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MASTER OF SCIENCE IN APPLIED GEOPHYSICS

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Data driven Orientation Determination of Multicomponent Seismic Sensors

Fabian Patterson

August 12, 2016

Data driven Orientation Determination of Multicomponent Seismic Sensors

MASTER OF SCIENCE THESIS

for the degree of Master of Science in Applied Geophysics at
Delft University of Technology
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by

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Department of Geoscience & Engineering	·	Delft University of Technology
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Delft University of Technology

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Abstract

The orientation of a three component sensor in seabed seismic needs to be known to successfully process the acquired data. Although it is often measured, the data processing centers are requested to estimate it when these measurements are corrupted or to validate them. A large class of methods used in the industry perform well in the presence of uniform shot coverage but suffer when shots are partly missing, for example because of obstructions.

I develop multiple novel algorithms that either make use of the source-receiver geometry or the knowledge of the seabed elastic properties around the receiver to determine the orientation of the three sensor axes. The seabed properties are used to model the response of the seabed to an incident pressure wave. The new routines are tested on real and synthetic data. All the new approaches perform better than Schlumbergers legacy proprietary processing software in retrieving a known receiver orientation from synthetic data, if the shot coverage is non-uniform. The application to real data also indicates that the new algorithm matches the results of the existing methods when the shot coverage is uniform and outperform them when there are large gaps. Especially the model based approach solves the problem of determining the orientation of a receiver entirely if the knowledge of the compressional and shear wave velocities of the seabeds is good enough.

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Fabian Patterson

Table of Contents

Abstract	v
Acknowledgements	vii
Nomenclature	xix
1 Introduction	1
1-1 Objectives of this thesis and main results	4
1-2 Motivation	5
I Theoretical background	7
2 Coordinate systems, Orientation and Rotations	9
2-1 Coordinate systems, Orientation and Rotations	9
2-1-1 Coordinate Systems	9
2-1-2 Orientation	9
2-1-3 Euler Rotation Theorem	11
Rotation convention	11
Euler-Rodrigues-formula	12
2-1-4 Quaternions	13
Quaternion rotation	14
2-1-5 The relationship between Quaternions and Euler angles	14
3 Wave propagation and polarization	17
3-1 Wave propagation and polarization	17
3-1-1 P and S waves in a marine environment	17
3-1-2 The seismic signal	18

3-2	Properties of the seabed	19
3-2-1	Boundary conditions	19
	Acoustic boundary conditions	19
	Elastic boundary conditions	19
3-2-2	Reflectivity and Transmission	19
	Acoustic Transmission coefficients	20
	Elastic Transmission coefficients	22
3-2-3	Interface polarization conversion	22
	Acoustic – Fluid-fluid boundary	22
	Elastic – fluid-solid	25
II	Orientation estimation	27
4	Data sets	29
4-1	Data sets	29
4-2	Real Data	29
4-3	Synthetic data	30
5	Data driven Orientation Determination	33
5-1	Introduction to ROTATE_3C_ANALY	34
5-1-1	Reproduction of ROTATE_3C_ANALY	36
5-2	Development and application of the algorithmic enhancements	37
5-2-1	Simultaneous Estimation	37
5-2-2	Center of Gravity method	38
5-2-3	Least Squares	40
5-2-4	Synthetic data	41
	Full coverage and opposite quadrants	41
	One sided shot coverage	43
5-2-5	Real data	46
5-2-6	Discussion	48
6	Model-based Orientation Determination	51
6-1	Orientation determination	51
6-2	Synthetic data	53
6-2-1	Fluid- Fluid boundary	53
6-2-2	Fluid- Solid boundary	55
6-2-3	Results Synthetic	55
6-3	Real data	56
6-3-1	Acoustic seabed	56
6-3-2	Elastic Seabed	58
6-3-3	Results Real data	59
6-4	Discussion	60
6-4-1	Sensitivity to the knowledge of the seabed elastic properties	60
6-4-2	The results	62

7	Conclusions and Outlook	65
7-1	Conclusions	65
7-2	Outlook	67
	Bibliography	69
A	Orientation Estimation in OMEGA	71
A-1	Orientation estimation in OMEGA	71
A-1-1	Finding the polarization angle	71
A-1-2	Correction of the polarization angle	73
B	Reproducing OMEGA	77
B-1	MATLAB routine reproducing OMEGA	77
B-1-1	MATLAB routine reproducing OMEGA results	77
B-1-2	MATLAB routine applied to synthetic data	79
B-1-3	MATLAB routine applied to real data	80
C	Algorithm flow	83
D	The Center of Gravity Approach	87

List of Figures

1-1	Different orientations of the three component receivers that have to be considered (Gaiser, 1999).	2
1-2	Dots show receiver positions for a seismic survey around a platform (receiver gap). The color code of the receivers shows the estimated inclination of the z- axis after applying <i>OMEGAs</i> orientation estimation.	5
2-1	Rotating the coordinate frame of a right handed coordinate system around the z axis (pointing towards the viewer).	10
2-2	Roll, pitch and yaw axis (x, y and z axis) in a right handed coordinate system. . .	11
2-3	The red coordinate frame is the result of applying different rotation operations to the black coordinate frame. Left: $R_x(\phi = 10^\circ)$ Middle: $R_y(\psi = -10^\circ)R_x(\phi = 10^\circ)$ Right: $R_z(\theta = 90^\circ)R_y(\psi = 10^\circ)R_x(\phi = 10^\circ)$	12
2-4	Rotation in the imaginary plane.	14
3-1	Reflection, transmission and mode conversions of an incident P wave at a fluid-solid interface.	18
3-2	Transmission and reflection coefficients for an incident wave I generated either in medium 1 or medium 2 (after Chapman (2004)).	21
3-3	Discrepancy between Angle of incidence and polarization angle for two acoustic media. Medium 1 (water) has a velocity $v_P = 1.5 \text{ km/h}$ and density $\rho = 1 \text{ kg/m}^3$. Medium 2 (seabed) has the same density but v_P is varied.	23
3-4	2D angle discrepancy for a centered receiver (red dot). The receiver is 90 m below the sea surface. The following properties were used: $v_{P1} = 1.5 \text{ km/s}$, $\rho_1 = 1 \text{ kg/V}$, $v_{P2} = 2 \text{ km/s}$, $v_{S2} = 1.3 \text{ km/s}$, $\rho_2 = 1.3 \text{ kg/V}$	24
4-1	Shot coverage (empty small circles) for the 21 receivers (filled teal circles) in the North Sea data set. The large circles around the receiver position indicate a radial distance of 100 m.	30
4-2	Common receiver gather of the x (and y) component as computed in ANIVEC and displayed in Omega.	31

4-3	Common receiver gather of the z component as computed in ANIVEC and displayed in Omega.	32
5-1	Schematic plot of the orientation determination approaches that are presented in Chapter 5 (right branch) and Chapter 6 (left branch)	33
5-2	Estimation of ϕ_0 based on the ϕ estimation of two reciprocal traces.	35
5-3	Left: ϕ_0 (a) and ψ_0 (b) estimation and comparison in Omega and the Matlab environment. The Matlab-receiver line is shifted for displaying purposes. Right: Difference between the Omega and Matlab.	36
5-4	Exemplary objective functions as used in the simultaneous approach. The value of the objective function varies with the trial angles ϕ and ψ and it is also dependent on the source- receiver pair used as input for the routine. The colorbar represents the summed energy of the x and y component as used in the objective function.	38
5-5	Visualization of the CoG approach.	39
5-6	Superposition (blue,dashed) of a sine (black) and cosine (red). The superposition is a cosine shifted by β	41
5-7	Uniform shot coverage.	42
5-8	Opposite quadrant shot coverage.	42
5-9	Two one sided acquisition setups.	43
5-10	The panels show the seismic traces after the receiver was numerically flattened by rotating the data with the angles approximated with the sequential approach (first row), the sequential approach with pre-rotation (second row) and the Least Squares approach with pre-rotation (third row).	45
5-11	z axis inclinations for 4 different methods applied to the North Sea data set receiver line.	46
5-12	Common receiver gathers as recored by receiver number 10 of the North Sea data set. First row: raw data, second row: ϕ_0 and ψ_0 applied to raw data, data is flattened now, third row: flattened data turned from x,y,z coordinates into r,t,z coordinates.	47
5-13	(a) Visualization of missing reciprocals around y axis (situation before CoG). The x axis however has reciprocal traces to its left and right. (b) After reorienting the receiver orientation we are left with this situation. Now both axis have sufficient reciprocal traces.	48
5-14	Pre- and Post rotation of the z component of the receiver.	49
5-15	Correction of Center of Gravity pre-rotation.	49
6-1	Schematic procedure of the seabed approach.	52
6-2	Determination of ϕ_0 : Trial rotation applied to the data is $\phi = 10^\circ$. See text for explanation of the three panels.	54
6-3	Determination of ψ_0 : Trial rotation applied to the data is $\psi = -10^\circ$	54
6-4	Determination of ϕ_0 : Trial rotation applied to the data is $\phi = 10^\circ$. The boundary region was excluded from the standard deviation calculation.	55
6-5	Determination of ψ_0 : Trial rotation applied to data is $\psi = -10^\circ$. The boundary region was excluded from the standard deviation calculation.	55
6-6	Determination of ϕ_0 : seabed is acoustic with $v_p = 2000$ ms.	56
6-7	Determination of ψ_0 : seabed is acoustic with $v_p = 2000$ ms.	57

6-8	Detail comparison of results for the trace by trace orientation determination with and without the correction term applied.	57
6-9	Determination of ϕ_0 : seabed is elastic with $v_p = 2000$ ms and $v_p/v_s = 3$	58
6-10	Determination of ψ_0 : seabed is elastic with $v_p = 2000$ ms and $v_p/v_s = 3$	58
6-11	Detail comparison of results for the trace by trace orientation determination with and without the correction term applied.	59
6-12	Comparison of the z inclination after ϕ_0 and ψ_0 are determined using ROTATE_3C_ANALY, the acoustic and the elastic seabed approach.	59
6-13	Left: ϕ -distribution determined with the sequential approach. Middle: angle discrepancy for $v_{P2} = 2.2$ km/s and $v_{S2} = 0$. Right: ϕ -distribution after it is corrected for angle discrepancy. From this we find $\phi_0 = 28.7^\circ$	60
6-14	Standard deviation and shot coverage of synthetic experiment.	61
6-15	ϕ_0 as a function of v_{p2} and v_{s2} , which are used to model the angle discrepancy correction.	62
A-1	Multicomponent traces after the application of a rotation around the x axis	73
A-2	The blue coordinate system shows the orientation of the MC receiver axis. The red coordinate system shows the orientation of the receiver axis after minimizing the energy in the x-y-plane. Observe that x'' is parallel to the source receiver vector i.e., x'' has the same inclination as the angle of incidence.	74
A-3	The blue coordinate system shows the orientation of the MC receiver axis. The red coordinate system shows the orientation of the receiver axis after minimizing the energy in the x-y-plane. Observe that x'' is parallel to the source receiver vector i.e., x'' has the same inclination as the angle of incidence.	75
B-1	Left: ϕ_0 estimation and comparison in OMEGA and the MATLAB environment. The MATLAB receiver line is shifted for displaying purposes. Right: Difference between the OMEGA and MATLAB.	78
B-2	Left: ψ_0 estimation and comparison in OMEGA and the MATLAB environment. The MATLAB receiver line is shifted for displaying purposes. Right: Difference between the OMEGA and MATLAB.	78
B-3	Values of ϕ and ψ used for the computation of ϕ_0 and ψ_0 of the most west receiver in Figure B-1 and B-2.	79
B-4	Left: ϕ . Right: ψ	81
B-5	The seismic traces as they change during the application of different rotations in MATLAB routine reproducing ROTATE_3C_ANALY. Time window shows first breaks only. Row 1: original multicomponent common receiver gather as acquired in North Sea data set. Row 2: multicomponent data after $\phi_0 =$ and $\psi_0 =$ were applied. Row 3: multicomponent data after rotation with θ_0 - SRAZ. Now the x-y-data is in radial-transversal- coordinates. Observe that most of the energy is on the radial component.	82
D-1	Center of Gravity method	88

List of Tables

1-1	Table showing visualizing how problematic it is to estimate the orientation and position of different land and seabed acquisition systems. Also the quality of the coupling with the subsurface for every system is given. * = major problems, *** = minor problems, DoF = degrees of freedom = number of possible rotations. . .	3
4-1	Parametrization of the layered, isotropic earth model used to create the synthetic data	30
5-1	Full shot coverage. * indicates that pre-rotated data was used as input.	42
5-2	Opposing quadrant shot coverage. * indicates that pre-rotated data was used as input.	42
5-3	One sided West. * indicates that pre-rotated data was used as input.	44
5-4	One sided North. * indicates that pre-rotated data was used as input.	44
B-1	Orientation estimation for a receiver with an Z axis perpendicular to seabed. Table shows estimated orientation for different shot coverages.	79
B-2	Orientation estimation for a receiver with an Z axis inclined by 72°. Table shows estimated orientation for different shot coverages.	80

Nomenclature

ϕ	Rotation angle around x (trace dependent)[°]
ψ	Rotation angle around y axis (trace dependent)[°]
θ	Rotation angle around z axis (trace dependent)[°]
ϕ_0	Rotation angle around x (global) [°]
ψ_0	Rotation angle around y axis (global) [°]
θ_0	Rotation angle around z axis (global) [°]
Rx, Ry, Rz	Rotation matrices
v	Particle movement vector [$L T^{-1}$]
v_P	P wave velocity [$L T^{-1}$]
v_S	S wave velocity [$L T^{-1}$]
P	Pressure [-]
p	Slowness vector [$L^{-1} T$]
q_α	Normal slowness [$L^{-1} T$]
q_β	Horizontal slowness [$L^{-1} T$]
w	Vector containing Boundary conditions [-]
t	Time [T]
ρ	Density [$M L^{-3}$]
κ	Bulk modulus [$M L^{-1} T^{-2}$]
τ	Transmission coefficient [-]
g_p	Particle movement vector of incident wave [$L T^{-1}$]
h_p	Particle movement vector of transmitted wave [$L T^{-1}$]
$\vec{g}$	Receiver- center of gravity vector [L]
$\vec{z}$	Direction of receivers z component [L]
q_α, q_β	Normal slowness of a P and S wave respectively [$L^{-1} T$]
p_l	Horizontal slowness [$L^{-1} T$]

Chapter 1

Introduction

In seismic surveying a source (eg. a explosion or a airgun) creates a source signal. This signal travels through the subsurface and is reflected, refracted and transmitted once the source signal propagates over a interface separating two media with different rock properties. Depending on the objective of the survey the receiver geometry is chosen. The receiver records the source signal that has been altered by traveling through the subsurface. For every source-receiver pair a seismic trace is recorded. The trace shows the ground movement induced by the source of the respective soure-receiver pair. On the receiver side of seismic experiment one has to decide on the geometry of the receivers, the spatial sampling density and on the type of receiver (OGP, 2011). By using 3 component (3C) receivers instead of 1 component receivers we are able to measure the entire seismic wave field instead of only its vertical component. This means that not only the P waves are recorded but also the S waves (Barkved et al., 2007). The S wave information can for example be used to detect vertical fracture zones, determine lithology and type of pore filling by analyzing the amplitude-variations-with offset (Gaiser, 1999). In land seismics however, the 3C are individually planted and therefore coupling and orientation can be better controlled.

One of the biggest problems in multicomponent seabed seismic is vector infidelity. Vector infidelity is the inability of the multicomponent (MC) receiver to correctly record the incoming wave field. Infidelities in the seismic data are due to component crosstalk, imperfect coupling, gain misbalance and mismatched frequency responses. A fifth problem that can degrade the recorded data comes from the fact that the orientation (i.e., the orientation of the 3D receiver axis) of the receiver is not known exactly (Dellinger et al., 2001).

There are many different systems for recording MC sea bottom seismic data such as ocean bottom cables (OBC) or nodal acquisition systems i.e., single receivers planted on the seabed. These receivers record the particle velocity of the 3D wave field (x-,y-,z- component) and the pressure. I will refer to this as a 3D four-component acquisition system (Gaiser, 1999). Due to the rough sea bottom, currents and limited sight during the deployment the orientation of the receivers is mostly unknown. Even when the orientation of the MC seismic sensor is measured by the acquisition contractor using tiltmeters, accelerometers and compasses the processing center is required to verify it with data-driven methods. Here it has to be mentioned that

the orientation estimation for an OBC is simpler than for a seabed node. This is because one axis of the multicomponent sensor axis is always parallel to the cable direction in OBC system and its orientation is therefore known or at least constrained. For a nodal system all the three orientation angles (i.e., axis directions) are unknown or have to be verified.

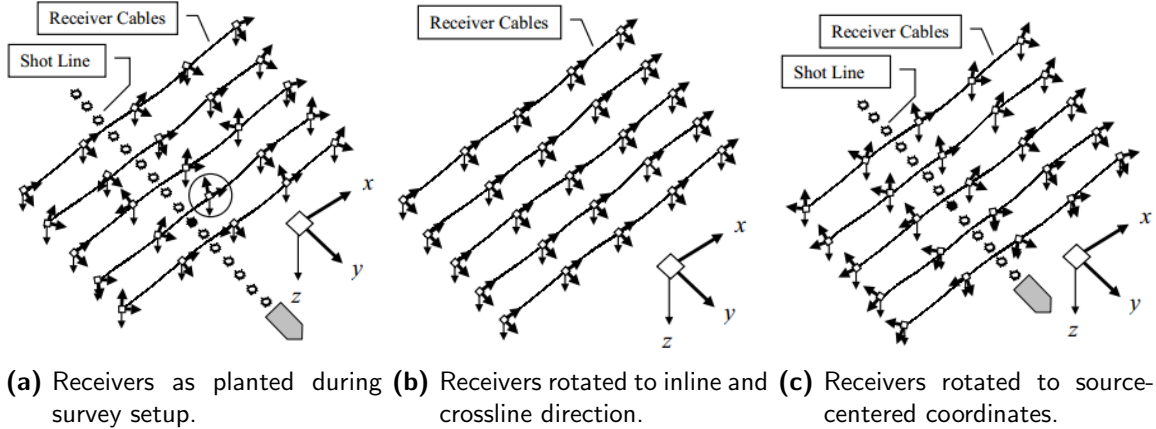


Figure 1-1: Different orientations of the three component receivers that have to be considered (Gaiser, 1999).

Because of the complications that can occur during the setup of the data acquisition we have to assume that the seismic data was recorded with non-gimbaled receivers (eg. OBC is twisted, node is planted upside down). Non-gimbaled in this context means that the receiver axis are not uniformly pointing in the same direction. This non-gimbaled situation is shown in Figure 1-1a. However, if the orientation of each receiver is known we can mathematically rotate the receiver in any desired orientation and correct for the tilt of the receiver. After applying the orientation correction, the recorded data will be orientated as if it was recorded in the desired survey setup. In Figure 1-1b the x- components are all pointing into one direction (North-West). Another possibility would be to rotate the recorded data in a way such that the x component would point along the source- receiver azimuth. In this case the x component becomes the **radial** (r) and the y component becomes the **transverse** (t) component of the wave field. This source centered survey setup is shown in Figure 1-1c. By rotating the x and y component into the r and t coordinate system we separate the P wave from the S wave component propagating in an isotropic elastic medium. After the rotation we expect all the P wave energy to be on the radial component where as all the S wave energy is on the transversal component (Gaiser, 1999).

Table 1-1 shows a general comparison of challenges in land and marine seismic. I will mainly focus on positioning, orientation and coupling of the sensor since these will play a role in the orientation determination. Mugenot (2012) writes that the problem of positioning land receivers -both, cable and autonomous node systems- is no longer a big problem. The use of Differential GPS allows to determine the position of a receiver with an accuracy of ± 1 m. Also the the position of ocean bottom cables (OBC) and autonomous nodes in in marine seismics are mostly well known. Nodal systems are deployed using rovers which allow a positioning of the node with an accuracy of ± 2 m away from the desired location (Bagaini, 2016). About the same accuracy is reached for OBC systems. With Multiwave's near-seabed

	cable system	autonomous node
	land acquisition	
positioning	***	***
orientation [dof]	3	3
coupling	** - *** (soil dependent)	
	seabed acquisition	
positioning	**	**
orientation [dof]	1	3
coupling	* (crossline) - ** (inline)	**

Table 1-1: Table showing visualizing how problematic it is to estimate the orientation and position of different land and seabed acquisition systems. Also the quality of the coupling with the subsurface for every system is given. * = major problems, *** = minor problems, DoF = degrees of freedom = number of possible rotations.

deployment system a cable can be deployed with an accuracy of 1 % of the water depth at the pre-plotted coordinates (Hodge, 2004). The node or cable position can also be determined after the node was deployed. Here methods are applied that are similar to the inversion algorithms that are used in earthquake hypocenter determination in seismology. The position of the receiver is found by measuring travel times of acoustic signals. These methods allow a positioning with an accuracy of magnitude 1 m (eg. Shiobara et al. (1997)). The coupling mechanism for ocean bottom systems and land systems are very different. Where as land systems mostly have spiking coupling (i.e., sensor is pushed in the soil) the seabed systems have gravitational coupling (i.e., sensor is placed on the seabed) (Bagain and Barajas-Olade, 2007). Especially for seabed seismics inconsistent coupling is a major problem. It has been experimentally verified that when cable systems are used, the geophone-ground coupling is often better in the inline direction. This can be intuitively understood because the cable and the way in which the cable is assembled creates a mechanical system which is not radially symmetric. This problem can be compensated for by using filter operations that recover the true particle movement instead of the coupling biased particle movement (eg. Gaiser (1998)). Nodal acquisition systems on the other hand, do not suffer from a inline-crossline-dependent coupling and thus the vector fidelity of these systems is typically higher than for cable based systems (Hokstad et al., 2002). Land systems do not suffer from coupling problems as much as seabed systems do. Since the geophone is directly coupled to the ground via a spike. It will follow the movement of the ground and recover the particle movement more accurate then a loosely deployed node. The biggest coupling issues for land data arises from the properties of the soil. Rather than coupling the receiver with its spike to a loose soil (eg. sand) it is recommended to bury the receiver to increase the coupling (Krohn, 1984).

I will assume that the position of the receiver is correct and that the coupling of the receiver to the ground was sufficient to record the correct particle movement. We will see that these two parameters are essential in estimating the orientation of the receiver. More over I will only investigate the orientation of seabed systems. The orientation determination of a seabed receiver is both, a processing tool as well as a quality controlling (QC) tool. It is obvious that MC data is useless if it is not sufficiently clear how the receiver axis were positioned when the particle movement was recorded. However, if the orientation is known the recorded data of every sensor can be rotated as if it was recorded in another position or source-receiver setup. This enables the processor to use the full S wave information content in the recorded data.

1-1 Objectives of this thesis and main results

The objective of this thesis is to assess the performance of the Schlumberger Omega module ROTATE_3C_ANALY, which is used for orientation estimation, in a variety of acquisition scenarios and to develop and implement some algorithmic improvements. I first develop a MATLAB program and verify that it replicates the results of ROTATE_3C_ANALY. I then observe that ROTATE_3C_ANALY is not sufficiently able to determine the orientation of a seabed sensor if the shot coverage around the receiver of interest is not uniform. Uniform is here defined as an acquisition setup where every trace at the radial distance $\vec{r}(x,y)$ has a reciprocal trace at $\vec{r}(-x,-y)$. I have implemented and applied to synthetic and real data two novel types of algorithmic improvements based on [Bagaini \(2016\)](#). The first, similarly to ROTATE_3C_ANALY, does not assume any knowledge of the seabed properties. It is based on the expected radial symmetry of the difference between arrival direction and polarization angle (geometry-based approach). The second approach makes use of the (approximate) knowledge of the seabeds elastic properties by modeling the seabed response to an incident pressure wave. Both of the novel approaches improve the estimation of the receivers orientation. Especially the model based delivers very good results.

Before I will introduce the existing and new algorithms used for the orientation determination I will review the theoretical background that is needed to understand the different approaches and how they were implemented (Part I). I will first review the mathematical constructs that allow us to describe the rotation of any body in a 3D space. This is necessary to describe the orientation of a receiver and, once the orientation is known, rotate the receiver and the recorded data in the desired direction. In the second part of the theoretical background I describe the conversion processes that occur at the seabed. This will be of big importance for the model based orientation determination approach. In Part II of the thesis I will first review the existing routine as it is implemented in Omega and show that the results obtained with Omega can be retrieved with the Matlab routine. We will use the reproduction of the Omega routine as a benchmark and compare the new routines with it. In [Chapter 5](#) the geometry-based algorithms are introduced. In [Chapter 6](#) the seabed approach is described. In newly developed algorithms are applied to real and synthetic data in the respective Chapter.

1-2 Motivation

Schlumberger's seismic processing package Omega gives good results in recovering the orientation of a receiver in the presence of uniform azimuthal shot illumination of the receiver. However, insufficient shot coverage (eg. because of obstructions) can lead to anomalous estimations. This problem is shown in Figure 1-2. The routine determining the orientation of a receiver in Omega is ROTATE_3C_ANALY.

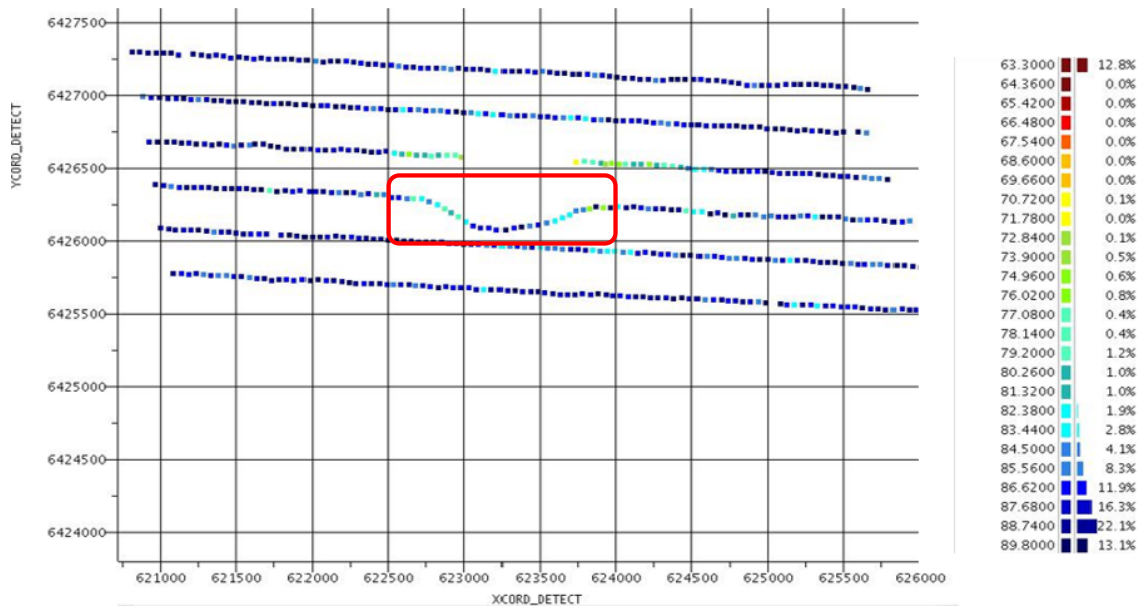


Figure 1-2: Dots show receiver positions for a seismic survey around a platform (receiver gap). The color code of the receivers shows the estimated inclination of the z- axis after applying *OMEGA*s orientation estimation.

