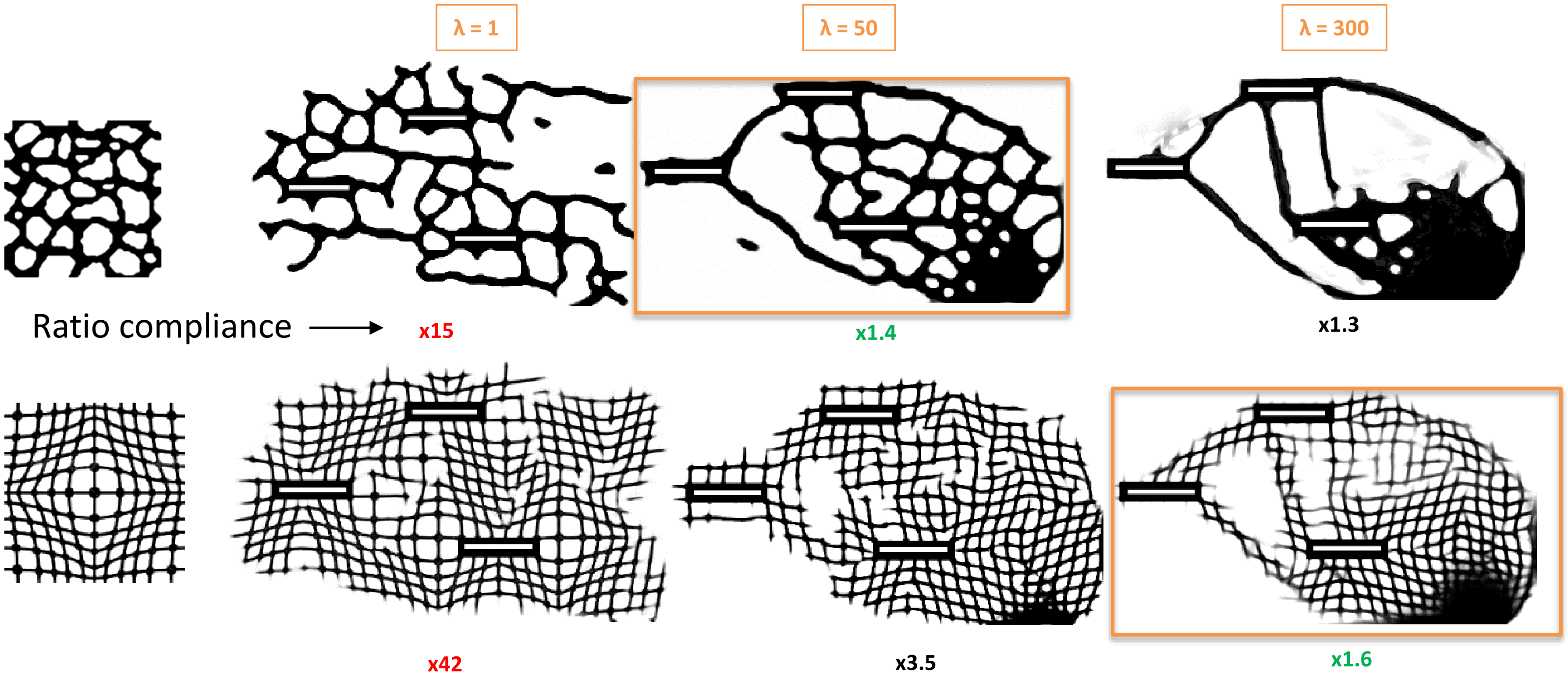


Weighted sum





~~Minimize $G(x) + \lambda\, C(x)$~~



Appearance + rigidity

minimise

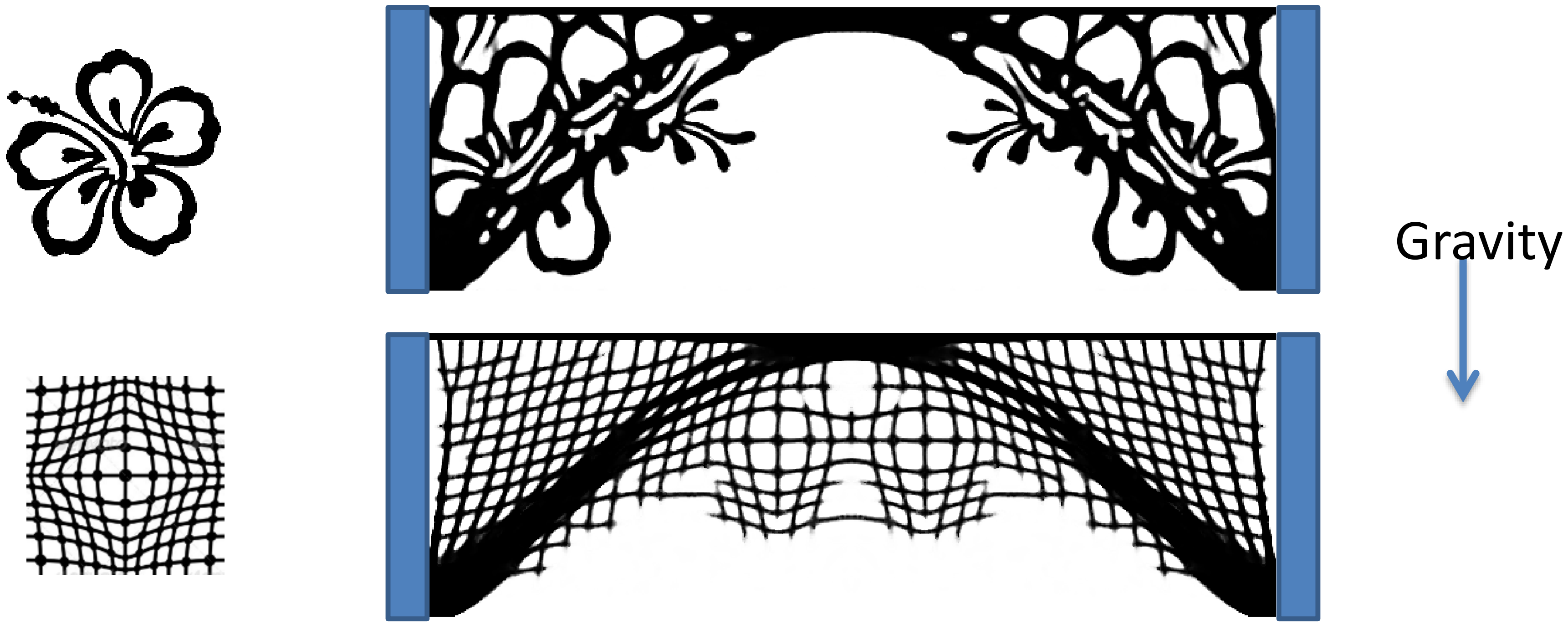
$$E_0(\Omega) = \int_{p\in\partial\Omega} \min_{q\in\partial X} D(N_\Omega(p), N_X(q))$$

appearance

such that

$$E_1(\Omega) = \left(\int_{\partial\Omega} g.u_g(\Omega)dx \right) < \alpha.C_{\max}$$

