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Multivariate Interactive Visualization of Data in Generative Design

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ABSTRACT

We describe our work on providing support for design decision making in generative design systems producing large quantities of data, motivated by the continuing challenge of making sense of large design and simulation result datasets. Our approach provides methods and tools for multivariate interactive data visualization of the generated designs and simulation results, enabling designers to focus not only on high-performing results but also examine suboptimal designs' attributes and outcomes, thus discovering relationships giving greater insight to design performance and facilitating guidance of further design generation. We illustrate this by an example exploring building massing and envelope design (fenestration arrangement and external shading) with simulations of daylighting and heat gain. We conclude that the visualization techniques investigated can help designers better comprehend inter-relationships between variable parameters, constraints and outcomes, with consequent benefits of: finding good design outcomes; verifying that simulation results are reliable and; understanding characteristics of the fitness landscape.

Author Keywords

parametric; performance design; optimization; exploration; visualization; multi-objective; multivariate; evolutionary computing.

ACM Classification Keywords

B.4.4: Performance Analysis and Design Aids
H.2.8: Database Applications
I.6.3 Applications
I.6.6 Simulation Output Analysis

1 INTRODUCTION

While increasingly powerful computational resources are permitting designers to explore increasing quantities of design variations, challenges remain in making sense of the correspondingly large volume of data involved [3, 16, 17, 18, 19]. Generative design systems in particular pose such challenges. In this paper we present our research on using multivariate interactive visualization of data to support design space exploration, optimization and design decision making via generative design systems. The method employed in this research combines parametric form generation with performance analysis using simulation

software guided by a multi-objective genetic algorithm to fill a database with representative solutions from the design space. This design space can subsequently be explored using SQL search and filter commands. The results can be compared through graphic images as well as data depictions: scatter plots or parallel coordinate graphs.

Optimization and design space exploration via generative algorithms and other evolutionary techniques is becoming a widely adopted approach in design, engineering and other fields. Their use offers the opportunity to identify sets of well performing design alternatives, which can be used as design solutions and also to suggest further improvements for new design solutions. However, the complexities of problem-definition and of results-interpretation pose persistent challenges to effective use of evolutionary computing for design applications. One reason is the large volume of data involved, which needs to be digested by users. Another reason is that such systems can be 'black boxes' which tend to obscure their inner workings and thus the important relationships among the variables involved and between these variables and the outcomes. In addition, due to the progressive generational characteristic of most evolutionary methods, it is ineffective to change the search algorithm (the fitness function) without essentially starting over. This limits the "exploratory" quality of the method. The method we used instead employs the ParaGen model: a Non-Destructive Dynamic Population Genetic Algorithm (NDDP GA) which maintains a database of all solutions and creates breeding populations dynamically, on demand, differing from e.g. [22]. In this way the search direction can be changed at any instant without restarting the process. This in turn allows a truer *exploration* of the design space.

In the next sections we first briefly recount relevant precedents to our work, followed by a description of our experimental apparatus and a specific design task case. We then present some results of the experiments and conclude with a summary and pointers for future work.

2 PRECEDENT WORK

Data visualization provides a body of knowledge and set of techniques which can alleviate both the large-volume and obscured-workings problems mentioned above. Ranging from Scientific Visualization through Interactive Information Visualization and Visual Analytics, much work has been done and results achieved assuming that humans'

visual pattern recognition can provide powerful insights to help unravel complex situations [1, 2, 3, 7]. In the work we present here, we have been motivated to apply some of these visualization techniques – especially those with potential for extension via interactivity – to the problems of making sense of the data from evolutionary computing.

2.1 Optimization processes as exploration tools

In traditional optimization, a single best solution (or a front of best solutions) is found for a given set of objectives applied to a specific problem. Optimization techniques have been mostly used in order to solve a specific (mono-disciplinary or interdisciplinary; single-objective or multi-objective) design problem by searching for an optimum solution. In this light, the role of optimization in design is to find within the design space the configuration that best matches desired performance goals [11]. This is unquestionably one of the major potentials of optimization techniques. However, this does not support a fully informative exploration of design solutions. In fact, there is a key difference between the process of design exploration and the search for an optimum solution. [4] presents this difference by distinguishing the design process in three possible types: routine process, innovative process, and creative process. From routine to creative, the attempt to describe a solution search loses potentials of precise and predetermined definition. A routine process is close to the concept of search, a creative one to a process of exploration. A key need in design exploration is the learning process of the designer [2, 3, 5, 13, 18].