The acquisition shown above was done in the North Sea and it is known that the seabed in the area of this acquisition is relatively hard and flat [Bagaini \(2016\)](#). Based on this knowledge it is assumed that the receivers should have a z axis inclination close to 90° (or -90°) since they drop flat on the ground. Observe the gap in the receiver line. This gap is due to a platform, here no shots were fired neither receiver were planted. We can observe that the inclinations for receivers far away from the gap are very close to 90° . These shots have uniform shot coverage. However, in the vicinity of the obstruction uniform shot coverage could not be realised. We can observe that for receivers close to the gap the estimated z axis orientation is about 75° to 80° . These anomalous values are explained by the insufficient shot coverage. We have seen how the z axis orientation is influenced by the x and y axis rotation (Figure 2-3). Following from this we can conclude that ROTATE_3C_ANALY's estimation of ϕ_0 and ψ_0 is not sufficiently accurate to rotate the receiver in a flat position.

Part I

Theoretical background

Chapter 2

Coordinate systems, Orientation and Rotations

2-1 Coordinate systems, Orientation and Rotations

In this chapter I will introduce the conventions for the used coordinate system, clarify the term orientation and present two mathematical constructs used to represent the 3D rotation of a rigid body.

2-1-1 Coordinate Systems

There are two types of three dimensional Cartesian coordinate systems following the x-y-z convention: left-handed and right-handed. The coordinate system used in seismic acquisitions and in the routines used to produce the results in this work is right handed. This defines a axis system with an x axis pointing to the right, an y axis pointing towards the viewer and an z axis pointing downwards. In addition to this, a clockwise rotation of any of these rotations (when the axis points to the viewer) is defined by a positive value. Anti-clockwise rotations are described by negative rotation angles.

2-1-2 Orientation

I will refer to *orientation* as the axis directions of one orthonormal basis with respect to another one. The coordinate systems defined by these bases have the same origin.

Figure 2-1 shows two bases formed by (x,y,z) and (x',y',z') . The (x,y,z) coordinate system was rotated around the z axis by the angle θ resulting in the (x',y',z') coordinate system. A rotation around the z axis is also known as a yaw rotation. A yaw rotation changes the pointing direction (i.e., direction of x axis also known as roll axis). Figure 2-2 shows that there are another two possible rotations that change the orientation of an object, namely a x axis rotation or roll rotation and a y axis rotation or pitch rotation. Any of these two rotations

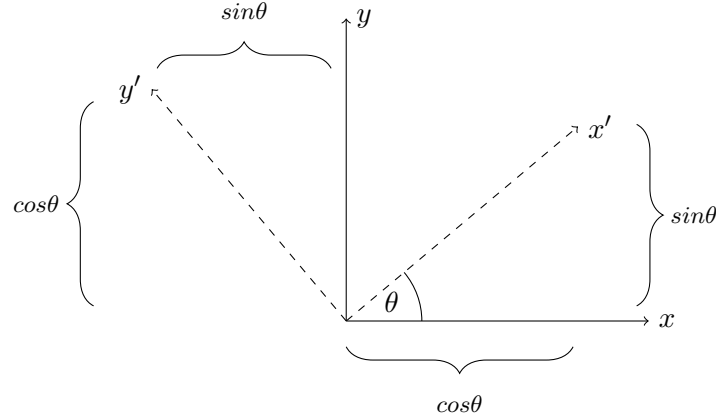


Figure 2-1: Rotating the coordinate frame of a right handed coordinate system around the z axis (pointing towards the viewer).

will tilt the z or yaw axis and thus tilt the orientation of the previously horizontal x-y- plane into a non-horizontal position. Coming back to the situation shown in Figure 2-1, we can now observe that only a yaw rotation was applied and the pitch and roll stay unchanged. We further observe that the x axis has now the direction of $(\cos\theta, \sin\theta)$ and the y axis direction is given by $(-\sin\theta, \cos\theta)$. From this it can be seen that the following relations maps every point (x, y) to (x', y') :

$$\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \cos(\theta) - y \sin(\theta) \\ x \sin(\theta) + y \cos(\theta) \end{bmatrix} \quad (2-1)$$

Equation 2-1 describes an anticlockwise rotation. A clockwise rotation is given by Equation 2-2. In the following I will use clockwise rotations as the rotation direction of choice.

$$\begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x' \\ y' \end{bmatrix} \quad (2-2)$$

In the context of sensor orientations we are mostly interested in the roll and pitch of the receiver, since these two have to be determined in order to mathematically rotate the recorded data as if they were recorded with a receiver laying flat on the seabed. A flat receiver is characterised by a z axis perpendicular to the seabed and a x and y axis parallel to the seabed. The orientation of a flat receiver will be the reference coordinate system used for a tilted receiver, this means that roll, pitch and yaw of the tilted receiver are described in respect to a flat receiver. The rotation around the yaw rotation of a flat receiver corresponds to turning the data into a uniform source-receiver geometry (eg. source centered or x axis pointing to desired azimuth). From this point on I will no longer use roll, pitch and yaw but ϕ , ψ and θ to describe the rotation around the x, y and z axis respectively. In the next section we will see that ϕ , ψ and θ correspond to the so called Euler angles.

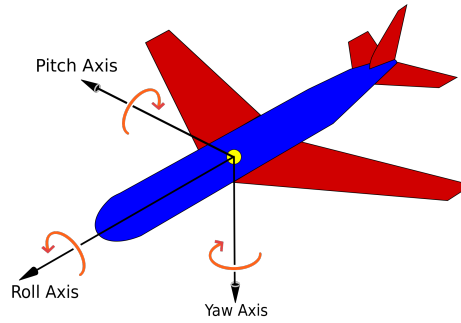


Figure 2-2: Roll, pitch and yaw axis (x, y and z axis) in a right handed coordinate system.

2-1-3 Euler Rotation Theorem

Euler's Rotation theorem states that any displacement of a rigid body in a three dimensional space where one point of the body stays fixed can be expressed as a single rotation around this point. It also states that a sequence of two rotations or more is also a rotation.

In the prior section a more intuitive approach was presented by introducing the three rotation angles ϕ , ψ and θ . In fact the orientation of a body is fully defined by these three angles (Palais et al., 2009). These three angles are applied to a reference coordinate frame as a sequence of rotation operations. The rotation operators are the rotation matrices $\mathbf{R}_x(\phi)$, $\mathbf{R}_y(\psi)$ and $\mathbf{R}_z(\theta)$.

Rotation convention

Any rotation of a rigid 3 dimensional body can be expressed in terms of three single rotations. Each rotation acts around one of the tree coordinate axis.

$$\underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{\text{rotated data}} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\phi) & \sin(\phi) \\ 0 & -\sin(\phi) & \cos(\phi) \end{bmatrix}}_{\mathbf{R}_x(\phi)} \underbrace{\begin{bmatrix} \cos(\psi) & 0 & -\sin(\psi) \\ 0 & 1 & 0 \\ \sin(\psi) & 0 & \cos(\psi) \end{bmatrix}}_{\mathbf{R}_y(\psi)} \underbrace{\begin{bmatrix} \cos(\theta) & \sin(\theta) & 0 \\ -\sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\mathbf{R}_z(\theta)} \underbrace{\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix}}_{\text{unrotated data}} \quad (2-3)$$

Here R_z , R_y and R_x denote the rotations around the z-, y- and x-coordinate axis respectively. θ , ψ and ϕ are the rotation angles that are associated with rotations around the axis. The three rotation matrices are applied in a right to left order (i.e., first R_z is applied to the (x', y', z') vector). Observe that the rotation matrix R_z contains the rotation relation as given by equation 2-2. Figure 2-3 visualizes the sequential application of the three matrices R_z , R_y and R_x .

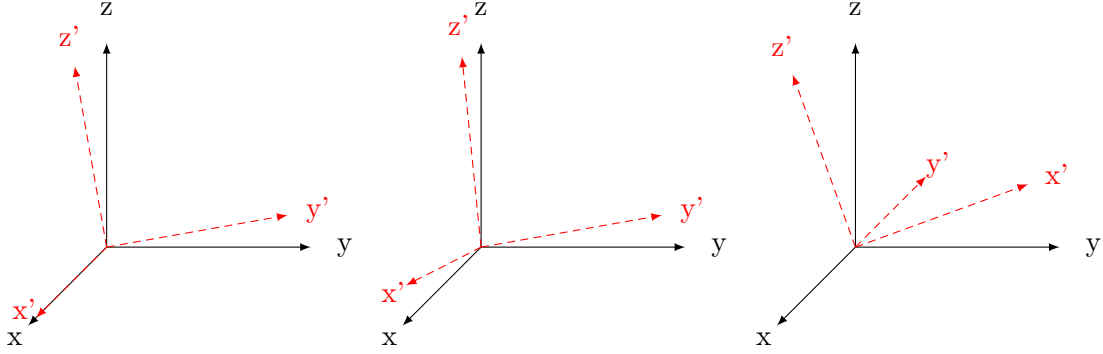


Figure 2-3: The red coordinate frame is the result of applying different rotation operations to the black coordinate frame. Left: $R_x(\phi = 10^\circ)$ Middle: $R_y(\psi = -10^\circ)R_x(\phi = 10^\circ)$ Right: $R_z(\theta = 90^\circ)R_y(\psi = 10^\circ)R_x(\phi = 10^\circ)$.

The inverse of the rotation matrices is simply obtained by applying the opposite of the rotation angle. The inverse of a rotation matrix corresponds to anticlockwise rotation. It can easily be shown that the following relation is valid:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(-\phi) & \sin(-\phi) \\ 0 & -\sin(-\phi) & \cos(-\phi) \end{bmatrix} = R_x^{-1} \quad (2-4)$$

The un-rotated data can be retrieved by applying the inverse of the rotation matrices in the the inverse order:

$$R_z(-\theta) R_y(-\psi) R_x(-\phi) \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{\text{rotated data}} = \underbrace{\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix}}_{\text{unrotated data}} \quad (2-5)$$

Euler-Rodrigues-formula

In the prior section we showed how to rotate a rigid body by applying three rotation matrices sequentially. The Euler theorem states that a sequence of rotations is also a rotation and the Euler-Rodrigues-formula allows to formulate such a rotation. Instead of the three Euler angles ϕ , ψ and θ a 3D rotation with angle Θ around the rotation axis given by the unit vector $\vec{k}$ can also be expressed with the Euler parameters a , b , c and d . The rotation of the point $\vec{x}$ to $\vec{x}'$ is given by

$$\vec{x}' = \begin{bmatrix} a^2 + b^2 - c^2 - d^2 & 2(bc - ad) & 2(bd + ac) \\ 2(bc + ad) & a^2 - b^2 + c^2 - d^2 & 2(cd - ab) \\ 2(bd - ac) & 2(cd + ab) & a^2 - b^2 - c^2 + d^2 \end{bmatrix} \vec{x} \quad (2-6)$$

The Euler parameters are given by

$$a = \cos\left(\frac{\Theta}{2}\right) \quad (2-7)$$

$$b = k_x \sin\left(\frac{\Theta}{2}\right) \quad (2-8)$$

$$c = k_y \sin\left(\frac{\Theta}{2}\right) \quad (2-9)$$

$$d = k_z \sin\left(\frac{\Theta}{2}\right) \quad (2-10)$$

The Euler parameters can be used to formulate the Euler-Rodrigues-formula in a more compact vector notation, where $\vec{\omega} = (b, c, d)$.

$$\vec{x}' = \vec{x} + 2a(\vec{\omega} \times \vec{x}) + 2(\vec{\omega} \times (\vec{\omega} \times \vec{x})) \quad (2-11)$$

2-1-4 Quaternions

Quaternions are an extension to the complex numbers and were first introduced by Hamilton. More precisely, a quaternion is a 4D complex numbers of the form

$$q = a + ib + jc + kd = \begin{bmatrix} r \\ \vec{v} \end{bmatrix} \quad (2-12)$$

where a, b, c, d are real numbers and i, j, k are defined as square root of -1 and define a coordinate system of imaginary axes. We can also represent the quaternion in terms of its real part a and the imaginary part $ib + jc + kd$. The real part is also referred to as scalar part (r), analogously the imaginary part becomes the vector part ($\vec{v}$) of q . The vector part $\vec{v}$ is a column vector containing containing (b,c,d). The multiplication of two quaternions $q_0 = (r_0, \vec{v}_0)$ and $q_1 = (r_1, \vec{v}_1)$ is defined as

$$q_1 q_0 = \begin{bmatrix} r_1 \\ \vec{v}_1 \end{bmatrix} \begin{bmatrix} r_0 \\ \vec{v}_0 \end{bmatrix} = \begin{bmatrix} r_1 r_0 - \vec{v}_1 \cdot \vec{v}_0 \\ r_1 \vec{v}_0 + r_0 \vec{v}_1 + \vec{v}_1 \times \vec{v}_0 \end{bmatrix} \quad (2-13)$$

Observe that $q_1 q_0 \neq q_0 q_1$ because of the non-commutative properties of the cross product in in vector part. Another important property of the quaternion is the definition of the inverse of a quaternion q . To define the q^{-1} we will have to define the norm $\|q\|$ and the adjoint $\bar{q}$ of q first.

$$\bar{q} = \begin{bmatrix} a \\ -\vec{v} \end{bmatrix} \quad (2-14)$$

$$\|q\| = \sqrt{a^2 + b^2 + c^2 + d^2} \quad (2-15)$$

$$q^{-1} = \frac{\bar{q}}{\|q\|} \quad (2-16)$$

Quaternion rotation

In a first step I will show how complex numbers can be used to perform rotations. Let us take a general complex number given by $x + iy$ and multiply it with a unit complex number given by $\cos(\theta) + i \sin(\theta)$.

$$(x + i y) (\cos(\theta) + i \sin(\theta)) = \underbrace{(x \cos(\theta) - y \sin(\theta))}_{\text{Re}} + i \underbrace{(x \sin(\theta) + y \cos(\theta))}_{\text{Im}} \quad (2-17)$$

In the case of $x + i y = 1$ the multiplication with $\cos(\theta) + i \sin(\theta)$ is equal to a rotation with angle θ . In fact, we have seen this kind of a 2D rotation already in a very similar form in Equation 2-1. Observe that the rotation in this case was performed by simply multiplying two (complex) numbers instead of performing a matrix vector multiplication. The quaternion multiplication is computationally less expensive which makes it an interesting option when a lot of (data) points have to be rotated. The 2D complex number can now be extended to a 4D complex number by using the definition of a quaternion. The extension to a 4D complex number is equivalent to adding two additional rotation axes. The rotation of a point $\vec{z}$ defined in a global coordinate system with a quaternion q is defined as (Diebel, 2006):

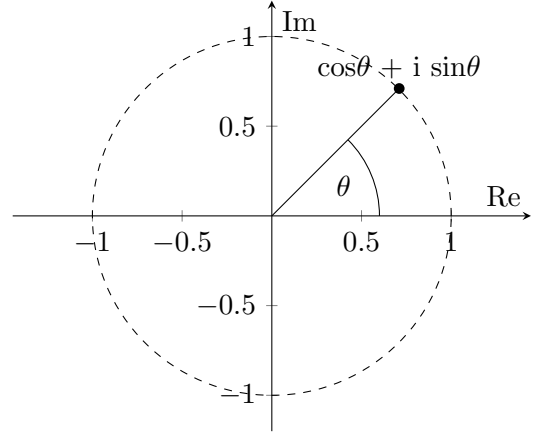


Figure 2-4: Rotation in the imaginary plane.

$$\vec{p}' = \begin{bmatrix} 0 \\ \vec{z} \end{bmatrix} = q \begin{bmatrix} 0 \\ \vec{z} \end{bmatrix} q^{-1} = q p q^{-1} \quad (2-18)$$

Where $\vec{v}^t$ denotes the transposed imaginary part of q_1 . Observe that the point $\vec{z}$ is treated as a quaternion by adding a zero to as the real part of $\vec{z}$. Equation 2-18 is found to be equal to Equation 2-19 after all the quaternion multiplications are carried out.

$$\vec{p}' = \vec{p} + 2a(\vec{z} \times \vec{v}) + \vec{v} \times (\vec{v} \times \vec{z}) \quad (2-19)$$

This equation has the properties and form as the Euler-Rodrigues- formula (Equation 2-11). This proves that quaternions rotations are proper 3D rotations (v Verth, 2013).

2-1-5 The relationship between Quaternions and Euler angles

In order to use quaternions and Euler rotations side-by-side we need to be able to convert them into each other. I have shown earlier that the rotation of a point $p(x, y, z)$ with the angle α around the rotation axis A given by the vector (x_r, y_r, z_r) can also be performed by multiplying the quaternion q with p . The quaternion q can also be written in the form of a unit quaternion or versor (i.e., a 4D complex number).

$$q = \cos\left(\frac{\alpha}{2}\right) + (x_r i + y_r j + z_r k) \sin\left(\frac{\alpha}{2}\right) \quad (2-20)$$

The same quaternion can be written in the Hamilton form.

$$q = e_0 + e_1 i + e_2 j + e_3 k \quad (2-21)$$

We can now use these two ways of writing the quaternion q to identify the e pre-factors.

$$\begin{aligned} e_0 &= \cos\left(\frac{\alpha}{2}\right) \\ e_1 &= x_r \sin\left(\frac{\alpha}{2}\right) \\ e_2 &= y_r \sin\left(\frac{\alpha}{2}\right) \\ e_3 &= z_r \sin\left(\frac{\alpha}{2}\right) \end{aligned}$$

Now the conversion from quaternion rotation to Euler angles following the z - y - x rotation convention is given by the following expressions ([Bermer, 2008](#)).

$$\phi = \text{atan2}((e_2 e_3 + e_0 e_1), (0.5 - (e_1^2 + e_2^2))) \quad (2-22)$$

$$\psi = \text{asin}(-2(e_1 e_3 - e_0 e_2)) \quad (2-23)$$

$$\theta = \text{atan2}((e_1 e_2 + e_0 e_3), (0.5 - (e_2^2 + e_3^2))) \quad (2-24)$$

Where atan2 is defined as following:

$$\frac{a}{b} = \tan(\gamma) \quad (2-25)$$

$$\gamma = \text{atan2}(a, b) \quad (2-26)$$

The atan2 is very similar to the atan . The reason why atan2 uses two input arguments rather than one is to be able to find the correct quadrant of γ . The arctan2 is fully defined for γ in the interval $[-\pi, \pi]$, whereas the $\tan$ is only defined for γ in the interval $[-\pi/2, \pi/2]$.

Chapter 3

Wave propagation and polarization

3-1 Wave propagation and polarization

In this section I will review the most fundamental concepts of wave propagation in a marine environment that will be needed to introduce and explain the assumptions and different approaches to determine the orientation of a sensor.

3-1-1 P and S waves in a marine environment

When a seismic source (typically an airgun) is activated in marine seismic survey, a pressure wave is generated. This pressure wave propagates as a P wave. The P wave has a back and forth particle movement in the direction of propagation. Since shear waves can not propagate through liquid media there is no shear component in the source signal. However, S waves can be generated when the source P wave hits the sea bottom or any other reflective, elastic surface. The S wave has a particle movement that is vertical (SV wave) and horizontal (SH wave) to the direction of propagation. The S wave generation process is called mode conversion and can be described by the Zoeppritz Equations (Zoeppritz, 1919). The four Zoeppritz Equations connect the amplitudes of the generated waves with the angle of incidence and the medium properties (v_P, v_S, ρ) of the two homogenous halfspaces in welded contact.

$$\begin{bmatrix} R_P \\ R_S \\ T_P \\ T_S \end{bmatrix} = \begin{bmatrix} -\sin(\theta_1) & -\cos(\phi_1) & \sin(\theta_2) & \cos(\phi_2) \\ \cos(\theta_1) & -\sin(\theta_1) & \cos(\theta_2) & -\sin(\phi_2) \\ \sin(2\theta_1) & \frac{v_{P1}}{v_{S1}} \cos(2\phi_1) & \frac{\rho_2 v_{S2}^2 v_{P1}}{\rho_1 v_{S1}^2 v_{P2}} \cos(2\phi_1) & \frac{\rho_2 v_{S2} v_{P1}}{\rho_1 v_{S1}^2} \cos(2\phi_2) \\ -\cos(2\phi_1) & \frac{v_{S1}}{v_{P1}} \sin(2\phi_1) & \frac{\rho_2 v_{P2}}{\rho_1 v_{P1}} \cos(2\phi_2) & \frac{\rho_2 v_{S2}}{\rho_1 v_{P1}} \sin(2\phi_2) \end{bmatrix}^{-1} \begin{bmatrix} \sin(\theta_1) \\ \cos(\theta_1) \\ \sin(2\theta_1) \\ \cos(2\phi_1) \end{bmatrix} \quad (3-1)$$

Where R and T are the reflection and transmission coefficient of the incident wave for either a P or S wave. Despite the difference in particle movement direction both, the P and S wave, are linearly polarized events (Bormann et al., 2008). Linear polarization is a displacement

property in which the particle motion of the seismic wave oscillates in one direction and traces out a line (Gaiser, 1999). One could also think of elliptical polarized events such as surface waves which oscillate in the horizontal and vertical direction at the same time. The properties of linear polarizations will be used to rotate the recorded x and y data of a MC sensor to its radial and transversal component for a source- centered acquisition.

3-1-2 The seismic signal

The essentials of MC receivers were reviewed earlier. I will shortly describe what we expect to see on the recorded seismograms and what can be deduced from them. Figure 3-1 shows the mode conversion processes at a reflective interface between two isotropic, elastic media. We can observe how the incident P wave is reflected and converted into an S wave with an SV component and a transmitted P wave. The **angle of incidence** $\theta_{inc} = \theta_{refl}$ at the receiver depends on radial offset and the vertical distance between source and receiver. We expect the MC receiver to record the ground movement and the pressure. The recorded pressure comes from the incident and reflected P wave. The recorded ground movement is induced by the conversion of the incident P wave to the transmitted S and P wave (Barkved et al., 2007). The conversion induces both, a vertical and a horizontal (longitudinal) particle movement denoted as v_n and v_l . These particle movements are detected by the sensor. The **polarization angle** with respect to the normal of the seabed is now defined as following:

$$\theta_{pol} = \text{atan} \left(\frac{v_n}{v_l} \right) \quad (3-2)$$

We can also observe that $\theta_{inc} \neq \theta_{pol}$. This observation is essential and will be a key issue in the orientation determination algorithms that will be introduced. I will derive the expressions for the particle movement of the P wave and the converted wave and the resulting angles in the next section.

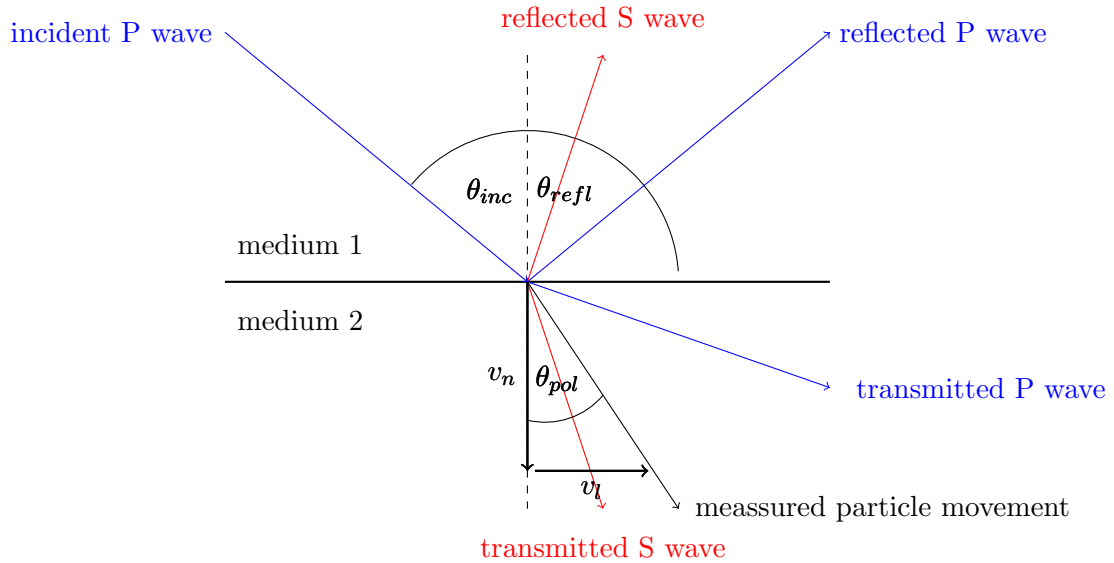


Figure 3-1: Reflection, transmission and mode conversions of an incident P wave at a fluid-solid interface.

3-2 Properties of the seabed

Above I have described the mode conversion that occurs at the seabed. I also defined the polarization angle and the angle of incidence. In the following I will derive expressions that describe the particle movement of the incident ray and the angle of incidence. Once the particle movement is known we are able to infer the propagation angle. In Chapter 6 we will use the difference of the propagation angle (deduced from recorded data) and the angle of incidence to improve the orientation estimation of a 3C receiver. I will introduce the equations needed to describe the elastic and acoustic behavior of a medium. Most of the theory explained in the following is based on [Chapman \(2004\)](#).

3-2-1 Boundary conditions

The boundary conditions (BC) set the conditions that the solution of the elastic or acoustic wave equation must satisfy at the interface of two media. In the context of orientation determination, the interface of interest is the water- seabed transition. We can treat the seabed as either acoustic or elastic which will lead to different boundary conditions.

Acoustic boundary conditions

The normal particle velocity $v_n = \hat{n} \cdot \vec{v}$ and P pressure are continuous. The longitudinal particle velocity v_l is discontinuous. v is a vector containing the particle velocities in x, y and z direction. $\hat{n}$ denotes the normal of the seabed. The continuous components are collected in the vector 2x1 vector w .

$$w = \begin{bmatrix} v_n \\ P \end{bmatrix} \quad (3-3)$$

The acoustic boundary condition holds true for a fluid-fluid interface.