rigidity



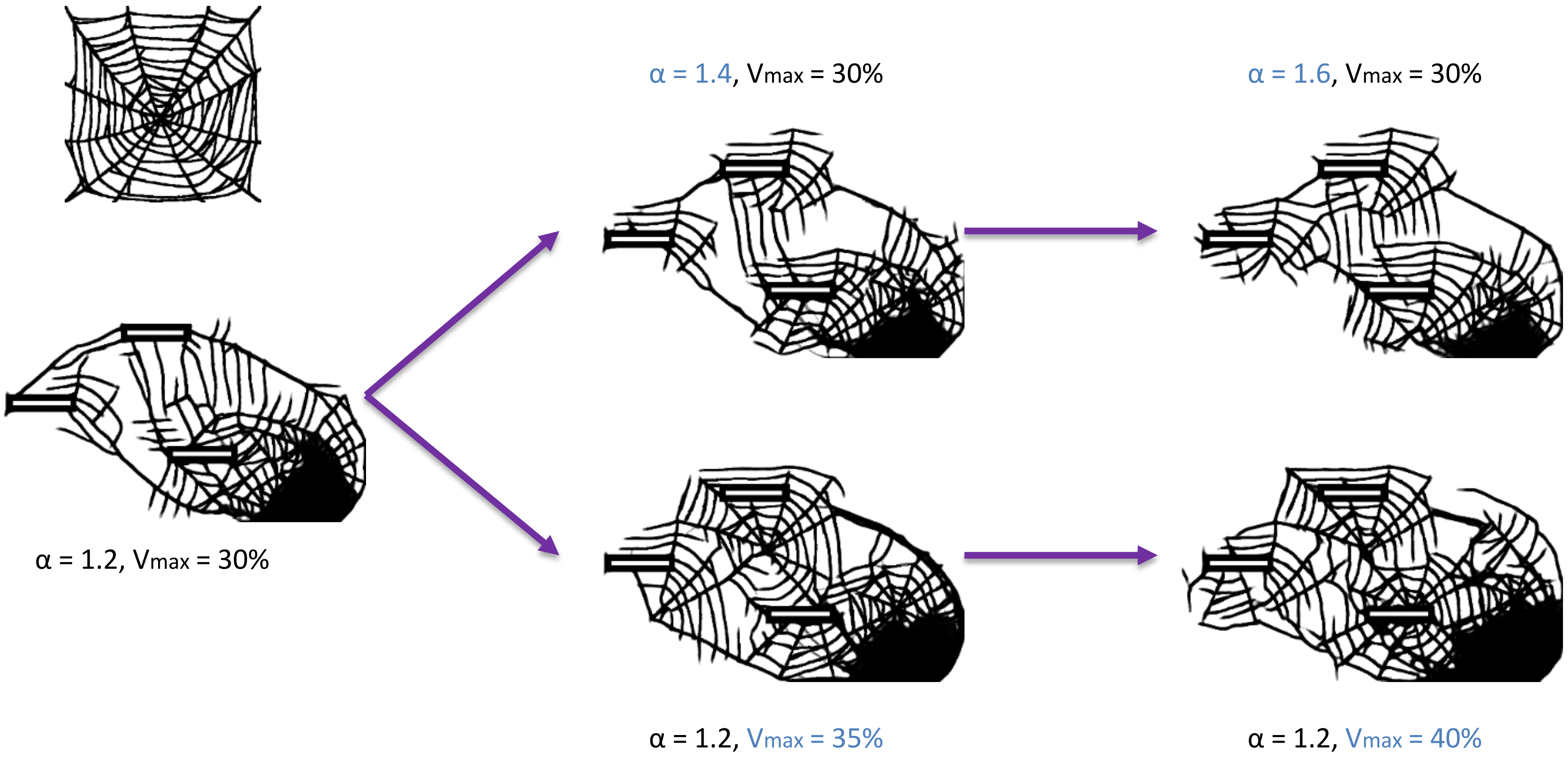
Solver

Not great due to combinatorial matching

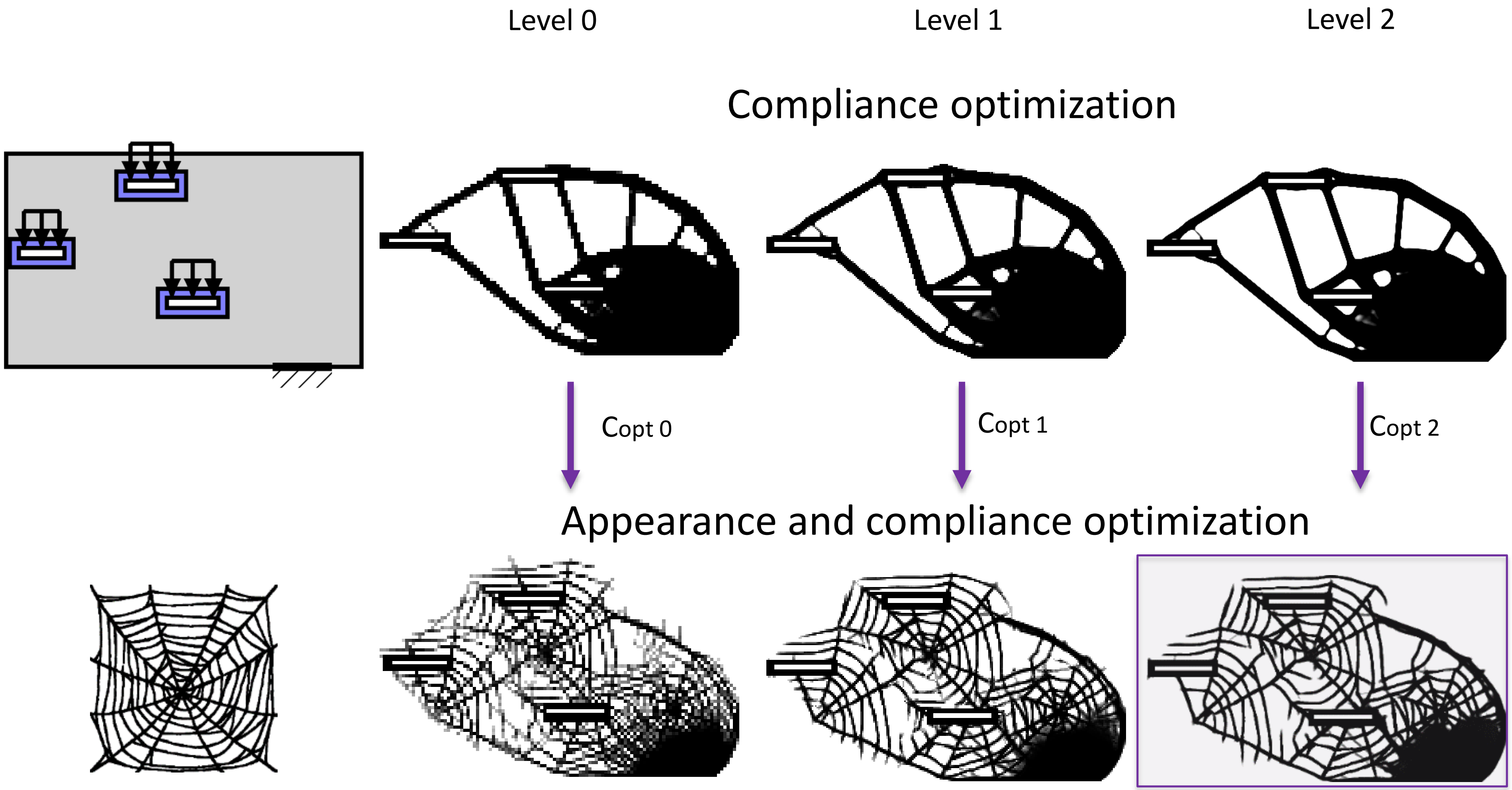
Appearance objective	- Neighborhood matching [Barnes09, Busto10, Kaspar15] - Derivatives **A(x)**
Compliance constraint	- Linear elasticity (FEM) - Derivatives **C(x)**
Volume constraint	- Derivatives **sum(x)**

Gradient-based Optimization
GCMMA [Svanberg95]