There are some precedents addressing this point, promoting optimization processes as exploration tools rather than as search for a specific optimum, e.g. [8, 18]. Some of the precedents propose solutions by focusing on the design objectives to be evolved during the process; the design variables to be set more freely; the consideration of the importance of the sub-optimal solutions. Among those focusing on the design objectives, [9] proposes a GA system which allows the co-evolution of the fitness function and design solution by representing the fitness as part of the genotype. Focusing on the exploration of different design variables, some precedents allow the user to intervene in the process by modifying, adding and/or deleting variables, re-defining the range of modification for variables, and modifying their constraints. It is for example the case of the work by [11], in which simulated annealing is used to assist in the preliminary design phase for acoustic measures, in which the user can optimize over materials and geometry either separately or simultaneously. Still, in these examples, once the optimization is initiated, no relevant attention seems to be given to sub-optima. As [12] points out, sub-optimal solutions are usually discarded and in most of the precedents they do not contribute to decision making after optimization runs. They stress instead that ‘the discarded “inferior” solutions and their fitness contain useful information about underlying sensitivities of the system and

can play an important role in creative decision making’, a view supported also by [14]. Based on this consideration, [12] proposes a visual method to analyze sub-optimal solutions. These are retained during optimization and represented in a fitness array visualization system called phi-array. Another precedent worth mentioning concerning a certain attention given to sub-optimal solutions is BGRID [10], a decision support system for the conceptual design of commercial office type buildings, employing GAs. It searches for viable design options in light of both structural and architectural criteria. Specifically, it is meant to support defining the layout of columns in floor plans including also lighting requirements and ventilation criteria. BGRID provides the user with a selection of optimal solutions to enhance the understanding of the underlying processes.

In contrast with the above methods, we wish to investigate more generalized visualization methods which both allow thorough examination of all data points – not only high-fitness ones – and also support this examination with interactivity. Statistical exploration tools such as X-/G-/R-Gobi [15] have such capabilities; however, we decided to work here with a more design-oriented platform, ParaGen [20, 21], described further in Section 3.

2.2 Interactive visualization of multi-variate data

The techniques existing for visualizing multi-variate data are many and various [3, 6, 7], some of which are also indicated in the section above. We have chosen to focus on ‘parallel coordinates’ and ‘scatterplots’ for our present investigation, emphasizing the potential contributions of interactivity to these graphing techniques.

To review briefly: parallel coordinates (or ‘parallel axes’) are a more than century old method which consists of plotting all independent and dependent dimensions of data onto a series of parallel lines representing each dimension, and connecting the corresponding points with lines. Thus, each ‘solution’ is represented by a line comprising a visual ‘signature’ of its inputs and outputs. A variant of this – sometimes referred to as ‘radar’ or ‘spiderweb’ plots – places the axes radially, forming a polar plot, with an advantage of compactness, but a disadvantage of legibility, especially for values near the pole. In our investigations interactivity was introduced to parallel coordinate graphing by enabling: a) rearrangement of the coordinate axes’ sequence; b) highlighting or coloring of lines; and c) restriction of displayed data ranges via graphical selection and text-field based filtering.

Scatterplots are of course very commonly used for two- and sometimes higher-dimensional data, for example by plotting data points onto three ‘physical’ axes (e.g. X, Y and Z) or by plotting multiple series of points (connected by lines or indicated by bars, etc.) differentiated by hatching, shading and/or symbols. Interaction for scatterplots was achieved by a) giving control of the data dimensions to be

plotted on two axes, b) enabling highlighting of data ranges and c) restriction of data ranges via text fields.

3 EXPERIMENTAL APPARATUS

Our research involved application of supplemental data visualization techniques to a previously existing interactive evolutionary generative design system, as described below.

3.1 ParaGen

ParaGen is a framework which utilizes a variety of commercial and custom written software to aid the designer in the exploration of good solutions [20, 21]. The system uses heuristic methods to direct the search toward areas of the solution space which the designers defines as most desirable. The designer can describe “desirable” either through multiple performance objectives or by qualitative, visual selections. Applications of ParaGen to date have been focused on architectural design problems, and as such combine both qualitative and quantitative performance aspects of a problem solution. In order to deal with both qualitative and quantitative information, ParaGen harvests both a wide range of performance data (the quantitative) as well as an array of visual images and data depictions (the qualitative). Both types of information together can aid designers particularly in the early phases of design.

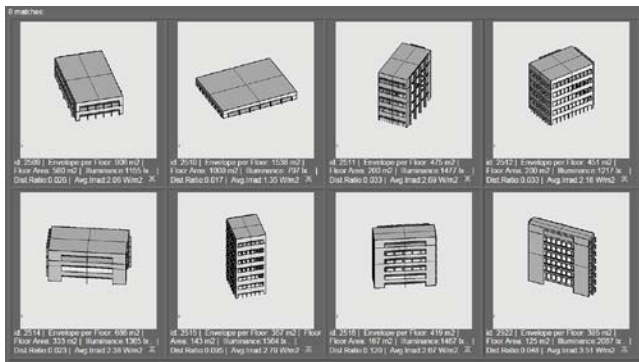


Figure 1. Examples of different design solution images. Any images saved during analysis can be alternately displayed.