Elastic boundary conditions

The elastic boundary conditions for the transition from one elastic to another elastic solid medium require the continuity of the particle movement $\vec{v}$ (i.e., particle movement is entirely preserved) and the normal traction $t_n = \hat{n} \cdot t$. The 6x1 vector w holds the boundary conditions.

$$w = \begin{bmatrix} \vec{v} \\ t_n \end{bmatrix} \quad (3-4)$$

3-2-2 Reflectivity and Transmission

We will now use the continuity of the boundary conditions to derive the reflection/transmission that connects the incident and generated waves. Further we will make use of the asymptotic ray theory to describe the propagation of the seismic wave. Even though the Zoeppritz equations as already introduced in Equation 3-1 I will derive the reflection and transmission coefficient in the following. This is done to show that the transmission coefficient can be complex which will rule out certain shot-receiver configurations for the use in the

seabed orientation approach. More over I will focus on the acoustic transmission coefficients. We will see in Chapter 6 that the acoustic approximation of the seabed is a good enough approximation to describe the physical behavior of the seabed as it is needed in the seabed approach.

Acoustic Transmission coefficients

The equations of motion and constitutive relation for acoustic waves are given by

$$\frac{\delta v}{\delta t} = -\frac{1}{\rho} \nabla P + \frac{1}{\rho} f \quad (3-5)$$

$$\frac{\delta P}{\delta t} = -\kappa \nabla v \quad (3-6)$$

We can find that these equations simplify to the following expressions once the ansatz of the ray is substituted in the equation of motion and the constitutive relation. κ is the bulk modulus which describes a materials resistance to uniform compression. Further the source term f is dropped.

$$\rho v = P p \quad (3-7)$$

$$P = \kappa v \quad (3-8)$$

From these four equations we will now only consider the two equations that fulfill the boundary conditions for acoustic interfaces. As a consequence we drop the terms of the longitudinal particle movement because it is not continuous. Equation 3-7 and 3-8 can be rewritten as (Chapman, 2004):

$$A w = \begin{bmatrix} 0 & -q_\alpha^2/\rho \\ -\rho & 0 \end{bmatrix} w = p_n w \quad (3-9)$$

Here q_α denotes the normal slowness. The horizontal slowness is denoted as p_l . The normal and horizontal slowness can be related via the following expression:

$$q_n = \sqrt{1/n^2 - p_l^2} \quad (3-10)$$

Here n stands for the wave type and its velocity. If $n = \alpha$ a P wave with the velocity α is considered. A $n = \beta$ symbolizes a S wave with the velocity β .

We can find that Equation 3-9 can also be written as:

$$A W = W p_n \quad (3-11)$$

Where W contains the eigenvectors of w and p_n is a diagonal matrix of eigenvalues. In the acoustic case we collect the transmission coefficients in the 2x2 matrix $\tau_{i,j}$. Where j indicates the origin of the incident wave (medium 1 or 2) and i indicates the medium of the generated wave. Now the transmission coefficient can be expressed by the following matrix equation which has to fulfill the boundary conditions w at the interface between medium 1 and 2.

$$W_1 \begin{bmatrix} \tau_{1,1} & \tau_{1,2} \\ 1 & 0 \end{bmatrix} = w = W_2 \begin{bmatrix} 0 & 1 \\ \tau_{2,1} & \tau_{2,2} \end{bmatrix} \quad (3-12)$$

Where W_1 and W_2 are the eigenvectors of medium 1 and 2 at the interface. Using Equation 3-11 and eigenvectors of A being equal to

$$w = \begin{bmatrix} \pm q_\alpha \\ -\rho \end{bmatrix} \quad (3-13)$$

We can now solve Equation 3-12 for all 4 components of τ which will describe the behavior of an incident and the generated rays at an interface. We find the following transmissions coefficients:

$$\tau_{1,1} = -\tau_{2,2} = \frac{\rho_2 q_{\alpha 1} - \rho_1 q_{\alpha 2}}{\rho_2 q_{\alpha 1} + \rho_1 q_{\alpha 2}} \quad (3-14)$$

$$\tau_{1,2} = -\tau_{2,1} = \frac{2\sqrt{\rho_2 q_{\alpha 1} \rho_1 q_{\alpha 2}}}{\rho_2 q_{\alpha 1} + \rho_1 q_{\alpha 2}} \quad (3-15)$$

Figure 3-2 shows the relation between the incident ray I and the generated rays which annotated as $\tau_{i,j}$ where i and j denote the medium in which the source is located and the medium in which the generated wave is traveling.

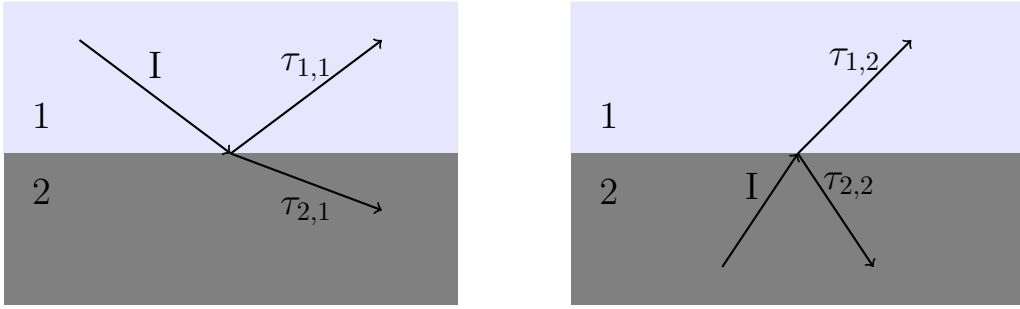


Figure 3-2: Transmission and reflection coefficients for an incident wave I generated either in medium 1 or medium 2 (after Chapman (2004)).

We can observe an interesting property of the transmissions coefficients.

$$\tau_{1,1}^2 + \tau_{2,1}^2 = 1 \quad (3-16)$$

This will be true for real valued normal slownesses $q_{\alpha i}$. However, this is not necessarily the case. If we consider a ray with an incident angle equal to the critical angle, the generated ray will be refracted and will have a vertical slowness equal to zero (i.e., the wave travels parallel to the interface). If the incident angle becomes larger than the critical angle the transmitted slowness becomes imaginary and the transmission coefficient becomes complex valued. This can be shown if we use the definition of the transmitted, vertical slowness $p_{z,trans}$:

$$p_{z,trans} = -\sqrt{\frac{1}{c^2} - p_x^2} = -\sqrt{\frac{1}{c^2} - \underbrace{\frac{\sin^2 \theta_{inc}}{c^2}}_{> \frac{1}{c^2}}} = -i\sqrt{\frac{\sin^2 \theta_{inc}}{c^2} - \frac{1}{c^2}} \quad (3-17)$$

The complex transmission coefficient can now be written as

$$\tau_{1,1} = \frac{\rho_2 q_{\alpha 1} - i\rho_1 |q_{\alpha 2}|}{\rho_2 q_{\alpha 1} + i\rho_1 |q_{\alpha 2}|} = e^{-2i\phi} \quad (3-18)$$

where

$$\tan\phi = \frac{\rho_1 |q_{\alpha 2}|}{\rho_2 q_{\alpha 1}} \quad (3-19)$$

Elastic Transmission coefficients

For elastic waves at an interface we can use the same approach to compute the transmission coefficients with the difference that an incident elastic ray will not only create one reflected and one transmitted ray but three reflected and three transmitted rays, if the interface is between two anisotropic media. The incident ray will convert into a P, SH and SV ray in both media. This conversion results in a 6x6 τ matrix as instead of 2x2 τ for the acoustic case. The derivation of the elastic transmission coefficient is lengthy and needs a lot of notation. Since it is not strictly needed for the content of this thesis I will not derive it here. The elastic transmission coefficients at a **fluid-solid interface** as we encounter it at the water-seabed interface is somewhat simpler to derive than the elastic-elastic interface. This is because in a fluid only a acoustic wave is possible (because there is no shear) which will convert into a reflected acoustic wave and three transmitted waves. For this case the elastic boundary conditions (Equation 3-4) simplify and require the continuity of the normal particle movement and the normal traction. The transverse traction (t_{l1} and t_{l2}) is equal to zero.

$$w = \begin{bmatrix} v_n \\ 0 \ (t_{l1}) \\ 0 \ (t_{l2}) \\ t_n \end{bmatrix} \quad (3-20)$$

3-2-3 Interface polarization conversion

The derived transmission coefficients can now be used to derive the particle movement in the second medium after an incident wave from medium one was converted at the interface.

Acoustic – Fluid-fluid boundary

The solution of w on the interface can be split in two components: the wave traveling in medium 1 and 2. The incident wave and the reflected wave are traveling in medium 1, the transmitted wave travels in medium 2. The sum of the incident and reflected wave gives the following boundary term:

$$\begin{bmatrix} v_l \\ v_n \\ P \end{bmatrix} = \frac{1}{\sqrt{2\rho_1 q_{\alpha,1}}} \begin{bmatrix} p \\ -q_{\alpha,1} \\ -\rho_1 \end{bmatrix} + \tau_{11} \frac{1}{\sqrt{2\rho_1 q_{\alpha,1}}} \begin{bmatrix} p \\ q_{\alpha,1} \\ -\rho_1 \end{bmatrix} \quad (3-21)$$

The transmitted wave gives the following boundary term:

$$\begin{bmatrix} v_l \\ v_n \\ P \end{bmatrix} = \tau_{21} \frac{1}{\sqrt{2\rho_2 q_{\alpha 2}}} \begin{bmatrix} p \\ -q_{\alpha 2} \\ -\rho_2 \end{bmatrix} \quad (3-22)$$

From these expressions we can find the polarizations of the incident wave and the transmitted wave. Both polarizations are described by the particle movements in either the first or second medium which are fixed by the boundary terms. The incident wave has the polarization

$$g_p = \begin{bmatrix} v_l \\ v_n \end{bmatrix} = \frac{1}{\sqrt{2\alpha_1 q_{\alpha_1}}} \begin{bmatrix} p \\ -q_{\alpha_1} \end{bmatrix} \quad (3-23)$$

The transmitted wave has the polarization

$$h_p = \begin{bmatrix} v_l \\ v_n \end{bmatrix} = \tau_{21} \frac{1}{\sqrt{2\alpha_2 q_{\alpha_2}}} \begin{bmatrix} p \\ -q_{\alpha_2} \end{bmatrix} \quad (3-24)$$

From the particle movement we are able to find the propagation direction of the wave since the particle movement for every considered wave type is known. The propagation direction of the incident wave will now be referred to as incidence angle θ_g where as the propagation direction of the transmitted wave will be referred to as polarization angle θ_h . The respective angles can be found by taking the arctan of the horizontal and vertical slowness as they appear in the boundary terms. Compare Equation 3-2. We can find that these two angles are not the same. The difference between these two angles is a function of the vertical and horizontal slowness and by this the difference in polarization angle and angle of incidence depends on the offset between shot and receiver position. I will refer to this difference as **angle discrepancy**.

$$\tan(\theta_g - \theta_h) = p \frac{q_{\alpha_1} - q_{\alpha_2}}{p^2 + q_{\alpha_1} q_{\alpha_2}} \quad (3-25)$$

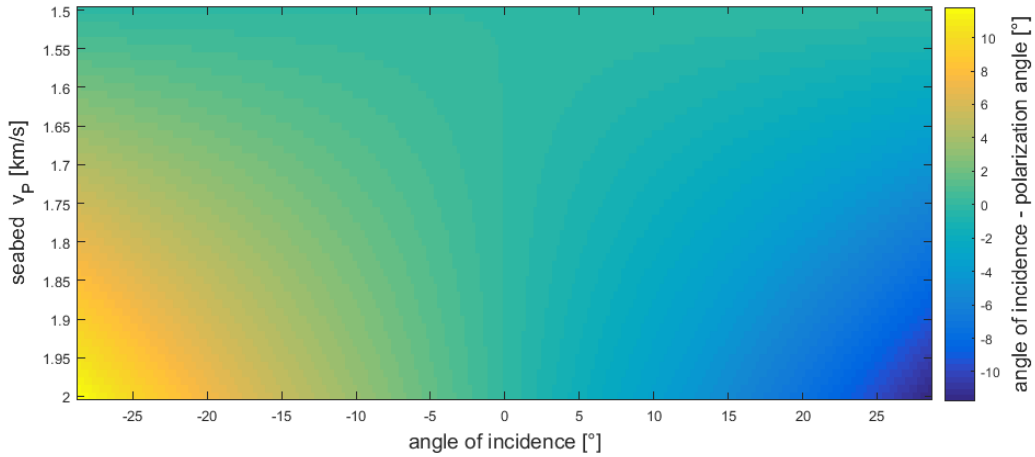


Figure 3-3: Discrepancy between Angle of incidence and polarization angle for two acoustic media. Medium 1 (water) has a velocity $v_P = 1.5 \text{ km/h}$ and density $\rho = 1 \text{ kg/m}^3$. Medium 2 (seabed) has the same density but v_P is varied.

Figure 3-3 shows the discrepancy between the angle of incidence and the polarization angle as a function of the seabed velocity. If the seabed velocity is equal to the water velocity the angle discrepancy is zero (upper edge in Figure 3-3). This is intuitive because in this situation no reflection will occur since the two acoustic media are the same. But whenever the velocity of the seabed is larger there will be an angle discrepancy which is a function

of the difference in v_p and the angle of incidence. Observe that in the case of an angle of incidence equal to zero the angle discrepancy will always be zero. Or in other words: if the source is located above the receiver, the polarization angle deduced from the recorded particle movements is equal to the angle of incidence. We can also observe that for negative angles of incidence the angle discrepancy is positive. This means that the polarization angle overestimates the angle of incidence. In general we can find that the bigger the difference between the P wave velocities of the seabed and the water is, the bigger will be the difference between the polarization angle and the incident angle. These observations match the results of Li and Yuan (1999).

Figure 3-3 with an algorithm written by Chapman (2004). In the following Chapters I will be using a modified version of this algorithm. The Chapman algorithm gives the angle discrepancy as a function of angle of incidence and velocity. This can also be understood as computing the angle discrepancy for a line of sources above a receiver as a function of the seabed velocity. In order to make use of the algorithm of Chapman it is necessary to consider a 2D source coverage rather than a 1D line coverage. The results for the 2D angle discrepancy error for an elastic and acoustic seabed is shown in Figure 3-4. The 2D discrepancy distribution shows the difference between the polarization angle and incident angle for one set of v_{P1} , ρ_1 and v_{P2} , v_{S2} , ρ_2 values. Observe the symmetry around the x axis. Also note that values outside of the circle corresponds to shot positions for which the reflectivity coefficient is complex and thus the angle discrepancy can not be computed. The results from the original Chapman algorithm would correspond to a vertical profile along the y axis going through the source position.

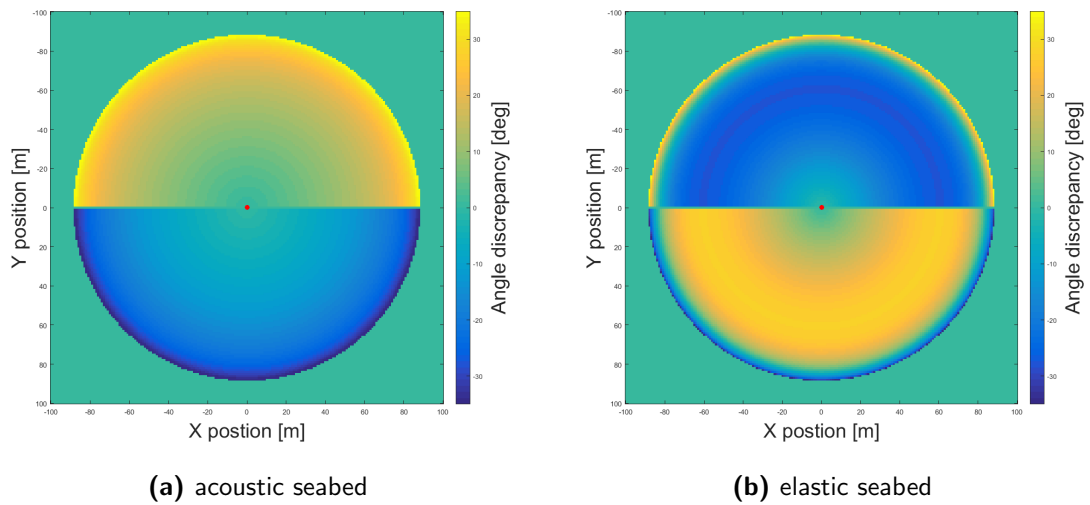


Figure 3-4: 2D angle discrepancy for a centered receiver (red dot). The receiver is 90 m below the sea surface. The following properties were used: $v_{P1} = 1.5$ km/s, $\rho_1 = 1$ kg/V, $v_{P2} = 2$ km/s, $v_{S2} = 1.3$ km/s, $\rho_2 = 1.3$ kg/V.

Elastic – fluid-solid

In the elastic case the resulting surface polarization depends on the incident wave. Or in other words: the seabed will behave differently if it is hit by an S or P wave. However, I will only look at incident P waves (no S wave in fluid). The polarization of the transmitted wave for a incident P wave is given by

$$h_p = \frac{1}{\omega_3 \mu \Delta} \begin{bmatrix} 2p \ q_\beta \\ 0 \\ \Omega \end{bmatrix} \quad (3-26)$$

where

$$\Omega = q_\beta^2 - p^2 = \frac{\rho - 2\mu p^2}{\mu} \quad (3-27)$$

$$q_\beta = \sqrt{\frac{1}{\beta^2} - p_l^2} \quad (3-28)$$

$$\omega_3 = \text{arbitrary normalization factor} \quad (3-29)$$

$$\Delta = 4p^2 q_{\alpha,2} q_{\beta,2} + \Omega^2 \quad (3-30)$$

From Equation 3-26 we can observe that for elastic media the mode conversion does depend on the same parameters as in the acoustic case plus the horizontal slowness q_β (i.e., the inverse shear velocity) of the seabed.

Part II

Orientation estimation

Chapter 4

Data sets

4-1 Data sets

In this Chapter I will introduce the two data sets that are analyzed and used to check the performance of the orientation determination algorithms. All the orientation estimations were performed on either one or both of them.

4-2 Real Data

The *real* data set was acquired in the North Sea. The data was recorded with seabed receivers at a water depth of approximately 100 m. The receiver line spacing is 300 m and the inline node spacing is 50 m. The shot spacing (flip-flop) is 25 by 25 m. The receiver node line azimuth is 93° with respect to the north. The seabed in the acquisition area is known to be relatively hard and flat. A subset composed of 21 common receiver gathers (CRG) which was acquired near the obstruction that resulted in the shot gap discussed in prior section. The 21 CRGs were extracted from the red box in Figure 1-2. I will refer to the most western CRG as common receiver gather 1. The next CRG to the east is common receiver gather 2 and so on. We can observe that the shots used for the orientation determination (gray circle) partly overlaps for neighboring receiver positions. In the orientation determination we will only consider pre-critical shots, that is why the shot positions are limited to a radius equal to the water depth ie. the receiver depth. We can also observe that the eastern and western receivers are generally illuminated with less shots than the receivers in the middle. But all the receivers are affected by the obstruction and thus have are predominantly missing uniform shot coverage in the south.

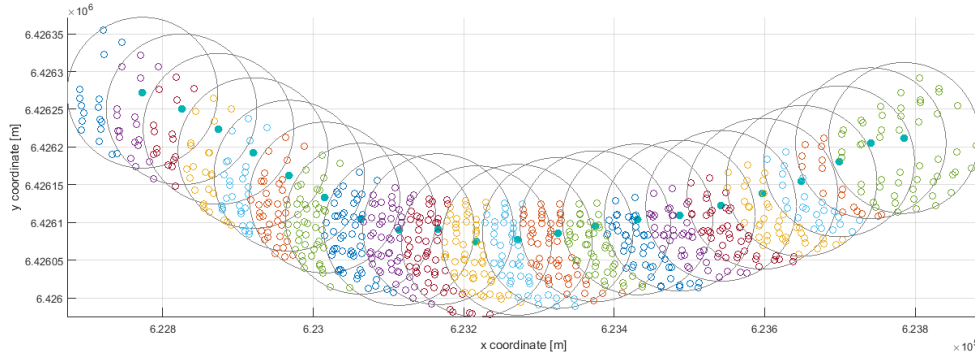


Figure 4-1: Shot coverage (empty small circles) for the 21 receivers (filled teal circles) in the North Sea data set. The large circles around the receiver position indicate a radial distance of 100 m.

4-3 Synthetic data

The synthetic data set was generated with the reflectivity modelling software ANIVEC by [Mallick and Frazer \(1987\)](#). ANIVEC can generate complete three component seismogram's for stratified, layered media. Both, the response of a fluid (acoustic) layer and a elastic layer can be modelled. The seismic source signal is generated by a point source. Since ANIVEC outputs common shot gathers, the source-receiver- reciprocity properties were used to obtain a common receiver gather [Vermeer \(2012\)](#).

We created two data sets that have the same medium parameters except for the thickness of the first layer. The first layer corresponds to the water above the stratified earth. We consider a water depth of 30 and 100 m, modeling a shallow and medium water depth acquisition respectively. Originally a common shot gather was generated, by using source-receiver reciprocity the shot gather was turned into a common receiver gather. The synthetic data set consists of a very dense shot coverage of 6.25 m by 6.25 m. The receiver (former source) is in the very center of the survey. The receiver is oriented flat on the seabed (i.e., $\phi = 0^\circ$ and $\psi = 0^\circ$). The water depth at the receiver is either 30 or 100 m and the sources are at a water depth of 10 m. Since the model is isotropic the x and y component gathers are equivalent. The obtained common receiver gather is displayed in Figure 4-2. Observe the polarity reversal at the source position. The z component gather is displayed in Figure 4-3.

v_p [m/s]	v_s [m/s]	Qp	Qs	Density [g/cm^3]	Thickness [m]
1500	1000	1000	1000	1.0	100/30
2000	1000	1000	1000	1.3	400
3000	1500	1000	1000	2.5	1570

Table 4-1: Parametrization of the layered, isotropic earth model used to create the synthetic data

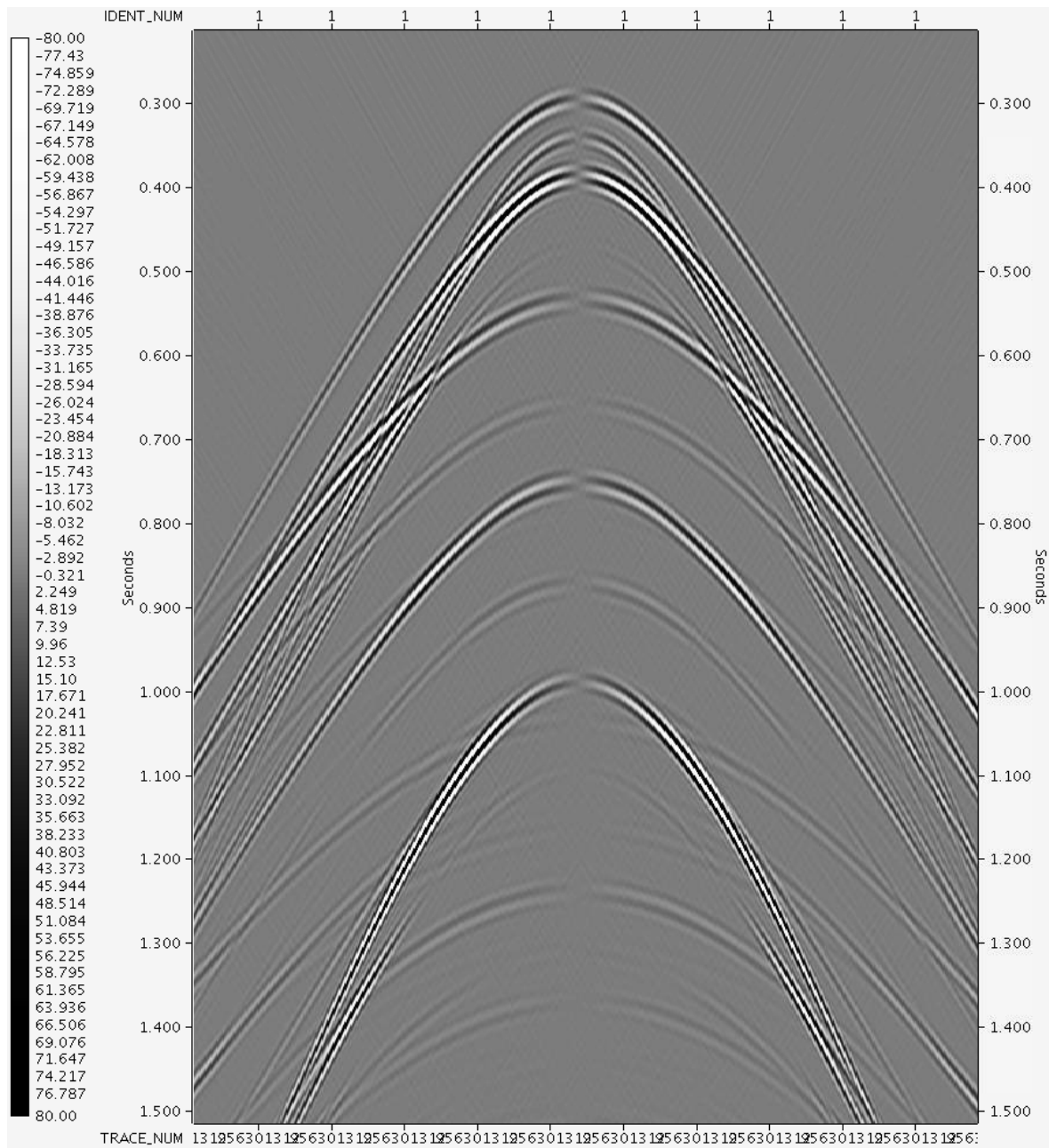
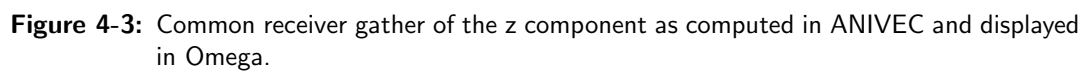


Figure 4-2: Common receiver gather of the x (and y) component as computed in ANIVEC and displayed in Omega.



Chapter 5

Data driven Orientation Determination

In this and the next Chapter I will introduce new algorithms and approaches to determine the orientation of seabed receiver. Chapter 5 describes a novel method for the determination of the orientation of a multicomponent sensor based upon the radial symmetry property of the particle velocity underneath the seabed. Chapter 6 follows an entirely different approach. Here the approximate knowledge of the seabed is used to model its response to a incident wave. We will see that by modeling the response of the seabed we are able to handle the orientation determination problem better then by using the source-receiver geometry.