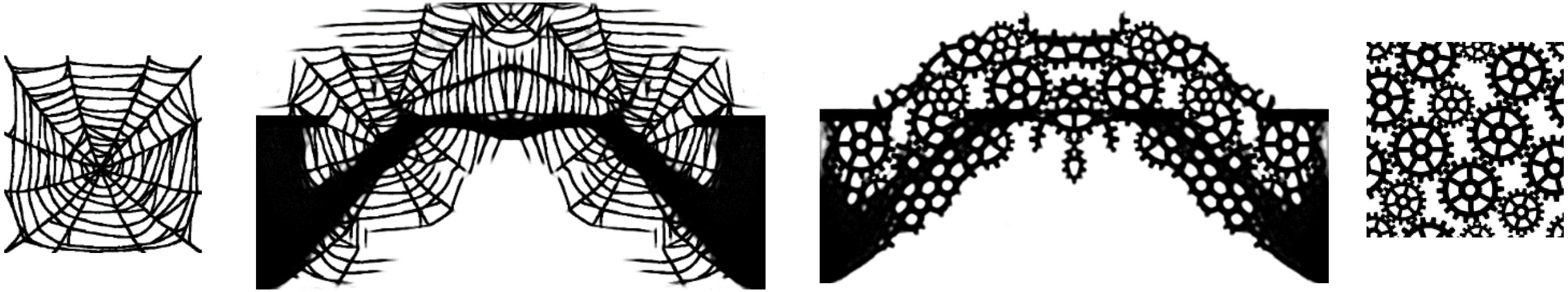
Compliance Relaxation



Multiresolution



Fabricated Objects



Contour extraction

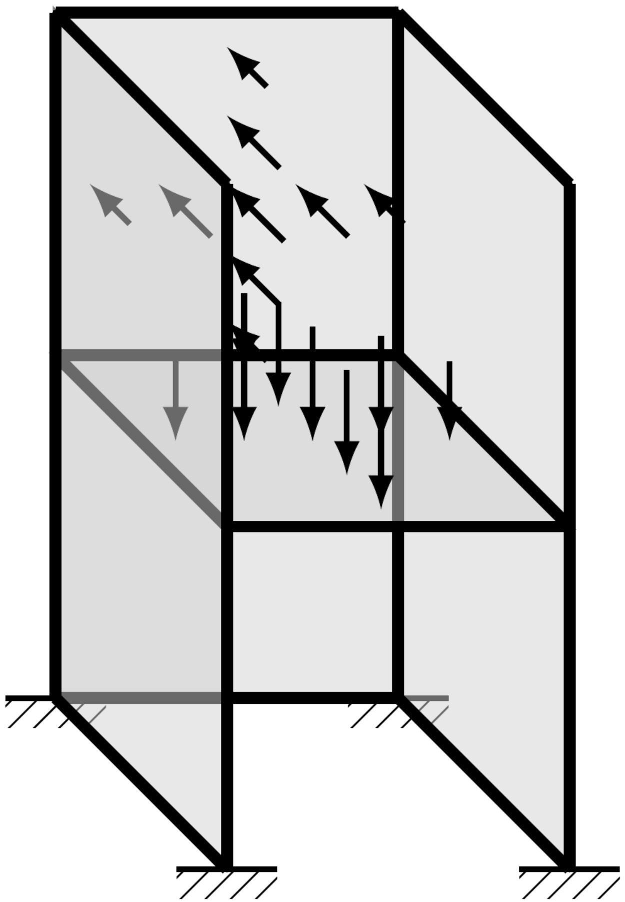
Fabricated Objects: Shelves

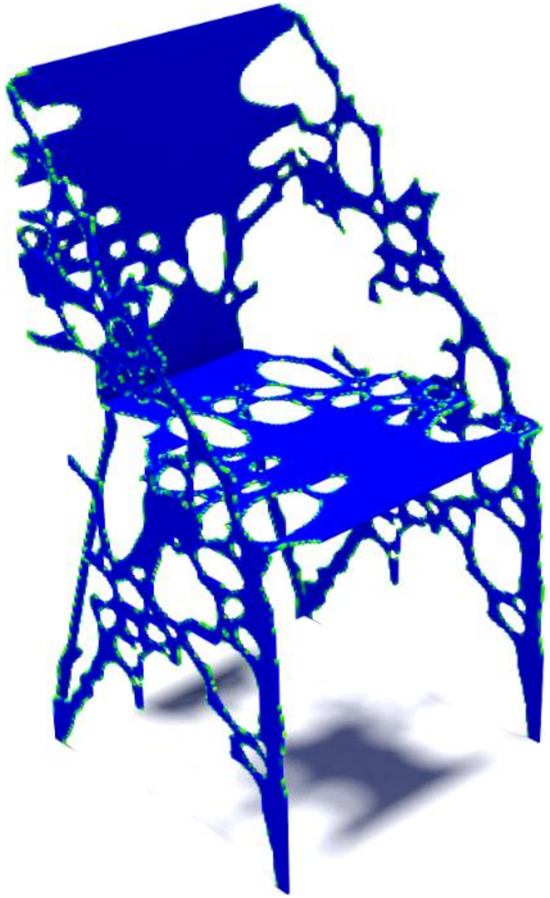
Floor attachment

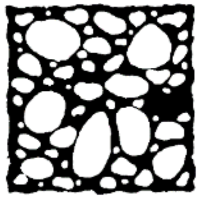
Fabricated Objects: Tables

Fabricated Objects: Phone Stands

3D Structures

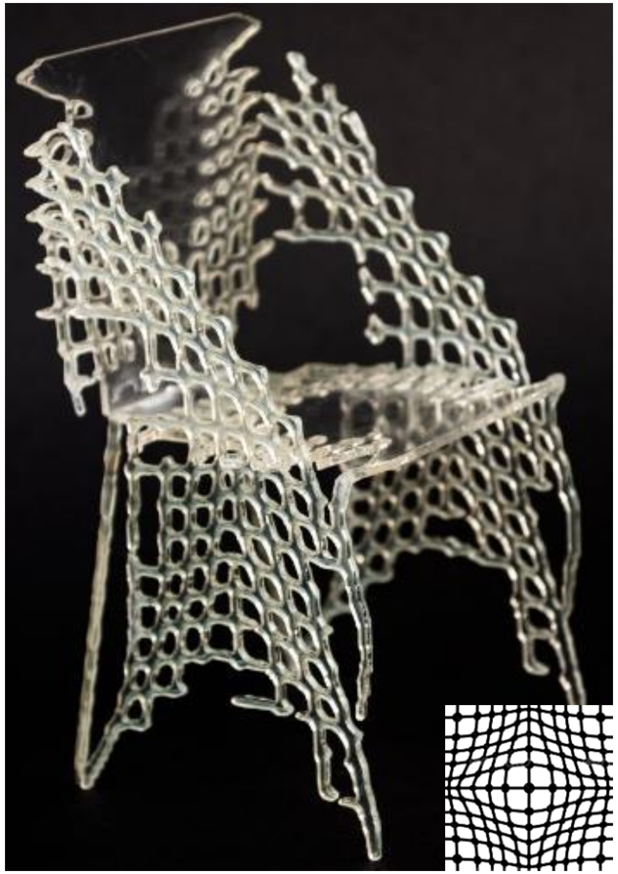






Fabricated Objects: Chairs






Texture Synthesis

- Three main directions
  - By-example synthesis
  - Procedural synthesis
  - Simulation (e.g. erosion)

We will see both in the context of fabrication

Texture Synthesis

- Three main directions
  - By-example synthesis
  - Procedural synthesis
  - Simulation (e.g. erosion)

We will see both in the context of fabrication

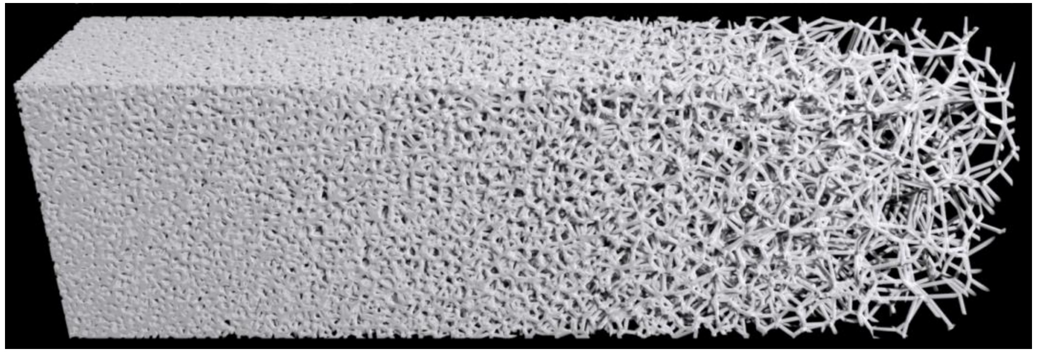
Foams in nature

Coral reef

Metallic foam (chemical reaction)

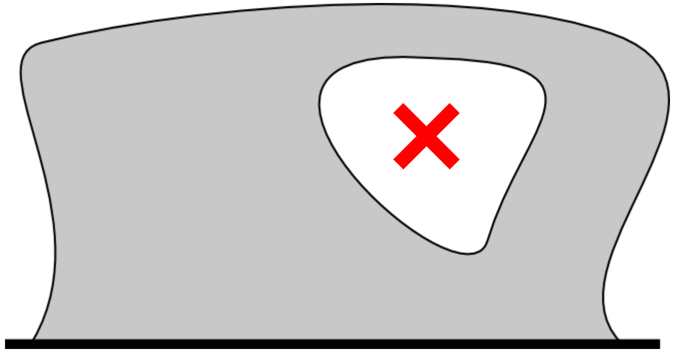
Challenges: scale, fabricability, mechanical properties