ParaGen makes use of parametric-associative modeling software to generate the geometry of the solutions, and then passes the digital model on to one or more simulation and analysis packages to determine the performance values. In the present work DIVA was used to evaluate daylighting and heat gain potential. Most modern simulation software also has the capability to display performance results in graphic form. For example Rhino/Grasshopper has many options for visual rendering that can include light level maps, interior and exterior perspectives, or other displays from various plugins. Most structural finite element software can image the deformed geometry under load or color coded stress information. Such images can convey valuable information to the designer in a qualitative way. Of course this same simulation software also produces quantitative performance values. ParaGen harvests both types of information and uploads it to a server where all

quantitative data is stored in a database and linked with the various images that are also saved. In addition the different simulation models themselves as well as small animations or 3D VRML models can be saved and linked to the solution for more detailed inspection by the designer.

ParaGen makes all of this information accessible for exploration through a web interface. The main page shows an array of design/solution images that can be switched out for other views (any set of the saved images) by selecting the image type from a simple pull down menu. Figure 1 shows ten randomly selected design variations. The solutions shown can be filtered and sorted using another series of pull down menus on the web page. Using any of the design input parameters or performance values the designer can interactively select a desired set of solutions from the database. This selection can take the form of a simple max/min sort or a complex, multi-variable SQL query. The results are immediately displayed on the web page for inspection and further modification.

The search method used by ParaGen is a Non-Destructive Dynamic Population Genetic Algorithm (NDDP GA). It is non-destructive in that all solutions are maintained in a database and can be recalled or searched at any time by the designer. Being able to see both what makes a good solution as well as a poor solution aids the designer in learning about the problem. This is a valuable capability for early design exploration. Dynamic populations also aid exploration by allowing instantaneous or interactive production of breeding populations for the GA. Very importantly, in this way fitness functions can be interactively altered by the designer in order to explore different regions of the design space. This again offers the designer a true exploration of the solution space. In addition, other means to explore the performance data are also available in the form of interactive graphing tools.

3.2 Interactive Graphing Methods

Two basic graphing methods were added to ParaGen for our present work: x-y scatter plots (with glyph control for a third level of information) and parallel coordinate plots, as described already in more detail above. The data plotted in both cases is controlled using the same sorts and filters which control the design/solution images. By plotting conflicting performance values, Pareto front optimality can be investigated. Figure 2 shows a plot of designs/solutions with average illuminance > 1500 lux and average irradiation < 2.8 W/m². Average illuminance (nominally to be maximized) is plotted against average irradiation (to be minimized). The red dots indicate the solutions with floorplate area above 200 m². The Pareto frontier is marked and two “red” solutions near the frontier are indicated. By clicking on any of the dots an image of the solution comes up. In this way the qualitative begins to be combined with the quantitative, and the graphing tools’ interactivity helps users of the system explore the data [3, 7] and test their ideas about underlying relationships within the data.

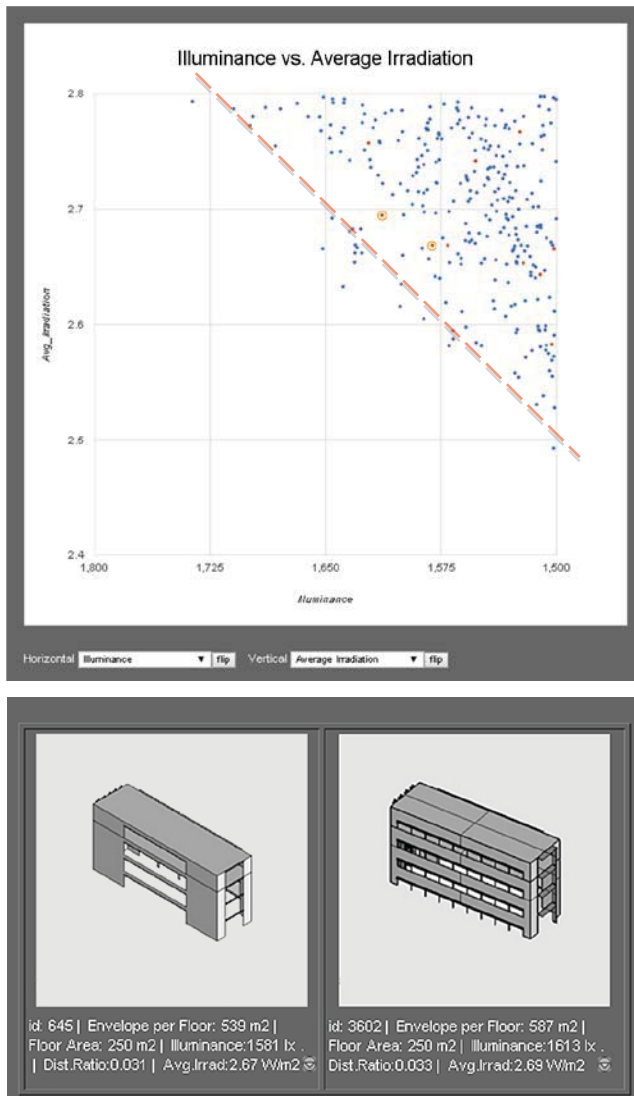


Figure 2. Scatterplot of illuminance vs. average irradiation, with Pareto frontier, highlighting of floorplates over 200 m² and isometric view of two selected solutions

4 CASE STUDY AND EXPERIMENT

To test our hypotheses regarding usefulness of interactive data visualization in design space exploration and optimization, we constructed an example design problem illustrating a multi- (or many-) objective search for design solutions selecting among parameters of building massing, fenestration and shading, and evaluating these against criteria relating to (interior) daylighting, thermal gain and construction quantity/cost. To support early-stage design and conceptualization, emphasis was on relatively rapid modelling and analyses rather than more detailed ones.