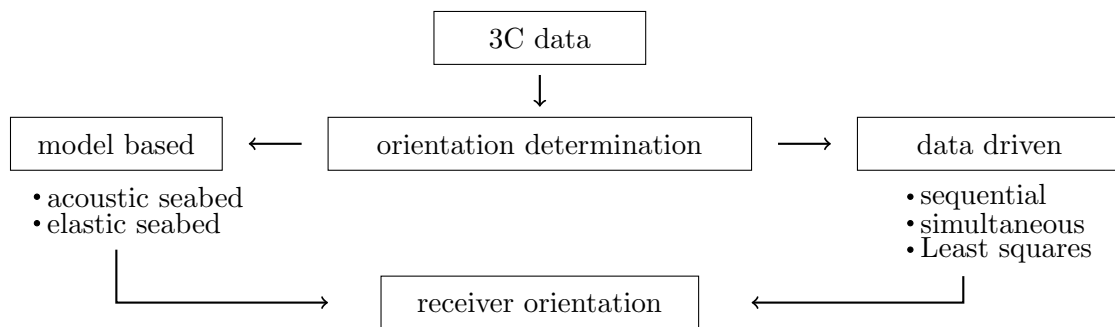


Figure 5-1: Schematic plot of the orientation determination approaches that are presented in Chapter 5 (right branch) and Chapter 6 (left branch) .

5-1 Introduction to ROTATE_3C_ANALY

The orientation determination in ROTATE_3C_ANALY (Gaiser and Jackson, 2003) is based on finding the angle of incidence for every shot-receiver pair in a common receiver gather. In ROTATE_3C_ANALY the angle of incidence is approximated by the polarization angle (compare Section 3-1-2). We have seen that this is not necessarily a good approximation (compare Angle discrepancy in Figure 3-3). However, we will shortly see that the error introduced by this assumption cancels out if two reciprocal traces are considered. It is known that the first arrivals are P waves with a particle movement in the direction of propagation. To make sure that no refracted energies are used for the orientation estimation, the radial distance from the receiver to a shot position is restricted to the water depth at the receiver. For this offset range the first arrivals will be pre-critical and the direct wave will be the first wave recorded by the receiver. Further it is assumed that the seabed and the sea surface are parallel.

The polarization angle approximating the incidence angle is found in two steps. First, the first break energy of a single trace is rotated around the x axis. The rotation angle minimizing the energy on the y component is found. This angle will be called ϕ and is determined for every trace in the common receiver gather. Then all the ϕ values are summed up and divided by the number of traces. The obtained value ϕ_0 gives the orientation around the x axis. The angle ϕ_0 is then applied to the data and the first break energy is rotated again. This time around the y axis in order to minimize the energy on the x component. The angle minimizing the x component is ψ . From all the ψ estimations ψ_0 is found. The two angles ϕ and ψ give the rotations needed to turn the receiver such that the plane given by its x-y components is perpendicular to the ray approximating the incident wave for one source-receiver pair. After these trace dependent angles are found, they are corrected for the inclination and azimuth given by the acquisition geometry. The angles ϕ_0 and ψ_0 describe the orientation of the receiver with respect to the axes of a flat receiver. In other words, they are the rotations around the x and y axis required to rotate flat the sensor at the seabed. In a last step θ_0 , which is the clockwise rotation around the vertical measured from the North, is found. The orientation determination in three steps is referred as **sequential**. A into depth description of ROTATE_3C_ANALY can be found in Appendix A.

In the following I will give a more intuitive description of ROTATE_3C_ANALY. Figure 5-2 shows two reciprocal shots and a receiver which is tilted by 45° around the x axis. The orientation of the y axis (ϕ_0) is estimated from the first break energy of the two traces. The receiver will register the transmitted wave. We will now find the trace dependent rotations around the x axis that will minimize the registered energy on the y component. Recall that it is assumed that most of the first arrival energy is found in the direction of propagation. Based on this the expected value for ϕ of the transmitted wave of shot 2 is 0° , since the x-y-plane of the receiver is already perpendicular to the transmitted ray. The angle ϕ for the first shot is 90° . After the ϕ values are summed and divided by two, the correct and expected value for ϕ_0 is found to be 45° . Observe that even though the wrong angles of incidence were estimated, the correct orientation was found. The inclination and azimuth corrections are not applied in this 2D example.

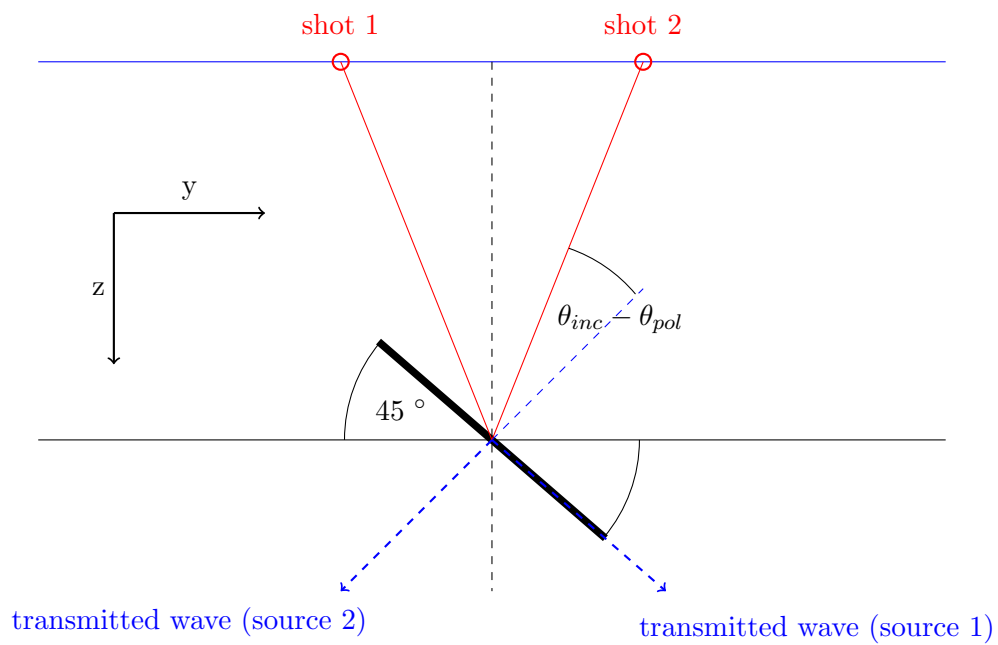


Figure 5-2: Estimation of ϕ_0 based on the ϕ estimation of two reciprocal traces.

5-1-1 Reproduction of ROTATE_3C_ANALY

In order to be able to compare the results of the new, enhanced algorithms with the results obtained with Omega's ROTATE_3C_ANALY, the latter was reproduced in Matlab. I will use the North Sea data set to show that the Matlab routine finds the same values for ϕ_0 and ψ_0 as ROTATE_3C_ANALY. Figure 5-3 shows the estimation of ϕ_0 and ψ_0 in Matlab and Omega. We find that the Matlab routine successfully reproduces the results of ROTATE_3C_ANALY. A more in depth comparison of the two routines can be found in Appendix B. The now algorithmic enhancements introduced in the following section are implemented on top of the Matlab environment.

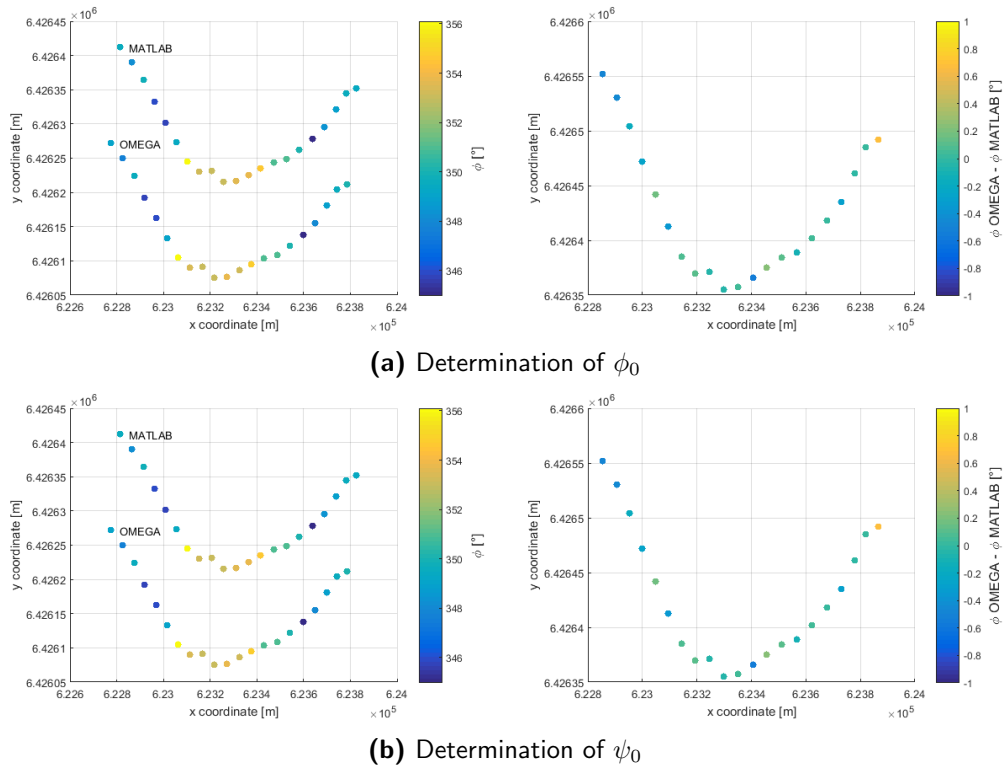


Figure 5-3: Left: ϕ_0 (a) and ψ_0 (b) estimation and comparison in Omega and the Matlab environment. The Matlab-receiver line is shifted for displaying purposes. Right: Difference between the Omega and Matlab.

Here only the results for the ϕ_0 and ψ_0 are shown. The third angle that determines the orientation of the receiver is θ_0 . This angle is easily found and does not pose a problem once ϕ_0 and ψ_0 are found.

5-2 Development and application of the algorithmic enhancements

It is clear that the estimations from the existing routine (ROTATE_3C_ANALY) are wrong if the reciprocal shot condition is not satisfied. In this section I describe three approaches Bagaini (2016) that, as we will see, outperform ROTATE_3C_ANALY in the presence of missing shots. The new routines will then be applied to synthetic and real data to check their performance. A flowchart showing the routines and functions called in the orientation determination can be found in Appendix C.

5-2-1 Simultaneous Estimation

Instead of estimating ϕ_0 and ψ_0 in two steps the simultaneous approach estimates the two angles at the same time. The simultaneous approach uses a 2D gradient method to minimize the energy in the x-y plane. The inverse problem is solved by using Matlabs non-linear optimization solver *fmincon*. In order to make use of such an algorithm we need to have an expression that has to be minimized. It is given as follows:

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\phi) & \sin(\phi) \\ 0 & -\sin(\phi) & \cos(\phi) \end{bmatrix}}_{\text{Rx}} \underbrace{\begin{bmatrix} \cos(\psi) & 0 & -\sin(\psi) \\ 0 & 1 & 0 \\ \sin(\psi) & 0 & \cos(\psi) \end{bmatrix}}_{\text{Ry}} \underbrace{\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix}}_{\text{data}} = \begin{bmatrix} x' \cos(\psi) - z' \sin(\psi) \\ y' \cos(\phi) + z' \cos(\psi) \sin(\phi) + x' \sin(\phi) \cos(\psi) \\ z' \cos(\phi) \cos(\psi) - y' \sin(\psi) + x' \cos(\phi) \sin(\psi) \end{bmatrix} \quad (5-1)$$

We now simply take the sum of x- and y- component after applying the rotation matrices Rx and Ry to obtain the objective function F that can be used to find the minimum that depends on all trial ϕ and ψ values at the same time. This term is squared since we are interested in the seismic energy, not the amplitude.

$$F(\psi, \phi) = [x'(\cos(\psi) + \sin(\phi)\cos(\psi)) + y'\cos(\phi) + z'(\cos(\psi)\sin(\phi) - \sin(\psi))]^2 \quad (5-2)$$

The function F will be referred to as the objective function and is be the expression that has to be minimized.

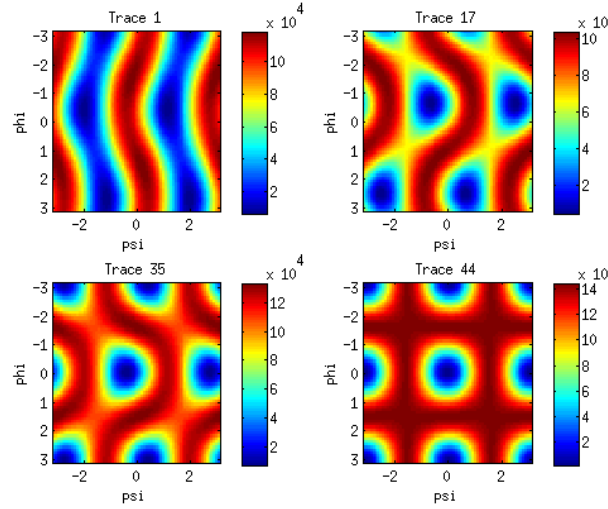


Figure 5-4: Exemplary objective functions as used in the simultaneous approach. The value of the objective function varies with the trial angles ϕ and ψ and it is also dependent on the source- receiver pair used as input for the routine. The colorbar represents the summed energy of the x and y component as used in the objective function.

The minimum of F is found by computing the gradient around a given starting point from this point *fmincon* follows the steepest gradient until a minimum is reached. The algorithm uses the gradient and the Hessian to check if the current position of the gradient search is at a minimum or not. Unfortunately F is a non-linear function which increases the difficulty of finding the minimum since local minima are possible (Chinneck, 2015). However, we can add constraints to the search domain. From the behavior of the sine and cosine we know that the objective function is periodic and will have a minima in the range of $[-2\pi, 2\pi]$ for both trial rotations ϕ and ψ . The trace dependent periodicity of the objective function can be seen in Figure 5-4. The simultaneous approach is a trace by trace operation similar to the sequential approach. All the ϕ and ψ values are averaged to obtain ϕ_0 and ψ_0 .

5-2-2 Center of Gravity method

As mentioned earlier the methods described so far assume that the reciprocal trace condition is satisfied. However, this assumption is not always true. If non-uniform shot coverage is present the inverted orientations will exhibit anomalous results, as shown in Figure 1-2. The *Center of Gravity method* (CoG) (Bagaini, 2016) corrects for these situations by introducing an additional rotation to the data. The idea is to numerically rotate the sensors vertical axis and by this the entire recorded data towards the center of gravity of the existing source grid. By this rotation the receiver is rotated towards a more uniform source distribution which should result in a more reliable orientation estimation that is less biased by non-uniform shot coverage. A visualization of this pre-rotation can be found in Figure 5-5b. A derivation of the CoG method can be found in Appendix D.

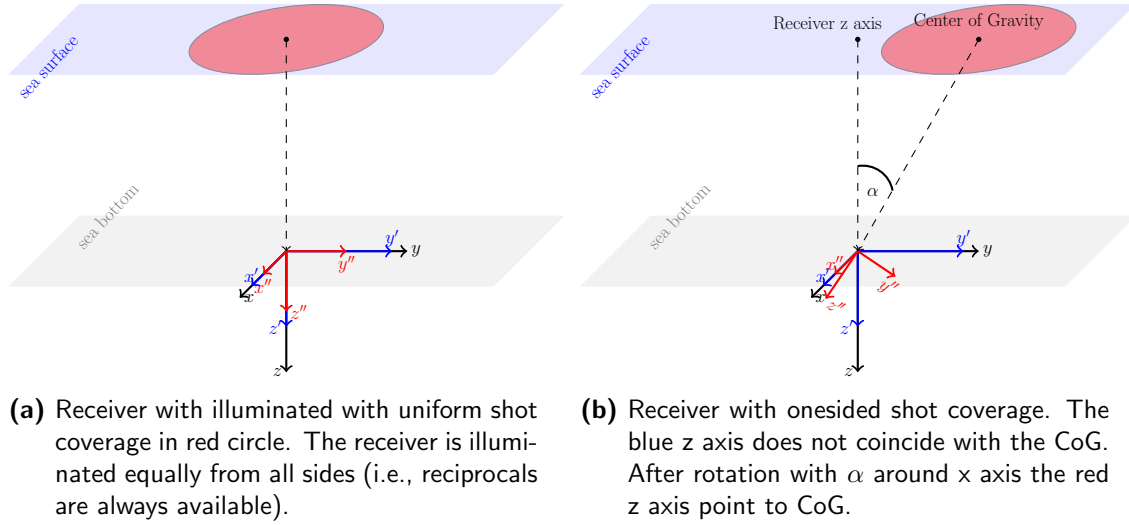


Figure 5-5: Visualization of the CoG approach.

The CoG method is implemented as an extension to the sequential, least squares and simultaneous approach. The way of determining ϕ_0 and ψ_0 is not affected by the pre-rotation of the data. The pre-rotation consists of three major steps:

1. **Determining α :** α is the angle between the z axis direction of a flat receiver and the vector connecting the receiver with the source positions center of gravity. The direction of the z axis is denoted by $\vec{z}$ and the receiver-center of gravity vector will be $\vec{g}$. The angle between two normalized vectors is defined by their dot product:

$$\cos\alpha = \frac{\vec{z} \cdot \vec{g}}{||\vec{z}|| ||\vec{g}||} \quad (5-3)$$

The rotation axis around which α is applied is given by the cross product of $\vec{z}$ and $\vec{g}$. This axis is denoted as $\vec{A}$

2. **3D angle to Euler angles:** Since the rotation axis $\vec{A}$ does not necessary coincide with one of the axes used to rotate the data, the angle α has to be split into the respective rotations around x, y and z. The pre-rotation angles are ϕ_{CoG} , ψ_{CoG} and θ_{CoG} . The transformation of the 3D angle to the three Euler angles is described in the Section 2-1-4.
3. **Application:** The three Euler angles are applied using the rotation matrices Rx, Ry and Rz (compare above). Now the recorded data is reorientated towards the center of gravity.

The pre-rotated data is then used as modified input for the sequential, least squares and/or simultaneous approach.

5-2-3 Least Squares

Other than the sequential approach the Least squares method is based on the cumulative energy of all the traces rather than on individual traces. Similar to the sequential approach the ϕ and ψ rotations are estimated after one another. First the recorded data is rotated around the x axis then around the y axis. I will shortly outline how this approach works. In a first step a trial rotation with angle ϕ is applied to all the traces, the first break energy of the rotated y data is extracted and then summed up. This is the cumulative the y energy for the first trial angle ϕ . After computing the y- energies (E_y) for n trial angles a sine is fitted through the energies of the trial rotated traces. The fit minimizes the energy for all the traces in a least squares sense rather than in a mean sense as the sequential approach does. We can describe this least squares problem with the following over-determined system of linear equations (Maurer, 2015):

$$\underbrace{\begin{bmatrix} E_{y \text{ rot}_1} \\ \dots \\ E_{y \text{ rot}_n} \end{bmatrix}}_d = \underbrace{\begin{bmatrix} 1 & \sin(2\phi_1) & \cos(2\phi_1) \\ \dots & \dots & \dots \\ 1 & \sin(2\phi_n) & \cos(2\phi_n) \end{bmatrix}}_G \underbrace{\begin{bmatrix} A \\ B \\ C \end{bmatrix}}_m \quad (5-4)$$

The observed data denoted as d that we want to explain with a certain model or fit with a curve. The model parameters for the best fit model are denoted as m . The expected behavior of the model is encapsulated in the G matrix. So for the MC receiver inversion we assume a energy distribution that can be described with a superposition of a sine (B) and a cosine (C) plus an initial phase shift (A) (OMEGA documentation). The d -vector contains the energy over a time window of a single trace for one specific trial rotation around the axis of interest. The length of d is the same as the amount of trial rotations. G is $[trial \text{ rotations} \times 3]$ large. Equation 5-4 is only valid for energies that belong to linear polarized events (eg. direct wave). In order to compute m we have to solve the following system of equations (Maurer, 2015):

$$(G^T G) m = G^T d \quad (5-5)$$

This set of equations is often referred to as *normal equations*. The linear superposition that fits the data the best is given by the coefficients A, B and C.

Any linear combination of a cosine and a sine of equal periods is equal to a single sine with the same period but with a phase shift and a different amplitude. And we know that the trace dependent energy distribution is exactly of such a form because it originates from a linear polarized event. We express it as follows:

$$A + B \sin(2\alpha) + C \cos(2\alpha) = X \sin(2\alpha + \beta) \quad (5-6)$$

So the linear combination of the two functions is described by its amplitude X and the phase shift β . The phase shift is what we are interested in. The phase shift and the amplitude X is given by

$$\beta = \text{atan} \left(\frac{C}{B} \right) \quad (5-7)$$

$$X = A + \sqrt{C^2 + B^2} \quad (5-8)$$

and it is equal to $\pm 90^\circ$ away from the polarization angle of the incident wave field (compare Figure 5-6). This is because the polarization angle is found when the y and x energy is at a

minimum. Using the periodicity of the trigonometric functions we can find that the minimum is 90° away from the maximum. This is shown in Figure 5-6 below.

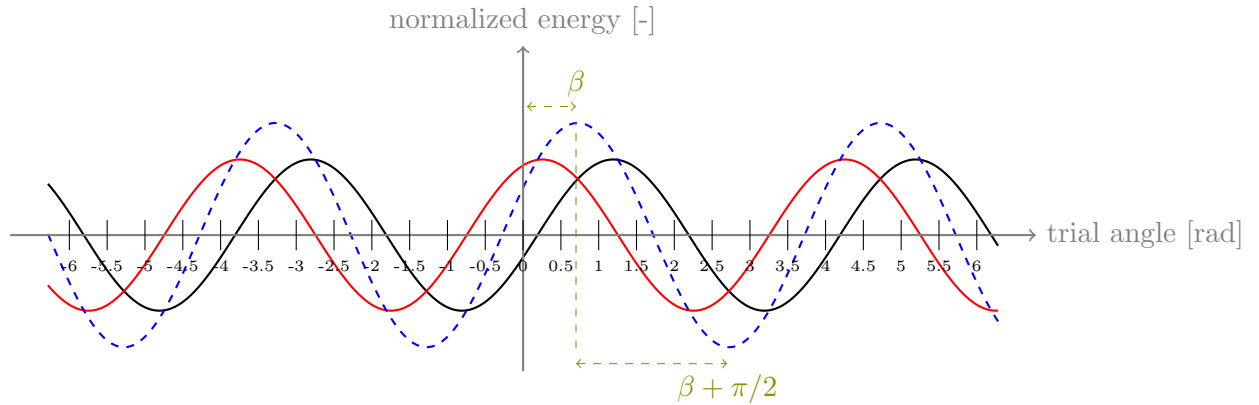


Figure 5-6: Superposition (blue,dashed) of a sine (black) and cosine (red). The superposition is a cosine shifted by β .

5-2-4 Synthetic data

The shallow water synthetics will be used to test the algorithmic enhancements. I will show the estimated receiver orientations obtained with the existing and new routines for different acquisition setups. The receiver in the synthetic common receiver gather is located at a depth of 30 m. In order to only include pre-critical first arrivals the radial distance for shots accepted in the orientation determination is limited to 25 m. The medium properties of the layered earth model can be found in Table 4-1.

Full coverage and opposite quadrants

To test the performance of the new algorithms in comparison to the existing Omega routine, we applied two trial rotations to the three components of the synthetic data set. The rotations were applied around the x and y axes and are referred to as *initial values* in Table 5-1. No θ_0 rotation was applied, so the heading of the x and y axis is unchanged. These initial values of ϕ and ψ had to be retrieved by the respective inversion approach. We will test the algorithms with four different shot coverage setups. In a first step we will use perfect uniform shot coverage as it is shown in Figure 5-7. The estimated orientations using the sequential, Center of Gravity (CoG), simultaneous and Least Squares approach are shown in Table 5-1. We can observe that all the methods are able to perfectly reproduce the the values of the trial rotations ψ_0 and ϕ_0 . Also observe that the sequential and the pre-rotated sequential (CoG) give the exact same result. This is expected and can easily be explained with the fact that the center of gravity is located directly above the receiver, so no rotation has to be applied to orient the z axis of the receiver to the center of gravity. This is equal to applying $\phi_{CoG} = \psi_{CoG} = \theta_{CoG} = 0$ to the data.

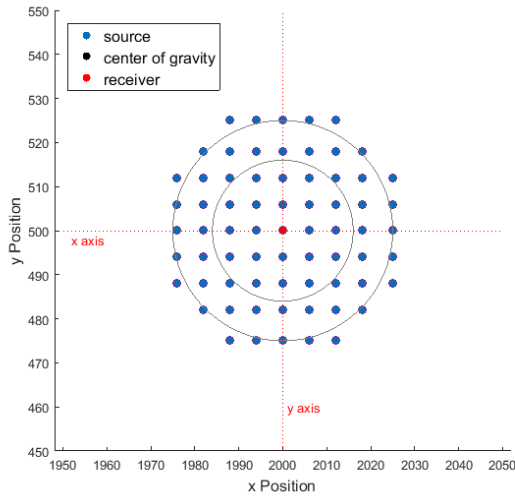


Figure 5-7: Uniform shot coverage.

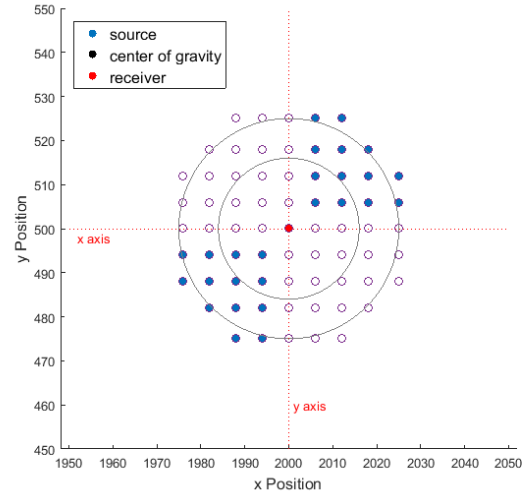


Figure 5-8: Opposite quadrant shot coverage.

	ϕ_0 [°]	ψ_0 [°]	Z Inclination [°]
initial values	15	-20 (=340)	65.1
sequential	16.1	337.8	62.8
sequential *	16.1	337.8	62.8
simultaneous *	15	340	65.1
Least Squares *	14.7	340	65.3

Table 5-1: Full shot coverage. * indicates that pre-rotated data was used as input.

Comparably good results as for perfect uniform coverage are obtained if only the shot receiver pairs of two opposite quadrants are used. The geometry for this setup is displayed in Figure 5-8. The orientation estimations are shown in Table 5-2. All the methods succeed because the "reciprocal trace condition" is satisfied.