- Data size

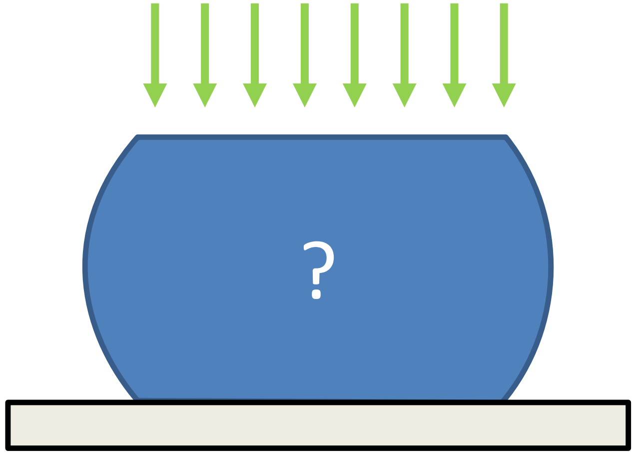


4 GB (.ply)

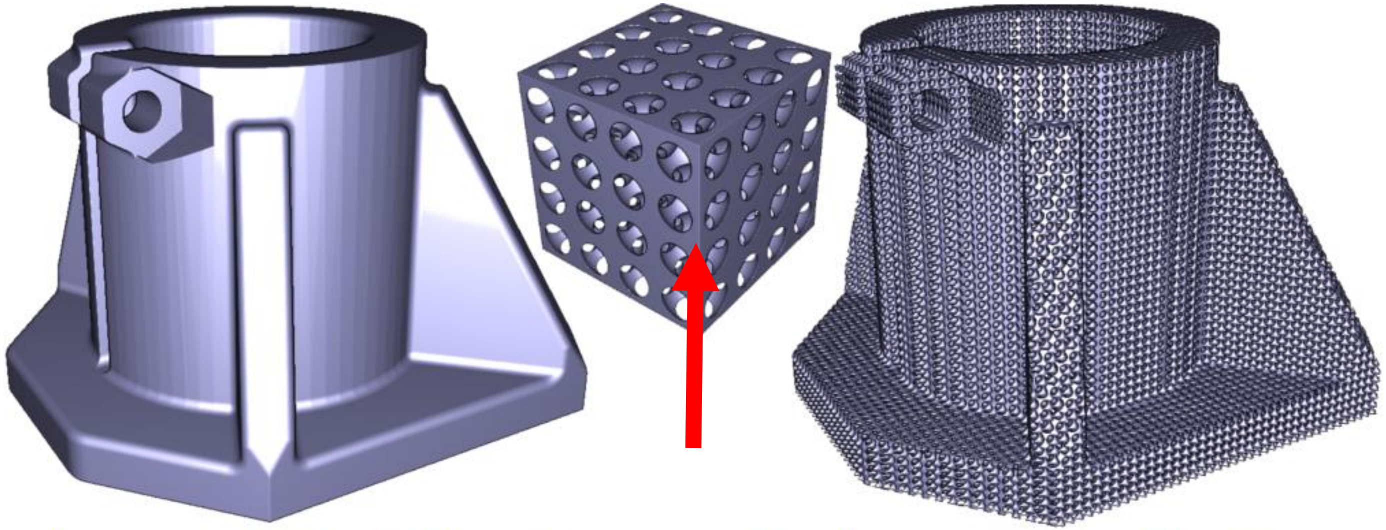
- Fabrication



- Mechanical properties

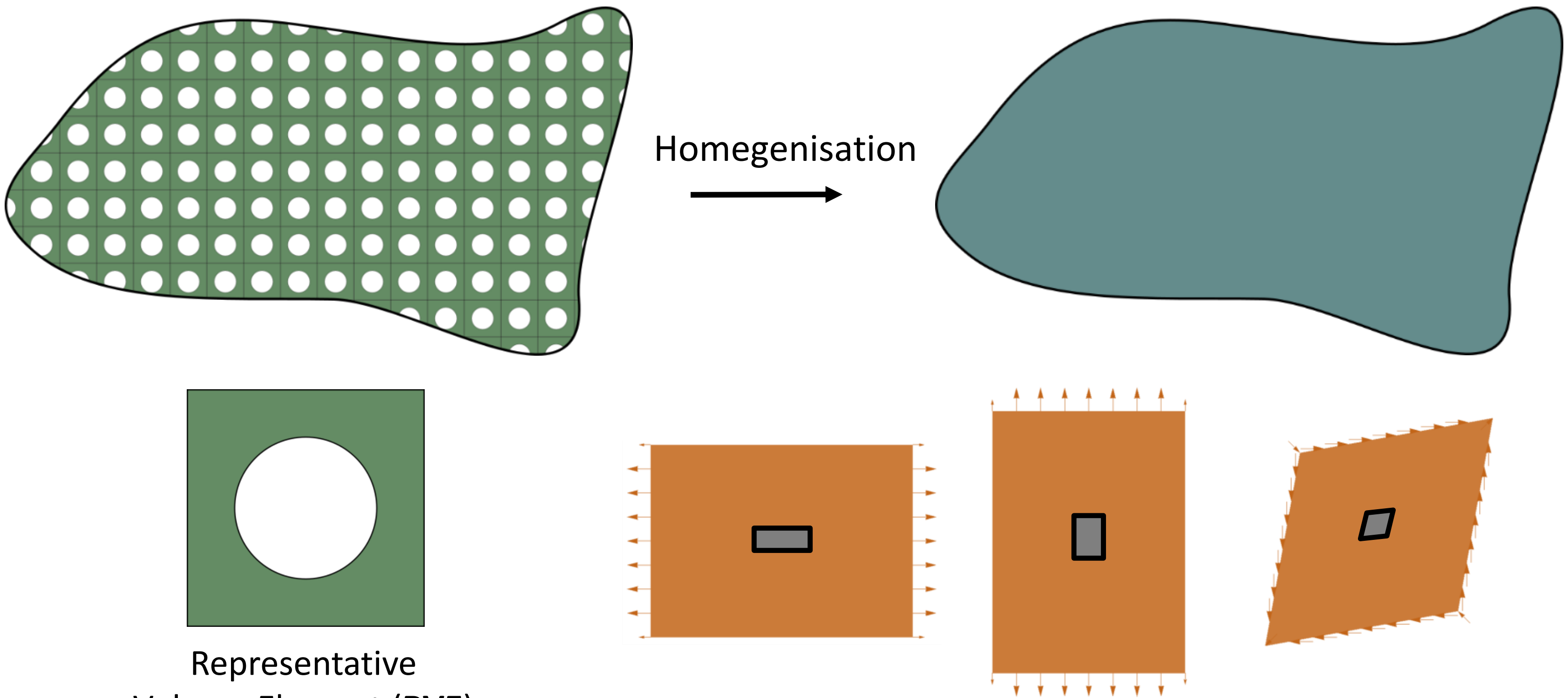


Standard approach: periodic structures



```
spheres = implicit(bx:min_corner(), bx:max_corner(), [[
float distanceEstimator(vec3 p)
{
  const float diameter = 1.0;
  p = p / diameter;
  float d = length(p-ivec3(floor(p))-vec3(0.5,0.5,0.5))-0.6;
  return -d*diameter;
}
]])
```

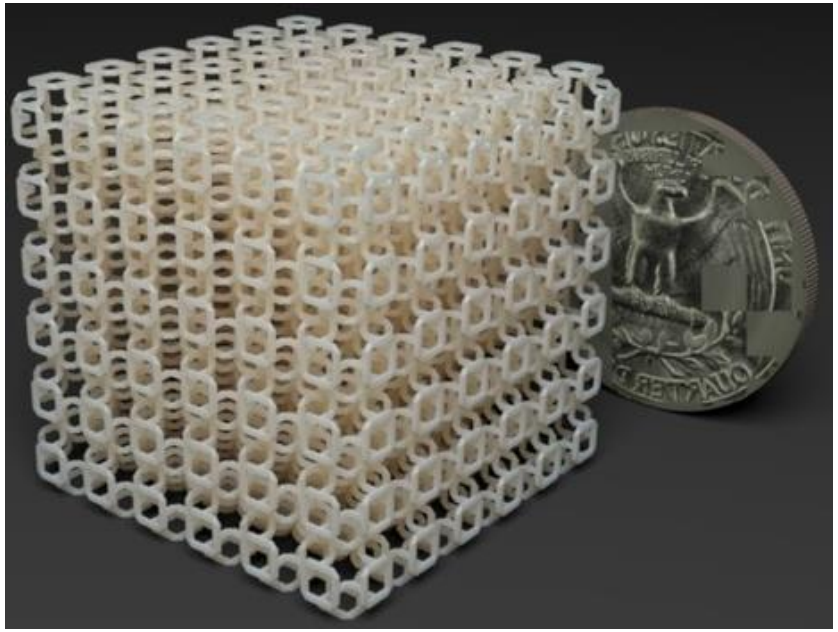
Homogenisation



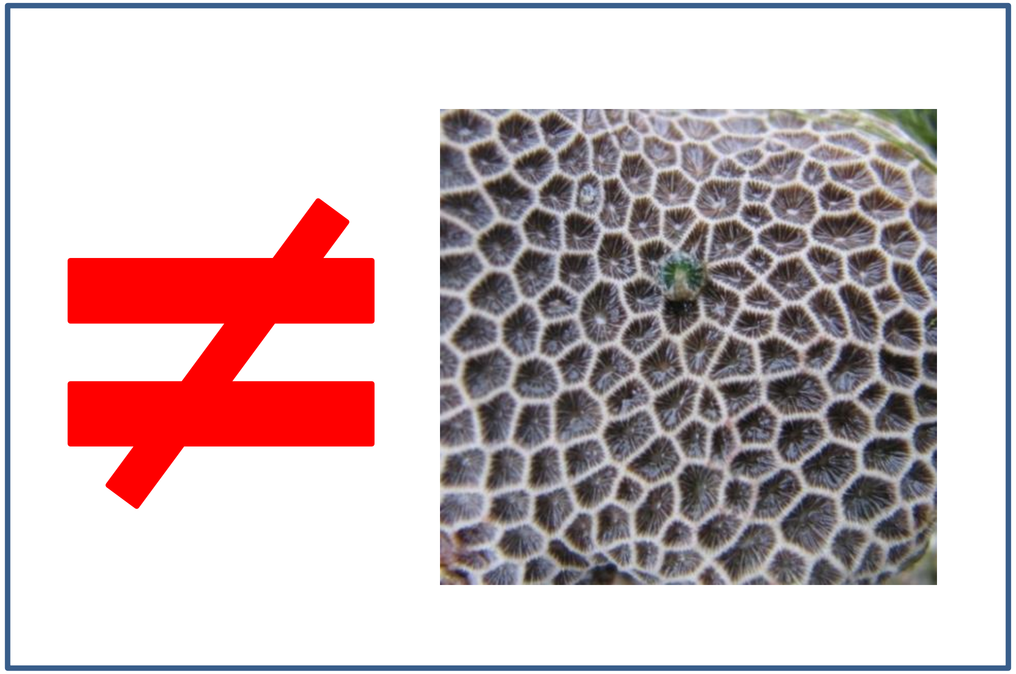
Representative Volume Element (**RVE**)

Homogenized elasticity tensor [Andreassen and Andreasen 2014]