4.1 Design Problem

The building model to be analyzed comprised a number of fixed and also variable parameters. These parameters correspond to design features, assumptions and constraints which would normally be considered within the process of

design. For example, the total floor area of the building is constrained to a fixed value of 1000 m² (an approximation neglecting the small variation in size of vertical circulation and other shaftways for varying building heights). The number of stories in which the 1000 m² are distributed is an independent variable of the model. The floor plan shape is set to be a rectangle; one dimension of the floor is also an independent variable. The other dimension is based on the first dimension and on the distribution of the 1000 m² on the (independently variable) number of stories. Four independent variables regarding the number of windows regulate on each façade both the number of glazed modules and the number of shading elements. Specifically, each glazed module is connected with one horizontal and one vertical shading element. Total glazed surface does not increase proportionally to the number of glazed modules; instead, the proportion between the glazed surface and the opaque surface is constrained to a fixed value for each façade, but this could also be made variable. The dimensions of the shading elements are varied based on additional independent parameters, resulting – even when discretized – in a total number of possible designs on the order of 1×10^{16} , making a brute force search infeasible. (The complete list of parameters is omitted here due to lack of space but is available from the authors upon request.)

Performance criteria for evaluating the fitness of generated designs were defined in three main categories with sub-dimensions: daylighting, solar gain, and quantity/cost of construction. Daylight and solar gain were simulated for the location of Guangzhou (China). Daylight was measured based on illuminance values on the 21st September, h.14.00, with overcast sky; on a grid located on the entire top floor. It was evaluated for intensity and homogeneity (to be maximized). For evaluating the intensity, the average illuminance was calculated. For evaluating homogeneity, the ratio of maximum to minimum illuminance levels was calculated. Solar gain was assessed based on irradiation in a typical week, measured on the same grid as the illuminance. The solar irradiation was to be minimized. Costs were considered in reference to surfaces of the standard floor and its envelope (without differentiation of unit or area costs for different construction element types). Maximizing the floor areas, while minimizing the envelope area (including also shading elements) was one aim, and other performance criteria were also introduced, such as minimizing heat gain while maximizing homogeneity of daylighting. While recognizing that these represent a quite simplified assessment of a rather limited set of design variations, our aim was not to carry out a real architectural design investigation, but rather experiment with data visualization techniques applied to a plausible design investigation.

4.2 Design Process and Data Visualization

The data for the design problem described above were generated with Rhino/Grasshopper, DIVA, and ParaGen. ParaGen uses an NDDP GA (see section 3.1) to explore and

expand the data in different sections of the solution space in response to fitness criteria set by the designer. Changing the fitness function in a traditional GA is not generally an effective technique since once a population converges on one fitness function, diversity is lost which limits, or at least delays, the development of solutions in the direction of the new fitness. Because in ParaGen all solutions are retained, new populations can be dynamically created on the fly in response to changing fitness criteria [20]. In the problem described above, the GA was initialized with the generation of a little over 250 random solutions. This was followed by a series of explorations using different single and multiple fitness functions. After about 6000 solutions had been explored, it was decided to focus on a fitness function which described the desired illuminance and the distribution ratio levels. This was continued for the remainder of the run which generated some 8000 solutions. ParaGen can also adjust the breeding algorithm in a couple of ways to produce either more diversity or stronger focus on the fitness function. The basic breeding technique used is half uniform crossover, HUX, which uses a Gaussian distribution to find a new variable value between two being crossed [20, 21]. The sigma value of this distribution can be adjusted to allow either a looser distribution around two parent values or to focus the child solution more tightly between the parent values. The level of disruption caused by the HUX breeding can also be reduced by adjusting the ratio of crossed to non-crossed genes (variables) from the regular ratio of 1 out of 2 (50/50) to 1 out of 5. Both of these techniques were used in the later generations which focused on illuminance and the distribution ratio.

5 RESULTS

By assisting the designer in analyzing the population of design variations, the system helps to reveal relations between parameters and performance. This includes relations that are fairly obvious based on common sense or basic domain knowledge and also relations that are more difficult to predict. Various examples are provided below.