	ϕ_0 [°]	ψ_0 [°]	Z Inclination [°]
initial values	15	-20 (=340)	65.1
sequential	17.3	337.9	62.2
sequential *	17.3	337.9	62.2
simultaneous *	15	340	65.1
Least Squares *	14.5	340.1	65.5

Table 5-2: Opposing quadrant shot coverage. * indicates that pre-rotated data was used as input.

One sided shot coverage

Now we will turn to a acquisition where no reciprocal traces are available. Figure 5-9 shows a two sided acquisition setup. I will start by considering one sided shot coverage from the west. This could simulate a acquisition with an obstruction east of the y axis of the receiver. This setup is interesting since it is similar to the shot coverage that causes ROTATE_3C_ANALY to estimate anomalous orientations. Note that the y axis is not illuminated with shots from both sides, nevertheless the shot distribution in the west is homogeneous (ie. every shot has a reciprocal). Table 5-3 shows the results obtained for this setup. Observe that the sequential approach fails to determine the ϕ_0 orientation whereas the ψ_0 orientation is off by 10° . We can also see that the sequential approach performs better if the data is pre-rotated towards the center of gravity. Here ϕ is estimated with an error of 5° . The value for the ψ_0 is nearly the same as for the un-rotated estimation. Even though the sequential approach is not able to retrieve ϕ_0 the value for the z inclination is approximately equal to the z inclination of the pre-rotated sequential (CoG) approach. This makes sense if we compare the estimations of ϕ_0 for both the algorithms. Note that $340^\circ (= -20^\circ)$ will result in the same tilt of the z axis as a rotation of 20° if ψ_0 is the same. This shows that the z inclination is not necessarily a good quality controlling tool for the orientation estimations since it can not resolve the angle ambiguity that was shortly outline. Both, the Least Squares and simultaneous approach are able to retrieve the trail rotations with an accuracy of $\pm 2^\circ$.

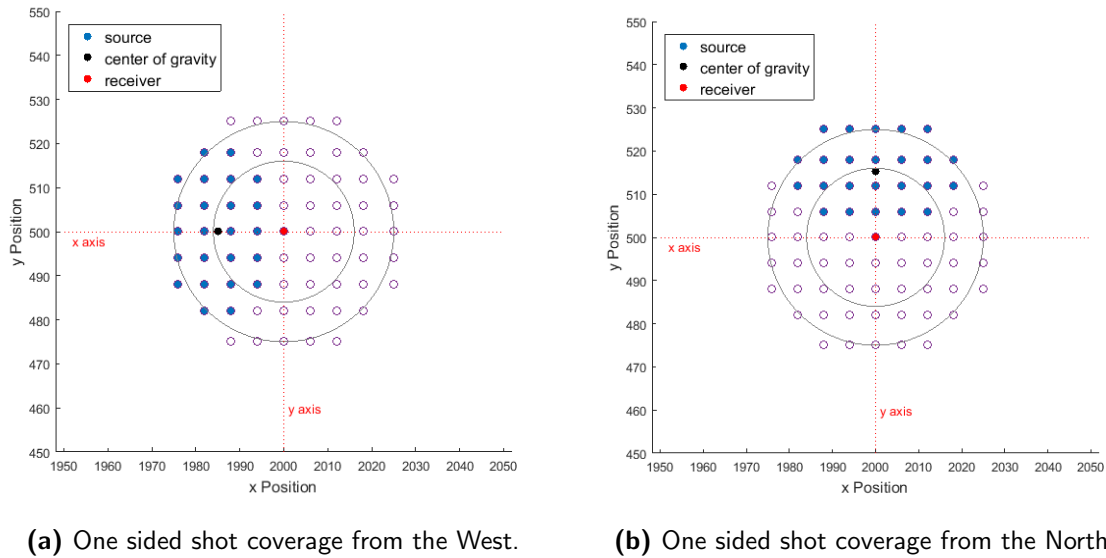


Figure 5-9: Two one sided acquisition setups.

Similar results as for the one sided shot coverage west of y axis are obtained for shot coverage north of the x axis (Figure 5-9b). Table 5-4 shows the orientation estimations for this acquisition setup. We can again observe that the sequential approach fails in determining the orientation of the receiver axis with one sided shot coverage (ψ_0) whereas the other orientation (ϕ_0) is much more accurate. Different to the one sided shot coverage from the west we can observe that the other three methods retrieve the initial orientation of ϕ_0 and ψ_0 with an error of $\pm 1^\circ$.

	ϕ_0 [°]	ψ_0 [°]	Z Inclination [°]
initial values	15	-20 (=340)	65.1
sequential	341.8	330.8	56.1
sequential *	20.1	331.2	55.4
simultaneous *	17.9	336.9	62.5
Least Squares *	17.4	337.1	61.5

Table 5-3: One sided West. * indicates that pre-rotated data was used as input.

	ϕ_0 [°]	ψ_0 [°]	Z Inclination [°]
initial values	15	-20 (=340)	65.1
sequential	14.9	306.3	34.9
sequential *	13.5	341	67.1
simultaneous *	13.9	340	66.1
Least Squares *	13.7	340	65.8

Table 5-4: One sided North. * indicates that pre-rotated data was used as input.

Figure 5-10 shows the seismic data after the orientations were estimated for a one-sided experiment and rotated to the radial-transversal coordinate system. Observe that sequential approach (first row) without pre-rotation is not able to separate the S and P wave from on another. The energy on the radial is still very high. The pre-rotated sequential and Least Squares approach (second and third row respectively) are able to minimize the energy on the radial much better. This indicates that these two algorithms indeed outperform the sequential approach in finding the orientation of a receiver in non-uniform shot coverage.

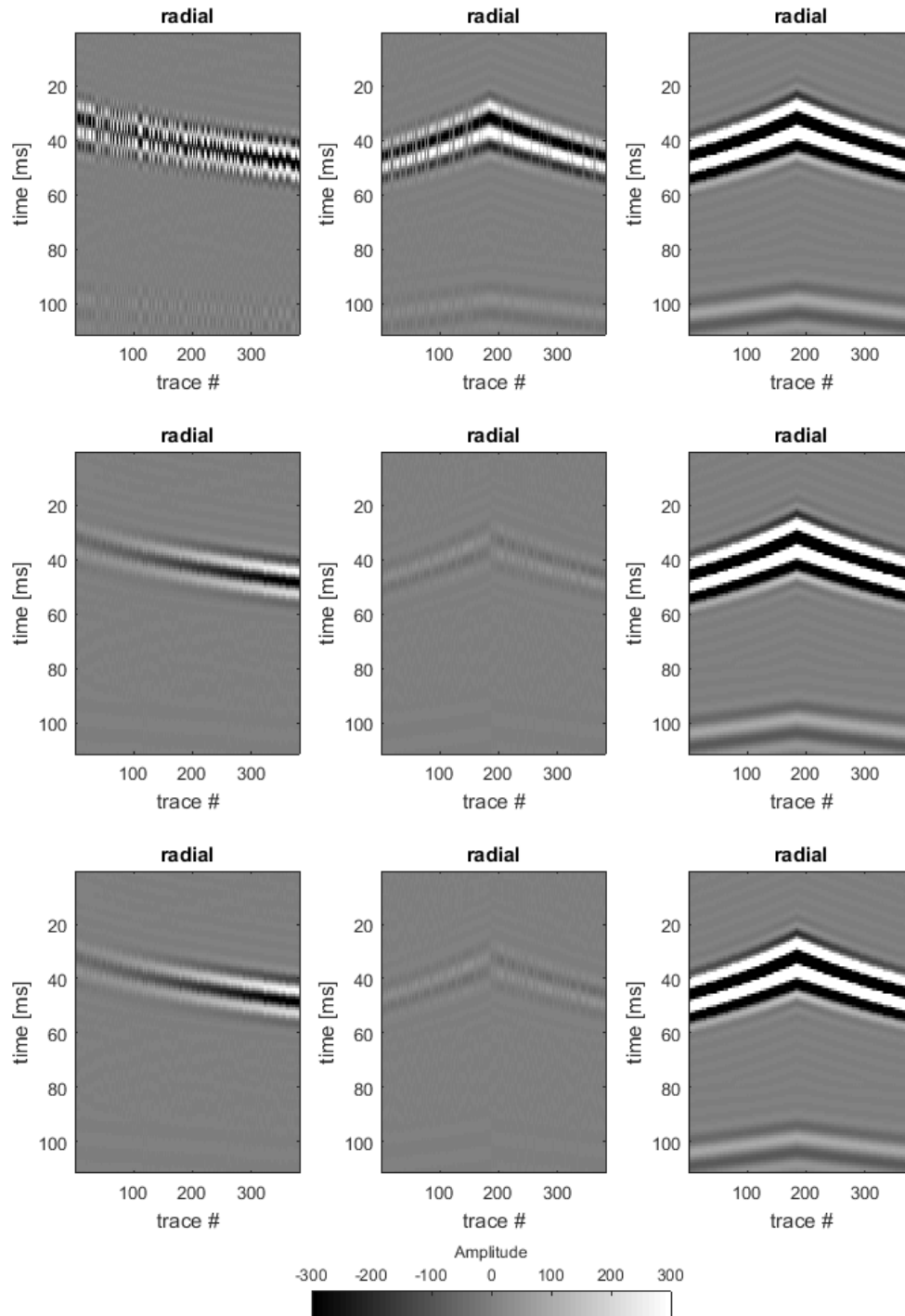


Figure 5-10: The panels show the seismic traces after the receiver was numerically flattened by rotating the data with the angles approximated with the sequential approach (first row), the sequential approach with pre-rotation (second row) and the Least Squares approach with pre-rotation (third row).

5-2-5 Real data

Figure 5-11 shows the inclination of the receivers z axis after the orientation estimations were applied to the 21 common receiver gathers from the North Sea data set which was introduced earlier (Figure 4-1). After the orientations were applied the receiver are expected to be flat on the seabed. Recall that z axis inclinations of 90° are expected for a flat receiver, since the seabed is relatively hard. The first thing to note is the good match between the results obtained using Omega's and Matlab's ROTATE_3C_ANALY.

The (pre-rotated) sequential CoG approach does a better job than ROTATE_3C_ANALY in rotating the receiver in a horizontal position i.e., inclination of 90° . But it is apparent that an inclination of 90° is not yet obtained for all the receivers. Also the variability of the z inclination is relatively high. We can also observe that the Least squares approach has a very high variability in the estimated z inclinations. This variability is high undesirable since it does not resolve the uncertainty of the unknown orientation.

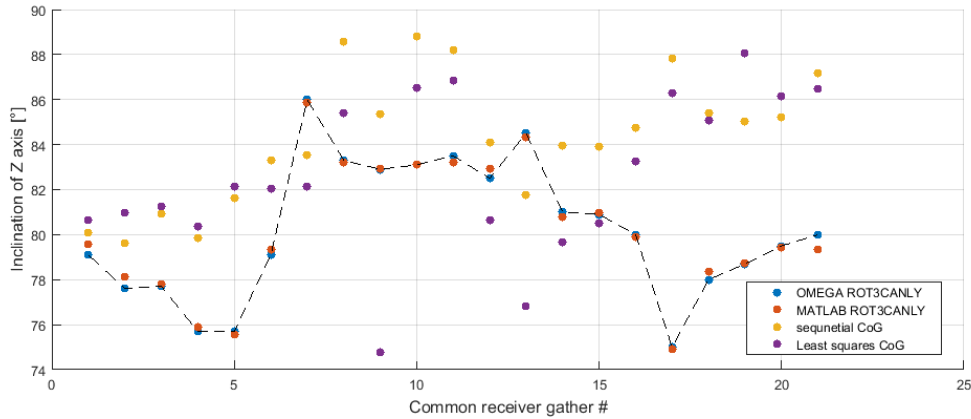


Figure 5-11: z axis inclinations for 4 different methods applied to the North Sea data set receiver line.

The simultaneous approach was also applied to the real data. However, I will not present the obtained results here. The orientation estimations obtained by the simultaneous approach seem to depend very strongly on the choice of the starting point for the gradient method. Different starting points were leading to different estimations for ϕ_0 and ψ_0 . This behaviour is highly undesirable since it includes a subjective choice of the starting point and the estimation of the polarization angle is no longer data driven. A possible explanation for this behavior of the simultaneous approach is the shape of the objective function. Further work on this, beyond the scope of this thesis, must be done.

Figure 5-12 shows the common receiver gather for the x, y and z component of receiver number 10 of the North Sea dataset. The first row shows the raw data as they were recorded in during the survey. The second row shows the same data after they were rotated with ϕ_0 and ψ_0 as they were determined with the pre-rotated sequential approach. The receiver should now be rotated flat on the seabed. The third row shows the flattened data after they were turned into a source-centered coordinate system. We can observe that nearly all the energy is on the radial component.

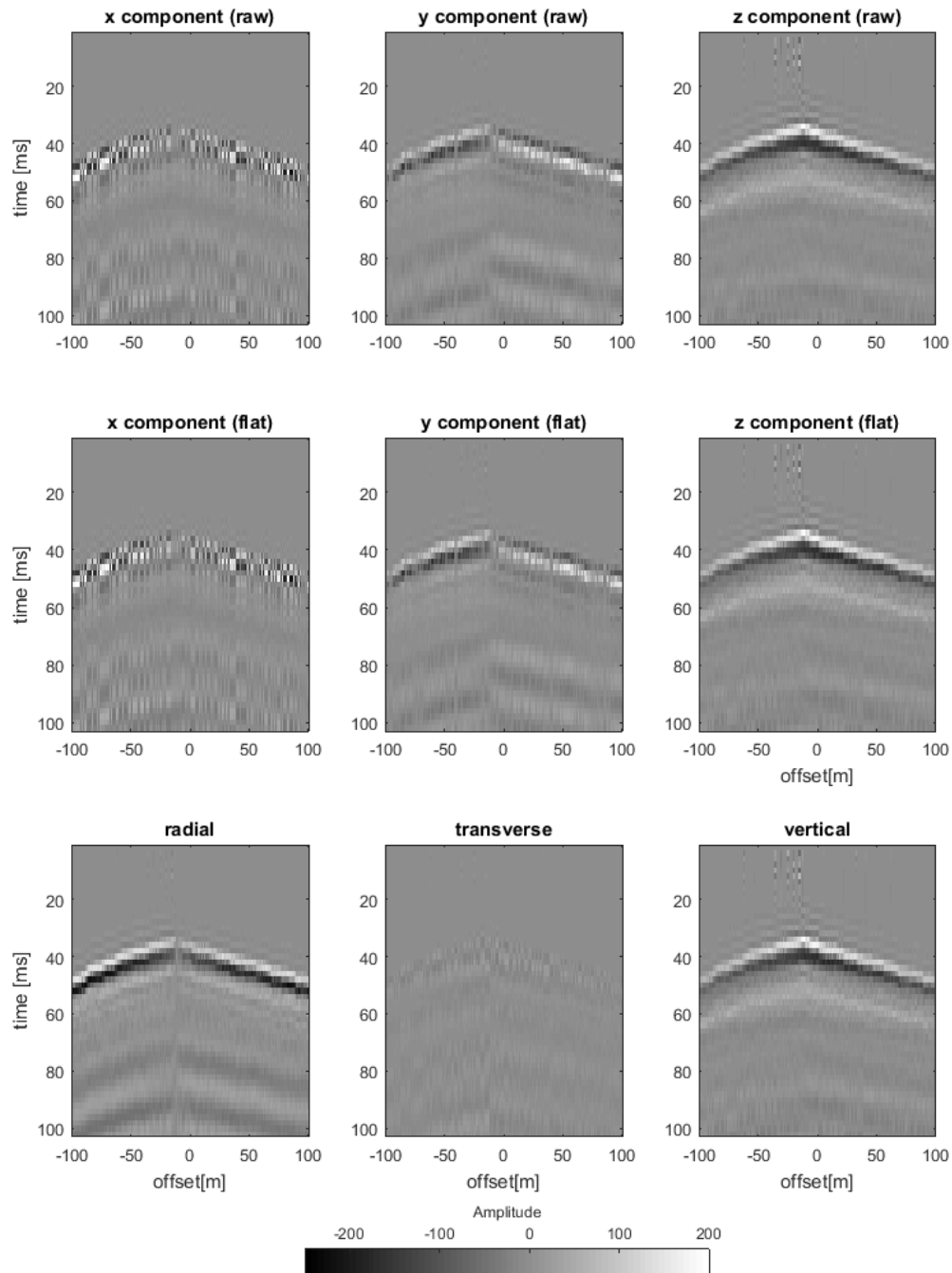


Figure 5-12: Common receiver gathers as recorded by receiver number 10 of the North Sea data set. First row: raw data, second row: ϕ_0 and ψ_0 applied to raw data, data is flattened now, third row: flattened data turned from x,y,z coordinates into r,t,z coordinates.

5-2-6 Discussion

From the synthetic data we can see that the sequential approach (ROTATE_3C_ANALY) estimates a wrong orientation if the reciprocal trace condition is not satisfied. Let us consider the results obtained for the one sided shot coverage from the north. It can be seen that the y axis of the receiver has shots to the west and east. The x axis on the other hand is only covered from the north, there are no shots in the south. Recall that ϕ_0 gives the direction of the y axis and ψ_0 the direction of the x axis of the receiver. After the data was pre-rotated towards the center of gravity the orientation estimation for the one sided illuminated x axis (i.e., ϕ_0) is much closer to the trial rotation that had to be retrieved. So indeed, by rotating the data towards an area of more homogeneous shot coverage, the problem of missing reciprocals seems to be solved. The post- and pre- CoG- rotation situation for the x and y axis of the receiver is visualized in Figure 5-13. The results obtained with the pre-rotated approaches seem to confirm the validity of simulating this new acquisition setup. More over we have found that a Least Squares fit of the trial rotated data is gives a more accurate orientation than using the mean of the trial rotated data (sequential).

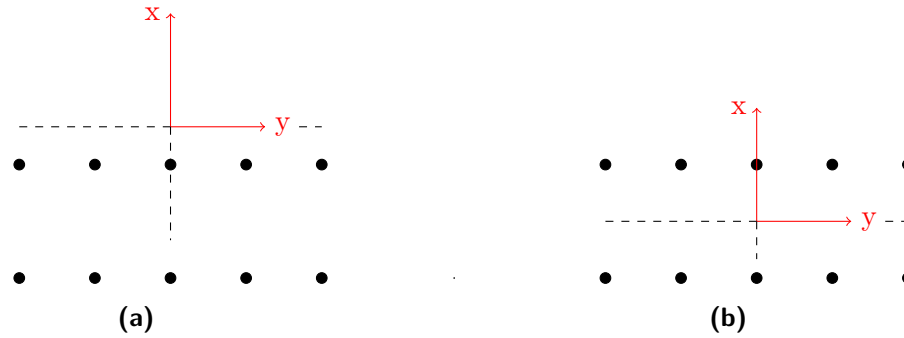


Figure 5-13: (a) Visualization of missing reciprocals around y axis (situation before CoG). The x axis however has reciprocal traces to its left and right. (b) After reorienting the receiver orientation we are left with this situation. Now both axis have sufficient reciprocal traces.

We have seen that the Center of Gravity, simultaneous and Least squares approach give better results than the ROTATE_3C_ANALY in the case of nonuniform shot coverage if synthetic data is used. Even though the orientation determination is improved we can still observe deviations of $\pm 2^\circ$ from the expected values. In the following I will try to give explanations for this behaviour. Recall that the three new algorithms all take into account the acquisition geometry in some way. By choosing the center of gravity- receiver vector as the new direction of the z axis we effectively consider a different seismic experiment. Since the determined orientations are deduced from a energy criterion (minimizing the energy of one receiver axis) it is critical that the energies used to determine the minimum are correct. Figure 5-14 shows a one sided seismic experiment before the pre-rotation (a) and after (b) the pre-rotation is applied. Observe that the z axis of the receiver is aligned with the CoG-receiver vector after the pre-rotation is applied.

Figure 5-15 shows the one sided experiment (b) and a uniform seismic experiment (a). It can easily be seen that the distance from shot position 1 and 4 to the receiver is the same for the uniform experiment. Assuming that the source energy is subject to geometrical spreading,

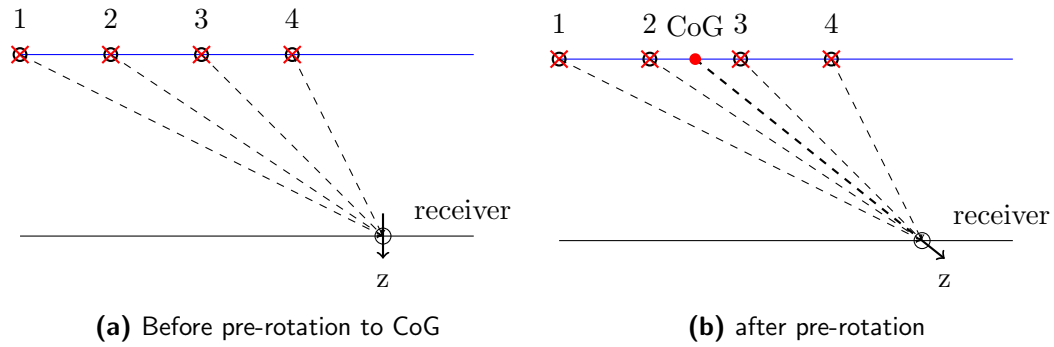


Figure 5-14: Pre- and Post rotation of the z component of the receiver.

the energy recorded by the receiver coming from source 1 and 4 will be the same. Both these statements are not true for the one sided experiment. Observe that the the distance from source position 1 to the receiver is much longer than from source position 4. This will also lead to higher energies recorded from shot 4 than 1. This is problematic since it is assumed that after the pre-rotation the z axis is illuminated as if it was covered uniformly as shown in Figure 5-15(a). The desired uniform shot coverage is shown by the blue dashed line in Figure 5-15(b). The distance between the desired shot position and the actual shot position is marked in orange. We have to correct for this difference in order to obtain the correct source energies such that shot 1 and 4 have the same energy content. If we do not correct this, the ϕ and ψ estimations will get more incorrect the further the shot is away from the center of gravity. This is because the correction that is needed to obtain the correct energy is a function of distance to the center of gravity. This inaccuracy in the energies used to determine the orientation might lead to orientation errors.

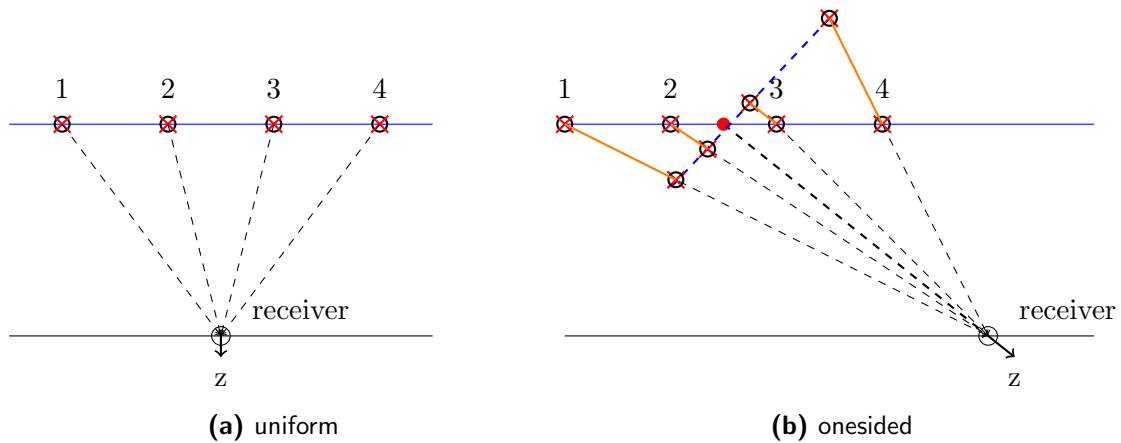


Figure 5-15: Correction of Center of Gravity pre-rotation.

Chapter 6

Model-based Orientation Determination

6-1 Orientation determination

The (approximate) knowledge of the physical properties of the seabed can be used to determine the orientation of a receiver laying on the seabed. One could also think of applications where the knowledge of the exact orientation can be used to deduce the physical properties of the seabed. But I will restrict myself to the first case. In Chapter 3 I have shown that the angle of incidence is not equal to the polarization angle which we deduce from the recorded data. Unfortunately the angle of incidence is used to find the orientation of a receiver on the seabed. We have also seen, that the error introduced by assuming the angle of incidence is equal to the polarization cancels out if uniform shot coverage is given (Table 5-1). I have also shown that we are able to compute the difference between the estimated polarization angle and the angle of incidence. The proposed method includes the knowledge of the seabed via the following operations:

1. **Compute** the polarization angle (and its respective ϕ_0 and ψ_0 component) for every shot receiver pair using the sequential approach and 2D interpolate the estimated polarization angles. The inclination and azimuth correction term that is applied during the orientation estimation of the sequential approach (compare Appendix A-1-2) is additionally weighted by the v_P velocity ratio of the water and the seabed $\left(\frac{v_{p,water}}{v_{p,seabed}}\right)$. Remember that the correction term in the sequential approach subtracts the inclination from the the estimated polarization angle, this will be of importance. The weighting factor is not needed when the sequential approach is used to estimate the orientation because the mode conversions are not taken into account. Not considering mode conversions is the same as weighting by a velocity ratio equal to 1.

2. **Model** the difference between the angle of incidence and the polarization angle (i.e., the angle discrepancy) for either a fluid-fluid (acoustic) boundary or a fluid-solid (elastic) boundary. The angle discrepancy is computed using the geometry of the acquisition (receiver depth, source depth, source-receiver offset) and the approximate physical properties of the seabed and the water (v_p , v_s , ρ). The angle difference is computed for the same 2D surface that was introduced in the computation of the polarization angle. Now the difference of the incident and polarization angle for every shot point of the survey can be deduced from this 2D angle discrepancy distribution. Here it is assumed that the angle of incidence is $\text{atan}(\text{offset}/\text{receiver depth})$.

The response of the seabed is modeled using the modified version of the algorithm developed by Chapman (2004) as it was introduced in Chapter 3-2-3. Here the error discrepancy for two layers (water and seabed) over a semi-infinite half space is modeled. The error discrepancy as it used here was derived earlier. It is given by Equation 3-25.

3. The 2D angle discrepancy distribution (compare Figure 3-4) is weighted by a cosine evaluated with the source-receiver azimuth. This is done to have a smoother angle discrepancy distribution around axis off zero azimuth (i.e., the preferential x-axis direction of the receiver).
4. The angle discrepancy term is applied to the polarization angle distribution to obtain the angle of incidence distribution.

The procedure of algorithm described above is visualized in Figure 6-1.