Drawbacks

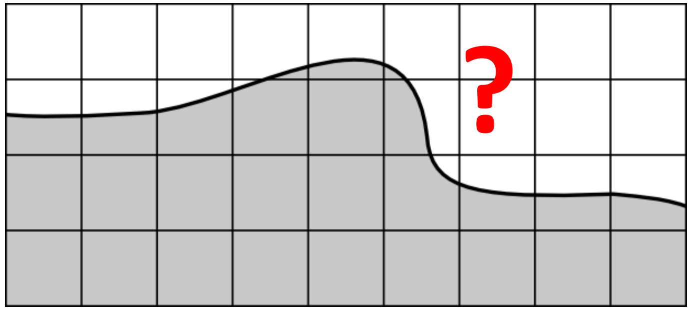


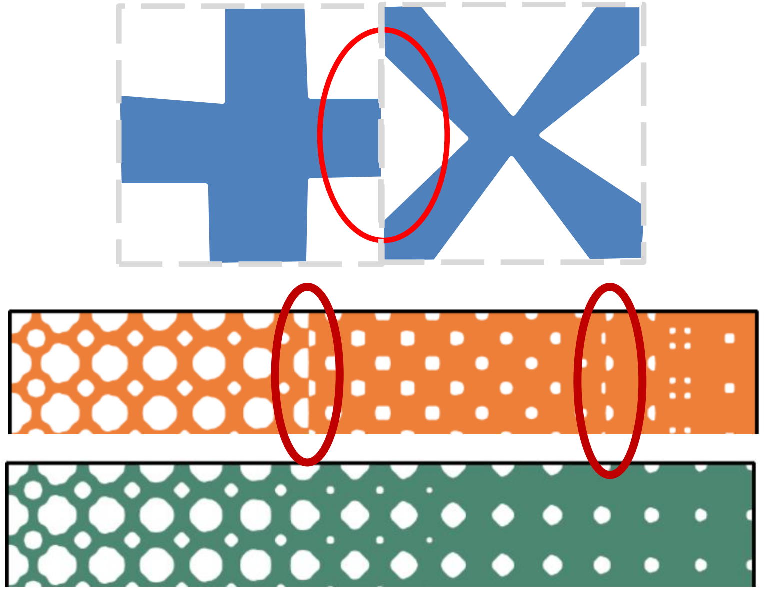
[Pannetta et al. SIGGRAPH 2015]



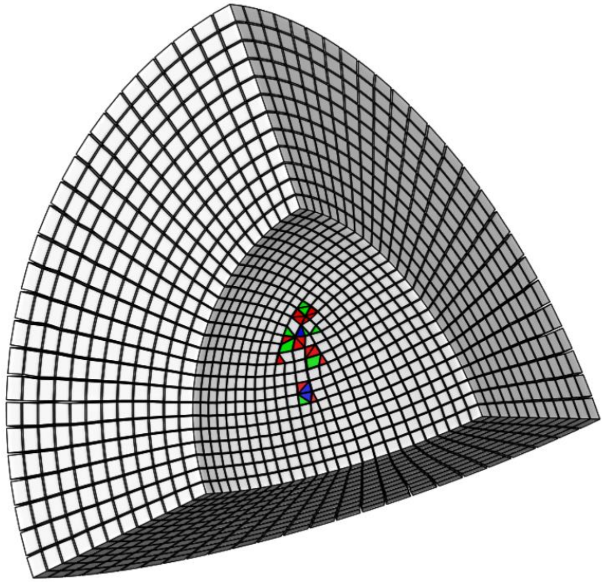
Periodic grid

- Mapping?
  - Hard problem
- Graded properties:
  - Possible, but transitions?





[Schumacher et al. SIGGRAPH 2015]



Hexahedral-dominant meshing
[Sokolov et al. 2015]

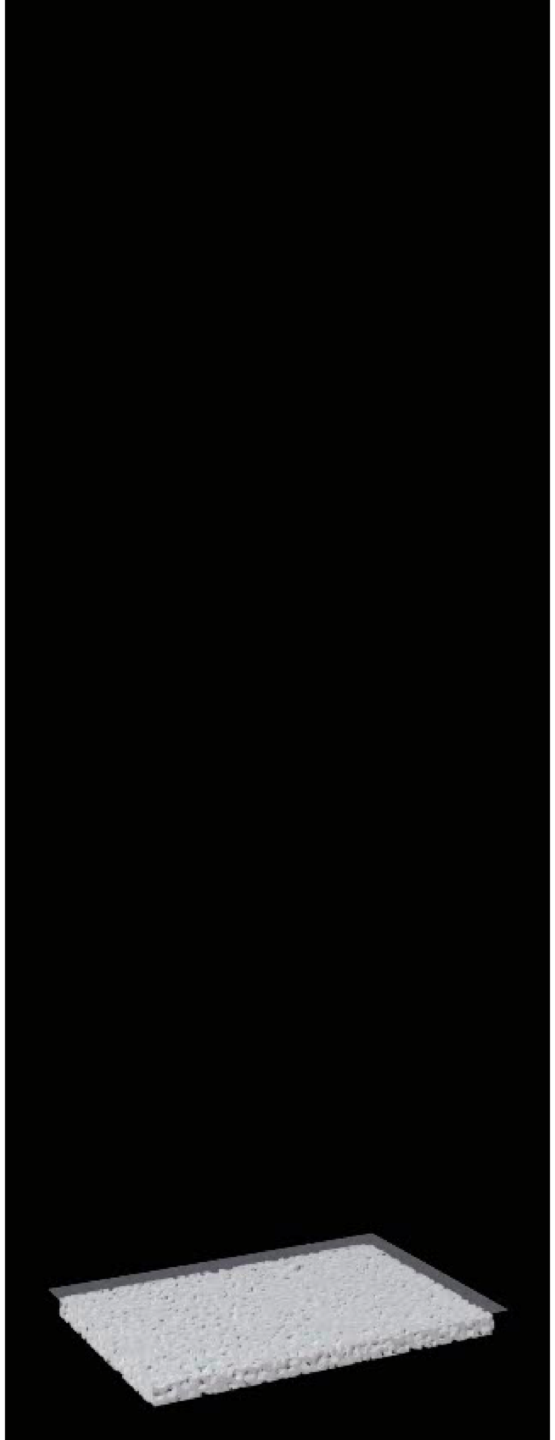
Procedural Voronoi Foams

- **Aperiodic, stochastic, stationary**
  Mimics nature.

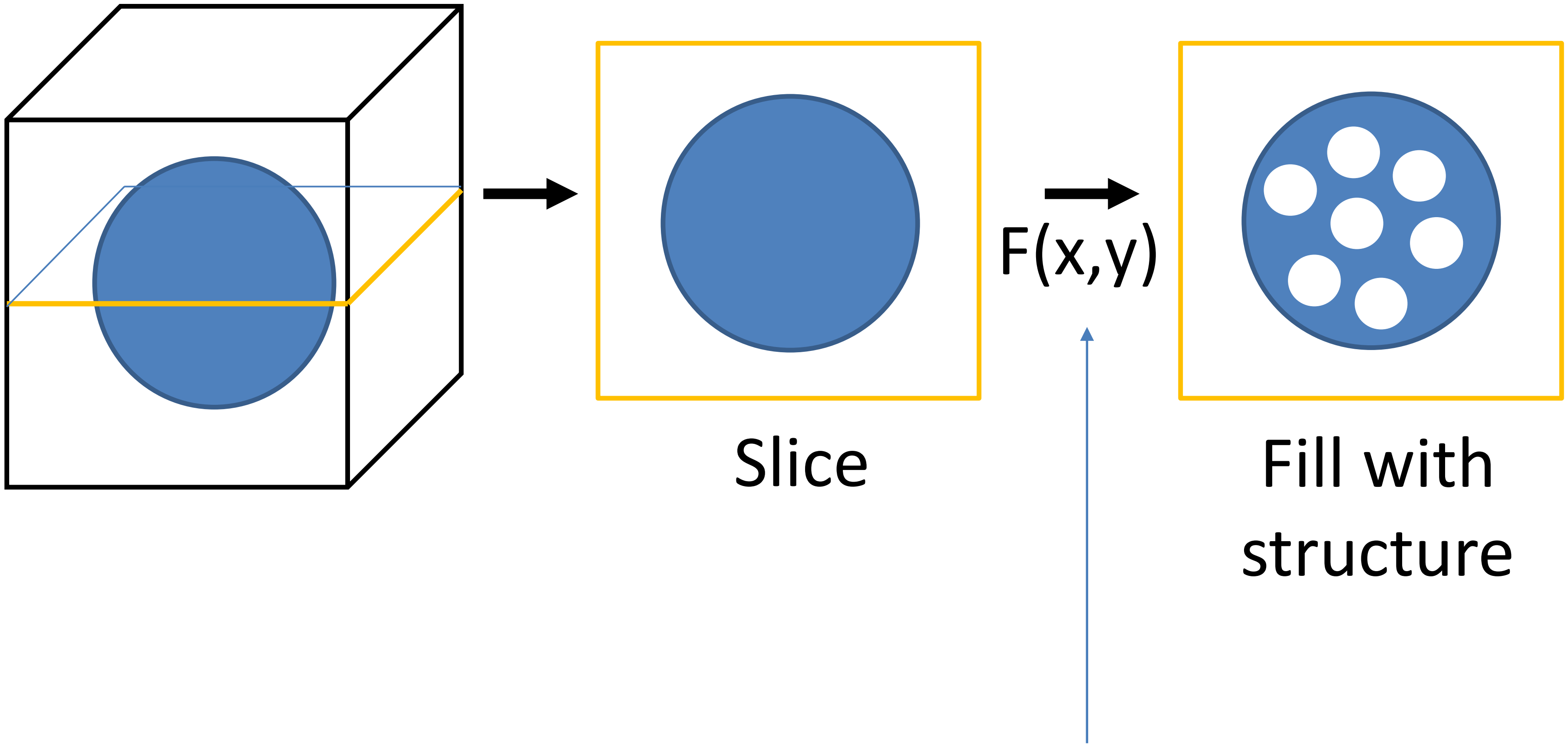
- **Trivially scales.**
  O(1) time + memory.