One basic relation is the association between the lowest average illuminance with the maximum depth of the floor and the minimum height of the story and windows. The system confirms this. Sorting the solutions based on ascending illuminance, the database shows that the first 52 solutions have one story only, with floor area of 1000 m²; while the first multi-story solution appears in position 53 (2 stories, each of which having a floor area of 500 m²). Most of the 52 solutions have a square floor plan. This is exemplified in Fig. 3, a screen capture of some of the sorted solutions and the parallel axis graph plotted for the solutions with illuminance lower or equal to 450 lux. Out of 20 solutions having the illuminance lower or equal to 450 lux, all have one story and 17 have the length of the building equal to 30 m, which is the length that gives a nearly square floor; while all the others have the length of the building near to 20 m. Looking at the parallel point

graph, this is clearly shown. The parallel point graph makes visually evident also that all of the solutions have the lowest building height and number of stories (because the 1000 m² are in one floor only); shading elements set to high horizontal and vertical lengths; an orientation of -90° or -80°. Plotting then only the 5 solutions with illuminance lower or equal to 300 lux it also becomes evident that these are the ones having all shading elements set to their maximum horizontal and vertical lengths (all equal to 1 m) in all directions; and also with lower amounts of glazed surface facing south. (S orientation = 0°, to lower right.)

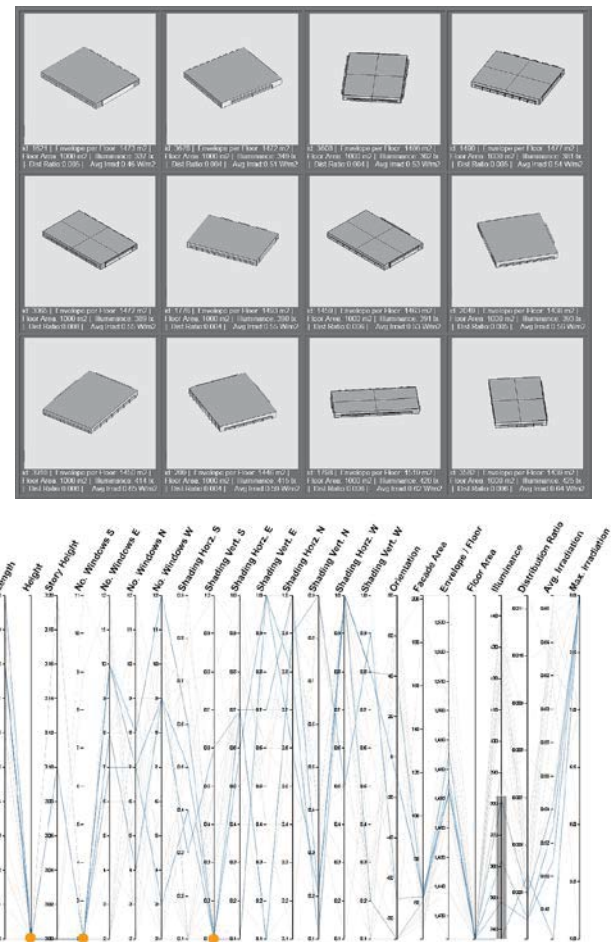


Figure 3. 15 solutions with lowest average illuminance. Top: screenshot of database, showing 15 solutions with lowest average illuminance, in isometric view. Bottom: parallel point graph plotted for 20 solutions having average illuminance lower than or equal to 450 lux. Height variable = 1 for all 20 solutions (circled).

Another basic relation confirmed by the system is the association between the highest average illuminance with the minimum depth of the floor and the maximum height of the story and windows, as shown in Figure 4. Plotting the solutions with illuminance higher than 2700 lux, the parallel point graph clearly shows that most of the solutions have the length of the building equal to 30 m; the others are very near to 30 m, which – when all solutions have a high number of stories (9) – leads to very narrow floors. Each

one of all stories has a resulting floor area of 111 m²; has high height (4 m); and an averagely high number of shading elements in all directions. Figure 4 shows all of this. Further evidence regarding this relation can be gained by looking at the scatterplots, by plotting the generated data in pairs. For the 27 solutions with illuminance higher than 2700 lux, Fig. 5 shows the examples of illuminance vs number of windows facing south; and illuminance vs orientation.