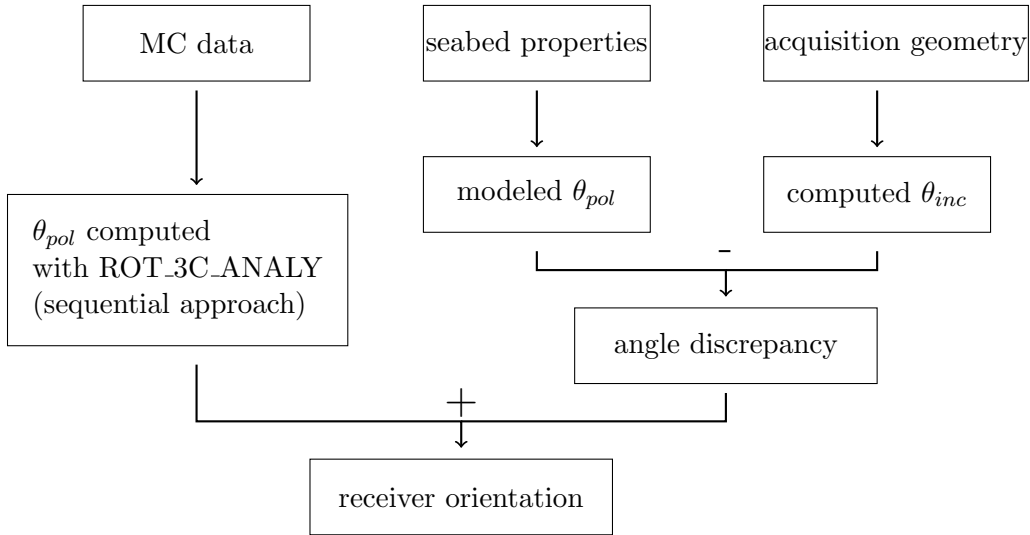


Figure 6-1: Schematic procedure of the seabed approach.

6-2 Synthetic data

We will use the medium water depth synthetic data set created in ANIVEC to understand the influence of an acoustic and an elastic seabed on the difference between the polarization and incident angle, here on referred to as angle discrepancy. The synthetic parametrization of the model can be found in Table 4-1. The synthetic data set simulates the results for one common shot gather with a receiver at a depth of 100 m. The angle discrepancy will then be used to correct the estimated polarization angle to find the correct angle of incidence. In an ideal case we expect that every single source- receiver pair should be able to retrieve the correct orientation of the receiver. We will analyze the different behavior of a seabed with different character (i.e., elastic or acoustic).

6-2-1 Fluid- Fluid boundary

Figures 6-2 and 6-3 show the results of the orientation estimation if the seabed is treated as an **acoustic**, rather than an elastic boundary. The most **left panels** in Figure 6-2 and 6-3 show the ϕ and ψ component of the polarization angles as they are determined with the sequential approach, respectively. Note that the data was rotated with $\phi = 10^\circ$ and $\psi = -10^\circ$. The red dots indicate the shot positions and the yellow dot indicates the receiver position of the common receiver gather data. After the shot-receiver dependent polarization angles and the corresponding ϕ and ψ values are determined, they are interpolated to the entire area covered by shots. The interpolated ϕ and ψ distributions are shown as the color coded circle underlying the shot positions.

The **middle panel** shows the correction term (i.e., angle discrepancy) that is needed to correct the polarization angle to obtain the angle of incidence. It was computed using the modified version of an algorithm introduced Chapman (2004). Since the correction term depends on the properties of the seabed, the same v_P and ρ values were used for the computation of the correction as for the generation of the synthetic data. Note that the area outside of the radial angle distribution corresponds to shot-receiver pairs with an angle of incidence that are beyond the critical angle and thus has a complex reflection coefficient. The condition for complex valued coefficients is given by Equation 3-17. The angle discrepancy can not be computed for these points. Shot-receiver pairs with a shot positions beyond the critical angle will not be considered and are discarded. The most **right panel** in 6-2 and 6-3 show the results after the correction term is added to the interpolated polarization angle distribution. These panels correspond to a angle of incidence distribution. Remember that the polarization angle estimation is already corrected for the inclination. So these panels rather show the **orientation of the receiver** than the angle of incidence! Or in other words: Panel 1 + Panel 2 = Panel 3. We can observe that the angle of incidence distribution is more balanced and generally does a much better job in directly recovering the applied rotations ϕ and ψ for every shot receiver pair. We find that both the most left and most right panel have about the same average value of around $\phi_0 = 10^\circ$ and $\psi_0 = -10^\circ$. So both approaches recover the trial rotations. This is expected since the shot coverage is uniform and errors in the sequential approach are canceled. However, a big difference can be observed in the standard deviation of the values in panels 1 and 3. The angles ϕ_0 and ψ_0 as estimated with the sequential approach have a standard deviation of 13.6° and 19.9° respectively. After the correction term is applied they decrease to 4.6° and 5.8° .

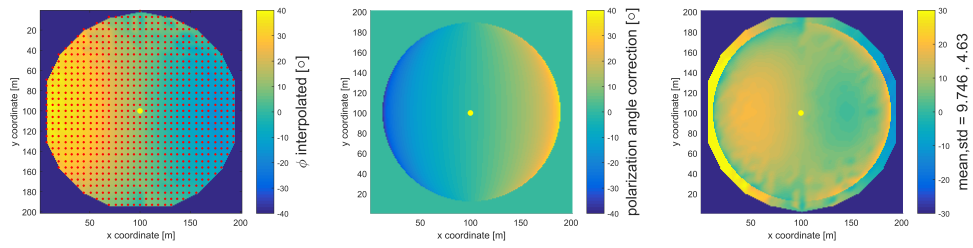


Figure 6-2: Determination of ϕ_0 : Trial rotation applied to the data is $\phi = 10^\circ$. See text for explanation of the three panels.

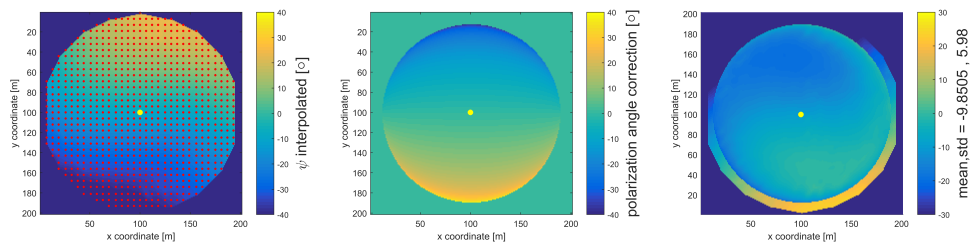


Figure 6-3: Determination of ψ_0 : Trial rotation applied to the data is $\psi = -10^\circ$.

6-2-2 Fluid- Solid boundary

Figures 6-4 and 6-5 present the results for an elastic seabed in the same way as for the acoustic seabed. By considering an elastic seabed the shear properties are additionally included in the computation of the correction term (i.e., angle discrepancy). Observe the different shape of the correction term in the middle panel in comparison to the correction terms used in the acoustic case. The correction term shows a polarity reversal before the reflectivity coefficient becomes complex. This behavior is also described by Chapman (2004). As v_P , v_S and ρ are the same values as in the ANIVEC model.

Again, the estimation of ϕ_0 and ψ_0 gives the expected values of 10° and -10° for the sequential and seabed approach. Again, this is because of uniform shot coverage the sequential approach is able to recover the correct orientation of the receiver. We can also observe a decrease of the standard deviation from 13.6° and 19.9° to 3.8° and 3.9° for ϕ_0 and ψ_0 respectively. In the computation of the standard deviation of the seabed corrected orientations of ϕ_0 and ψ_0 the shot-receiver pairs corrected with a polarity flipped correction term were excluded.

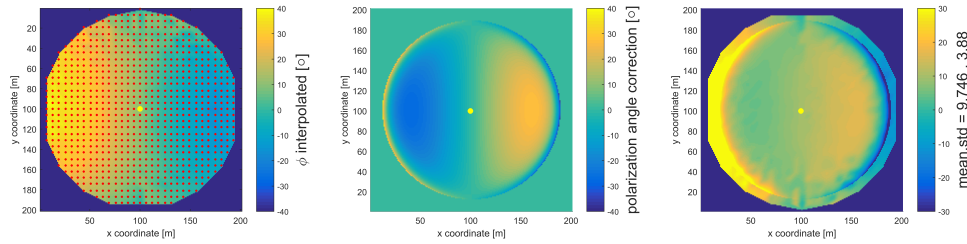


Figure 6-4: Determination of ϕ_0 : Trial rotation applied to the data is $\phi = 10^\circ$. The boundary region was excluded from the standard deviation calculation.

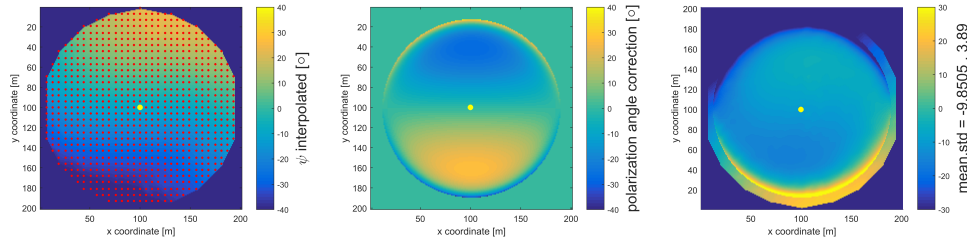


Figure 6-5: Determination of ψ_0 : Trial rotation applied to data is $\psi = -10^\circ$. The boundary region was excluded from the standard deviation calculation.

6-2-3 Results Synthetic

We have seen that by including the seabed properties into the orientation determination were where able to obtain better results than with the Omega implementation of the sequential approach. Both, the inclusion of the elastic and acoustic seabed properties resulted in smaller standard deviations for ϕ_0 and ψ_0 . In terms of the standard deviation the elastic seabed approach outperformed the acoustic seabed approach. The implication of a small standard deviation is that a single trace is more likely to give the exact orientation- or at least a close estimation- directly.

6-3 Real data

Now the seabed approach is applied to the North Sea data set. It is known that the seabed around the receiver line has a $v_p \approx 2000$ m/s. The v_p/v_s ratio is about 3-4 (Bagaini, 2016). Recall that because of the hard seabed a inclination of around 90° is expected for the z component of the receiver. This corresponds to values of around 0° for ϕ_0 and ψ_0 . The water depth is around 100 m. I will first approximate the seabed as an acoustic interface. Secondly, I will include the v_s information and treat the seabed as an elastic surface. I will show the results obtained for a single common receiver gather as well as for the entire receiver line. The common receiver gather shown for both, the elastic and acoustic case, was extracted from the middle of the receiver line (common receiver gather number 9 of the North Sea data set).

6-3-1 Acoustic seabed

Figures 6-6 and 6-7 show the results after the seabed properties approach was applied to a common receiver gather from the North Sea data set. The most left panel in Figures 6-6 and 6-7 show the single trace estimations of ϕ and ψ for every shot position (red dots). The estimations are then interpolated within the bounds of shot positions with the biggest offset. Observe that the shot coverage is not uniform around the receiver (yellow dot). The middle panel visualizes the correction term (i.e., the angle discrepancy). Observe that the computed angle discrepancy is adjusted to the non-uniform shot coverage. The most right panel shows the corrected polarization angle estimations. Remember that the corrected polarization angle is not the angle of incidence but the orientation of the receiver (polarization angle is already corrected for the inclination). From the corrected ϕ and ψ estimations, ϕ_0 and ψ_0 are determined. We find $\phi_0 = 0.6^\circ$ with a standard deviation of 4.85° and $\psi_0 = 1.3^\circ$ with an standard deviation of 5.1° . The standard deviation is computed from all the ϕ and ψ estimations (Figure 6-8) used for computing ϕ_0 and ψ_0 .

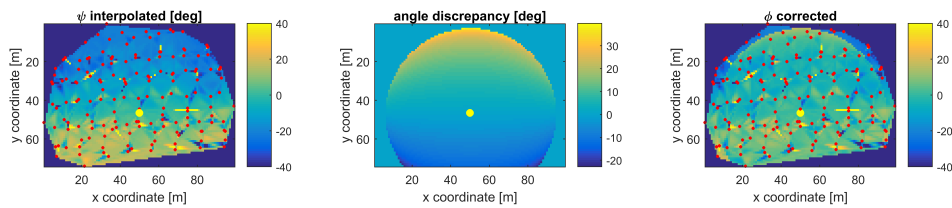


Figure 6-6: Determination of ϕ_0 : seabed is acoustic with $v_p = 2000$ ms.

Figure 6-8 shows the trace by trace orientation estimation of ϕ and ψ for the approximately 160 traces in the common receiver gather. In Figures 6-7 and 6-6 the shots corresponding to these traces are marked by red dots. We can observe that the single trace determination is much closer to 0° after we corrected for the difference between the polarization angle and angle of incidence. As a consequence, the standard deviation for the ϕ estimation decreases from 13.7° (Omega) to 4.85° . For ψ the standard deviation decreases from 18.2° (Omega) to 5.1° .

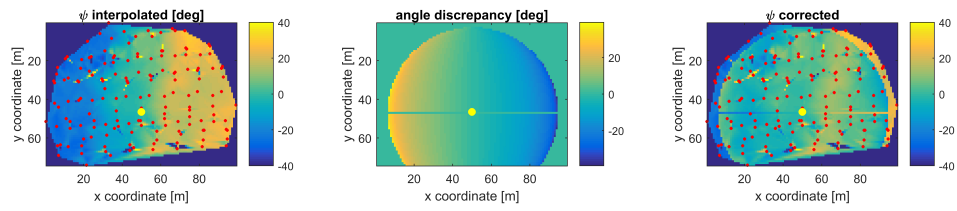


Figure 6-7: Determination of ψ_0 : seabed is acoustic with $v_p = 2000$ ms.

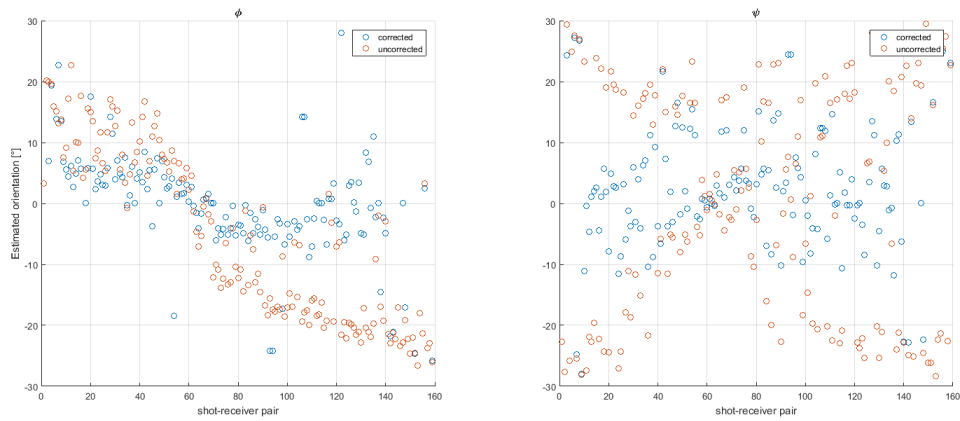


Figure 6-8: Detail comparison of results for the trace by trace orientation determination with and without the correction term applied.

6-3-2 Elastic Seabed

Instead of modeling the response of an acoustic seabed I will now consider an elastic seabed with $v_p = 2000$ ms and $v_p/v_s = 3$. Figures 6-9 and 6-10 have the same setup as Figures 6-6 and 6-7. Observe the difference in the correction term (middle panel) that arises from the different boundary conditions that are implied by considering an elastic seabed rather than a acoustic seabed. The estimated ϕ_0 value is -4.5° with a standard deviation of 5.6° . ψ_0 is equal to 1.1° with a standard deviation of 5.3° .

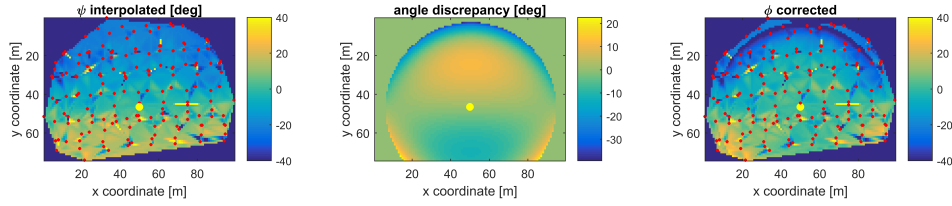


Figure 6-9: Determination of ϕ_0 : seabed is elastic with $v_p = 2000$ ms and $v_p/v_s = 3$.

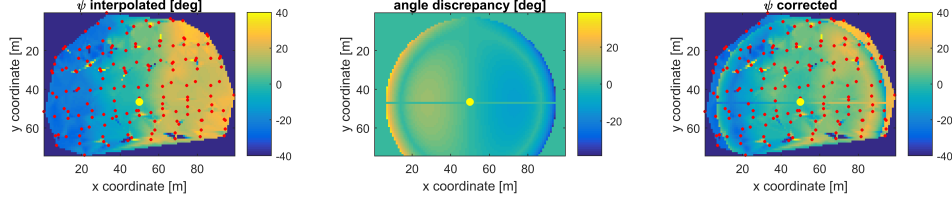


Figure 6-10: Determination of ψ_0 : seabed is elastic with $v_p = 2000$ ms and $v_p/v_s = 3$.

Figure 6-11 shows the effect of the seabed properties correction on the trace by trace dependent estimation of ϕ and ψ . The standard deviation of ϕ and ψ decreases from 13.7° and 18.2° to 5.62° and 5.3° respectively.

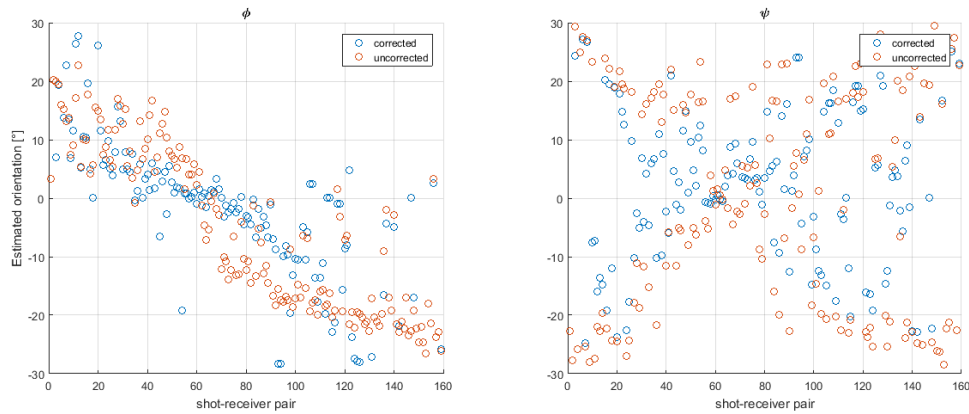


Figure 6-11: Detail comparison of results for the trace by trace orientation determination with and without the correction term applied.

6-3-3 Results Real data

As mentioned above, a z inclination of about 90° is expected for the receivers of the North Sea data set. We will now use this expectation to compare the obtained results. Figure 6-12 shows the inclination of the z axis of the receiver after ϕ_0 and ψ_0 are determined and applied to the receiver. We can find that the acoustic seabed approach recovers z inclinations centered around approximately 88° with deviation of $\pm 2^\circ$. The elastic seabed approach on the other hand recover steeper z inclinations than ROTATE_3C_ANALY does. The results seem to follow the results obtained in Omega.

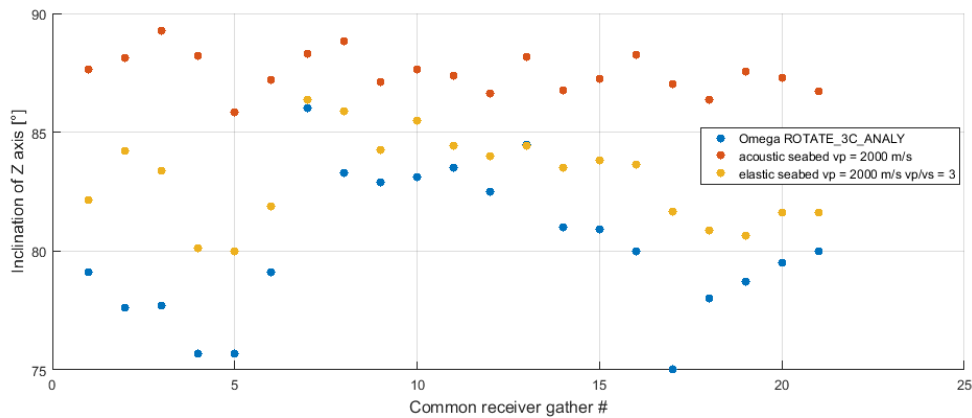


Figure 6-12: Comparison of the z inclination after ϕ_0 and ψ_0 are determined using ROTATE_3C_ANALY, the acoustic and the elastic seabed approach.

6-4 Discussion

I will first discuss the sensitivity of the elastic seabed approach to different v_p/v_s ratios of the seabed. I will use the findings from this discussion and the presented seismic experiment to discuss the results obtained above.

6-4-1 Sensitivity to the knowledge of the seabed elastic properties

To understand the influence of the seabeds **elastic** properties used for modeling the angle discrepancy, another experiment was performed. The goal of the experiment is to see how exact the knowledge of the seabed velocities v_P and v_S needs to be to recover the correct orientation of the receiver. I have shown that for uniform coverage we are able to always retrieve the correct orientation. That is why the synthetic data set (water depth= 100 m) was cropped to simulate one sided shot coverage (compare Figure 6-14b). Then ϕ and ψ were determined using the sequential approach (no pre-rotation towards center of gravity applied). These estimations are corrected by including the modeled angle discrepancy of an elastic seabed. From the corrected ϕ and ψ estimations, ϕ_0 and ψ_0 are determined. These three steps are shown exemplary in Figure 6-13. Since we are dealing with one sided shot coverage we expect that the sequential approach will fail to estimate the correct ϕ_0 and ψ_0 values. This is because the errors committed by assuming the angle of incidence is equal to the polarization angle will not cancel out. However, it is expected that the modeled correction term should be able to make up for the error committed by this assumption. In the most ideal case every shot-receiver pair should be able to retrieve the orientation of the receiver. We expect to be able to retrieve the correct orientation of the receiver if we have perfect knowledge of the seabed. This is because with perfect knowledge of the seabed we can compensate for the difference between the angle of incidence and the polarization angle.

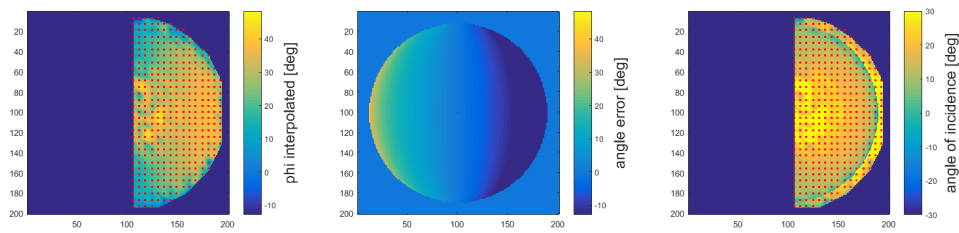


Figure 6-13: Left: ϕ -distribution determined with the sequential approach. Middle: angle discrepancy for $v_{P2} = 2.2$ km/s and $v_{S2} = 0$. Right: ϕ -distribution after it is corrected for angle discrepancy. From this we find $\phi_0 = 28.7^\circ$.

Figure 6-15 shows ϕ_0 as a function of the seabed velocities. This means that the trace dependent ϕ estimations were corrected with different angle discrepancies based on different seabed velocities v_p and v_s . The exact seabed velocities from the synthetic data set are known to be $v_p = 2000$ m/s and $v_s = 1000$ m/s. Additionally, the data was rotated with $\phi_0 = 10^\circ$ and $\psi_0 = -10^\circ$. These are the orientations that have to be retrieved.

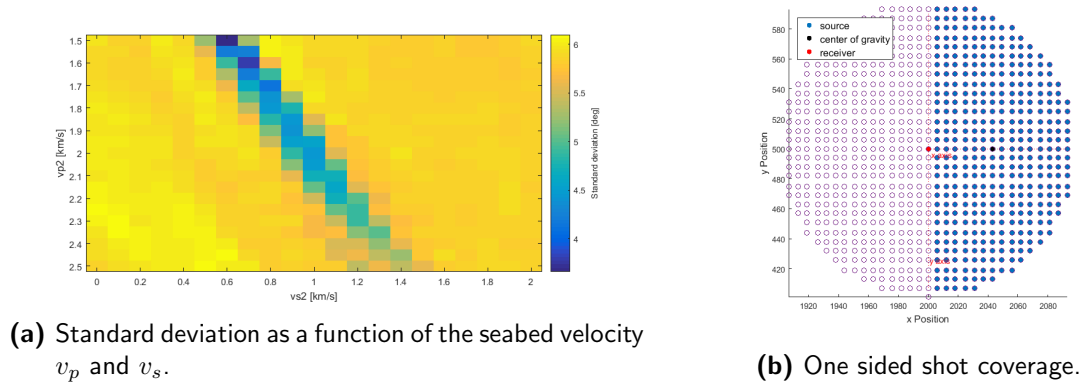


Figure 6-14: Standard deviation and shot coverage of synthetic experiment.

We can observe in Figure 6-15 that $\phi_0 = 10^\circ$ is retrieved if the velocities used to model the angle discrepancy are the same as in layered earth model that was used to generate the synthetic data ($v_p = 2$ km/s $v_s = 1$ km/s). But we can also observe that the value of $\phi_0 = 10^\circ$ is also found if the ratio of seabed velocities v_p/v_s used to model the seabed response is equal to the ratio used to generate the synthetic data ($v_p/v_s = 2$). The estimation of ψ_0 is not affected by the different choices for v_p and v_s and the correct value for $\psi_0 \approx -10^\circ$ is always retrieved-independent of the velocity ratio. This is the case because both, the correction term and the polarization angle estimation are symmetrical around the x axis. This symmetry will cancel the errors committed even if v_p and v_s used to model the angle discrepancy is not equal to v_p and v_s used to generate the synthetic data. We have observed a similar behavior already. Compare Tables 5-1 through 5-4 where the results for different shot coverages and different inversion approaches were shown earlier.

This shows that the exact seabed velocities do not need to be known, it seems to be sufficient that the ratio of v_P/v_S is known to compute a good angle discrepancy correction. We can find that this ratio also governs the Poisson's ratio ν :

$$\nu = \frac{1}{2} \frac{(v_p/v_s)^2 - 2}{(v_p/v_s)^2 - 1} \quad (6-1)$$

Going back to Figure 6-15 it seems to be very important to have a very good understanding of the seabeds Poisson's Ratio in order to use the angle discrepancy correction term. We have seen that for a $v_P/v_S = 2$ the correct values of $\phi_0 \approx 10^\circ$ and $\psi_0 \approx -10^\circ$ can be recovered. But if a wrong v_P/v_S of 1.7 is used, the estimated ϕ_0 value becomes $\approx 33^\circ$ (compare Figure 6-15). This is a mismatch of 23° .