- **Fabricable.**
  Few pockets, connected, thickness ok.

- ***Controllable elasticity***



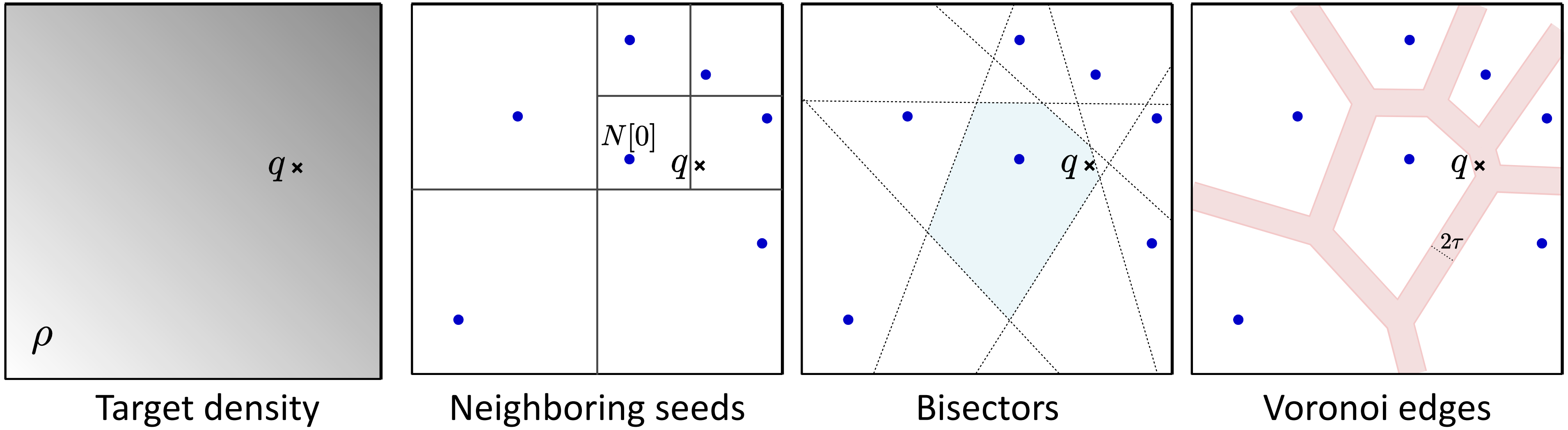
Procedural synthesis



F(x,y) called in every slice 'pixel'

Procedural synthesis

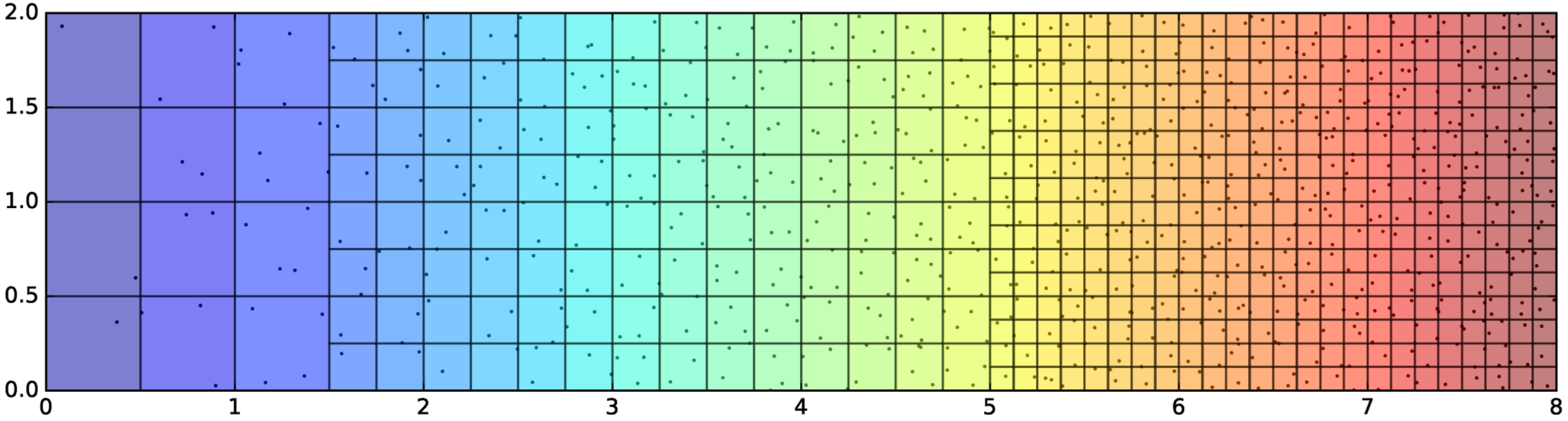
F(x,y): is q=(x,y) inside?



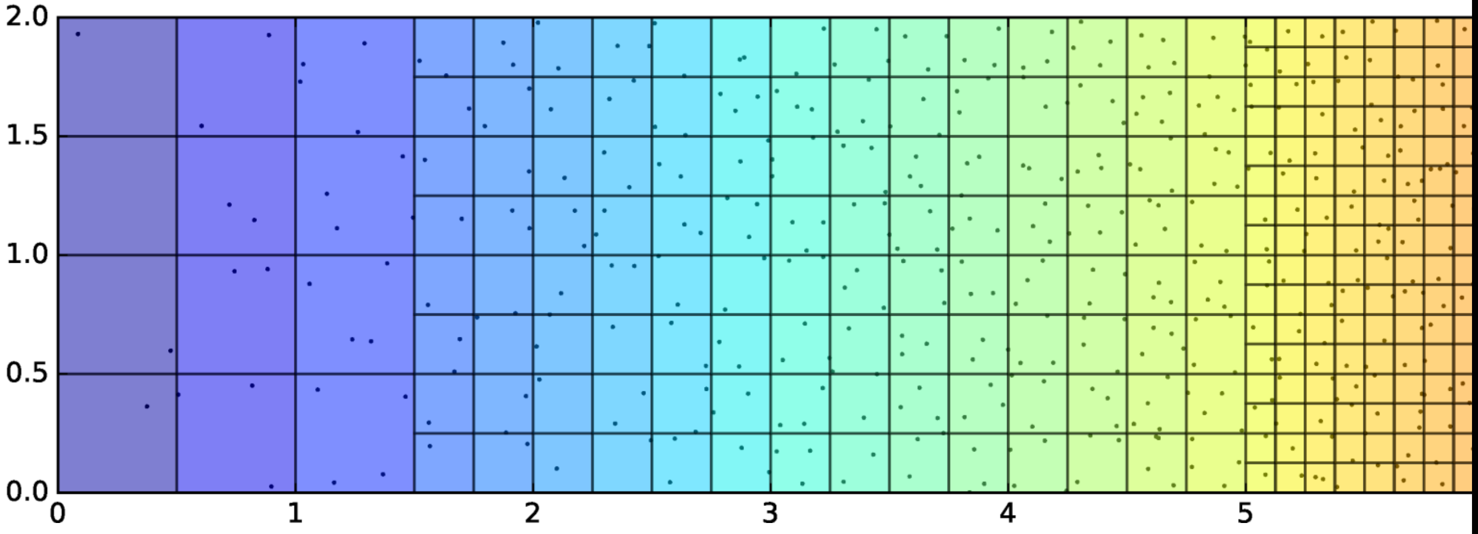
Local computations, O(1)