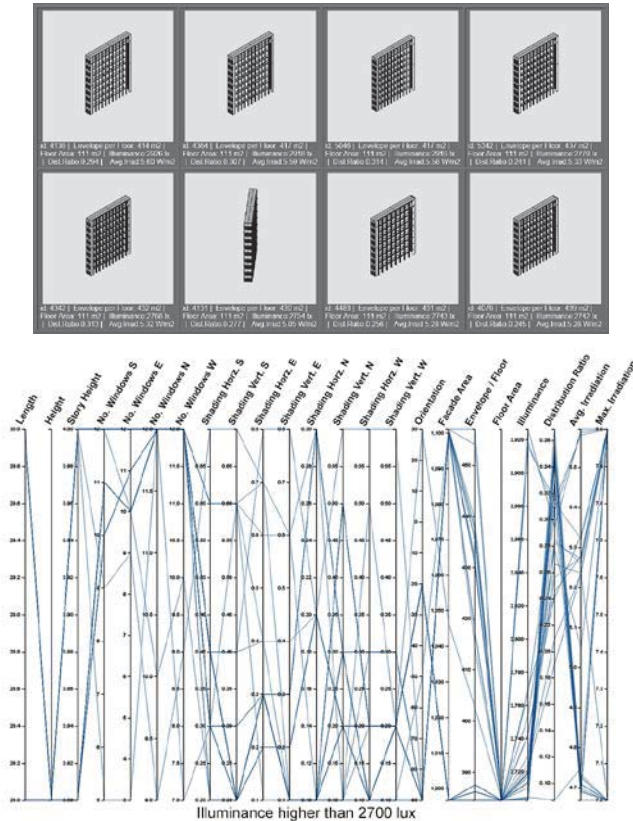


Figure 4. Solutions with highest average illuminance. Top: The 8 solutions with highest average illuminance. Bottom: parallel point graph plotted for the 27 solutions having average illuminance higher than 2700 lux

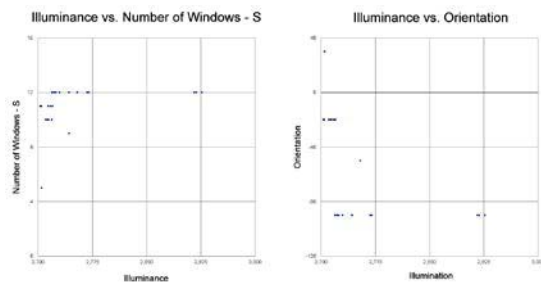


Figure 5. Scatterplots of the 27 solutions with illuminance higher than 2700 lux. Left: illuminance vs number of windows facing south. Right: illuminance vs orientation

Typically, in order to combine very high illuminance with maximum floor area, the only option (absent skylights, clerestories, etc.) is to have a very long and narrow

floorplan of 1000 m² in one story only. This predictable relation is made evident both when sorting and filtering the views of the solutions and when using the parallel point graph, as shown in Fig. 6.

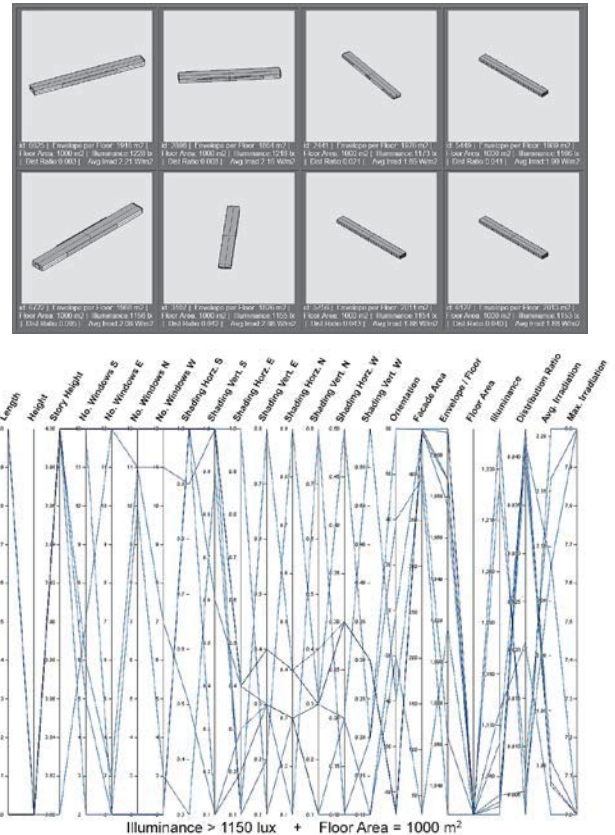


Figure 6. The image shows the solutions with average illuminance higher than 1150 lux and floor area of 1000 m². Top: screenshot of database, illustrating 8 of the 38 solutions matching this criteria, in isometric view. Bottom: parallel axis graph of the 38 solutions

Using the scatterplots, the fact that also shading on the north façade has some relevance in obstructing daylight emerges; the major relevance of high stories also is evident (having all points except one aligned on the maximum height of 4). This is illustrated in Fig. 7.

Similarly, looking at the highest average irradiation, the parallel point graph (plotted for the 17 solutions with average irradiation higher than 13.5 W/m² and illustrated in Fig. 8) shows that 13 solutions have the length of the building equal to 30 m, three solutions near to 30 m (29 m and 28 m) and one solution 25 m; all the solutions have the height of the building equal to 9 stories and the height of each story is 4 m for all except one (3.8 m); have an averagely high number of windows in all directions; and an averagely low shading; mostly an orientation of -90° to -80°. The resulting floor area per story is 111 m² for each.