The importance of the correct Poisson's Ratio is also confirmed if we take a look at the standard deviation of ϕ_0 as a function of v_p and v_s as shown in Figure 6-14a. We can find that the standard deviation increases relatively fast once other v_p/v_s -ratios than the ANIVEC model are used to compute the angle discrepancy. The standard deviation of ψ_0 is not shown, but a very similar result is obtained.

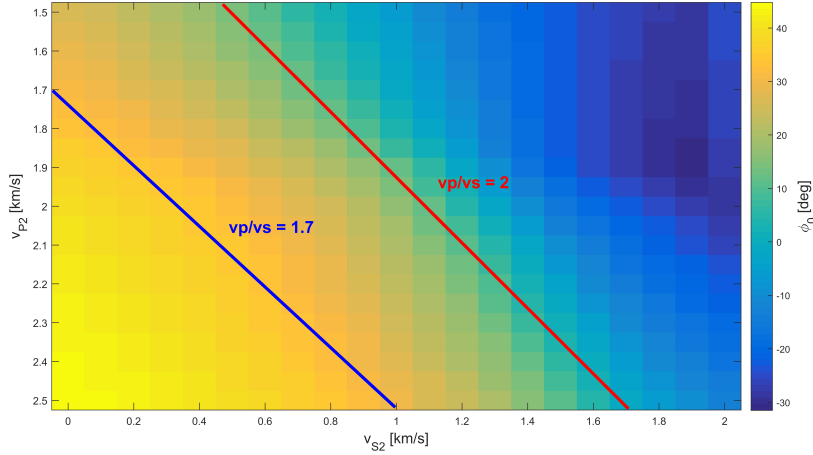


Figure 6-15: ϕ_0 as a function of v_{p2} and v_{s2} , which are used to model the angle discrepancy correction.

6-4-2 The results

We have applied the seabed approach for an elastic and an acoustic seabed to synthetic and real data. We have seen that for the synthetic data we were able to retrieve the correct orientation of a tilted receiver with uniform shot coverage. The determined orientation was approximate equal to the orientation determined by the sequential approach but with a much smaller standard deviation. We have seen in the above section that a smaller standard deviation indicates that we are able to retrieve the orientation of a receiver with the seabed approach even in the situation of onesided shot coverage.

In the most ideal case the standard deviation of ϕ_0 and ψ_0 estimated for **synthetic** data is expected to be equal to 0° since the exact knowledge of the seabed is used for the computation of the angle discrepancy. The fact that the standard deviation is not equal to 0° is explained with the inaccuracies in picking the first break window. Imperfect first break windows will result in slightly wrong energies that are used in the estimation of the polarization angle. Recall that the polarization angle in the sequential approach is found by determining the minimal energy of the horizontal first break energies. Another factor leading to the relatively small standard deviation as we observed it could be founded in the way how ANIVEC and the modified Chapman algorithm work. In ANIVEC the seabed response is computed with a model of three layers. Chapman determines the response of the seabed from one layer over an infinite half-space.

We have seen that by modeling a elastic seabed rather than a elastic seabed we were able to obtain better results in terms of the standard deviation. This is somewhat expected: the better we are able to compute the response of the seabed, the better the seabed approach is able to compute the correct angle discrepancy.

Opposite to the results observed for the synthetic data, we find that the acoustic approximation of the seabed gives better results than the elastic approximation when applied to the North Sea data set. The z inclinations for the acoustic seabed approach are much closer to the expected 90° (compare Figure 6-12) than for the elastic seabed approach. If v_P/v_S is not

sufficiently well known it is even more likely that an additional error is introduced to the orientation estimation if the ratio is roughly estimated. The elastic seabed approach however performs better than ROTATE_3C_ANALY in retrieving inclinations close to 90° .

The superiority of the acoustic to the elastic approach for real data is counterintuitive. But recall that the v_P/v_S ratio was given as 3-4. We have seen above that the accuracy of this ratio is crucial for the successful use of the elastic seabed approach. We have seen above that a v_P/v_S ratio difference of 0.3 from the actual ratio might lead to an orientation error of about 20° (compare Figure 6-15). So it might be very likely that a v_P/v_S ratio estimated as 3-4 is not accurate enough. I would expect a range for v_P/v_S ratio of eg. 3.1- 3.3 as accurate enough to successfully use the elastic seabed approach.

Another error source might be the insufficient approximation of the seabed in the modified Chapman algorithm. The angle discrepancy used to correct the ϕ and ψ distribution is modeled with a very simple seabed of one layer with one v_P and one v_S . In reality we have to deal with a seabed that has varying v_P and v_S with depth (Dong et al., 2013). A seabed like this is not sufficiently represented by a simple one layer model.

It is clear that the seabed approach is a very powerful processing tool for determining the correct orientation of a receiver. But it comes with the computational cost of modeling the angle discrepancy for every common receiver gather. For the common receiver gather of receiver number 9 (eg. Figure 6-9) the overall orientation determination lasts approximately 100 seconds. The estimation of the ϕ and ψ lasts about 55 seconds, the computation and application of the modeled angle discrepancy lasts about 45 seconds. So the computational cost is doubled.

Conclusions and Outlook

In this thesis I presented novel model and geometry based algorithms to determine the orientation of a 3C seabed sensor. Both methods were applied to synthetic and real data. The results obtained for the model and geometry based approaches were discussed earlier on in the respective Chapter.

7-1 Conclusions

In Chapter 5, I presented different algorithms that make use of a pre-rotation of 3C data to the source center of gravity. We have seen that the unknown orientation of a receiver can always be nearly perfectly retrieved if uniform shot coverage is given. This was shown by using synthetic 3C data which were permuted with a known rotation. In the case of uniform shot coverage a pre-rotation to the center of gravity does not effect the orientation estimations since the center of gravity is directly above the receiver.

However, in the case of non-uniform shot coverage (i.e., one sided coverage) the pre-rotation of the data showed major improvements in comparison to the results obtained with Omegas ROTATE_3C_ANALY (sequential approach) when un-rotated input data is used. The orientation of the receiver is retrieved with an accuracy of $\pm 2^\circ$ when Omegas sequential approach is used with pre-rotated input data. The newly implemented simultaneous and Least Squares approaches are able to retrieve the orientation of the receiver with an accuracy of $\pm 1^\circ$.

From the synthetic seismic experiments we can conclude that a pre-rotation to a more homogeneous shot distribution improves the orientation estimation.

The algorithms were also tested on real data acquired in the North Sea. The use of pre-rotated data also led to better orientation estimations. The sequential approach gave the best results in retrieving the expected value for the z axis inclination, after the receiver was corrected for its orientation and was turned flat. The simultaneous and Least Squares approach did not show major improvements when applied to the real data. The simultaneous approach is suffering from choosing a good starting point for the gradient search. The Least Squares approach showed a very high variability in estimated z inclinations which is not desirable. Overall the

application to the real data shows that ROTATE_3C_ANALY might be easily improved by implementing the pre-rotation in the existing FORTRAN code. The implementation should be relatively straight forward since the Matlab routines used to determine the orientations are very similar to the FORTRAN routines. One question that has to be addressed and clarified is how dense does the shot coverage need to be in order to be sufficient as input to ROTATE_3C_ANALY.

I also found that the using quaternion multiplication and matrix multiplication using Euler angles as input lead to the exact same results. Also the computational efficiency is not increased. It seems that the Euler rotations used in ROTATE_3C_ANALY are sufficient for the use rotation operations applied to the seismic data. These results are not shown in the thesis.

In Chapter 6, I present the model based seabed approach. We have seen that the seabed approach is a very powerful processing approach and we are able to retrieve the correct orientation nearly perfectly. The residual uncertainty is expected to be caused by the insufficient accuracy of the first break picks. Using synthetic data I was able to confirm the intuitive expectation that by modeling an elastic seabed rather than an acoustic seabed, the orientation estimation and its respective standard deviation is improved. Nevertheless, the acoustic approximation of the seabed seems to be a good one and sufficient as it is used in the orientation determination. We find that the major advantage from the model based approach to the data driven approaches (Chapter 5) is that the orientation can be determined from a single trace, since all the single trace estimations of ϕ and ψ are almost equal to ϕ_0 and ψ_0 . In order to reduce the inaccuracy in the estimation, introduced by insufficient first break picks, it is still important that all trace dependent estimations are averaged.

The seabed approach gives really good results when applied to the North Sea data set. We find that by using the acoustic approximation of the seabed, nearly all the receivers can be rotated flat on the seabed and exhibit the expected z inclination of 90° . The use of the elastic seabed approach only shows minor improvements. This is counter-intuitive and opposite to the findings made on synthetic data. This unexpected behavior is explained with the accuracy of the v_p/v_s -ratio that is needed to successfully model the response of an elastic seabed. It is very important that this ratio is determined and given as close as possible to the actual value. If this is not possible errors might be introduced and the elastic seabed approach might even degrade the estimated orientation. Anyhow, the acoustic approximation seems to be sufficient for the North Sea data set and the given v_P . The improved orientation determination comes with the price of a nearly double computational cost. Additionally, the approach seabed requires major changes in the ROTATE_3C_ANALY since the entire modeling procedure of the seabed approach would have to be included in the algorithm.

7-2 Outlook

In this thesis orientation determination algorithms were discussed and introduced that assume that the sea surface and the seabed are parallel. This assumption is not valid when the seabed is very rough and characterized by a variable topography. In this case the entire routines will have to be reviewed, since the angle of incidence based orientation algorithms are likely to fail if not modified for this new situation. Another open question is in how far the good results obtained for seabed seismics might be transferred to the data acquisition on land. Another interesting question could be if the results obtained for a hard seabed are also valid for a soft seabed with a much higher v_p/v_s -ratio. The probably biggest step would be to implement the pre-rotation into the existing Omega routine, this relatively small change might hold the key to major improvements. This would require more testing of the pre-rotation approach on real data to make sure that the desired results are obtained.

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Orientation Estimation in OMEGA

A-1 Orientation estimation in OMEGA

In this section the orientation determination algorithm as used in Schlumberger's seismic processing package OMEGA will be explained. The seismic function module (SFM) that determines the orientation of a receiver is ROTATE_3C_ANALY. This SFM will be the reference and all algorithmic improvements will be compared with it. The estimated orientations are applied to the data set with the ROTATE_3C_APPLY SFM.

A-1-1 Finding the polarization angle

In OMEGA the orientation of the receiver is determined in three steps. In a first step the data is rotated about the x axis to find the rotation angle ϕ_0 that minimizes the energy on the y component. In the second step ψ_0 is found by rotating the data about the y axis minimizing the x energy. In the last step the orientation of the x- axis with respect to an azimuth of choice is found, this angle is θ_0 . This behavior is referred to as sequential. The orientation estimation in OMEGA is based on three assumptions:

1. The included MC data has been restricted to a time window around the first arrivals. This restricts the recorded energy to the direct arrivals with a propagation direction equal to the source-receiver azimuth (SRAZ) direction. Reflected, upgoing energy is excluded.
2. By not including the properties of the seabed we assume that the **angle of incidence of the P wave is equal to the polarization angle** found from the recorded data. This is not the case because of refraction and mode conversion at the seabed. However, when several traces are used, these errors on individual traces tend to compensate each other and, if the shot coverage is uniform, the estimated orientation is correct.

3. Due to the polarization of the P wave we know that the energy in the x-y plane and thus the energy recorded by the x- and y-component of the receiver is at a minimum if the z component is parallel to the angle of incidence of the P wave.

In OMEGA the orientation estimation is a trace by trace operation acting on a common receiver gather data. This means that the orientation for one shot- receiver pair is estimated one at a time. All these trace dependent estimations are then summed up and averaged to give the overall (or global) orientation. In the following I will describe the orientation estimation of a single shot-receiver pair.

In a first step the first breaks on the x , y and z component are extracted. Now the first break energy is rotated around the x- axis by applying trial rotations ϕ . The trial rotations are applied with the rotation matrix R_x as defined in Equation 2-3. The goal of the rotation is to find the angle ϕ that minimizes the energy (= seismic amplitude squared) of the rotated y component. The angle ϕ is equal to the polarization angle (i.e., angle of incidence of the P wave) for the single shot-receiver pair but it still has to be corrected for the geometry of the source-receiver pair (compare below).

This operation is performed on every source-receiver pair in the common receiver gather. The estimated ϕ values for every source-receiver pair are then summed up and divided by the amount of pairs. This global rotation angle is ϕ_0 . The angle ϕ_0 is then applied to the data and it re-orientes the y- and z- axis of the receiver. This finalizes the first step in the sequential orientation estimation. The ψ_0 estimation works similarly to this with the difference that trial angles ψ around the y axis are applied to minimize the x energy.

After ϕ_0 and ψ_0 are applied the receiver is numerically rotated with a flat base at the seabed. This means that the energy on the traces is now distributed in such a way as if the seismic signal would have been recorded with a receiver that laid flat on the seabed. Problematic at this point is the 180 °ambiguity of ϕ_0 and ψ_0 . A rotation of ϕ_0+180° for example, results in the same minimal energy (seismic amplitude squared) as a rotation of ϕ_0 . This can be observed in Figure A-1a and A-1b. This ambiguity comes from the fact that the receiver might be planted up-side down. This means that the z axis of the receiver is pointing towards the sea surface rather than towards the seabed. This ambiguity is resolved by cross-correlating the recorded pressure with the seismogram of the z component.

After ϕ_0 and ψ_0 are estimated the algorithm estimates the rotation around the z axis with angle θ_0 to direct the x axis in a direction of choice. This could either be the geographical north or the source-receiver direction. The angle θ_0 is found on a similar way as ϕ_0 was estimated. Again, the first break energy of the y component is minimized after it was rotated with some trial angle. Now the trial angle is θ and it is applied to the data using the rotation matrix R_z . Again, all the θ rotations that minimize the energy on a single trace are summed and divided by the number of used traces. The averaged angle is θ_0 and it is the angle difference between the current direction of the x axis of the receiver and the north direction which has an azimuth of 0 °. If θ_0 is applied to the data, an acquisition with a receiver axis pointing to the north (or north/west eg. θ_0+45°) is simulated, compare Figure 1-1a) is simulated. If a source-centered acquisition is simulated every shot receiver has to be rotated by a trace dependent θ_{r-t} value. The angle $\theta_{r-t} = -\theta_0 + SRAZ$ will minimize the energy on the radial component of the shot receiver pair with a source-receiver azimuth $SRAZ$.

The following plot should give a more visual explanation of what is happening inside the the OMEGA routine. Figure A-1 shows 4 triplets of multicomponent data. Figure A-1a shows the original x,y and z data as it was recorded. The other three triplets show rotated versions

of A-1a. The rotation was applied around the x axis using Rx and the data belongs to a time window around the first breaks. So the rotation convention and the input data are the same as in the OMEGA routine. Observe that a rotation of 180° results in a polarity flip of the two rotated components y and z. The x component is not effected by the rotation at all since it is the rotation axis. Recall that the OMEGA routine searches the trial rotation angle that minimizes the energy on the y component in order to find the polarization angle (i.e., in this case: the angle of incidence). For the 4 rotations in Figure A-1 we can find that a rotation of $-20^\circ (= 340^\circ)$ results in the smallest overall energy on the y component.

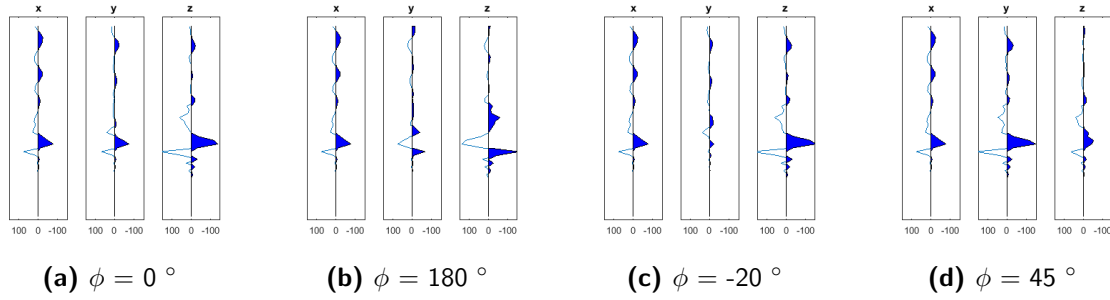


Figure A-1: Multicomponent traces after the application of a rotation around the x axis

A-1-2 Correction of the polarization angle

Recall that in ROTATE_3C_ANALY the assumption is made that the polarization angle is equal to the angle of incidence. So the angles ϕ_0 and ψ_0 are the angles that rotate that rotate the x-y plane of the receiver perpendicular and the z component parallel to the angle of incidence. This situation has to be corrected for to turn the z component perpendicular to the seabed. The needed correction term can be found from geometrical considerations of a shot-receiver pair.

The corrections contain a term for the **inclination** and the relative **azimuth** between the source-receiver- direction and the cable azimuth (i.e., the expected x- axis orientation).

- **Step 1:**

The following correction term is computed:

$$correct_1 = abs(atan(SSDD/(DD - SD))) \quad (A-1)$$

Where $SSDD$ is the source-receiver offset, and $DD - SD$ is the height difference between the source and the ocean bottom receiver. This correction computes the inclination of the incoming wave field recorded by the receiver. Or in other words: If the source would be located directly above the receiver, $SSDD$ would be equal to zero and thus the whole correction term would go to zero. So if the source is directly located above the receiver we don't have to correct the recorded data. Physically this makes sense, since we assume that the incoming wave field has no energy in the x-y plane and all the energy is already focused on the z component.

- **Step 2:**

In this step we compute the final correction that will be applied by simply adding it to the inverted orientations of the source-receiver pairs (figure above).

Correction applied to y-axis orientation (1st rotation around x- axis with R_x):

$$correct_2 = correct_1 * \sin((SRAZ - RCLD) * \pi/180) * 180/\pi \quad (A-2)$$

Correction applied to x-axis orientation (2nd rotation around y- axis with R_y):

$$correct_2 = correct_1 * \cos((SRAZ - RCLD) * \pi/180) * 180/\pi \quad (A-3)$$

Where $SRAZ$ is the computed source-receiver azimuth and $RCLD$ is the prescribed and/or expected cable orientation and thus the x direction of the MC receiver. If $correct_1$ is equal to zero (source above receiver) also the overall correction term $correct_2$ will be equal to zero.

In the following I will explain and clarify the need and the physical meaning of these different correction terms by considering different constellations that we could face in terms of receiver orientation and source- receiver geometry constellation.

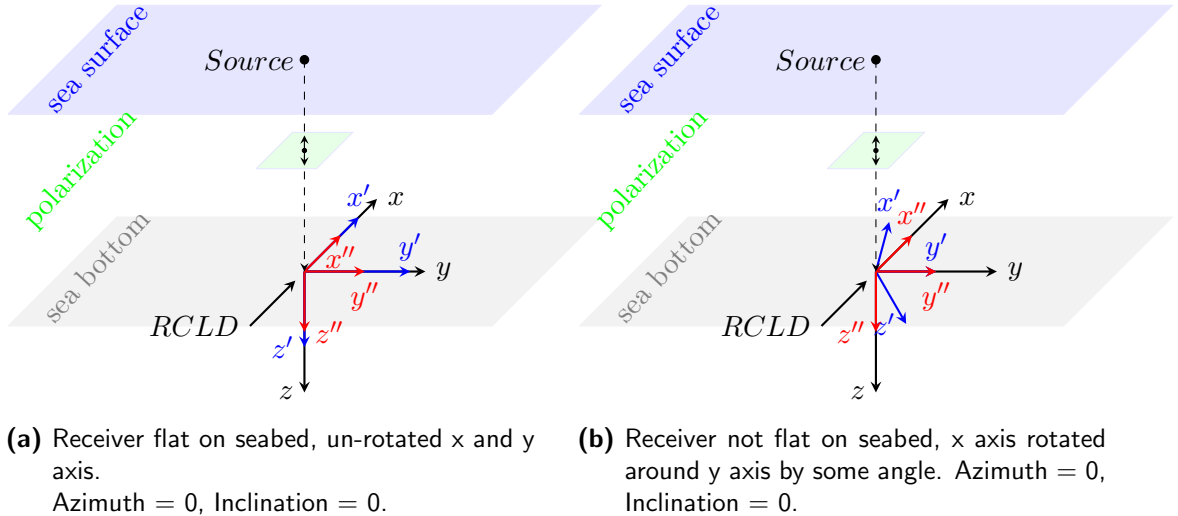


Figure A-2: The blue coordinate system shows the orientation of the MC receiver axis. The red coordinate system shows the orientation of the receiver axis after minimizing the energy in the x-y-plane. Observe that x'' is parallel to the source receiver vector i.e., x'' has the same inclination as the angle of incidence.

I will start by considering a setup as in Figure A-2a. It is intuitive that there is no need for any correction since the receiver is already flat on the seabed. However, the physical reason for this is that the both the inclination and the azimuth are equal to zero. More over, the inversion algorithm will be able to find the correct angles ϕ_0 and ψ_0 because the x- and y- component of the receiver are already oriented perpendicular to the incident wave (here shown as a ray).

In Figure A-2b we change the setup a bit by rotating the x axis of the receiver by some angle around the y axis. Again the inversion algorithm will be able to find the correct orientation. The inversion algorithm will determine the angle needed to flatten the receiver by rotating x' around y' . This angle has not to be corrected.

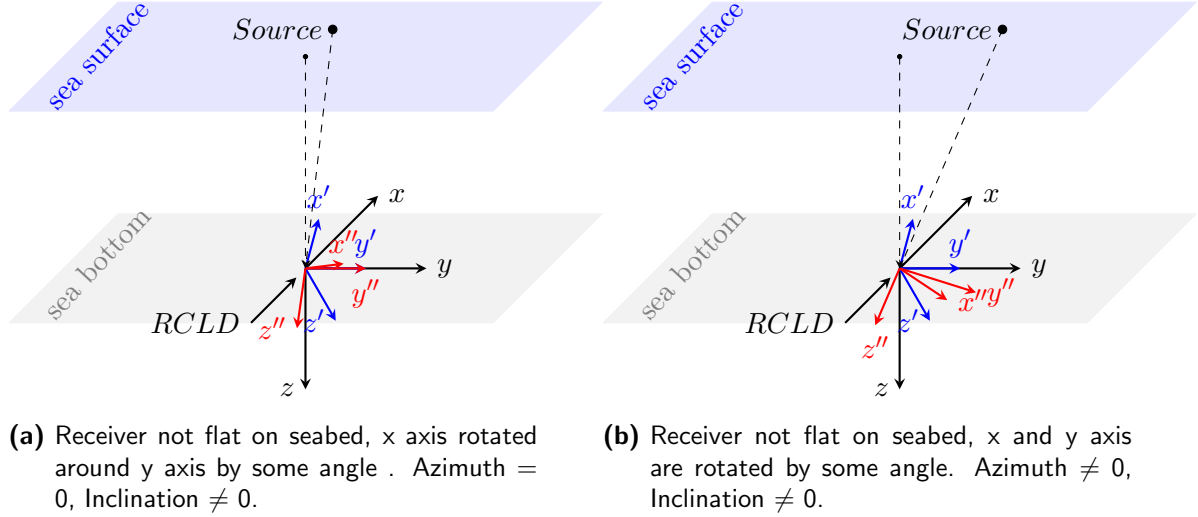


Figure A-3: The blue coordinate system shows the orientation of the MC receiver axis. The red coordinate system shows the orientation of the receiver axis after minimizing the energy in the x-y-plane. Observe that x'' is parallel to the source receiver vector i.e., x'' has the same inclination as the angle of incidence.

Now we will have a look at more complex setups. First we will change the source position, such that the inclination is not equal to zero any longer. This is shown in Figure A-3a. In this situation we will have to correct the inverted orientations. In this setup only the x axis is rotated. The y axis is perpendicular to the incident wave field (here approximated by its ray). The outcome of the inversion will be such that the x axis will also be perpendicular to the incident wave front (again approximated by an ray). This orientation satisfies the conditions given by the polarization angle. The z axis will be parallel to the ray. It is obvious that in this case we will have to correct the estimated rotation around the y axis by adding the inclination of the wave field in order to rotate the x-axis of the detector onto the seabed. As a consequence the z- axis will be perpendicular to the seabed.

In terms of equations:

$$inclination = \text{abs}(\text{atan}(SSDD/(DD - SD))) \neq 0 \quad (\text{A-4})$$

Correction applied to y-axis orientation (1st rotation around x- axis with R_x):

$$correct_2 = inclination * \underbrace{\sin((0) * \pi/180)}_{=0} * 180/\pi = 0 \quad (\text{A-5})$$

Correction applied to x-axis orientation (2nd rotation around y- axis with R_y):

$$correct_2 = inclination * \underbrace{\cos((0) * \pi/180)}_{=1} * 180/\pi \neq 0 \quad (\text{A-6})$$

From the correction terms we can observe the predicted corrections. The y-axis orientation (ϕ) does not need to be corrected where as ψ needs to be corrected by adding the *correct*₂ term, which is equal to the inclination.

As a last example we will consider the setup shown in Figure A-3b. This is also the most 'likely' situation that we will have to deal with. Here the x and y axis are rotated and inclination and azimuth are unequal to zero. Since this is a very abstract situation I will directly use the equations to give explain the corrections needed.

Again, we will start by taking a look at the inclination.

$$inclination = abs(atan(SSDD/(DD - SD))) \neq 0 \quad (A-7)$$

Correction applied to y-axis orientation (1st rotation around x- axis with Rx):

$$correct_2 = inclination * \underbrace{\sin((0) * \pi/180)}_{\neq 0} * 180/\pi \neq 0 \quad (A-8)$$

Correction applied to x-axis orientation (2nd rotation around y- axis with Ry):

$$correct_2 = inclination * \underbrace{\cos((0) * \pi/180)}_{\neq 0} * 180/\pi \neq 0 \quad (A-9)$$

So what happens is that the inclination is weighted by the azimuth term which will correct the estimated ϕ_0 and ψ_0 in order to turn the receiver flat on the receiver.

Appendix B

Reproducing OMEGA

B-1 MATLAB routine reproducing OMEGA

To be able to investigate and correct the anomalous orientation estimations in OMEGA, the FORTRAN routine ROTATE_3C_ANALY had to be recreated in MATLAB. To obtain meaningful results it is important that the MATLAB routine behaves like ROTATE_3C_ANALY. This is tested on the North Sea data set and synthetic data. Both data sets were introduced in Chapter 4-1.