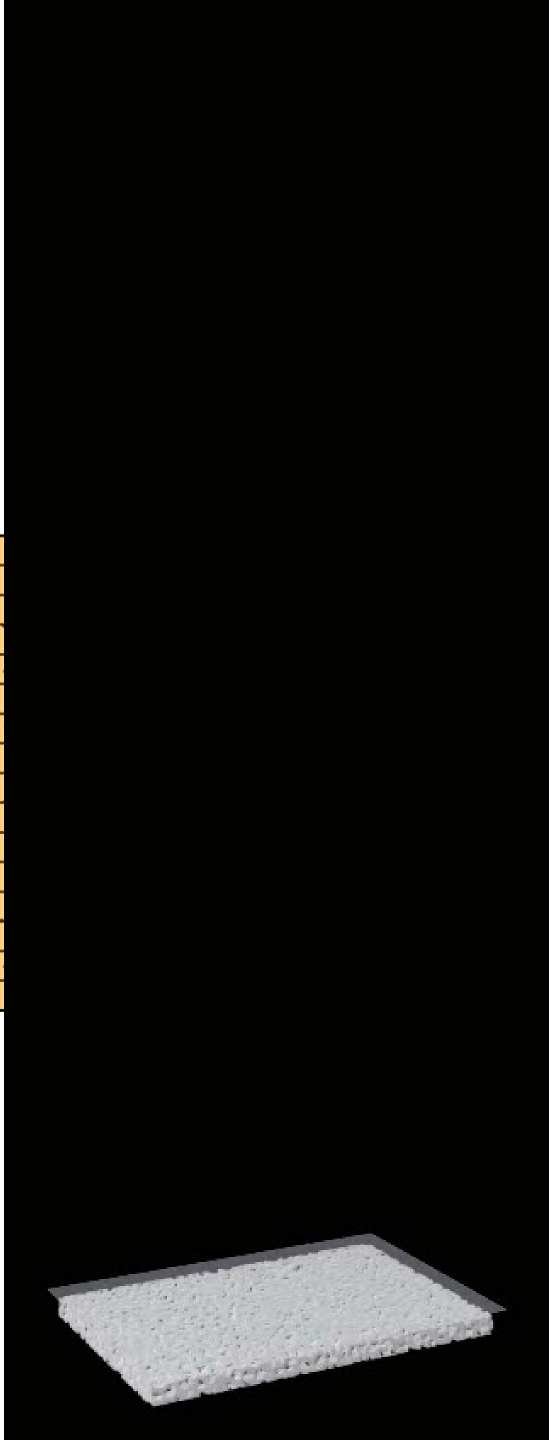
Trivialy parallel (GPU)

Gradation (stackless)

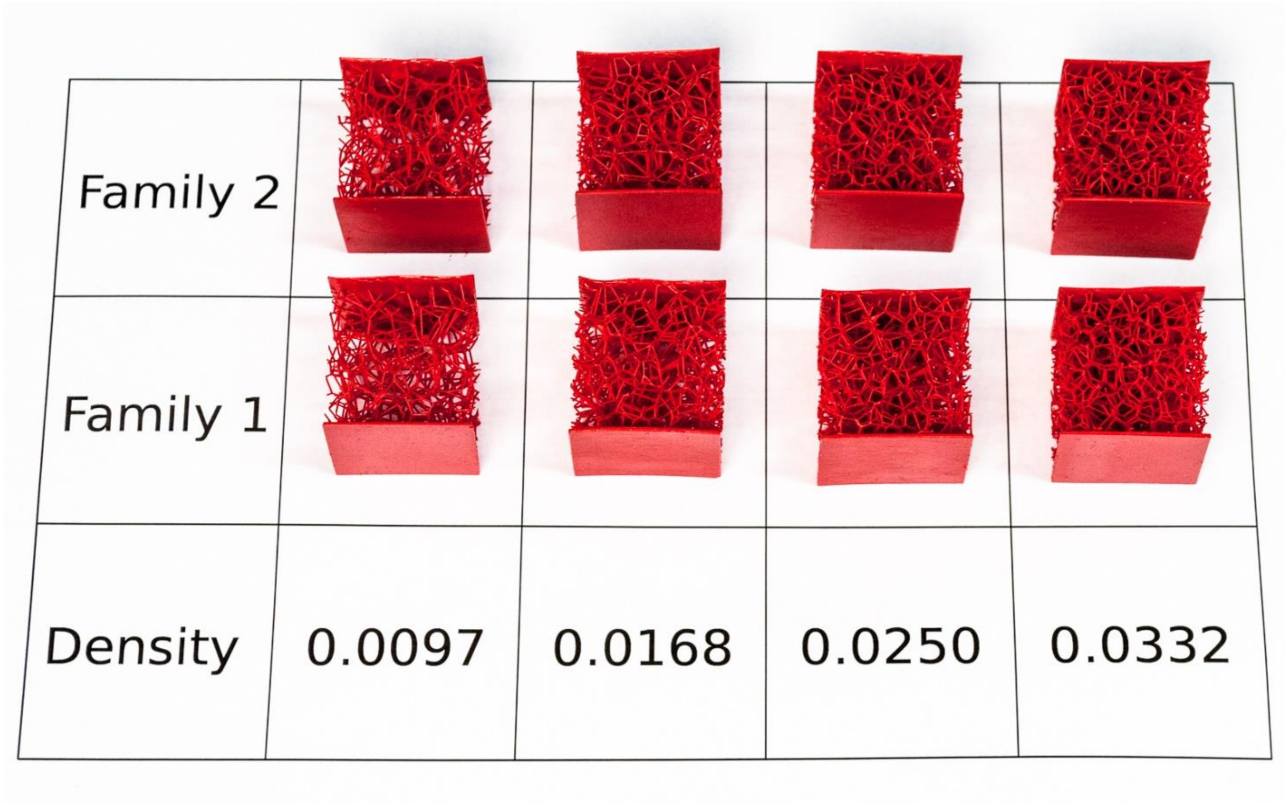


Gradation (stackless)