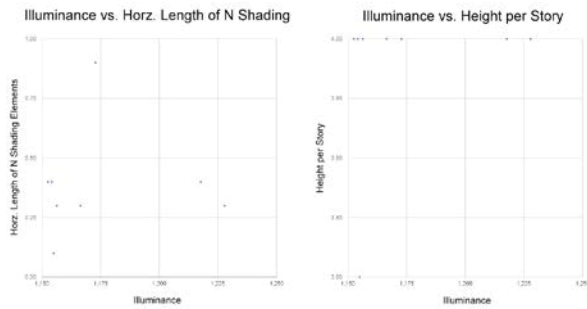


Figure 7. Scatterplots of 8 solutions with illuminance > 1150 lux, floor area of 1000m². Left: illuminance vs horizontal length of north-facing shading elements. Right: illuminance vs story height

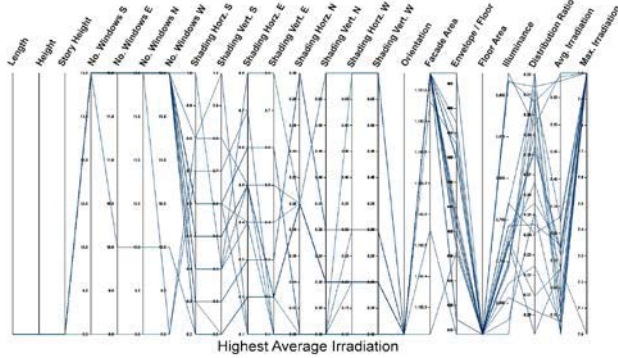


Figure 8. Solutions with highest average irradiation, with parallel point graph plotted for the 19 solutions having average irradiation higher than 5.1 W/m². (Note: solutions similar to Figure 4).

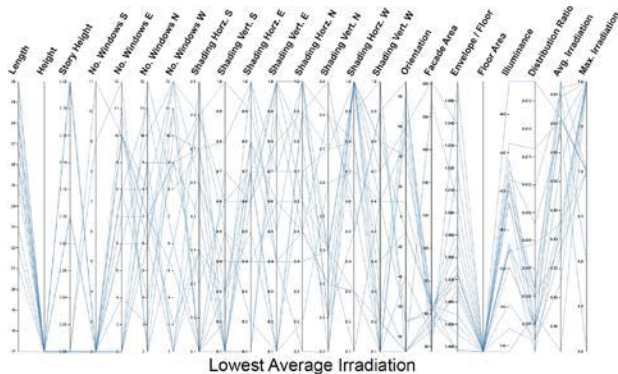
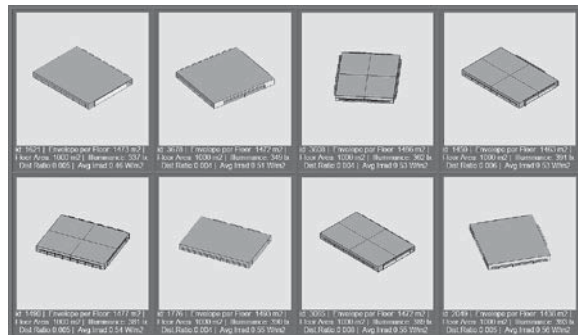


Figure 9. Solutions with lowest average irradiation. Top: The 8 solutions with lowest average irradiation. Bottom: parallel point graph plotted for the 18 solutions having average irradiation lower than 0.65 lux

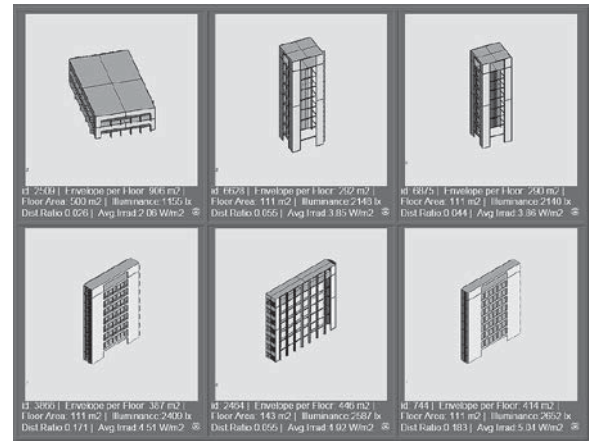


Figure 10. The image shows examples of solutions having maximum irradiation higher than 8 W/m². Top: screenshot of the database, illustrating 6 examples, in isometric view. Bottom: parallel point graph plotted for the 6 solutions.

Looking at the minimum average irradiation, the parallel point graph (plotted for the 18 solutions with average irradiation lower than 0.65 W/m² and illustrated in Fig. 9) shows that all the solutions have floor area of 1000 m² and one story of low height (3 m); four have the length of the building equal to 20 m, three equal to 22 m, while the other 11 solutions have the length of the building equal to 30 m, which leads to a square plan; low number of windows facing south; mostly an orientation of -90° to -80°.