B-1-1 MATLAB routine reproducing OMEGA results

In a first step the OMEGA routine that estimates the receiver orientation is re-created in MATLAB. From this MATLAB environment it is easier to prototype algorithms that might perform better in estimating the receiver orientation if uniform shot coverage is not available. To be able to compare new algorithms with the existing OMEGA routine it is of crucial importance that the MATLAB environment is able to reproduce the OMEGA results as close as possible. The MATLAB routine is tested on a seismic line containing 21 common receiver gathers. The seismic data was acquired in the North Sea using ocean bottom cables. The receiver positions and the overall orientations ϕ_0 and ψ_0 of the common receiver gathers are shown in Figure 4-1.

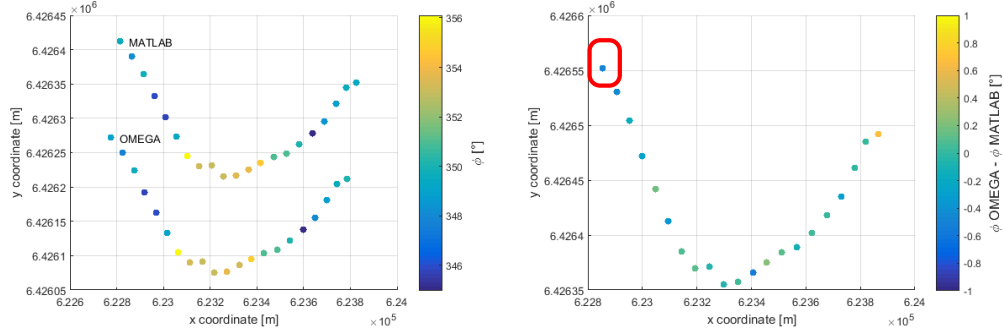


Figure B-1: Left: ϕ_0 estimation and comparison in OMEGA and the MATLAB environment. The MATLAB receiver line is shifted for displaying purposes. Right: Difference between the OMEGA and MATLAB.

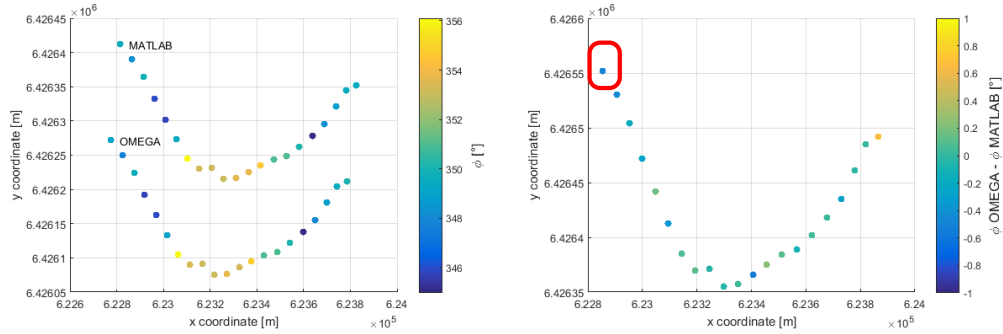


Figure B-2: Left: ψ_0 estimation and comparison in OMEGA and the MATLAB environment. The MATLAB receiver line is shifted for displaying purposes. Right: Difference between the OMEGA and MATLAB.

It can be observed that the maximum difference of the ϕ_0 (i.e., the trace averaged orientation estimation) estimated in MATLAB and in OMEGA is 0.6° . The maximum difference for ψ_0 is 1° . Overall the global orientations are nearly perfectly reproduced.

Since ϕ_0 and ψ_0 are estimated from the ϕ and ψ values estimated for every shot-receiver pair of a common receiver gather, it is also of importance that the MATLAB routine reproduces the trace dependent ϕ and ψ values. These trace by trace estimations are then summed up and averaged. This gives ϕ_0 and ψ_0 . Figure B-3 shows the ϕ and ψ values estimated for every source-receiver pair that was used in estimating the orientation of the most east receiver in Figure B-1 and B-2, which are highlighted by a red rectangle. It can be observed that also the trace by trace estimations match very well. The difference between the MATLAB and OMEGA ϕ and ψ is very close to zero for all the source-receiver pairs in the common receiver gather.

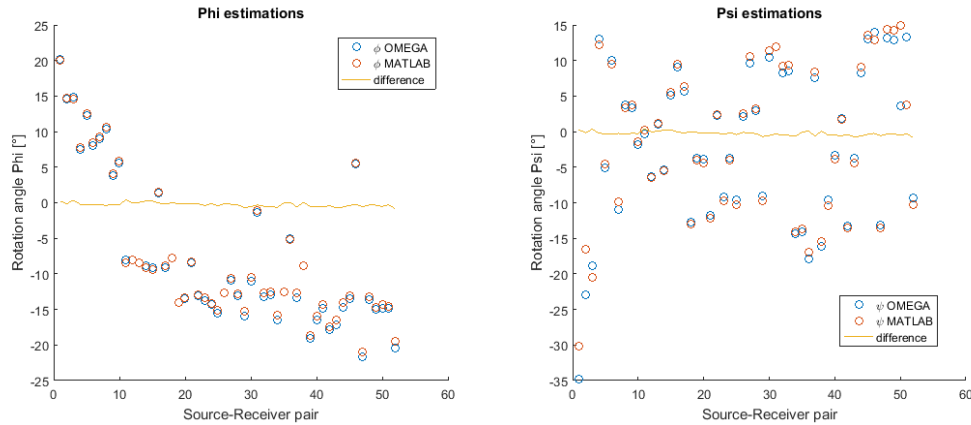


Figure B-3: Values of ϕ and ψ used for the computation of ϕ_0 and ψ_0 of the most west receiver in Figure B-1 and B-2.

B-1-2 MATLAB routine applied to synthetic data

In the following we will use synthetic data to test algorithms that should yield better results in the case of non-uniform shot coverage. To make sure that the MATLAB routine behaves the same for real and synthetic data it is first tested on synthetic data generated with a reflectivity modeling code called ANIVEC (Mallick and Frazer, 1987). In the synthetic data set the orientation of the receiver is known and can be altered by applying trial rotations around the x and y axis to simulate that the receiver is not flat on the seabed. The application of trial rotations we result in differently inclined z axis. Additionally to applying trial rotations different shot coverages are considered. Full and Opposite quadrant coverage (compare Figure 5-7 and 5-8) are exemplary for uniform, reciprocal shot coverage. West and North shots only simulates one sided shot coverage (compare Figure 5-9b and 5-9a).

	Phi [°]	Psi [°]	Z Inclination [°]
<i>initial values</i>	<i>0</i>	<i>0</i>	<i>90</i>
Full coverage	359.9	0.1	89.9
Opposite quadrant	0.1	0.1	89.8
West shots only	326	355.3	55.7
North shots only	357.2	325.2	55.1

Table B-1: Orientation estimation for a receiver with an Z axis perpendicular to seabed. Table shows estimated orientation for different shot coverages.

	Phi [°]	Psi [°]	Z Inclination [°]
<i>initial values</i>	15	350	72
Full coverage	16.2	347.3	69.5
Opposite quadrant	17.5	346.5	68
West shots only	342.6	342	65.2
North shots only	15.1	315.5	43.5

Table B-2: Orientation estimation for a receiver with an Z axis inclined by 72°. Table shows estimated orientation for different shot coverages.

The two tables above confirm that the MATLAB routine behaves the same for synthetic and real data. In the the case of reciprocal traces (Full and Opposite quadrant) the initial values of ϕ_0 and ψ_0 can be retrieved quite accurately. Once the shot coverage is one sided (i.e., non-uniform) the retrieved orientations are wrong.

B-1-3 MATLAB routine applied to real data

In section B-1-1 I compared the orientations determined in OMEGA with the orientations estimated by the MATLAB routine. I used the North Sea data set as input data. Now I will demonstrate how the seismic traces are effected by the different operations applied to them. I will also show the symmetric properties of the ϕ and ψ estimations.

This symmetry is essential for the sequential approach as it is implemented in OMEGA. Figure B-4 shows the shot positions and the estimation of ϕ and ψ associated with the respective shot-receiver pair. The data comes from the common receiver gather 9 of the North Sea data set. Observe that the shot coverage for this receiver is not uniform and is missing shots in the northern part. Also observe that the estimation of the ψ values in Figure B-4 (right) has mostly negative values to the west and positive values to the east of the y axis. The same is true for the north and south of the x axis. We can find that following odd symmetry exists:

$$\phi(x, y) = -\phi(-x, -y) \quad (\text{B-1})$$

$$\psi(x, y) = -\psi(-x, -y) \quad (\text{B-2})$$

Where $\phi(x, y)$ denotes the estimated ϕ value at the in-line and cross line offsets x and y. For a symmetry like we can easily show that in the case of uniform shot coverage the estimation of ϕ_0 and ψ_0 is not influenced by the error introduced with the assumption that the polarization angle is equal to the angle of incidence. The error effects $\phi(x, y)$ and $-\psi(x, y)$ equally. The error will cancel out once all the traces are summed and averaged to compute ϕ_0 and ψ_0 . It is also obvious that in the case of non-uniform shot coverage (as shown here) the averaging will not cancel out the introduced error entirely. This is especially true for the determination of ϕ_0 since there are no shots to the north of the x axis that could cancel the biased ϕ estimations in the south (Figure B-4 (left)). ϕ_0 will become negative valued. ψ_0 on the other hands is estimated from traces that have negative values for ϕ to the west and positive values in the east. There are reciprocal traces, the error will cancel out.

It has to be mentioned that other symmetries can occur for different acquisition geometries or tilted receivers. However, the cancellation of the errors always holds true for uniform shot coverage.

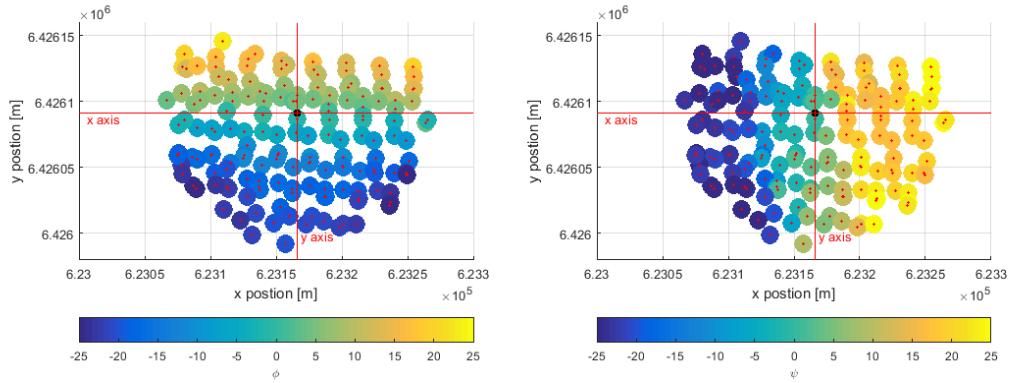


Figure B-4: Left: ϕ . Right: ψ .

Figure B-5 shows the raw data (Row 1), the flattend data (Row 2) and the data in the radial-transverse coordinates (Row 3). The data shown here is extracted from the common receiver gather 9 in the North Sea data set. The raw data is rotated with $\phi_0 = 3.5^\circ$ and $\psi_0 = -7.5^\circ$. After this rotation the data is flattend. The flattend data in the x-y-z coordinate system is then rotated into the radial-transverse-z coordinate system. After this rotation around the z axis the energy on the radial component is maximized and the transverse component is left with small residual energy. The vertical traces stay unchanged.

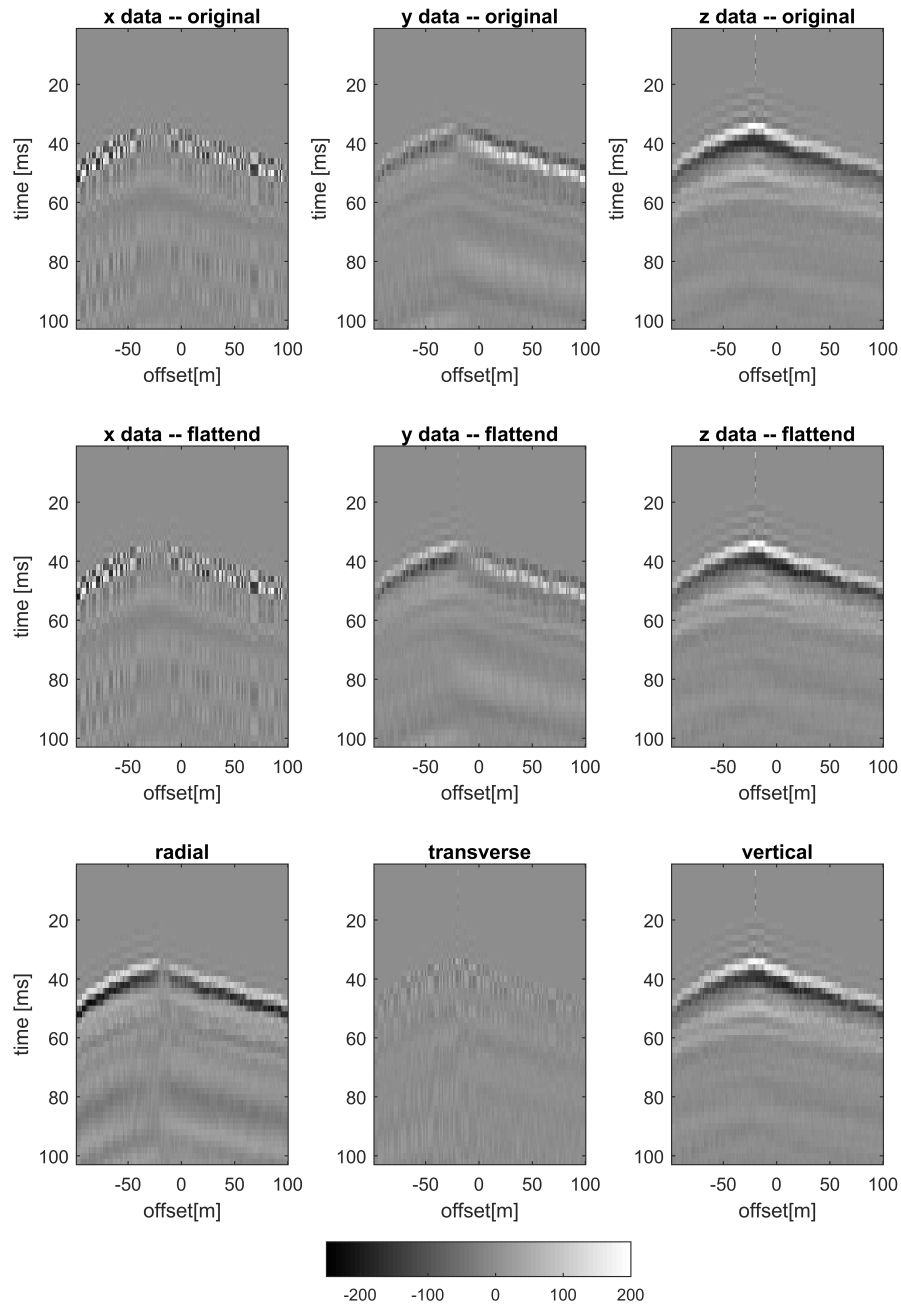
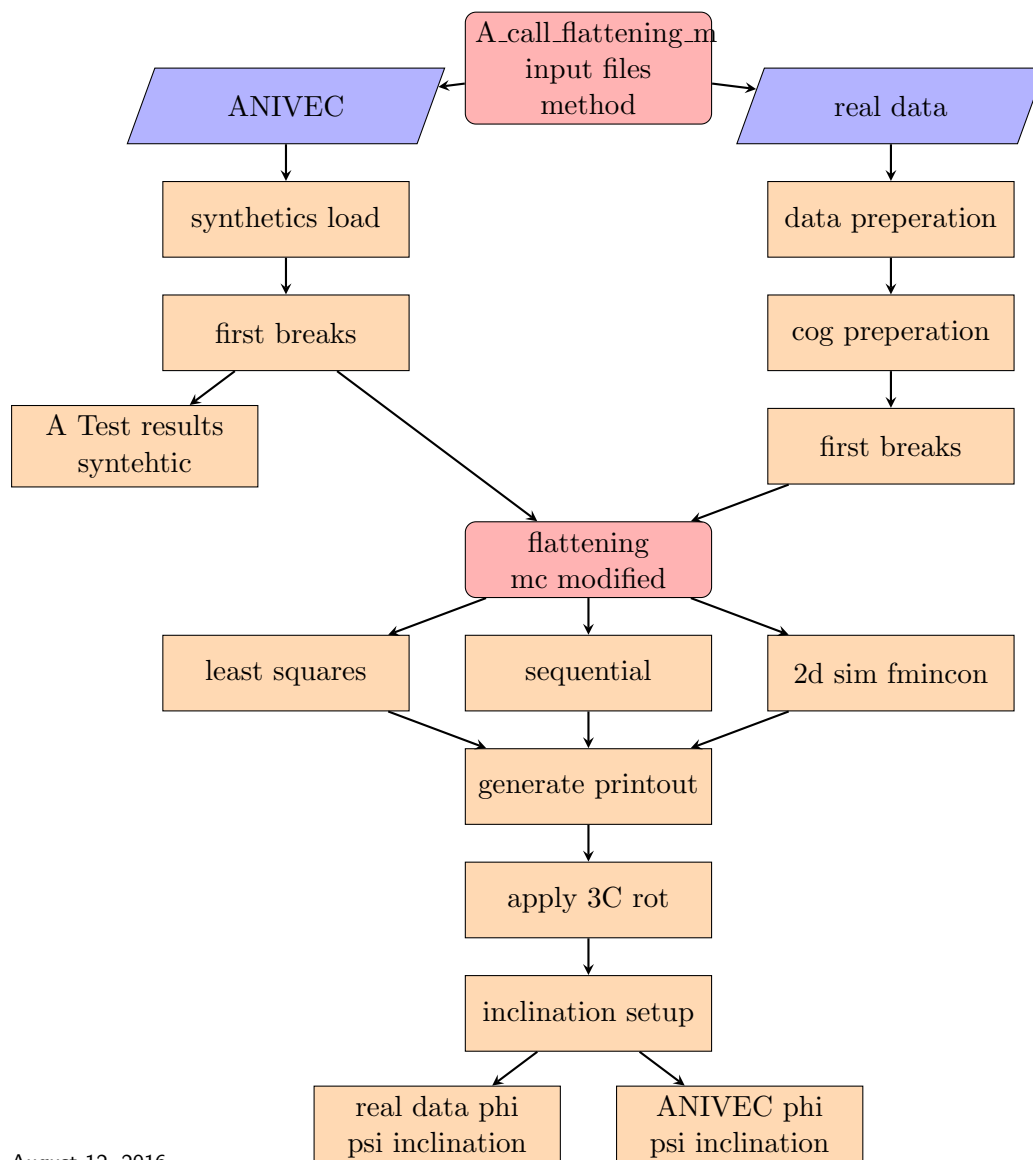


Figure B-5: The seismic traces as they change during the application of different rotations in MATLAB routine reproducing ROTATE_3C_ANALY. Time window shows first breaks only. Row 1: original multicomponent common receiver gather as acquired in North Sea data set. Row 2: multicomponent data after $\phi_0 =$ and $\psi_0 =$ were applied. Row 3: multicomponent data after rotation with θ_0 - SRAZ. Now the x-y-data is in radial-transversal- coordinates. Observe that most of the energy is on the radial component.

Appendix C

Algorithm flow



Short description of routines:

1. **A_call_flattening:** Here the input SEGY file(s) is/ are chosen. Additional parameters like the inversion method and acquisition specific parameters are set.
2. **ANIVEC**
 - (a) **synthetic_load:** Here the 3 SEGY files (x,y,z seismograms) of the ANIVEC model are loaded. Specific headers and entries need to be provided and written, because they are not provide by ANIVEC (eg. source depth).
 - (b) **first_breaks:** The first break windows are either picked or imported. The energy in this window will be used to find ϕ and ψ .
 - (c) **A_Test_results_synthetic:** After seismic data is loaded this script allows easy testing of new algorithms. It is more flexible then the entire orientation estimation flow.
3. **real_data**
 - (a) **data_preperation:** Here the SEGY from Omega is loaded. Data is then cropped to a portion of its size. Not all the traces are needed and time samples are needed. They are discarded, so less memory is occupied.
 - (b) **cog_preperation:** Here the rotations for the Center of gravity pre-rotation is computed. This is only used if specified earlier.
 - (c) **first_breaks:** same routine as on the ANIVEC branch.
4. **flattening_mc_modified:** This function contains the major operation in which ϕ_0 , ψ_0 and θ_0 are determined. One has to choose from the following three methods: least_squares, sequential and 2d_sim_fmicon.
5. **generate_printout:** Here input files are written. They contain all the main information (eg. single trace estimations of ϕ etc.) which were used and computed in the prior routines. The printout can be used for QC-ing or further operation.
6. **apply_3C_rot:** Here the estimated values of ϕ_0 , ψ_0 and θ_0 are applied to the data. The receiver is now flat on the seabed and the x component points away from the source.
7. **inclination_setup:** Here the seabed properties are included. This routine consist of a modified routine by Chapman (2004). The correction term for the flattend data set is computed.
8. **Application of correction terms**
 - **real_data_phi_psi_inclination:** Here the seabed correction term is applied to a real data set.
 - **ANIVEV_phi_psi_inclination:** Here the seabed correction is applied to a synthetic data set.
Two different routines are needed because of different coordinate systems used by Omega and ANIVEC.

Appendix D

The Center of Gravity Approach

The Center of Gravity (CoG) approach includes a pre-rotation of the recorded data before the angles ϕ_0 , ψ_0 and θ_0 are estimated. The CoG is found by averaging the x, y and z coordinates of all shots. Then the z axis of the of the receiver -which is assumed to be vertical- is rotated towards the CoG.

For the simple 2D example shown in Figure D-1 the position of the CoG is found by adding the source position offsets x_1 and x_2 and dividing them by 2. Then the inclination of the CoG-receiver vector is given by

$$\theta_{CoG} = \arctan\left(\frac{\text{off}_2 + \text{off}_1}{2d}\right) = \beta \quad (\text{D-1})$$

where d denotes the depth of the receiver and off_i is the shot-receiver offset.

The Addition Theorem (source) states that

$$\theta_1 + \theta_2 = \arctan\left(\frac{\text{off}_1}{d}\right) + \arctan\left(\frac{\text{off}_2}{d}\right) = \arctan\left(\frac{\frac{\text{off}_2}{d} + \frac{\text{off}_1}{d}}{1 - \frac{\text{off}_2}{d} \frac{\text{off}_1}{d}}\right) \quad (\text{D-2})$$

Because of the acquisition geometry and the constraints on the maximum distance for a shot-receiver pair it is save to assume that $\frac{\text{off}_2}{d} \frac{\text{off}_1}{d} \ll 1$ such that above expression is equal to

$$\theta_1 + \theta_2 \approx \arctan\left(\frac{\text{off}_2 + \text{off}_1}{d}\right) \quad (\text{D-3})$$

Using the Half angle formula Equation D-3 can be written as following:

$$\arctan\left(\frac{\text{off}_2 + \text{off}_1}{d}\right) = 2\arctan\left(\frac{\frac{\text{off}_2 + \text{off}_1}{d}}{1 + \sqrt{1 + \frac{\text{off}_2 + \text{off}_1}{d}^2}}\right) \quad (\text{D-4})$$

Here, again, it is assumed that the offset terms will be much smaller than the depth and thus the square root will be approximately 1. This allows us to rewrite D-2.

$$\theta_1 + \theta_2 \approx 2 \arctan\left(\frac{\text{off}_2 + \text{off}_1}{2d}\right) = 2 \theta_{CoG} = 2 \beta \quad (\text{D-5})$$

Recall that the orientation estimation in OMEGA is based on the averaging of inclination corrected polarization angles for every shot-receiver pair. The expected y axis orientation for the shot-receiver pair with the source at x_1 is $\alpha - \theta_1$. The expected y axis orientation after applying the pre-rotation around the x axis with the angle β is equal to $\alpha + \beta - \theta_1$. The expected values for x_2 can be written analogous. The averaging of these two shot-receiver pairs yields the following expression:

$$y_{orient} = \frac{\alpha + \beta - \theta_1 + \alpha + \beta - \theta_2}{\text{traces}} \quad (\text{D-6})$$

$$= \frac{2\alpha + 2\beta - \theta_1 - \theta_2}{2} \quad (\text{D-7})$$

If we now insert the result from D-5 we find that

$$y_{orient} = \frac{2\alpha + 0}{2} = \alpha \quad (\text{D-8})$$

After applying the pre-rotation around the x axis the estimated y axis orientation is able to recover the y axis orientation of the receiver. Without the pre-rotation the correction terms would not cancel out and thus the estimated y axis orientation would be biased.

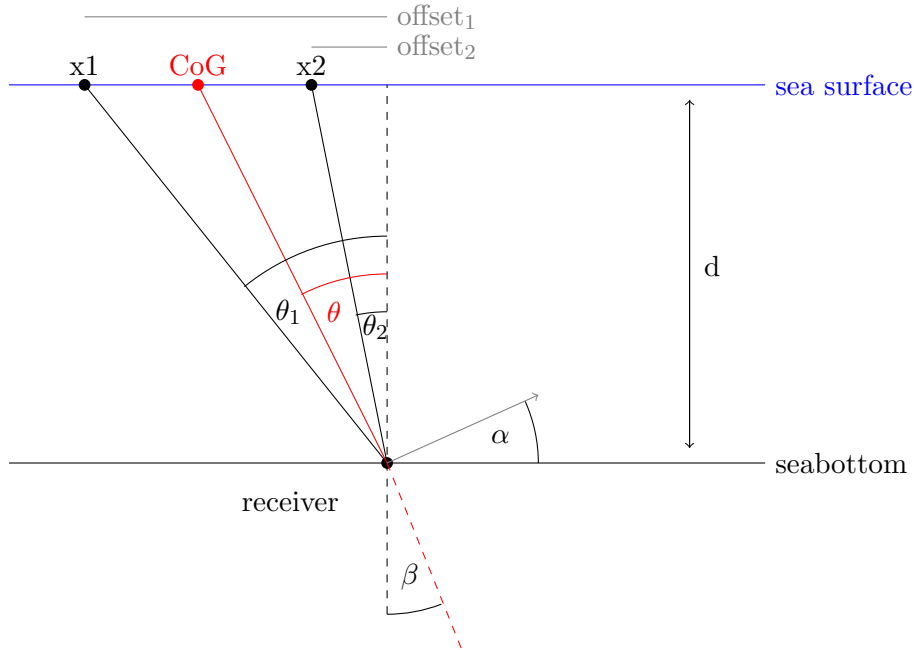


Figure D-1: Center of Gravity method