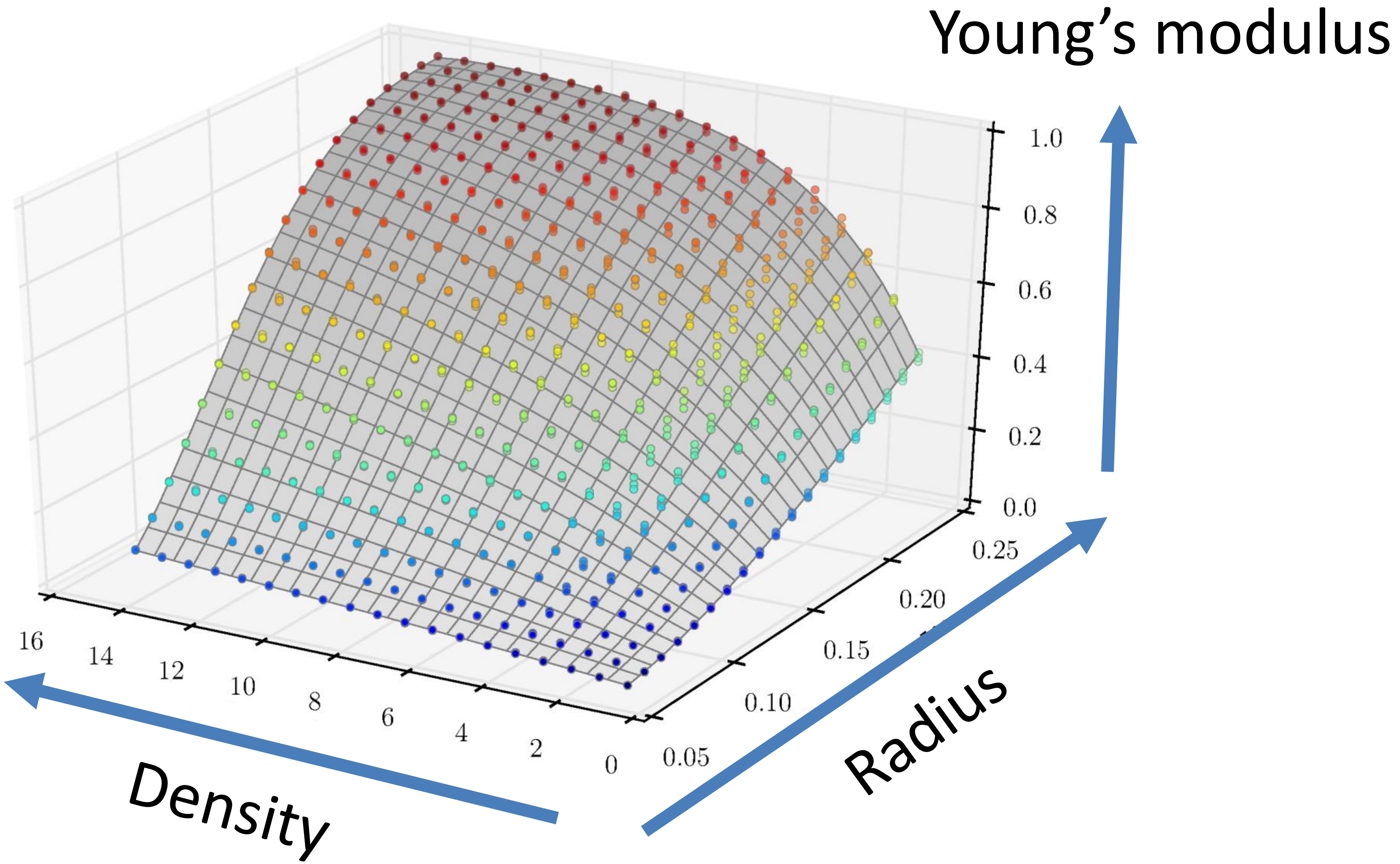




Elasticity control



Homogenisation

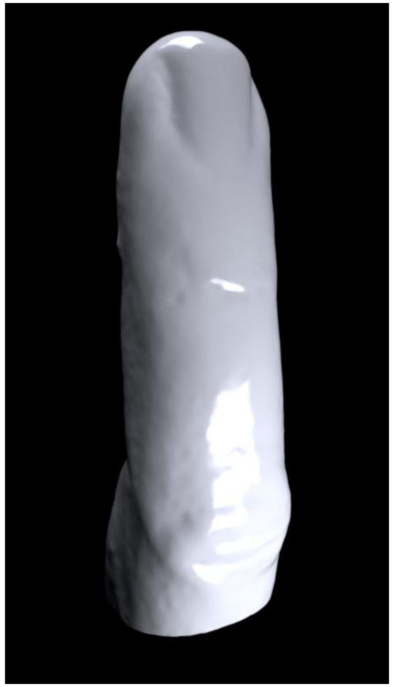


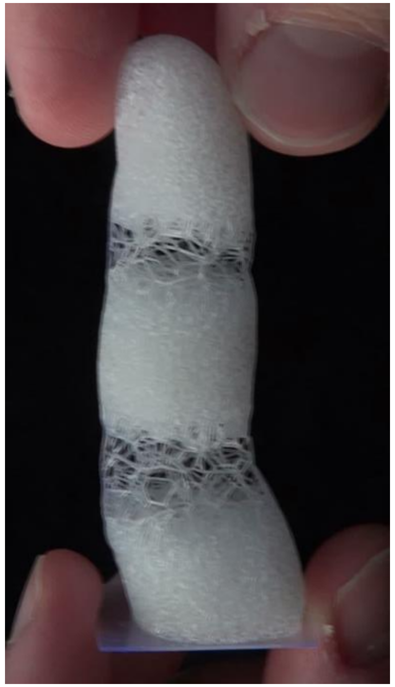
- **Results**

Crusty Knight

- **Results**

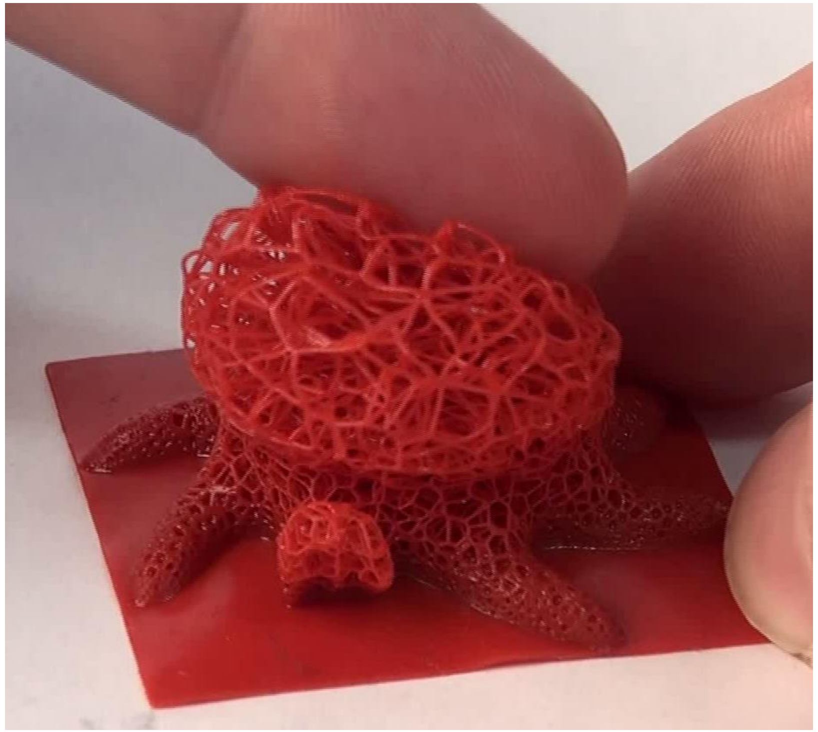
Articulated Finger





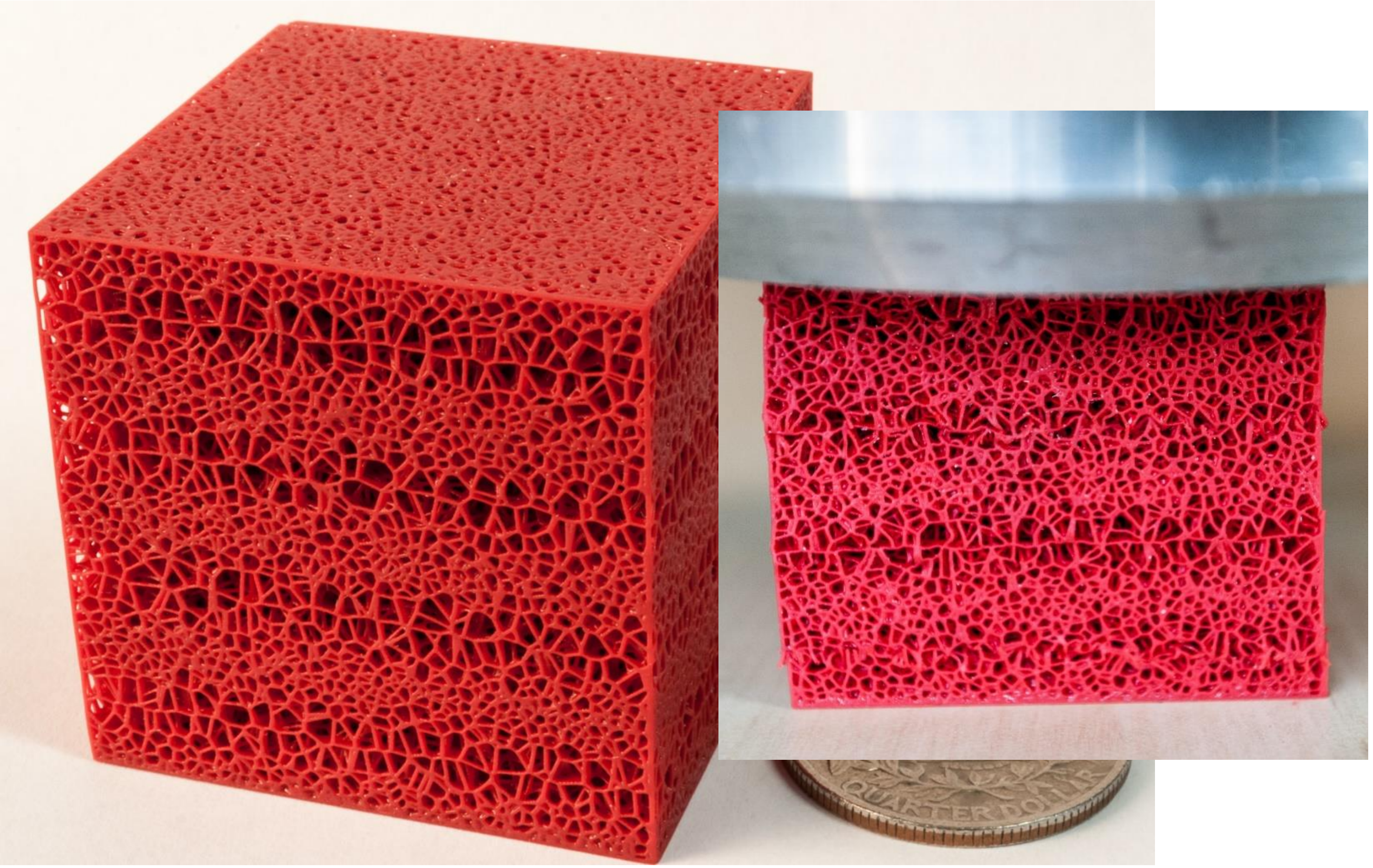
Cute Octopus

- Results



- **Results**

Anisotropy



Performances

Example		Extent (mm)	# Voxels	Volume	% Filtered	Time per slice (ms)
Moomin	fig. 1	26.7 × 40.8 × 51.9	534 × 815 × 1038	6.44%	0.005%	68.34
Ellipsoid	fig. 13	30.9 × 30.9 × 41.1	617 × 617 × 822	6.30%	0.001%	37.28
Knight	fig. 14	26.1 × 30.0 × 50.55	521 × 600 × 1011	12.50%	0.023%	20.25
Finger	fig. 15	25.0 × 23.25 × 70.5	500 × 465 × 1410	23.35%	0.006%	28.03
SIGGRAPH logo	fig. 16	20.0 × 40.0 × 80.0	400 × 800 × 1600	5.73%	0.003%	69.18
Half-dome	fig. 17	25.0 × 50.0 × 25.0	500 × 1000 × 500	19.49%	0.025%	71.22
Octopus	fig. 18	41.7 × 41.1 × 28.8	833 × 822 × 576	17.27%	0.009%	150.22
Anisotropic cube	fig. 19	40.0 × 40.0 × 40.0	800 × 800 × 800	26.86%	0.005%	113.52
Forest dragon	fig. 20	770.1 × 990.7 × 961.7	15402 × 19814 × 19234	N/A	N/A	1666.91

Thank you for your attention!

Questions?

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Topology Optimization for Computational Fabrication

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Abstract

Additive manufacturing (AM) and topology optimization (TO) form a pair of complementary techniques in transforming digital models into physical replicas: AM enables a cost-effective fabrication of geometrically complex shapes, while TO provides a powerful design methodology for generating optimized models, which are typically complex from a geometric perspective. The potential of both techniques has recently been explored in graphics, resulting in fantastic applications especially regarding structural and aesthetic properties of fabricated models. In this tutorial, we start from the fundamentals of AM and TO, and proceed to advanced TO techniques which steer the optimization process, i.e., taking into account the manufacturing as well as aesthetic appearance.