6 SUMMARY AND FUTURE WORK

We have presented a description of our work on applying various multi-variate/-dimensional data visualization techniques to a complex parametric-associative building model for design space exploration and multi-objective optimization. (Note that due to limitations of space, illustrations herein are reproduced at small sizes; for full images see: <http://hdl.handle.net/2027.42/117408>.) Our findings indicate that such visualization techniques do offer useful feedback in the use of such a model, aiding comprehension and modification of the design space [5, 9, 13] by introducing additional interactive data visualization

components to the otherwise mainly automated operations of evolutionary algorithms. We see too that multivariate interactive visualization aids in model verification, which is also important – even if not as glamorous as the discovery of unanticipated valid relationships in the data.

Further investigation of this approach should include development of improved and additional visualization and interaction capabilities, as well as experiments to compare the relative efficacy of the different visualizations and their combinations. These would be applicable both to conventional building types and to non-standard ones requiring more innovative designs for more ‘wicked’ situations. For example, one experiment could test the ability of the system’s users – e.g. students, practitioners – to predict the outputs (i.e. performance) of some solutions given only their inputs and a larger set of solutions evaluated also for performance. Another experiment would test the system’s support for reasoning from available patterns of inputs and outputs to interpolate or extrapolate other designs. In any case, we expect the graphing to aid pattern recognition, and such abilities would in turn increase the value of the system as a learning device for pedagogic purposes as well as more generally for reflective practice.

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REFERENCES

1. Arentze, T.A. Spatial data mining, cluster and pattern recognition, *International Encyclopedia of Human Geography*, (2009), 325–331.
2. Chaszar, A. Bridging the gap with collaborative design programs. In: *Blurring the Lines*. Wiley-Academy (2006), 158-167.
3. Chaszar, A. Finding rhythms in a tempest. In: J. Burry (ed.) *Designing the Dynamic: High-performance Sailing and Real-time Feedback in Design*, (2013) Melbourne Books, 118-123.
4. Grierson, D.E., Conceptual design using emergent computing techniques. *Emergent Computing Methods in Design; Applications of genetic algorithms and neural networks*, NATO ASI Series F, Computer and Systems Sciences, 149, Springer, (1996), 150–161.
5. Hatchuel, A., Le Masson, P., & Weil, B. The development of science-based products: managing by design spaces. *Creativity and innovation management*, 14, 4 (2005), 345-354.
6. Holmes, J.H. Knowledge discovery in biomedical data: theory and methods, *Methods in Biomedical Informatics A Pragmatic Approach*, (2014), 179–240.
7. Keim, D., Kohlhammer, J., Ellis, G. and Mansmann, F. (eds.) *Mastering the Information Age: Solving Problems with Visual Analytics*. Eurographics, Goslar, 2010.
8. Kilian, A. *Design Exploration through Bi-directional Modeling of Constraints*. Doctoral dissertation, Massachusetts Institute of Technology, 2006.
9. Maher, M.L., Poon, J., Co-evolution of the fitness function and design solution for design exploration. *Proc. IEEE International Conference Evolutionary Computing*, (1995), 240-244.
10. Miles, J.C., Sisk, G., Moore, C.J The conceptual design of commercial buildings using a genetic algorithm. *Computers & Structures* 79, (2001), 1583-1592.
11. Monks, M., Oh, B., Dorsey, J., Audiopitimization: Goal based acoustic design. *Computer Graphics and Applications, IEEE Journal* 20, 3, (2000), 76–90.
12. Mourshed, M., Shikder, S., Price, A., ϕ -array: a novel method for fitness visualization and decision making in evolutionary design optimization. *Journal of Advanced Engineering Informatics* 25, (2011), 676–687.
13. Schon, D. *The Reflective Practitioner: How Professionals Think in Action*, Basic Books, New York, 1983.
14. Sosa R., Gero, J.S. The creative value of bad ideas. In: *Proc. CAADRIA 2013*.
15. Swayne, D., Cook, D., Buja, A., Lang, D.T., Wickham, H., Lawrence, M. (2006). GGobi manual. Sourced from www.ggobi.org/docs/manual.pdf
16. Turrin, M., von Buelow, P., Stouffs, R. Design explorations of performance driven geometry in architectural design using parametric modeling and genetic algorithms. *AEI* 25, 4 (2011), 656-675.
17. Turrin, M., von Buelow, P., Kilian, A., Stouffs, R. Performative skins for passive climatic comfort: a parametric design process. *Automation in Construction* 22 (2011), 36-50.
18. Woodbury, R., Burrow, A.L. Whither design space? *AIEDAM* 20, 2 (2006), 63–82.
19. Woodruff M.J., Reed P.M., Simpson T.W. Many objective visual analytics: Rethinking the design of complex engineered systems. *Struct Multidisc Optim* 48 (2013), 201-219.
20. von Buelow, P. ParaGen: Performative exploration of generative systems. *J. IASS* 53, 4 (2012), 271-284.
21. von Buelow, P. Using database storage to improve explorative optimization of form critical structures. In: *Proc. IASS-SLTE* (2014).
22. YAshour, Y. Kolarevic, B. Optimizing creatively in multi-objective optimization. In: *Proc. SimAUD 2015*, 128-135.